The Term Structure of Forward Exchange Rates and the Forecastability of Spot Exchange Rates: Correcting the Errors

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Abstract

This paper revisits one of the oldest questions in international finance: does the forward exchange rate contain useful information about of the future path of the spot exchange rate? We present a theoretical framework and provide evidence that challenges the common view (Mussa (1979); Dornbusch (1980); Frenkel (1981); Cumby-Obstfeld (1984)) that forward premia contain little information regarding subsequent changes in the spot exchange rate. Using weekly dollar-DM and dollar-sterling data on spot exchange rates and 1, 3, 6, and 12 month forward exchange rates, we find that, as predicted by the theoretical framework the term structure of forward exchange rates together with the spot exchange rate comprise a system that is well represented by a vector error correction model. Employing Johansen's (1991) maximum likelihood approach, we test and confirm for each country the existence of 4 cointegrating relationships as predicted by the theory. We then test and confirm for each country the joint hypothesis that a basis for this cointegrating space is the vector of 4 forward premia. We next test, and reject for each country, the hypothesis that the spot exchange rate is weakly exogenous with respect to the term structure of forward rates. Out-of-sample simulations indicate that the information contained in the term structure of forward premia can be used to reduce the mean squared error in forecasting the spot rate by at least 33 percent at a 6 month horizon and 50 percent at a 1 year horizon.

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1. Introduction

This paper revisits one of the oldest questions in international finance: does the forward exchange rate contain useful information about of the future path of the spot exchange rate? Professional thinking on this subject has undergone a significant shift over the past twenty years. According to the uncovered interest parity theorem (Fisher (1930)) and the efficient markets hypothesis, the equilibrium forward exchange rate established at date t for delivery of foreign exchange at date t + n, $F_{n,t}$, should be the best available predictor of the level of the spot exchange rate realized at date t + n, S_{t+n}. In an influential paper, Frenkel (1981) tested this hypothesis using data for the 1970s. Running log-linear regressions of the form:

(1) $$s_{t+1} = \alpha + \beta f_{1,t} + \gamma z_t + \epsilon_{t+n};$$

he found that he could not reject the hypothesis that $\beta = 1$ and $\gamma = 0$, where z_t is a vector of information variables known at time t. These results were taken to be supportive of the efficient markets - interest parity hypothesis that:

(2) $$E(s_{t+n}|\Omega_t) = f_{n,t};$$

where Ω_t is the set of information available to market st time t.

In the early 1980s, researchers such as Hansen and Hodrick (1980), Cumby and Obstfeld (1980;1984), Meese and Singleton (1982), Fama (1984), and Meese (1986) began to recognize that a potential problem with regressions such as (1) is that if - as appears to be the case - s_{t+n} and $f_{n,t}$ are nonstationary variables, the usual asymptotic theory invoked to construct hypothesis tests becomes inapplicable. For this reason, researchers interested in uncovering the information contained in the forward exchange rate have in recent years estimated regressions of the variety:

$$s_{t+1} - s_t = \alpha + \beta(f_{1,t} - s_t) + \epsilon_{t+1}. \tag{3}$$

In distinct contrast to the findings reported for the levels regressions run by Frenkel (1981) and others, Fama (1984) and Cumby-Obstfeld (1984) find that the forward premium *mispredicts* the direction of the subsequent change in the spot rate. That is, when foreign exchange is selling at a forward premium, the dollar tends on average to appreciate over the length of the forward contract, not depreciate as would be implied by interest parity. Equivalently via the covered interest arbitrage condition:

$$i^{us}_{n,t} - i^{*}_{n,t} = f_{n,t} - s_{n,t}; \tag{4}$$

these findings indicate that, when US interest rates exceed foreign interest rates, the dollar tends on average to appreciate over the holding period, not depreciate so as to offset on average the interest differential in favor of the US. Not only do these results indicate that interest parity is violated, the inability of projection equations such as (3) to account for much of the observed variance in actual exchange rate changes has convinced most, if not in fact virtually all, researchers to conclude that:

> . . . forward premia contain little information regarding subsequent exchange rate changes. As emphasized by Dornbusch (1980), Mussa (1979), and Frenkel (1981), exchange rate changes over the recent period of floating seem to have been largely unanticipated. (Cumby and Obstfeld (1984), p. 139).

In this paper, we present a theoretical framework and provide evidence that challenges this view - a view we shared until completing this project - that forward premia contain little information regarding subsequent changes in the spot exchange rate. Our theoretical framework - which draws upon a similar such framework developed recently by Hall, Anderson, and Granger (1992) to study the term structure of treasury bill yields - predicts that in a $j + 1$ variable system of j forward rates and 1 spot exchange rate, there should exist j cointegrating vectors and exactly 1 common trend which propels the nonstationary component of each of the j forward rates and the 1 spot exchange rate. In fact, the theoretical framework predicts that a basis for the space of cointegrating relationships is just the vector of the j forward exchange rate premia.

Using weekly data on the spot exchange rate and 1, 3, 6, and 12 month forward exchange rates for Germany and Britain, we find for each country that, as predicted by the theoretical framework and the Granger Representation Theorem (Granger and Engle (1987)), the term structure of forward exchange rates together with the spot exchange rate comprise a system that is well represented by a vector error correction model. Employing Johansen's (1991) maximum likelihood approach, we test and confirm for each country the existence of $j = 4$ cointegrating relationships as predicted by the theory. We then test and confirm for each country the joint hypothesis that a basis for this cointegrating space is the vector of 4 forward premia. We next test, and reject for each country, the hypothesis that the spot exchange rate is weakly exogenous with respect to the term structure of forward rates. Out-of-sample simulations indicate that the information contained in the term structure of forward premia can be used to reduce the mean squared error in forecasting the spot rate by more than 33 percent at a 6 month horizon and 50 percent at a 1 year horizon.

2. Theoretical Framework

Consider a vector y_t comprised of the log spot exchange rate s_t and j log forward exchange rates at horizons h(1), . . ., h(j):

$$(5) \qquad y_t = [s_t,\ f_{h(1),t},\ f_{h(2),t},\ \ldots, f_{h(j),t}]'.$$

We suppose, and confirm empirically below, that the spot exchange rate possesses a unit root and evolves according to:

$$(6) \qquad s_t = z_t + v_t;$$

where v_t is a zero mean stationary stochastic process and z_t is a random walk:

$$(7) \qquad z_t = \mu + z_{t-1} + e_t$$

Using equation (2), we define the risk premium at horizon h(j):

$$(8) \qquad \varphi_{h(j),t} \equiv f_{h(j),t} - E(s_{t+h(j)}|\Omega_t).$$

Combining (6) and (8) we obtain an expression for the forward exchange rate at horizon h(j):

$$(9) \qquad f_{h(j),t} = h(j)\mu + z_t + E_t v_{t+j} + \varphi_{h(j),t}.$$

If, as is suggested by asset pricing theory, the risk premium $\varphi_{h(j),t}$ is a stationary stochastic process, then the forward exchange rate at horizon h(j) and the spot rate share a common stochastic trend z_t and are cointegrated such that the forward premium at horizon h(j) is a stationary stochastic process:

$$(10) \qquad f_{h(j),t} - s_t = h(j)\mu + E_t(v_{t+j} - v_t) + \varphi_{h(j),t}.$$

It follows that, among the j forward rates and the spot exchange rate contained in y_t, there will exist at least j cointegrating vectors that are defined by the j forward premia $f_{h(1),t} - s_t$, $f_{h(2),t} - s_t$, . . ., $f_{h(j),t} - s_t$, so long as the departures from interest parity at all horizons are stationary stochastic processes. However, since (6) and (9) imply that all j + 1 variables in y_t share a common stochastic trend z_t, we know from the results of Stock and Watson (1988) that there will exist *exactly* j independent cointegrating vectors among the j + 1 variables in y_t. Thus, this theoretical framework has the following empirical implications.

First, a vector comprised of the spot exchange rate and j forward exchange rates should be well represented by a vector error-correction model. This follows from (6), (9), (10) and the Granger Representation Theorem. Second, there should exist exactly 1 common trend and thus exactly j cointegrating vectors in such a system, an implications that follows from (6) and (9) and the Stock-Watson Common Trends Representation Theorem. Third, a basis for this space of j cointegrating vectors should be defined by the j forward premia in this system $f_{h(1),t} - s_t$, $f_{h(2),t} - s_t$, . . ., $f_{h(j),t} - s_t$. This follows from (10). We also note that the results obtained in Phillips (1990) imply that it must be possible to select a triangular representation of the cointegration space of this system such that each of j variables in the system is cointegrated with the 1 remaining "right-hand-side" variable not included among these j variables.

If exchange changes Δs_{t+1} are not Granger caused by any other available information - so that $E(\Delta s_{t+1}|\Omega_t) = E(\Delta s_{t+1}|\Delta s_t, \Delta s_{t-1}, . . .)$ - our theoretical framework implies that the term structure of forward premia should not contain any information that helps to improve a forecast of the spot exchange rate given the history of the spot exchange rate itself. Thus, merely establishing that spot and forward exchange rates are cointegrated does not guarantee that the term structure of forward premia contains useful information about the future path of the spot exchange rate. If exchange changes Δs_{t+1} are Granger caused by available information other than the history of the spot exchange rate, ρur theoretical framework implies that the term structure of forward premia should contain information that helps to improve a forecast of the spot exchange rate given the history of the spot exchange rate itself. This hypothesis - that the spot exchange rate is weakly exogenous with respect to the term structure of forward rates - is testable.

Before moving on to the empirical results, we should comment on an alternative theoretical framework that can be employed to interpret the joint behavior of the spot exchange rate and term structure of forward exchange rates. Our framework imposes the testable restriction that $\varphi_{h(j),t}$, the departure from interest rate parity, is a stationary stochastic risk premium. If instead $\varphi_{(j),t}$ possesses a unit root, we should be able to reject the hypothesis that each of the j forward premia $f_{h(1),t} - s_t$, $f_{h(2),t} - s_t$, . . ., $f_{h(j),t} - s_t$ is stationary. To see this point, one which has been made recently by Evans and Lewis (1992), suppose that:

$$(11) \qquad \varphi_{(j),t} = \varphi(j)x_t + w_{jt};$$

where w_t is a zero mean, stationary stochastic process, and x_t is a random walk. Using (9) we see that:

$$(12) \qquad f_{h(j),t} = h(j)\mu + z_t + \varphi(j)x_t + E_t v_{t+j} + w_{jt}.$$

From (6), we see that $f_{\varphi(j),t} - s_t$ inherits the unit root present in x_t. Thus, if this alternative interpretation of the data is correct, we should be able to reject the hypothesis that each of the j forward premia is stationary. Moreover, unless x_t is proportional to z_t, this alternative interpretation of the data also implies that among the j + 1 variables in the system, there are 2 common trends and thus j - 1 cointegrating vectors. Thus, a finding of j - 1 or fewer cointegrating vectors in a system comprised of j forward exchange rates and the spot exchange rate is evidence in favor of this alternative interpretation.

3. The Data and Empirical Preliminaries

We investigate weekly data on spot and 4, 13, 26, and 52 week forward exchange rates for West Germany and Britain obtained from the Harris Bank data base maintained by Richard Levich. The sample runs from 1977:1 through 1990:26. The choice of starting date reflects the view, first expressed by Hansen and Hodrick (1982) in their classic study of the forecastability of excess returns in the foreign exchange market, that during the early years of floating and until the Rambouillet Agreement in February 1976, market participants may very well have believed that a return to fixed parities was imminent. If this was in fact the case, then departures from interest parity during these years would have reflected not only a risk premium, but also an extra component incorporating the effect of a return to fixed parities on expected payoffs to foreign exchange speculation.

To see this, suppose that in the absence of a return to fixed exchange rates, the equilibrium spot rate is governed by:

$$s_t = z_t. \tag{13}$$

If the probability of a return to fixed rates is constant and equal to $1 - \pi$, interest parity implies:

$$f_{1,t} = \pi z_t + (1 - \pi)E\underline{s}_{t+1}; \tag{14}$$

where $\underline{s}_{t+1}$ is the spot rate next period if floating exchange rates are abandoned. Note that even if $\underline{s}_t = \underline{s}$, the forward premium will not be stationary during a sample in which a return to fixed exchange rates never occurs, since from (13) and (14) we have:

$$f_{1,t} - s_t = (\pi - 1)z_t + (1 - \pi)\underline{s}. \tag{15}$$

If $\underline{s}_{t+1} = x_{t+1}$ with x_{t+1} a unit root process, then during a sample in which a return to fixed exchange rates never occurs, forward and spot exchange rates will not even be cointegrated and at most only j - 1 cointegrating relationships can exist among a set of j forward exchange rates and a spot exchange rate. This is of course just one example of a "peso problem" (Rogoff (1977)). As demonstrated by Evans and Lewis (1992), it will often be the case that a peso problem introduces an extra common trend into a system of spot and forward asset prices.

Table 1 reports the results of Dickey-Fuller tests of the null hypotheses that spot and forward exchange rates in Britain and Germany possess a unit root. Three tests are reported, a Dickey-Fuller t-test $Z(\tau_\tau)$ in which a trend and a constant is included in the regression, a Dickey-Fuller t-test $Z(\tau_\mu)$ in which only a constant is included, and a Dickey-Fuller F-test $Z(\Phi_3)$ of the hypothesis that the change in each spot and forward rate is stationary about a constant drift. We present Phillips and Perron (1986) modified statistics which make a nonparametric correction for serial correlation.

The results in Table 1 confirm the findings, reported in many earlier studies, that spot and forward DM and sterling exchange rates appear to possess a unit root. In no instance can the hypothesis of a unit root in the level of a spot or forward rate be rejected at even the 15 percent level, while in all instances can the hypothesis that the change in a spot or forward exchange rate is nonstationary be rejected at the 1 percent level. Having verified that these spot and forward exchange rates possess a unit root, we now proceed to estimate and investigate vector error correction models of the time path of the spot exchange rate and the term structure of forward exchange rates for Britain and Germany.

4. Empirical Results

Based upon the theoretical framework developed in Section 2 and our finding that the variables under study are integrated of order 1, we investigate a dynamic vector error correction model (Engle and Granger (1987); Johansen (1991)) for the spot exchange rate and the term structure forward exchange rates in Britain and Germany. Letting $y_t = [s_t, f_{4,t}, f_{13,t}, f_{26,t}, f_{52,t}]'$ denote the $j + 1 = 5$ by 1 vector of the system's variables for a particular currency, the vector error correction model can be written:

$$(16) \qquad \Delta y_t = \mu + \Gamma_1 \Delta y_{t-1} + \ . \ . + \Gamma_{k-1} \Delta y_{t-k+1} + \pi y_{t-k} + \zeta_t .$$

If the matrix π is of full rank $r = 5$, the VECM reduces to the usual VAR in the levels of stationary variables. If π is the null matrix so that $r = 0$, the VECM represents a VAR in first-differences. The VECM differs from the usual VAR in that it allows for the existence of long-run "equilibrium" relationships among a system's variables. If the matrix π is of reduced rank $r < 5$, it can be factored into the product of two 5 by r matrices α and β such that:

$$(17) \qquad \pi = \alpha\beta';$$

where β' is the r by 5 matrix of the system's r cointegrating vectors, and α is the 5 by r matrix of r adjustment coefficients for each of the system's n equations.

Each cointegrating relationship defines a *long run equilibrium* to which the system ultimately returns after a shock. The parameters in the α matrix determine the rates at which each of the system's variables adjust in response to lagged deviations from the r cointegrating relationships. Stock and Watson (1988) prove that the long-run behavior of a system of n variables with $r < n$ cointegrating relationships is governed by $n - r$ common stochastic trends. Thus a test for the cointegration rank r is also test for the number common trends.

Table 2 presents the results of two tests developed by Johansen (1991) to investigate the hypothesis that the number of cointegrating vectors in a system of n variables is less than or equal to equal to r. Note that the Stock and Watson results cited above imply that this also test of the hypothesis that the number of stochastic trends in the n variable system is greater than or equal to n - r. According to both the *trace* and the λ-*max* statistic, we cannot reject for either the DM or sterling the hypothesis that $r \leq 4$, but, we can reject at the 5% level the hypothesis that $r \leq 3$. Thus, for both sterling and the DM, these findings are consistent with the predictions of the theoretical framework that, in a system comprised of a spot exchange rate and j forward exchange rates, exactly 1 common trend and 4 cointegrating relations are needed to account for the dynamic behavior of the system.

Another prediction of the theoretical framework is that a basis for the space of cointegrating relationships is defined by the vector of j = 4 forward premia $[f_{4,t} - s_t, f_{13,t} - s_t, f_{26,t} - s_t, f_{52,t} - s_t]'$. A likelihood ratio statistic is employed to test this hypothesis. Conditional on there being 4 cointegrating vectors in the system, the likelihood ratio statistic is distributed as $\chi^2(4)$ under the null. The results of this test are reported in Table 3. As can be seen from the table, for neither sterling nor the DM is it possible to reject the hypothesis that the vector of forward premia defines a basis for the space of j = 4 linearly independent cointegrating relationships implied by the estimated VECMs for Britain and Germany.

We conclude from this evidence that the theoretical framework outlined in Section 2 is well supported by the data. In particular, both DM and sterling systems of the spot exchange rate and the term structure of forward rates are well modeled by VECM. In both systems, exactly 1 common trend and thus 4 cointegrating vector are required to explain the dynamic behavior of spot and forward exchange rates. These 4 cointegrating relations are, as predicted by the analysis, defined by the vector of 4 forward premia for each currency. We now investigate whether or not the term structure of forward premia contains incremental predictive content for the time path of the spot exchange rate.

Tables 4 and 5 present FIML estimates of the 5 equation VECMs for the sterling and DM systems, respectively. Of particular interest are the results for the Δs_t equation reported in the first two columns of the tables. As can be seen in Table 4, the spot dollar-sterling exchange rate is not exogenous with respect to lagged information contained in the term structure of forward premia. Indeed, the lagged 13, 26, and 52 week forward premia contain statistically significant information about the future path of the dollar-sterling spot exchange rate that is not contained in the lagged change in the spot rate. Similarly, Table 5 reports that the spot dollar-DM exchange rate is not exogenous with respect to lagged information contained in the term structure of forward premia. The entire lagged term structure of forward premia contains statistically significant information about the future path of the dollar-DM spot exchange rate that is not contained in the lagged change in the spot dollar-DM rate. These results suggest that, at least for dollar-DM and dollar-sterling spot and forward exchange rates since 1977, the answer to the question that began this paper is "yes".

In order to assess the usefulness of the information in the term structure of forward exchange rates, the following out-of-sample forecasting exercise was conducted. The full VECM for each currency was estimated through 1989:26 and a forecast of the spot exchange rate for 1989:27 through 1990:30 was computed using only information available though the estimation period 1977:1-1989:26.

The model was then re-estimated through 1989:27, a forecast of the spot exchange rate in each week from 1989:28-1990:30 was computed. This process was continued until the model was re-estimated through 1990:29 and a single 1 step-ahead forecast was computed. Figure 1 depicts at each horizon the ratio of the root-mean-squared error of these forecasts to the RMSE obtained from a naive random walk forecast $E_t s_{t+j} = s_t$, the standard against which exchange rate forecasts have been compared since the original work of Meese and Rogoff (1981). As can be seen in the figure, the term structure of forward premia contains information that can substantially improve forecasts of the dollar-DM and dollar sterling spot exchange rates. At a 10 week horizon, the forecast obtained from the estimated VECM for the dollar-DM system has a RMSE that is 14 percent smaller than that derived from the random walk forecast. At a horizon of 25 weeks, the VECM forecast has a 33 percent smaller RMSE than does the random walk forecast. At horizons of 40 weeks and longer, the VECM forecast for the dollar-DM exchange rate has nearly a 50 percent smaller RMSE than the random walk forecast.

At forecast horizons under 20 weeks, the VECM for the dollar-sterling exchange rate is dominated by the random walk forecast. However, at longer horizons, the VECM forecast substantially improves upon the random walk forecast. At a 26 week horizon, the forecast obtained from the estimated VECM for the dollar-sterling system has a RMSE that is 33 percent smaller than that derived from the random walk forecast. This forecasting advantage is maintained at successively longer horizons. At horizons in excess of 47 weeks, the forecast for the dollar-sterling exchange rate obtained from the VECM bests the random walk forecast by more than 50 percent.

References

Campbell, J. and R. Clarida, "The Term Structure of Euromarket Interest Rates: An Empirical Investigation," Journal of Monetary Economics, 19, 1987.

Campbell, J. and R. Shiller, "Cointegration and Tests of Present Value Models," Journal of Political Economy, 95, 1987.

Cumby, R. and M. Obstfeld, "A Note on Exchange Rate Expectations and Nominal Interest Differentials," Journal of Finance, 36 (June), 1980.

--------, "International Interest Rate and Price Level Linkages," in J. Bilson and R. Marston eds., Exchange Rate Theory and Practice, Chicago: University of Chicago Press, 1984.

Dornbusch, R., "Exchange Rate Economics: Where Do We Stand?" Brookings Papers on Economic Activity, 11, 1980.

Engle, R. and C. Granger, "Cointegration and Error Correction," Econometrica, 55, 1987.

Evans, M., and K. Lewis, "Trends in Expected Returns in Currency and Bond Markets," NBER Working Paper No. 4116, 1992

Fama, E., "Forward and Spot Exchange Rates," Journal of Monetary Economics, 14, 1984.

Fisher, I., The Theory of Interest, New York: Macmillan, 1930.

Frenkel, J., "Flexible Exchange Rates, Prices, and the Role of News," Journal of Political Economy, 89, 1981.

Hall, A., H. Anderson, and C. Granger, "A Cointegration Analysis of Treasury Bill Yields," Review of Economics and Statistics, 44, 1992.

Hansen, L., and R. Hodrick, "Forward Exchange Rates as Optimal Predictors of Future Spot Exchange Rates," Journal of Political Economy, 88, 1980.

Hansen, L. and R. Hodrick, "Risk Averse Speculation in the Forward Foreign Exchange Market," in J. Frenkel, ed., Exchange Rates and International Macroeconomics, Chicago: University of Chicago Press, 1982.

Johansen, S., "Estimation and Hypothesis Testing of Cointegration Vectors," Econometrica, 59, 1991.

Meese, R. and K. Singleton, "A Note on Unit Roots and the Empirical Modeling of Exchange Rates," Journal of Finance, 37, 1982.

Meese, R., "Empirical Assessment of Foreign Currency Risk Premiums," in Financial Risk: Theory, Evidence, and Implications, Federal Reserve Bank of St. Louis, 1988.

Mussa, M., "Empirical Regularities in the Behavior of Exchange Rates," Carnegie-Rochester Conference on Public Policy, 11, 1979.

Osterwald-Lenum, M., "Fractiles of the Asymptotic Distribution of the Cointegration Rank Test Statistic," Institute of Economics, Copenhagen, 1990

Phillips. P., "Optimal Inference in Cointegrated Systems," Cowles Foundation Discussion Paper No. 866, 1990.

Phillips, P. and P. Perron, "Testing for Unit Roots in Time Series Regression," Cowles Foundation Discussion Paper, 1986.

Rogoff, K., Essays on Expectations and Exchange Rate Volatility, unpublished Ph. D. Dissertation, MIT, 1980.

Stock, J. and M. Watson, "Testing for Common Trends," Journal of the American Statistical Association, 83, 1988.

Table 1

Unit Root Tests

	$Z(\tau_\mu)$	$Z(\tau_\tau)$	$Z(\Phi_3)$
<u>Dollar-Sterling</u>			
s_t	-1.27	-1.37	0.95
Δs_t	-26.32*	-26.31*	346.19*
$f_{4,t}$	-1.27	-1.39	0.98
$\Delta f_{4,t}$	-26.32*	-26.30*	346.13*
$f_{13,t}$	-1.28	-1.45	1.06
$\Delta f_{13,t}$	-26.39*	-26.38*	347.97*
$f_{26,t}$	-1.31	1.55	1.20
$\Delta f_{26,t}$	-26.35*	-26.34*	347.05*
$f_{52,t}$	-1.37	-1.74	1.55
$\Delta f_{52,t}$	-26.78*	-26.77*	358.50*
<u>Dollar-mark</u>			
s_t	-0.88	-1.01	0.72
Δs_t	-25.72*	-25.72*	330.68*
$f_{4,t}$	-0.89	-1.01	0.71
$\Delta f_{4,t}$	-25.71*	-25.70*	330.36*
$f_{13,t}$	-0.91	-1.02	0.69
$\Delta f_{13,t}$	-25.86*	-25.86*	334.31*
$f_{26,t}$	-0.96	-1.06	0.69
$\Delta f_{26,t}$	-25.82*	-25.81*	333.22*
$f_{52,t}$	-1.08	-1.14	0.73
$\Delta f_{52,t}$	-26.40*	-26.39*	348.16*

The sample is 1977:1 - 1990:26. The Phillips-Perron Statistics were constructed using a lag truncation parameter of 13 and a Newey-West (1987) lag window. * indicates significance at the 1% level; otherwise, not significant at the 15% level.

Table 2

Tests of Cointegrating Rank of $y_t = [s_t,\ f_{4,t},\ f_{13,t},\ f_{26,t},\ f_{52,t}]'$

		λ-max Statistic	5% Critical Value	trace Statistic	5% Critical Value
Dollar-Sterling					
	$Ho:\ r \leq 4$	*3.729*	*3.762*	*3.729*	*3.762*
	$Ho:\ r \leq 3$	*16.129*	*14.069*	*19.858*	*15.410*
Dollar-Mark					
	$Ho:\ r \leq 4$	*0.196*	*3.762*	*0.196*	*3.762*
	$Ho:\ r \leq 3$	*16.245*	*14.069*	*16.441*	*15.410*

Note: Critical values are from Osterwald-Lenum (1990) Table 2. These values are correct if $\mu > 0$. If, in truth $\mu = 0$, the appropriate critical values are those reported in Osterwald-Lenum Table 3. Using these more conservative values and the λ-max statistic, we still reject at the 5% level for both currencies $r \leq 3$ in favor of $r = 4$. For the trace statistic, we reject at the 5% level for sterling and at the 10% level for the DM the hypothesis $r \leq 3$ in favor of $r = 4$.

Table 3

Tests of the Null Hypothesis that Four Linearly Independent Forward Premiums Comprise a Basis for the Cointegration Space

	$\chi^2(4)$	Marginal Significance Level
Dollar-Sterling	*2.88*	*58%*
Dollar-Mark	*5.32*	*26%*

The test is conditional on there being four linearly independent co-integrating vectors.

Table 4

FIML Error Correction Model for the Five-Variable System: Dollar-Sterling

Explanatory Variable	Model for Δs_t		Model for $\Delta f_{4,t}$		Model for $\Delta f_{13,t}$		Model for $\Delta f_{26,t}$		Model for $\Delta f_{52,t}$	
	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
Δs_{t-1}	-1.701	1.026	-1.749	1.023	-2.044	1.023	-2.374	1.034	-3.295	1.074
$\Delta f_{4,t-1}$	1.847	1.030	1.917	1.027	2.432	1.029	2.827	1.042	3.472	1.078
$\Delta f_{13,t-1}$	-	-	-0.058	0.031	-0.407	0.061	-0.467	0.069	-	-
$\Delta f_{26,t-1}$	-0.153	0.041	-0.130	0.032	-	-	-	-	-	-
$\Delta f_{52,t-1}$	-0.011	0.007	-	-	-	-	-	-	-0.189	0.030
$(s-f_4)_{t-1}$	-	-	-0.143	0.045	-0.643	0.115	-0.807	0.193	-0.723	0.324
$(s-f_{13})_{t-1}$	0.203	0.071	-0.921	0.436	-0.981	0.306	0.197	0.072	0.300	0.076
$(s-f_{26})_{t-1}$	-0.178	0.059	0.178	0.059	-0.184	0.093	-0.123	0.060	-0.248	0.062
$(s-f_{52})_{t-1}$	0.521	0.175	0.519	0.174	0.485	0.174	0.339	0.176	-0.734	-0.182
Constant	-0.002	0.001	-0.002	0.001	0.002	0.001	0.003	0.001	0.004	0.001
	Q(77) = 74.22 (0.57)		Q(77) = 73.16 (0.60)		Q(77) = 75.91 (0.51)		Q(77) = 72.22 (0.63)		Q(77) = 80.19 (0.38)	

H(325) = 343.33
(0.24)

REST(18) = 9.61
(0.94)

Notes: A "-" indicates that the coefficient was found to be insignificant in the reduction process; the Q-Statistics are Ljung-Box Statistics computed at 77 autocorrelations of the residual series; H is Hosking's (1980) multivariate portmanteau statistic computed at 13 autocorrelations; REST is a likelihood ratio statistic for the exclusion restrictions. All statistics are distributed as central chi-square under the null hypothesis, with the degrees of freedom indicated; figures in parentheses are marginal significance levels.

Table 5

FIML Error Correction Model for Five-Variable System: Dollar-DM

Explanatory Variable	Model for Δs_t		Model for $\Delta f_{4,t}$		Model for $\Delta f_{13,t}$		Model for $\Delta f_{26,t}$		Model for $\Delta f_{52,t}$	
	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
Δs_{t-1}	-1.789	0.710	-1.734	0.705	-1.876	0.700	-2.284	0.702	-2.344	0.739
$\Delta f_{4,t}$	3.267	0.746	3.075	0.736	2.104	0.722	2.589	0.415	2.604	0.740
$\Delta f_{13,t}$	-1.799	0.229	-1.6433	0.212	-1.596	0.182	-0.985	0.143	-0.252	0.033
$\Delta f_{26,t}$	0.328	0.067	0.310	0.057	0.277	0.040	-	-	-	-
$\Delta f_{52,t}$	-	-	-	-	-	-	-	-	-	-
$(s-f_4)_{t-1}$	-0.981	0.213	-0.939	-0.242	0.601	0.171	0.597	0.171	0.596	0.173
$(s-f_{13})_{t-1}$	-0.743	0.202	-0.361	-0.128	-0.261	0.088	-0.283	0.087	-0.167	0.089
$(s-f_{26})_{t-1}$	-0.376	0.143	-	-	-0.224	0.024	-	-	-0.18	0.010
$(s-f_{52})_{t-1}$	-0.061	0.018	0.408	0.135	0.412	0.135	0.366	0.135	-0.712	0.130
Constant	0.006	0.002	0.005	0.002	0.005	0.002	0.006	0.002	0.008	0.002
	Q(77) = 72.21 (0.63)		Q(77) = 73.16 (0.60)		Q(77) = 74.91 (0.55)		Q(77) = 78.65 (0.43)		Q(77) = 81.22 (0.35)	

H(325) = 337.36
(0.31)

REST(18) = 5.8
(0.99)

Notes: A (-) indicates that the coefficient was found to be insignificant in the reduction process; the Q-Statistics are Ljung-Box Statistics computed at 77 autocorrelations of the residual series; H is Hosking's (1980) multivariate portmanteau statistic computed at 13 autocorrelations; REST is a likelihood ratio statistic for the exclusion restrictions. All statistics are distributed as central chi-square under the null hypothesis, with the degrees of freedom indicated; figures in parentheses are marginal significance levels.

Figure 1

Ratio of VECM RMSE to Random Walk RMSE

(Rolling Estimation, Out of Sample Forecast)