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Endowments, Preferences, Abatement and Voting:
Microfoundations of Environmental Kuznets Curves

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Endowments, Preferences, Abatement and Voting:

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Abstract

Will economic growth inevitably degrade the environment, throughout development? This paper presents a simple household-choice framework that emphasizes the tradeoff between pollution-causing consumption and pollution-reducing abatement expenditures. The framework yields a simple explanation for Environmental Kuznets Curves (EKCs, i.e. non-monotonic, upward-turning paths of environment while development continues) and facilitates analysis of household voting decisions that lead to public regulation of environmental externalities. Our sufficient conditions, more general than the literature, make clear that an asymmetric endowment (i.e. positive environmental quality but zero consumption at zero income) is sufficient for an EKC given standard preferences and a wide range of abatement technologies. The key is that the MRS leads the household to prefer not to abate (or to vote for whatever 'abatement' implies) at low income levels. Without the endowment, abatement technologies alone are insufficient for an EKC path. For a multi-agent setting with externalities, an analogous result is derived in which the chosen tax rate rises with income and environmental quality at first falls and later rises.

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1. Introduction

Whether environmental quality will inevitably fall during economic development has spurred immense empirical, theoretical and policy debate. Early empirical analyses suggested the existence of so-called Environmental Kuznets Curves, or EKCs, i.e. non-monotonic, U-shaped relationships at an aggregate level between per-capita income and environmental quality. Thus, environmental quality would rise during later stages of development.[1] To caricature the early literature, such relationships seemed empirically clear but theoretically puzzling. The puzzle only helped to fuel debate about the appropriate interpretation of any such relationship: is it evidence that regulation is unnecessary, or instead of increased regulation as income rises?

This puzzle spawned EKC models, in two groups: 1) external effects among households are not internalized; and 2) planner or single-household models. The latter are relevant when degradation features a significant private component[2], and as a foundation for analysis of household voting when externalities are not internalized. For both reasons, this paper is based on a single-household model, in which any household (even the poorest) has an initial endowment of environmental quality.

In addition to clarity and transparency, this model permits two advances relative to the literature. First, our EKC sufficient conditions permit easy evaluation of whether a given combination of preferences and abatement technologies gives rise to an EKC. While existing literature has tended to focus either on preferences or on abatement technologies[3], we allow the effect of a given technology to depend upon the preferences, and vice versa. Second, with our model as a base we can join the two model categories by modeling voting for regulation by a set of households, i.e. an explicit mechanism by which externalities can be internalized as incomes rise.

Why join the categories for modeling EKCs? Consider the first, often relevant category: regulations do not exist, households do not fully internalize their effects, and it is neither surprising nor new that environment can fall with rising income.[4] What is hard to explain about an EKC in this setting is how environmental quality rises with later increases in income. Any actual mechanism must involve households caring enough about the environment to coordinate and aggregate preferences, such as through household voting. Thus, insights from household modeling with at least partial internalization, i.e. the second model category, should be brought to a multi-agent setting with externalities, i.e. the first category. We provide such a model.

Why is this a contribution, given the many planning models already in the second category? Neoclassical growth models that consider pollution and growth have provided one approach when externalities are assumed to be internalized, and

[1] See, for instance, World Bank 1992, Selden and Song 1994, Shafik 1994, Holtz-Eakin and Selden 1995, Grossman and Krueger 1995, and more recently, special issues of both Environment and Development Economics, in November 1997, and Ecological Economics, in May 1998.

[2] For example, Chaudhuri and Pfaff 1998 and 2002 consider empirically how household fuel choice in Pakistan changes with income, in light of fuels' effects on indoor air quality, a private good. While stove emissions have external effects as well, private environmental quality is significantly degraded. More generally, other forms of degradation of the environment also feature private components, and there exists significant private provision of environmental abatement in the absence of regulations.

[3] For instance, Stokey 1998 emphasizes the role of elasticity of preferences, while Andreoni and Levinson 2001 focuses upon a role for a very particular type of increasing returns to abatement.

[4] Environmental economics textbooks feature environmentally damaging external emissions that rise with the scale of production of the polluting good. If regulations or Coasian bargaining do not lead to internalization, these emissions will surely lower environmental quality as incomes rise.

can provide results similar to ours.[5] But they will not easily explain regulatory choice given heterogeneous voters.[6] The dynamic representative agent framework lacks a realistic political economic mechanism through which degradation might in reality be reversed. In contrast, household models can yield insights within a setting of internalization and, as they can be applied in a multi-agent setting in the presence of externalities, can permit explicit modeling of how environmental preferences might be aggregated through voting in order to produce regulation.

For each household, our model focuses upon the asymmetric endowment of consumption and environment, i.e. positive environmental quality but zero consumption at zero income. This is so natural an assumption as to appear obvious. We show that it is nonetheless crucial. Given standard preferences for consumption and environment, such that if households could purchase them separately and independently both would be normal goods, an endowment is sufficient for an EKC given a wide range of abatement technologies encompassing fixed costs and decreasing returns.[7] Thus, the model can account for the initial decrease in environmental quality as income rises and for increasing environmental quality as income continues to grow.

The key intuition is that, given an endowment of environmental quality that is degraded by consumption, and given convex preferences for consumption and environment, at low incomes the marginal rate of substitution implies that the household prefers not to spend to abate the effects of consumption. This results in a corner solution where no resources are expended on abatement, but consumption will rise with income. This causes environmental quality to fall with income.

However, as income continues to rise, and the environment is degraded, the marginal rate of substitution between environmental quality and consumption increases in favor of the environment, until it is desirable to abate. This moves the household to an interior solution, in which it both consumes and abates, and for a wide range of abatement technologies environmental quality will increase with income because both goods are normal. We provide a condition which determines whether as income gets high enough environmental quality will rise with income. This compares the change in the marginal rate of substitution as incomes rise to the change in the marginal rate of transformation implied by the abatement technology.[8] As noted, such a general condition simplifies consideration of the implications for EKCs of combinations of whatever preferences and abatement technologies are of interest.

We also demonstrate that this household analytical approach permits explicit consideration of voting for enviromental policies when externalities exist. For a

[5]See Plourde 1972, Keeler et al. 1972, D'Arge and Kogiku 1973, Forster 1973, Gruver 1976, Stephens 1976, Asako 1980, Becker 1982, Tahvonen and Kuuluvainen 1993, John and Pecchenino 1994, Selden and Song 1995, Jones and Manuelli 1995, Stokey 1998, and Chimeli 2001.

[6]Note that while Jones and Manuelli 1995 features a representative agent at each point in time, the paper considers the problem of intertemporal collective decisionmaking.

[7]Such an endowment assumption (which we argue is hard to refute) is implicit in some existing papers (e.g., John and Pecchenino 1994). However, its truly central role has not been highlighted. Further, such endowments can be thought of more broadly if we consider not only preferences and the MRS but also technologies and the MRT (again, our framework easily permits their comparison). For instance, Chimeli 2001 suggests that an off-equilibrium-path 'endowment' of capital may exist for economies in transition and, given that, traces the optimal path of the MRT as income rises.

[8]This part of the paper significantly generalizes our related work in Chaudhuri and Pfaff (1997 a and b) and Pfaff, Chaudhuri and Nye 2001. The theoretical analyses they present develop in detail the case of input substitution as an abatement technology. That case, in turn, corresponds to the empirical work on "household EKCs" for indoor air quality in Chaudhuri and Pfaff 1998 and 2002.

distribution of households who do not fully internalize their effects, we give the conditions for the chosen environmental tax rate to be rising with income, after a low-income range in which no abatement spending is chosen. Thus we provide an analogous result, which is based on voting, to the EKC without externalities.

Our final point is motivated by this issue: what is hard to explain for EKCs, the rise or the fall of environment, differs by model category. As noted, with externalities a fall in environmental quality is the default while a rise is harder to explain. We show that without externalities, though, rising environmental quality is the default. As long as environment is a normal good, for a wide range of abatement technologies the Engel curves for environment ought to be positively sloped at *all* incomes.

However, why environment might fall with income is unclear. The need for an *explicit* explanation of this otherwise puzzling fall (in the case of no externalities or, more relevant, significant regulation leading to internalization) has been overlooked, as suggested by the main theorem in Andreoni and Levinson 2001 [paraphrases inserted in brackets]: [assuming consumption and environment are normal goods, and a particular increasing returns abatement technology] "for any combination of utility and abatement technology that yields positive pollution [lowers environmental quality] for some level of income, optimal pollution will eventually decline back to zero [environmental quality will eventually rise] for some sufficiently large income." As why environmental quality would fall is not specified, we show that lacking our asymmetric endowment, for standard convex preferences neither the abatement technologies we consider nor the increasing returns technology in Andreoni and Levinson accounts for a range of income in which environment is degraded. Hence, these abatement technologies alone cannot generate EKCs, and an additional explicit rationale for falling quality, such as our asymmetric endowment, is needed.

Below, Section 2 briefly presents our simple model and, retaining its generality, works through to our sufficient conditions for an EKC. Section 3 adds intuition through specific cases that show the robustness of our endowment-based result to a range of abatement technologies. It also explores cases of abatement technologies without an environmental endowment, finding that even increasing returns is not sufficient for an EKC. Section 4 demonstrates that this static household modeling approach easily accomodates an application to a multi-agent setting with externalities, and provides an example with conditions for stronger environmental policy as incomes rise, and for first decreasing and then increasing environmental quality. Section 5 concludes with a brief discussion and implications for further research.

2. Household Income and Environmental Quality

2.1. Preferences, Abatement Technology and Environmental Quality

A household gets utility from two goods, a marketed consumption good, denoted by c, and environmental quality, denoted q, so that utility can be written as:

$$U = U(c, q) \tag{2.1}$$

where $U_c > 0$, $U_q > 0$, and U is concave in c and q. Households enjoy an initial endowment of environmental quality ($q_0 \geq 0$) that is degraded by pollution, which as a byproduct of consumption rises with c. However, the household can choose to expend resources to "abate" the effects of pollution on the environment, i.e. to make consumption less damaging, for instance by using cleaner but more expensive inputs or by cleaning up pollution already generated. Denoting such expenditures on environmental investment as e, we write environmental quality as:

$$q = q(c, e) \tag{2.2}$$

where $q_c < 0$–environmental quality falls with rising consumption–and $q_e > 0$.

The general household problem, then, is to choose c and e to maximize (2.3) subject to the budget constraint (2.4) and, since a household can choose to expend zero resources on either c or e, also the non-negativity constraints (2.5):

$$U = U(c, q(c, e)) \tag{2.3}$$

$$p_c c + p_e e = y \tag{2.4}$$

$$c \geq 0, e \geq 0 \tag{2.5}$$

where y is household income, and p_c and p_e are, respectively, the prices of c and e.

2.2. Sufficient Conditions for an EKC

Before providing specific results for particularly interesting cases (see Section 3), we derive general conditions for the two parts of an EKC, i.e. environmental quality falling with income at low incomes, and rising with income at higher incomes. For these general results, we start with a few assumptions about preferences:

$$\begin{array}{llll} \text{i) } U_c > 0 & \text{(ii) } U_{cc} < 0 & \text{(iii) } U_q > 0 & \text{(iv) } U_{qq} < 0 \\ \text{(v) } U_{qq}U_{cc} - U_{cq}^2 \geq 0 & \text{(vi) } \lim\limits_{c \to 0} U_c(c, q) = +\infty & & \text{(vii) } \lim\limits_{q \to 0} U_q(c, q) = +\infty \end{array} \tag{2.6}$$

We assume further that preferences are such that the demand for both of these goods, c and q, would be normal if these goods could both be purchased separately and independently:

$$\text{(i) } U_c U_{cq} - U_q U_{cc} > 0 \qquad \text{(ii) } U_q U_{cq} - U_c U_{qq} > 0 \tag{2.7}$$

We also make the following assumptions about the relationships between consumption, environmental degradation and the abatement technology:

$$\begin{array}{llll} \text{(i) } q_e > 0 & \text{(ii) } q_{ee} \leq 0 & \text{(iii) } q_c < 0 & \text{(iv) } q_{cc} \leq 0 \\ \text{(v) } q_{ce} \geq 0 & \text{(vi) } \lim\limits_{e \to 0} q_e(c, e) = m < +\infty & & \text{(vii) } q(0, 0) = q_0 > 0 \end{array} \tag{2.8}$$

To simplify the notation, we set $p_c = p_e = 1$.

Given these conditions, we ask whether an asymmetric endowment ($q_0 > 0$) leads to a low income range in which nothing is spent on abatement but consumption occurs, such that environmental quality falls. Also, we examine whether such a range is followed by one in which consumption *and* abatement occur, and both rise with income such that environmental quality also rises, yielding an EKC. Here we do so for the general model, while in Section 3 we provide specific results (for instance income ranges for environmental quality falling and rising) for some cases of interest.

2.2.1. No Abatement at Low Incomes

From (2.6, vi), we know the non-negativity constraint on c will never bind, and from (2.8, i) we know the budget constraint will always bind. Hence we can write the first-order condition for maximization of (2.3) subject to (2.5) and the budget as:

$$U_c(c, q(c, y-c)) + U_q(c, q(c, y-c))q_c(c, y-c) \geq U_q(c, q(c, y-c))q_e(c, y-c) \quad (2.9)$$

which holds with equality only if $e = y - c > 0$. On the left is the net marginal utility from additional consumption, including the marginal disutility from the loss of environmental quality brought about by additional consumption. The term on the right represents the marginal utility from additional abatement expenditures.

Let $c^*(y)$ and $e^*(y)$ denote the optimal choices of c and e from the maximization problem above. Given the above assumptions regarding preferences and technology, we will show here that there exists $\widehat{y} > 0$ such that for all $y < \widehat{y}$:

$$c^*(y) = y \qquad e^*(y) = 0 \qquad \frac{dq}{dy} = q_c \frac{dc^*}{dy} + q_e \frac{de^*}{dy} = q_c < 0$$

To see this, we can by start by defining:

$$\begin{aligned} g(y) &\equiv U_c(y, q(y,0)) + U_q(y, q(y,0))q_c(y,0) \\ l(y) &\equiv U_q(y, q(y,0))q_e(y,0) \end{aligned} \qquad (2.10)$$

For income y, $g(y)$ is the net marginal gain from devoting all income to consumption, whereas $l(y)$ is the marginal loss from doing so. Differentiation of g and l shows that (2.6), (2.7) and (2.8) imply that $g(y)$ declines but $l(y)$ increases with y. Further, since from (2.6, vi) along with (2.8, vi and vii) we know that:

$$\lim_{y \to 0} g(y) = +\infty \qquad \lim_{y \to 0} l(y) = K < +\infty$$

It follows that there exists $\widehat{y} > 0$ such that:

$$\begin{array}{ll} g(y) > l(y) & \forall y < \widehat{y} \\ g(\widehat{y}) = l(\widehat{y}) & \\ g(y) < l(y) & \forall y > \widehat{y} \end{array} \qquad (2.11)$$

The result follows from (2.11) given (2.9). Note the crucial role here of the assumption that $q_0 > 0$–without the environmental endowment the three ranges in (2.11) may not exist. Given the endowment, when $y < \widehat{y}$ the household will not spend on abatement because the net marginal utility of consumption, taking into account environmental degradation, is greater than the gain from abatement spending. This dictates the corner solution in which environmental quality must fall with income.

2.2.2. Rising Consumption and Abatement at Higher Incomes

Here we will show that under assumptions (2.6), (2.7) and (2.8), for all $y > \widehat{y}$:

$$0 < c^*(y) < y \qquad 0 < \frac{dc^*}{dy} < 1 \qquad 0 < e^*(y) < y \qquad 0 < \frac{de^*}{dy} < 1$$

By totally differentiating (2.9) and rearranging terms, we can see that:

$$\frac{dc^*}{dy} = \frac{\Omega_c}{\Omega_e + \Omega_c}$$

where:

$$\begin{aligned} \Omega_c &= U_q(q_{ee} - q_{ce}) - U_{cq}q_e - U_{qq}q_e(q_c - q_e) < 0 \\ \Omega_e &= U_{cc} + U_{cq}q_c + U_{qc}(q_c - q_e) + U_{qq}q_c(q_c - q_e) + U_q(q_{cc} - q_{ce}) < 0 \end{aligned} \tag{2.12}$$

Hence we can immediately see that:

$$0 < \frac{dc^*}{dy} < 1$$

$$0 < \frac{de^*}{dy} = (1 - \frac{dc^*}{dy}) = \frac{\Omega_e}{\Omega_e + \Omega_c} < 1$$

These expressions indicate that, with rising income, eventually the household will want to spend on both consumption and environment. Further, we can see that the expenditures on each will rise with income. The question, then, is whether the simultaneous increases in pollution-causing consumption and pollution-reducing abatement expenditures will permit environmental quality to rise with income.

2.2.3. Falling Then Rising Environmental Quality

That the abatement expenditures will rise with income once $y > \widehat{y}$ does not by itself guarantee that environmental quality will rise with income beyond the threshold $\widehat{y}$. Because consumption is rising as well, the increase in e has to be large enough to offset the additional pollution caused by increased consumption. Under what combinations of preferences and abatement technologies is that likely to occur?

Note that the assumptions we have made thus far are *not* sufficient to ensure environmental quality rising with income. To see that this is the case, by way of contrast consider first the familiar case from basic consumer theory, in which the marginal rate of transformation (MRT) the consumer faces—i.e., the rate at which the consumer is able to exchange one marketed commodity for another—is fixed by exogenously given market prices and hence is independent of the consumer's income. In that case simple restrictions on preferences, e.g. of the sort we have imposed, do suffice to guarantee that the demand for these *marketed* commodities is normal.

We require further assumptions because environment is a *non-marketed* commodity. This implies that the relative shadow price of environmental quality, i.e. the MRT along the c-q consumption possibility frontier, will generally (though not always) depend on the household's income. Whether non-marketed environmental quality falls or rises with income will, therefore, depend not just on preferences, i.e. how the marginal rate of substitution (MRS) of c for q changes, but instead on how both the MRS and the MRT change as we move between optima as income rises. The assumptions we have made so far pin down the changes in the MRS both along an indifference curve and in moving between indifference curves within a shift to a new optimum. They also pin down the change in MRT along a given consumption possibility frontier. The proposition below determines what we need to assume in addition, in order for EKCs to arise, about the change in the MRT in moving from one consumption possibility frontier to another within shifts to new optima, conditional on and specifically relative to the change in the MRS.

Proposition 2.1. *Let:*

$$
\begin{aligned}
MRS(c,q) &\equiv \frac{U_c}{U_q} \\
MRT(c,q) &\equiv q_e - q_c
\end{aligned}
$$

If assumptions (2.6), (2.7) and (2.8) hold and there exists $\widetilde{y}$ *such that for all* $y > \widetilde{y}$ *:*

$$
\left.\frac{\partial MRS(c^*(y),q^*(y))}{\partial c}\right|_{q=q^*} - \left.\frac{\partial MRT(c^*(y),q^*(y))}{\partial c}\right|_{q=q^*} < 0 \tag{2.13}
$$

then:

$$
\begin{aligned}
\frac{dq^*}{dy} &< 0 \quad \text{for all } y < \widehat{y} \text{ where } \widehat{y} \text{ is implicitly defined by } g(\widehat{y}) = l(\widehat{y}) \\
\frac{dq^*}{dy} &> 0 \quad \text{for all } y > \max\{\widehat{y}, \widetilde{y}\}
\end{aligned}
$$

Proof: That environment decreases with rising income until $\widehat{y}$, given $q_0 > 0$, follows from Section 2.2.1. To see that adding (2.13) is sufficient for there to exist an income level beyond which environmental quality increases with income, note that for income above $\widehat{y}$, when the non-negativity constraint on e is no longer binding:

$$
\begin{aligned}
\frac{dq(c^*(y),e^*(y))}{dy} &= q_c\frac{dc^*}{dy} + q_e\frac{de^*}{dy} \\
&= q_c\frac{\Omega_c}{\Omega_e+\Omega_c} + q_e\frac{\Omega_e}{\Omega_e+\Omega_c} = \frac{q_c\Omega_c + q_e\Omega_e}{\Omega_e+\Omega_c}
\end{aligned}
$$

where Ω_e and Ω_c are defined as in (2.12). Since $(\Omega_e+\Omega_c) < 0$, we have that $\frac{dq^*}{dy} > 0$ if and only if $(q_e\Omega_e + q_c\Omega_c) < 0$. Substituting (2.9) and (2.12) above and rearranging:

$$
q_e\Omega_e + q_c\Omega_c = U_q[(\frac{U_{cc}}{U_q} - \frac{U_{qc}U_c}{U_q^2}) + \frac{1}{q_e}(q_e(q_{cc}-q_{ce}) + q_c(q_{ee}-q_{ce}))]
$$

But we also know that:

$$
\begin{aligned}
\left.\frac{\partial MRS(c^*(y),q^*(y))}{\partial c}\right|_{q=q^*} &= \frac{U_{cc}}{U_q} - \frac{U_{qc}U_c}{U_q^2} \\
\left.\frac{\partial MRT(c^*(y),q^*(y))}{\partial c}\right|_{q=q^*} &= \frac{1}{q_e}(q_e(q_{ce}-q_{cc}) + q_c(q_{ce}-q_{ee}))
\end{aligned}
$$

and thus we can see directly that:

$$
q_e\Omega_e + q_c\Omega_c = U_q[\left.\frac{\partial MRS(c^*(y),q^*(y))}{\partial c}\right|_{q=q^*} - \left.\frac{\partial MRT(c^*(y),q^*(y))}{\partial c}\right|_{q=q^*}]
$$

Clearly then, $\frac{dq^*}{dy} > 0$ if and only if $[\frac{\partial MRS(c^*(y),q^*(y))}{\partial c} - \frac{\partial MRT(c^*(y),q^*(y))}{\partial c}] < 0$, as the sufficient condition in the proposition suggests. If $\widetilde{y} < \widehat{y}$, then $\frac{dq^*}{dy} > 0$ from the moment that abatement expenditures are positive. If on the other hand $\widetilde{y} > \widehat{y}$, then even after households start to spend on abatement, environmental quality may fall with rising income, although only up to the threshold level of income $\widetilde{y}$. Beyond that income level, environmental quality will improve with increases in income.

This result completes the intuition for sufficiency of an asymmetric endowment for an EKC (since Section 2.2.1 showed falling environment at low incomes, i.e. the first part of an EKC). In light of (2.13), see that an endowment yields a falling MRS as the scale of income and consumption rises, because with convex preferences the marginal gain from consumption falls as consumption rises. Thus, even were the MRT not to change with scale, given an endowment the conditions would exist for rising $q^*(y)$ once y is high enough, i.e. for the second part of the EKC. In fact, for a wide set of technologies the endowment will be sufficient for an EKC.

This result also permits the direct evaluation of whether a particular combination of preferences and abatement technologies can be expected to generate an EKC. Constant returns (unchanging MRT) leaves things to the preferences, such that an asymmetric endowment yields an EKC. Increasing returns to abatement spending (e.g. $q_{ee} > 0$, $q_{cc} = q_{ce} = 0$) should help the second part of the EKC, i.e. rising environment, because raising q through abatement is easier as scale rises with income. In light of (2.13), note that this makes the change in MRT as scale rises positive. Thus, as per Proposition 2.1 even if the MRS were unchanged with scale eventually environmental quality would rise with income, i.e. increasing returns abatement technologies do help generate the second part of the EKC, rising environment.

As noted earlier, though, without an asymmetric endowment we lack an explicit story for why environment falls in the low income range, i.e. for the first part of an EKC. Thus, despite its role in raising environmental quality at higher incomes, increasing returns shifting the abatement MRT is not sufficient for an EKC.

3. Robustness and Sufficiency

We now work through several illustrative examples in some detail, for two purposes: first, to demonstrate that an environmental endowment is sufficient for an EKC under a broad range of abatement technologies; and second, to show that even increasing returns to abatement is not sufficient, as without externalities an additional explicit story is necessary for why environmental quality falls with income.

3.1. The Sufficiency of Asymmetric Endowments

3.1.1. Constant Returns to Abatement

For a first simple but in many ways quite representative general example, we assume Cobb-Douglas preferences for consumption and environmental quality:

$$U(c, q) = c^{\alpha} q^{\beta} \qquad \alpha + \beta = 1 \tag{3.1}$$

We assume an asymmetric endowment $q_0 > 0$, i.e. positive environmental quality but zero consumption at zero income. This is a natural assumption (again, below we argue that it is hard to see when it is *not* reasonable, for people who are able to stay alive and thus face this optimization problem). For simplicity and transparency, we specify in (3.2) a class of simple constant-returns abatement functions[9]:

$$q = q_0 - \gamma c + \delta e \qquad \gamma, \delta > 0 \tag{3.2}$$

Given this expression for q, the household chooses c and e to maximize (3.1) subject to the budget constraint (2.4) and the non-negativity constraints (2.5). This gives rise to a non-linear programming problem, the first-order Kuhn Tucker conditions of which lead one to consider the following two cases: 1) $c > 0, e = 0$; and 2) $c > 0, e > 0$.[10] The first case corresponds to a corner solution in which the household chooses not to abate, but does consume, and thus environment falls with income.

The $e^* = 0$ result is optimal for poorer households, i.e. those satisfying:

$$y \leq \frac{q_0 \alpha p_c p_e}{\gamma p_e + \beta \delta p_c} \tag{3.3}$$

For a household in this income range, the optimal level of consumption will rise with income (so that pollution will rise with income as well). Since nothing is spent on abatement, the optimal level of environmental quality must fall with income:

$$c^* = \frac{y}{p_c}; e^* = 0; q^* = q_0 - \frac{\gamma y}{p_c}; \frac{dq^*}{dy} = -\frac{\gamma}{p_c} < 0 \tag{3.4}$$

While abatement is feasible, at low incomes it is not desirable. The household devotes all of its resources to consumption (expenditure on consumption, $p_c c^*$, equals y). If there were no environmental endowment ($q_0 = 0$), though, there would be no income range in which abatement is zero. It is the asymmetric endowment that leads to the boundary solution in which environmental quality falls with income.

The case where $e^* > 0$ is optimal for richer households, those satisfying:

$$y > \frac{q_0 \alpha p_c p_e}{\gamma p_e + \beta \delta p_c} \tag{3.5}$$

Under the linear technology in (3.2), the MRT faced by the household does not vary with income. From Proposition 2.1, we know then that all that matters is whether the MRS falls with increases in income (and consumption). But with Cobb-Douglas preferences, which ensure that q is a normal good, this is guaranteed. Hence, even though consumption (and pollution) will rise with income, the household spends enough on abatement to ensure that environmental quality also increases:

$$c^* = \frac{y\alpha\delta + q_0 \alpha p_e}{\delta p_c + \gamma p_e}; e^* = \frac{y(\gamma p_e + \beta \delta p_c) - q_0 \alpha p_c p_e}{p_e(\gamma p_e + \delta p_c)}; \frac{dq^*}{dy} = \frac{\delta \beta}{p_e} > 0 \tag{3.6}$$

The derivative of optimal environmental quality with respect to income in these results conveys that the weight on the environment within the preferences matters. These results (see 3.5) also confirm that the asymmetric endowment is crucial. Were $q_0 = 0$ (i.e., the standard, zero-endowments case in which normal goods are defined), the solution in (3.6) would always obtain. Thus, as normal goods, both consumption and environmental quality would increase with income at *all* income levels.

[9] Note that the input-substitution technology in Pfaff, Chaudhuri and Nye 2001 is constant returns. As that paper argues, there are many cases in which input substitution is the relevant abatement technology. Note also, including as motivation for Section 3.1.3, that the existence of a cleanest input may imply that at highest incomes environmental quality will again fall with income. At the highest incomes only the cleanest input is used, and its use rises with income.

[10] In all, the Kuhn-Tucker conditions allow for four cases: 1) $c > 0, e = 0$; 2) $c > 0, e > 0$; 3) $c = e = 0$; and 4) $c = 0, e > 0$. Given our assumption on preferences (2.7, vi), as long as $y > 0$ the non-negativity constraint on c will never be binding, ruling out 3) and 4).

3.1.2. Decreasing Returns

Since increasing returns to abatement spending was seen above (see discussion of Proposition 2.1) to support the second part of an EKC (environmental quality rising with income at higher incomes), and since constant returns to abatement leaves things to the preferences, it is worth considering whether decreasing returns to abatement prevents an environmental endowment from leading to an EKC. With the preferences in (3.1), we know from Section 2.2.1 that the endowment will be sufficient for the fall in environmental quality within the low income range. Thus, the question is whether, with an endowment but also decreasing returns to abatement, the quality of the environment can still rise with income at higher incomes.

Demonstrating the utility of Proposition 2.1, we can simply check whether a particular combination of preferences and an abatement technology satisfy the conditions provided there for environmental quality rising with income, once income is above a given level. Consider, then, (3.1)'s preferences and (3.7)'s technology:

$$q = q_0 + (1 - \exp[\gamma c]) + (1 - \exp[-\delta e]) \qquad \gamma > \delta > 0 \tag{3.7}$$

For these specifics, all of (2.6), (2.7) and (2.8) hold. In terms of (2.13), we have:

$$\begin{aligned} MRS(c, q) &\equiv \frac{\alpha}{(1-\alpha)} \frac{q}{c} \\ MRT(c, q) &\equiv \delta \exp[-\delta e] + \gamma \exp[\gamma c] \end{aligned}$$

$$\frac{\partial MRS}{\partial c} - \frac{\partial MRT}{\partial c} = \frac{-\alpha}{(1-\alpha)} \frac{q}{c^2} - \gamma \exp[\gamma c](\gamma - \delta) < 0 \tag{3.8}$$

Thus, given (3.1), the asymmetric environmental endowment remains sufficient for an EKC even for the decreasing returns to abatement technologies in (3.7).

3.1.3. Extreme Decreasing Returns to Abatement

Consider again the constant-returns abatement function (3.2), except now add an extreme diminishing returns component, such that actual abatement, denoted a, rises with abatement expenditures e only up to $e_{\max}$. After that point, actual abatement a equals $e_{\max}$ no matter how high the abatement expenditures e:

$$a(e) = \begin{cases} e, & \text{if } e \leq e_{\max} \\ e_{\max}, & \text{if } e > e_{\max} \end{cases} \tag{3.9}$$

Going from a level of abatement spending that is below $e_{\max}$ to one above it, the marginal abatement per unit of spending decreases discretely from 1 to 0.

The household then maximizes (3.1) subject to (2.4) and (2.5), given the technology $q = q_0 - c + a(e)$, where $a(e)$ is as defined in (3.9) (and the γ and δ from (3.2) are dropped to avoid unnecessary clutter). The optimization problem yields three active cases: 1) $c > 0, e = 0$; 2) $c > 0, 0 < e < e_{\max}$; and 3) $c > 0, e = e_{\max}$.

The first two cases are essentially identical to the two cases in Section 3.1.1, with households in the low income range (as in (3.3)) spending nothing on environmental investment and lowering the quality of the environment as income rises. Those with higher incomes (as in (3.5), though in this case also bounded above by the expression in (3.10)) spend on both consumption and environmental investment, and improve the quality of the environment as income rises. Thus, the basic EKC result from Section 3.1.1 is seen to hold with this decreasing returns abatement technology.

The new feature is case 3), which is optimal for the richest households:

$$y \geq \frac{e_{\max} p_e (p_e + p_c) + q_0 \alpha p_c p_e}{p_e + \beta p_c} \tag{3.10}$$

Although environmental quality is still normal, households cease investing in the environment through abatement spending because the marginal abatement from environmental investment is zero after e exceeds $e_{\max}$. However, consumption continues to increase with income, such that pollution increases and environmental quality must fall with income, as seen in the following optimal values for this income range:

$$c^* = \frac{y - p_e e_{\max}}{p_c}; e^* = e_{\max}; q^* = q_0 - \frac{y}{p_c} + \frac{(p_c + p_e)}{p_c} e_{\max}; \frac{dq^*}{dy} = -\frac{1}{p_c} < 0 \tag{3.11}$$

Thus with decreasing returns to abatement, both poor and rich households can arrive at corner solutions where environmental quality falls with income because of a lack of additional abatement effort to offset rising consumption. The relationship between income and environmental quality can then become an "inverted N" or "sideways S", as quality decreases, increases, and then decreases again with income.

This is an interesting result at the least because of related findings in the empirical literature on EKCs, where some fitted aggregate relationships take this shape.[11] Also, such an empirical finding might even be expected, given a finite set of feasible abatement technologies to choose from (as opposed to a technology within which one can invest continuously in abatement without limit), such that the rich, upon using only the "cleanest" technology, may not have further scope for abatement.[12]

3.2. The Insufficiency of Increasing Returns

3.2.1. Fixed Costs of Abatement

Now we modify (3.2) again, but instead of facing decreasing productivity of abatement spending on the margin as in (3.9) now a household can choose from two types of environmental investment: e_1, with no fixed cost but a relatively high marginal cost p_1; and e_2, with a fixed cost, $f > 0$, but a relatively lower marginal cost p_2.[13] Together, these abatement choices $\overrightarrow{e} = (e_1, e_2)$ form the simple increasing returns abatement technology in (3.12), the last part of a $q = q_0 - c + a(e)$ technology:

$$a(\overrightarrow{e}) = e_1 + e_2 \tag{3.12}$$

[11] See, for example, Grossman and Krueger 1995 (page 361, Figures 1,3 and 4, and page 369), Torras and Boyce 1998 (pages 152-3, 157) and Hill and Magnani 2001 (Table 1).

[12] See Pfaff, Chaudhuri and Nye 2001 for a formalized theoretical result. One example they mention is switching among a finite set of fuels in order to shift the consumption - air quality MRT. Note also the discussion in Jones and Manuelli 1995 and Torras and Boyce 1998.

[13] Fixed costs may well exist. Further, lower-marginal-cost options may have higher fixed costs. Andreoni and Levinson 2001 provides useful evidence that abatement technologies with higher fixed costs may have lower marginal costs. They cite EPA studies of the emission control from large coal-fired burners, and they also regress pollution abatement operating costs by industry and by U.S. state on a measure of the size of the industry's contribution to gross state product.

where the household is faced with the piecewise defined budget constraint,

$$y = \begin{cases} p_c c + p_1 e_1 & \text{if } e_2 = 0 \\ p_c c + p_1 e_1 + p_2 e_2 + f & \text{if } e_2 > 0 \end{cases} \quad (3.13)$$

where $p_2 < p_1$. The household is also faced with the non-negativity constraints:

$$c \geq 0, e_1 \geq 0, e_2 \geq 0 \quad (3.14)$$

and picks c and $\overrightarrow{e}$ to maximize (3.1) subject to (3.13), (3.14) and (3.15):

$$q = q_0 - c + e_1 + e_2 \quad (3.15)$$

Assuming that the first type of abatement investment (i.e., e_1) is not dominated[14], the optimization problem leads one to consider three cases[15]:

$$1)\ c > 0, e_1 = e_2 = 0;\ 2)\ c > 0, e_1 > 0, e_2 = 0;\ 3)\ c > 0, e_1 = 0, e_2 > 0 \quad (3.16)$$

The $c > 0, e_1 = e_2 = 0$ result is optimal for the poorest households:

$$y \leq \frac{q_0 \alpha p_c p_1}{p_1 + \beta p_c} \quad (3.17)$$

For these households, the optimal values c^* and q^* are like those for the poorer households in Section 3.1.1, and so $\frac{dq^*}{dy}$ here is equal to $-\frac{1}{p_c} < 0$. Thus, this is again an income range in which environmental quality falls with increasing income (and, as above, this is an income range which does not exist if $q_0 = 0$).

The $c > 0, e_1 > 0, e_2 = 0$ result is optimal for middle incomes[16]:

$$\frac{q_0 \alpha p_c p_1}{p_1 + \beta p_c} < y \leq \frac{q_0 \alpha p_c p_2}{p_2 + \beta p_c} + f \quad (3.18)$$

The optimal values c^*, q^*, and e_1^* for households in this income range are like those for the richer households in Section 3.1.1 (substituting e_1 for e and p_1 for p_e). Thus, much as in that setting, $\frac{dq^*}{dy} = \frac{\beta}{p_1} > 0$, i.e. environmental quality rises with income.

[14] Specifically, this is the assumption that:

$$f > \frac{q_0 \alpha p_c^2 \beta (p_1 - p_2)}{p_1 + \beta p_c}$$

[15] Consider two non-linear programming problems, one for $e_2 = 0$ and one for $e_2 > 0$. The $e_2 = 0$ problem yields four cases: 1) $c > 0, e_1 = e_2 = 0$; 2) $c > 0, e_1 > 0, e_2 = 0$; 3) $c = e_1 = e_2 = 0$; 4) $c = 0, e_1 > 0, e_2 = 0$. However, given (2.7, vi), such that when $y > 0$ the non-negativity constraint on c will not be binding, 3) and 4) are ruled out. The $e_2 > 0$ problem also yields four cases: 1) $c = e_1 = 0, e_2 > 0$; 2) $c = 0, e_1 > 0, e_2 > 0$; 3) $c > 0, e_1 = 0, e_2 > 0$; 4) $c > 0, e_1 > 0, e_2 > 0$. As above, 1) and 2) are ruled out by (2.7, vi) when $y > 0$. Also, it is easily shown that once $e_2 > 0$, i.e. if the fixed cost has been incurred, given $p_1 > p_2$ case 4) is ruled out. From both problems together, then, we are left with the three cases considered in the text.

[16] The assumption of the condition under which e_1 is not dominated, specified earlier, ensures that this income range exists, i.e. that:

$$\frac{q_0 \alpha p_c p_1}{p_1 + \beta p_c} < \frac{q_0 \alpha p_c p_2}{p_2 + \beta p_c} + f$$

Lastly, the $c > 0, e_1 = 0, e_2 > 0$ result is optimal for the richest households:

$$y > \frac{q_0 \alpha p_c p_2}{p_2 + \beta p_c} + f \tag{3.19}$$

This is much like just above (but now substitute e_2 and p_2 for e and p_e in Section 3.1.1). Thus, $\frac{dq^*}{dy} = \frac{\beta}{p_2} > 0$, and environmental quality rises with income. While q rises in both the middle and the highest income ranges, because $p_2 < p_1$ the derivative of environmental quality with respect to income is greater for the higher income range. Note, then, that the transition between environmental investments, which raises the fixed costs but lowers the marginal cost of abatement, discretely increases the rate at which environmental quality rises with income.

In any case, these results further demonstrate the robustness of the endowment-based EKC result, for an increasing returns technology. More importantly, though, they show the insufficiency of the increasing returns abatement technology on its own. If $q_0 = 0$, the income range in (3.17) vanishes, and environmental quality always rises with income, as the middle income range in (3.18) becomes simply $y < f$, and the high income range in (3.19) becomes $y \geq f$. As $\frac{dq^*}{dy} > 0$ for both ranges, we can see that without the asymmetric endowment the quality of the environment will rise with income for *all* incomes, i.e. there will not be an EKC.

To consider the validity of the asymmetric endowment, note the results when even e_1 has a fixed cost, but there is no endowment. If a household is rich enough (given this fixed cost) to both consume and abate, then outcomes are as just described: the income range in (3.17) vanishes and environmental quality rises with income. However, until that point, the household *neither abates nor consumes.* Thus, a starving household will choose not to consume because of the implications for the environment. In our minds this is so generally unrealistic, when thinking of actual low-income households, as to lead us to seek the source of the lack of relevance of the result. Our conclusion is that households would be dead at $q = 0$. Thus, the almost-starving household (low c due to low y) in which people can stay alive and consume (given that many die of, e.g., lack of potable water) clearly has an endowment of environmental quality, e.g. water to drink and air to breathe.

3.2.2. 'Explosive' Increasing Returns to Abatement

Andreoni and Levinson posit a particular increasing returns abatement technology which depends upon consumption directly. Their specification of the technology $q = q_0 - c + a$ assumes $q_0 = 0$ and an $a(c, e)$ where $a_c > 0, a_e > 0$, and a is homogenous of degree k where $k > 1$. We call this 'explosive' increasing returns because as the scale of income and consumption rise, the returns to abatement investments in e increase *ad infinitum*. Their motivating example, however, is small-scale: a broom can for the same level of effort accomplish more abatement when sweeping up a quarter inch of dust, e.g., than when sweeping up an eighth of an inch. It may not be appropriate to generalize from this small scale to unlimited scale.[17]

[17] As in the broom example, it may often be the case that rising c increases $\frac{da}{de}$ near $c = 0$: no matter how hard you try ($e > 0$), vacuuming a clean ($c = 0$) rug accomplishes nothing ($a = 0$). However, often a capacity constraint (given e) may arise well within the relevant scale of c. Consider a single broom, thought of as a single unit of e spending. A sweep of a clean floor accomplishes nothing, while a sweep of a floor with a half-inch of dirt accomplishes more than a sweep of a floor

The point here is that this technological assumption cannot by itself generate an EKC. It implies that as income and c rise, marginal productivity of e also rises. A given investment in e yields more a. That supports the upward-sloping part of an EKC, as per Proposition 2.1, but does not substitute for the asymmetric endowment in explaining (as in Section 2.2.1) the downward-sloping part of an EKC.

Consider $a(c, e) = ce$. Here $a_c > 0, a_e > 0$, and a is homogenous of degree k where $k > 1$. The household's problem is to pick c and e to maximize (3.1) subject to (2.4), (2.5) and, of course, this specification of $a(c, e)$ and thus also of $q(c, e)$. As in some of the problems above, the cases to consider are: 1) $c > 0, e = 0$; and 2) $c > 0, e > 0$. The $e^* = 0$ result is optimal for poorer households, satisfying:

$$y \leq \frac{\sqrt{p_e^2 + 4\beta q_0 \alpha p_c p_e} - p_e}{2\beta} \tag{3.20}$$

The key point here can already be made, with reference to this expression: with no environmental endowment ($q_0 = 0$), this income range in which environmental quality will fall with income (as in (3.4) and (3.11)) simply vanishes. Since elsewhere environmental quality rises with income (as discussed above, increasing returns makes this more likely), lacking an endowment this technology does not generate an EKC. Formally, the $e^* > 0$ case is optimal for richer households:

$$y > \frac{\sqrt{p_e^2 + 4\beta q_0 \alpha p_c p_e} - p_e}{2\beta} \tag{3.21}$$

so that if environmental quality is in fact rising with income within this range, then for the $q_0 = 0$ case it will always rise with income. And in fact[18]:

$$e^* = \frac{2yp_e\beta + p_e y + p_e^2 - \sqrt{p_e^2(y - p_e)^2 + q_0 \alpha p_e^3 p_c [\beta + p_e^2]}}{2(2 - \alpha)p_e^2}$$

$$\frac{dq^*}{dy} = \frac{\sqrt{\Phi + D}\,[4yp_e(2 - \alpha)\beta + 2yp_e\beta + yp_e + p_e^2] + p_e^2\beta(y - 2p_e)^2 + D}{2(2 - \alpha)p_c\sqrt{\Phi + D}} > 0$$

where $\Phi = p_e^2(y - p_e)^2$ and $D = q_0 \alpha p_e^3 p_c[\beta + p_e^2]$ are used to simplify. Without an environmental endowment, even this 'explosive' increasing returns to scale technology explains only environmental quality increasing with income, not an EKC.

4. Income and Environmental Quality in a Multi-agent Setting

Many environmental amenities are public goods in that the polluting activities of a household degrade the environment for others. Here we demonstrate the utility of our static household context by showing how the EKC mechanisms we have shown

with a quarter-inch. But then consider a floor with two feet of dirt, a scale likely to be beyond the capacity of a single sweep of the broom. At four inches per sweep, e.g., simple division suggests that it will take six sweeps to eliminate the dirt. But simple division is precisely a statement of capacity and, by implication, constant returns over large scales. Thus, for a scale of c well beyond the capacity of the e in question, abatement will become effectively constant returns to scale.

[18] This is one of two roots of a quadratic equation. It is the one in which a higher environmental endowment implies lower optimal abatement expenditures (as makes intuitive sense, given the effect of "free" environment on the MRS and given our previous results, e.g. in Section 3.1.1).

can carry over to the multi-agent context. This permits explicit consideration of how externalities might be addressed through choice of government policies.

Consider a situation where the environmental quality that a particular household enjoys depends not just on its own consumption level c and its own abatement expenditures e—the situation we have been considering thus far—but also on the consumption levels and abatement expenditures of other households. Let C denote the overall, or total, level of consumption in the economy, and E denote aggregate, or total, abatement expenditures. The environmental quality enjoyed by any household is then expressed as $q = q(C, E)$ where as before (in (2.8)) we assume that:

$$\begin{array}{cccc} \text{(i) } q_E > 0 & \text{(ii) } q_{EE} \leq 0 & \text{(iii) } q_C < 0 & \text{(iv) } q_{CC} \leq 0 \\ \text{(v) } q_{CE} \geq 0 & \text{(vi) } q_{EE}q_{CC} - q_{EC}^2 > 0 & \text{(vii) } q(0,0) = q_0 > 0 & \end{array} \tag{4.1}$$

We assume that household preferences continue to be described by a utility function $U(c, q)$ satisfying assumptions (2.6) and (2.7). But in addition we assume that[19]:

$$\frac{-cU_{cc}}{U_c} > 1 \qquad \text{for all } c \tag{4.2}$$

When the number of households is large enough that each individual household ignores the effects of its own consumption and abatement expenditures on environmental quality, it is clear that no single household will choose to independently incur any abatement expenditures. And in that case, in the absence of any collective choice mechanism, as incomes grow and consumption levels increase environmental quality will continually and monotonically decline with income.

However, as the literature on local public goods amply highlights, simple voting mechanisms can provide a way of coordinating individual decisions. We consider voting on a proportional income tax rate, where it is understood that the proceeds of the tax will be used to finance public abatement expenditures.

4.1. Rising Preferred Tax Rates

Imagine that each household calculates its preferred tax rate by solving the following:

$$\max_{0 \leq t \leq 1} V(t; y) = U((1-t)y, q((1-t)Y, tY)$$

where y is the household's income, t is the proportional tax rate, and Y is aggregate income. The first-order condition for this maximization problem is:

$$yU_c \geq U_q Y[q_E - q_C]$$

which holds with equality if $t > 0$. Let $t^*(y; Y, q_0) \in [0, 1]$ denote the preferred tax rate of a household with income y. Assumptions (2.6), (2.7), (4.1) and (4.2) imply:

$$t^*(y; Y, q_0) = 0 \qquad \text{for all } y \leq \widehat{y}(Y, q_0)$$

[19] This says that preferences for consumption should be sufficiently elastic. Then, as incomes and pollution increase, households will devote a higher *share* of their incomes to abatement, by voting for higher proportional income tax rates. Stokey 1998 requires such an assumption for an EKC even in the single-agent case, given a specific abatement technology. We did *not* require this above.

where $\widehat{y}(Y, q_0)$ is implicitly defined by:

$$\widehat{y}U_c(\widehat{y}, q(Y,0)) = U_q(\widehat{y}, q(Y,0))Y[q_E(Y,0) - q_C(Y,0)]$$

Furthermore, for $y > \widehat{y}(Y, q_0)$, we have that $\frac{\partial t^*}{\partial y}$ is equal to:

$$\frac{U_c[\frac{(1-t)yU_{cc}}{U_c} + 1] - U_{qc}(1-t)Y[q_E - q_C]}{U_{qq}Y^2[q_E - q_C]^2 - 2yYU_{cq}[q_E - q_C] + y^2U_{cc} + U_qY^2[q_{EE} - 2q_{EC} + q_{CC}]} \tag{4.3}$$

This is positive, since the numerator is negative, given (2.6) and (4.2), and the denominator is also negative, given the assumed concavity of $U(c, q)$ and (4.1). Hence, the preferred tax rate rises (weakly) monotonically with household income.

4.2. An Illustrative Example

To provide a sense of the sorts of results that this framework can potentially yield, we work through a simple example in some detail. We assume that (much like in (3.2), but with aggregate C and E drivers) environmental quality is given by:

$$q = q_0 - \gamma C + \delta E \qquad \gamma, \delta > 0 \tag{4.4}$$

To keep the analysis especially simple we assume that all N households in the economy have the same level of household income, y, and that aggregate income is therefore simply Ny. Household preferences are as described above.

As before, let $\widehat{y}(Y, q_0)$, the threshold level of income below which a household will prefer a tax rate of zero, be implicitly defined by:

$$\widehat{y}U_c(\widehat{y}, q_0 - \gamma Y) = U_q(\widehat{y}, q_0 - \gamma Y)Y(\gamma + \delta) \tag{4.5}$$

Our assumptions about preferences imply that:

$$\begin{aligned} \lim_{y \to 0} \widehat{y}(Ny, q_0) &= +\infty \\ \lim_{y \to +\infty} \widehat{y}(Ny, q_0) &= 0 \\ \frac{\partial \widehat{y}(Y, q_0)}{\partial Y} &= \frac{U_q(\gamma + \delta) + \gamma \widehat{y}U_{cq} - \gamma(\gamma + \delta)U_{qq}}{U_c(\widehat{y}\frac{U_{cc}}{U_c} + 1) - Y(\gamma + \delta)U_{qc}} < 0 \end{aligned} \tag{4.6}$$

As aggregate income (and hence aggregate consumption) falls to zero, $\widehat{y}(Y, q_0)$ goes to infinity. At the other extreme, as aggregate income goes to infinity $\widehat{y}(Y, q_0)$ falls to zero and a positive tax rate is preferred by all households with positive incomes. Between these two extremes $\widehat{y}(Y, q_0)$ falls monotonically with aggregate income. Together, the results in (4.6) imply that there exists a y_l defined implicitly by:

$$y_l = \widehat{y}(Ny_l, q_0)$$

such that for all levels of per-household income $y < y_l$:

$$\begin{aligned} y &< \widehat{y}(Ny, q_0) \\ t^*(y, Y, q_0) &= 0 \\ \frac{\partial q(Ny, 0)}{\partial y} &= -\gamma N < 0 \end{aligned} \tag{4.7}$$

When per-household income is below y_l, the (identical) incomes of all households are lower than the threshold $\hat{y}(Ny, q_0)$ below which a tax rate of zero is preferred. Because the preferred tax rate is zero for everybody, there are no tax-financed public abatement expenditures and the only effect of increasing incomes is a rise in consumption levels and a corresponding decrease in environmental quality.

To characterize the impact of increases in income beyond the threshold y_l:

$$\frac{\partial t^*(y, Y, q_0)}{\partial Y} = \frac{y U_{cq} \Delta - U_q(\gamma + \delta) - Y(\gamma + \delta) U_{qq} \Delta}{U_{cc} y^2 - 2yY(\gamma + \delta) U_{qc} + U_{qq} Y^2 (\gamma + \delta)^2} \tag{4.8}$$

where $\Delta \equiv \delta t - \gamma(1 - t)$ represents the direct impact of an increase in aggregate income on environmental quality given an initial tax rate of t. Inspecting (4.8), we see that if $\Delta < 0$, which will be the case at low values of t, the preferred tax rate for a household rises with aggregate income. Each household recognizes that at the existing tax rate an increase in aggregate income has a negative net impact on the environment, and is therefore willing to at least partially offset the deterioration in environmental quality through an increase in the tax rate. Starting from $t = 0$ at the threshold y_l, the preferred tax rate will rise unambiguously with increases in income until income reaches an upper threshold y_h defined implicitly by:

$$t^*(y_h, Ny_h, q_0) = \frac{\gamma}{\gamma + \delta} \tag{4.9}$$

What happens beyond this point—whether the preferred tax rate continues to rise monotonically or exhibits a more complicated non-monotonic relationship with respect to income—depends on the particular specification of preferences and the particular values of γ and δ. However, the preferred tax rate cannot fall below the level in (4.9) as long as per-household income remains above y_h.

Turning next to the implications of increases in income on environmental quality, and keeping in mind that $Y = Ny$, the relevant expression to consider is:

$$\frac{dq((1 - t^*)Y, t^*Y)}{dy} = (\gamma + \delta) Y \frac{\partial t^*}{\partial y} + N \left(\Delta + (\gamma + \delta) Y \frac{\partial t^*}{\partial Y} \right) \tag{4.10}$$

From (4.3), the first term on the right hand side is unambiguously positive. As per-household income rises, *holding fixed aggregate income*, households prefer higher tax rates. And a higher tax rate, by lowering aggregate consumption and increasing public abatement expenditures, unambiguously improves environmental quality. But an increase in per-household income also implies an increase in aggregate income and the impact of this increase on environmental quality is captured by the second term on the right. The impact can be decomposed into the direct impact, Δ, and the indirect impact which is realized through the change in the preferred tax rate induced by an increase in aggregate income. When per-household income is below the upper threshold y_h, the net impact is ambiguous and hence the sign of (4.10) cannot be determined without further specifying preferences and parameters. However, by substituting in from (4.8) into (4.10) it is easily verified that once incomes cross the upper-threshold y_h, subsequent increases in per-household (and hence aggregate) income unambiguously lead to improvements in environmental quality.

Combining this last result with (4.7) we have that, except for an intermediate region where the relationship between income growth and environmental quality is indeterminate, the overall relationship broadly mirrors the non-monotonic relationship emphasized by the empirical work on environmental Kuznets curves. That is, there exist two thresholds, y_l and y_h where $0 < y_l < y_h$ such that:

$$\begin{aligned} \frac{dq}{dy} &< 0 \quad \text{for all } y < y_l \\ \frac{dq}{dy} &> 0 \quad \text{for all } y > y_h \end{aligned}$$

5. Conclusion

Using a static household-choice framework, this paper provided a simple explanation for Environmental Kuznets Curves (EKCs). The very natural assumption of an asymmetric endowment (i.e., positive environmental quality but zero consumption at zero income) is sufficient to yield an EKC. The intuition is that the MRS between consumption and environment at low incomes, given the endowment, makes abatement undesirable for households. However, as consumption increases with income and the endowment is degraded by consumption, this corner solution gives way to interior solutions in which both consumption and abatement expenditures rise with income. We provide useful sufficient conditions on preferences and abatement technologies that ensure that the increase in abatement expenditures is large enough to offset the increase in pollution caused by increased consumption.

This endowment-based result is robust to a wide range of abatement technologies of interest, including fixed costs and decreasing returns. Our decreasing returns case, which also stands in for a finite set of abatement options, stimulates further scrutiny of the empirical literature for results other than U shapes. Also, the abatement technologies themselves do not generate EKCs. The reason is that they do not generate an income range in which environmental quality falls with income.

Finally, we demonstrated the utility of the household approach by applying our approach to a setting of externalities and multiple agents who vote for taxation and environmental spending. We derive an EKC result analogous to our previous results, in which the tax rate chosen rises with income, after a range in which no abatement occurs, and environmental quality at first falls and later rises.

This area of aggregation of heterogeneous preferences (given whatever abatement technologies seem most relevant) would appear to merit much additional consideration, given the set of stories now present in the EKC literature. This suggests additional value to starting with the household model in order to pursue additional complications within the voting arena, as we plan to do in future research.

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