EURASIP Journal on Wireless Communications and Networking 2005:3, 343–353

Adaptive Mobile Positioning in WCDMA Networks

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Received 6 November 2004; Revised 14 March 2005

We propose a new technique for mobile tracking in wideband code-division multiple-access (WCDMA) systems employing multiple receive antennas. To achieve a high estimation accuracy, the algorithm utilizes the time difference of arrival (TDOA) measurements in the forward link pilot channel, the angle of arrival (AOA) measurements in the reverse-link pilot channel, as well as the received signal strength. The mobility dynamic is modelled by a first-order autoregressive (AR) vector process with an additional discrete state variable as the motion offset, which evolves according to a discrete-time Markov chain. It is assumed that the parameters in this model are unknown and must be jointly estimated by the tracking algorithm. By viewing a nonlinear dynamic system such as a jump-Markov model, we develop an efficient auxiliary particle filtering algorithm to track both the discrete and continuous state variables of this system as well as the associated system parameters. Simulation results are provided to demonstrate the excellent performance of the proposed adaptive mobile positioning algorithm in WCDMA networks.

Keywords and phrases: mobility tracking, Bayesian inference, jump-Markov model, auxiliary particle filter.

1. INTRODUCTION

Mobile positioning [1, 2, 3, 4], that is, estimating the location of a mobile user in wireless networks, has recently received significant attention due to its various potential applications in location-based services, such as location-based billing, intelligent transportation systems [5], and the enhanced-911 (E-911) wireless emergence services [6]. In addition to facilitating these location-based services, the mobility information can also be used by a number of control and management functionalities in a cellular system, such as mobile location indication, handoff assistance [3], transmit power control, and admission control.

Various mobile positioning schemes have been proposed in the literature. Typically, they are based on the measurements of received signal strength [7], time of arrival (TOA) or time difference of arrival (TDOA) [8], and angle of arrival (AOA) [4]. In [4], a hybrid TDOA/AOA method is proposed and the mobile user location is calculated using a two-step least-square estimator. Although this scheme offers a higher location accuracy than the pure TDOA scheme, there is still a gap between its performance and the optimal performance since it is based on a linear approximation of the highly nonlinear mobility model. Moreover, that work deals with the static scenario only and does not address mobility tracking in a dynamic environment. In [2, 9, 10], the extended Kalman filter (EKF) is used to track the user mobility. It is well known that the EKF is based on linearization of the underlying nonlinear dynamic system and often diverges when the system exhibits strong nonlinearity.

On the other hand, the recently emerged sequential Monte-Carlo (SMC) methods [11, 12] are powerful tools for online Bayesian inference of nonlinear dynamic systems. The SMC can be loosely defined as a class of methods for solving online estimation problems in dynamic systems, by recursively generating Monte-Carlo samples of the state variables or some other latent variables. In [3], an SMC algorithm for mobility tracking and handoff in wireless cellular networks is developed. In [8], several SMC algorithms for positioning, navigation, and tracking are developed, where the mobility model is simpler than the one used in [3]. Note that in both works, the trial sampling density is based only on the prior distribution and does not make use of the measurement information, which renders the algorithms less efficient. Moreover, the model parameters are assumed to be perfectly known, which is not realistic for practical mobile positioning systems.

In this paper, we propose to employ a more efficient SMC method, the auxiliary particle filter, to jointly estimate both the mobility information (location, velocity, acceleration, and the state sequence of commands) and the unknown system parameters. We assume the mobility estimation is based on TDOA measurements at the mobile station (MS) and AOA measurements as well as the received signal strength measurements in the neighbor base stations (BSs). All these measurements are available in WCDMA networks. The remainder of this paper is organized as follows. In Section 2, we describe the nonlinear dynamic system model under consideration, and present the mathematical formulation for the problem of mobility tracking in a WCDMA wireless network. In Section 3, we briefly introduce some background materials on sequential Monte-Carlo techniques. The new mobility tracking algorithms are developed in Section 4. Section 5 provides the simulation results; and Section 6 contains the conclusions.

2. SYSTEM DESCRIPTIONS

2.1. Mobility model

Assume that a mobile of interest moves on a two-dimensional plane, and the motion state $\mathbf{x}_k \triangleq [x_k, v_{x,k}, r_{x,k}, y_k, v_{y,k}, r_{y,k}]^T$ corresponds to the observation measurements at $t_k = t_0 + \Delta t \cdot k$, where Δt is the sampling time interval; x_k and y_k are, respectively, the horizontal and vertical Cartesian coordinates of the mobile position at time instance k; $v_{x,k}$ and $v_{y,k}$ are the corresponding velocities; $r_{x,k}$ and $r_{y,k}$ are the corresponding accelerators. The discrete-time moving equation can be expressed as [2, 3]

$$\begin{bmatrix} x_k \\ v_{x,k} \\ y_k \\ v_{y,k} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ v_{x,k-1} \\ y_{k-1} \\ v_{y,k-1} \end{bmatrix} + \begin{bmatrix} \frac{\Delta t^2}{2} & 0 \\ \Delta t & 0 \\ 0 & \frac{\Delta t^2}{2} \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} a_{x,k-1} \\ a_{y,k-1} \end{bmatrix}, \tag{1}$$

where $\mathbf{a}_k \triangleq [a_{x,k}, a_{y,k}]^T$ is the driving acceleration vector at time k. Note that in mobility tracking applications, the time interval Δt between two consecutive update intervals is typically on the order of several hundred symbol intervals to allow for the measurements of TDOA, AOA, and RSS. Such a relatively large time scale also makes it possible to employ more sophisticated signal processing methods for more accurate mobility tracking.

In practical cellular systems, a mobile user may have sudden and unexpected changes in acceleration caused by traffic lights and/or road turn; on the other hand, the acceleration of the mobile may be highly correlated in time. In order to incorporate the unexpected as well as the highly correlated changes in acceleration, we model the motion of a user as a dynamic system driven by a command $\mathbf{s}_k \triangleq [s_{x,k}, s_{y,k}]^T$ and a correlated random acceleration $\mathbf{r}_k \triangleq [r_{x,k}, r_{y,k}]^T$, that is, $\mathbf{a}_k = \mathbf{s}_k + \mathbf{r}_k$. Following [2, 3], the command $\mathbf{s}_k$ is modelled as a first-order discrete-time Markov chain with finite state $\mathcal{S} = \{S_1, S_2, \dots, S_N\}$ and the transition probability matrix $\mathbf{A} \triangleq [a_{i,j}]$, $a_{i,j} \triangleq P(\mathbf{s}_k = S_j \mid \mathbf{s}_{k-1} = S_i)$. It is assumed that $a_{i,j} = p$ for $i = j$ and $a_{i,j} = (1-p)/(N-1)$ for $i \neq j$, where N is the total number of states. The correlated random accelerator $\mathbf{r}_k$ is modelled as the first-order autoregressive (AR) model, that is, $\mathbf{r}_k = \alpha \mathbf{r}_{k-1} + \mathbf{w}_k$, where α is the AR coefficient, $0 < \alpha < 1$, and $\mathbf{w}_k$ is a Gaussian noise vector with covariance matrix $\sigma_w^2 \mathbf{I}$.

Based on the above discussion, the motion model can be expressed as

$$\underbrace{\begin{bmatrix} x_k \\ v_{x,k} \\ r_{x,k} \\ y_k \\ v_{y,k} \\ r_{y,k} \end{bmatrix}}_{\mathbf{x}_k} = \underbrace{\begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} & 0 & 0 & 0 \\ 0 & 1 & \Delta t & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & \alpha \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} x_{k-1} \\ v_{x,k-1} \\ r_{x,k-1} \\ y_{k-1} \\ v_{y,k-1} \\ r_{y,k-1} \end{bmatrix}}_{\mathbf{x}_{k-1}} + \underbrace{\begin{bmatrix} \frac{\Delta t^2}{2} & 0 \\ \Delta t & 0 \\ 0 & 0 \\ 0 & \frac{\Delta t^2}{2} \\ 0 & \Delta t \\ 0 & 0 \end{bmatrix}}_{\mathbf{C}_s} \underbrace{\begin{bmatrix} s_{x,k} \\ s_{y,k} \end{bmatrix}}_{\mathbf{s}_k} + \underbrace{\begin{bmatrix} \frac{\Delta t^2}{2} & 0 \\ \Delta t & 0 \\ 1 & 0 \\ 0 & \frac{\Delta t^2}{2} \\ 0 & \Delta t \\ 0 & 1 \end{bmatrix}}_{\mathbf{C}_w} \underbrace{\begin{bmatrix} w_{x,k} \\ w_{y,k} \end{bmatrix}}_{\mathbf{w}_k}. \tag{2}$$

In short,

$$\mathbf{x}_k = \mathbf{B}\mathbf{x}_{k-1} + \mathbf{C}_s \mathbf{s}_k + \mathbf{C}_w \mathbf{w}_k. \tag{3}$$

2.2. Measurement model

Some new features in WCDMA systems (e.g., cdma2000) such as network synchrony among the BSs, dedicated reverse-link for each MS, adaptive antenna array for AOA estimation, and forward link common broadcasting channel, make several measurements available in practice for mobile tracking.

First of all, methods for determining the time difference of arrival (TDOA) from the spread-spectrum signal, including the coarse timing acquisition with a sliding correlator or matched filter, and fine timing acquisition with a delay-locked loop (DLL) or tau-dither loop (TDL) [13, 14], can be applied in WCDMA systems. Coarse timing acquisition can achieve the accuracy within one chip duration whereas fine synchronization by the DLL can achieve the accuracy within fractional portion of chip duration. Moreover, in WCDMA systems, much higher chip rate is used than that in IS-95 systems with shorter chip period, thereby improving the precision of timing. Furthermore, with multiple antennas to collect the radio signal at the base station (in particular, phaseinformation), we could apply array signal processing algorithms (e.g., MUSIC or ESPRIT) [15] to estimate the angle of arrival (AOA). In addition, the received signal strength indicator (RSSI) signal in WCDMA systems contains the distance information between a mobile and a given base station, which is quantified by the large-scale path-loss model with lognormal shadowing [16]. Note that by averaging the received pilot signal, the rapid fluctuation of multipath fading (i.e., small-scale fading effect) is mitigated. Based on the above discussion, we consider the measurements for mobility tracking to include RSS $p_{k,i}$, AOA $\beta_{k,i}$ at the BSs, and TDOA $\tau_{k,i}$ fed back from MS. Denote $D_{k,i} = [(x_k - a_i)^2 + (y_k - b_i)^2]^{1/2}$, where (a_i, b_i) is the position of the ith BS. We have

$$\begin{aligned} p_{k,i} &= p_{0,i} - 10\eta \log D_{k,i} + n_{k,i}^{p}, \quad i = 1,2,3, \\ \tau_{k,i} &= \frac{1}{c}\left(D_{k,i} - D_{k,1}\right) + n_{k,i}^{\tau}, \quad i = 2,3, \\ \beta_{k,i} &= \tan^{(-1)}\left(\frac{y_k - b_i}{x_k - a_i}\right) + n_{k,i}^{\beta}, \quad i = 1,2,3, \end{aligned} \tag{4}$$

where i is the BS index; $p_{0,i}$ is a constant determined by the wavelength and the antenna gain of the ith BS; $n_{k,i}^{p} \sim \mathcal{N}(0, \eta_d)$ is the logarithm of the shadowing component, which is modelled as Gaussian distribution; c is the speed of light and η is the path-loss factor; $n_{k,i}^{\tau} \sim \mathcal{N}(0, \eta_\tau)$ is the measurement noise of TDOA between the ith BS and the serving BS; and $n_{k,i}^{\beta} \sim \mathcal{N}(0, \eta_\beta)$ is the estimation error of AOA at the ith BS. The noise terms in (4) are assumed to be white both in space and in time.

Denote the measurements at time instance k as $\mathbf{z}_k \triangleq [p_{k,1}, p_{k,2}, p_{k,3}, \tau_{k,2}, \tau_{k,3}, \beta_{k,1}, \beta_{k,2}, \beta_{k,3}]^T$. Then, we have the following measurement equation of the underlying dynamic model:

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \tag{5}$$

where $\mathbf{v}_k \triangleq [n_{k,1}^{p}, n_{k,2}^{p}, n_{k,3}^{p} n_{k,1}^{\tau}, n_{k,2}^{\tau}, n_{k,1}^{\beta}, n_{k,2}^{\beta}, n_{k,3}^{\beta}]$ with covariance matrix $\mathbf{Q} = \operatorname{diag}(\eta_d \mathbf{I}, \eta_t \mathbf{I}, \eta_\beta \mathbf{I})$; and $\mathbf{h}(\mathbf{x}_k) \triangleq [h_1(\mathbf{x}_k), \ldots, h_8(\mathbf{x}_k)]$ where the form of each $h_i(\cdot)$, is given by (4). Note that the availability of TDOA and AOA will enhance the mobility tracking accuracy. In practice, if in some served mobiles such information is not available, mobility tracking can still be performed based only on the RSS measurement, with less accuracy.

2.3. Problem formulation

Based on the discussions above, the nonlinear dynamic system under consideration can be represented by a jump-Markov model as follows:

$$\mathbf{s}_k \sim \mathcal{MC}(\boldsymbol{\pi}, \mathbf{A}), \quad \mathbf{x}_k = \mathbf{B}\mathbf{x}_{k-1} + \mathbf{C}_s \mathbf{s}_k + \mathbf{C}_w \mathbf{w}_k, \quad \mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \tag{6}$$

where $\mathcal{MC}(\boldsymbol{\pi}, \mathbf{A})$ denotes a first-order Markov chain with initial probability vector $\boldsymbol{\pi}$ and transition matrix $\mathbf{A}$. Denote the observation sequence up to time k as $\mathbf{Z}_k \triangleq [\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_k]$, the corresponding discrete state sequence $\mathbf{S}_k \triangleq [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_k]$, and the continuous state sequence $\mathbf{X}_k \triangleq [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k]$. Let the model parameters be $\boldsymbol{\theta} = \{\boldsymbol{\pi}, \mathbf{A}, \eta_w, \eta_d, \eta_t, \eta_\beta\}$. Given the observations $\mathbf{Z}_k$ up to time k, our problem is to infer the current position and velocity. This amounts to making inference with respect to

$$\begin{aligned} p(\mathbf{x}_k \mid \mathbf{Z}_k) &= \sum_{\mathbf{S}_k \in \mathcal{S}^k} \int \cdots \int p(\mathbf{s}_1, \ldots, \mathbf{s}_k, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1} \mid \mathbf{Z}_k) d\mathbf{x}_1, \ldots, d\mathbf{x}_{k-1} \\ &\propto \sum_{\mathbf{S}_k \in \mathcal{S}^k} \int \cdots \int \prod_{i=1}^{k} \Big[p(\mathbf{z}_i \mid \mathbf{x}_i) p(\mathbf{x}_i \mid \mathbf{x}_{i-1}, \mathbf{s}_i) p(\mathbf{s}_i \mid \mathbf{s}_{i-1}) \Big] d\mathbf{x}_1, \ldots, d\mathbf{x}_{k-1}. \end{aligned} \tag{7}$$

The above exact expression of $p(\mathbf{x}_k \mid \mathbf{Z}_k)$ involves very high-dimensional integrals and the dimensionality grows linearly with time, which is prohibitive to compute in practice. In what follows, we resort to the sequential Monte-Carlo techniques to solve the above inference problem.

3. BACKGROUND ON SEQUENTIAL MONTE CARLO

Consider the following jump-Markov model:

$$\begin{aligned} \mathbf{x}_k &= \mathbf{A}(\mathbf{s}_k)\mathbf{x}_{k-1} + \mathbf{B}(\mathbf{s}_k)\mathbf{v}_k, \\ \mathbf{z}_k &= \mathbf{C}(\mathbf{s}_k)\mathbf{x}_k + \mathbf{D}(\mathbf{s}_k)\boldsymbol{\varepsilon}_k, \end{aligned} \tag{8}$$

where $\mathbf{v}_k \overset{\text{i.i.d.}}{\sim} \mathcal{N}_c(0, \eta_v \mathbf{I})$, $\boldsymbol{\varepsilon}_k \overset{\text{i.i.d.}}{\sim} \mathcal{N}_c(0, \eta_\varepsilon \mathbf{I})$, and $\mathbf{s}_k$ is the discrete hidden state evolving according to a discrete-time Markov chain with initial probability vector $\boldsymbol{\pi}$ and transition probability matrix $\mathbf{A}$. Denote $\mathbf{y}_k \triangleq \{\mathbf{x}_k, \mathbf{s}_k\}$ and the system parameters $\boldsymbol{\theta} = \{\boldsymbol{\pi}, \mathbf{A}, \eta_v, \eta_\varepsilon\}$. Suppose we want to make an online inference about the unobserved states $\mathbf{Y}_k = (\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k)$ from a set of available observations $\mathbf{Z}_k = (\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_k)$. Monte-Carlo methods approximate such inference by drawing m random samples $\{\mathbf{Y}_k^{(j)}\}_{j=1}^{m}$ from the posterior distribution $p(\mathbf{Y}_k \mid \mathbf{Z}_k)$. Since sampling directly from $p(\mathbf{Y}_k \mid \mathbf{Z}_k)$ is often not feasible or computationally too expensive, we can instead draw samples from some trial sampling density $q(\mathbf{Y}_k \mid \mathbf{Z}_k)$, and calculate the target inference $E_p\{\varphi(\mathbf{Y}_k) \mid \mathbf{Z}_k\}$ using samples drawn from $q(\cdot)$ as

$$E_p\{\varphi(\mathbf{Y}_k) \mid \mathbf{Z}_k\} \cong \frac{1}{W_k}\sum_{j=1}^{m} w_k^{(j)}\varphi\left(\mathbf{Y}_k^{(j)}\right), \tag{9}$$

where $w_k^{(j)} = p(\mathbf{Y}_k^{(j)} \mid \mathbf{Z}_k)/q(\mathbf{Y}_k^{(j)} \mid \mathbf{Z}_k)$, $W_k = \sum_{j=1}^m w_k^{(j)}$, and the pair $\{\mathbf{Y}_k^{(j)}, w_k^{(j)}\}_{j=1}^m$ is called a set of *properly weighted samples* with respect to the distribution $p(\mathbf{Y}_k \mid \mathbf{Z}_k)$ [17].

Suppose a set of properly weighted samples $\{\mathbf{Y}_{k-1}^{(j)}, w_{k-1}^{(j)}\}_{j=1}^m$ with respect to $p(\mathbf{Y}_{k-1} \mid \mathbf{Z}_{k-1})$ has been drawn at time $(k-1)$, the sequential Monte-Carlo (SMC) procedure generates a new set of samples $\{\mathbf{Y}_k^{(j)}, w_k^{(j)}\}_{j=1}^m$ properly weighted with respect to $p(\mathbf{Y}_k \mid \mathbf{Z}_k)$. In [18], it is shown that the optimal trial distribution is $p(\mathbf{y}_k \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_k)$, which minimizes the conditional variance of the importance weights. The SMC recursion at time k is as follows [17, 19].

For $j = 1, \dots, m$,

(i) draw a sample $\mathbf{y}_k^{(j)}$ from the trial distribution

$$\begin{aligned} p\left(\mathbf{y}_k \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_k\right) &\propto p(\mathbf{z}_k \mid \mathbf{y}_k)\, p\left(\mathbf{y}_k \mid \mathbf{y}_{k-1}^{(j)}\right) \\ &= p(\mathbf{z}_k \mid \mathbf{x}_k, \mathbf{s}_k)\, p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k\right) p\left(\mathbf{s}_k \mid \mathbf{s}_{k-1}^{(j)}\right), \end{aligned} \tag{10}$$

and let $\mathbf{Y}_k^{(j)} = (\mathbf{Y}_{k-1}^{(j)}, \mathbf{y}_k^{(j)})$,

(ii) update the importance weight

$$\begin{aligned} w_k^{(j)} &\propto w_{k-1}^{(j)} p\left(\mathbf{z}_k \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1}\right) \\ &= w_{k-1}^{(j)} \sum_{\mathbf{s}_k=1}^{N} p\left(\mathbf{s}_k \mid \mathbf{s}_{k-1}^{(j)}\right) \int p(\mathbf{z}_k \mid \mathbf{x}_k, \mathbf{s}_k, \mathbf{Z}_{k-1})\, p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k\right) d\mathbf{x}_k. \end{aligned} \tag{11}$$

Apparently, it is difficult to use such an optima trial sampling density because the importance weight update equation does not admit a closed-form and involves a high-dimension integral for each sample stream [19]. To approximate the integral in (11), we use

$$\begin{aligned} &\int p(\mathbf{z}_k \mid \mathbf{x}_k, \mathbf{s}_k, \mathbf{Z}_{k-1})\, p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k\right) d\mathbf{x}_k \\ &\quad\approx \int p(\mathbf{z}_k \mid \mathbf{x}_k, \mathbf{s}_k, \mathbf{Z}_{k-1})\, \delta\left(\mathbf{x}_k = \boldsymbol{\mu}_k^{(j)}\left(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k\right)\right) d\mathbf{x}_k \\ &\quad= p\left(\mathbf{z}_k \mid \mathbf{Z}_{k-1}, \boldsymbol{\mu}_k^{(j)}\left(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k\right)\right), \end{aligned} \tag{12}$$

where $\boldsymbol{\mu}_k(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k)$ is the mean of $p(\mathbf{x}_k \mid \mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k)$. Using (12), the importance weight update is approximated by

$$w_k^{(j)} \approx w_{k-1}^{(j)} \underbrace{\sum_{\mathbf{s}_k=1}^{N} p\left(\mathbf{z}_k \mid \boldsymbol{\mu}_k^{(j)}\left(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k\right), \mathbf{Z}_{k-1}\right) p\left(\mathbf{s}_k \mid \mathbf{s}_{k-1}^{(j)}\right)}_{\psi\left(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}_{k-1}^{(j)}, \mathbf{z}_k\right)}. \tag{13}$$

To make the SMC procedure efficient in practice, it is necessary to use a *resampling* procedure as suggested in [17, 18]. Roughly speaking, the aim of resampling is to duplicate the sample streams with large importance weights while eliminating the streams with small ones. In [19], it is suggested that we resample $\{\mathbf{Y}_{k-1}^{(j)}\}$ according to the weights $\rho_k^{(j)} \propto w_{k-1}^{(j)}\psi(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}_{k-1}^{(j)}, \mathbf{z}_k)$. Since the term $\psi(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}_{k-1}^{(j)}, \mathbf{z}_k)$ is independent of $\mathbf{s}_k^{(j)}$ and $\mathbf{x}_k^{(j)}$, we use it as the p.d.f. for generating the auxiliary index κ_k before we sample the state variables $(\mathbf{s}_k, \mathbf{x}_k)$. Such a scheme is termed as the auxiliary particle filter [20], where some auxiliary variable is introduced in the sampling space such that the trial distribution for the auxiliary variable can make use of the current measurement $\mathbf{z}_k$. In order to utilize the observation in the trial sampling density of $\mathbf{s}_k$, we sample $\mathbf{s}_k$ according to $p(\mathbf{z}_k \mid \boldsymbol{\mu}_k(\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k))p(\mathbf{s}_k \mid \mathbf{s}_{k-1}^{(\kappa_k^{(j)})})$ and sample $\mathbf{x}_k$ according to $p(\mathbf{x}_k \mid \mathbf{X}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)})$. The importance weights are then updated according to

$$\begin{aligned} w_k^{(j)} &\propto \frac{p\left(\mathbf{z}_k \mid \mathbf{x}_k^{(j)}\right) p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)}\right) p\left(\mathbf{s}_k^{(j)} \mid \mathbf{s}_{k-1}^{(\kappa_k^{(j)})}\right)}{p\left(\mathbf{z}_k \mid \boldsymbol{\mu}_k\left(\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)}\right)\right) p\left(\mathbf{s}_k^{(j)} \mid \mathbf{s}_{k-1}^{(\kappa_k^{(j)})}\right) p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)}\right)} \\ &= \frac{p\left(\mathbf{z}_k \mid \mathbf{x}_k^{(j)}\right)}{p\left(\mathbf{z}_k \mid \boldsymbol{\mu}_k\left(\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)}\right)\right)}. \end{aligned} \tag{14}$$

Considering the jump-Markov model (8), we have $\boldsymbol{\mu}_k(\mathbf{x}_{k-1}^{(\kappa^{(j)})}, \mathbf{s}_k^{(j)}) = E\{\mathbf{x}_k \mid \mathbf{s}_k, \mathbf{X}_{k-1}^{(\kappa_k^{(j)})}\} = \mathbf{A}(\mathbf{s}_k)\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}$. The auxiliary particle filter algorithm at the kth recursion is summarized in Algorithm 1.

If the system parameter $\boldsymbol{\theta}$ is unknown, we need to augment the unknown parameter $\boldsymbol{\theta}$ to the state variable $\mathbf{y}_k$ as

(i) For $j = 1, \dots, m$ and $\mathbf{s}_k = 1, \dots, N$, calculate the trial sampling density $\rho_k^{(j)} \propto w_{k-1}^{(j)} \psi(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}_{k-1}^{(j)}, \mathbf{z}_k)$.
(ii) For $j = 1, \dots, m$,
 (a) draw the auxiliary index $\kappa_k^{(j)}$ with probability $\rho_k^{(j)}$,
 (b) draw a sample $\mathbf{s}_k$ from the trial distribution $p(\mathbf{z}_k \mid \boldsymbol{\mu}_k(\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k)) p(\mathbf{s}_k \mid \mathbf{s}_{k-1}^{(\kappa_k^{(j)})})$ and let $\mathbf{S}_k^{(j)} = (\mathbf{S}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)})$,
 (c) draw a sample $\mathbf{x}_k$ from the trial distribution $p(\mathbf{x}_k \mid \mathbf{X}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)})$ and let $\mathbf{X}_k^{(j)} = (\mathbf{X}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{x}_k^{(j)})$,
 (d) update the importance weight $w_k^{(j)} \propto p(\mathbf{z}_k \mid \mathbf{x}_k^{(j)}) / p(\mathbf{z}_k \mid \boldsymbol{\mu}_k(\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)}))$.

Algorithm 1: The auxiliary particle filter algorithm at the kth recursion.

the new state variable. Therefore, we have to sample from the joint density

$$p\left(\mathbf{y}_k, \boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_k\right) = p\left(\mathbf{y}_k \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{z}_k, \boldsymbol{\theta}\right) p\left(\boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1}\right) \propto p(\mathbf{z}_k \mid \mathbf{y}_k, \boldsymbol{\theta}) p\left(\mathbf{y}_k \mid \mathbf{y}_{k-1}^{(j)}, \boldsymbol{\theta}\right) \times p\left(\boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1}\right). \tag{15}$$

And the importance weights are updated according to

$$w_k^{(j)} \propto w_{k-1}^{(j)} p\left(\mathbf{z}_k \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1}, \boldsymbol{\theta}^{(j)}\right) \approx w_{k-1}^{(j)} \sum_{\mathbf{s}_k=1}^{N} p\left(\mathbf{z}_k \mid \boldsymbol{\mu}_k\left(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k\right), \boldsymbol{\theta}^{(j)}\right) p\left(\mathbf{s}_k \mid \mathbf{s}_{k-1}^{(j)}, \boldsymbol{\theta}^{(j)}\right). \tag{16}$$

For each sample stream j, the trial sampling density for the state variable $(\mathbf{s}_k, \mathbf{x}_k)$ and the importance weight update are both based on the sampled unknown parameter $\boldsymbol{\theta}^{(j)}$. At the end of the kth iteration, we update the trial sampling density $p(\boldsymbol{\theta} \mid \mathbf{Y}_k^{(j)}, \mathbf{Z}_k)$ based on $p(\boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1})$, $\mathbf{y}_k^{(j)}$ and $\mathbf{z}_k$. The auxiliary particle filter algorithm at the kth recursion for the case of unknown parameters is summarized in Algorithm 2.

4. NEW MOBILITY TRACKING ALGORITHM

4.1. Online estimator with known parameters

We next outline the SMC algorithm for solving the problem of mobility tracking based on the jump-Markov model given by (6). Let $\mathbf{y}_k = (\mathbf{x}_k, \mathbf{s}_k)$, $\mathbf{X}_k = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_k)$, $\mathbf{S}_k = (\mathbf{s}_1, \dots, \mathbf{s}_k)$, $\mathbf{Y}_k = (\mathbf{y}_1, \dots, \mathbf{y}_k)$, and $\mathbf{Z}_k = (\mathbf{z}_1, \dots, \mathbf{z}_k)$. The aim of mobility tracking is to estimate the posterior distribution of $p(\mathbf{Y}_k \mid \mathbf{Z}_k)$. Using SMC, we can obtain a set of Monte-Carlo samples of the unknown states $\{\mathbf{Y}_k^{(j)}, w_k^{(j)}\}_{j=1}^m$ that are properly weighted with respect to the distribution $p(\mathbf{Y}_k \mid \mathbf{Z}_k)$. The MMSE estimator of the location and velocity at time k can then be approximated by

$$E\{\mathbf{x}_k \mid \mathbf{Z}_k\} \cong \frac{1}{W_k} \sum_{j=1}^{m} \mathbf{x}_k^{(j)} \cdot w_k^{(j)}, \quad k = 1, 2, \dots, \tag{17}$$

(i) For $j = 1, \dots, m$,
 (a) draw samples of the unknown parameter $\{\boldsymbol{\theta}^{(j)}\}_{j=1}^m$ from $p(\boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1})$,
 (b) calculate the auxiliary variable sampling density $\rho_k^{(j)} \propto w_{k-1}^{(j)} \sum_{\mathbf{s}_k=1}^N p(\mathbf{z}_k \mid \boldsymbol{\mu}_k(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k), \boldsymbol{\theta}^{(j)}) p(\mathbf{s}_k \mid \mathbf{s}_{k-1}^{(j)}, \boldsymbol{\theta}^{(j)})$.
(ii) For $j = 1, \dots, m$,
 (a) draw the auxiliary index $\kappa_k^{(j)}$ with probability $\rho_k^{(j)}$,
 (b) draw a sample $\mathbf{s}_k^{(j)}$ from the trial distribution $p(\mathbf{z}_k \mid \boldsymbol{\mu}_k(\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k), \boldsymbol{\theta}^{(j)}) p(\mathbf{s}_k \mid \mathbf{s}_{k-1}^{(\kappa_k^{(j)})}, \boldsymbol{\theta}^{(j)})$,
 (c) draw a sample $\mathbf{x}_k^{(j)}$ from the trial distribution $p(\mathbf{x}_k \mid \mathbf{X}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)}, \boldsymbol{\theta}^{(j)})$ and let $\mathbf{y}_k^{(j)} = (\mathbf{s}_k^{(j)}, \mathbf{x}_k^{(j)})$ and let $\mathbf{Y}_k^{(j)} = (\mathbf{Y}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{y}_k^{(j)})$,
 (d) update the importance weight $w_k^{(j)} \propto p(\mathbf{z}_k \mid \mathbf{x}_k^{(j)}, \boldsymbol{\theta}^{(j)}) / p(\mathbf{z}_k \mid \boldsymbol{\mu}_k(\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)}), \boldsymbol{\theta}^{(j)})$,
 (e) update the sampling density $p(\boldsymbol{\theta} \mid \mathbf{Y}_k^{(j)}, \mathbf{Z}_k)$ based on $p(\boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1}), \mathbf{y}_k^{(j)}$ and $\mathbf{z}_k$.

Algorithm 2: The auxiliary particle filter algorithm of the kth recursion for the case of unknown parameters.

where $W_k = \sum_{j=1}^m w_k^{(j)}$. Following the auxiliary particle filter framework discussed in Section 3, we choose the sampling density for generating the auxiliary index κ_k as

$$q(\kappa_k = j) \propto w_{k-1}^{(j)} \sum_{\mathbf{s} \in \mathcal{S}} p\left(\mathbf{z}_k \mid \boldsymbol{\mu}_k\left(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}\right)\right) p\left(\mathbf{s} \mid \mathbf{s}_{k-1}^{(j)}\right), \quad j = 1, \dots, m. \tag{18}$$

Considering the motion equation (3) and the measurement equation (5), we have $\boldsymbol{\mu}_k(\mathbf{x}_{k-1}^{(j)}, \mathbf{s}) = \mathbf{B}\mathbf{x}_{k-1}^{(j)} + \mathbf{C}_s\mathbf{s}$. Next we draw a sample of state $\mathbf{s}_k$ from the trial distribution

$$q(\mathbf{s}_k = \mathbf{s}) \propto p\left(\mathbf{z}_k \mid \boldsymbol{\mu}_k\left(\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}\right)\right) \cdot p\left(\mathbf{s} \mid \mathbf{s}_{k-1}^{(\kappa_k^{(j)})}\right) = \phi\left(\mathbf{h}\left(\boldsymbol{\mu}_k\left(\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}\right)\right), \mathbf{Q}\right) \cdot a_{\mathbf{s}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}}, \tag{19}$$

where $\phi(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes the p.d.f. of a multivariate Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$. The trial sampling density for $\mathbf{x}_k$ is given by

$$p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k^{(j)}\right) = \phi\left(\mathbf{B}\mathbf{x}_{k-1}^{(\kappa_k^{(j)})} + \mathbf{C}_s\mathbf{s}_k^{(j)}, \eta_w \mathbf{C}_w \mathbf{C}_w^T\right). \tag{20}$$

And the importance weight is updated according to

$$w_k^{(j)} \propto \frac{p\left(\mathbf{z}_k \mid \mathbf{x}_k^{(j)}\right)}{p\left(\mathbf{z}_k \mid \boldsymbol{\mu}_k\left(\mathbf{x}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{s}_k\right)\right)}, \tag{21}$$

where $p(\mathbf{z}_k \mid \mathbf{x}_k) = \phi(\mathbf{h}(\mathbf{x}_k), \mathbf{Q})$. Finally, we summarize the adaptive mobile positioning algorithm with known parameters in Algorithm 3.

(I) Initialization: for $j = 1, \ldots, m$, draw the state vector $\mathbf{x}_0^{(j)}$ from the multivariate Gaussian distribution $\mathcal{N}(\mathbf{x}_0, 10\mathbf{I})$ and draw $\mathbf{s}_0^{(j)}$ uniformly from $\mathcal{S}$; all importance weights are initialized as $w_0^{(j)} = 1$.
(II) For $k = 1, 2, \ldots,$
  (a) for $j = 1, \ldots, m$, calculate the trial sampling density for the auxiliary index according to (18),
  (b) for $j = 1, \ldots, m$,
    (i) draw an auxiliary index $\kappa_k^{(j)}$ with the probability $q(\kappa_k = j)$,
    (ii) draw a sample $\mathbf{s}_k^{(j)}$ according to (19),
    (iii) draw a sample $\mathbf{x}_k^{(j)}$ according to (20),
    (iv) update the importance weight $w_k^{(j)}$ according to (21),
    (v) append $\mathbf{y}_k^{(j)} = \{\mathbf{x}_k^{(j)}, \mathbf{s}_k^{(j)}\}$ to $\mathbf{Y}_{k-1}^{(\kappa^{(j)})}$ to form $\mathbf{Y}_k^{(j)} = \{\mathbf{Y}_{k-1}^{(\kappa^{(j)})}, \mathbf{y}_k^{(j)}\}$.

Algorithm 3: Adaptive mobile positioning algorithm with known system parameters.

Complexity

The major computation involved in Algorithm 3 includes evaluations of Gaussian densities (i.e., mN evaluations in (18), mN evaluations in (19), and m evaluations in (21))), and simple multiplications (i.e., mN multiplications in (18) and mN multiplications in (19)). Note that Algorithm 3 is well suited for parallel implementations.

4.2. Online estimator with unknown parameters

We next treat the problem of jointly tracking the state $\mathbf{Y}_k$ and the unknown parameters $\boldsymbol{\theta} = \{\boldsymbol{\pi}, \mathbf{A}, \eta_w, \eta_d, \eta_t, \eta_\beta\}$. We first specify the priors for the unknown parameters. For the initial probability vector $\boldsymbol{\pi}$ and the ith row of the transition probability matrix $\mathbf{A}$, we choose a Dirichlet distribution as their priors:

$$\begin{gathered} \boldsymbol{\pi} \sim \mathcal{D}(\alpha_1, \alpha_2, \ldots, \alpha_N), \\ \mathbf{a}_i \sim \mathcal{D}(\alpha_1, \alpha_2, \ldots, \alpha_N), \quad i = 1, \ldots, N. \end{gathered} \tag{22}$$

For the noise variances, η_w, η_d, η_t, and η_β, we use the inverse chi-square priors:

$$\begin{aligned} \eta_w &\sim \chi^{-2}(\nu_{0,w}, \lambda_{0,w}), & \eta_d &\sim \chi^{-2}(\nu_{0,d}, \lambda_{0,d}), \\ \eta_t &\sim \chi^{-2}(\nu_{0,t}, \lambda_{0,t}), & \eta_\beta &\sim \chi^{-2}(\nu_{0,\beta}, \lambda_{0,\beta}). \end{aligned} \tag{23}$$

Suppose that at time $(k-1)$, we have m sample streams of state $\mathbf{Y}_{k-1}$ and parameter $\boldsymbol{\theta}$, $\{\mathbf{Y}_{k-1}^{(j)}, \boldsymbol{\theta}_{k-1}^{(j)}\}_{j=1}^m$, and the associated importance weights $\{w_{k-1}^{(j)}\}_{j=1}^m$, representing an important sample approximation to the posterior distribution $p(\mathbf{Y}_{k-1}, \boldsymbol{\theta} \mid \mathbf{Z}_{k-1})$ at time $(k-1)$. Note that here the index k on the parameter samples indicates that they are drawn from the posterior distribution at time k rather than implying that $\boldsymbol{\theta}$ is time-varying. By applying Bayes' theorem and considering the system equations (6), at time k, we sample the state variable and the unknown parameter from

$$\begin{aligned} p\big(\mathbf{y}_k, \boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_k\big) & \\ \propto p(\mathbf{z}_k \mid \mathbf{y}_k, \boldsymbol{\theta})\, & p\big(\mathbf{y}_k \mid \mathbf{y}_{k-1}^{(j)}, \mathbf{z}_k, \boldsymbol{\theta}\big)\, p\big(\boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1}\big), \end{aligned} \tag{24}$$

where $p(\boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1})$ is the trial sampling density for the unknown parameter at time $(k-1)$ and can be decomposed as

$$\begin{aligned} p\big(\boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1}\big) &= p\big(\boldsymbol{\pi}, \mathbf{A}, \eta_w, \eta_v, \eta_t, \eta_\beta \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1}\big) \\ &= p\big(\boldsymbol{\pi} \mid \mathbf{s}_0^{(j)}\big)\left[\prod_{i=1}^N p\big(\mathbf{a}_i \mid \boldsymbol{\pi}, \mathbf{S}_{k-1}^{(j)}\big)\right] p\big(\eta_w \mid \mathbf{X}_{k-1}^{(j)}, \mathbf{Z}_k\big) \\ &\quad \times p\big(\eta_d \mid \mathbf{X}_{k-1}^{(j)}, \mathbf{Z}_{k-1}\big)\, p\big(\eta_t \mid \mathbf{X}_{k-1}^{(j)}, \mathbf{Z}_{k-1}\big)\, p\big(\eta_\beta \mid \mathbf{X}_{k-1}^{(j)}, \mathbf{Z}_{k-1}\big). \end{aligned} \tag{25}$$

Suppose we have updated the trial sampling density for $\boldsymbol{\theta}$ at the end of time $(k-1)$. Based on the sampled parameters $\boldsymbol{\theta}_k^{(j)} = \{\boldsymbol{\pi}^{(j)}, \mathbf{A}^{(j)}, \eta_w^{(j)}, \eta_d^{(j)}, \eta_t^{(j)}, \eta_\beta^{(j)}\} \sim p(\boldsymbol{\theta} \mid \mathbf{Y}_{k-1}^{(j)}, \mathbf{Z}_{k-1})$ at time k, we draw samples of the auxiliary index κ_k, the discrete state $\mathbf{s}_k$, and the continuous state $\mathbf{x}_k$ according to (18), (19), and (20) and update the importance weight using (21). In (18), (19), (20), and (21), the known system parameter $\boldsymbol{\theta}$ is replaced by $\boldsymbol{\theta}_k^{(\kappa_k^{(j)})}$ and the noise covariance matrix $\mathbf{Q}$ is substituted by $\mathbf{Q}^{(\kappa_k^{(j)})} = \mathrm{diag}(\eta_d^{(\kappa_k^{(j)})}\mathbf{I}, \eta_t^{(\kappa_k^{(j)})}\mathbf{I}, \eta_\beta^{(\kappa_k^{(j)})}\mathbf{I})$. The location and velocity are estimated through (17) and the minimum mean-squared error (MMSE) estimate of the unknown parameter $\boldsymbol{\theta}$ at time k is given by $\hat{\boldsymbol{\theta}}_k = (1/W_k)\sum_{j=1}^m \boldsymbol{\theta}_k^{(j)} w_k^{(j)}$, where $W_k = \sum_{j=1}^m w_k^{(j)}$. At the end of time k, we update the trial sampling density for $\boldsymbol{\theta}$ as follows.

At the end of time k, we update the trial sampling density for the initial state probability vector $\boldsymbol{\pi}$ as

$$p\big(\boldsymbol{\pi} \mid \mathbf{s}_0^{(j)}\big) \sim \mathcal{D}\big(\alpha_1 + \delta_{\mathbf{s}_0^{(j)} - 1}, \alpha_2 + \delta_{\mathbf{s}_0^{(j)} - 2}, \ldots, \alpha_N + \delta_{\mathbf{s}_0^{(j)} - N}\big). \tag{26}$$

Given the prior distribution of the ith row $\mathbf{a}_i$ of the transition probability matrix $\mathbf{A}$ at the end of time $(k-1)$, that is, $p(\mathbf{a}_i \mid \boldsymbol{\pi}, \mathbf{S}_{k-1}^{(j)}) \sim \mathcal{D}(\alpha_{i,1}^{(k-1,j)}, \alpha_{i,2}^{(k-1,j)}, \ldots, \alpha_{i,N}^{(k-1,j)})$, at time k, the trial sampling density for $\mathbf{a}_i$ is updated according to

$$\begin{aligned} p\big(\mathbf{a}_i \mid \boldsymbol{\pi}, \mathbf{S}_k^{(j)}\big) &\propto p\big(\mathbf{s}_k^{(j)} \mid \boldsymbol{\pi}, \mathbf{S}_{k-1}^{(j)}, \mathbf{a}_i\big)\, p\big(\mathbf{a}_i \mid \boldsymbol{\pi}, \mathbf{S}_{k-1}^{(j)}\big) \\ &\sim \mathcal{D}\Bigg(\underbrace{\alpha_{i,1}^{(k-1,j)} + \delta_{\mathbf{s}_{k-1}^{(j)} - i}\delta_{\mathbf{s}_k^{(j)} - 1}}_{\alpha_{i,1}^{(k,j)}}, \underbrace{\alpha_{i,2}^{(k-1,j)} + \delta_{\mathbf{s}_{k-1}^{(j)} - i}\delta_{\mathbf{s}_k^{(j)} - 2}}_{\alpha_{i,2}^{(k,j)}}, \ldots, \\ &\qquad \underbrace{\alpha_{i,N}^{(k-1,j)} + \delta_{\mathbf{s}_{k-1}^{(j)} - i}\delta_{\mathbf{s}_k^{(j)} - N}}_{\alpha_{i,N}^{(k,j)}}\Bigg). \end{aligned} \tag{27}$$

And given the noise variance sampling density at time $(k-1)$, $p(\eta_w \mid \mathbf{X}_{k-1}^{(j)}, \mathbf{Z}_{k-1}) \sim \chi^{-2}(\nu_{k-1,w}, \lambda_{k-1,w}^{(j)})$, at time k, the trial sampling density for η_w is updated according to

$$p\left(\eta_w \mid \mathbf{Y}_k^{(j)}, \mathbf{Z}_k\right) \propto p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}^{(j)}, \mathbf{s}_k^{(j)}, \eta_w\right) p\left(\eta_w \mid \mathbf{X}_{k-1}^{(j)}, \mathbf{Z}_{k-1}\right) \sim \chi^{-2}\left(\nu_{k-1}+1, \lambda_{k,w}^{(j)}\right), \tag{28}$$

where $\lambda_{k,w}^{(j)} = (\nu_{0,w}+k-1)/(\nu_{0,w}+k)\lambda_{k-1,w}^{(j)} + \sum_{i=1}^{2}(x_{k,3i} - \alpha x_{k-1,3i})^2/2(\nu_{0,w}+k)$. Similarly, we have

$$p\left(\eta_d \mid \mathbf{Y}_k^{(j)}, \mathbf{Z}_k\right) \sim \chi^{-2}\left(\nu_{k-1,d}+1, \lambda_{k,d}^{(j)}\right), \tag{29}$$

$$p\left(\eta_t \mid \mathbf{Y}_k^{(j)}, \mathbf{Z}_k\right) \sim \chi^{-2}\left(\nu_{k-1,t}+1, \lambda_{k,t}^{(j)}\right), \tag{30}$$

$$p\left(\eta_\beta \mid \mathbf{Y}_k^{(j)}, \mathbf{Z}_k\right) \sim \chi^{-2}\left(\nu_{k-1,\beta}+1, \lambda_{k,\beta}^{(j)}\right), \tag{31}$$

where $\lambda_{k,d}^{(j)} = (\nu_{0,d}+k-1)/(\nu_{0,d}+k)\lambda_{k-1,d}^{(j)} + \sum_{i=1}^{3}(p_{i,k} - h_i(\mathbf{x}_k^{(j)}))^2/3(\nu_{0,d}+k)$, $\lambda_{k,t}^{(j)} = (\nu_{0,t}+k-1)/(\nu_{0,t}+k)\lambda_{k-1,t}^{(j)} + \sum_{i=1}^{2}(\tau_{i,k} - h_{i+3}(\mathbf{x}_k^{(j)}))^2/2(\nu_{0,t}+k)$ and $\lambda_{k,\beta}^{(j)} = (\nu_{0,\beta}+k-1)/(\nu_{0,\beta}+k)\lambda_{k-1,\beta}^{(j)} + \sum_{i=1}^{3}(\beta_{k,i} - h_{i+5}(\mathbf{x}_k^{(j)}))^2/3(\nu_{0,\beta}+k)$. Finally, we summarize the adaptive mobile positioning algorithm with unknown system parameters Algorithm 4.

Complexity

Compared with the known parameter case, that is, Algorithm 3, the additional computation in Algorithm 4 is introduced by the updates of the trial densities of the unknowns and the draws of these parameters, which at iteration, involve $4m$ simple multiplications, as well as the $m(N+1)$ samplings from the Dirichlet distribution and $4m$ samplings from the inverse chi-square distribution. As noted previously, since in mobility tracking applications the update is performed at a time scale of several hundred symbols, the above SMC-based tracking algorithm is feasible to implement in practice.

(I) Initialization: for $j = 1,\ldots,m$, draw the samples of the initial probability vector $\boldsymbol{\pi}$, the ith row $\mathbf{a}_i$ of the transition probability matrix, the noise variance η_w, η_d, η_t, and η_β according to their prior distributions in (22) and (23), respectively. Draw the state vector $\mathbf{x}_0^{(j)}$ from the multivariate Gaussian distribution $\mathcal{N}(\mathbf{x}_0, 10\mathbf{I})$, and draw $\mathbf{s}_0^{(j)}$ uniformly from $\mathcal{S}$, all importance weights are initialized as $w_0^{(j)} = 1$.

(II) For $k = 1, 2, \ldots$,
- (a) for $j = 1, 2, \ldots, m$, calculate the trial sampling density for the auxiliary index according to (18), where the actually unknown parameter $\boldsymbol{\theta}$ is replaced by $\boldsymbol{\theta}_{k-1}^{(j)}$,
- (b) for $j = 1, 2, \ldots, m$,
  - (i) draw an auxiliary index $\kappa_k^{(j)}$ with the probability $q(\kappa_k = j)$,
  - (ii) draw a sample $\mathbf{s}_k^{(j)}$ according to (19),
  - (iii) draw a sample $\mathbf{x}_k^{(j)}$ according to (20),
  - (iv) update the importance weights $w_k^{(j)}$ according to (21),
  - (v) append $\mathbf{y}_k^{(j)} = \{\mathbf{x}_k^{(j)}, \mathbf{s}_k^{(j)}\}$ and $\mathbf{Y}_{k-1}^{(\kappa_k^{(j)})}$ to form $\mathbf{Y}_k^{(j)} = \{\mathbf{Y}_{k-1}^{(\kappa_k^{(j)})}, \mathbf{y}_k^{(j)}\}$,
  - (vi) update the trial sampling density for $\boldsymbol{\theta}$ according to (26), (28), (29), (30), and (31),
  - (vii) sample the unknown system parameters $\boldsymbol{\theta}_k^{(j)} = (\boldsymbol{\pi}^{(j)}, \mathbf{A}^{(j)}, \eta_w^{(j)}, \eta_d^{(j)}, \eta_t^{(j)}, \eta_\beta^{(j)})$ according to (26), (28), (29), (30), and (31), respectively.

ALGORITHM 4: Adaptive mobile positioning algorithm with unknown system parameters.

5. SIMULATION

Computer simulations are performed on a WCDMA hexagon cellular network to assess the performance of the proposed adaptive mobile positioning algorithms. The network under investigation contains 64 BSs with cell radius 2 km. The mobile trajectories within the network are generated randomly according to the mobility model described in Section 2.1 and fixed for all simulations. On the other hand, the pilot signals are generated randomly according to the observation model (5) for each simulation realization. Some parameters used in the simulations are the sampling interval $\Delta t = 0.5$ seconds; the correlation coefficient of the random accelerator in (3) is $\alpha = 0.6$; the variance of each random variable in $\mathbf{w}_k$ is $\eta_w = 1$; the standard deviation of lognormal shadowing $\sqrt{\eta_d} = 5$ dB. We consider two scenarios. In scenario 1, the standard deviation of AOA $\sqrt{\eta_\beta} = 4/360$, the standard deviation of TDOA $\sqrt{\eta_t} = 100/c$; whereas in scenario 2, the standard deviation of AOA $\sqrt{\eta_\beta} = 2/360$, the standard deviation of TDOA $\sqrt{\eta_t} = 50/c$; where $c = 3 \cdot 10^8$ m/s is the speed of light. In both scenarios, the base station transmission power $p_{0,i} = 90$ mW, the path-loss index $\eta = 3$, and the number of samples $m = 250$. All simulation results are obtained based on $M = 50$ random realizations.

5.1. Performance comparison with existing techniques

We first compare the performance of the extended Kalman filter (EKF) mobility tracker [2], the standard particle filter mobility tracker [3], and the proposed auxiliary particle filter (APF) mobility tracker (Algorithm 3) in terms of the normalized mean-squared error (NMSE) assuming that the system parameters are known. The NMSE is defined as $\text{NMSE} = (1/L)\sum_{k=1}^{L}((\hat{x}_k - x_k)^2 + (\hat{y}_k - y_k)^2)/(x_k^2 + y_k^2)$, where L is the observation window size. The NMSE results based on the different observations (i.e., RSS only, RSS/AOA and RSS/AOA/TDOA) for scenarios 1 and 2 are reported in Tables 1 and 2, respectively. It is seen that both the standard PF and the APF significantly outperform the EKF in the above two scenarios under the same observations. In fact, the performance gain varies from 5–10 dB for different scenarios and observations. Moreover, by utilizing the current observations in the trial sampling density, the APF demonstrates further improvement over the standard PF (roughly 3 dB).

Table 1: Performance comparisons between EKF, standard PF, and APF in terms of NMSE based on different observations for scenario 1.

Mobility tracker	RSS	RSS/AOA	RSS/AOA/TDOA
EKF (known)	−27.24 dB	−29.47 dB	−31.62 dB
Standard PF (known)	−33.47 dB	−41.75 dB	−48.64 dB
APF (known)	−36.63 dB	−44.87 dB	−52.21 dB
Standard PF (unknown)	−31.47 dB	−39.88 dB	−45.17 dB
APF (unknown)	−34.92 dB	−43.33 dB	−49.18 dB

Table 2: Performance comparisons between EKF, standard PF, and APF in terms of NMSE based on different observations for scenario 2.

Mobility tracker	RSS	RSS/AOA	RSS/AOA/TDOA
EKF (known)	−27.24 dB	−31.01 dB	−33.95 dB
Standard PF (known)	−33.47 dB	−43.51 dB	−51.37 dB
APF (known)	−36.63 dB	−46.72 dB	−55.37 dB
Standard PF (unknown)	−31.47 dB	−41.96 dB	−47.71 dB
APF (unknown)	−34.92 dB	−45.21 dB	−52.79 dB

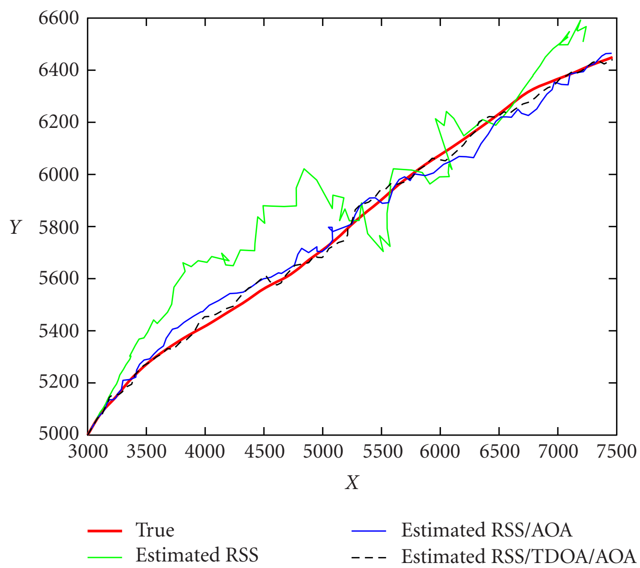

Figure 1: Estimated trajectories based on different observations for scenario 1.

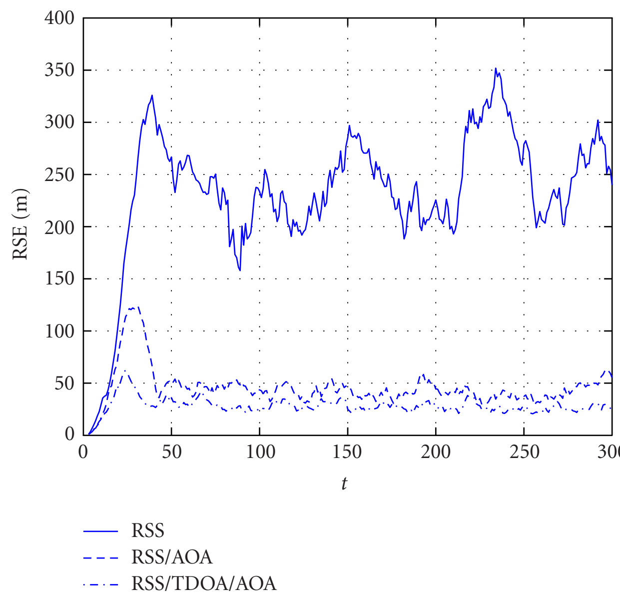

Figure 2: The root-squared error as a function of time for different mobile positioning schemes for scenario 1.

We also compare the APF mobility tracker (Algorithm 4) with the standard PF mobility tracker assuming that the system parameters are unknown. The NMSE results for scenarios 1 and 2 are reported in Tables 1 and 2, respectively. It is seen that performance penalty due to unknown system parameters is less than 3 dB whereas the APF is still 3-4 dB better than the standard PF.

5.2. Tracking performance of the proposed algorithm

In Figure 1, we compare the trajectories estimated by the APF algorithm (Algorithm 4) based on RSS only, RSS/AOA, and RSS/AOA/TDOA, respectively for scenario 1. It is seen that the online estimation algorithm based on the combined observations RSS/AOA/TDOA achieves the best performance and the one based on RSS only performs the worst. We report the corresponding root-squared error (RSE) as a function of time for scenarios 1 and 2 in Figures 2 and 3, respectively. RSE is defined as RSE $= \sqrt{((\hat{x}_k - x_k)^2 + (\hat{y}_k - y_k)^2)}$. It is observed that by incorporating the AOA measurements into the observation function, the RSE is significantly reduced. Further RSE reduction is achieved by using additional TDOA measurements. Figures 4 and 5 show the empirical cumulative distribution function (CDF) of root-squared error (RSE) based on different observations (i.e., RSS only, RSS/AOA, and RSS/TDOA/AOA) measurements. It is seen that the estimated location based on RSS only is most likely to have large deviation from the

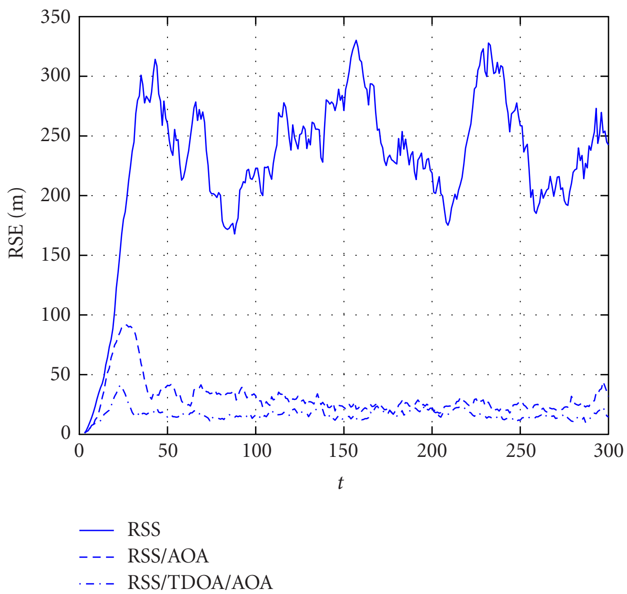

FIGURE 3: The root-squared error as a function of time for different mobile positioning schemes for scenario 2.

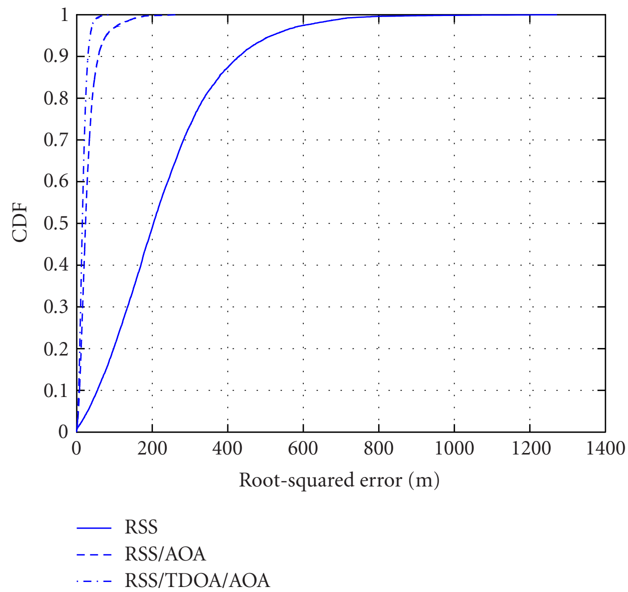

FIGURE 5: CDF of root-squared error based on different mobile positioning schemes for scenario 2.

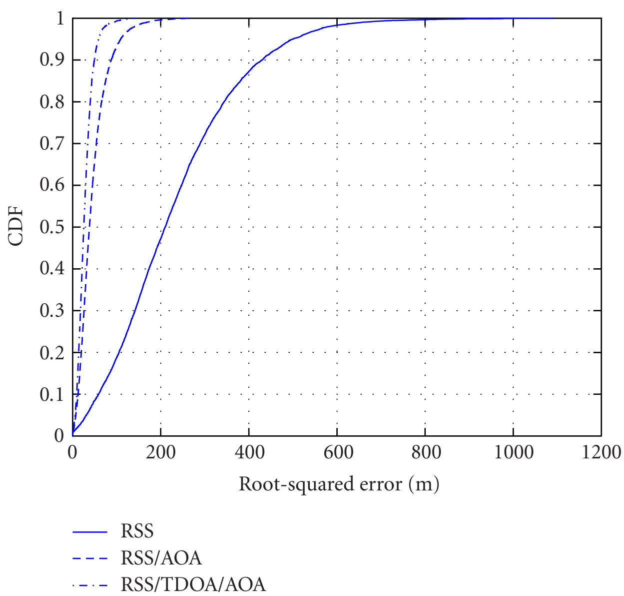

FIGURE 4: CDF of root-squared error based on different mobile positioning schemes for scenario 1.

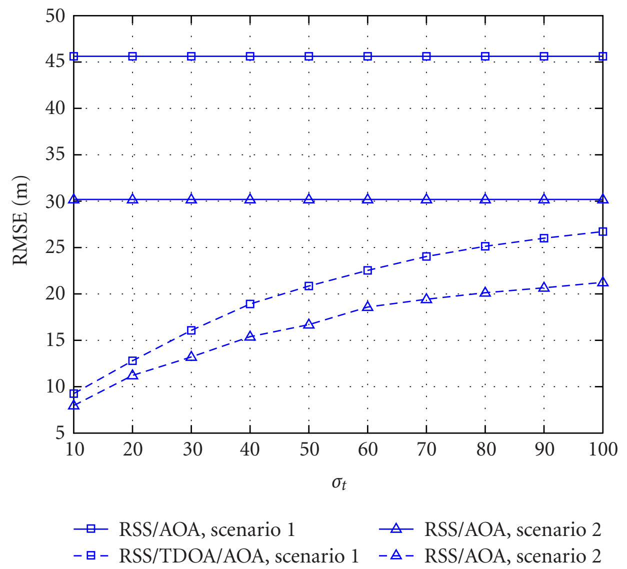

FIGURE 6: RMSE as a function of $\sigma_t \triangleq \sqrt{\eta_t}$ in **Algorithm 4** using different observations.

actual location whereas that based on RSS/TDOA/AOA has the smallest outage probability. By comparing the estimation performance in scenarios 1 and 2, it is seen that the algorithm achieves better performance for scenario 2 due to the smaller measurement noise.

We also report the effect of the variance of TDOA measurement on the estimation performance in Figure 6 in terms of root mean-squared error (RMSE) defined as RMSE $= \sqrt{(1/L)\sum_{k=1}^{L}((\hat{x}_k - x_k)^2 + (\hat{y}_k - y_k)^2)}$. It is seen that the RMSE with RSS/TDOA/AOA monotonically increases in both scenarios and the performance gain over that of RSS/AOA diminishes as the variance of TDOA measurements increases. When the TDOA measurement noise variance is small, a large performance improvement by the TDOA/AOA is achieved. However, when the AOA measurement error increases above a certain level, the performance improvements become negligible. The RMSE in scenario 2 is smaller than that in scenario 1 in both RSS/AOA and RSS/TDOA/AOA location because of a better accuracy in AOA and TDOA measurements.

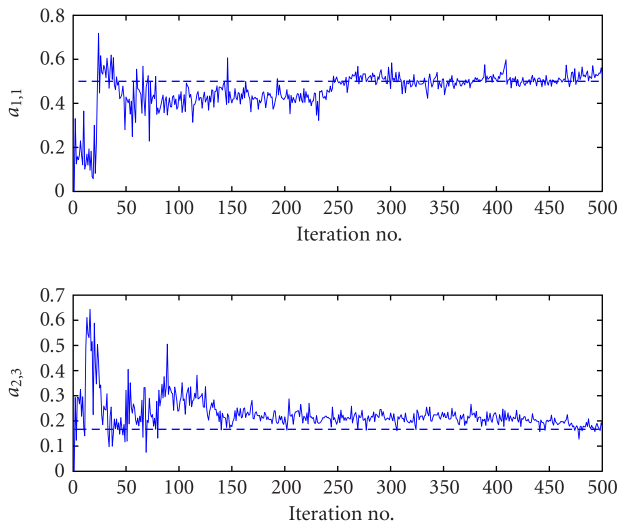

Figure 7: Parameter tracking performance of the transition probability matrix **A** as a function of the iteration number for scenario 1.

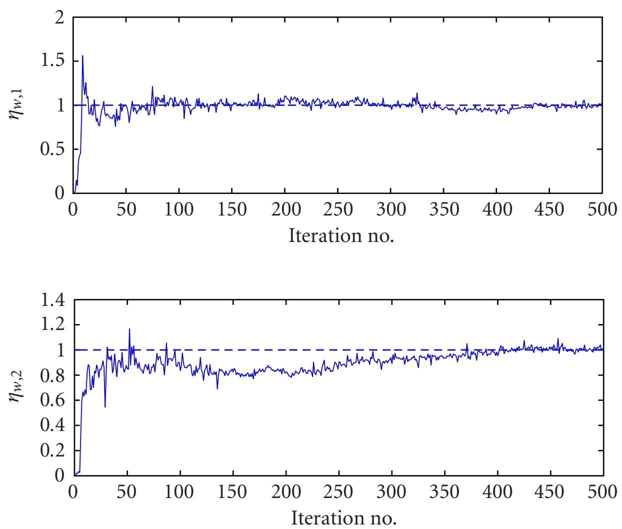

Figure 8: Parameter tracking performance of the motion variance η_w as a function of the iteration number for scenario 1.

We next illustrate the parameter tracking behavior of the proposed adaptive mobile positioning algorithm with unknown parameters in scenario 1. The estimates of the parameters $a_{1,1}$ and $a_{2,3}$ as a function of time index k for one vehicle trajectory are plotted in Figure 7. We also plot the estimates of the noise variances $\eta_{w,1}$ and $\eta_{w,2}$ in Figure 8. It is observed that although the initial estimates of the unknown parameters are far from the actual value, after a short period of time, the estimates of these unknown parameters converge to the true values, demonstrating the excellent tracking performance of the proposed algorithm.

6. CONCLUSIONS

We have considered the problem of mobile user positioning under the sequential Monte-Carlo Bayesian framework. We have developed a new adaptive mobile positioning algorithm based on the auxiliary particle filter algorithm. The algorithm makes use of the measurements of time difference of arrival, angle of arrival as well as received signal strength, all of which are available in practical WCDMA networks. The proposed algorithm jointly tracks the unknown system parameters as well as the mobile position and velocity. Simulation results show that the proposed algorithm has an excellent mobility tracking and parameter estimation performance and it significantly outperforms the existing mobility estimation schemes.

ACKNOWLEDGMENTS

This work was supported in part by the US National Science Foundation (NSF) under Grants DMS-0225692 and CCR-0225826, and by the US Office of Naval Research (ONR) under Grant N00014-03-1-0039.

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