The relative merits of a tariff and a quota at the border for achieving a government objective have been discussed a good deal in the literature. It has long been recognized that, provided the government auctions off the quota, the optimum pure tariff and the optimum pure quota are equivalent in a competitive world with no uncertainty. The proposition continues to hold if there is uncertainty, but where every agent, including the government, can monitor (and therefore distinguish) costlessly every state of nature. However, in such a world the equivalence is between a pure tariff and a pure quota that are both functions of the state of nature. It is this last observation that leads us to suspect that the equivalence result is of rather limited practical use. One would imagine that the possible states of nature are large in number. It is then difficult to envisage a government announcing a trade policy that is contingent entirely on the state of nature. Such a policy would be costly to calculate and difficult to comprehend. We are therefore encouraged to simplify a good deal and to restrict the set of admissible trade policies. But this is a difficult matter. It is by no means immediate what restrictions would appear as being natural to contemplate. The border policies that are most commonly resorted to by governments are fixed tariff rates and fixed quantity restrictions. They are often regarded as polar forms of trade restrictions (one involving prices and the other involving quantities). They have very different effects: a fixed tariff on a commodity stabilizes its domestic price in the face of random domestic demand and supply but a fixed international price; while a quota stabilizes its domestic price if its international price is random but its domestic demand and supply functions are fixed.

The major purpose of this paper is to examine the relative merits of these two trade policies in the presence of uncertainty. The central result that we shall present here came somewhat as a surprise to us. Under the conventional criterion of maximizing the expected value of net consumer’s surplus, the optimum fixed tariff is superior to the optimum fixed quota. The result continues to hold if instead the maxi-min criterion is followed. Section I is concerned with this issue. In Section II we discuss in what sense both tariffs and quotas may be viewed as special cases of a more general class of trade policies, and how the problem of the choice of an optimum trade policy may be viewed as an example of a general class of problems arising out of imperfect information.

I. Tariffs versus Quotas

Imagine a commodity that can be both produced domestically under competitive conditions and at the same time imported. Denoting by \( q \) its domestic price and by \( D \) its domestic demand, we suppose that the market demand curve can be represented as

\[
q = \alpha - \beta D
\]

where \( \beta \) is a positive constant and where \( \alpha \)

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is a random variable with a known distribution. Writing $S$ as the domestic supply of the commodity, we take it that the domestic supply function can be represented as:

$$q = \gamma + \delta S$$

where $\delta$ is a positive constant and where $\gamma$ is a random variable. In short, we are postulating linear demand and supply curves, each of which possesses an unbiased shift parameter. (See Figure I.) Alternatively, one could suppose that all the random variables have small variances, so that we would be justified in taking linear approximations of the domestic demand and supply functions at the optimum tariff point.

The economy in question is assumed to be small in that its import requirement does not influence the foreign price. The import price $p$ is not known with certainty but is random with a known distribution.

Given that the commodity is domestically produced under competitive conditions the domestic cost function $C(S)$ is the integral of supply curve:

$$C(S) = \gamma S + \frac{\delta}{2} S^2 = (\gamma^2 - \gamma^2)/2\delta$$

(Without loss of generality we are setting the constant of integration at zero.) Likewise, consumer's gross benefit $B(D)$ from (1) can be expressed as:

$$B(D) = \alpha D - \frac{\beta}{2} D^2 = (\alpha^2 - \gamma^2)/2\beta$$

Given that we are concerned here with the relative merits of a pure tariff and a pure quota we need an account of the rationale for introducing a trade restriction. Assume then that the government desires to introduce such a restriction with a view to raising a given expected level of revenue $R$, and to keep matters simple we take it that the government is risk neutral. Now it is plain that there are various manners in which the government can introduce trade restrictions in order to ensure an expected level of revenue $R$. Let $E$ denote the expectation operator. We suppose that it ranks various feasible policies in accordance with the function

$$W = E(B(D) - C(S) - \bar{p}l)$$

where $l$ is the equilibrium level of import.

By way of contrast let us look at the first best formulation of the problem initially. For this economy a state of nature is characterized by a triplet of numbers $(\alpha, \gamma, \bar{p})$.

In order to raise the expected level of revenue $R$, the government would announce an ad valorem tariff $t$, contingent on the state of nature. Writing by $I(\alpha, \gamma, \bar{p})$ the quantity imported in the state of nature $(\alpha, \gamma, \bar{p})$, and using (1) and (2), a market equilibrium would be characterized by the condition

$$I(\alpha, \gamma, \bar{p}) = \frac{\alpha}{\beta} + \frac{\gamma}{\delta} - \bar{p}(1 + t(\alpha, \gamma, \bar{p}) \frac{(\beta + \delta)}{\beta \delta})$$

In what follows we shall take it for sim-

5It is, of course, possible to present the analysis by postulating directly the excess demand function, rather than working separately with the demand and supply functions. The final result that we shall present subsequently is, however, easier to dissect if we consider them separately.

6In other words, the government ranks policies in accordance to their contributions to the expected value of the sum of the surpluses accruing to consumers and producers. (Note that it makes no difference whether the government ranks projects according to $E[B - C - p\bar{l}]$ or $E[B - C - q\bar{l}]$ since they differ by exactly $R$.)
licity that the ranges of the random variables $\bar{a}$, $\bar{y}$, and $\bar{p}$ are small in the sense that in the absence of any trade restrictions (i.e., $t(\bar{a}, \bar{y}, \bar{p}) = 0$), one has for all realizations of $\bar{a}$, $\bar{y}$, and $\bar{p}$, $I(\bar{a}, \bar{y}, \bar{p}) > 0$. To keep the analysis uncomplicated we shall subsequently assume as well that $R$ is small in a sense that will be made precise.

The government's problem would then consist of determining a tariff schedule $t(\bar{a}, \bar{y}, \bar{p})$ that will maximize $E(B(D) - C(S) - pI)$, subject to the constraint (5) and the condition

\begin{equation}
E(pI(\bar{a}, \bar{y}, \bar{p})I(\bar{a}, \bar{y}, \bar{p})) = R
\end{equation}

Assume for the moment that an optimum exists. It is of course plain that whether the government announced the resulting optimal tariff schedule $t^*(\bar{a}, \bar{y}, \bar{p})$ or instead uses (5) to auction off the corresponding import quota schedule $I^*(\bar{a}, \bar{y}, \bar{p})$ is a matter of indifference. This is the classical equivalence between tariffs and quotas.

More generally, from equations (3), (4), and (5), we can express expected net benefits as

\begin{equation}
W = \frac{E(\bar{a}^2)}{2\beta} + \frac{E(\bar{y}^2)}{2\delta} - \frac{\beta + \delta}{\beta\delta} \left[ \frac{E(\bar{q}^2)}{2} - E(\bar{p}\bar{q}) \right] - \left[ \frac{E(\bar{a}\bar{p})}{\beta} + \frac{E(\bar{y}\bar{p})}{\delta} \right]
\end{equation}

It follows that policies will be ranked simply on the basis of the value of

\begin{equation}
Z = E(\bar{p}\bar{q}) - E(\bar{q})^2/2
\end{equation}

We are now concerned with the second best problem, where the admissible set of trade policies is severely restricted. We suppose that the government can costlessly monitor the total volume of imports, $\bar{I}$, and can therefore base its trade restriction on $\bar{I}$; but that $\bar{a}$, $\bar{y}$, and $\bar{p}$ are separately unobservable, and therefore it cannot base trade policies on them.\(^7\) In this section we focus attention on two of the simplest of such trade policies, namely: 1) a pure tariff $t(\bar{I}) = t$ (a constant); and 2) a pure quota $I$, which is equivalent to an implicit specific tariff $\tau$ given by the rule $\tau = \bar{q} - \bar{p}$ for $\bar{q} > \bar{p}$ and $I(\bar{a}, \bar{y}, \bar{p}) \leq I$, and $\tau = \infty$ for $I(\bar{a}, \bar{y}, \bar{p}) > I$.\(^8\) In what follows we analyze the two sequentially. (See the accompanying figure.)

A. Pure Tariff

Denote by $t$ the pure ad valorem tariff. It follows from equation (5) that the import function is of the form

\begin{equation}
\bar{I} = \frac{\bar{a}}{\beta} + \frac{\bar{y}}{\delta} - \frac{\bar{p}(1 + t)(\beta + \delta)}{\beta\delta}
\end{equation}

and from the revenue constraint (6) that

\begin{equation}
R = E(\bar{p}\bar{I}) = tE(\bar{p}\bar{I})
\end{equation}

To analyze the issues involved it is simplest (though not essential) to suppose that the random variables $\bar{a}$, $\bar{y}$, $\bar{p}$, are all pair-wise independent of one another. We can then use equation (9) in equation (10) to obtain a quadratic equation in $t$:

\begin{equation}
t^2 - \left[ \frac{\bar{p}(\alpha\delta + \gamma\beta)}{(\beta + \delta)E(\bar{p}\bar{q})} - 1 \right] t
+ \frac{\beta\delta R}{(\beta + \delta)E(\bar{p}^2)} = 0
\end{equation}

If $R$ is too large, there will be no real solution for $t$. There is then no feasible pure tariff policy. Consequently we take it that $R$ is small. Of the two real solutions of (11) it is the smaller tariff rate which yields a higher level of expected net benefits. It is this smaller value we are interested in. Denote it by $t^*$. To get a tidy expression for $t^*$ it will be convenient to assume that $R$ is small enough so as to enable one to ignore

\(^7\)Thus we rule out by assumption the possibility of smuggling. This raises rather different issues.

\(^8\)In what follows we shall suppose that the ranges of $\bar{a}$, $\bar{y}$, and $\bar{p}$ are sufficiently small and the quota is not overly large. Consequently we shall take it that $\bar{q} > \bar{p}$ and hence the quota is binding. It follows that strictly speaking one does not need to set $\tau = \infty$ for $I(\bar{a}, \bar{y}, \bar{p}) > I$. A large enough $\tau$ will do. Notice that as the quota is, by assumption, auctioned off, the implicit tariff associated with a quota is random.
all the second and higher powers of $R$. Consequently from (11) one has

\[(12) \quad t^* \sim \frac{\beta \delta R}{\bar{p}(\bar{\alpha} + \bar{\gamma} + \bar{\beta} + \delta) E(\bar{p}^2)} \]

It follows then that

\[(13) \quad \bar{q} \sim \bar{p} \left(1 + \frac{\beta \delta R}{\bar{p}(\bar{\alpha} + \bar{\gamma} + \bar{\beta} + \delta) E(\bar{p}^2)} \right) \]

Under a pure tariff scheme the market-clearing price is random as long as the foreign price is random.

Thus, on using equation (8) one obtains (for small $R$)

\[(14) \quad Z_{\text{tariff}} = E(\bar{p}^2)/2 \]

### B. Pure Quota

If a pure import quota $q$ is announced and auctioned off, the resulting domestic price $q$ in equilibrium is obtained from equation (5):

\[(15) \quad \bar{q} = \frac{\beta \delta}{\beta + \delta} \left(\frac{\bar{\alpha} + \bar{\gamma}}{\beta} + \frac{\bar{\beta}}{\delta} - 1\right) \]

Thus, the uncertainty in the equilibrium price resulting from a quota is due solely to the uncertainty in the domestic supply and demand conditions. The uncertainty in the foreign price has no bearing on this. All this is, of course, obvious. Now if $\bar{q}$ is the equilibrium market price, the implicit specific tariff $\bar{r}$ due to the quota in equilibrium is

\[(16) \quad \bar{r} = \bar{q} - \bar{p} \]

which, on using in (15) yields

\[(17) \quad \bar{r} = \frac{\bar{\alpha} \delta + \bar{\gamma} \beta}{\beta + \delta} - \bar{p} - \frac{\beta \delta I}{\beta + \delta} \]

$E(\bar{r}I)$ is the expected revenue by an auction of the quota $I$ in a risk-neutral market. Therefore condition (6) implies that

\[(18) \quad R = E(\bar{r}I) = IE(\bar{r}) \]

Notice that the validity of the approximation for $t^*$ depends on the magnitude of the right-hand side of equation (12), while the requirement that $\bar{q} > \bar{p}$ (see fn. 8) implies restriction of the range of $(\bar{\alpha}, \bar{\gamma}, \bar{p})$.

Using equation (17) and (18) yields a quadratic expression in $I$:

\[(19) \quad I^2 - \left(\frac{\bar{\alpha} \delta + \bar{\gamma} \beta - \bar{p}(\beta + \delta)}{\beta \delta} \right) I \]

\[+ \frac{R(\beta + \delta)}{\beta \delta} = 0 \]

There are then two feasible quota specifications. Plainly the larger of the two solutions of equation (19) yields higher expected net benefits: denote it by $I^*$. It follows that

\[(20) \quad I^* \sim \frac{\bar{\alpha} \delta + \bar{\gamma} \beta - \bar{p}(\beta + \delta)}{\beta \delta \left(\bar{\alpha} \delta + \bar{\gamma} \beta - \bar{p}(\beta + \delta)\right)} \]

\[+ \frac{R(\beta + \delta)}{\bar{\alpha} \delta + \bar{\gamma} \beta - \bar{p}(\beta + \delta)} \]

Using equations (15) and (20) now yields the equilibrium price under the pure quota scheme as

\[(21) \quad \bar{q} = \bar{p} + \frac{R\beta \delta}{\bar{\alpha} \delta + \bar{\gamma} \beta - \bar{p}(\beta + \delta)} \]

\[+ \left(\frac{\beta \delta I}{\beta + \delta} \right) \frac{R(\beta + \delta)}{\beta \delta} \]

As before, we are concerned with the level of expected net benefits under the optimum pure quota level $I^*$. From equation (8) and (21) one then obtains

\[(22) \quad Z_{\text{quota}} = \left(\bar{p}^2 - \frac{\sigma^2_{\alpha} \delta^2}{\beta \delta} + \frac{\sigma^2_{\gamma} \beta^2}{(\beta + \delta)^2} + 2 \right) \]

where $\sigma^2_{\alpha}$ and $\sigma^2_{\gamma}$ are the variances of $\bar{\alpha}$ and $\bar{\gamma}$, respectively.

### C. Comparison

We have finally to compare (14) and (22) to determine which of the two schemes is superior. Since $\sigma^2_{\alpha} = E(\bar{p}^2) - \bar{p}^2$, it follows that

\[(23) \quad W_{\text{tariff}} - W_{\text{quota}} = \frac{\beta + \delta}{\beta \delta} \left(Z_{\text{tariff}} - Z_{\text{quota}}\right) \]

\[= \frac{\delta \sigma^2_{\alpha}}{2\beta(\beta + \delta)} + \frac{\beta \sigma^2_{\gamma}}{2\delta(\beta + \delta)} + \frac{(\beta + \delta) \sigma^2_{\beta}}{2\beta \delta} \]

Equation (23) is the basic result of this
paper and in what follows we comment on it. Notice first that the reason why moments of order higher than the variance do not appear in (23) is the fact that the government's objective function is quadratic. Notice as well that if \( \sigma^2 = \sigma_1^2 = \sigma_2^2 = 0 \), then \( W_{\text{tariff}} = W_{\text{quota}} \). This last is, of course, the classical equivalence result. However, the interesting feature of equation (23) is that so long as at least one of the variances is positive, \( W_{\text{tariff}} > W_{\text{quota}} \). In other words, a pure tariff is unambiguously superior to a pure quota in generating a given expected level of government revenue.

We had not anticipated this result. Indeed, we had supposed that the relative merits of a tariff and a quota would depend on the relative steepnesses of the demand and supply functions, and possibly also on the relative magnitudes of the coefficients of variation of the different random variables. No doubt under more general formulations of the excess demand function there are circumstances in which a pure quota is a superior policy measure to a pure tariff. But a linear excess demand function is the simplest laboratory in which to raise this question. At any rate, within the confines of such a formulation the answer emerges as being unambiguous.

The result is particularly telling for the situation where \( \sigma^2 > \sigma_1^2 = \sigma_2^2 = 0 \). It is under this circumstance that a pure quota is a stabilizing policy. Under such a regime there is no uncertainty in the domestic equilibrium price (see equation (21)). Consequently there is no uncertainty in the sum of the surpluses accruing to producers and consumers. However, with a pure tariff the domestic equilibrium price is random (see equation (13)). It might then be thought that given risk aversion on the part of the private sector, a quota would be a superior policy measure to a tariff in generating the required government revenue. What this argument overlooks is the fact that the quota does not allow for variations in imports, say, when the social cost of imports is low (because \( \bar{p} \) is low) or when the value of imports is high (for example, because \( \bar{\alpha} \) is large). This relative adaptability of tariffs has long been argued as one of its advantages. The result here makes precise the sense in which this is true.

Remarkably enough the result does not depend on the assumption of risk neutrality with respect to net consumer's surplus. Suppose instead infinite risk aversion and consequently that policies are ranked by the function

\[
W = \min(\bar{B} - \bar{C} - \bar{p} \bar{I})
\]

With the max-min criteria the same result obtains, for on using equations (12) and (20), routine calculations yield

\[
W_{\text{tariff}} - W_{\text{quota}} = \frac{\beta + \delta}{2\beta \delta} \left[ \left( p_{\text{max}} - \bar{p} - \frac{R \bar{\beta} \delta}{\bar{\alpha} \delta + \bar{\gamma} \beta - \bar{p} (\beta + \delta)} \right)^2 \right] > 0
\]

II. Second Best Optimum Tariff Schedules

The analysis of the previous section could be viewed as one concerning the optimum tariff schedule when the admissible set of policies is restricted to the fixed tariff and the fixed quota. It was noted that this was an immense restriction. It emerged that even for small expected revenue requirements there are only four feasible policies in all (see equations (11) and (19)). Computing the optimum border policy was then an easy enough matter. The central result of this paper was that the fixed ad valorem tariff \( t^* \) in equation (12) is the optimum of this restricted set of policies.

Now, there is of course no reason why in principle we should restrict our attention to the fixed tariff and quota as admissible policies. The tariff schedules need to be based on observables, and ought to be administratively simple. For instance, it may be relatively easy to monitor the volume of imports but difficult to monitor the true

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10 This may well be an implicit argument in justifying the European Economic Community (EEC) policy of stabilizing domestic price by setting a tariff contingent on the imported price of commodities which is precisely what a quota achieves.
price (for example, because of kickbacks, special credit facilities, etc.). Thus we could imagine the government announcing an ad valorem tariff rate \( t(I) \). In order that such a function yields an outcome it must result in a real valued random variable \( \tilde{I} \), satisfying the conditions

\[
\tilde{I} = \frac{\alpha}{\beta} + \frac{\gamma}{\delta} - \tilde{p}(1 + t(\tilde{I}))(\beta + \delta)/\beta \delta
\]

and

\[
R = E(\tilde{p}t(\tilde{I})\tilde{I})
\]

The aim may then be to select a schedule \( t^*(I) \) that maximizes

\[
W = E(B(D) - C(S) - \tilde{p})
\]

It is clear from this formulation that the problem of optimum tariff structure, as we have posed it, is yet another example of a wide class of optimum control problems with imperfect information which share a common structure. Other examples include the analysis of sharecropping with risk and incentive effects (for example, Stephen Cheung; Stiglitz, 1974); the choice of an optimum income tax structure (for example, Ray Fair; James Mirrlees, 1971, 1974, 1976; Eytan Sheshinski; Anthony Atkinson and Stiglitz; the analysis of insurance markets (for example, Kenneth Arrow; A. Michael Spence and Richard Zeckhauser; Mark Pauly; and Stiglitz, 1975c); the use of piece rates versus time rates in labor contracts (for example, John Pencavel, Stiglitz, 1975b); the determination of the optimum tariff structure for utilities (for example, Martin Weitzman, 1974a; Spence, 1975); the analysis of education as a screening device (for example, Spence, 1974; Stiglitz, 1975a); the question of using prices or quantities in planning (for example, Weitzman, 1974b; Mark Roberts and Spence).

We have gone into the nature of the common structure of these seemingly diverse problems in Dasgupta and Stiglitz.

We should perhaps emphasize that we have restricted ourselves to the pure tariff and the pure quota cases not only because of ease of calculation. There is in addition the question of whether it is reasonable to imagine a government announcing and administering hopelessly complicated tariff schedules. Approximations to the optimum are therefore required, and it is this that is achieved by restricting the admissible set of tariff schedules to those that are simple in form. But still a fixed tariff may be unduly restrictive. One could well imagine, for example, that an appropriately chosen two-tier tariff structure would prove superior to the pure tariff rate \( t^* \). Formally, such a schedule would read \( t(I) = t_1 \) for \( I < \tilde{I} \), and \( t(I) = t_2 \) for \( I \geq \tilde{I} \); normally \( t_1 \neq t_2 \) for an optimum; that is, an optimum two-tier tariff would be superior to a single tariff rate.

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