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Abstract

In a discrete choice model of product differentiation, the symmetric duopoly price may be lower than, equal to, or higher than the single-product monopoly price. While the *market share effect* of competition encourages a firm to charge less than the monopoly price because a duopolist serves fewer consumers, the *price sensitivity effect* of competition motivates a higher price when more consumer choice steepens the firm's demand curve. The joint distribution of consumer values for the two conceivable products determines the relative strength of these effects, and whether presence of a symmetric competitor results in a higher or lower price compared to single-product monopoly. The analysis reveals that price-increasing competition is unexceptional from a theoretical perspective.

Keywords: product differentiation, entry, duopoly, monopoly

JEL classification: D4, L1

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1. INTRODUCTION

A fundamental insight of economics is that competition usually lowers prices. The strength of this conclusion, however, is called into question by scattered evidence from assorted industries. For example, Pauly and Sattherthwaite (1981) argue that physician services are priced higher in urban areas with more physicians per capita; Bresnaham and Reiss (1991) present survey evidence showing that automobile tire prices are somewhat higher in local markets with two dealers rather than one, although the difference is not statistically significant; Bresnahan and Reiss (1990) infer from the structure of local auto retail markets that profit margins might be higher under duopoly than monopoly; Perloff, Suslow, and Seguin (2005) conclude that new entry raises prices in the anti-ulcer drug market; Ward et al. (2002) provide evidence that new entry of private labels raises prices of name-brand goods in the food industry;[1] Goolsbee and Syverson (2004) show that airlines raise route prices when Southwest opens new routes to the same destination from a nearby airport; and Thomadsen (2005) simulates with estimated parameters from the fast food industry how prices may rise with closer geographic positioning of competitors. A theoretical re-consideration of how competition affects prices thus seems appropriate.

In this paper, we study a discrete choice model of product differentiation in which consumers' values for two substitute products have an arbitrary symmetric joint distribution. Each firm produces a single product, and the market structure is either monopoly or duopoly. We characterize under weak assumptions a necessary and sufficient condition for the symmetic duopoly price to be higher than, equal to, or lower than monopoly price. This condition balances two economic effects. At the monopoly price, a duopoly firm sells to fewer consumers than the monopolist. The larger is this difference, the greater is the incentive of a duopolist to reduce price below the monopoly level. We call this the *market share effect*. On the other hand, under product differentiation a duopoly firm's demand curve

[1] More precisely, the entry of private labels raises the price of the national brands, with a possible increase of average market price. See also Caves, Whinston, and Hurwicz (1991) and Grabowski and Vernon (1992) for evidence that generic entry triggers higher prices for corresponding brand-name drugs in the U.S. pharmaceutical industry.

may be steeper than the monopolist's, because consumers have a choice of products in the duopoly case, and, therefore, are less keen to purchase the duopolist's product in response to a price cut. The steeper is the duopolist's demand curve, relative to the monopolist's, the greater is the incentive of the duopolist to raise price above the monopoly level. We call this the *price sensitivity effect*. When the second effect dominates, as, for example, if consumer values for the two products are drawn from a (Gumbel) bivariate exponential distribution, duopoly competition increases price compared to monopoly. We derive from the general necessary and sufficient condition various particular conditions under which price is higher under symmetric single-product duopoly than under single-product monopoly. For example, if consumer values for the two products are independent, the duopoly price is higher if the hazard rate of the marginal distribution function is decreasing.

We further analyze a class of special cases in which the calculation of monopoly and duopoly prices is straightforward. In these special cases, consumer preferences for two products have a joint uniform distribution on a varying support allowing different degrees of negative or positive correlation. This analysis includes a new and unified treatment of two familiar models in oligopoly analysis. The Hotelling duopoly model (Hotelling, 1929) is a limiting case in which the preferences are perfectly negatively correlated, and the Bertrand duopoly model is a limiting case when the preferences are perfectly positively correlated. Duopoly competition raises price if consumers' preferences for the two products are sufficiently diverse and negatively correlated, as for instance in the Hotelling model when the market is fully served under duopoly but not under monopoly.

The standard view of relationship between market structure and price has been challenged also by several other theoretical studies. For instance, when consumers must search to find firms' prices, the presence of more firms makes it more difficult to find the lowest price in the market, reducing consumers' incentives to search. This can cause the equilibrium market price to increase with the number of competitors (Stiglitz, 1987).[2] An alternative

[2] Also, Satterthwaite (1979) considers a model where firms produce reputation goods and an increase in the number of firms can reduce the efficieny of search by consumers. On the other hand, Schulz and Stahl (1996) considers a model where consumers have uncertain product valuations that must be discovered

approach assumes that each seller faces two groups of consumers, a captured loyal group and a switching group. With more sellers, each seller's share of the switching group is reduced, increasing its incentive to exploit the captured consumers through a higher price; but equilibrium prices under competition are in mixed strategies (Rosenthal, 1980).[3] In contrast, in our analysis here, consumers have perfect information, and firms' prices are in pure strategies. While Chen and Riordan (2005) and Perloff, Suslow, and Seguin (2005) have also shown that competition can increase price under perfect information and pure strategies, these papers rely on specific spatial models where consumer valuations for two products are perfectly negatively correlated.[4] Our present paper goes further by developing a necessary and sufficient condition for price-increasing competition in a more general symmetric model of product differentiation, and clarifying the economic effects associated with this condition.

The rest of the paper is organized as follows. Section 2 formulates and analyzes the general model. Section 3 analyzes the special cases of a uniform distribution of preferences on a varying support, thus generalling limiting cases of Hotelling duopoly and Bertrand duopoly. Section 4 compares duopoly competition with a multiproduct monopoly producing both products, demonstrating the expected result that the symmetric multiproduct monopoly price exceeds the duopoly price. Section 5 draws conclusions. Detailed calculations for Section 3 are in the Appendix.

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through search. An increase in the number of firms in their model can increase the returns from search and market demand, resulting in higher prices.

[3] In a similar setting with high- and low-valuation types of consumers, entry and endogenous product selection can result in market segmentation with the incumbent raising its price to sell only to high-valuation customers, if the game is solved with a solution concept that is not Nash equilibrium. See Davis, Murphy, and Topel (2004).

[4] Perloff, Suslow, and Seguin (2005) study a variant of the Hotelling duopoly model, while Chen and Riordan (2005) generalize the Hotelling model to an arbitrary number of single-product firms.

2. DISCRETE CHOICE MODEL OF PRODUCT DIFFERENTIATION

Preferences

Each consumer desires to purchase at most one of two goods. The preferences of a consumer are described by reservation values for the two goods, (v_1, v_2), where $v_i \in [\underline{v}, \bar{v}]$, and $0 \leq \underline{v} < \bar{v} \leq \infty$. The distribution of preferences over the population of consumers is assumed to be nondegenerate and symmetric. Thus, the population of consumers, the size of which is normalized to one, is described by a marginal distribution function $F(v_1)$ and a conditional distribution function $G(v_2 \mid v_1)$. These distribution functions are assumed to be differentiable on $[\underline{v}, \bar{v}]$, with associated density functions $f(v_1)$ and $g(v_2 \mid v_1)$. The joint density function, therefore, is $h(v_1, v_2) \equiv f(v_1)\, g(v_2 \mid v_1)$. The support of the corresponding joint distribution function, $\Omega \subseteq [\underline{v}, \bar{v}]^2$, is symmetric about the 45^{o}line.

Monopoly

First, consider a single firm producing one of the two goods with a constant unit cost $c \in [\underline{v}, \bar{v})$. The firm sets a price to solve the "monopoly problem":

$$\max_{p \geq c} \; (p - c)\left[1 - F(p)\right]. \tag{1}$$

Assumption 1. There exists a unique interior solution to the monopoly problem, $p^m \in (c, \bar{v})$.

The necessary first-order condition for the solution to the monopoly problem is

$$\left[1 - F(p^m)\right] - (p^m - c)\, f(p^m) = 0. \tag{2}$$

The first-order condition can also be written as

$$(p^m - c)\, \lambda(p^m) = 1, \tag{3}$$

where

$$\lambda(v) \equiv \frac{f(v)}{1 - F(v)} \tag{4}$$

denotes the hazard rate.

Sufficient primitive conditions for Assumption 1 are: $(\bar{v}-c)\,\lambda(\bar{v})>1$;[5] and $\lambda(v)$ continuously increasing on $[c,\bar{v}]$. The familiar monotone hazard rate condition, however, is not necessary. For example, Assumption 1 holds if $F(v)$ is a standard exponential distribution and $\lambda(v)=1$, and, therefore, by continuity, also holds for sufficiently small departures from the exponential case. More generally, if $\lambda(v)$ is differentiable, then sufficient conditions for Assumption 1 are: (i) $(\bar{v}-c)\,\lambda(\bar{v})>1$; and (ii) $\frac{d\ln\lambda(p)}{dp}>-\frac{1}{p-c}$ for $p\in[c,\bar{v}]$. Thus a decreasing or non-monotonic hazard rate is consistent with our analysis, although a uniformly decreasing hazard rate requires $\bar{v}=\infty$.

Duopoly

Second, consider two firms, each producing one of the two products with constant unit cost c. The two firms play a duopoly game, setting prices simultaneously and independently and each maximizing its own profit given equilibrium beliefs about the rival's action.

Assuming its rival sets $\bar{p}$, each firm sets a price to solve the "duopoly problem":

$$\max_{p}\;(p-c)\,q(p,\bar{p}) \tag{5}$$

with

$$q(p,\bar{p})\equiv\int_{\underline{v}}^{\bar{p}}\left[1-G\left(p\mid v\right)\right]f\left(v\right)dv+\int_{\bar{p}}^{\bar{v}+\min\{0,\bar{p}-p\}}\left[1-G\left(v-\bar{p}+p\mid v\right)\right]f\left(v\right)dv. \tag{6}$$

Assumption 2. There exists a unique interior symmetric equilibrium of the duopoly game, $p^{d}\in(c,\bar{v})$.

A best response function, $p=R\left(\bar{p}\right)$, describes a solution to the duopoly problem for each $\bar{p}$. The symmetric equilibrium of the duopoly game is a unique interior fixed point of a best response function, $p^{d}=R\left(p^{d}\right)$, on the domain $[c,\bar{v}]$. Sufficient conditions for Assumption 2 are: (iii) $R\left(\bar{p}\right)$ is unique for all $\bar{p}\in[c,\bar{v}]$; (iv) $R\left(c\right)>c$; (v) $R\left(\bar{v}\right)<\bar{v}$; (vi) $R\left(\bar{p}\right)$ continuous;

[5] We adopt the notational convention $(\bar{v}-c)\,\lambda(\bar{v})=\lim_{v\to\infty}(v-c)\,\lambda(v)$ if $\bar{v}=\infty$.

and (vii) $R(\bar{p})$ crosses the 45^o degree line but once (from above). Note that $R(\bar{v}) = p^m$ means that condition (v) follows from Assumption 1.

An alternative formulation of sufficient conditions for Assumption 2 is based on the function

$$\overline{\lambda}(p, \bar{p}) = \frac{\left|\frac{\partial q(p,\bar{p})}{\partial p}\right|}{q(p, \bar{p})}. \tag{7}$$

The first order condition for a solution to the duopoly problem is

$$(p - c)\,\overline{\lambda}(p, \bar{p}) = 1. \tag{8}$$

Therefore, $p = R(\bar{p})$ is the unique solution to this equation if, for all $c \leq \bar{p} \leq \bar{v}$: (i') $(\bar{v} - c)\,\overline{\lambda}(\bar{v}, \bar{p}) > 1$; and (ii')

$$\frac{\partial \ln \overline{\lambda}(p, \bar{p})}{\partial p} > -\frac{1}{p - c}$$

for $p \in [c, \bar{v}]$. These primitive conditions are analagous to those that support Assumption 1 and are sufficient for Assumption 2. In fact, the conditions are the same when $\bar{p} = \bar{v}$. Alternatively, Assumption 2 is satisfied if p^d uniquely solves

$$(p - c)\,\overline{\lambda}(p, p) = 1$$

and $(p - c)\,\overline{\lambda}(p, p^d)$ is strictly increasing in p.

An important special case is the independent exponential case with $\bar{v} = \infty$ and

$$G(v_1|v_2) = F(v_1) \equiv 1 - e^{-\lambda v_1}$$

for some $\lambda > 0$. In this case,

$$\overline{\lambda}(p, \bar{p}) = \frac{\lambda^2 e^{-\lambda(p-\bar{p})} \int_p^\infty e^{-2\lambda v} dv + \lambda e^{-\lambda p}\left(1 - e^{-\lambda \bar{p}}\right)}{\lambda \int_p^\infty e^{-\lambda(v-\bar{p}+p)} e^{-\lambda v} dv + e^{-\lambda p}\left(1 - e^{-\lambda \bar{p}}\right)} = \lambda.$$

Therefore, conditions (i')-(ii') hold strictly, and $p^d = p^m = c + \frac{1}{\lambda}$. Because conditions (i')-(ii') hold strictly for the case of a constant hazard rate, it is possible to perturb $h(v_1, v_2)$ around the independent exponential case and still satisfy the assumptions. Thus, our analysis below applies to cases of increasing, decreasing, and non-monotonic hazard rates.

Comparison

Third, compare the duopoly price and the monopoly price. A useful restatement of the first-order condition of the monopoly problem is

$$\int_{\underline{v}}^{\bar{v}} [1 - G(p^m|v)] f(v)\, dv - (p^m - c) \int_{\underline{v}}^{\bar{v}} g(p^m|v) f(v)\, dv = 0. \tag{9}$$

For the purpose of comparing monopoly and duopoly, it is useful to define the function

$$\begin{aligned} \Psi(p) \equiv & \left\{ \int_{\underline{v}}^{p} [1 - G(p|v)] f(v)\, dv + \int_{p}^{\bar{v}} [1 - G(v|v)] f(v)\, dv \right\} \\ & - (p - c) \left\{ \int_{\underline{v}}^{p} g(p|v) f(v)\, dv + \int_{p}^{\bar{v}} g(v|v) f(v)\, dv \right\}. \end{aligned} \tag{10}$$

The equilibrium condition $p^d = R(p^d)$ implies $\Psi(p^d) = 0$, which at equilibrium is the necessary first-order condition for the duopoly problem. The comparison of monopoly and duopoly relies on the following additional assumption.

Assumption 3. $\Psi(p) \geq 0$ if and only if $c \leq p \leq p^d$, and $\Psi(p) \leq 0$ if and only if $p^d \leq p \leq \bar{v}$.

Assumption 3 holds for the same sufficient conditions stated above for Assumption 2. In particular, the assumption holds if $R(\bar{p})$ crosses the 45^{o} degree line once from above.

If follows from $\Psi(p^d) = 0$ and Assumption 3 that $p^d \geq p^m$ if $\Psi(p^m) \geq 0$, and, conversely, $p^d \leq p^m$ if $\Psi(p^m) \leq 0$. Note further that the first-order condition for the monopoly problem implies

$$\Psi(p^m) \equiv \left\{ - \int_{p^m}^{\bar{v}} [G(v|v) - G(p^m|v)] f(v)\, dv \right\} - (p^m - c) \left\{ \int_{p^m}^{\bar{v}} [g(v|v) - g(p^m|v)] f(v)\, dv \right\}. \tag{11}$$

Therefore, we have the following comparison.

Theorem 1 *Under Assumption 1-3, $p^d \geq p^m$ if and only if*

$$\left\{ \int_{p^m}^{\bar{v}} [G(v|v) - G(p^m|v)] f(v)\, dv \right\} \leq (p^m - c) \left\{ \int_{p^m}^{\bar{v}} [g(p^m|v) - g(v|v)] f(v)\, dv \right\}, \tag{12}$$

and the converse.

The theorem is explained with Figure 1 below.

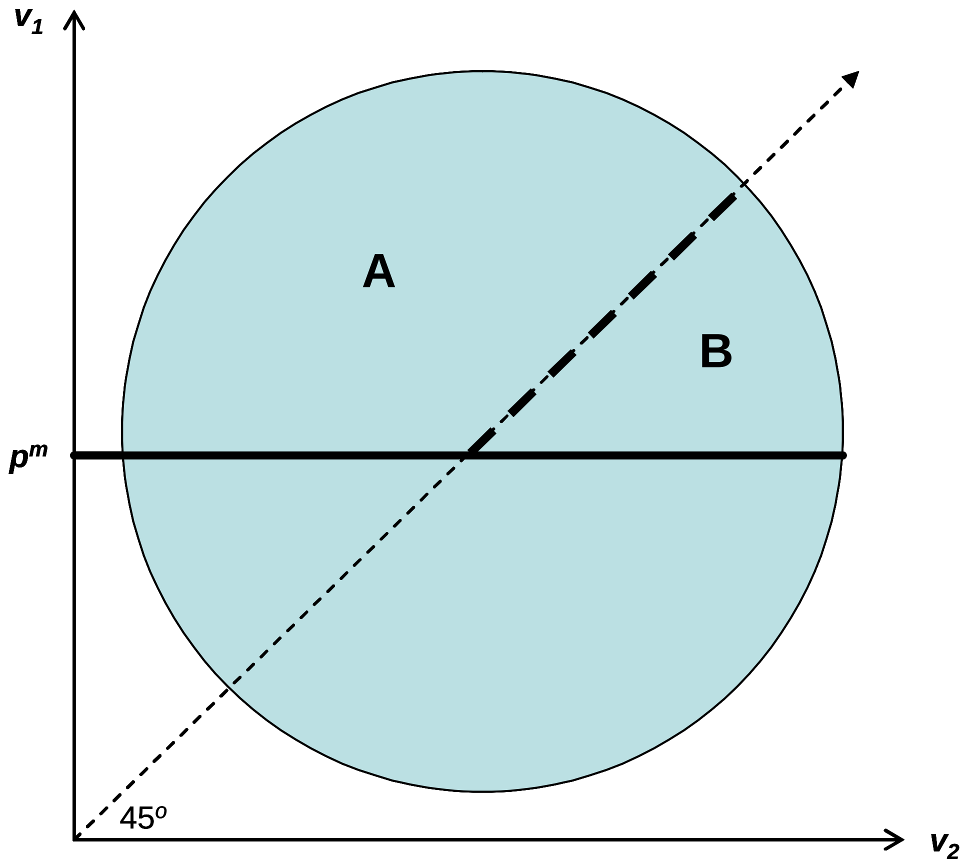

Fig. 1. Comparing single-product monopoly and symmetric duopoly

The set of possible preferences, Ω, is represented by the shaded circle. The line dividing the circle, labelled p^m, represents the monopoly price. The area of the circle above this line (region A + region B) represents the market share[6] of monopolist selling good 1. Region A alone represents the market share of a duopolist selling product 1 when both duopolists charge p^m. Thus, the wedge-shaped region B is equal to difference in market share for a monopolist charging p^m and for a duopolist when both firms charge p^m. The larger is this difference, the greater is the incentive of a duopolist to reduce price. Call this the "market share effect" of competition. The probability that preferences lie within region B is $\int_{p^m}^{\bar{v}} [G(v \mid v) - G(p^m \mid v)] f(v)\, dv$, which is the left-hand side of the expression in Theorem 1. Next consider the two straight edges of the wedge. Difference in the density of preferences along these edges is $\int_{p^m}^{\bar{v}} [g(v \mid v) - g(p^m \mid v)] f(v)\, dv$, which is part of the right-hand side of the expression in Theorem 1. This amount is difference in the slope of the duopolist's (residual) demand curve and the monopolist's demand curve at price p^m. The steeper is the duopolist's demand curve, relative to the monopolist's, the greater is the incentive of the duopolist to raise price above p^m. Call this the "price sensitivity effect" of competition. In order for $p^d > p^m$, it is thus necessary, but not sufficient, that the duopolist's demand curve is steeper than the monopolist's. This necessary condition can often hold under product differentiation, because consumers have a choice of products in the duopoly case, and, therefore, are less keen to purchase the duopolist's product in response to a price cut.[7] To sum up, Theorem 1 states that the duopolist will have incentive to raise price above the monopoly level if the price sensitivity effect is sufficiently important compared to the market share effect.

A provocative application of the theorem is the case of a (Gumbel) bivariate exponential distribution:

$$h(v_1, v_2) = [(1 + \theta v_1)(1 + \theta v_2) - \theta] \exp\{-v_1 - v_2 - \theta v_1 v_2\}$$

[6] Here “market share" means the portion of the consumer population who are purchasing the product, or market coverage.

[7] This condition is more likely to hold when consumers' preferences for the two prodcuts are negatively correlated; but negative correlation is not required.

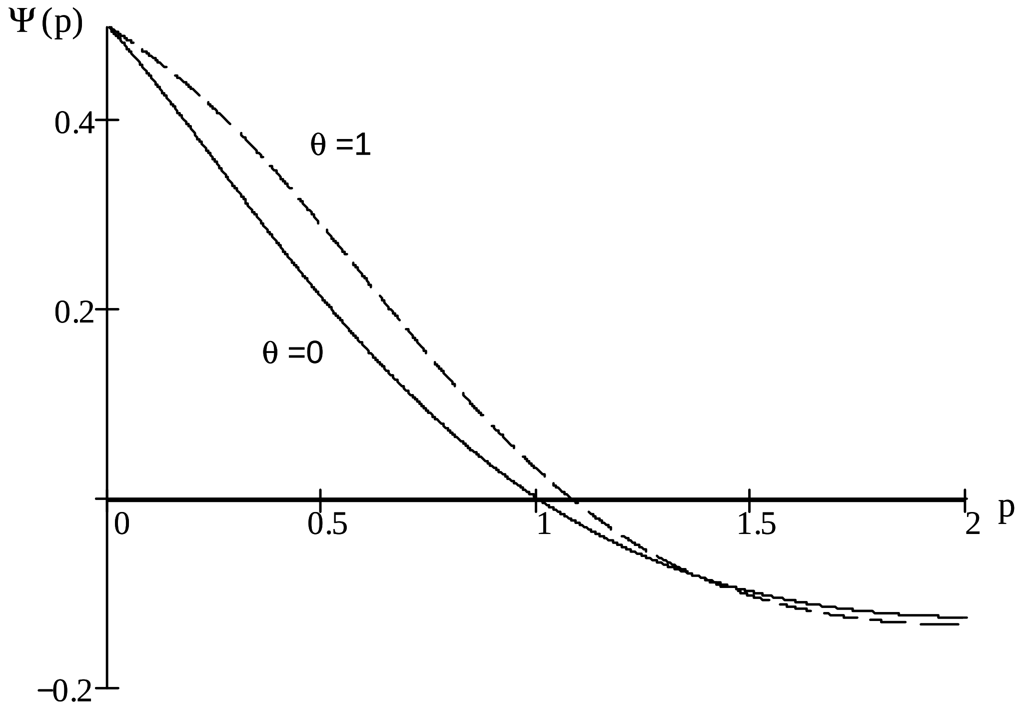

FIG. 2. Bivariate exponential case

with $0 \leq \theta \leq 1$. The v_i are independent for $\theta = 0$, and negatively correlated for $0 < \theta \leq 1$. The marginal distributions of v_1 and v_2 are standard exponential distributions. Therefore, $p^m = 1$ if $c = 0$. In this case, a straightforward numerical analysis establishes that $p^d = 1$ if $\theta = 0$, and $p^d > 1$ if $\theta > 0$. Fig. 2 graphs the function $\Psi(p)$ for $\theta = 0$ (solid line), and $\theta = 1$ (dashed line). The curve is downward sloping as required by Assumption 3,[8] and the duopoly solution occurs where $\Psi(p) = 0$. The diagram shows that $\Psi(1) = 0$ for $\theta = 0$ and $\Psi(1) > 0$ for $\theta = 1$. The graph of $\Psi(p)$ for intermediate cases $0 < \theta < 1$ is between these two extremes. Therefore, competition increases price in the bivariate exponential case.

[8]It is straightforward to verify numerically that the profit function of duopolist is quasi-concave when the rival charges p^d satisfying $\Psi(p^d) = 1$, implying Assumption 2.

Proceeding more generally, let

$$\mu(p|v) = \frac{g(p|v)}{1 - G(p|v)} \tag{13}$$

denote the conditional hazard rate. Then the condition of Theorem 1 can be written as

$$\int_{p^m}^{\bar{v}} \left[p^m - c - \frac{1}{\mu(v|v)} \right] g(v|v) f(v) dv \leq \int_{p^m}^{\bar{v}} \left[p^m - c - \frac{1}{\mu(p^m|v)} \right] g(p^m|v) f(v) dv, \tag{14}$$

which leads to the following conclusion.[9]

Corollary 1 *Assume $\bar{v} = \infty$. Then $p^d > p^m$ if $\mu(v|v) < \mu(p^m|v)$ for $v > p^m$.*

The bivariate exponential distribution has these properties for $\theta > 0$, and, therefore, provides a special case of the corollary.

Next, consider the independence case from a more general perspective. If v_1 and v_2 are independent, then $g(v|v) = f(v)$ and $\mu(v|v) = \lambda(v)$, and the condition of Theorem 1 becomes

$$\int_{p^m}^{\bar{v}} \left[p^m - c - \frac{1}{\lambda(v)} \right] f(v)^2 dv \leq \int_{p^m}^{\bar{v}} \left[p^m - c - \frac{1}{\lambda(p^m)} \right] f(p^m) f(v) dv,$$

which is equivalent to

$$\int_{p^m}^{\bar{v}} \left[\frac{1}{\lambda(p^m)} - \frac{1}{\lambda(v)} \right] f(v)^2 dv \leq 0.$$

Therefore, we have the following comparison.

Corollary 2 *If v_1 and v_2 are independent, then $p^d \geq p^m$ if and only if*

$$\int_{p^m}^{\bar{v}} \left[\frac{1}{\lambda(v)} - \frac{1}{\lambda(p^m)} \right] f(v)^2 dv \geq 0, \tag{15}$$

and the converse.

Therefore, if v_1 and v_2 are independently and identically distributed and Assumption 1-3 hold, then $p^d < p^m$ if $\lambda(v)$ is increasing, and, conversely $p^d > p^m$ if $\lambda(v)$ is decreasing. We next provide an example where $p^d > p^m$ for independent valuations.

[9] Note that $\mu(v|v) < \mu(p^m|v)$ implies $g(v|v) < g(p^m|v)$ for $v > p^m$.

Example 1 *Consider the following case for* $\bar{v} = \infty$ *and* $\alpha > 0$*:* $c > 0$*,* $\lambda(v) = \frac{\beta}{v}$*,* $F(v) = 1 - e^{-\beta[\ln v - \ln \alpha]}$*, and* $f(v) = \frac{\beta}{v} e^{-\beta[\ln v - \ln \alpha]}$*, with* $\beta > 1$*. Then,* $(\bar{v} - c)\,\lambda(\bar{v}) > 1$ *and* $\frac{\lambda'(p)}{\lambda(p)} = \frac{-\frac{\beta}{p^2}}{\frac{\beta}{p}} = -\frac{1}{p} > -\frac{1}{p-c}$*. To be concrete, assume that* $\beta = 3$ *and* $\alpha = c = 2$*. It follows that* $p^m = 3$*. Therefore, for* $p > c$*,*

$$\begin{aligned}
\bar{\lambda}(p,p) &= \frac{\int\limits_p^\infty \frac{\beta^2}{v^2} e^{-2\beta[\ln v - \ln \alpha]} dv + \frac{\beta}{p} e^{-\beta[\ln p - \ln \alpha]}\left[1 - e^{-\beta[\ln p - \ln \alpha]}\right]}{\int\limits_p^\infty \frac{\beta}{v} e^{-2\beta[\ln v - \ln \alpha]} dv + e^{-\beta[\ln p - \ln \alpha]}\left[1 - e^{-\beta[\ln p - \ln \alpha]}\right]} \\
&= \frac{3}{p} \frac{p^3 - \frac{32}{7}}{p^3 - 4}
\end{aligned}$$

It can be checked numerically that $(p-c)\bar{\lambda}(p,p)$ *is strictly increasing in* p*. Solving* $(p - c)\bar{\lambda}(p,p) = 1$ *yields* $p^d = 3.037$*. Furthermore, it can be checked numerically that* $(p - c)\bar{\lambda}(p, p^d)$*, evaluated at* $\beta = 3$*,* $c = 2$ *and* $p^d = 3.037$*, is strictly increasing in* p*. Thus* p^d *is indeed the equilibrium price, and we have* $p^d > p^m$ *in this example.*

Finally, consider Theorem 1 in terms of the shape of $g\,(v_2|v_1)$ for $v_1 \geq p^m$ and $\underline{v} \leq v_2 \leq v_1$. The condition of Theorem 1 is equivalent to $\Psi\left(p^M\right) \geq 0$, and can be rewritten as

$$\begin{aligned}
& \int_{p^m}^{\bar{v}} \left[\int_{p^m}^{v_1} g\,(v_2|v_1)\,dv_2\right] f\,(v_1)\,dv_1 + (p^m - c)\int_{p^m}^{\bar{v}} g\,(v_1|v_1)\,f\,(v_1)\,dv_1 \\
\leq \;& (p^m - c)\left\{\int_{p^m}^{\bar{v}} g\,(p^m|v)\,f\,(v)\,dv\right\}.
\end{aligned}$$

As a benchmark, consider the independent exponential case, in which this condition holds with equality. Now, starting from the independent exponential case, for each $v_1 \geq p^m$, rotate $g\,(v_2|v_1)$ around p^m by shifting probability density from $v_2 > p^m$ to $v_2 < p^m$. These rotations do not alter $f\,(v_1)$, and, therefore, do not effect p^m or the right-hand side of the above inequality. The rotations, however, decrease $\int_{p^m}^{v_1} g\,(v_2|v_1)\,dv_2$ and $\int_{p^m}^{v_1} g\,(v_1|v_1)\,dv_1$ for $v_1 > p^m$, thus decreasing the left-hand side of the inequality. Consequently, starting from the independent exponential case, "negative rotations" of $g\,(v_2|v_1)$ around p^m result in $p^d > p^m$. Conversely, "positive rotations" result in $p^d < p^m$. This demonstrates that there are large families of cases for which the symmetric duopoly price either exceeds or falls short of the monopoly price.

The following result formalizes these observations.

Corollary 3 *Let* $f(v_1) = \lambda e^{-\lambda v_1}$ *and* $g(v_2|v_1) = \alpha\phi(v_1, v_2) + f(v_2)$ *for* $\alpha > 0$*, and*

$$\phi(v_1, p^m) = 0$$

for all $v_1 \in [\underline{v}, \bar{v}]$.[10] *If, in addition,*

$$\phi(v_1, v_2) > 0 \text{ for } \underline{v} \leq v_2 \leq p^m$$

and

$$\phi(v_1, v_2) < 0 \text{ for } p^m \leq v_2 \leq \bar{v},$$

then $p^d > p^m$. *Conversely, if*

$$\phi(v_1, v_2) < 0 \text{ for } \underline{v} \leq v_2 \leq p^m,$$

and

$$\phi(v_1, v_2) > 0 \text{ for } p^m \leq v_2 \leq \bar{v},$$

then $p^d < p^m$.

Note that Assumptions 1-3 necessarily hold for $\alpha > 0$ sufficiently small, because sufficient conditions for these assumptions hold strictly when $\alpha = 0$. Furthermore, the result allows for either negative or positive correlation of v_1 and v_2.

3. UNIFORM DISTRIBUTIONS OF PREFERENCES

We next consider an analytically more tractable model of preferences, where (v_1, v_2) are uniformly distributed, to illustrate the comparison of prices under monopoly and duopoly.

[10] Obviously,

$$\int_{\underline{v}}^{\bar{v}} \phi(v_1, v_2) dv_2 = 0,$$

since $g(v_2|v_1)$ is a conditional density. Furthermore, the symmetry of $h(v_1, v_2)$ also constrains $\phi(v_1, v_2)$. It is, however, possible to construct $\phi(v_1, v_2)$ that satisfy these conditions, as well as the following rotation conditions.

This model allows each consumer's valuations for the two products to have various forms of negative or positive correlations, with the familiar Hotelling and Bertrand models as limiting cases.

Suppose the support for (v_1, v_2), Ω, is a rectangular area on the v_1-v_2 space that is formed by segments of four lines with the following inequalities:

$$\begin{gathered} 2(1+a) \geq v_1 + v_2 \geq 2; \\ b \geq v_1 - v_2 \geq -b, \end{gathered} \tag{16}$$

where $a \in (0, \infty)$ and $b \in (0, 1]$.[11] Suppose that (v_1, v_2) is uniformly distributed on Ω. Then

$$\phi(v_1, v_2) = \frac{1}{2ab}, \ \ (v_1, v_2) \in \Omega. \tag{17}$$

When $b \to 0$, Ω converges to an upward sloping line; in the limit, v_1 and v_2 have perfect positive correlation and the model becomes the standard model of Bertrand competition with a downward sloping demand curve. On the other hand, when $a \to 0$, Ω converges to a downward sloping line; in the limit v_1 and v_2 have perfect negative correlation and the model becomes one of Hotelling competition with the unit transportation cost being 1, the length of the Hotelling line being $\sqrt{2}b$, and consumers valuing either product variety at $\frac{2+b}{2}$ (not including the transportation cost). In fact, we may consider Ω as consisting of a dense map of lines $v_1 + v_2 = x$, $x \in [2, 2(1+a)]$, each having length $\sqrt{2}b$ and being parallel to line $v_1 + v_2 = 2$. Each of these line segments corresponds to a Hotelling line with length equal to $\sqrt{2}b$, unit transportation cost equal to 1, and consumer valuation equal to $\frac{x+b}{2}$. Fig. 3 illustrates Ω for representative values of a and $b = 1$.

We first obtain the prices under duopoly and under monopoly. We have:

Lemma 1 *With Uniform Distributions, the (symmetric) equilibrium price under duopoly*

[11] The parameter restriction $b \leq 1$ is imposed to ensure that, when $a \to 0$, in the duopoly equilibrium all consumers will purchase with positive surpluses. As we will see shortly, the equilibrium duopoly price will always be $p^d = b$. If $a \to 0$, $v_1 + v_2 \to 2$; and $b \leq 1$ is needed so that the consumer with $v_1 = v_2$ will still purchase with a positive surplus. When a increases, the maximum allowed value of b also increases.

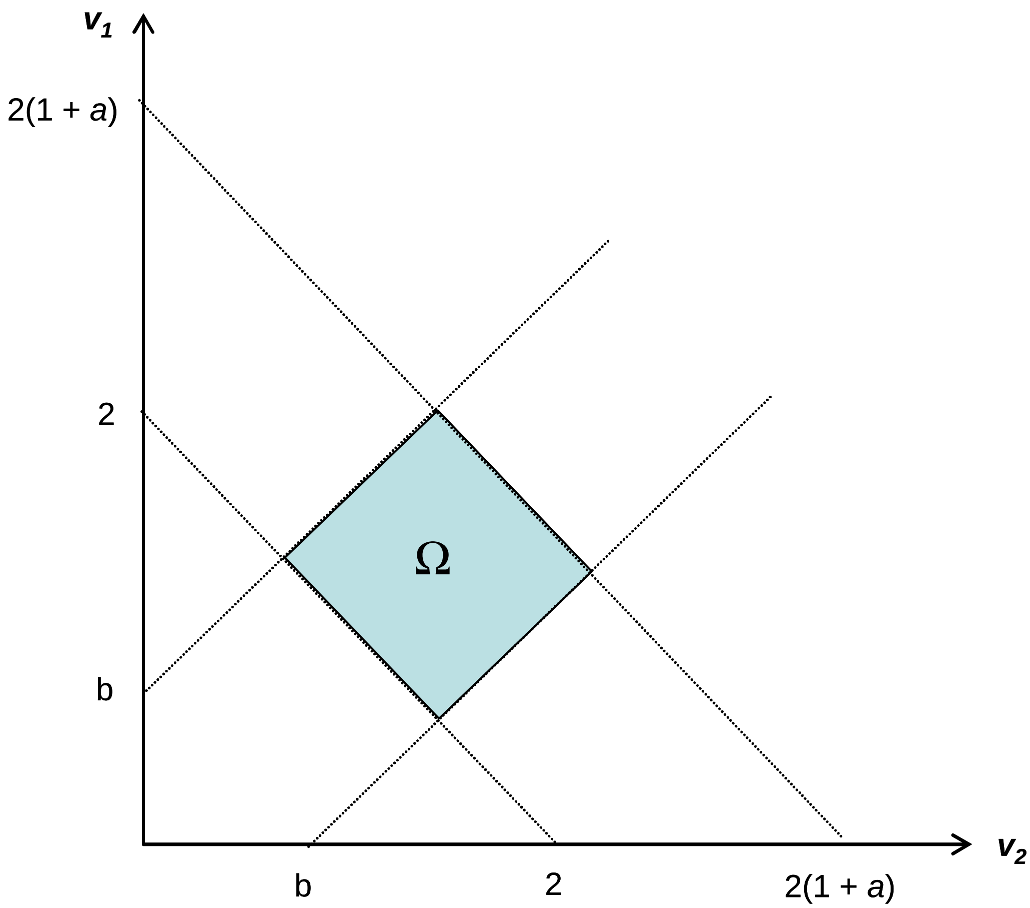

Fig. 3. Ω is an oriented rectangle

is $p^d = b$, *and the optimal price for the monopolist is*

$$p^m = \begin{cases} \frac{a+b+2}{4} & if \quad 0 < a < b - \frac{2}{3} \\ \frac{2-b}{3} + \frac{1}{6}\sqrt{24ab - 4b + b^2 + 4} & if \quad \max\{b - \frac{2}{3}, 0\} \leq a < 1 + b \\ \frac{1+a}{2} & if \quad 1 + b \leq a \end{cases} . \tag{18}$$

Notice that the equilibrium duopoly price, $p^d = b$, is simply the equilibrium price of the Hotelling model when $a \to 0$ and Ω collapses to a single downward sloping line. The Hotelling solution generalizes because Ω is essentially a collection of stacked Hotelling lines, for each of which b is a best response to b. Calculation of the monopoly price is slightly more complicated. Details of the calculations that establish Lemma 1 as well as Proposition 1 below are contained in an appendix.

The variance and correlation coefficient of v_1 and v_2 are also calculated in the Appendix, and they are

$$\begin{aligned} Var(v_1) &= \frac{1}{12}\left(a^2 + b^2\right) = Var(v_2), \\ \rho &= \frac{(a-b)(a+b)}{a^2 + b^2}. \end{aligned}$$

Therefore, given the parameter restrictions $a > 0$ and $0 < b \leq 1$, the case $a < b$ corresponds to negative correlation, $a = b$ to independence, and $a > b$ to positive correlation.

The comparison of the duopoly and monopoly prices is straight forward:

$$p^m - p^d = \begin{cases} \frac{a-3b+2}{4} & if \quad 0 < a < b - \frac{2}{3} \\ \frac{2-4b}{3} + \frac{1}{6}\sqrt{24ab - 4b + b^2 + 4} & if \quad \max\{b - \frac{2}{3}, 0\} \leq a < 1 + b \\ \frac{1+a}{2} - b & if \quad 1 + b \leq a \end{cases} . \tag{19}$$

Analysis of this equation yields the following comparison:

Proposition 1 *Under uniform Distributions,*

$$p^m - p^d \begin{cases} < 0 & if \quad 0 < a < \frac{(3b-2)(7b-2)}{8b} \text{ and } b > \frac{2}{3} \\ = 0 & if \quad 0 < a = \frac{(3b-2)(7b-2)}{8b} \text{ and } b > \frac{2}{3} \\ > 0 & if \quad otherwise \end{cases} . \tag{20}$$

We note that when $b > \frac{2}{3}$, $b - \frac{2}{3} < \frac{(3b-2)(7b-2)}{8b} < b$, and $\frac{(3b-2)(7b-2)}{8b}$ increases in b. Thus in this model of uniform distributions, duopoly price is higher than monopoly price if a is sufficiently small relative to b and b is above certain critical value, or if ρ is small and $Var(v_i)$ is high enough. This amounts to stating that competition increases price if consumer preferences are sufficiently negatively correlated and diverse. The parameter values with this feature represent a plausible but relatively small region of the preference space considered.

Unlike under more general distributions, under uniform distributions competition increases price only if preferences are negatively correlated ($a < b$).

4. MULTIPRODUCT MONOPOLY

As other studies in the literature concerning the effects of market structure on prices, our main interest in this paper is to compare duopoly and monopoly prices for single-product firms. This comparion sheds light, for instance, on situations where a competitor with a differentiated product enters a market that is initially monopolized. For completeness, we now also compare the price of a multi-product monopolist who sells both products and a pair of duopolists. The result is the expected one: the symmetric multiproduct monopoly price exceeds the duopoly price, confirming the intuition that a multiproduct monopolist internalizes the externality between products that is ignored by competing firms under independent pricing. Hence, from the perpective of the guidelines used by the U.S. government to evaluate horizontal mergers, a significantly higher multiproduct monopoly price indicates that the two products are in the same antitrust market.[12]

A single firm producing both products solves the "multiproduct monopoly problem":

$$\max_{(p_1,p_2)\in[c,\bar{v}]^2} \ (p_1 - c)\, q(p_1, p_2) + (p_2 - c)\, q(p_2, p_1). \tag{21}$$

Assumption 4. There exists a unique interior symmteric solution to the multiproduct monopoly problem, $p^{mm} \in (c, \bar{v})$.

[12] U.S. Department of Justice and Federal Trade Commission, *Horizontal Merger Guidelines*, issued April 2, 1992, revised April 9, 1997, http://www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html. .

The first-order condtion for a symmetric solution is

$$
\begin{aligned}
& \int_{\underline{v}}^{p^{mm}} \left[1 - G\left(p^{mm} \mid v\right)\right] f(v)\, dv + \int_{p^{mm}}^{\bar{v}} \left[1 - G(v \mid v)\right] f(v)\, dv \\
& - \left(p^{mm} - c\right) \left[\int_{\underline{v}}^{p^{mm}} g\left(p^{mm} \mid v\right) f(v)\, dv + \int_{p^{mm}}^{\bar{v}} g(v \mid v) f(v)\, dv\right] \\
& + \left(p^{mm} - c\right) \left[\left[1 - G\left(p^{mm} \mid p^{mm}\right)\right] f\left(p^{mm}\right) + \int_{p^{mm}}^{\bar{v}} g(v \mid v) f(v)\, dv\right] \\
= \; & 0.
\end{aligned}
$$

Recalling the definition of $\Psi(\cdot)$ from equation (10), the first-order condition is equivalent to

$$
\Psi\left(p^{mm}\right) + \left(p^{mm} - c\right) \left[\left[1 - G\left(p^{mm} \mid p^{mm}\right)\right] f\left(p^{mm}\right) + \int_{p^{mm}}^{\bar{v}} g(v \mid v) f(v)\, dv\right] = 0, \qquad (22)
$$

which implies

$$
\Psi\left(p^{mm}\right) < 0.
$$

Since $\Psi\left(p^{d}\right) = 0$, and by Assumption 3, $\Psi(p) < 0$ if and only if $p > p^{d}$, we have

Theorem 2 *Under Assumption 2-4,* $p^{mm} > p^{d}$.

This result is familiar, and the intuition is well known: the price change of one product affects the profit of another product; this effect is not taken into account when the duopolists set prices independently, but the multiproduct monopolist internalizes this effect when setting prices jointly for the two products. Consequently, since the products are substitutes here, the multiproduct monopolist charges a higher price for the two products than the duopolists.[13]

Therefore, comparing prices between a multiproduct monopolist and single-product competitors is very different from comparing prices under different market structures with

[13] If the two products were complements (and hence consumers might buy both products), prices would again be lower under multiproduct monopoly than under duopoly competition, More generally, whether or not prices are higher under the multiproduct monopoly depends on the nature of relations between products (e.g., Chen, 2000; and Davis and Murphy, 2000).

single-product firms. In the former, the results are based on the familiar idea of a monopolist internalizing the externalities between different products. In the latter, the forces at work have not been well understood. The contribution of our analysis is to explain the effects determining how prices change from monopoly to duopoly for single-product firms, and identify precise conditions for price-increasing competition.

5. CONCLUSION

The relationship between market structure and price is a central issue in economics. This paper has provided a complete comparison of equilibrium prices under single-product monopoly and symmetric duopoly in an otherwise general discrete choice model of product differentiation. The necessary and sufficient condition for price-increasing competition balances two effects of entry by a symmetric firm into a monopoly market, the market share effect and the price sensitivity effect. The market share effect is that a reduced quantity per firm under duopoly provides an incentive for the firms to cut price below the monopoly level. The price sensitivity effect is that a steeper demand curve resulting from greater consumer choice provides an incentive to raise price. Under certain conditions the price sensitivity effect outweighs the market share effect, resulting in a higher symmetric duopoly price compared to monopoly. For example, the symmetric duopoly price is higher than the single-product monopoly price if consumers' values for the two products are independently drawn from a distribution function with a descreasing hazard rate. A class of special cases is when consumer values for two products have a joint uniform distribution on a varying oriented rectangular support. This framework nests the familiar Hotelling and Bertrand models. Competition increases prices in these cases if valuations are sufficiently diverse and negatively correlated. More generally, however, competition can lead to higher prices even with independent or positively correlated values. In summary, our analysis shows that the consumer preferences leading to price-increasing competition are by no means exceptional.

The theoretical possibility of price-increasing competition potentially has important implications for emprical industrial organization. For instance, it is a standard procedure of the new empirical industrial organization to estimate a discrete choice differentiated products demand model, and infer unobservable marginal costs from the corresponding first-order conditions for equilibrium pricing (Berry, 1994). If the demand model presumes restrictions on consumer preferences that are inconsistent with price-increasing competition, then the analysis might mistakenly conclude that marginal costs are higher in duopoly markets than monopoly markets.[14]

There are several promising directions for further theoretical research. Under product differentiation, higher price in the presence of an additional firm need not mean that consumers are worse off; consumers' benefit from the additional variety should be considered. It is thus desirable to extend our analysis to understand fully how competition affects consumer welfare and social welfare.[15] Another direction for future research is to consider the relationship between market structure and prices with an arbitrary number of firms. Chen and Riordan (2005) provides an analysis within a spatial setting, the spokes model. It would be interesting to study the relationship in a more general framework of preferences.

Finally, as discussed in the introduction, there is scattered empirical evidence pointing to the phenomenon of price-increasing competition in several industries. We hope that our theory stimulates new empirical research on the topic.

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[14] Goosbee and Petrin (2004) demonstrate that a more flexible model, allowing for negative or positive correlations of unobservable components of consumer values, also matters for estimated substitution patterns.

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APPENDIX

This appendix calculates and compares single-product monopoly and symmetric duopoly prices for the uniform-distribution cases of Section 3.

Case 1: $a < b$

In this case, Ω is described by

$$v_1 \in (\tfrac{2-b}{2}, 1 + \tfrac{b}{2} + a), \text{ and}$$
$$v_2 \in \begin{cases} (2 - v_1,\ v_1 + b) & if \quad \frac{2-b}{2} \le v_1 < \frac{2-b}{2} + a \\ (2 - v_1,\ 2(1+a) - v_1) & if \quad \frac{2-b}{2} + a \le v_1 < \frac{2+b}{2} \\ (v_1 - b,\ 2(1+a) - v_1) & if \quad \frac{2+b}{2} \le v_1 < \frac{2+b}{2} + a \end{cases}.$$

We have

$$f(v_1) = \begin{cases} \int_{2-v_1}^{v_1+b} \frac{1}{2ab} dv_2 = \frac{2v_1+b-2}{2ab} & if \quad \frac{2-b}{2} \le v_1 < \frac{2-b}{2} + a \\ \int_{2-v_1}^{2(1+a)-v_1} \frac{1}{2ab} dv_2 = \frac{1}{b} & if \quad \frac{2-b}{2} + a \le v_1 < \frac{2+b}{2} \\ \int_{v_1-b}^{2(1+a)-v_1} \frac{1}{2ab} dv_2 = \frac{2(1+a)+b-2v_1}{2ab} & if \quad \frac{2+b}{2} \le v_1 < \frac{2+b}{2} + a \end{cases},$$

$$F(v_1) = \begin{cases} \int_{\frac{2-b}{2}}^{v_1} \frac{2v+b-2}{2ab} dv = \frac{(2v_1+b-2)^2}{8ab} & if \quad \frac{2-b}{2} \le v_1 < \frac{2-b}{2} + a \\ \int_{\frac{2-b}{2}+a}^{v_1} \frac{1}{b} dv + \frac{\left(2\left(\frac{2-b}{2}+a\right)+b-2\right)^2}{8ab} = \frac{1}{b} v_1 - \frac{a+2-b}{2b} & if \quad \frac{2-b}{2} + a \le v_1 < \frac{2+b}{2} \\ \int_{\frac{2+b}{2}}^{v_1} \frac{2(1+a)+b-2v}{2ab} dv + \left(\frac{2+b}{2b} - \frac{a+2-b}{2b}\right) = \frac{(4a+b-2v_1+2)(2v_1-b-2)}{8ab} + \frac{2b-a}{2b} & if \quad \frac{2+b}{2} \le v_1 < \frac{2+b}{2} + a \end{cases},$$

$$g(v_2 \mid v_1) = \frac{\phi(v_1, v_2)}{f(v_1)} = \begin{cases} \frac{1}{2v_1+b-2} & if \quad \frac{2-b}{2} \le v_1 < \frac{2-b}{2} + a,\ 2 - v_1 < v_2 < v_1 + b \\ \frac{1}{2a} & if \quad \frac{2-b}{2} + a \le v_1 < \frac{2+b}{2},\ 2 - v_1 < v_2 < 2(1+a) - v_1 \\ \frac{1}{2(1+a)+b-2v_1} & if \quad \frac{2+b}{2} \le v_1 < \frac{2+b}{2} + a,\ v_1 - b < v_2 < 2(1+a) - v_1 \end{cases},$$

and

$$G\left(v_2 \mid v_1\right)=\begin{cases} \int_{2-v_1}^{v_2} \frac{1}{2v_1+b-2} dv = \frac{v_2+v_1-2}{2v_1+b-2} & if \quad \begin{array}{c} \frac{2-b}{2} \leq v_1 < \frac{2-b}{2}+a \\ 2-v_1 < v_2 < \ v_1+b \end{array} \\ \int_{2-v_1}^{v_2} \frac{1}{2a} dv = \frac{v_2+v_1-2}{2a} & if \quad \begin{array}{c} \frac{2-b}{2}+a \leq v_1 < \frac{2+b}{2} \\ 2-v_1 < v_2 < \ 2\left(1+a\right)-v_1 \end{array} \\ \int_{v_1-b}^{v_2} \frac{1}{2(1+a)+b-2v_1} dv_2 = \frac{v_2-v_1+b}{2(1+a)+b-2v_1} & if \quad \begin{array}{c} \frac{2+b}{2} \leq v_1 < \frac{2+b}{2}+a \\ v_1-b < v_2 < \ 2\left(1+a\right)-v_1 \end{array} \end{cases}.$$

First, the monopoly price, p^m, satisfies

$$1-F\left(p^m\right)-p^m f\left(p^m\right)=0.$$

(1) If $a<b-\frac{2}{3}$, since for $p \leq \frac{2-b}{2}+a$,

$$\begin{aligned} 1-F\left(p\right)-pf\left(p\right) &= 1-\frac{\left(2p+b-2\right)^2}{8ab}-p\frac{2p+b-2}{2ab} \\ &\geq 1-\frac{\left(2\left(\frac{2-b}{2}+a\right)+b-2\right)^2}{8ab}-\left(\frac{2-b}{2}+a\right)\frac{2\left(\frac{2-b}{2}+a\right)+b-2}{2ab} \\ &= \frac{1}{2b}\left(3b-2-3a\right)>0, \end{aligned}$$

we have $p^m>\frac{2-b}{2}+a$. For $\frac{2-b}{2}+a \leq p^m < \frac{2+b}{2}$, p^m satisfies

$$1-\left(\frac{1}{b}p-\frac{a+2-b}{2b}\right)-p\frac{1}{b}=0,$$

or $p^m=\frac{a+b+2}{4}$, and indeed

$$\frac{2-b}{2}+a \leq \frac{a+b+2}{4} < \frac{2+b}{2}$$

for $a<b-\frac{2}{3}$. Note that

$$q^m=1-\left(\frac{a+b+2}{4b}-\frac{a+2-b}{2b}\right)=\frac{\left(a+b+2\right)}{4b}<1.$$

For $p \in [\frac{2+b}{2}, \frac{2+b}{2}+a)$,

$$\begin{aligned} 1-F\left(p\right)-pf\left(p\right) &= 1-\left(\frac{\left(4a+b-2p+2\right)\left(2p-b-2\right)}{8ab}+\frac{2b-a}{2b}\right)-p\frac{2\left(1+a\right)+b-2p}{2ab} \\ &= \frac{\left(6p-b-2a-2\right)\left(2p-b-2a-2\right)}{8ab}<0. \end{aligned}$$

since

$$2p-b-2a-2<2\left(\frac{2+b}{2}+a\right)-b-2a-2=0$$

and

$$6p-b-2a-2\geq 6\left(\frac{2+b}{2}\right)-b-2a-2=2\left(b-a+2\right)>0.$$

Thus $p^m=\frac{a+b+2}{4}$ if $a<b-\frac{2}{3}$.

(2) If $\max\left\{b-\frac{2}{3},0\right\}\leq a<b$, for $p\leq\frac{2-b}{2}+a$, we have

$$1-F\left(p\right)-pf\left(p\right)=1-\frac{\left(2p+b-2\right)^2}{8ab}-p\frac{2p+b-2}{2ab}=-\frac{8bp-16p-8ab-4b+b^2+12p^2+4}{8ab},$$

which is decreasing in p if $p>\frac{2-b}{3}$. Thus the solution to

$$1-F\left(p\right)-pf\left(p\right)=1-\frac{\left(2p+b-2\right)^2}{8ab}-p\frac{2p+b-2}{2ab}=-\frac{8bp-16p-8ab-4b+b^2+12p^2+4}{8ab}=0$$

is

$$p^m=\frac{2-b}{3}+\frac{1}{6}\sqrt{24ab-4b+b^2+4}.$$

Note that

$$\frac{2-b}{3}+\frac{1}{6}\sqrt{24ab-4b+b^2+4}\geq\frac{2-b}{3}+\frac{1}{6}\sqrt{-4b+b^2+4}=\frac{2-b}{3}+\frac{2-b}{6}=\frac{2-b}{2},$$

and, since

$$\left(24a\left(a+\frac{2}{3}\right)-4b+b^2+4\right)-\left(6a+2-b\right)^2=4a\left(3b-3a-2\right)\leq 0$$

for $b-\frac{2}{3}\leq a$,

$$\begin{aligned}\frac{2-b}{3}+\frac{1}{6}\sqrt{24ab-4b+b^2+4} &\leq \frac{2-b}{3}+\frac{1}{6}\sqrt{24a\left(a+\frac{2}{3}\right)-4b+b^2+4}\\ &\leq \frac{2-b}{3}+\frac{1}{6}\sqrt{\left(6a+2-b\right)^2}=\frac{2-b}{2}+a.\end{aligned}$$

Next, consider the duopoly price.

Suppose first that $\frac{2-b}{2}+a<1$. If $\frac{2-b}{2}<p^d<\frac{2-b}{2}+a<1$, or $a<\frac{b}{2}$, we have

$$\begin{aligned}
&\left\{\int_{\underline{v}}^{p^d}\left[1-G\left(p^d\mid v\right)\right]f(v)\,dv+\int_{p^d}^{\bar{v}}\left[1-G(v\mid v)\right]f(v)\,dv\right\}\\
&-(p^d-c)\left\{\int_{\underline{v}}^{p^d}g\left(p^d\mid v\right)f(v)\,dv+\int_{p^d}^{\bar{v}}g(v\mid v)f(v)\,dv\right\}\\
=&\left(\begin{array}{c}\int_{\frac{2-b}{2}}^{p^d}(1-0)\frac{2v-1}{2a}dv+\int_{p^d}^{\frac{2-b}{2}+a}(1-0)\frac{2v-1}{2a}dv\\+\int_{\frac{2-b}{2}+a}^{1}(1-0)\,dv+\int_1^{1+a}\left(1-\frac{v+v-2}{2a}\right)dv\end{array}\right)-p^d\left\{\int_1^{1+a}\frac{1}{2ab}dv\right\}\\
=&\ \frac{1}{2}-p^d\frac{1}{2b}=\frac{1}{2}\left(1-p^d\frac{1}{b}\right)=0.
\end{aligned}$$

Or $p^d=b$. Thus $p^d=b$ if $b<\frac{2-b}{2}+a$.

If $\frac{2-b}{2}+a\le p^d\le 1$, p^d solves

$$\begin{aligned}
&\int_{\frac{2-b}{2}}^{\frac{2-b}{2}+a}\frac{2v+b-2}{2ab}dv+\int_{\frac{2-b}{2}+a}^{p^d}(1-0)\frac{1}{b}dv+\int_{p^d}^{1}(1-0)\frac{1}{b}dv\\
&+\int_1^{1+a}\left(1-\frac{v+v-2}{2a}\right)\frac{1}{b}dv-p^d\left\{\int_1^{1+a}\frac{1}{2ab}dv\right\}=\frac{1}{2}-p^d\frac{1}{2b}=0,
\end{aligned}$$

or $p^d=b$.

Now suppose that $\frac{2-b}{2}+a\ge 1$, or $a>\frac{b}{2}$, provided $p^d\le 1$,

$$\begin{aligned}
&\left\{\int_{\underline{v}}^{p^d}\left[1-G\left(p^d\mid v\right)\right]f(v)\,dv+\int_{p^d}^{\bar{v}}\left[1-G(v\mid v)\right]f(v)\,dv\right\}\\
&-(p^d-c)\left\{\int_{\underline{v}}^{p^d}g\left(p^d\mid v\right)f(v)\,dv+\int_{p^d}^{\bar{v}}g(v\mid v)f(v)\,dv\right\}\\
=&\left(\begin{array}{c}\int_{\frac{2-b}{2}}^{p^d}(1-0)\frac{2v+b-2}{2ab}dv+\int_{p^d}^{1}(1-0)\frac{2v+b-2}{2ab}dv+\int_1^{\frac{2-b}{2}+a}\left(1-\frac{v+v-2}{2v+b-2}\right)\frac{2v+b-2}{2ab}dv\\+\int_{\frac{2-b}{2}+a}^{1+a}\left(1-\frac{v+v-2}{2a}\right)\frac{1}{b}dv\end{array}\right)\\
&-p^d\left\{\int_1^{\frac{2-b}{2}+a}\frac{1}{2v+b-2}\frac{2v+b-2}{2ab}dv+\int_{\frac{2-b}{2}+a}^{1+a}\frac{1}{2ab}dv\right\}=\frac{1}{2}-p^d\frac{1}{2b}=0.
\end{aligned}$$

Again $p^d=b$.

To summarize, for $a<b$, we have

$$p^d=b;$$

$$p^m=\begin{cases}\frac{a+b+2}{4} & if \quad 0<a<b-\frac{2}{3}\\ \frac{2-b}{3}+\frac{1}{6}\sqrt{24ab-4b+b^2+4} & if \quad \max\{b-\frac{2}{3},0\}\le a<b\end{cases}. \tag{A1}$$

If $0 < a \leq b - \frac{2}{3}$,

$$p^m - p^d = \frac{a+2-3b}{4} \leq \frac{b-\frac{2}{3}+2-3b}{4} = \frac{2-3b}{6} < 0;$$

and if $0 = \max\{b - \frac{2}{3}, 0\} < a < b$,

$$p^m - p^d > \frac{2-4b}{3} + \frac{1}{6}\sqrt{-4b+b^2+4} = \frac{2-3b}{2} > 0.$$

Further, $a = \frac{(3b-2)(7b-2)}{8b}$ solves

$$\frac{2-b}{3} + \frac{1}{6}\sqrt{24ab-4b+b^2+4} = b$$

Thus, noticing that $b - \frac{2}{3} < \frac{(3b-2)(7b-2)}{8b} < b$ for $b - \frac{2}{3} \geq 0$, we have

$$p^m - p^d \begin{cases} < 0 \quad if & 0 < a < \frac{(3b-2)(7b-2)}{8b} \text{ and } b > \frac{2}{3} \\ = 0 \quad if & 0 < a = \frac{(3b-2)(7b-2)}{8b} \text{ and } b > \frac{2}{3} \\ > 0 \quad if & \text{either } \frac{(3b-2)(7b-2)}{8b} < a < b \text{ and } b > \frac{2}{3}, \text{ or } 0 < a < b \leq \frac{2}{3}. \end{cases} \tag{A2}$$

In other words, $p^m < p^d$ if *both* a is relatively small and b relatively large.

We can also find the correlation between v_1 and v_2:

$$\mu_1 = \int_{\frac{2-b}{2}}^{\frac{2-b}{2}+a} x\frac{2x+b-2}{2ab}dx + \int_{\frac{2-b}{2}+a}^{\frac{2+b}{2}} x\frac{1}{b}dx + \int_{\frac{2+b}{2}}^{\frac{2+b}{2}+a} x\frac{2(1+a)+b-2x}{2ab}dx = \frac{a+2}{2}$$

$$\begin{aligned} Var(v_1) &= \int_{\frac{2-b}{2}}^{\frac{2-b}{2}+a} \left(x - \frac{a+2}{2}\right)^2 \frac{2x+b-2}{2ab}dx + \int_{\frac{2-b}{2}+a}^{\frac{2+b}{2}} \left(x - \frac{a+2}{2}\right)^2 \frac{1}{b}dx \\ &+ \int_{\frac{2+b}{2}}^{\frac{2+b}{2}+a} \left(x - \frac{a+2}{2}\right)^2 \frac{2(1+a)+b-2x}{2ab}dx = \frac{1}{12}\left(a^2+b^2\right) = Var(v_2), \end{aligned}$$

$$\begin{aligned} Cov(v_1, v_2) &= \int_{\frac{2-b}{2}}^{\frac{2-b}{2}+a} \left(v_1 - \frac{a+2}{2}\right)\left(\int_{2-v_1}^{v_1+b} \frac{1}{2ab}\left(v_2 - \frac{a+2}{2}\right)dv_2\right)dv_1 \\ &+ \int_{\frac{2-b}{2}+a}^{\frac{2+b}{2}} \left(v_1 - \frac{a+2}{2}\right)\left(\int_{2-v_1}^{2(1+a)-v_1} \frac{1}{2ab}\left(v_2 - \frac{a+2}{2}\right)dv_2\right)dv_1 \\ &+ \int_{\frac{2+b}{2}}^{\frac{2+b}{2}+a} \left(v_1 - \frac{a+2}{2}\right)\left(\int_{v_1-b}^{2(1+a)-v_1} \frac{1}{2ab}\left(v_2 - \frac{a+2}{2}\right)dv_2\right)dv_1 \\ &= \frac{(a-b)(a+b)}{12}. \end{aligned}$$

$$\rho = \frac{Cov\left(\varepsilon_1, \varepsilon_2\right)}{\sqrt{Var\left(\varepsilon_1\right) Var\left(\varepsilon_1\right)}} = \frac{\frac{(a-b)(a+b)}{12}}{\frac{1}{12}\left(a^2+b^2\right)} = \frac{(a-b)(a+b)}{a^2+b^2}.$$

We have $\rho < 0$ for $a < b$, and $\rho \to -1$ if $a \to 0$. Thus, if v_1 and v_2 are sufficiently negatively correlated, and $Var\left(v_i\right)$ or b is relatively large (or preference is sufficiently diverse), we will have $p^m < p^d$.

Case 2: $a \geq b$.—

In this case,

$$f\left(v_1\right) = \begin{cases} \int_{2-v_1}^{v_1+b} \frac{1}{2ab} dv_2 = \frac{2v_1+b-2}{2ab} & if \quad \frac{2-b}{2} \leq v_1 < \frac{2+b}{2} \\ \int_{v_1-b}^{v_1+b} \frac{1}{2ab} dv_2 = \frac{1}{a} & if \quad \frac{2+b}{2} \leq v_1 < \frac{2-b}{2} + a \\ \int_{v_1-b}^{2(1+a)-v_1} \frac{1}{2ab} dv_2 = \frac{2(1+a)+b-2v_1}{2ab} & if \quad \frac{2-b}{2} + a \leq v_1 < \frac{2+b}{2} + a \end{cases},$$

$$F\left(v_1\right) = \begin{cases} \int_{\frac{2-b}{2}}^{v_1} \frac{2v+b-2}{2ab} dv = \frac{\left(2v_1+b-2\right)^2}{8ab} & if \quad \frac{2-b}{2} \leq v_1 < \frac{2+b}{2} \\ \int_{\frac{2+b}{2}}^{v_1} \frac{1}{a} dv + \frac{(2+b+b-2)^2}{8ab} = \frac{1}{a}\left(v_1 - 1\right) & if \quad \frac{2+b}{2} \leq v_1 < \frac{2-b}{2} + a \\ \frac{\left(2v_1-2a+b-2\right)\left(3b+2-2v_1+2a\right)}{8ab} + 1 - \frac{b}{2a} & if \quad \frac{2-b}{2} + a \leq v_1 < \frac{2+b}{2} + a \end{cases},$$

$$g\left(v_2 \mid v_1\right) = \frac{\phi\left(v_1, v_2\right)}{f\left(v_1\right)} = \begin{cases} \frac{1}{2v_1+b-2} & if & \begin{array}{c} \frac{2-b}{2} \leq v_1 < \frac{2+b}{2} \\ 2 - v_1 < v_2 < v_1 + b \end{array} \\ \frac{1}{2b} & if & \begin{array}{c} \frac{2+b}{2} \leq v_1 < \frac{2-b}{2} + a \\ v_1 - b < v_2 < v_1 + b \end{array} \\ \frac{1}{2(1+a)+b-2v_1} & if & \begin{array}{c} \frac{2-b}{2} + a \leq v_1 < \frac{2+b}{2} + a \\ v_1 - b < v_2 < 2\left(1+a\right) - v_1 \end{array} \end{cases},$$

and

$$G\left(v_2 \mid v_1\right)=\begin{cases}\int_{2-v_1}^{v_2}\frac{1}{2v_1+b-2}dv=\frac{v_2+v_1-2}{2v_1+b-2} & if \quad \begin{array}{c}\frac{2-b}{2}\leq v_1<\frac{2+b}{2}\\ 2-v_1<v_2<\ v_1+b\end{array}\\ \int_{v_1-b}^{v_2}\frac{1}{2b}dv=\frac{v_2-v_1+b}{2b} & if \quad \begin{array}{c}\frac{2+b}{2}\leq v_1<\frac{2-b}{2}+a\\ v_1-b<v_2<v_1+b\end{array}\\ \int_{v_1-b}^{v_2}\frac{1}{2(1+a)+b-2v_1}dv_2=\frac{v_2-v_1+b}{2(1+a)+b-2v_1} & if \quad \begin{array}{c}\frac{2-b}{2}+a\leq v_1<\frac{2+b}{2}+a\\ v_1-b<v_2<\ 2\left(1+a\right)-v_1\end{array}\end{cases}.$$

Under monopoly:

For $b\leq a<1+b$, $p^m<\frac{2+b}{2}$ and solves:

$$\begin{aligned}1-F\left(p\right)-pf\left(p\right) &= 1-\frac{\left(2p+b-2\right)^2}{8ab}-p\frac{2p+b-2}{2ab}\\ &= -\frac{8bp-16p+12p^2-8ab-4b+b^2+4}{8ab}=0,\end{aligned}$$

or

$$p^m=\frac{2-b}{3}+\frac{1}{6}\sqrt{24ab-4b+b^2+4},$$

and indeed $\frac{2-b}{3}<p^m<\frac{2+b}{2}$.

For $a\geq 1+b$, $\frac{2+b}{2}\leq p^m<\frac{2-b}{2}+a$, and it solves

$$1-F\left(p\right)-pf\left(p\right)=1-\frac{1}{a}\left(p-1\right)-p\frac{1}{a}=\frac{a-2p+1}{a}=0,$$

or

$$p^m=\frac{1+a}{2}.$$

Under duopoly, we again have $p^d=1$ from straightforward calculations.

In summary: if $a\geq b$, we have

$$p^d=b$$

$$p^m=\begin{cases}\frac{2-b}{3}+\frac{1}{6}\sqrt{24ab-4b+b^2+4} & if \quad b\leq a<1+b\\ \frac{1+a}{2} & if \quad 1+b\leq a\end{cases}. \tag{A3}$$

The correlation coefficient when $a\geq b$ is also

$$\rho = \frac{(a-b)(a+b)}{a^2+b^2} > 0,$$

and $\rho > 0$ for $a > b$.

Since

$$\frac{2-4b}{3} + \frac{1}{6}\sqrt{24ab - 4b + b^2 + 4} > \frac{2-4b}{3} + \frac{2-b}{6} = \frac{2-3b}{2} \geq 0 \text{ if } b \leq \frac{2}{3},$$

and

$$\frac{2-4b}{3} + \frac{1}{6}\sqrt{24ab - 4b + b^2 + 4} = 0$$

when $a = \frac{(3b-2)(7b-2)}{8b}$, but $\frac{(3b-2)(7b-2)}{8b} > b$ only when $b \leq 0.236$, we have

$$p^m - p^d = \begin{cases} \frac{2-4b}{3} + \frac{1}{6}\sqrt{24ab - 4b + b^2 + 4} > 0 & if \quad b \leq a < 1 + b \\ \frac{1+a-2b}{2} > 0 & if \quad 1 + b \leq a \end{cases}, \tag{A4}$$

or $p^m > p^d$ if $a \geq b$.

Combining (A1) and (A3), we have $p^d = b$ and

$$p^m = \begin{cases} \frac{a+b+2}{4} & if \quad 0 < a < b - \frac{2}{3} \\ \frac{2-b}{3} + \frac{1}{6}\sqrt{24ab - 4b + b^2 + 4} & if \quad \max\{b - \frac{2}{3}, 0\} \leq a < 1 + b \\ \frac{1+a}{2} & if \quad 1 + b \leq a \end{cases},$$

establishing Lemma 1.

Combining (A2) and (A4), we have

$$p^m - p^d \begin{cases} < 0 & if \quad 0 < a < \frac{(3b-2)(7b-2)}{8b} \text{ and } b > \frac{2}{3} \\ = 0 & if \quad 0 < a = \frac{(3b-2)(7b-2)}{8b} \text{ and } b > \frac{2}{3} \\ > 0 & if \quad \text{otherwise} \end{cases}, \tag{A5}$$

establishing Proposition 1.