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Labor-Turnover Model of the 'Natural Rate'**

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Abstract

This paper studies two kinds of wage subsidy in a model of the natural rate having a continuum of workers ranked by their productivity—a flat wage subsidy and a graduated wage subsidy, each program financed by a proportional payroll tax. We show that in the model's small open economy version, both subsidy schemes expand employment throughout the distribution; for those whose productivity is sufficiently far below the mean, take-home pay is unambiguously up, though the tax financing lowers take-home pay at the mean and above. For any particular class of workers paid the same amount of the wage subsidy under the two plans, the graduated plan expands employment more. In the closed-economy case, the interest rate is pulled up, and employment is increased for workers whose productivity levels are below or equal the mean. A hiring subsidy is also studied. (*JEL* E24, H22)

Key words: Natural rate of unemployment, low-wage employment subsidy.

There is considerable agreement that the extraordinarily low commercial productivity of active-age persons in the lower reaches of the distribution relative to median productivity is the number one social problem of our time. In creating a huge wage gap it makes the less productive incapable of supporting a family or in some cases themselves (in a way meeting community standards of decency at any rate) and having access to mainstream community life. In reducing the wage incentives that private enterprise can afford to offer low wage workers relative to their other resources and attractions, it worsens unemployment and nonparticipation. Both sets of effects operate in turn, especially in areas where there is a high concentration of these effects, to increase dependency on welfare and property crime, spread drug use and violence, widen illegitimacy and blight the upbringing of children (Freeman, 1996; Murray, 1984; Phelps, 1994b; Wilson, 1996.)

There is far less agreement on what, if anything, would be useful to do about it. An important line of thinking, however, looks to wage subsidies of one kind or another. The pioneers were Arthur Pigou (1933) and Nicholas Kaldor (1936), who sought the conditions for employment subsidies to be self-financing. Targeted hiring subsidies were championed by Daniel Hamermesh (1978), Michael Hurd and John Pencavel (1981) and by Robert Haveman and John Palmer (1982). The employment-expanding effects of a constant employment subsidy were studied by Richard Jackman and Richard Layard (1986). One of us argued informally for a graduated employment subsidy to raise low-end wage rates (Phelps, 1994a) and to reduce unemployment (Phelps, 1994b) as a counterweight to the welfare system. Employment subsidies were urged to counter the effects of payroll taxes by Jacques Dréze and Edmond Malinvaud (1994). A hiring subsidy targeted at the long-term unemployed has been championed by Dennis Snower (1994). Christopher Pissarides (1996) has studied employment-tax relief, which is apt to have a still different character.

These analyses focus on the subsidies' near-term effects. None of the papers expressly argues that there would be a permanent effect on unemployment. Some of the authors may have thought the effect was only temporary but a way to buy valuable time. To study the long-term effects, however, requires an intertemporal model in which workers accumulate wealth and firms invest in capital of one or more kinds according to expectations of the future and interest rates.

The present paper analyzes some wage subsidies in the steady state in our labor-turnover model of the natural rate (Phelps, 1968, 1994c; Hoon and Phelps, 1992, 1996). In this theory, quitting by employees poses an incentive problem for the firm, since it must invest in the firm-specific training of workers to make them functioning employees and such an investment is lost whenever an employee quits. The problem prompts firms to drive up the going wage. This leads in turn to involuntary unemployment in labor-market equilibrium. Our 1992 paper posited worker-savers in overlapping cohorts to obtain a general-equilibrium framework with which to endogenize the rate of interest or the accumulation of net foreign assets. The present paper introduces a continuum of workers differentiated by productivity in each cohort.

We mainly study two employment-subsidy programs offering tax credits to bona-fide firms for every employee earning a qualifying wage: first, a flat (constant) subsidy and, next, a graduated subsidy that decreases with the wage rate and vanishes at the top—each program financed by a flat-rate payroll tax (as if no reflow of budgetary savings and revenue gains resulted).

The gist of our findings can be indicated. The impact of both subsidies, of course, is to reduce the hourly labor cost to firms of employing those workers bringing in a subsidy—all workers under the flat subsidy, low-wage workers under the graduated plan. Given net wealth and the interest rate, this operates to reduce the unemployment rates of those workers and to pull up their wage rates. On the other hand, in the same “short run,” the

financing via a flat-rate payroll tax operates to reduce the after-tax pay rates of all employees and to increase their unemployment rates. At the low end of the scale, where the tax will be much smaller than the subsidy, the net results are a rise in the after-tax wage and a rise in employment—wealth and interest held constant.

The long-run question in the open-economy case is the effect of wealth adjustment. Won't the workers whose employment and wage are increased accumulate more wealth, sending their propensity to quit higher, thus weakening firms' demand for their labor, until their unemployment rate is back to its previous equilibrium level? We show that wealth decumulation serves to eliminate all the employment declines brought by the tax while wealth accumulation operates to moderate the gains to employment. The net result, then, is that employment is increased throughout the distribution.

The long-run question in the closed-economy case is the subsidies' effect on the rate of interest and the effect in turn on wages and employment. Here we find that if the labor demand curve is elastic, aggregate wealth supply is increased, but it increases by less than the increase in asset demand. The result is a rise in the rate of interest. However, for workers whose productivity levels are below or equal the mean, employment is expanded; at productivities far enough below the mean, take-home wages will rise.

It is also found that the graduated scheme, besides having (for the same subsidy rate at the bottom) a lighter budgetary burden than the constant subsidy, has an extra downward impact on hourly labor cost, as firms moderate wage rates above the bottom to win a larger subsidy, with the result that employment receives an extra boost. Such an effect raises the fear that some middle-wage workers would see their wage reduced on balance. We show, however, that unless the subsidy tapers off too fast no such wage effect occurs. Finally, the effects of a hiring subsidy are briefly examined.

The paper is organized as follows. Section I presents the basic features of the economy having a continuum of workers. Section II studies the incidence

of the subsidies in the steady-state, general-equilibrium model of the small open economy while Section III studies the closed-economy case. Section IV concludes.

I. Basic Features Of The Economy

In each cohort, the workers form a continuum when ranked by their respective potential productivity levels. The productivity, or ability, of worker input at location i in this continuum is measured by a labor-augmenting, hence Harrod-neutral, parameter denoted Λ_i . There is a known and unvarying distribution of Λ_i in the working population, which we normalize to one. The proportion of workers with productivity level Λ_i or less is $F(\Lambda_i)$. We assume that F is differentiable, so that there is a density function for productivity levels, $f(\Lambda_i) = F'(\Lambda_i)$. We call a worker with productivity level Λ_i a type- i worker.

The price paid by consumers and received by firms is set equal to unity. The real household wage received by a type- i worker is denoted v_i^h . Then, if the ad valorem payroll tax rate is τ , and the subsidy for a type- i worker is s_i , the hourly labor cost to the firm of a type- i worker is $v_i^f = (1 + \tau)v_i^h - s_i$.

Under the flat or specific subsidy scheme, s_i equals s^F and hence $v_i^h = (v_i^f + s^F)/(1 + \tau)$. Under a graduated scheme, s_i is a decreasing function of the wage paid by the firm to each type- i worker, v_i^f . We choose to write $s_i = S(v_i^f)$ such that $S'(v_i^f) < 0$ and $|S'(v_i^f)| < 1$.

There are many identical firms. For convenience we may think of them as fixed in number (normalized to one) and equal in size. Consider the representative firm j . Its problem is to choose the wage and hiring-training

policies that maximize¹

$$\int_0^\infty \int_{\underline{\Lambda}}^\infty N_{jit} \{ \Lambda_i [1 - \Phi(h_{jit})] - v_{jit}^f \} f(\Lambda_i) \exp^{-\int_0^t r_\nu d\nu} d\Lambda_i dt,$$

which is the present value of the stream of real quasirents, subject to

$$\dot{N}_{jit} = N_{jit} [h_{jit} - \zeta(z_{it}^{he}/v_{jit}^h, y_{it}^w/v_{jit}^h) - \theta]$$

and given N_{ji0} . Note that s_i is implicit in v_{ji}^h and v_{ji}^f , given τ . (Since to simplify we will later work with constant marginal training cost, we also assume that h_{jit} is bounded, $0 \leq h_{jit} \leq \bar{h}$.) Here $\underline{\Lambda}$ is the minimum productivity level, N_{jit} is the stock of type- i employees at the representative firm j taken as a ratio to the type- i workforce (equivalently, the rate of employment among type- i workers, that is, the ratio of employed type- i workers to the total number of type- i workers), $\Phi(h_{jit})$ is the fraction of their working time type- i employees devote to training new hires, h_{jit} is the gross hiring rate of new type- i recruits, ζ similarly measured is the quit rate, r_t is the instantaneous rate of interest, θ is the mortality rate, z_{it}^{he} is a proxy for the expected value of real wage earnings of a type- i worker employed at firm j if he quits, and y_{it}^w is the average income from wealth.²

Setting $\Phi(h_{jit}) = \beta h_{jit}$ where β , the marginal training cost in manhours, is a constant, we may write the current-value Hamiltonian as

$$\int_{\underline{\Lambda}}^\infty \{ \Lambda_i [1 - \beta h_{jit}] - v_{jit}^f + q_{jit} [h_{jit} - \zeta(z_{it}^{he}/v_{jit}^h, y_{it}^w/v_{jit}^h) - \theta] \} N_{jit} f(\Lambda_i) d\Lambda_i,$$

¹Using the notation $\tilde{N}_{jit} = \bar{L}_i N_{jit}$ where $\bar{L}_i \equiv f(\Lambda_i)$ is the fixed workforce of type- i workers (the total workforce being normalized to one), the problem solved by firm j could also be re-expressed as: Maximize $\int_0^\infty \int_{\underline{\Lambda}}^\infty \tilde{N}_{jit} \{ \Lambda_i [1 - \Phi(h_{jit})] - v_{jit}^f \} \exp^{-\int_0^t r_\nu d\nu} d\Lambda_i dt$ subject to $\dot{\tilde{N}}_{jit} = \tilde{N}_{jit} [h_{jit} - \zeta(z_{it}^{he}/v_{jit}^h, y_{it}^w/v_{jit}^h) - \theta]$ given \tilde{N}_{ji0} . Since all firms are equal in size and the total number is normalized to one, employment at the representative firm j , \tilde{N}_{jit} , equals $(1 - u_{it})\bar{L}_i$, where u_{it} is the fraction of type- i workforce that is unemployed.

²The quit rate function has the following first derivatives: $\zeta_1 > 0$ and $\zeta_2 > 0$. By virtue of the firm's second-order condition for maximization, $\zeta_{11} > 0$ and $\zeta_{22} > 0$. We also make the assumption that an increase in the nonwage income raises a worker's marginal propensity to quit with respect to wage prospects elsewhere, that is, $\zeta_{12} > 0$.

where q_{jit} is the co-state variable.³ It measures the shadow value of a type- i worker after training by the employer. First-order necessary conditions (which are also sufficient under our assumptions) are given by:

$$(1) \quad \begin{cases} h_{jit} = \bar{h} & \text{if } q_{jit} > \Lambda_i \beta; \\ h_{jit} = 0 & \text{if } q_{jit} < \Lambda_i \beta; \\ h_{jit} \in [0, \bar{h}] & \text{if } q_{jit} = \Lambda_i \beta; \end{cases}$$

$$(2) \quad N_{jit} \left\{ -1 + q_{jit} \left[\frac{\zeta_1 z_{it}^{he}}{v_{jit}^{h2}} + \frac{\zeta_2 y_{it}^w}{v_{jit}^{h2}} \right] \frac{dv_{jit}^h}{dv_{jit}^f} \right\} = 0;$$

$$(3) \quad \dot{q}_{jit} - r_t q_{jit} = -[(\Lambda_i - v_{jit}^f) - q_{jit} \zeta \left(\frac{z_{it}^{he}}{v_{jit}^h}, \frac{y_{it}^w}{v_{jit}^h} \right) - \theta];$$

$$(4) \quad \lim_{t \rightarrow \infty} \exp^{-\int_0^t r_\nu d\nu} q_{jit} N_{jit} f(\Lambda_i) = 0.$$

The equations represented by (1) characterize the optimal number of new hires. In the case arising in the steady-state analysis below, the shadow value of a trained worker is equal to the marginal training cost in output terms. Equation (2) gives the optimal real wage-turnover cost trade-off, equating the marginal cost of raising v_{jit}^f to the marginal benefit. Equation (3) relates the shadow value of functional employees to the total marginal benefit of having one more employee. The transversality condition is in (4). These equations summarize the conditions that have to be satisfied for the typical firm.

To move to the equilibrium conditions, we use the Salop-Calvo approximation for z_i^{he} ,

$$(5) \quad z_{it}^{he} = N_{it}^e v_{it}^{he}.$$

(Using instead the exit rate from the unemployment pool would not differ in the steady state.) On any equilibrium (correct-expectations) path with identical firms, $v_{jit}^h = v_{it}^h = v_{it}^{he}$ and $N_{jit} = 1 - u_{it} \equiv N_{it} = N_{it}^e$. Hence we obtain a subsystem of equations in the equilibrium path of the economy.

³The flow of output at firm j is then given by $\int_{\Lambda}^{\infty} \Lambda_i [1 - \beta h_{jit}] N_{jit} f(\Lambda_i) d\Lambda_i$.

For any exogenously given path of the instantaneous real interest rates, this subsystem is

$$(6) \quad \dot{q}_{it} = q_{it}[\zeta(N_{it}, \frac{y_{it}^w}{v_{it}^h}) + \theta + r_t] - [\Lambda_i - v_{it}^f];$$

$$(7) \quad \dot{N}_{it} = N_{it}[h_{it} - \zeta(N_{it}, \frac{y_{it}^w}{v_{it}^h}) - \theta];$$

$$(8) \quad N_{it} \left\{ -1 + q_{it} \left[\frac{\zeta_1 N_{it}}{v_{it}^h} + \frac{\zeta_2 y_{it}^w}{v_{it}^{h2}} \right] \frac{dv_{it}^h}{dv_{it}^f} \right\} = 0.$$

The nonwage income-wage ratio per type- i worker, y_{it}^w/v_{it}^h , can be written $(r_t + \theta)W_{it}/v_{it}^h$ where W_{it} is the nonhuman wealth per type- i worker, θW_{it} being the actuarial dividend. Note that θ is the probability of death in the Blanchard-Yaari set-up adopted here. In the small open economy case, $\int_{\underline{\Lambda}}^{\infty} W_{it} f(\Lambda_i) d\Lambda_i = \int_{\underline{\Lambda}}^{\infty} [q_{it} N_{it} + F_{it}] f(\Lambda_i) d\Lambda_i$ where F_{it} is the amount of net foreign assets held per type- i worker whereas in the closed-economy case, $\int_{\underline{\Lambda}}^{\infty} W_{it} f(\Lambda_i) d\Lambda_i = \int_{\underline{\Lambda}}^{\infty} [q_{it} N_{it}] f(\Lambda_i) d\Lambda_i$.

Using the Blanchard-Yaari demographic set-up, and assuming that all workers have identical rate of time preference and mortality rate, and adopting the log utility function, we obtain an equation for the motion of real consumption expenditure per member of the type- i workforce, C_{it} ,

$$(9) \quad \dot{C}_{it} = (r_t - \rho)C_{it} - \theta(\rho + \theta)W_{it},$$

where ρ is the rate of time preference.

In the small open economy case, the path of the domestic interest rate conforms to the exogenously given world interest rate, r^* :

$$(10) \quad r_t = r^*, \quad r^* \text{ a constant } > 0.$$

The level of net external assets adjusts endogenously to bring about this condition. In contrast, in the closed-economy case, the interest rate is endogenous, reaching the level that equates the demand for “capital” to wealth supply.

II. Incidence In The Open Economy

Steady-state Conditions

In steady-state, $\dot{N}_{it} = 0$. This and (7) give the steady-state employment (SSE) condition that hires balance quits and mortality:

$$(11) \quad h_i = \zeta(N_i, \frac{y_i^w}{v_i^h}) + \theta.$$

This implies that $q_i = \Lambda_i \beta$.

With $\dot{q}_{it} = 0$ in (6) and $q_i = \Lambda_i \beta$, the zero-profit (ZP) condition that quasirents cover interest and depreciation on training becomes

$$(12) \quad \Lambda_i - v_i^f = \Lambda_i \beta [\zeta(N_i, \frac{y_i^w}{v_i^h}) + r^* + \theta],$$

where by (10) r^* is substituted for the domestic interest rate or, dividing (12) throughout by Λ_i ,

$$(12') \quad 1 - \frac{v_i^f}{\Lambda_i} = \beta [\zeta(N_i, \frac{y_i^w}{v_i^h}) + r^* + \theta].$$

Next, the Blanchard-Yaari set-up and (10) again require

$$(13) \quad r^* = \rho + \frac{\theta}{1 + (v_i^h/y_i^w)N_i}.$$

This condition is obtained by noting that in steady-state, with $\dot{C}_{it} = 0$, (9) and (10) give $r^* = \rho + [\theta(\theta + \rho)W_i/C_i]$. Then, using $C_i = (\theta + \rho)[H_i + W_i]$ where H_i is human wealth, and noting that in the steady state $H_i = v_i^h N_i / (r^* + \theta)$ and $y_i^w = (r^* + \theta)W_i$, we obtain (13). This relationship gives us a constant-interest RR curve.

Finally, assuming that the employment rate is always strictly positive, we obtain from (8) the incentive wage (IW) condition:

$$(14) \quad \Lambda_i \beta [\zeta_1 N_i + \zeta_2 (\frac{y_i^w}{v_i^h})] = \frac{v_i^h}{dv_i^h/dv_i^f}$$

or, dividing throughout (14) by Λ_i ,

$$(14') \quad \beta[\zeta_1 N_i + \frac{\zeta_2 y_i^w}{v_i^h}] = \frac{v_i^h / \Lambda_i}{dv_i^h / dv_i^f}.$$

Equations (12'), (13) and (14') give us three equations in the three unknowns: v_i^f , N_i and y_i^w / v_i^h .⁴ From (13), we can express

$$(15) \quad \frac{y_i^w}{v_i^h} = \Psi(r^* - \rho, N_i); \quad \Psi_1 > 0, \Psi_2 > 0.$$

Substituting into (12'), we obtain a reduced form zero-profit condition:

$$(16) \quad 1 - \frac{v_i^f}{\Lambda_i} = \beta[\zeta(N_i, \Psi(r^* - \rho, N_i)) + r^* + \theta],$$

whose slope is given by

$$\left. \frac{d(v_i^f / \Lambda_i)}{dN_i} \right|_{\text{ZP}} = -\beta[\zeta_1 + \zeta_2 \Psi_2] < 0.$$

Further substituting (15) into (14'), we obtain a reduced form incentive-wage condition

$$(17) \quad \begin{aligned} & \beta[\zeta_1(N_i, \Psi(r^* - \rho, N_i))N_i + \zeta_2(N_i, \Psi(r^* - \rho, N_i))\Psi(r^* - \rho, N_i)] \\ & = \frac{v_i^h / \Lambda_i}{dv_i^h / dv_i^f}. \end{aligned}$$

Suppose that initially the ad valorem payroll tax rate is zero and the subsidy is also zero. (In that case, with $\tau = s^f = 0$, v_i^f is identically equal to v_i^h .) Equation (16) can be represented as a downward-sloping demand wage schedule while (17) can be depicted as an upward-sloping wage curve in the

⁴Given τ and s_i , $v_i^h (dv_i^h / dv_i^f)^{-1}$ can be expressed as a function of v_i^f . Also, note that once the equilibrium values of N_i and y_i^w / v_i^h are determined, substitution into (11) gives the equilibrium rate of hiring in steady state, h_i .

Marshallian plane shown in Figure 1. Examining (13), and recalling that in the absence of the tax-subsidy scheme $v_i^h \equiv v_i^f$, notice that we can also draw a family of hyperbolas in Figure 1 with each hyperbola lying north-east corresponding to a higher level of y_i^w . Note also that when the ZP curve cuts the hyperbola from below, as we have drawn in Figure 1, the labor-cost elasticity of labour demand is implied to exceed one. (In that case, as we shall see, the proportionate increase of N_i effected by the subsidy exceeds the proportionate decrease of v_i^f/Λ_i that the increased N_i induces so that, on balance, the product $(v_i^f/\Lambda_i)N_i$ is up.) The derivative immediately following (16) gives the slope of the demand wage schedule, which depends on how sensitive the quit rate function is to a change in the economy-wide rate of employment. If we accept that, in the equilibrium steady-state scenario we are considering, the quit rate does not vary much with movements in the employment rate, the zero-profit curve will be somewhat flat, that is, the labor-cost elasticity of labor demand will be high⁵. We also notice that the same diagram (Figure 1) represents the equilibrium for every type- i worker. The employment rate, N_i , real effective wage, v_i^f/Λ_i , and the nonwage income taken as a ratio to productivity level, y_i^w/Λ_i , are the same for every type- i worker so the real wage, v_i^f , is twice as high for a worker who is twice as productive as another worker.⁶ The nonwage income, y_i^w , corresponding to the hyperbola passing through \hat{E}_0 , is also twice as high for a worker who is twice as productive as another worker.

⁵For the United States over the period 1931-62, Eagly (1965) obtains an estimate of the elasticity of the quit rate with respect to the unemployment rate that is equal to -0.634 .

⁶The equalization of unemployment rate result depends on the assumption that the marginal training cost in *manhours*, β , is the same across all types of workers. If we have $\beta_i > \beta_j$, then it can be shown that the unemployment rate for type- i workers will be higher than that for type- j workers. Note that this assumption is consistent with $\Lambda_i\beta_i < \Lambda_j\beta_j$, that is, although the marginal training cost for type- i workers is higher when measured in manhours, it could be lower when measured in terms of output.

Flat subsidy

Since the proportional payroll tax leads to declines in employment and take-home wages, hence working against the expansionary effects of the subsidy, it may seem that the net effects are uncertain. We show, however, that in our long-run model employment unambiguously expands throughout the distribution; and take-home pay is increased for a worker whose productivity is sufficiently far below the mean.

With a flat subsidy, recall that $v_i^h = (v_i^f + s^F)/(1 + \tau)$. Hence, with $dv_i^h/dv_i^f = 1/(1 + \tau)$, we write the incentive-wage condition in (17) as

$$(18) \quad \begin{aligned} & \beta[\zeta_1(N_i, \Psi(r^* - \rho, N_i))N_i + \zeta_2(N_i, \Psi(r^* - \rho, N_i))\Psi(r^* - \rho, N_i)] \\ & = \frac{v_i^f}{\Lambda_i} + \frac{s^F}{\Lambda_i}, \end{aligned}$$

whose slope is given by

$$\left. \frac{d(v_i^f/\Lambda_i)}{dN_i} \right|_{IW} = \beta[\zeta_{11}N_i + \zeta_{12}\Psi_2N_i + \zeta_1 + \zeta_{21}\Psi + \zeta_{22}\Psi_2\Psi + \zeta_2\Psi_2] > 0.$$

Raising the common ad valorem payroll tax to finance a flat subsidy leaves the zero-profit curve given by (16) unshifted in the N_i — (v_i^f/Λ_i) plane but shifts down the wage curve given by (18) vertically downward by s^F/Λ_i . The result is that employment unambiguously expands for workers of any type- i . Notice also that the before-payroll-tax wage paid out on each type- i worker, $(v_i^f/\Lambda_i) + (s^F/\Lambda_i)$, accordingly rises. Mathematically, the effects of an increase in (s^F/Λ_i) on $(v_i^f/\Lambda_i) + (s^F/\Lambda_i)$ and N_i are given, respectively, by

$$\begin{aligned} \frac{d[(v_i^f/\Lambda_i) + (s^F/\Lambda_i)]}{d(s^F/\Lambda_i)} &= \frac{\eta_{ZP}}{\eta_{ZP} + \eta_{IW}}; \\ \frac{d \log N_i}{d(s^F/\Lambda_i)} &= \left(\frac{v_i^f}{\Lambda_i}\right)^{-1} \left[\frac{\eta_{IW}\eta_{ZP}}{\eta_{ZP} + \eta_{IW}} \right], \end{aligned}$$

where η_{zP} and η_{IW} are the elasticities of the zero-profit curve and incentive-wage curve, respectively. When the zero-profit curve is relatively flat, each percent increase in s^F/Λ_i has a large effect on the before-payroll-tax wage paid out on each type- i worker as well as a large employment-creating effect. When the zero-profit curve is horizontal, the worker's before-tax wage is raised by the full amount of the subsidy.

Inspecting (16) and (18), we see that the common ad valorem payroll tax rate does not appear. Thus, τ has no effect on v_i^f and N_i for workers of any type i in the long run. Why is this so? The reason is that although in the "short run" (when nonwage income is given) a rise in τ alone, in lowering the demand wage by more than the supply wage, lowers each N_i , the downward adjustment of nonwage income resulting from wealth decumulation completely neutralizes the effects of τ on v_i^f and N_i in the long run. (Of course, the effect of the subsidies, holding the tax rate constant, would cause wealth accumulation. We show in the next subsection that this wealth accumulation does act to moderate employment gains but not to eliminate them.)

We can also see that, with the same dollar amount of wage subsidy given to each type- i worker, a less productive worker enjoys a higher subsidy relative to his productivity level. In Figure 2 we show that the employment effect is larger for a less productive worker as his wage curve is shifted further down than that of a more productive worker.⁷

An important question to ask is how the tax-subsidy scheme affects the *take-home* wage of each type- i worker, $v_i^h \equiv (v_i^f + s^F)/(1 + \tau)$. As we have shown, the value $(v_i^f + s^F)$ unambiguously rises for workers of any type i . To see how the scheme affects v_i^h for each type- i worker, an alternative characterization of the equilibrium that plots the worker's take-home wage, v_i^h ,

⁷Mathematically, $\frac{d \log N_i}{ds^F} = \left(\frac{1}{v_i^f}\right) \left[\frac{\eta_{IW}\eta_{zP}}{\eta_{zP} + \eta_{IW}}\right]$.

against the employment rate, N_i , is instructive. The demand wage schedule in this plane is written as

$$(19) \quad (1 + \tau)v_i^h = \Lambda_i \{1 - \beta[\zeta(N_i, \Psi(r^* - \rho, N_i)) + r^* + \theta]\} + s^F,$$

while the incentive-wage schedule is now written as

$$(20) \quad (1 + \tau)v_i^h = \Lambda_i \beta [\zeta_1(N_i, \Psi(r^* - \rho, N_i)) N_i + \zeta_2(N_i, \Psi(r^* - \rho, N_i)) \Psi(r^* - \rho, N_i)].$$

We show in Figure 3 an example of a flat subsidy applied to a worker whose Λ_i is sufficiently below the mean level, $\Lambda_{\text{mean}} \equiv \int_{\underline{\Lambda}}^{\infty} \Lambda_i f(\Lambda_i) d\Lambda_i$, that after-tax wage is increased. The effects of the tax-subsidy scheme can be decomposed diagrammatically into two separate movements. First, the effect of the common ad valorem payroll tax is to shift both the ZP curve and IW curve vertically downward to the same extent⁸. Second, the ZP curve shifts vertically up on account of the s^F term in (19).

Under a balanced-budget policy, the following relationship holds:

$$(21) \quad \int_{\underline{\Lambda}}^{\infty} s^F N_i f(\Lambda_i) d\Lambda_i = \int_{\underline{\Lambda}}^{\infty} \tau v_i^h N_i f(\Lambda_i) d\Lambda_i.$$

As we noted earlier, around an equilibrium with no tax-subsidy, N_i is equal for every type i . It follows that (21) can be simplified to

$$(22) \quad \tau = \frac{s^F}{v_{\text{mean}}^h},$$

where $v_{\text{mean}}^h = \int_{\underline{\Lambda}}^{\infty} v_i^h f(\Lambda_i) d\Lambda_i$. For an employee whose $\Lambda_i < \Lambda_{\text{mean}}$, the tax liability (τv_i^h) is therefore less than the subsidy (s^F). Hence the second (upward) shift will dominate the first (downward) shift of the ZP curve for a worker whose productivity level, Λ_i , is below the mean; and vice versa for

⁸This shift of the two curves corresponds to the result noted earlier that in the long run, v_i^f and N_i are independent of τ .

Λ_i above the mean⁹. For Λ_i sufficiently below Λ_{mean} , v_i^h is increased. To see this, use (22) in (19) to obtain

$$(23) \quad v_i^h = \frac{\Lambda_i \{1 - \beta[\zeta(N_i, \Psi(r^* - \rho, N_i)) + r^* + \theta]\} + s^F}{1 + \frac{s^F}{v_{\text{mean}}^h}}.$$

Taking the derivative of v_i^h with respect to s^F in (23), and evaluating at $s^F = 0$, we have

$$\left. \frac{dv_i^h}{ds^F} \right|_{s^F=0} = 1 - \frac{v_i^h}{v_{\text{mean}}^h} - \Lambda_i \beta (\zeta_1 + \zeta_2 \Psi_2) \frac{dN_i}{ds^F}.$$

However, noting that

$$\frac{dN_i}{ds^F} = \frac{1}{\Lambda_i \beta [2(\zeta_1 + \zeta_2 \Psi_2) + (\zeta_{11} + \zeta_{12} \Psi_2) N_i + (\zeta_{21} + \zeta_{22} \Psi_2) \Psi]},$$

and that around a zero-subsidy equilibrium, $(v_i^h/v_{\text{mean}}^h) = (\Lambda_i/\Lambda_{\text{mean}})$, we finally obtain

$$(24) \quad \left. \frac{dv_i^h}{ds^F} \right|_{s^F=0} = \mu - \frac{\Lambda_i}{\Lambda_{\text{mean}}},$$

where

$$0 < \mu \equiv \frac{(\zeta_1 + \zeta_2 \Psi_2) + (\zeta_{11} + \zeta_{12} \Psi_2) N_i + (\zeta_{21} + \zeta_{22} \Psi_2) \Psi}{2(\zeta_1 + \zeta_2 \Psi_2) + (\zeta_{11} + \zeta_{12} \Psi_2) N_i + (\zeta_{21} + \zeta_{22} \Psi_2) \Psi} < 1.$$

Examining (24), it is clear that for a worker whose Λ_i is sufficiently low, say $\Lambda_i \rightarrow \underline{\Lambda} \rightarrow 0$, the derivative of v_i^h with respect to s^F is unambiguously positive. But employment is expanded everywhere, as found earlier.

⁹At the original $N_i = N_{i0}$, the net vertical shift of the ZP curve is calculated by taking the derivative of (19) after substituting for τ using (22) to yield: $dv_i^h/ds^F|_{s^F=0} = 1 - (v_i^h/v_{\text{mean}}^h)$. Evaluated at the zero-subsidy equilibrium, $(v_i^h/v_{\text{mean}}^h) = (v_i^f/v_{\text{mean}}^f) = (\Lambda_i/\Lambda_{\text{mean}})$, where the first equality uses the result that with balanced budget, $v_{\text{mean}}^h = v_{\text{mean}}^f$, while the second equality uses the demand wage relation and the result that ζ is equal across all types of workers around a zero-subsidy equilibrium. Hence this derivative is positive for $\Lambda_i < \Lambda_{\text{mean}}$.

Note that a sort of wedge analysis can be used if we juxtapose the wage curve expressed in terms of $(1 + \tau)v_i^h$ in (20) and the zero-profit curve in (16) in terms of v_i^f . Then the subsidy drives a wedge between these curves, implying that employment is unambiguously increased and v_i^h is increased for small Λ_i .

Characterization in terms of iso- y_i^w curves

To contrast the “short-run” and the long-run effects of the tax-subsidy scheme, we now conduct our analysis in terms of the iso- y_i^w ZP, iso- y_i^w IW and iso- y_i^w RR curves. By proceeding in this manner, we are able to highlight the role played by wealth or correspondingly, nonwage income, in the adjustment to the tax-subsidy scheme in the small open economy. We show that wealth accumulation does operate to moderate the gains to employment and wages achieved in the “short run” but not to eliminate those gains in the long run.

Using the relationships $v_i^f = (1 + \tau)v_i^h - s^F$ and $v_i^h(dv_i^h/dv_i^f)^{-1} = (1 + \tau)v_i^h$ with a flat subsidy, (12) and (14) can be re-expressed, respectively, as

$$(25) \quad (1 + \tau)v_i^h = \Lambda_i \{1 - \beta[\zeta(N_i, \frac{y_i^w}{v_i^h}) + r^* + \theta]\} + s^F$$

$$(26) \quad (1 + \tau)v_i^h = \Lambda_i \beta [\zeta_1(N_i, \frac{y_i^w}{v_i^h})N_i + \zeta_2(N_i, \frac{y_i^w}{v_i^h})(\frac{y_i^w}{v_i^h})].$$

In these terms, (25) and (26) give two equations relating v_i^h to N_i for given y_i^w . In the N_i — v_i^h plane, we can derive a set of iso- y_i^w zero-profit (ZP), iso- y_i^w incentive-wage (IW) and iso- y_i^w constant interest (RR) curves¹⁰. The iso- y_i^w RR curve is given by (13).

Around the equilibrium, we obtain

$$\frac{dv_i^h}{dN_i} \Big|_{\text{ZP}} = \frac{-\Lambda_i \beta \zeta_1 v_i^h}{(1 + \tau)v_i^h - \Lambda_i \beta \zeta_2 (y_i^w / v_i^h)} = -\left(\frac{v_i^h}{N_i}\right)$$

¹⁰The iso- y_i^w ZP and iso- y_i^w IW curves are related to, but are distinct from, the reduced-form ZP and reduced-form IW curves used in the previous subsection.

and so the iso- y_i^w zero-profit curve is negatively-sloped in this plane.¹¹ The slope of the iso- y_i^w incentive-wage curve is

$$\left. \frac{dv_i^h}{dN_i} \right|_{IW} = \frac{\Lambda_i \beta [\zeta_1 + N_i \zeta_{11} + (\frac{y_i^w}{v_i^h}) \zeta_{21}]}{(1 + \tau) + \Lambda_i \beta [\zeta_2 + N_i \zeta_{12} + (\frac{y_i^w}{v_i^h}) \zeta_{22}] (\frac{y_i^w}{v_i^{h2}})},$$

which is positive. Finally, we note from the Blanchardian equation, (13), that we can express an iso-interest curve denoted RR in Figure 4 as a rectangular hyperbola. Its slope is given by

$$\left. \frac{dv_i^h}{dN_i} \right|_{RR} = -\left(\frac{v_i^h}{N_i}\right).$$

Consider an initial situation in the absence of the tax-subsidy scheme. Figure 4 shows that, initially, the three iso- y_i^w curves intersect at a point E_0 ¹². (This point is drawn to correspond to E_0 in Figure 3.) What is the effect of introducing a flat subsidy financed by a common flat rate payroll tax? At the initial level of y_i^w , inspection of (25) and (26) reveals that the iso- y_i^w ZP and iso- y_i^w IW curves are shifted away from point E_0 . (Inspection of (13), however, reveals that the iso- y_i^w Blanchardian RR curve does not shift at the initial level of y_i^w .) For a worker of type- i whose productivity level is sufficiently below the mean, the new intersection point between the iso- y_i^w ZP and iso- y_i^w IW curves occurs at a point to the northeast of point E_0 , namely point E'_1 —a sort of Marshallian “short run” where wealth is held constant¹³.

¹¹We have used the result that at the equilibrium, the denominator can be simplified using the relationship describing the incentive-wage condition, re-expressed as $(1 + \tau)v_i^h - \Lambda_i \beta \zeta_2 (y_i^w / v_i^h) = \Lambda_i \beta \zeta_1 N_i$.

¹²Notice that around an equilibrium, the derivatives $dv_i^h/dN_i|_{ZP}$ and $dv_i^h/dN_i|_{RR}$ describing the slopes of the iso- y_i^w ZP and iso- y_i^w RR curves, respectively, are equal.

¹³At the original N_{i0} , and evaluated around the initial zero-subsidy equilibrium, we obtain the algebraic vertical shift of the iso- y_i^w ZP curve as $dv_i^h/ds^F|_{s^F=0} = v_i^h [1 - (v_i^h/v_{\text{mean}}^h)] [\Lambda_i \beta \zeta_1 N_i]^{-1}$, which is positive for $\Lambda_i < \Lambda_{\text{mean}}$. A similar calculation for the iso- y_i^w IW curve yields $dv_i^h/ds^F|_{s^F=0} = -(v_i^h/v_{\text{mean}}^h) \{1 + \Lambda_i \beta [\zeta_{12} N_i + \zeta_{22} (y_i^w / v_i^h) + \zeta_2] (y_i^w / v_i^{h2})\}^{-1}$, which is negative for each type i worker.

Long-run general equilibrium is then restored by an upward adjustment of y_i^w as the workers, finding their human wealth increased, raise their supply of nonhuman wealth. From (13), it is apparent that, at any given N_i , and in particular at N_{i0} , a one-percent increase in y_i^w leads to a one-percent increase in v_i^h . In the Appendix, we show, however, that a one-percent increase in y_i^w shifts up the IW curve in the $N_i-v_i^h$ plane by less than one percent. (The rise of the nonwage income relative to the wage increases the effectiveness of raising the wage to combat quitting. So the optimal incentive wage rises. However, it rises by less than the proportion by which nonwage income rises due to the diminishing marginal effectiveness of the incentive pay in combatting quitting.) This result implies that the long-run capital market equilibrium as well as labor market equilibrium can be restored only at an employment rate above N_{i0} . If the subsidy were proportional, and only then, the subsidy would be neutral like the proportional tax. At the same time, the rise of y_i^w , shifts down the iso- y_i^w ZP curve as the demand wage is reduced on account of the rise in the quit propensity. This downward shift of the ZP curve acts in conjunction with the above mentioned upward shift of the IW curve to cause a fall in N_i relative to the “short-run” level denoted N'_{i1} in Figure 4. Hence the upward adjustment of nonhuman wealth does act to moderate the gains to employment achieved in the “short run” but not to eliminate those gains in the long run. In Figure 4, we show that the new post-tax-subsidy long-run equilibrium point settles at E_1 , which corresponds to point E_1 in Figure 3, with both N_i and v_i^h raised for the low- Λ_i worker.

What is the effect of the tax-subsidy policy on the aggregate supply of wealth in the small open economy? To reach an answer, we note that an expression for the average supply of wealth per member of the type- i workforce is obtained by substituting $y_i^w \equiv (r^* + \theta)W_i$ in (13):

$$(27) \quad W_i = \left(\frac{v_i^h N_i}{r^* + \theta} \right) \left[\frac{r^* - \rho}{\theta + \rho - r^*} \right].$$

As argued in Blanchard (1985), if r^* were larger than $\theta + \rho$, so that $r^* - \rho > \theta$, individual consumption would increase at a rate higher than the rate of death; and aggregate consumption would therefore increase forever. To have a well-defined steady state, we exclude this case.

Using (27), the aggregate supply of wealth can be expressed as

$$(28) \quad W \equiv \int_{\underline{\Lambda}}^{\infty} W_i f(\Lambda_i) d\Lambda_i = \left[\frac{r^* - \rho}{(r^* + \theta)(\theta + \rho - r^*)} \right] \int_{\underline{\Lambda}}^{\infty} v_i^h N_i f(\Lambda_i) d\Lambda_i.$$

But since $v_i^f = (1 + \tau)v_i^h - s^F$, budget balance implies

$$(29) \quad \begin{aligned} & \int_{\underline{\Lambda}}^{\infty} v_i^f N_i f(\Lambda_i) d\Lambda_i - \int_{\underline{\Lambda}}^{\infty} v_i^h N_i f(\Lambda_i) d\Lambda_i = \\ & \int_{\underline{\Lambda}}^{\infty} \tau v_i^h N_i f(\Lambda_i) d\Lambda_i - \int_{\underline{\Lambda}}^{\infty} s^F N_i f(\Lambda_i) d\Lambda_i = 0. \end{aligned}$$

Using (29) in (28), we get

$$(30) \quad W = \left[\frac{r^* - \rho}{(r^* + \theta)(\theta + \rho - r^*)} \right] \int_{\underline{\Lambda}}^{\infty} v_i^f N_i f(\Lambda_i) d\Lambda_i.$$

Around the zero-subsidy equilibrium, v_i^f/Λ_i and N_i are identical across all types of workers. Thus, (30) can be re-expressed as

$$(31) \quad W = \left[\frac{r^* - \rho}{(r^* + \theta)(\theta + \rho - r^*)} \right] \left(\frac{v_i^f}{\Lambda_i} \right) N_i \int_{\underline{\Lambda}}^{\infty} \Lambda_i f(\Lambda_i) d\Lambda_i.$$

When the labor-cost elasticity of labor demand exceeds unity, the tax-subsidy policy raises $(v_i^f/\Lambda_i)N_i$ for all type- i workers¹⁴. Consequently, the aggregate supply of wealth is increased.

To determine how the tax-subsidy scheme affects the small open economy's net external asset position, note that the total value of net external

¹⁴In terms of Figure 2, observe that the hyperbola passing through the post-subsidy intersection point between the IW and ZP curves is to the north-east of the original hyperbola.

assets is given by the difference between the aggregate supply of wealth and the total value of domestic assets (which consists here of domestic “capital”):

$$(32) \quad \int_{\underline{\Lambda}}^{\infty} F_i f(\Lambda_i) d\Lambda_i \equiv W - \int_{\underline{\Lambda}}^{\infty} \Lambda_i \beta N_i f(\Lambda_i) d\Lambda_i.$$

Suppose that the small open economy is initially neither in a net creditor nor net debtor position. It is apparent that the tax-subsidy policy, through raising N_i and the product $(v_i^f/\Lambda_i)N_i$, raises both W and the second term on the righthand side of (32). However, it raises the second term of the righthand side of (32) by more. Consequently, the small open economy’s net foreign asset position moves from zero to negative as a result of the tax-subsidy policy¹⁵.

That the economy ends up in a net debtor position in the new steady-state can be explained as follows: The tax-subsidy scheme generates an increase of total consumption demand, fueled by the higher expected earnings of workers. But consumption supply is not increased as much since each employee devotes a larger fraction of his time to training rather than production activity as a result of the tightening of the labor market. A decrease of net external assets (increased overseas debt) serves to trim consumption demand and boost consumption supply to reequat the two.

Graduated wage subsidy

With a graduated wage subsidy scheme, $v_i^h = (v_i^f + S(v_i^f))/(1 + \tau)$ where $S'(v_i^f) < 0$ and $|S'(v_i^f)| < 1$. Hence, with $dv_i^h/dv_i^f = (1 + S'(v_i^f))/(1 + \tau)$, we write the incentive-wage condition as

$$(33) \quad \frac{\beta[\zeta_1(N_i, \Psi(r^* - \rho, N_i))N_i + \zeta_2(N_i, \Psi(r^* - \rho, N_i))\Psi(r^* - \rho, N_i)]}{\Lambda_i[1 + S'(v_i^f)]},$$

¹⁵When the labor-cost elasticity of labor demand is less than one, the aggregate supply of wealth would actually fall. The tax-subsidy scheme would then lead to an even larger decline in the net foreign asset position.

whose slope is given by

$$\left. \frac{d(v_i^f/\Lambda_i)}{dN_i} \right|_{\text{IW}} = \frac{\beta[\zeta_{11}N_i + \zeta_{12}\Psi_2N_i + \zeta_1 + \zeta_{21}\Psi + \zeta_{22}\Psi_2\Psi + \zeta_2\Psi_2]}{1 + S'(v_i^f) - [(v_i^f + S(v_i^f))/(1 + S'(v_i^f))]S''(v_i^f)}.$$

If we assume that $|S''(v_i^f)|$ is not too large, the wage curve continues to be upward-sloping in the Marshallian plane.

Comparing the wage curve under the graduated subsidy scheme in (33) with (18) under the flat subsidy scheme, we notice that if the same dollar amount is given under both schemes to an arbitrarily chosen type- i worker so that $s^F = S(v_i^f)$, the wage curve is shifted further down under the graduated scheme on account of the term, $[1 + S'(v_i^f)]$, on the righthand side of (33). This is because a graduated subsidy scheme induces firms to moderate wage rates above the bottom to gain a larger subsidy. Thus, under the proviso that the wage curve under a graduated subsidy scheme is not much steeper than the corresponding wage curve under a flat subsidy scheme, there is a further employment-creating effect present in a graduated scheme that is not available in a flat subsidy scheme. Note that a wedge analysis is now more complex since there is a shift of the wage curve in terms of $(1 + \tau)v_i^h (=v_i^f + S(v_i^f))$.

While the tax-graduated-subsidy scheme unambiguously expands equilibrium employment, under what conditions will workers' take-home wages (particularly, low-wage workers' wages) rise? To work toward obtaining the answer, let us express (12) and (33) in the following respective implicit functional forms:

$$(34) \quad N_i^d = \Upsilon(v_i^f); \quad \Upsilon'(\cdot) < 0$$

$$(35) \quad N_i^s = \Gamma\left(\frac{v_i^f + S(v_i^f)}{1 + S'(v_i^f)}\right); \quad \Gamma'(\cdot) > 0.$$

Around the equilibrium for a particular type- i worker,

$$(36) \quad \Upsilon(v_i^f) = \Gamma\left(\frac{v_i^f + S(v_i^f)}{1 + S'(v_i^f)}\right).$$

Let the size of the wage subsidy given to the particular type- i worker be s^* where $s^* = S(v_i^{f*})$, v_i^{f*} being the wage paid by the firm to this particular type- i worker. Around a zero-tax-subsidy equilibrium, the response of v_i^f to a small change in s^* is given by

$$(37) \quad \left. \frac{dv_i^f}{ds^*} \right|_{(36)} = \frac{-\eta_{IW}}{\eta_{IW} + \eta_{ZP} - \left[\frac{v_i^{f*} S''}{(1+S')}\right] \eta_{IW}}.$$

In view of (37), the necessary and sufficient condition for $v_i^f + S(v_i^f)$ to rise is that

$$\left[\frac{v_i^f S''}{(1+S')}\right] \eta_{IW} < \eta_{ZP}.$$

For finite values of η_{ZP} and η_{IW} , we see that this inequality is satisfied if $S'' \leq 0$ as well as $-1 < S' < 0$. For the before-tax wage received by workers not to fall, the scheme should not make the subsidy rise too rapidly as v_i^f is reduced and not make their decline accelerate as v_i^f is increased. These conditions will prevent firms from lowering wage rates too far in order to benefit from a larger subsidy.

Letting $\mathcal{S} \equiv \int_{\underline{\Lambda}}^{\infty} S(v_i^f) f(\Lambda_i) d\Lambda_i$, and using (37), we can show that around a zero-tax-subsidy equilibrium, the following derivative holds:

$$\left. \frac{dv_i^h}{ds^*} \right|_{\tau=0} = \left\{ \frac{\eta_{ZP} - \left[\frac{v_i^{f*} S''}{(1+S')}\right] \eta_{IW}}{\eta_{IW} + \eta_{ZP} - \left[\frac{v_i^{f*} S''}{(1+S')}\right] \eta_{IW}} \right\} - \left(\frac{\Lambda_i}{\Lambda_{\text{mean}}} \right) \left(\frac{d\mathcal{S}}{ds^*} \right),$$

where $d\mathcal{S}/ds^* > 0$. So long as the subsidy does not rise too rapidly as v_i^f is reduced, the above derivative is positive for a worker whose productivity is sufficiently low. Moreover, by designing a subsidy plan such that the subsidy

asymptotically reaches zero as v_i^f is increased (as in the illustrative graduated plan shown below), we ensure that employment is raised throughout the distribution although the expansionary effect is smaller at higher v_i^f .

An illustrative graduated scheme is shown in Table 1 with $S(v_i^f) = A \exp\{-b(v_i^f)^2\}$ where $A = 4.85$ $b = 0.03$. Under this scheme, a worker earning \$4 an hour receives a wage subsidy of \$3 per hour while a worker earning \$10 per hour receives a subsidy of \$0.24 per hour.

Hiring subsidy

Before concluding our analysis of the small open economy, let us examine the effects of a hiring subsidy in our model. Suppose that an ad valorem payroll tax is used to finance a flat hiring subsidy of s^{HF} for each new recruit hired. It is straightforward to show that our two fundamental equations giving the reduced-form ZP and IW schedules become, respectively,

$$(38) \quad \frac{v_i^f}{\Lambda_i} = 1 - \left[\beta - \frac{s^{\text{HF}}}{\Lambda_i} \right] \times [\zeta(N_i, \Psi(r^* - \rho, N_i)) + r^* + \theta]$$

$$(39) \quad \frac{v_i^f}{\Lambda_i} = \left[\beta - \frac{s^{\text{HF}}}{\Lambda_i} \right] \times [\zeta_1(N_i, \Psi(r^* - \rho, N_i))N_i + \zeta_2(N_i, \Psi(r^* - \rho, N_i))\Psi(r^* - \rho, N_i)].$$

In Figure 5, we show that the policy shifts up the ZP curve but shifts down the IW curve leading to an unambiguous expansion of equilibrium employment but possible decline of the product wage, v_i^f . (In contrast, under both the flat and graduated subsidy plans, the before-tax wage of the workers, $v_i^f + s_i$, unambiguously rises.) The take-home wage would accordingly fall further as the payroll tax is applied. Moreover, the uncertainty regarding v_i^h

does not disappear as $\Lambda_i \rightarrow \underline{\Lambda} \rightarrow 0$. This can be seen from the following derivative¹⁶:

$$\left. \frac{dv_i^h}{ds^{\text{HF}}} \right|_{s^{\text{HF}}=0} = (\zeta + \theta) \left[\mu - \left(\frac{\Lambda_i}{\Lambda_{\text{mean}}} \right) \right] + \mu r^* - (1 - \mu) [\zeta_1 N_i + \zeta_2 \Psi]$$

where μ , defined before, lies between zero and one.

III. Incidence in The Closed Economy

The essential task here is to endogenize the steady-state rate of interest. We confine our analysis to a flat subsidy in the closed economy financed by a proportional payroll tax.

For any particular r , our reduced-form ZP and IW curves are written respectively as

$$(40) \quad 1 - \frac{v_i^f}{\Lambda_i} = \beta [\zeta(N_i, \Psi(r - \rho, N_i)) + r + \theta]$$

$$(41) \quad \frac{v_i^f}{\Lambda_i} + \frac{s^F}{\Lambda_i} = \beta [\zeta_1(N_i, \Psi(r - \rho, N_i)) N_i + \zeta_2(N_i, \Psi(r - \rho, N_i)) \Psi(r - \rho, N_i)]$$

where we have again substituted for y_i^w/v_i^h the function $\Psi(r - \rho, N_i)$ obtained from the Blanchardian relationship expressed as

$$(42) \quad r = \rho + \frac{\theta}{1 + (v_i^h/y_i^w) N_i}$$

We note from (40) and (41) that by equating the required incentive wage to the demand wage, we can express the employment rate of any type- i

¹⁶The balanced budget condition with a hiring subsidy simplifies to $r = [(\zeta + \theta) s^{\text{HF}}/v_{\text{mean}}^h]$ around a zero-hiring-subsidy equilibrium, noting that in the steady-state, the hiring rate equals $\zeta + \theta$ for every type of worker.

worker as an implicit function of the interest rate and the subsidy relative to productivity level, namely,

$$(43) \quad N_i = \varepsilon(r; (s^F/\Lambda_i)); \quad \varepsilon_1 < 0; \quad \varepsilon_2 > 0.$$

The function ε is interpretable as the demand for the stock of employees in steady state. The value of the total stock of employees, which are the only form of asset in the closed economy, is $A \equiv \int_{\underline{\Lambda}}^{\infty} \beta \Lambda_i N_i f(\Lambda_i) d\Lambda_i$ since each employee is worth $\beta \Lambda_i$. By (43), A is a decreasing function of the rate of interest:

$$(44) \quad A = \int_{\underline{\Lambda}}^{\infty} \Lambda_i \beta \varepsilon(r; (s^F/\Lambda_i)) f(\Lambda_i) d\Lambda_i.$$

For general equilibrium, the interest rate must equate the value of the assets demanded to the quantity of wealth supplied. As before, an expression for the average supply of wealth per member of the type- i workforce is obtained by substituting $y_i^w \equiv (r + \theta)W_i$ in (42):

$$(45) \quad W_i = \left(\frac{v_i^h N_i}{r + \theta} \right) \left[\frac{r - \rho}{\theta + \rho - r} \right].$$

Excluding the case where $r - \rho > \theta$, we have a well-defined steady state with the righthand side of (45) being unambiguously positive. Observe that the first bracketed term in (45) is simply human wealth per type- i worker, and for a given expected after-tax real wage ($v_i^h N_i$), H_i is decreasing in r . On this account, W_i falls as r rises. On the other hand, a rise of r has a positive effect on W_i on account of the second bracketed term, W_i/H_i .

The total supply of wealth per worker is given by $W \equiv \int_{\underline{\Lambda}}^{\infty} W_i f(\Lambda_i) d\Lambda_i$. Using (45), we obtain

$$(46) \quad W = \left[\frac{r - \rho}{(\theta + \rho - r)(r + \theta)} \right] \int_{\underline{\Lambda}}^{\infty} v_i^h N_i f(\Lambda_i) d\Lambda_i.$$

Assuming that the budget is balanced, we get

$$(47) \quad W = \left[\frac{r - \rho}{(\theta + \rho - r)(r + \theta)} \right] \int_{\underline{\Lambda}}^{\infty} v_i^f N_i f(\Lambda_i) d\Lambda_i.$$

Further using (40) and (43) in (47), we obtain an expression giving us total desired supply of wealth as a function of the rate of interest:

$$W = \left[\frac{r - \rho}{(\theta + \rho - r)(r + \theta)} \right]$$

$$(48) \times \int_{\underline{\Lambda}}^{\infty} \left\{ 1 - \beta \left[\zeta \left(\varepsilon \left(r; \frac{s^F}{\Lambda_i} \right), \Psi \left(r - \rho, \varepsilon \left(r; \frac{s^F}{\Lambda_i} \right) \right) \right) + r + \theta \right] \right\} \varepsilon \left(r; \frac{s^F}{\Lambda_i} \right) \Lambda_i f(\Lambda_i) d\Lambda_i.$$

Suppose that initially the flat subsidy and payroll tax rate are zero. In that case, we note from (40) and (41) that setting $s^F = 0$ implies that N_i and y_i^w/v_i^h are equal across all types of workers. Consequently, the quit rate is initially identical across all types of workers. What is the shape of the supply of wealth curve in the absence of the tax-subsidy? There are two opposing forces. In the general equilibrium, an increase of r lowers the real wage as well as the probability of obtaining employment (as each N_i is reduced); and, as remarked above, it lowers the present value of these expected earnings¹⁷. So human wealth is reduced. However, the second bracketed term in (45) works to increase desired supply of wealth as r rises. At r sufficiently low that W_i is at or near zero, the former effects are outweighed by the latter though at sufficiently high r the opposite may occur. Hence the per worker supply of wealth schedule is upward-sloping initially but at very high r may bend backward in Figure 6. In the same plane, per worker demand for the domestic assets in value terms is downward sloping. In what follows we suppose that the equilibrium r is unique or that only the lowest equilibrium r is empirically relevant.

To see how the tax-subsidy policy affects the rate of interest, it will help to have a sharper characterization of this equilibrium. Since the quit rate

¹⁷Although the increase of r leads directly to a decline in the real demand wage, the fall in N_i acts to lower the quit propensity and hence indirectly acts to offset the fall in wage. We assume that the direct effect dominates.

is equal across all types of workers in the neighborhood of the zero-subsidy equilibrium, we can simplify the equilibrium condition to

$$\begin{aligned}
W &\equiv \left[\frac{r - \rho}{(\theta + \rho - r)(r + \theta)} \right] \\
&\times \left\{ 1 - \beta \left[\zeta \left(\varepsilon(r; \frac{s^F}{\Lambda_i}), \Psi(r - \rho, \varepsilon(r; \frac{s^F}{\Lambda_i})) \right) \right] + r + \theta \right\} \int_{\underline{\Lambda}}^{\infty} \varepsilon(r; \frac{s^F}{\Lambda_i}) \Lambda_i f(\Lambda_i) d\Lambda_i \\
(49) &= \int_{\underline{\Lambda}}^{\infty} \Lambda_i \beta \varepsilon(r; (s^F / \Lambda_i)) f(\Lambda_i) d\Lambda_i \equiv A.
\end{aligned}$$

The equilibrium r is therefore given by

$$\begin{aligned}
&\left[\frac{(r - \rho)}{(\theta + \rho - r)(r + \theta)} \right] \\
(50) \quad &\times \left\{ 1 - \beta \left[\zeta \left(\varepsilon(r; \frac{s^F}{\Lambda_i}), \Psi(r - \rho, \varepsilon(r; \frac{s^F}{\Lambda_i})) \right) \right] + r + \theta \right\} = \beta.
\end{aligned}$$

Thus we see that a tax-subsidy policy involving a small change in s^F financed by a proportional payroll tax has ultimately an influence on the interest rate only via its influence on the quit rate. The effects of introducing a small subsidy are as follows: At the original r , the subsidy, in expanding the demand for employees of all types, shifts the domestic asset demand schedule in Figure 6 to the right. (See the righthand side of (49).) As workers, finding the probability of obtaining employment improved, step up their saving accordingly, the supply of wealth schedule is also shifted to the right. (See the lefthand side of (49).) In fact, both rightward shifts are equal in magnitude, leaving the interest rate unchanged. But the rise in each N_i acts in (49) to tighten the labor market of each type- i worker. The resulting increase in the propensity to quit reduces the demand wage. This leads to a leftward shift of the supply of wealth schedule causing the interest rate to rise.¹⁸ (When the zero profit curve is horizontal, however, this effect would

¹⁸In the Appendix, we calculate the extent of the horizontal shifts of the total supply of wealth and total asset demand schedules. When the labor cost elasticity of labor demand is greater than one, the net shift of the total supply of wealth schedule is rightward. When this elasticity is less than one, the net shift is leftward.

be zero.) But clearly this effect can only moderate the net expansionary effect on employment of low- Λ_i workers. To show this we may calculate the total derivative of $\varepsilon(r, s^F)$ evaluated at a low Λ_i with respect to s^F .

Taking the total derivative in (49), we obtain

$$(51) \quad \frac{dr}{ds^F} = \frac{(\zeta_1 + \zeta_2 \Psi_2) \varepsilon_2}{\left\{ \left[\frac{\theta(\theta + \rho)}{(r - \rho)^2} - \zeta_2 \Psi_1 \right] - (\zeta_1 + \zeta_2 \Psi_2) \varepsilon_1 \right\} \Lambda_{\text{mean}}},$$

where $\Lambda_{\text{mean}} = \int_{\underline{\Lambda}}^{\infty} \Lambda_i f(\Lambda_i) d\Lambda_i$ is the mean productivity level. In the Appendix, we show that a necessary condition for the aggregate supply of wealth schedule to be positively sloped under the proviso that the labor-cost elasticity of labor demand exceeds unity is that $[\theta(\theta + \rho)/(r - \rho)^2] > \zeta_2 \Psi_1$.¹⁹ Hence the tax-subsidy scheme raises the rate of interest. To prove that for low- Λ_i workers, the rise of r only moderates but does not overturn the expansionary employment effect of s^F , we calculate the following derivative:

$$(52) \quad \begin{aligned} \frac{dN_i}{ds^F} &= \varepsilon_1 \left(\frac{dr}{ds^F} \right) + \frac{\varepsilon_2}{\Lambda_i} \\ &= \frac{-\left(\zeta_1 + \zeta_2 \Psi_2\right) \varepsilon_1 \varepsilon_2 \left(\Lambda_i^{-1} - \Lambda_{\text{mean}}^{-1}\right) + \left[\frac{\theta(\theta + \rho)}{(r - \rho)^2} - \zeta_2 \Psi_1\right] \varepsilon_2 \Lambda_i^{-1}}{\left\{ \left[\frac{\theta(\theta + \rho)}{(r - \rho)^2} - \zeta_2 \Psi_1 \right] - (\zeta_1 + \zeta_2 \Psi_2) \varepsilon_1 \right\}}. \end{aligned}$$

In the homogeneous case, the tax-subsidy scheme unambiguously expands employment for everyone. In the heterogeneous case, all workers whose Λ_i is either below or equal to the mean must find their employment expanded. It is straightforward to obtain an expression for the derivative of v_i^h with respect to s^F :

$$(53) \quad \left. \frac{dv_i^h}{ds^F} \right|_{s^F=0} = 1 - \frac{\Lambda_i}{\Lambda_{\text{mean}}} - \Lambda_i \beta \left[(\zeta_1 + \zeta_2 \Psi_2) \frac{dN_i}{ds^F} + \zeta_2 \Psi_1 \frac{dr}{ds^F} \right].$$

¹⁹The inequality could (but need not necessarily) be reversed when the labor cost elasticity of labor demand is less than one. If the inequality is reversed but the aggregate supply of wealth schedule remains positively sloped, there is an increased upward pressure on the interest rate. If the inequality is reversed and the aggregate supply of wealth becomes negatively sloped, there is the theoretical possibility that the rate of interest is lowered as a result of the tax-subsidy scheme. In such a case, employment is unambiguously expanded for workers of all types.

We see that for a worker whose Λ_i is sufficiently low, his v_i^h will rise as well.

IV. Concluding Remarks

The pay open to the less advantaged is now so inadequate for meaningful self-support and their participation rates and job attachment, especially among men, are now so far from integrating poor communities in the nation's business life that, arguably, any remedy will require novel intervention. (If the goal is now far, just raising the level of familiar instruments may suffice to reattain it, but if the goal is far, designing de novo a more tailored instrument may be cheaper.) Any such innovation, however, may obey the law of unintended consequences, since we do not know the scale and perhaps the nature of all the effects. This uncertainty leads to hesitation and disagreement over the intervention to select. An investment in education that would hypothetically restore low-end wages to their late-1970s level has been calculated to cost nearly two trillion dollars (Heckman, 1993). But the radical uncertainty over exactly what education reforms and expenditures to make may be a bigger drawback (along with the needed one-generation lead-time).

The employment subsidy instrument has the advantage that economists are familiar with the workings of corrective taxes and subsidies—but mainly at the partial-equilibrium level of the individual industry. Massive and perhaps permanent low-wage employment subsidies would not likely prove an exception to the law of unanticipated effects. This paper has been addressed to the doubts over such subsidies that might arise at the level of general equilibrium. Is it theoretically possible in the context of our model of the natural rate that the rise of the wage rate relative to nonwage income initially achieved by the subsidies—recall that the increased payroll tax rate is ultimately neutral for that ratio—will induce worker-savers to build up nonwage income relative to the wage rate until incentive wages have been driven up and the demand-wage rate driven down by enough to nullify the expansion of

employment? As the paper has shown, the adjustment of wealth in the small open economy does act to moderate the expansion of employment achieved by the subsidies in the “short run” but in the long run employment is increased throughout the distribution. In the closed-economy case, the interest rate is pulled up, which moderates employment expansion. Nevertheless, employment unambiguously expands for all workers whose productivity level is below the mean.

Other uncertainties must be left for future work. One of these, obviously, is the net budgetary cost of wage subsidies. In principle, employment subsidies could be targeted on groups who, if their employment were not subsidized, would otherwise cost the government as much or more in public support—single parent, generally mothers, with dependent children, for example. In America, however, it may be the increased difficulty of self-support and the increased disengagement from business life among *men* that is fundamental, since that may lie behind the rise of single-parenting as well as the rise of crime, violence and drug abuse. And men are not as eligible as women for most welfare outlays. So employment subsidies had better be untargeted. And the argument that their net budgetary cost will be small enough to satisfy taxpayers has to rest on estimates of the indirect savings and revenues achieved when entire poor communities are made self-supporting through work: the savings in welfare, crime prevention, administration of justice, unemployment compensation and other social insurance programs (under existing benefit schedules), and the revenues from the additional collection of income and sales taxes (under existing rates).

An attractive feature of hiring subsidies is that they can be targeted at those potential workers currently depending on unemployment compensation or welfare benefits for their support. So the budgetary savings achieved by stimulating their employment may equal or exceed the gross budgetary outlay for the subsidies. This feature has been used by Dennis Snower in designing a program whereby the unemployed worker creates his own hiring subsidy

by trading in his unemployment benefits in return for a job. We found, however, that subsidies to hiring might actually reduce wage rates at the low end, perhaps, appreciably so, and this would be a serious drawback in the American context where, among the disadvantaged, low wages are as much in need of remedying as depressed employment. Furthermore, jobless American men receive little in entitlements that they could exchange for a job other than their unemployment compensation and those benefits are not long-term and not broad-based.

Uncertainty also hangs over the amount of abuse and fraud that wage subsidy programs would lead to. Hiring subsidies would apparently invite employers to swap employees, perhaps after the spell of unemployment required for eligibility, and to move employees more freely from corporation to corporation under the same parent company—all in order to collect increased hiring subsidies. An advantage of the employment subsidies studied here is that they would not encourage those abuses. However, employment subsidies (and possibly hiring subsidies too) would inspire firms, especially single-proprietor firms, to featherbed the payroll with phantom employees under the names of persons, such as family members, whose silence would be trusted. On balance, it might be advantageous for this as well as other reasons to restrict the subsidies to full-time jobs, to good-sized firms where whistleblowers would be a deterrent, and to limit the subsidies to credits against the firms' tax liabilities. In another sort of abuse, the employer and employee would agree to a reduced wage, which would add to the subsidy earned, and a compensating increase in nonwage benefits, which, if undetected or not counted as compensation, would not add to the subsidy earned. For this reason, a graduated subsidy must decrease slowly with the wage rate so that this temptation is not too strong in relation to the monitoring powers of the tax authorities. Yet another abuse would draw upon the collusion of third parties. To earn increased employment subsidies an employer might reduce the wage rate of employees and compensate them with side jobs above their

normal pay rates at a cooperating firm, which might do the same with the first firm or with other firms. Similarly, under the existing Earned Income Tax Credit program awarding subsidies directly to the taxpayer reporting low earnings, the wage can be reduced and the employee compensated through special discounts obtained from third parties. It may be, however, that such abuses could be deterred by punishing them with the same severity meted out to other kinds of tax fraud.

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Appendix

1. The vertical shifts of the iso- y_i^w ZP and iso- y_i^w IW curves in response to a change in y_i^w are given respectively by

$$\begin{aligned} \left. \frac{y_i^w}{v_i^h} \frac{dv_i^h}{dy_i^w} \right|_{\text{ZP}} &= \frac{-\Lambda_i \beta \zeta_2 (v_i^h / y_i^w)^{-1}}{(1 + \tau) v_i^h - \Lambda_i \beta \zeta_2 (y_i^w / v_i^h)} \\ &= -\frac{\zeta_2}{\zeta_1} \left(\frac{y_i^w}{v_i^h} \right) \left(\frac{1}{N_i} \right) \\ \left. \frac{y_i^w}{v_i^h} \frac{dv_i^h}{dy_i^w} \right|_{\text{IW}} &= \frac{\Lambda_i \beta [\zeta_2 + N_i \zeta_{12} + (y_i^w / v_i^h) \zeta_{22}] (y_i^w / v_i^h)}{(1 + \tau) v_i^h + \Lambda_i \beta [\zeta_2 + N_i \zeta_{12} + (y_i^w / v_i^h) \zeta_{22}] (y_i^w / v_i^h)}. \end{aligned}$$

2. To prove that at given r , the tax subsidy policy shifts the aggregate supply of wealth schedule to the right, we express (48) as $W = \Omega(r, s^F)$. Taking a total derivative through (48) with respect to s^F we obtain

$$\Omega_2 = R \{ [1 - \beta(\zeta + r + \theta)] - \beta(\zeta_1 + \zeta_2 \Psi_2) \varepsilon \} \varepsilon_2$$

where $R = (r - \rho) / [(\theta + \rho - r)(r + \theta)]$. The assumption that the labor-cost elasticity of labor demand exceeds one implies that the reduced-form ZP curve cuts the hyperbola from below in the $N_i - (v_i^f / \Lambda_i)$ plane. The slope of the hyperbola is given by $-(v_i^f / \Lambda_i) N_i^{-1}$, which equals $-[1 - \beta(\zeta + r + \theta)] N_i^{-1}$ around the equilibrium while the slope of the reduced-form ZP curve is given by $-\beta(\zeta_1 + \zeta_2 \Psi_2)$. Hence,

$$[1 - \beta(\zeta + r + \theta)] > \beta(\zeta_1 + \zeta_2 \Psi_2) \varepsilon.$$

Accordingly, $\Omega_2 > 0$.

From (44), we can express total asset demand as $A = \Theta(r, s^F)$. We can obtain the following derivative:

$$\Theta_2 = \beta \varepsilon_2,$$

which is positive. Noting (50), it is clear that $\Theta_2 > \Omega_2$.

3. It is straightforward to show that

$$\Omega_1 = \beta R \left\{ \varepsilon \left[\frac{\theta(\theta + \rho)}{(r - \rho)^2} - \zeta_2 \Psi_1 \right] - \varepsilon_1 R^{-1} [R\varepsilon(\zeta_1 + \zeta_2 \Psi_2) - 1] \right\} \Lambda_{\text{mean}}.$$

Noting (50), the condition

$$[1 - \beta(\zeta + r + \theta)] > \beta(\zeta_1 + \zeta_2 \Psi_2) \varepsilon$$

can be re-expressed as

$$R\varepsilon(\zeta_1 + \zeta_2 \Psi_2) < 1.$$

Thus for Ω_1 to be positive, it is required that

$$\left[\frac{\theta(\theta + \rho)}{(r - \rho)^2} - \zeta_2 \Psi_1 \right] > \frac{\varepsilon_1}{\varepsilon} \left[\frac{R\varepsilon(\zeta_1 + \zeta_2 \Psi_2) - 1}{R} \right] > 0.$$

Hence a necessary condition for the aggregate supply of wealth curve to be positively sloped when the labor-cost elasticity of labor demand exceeds one is that $[\theta(\theta + \rho)/(r - \rho)^2] > \zeta_2 \Psi_1$.

Table 1

Table of Wage Subsidy Schedule: $s = A \exp\{-b(v^f)^2\}$

$A = 4.85, b = 0.03$

v^f	s
4	3.00
5	2.29
6	1.65
7	1.12
8	0.71
9	0.43
10	0.24
11	0.13
12	0.06

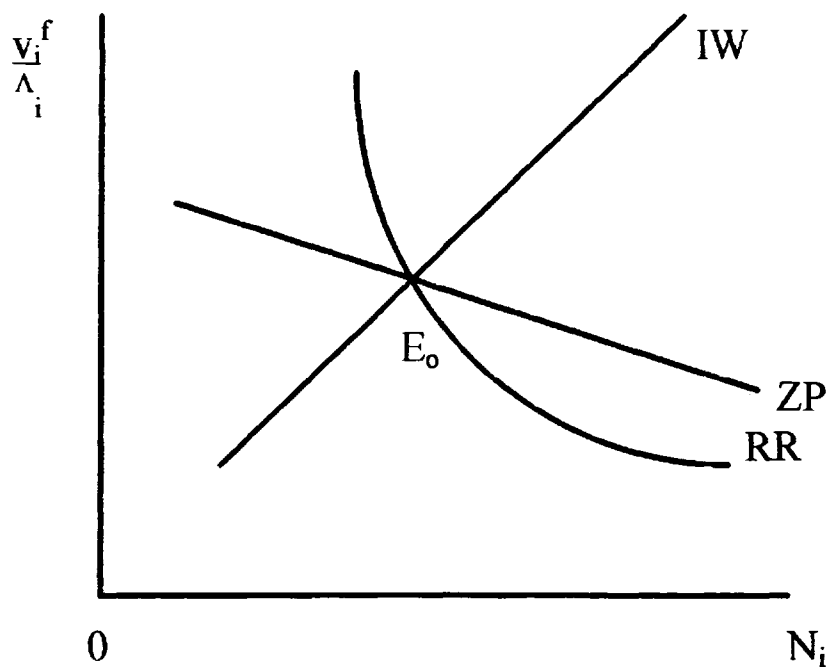


Figure 1
Labor, Product and Capital Market Equilibrium Conditions

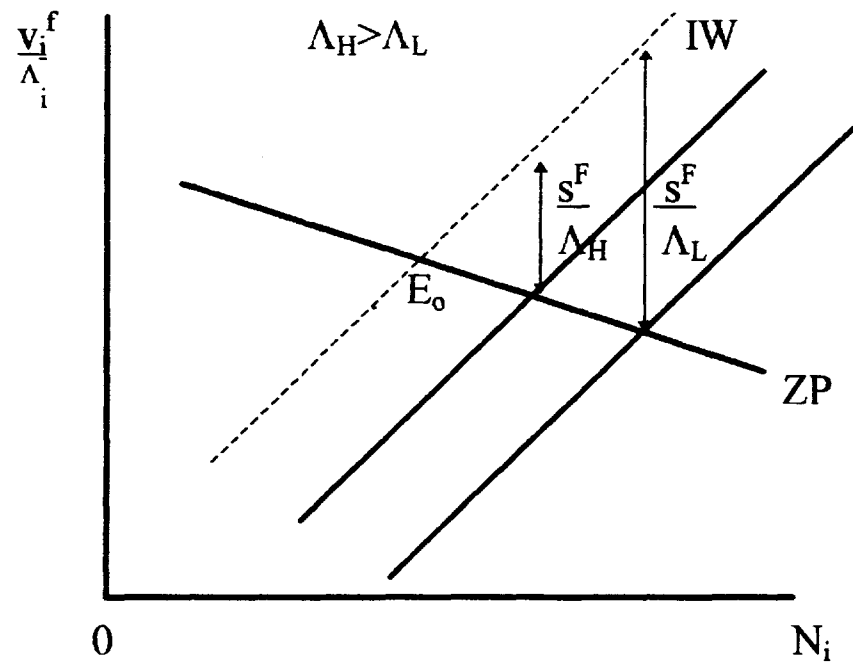


Figure 2
Effects of Flat Subsidy

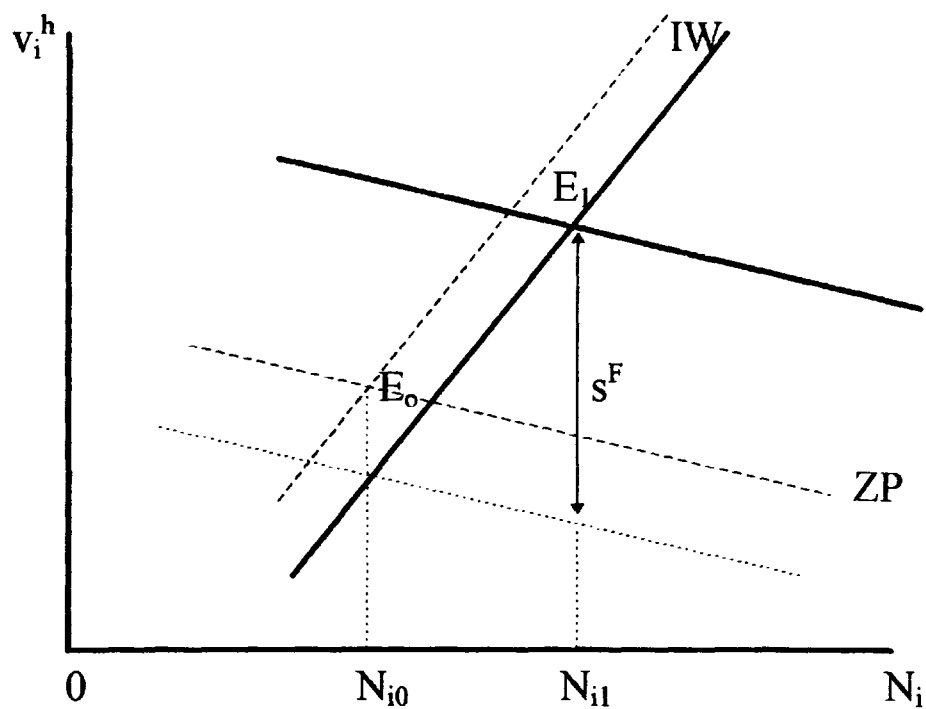


Figure 3

Alternative Characterization Using Reduced Form Curves

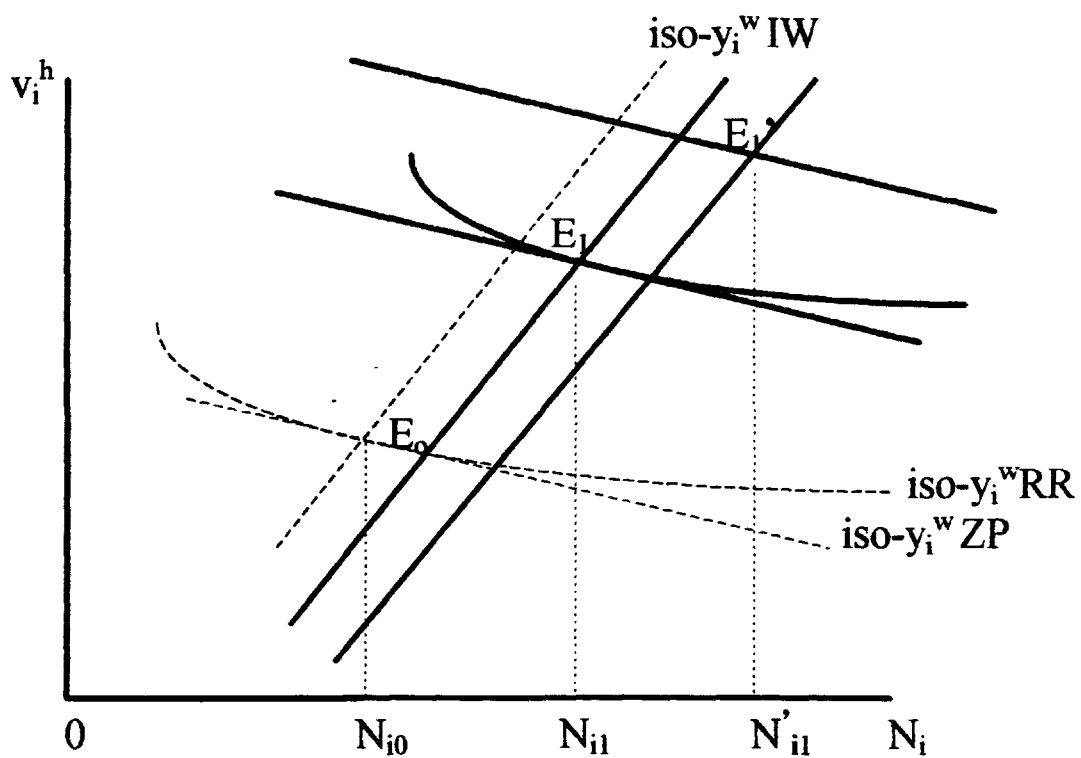


Figure 4

Alternative Characterization Using $iso-y_i^w$ Curves

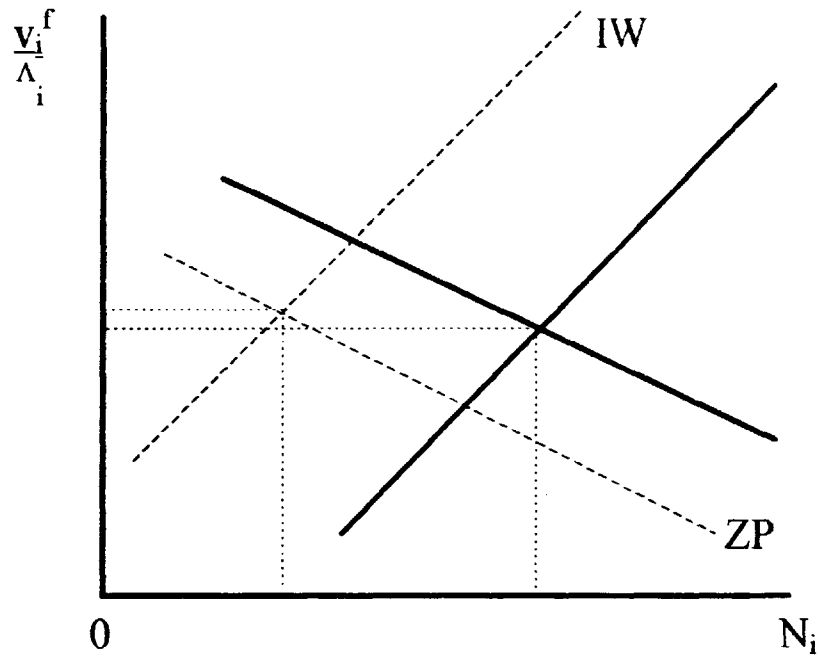


Figure 5
Effects of Hiring Subsidy

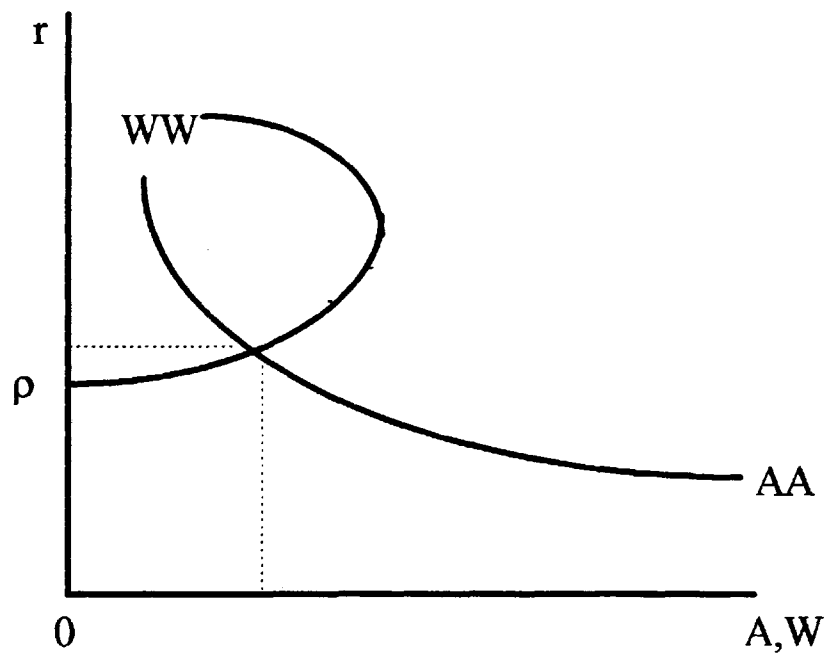


Figure 6
Wealth Supply and Asset Demand

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