Transients from the Birth and Death of Compact Objects

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ABSTRACT

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Astrophysical compact objects — white dwarfs (WDs), neutron stars (NSs), and stellar mass black holes (BHs) — mark the endpoints of normal stellar evolution. Their birth is often associated with dramatic explosions known as core-collapse supernovae (SNe). Such SNe are archetypal “transients” — astronomical events which produce detectable emission for only a limited period of time (measurable over human timescales). This dissertation investigates the astrophysical implications of the formation and destruction of compact objects with particular focus on the transient phenomena that may be produced in such events.

Part I is devoted to the “death” of compact objects by their coalescence with a binary companion. Such compact object binaries are driven towards merger by the extraction of orbital energy in the form of gravitational-waves (GW), and are thus prime targets for current and future GW detectors. In the first two chapters of Part I we consider the merger of a WD with a NS companion, beginning with Chapter 2, in which we explore the nuclearly-reactive accretion flow produced in the aftermath of such mergers and the possible ‘SN-like’ transient it may give rise to. We continue in Chapter 3 by proposing that the late-time evolution of this post-merger accretion disk may result in terrestrial planet formation, broadly consistent with the mysterious “pulsar planets” observed orbiting PSR B1257+12. We shift our attention
in the next couple chapters of this first part of the dissertation to binary NS mergers. In Chapter 4 we address the question of disk formation in the aftermath of the collapse of a rigidly-rotating supramassive NS, which is directly applicable to various models of gamma-ray bursts (GRBs). In Chapter 5 we utilize both GW and electromagnetic signatures of the first observed NS merger GW170817 to place new constraints on the NS equation of state.

Finally, in Part II of this dissertation, we explore the connection between transient phenomena ranging from long- and ultra-long- GRBs, to energetic super-luminous SNe (SLSNe) and fast radio bursts (FRB), and relate these to the “birth” of a rapidly rotating highly-magnetized NS, a millisecond “magnetar”. In Chapter 6 we show that both jetted and thermal transients (namely a GRB and a SLSN) can be powered simultaneously by such magnetars, and explore the various observational implications of this connection. We end with Chapter 7 in which we study the photo-ionization of the medium surrounding a newly born magnetar, discussing the observational signatures related to the escape of this ionizing radiation. We additionally address the propagation of radio waves and the dispersion measure induced by such photo-ionization and apply these to show that FRBs are broadly consistent with having young magnetars as their progenitors.
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2.14 Velocity distribution of the wind ejecta, from the fiducial helium WD model, He\textsubscript{Fid}, evaluated at the simulation end time. The distribution of different isotopes are colored as in Fig. 2.8(a). A solid black curve shows the total mass distribution (summed over all elements). The \(^{56}\text{Ni}\) distribution extends up to the largest velocities (smallest radius) captured by our numerical grid, indicating that we do not resolve the entire \(^{56}\text{Ni}\) outflow, and thus that our model provides only a lower limit on the nickel mass in the ejecta.
2.15 Evolution of the nuclear composition for the model $\text{He}_{\text{Alpha}}$ (colored curves), as in Fig. 2.12, compared to the fiducial model, $\text{He}_{\text{Fid}}$ (grey curves). Different panels correspond to snapshots defined by $t_{\text{He}_{\text{Fid}}} = \left(\frac{\alpha_{\text{He}_{\text{Alpha}}}}{\alpha_{\text{He}_{\text{Fid}}}}\right) \times t_{\text{He}_{\text{Alpha}}}$, where $\left(\frac{\alpha_{\text{He}_{\text{Alpha}}}}{\alpha_{\text{He}_{\text{Fid}}}}\right) = 0.1$, and are equivalent to the panels in Fig. 2.12 for the fiducial model. Note that the composition profiles are qualitatively different in model $\text{He}_{\text{Alpha}}$ due to the density sensitivity of the limiting triple-$\alpha$ reaction. This is in contrast to C/O models, in which scaling the viscosity parameter, $\alpha$, changes (to first order) only the overall timescale of disk evolution (Fig. 2.10).

2.16 Composition profiles of hybrid WD model $\text{CO}_{\text{He2}}$. The background grey curves plot the composition of the fiducial C/O WD model ($\text{CO}_{\text{Fid}}$). This model, which contains only 5\% initial $^4\text{He}$ abundances is comparatively similar to the fiducial C/O composition profiles except for $^{16}\text{O} \alpha$-captures which fuse $^{20}\text{Ne}$ and $^{24}\text{Mg}$ at large radii $\sim 10^9$ cm.

2.17 Nuclear heating rate relative to the viscous heating rate, $|\dot{q}_{\text{nuc}}/\dot{q}_{\text{visc}}|$, for the hybrid WD models $\text{CO}_{\text{He1}}$ (a), and $\text{CO}_{\text{He2}}$ (b). In contrast to previous plots of this quantity for other models (see Figs. 2.7(b), 2.13(b)), in this case the colorbar is logarithmically spaced. The nuclear heating rate exceeds the viscous heating rate by more than an order of magnitude at early times and around the $^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$ burning front ($\sim 10^9$ cm).

2.18 Total ejecta mass of each isotope at the end of the simulation for the C/O WD models (a), and He WD models (b). Tabulated data is also presented in Table E1.

3.1 Temporal evolution of the remnant accretion disk from a WD-NS merger, corresponding to the fiducial model of a $0.6M_\odot$ C/O WD for a viscosity $\alpha = 0.1$, with initial conditions from (alias?). Panels show the outer disk radius (top), surface density (middle), and temperature (panel). Vertical dashed lines indicate important transitions in the disk accretion regime, whereas vertical dotted lines show transitions in the opacity law (which only affect the evolution during the viscously heated radiative phase). The top panel also shows important transition radii $R_{\text{rad}}$, $R_{\text{irr}}$, and $R_{\text{evap}}$ (see description in §3.2.1).
3.2 Local mass and temperature at the presumptive planet forming radius, $R_p = 0.4$ AU, for the fiducial model (solid) and a variation where irradiation of the outer disk only begins once the accretion rate becomes sub-Eddington (dashed line). The grey arrow indicates the temporal evolution direction. The disk first reaches $R_p$ in the super-Eddington irradiated regime. For the fiducial model, the temperature remains constant during this phase (equation 3.22), until the accretion rate drops below $\dot{M}_{\text{Edd}}$ and the disk transitions to the sub-Eddington irradiated regime. In the delayed irradiation model, the disk is viscously heated when it first reaches $R_p$. Incident radiation heats the disk once $\dot{M} = \dot{M}_{\text{Edd}}$ and briefly maintains the disk at a constant accretion rate, after which $\dot{M}$ decreases below $\dot{M}_{\text{Edd}}$ and the solution evolves similarly to the fiducial model. The disk is viscously heated when it first reaches $R_p$. Incident radiation heats the disk once $\dot{M} = \dot{M}_{\text{Edd}}$ and briefly maintains the disk at a constant accretion rate, after which $\dot{M}$ decreases below $\dot{M}_{\text{Edd}}$ and the solution evolves similarly to the fiducial model. The disk is viscously heated when it first reaches $R_p$. Incident radiation heats the disk once $\dot{M} = \dot{M}_{\text{Edd}}$ and briefly maintains the disk at a constant accretion rate, after which $\dot{M}$ decreases below $\dot{M}_{\text{Edd}}$ and the solution evolves similarly to the fiducial model. The thick markings on the horizontal (vertical) axis mark the graphite condensation temperature (PSR B1257+12 combined planetary mass), respectively.

3.3 Mass and temperature conditions when the disk first spreads to the planet formation radius $R_d = R_p = 0.4$ AU in the parameter space of the viscosity ($\alpha$) and wind mass-loss exponent ($p$). Black contours show the local mass at $R_p$, $M_p$ (labeled in units of $M_\oplus$) calculated from equation (3.20) assuming $A = 3$ (see equation 3.21). The local temperature at the same position, $T_p$ is labeled in units of Kelvin and plotted in red (brown) for an irradiation (viscously)-heated disk, as calculated via equations (3.22-3.24). Larger values of $p$ result in strong initial outflows which therefore decrease the available mass at late times, whereas lower values of $\alpha$ decrease the viscous heating and accretion rate, leading to smaller temperatures. The shaded blue regions show the allowed parameter space if a mass of $100M_\odot$ when the temperature first reaches 2000 K is required to produce the observed pulsar-planets assuming a formation efficiency of 8% (3.4). Despite this conservative assumption, a reasonably wide parameter-space satisfies these constraints.

3.4 Observational constraints on NS planetary systems. Pulsar planets of mass $M_p \sin i$ at radii $R_p$ are ruled out with 95% confidence limits in the blue shaded region for a large sample of young pulsars (Kerr et al. 2015). Brown circles show the earth-mass PSR B1257+12 planets considered in this work (Konacki & Wolszczan 2003), while the green square shows a best-fit model for the 4U 0142+61 pulsar debris disk (Wang et al. 2006), a SN fall-back disk candidate. Lines of constant angular momentum are plotted as dashed grey curves. The dearth of observed pulsar planetary systems, $\lesssim 1:150$, is consistent with the expected low rate of WD-NS mergers, $\sim 10^{-2}$ of the core-collapse SN rate.
3.5 Pulsar spin period following WD-NS merger accretion phase, $P_0$, versus the RIAF phase mass-loss exponent, $p$ (equation 3.30). More mass reaches the NS surface for low values of $p$, leading to more significant spin-up, until the NS rotates at breakup frequency $\approx \Omega_k(R_{\text{NS}})$. The dashed red curve indicates the currently observed PSR B1257+12 period $P_{\text{obs}} \sim 6.2$ ms. If PSR B1257+12 has only spun-down since its presumptive initial rapid accretion event, then $P_0$ must be $\lesssim P_{\text{obs}}$, constraining the mass-loss exponent to $p \lesssim 0.4$.

4.1 Sequence of neutron star mass $M$ and spin parameter $a$ for three sample EOSs, illustrating our method for assessing the possibility of disk formation following SMNS collapse. The top portion of the figure shows the maximal mass sequence (triangles/squares/circles) and mass shed limits (small points) for each EOS. Masses are normalized to the maximal value for a non-rotating star corresponding to each EOS. The bottom portion shows the (dimensionless) specific angular momentum of a test particle at the SMNS equator, $j_{\text{e}}/GM$, along the maximal sequence curves. A solid red line denotes the minimal angular momentum required to orbit the resulting Kerr black hole with spin parameter $a$, $j_{\text{isco}}(a)$. According to the criterion (4.1), disk formation is ruled out as long as $j_{\text{e}}$ lies below this red curve. These three EOS are also marked in Figure 4.2.

4.2 Regions of allowed and forbidden disk formation in the EOS parameter space. Dashed purple curves show contours of constant maximum mass for non-rotating neutron stars, while dotted black lines indicate constant radius values for a $1.4M_\odot$ non-rotating star. The green region shows the $2\sigma$ allowed parameter space based on observed neutron star masses (Antoniadis et al. 2013a) (bottom boundary) and constraints on neutron star radius (Steiner et al. 2013) (left and right side boundaries).

5.1 The strength of the red and blue KN signatures of a BNS merger depends on the compact remnant which forms immediately after the merger; the latter in turn depends on the total mass of the original binary or its remnant, $M_{\text{tot}}$, relative to the maximum NS mass, $M_{\text{max}}$. A massive binary ($M_{\text{tot}} \gtrsim 1.3 - 1.6M_{\text{max}}$) results in a prompt collapse to a BH; in such cases, the polar shock-heated ejecta is negligible and the accretion disk outflows are weakly irradiated by neutrinos, resulting in a primarily red KN powered by the tidal ejecta (left panel). By contrast, a very low mass binary $M_{\text{tot}} \lesssim 1.2M_{\text{max}}$ creates a long-lived SMNS, which imparts its large rotational energy $\gtrsim 10^{52}$ erg to the surrounding ejecta, imparting relativistic expansion speeds to the KN ejecta or producing an abnormally powerful GRB jet (right panel). In the intermediate case, $1.2M_{\text{max}} \lesssim M_{\text{tot}} \lesssim 1.3 - 1.6M_{\text{max}}$ a HMNS or short-lived SMNS forms, which produces both blue and red KN ejecta expanding at mildly relativistic velocities, consistent with observations of GW170817.
5.2 The maximum extractable rotational energy of the merger remnant $\Delta T = T_0 - T_\bullet$ (Eq. 5.1) is shown as a dark-blue solid curve for a sample EOS. Vertical dashed curves demarcate the range of baryonic remnant masses $M_{\text{rem}}^p$ for which the immediate post-merger compact object is a stable NS, SMNS, HMNS, or a BH (prompt-collapse). A horizontal red dashed curve shows the maximal energy transferred to the environment of the merger consistent with EM observations of GW170817 for the GRB and KN emission. The parameter space where $\Delta T \gg E_{\text{EM}}$ is thus ruled-out. The prompt-collapse scenario is also ruled out (see text), such that $M_{\text{rem}}^p$ is constrained within an ‘allowed’ region shown by red arrows. The grey curve shows the remnant mass probability distribution function (Eq. 5.4), and the consistency is the integral over this distribution within the allowed region (Eq. 5.6 shaded-gray area).

5.3 Parameter-space of external dipole magnetic field $B_d$, responsible for EM spin-down (Eq. 5.2), and internal toroidal field $B_t$, which can deform the NS causing GW-driven spin-down. Contours show the spin-down timescale (blue) and ratio of EM to GW extracted spin-down energy (black) calculated by integrating equations for the spin frequency $\Omega$ and misalignment angle $\chi$ as a function of time [Cutler & Jones 2001; Dall’Osso et al. 2009]. The region where GWs could dominate over EM emission falls below the thick black curve, but this region is: (a) susceptible to magnetic instabilities (grey shaded areas Braithwaite 2009; Akgün et al. 2013), (b) implies long spin-down timescales $\gtrsim 100$ s at odds with the detection of a GRB only 2 s after the merger, and (c) would produce a strong GW signal.

5.4 Constraints on properties of the NS EOS — radius of a 1.3$M_\odot$ NS, $R_{1.3}$, and maximal non-rotating gravitational mass, $M_{\text{max}}^g$ — based on joint GW-EM observations of GW170817. Different EOS are represented as points, the color of which corresponds to the consistency of the given EOS with observational constraints. The similarly colored diagonal curves represent polytropic EOSs of index $n$, while the grey shaded regions to the bottom right are ruled out by the requirement of causality (see text). Clearly, a low NS maximal mass is preferred due to constraints ruling out SMNS formation. The background grey curve shows the cumulative probability distribution function that the maximum mass $M_{\text{max}}^g$ is less than a given value (see text), from which we find $M_{\text{max}}^g \lesssim 2.17 M_\odot$ at 90% confidence. The bottom panel shows masses of observed Galactic NSs, from which a lower limit on $M_{\text{max}}^g$ can be placed (vertical dashed line).
6.1 Schematic diagram (not to scale) showing how the same millisecond magnetar engine can power both a relativistic GRB jet and a SLSN via isotropic radiative diffusion. A magnetar (grey) with a non-zero misalignment between the rotation and magnetic dipole axes develops a striped-wind configuration in a wedge near the equatorial plane. The fraction of the spin-down energy carried by the striped wind is thermalized when the alternating field undergoes magnetic reconnection near the wind termination-shock, heating the pulsar-wind nebula (PWN; yellow). This thermal energy diffuses through the spherical SN ejecta (blue), powering luminous SN emission. By contrast, the spin-down power at high latitudes is channeled into a bi-polar collimated jet (orange; §6.2). Even once the jet has escaped from the star, a fraction of its power will continue to be thermalized at the interface between the jet and the ejecta walls, driving a hot mildly-relativistic wind of velocity $v_w$. Thermal radiation from this wind may give rise to relatively isotropic optical/UV emission viewable off the jet axis, producing a pre-maximum peak in the light curves of SLSNe (§6.4.2).

6.2 Fraction of the spin-down luminosity of the magnetar available for powering an ordered, magnetically-dominated jet $f_j$ (solid red; equation 6.5) versus the complementary fraction $f_{th} = 1 - f_j$ (dashed black) which is thermalized due to forced reconnection in the striped wind, shown as a function of the misalignment angle between rotation and magnetic axes, $\alpha$. $f_{th}$ is well approximated by equation 6.6 (solid blue). In this model, a mis-aligned magnetar can simultaneously power both a luminous SN and a jetted GRB.

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Chapter 1

Introduction

I begin by deconstructing the title of this dissertation, breaking down and elaborating on each of its components. I first give a brief introduction to compact objects, followed by a discussion of the “death” of such objects through mergers, in which two compact objects in a binary orbit coalesce. I continue by giving an overview of transient astrophysical phenomena, and finish by outlining the structure of this dissertation.

Much more detail and references can be found within the introduction sections of each chapter in this thesis. This current introduction chapter is therefore intended to give a broader overview for the untrained reader, and derive a few basic and useful results that would normally not be included in published articles. This is by no means exhaustive or comprehensive, rather a small collection of things I have found relevant and interesting. The interested reader is invited to learn more from the many useful and canonical textbooks covering such topics, including: Shapiro & Teukolsky (1986) for compact object physics, Frank et al. (2002) for the physics of accretion, Rybicki & Lightman (1986) for EM radiation in
astrophysical context, [Hansen et al. (2004)] and [Kippenhahn et al. (2012)] for stellar structure and evolution, and [Armitage (2013)] for the physics of planet formation.

1.1 Astrophysical ‘Compact Objects’

As with most things in the realm of Astrophysics, ‘compact objects’ are by no means “small” judging by any day-to-day human scales. Indeed, the term refers to gravitationally-bound stellar mass objects, i.e. ones with masses comparable to that of the Sun, $M_{\odot} \simeq 1.99 \times 10^{33}$ g. The naming therefore does not reflect their mass, but rather their density. While the average density of the Sun is similar to that of water, only $\sim 1$ g cm$^{-3}$, the density inside compact objects such as white dwarfs (WDs), whose typical size is comparable to the Earth, may reach $\sim 10^6$ g cm$^{-3}$, and inside neutron stars (NSs), which have radii of order 10 km still much higher, $\sim 10^{15}$ g cm$^{-3}$.

Matter behaves quite differently at such extreme densities, as parameterized via the equation of state (EOS) relating the local thermodynamic variables such as pressure, density and temperature. WDs are supported against gravitational collapse through the pressure provided by degenerate electrons. While this EOS for WDs is well understood and easily computable, the EOS in the interior core of NSs is not well known — densities are high enough in this regime that free nucleons become degenerate and provide some pressure support, however it is clear that an ideal gas of non-interacting nucleons cannot provide sufficient support to explain observed NS masses, and therefore repulsive nucleonic interactions must become important at such densities.

A final class of compact objects, black holes (BHs) represents the most extreme state
of matter and space-time. Such objects are solutions to the Einstein equations for a point source, and possess an event horizon at $r_s = 2GM/c^2$ within which all signals, including any emitted light or radiation, are trapped — hence their naming. This event horizon defines a characteristic dimension for BHs, $r_s \approx 3 \text{ km} \left(M/M_\odot\right)$. Astrophysical stellar mass BHs are thought to form in the core-collapse of the most massive stars, but much larger supermassive BHs with masses $\sim 10^5 - 10^{10} M_\odot$ are also known to exist at the centers of most galaxies and are thought to grow by vigorous accretion as well as hierarchical galaxy mergers.

### 1.1.1 A Brief History — Discovery, Early Theory, and the Galactic Population

The first discovered WD, Sirius B, was found indirectly in 1844 by Friedrich Bessel who noticed that its binary companion, Sirius A, exhibited periodic motion which could be explained by a binary system. Since Bessel did not detect Sirius B (due to its optical faintness compared with Sirius A), he suggested that there was a “dark star” binary companion. In 1863 Alvan Clark was the first to observe Sirius B directly. From spectra acquired later on it became apparent that Sirius B was extremely hot. To explain its faintness therefore required it to have a very small radius ($L \propto R^2 T^4$ for a black-body), hence the terminology white dwarf. It was only much later, during the first half of the 20th century, that theoretical understanding of WDs evolved, through the work of Eddington, Fowler, and Chandrasekhar (for which he received the 1983 Noble Prize in Physics, which, coincidentally, was shared with Fowler for the latter’s contributions to nuclear astrophysics; see [Holberg 2009](#) for a more elaborate historical review of WD discovery).
In contrast, NSs were theoretically proposed long before they had been observed. These stars were first theorized by Baade and Zwicky in a number of bold papers submitted not long after interest in supernovae (SNe) began to grow. The term supernova was first coined and promoted by Zwicky at the same time. This incredible hypothesis, proposed merely two years after the discovery of the neutron, was based on little direct evidence at the time. In fact, Baade and Zwicky themselves were well aware of the boldness of their hypothesis, and wrote “With all reserve we advance the view that a supernova represents the transition of an ordinary star into a neutron star, consisting mainly of neutrons” and “We are fully aware that our suggestion carries with it grave implications regarding the ordinary views about the constitution of stars and will therefore require careful studies”. By means of numerous observational tests, including the association of NSs with young SNe remnants, Baade and Zwicky’s model has now become established as near fact.

Upwards of one thousand NSs have nowadays been observed within our Galaxy. These are primarily, though not exclusively, found as radio pulsars which emit regular periodic radio pulses. The history of pulsars is itself quite interesting — first detected accidentally in 1967 by Jocelyn Bell and Anthony Hewish, who were trying to observe distant quasars, the periodic signal was a complete mystery and even hypothesized as being a form of extraterrestrial communication from some distant civilization, dubbed “little green men”. As more pulsars were discovered, it became clear that these were naturally occurring astrophysical sources, and later understood as originating from rotating NSs. The discovery was so significant, it was awarded the 1974 Noble Prize in Physics.

NSs are also regularly detected when they accrete matter from a binary companion,
and in rare cases, young isolated NSs can be observed through their thermal emission in the X-ray band. Finally, young NSs with extreme magnetic fields are observed through their flaring magnetic activity. This class of NSs is commonly referred to as “magnetars”, which are also observed as steady or pulsing X-ray sources (due to anomalous heating from magnetic dissipation).

1.1.2 Stellar Evolution in a Nutshell and Compact Object Birth

Modern understanding links the birth of compact objects with the end-points of normal stellar evolution. Stars begin their lives from the gravitational collapse of massive gas clouds, “giant molecular clouds”, triggered if the cloud exceeds the so-called Jean’s mass. The subsequent fragmentation of the cloud creates smaller contracting regions, protostars, which heat-up due to the gravitational binding energy released during contraction. Glossing over many physical subtleties, the protostar eventually reaches temperatures sufficient for nuclear fusion to commence. This point marks the end of the “labor” period and birth of the star.

Depending on the initial mass of the star, it may undergo different evolutionary pathways throughout its lifetime. Low mass stars like our Sun burn hydrogen in their core for billions of years till the helium core mass accumulates and hydrogen burning stops. The star reacts by contracting its core which, depending on the star’s initial mass, can heat the core sufficiently to commence helium burning (and also ignite shell hydrogen burning outside the helium core). If this occurs, the star’s envelope swells into the so-called red-giant phase. More massive stars can continue this iterative process — after helium burning (through the triple-α process) creates a sufficiently massive carbon core, helium burning stops, the core
contracts, and (if the star is massive enough) carbon burning may ignite in which case the envelope expands and the star enters the red-supergiant phase. The more massive the star, the higher up the chain of elements it will be able to burn.

Stars with initial mass $\lesssim 8M_\odot$ cannot fuse elements above oxygen. As their carbon/oxygen core contracts, the star’s envelope expands. Since core burning cannot initiate, the core continues to contract until it is eventually halted when its density is so high that electron degeneracy pressure sets in and provides sufficient pressure to support against further collapse. This marks the birth of a carbon/oxygen WD, surrounded by the dissipated envelope of the former star, in what is known as a planetary nebula. We note that there exists a similar lower mass cutoff for forming He WDs, though the observed population of He WDs could not have actually formed in this manner since the timescale for this process to have occurred in isolation (without binary interactions) is longer than the age of the Universe.

Stars that start their lives with mass $\gtrsim 8M_\odot$ are able to continue successively fusing elements heavier than oxygen up the $\alpha$-chain. This process can continue all the way to up to iron, the most tightly bound nucleus, at which point it becomes energetically unfavorable to continue fusion. The star therefore looses its nuclear heat source supporting it against collapse. The mass of the core in these cases is near the Chandrasekhar mass $\sim 1.4M_\odot$ meaning that electron degeneracy pressure cannot support the collapsing core. This results in a further collapse of the core until neutron degeneracy pressure and nucleonic interactions provide enough pressure to stop further collapse. This marks the birth of a NS (usually referred to as a proto-NS while still in this hot, un-relaxed configuration). By processes that
are still widely debated, the onset of neutron-pressure and formation of the proto-neutron star create an inflection and generate an outgoing shock wave that sweeps up and expels the star’s envelope, generating a supernova.

Higher mass stars may in fact fail to explode as SNe, in which case the stellar envelope will rain down on the core increasing its mass far above the maximum supportable mass of a NS and forming a BH instead. Observational searches for the disappearance of extremely massive stars by such a process (sometimes referred to as an “un-nova”) have so far identified only one candidate event, yet its interpretation as a true un-nova is still highly-contentious (Kochanek et al. 2008; Gerke et al. 2015).

Finally, we note that the story described above is grossly over-simplified and neglects all of the complications, intricacies, and dirty details that can conspire to alter these outcomes in various ways. In particular, modern simulations of massive star’s stellar structure and explodability show that the historically accepted paradigm outlined above fails qualitatively, not just quantitatively — not only are the precise mass cuts between various outcomes (WD, NS or BH) uncertain, but the whole notion of a monotonic function relating the initial mass to WDs, NSs or BHs may be mistaken. Instead, these works shows “islands”, or regions in the initial mass parameter-space, where NSs or BHs form, with no apriori easily predictable behavior (e.g. Sukhbold et al. 2016).

1.1.3 the NS Equation of State

In contrast to the case for WDs, the EOS of cold dense nuclear matter governing the structure of NSs is, unfortunately, not well known. Laboratory experiments are only able to probe
densities \( \lesssim \rho_{\text{sat}} \approx 2.6 \times 10^{14} \text{g cm}^{-3} \), the nuclear saturation density of nuclei (equivalent to a number density \( n_{\text{sat}} \approx 0.16 \text{fm}^{-3} \)). However, central densities of NSs can exceed the saturation density by factors of \( \sim 5 \) or more. This can easily be shown by estimating the average density of a typical NS. A NS with characteristic mass \( \approx 1.4M_\odot \) and radius \( \approx 10 \text{ km} \) implies a mean density of \( \langle \rho \rangle \approx 2\rho_{\text{sat}} \), so that the central density within the NS must be even larger.

Furthermore, beta-equilibrium in the NS interior implies a low proton to neutron ratio, far from the roughly symmetric state of nuclear matter accessible in terrestrial laboratory experiments. Finally, theoretical computation of the EOS from first principles is currently not possible due to numerical limitations of lattice QCD at larger densities (the so-called “numerical sign problem”). Conversely, perturbative QCD approaches are doomed at densities of several times \( \rho_{\text{sat}} \) because the coupling constant becomes too large at such ‘low’ energies (a phenomena well known as “confinement”). These circumstances mean that extrapolations of nuclear properties far from their constrained parameter regions are required in modeling NSs.

An interesting effect due to the importance of General Relativity in NSs leads to the fact that there is a maximum allowed NS mass which is supported against gravitational collapse. This is akin to the Chandrasekhar mass for WDs, except that the instability is driven by different physics. For WDs, in which GR is to zeroth-order negligible, the instability is triggered due to EOS effects once the degenerate electrons which provide the star’s pressure support become relativistic and the effective adiabatic index is \( \Gamma = 4/3 \) (here the adiabatic index is defined for polytropic EOSs by \( P \propto \rho^\Gamma \)). In NSs however the instability is driven due
to the fact that the star’s pressure also exerts a gravitational pull, because the gravitational mass in GR is set by the total energy density, not the rest mass density. While the precise value of the maximal mass $M_{\text{max}}$ above which this instability sets in depends on the unknown high-density EOS, the mere existence of the instability is independent of the assumed EOS, in contrast to the WD case.

In recent years, two pulsars with well constrained masses $\approx 2M_\odot$ have been detected \cite{Demorest2010, Antoniadis2013}, putting a clear lower-limit on the maximal mass of a NS, $M_{\text{max}} > 2.01 \pm 0.04 M_\odot$. An upper limit on $M_{\text{max}}$ can be set from causality, i.e. the requirement that the local sound speed inside the star be sub-luminal, however this yields relatively weak constraints, $M_{\text{max}} \lesssim 4.1 M_\odot \left( \frac{\epsilon_0}{\epsilon_{\text{sat}}} \right)^{-1/2}$, where $\epsilon_{\text{sat}} \simeq 150 \text{ MeV fm}^{-3}$ is the energy density at $\rho_{\text{sat}}$ and $\epsilon_0$ is a free-parameter of the so-called “maximally-compact” EOS, and sets a density scale such that $P = 0$ for $\epsilon < \epsilon_0$ and $P = \epsilon - \epsilon_0$ for $\epsilon \geq \epsilon_0$ \cite{Rhoades1974, Koranda1997}.

The two most important NS global quantities from which we expect to constrain the micro-physical EOS are the maximal mass, which is set primarily by the pressure at the highest densities in the NS core, and the “typical” NS radius, which is instead governed by the EOS at roughly $\sim 2\rho_{\text{sat}}$ \cite{Lattimer2001}. Although the radius of a NS really depends on its mass, in practice there is a wide range of masses over which the radius changes very little, i.e. the $M(R)$ relationship for NSs has a $\sim$vertical section, an attribute that arises universally for all hadronic EOSs. Thus, accurate radii measurement of “typical” NSs which lie in this parameter space region will yield strong constraints on the EOS even if their masses are known to lesser precision.
1.2 Compact Object Mergers

There is an observed strong preference for stars to be born in multiplicity, and binary stellar systems are extremely common in the Universe. Furthermore, stars in such binary systems are biased towards having similar masses (as opposed to independent random variables drawn from the initial-mass function). At the simplest level, these facts alone would imply the existence of compact binary systems — stellar systems in which both stars evolved to produce a compact object, similar to the discussion in the previous section. The reality is complicated by the fact that binary stellar systems do not always evolve as independent isolated stars, and interaction induced by mass transfer or common envelope phases can alter the evolutionary track of each star significantly (Sana et al. 2012). Additionally, mass lost during the nebular phase or SN producing one of the compact objects, as well as NS birth-kicks (which can easily exceed $\gtrsim 100 \text{ km s}^{-1}$) can easily unbind the original binary.

Despite these apparent theoretical difficulties in our understanding of the formation of compact binary systems, we know that Nature does produce such systems. Binary WD systems are directly observed in our local Solar neighborhood, WD-NS systems have also been observed where the NS is an active pulsar, and Galactic NS-NS binaries like the double-pulsar system are also known to exist. NS-BH systems, though not yet detected, are hypothesized to exist based on similar grounds.

Such compact binary systems have led to exciting and remarkable discoveries and measurements: WD-NS systems have just recently proved the existence of $\simeq 2M_\odot$ NSs putting strong lower-limits on the maximum mass supportable by a NS; beforehand, the first dis-
covered binary pulsars PSR B1913+16 indirectly confirmed for the first time the existence of gravitational-waves, a discovery which was awarded the 1993 Noble Prize in Physics (Hulse & Taylor 1975); the first direct detection of gravitational waves was made in September 2015, when the Laser Interferometer Gravitational-Wave Observatory (LIGO) discovered two BHs mergering, a groundbreaking discovery culminating decades of efforts and which was awarded the 2017 Noble Prize in Physics; finally, as of August 2017, a merging binary NS system was first discovered directly via its GW emission (Abbott et al. 2017a) and later by its accompanying electromagnetic signature.

Such mergers are driven by the emission of gravitational-waves from the binary system — these waves extract energy and angular momentum from the binary’s orbit which cause the orbit to shrink (this is also known as “hardening” the binary) until the two compact objects eventually merge (by which we mean the two stars either physically come into contact, or one of the objects is disrupted due to tidal forces). In the following section we briefly derive some basic results regarding such GW emission.

### 1.2.1 GW Radiation from a Binary System

At large enough separations the two compact objects orbiting one another can be treated as point particles. Remembering that the lowest order GW radiation term enters at the quadrupole level in the multipolar expansion (the monopole has vanishing time derivative because of mass conservation, while the dipole time-derivative is zero due to momentum conservation) implies a strain

$$ h_{ij} \sim \frac{2G\dot{Q}_{ij}}{c^2r} \quad (1.1) $$
and corresponding gravitational-wave power

\[ P \sim \frac{G}{c^5} \left| \ddot{Q} \right|^2. \quad (1.2) \]

For a binary system of masses \( m_1 \) and \( m_2 \) orbiting with orbital frequency \( \Omega \) at a semi-major axis \( a \), the quadrupole moment is \( Q \sim \mu a^2 \), and this corresponds to

\[ P \sim \frac{G}{c^5} \left( \Omega^3 \mu a^2 \right)^2 \sim \frac{G^4 \mu^2 M^3}{a^5}, \quad (1.3) \]

where \( \mu = m_1 m_2 / M \) is the reduced mass and \( M = m_1 + m_2 \) the total mass of the system, and in the second equality we have assumed a Keplerian orbit (valid at large separation \( a \)) so that \( \Omega = \left( GM / a^3 \right)^{1/2} \). The combination of masses appearing in the equation above is usually called the 'chirp mass', \( M_c = \mu^{2/5} M^{3/5} \). The full analysis reproduces this exact same result with the addition of a prefactor of \( 32/5 \) [Peters & Mathews, 1963].

The GW power is a strongly decreasing function of the binary separation \( a \), so that the inspiral timescale is set by the largest (initial) separation \( a_0 \),

\[ t_{GW} \sim \frac{GM \mu}{a_0 P_{GW}(a_0)} = \frac{5}{32} \frac{a_0^4 c^5}{G^3 \mu M^2} \]

\[ \approx t_{\text{Hubble}} \left( \frac{a_0}{2 \times 10^{11} \text{ cm}} \right)^4 \left( \frac{m}{1.4 M_\odot} \right)^{-3} \left( \frac{q(1 + q)}{2} \right)^{-1}. \quad (1.4) \]

Here \( t_{\text{Hubble}} \approx 13.8 \text{ Gyr} \) is the age of the Universe and \( m_1 \equiv m, m_2 \equiv qm \). Thus, for a typical binary NS system to merger within a Hubble time would require initial separations less than a few \( R_\odot \) (\( \approx 7 \times 10^{10} \text{ cm} \)), indicating that common envelope phases during the
binary evolution are essential in forming observed merging systems.

1.2.2 Implications — \(r\)-process Nucleosynthesis, Gamma-ray Bursts, Kilonovae, Type Ia SNe, and More

GW emission from two \(\sim 30M_\odot\) merging BHs detected on September 14th 2015 by the Laser Interferometer Gravitational-Wave Observatory (LIGO) at Hanford WA and Livingston LA marked dawn of the GW-astronomy era. This first direct detection of GWs garnered the 2017 Noble Prize in Physics awarded to Rainer Weiss, Barry Barish and Kip Thorne for their contributions to LIGO. It also provides the strongest observational evidence yet for the existence of BHs.

To date (as of the end of the LIGO O2 observing run), 5 binary BH merging systems have been detected by LIGO with combined masses ranging between \(\sim 30M_\odot\) to \(\sim 60M_\odot\). These discoveries were, and to some extent still remain, a theoretical surprise due to the large BH masses involved — larger by factors of several than previously known BHs detected as X-ray binaries. The formation mechanisms of such binaries are still widely debated in the literature, with prominent camps advocating for either dynamical assembly of such binaries in dense stellar systems such as globular clusters (e.g. Rodriguez et al. 2016), unique binary stellar evolutionary pathways such as the so-called homogeneous evolution model (e.g. Marchant et al. 2016; Mandel & de Mink 2016; de Mink & Mandel 2016), or formation within active-galactic nuclei disks (Stone et al. 2017; Bartos et al. 2017).

On August 17th 2017 the face of the field would change with the first detection of a binary NS merger. These systems were always envisioned as prime targets for LIGO
which was largely designed with binary NS mergers in mind. One reason these systems received significant attention in comparison to BH mergers is there expected flurry of possible associated electromagnetic (EM) transients.

Binary NS mergers and NS-BH mergers first gained attention when they were proposed by Lattimer & Schramm (1974) as a site of \( r \)-process nucleosynthesis in the Universe. The \( r \)-process, or “rapid” neutron-capture process, is a nucleosynthesis pathway for forming most of the heaviest elements in the Universe (those heavier than iron). First identified in the seminal work of Burbidge et al. (1957), the \( r \)-process requires an immense flux of neutrons so that neutrons may capture onto initial seed nuclei at a faster rate than the \( \beta \)-decay rate (this defines the distinction between “rapid” and “slow” neutron capture). This allows formation of extremely neutron-rich nuclei far from the nuclear “valley of stability” (along which \( N \gtrsim Z \), where \( Z \) is the elemental/charge number and \( N \) the number of neutrons in the nucleus) which subsequently \( \beta \)-decay back to stability after the flux of neutrons “freezes-out”. While the \( s \)-process is widely accepted as occurring in massive stars, the astrophysical origin of the \( r \)-process has been debated for many decades.

Initially thought to occur in standard core-collapse SNe, today we have several lines of evidence pointing towards NS mergers as (at least a dominant) site of \( r \)-process in the Universe, giving credence to the (e.g. Lattimer & Schramm 1974) proposition (Hotokezaka et al. 2015, 2018).

For many years following the early works by Lattimer, Schramm and collaborators (Lattimer & Schramm 1974, 1976, Lattimer et al. 1977), NS mergers remained largely unstudied and under-appreciated. These systems garnered renewed interest following the work of Eich-
ler et al. (1989) which suggested that binary NS mergers may be responsible for the class of transients known as gamma-ray bursts (GRB; note that Paczynski (1986) had previously remarked on the possibility of GRBs arising from NS mergers). This suggestion came at a time when it was still unclear that GRBs were in fact cosmological in origin, and paved the way towards much of our current understanding of GRBs and NS mergers.

Later work investigated optical ‘SN-like’ counterparts associated with NS mergers. The idea rests on the fact that, similar to SNe, NS mergers eject some radioactive material. As this material expands, its optical depth decreases, and photons produced in the ejecta thermal bath heated by the radioactive decays can escape and produce a thermal transient (see following section for more detailed derivation and discussion of the physics of such thermal transients). This idea was first suggested and explored by Li & Paczyński (1998). These authors solved the problem for a parameterized nuclear energy deposition rate and found transients that may peak at luminosities of \( \sim 10^{44} \text{erg s}^{-1} \) on \( \sim 2 \text{ day} \) timescales. This model, though extremely useful in outlining the basic theoretical constructions, was lacking the details of nuclear energy release to provide quantitatively accurate predictions.

The problem was later revisited by Kulkarni (2005) who, neglecting \( r \)-process nucleosynthesis, envisaged radioactive heating by decay of free neutrons, and coined the term “macronova” for the resulting transient. More significant progress was made when Metzger et al. (2010a) re-approached the problem with the addition of \( r \)-process nuclear physics calculations. This produced the first micro-physically motivated nuclear heating rates and allowed a more quantitative estimate of the transient luminosity. Finding a peak luminosity of \( \sim 10^{41} - 10^{42} \text{erg s}^{-1} \), or roughly \( \sim 10^3 \) more luminous than typical novae, Metzger et al.
proposed the term ‘kilonova’ for the resulting transient.

Significant work followed immediately after, with the most valuable contributions given by Barnes & Kasen (2013) and Tanaka & Hotokezaka (2013) who accounted for the high opacity of the NS merger ejecta due to the significant abundance of lanthanides freshly synthesized by the $r$-process (the Metzger et al. 2010a model had previously assumed an opacity of $\kappa = 0.1 \text{cm}^2 \text{g}^{-1}$, similar to iron-peak elements). These works showed that the opacity due to lanthanide elements is significantly larger, $\kappa \sim 10 - 100 \text{cm}^2 \text{g}^{-1}$, due to the more complex atomic shell structure of these elements. The impact on the kilonova light-curve is an increase in the peak time of the transient and a corresponding decrease in peak luminosity, additionally shifting the emission to ‘redder’ (colder) bands.

In parallel with these developments, numerical simulations of the merger process developed rapidly. Starting with Newtonian smooth-particle hydrodynamics (SPH) simulations in the 90s, and progressing all the way to state-of-the-art general-relativistic magneto-hydrodynamic (GRMHD) simulations with approximate neutrino-transport run nowadays.

We have outlined the major implications and historical developments in the theory of binary NS mergers preceding the detection of GW170817, however it is worth pointing out a separate yet related line of inquiry into another class of compact object mergers which has been ongoing in parallel. This is the investigation of binary WD mergers (i.e. WD-WD mergers). The motivation for these investigations is largely due to their possible link to Type Ia SNe (and see next section for further details on SNe classification systems). Due to lack of space, and the fact that this thesis does not directly address binary WD mergers or Type Ia SNe, I omit an historical overview of such studies from this introduction.
Despite the extensive body of literature on both NS-NS and WD-WD mergers, there has been little exploration of the consequences of a WD-NS merger. Such investigations form a significant chapter in this dissertation, where we show that such mergers might synthesize intermediate-mass elements, power rapidly-evolving ‘peculiar-SNe’, be a pathway for forming isolated recycled pulsars, and can produce planetary systems around such pulsars, broadly consistent with the PSR B1257+12 planetary system. Similarly to WD-WD binary systems, close Galactic WD-NS binaries should produce GW emission observable by the proposed Laser Interferometer Space Antenna (LISA) mission, adding an additional motivation for further studying such systems.

1.3 Transients

The field of time-domain astronomy has rapidly evolved over past decades. We have transitioned from a static picture of the stars to a highly-dynamic time-varying image of the sky. Transients, or ephemeral astrophysical signals, are detected across the entire EM spectrum, from gamma-rays, down to X-rays, optical, and radio wavelengths, featuring a wide range of timescales, luminosities and astrophysical origin. Below, I briefly outline and review some of the main classes of transients regularly observed on the sky.

1.3.1 Gamma-ray Bursts

Gamma-ray bursts (GRB) are a class of high-energy transients first detected in the late 60s by cold-war espionage satellites. At the time, US gamma-ray satellites were developed and deployed for military applications, in an effort to uncover any Soviet nuclear tests conducted
in violation of the nuclear test ban treaty of 1963. It soon became apparent however that the satellites were detecting daily bursts coming from directions pointing away from the Earth, ruling out a terrestrial origin.

The physical origin and diversity in the class of gamma-ray bursts sparked heated debates in the decades to come. In the early 90s, BATSE was launched and showed that the sky distribution of bursts is isotropic. This finally led the community to adopt the viewpoint that these bursts were cosmological in origin (if they were Galactic in origin then one would expect an anisotropy of bursts preferentially distributed towards the Galactic plane). This discovery implied that the energy emitted in such bursts must be enormous.

We now know of two distinct sub-classes of GRBs separated primarily by their duration (though they also exhibit a difference in “hardness” — the ratio of high-energy to lower-energy X-ray flux), which is usually characterized via the parameter $T_{90}$ defined as the time since trigger over which 90% of the burst energy is emitted. Short hard GRBs (SGRBs) have observed durations of $\sim 0.1 - 2$ s, while long GRBs (LGRBs) have durations $\sim 10 - 100$ s. A possible new class named ultra-long GRBS (ULGRBs), currently consists of only a few detected events which exhibit durations of order $\sim 10^3 - 10^4$ s (Levan et al. 2014). Though only a few ULGRBs have currently been detected, it is not clear that this means the volumetric rate of ULGRBs is lower than LGRBs since there are significant biases and selection effects making ULGRBs harder to detect.

The total “isotropic-equivalent” energy emitted in $\gamma$-rays, that is the total emitted energy assuming the source emits isotropically, is typically $\sim 10^{49} - 10^{51}$ erg for SGRBs and $\sim 10^{52} - 10^{54}$ erg for LGRBs (Nakar 2007). The bursts are thought to originate from ultra-
relativistic collimated jets, hence the true GRB energy is a factor of $f_b = \Omega/4\pi$ smaller than the isotropic equivalent value. Here $f_b \sim 1/20 - 1/100$ is the beaming fraction, $\Omega \approx \theta^2/2$, and $\theta$ is jet the opening angle.

LGRBs and SGRBs also differ in their host environments — while SGRBs occur in all galaxy types, LGRBs occur preferentially in late-type, low-luminosity (dwarf) low-metalicity, high-specific star-formation rate galaxies and are known to be associated with energetic Type Ic broad-lined SNe (see following subsection for more on SN classification).

The standard theory nowadays posits that LGRBs originate from the collapse of massive, rapidly-rotating stars (MacFadyen & Woosley 1999a), while SGRBs are produced in binary NS or BH-NS mergers (Eichler et al. 1989). The mechanism, or “engine”, responsible for launching the ultra-relativistic jet which produces the GRB is still debated. Early models proposed accretion onto a BH as the energy source (e.g. through the Blandford-Znajek process; Blandford & Znajek 1977), in what became known as the “collapsar” model in the context of LGRBs (MacFadyen & Woosley 1999a). However, certain peculiarities, including extended X-ray emission observed in some GRBs have led to the consideration of an alternative model by which the burst is powered by the rotational energy of a rapidly-rotating highly-magnetized NS, a magnetar (e.g. Metzger et al. 2011). In the part II of this dissertation we explore such magnetar models for LGRBs (and ULGRBs), and we discuss further details of this model in those chapters.

A final remark should be made as to the “afterglow” of GRBs. This signature detected following most LGRBs and some SGRBs is distinct from the “prompt” emission responsible for the observed $\gamma$-rays, and is produced as the ultra-relativistic jet plows into the surrounding
circum-burst medium. Typically detected in X-rays and optical, this afterglow emission is well understood as resulting from a non-thermal population of synchrotron emitting electrons accelerated at the blast-wave shock front (e.g. Sari et al. 1998a). This is conceptually similar to the mechanism powering radio-SNe, which emit preferentially at radio frequencies due to the non-relativistic shock velocities in that case.

I have skipped over any detailed theoretical discussion or modeling of GRB hydrodynamics and emission mechanisms (the prompt-emission mechanism is still debated), but interested readers should consult the review articles by Piran (2004); Nakar (2007); Buborodov & Mészáros (2017) for further details and references.

1.3.2 Supernovae

Known references of SNe observations date back to 185AD, when Chinese astronomers recorded a “guest star” which appeared on the sky and remained visible for several months. Numerous historical accounts of similar nature appeared since then, including of SN 1054 which produced the Crab pulsar and nebular, of SN 1572 which was observed by Tycho Brahe, and of SN 1604, which is the most recent documented SN to occur within the Milky Way, and was observed and studied by Kepler.

The number of detected SNe has increased exponentially with the advent of modern observational facilities, especially wide-field high-cadence facilities specifically designed for transient astronomy. We now know that SNe come in a variety of flavors, or “types”, which have historically been classified based on their observational signatures, most notably their spectra at near-maximum light. The taxonomy of SN classification has become so complex,
that I cannot possibly review it in its entirety in this brief introduction. I will however attempt to outline the most important, and in other cases most relevant to this dissertation, types of SNe.

SNe are most broadly classified into one of two categories: Type II SNe, which show hydrogen in their spectra, and Type I which do not. These categories are then subdivided as shown below. Before delving into such details, we remark on the physical origin of SNe — apart from SN-Ia, which are believed to arise from thermonuclear explosions associated with WDs, most other SNe types are related to the deaths of massive stars in what is known as “core-collapse” events (see discussion in previous section regarding the remarkable early hypothesis by Baade & Zwicky 1934a that such processes produce observed SNe).

As discussed in the previous section, massive stars end their lives by exhausting their nuclear fuel, instigating the collapse of their core and launching an outgoing shock wave which, through some process which is still debated in the literature, is revitalized and sweeps up the surrounding stellar envelope, expelling it in a so-called SN ejecta. An important detail in this process is the fact that higher-mass elements are fused by the extreme post-shock temperatures, where radioactive isotopes such as $^{56}\text{Ni}$ in particular may be synthesized. It is this expanding ejecta which ultimately emits the optical light which we observe as a SN. Some details and the basic physics behind this process are derived in a subsequent subsection, but the main point is that radioactive decay of $^{56}\text{Ni}$ deposits energy in the ejecta at times when its optical depth first allows photons to escape ($\tau = c/v$). It is this process which sets the peak time and luminosity of the resulting SN (although, as explained below, some types of SNe are thought to be powered by interaction of the expanding ejecta with some
circum-stellar material, or by a central engine such as a magnetar instead).

The class of Type I SNe are usually sub-divided according to the following criteria: “normal” Type Ia SNe show strong signatures of Si absorption and have relatively homogeneous and universal photometric properties that differ from Type Ib or Ic SNe; Type Ib SNe show He in their spectra, distinguishing them from Type Ic (in many cases it is difficult to distinguish between the two types and it is common to group the two classes, calling them simply Type Ibc SNe); An important rare-class of the latter SNe are broad-lined Type Ic SNe (or Type Ic-BL; also some times referred to as “hypernovae”) which show very broad emission lines indicating extremely fast expansion velocities $\sim 30,000 \text{ km s}^{-1}$ (at least a factor of $\gtrsim 3$ larger than regular Type Ic SNe). This class is particularly important as Type Ic-BL SNe are known to be associated with LGRBs (Woosley & Bloom 2006), supporting the SN-GRB connection and giving further evidence suggesting that a central engine which deposits significant energy is at the heart of such events.

Type II SNe have unambiguous strong hydrogen absorption lines, and are sometimes divided into several subclasses based on both photometric and spectroscopic criteria: SNe II-P exhibit a plateau in their early light-curve; SNe II-L which show an rapid $\sim$linear decline in their light-curve; SNe IIb, which are in all respects similar to Type Ib SNe, apart from the presence of hydrogen absorption lines. These hydrogen detections are typically limited to only the fastest ejecta layers, indicating only residual amounts of hydrogen in the ejecta, and hinting that Ib and IIb SNe have no significant difference in progenitors; Type IIn SNe which show narrow emission lines indicative of circum-stellar interaction (note that there is also a hydrogen-poor analog of interacting SNe called Type Ibn).
Super-luminous SNe (SLSNe) are a newly discovered rare-class of SNe with peak luminosities $\sim 10^{44} - 10^{45}$ erg s$^{-1}$, ten to a hundred times brighter than normal SNe (Quimby et al. 2011; Gal-Yam 2012). Like regular SNe, these events are classified into hydrogen-poor Type I SLSNe and hydrogen-rich Type II SLSNe (often called SLSNe-I and SLSNe-II for shorthand). These SNe are so bright that they cannot be powered by radioactivity — even if one assumed that the entire ejecta mass were composed of radioactive $^{56}$Ni (the most extreme and optimistic case, i.e. one which yields the highest radioactive heating-rate) the predicted luminosity would not be sufficient to explain observed SLSNe. Furthermore, one would expect a significant abundance of iron-group elements in the ejecta in such an extreme radioactively-powered scenario, in tension with the iron-poor spectroscopic signatures. This has led to the suggestion that SLSNe are powered by either circum-stellar interaction, similar to Type IIIn models (Chevalier & Irwin 2011; Ginzburg & Balberg 2012), or by a central engine, typically a millisecond magnetar (Maeda et al. 2007; Kasen & Bildsten 2010; Woosley 2010) though possible alternatives include fall-back accretion from a radially-extended star (Quataert & Kasen 2012; Dexter & Kasen 2013). The host-galaxy environment of SLSNe, broadly consistent with that of the population of LGRBs provides further evidence supporting the central engine hypothesis, since LGRBs are thought to require similar engines to be produced (see previous subsection of GRBs).

Finally we point out a growing class of so-called “peculiar” SNe. A non-exhaustive list includes calcium-rich SNe, which are characterized by relatively short durations, low-luminosities, fast expansion velocities, and extremely strong calcium emission lines in the nebular phase (Kasliwal et al. 2012b). These SNe also occur preferentially at extremely large
distances from their host galaxies and have been proposed to originate from some process involving a WD (e.g. Perets et al. 2010b; Metzger 2012a). Additionally interesting are a small number of rapidly-evolving SNe which rise and decline over very short timescales (e.g. Drout et al. 2014). The fast rise time indicates a low ejecta mass (see subsequent subsection deriving thermal transients’ peak time), which taken in conjunction with the transients’ high-luminosity, mean these SNe cannot be radioactively-powered (a similar argument to the case for SLSNe). For more on SN classification, I refer readers to Gal-Yam (2017) and references therein.

1.3.3 Fast-radio Bursts

Fast-radio bursts (FRBs) are a recently discovered class of \( \sim \) ms duration radio bursts, the astrophysical origin of which is still largely unclear. These bursts are characterized by large dispersion-measures (DM) — the frequency-dependent time-delay in signal arrival induced by propagation of radio waves through an ionized plasma — far exceeding the maximal DM contribution by the Milky-Way ISM, indicating an extra-Galactic origin (in which case a significant portion of the observed DM would be due to the inter-galactic medium).

Typical bursts have millisecond durations (some showing complex, intra-burst structure), \( \sim \) Jy peak fluxes (1 Jy = \( 10^{-23} \) erg s\(^{-1}\) Hz\(^{-1}\)), dispersion-measures of \( \sim 300 - 2000 \) pc cm\(^{-3}\), and show evidence for scattering. Furthermore, these bursts are extremely common, with estimated all-sky rates as high as \( 10^3 - 10^4 \) per day above 1 Jy.

First found in archival data at the Parkes radio telescope (Lorimer et al. 2007), these bursts quickly garnered much attention even before it became evident that the bursts were
in fact naturally occurring and astrophysical in nature (see similarities with initial reactions
to the detection of pulsars by Bell and Hewish). Indeed, to further muddy the waters, a
sub-class of FRBs was flagged as possible imposters due to their detection in the Parkes
side-lobes. Dubbed ‘Perytons’, these bursts were eventually identified as being caused by a
faulty microwave on site at Parkes whenever it was prematurely opened (Petroff et al. 2015).

The situation became more reassuring following first detections of FRBs by other radio
facilities. By that time (and likely still true today), there were more theoretical models for
FRBs than actual detected bursts. If I may take this one step further, and in the spirit of
a common Jewish saying, I might muse that there were in fact more theoretical models for
FRBs than theorists working on the subject.

Proposed models range from cataclysmic or ‘one-off’ events such as collapse of supra-
massive NSs (Falcke & Rezzolla 2014a), collisions between asteroids and NSs (Geng & Huang
2015), binary NS mergers (Totani 2013) to progenitors such as non-cosmological magnetars
(Pen & Connor 2015), planetary bodies orbiting pulsars (Mottez & Zarka 2014), flaring stars
within our own Galaxy (Loeb et al. 2014), distant active-galactic nuclei, or rare classes of
giant-magnetar flares (e.g. Lyubarsky 2014).

A major development came with the discovery of FRB 121102 with Arecibo (Spitler
et al. 2014). Its significance was that this FRB was observed to repeat (and has been
continuously repeating since its discovery, albeit with sporadically interspersed non-periodic
“bunches” of bursts clustered close to one another). Currently the only known repeating
burst, FRB 121102 is also commonly referred to simply as “the repeater”. The repetition
of FRB 121102 allowed for extensive follow-up by the Very Large Baseline Interferometer

25
(VLBI) which eventually detected and localized bursts from FRB 121102 to a dwarf star-forming galaxy at a redshift of $z \approx 0.2$ \cite{luminosity distance}, This was the first direct proof that, at least some FRBs, are extra-Galactic in origin. Furthermore, the unusual host-galaxy environment of FRB 121102, consistent with the host-galaxies of LGRBs and SLSNe, provides new insight into the nature and progenitors of FRBs, and supports the magnetar hypothesis \cite{magnetar hypothesis}.

Localization of the repeater also revealed a $\approx 10^{39}$ erg s$^{-1}$ luminous quiescent radio source coincident within $\lesssim 0.8$ pc of the FRB location \cite{luminous source}. This unusually luminous radio emission has been interpreted by some as a nascent nebula surrounding a newly born magnetar which is powered either by its rotational \cite{rotation} or magnetic energy reservoir \cite{magnetic energy reservoir}. Finally, in recent unfolding events, FRB 121102 was shown to have an extremely large rotation-measure, $RM \sim 10^5$ rad m$^{-2}$ \cite{rotation measure}, indicating a highly-magnetized electron-ion plasma environment. In the final chapter of this dissertation, we show that such an environment can potentially be produced in the vicinity of a young magnetically-active magnetar.

The book is far from closed on FRBs, as we have only begun to scratch the surface of potential observational tests. Despite the importance and necessity of much theoretical work in trying to better understand the possible progenitors and emission mechanisms responsible for FRBs, it is clear the further observational data will lead the unfolding tale. It remains unclear at this stage for example, whether there are sub-classes of FRBs, e.g. repeating and
non-repeating, or whether all bursts might repeat yet infrequently enough and with such a luminosity distribution that only radio facilities like Arecibo would be sensitive enough to detect repetitions. Our entire current knowledge regarding FRB host environments is based off of a single data point, FRB 121102, so we clearly need to localize additional bursts to move forward.

1.3.4 Physics of Thermal Transients

The fundamental physics governing the observed SNe light-curves apply more broadly to any type of ‘thermal’ transient, i.e. one which is produced by thermal radiation escaping an expanding ejecta. Consider an expanding ball of gas of mass $M$ (the ‘ejecta’; e.g. the outer layers of a massive star which are expelled and become unbound due to the SN shock wave). In the simplest, ‘one-zone’, treatment of the problem we do not worry about any internal structure of this ejecta and express all relevant variables in terms of bulk macroscopic quantities of the ejecta. In practice there will be a radial stratification of these local quantities (and additionally possible non-spherical deviations, though those are generally small). We begin with the assumption that the ejecta is expanding homologously with velocity $v$, such that $v = R/t$ and $R$ and $t$ are the distance and time from the explosion. This assumption applies at times later than the initial expansion timescale $R_0/v$ which, for typical Type II-P core-collapse SNe is of order $\sim$-day. The total thermal energy of the ejecta $E$ then evolves as a function of time according to the simple differential equation

$$\frac{dE}{dt} = -P\frac{dV}{dt} - L_{\text{rad}} + \dot{E}, \quad (1.5)$$
where \( V = 4\pi R^3 / 3 \) is the ejecta’s volume, \( P \approx E / 3V \) is the pressure here assumed to be radiation dominated, \( \dot{E} \) is an energy deposition source and

\[
L_{\text{rad}} \approx \frac{E}{t_{\text{diff}}} \tag{1.6}
\]

is the radiated thermal energy which escapes and can be observed as the visible SN. In the last equation, \( t_{\text{diff}} \) is the photon diffusion timescale out of the ejecta. In the regime where the photon mean-free path \( l_{\text{mfp}} \) is small and the optical depth \( \tau \equiv R / l_{\text{mfp}} \gg 1 \), photons random walk and undergo \( \tau^2 \) scatterings before escaping. The diffusion time is therefore larger than the scattering time \( l_{\text{mfp}} / c \) by this factor. Expressing the mean-free path in terms of the opacity \( \kappa \) (the mass weighted photon cross section), we obtain

\[
\tau = \frac{R}{1 / \kappa \rho} = \frac{3\kappa M}{4\pi R^2} \tag{1.7}
\]

and therefore

\[
t_{\text{diff}} = \tau^2 \frac{l_{\text{mfp}}}{c} = \tau \frac{R}{c} = \frac{3\kappa M}{4\pi c R}. \tag{1.8}
\]

The diffusion time, which is initially very large, later decreases as the ejecta expands and becomes more dilute.

We can now, using also the homologous expansion assumption, rewrite the ODE for the thermal energy \( E(t) \) as

\[
\frac{dE}{dt} = -\frac{E}{t} - \frac{4\pi Evt}{3\kappa M} + \dot{E}(t). \tag{1.9}
\]

At early times \( t_{\text{diff}} \gg t \) and photons cannot efficiently escape before the ejecta doubles in
size. In this regime the first term in the equation above, which reflects energy loss due to adiabatic expansion, dominates over the radiative losses, and the emitted luminosity is small. Conversely, once $t_{\text{diff}} \ll t$ the photons can easily escape and the radiative losses dominate. Thermal radiation will be lost and $L_{\text{rad}}$ will decay with time. It is clear from these simple arguments that the transient peaks on the timescale at which $t_{\text{diff}} \sim t$. Solving for the time $t_{pk}$ at which this requirement is satisfied, we obtain

$$t_{pk} = \left( \frac{3\kappa M}{4\pi cv} \right)^{1/2}. \quad (1.10)$$

Stated another way, the optical depth at this peak time when most of the radiation escapes is $\tau = c/v$. At late times $t \gg t_{pk}$ the ejecta is transparent and the deposited energy is nearly instantaneously radiated away. This implies that $L_{\text{rad}} \approx \dot{E}$ at late times. Using this result we can estimate the peak luminosity of the thermal transient,

$$L_{pk} \equiv L_{\text{rad}}(t_{pk}) \approx \dot{E}(t_{pk}), \quad (1.11)$$

which is known as “Arnet’s Rule”.

The homologous assumption is generally valid only if the external energy deposition is small, $\int \dot{E} dt \lesssim Mv^2/2$. In the case of central engine powered SNe such as Type-I SLSNe this however is usually not the case. We can easily generalize the one-zone equations to account
for this by adding two dynamical equations:

\[
\frac{dE}{dt} = -E(t) \frac{v(t)}{R(t)} - E(t) \frac{4\pi c R(t)}{3\kappa M} + \dot{E}(t), \tag{1.12}
\]

\[
\frac{dv}{dt} = \frac{E(t) M}{R(t)}, \tag{1.13}
\]

\[
\frac{dR}{dt} = v(t). \tag{1.14}
\]

1.4 This Dissertation

This dissertation explores the astrophysical implications of the birth and death of compact objects, with particular focus on transients. The thesis is thus broadly organized into two containing themes — the death, or transmutation, of compact objects, and the birth of such objects. The former discusses compact object mergers and is further divided into two subcategories — WD-NS mergers and binary NS mergers, while the latter focuses on magnetar birth in core-collapse SNe.

In this second part of the dissertation, we advocate towards a unification of seemingly distinct classes of transients under a single theoretical model. We show that newly born, rapidly rotating magnetars can explain certain characteristics of long- and ultra-long-GRBs, broad-lined Type Ic SNe, SLSNe, and FRBs.

This viewpoint is motivated by several observational clues. Long GRBs, SLSNe, and most recently, following the localization and host galaxy identification of FRB121102, also FRBs, have been found to share similar host galaxy environments (Lunnan et al. 2014; Metzger et al. 2017a). These occur preferentially in low-luminosity metal-poor dwarf galaxies.
with high specific star formation rate, a rare environment with respect to where most of the stellar mass or star formation occur in the Universe. Long GRBs have also been observed to be accompanied by energetic broad-lined Type Ic SNe (e.g., Woosley & Bloom 2006), giving credence to the theoretically proposed GRB-SN connection (MacFadyen & Woosley 1999b). More recently, Greiner et al. (2015) claimed to detect a highly luminous (near SLSN) SN in coincidence with an ultra-long GRB. The peculiar ‘doubly-unique’ nature of this event, is in fact a natural consequence within the magnetar framework for LGRBs and SLSNe, additionally strengthening the connection Metzger et al. (2015).
Part I

Death or Transmutation of Compact Objects — Mergers
Subpart A

WD-NS Mergers
Chapter 2

Time dependent models of accretion disks with nuclear burning following the tidal disruption of a white dwarf by a neutron star

2.1 Introduction

The gravitational wave (GW)-driven coalescence of binary compact objects, including white dwarfs (WD), neutron stars (NS), and stellar mass black holes (BH), are widely studied as models for luminous transients. NS-NS and NS-BH binary mergers are potential central

\footnote{This chapter is a reproduction of a paper that has been published by Monthly Notices of the Royal Astronomical Society. It can be found at \url{https://academic.oup.com/mnras/article/461/2/1154/2608518}. The article has been reformatted for this section.}
engines of short duration gamma-ray bursts (GRBs; e.g., Eichler et al. 1989; Berger 2014 for a review) and other electromagnetic counterparts to the GW signal (e.g., Metzger et al. 2010b; Margalit & Piran 2015). The coalescence of WD-WD binaries are likewise believed to be one of the primary channels for producing Type Ia supernovae (SNe; Webbink 1984).

In this paper we explore the outcome of WD-NS or WD-BH mergers, a class of events which have thus far received far less attention than their NS-NS, NS-BH or WD-WD counterparts. Roughly twenty WD-NS binaries are known in our Galaxy, of which four are on sufficiently tight orbits that they will merge completely due to GW radiation within a Hubble time. This population results in an estimated coalescence rate of $R \sim 10^{-5} - 10^{-4}$ yr$^{-1}$ per galaxy (O’Shaughnessy & Kim 2010a), comparable within uncertainties to the rate of NS-NS mergers (Kim et al. 2015).

WD-BH mergers were first studied by Fryer et al. (1999a) as a model for long duration GRBs (see also King et al. 2007). They showed that a sufficiently massive WD is tidally disrupted by its BH companion as the binary orbit shrinks due to unstable mass transfer. The WD debris is then sheared into an accretion disk with an initial size which is comparable to that of the initial binary at the time of Roche lobe overflow. Subsequent accretion of this massive torus was proposed to power a collimated relativistic jet and GRB via $\nu - \bar{\nu}$ annihilation or the Blandford-Znajek process (Fryer et al. 1999a). Paschalidis et al. (2011) explored the merger of WD-NS mergers using general relativistic hydrodynamical simulations. They also found that the final state is a NS surrounded by a massive torus, which they argued evolves into a Thorne-Zytkow-like object following the transport of angular momentum outwards. They described the GW signal that would occur if the central NS collapses to a BH.
following the cooling and accretion by the NS of the envelope.

The outcome of WD-NS and WD-BH mergers were revisited by Metzger (2012b, hereafter M12), who focused on the steady-state structure of the remnant accretion disk. M12 pointed out the importance of nuclear reactions on the structure and dynamics of the accretion flow. As matter accretes onto the central NS or BH, gravitational energy is converted to internal energy. This increases the midplane temperature to the point that nuclear fusion converts the inflowing WD matter into increasingly heavier elements at sequentially smaller radii. Moving inwards through the disk, nucleosynthesis proceeds up to Fe-group elements until, at even higher temperatures, inflowing matter is photodisintegrated into \( \alpha \)-particles and free nuclei. M12 showed that the rate of nuclear energy generated exceeds that of gravity in the outer regions of the disk at hundreds to thousands of gravitational radii, modifying the disk dynamics from those of a standard radiatively inefficient accretion flow. This novel accretion regime is termed a ‘nuclear dominated accretion flow’, or ‘NuDAF’ (M12).

The high densities and optical depths of the accretion flow following a WD-NS merger prevents matter from efficiently cooling through photon radiation, while the temperatures throughout most of the disk are not enough for neutrino cooling to be dynamically relevant (Popham et al. 1999, Di Matteo et al. 2002, Chen & Beloborodov 2007). One dimensional models of such ‘radiatively inefficient accretion flows’ are characterized by positive Bernoulli parameters (Narayan & Yi 1995), indicating the potential importance of unbound outflows on the disk dynamics. We follow the general framework of Blandford & Begelman (1999a), who postulate that disk winds provide an important cooling mechanism which offsets gravitational (viscous) and nuclear heating (see Yuan & Narayan 2014 for a review).
Depending on the radial profile of nucleosynthesis within the disk, outflows from the inner regions, where Fe-group elements form, can contain varying amounts of radioactive $^{56}$Ni. However, the total nickel yield integrated over the lifetime of the torus is generally much less than that produced through shock heating in standard core collapse or Type Ia SNe (M12). This does not exclude WD-NS or WD-BH mergers as progenitors of subluminous, or otherwise exotic, supernova-like transients.

The ‘Ca-rich gap transients’ (Perets et al. 2010a; Kasliwal et al. 2012a) are a class of recently discovered SNe which are characterized by low luminosities (indicating a small $^{56}$Ni ejecta mass), an ejecta composition rich in calcium (and poor in oxygen), fast temporal evolution (indicating a low ejecta mass of a few tenths of a solar mass), and a puzzling tendency to occur outside the disks of their host galaxies (Perets et al. 2010a; Kasliwal et al. 2012a). Their locations show no evidence for star formation or the presence of an underlying quiescent stellar population, such as a dwarf galaxy or globular cluster (Lyman et al. 2014, 2016a). Nuclear burning of helium rich matter is a natural explanation for their high Calcium abundances (Perets et al. 2010a), leading M12 to propose the mergers of a helium WD with a NS as their progenitors. A large fraction of WD-NS binaries could occur in remote locations if they receive a natal kick from the SN which births the NS (M12 Lyman et al. 2014).

Fernández & Metzger (2013a, hereafter FM13) followed the 1D steady-state model of M12 with 2D (axisymmetric) hydrodynamical simulations of radiatively inefficient accretion flows with nuclear burning. These calculations explored the vertical dynamics of the disk and its interplay with radially-steady burning front, e.g. at which carbon is synthesized to magnesium. FM13 found that if the nuclear energy released at the burning front is large
compared to the local thermal energy, then the burning fronts can spontaneously transition into outwards-propagating detonations due to the mixing of hot downstream matter (ash) with cold upstream gas (fuel). These detonations either falter as the shock propagates into the outer regions of the disk, or completely disrupt the large-scale accretion flow. Despite this intriguing finding, FM13 note that the detonations they observe could be an artifact of their simplified equation of state, which included only gas pressure and neglected radiation pressure (thus artificially accentuating the temperature discontinuity at the burning front). FM13 also employed only a single nuclear reaction, which prevented them from making detailed predictions for the composition of the disk outflows and their electromagnetic signatures.

This paper extends the work of M12 and FM13 by developing a one-dimensional time-dependent $\alpha$-disk model (with outflows) for the remnant accretion disks produced by WD-NS mergers. Although we focus primarily on WD-NS mergers, our analysis applies equally to WD mergers with stellar mass BHs. We use this model to explore the response of the disk to nuclear burning and the resulting time-dependent outflow properties (mass, composition, velocity). These details bear significantly on the optical light curves and spectra of WD-NS mergers, as well as their radio emission from the interaction of the ejecta with the interstellar medium. These observational signatures will be investigated in a companion paper.

The paper is structured as follows. We begin with a brief discussion of the conditions and processes which lead up to the disruption of the WD by its binary companion, and subsequent formation of an accretion disk ($\S$2.2). We continue in $\S$2.3 by describing the disk and outflow model adopted in our work. We present analytic results in $\S$2.4 the details
of which are developed in Appendices B and C. Results of our numerical simulations are presented in §2.5. We begin with a detailed analysis of our fiducial C/O WD model (§2.5.1), followed by a parameter study of variations about the fiducial model (§2.5.2). In §2.5.3 we explore models for disrupted He WDs, and in §2.5.4 we explore ‘hybrid’ C/O/He WDs. We discuss our results in §6.5 and conclude in §6.6.

2.2 WD Disruption and Disk Formation

We are interested in binary systems consisting of a WD secondary of mass $M_{\text{WD}}$ and a NS primary of mass $M$. The formation channels for such binaries are extremely rich, and we refer the interested reader to Bobrick et al. (2016) and references therein for further details. To briefly summarize, WD-NS binaries can form in the field through standard binary stellar evolution. There is an expected observational bias towards detecting low-mass WD systems, since the NS in this case likely forms before the WD, which provides a mechanism by which the NS can be recycled and observed as a millisecond pulsar. This fact implies that the observationally inferred WD-NS merger rates (that are based on the pulsar population) are in fact only lower-bounds on the true rates, which might be preferentially higher for more massive WDs. Globular clusters are also important sites for WD-NS binary formation, and are known to host a significant fraction of observed WD-NS systems. These binary systems are likely to form dynamically in globular clusters, either by tidal captures of NSs by red giant progenitors of WDs or via exchange interactions.

Regardless of the formation scenario, a compact WD-NS binary loses orbital energy through GW emission, causing the orbit to shrink and leading to an eventual contact. The
WD experiences Roche lobe overflow (RLOF) once the orbital separation reaches a value

$$a_{\text{RLOF}} \approx R_{\text{WD}} \frac{0.6q^{2/3} + \ln \left(1 + q^{1/3}\right)}{0.49q^{2/3}},$$  \hspace{1cm} (2.1)

where $q = M_{\text{WD}}/M$ and $R_{\text{WD}}$ is the WD radius. The latter is well approximated by

$$R_{\text{WD}} \approx 10^9 \text{ cm} \left(\frac{M_{\text{WD}}}{0.7 \, M_\odot}\right)^{-1/3} \left[1 - \left(\frac{M_{\text{WD}}}{M_{\text{ch}}}ight)^{4/3}\right]^{1/2},$$  \hspace{1cm} (2.2)

where $M_{\text{ch}} \approx 1.45 \, M_\odot$ is the Chandrasekhar mass assuming the mean molecular weight per electron of $\mu_e = 2$.

As mass is transferred from the WD to the more massive primary, conservation of angular momentum drives the binary semi-major axis to increase. On the other hand, as the WD loses mass its radius increases (equation 2.2), which increases the minimal separation for RLOF, $a_{\text{RLOF}}$ (equation 2.1). The competition between the two effects is ultimately determined by their timescales — if $a_{\text{RLOF}}$ increases faster than the binary’s semi-major axis, the system will progress into runaway mass transfer, effectively disrupting the WD on a dynamical timescale. Otherwise, the binary will slowly drift apart, maintaining stable mass transfer.

Conservative mass transfer, in which the orbital angular momentum remains constant, is unstable for binaries with mass ratios $q \gtrsim 0.43 - 0.53$. If, however, orbital angular momentum is deposited into an accretion disk which does not transfer it back into the binary (Lubow & Shu 1975), then significantly smaller mass ratios (as small as $q \sim 0.2$) can also lead to unstable mass transfer (Verbunt & Rappaport 1988a; Paschalidis et al. 2009). For a $1.4M_\odot$
Figure 2.1 Key parameters of the accretion disk produced by the tidal disruption of a WD by a 1.4\(M_\odot\) NS binary companion, as a function of the WD mass. The black curve depicts the circularization radius \(R_c\) (equation 2.3), which represents the characteristic initial radius of the disk, \(R_d\). A red dashed curve shows the initial disk surface density \(\Sigma_0(R_d)\) at \(r = R_d\), calculated for \(m = 2\), and \(n = 7\) (equation 2.4). The two green curves bracket the midplane temperature \(T(R_d)\) in the limits that radiation pressure (bottom, thicker curve) and gas pressure (top, lighter curve) dominate, respectively. The shaded background shows the expected WD composition based on its mass (e.g. Liebert et al. 2005a). Vertical grey dashed lines mark critical WD masses for unstable mass transfer (Paschalidis et al. 2009), corresponding to the lower limit set by conservative mass transfer (rightmost line). The leftmost dashed line provides an estimate of the lower limit on the WD mass allowing unstable mass transfer, in the more realistic case of non-conservative mass transfer.

NS primary, a conservative lower-limit on the WD mass necessary for disruption is therefore \(M_{WD} \gtrsim 0.66 M_\odot\). However, in the more realistic case that at least some orbital angular momentum is lost, lower mass WDs can also be disrupted. The stringent lower-limit of \(M_{WD} \gtrsim 0.23 M_\odot\) for disruption can in principle extend into the mass range of helium WDs (Bobrick et al. 2016 Fig. 2.1). Low-mass WDs which are not disrupted by RLOF will slowly increase their orbital separation as they loose mass, and will likely form ultra-compact X-ray binaries (e.g. van Haaften et al. 2012).

If the WD is disrupted by unstable mass transfer at \(a_{\text{RLOF}}\), its debris will quickly be sheared into an accretion disk of characteristic dimensions proportional to the circularization
radius,

$$R_c = a_{\text{RLOF}}(1 + q)^{-1}. \quad (2.3)$$

This circularization radius is defined as the semi-major axis of a point mass $M_{\text{WD}}$ orbiting the central NS/BH, with an angular momentum equal to that of the binary at the time of disruption.

Detailed hydrodynamical simulations of the WD disruption are required to determine the disk configuration following the disruption (Fryer et al. 1999a; Paschalidis et al. 2011). Such a detailed numerical calculation is beyond the scope or purpose of the present work. We instead adopt a flexible analytic description for the “initial” disk surface density formed by the disruption,

$$\Sigma_0(r) = N(m,n) \frac{M_d}{2\pi R_d^2} \left( \frac{r}{R_d} \right)^m \left[ 1 + \frac{m + 2}{n - 2} \left( \frac{r}{R_d} \right) \right]^{-(m+n)}. \quad (2.4)$$

Here $R_d = R(m,n)R_c$ is the characteristic disk radius, at which the local mass $\propto \Sigma_0 r^2$ peaks, $r$ is the cylindrical radial coordinate centered on the NS/BH, and $N(m,n)$, $R(m,n)$ are constants $\lesssim 1$ given explicitly in Appendix A. The latter are calculated assuming that mass and angular momentum are conserved in the disruption process, in which case the total disk mass is $M_d = M_{\text{WD}}$.

We further assume that energy is conserved during the process of disk formation because the timescale for energy transport via convection or radiation is orders of magnitude longer than the dynamical timescale over which the disruption occurs. Equating the orbital energy at disruption to the total initial disk energy (internal+kinetic+gravitational) and
using equation (2.4), we solve for the disk aspect ratio at formation,

\[ \theta_{\text{initial}} = \left( \frac{H}{r} \right)_{\text{initial}} = \sqrt{\frac{\gamma - 1}{2} \left( 1 - \frac{1}{(1 + q) T(m, n)} \right)} \],

(2.5)

which we assume is radially constant. Here \( H \) is the isothermal scaleheight of the disk at radius \( r \), \( \gamma \) is the adiabatic index (equation 2.17), \( T(m, n) \) is a constant (Appendix A), and we have assumed that the disk orbits at the Keplerian rate, \( \Omega = \Omega_k = (GM/r^3)^{1/2} \). Fig. 2.2 shows \( \theta_{\text{initial}} \) as a function of the binary mass ratio, \( q \), for the physically allowed range of the parameters \( m \) and \( n \). For comparison a horizontal solid purple line shows the characteristic

Figure 2.2 Initial disk aspect ratio, \( \theta \), as a function of the binary mass ratio, \( q \), for \( \gamma = 5/3 \) and a range of values of the power-law index parameters \( (m, n) \) used to define the initial surface density (labeled along each curve; equation [2.4]). The dashed purple curves bracket the permissible range of \( \theta_{\text{initial}} \). The horizontal solid purple curve depicts the steady-state value of \( \theta = \theta_{\text{ss}} \) to which the disk evolves (taking \( \text{Be}_{\text{crit}} = 0 \); equation 2.28). If the initial aspect ratio obeys \( \theta > \theta_{\text{ss}} \), then energy is quickly dissipated by strong outflows until \( \theta = \theta_{\text{ss}} \). Alternatively, initial disk configurations with \( \theta < \theta_{\text{ss}} \) will expand to \( \theta_{\text{ss}} \) due to viscous and nuclear heating without a significant prompt outflow (red arrows). Values of the Toomre parameter \( Q_0 \) are illustrated by black points and stars. The value of \( Q_0 \) decreases as one moves along each curve to larger \( q \). Only for very large values of \( q \) and \( (m, n) \) does \( Q_0 \) drop below unity, indicating that our disk configurations are stable to self-gravitational instabilities. The right (left) dashed vertical curve approximates the conservative (lower-limit) mass ratio above which the WD is tidally disrupted (see Fig. 2.1). The vertical axis is readily scaled to different adiabatic indexes; for \( \gamma = 4/3 \), the values of \( \theta \) decrease by a factor of \( \sqrt{2} \).
value of the disk thickness obtained once a steady inflow is achieved (§2.4.1).

The disk is sufficiently massive that we consider the possibility that it becomes susceptible to instabilities arising from self-gravity. The Toomre parameter,

$$Q = \frac{\Omega c_s}{\pi G \Sigma} = \frac{M \theta}{\pi r^2 \Sigma} \propto \frac{\theta}{q}$$

is less than unity for unstable configurations. The minimal value of this parameter, $Q_0$, is obtained at $t = 0$ and $r = R_d$. Using equations (2.4) and (2.5) for the initial density and disk aspect ratio, we find that $Q_0 > 1$ for most reasonable parameters, indicating that our disks are stable (Fig. 2.2).

The midplane densities and temperatures of WD-NS merger disks span a range of values for which ions, radiation, and (to a lesser extent) degenerate electrons can all contribute significantly to the pressure and energy density of the fluid (M12). At large radii in the disk, and at times soon after disruption, the entropy is relatively low and gas pressure dominates over radiation pressure. At smaller radii at early times (and for most radii at late times), radiation pressure instead becomes dominant. In the limits that gas or radiation dominate the midplane pressure, the midplane temperature is given by

$$T(r) = \begin{cases} 
(\mu m_p/k_B)\theta^2 \Omega_k^2 r^2, & \text{gas} \\
[(3/2a)\theta \Omega_k^2 r \Sigma]^{1/4}, & \text{radiation}
\end{cases}$$

(2.7)

where $\mu$ is the mean molecular weight. Fig. 2.1 shows the initial disk temperature at $R_d$ for various parameters.
2.3 Disk and Outflow Model

This section describes our numerical model for the disk evolution and outflows. We begin by summarizing the vertically averaged disk equations governing the dynamics, before continuing with details of the mass loss prescription, nuclear burning, and our numerical procedure.

2.3.1 Disk Equations

The vertically integrated continuity equation reads

\[ \partial_t \Sigma + \frac{1}{r} \partial_r (r v_r \Sigma) + \dot{\Sigma}_w = 0, \quad (2.8) \]

where \( v_r \) is the radial fluid velocity and \( \dot{\Sigma}_w \) is a sink term which accounts for mass loss from the disk via winds (§2.3.2). Vertical hydrostatic equilibrium is assumed, implying that \( H/r \approx c_s/v_k \), where \( c_s \equiv \sqrt{P/\rho} \) is the midplane isothermal sound speed, and \( v_k = r \Omega_k \) is the Keplerian orbital velocity.

The radial momentum equation can be manipulated to obtain the angular velocity,

\[ \Omega \approx \Omega_k \sqrt{1 + \theta^2 \left( \partial \ln \Sigma / \partial \ln r - 1 \right)}. \quad (2.9) \]

However, because in practice we find that in most cases \( \Omega \approx \Omega_k \) to an accuracy of \( \lesssim 10\% \), for simplicity we fix \( \Omega = \Omega_k \) throughout the remainder of this work.

The vertically-averaged azimuthal momentum equation can be rearranged to obtain the
radial velocity

\[ v_r \approx -3 \frac{\nu}{r} \frac{\partial \ln [r^2 \nu \Sigma \Omega]}{\partial \ln r} \]

\[ = -3 \alpha \theta^2 v_k \left[ 2 + \frac{\partial \ln \Sigma}{\partial \ln r} + 2 \frac{\partial \ln c_s}{\partial \ln r} \right], \quad (2.10) \]

where \( \nu \) is the kinematic ‘viscosity’, which physically is associated with an anomalous stress.

In the second equality we have adopted the standard Shakura & Sunyaev (1973a) alpha prescription,

\[ \nu = \alpha c_s^2 / \Omega_k. \quad (2.11) \]

As the magnetorotational instability (MRI) provides one physical mechanism for angular momentum transport (Balbus & Hawley 1991a), we adopt values of \( \alpha \sim 0.01 - 0.1 \), consistent with those measured by numerical simulations of the MRI (e.g. Davis et al. 2010a).

The surface density of the disk evolves on the characteristic viscous timescale,

\[ t_{\text{visc}} = \frac{r^2}{\nu} = \alpha^{-1} \theta^{-2} \Omega_k^{-1} \quad (2.12) \]

which is longer than the dynamical timescale \( \Omega_k^{-1} \) by a factor of \( \alpha^{-1} \theta^{-2} \gg 1 \).

Finally, the specific entropy \( s \) and internal energy \( u \) evolve according to the first law of thermodynamics,

\[ \dot{q}_{\text{tot}} = \Sigma T(D_t s) = \Sigma (D_t u) - c_s^2 (D_t \Sigma), \quad (2.13) \]
where \( D_t \equiv \partial_t + v_r \partial_r \) is the Lagrangian derivative and

\[
\dot{q}_{\text{tot}} = \dot{q}_{\text{visc}} + \dot{q}_{\text{nuc}} + \dot{q}_{\text{wind}}
\]  (2.14)

is the total disk heating rate per unit area, where

\[
\dot{q}_{\text{visc}} = \Sigma \nu \Omega^2 \left( \frac{\partial \ln \Omega}{\partial \ln r} \right)^2 = \frac{9}{4} \alpha \Sigma c_s^2 \Omega_k
\]  (2.15)

is the viscous heating rate, \( \dot{q}_{\text{nuc}} \) is the heating rate due to nuclear burning (§2.3.3), and \( \dot{q}_{\text{wind}} \) is the wind cooling rate (§2.3.2).

Using continuity (equation 2.8), equation (2.13) can be recast as

\[
\partial_t u = \frac{\dot{q}_{\text{tot}}}{\Sigma} - v_r \partial_r u + c_s^2 \left[ \frac{1}{r} \partial_r (rv_r) + \frac{\dot{\Sigma}_w}{\Sigma} \right].
\]  (2.16)

The above equations are closed by imposing an EOS which relates the isothermal sound speed to the internal energy and density \( c_s = c_s(u, \Sigma) \). For a ‘gamma-law’ EOS, this relation takes the form

\[
u = \frac{c_s^2}{\gamma - 1}.
\]  (2.17)

Although we incorporate a full EOS in our numerical calculations, equation (2.17) is used in analytic estimates.
2.3.2 Wind Prescription

Outflows launched from the disk represent an important sink of mass and energy, as represented by the terms \( \propto \dot{\Sigma}_w \) in equations (2.8) and (2.16). We assume that winds do not exert a net torque on the disk and hence neglect their effects on the angular momentum evolution of the disk.

Two parameters are required to prescribe the outflow. Following Kohri et al. (2005) and M12, we define a wind cooling efficiency \( \eta_w \), which is related to the asymptotic wind velocity by

\[
v_w = v_k \sqrt{2 \eta_w}.
\]

(2.18)

A value \( \eta_w \sim \mathcal{O}(1) \) corresponds to winds launched at velocities close to the local escape speed. The corresponding timescale for mass loss is \( t_w \sim H/v_w \sim \theta \Omega_k^{-1} \); for \( \eta_w \sim 1 \) this is a factor of \( \theta < 1 \) times smaller than the local dynamical timescale and a factor of \( \alpha \theta^3 \ll 1 \) smaller than the accretion timescale. This motivates a prescription for local wind cooling, which effectively acts instantaneously.

Another important quantity is the Bernoulli parameter of the disk midplane,

\[
Be_d = \frac{1}{2} \Omega^2 r^2 + \frac{1}{2} v_r^2 + u + c_s^2 - v_k^2,
\]

(2.19)

and its normalized value \( Be'_d = Be_d/v_k^2 \). The fact that this quantity is generally positive in one dimensional models of radiatively inefficient accretion flows (Narayan \\& Yi 1995, Blandford \\& Begelman 1999a) shows that matter in principle has sufficient thermal energy.
to adiabatically expand to infinity. Using a $\gamma$-law EOS (equation [2.17]), the normalized Bernoulli parameter can be written as

$$Be'_d \approx -\frac{1}{2} + \frac{\gamma}{\gamma - 1} \theta^2,$$

(2.20)

where the radial kinetic energy $\propto \alpha^2 \theta^4 \ll 1$ has been neglected.

We adopt a wind prescription which cools the disk when the Bernoulli parameter exceeds a fixed value, $Be'_\text{crit} \lesssim 0$. To conserve energy globally, this is tantamount to assuming that some mechanism (e.g. turbulence or wave damping) heats matter in the corona where the wind is launched at a specific rate exceeding that in the midplane. In other words, this preferential heating above the midplane allows some matter to become unbound at the expense of the rest of the disk maintaining $Be'_d \lesssim 0$. Although we do not presume to understand the details of the wind launching process, the properties of the disk/outflow structure that we find by making this assumption show qualitative agreement with global hydrodynamical (e.g. Stone et al. 1999) and MHD (e.g. Hawley & Balbus 2002) simulations of radiatively inefficient accretion flows. These simulations indeed find that the Bernoulli parameter in outflows from the disk at high latitudes are higher than its value in the disk midplane, where $Be'_d \lesssim 0$, due to a higher specific heating rate above the midplane.

The picture described above translates into the following functional form for the wind mass loss rate,

$$\dot{\Sigma}_w = \Sigma \Omega_k \theta^{-1} \sqrt{2 (\eta_w + 1)} \times \Theta (Be'_d - Be'_\text{crit}),$$

(2.21)

where $\Theta (x)$ is the Heaviside function. This prescription captures the qualitative expectation
that matter is only unbound if the Bernoulli parameter of the disk exceeds a threshold value of $B_{\text{crit}}'$. When outflows are present, it is also consistent with the order of magnitude estimate $\dot{\Sigma}_w \sim \Sigma/t_w$.

The wind efficiency parameter $\eta_w$ essentially equals the specific energy carried away in the wind. The cooling rate of the disk by the wind is therefore given by

$$\dot{q}_w = -\dot{\Sigma}_w \left( v_w^2/2 - B_{\text{d}} \right) = -\dot{\Sigma}_w v_k^2 \left( \eta_w - B_{\text{d}}' \right).$$  \hspace{1cm} (2.22)

This general cooling prescription does not depend on the less certain form of $\dot{\Sigma}_w$ (equation 2.21) in the common scenario of a quasi-steady-state disk evolution (see Appendix C). Also note that as long as the mass loss mechanism regulates the disk Bernoulli parameter to $B_{\text{crit}}'$, then we must require that $B_{\text{crit}}' < \eta_w$, as otherwise the wind cannot cool the disk. This condition is satisfied if the unbound material has been preferentially heated, as hypothesized above.

### 2.3.3 Nuclear Burning

The mass fraction of each isotope in the disk, $X_A$, evolves according to an equation of continuity,

$$\partial_t X_A + \frac{1}{r} \partial_r \left( rv_r X_A \right) - \frac{1}{r \Sigma} \partial_r \left[ r \Sigma \nu_{\text{mix}} \left( \partial_r X_A \right) \right] + X_{A}^{(\text{nuc})} = 0,$$  \hspace{1cm} (2.23)
where the second term accounts for the radial advection of the nuclear species with the accretion velocity $v_r$. The third term accounts for mixing of nuclear isotopes with a diffusion coefficient $\nu_{\text{mix}}$. Such mixing is expected due to the same turbulent motions in the disk which drive angular momentum transport, and hence $\nu_{\text{mix}}$ is intimately related to the ‘Shakura-Sunyaev’ viscosity $\nu$. We therefore assume

$$\nu_{\text{mix}} = \tilde{\alpha} \nu . \quad (2.24)$$

Numerical simulations of the MRI which follow the evolution of a passive scalar suggest that $\tilde{\alpha} \approx 0.1$ (Carballido et al. 2005), indicating that ‘chemical’ mixing is less efficient than angular momentum transport. We take this as the fiducial value of $\tilde{\alpha}$ throughout our work, but also vary the value of this parameter, examining its affect on the results.

The last term in equation (2.23) represents species-changing nuclear reactions. For purposes of analytic estimates it is convenient to approximate individual burning rates, $\dot{X}_A^{\text{(nuc)}}$, as power-laws near their burning temperature,

$$\dot{X}_A^{\text{(nuc)}} \propto \rho^\delta X_A^{\delta+1} T^\beta . \quad (2.25)$$

For carbon burning, $^{12}\text{C}(^{12}\text{C}, \gamma)^{24}\text{Mg}$, one can approximate the reaction rate around $\sim 10^9$ K with $\beta = 29$, $\delta = 1$.

Note that in steady-state, and neglecting the diffusive mixing term, the nuclear reaction rate at the burning front is determined entirely by the accretion velocity, $v_r$, which supplies unburned fuel to the burning front.
In addition to altering the disk composition, nuclear reactions provide a source of heating or cooling, \( \dot{q}_{\text{nuc}} \), which contributes to the net heating rate \( \dot{q}_{\text{tot}} \) in equation (2.16). This term is obtained by summing the energy production rates of all isotopes

\[
\frac{\dot{q}_{\text{nuc}}}{\Sigma} = \sum_{A, A'} \dot{X}_{A \rightarrow A'}^{(\text{nuc})} \frac{Q_{A \rightarrow A'}}{m_{A'}} ,
\]  

(2.26)

where \( m_{A'} \) is the mass of isotope \( A' \), and \( Q_{A \rightarrow A'} \) is the Q-value of the reaction turning isotope \( A \) into \( A' \). The latter neglects energy carried away by neutrinos, which are not trapped for the characteristic densities of the accretion flow.

### 2.3.4 Numerical Procedure

We numerically solve equations (2.8), (2.16), and (2.23), using expressions for the accretion velocity (2.10), mass loss rate (2.21), and wind cooling terms (2.22). We employ the Helmholtz EOS (Timmes & Swesty 2000) in relating the thermodynamic variables \( c_s, u, \Sigma, \) and \( T \) (the last of which is necessary to evaluate the nuclear burning rates). This EOS accurately and consistently accounts for an electron-positron gas with arbitrary degree of degeneracy and relativistic motion, an ideal gas of ions, and a Planckian distribution of photons.

The nuclear reaction rates, \( \dot{X}_A \), and nuclear heating term, \( \dot{q}_{\text{nuc}} \), are numerically evaluated using the publicly available\(^1\) 19-isotope \( \alpha \)-chain reaction network of Weaver et al. (1978a). This network effectively captures the main burning channels of WD matter (\( ^{12}\text{C}, ^{16}\text{O}, ^{4}\text{He}, ^{20}\text{Ne}, ^{24}\text{Mg} \)) up to \(^{56}\text{Ni} \). This network takes as input arguments a list of the abun-

\( ^1\)http://cococubed.asu.edu/code_pages/burn_helium.shtml
dances \( \{X_A\}_A \), the temperature \( T \), and the density \( \rho \) at a particular radial and temporal gridpoint, as well as the burning time \( dt \), and returns the updated abundances, and energy deposition.

The equations are converted into finite-difference form and solved on a logarithmic radial grid spanning two orders of magnitude above and below the initial peak-density radius \( = R_d \times [m(n - 2)] / [n(m + 2)] \). The initial conditions for \( \Sigma \) and \( \theta \) are taken according to equations (2.4) and (2.5). The variable timestep between each iteration is chosen based on a Courant condition

\[
dt = 0.1 \min \left[ \frac{dr^2}{\nu} , \frac{dr}{c_s} , \text{Eq. (2.16)} , \text{R.H.S.} \right],
\]

where the minimum runs also over the entire radial grid on which the three arguments implicitly depend. The third argument of the minimum function ensures that heat deposition in the disk is temporally resolved, which is particularly important considering nuclear heating contributions.

Since nuclear network calls are computationally expensive, we develop a numerical ‘steady-state scheme’. The basic principle is motivated by the fact that the accretion flow quickly (on a \( \lesssim \) viscous timescale) establishes a quasi-steady-state regime, after which, physical quantities vary only secularly with mass loss from the disk. This means that over short, dynamical, timesteps the temperature, density, and abundance profiles do not change significantly, and consequently neither do the nuclear reaction rates.

We utilize this property of the accretion flow by logging the nuclear reaction and heating rate at each gridpoint immediately after the nuclear network has been called. At later
timesteps, we use the same $X_A$ and $q_{\text{nuc}}$ at this gridpoint in evaluating equations (2.23) and (2.16), instead of calling the nuclear network. We continue using these logged rates until either the temperature, or one of the abundances has fractionally changed by more than $10^{-2}$ since the last network call, at which point we recalculate the rates using the nuclear network. This simple procedure retains nearly perfect fidelity with the full network calculation yet reduces the computational time by factors of several (the effective benefit of our method increases with time, as the accretion flow evolves over longer timescales).

### 2.4 Analytic Results

We begin by summarizing several key analytic results for the steady-state structure of the disk and outflows, the derivations of which are provided in Appendices B and C.

#### 2.4.1 Disk Winds

Outflows regulate the Bernoulli parameter of the disk midplane (equation 2.20) to a critical value, $Be'_d \simeq Be'_\text{crit}$. In steady-state, this condition yields a radially constant disk aspect ratio of

$$\theta_{\text{ss}} = \left( \frac{H}{r} \right)_{\text{ss}} \approx \sqrt{\frac{\gamma - 1}{2\gamma}(1 + 2Be'_\text{crit})}.$$  \hspace{1cm} (2.28)

Any disk structure will achieve this universal aspect-ratio on a short timescale set either by the outflow or thermal time (depending on whether initially $\theta > \theta_{\text{ss}}$ or $\theta < \theta_{\text{ss}}$, respectively). This result is independent of the specific implementation of our wind prescription as long as $Be'_d = Be'_\text{crit}$.  

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The aspect ratio provides a measure of the thermal energy of the disk (equation 2.20). If the disk is initially too hot, such that the initial aspect ratio (equation 2.5) exceeds its steady-state value, $\theta_{ss}$, then strong winds will act to quickly cool the flow, until $B_{ed}' \approx B_{crit}'$. The total mass lost from the disk during this brief ‘precursor’ phase is approximately (Appendix B)

$$M_{w}^{(\text{precursor})} \approx \frac{1 + 2B_{crit}'}{\gamma (\eta_{w} - B_{crit}')} \left( \frac{\Delta \theta}{\theta_{ss}} \right) \times M_d,$$

(2.29)

where $\Delta \theta = \theta_{\text{initial}} - \theta_{ss}$ is the difference between the initial and steady-state value of the disk aspect ratio.

Only for large binary mass ratio $q$ does $\theta_{\text{initial}}$ (solid red line in Fig. 2.2) exceed the steady-state value $\theta_{ss}$ (horizontal solid purple curve). Even in this case, however, $\Delta \theta / \theta_{ss} \lesssim 5 \times 10^{-2}$ is sufficiently small that $M_{w}^{(\text{precursor})} \lesssim 3 \times 10^{-2} M_d$ for fiducial values of the relevant parameters.

Following standard notation (e.g., Blandford & Begelman 1999a), we define the mass inflow exponent

$$p \equiv \frac{\partial \ln \dot{M}_{\text{in}}}{\partial \ln r},$$

(2.30)

where $\dot{M}_{\text{in}} = 2\pi rv_{r}\Sigma$ is the local mass inflow rate. The value of $p$ is constrained by energy and mass conservation to be in the range $0 \leq p < 1$ for normal accretion disks without nuclear burning as an additional source of energy.

As shown in Appendix C, combining the wind cooling prescription (equation 2.22) with mass and energy conservation (equations 2.8, 2.16) under steady-state conditions ($\partial_t = 0$) fully determines the value of $p = p(\eta_{w}, B_{crit}', \gamma)$. Fig. 2.3 shows that for physically reasonable
choices of $\eta_w \approx 1$ and $\text{Be}'_{\text{crit}} \approx 0$, one obtains values of $p \gtrsim 0.5$ which are in broad agreement with the results of hydrodynamical and MHD simulations of radiatively inefficient accretion flows (Stone et al. 1999; Igumenshchev & Abramowicz 2000; Hawley et al. 2001; Narayan et al. 2012; McKinney et al. 2012; Yuan et al. 2012).

Our analytic solution does not account for nuclear heating, $\dot{q}_{\text{nuc}}$, which breaks the self-similarity of the problem by introducing additional energy and time scales. Nuclear heating competes with viscous heating in locally balancing wind cooling (advective cooling is approximately a fixed fraction of $\dot{q}_{\text{visc}}$ in steady-state; Appendix C). Since more mass must be lost to winds to offset additional nuclear burning at fixed $\eta_w$, nuclear heating increases the value of $p$ locally near the burning front, thus decreasing the mass-inflow rate in this region accordingly.
2.4.2 Late-time Disk Evolution

Most of the disk mass accretes over a characteristic timescale equal to the viscous time $t_{\text{visc}}$ (equation 2.12) evaluated at the initial characteristic disk radius $\sim R_d$. At times $t \gg t_{\text{visc}}$ the disk evolution approaches a self-similar state. Following known solutions for accretion disks with outflows (e.g. Metzger et al. 2008a, and references therein), the characteristic disk radius expands as $R_d \propto t^{2/3}$, and the mass inflow rate scales as

$$\dot{M}_{\text{in}} \propto r^p t^{-4(p+1)/3}. \quad (2.31)$$

Note that the radial scaling applies only in the steady-state part of the disk ($r < R_d$) and that terms of order $(r*/R_d)^p \ll 1$ have been neglected, where $r_*$ is the inner boundary of the disk.

Combining 2.31 with equations (2.30), (2.10), and (2.7), the disk surface density evolves as

$$\Sigma \propto r^{p-1/2} t^{-4(p+1)/3}, \quad (2.32)$$

and the midplane temperature (equation 2.7) as

$$T \propto \begin{cases} r^{-1/4} t^0, & \text{gas} \\ r^{(p-5/2)/4} t^{-(p+1)/3}, & \text{radiation} \end{cases} \quad (2.33)$$

where the latter has been separated into gas and radiation pressure-dominated regimes.

Most nuclear reaction rates depend more sensitively on temperature than density (an
important exception sometimes being the triple-α reaction). Burning fronts therefore typically track the evolution of constant temperature surfaces. For the radiation dominated case of most relevance at late times and small radii, equation (2.33) is inverted to find

$$r \left( T = T_{\text{rad}} = \text{const.} \right) \propto t^{-\frac{3(p+1)}{3(5/2-p)}} . \quad (2.34)$$

Under the assumptions that (1) the burning front of an isotope \( A \) peaks around \( r(T_{\text{burn}}) \), and (2) the radial shape of the abundance profile \( X_A \) and its peak value are constant in time, then the mass ejection rate in this isotope is approximately given by

$$\dot{M}_w(X_A) \approx X_A \Sigma_w r^2 \left|_{r(T_{\text{burn}})} \right. \propto \dot{M}_{\text{in}} \left[ r(T_{\text{burn}}) \right] . \quad (2.35)$$

The total outflow rate (integrated across all radii) then evolves as

$$\dot{M}_w \propto t^{-(2p+4)/3} , \quad (2.36)$$

intimating that the factional mass loss rate in isotope \( X_A \) decreases at times \( t \gg t_{\text{visc}} \) as

$$\frac{\dot{M}_w(X_A)}{\dot{M}_w} \propto t^{-2p/3-4p(p+1)/3(5/2-p)} . \quad (2.37)$$

Physically, this temporal decrease of \( \dot{M}_w(X_A) \) is driven by the inward migration of the burning fronts as the disk temperature decreases with time. A lower temperature reduces the radius at which a particular isotope is first formed, thus reducing its contribution to the disk outflows. This result will prove useful later in extrapolating our numerical results to
times later than the end of the simulation.

2.5 Numerical Results

Following the procedure described in §2.3.4, we have performed a suite of accretion disk/outflow simulations, as summarized in Table 2.1, corresponding to different model parameters and compositions of the disrupted WD.

2.5.1 Fiducial Model

Our fiducial model, $\text{C0.Fid}$, corresponds to the merger of a $0.6M_\odot$ C/O WD with a $1.4M_\odot$ NS. The initial composition of the WD, and hence of the disk, is half (by mass) carbon and half oxygen, $X_{12C} = X_{16O} = 0.5$. We employ a Shakura-Sunyaev alpha viscosity parameter of $\alpha = 0.1$ and a composition mixing parameter (equation 2.24) of $\tilde{\alpha} = 0.1$ (Carballido et al. 2005). The fiducial wind efficiency parameter and critical (normalized) Bernoulli parameter are taken to be $\eta_w = 1$ and $B_{\text{crit}}' = 0$, respectively. In steady-state, these parameters describe a marginally bound disk with a mass inflow index of $p \approx 0.43$–$0.47$ for adiabatic indexes $\gamma = 1.33$–$1.67$ (Fig. 2.3). The power-law parameters of the initial disk density profile (see equation 2.4) are taken to be $m = 2$ and $n = 7$.

The characteristic initial radius of the disk is $R_d \simeq 1.8 \times 10^9$ cm, corresponding to an initial viscous timescale of $t_{\text{visc},0} \simeq 68$ s measured at the radius where the initial density distribution peaks. We terminate our simulations at the time $t_{\text{end}} = 2t_{\text{visc},0}$, at which point roughly half the initial mass of the disk has either been lost to outflows or has been accreted through the inner boundary of the grid. By $t = t_{\text{end}}$ the burning fronts creating Fe-group
Table 2.1 Model parameters of simulations performed in this paper. (a) Initial mass fractions $X_A$ of the WD or disk. (b) Initial mass of WD or disk in solar masses. (c) Mass of WD binary companion (NS or BH) in solar masses. (d) Shakura-Sunyaev alpha viscosity parameter (equation 2.11). (e), (f) Wind efficiency parameter and critical Bernoulli parameter respectively (§2.3.2). (g) Initial disk density power-law parameters (equation 2.4). (h) Normalized mixing efficiency parameter (equation 2.24).

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</tbody>
</table>
Figure 2.4 (a) Mass inflow rate, $\dot{M}_{\text{in}}(r_*)$, through the inner radial boundary at $r_* \simeq 7 \times 10^6$ cm (blue) and total wind outflow rate, $\dot{M}_w$ (purple), as a function of time following disk formation. The accretion rate peaks on a timescale of $t_{\text{visc}} \approx 8$ s. The late-time power-law evolution of the inflow rate predicted by a self-similar model (equations 2.31, 2.36) are shown as dashed red curves. The light pink curve is a direct power-law extrapolation of $\dot{M}_w$ from the simulation end time, while the dashed purple curve shows an intermediate power-law extrapolation based on mass conservation (equation 2.39). (b) Snapshot of the radial profile of the mass inflow rate, $\dot{M}_{\text{in}}$, at $t = 16$ s $\gtrsim t_{\text{visc}}$, for our fiducial model CO$_{\text{Fid}}$. The bottom panel shows the mass inflow exponent, $p \equiv \partial \ln \dot{M}_{\text{in}} / \partial \ln r$. The pink shaded region shows the range of $p$ predicted for a steady-state disk (see Appendix C). Grey curves in both panels show a model CO$_{\text{Nuc}}$ in which nuclear heating is manually turned off. Local peaks in $p(r)$, relative to the CO$_{\text{Nuc}}$ model, are caused by strong localized nuclear heating from, e.g., $^{12}$C and $^{16}$O burning fronts. The local minimum in $p(r)$ at $r \lesssim 2 \times 10^7$ cm is the result of cooling from endothermic photodisintegration.

elements begin crossing through the inner boundary of our grid.

2.5.1.1 Accretion/Outflow Rates

Fig. 2.4 shows the time evolution (top panel) and radial profile (bottom panel) of the total mass inflow and outflow rates. The maximum inflow rate at $R_d$ can be estimated by $\dot{M}_{\text{in}}(R_d) \sim M_d / t_{\text{visc},0} \sim 9 \times 10^{-3}$ $M_\odot$ s$^{-1}$. However, most of this inflow is ultimately lost to outflows, with only a fraction $\sim (r_*/R_d)^p \ll 1$ reaching the inner boundary at $r = r_*$. This is illustrated explicitly in Fig. 2.4(a) which shows that $\dot{M}_{\text{in}}(r_*) \ll \dot{M}_w$. Physically, $r_*$ represents the NS surface, but in our case it represents the inner boundary of our radial
grid at \( r_s \simeq 7 \times 10^6 \text{ cm} \). The wind outflow rate \( \dot{M}_w \sim \dot{M}_{\text{in}}(R_d) \) peaks at roughly the same value as the accretion rate, although it rises to a maximum on a timescale \( t_{\text{visc}} \simeq 8 \text{ s} \) which is shorter than \( t_{\text{visc},0} \).

Dashed lines show a range of power-law extrapolations of the mass inflow and outflow rates. A light pink line shows an extrapolation of \( \dot{M}_w(t_{\text{end}}) \) based on the best-fit logarithmic slope measured near the end of the simulation run. Red curves show the late-time self-similar evolution predicted by equations (2.31) and (2.36), which are generally steeper because they represent the asymptotic power-law towards which the solution is evolving. An intermediate extrapolation shown with a purple line is derived by requiring that the integrated mass loss rate obey mass conservation, viz.

\[
M_{\text{acc}}(t_{\text{end}}) + M_w(t_{\text{end}}) + \int_{t_{\text{end}}}^{\infty} \dot{M}(t > t_{\text{end}}) \, dt = M_d, \tag{2.38}
\]

where \( M_{\text{acc}}(t_{\text{end}}) \) and \( M_w(t_{\text{end}}) \) are the total mass accreted through the inner grid boundary and lost to wind outflows by the simulation end time, respectively. Solving for the appropriate wind mass loss exponent \( \zeta \), defined by

\[
\dot{M}_w(t > t_{\text{end}}) = \dot{M}_w(t_{\text{end}}) \times \left( \frac{t}{t_{\text{end}}} \right)^{-\zeta}, \tag{2.39}
\]

we obtain

\[
\zeta = 1 + \frac{\dot{M}_w(t_{\text{end}}) \times t_{\text{end}}}{M_d - M_{\text{acc}}(t_{\text{end}}) - M_w(t_{\text{end}})}. \tag{2.40}
\]

We employ this power-law scaling when we extrapolate the properties of outflows from the final timestep of our numerical simulations \( t_{\text{end}} \) to late times, \( t = \infty \).
Fig. 2.4(b) shows the radial profile of the inflow rate $\dot{M}_{\text{in}}$ at a fixed time, $t = 16\,s \sim 2t_{\text{visc}}$. As expected, a steady-state power-law scaling $\dot{M}_{\text{in}} \sim r^p$ is obtained for radii $r \lesssim R_d$ (equation 2.31). The local dip in $\dot{M}_{\text{in}}$ and the apparent discontinuity in its derivative near $r = 2 \times 10^9\,\text{cm}$ is an artifact of the absolute value and logarithmic scale of the vertical axis. This location corresponds to a turnover point, where the radial velocity $v_r$ passes through zero. Outside of this radius, where the radial velocity is positive, a small amount of mass carries angular momentum to large radii.

The bottom panel of Fig. 2.4(b) shows the radial profile of the mass loss index $p$ (equation 2.30). If no nuclear burning were present, then in the steady-state portion of the disk at $r \lesssim R_d$ we would expect $p$ to vary about the theoretically expected range, as depicted by the shaded pink region for adiabatic index in the range $\gamma = 1.33 - 1.67$ (equation C5). Indeed, this range is reasonably well matched by the grey curves, which show an otherwise identical model, $\text{CO}_\text{Nuc}$, but with the effects of nuclear burning artificially turned off. Localized spikes in $p(r)$, such as those located at $r \approx 2 \times 10^8\,\text{cm}$ and $r \approx 5 \times 10^7\,\text{cm}$ which break from the smooth trend exhibited by the grey $\text{CO}_\text{Nuc}$ solution, occur at the $^{12}\text{C}$ and $^{16}\text{O}$ burning fronts. The significant amounts of energy released by nuclear burning at these locations (Fig. 2.7) must be offset by greater cooling of the disk (stronger outflows) than in disks heated purely by viscosity. These local maxima in the mass outflow rate are accompanied by a decrease in $\dot{M}_{\text{in}}$ (as required by mass conservation), which reflect as local peaks in the mass inflow exponent $p$. 

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Figure 2.5 Snapshots in the evolution of the radial profile of the mass fraction $X_A$ of key isotopes. The second panel ($t = 8$ s) roughly corresponds to the time of peak accretion. The final panel ($t = 128$ s) approximately corresponds to the simulation end time. The composition profiles exhibit a self-similar evolution, with the overall abundance pattern shifting as a whole to larger (smaller) radii before (after) the peak accretion timescale, respectively. This ‘steady state’ self-similar behaviour characterizes the disk composition already from very early times $\ll t_{\text{visc}}$. 
2.5.1.2 Disk Composition

Fig. 2.5 shows snapshots of the radial profile of the mass fraction $X_A(r)$ of key isotopes. The disk composition assumes an onion-skin structure, reminiscent of that of evolved massive stars, in which successively heavier elements burn at sequentially smaller radii. At radii $r \lesssim 2 \times 10^8$ cm, the temperature of the disk midplane becomes sufficiently high, $T \gtrsim 10^9$ K, to initiate burning of the initial carbon/oxygen composition, generating $^{20}$Ne and $^{24}$Mg. At smaller radii, the temperature increases further, fusing these isotopes into $^{28}$Si. At $r \sim 6 \times 10^7$ cm, $^{32}$S is created, which quickly burns to $^{36}$Ar, $^{40}$Ca, $^{44}$Ti, $^{48}$Cr and $^{52}$Fe, and finally up to $^{54}$Fe and $^{56}$Ni. Near the innermost radii, $r \lesssim 3 \times 10^7$ cm, photo-disintegrations breaks these heavy elements apart into $^4$He ($\alpha$-particles) and free nucleons.

The same qualitative picture holds at each snapshot in time because the key nuclear reactions are temperature limited. The composition profiles at different times therefore remain nearly identical to one another, modulo rescaling of the radial axis. This apparent self-similarity in $X_A(r,t)$ is a direct consequence of the self-similarity in the temperature profile (equation 2.33), insofar as the burning fronts reside in regions of the disk dominated by radiation pressure and at radii $\lesssim R_d$ characterized by a steady inward accretion rate.

The composition profile in Fig. 2.5 is similar to that obtained by the steady-state model of M12. At any time the composition is well described by a steady-state model, with the mass feeding rate $\dot{M}_{\text{in}}(R_d)$ varying secularly in time.

At early times $t < t_{\text{visc}}$, the density and temperature at a fixed radius $r < R_d$ are small, with a correspondingly small burning front radius (first panel of Fig. 2.5). As gas fills the inner disk and accretes onto the NS, the temperature rises and the burning fronts
move outwards, reaching their peak values on a timescale $t \sim t_{\text{visc}}$ (second panel). Finally, at times $t > t_{\text{visc}}$, as the disk mass and density decrease, the constant temperature regions again move inwards to smaller radii, and the burning fronts and composition profiles shift steadily in the same fashion (third and fourth panels).

Fig. 2.6 further illustrates this evolution by showing contours of the mass fraction of two sample elements, $^{56}\text{Ni}$ and $^{20}\text{Ne}$, in the space of radius and time. The peak mass fractions of each element rise to larger radii at $t \lesssim t_{\text{visc}}$, and decrease after $t \gtrsim t_{\text{visc}}$. Contours of constant temperature are overplot with grey curves. The fact that the composition and temperature contours track one another again illustrates that the relevant nuclear reactions are temperature limited. The constant temperature curves at $r < R_d$ and $t \gg t_{\text{visc}}$ also agree well with the predicted late-time self-similar evolution in the radiation-dominated regime (equation 2.34), which we have overplot with a dashed red line.
Beyond generating a rich radial abundance distribution, nuclear burning can have dynamically important influence on the disk and its outflows. Fig. 2.7(a) compares contributions to the net heating $\dot{q}_{\text{tot}}$ in equation (2.16) at a snapshot around the time $t \sim t_{\text{visc}}$. The nuclear heating rate, $\dot{q}_{\text{nuc}}$, as a function of radius is shown with a solid red curve, in units of the viscous heating rate $\dot{q}_{\text{visc}}$ (equation 2.15). The two clear peaks, at around the radii $r \approx 2 \times 10^8$ cm and $r \approx 6 \times 10^7$ cm correspond to the carbon and oxygen burning fronts, respectively. In the first case nuclear heating rate is locally as important as viscous heating, i.e. $\dot{q}_{\text{nuc}} \sim \dot{q}_{\text{visc}}$ (M12; FM13). For a steady-state disk, the advective cooling rate $\dot{q}_{\text{adv}}$ (purple line) is a constant fraction of $\dot{q}_{\text{visc}}$ (equation C2), as depicted by the horizontal lightly shaded pink region. As in Fig. 2.4(a), our numerical results roughly agree with this expectation for $r \lesssim R_d$, especially in the comparison model, CO_Nuc, for which nuclear burning has been artificially turned off (grey curve).

Fig. 2.7(b) shows contours of $\dot{q}_{\text{nuc}}/\dot{q}_{\text{visc}}$ in the space of disk radius and time. Comparison with Fig. 2.5 shows that $\dot{q}_{\text{nuc}}$ follows the $^{12}\text{C}$ and $^{16}\text{O}$ burning fronts, and is most important relative to viscous heating at early times $t \lesssim t_{\text{visc}}$ when the burning fronts occur at larger radii in the disk. Despite the importance of nuclear burning near the burning fronts prior to peak accretion, it is subdominant to viscous heating across most radii (away from the burning fronts) and at late times $t \gg t_{\text{visc}}$.

### 2.5.1.3 Outflow Properties

Fig. 2.8(a) shows the cumulative mass distribution $M_w(< v_w)$ of the disk outflows below a given outflow velocity $v_w$, separately for each isotope. The horizontal blue axis along the top
Figure 2.7 (a) Snapshot of the radial profile of heating and cooling rates in the disk midplane (equation 2.16) at $t = 8$ s. Red and purple curves show, respectively, the nuclear heating rate and advective cooling rate, normalized to the viscous heating rate (equation 2.15). Nuclear burning has an order unity impact on the disk and outflow dynamics at locations where $\dot{q}_{\text{nuc}} \sim \dot{q}_{\text{visc}}$, specifically near the $^{12}\text{C}$ and $^{16}\text{O}$ burning fronts at $r \approx 2 \times 10^8 \text{ cm}$ and $r \approx 6 \times 10^7 \text{ cm}$ (Fig. 2.5). At small radii $r \lesssim 2 \times 10^7 \text{ cm}$, endothermic photodisintegrations provide a source of nuclear cooling. A grey curve shows the advective cooling rate for an otherwise identical model, CO$_{\text{Nuc}}$, with nuclear heating artificially turned off. The pink shaded region shows the theoretically expected range of $\dot{q}_{\text{adv}}$ (equation C2) for a steady-state disk with $\gamma = 1.33 - 1.67$. Wind cooling, which is not illustrated here, provides additional cooling of the disk, such that the net heating rate $\Sigma_i \dot{q}_i \approx 0$. (b) Contours of the nuclear heating rate normalized to the viscous heating rate in the space of disk radius $\log_{10}(r)$ and time $\log_{10}(t)$. Peaks in the nuclear heating rate again closely follow the carbon and oxygen burning fronts (cf. Fig. 2.6). Nuclear heating is most significant at early times $t \lesssim t_{\text{visc}}$ at the outermost $^{12}\text{C}$ burning front, where the gravitational potential well is shallow.

shows the corresponding radius $r = 2\eta_w G M/v_w^2$ from which matter leaves the disk. A solid black curve shows the total mass (all isotopes). Short horizontal curves extending beyond the axis depict extrapolated upper bounds on the total mass ejected in various isotopes at $t \to \infty$. For most isotopes, these extrapolations are only very small corrections to the ejected mass at $t_{\text{end}}$, apart from unburned carbon and oxygen (not shown) which increase by a factor of $\sim$ two (see total ejecta extrapolation; black curve).

Of the total mass $M_w = 0.31 M_\odot$ unbound by the end of the simulation, approximately $0.12 M_\odot$ is unburned carbon and $0.14 M_\odot$ is unburned oxygen. Heavier isotopes are ejected with smaller abundances and at higher velocities, which is understood by the fact that they...
Figure 2.8 (a) Cumulative mass distribution $M_w(< v_w)$ of the disk outflows below a given outflow velocity $v_w$ in the C/O fiducial model, evaluated at the final snapshot and shown separately for each isotope. Color and style conventions are the same as in Fig. 2.5, apart for the additional black curve illustrating the total (i.e. summed over all elements) wind distribution for the fiducial model. The horizontal blue axis across the top of the plot equivalently shows the distribution in the disk radii from which the outflow was ejected. Short horizontal curves outside the right axis show the total outflow mass in various elements, extrapolated from the end of the simulation to $t \rightarrow \infty$. (b) Fractional mass outflow rates of various elements $\dot{M}_w(X_A)/\dot{M}_w$, as a function of time. Intermediate mass isotopes are well approximated by the theoretically motivated power-law extrapolation given by equation (2.37), as illustrated by solid grey curves beginning near the simulation end time.

originating from smaller radii in the disk, where $v_w$ is larger. The average mass weighted outflow velocity of the ejecta is $\langle v_w \rangle \simeq 1.2 \times 10^9$ cm (vertical dotted line in Fig. 2.8(a)).

Fig. 2.8(b) shows the fraction of the total mass outflow rate in different isotopes, $\dot{M}_w(X_A)/\dot{M}_w$, as a function of time. In §2.4.2 we described an analytic method for extrapolating the mass outflow rates from the disk to times later than the endpoint of the simulation. Solid grey lines show this power-law extrapolation of the mass loss rates for different isotopes from equation (2.37). Although this provides a reasonable description for intermediate mass elements such as $^{40}$Ca and $^{36}$Ar, other isotopes do not fare as well. The abundances of the unburned isotopes carbon and oxygen obviously do not peak around a particular burning front, but rather extend to the outer edge of the disk. The lowest mass
isotopes, $^4\text{He}$ and free nucleons (not illustrated in Fig. 2.8(b)), which are only present at small radii, are plagued by a similar problem; their radial domain is broad and extends inside the range captured by our numerical grid.

Fig. 2.8(b) also shows that the mass fraction of $^{56}\text{Ni}$ decreases more rapidly with time near the end of our simulation than predicted by equation (2.37; see also Figs. 2.5 and 2.6). This disagreement stems from the assumption that the peak value of $X_A$ is constant in time, while for $^{56}\text{Ni}$ it decreases. The same issue affects the intermediate isotopes discussed previously, albeit to a lesser extent. For these reasons, our extrapolated values for the total ejecta are best taken as upper limits.

2.5.2 Variations about the Fiducial Model

2.5.2.1 Nuclear Heating

Fig. 2.4(a) shows clear differences between the accretion inflow rate in our fiducial model $\text{CO}_{\text{Fid}}$ (dark blue curve) and that with heating from nuclear burning turned off, $\text{CO}_{\text{Nuc}}$ (light grey curve). In the fiducial case $\dot{M}_{\text{in}}$ increases faster with radius than the smooth power-law decline of $\text{CO}_{\text{Nuc}}$, predominantly in two ‘steps’ at the carbon and oxygen burning fronts. As was already discussed, these differences are the result of nuclear burning increasing the wind outflow rate near the burning fronts.

Fig. 2.9(a) compares the disk composition in the $\text{CO}_{\text{Nuc}}$ model (colored curves) to the fiducial case (light grey curves). The composition profiles are nearly identical in shape, yet systematically shifted to slightly larger radii, as compared to the fiducial case. Because less mass is lost to outflows, the correspondingly larger inflow rate increases the disk temperature,
which in turn moves the burning fronts outwards. Despite the outflow rate being locally enhanced near the burning fronts, the total (radial- and time-integrated) mass loss rate is not affected significantly. This generic result is a consequence of the fact that if \( (r_*/R_d)^p \ll 1 \), then the total outflow rate is controlled by the outer feeding rate \( \dot{M}_{\text{in}}(R_d) \), which is unaffected by nuclear burning. The total ejecta mass and its velocity distribution are therefore nearly identical to the fiducial case.

2.5.2.2 Chemical Mixing Efficiency

Models \( \text{CO Mix1} \) and \( \text{CO Mix2} \) explore the effect of changing the dimensionless mixing parameter to values of \( \bar{\alpha} = 0 \) and \( \bar{\alpha} = 1 \), respectively, as compared to the fiducial model with \( \bar{\alpha} = 0.1 \). Mixing should have its greatest impact on the radial composition profile, as
diffusive mixing smooths out strong gradients and discontinuities in $X_A(r)$.

With mixing turned off ($\text{CO Mix1}$), the results are nearly indistinguishable from those of the fiducial case. From this we can conclude that if turbulence is indeed less efficient at mixing passive scalars (such as $X_A$) as compared to transporting angular momentum, i.e. $\tilde{\alpha} \ll 1$, then the effects of mixing can to high accuracy be neglected altogether. Although this is a trivial result for a truly passive scalar, in our case the composition $X_A$ enters the nuclear reaction rates, which feedback on the dynamical structure of the disk.

In the opposite case of strong mixing, the results change more significantly. Fig. 2.9(b) compares the composition profile for model $\text{CO Mix2}$ to the fiducial case. As expected, mixing smooths out sharp features in the composition and generally distributes the burning products across a wider range of radii. Matter is seen to diffuse upstream to larger radii, as shown most clearly in the case of $^{32}\text{S}$ and $^{56}\text{Ni}$. Diffusion downstream also occurs, but it is not readily observed in the composition profiles because the inner profile of the mass fraction is truncated by nuclear burning, which occur sharply inside a fixed radius in the disk, largely irrespective of $X_A$.

Although the total mass of the ejecta is also found to be insensitive to $\tilde{\alpha}$, the abundance of particular isotopes can be altered significantly, generally increasing in comparison with the fiducial model (except for $^{24}\text{Mg}$). Most significantly, the ejected $^{56}\text{Ni}$ mass increases by a factor of $\sim$four (see Table for numerical values for representative isotopes).
Figure 2.10 Evolution of the radial composition for the model \texttt{CO\_Alpha} (colored curves), as in Fig. 2.5 compared to the fiducial model, \texttt{CO\_Fid} (grey curves). The different panels are plotted at snapshots such that $t_{\text{CO\_Fid}} = (\alpha_{\text{CO\_Alpha}}/\alpha_{\text{CO\_Fid}}) \times t_{\text{CO\_Alpha}}$, where $(\alpha_{\text{CO\_Alpha}}/\alpha_{\text{CO\_Fid}}) = 0.1$, and are equivalent to the panels in Fig. 2.5 for the fiducial model. Notably, the composition profiles and evolution of both models scaled this way are identical, apart from a slight shift in radii, which is well explained by equation (2.41).

2.5.2.3 Strength of Turbulent Viscosity

Modifying the value of the viscosity parameter affects the evolution timescale of the disk, which is determined by the viscous timescale $t_{\text{visc}} \propto \alpha^{-1}$ (equation 2.12). Model \texttt{CO\_Alpha} is calculated for $\alpha = 0.01$, as compared to our fiducial model with $\alpha = 0.1$. 

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Fig. 2.10 compares the composition profile for the CO Alpha run to that of the fiducial model at four snapshots, taken at times normalized to the same fraction of the viscous time, i.e. $t_{\text{CO,Fid}} = 0.1 \times t_{\text{CO,Alpha}}$. Applying this mapping, the overall composition at a given time remains nearly identical, which is a non-trivial result because nuclear reactions break the self-similarity of the $\alpha$-disk. The smaller value of $\alpha$ does cause a small shift in the composition profiles of the CO Alpha model to larger radii, with the $^{12}$C burning front increasing by $\approx 20\%$. This shift causes the ratio of nuclear to viscous heating rates, $\dot{q}_{\text{nuc}}/\dot{q}_{\text{visc}}$, to increase by a modest factor of $\sim r_{\text{burn}}(\alpha = 0.01)/r_{\text{burn}}(\alpha = 0.1)$. However, this difference is much smaller than the factor of 10 difference one would expect if the burning fronts occurred at the same radius independent of $\alpha$ ($\dot{q}_{\text{visc}} \propto \alpha$, while $\dot{q}_{\text{nuc}}$ does not depend on $\alpha$).

Finally, the total mass and composition of the disk outflows are also nearly independent of $\alpha$, with the important exception of the $^{56}$Ni mass, which increases by a factor of $\sim$three for $\alpha = 0.01$ as compared to the fiducial model.

### 2.5.2.4 Initial Density Profile

We also explore the sensitivity of our results to the initial density profile of the disk, which is uncertain because it depends on the details of how the WD is disrupted. Model CO_Den explores the impact of increasing the radial power-law index of the inner initial density profile (equation 2.4) to $m = 4$ from its fiducial value of $m = 2$. This slightly increases the

\[ r_{\text{burn}} \propto \alpha^{1/[p-(5/2-p)\beta/4]} \]  

(2.41)

Using $\beta = 29$ as appropriate for the $^{12}$C($^{12}$C, $\gamma$)$^{24}$Mg reaction, we obtain $r_{\text{burn}}(\alpha = 0.01)/r_{\text{burn}}(\alpha = 0.1) \approx 1.17$, in perfect agreement with the numerical results.

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initial radius of the disk, $R_d$, and, more importantly, decreases the initial density at $r < R_d$. Although the composition profile of model CO_Den are nearly identical to those of the fiducial models at late times $\gtrsim 12 \text{ s} \sim t_{\text{visc}}$, the differences at early times are more pronounced. In CO_Den the burning fronts occur at smaller radii than the fiducial case because of the lower normalization of the temperature profile resulting from the lower initial density.

Although the initial density distribution of the disk impacts its evolution only at early times, $t \lesssim t_{\text{visc}}$, the final (integrated) outflow distribution does exhibit some significant differences, most notably in that the mass distributions of some isotopes extend to higher velocities. This is because, at early times when the burning fronts are located at smaller radii than in the fiducial model, nucleosynthesis occurs deeper in the potential well where the outflow velocity $v_w \propto r^{-1/2}$ is larger.

### 2.5.2.5 Initial Composition

Exploring the sensitivity of the disk composition and associated nuclear burning to variation of the initial C/O mixture (models CO_Comp1, CO_Comp2) revealed only very weak dependence on this parameter. Model CO_Comp2, which has slightly larger carbon abundances ($X_{^{12}\text{C}} = 0.6$) produced somewhat larger peak $^{20}\text{Ne}$ and $^{24}\text{Mg}$ abundances, although the composition profile morphology is otherwise identical to Fig. 2.5. Similarly, model CO_Comp1, which is slightly carbon deficient (and appropriately oxygen rich), $X_{^{12}\text{C}} = 0.4$, led to weaker carbon burning and subsequent $^{20}\text{Ne}$, $^{24}\text{Mg}$ abundances. These traits, in addition to the zeroth order effect of larger or smaller initial carbon/oxygen abundances in each model, were the only observable differences between the ejecta distribution of these models and the fiducial
model.

### 2.5.2.6 Wind Prescription

Finally, we explore the sensitivity of our results to the parameters of the wind outflow model. Models CO\_Wnd1, CO\_Wnd2 vary the fiducial critical Bernoulli parameter from zero, to $B_{\text{crit}}' = \pm 0.1$, and models CO\_Wnd3, CO\_Wnd4 alter the wind efficiency parameter from its nominal value of one, to $\eta_w = 0.5, 2$ respectively.

In model CO\_Wnd1 the initial aspect ratio of the disk (equation 2.5) is larger than its steady-state value, $\theta_{ss}$ (equation 2.28). As discussed in §2.4.1 and Appendix B, this initial configuration results in a strong transient wind phase lasting a short time $\sim t_w$ that cools the disk to its steady-state Bernoulli parameter, $B_{\text{crit}}'$. In this specific case, the initial aspect ratio of the disk is $\theta_{\text{initial}} = 0.418$ (same as for the fiducial model) is 10% larger than its steady-state value at $R_{d,0}$ of $\theta_{ss} \approx 0.38$ (evaluated numerically). From equation (2.29), we predict a prompt ejection of $\sim 3 \times 10^{-2}M_\odot$, in excellent agreement with the $2.8 \times 10^{-2}M_\odot$ outflow mass measured from the model at early times. Besides this precursor outflow, the evolution and final outflow composition for model CO\_Wnd1 is nearly identical to the fiducial model. Model CO\_Wnd2 also shows no significant deviations from the fiducial model at times $t \gtrsim t_{\text{visc}}$. This is not surprising because Fig. 2.3 shows that the mass-inflow index $p$ is not sensitive to the value of the critical Bernoulli parameter.

The results are more sensitive to the wind efficiency parameter, $\eta_w$. Increasing $\eta_w$ by a factor of two, as in model CO\_Wnd4, decreases the mass-inflow exponent $p$ noticeably (as also predicted by Fig. 2.3). This means that more material accretes inwards, at the expense of
Figure 2.11 Velocity distribution of wind ejecta for models CO_Wnd3, (a), and CO_Wnd4, (b). Different style/color curves represent the distribution in various isotopes (same as in Fig. 2.8(a)). Black curves plot the total outflow mass distribution, which can be compared with the solid grey curves, illustrating the same quantity for model CO_Fid (note though that the top x-axis does not apply to model CO_Fid).

weaker outflows. This causes the disk density and thereby temperature at any given radius to increase in comparison with the fiducial model, shifting the burning fronts to larger radii but preserving the shape and evolution of the composition profiles. The outflow distribution, on the other hand, changes qualitatively.

Fig. 2.11(b) shows the final wind distribution for model CO_Wnd4 in comparison to the fiducial model CO_Fid. The most noticeable change is that the wind distribution extends to larger velocities, and the total ejecta mass decreases (although particular isotope yields do increase). The first of these trends is straightforward to understand because the wind efficiency parameter directly determines the ejecta velocity. However, even by rescaling the velocity axis by $\eta_w^{1/2}$, the total CO_Wnd4 ejecta distribution curves shows an excess of mass at high velocities, due to the fact that more mass flows to smaller radii (the small value of $p$). A similar argument explains why most high mass isotopes, such as $^{56}$Ni are overproduced.
Quantitatively, the total ejecta mass for this model decreases by $\sim 25\%$ to $0.23M_\odot$, while the $^{56}\text{Ni}$ yield increases by a factor of two to $1.3 \times 10^{-3}M_\odot$. Model $\text{C}_0\_\text{Wnd3}$, in which the wind efficiency parameter is decreased, can be explained by similar arguments (Fig. 2.11(a)).

### 2.5.3 He WD Models

We additionally consider models for the accretion of a disrupted helium WD, the properties of which differ qualitatively from the C/O models discussed above. Our fiducial model, $\text{He}_\text{Fid}$, describes a typical $0.3M_\odot$ He WD which merges with a $1.2M_\odot$ NS companion. The model parameters, $\alpha = 0.1$, $\eta_w = 1$, $\text{Be}_{\text{crit}}' = 0$, $\bar{\alpha} = 0.1$, and $(m,n) = (2,7)$, are the same as for the fiducial C/O model (see Table 2.1).

The initial disk radius for this model, $R_d \simeq 3.4 \times 10^9$ cm, is larger than in the case of our C/O models due to the smaller mass of He WDs (Fig. 2.1). The peak accretion (or outflow) rate is $\simeq 8 \times 10^{-4} M_\odot$ s$^{-1}$ and is achieved on a timescale of $t_{\text{visc}} \approx 25$ s.

Fig. 2.12 shows snapshots of the disk composition at four representative timesteps, similar to Fig. 2.5 for $\text{C}_0\_\text{Fid}$. At early times (top panel), the density limited triple-$\alpha$ reaction begins fusing $^{12}\text{C}$ at $r \sim 4 \times 10^8$ cm. Due to the high $^4\text{He}$ abundance, rapid $\alpha$-captures onto the seed $^{12}\text{C}$ nuclei immediately fuse into higher mass elements, $^{28}\text{Si}$, $^{32}\text{S}$, $^{36}\text{Ar}$, $^{40}\text{Ca}$, and peaking at $^{56}\text{Ni}$. The intermediate elements $^{16}\text{O}$, $^{20}\text{Ne}$ and $^{24}\text{Mg}$, which have extremely high $\alpha$-capture rates serve as ‘stepping stones’ in this process, but are severely underproduced themselves, reaching peak abundances of only $10^{-4}$, $3 \times 10^{-4}$, and $10^{-3}$, respectively. As the density increases towards the peak accretion time (second panel), the triple-$\alpha$ reaction becomes more effective, increasing the seed carbon abundance and thereby
Figure 2.12 Evolution of the disk composition, similar to Fig. 2.5 but for the fiducial helium WD model, He_Fid.
the higher mass elements’ mass fractions as well ($^{56}$Ni reaches peak abundances of $\sim 1$). At later times (third panel from top) the disk density decreases, inhibiting the triple-$\alpha$ reaction and increasing the helium abundance while the high mass isotopes decrease, until at late times the disk reverts to a nearly pure helium composition (fourth panel).

Qualitatively, this nucleosynthesis is dramatically different than that of our previous C/O WD models, producing large $^{56}$Ni and $^{40}$Ca abundances (along of course with a large unburned $^4$He abundance) despite extremely low $^{16}$O mass fractions. Additionally, the evolution of the composition profiles differs qualitatively from the C/O WDs — the composition is set almost entirely by the triple-$\alpha$ reaction which, while effective at $\sim$peak-accretion time when the density is highest, becomes very inefficient at late times in the disk evolution. This causes the mass fraction $X_A$ profiles to steadily decrease after $t \gtrsim t_{\text{visc}}$, and essentially disappear at late times. In comparison, the C/O model composition profiles preserved their morphology and normalization in a self-similar manner, merely shifting inwards to smaller radii at late times.

The inefficiency of the triple-$\alpha$ reaction at early/late times is a direct consequence of its strong density dependence, and on the fact that the reverse reaction, $^{12}$C $\rightarrow$ 3$\alpha$, is in contrast a temperature sensitive reaction. The two balance each other at a fixed temperature, $T_{\text{lim}} \simeq 1.7 \times 10^9$ K (see Appendix D and in particular equation D3). At temperatures $\gtrsim T_{\text{lim}}$, the triple-$\alpha$ reaction cannot effectively fuse $^{12}$C, since any carbon would immediately be disintegrated back into $^4$He by the dominant reverse triple-$\alpha$ process. Although triple-$\alpha$ may successfully occur around $\sim t_{\text{visc}}$, at early (late) times the disk density rises (drops) at a faster rate than the disk temperature, so that at some point $T(\rho_{3\alpha}) > T_{\text{lim}}$, and carbon
Figure 2.13 (a) Contour plot of two representative isotope abundances, $^{12}$C and $^{56}$Ni, in the fiducial helium disk model as a function of $r$ and $t$. The carbon abundance traces the triple-$\alpha$ limiting reaction in the flow. The inset grey contours depict curves of constant density, $\rho$, logarithmically equally spaced by $\Delta \log_{10}(\rho \text{ [g cm}^{-3}\text{]}) = 0.2$, and labeled at $\rho = 10^4, 10^5, 10^6$ g cm$^{-3}$. The strongly density dependent triple-$\alpha$ reaction is seen to roughly track these curves, but effectively shuts off after $t \sim 200$ s, when the triple-$\alpha$ ‘burning density’, $\rho_{3\alpha} \sim 10^5$ g cm$^{-3}$, approaches the $T = T_{\text{lim}}$ constant temperature curve (thick red). Above this temperature (equation D3) the reverse triple-$\alpha$ reaction rate, $^{12}$C $\rightarrow$ $^{3}\alpha$, exceeds the forward $3\alpha \rightarrow ^{12}$C rate, and carbon cannot effectively be fused. (b) Nuclear heating rate in the $\log_{10}(r)$, $\log_{10}(t)$ plane. Nuclear reactions deposit a significant amount of energy in the disk in the outer radii at which the triple-alpha reaction commences, and are dynamically more important than in the C/O burning case (see Fig. 2.7(b)).

fusion effectively ceases. Here $\rho_{3\alpha}$ is the ‘burning density’ at which the triple-$\alpha$ process occurs. Since the seed carbon nuclei are key to forming successively heavier elements through rapid $\alpha$-captures, this affects the entire disk composition for elements above $^{4}$He.

Fig. 2.13(a) illustrates this point by showing a contour plot of the evolution of two representative isotopes — $^{56}$Ni and $^{12}$C, the second of which is a direct tracer of the triple-$\alpha$ burning. Additionally, curves of constant density are plotted in logarithmic spacings of $\Delta \log_{10}(\rho \text{ [g cm}^{-3}\text{]}) = 0.2$. To first order, the triple-$\alpha$ burning front tracks the density evolution and peaks around $\rho_{3\alpha} \sim 10^5$ g cm$^{-3}$. The thick red curve plots a constant temperature contour at $T = T_{\text{lim}}$. It is clear that at late times, $\gtrsim 200$ s, the condition $T(\rho_{3\alpha}) > T_{\text{lim}}$
is satisfied and the carbon abundance drops significantly. The $^{56}$Ni abundance also drops starting at this time, illustrating how the triple-$\alpha$ reaction effectively limits the entire disk composition.

As in our previous discussion of C/O WDs, nuclear burning deposits significant energy in the disk. Fig. 2.13(b) plots the nuclear heating rate relative to the viscous disk heating, similar to Fig. 2.7(b) for the C/O fiducial model. Nuclear heating is an important energy source as long as the triple-$\alpha$ reaction is effective, and dominates the total disk heating around $\sim 10^8$ cm for a significant portion of the disk evolution.

Finally, in Fig. 2.14 we plot the outflow velocity distribution at the simulation termination time, $t_{\text{end}} = 434$ s. At this time, 0.16$M_\odot$ or roughly half of the initial WD mass has been ejected (black curve), predominantly as unburnt $^4$He (solid blue curve), at characteristic velocities of $\langle v_w \rangle \approx 8.7 \times 10^8$ cm s$^{-1}$.

We note that the numerically obtained values of the $^{56}$Ni and $^{54}$Fe ejecta mass are only lower limits on their true values because these isotopes’ composition profile extends interior to our numerical inner boundary, and therefore their contributions to the ejecta are not entirely captured (see Fig. 2.12). Additionally, we do not extrapolate the outflow composition to $t \to \infty$ as we did for the C/O models, since the triple-$\alpha$ reaction which sets the disk composition does not obey the analytic scaling of equation (2.37) which was developed for temperature limited nuclear reactions. Despite this fact, the ejecta mass in various isotopes at time $t_{\text{end}}$ is likely a reliable estimator of the ejected mass at time $t \to \infty$ (except for the case of $^{56}$Ni and $^{54}$Fe discussed above, and for helium which tracks the total ejecta mass and is expected to reach values of $\sim M_d = 0.3M_\odot$). This is because
Figure 2.14 Velocity distribution of the wind ejecta, from the fiducial helium WD model, \( \text{He}_{\text{Fid}} \), evaluated at the simulation end time. The distribution of different isotopes are colored as in Fig. 2.8(a). A solid black curve shows the total mass distribution (summed over all elements). The \( ^{56}\text{Ni} \) distribution extends up to the largest velocities (smallest radius) captured by our numerical grid, indicating that we do not resolve the entire \( ^{56}\text{Ni} \) outflow, and thus that our model provides only a lower limit on the nickel mass in the ejecta.

by the simulation termination time, the peak mass fractions of isotopes heavier than \( ^4\text{He} \) decrease below \( \sim 10^{-2} \) (see Fig. 2.12), so that at subsequent times, the disk composition and accompanying outflow is essentially purely helium.

The total outflow distribution of model \( \text{He}_{\text{Nuc}} \) is overall very similar to the fiducial model, except that \( \text{He}_{\text{Fid}} \) exhibits a slight excess of ejected matter around \( \sim 10^9 \text{ cm s}^{-1} \). This occurs due to the significant (in fact dominant) contribution of nuclear burning to the disk heating rate (Fig. 2.13(b)), which is locally balanced by stronger wind cooling, i.e., larger outflows.

The lack of nuclear feedback in model \( \text{He}_{\text{Nuc}} \) causes an increase in the disk density (compared with the fiducial model) at radii \( \lesssim 2 \times 10^8 \text{ cm} \) (near the triple-\( \alpha \) burning front), which in turn increases the efficiency of the density-limited triple-\( \alpha \) burning. This changes the composition profiles somewhat more substantially than by merely shifting the burning.
fronts to larger radii (as was the case for the temperature limited reactions of the C/O disk, see Fig. 2.9(a)), and in particular, more \(^{56}\text{Ni}\) is synthesized.

As in the C/O WD scenario, the mixing parameter \(\tilde{\alpha}\) has little effect on the results. With mixing effectively turned off (model \(\text{He Mix1}\)), the results are essentially identical in every respect to the fiducial model, indicating once again that small mixing parameters can be well approximated by neglecting mixing altogether. For model \(\text{He Mix2}\), in which the mixing parameter is increased to \(\tilde{\alpha} = 1\), the composition profiles show prominent tails towards larger radii due to burned ash diffusing upstream, in parallel with the results of C/O WD mixing illustrated in Fig. 2.9(b). This does not have a significant effect on the outflow composition, except on the \(^{56}\text{Ni}\) yield, which increases by a modest factor of \(\sim 1.5\).

Similarly, varying the initial density distribution of the disk, as in model \(\text{He Den}\), has little effect on the outcome. Just as for the C/O models, the composition evolution changes slightly at early times \(\lesssim t_{\text{visc}}\), but is identical to the fiducial model at later times.

On the other hand, varying the alpha-viscosity parameter (model \(\text{He Alpha}\)) impacts the results much more significantly than for the C/O models. Fig. 2.15 shows the evolution of the composition profile for this model at four representative timesteps. These are usefully compared with the fiducial helium model composition (Fig. 2.12), which are overplot with light grey curves. In the case of C/O WDs, the composition profile preserved its radial shape in time. However, for helium accretion this is clearly not the case. At the time of peak accretion (second panel), the \(^4\text{He}\) abundance decreases below \(X_A \lesssim 10^{-2}\) at radii \(r \lesssim 8 \times 10^7\) cm, resulting in the significantly larger amounts of intermediate elements such as \(^{40}\text{Ca}\) and \(^{36}\text{Ar}\) being synthesized further in. The isotopes \(^{56}\text{Ni}\) and \(^{54}\text{Fe}\) are produced almost
entirely interior to our inner grid boundary, precluding a reliable prediction of their ejecta abundances.

The more prominent nucleosynthesis of this model is readily explained by equation (D6), which shows that the threshold (minimum) accretion rate required to sustain efficient triple-\(\alpha\) burning drops substantially with the Shakura-Sunyaev viscosity parameter \(\alpha\), so that \(^4\!\mathrm{He}\) burning is more effective for smaller values of \(\alpha\).

Changing the helium WD mass also changes the outcome significantly. Model He\_Mass corresponds to a 0.4\(M_\odot\) He WD with the same, nominal 1.2\(M_\odot\) binary companion. The results differ from the fiducial model and to some extent continue the trend apparent in He\_Alpha of strong triple-\(\alpha\) burning.

Finally, in model He\_Wnd4 we increase the wind efficiency parameter from its fiducial value of \(\eta_w = 1\). This also has a substantial affect on the results, mainly by decreasing the mass inflow exponent \(p\) (Fig. 2.3). The resulting higher mass inflow rate increases the disk density at each radius, which as previously discussed is intimately related to the efficiency of nuclear burning. For larger values of \(\eta_w\), the triple-\(\alpha\) process remains effective for a longer period of time, increasing the nucleosynthesis of heavy elements. The dynamical significance of helium burning is also increased accordingly, with \(|q_{\text{nuc}}/q_{\text{visc}}|\) reaching peak values of \(\sim 20\). The same reasoning explains why nucleosynthesis is less effective for model He\_Wnd3, for which \(\eta_w\) is smaller than its fiducial value.
Figure 2.15 Evolution of the nuclear composition for the model He\textsubscript{Alpha} (colored curves), as in Fig. 2.12 compared to the fiducial model, He\textsubscript{Fid} (grey curves). Different panels correspond to snapshots defined by $t_{\text{He Fid}} = \left(\frac{\alpha_{\text{He Alpha}}}{\alpha_{\text{He Fid}}}\right) \times t_{\text{He Alpha}}$, where $\left(\frac{\alpha_{\text{He Alpha}}}{\alpha_{\text{He Fid}}}\right) = 0.1$, and are equivalent to the panels in Fig. 2.12 for the fiducial model. Note that the composition profiles are qualitatively different in model He\textsubscript{Alpha} due to the density sensitivity of the limiting triple-$\alpha$ reaction. This is in contrast to C/O models, in which scaling the viscosity parameter, $\alpha$, changes (to first order) only the overall timescale of disk evolution (Fig. 2.10).
2.5.4 Hybrid WDs

We conclude by discussing results for WDs composed of both C/O and He, so-called ‘hybrid’ WDs (Han et al. 2000). Given the rather speculative nature of this type of WD, we run only a couple models and do not perform a full parameter space survey as was done for C/O and He WDs. The model parameters are identical to the fiducial C/O case (§2.5.1), except for the initial composition of $X_{12C} = X_{16O} = 0.4$, $X_{4He} = 0.2$ and $X_{12C} = X_{16O} = 0.475$, $X_{4He} = 0.05$ for models CO\_He1 and CO\_He2 respectively.

The composition profile of model CO\_He2 and its evolution are illustrated in Fig. 2.16. Grey curves show for comparison the results of the fiducial C/O WD model, CO\_Fid. The composition profiles are generally similar to the C/O model. The primary difference is at large radii, where $\alpha$-captures onto $^{16}O$ fuse $^{20}Ne$ and subsequently $^{24}Mg$ already at $\sim 10^{9}$ cm. This increases these isotopes’ abundances in the wind significantly, but does not alter the profiles at small radii appreciably. The composition profile of model CO\_He1, which has a large initial helium abundance, schematically extends the same trend apparent in CO\_He2. $\alpha$-captures efficiently burn the initial oxygen content into $^{20}Ne$, $^{24}Mg$ and even $^{28}Si$ at large radii. Carbon burning at $r \sim 10^{8}$ cm replenishes the depleted $^{16}O$ abundance, and at higher temperatures high mass isotopes are synthesized up to $^{56}Ni$.

One small but noticeable difference of the hybrid models is that the burning fronts of heavy isotopes shift slightly to smaller radii, indicating that the density, and hence temperature at a given radius are smaller than for the fiducial C/O model. The reason is the familiar argument — nuclear reactions, in this case at the $^{16}O(\alpha, \gamma)^{20}Ne$ burning front, deposit large amounts of energy at large radii, which launches substantial outflows and decreases the den-
Figure 2.16 Composition profiles of hybrid WD model CO\textsubscript{He2}. The background grey curves plot the composition of the fiducial C/O WD model (CO\textsubscript{Fid}). This model, which contains only 5\% initial \textsuperscript{4}He abundances is comparatively similar to the fiducial C/O composition profiles except for \textsuperscript{16}O $\alpha$-captures which fuse \textsuperscript{20}Ne and \textsuperscript{24}Mg at large radii $\sim 10^9$ cm.
sity at smaller radii. This is illustrated in Fig. 2.17, which shows contours of $|\dot{q}_{\text{nuc}}/\dot{q}_{\text{visc}}|$ (spaced logarithmically this time). The nuclear heating rate exceeds the viscous heating rate by over an order of magnitude at early times around $r \lesssim 10^9$ cm.

The short timescales and large energy release associated with the $^{16}$O $\alpha$-captures suggest that this burning may realistically produce a detonation instead of a steady inflow. Such a detonation would not be captured by our numerical scheme, and we therefore cannot resolve this in our present work. Estimating the ratio of the burning to dynamical timescales, we find for the hybrid WD models

$$\frac{t_{\text{nuc}}}{t_{\text{dyn}}} \sim \frac{u \Omega_K}{\dot{q}_{\text{nuc}} |_{r_{\text{burn}}}} < 1,$$

indicating that nuclear burning proceeds dynamically. Importantly, none of the other C/O or He WD models constructed in our work satisfy this criterion, illustrating that this is a direct feature of composite He/O matter burning.

### 2.6 Discussion

We have not included massive O/Ne WDs in our analysis due to the numerical difficulty in evolving the disk when the photodisintegration burning front extends to large radii. We expect such models to behave qualitatively similar to our C/O models, except that burning occurs at larger radii and is hence dynamically more important. For extremely massive WDs, nuclear burning can extend all the way out to $\sim R_d$, in which case it may be important already during circularization, and our model does not strictly apply.
Disk outflows from WD-NS mergers are capable of powering short lived supernova-like optical transients (M12). These fast transients peak on a characteristic timescale of (Arnett 1982)

$$t_{pk} = \left( \frac{3\kappa M_w}{4\pi c \langle v_w \rangle} \right)^{1/2} \approx 6.5 \text{ days} \left( \frac{M_w}{0.4M_\odot} \right)^{1/2} \left( \frac{\langle v_w \rangle}{10^9 \text{ cm s}^{-1}} \right)^{-1/2},$$

where $\kappa = 0.05 \text{ cm}^2 \text{ g}^{-1}$ is the opacity, normalized to a value appropriate for Fe-poor matter, and $\langle v_w \rangle$ is the mass-weighted average velocity of the ejecta. The peak luminosity of the transient approximately equals the rate of thermal heating of the ejecta at the peak time, $L_{pk} \approx \dot{E}(t_{pk})$. If radioactive decay of $^{56}\text{Ni}$ provides the dominant heating source, then the
optical transients are typically dim,

\[ L_{pk} \approx 3 \times 10^{40} \text{ erg s}^{-1} \frac{M_w(^{56}\text{Ni})}{10^{-3} M_\odot} \exp \left[ 0.8 \left( 1 - \frac{t_{pk}}{7 \text{day}} \right) \right], \tag{2.44} \]

given the modest $^{56}\text{Ni}$ yields of our disk wind solutions, $M_w(^{56}\text{Ni}) \sim 10^{-4} - 3 \times 10^{-3} M_\odot$ (Table E1). The amount of nickel in the ejecta could in principle be increased due to outflows from the very inner portions of the accretion disk near the central compact object. Here the midplane is composed of alpha particles and free nucleons, but the temperature is high enough that heavy elements are synthesized above the disk midplane, i.e. within the outflow itself (MacFadyen & Woosley 1999b).

The luminosity of the transient could also be enhanced by additional energy deposited within the wind ejecta by shocks. High velocity winds which are launched off the disk at late times ($\gg t_{v\text{isc},0}$) could collide with the bulk of ejecta shell launched earlier, thermalizing the kinetic energy of the late-time winds. The kinetic power released from the disk winds at late times and small radii $\sim r_*$ based on our analytic estimates in \ref{eq:2.42} may be crudely estimated as

\[ \dot{E}_w(r_*, t) \sim \frac{G M M_d}{r_* t_{v\text{isc},0}} \left( \frac{r_*}{R_{d,0}} \right)^p \left( \frac{t}{t_{v\text{isc},0}} \right)^{-\left(2p+4\right)/3}. \tag{2.45} \]

For characteristic parameters ($p = 0.5$), this yields peak transient luminosities of

\[ L_{pk} \sim 2 \times 10^{43} \text{ erg s}^{-1} \left( \frac{M}{1.4 M_\odot} \right)^{2/3} \left( \frac{M_d}{0.6 M_\odot} \right) \left( \frac{\alpha}{0.1} \right)^{-2/3} \times \left( \frac{\theta}{0.4} \right)^{-4/3} \left( \frac{R_{d,0}}{10^9 \text{ cm}} \right)^{1/2} \left( \frac{r_*}{5 \times 10^6 \text{ cm}} \right)^{-1/2} \left( \frac{t_{pk}}{7 \text{ day}} \right)^{-5/3}, \tag{2.46} \]
comparable to those of normal SNe. Such a scenario might give rise to more luminous, rapidly-evolving transients, such as SN 2002bj (Poznanski et al. 2010; Drout et al. 2014; Shivvers et al. 2016).

WD-NS mergers can in principle power high-energy transients, if a fraction of the accreted mass can be launched as a collimated jet from the inner accretion disk (although we do not identify any specific mechanism for this here). Such a relativistic outflow could carry as much as $\sim 10^{51}$ erg, comparable in energy to the non-relativistic disk winds. An additional, more robust, associated transient is a late-time radio flare produced as the disk winds (or the possible collimated jet) shock the surrounding interstellar medium (Nakar & Piran 2011; Margalit & Piran 2015).

The total nucleosynthetic yields of our models are summarized in Fig. 2.18 and Table E1. For the C/O WD models, the nucleosynthesis is relatively robust for lighter isotopes, varying by factors of a few between models. Heavier elements, in particular $^{56}$Ni, show a significant scatter of nearly an order of magnitude between models.$^3$

Although $^{40}$Ca is among the most abundant isotopes produced in our He WD models, its total mass within the ejecta of $\lesssim 10^{-2}M_\odot$ appears to be insufficient to explain the inferred calcium abundances of the Ca-rich transients of $\gtrsim 0.1M_\odot$ (e.g. Table 4 of Perets et al. 2010a). If these abundance measurements are robust, then WD-NS mergers as envisioned in this paper could represent at most only a subclass of these transients.

Explosive burning in the accretion disk if a detonation wave develops (FM13) could also produce significantly larger calcium yields. Indeed, the hybrid C/O/He WD models show

$^3$Note that in these models helium is only synthesized by photodisintegrations at small radii, which are not entirely resolved in the boundaries of our numerical grid, and therefore the helium abundances are only lower bounds.
strong, dynamical, nuclear burning indicative of their possible susceptibility to explosions. Their composition is similar to that found to give large Ca abundances following dynamical burning (e.g., Perets et al. 2010a; Waldman et al. 2011). Similar dynamical burning and detonation of a disk-like configuration may occur during the core collapse of rapidly rotating stars (Kushnir 2015), also as a result of the low temperature threshold of the $^{16}$O($\alpha$, $\gamma$)$^{20}$Ne reaction. Nuclear burning in this context may be particularly relevant to collapsar accretion disks and the source of $^{56}$Ni in gamma-ray burst SNe (M12).

We conclude with a discussion of the fate of the accreting NS in a WD-NS merger. For our fiducial model, only a small fraction of the disrupted WD mass $\lesssim (r_*/R_d)^p$ is accreted onto the NS, with the remainder unbound from the system by outflows. For our fiducial C/O WD model, only $2.7 \times 10^{-2}M_\odot$ crosses the numerical boundary of our grid at the simulation end time, which corresponds to a conservative upper bound of $\lesssim 4.8 \times 10^{-2}M_\odot$ reaching the NS surface at $t = \infty$. A more realistic value of $2 \times 10^{-2}M_\odot$ is obtained if we extrapolate...
the wind mass loss to small radii, so that only a portion of the matter crossing the inner boundary of our numerical grid reaches the NS surface. It is a reasonable assumption that the NS can accept this accreted matter because at the extremely super-Eddington accretion rates involved, photons are trapped and advected with the accreting matter (Chevalier 1989). The NS atmosphere can then efficiently cool and settle via neutrino emission (to which the flow is optically thin) from electron-positron and electron-capture Urca reactions.

Given the large maximum NS mass inferred from recent observations of $\sim 2M_\odot$ NSs (Demorest et al. 2010b; Antoniadis et al. 2013b), it is improbable that a less massive NS will accrete enough matter to collapse to a BH. In fact, even if winds are inefficient at cooling the disk and $p$ is small (in contradiction with global MHD simulations), nearly the entire mass of the disrupted WD must be accreted to induce a collapse. This contrasts with previous studies that neglect disk winds and nuclear burning (Paschalidis et al. 2011), which predict a collapse once the envelope sheds its angular momentum and cools.

Assuming that the final merger outcome is an isolated NS, a natural question is whether the small amount of mass accreted onto the NS surface can spin it up, forming a recycled millisecond pulsar. From our fiducial C/O WD model, we estimate that approximately $\approx 6 \times 10^{47}$ g cm$^{-2}$ s$^{-1}$ of angular momentum is accreted with the inflowing mass, which, for characteristic NS moments of inertia $\sim 10^{45}$ g cm$^{-2}$ (Lattimer & Schutz 2005a) is equivalent to spinning up the NS from rest to a rotation period of $P \sim 10$ ms. A more detailed analysis of the accretion process in the final region up to the NS surface is required to more accurately quantify this, but at the level of uncertainty of our current model, WD-NS mergers appear to provide another channel for producing isolated recycled millisecond pulsars (e.g. Lorimer).

2.7 Conclusions

We have presented a vertically averaged time-dependent model of the accretion disks from WD-NS mergers that incorporates nuclear burning. Such disks are expected as an outcome of unstable mass transfer between a WD and a binary NS companion, which may set in once GW emission drives the binary into Roche lobe contact. Note that besides characteristic masses, we have not assumed any parameters specific to the NS. As such, our model applies in its entirety to mergers of a WD with a stellar mass BH companion.

The extremely high density of the accretion flow renders it radiatively inefficient, necessitating an alternative means of cooling (other than photon radiation) to offset the nuclear and gravitational (viscous) heating. Following Blandford & Begelman (1999a) we have assumed that disk outflows provide this mechanism, and locally regulate the disk’s enthalpy. The properties of disk outflows predicted by our model (in particular, the mass loss coefficient, \( p \)) qualitatively agree with the results of global hydrodynamical and MHD disk simulations.

Nuclear burning plays a non-trivial role in both the dynamics and the nucleosynthesis in the accretion disk, as first described by M12. The radial composition profile resembles the ‘onion-skin’ structure of evolved stars, where the initial WD matter is successively synthesized into heavier elements at sequentially smaller radii. The temperature and density of the disk midplane at at any radius \( \lesssim R_d \) rise until the time of peak accretion \( \sim t_{\text{visc}} \), and subse-
quently decrease. This shifts the radial composition profiles in C/O models to larger/smaller radii, respectively, and effectively inhibits nuclear burning for He models (which is limited by the triple-$\alpha$ barrier) at early/late times.

Unbound outflows from the disk carry away the majority of the initial WD mass at velocities of $\langle v_w \rangle \sim 10^9$ cm s$^{-1}$. Most of the wind ejecta is unburned, with a composition matching that of the initial WD. However, the ejecta also contains a significant fraction of freshly synthesized intermediate-mass and heavy isotopes, including $\sim 10^{-3} M_\odot$ of $^{56}$Ni. These outflows may give rise to a short-lived $\sim$week long optical transient similar to SNe, a long-term radio relic due to the interaction of the fast ejecta with the interstellar medium, and more speculatively — a possible high-energy transient. We additionally find that accretion onto the NS surface is relatively limited ($\sim 10^{-2} M_\odot$); it is thus unlikely that the NS will collapse to a BH, but it might accrete sufficient angular momentum to be spun up to periods of $P \sim 10$ ms.

The qualitative features of our numerical models are summarized as follows:

1. The radial composition profiles of C/O WD models preserve a fixed morphology, which evolves self-similarly with time. This transcends any specific model parameter assumptions and applies globally to all our C/O WD simulations. In this sense, one can approximate the flow as a steady-state model at any given time, with only the outer mass feeding rate $\dot{M}_{\text{in}}(R_d)$ secularly changing between epochs.

2. For C/O models, nuclear burning only moderately impacts the disk dynamics at the carbon burning front (and to a lesser extent at the oxygen burning front and the photodisintegration region). He WD models are affected more significantly by nuclear
burning, especially for small $\alpha$ or large $\eta_w$, in which case nuclear heating becomes the dominant energy source in a large portion of the disk.

3. The outflow rate in isotopes other than the initial WD composition peaks on short timescales of $\sim t_{\text{visc}}$, and is therefore well captured by our simulations. Extrapolations of the ejecta composition to late times yields in most cases nearly identical results.

4. The results are robust to the initial density distribution, the ‘chemical’ mixing efficiency, and the (regulated) disk Bernoulli parameter. The C/O models are also relatively unaffected by changes to the Shakura-Sunyaev alpha-viscosity parameter, except by scaling the evolution time of the disk ($t_{\text{visc}} \propto \alpha^{-1}$). He WD models on the other hand are sensitive to the value $\alpha$. Both C/O and He WD models depend strongly on the mass inflow exponent, $p$, which is set primarily by the wind efficiency parameter $\eta_w$.

5. ‘Hybrid’ C/O/He WD models with even modest helium mass fractions exhibit strong, dynamical, nuclear burning, indicative of their possible explosive nature. None of our C/O or He WD models showed signs of dynamic burning.

The one-dimensional model presented here is only an approximate starting point to accurately modeling the aftermath of WD-NS/BH mergers. It is nevertheless justified given the limited number of previous studies of these systems, and the rich behavior even this simple model already reveals. Future work, including multi-dimensional hydrodynamic models, is needed to explore outstanding issues such as the role of dynamical burning and relative importance of convection in transporting energy outwards in the accretion flow (Narayan 97).
Despite its limitations, our approach has allowed us to extensively explore the parameter space of WD-NS merger accretion disks (which would be computationally prohibitive with multi-dimensional simulations), and to develop analytic estimates to aid future studies. In particular, we plan to pursue in future work more detailed models for the optical and radio transients of the disk outflows calculated here.
Chapter 3

Merger of a White Dwarf-Neutron Star Binary to $10^{29}$ Carat Diamonds:

Origin of the Pulsar Planets

3.1 Introduction

The millisecond pulsar PSR B1257+12 is famous for hosting the first known extra-solar planets, as discovered through its timing residuals (Wolszczan & Frail 1992). Continuing observations have confirmed the existence of two earth-mass exoplanets and one moon size body orbiting PSR B1257+12 with masses $3.9M_\oplus$, $4.3M_\oplus$, $0.02M_\oplus$ and semi-major axes 0.46 AU, 0.36 AU, 0.19 AU, respectively (Wolszczan 1994; Konacki & Wolszczan 2003). The

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nearly coplanar orbits of the earth-mass planets, along with their $\sim$3:2 resonance and small eccentricity ($e \simeq 0.02$), provide strong evidence for a disk formation scenario (Konacki & Wolszczan 2003). Although a number of general formation scenarios have been proposed (Phinney & Hansen 1993, Podsiadlowski 1993, and references therein), the origin of the pulsar planets remains a mystery.

The fundamental reason that the PSR B1257+12 planetary system poses such a theoretical challenge, and is inherently different from our own solar system or the multitude of exoplanetary systems discovered since, is the fact that the central body is a neutron star (NS). Standard planet formation theory focuses on bodies formed in the gaseous disk that exists concurrent with the birth of the stellar system (e.g., the progenitor of the NS). However, these theories encounter severe difficulties in explaining the PSR B1257+12 system because it is unclear how the observed planets could have survived the supernova (SN) explosion responsible for the NS, or the preceding giant phase during which the planets would have been engulfed in an extended stellar envelope (e.g. Nordhaus et al. 2010). It is thus conventionally believed that the PSR B1257+12 planets must have formed after the SN, thereby requiring a novel formation mechanism.

Podsiadlowski (1993) overviews the range of proposed origins for the pulsar planets. Beyond elucidating the planet formation mechanism itself, a successful model should explain why PSR B1257+12 is a recycled millisecond pulsar (Miller & Hamilton 2001), as well as the dearth of planets around the vast majority of other pulsars (Kerr et al. 2015, Fig. 3.4). Two leading explanations for the gaseous disk out of which the planets form are: (a) the fall-back accretion of bound stellar debris following the SN explosion (e.g. Menou et al. 2001), and (b)
the tidal disruption of a stellar object into a gaseous disk following a close encounter with
the NS. Variants of the latter include the disruption of a white dwarf (WD) by a binary NS
companion; a WD-WD merger in which one WD is disrupted and the other gains mass and
collapses to a NS; and the collision of the NS with a non-degenerate star, such as a binary
companion (following the SN birth kick received by the NS).

Phinney & Hansen (1993) explore the viability of a wide range of such models, using a
general analytic framework to describe the long-term viscous spreading of the comparatively
promptly formed gaseous disk (Pringle 1981). They found that both SN fallback and disrup-
tion models are at least marginally capable of placing sufficient mass on the correct radial
scale to explain the PSR B1257+12 system. Currie & Hansen (2007) built on this work by
including additional physical ingredients, such as irradiation and layered accretion, and by
numerically solving for the disk evolution. These authors found that tidal disruption models
typically underproduce the amount of solids required at $\sim 1$ AU compared with SN fallback
models, in large part due to the assumption of solar metallicity in the tidal disruption models
as compared to the metal-rich fall back case. In followup work, Hansen et al. (2009) study
the planet assembly process from smaller bodies, using initial conditions motivated by the
disk models of Currie & Hansen (2007).

In this work we explore the merger of WD-NS binaries as a formation channel for
the PSR B1257+12 planetary system. We describe the long-term evolution of the remnant
gaseous disks from such mergers using an approximate analytic approach, which extends the
work of Phinney & Hansen (1993), however incorporating several physical aspects unique
to this model. In particular, we impose initial conditions informed by our recent study of
The early phases of WD-NS disk evolution (Margalit & Metzger 2016, hereafter MM16). We also model the previously ignored radiatively-inefficient accretion flow (RIAF) phase at early times, which leads to an important mass sink in the form of disk outflows. We also account for the unique C/O composition predicted by this scenario.

The early phases of a WD-NS merger have been previously studied as sources of gamma-ray bursts (Fryer et al. 1999b; King et al. 2007; Paschalidis et al. 2011) and SN-like optical transients (Metzger 2012a; Fernández & Metzger 2013b; MM16). The stages in the evolution of such systems are briefly summarized as follows (e.g. MM16). A detached WD-NS binary slowly inspirals due to gravitational wave emission, before overflowing its Roche lobe at a separation of \( \sim 10^9 \) cm. If the ratio of the WD to the NS mass is sufficiently large (e.g., \( \gtrsim 0.2-0.5; \) Bobrick et al. 2016), then mass transfer from the WD is unstable, and the ensuing phase of runaway accretion results in the WD being tidally disrupted (Verbunt & Rappaport 1988b) into a thick and dense torus surrounding the NS (e.g. Fryer et al. 1999b). During the earliest stages of disk evolution (\( \sim \) minutes-hours), dynamically important nuclear burning occurs in the disk midplane (Metzger 2012a). Outflows from the disk reduce the mass reaching the central NS and provide a necessary source of cooling, which offsets gravitational (viscous) and nuclear heating.

As a pulsar planet formation scenario, a WD-NS merger is appealing for several reasons. First, it naturally explains the millisecond rotation period of PSR B1257+12, as the initially old NS is spun-up due to the accretion of debris from the disrupted WD (van den Heuvel & Bonsema 1984; Ruderman & Shaham 1985; MM16). Second, the rates of WD-NS mergers (a fraction \( \sim 10^{-3} - 10^{-2} \) of the core collapse SN rate; O'Shaughnessy & Kim 2010b; Kim et al. 2010c; Kim et al. 2012a).
Kerr et al. (2015) recently surveyed a sample of 151 young pulsars and found no evidence for additional planetary systems, implying that only a small fraction $\lesssim 10^{-2}$ of pulsars host planets. This agreement contrasts with SN fallback models, which at least naively would predict pulsar planetary systems to be common. Finally, the high metallicity of disks formed from WD debris can lead to a large fraction of the disk mass forming solid rocks, circumventing efficiency problems associated with solar metallicity models. One intriguing consequence of the predicted carbon-rich composition of the pulsar planets is that they are in essence enormous ($\sim 10^{29}$ Carat) diamonds (e.g., Kuchner & Seager 2005).

This paper is organized as follows. We begin by outlining our viscous disk model in §3.2. In the following section we present detailed results for a sample model, and investigate whether disk conditions are conducive to planet-formation by conducting a parameter-space survey (§3.3). We continue by discussing a possible planet-formation mechanism specific to our WD-NS merger scenario (§3.4). Finally, in §6.5 we conclude and discuss possible implications and observational signatures of our model.

### 3.2 Disk Evolution Model

The most basic feature of accretion disk evolution follows from angular momentum conservation — globally, the disk must spread as it loses mass in order to conserve angular momentum (Pringle 1981). If the angular momentum carried away by the lost mass is negligible (which is not true during the early RIAF phase following a WD-NS merger), then the mass-averaged
disk radius $R_d$ increases with decreasing total mass $M_d$ as

$$R_d \propto M_d^{-2}. \tag{3.1}$$

This process of ‘viscous spreading’ allows planetary bodies to form on $\sim$ AU radial scales from an initial disk of size $\sim 10^9$ cm $\approx 10^{-4}$ AU.

In greater detail, the disk surface density $\Sigma$ evolves according to a diffusion equation (Pringle 1981)

$$\partial_t \Sigma = \frac{3}{r} \partial_r \left[ \nu \frac{\partial \ln (\nu r^2 \Sigma \Omega)}{\partial \ln r} \right] - \dot{\Sigma}_w, \tag{3.2}$$

where $\dot{\Sigma}_w$ is a sink term accounting for local mass loss in the form of disk outflows. Outwards angular momentum transfer is commonly described by an effective kinematic viscosity

$$\nu = \alpha c_s H \approx \alpha c_s^2 / \Omega = \alpha \theta^2 r v_k, \tag{3.3}$$

where $c_s \equiv \sqrt{P/\rho}$ is the isothermal sound speed in the disk midplane, $H = c_s / \Omega$ the disk scale-height, $\Omega \approx \Omega_k = v_k / r = \sqrt{GM_{\text{NS}}/r^3}$ is the orbital angular velocity, and

$$\theta \equiv H / r = c_s / v_k, \tag{3.4}$$

is the disk aspect ratio. The dimensionless parameter $\alpha$ parameterizes the strength of the viscosity (Shakura & Sunyaev 1973b). When the disk is ionized, turbulence due to the magnetorotational instability (MRI) provides a physical mechanism driving angular momentum transport (Balbus & Hawley 1991b), with values of $\alpha \gtrsim 0.01 - 0.1$ motivated by simulations.
of the MRI (e.g. Davis et al. 2010b) and observations (King et al. 2007). Once the midplane temperature decreases to $\lesssim 1000$ K and the disk material recombines to become neutral, the MRI may be suppressed or confined to a thin ‘active’ ionized layer on the disk surface (Gammie 1996, Hansen 2002). However, the high intensity of cosmic rays and ionizing radiation in the vicinity of an accreting neutron star will substantially reduce the dead zone as compared to the standard proto-stellar case. Turbulent eddies in the active region may overshoot the active/dead boundary and transport mass also within the dead zone (e.g., Fleming & Stone 2003).

In thermal equilibrium, a local balance exists between the heating and cooling rates per unit surface area of the disk, $\dot{q}^+ = \dot{q}^-$. During the early phases of disk evolution, viscous dissipation provides the dominant heat source,

$$\dot{q}^+_{\text{visc}} = \nu \Sigma \Omega^2 \left( \frac{\partial \ln \Omega}{\partial \ln r} \right)^2 \approx \frac{9}{4} \alpha \theta^2 \Sigma \Omega_k^3 r^2, \quad (3.5)$$

while at later times irradiation of the outer disk from the central accretion flow dominates. The competing cooling rate $\dot{q}^-$ is set by disk outflows/radial advection at early times, and radiative cooling at late times.

Specifying an equation of state (EOS) closes the equations, allowing a full solution of the disk radial structure and temporal evolution in terms of $\Sigma$, $\theta$, and the midplane temperature $T$. Two limiting cases, corresponding to the midplane pressure being dominated by gas or
radiation, respectively, give temperatures of

\[
T = \begin{cases} 
(\mu m_p/k_B)\theta^2 \Omega^2 r^2 ; & \text{gas} \\
[(3/2a)\theta^2 \Omega^2 r \Sigma]^{1/4} ; & \text{radiation},
\end{cases}
\]

(3.6)

where \( \mu \) is the mean molecular weight. In analytic estimate we adopt a value of \( \mu = 13 \) corresponding to neutral gas of half carbon and half oxygen composition, although \( \mu \) can be lower at early times (high temperatures) when the gas is ionized or higher at later times (lower temperature) after molecular CO forms.

### 3.2.1 Evolution Stages

The dominant sources of heating and cooling change as the disk viscously spreads outwards.

Here, we overview the evolutionary stages of the disk, before describing each in greater detail (see Fig. 3.1 for an example solution presented later).

1. Immediately after its formation, the dense torus is unable to cool effectively via radiation because the photon diffusion timescale out of the disk midplane is much longer than the viscous timescale. During this ‘RIAF’ phase, heating due to viscous dissipation and nuclear burning are offset by cooling due to radial advection and by launching powerful disk outflows, which carry away both energy and mass.

2. The disk evolves in the RIAF regime until the midplane density decreases to the point that radiation is no longer trapped on the accretion timescale. After this point, radiative cooling takes over and the disk transitions to a standard thin configuration. In
this ‘Viscously-Heated Radiative’ phase, the disk cooling rate depends on its optical
dept and hence on the opacity law $\bar{\kappa}(\rho, T)$. The resulting evolution is rich, traversing
a diversity of phases as the decreasing midplane temperature/density evolution carry it through different opacity regimes.

3. As the disk continues to evolve and the accretion rate decreases, heating due to irra-
diation from the innermost accretion flow eventually comes to exceed internal viscous
heating. Once the disk enters this ‘Irradiated’ phase, it becomes vertically isothermal
and the midplane temperature no longer depends on $\bar{\kappa}$. The disk evolution no longer
depends on the opacity law, but a transition in its evolution can still occur when the
accretion rate drops below the Eddington rate.

4. The disk continues in the irradiated phase until finally dispersing at late times due
to solid condensation and photo-evaporation. Evaporation begins in full once the
disk spreads to a sufficiently large radius $R_{\text{evap}} \sim 10$ AU, where the sound speed of
irradiated surface layers of the disk exceeds about ten percent of the escape speed. In
this ‘Photo-evaporation’ phase, the disk expansion stalls at $R_{\text{evap}}$ and its gas content
drains exponentially on the viscous timescale, in contrast to the power-law evolution
which characterizes the earlier accretion stages.

3.2.1.1 RIAF Phase

The accretion torus formed by the tidal disruption of the WD is initially extremely hot and
sufficiently dense as to be unable to cool through radiative diffusion. The early stages of this
torus evolution were explored by MM16 who adopt an ADIOS type model (Blandford &
Figure 3.1 Temporal evolution of the remnant accretion disk from a WD-NS merger, corresponding to the fiducial model of a $0.6M_\odot$ C/O WD for a viscosity $\alpha = 0.1$, with initial conditions from MM16. Panels show the outer disk radius (top), surface density (middle), and temperature (panel). Vertical dashed lines indicate important transitions in the disk accretion regime, whereas vertical dotted lines show transitions in the opacity law (which only affect the evolution during the viscously heated radiative phase). The top panel also shows important transition radii $R_{\text{rad}}$, $R_{\text{irr}}$, and $R_{\text{evap}}$ (see description in §3.2.1).

Begelman (1999b) which presumes that a powerful wind of hot matter is blown off the disk. The mass loss rate and wind kinetic energy at each radius are determined by the requirement that the winds carry away sufficient energy to locally balance viscous and nuclear heating, such that the Bernoulli parameter of the disk midplane is regulated to a fixed value $\lesssim 0$ (bound disk).

The radial dependence of the mass inflow rate during the RIAF can be approximated
as a power-law,

\[ \dot{M}_{\text{in}} = \dot{M}_d \left( \frac{r}{R_d} \right)^p, \]

(3.7)

where hereafter all quantities with subscripts \( X_d \) are evaluated at the outer (characteristic) disk radius. The exponent \( 0 \leq p < 1 \) characterizes the disk mass loss, since continuity implies that the mass loss rate obey \( \dot{M}_{\text{out}} \propto [1 - (R_{NS}/R_d)^p] \). Hydrodynamical \( \alpha \)-disk and global MHD simulations of RIAFs typically find \( p \gtrsim 0.5 \) (Stone et al. 1999; Igumenshchev & Abramowicz 2000; Hawley et al. 2001; Narayan et al. 2012; McKinney et al. 2012; Yuan et al. 2012).

Importantly, the total angular momentum of the disk is not conserved during the RIAF phase due to the presence of disk winds. Under the assumption that disk winds exert zero net torque on the disk (i.e. mass is lost with the same specific angular momentum as that of the disk midplane from the wind launching point), Metzger et al. (2008b) show that

\[ R_d \propto M_d^{-2/(2p+1)}, \]

(3.8)

which reduces to equation (3.1) in the zero wind mass loss limit \( (p = 0) \).

As initial conditions for the present study, we make use of the ‘final’ disk configurations of MM16 calculated out to several times the initial viscous timescale of the torus formed from the merger. We extrapolate these ‘final’ configurations to much later times of interest here, using the RIAF self-similar evolution described by Metzger et al. (2008b) and summarized in Appendix F. Except at very early times, the disk is dominated by radiation pressure during the RIAF phase, and becomes increasingly so with time.
3.2.1.2 Viscously-Heated, Radiatively Cooled Phase

Once the disk enters the radiative phase, the midplane cools via radiation diffusion at a rate given by

\[ \dot{q}_{\text{rad}} = \frac{4\sigma T^4}{3f(\tau)}, \]  

(3.9)

where

\[ f(\tau) \approx \tau + 2\tau^{-1}/3 + 4/3 \approx \begin{cases} \tau; & \tau \geq \sqrt{2/3} \\ 2/3\tau; & \tau < \sqrt{2/3} \end{cases} \]

(3.10)

is a function of the vertical optical depth (\( \tau = \bar{\kappa}\Sigma/2 \)), which results from solving the vertical radiative transfer problem (e.g. Sirko & Goodman 2003), and \( \bar{\kappa} \) is the Rosseland mean opacity. Moving from high to low temperatures, relevant opacities include electron scattering, free-free/bound-free opacities of C/O-rich matter, and graphite dust. Appendix G describes the full opacity curve \( \bar{\kappa}(\rho,T) \) implemented in our analysis, which we approximate in our analytic estimates by a series of broken power-laws of the form \( \bar{\kappa} \propto \rho^l T^{-k} \) (the exponents \( l, k \) enter in the self-similar disk solutions described in Appendix F).

During the RIAF phase, viscous heating is offset by radial advection and wind cooling. However, as the disk evolves and its optical depth decreases, radiative losses eventually become dominant, and the disk becomes geometrically thin (\( \theta_d \ll 1 \)). This transition to the radiative phase occurs once \( \dot{q}_{\text{rad}} - \dot{q}_{\text{visc}} = \eta_{\text{rad}}\dot{q}_{\text{visc}} \) with \( \eta_{\text{rad}} \lesssim 1 \). In the optically-thick, radiation-pressure dominated regime appropriate for the RIAF phase, this condition is satisfied once

\footnote{Note that the photon diffusion timescale is shorter than the viscous inflow timescale when this criterion is satisfied, justifying treating the radiative energy losses as truly being local.}

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the disk radius exceeds the critical value

\[ R_{\text{rad}} \approx \left( \frac{3}{2} \right)^4 \eta_{\text{rad}}^2 \alpha^2 \theta^2 \tau \frac{G M_{\text{NS}}}{c^2}. \]  

Initially, the value of \( R_{\text{rad}} \) greatly exceeds the actual outer radius of the disk, \( R_d \). However, as \( R_{\text{rad}} \) decreases and \( R_d \) grows, the outer disk transitions from RIAF to radiative once \( R_d \approx R_{\text{rad}} \) (Fig. 3.1).

The radiative transition would be continuous, were it not for the fact that the radiation pressure-dominated, radiative disk solution is thought to be unstable (Lightman & Eardley 1974). Assuming this instability manifests, then following the radiative transition, the disk immediately (on a thermal timescale) collapses to the corresponding gas-pressure dominated solution satisfying \( \dot{q}_{\text{rad}}(T, \theta) = \dot{q}_{\text{visc}}(\theta) \) for the same \( \Sigma, R_d \) (see Shen & Matzner 2014, for a similar discussion in the context of tidal disruption events). Recent work by Jiang et al. (2016) suggests that opacity due to line transitions of iron may help stabilize the disk at temperatures \( T \sim 2 \times 10^5 \) K close to those which characterize the radiative transition in our models; however, additional theoretical work is required to determine whether the disk can truly remain stable for the tens or hundreds of thermal times relevant to the viscous timescale evolution.

Another important discontinuity occurs if the disk cools to \( T \sim 10^4 \) K while still in the optically-thick, viscously-heated radiative phase. At such temperatures the disk is susceptible to the recombination instability, induced by the sudden drop in opacity once the gas begins to recombine (see Fig. G1). Modeling the opacity during this transition as a steep power-law, \( \kappa \propto T^k \), one can show that (in the optically thick regime) \( \partial \ln \dot{q}_{\text{visc}}^+ / \partial \ln T > \partial \ln \dot{q}_{\text{rad}}^- / \partial \ln T \),
implying that the disk is thermally unstable (Piran 1978).

The recombination instability has been invoked as a possible origin of the limit-cycle behavior observed in dwarf novae (e.g. Cannizzo et al. 1988; Coleman et al. 2016). Indeed, as long as some fixed (and sufficiently large) accretion rate $\dot{M}_d$ is enforced as an external boundary condition, the instability region will exhibit cyclical behavior, oscillating between the ionized and neutral solutions. If however, as in our case, the instability occurs first at the outer disk, and external mass is not supplied to the system (i.e. $\dot{M}_d$ is not externally forced), then the result is a single transition between the ionized and neutral solutions.

### 3.2.1.3 Irradiation-Heated, Radiatively Cooled Phase

Irradiated disks have been widely studied in the case of protoplanetary disks illuminated by the central star or a surrounding cluster (e.g. Chiang & Goldreich 1997). In our scenario, the disk is illuminated primarily by the accretion luminosity from the inner disk onto the central NS.

Assuming that the inner disk is unobscured from the viewpoint of the outer disk, the heating rate per area due to irradiation is approximately given by

$$\dot{q}_{\text{irr}}^+ = \theta \left( \frac{\partial \ln \theta}{\partial \ln r} \right) \frac{GM_{\text{NS}}}{4\pi r^2 R_{\text{NS}}} \min \left( \dot{M}, \dot{M}_{\text{Edd}} \right),$$

(3.12)

where the factors including $\theta$ account for the geometric cross-section illuminated by the central source, and the minimum accounts for the fact that the NS cannot radiate above the Eddington limit, $\dot{M}_{Edd} \simeq 2 \times 10^{18} (R_{\text{NS}}/10^6 \text{ cm}) (\bar{\kappa}/0.2 \text{ cm}^2 \text{ g}^{-1})^{-1} \text{ g s}^{-1}$.

In practice, when the disk is accreting at highly super-Eddington rates, the accretion
luminosity from the inner disk could be channeled along the rotation axis by the thick inner
torus (e.g. Jiang et al. 2014, Sadowski & Narayan 2015), away from the outer disk. There is
also some evidence in AGN that the outer disk may be shielded from X-rays emitted by the
inner disk, even at sub-Eddington accretion rates (e.g. Luo et al. 2015). To account for the
possibility of shielding of the inner disk during the super-Eddington phase, we also consider
models in which we set \( \dot{q}_{\text{irr}} = 0 \) for \( \dot{M} \gtrsim \dot{M}_{\text{Edd}} \).

Solving the radiative transfer equation for an externally illuminated disk illustrates that
the disk develops a nearly isothermal vertical profile, in contrast to when the disk is heated
from the midplane (e.g. Kratter et al. 2010). As the radiative cooling rate no longer depends
on \( \tau \) (i.e. \( f(\tau) \to 1 \) in equation 3.9), irradiated disk solutions do not depend on the opacity.
However, from equation (3.12) note that the time evolution does depend on whether \( \dot{M} \) is
larger or smaller than \( \dot{M}_{\text{Edd}} \).

The transition from viscous- to irradiation-dominated occurs when \( \dot{q}_{\text{irr}} = \dot{q}_{\text{visc}} f(\tau) \), once
the disk spreads to a characteristic radius

\[
R_{\text{irr}} = \frac{3}{2} R_{\text{NS}} f(\tau) \max \left( 1, \frac{\dot{M}}{\dot{M}_{\text{Edd}}} \right) \theta^{-1} \left[ 1 + \left( \frac{\partial \ln \Sigma}{\partial \ln r} \right)^2 \right]^{-1},
\]

where we have used equation (3.12). Again, although initially \( R_{\text{irr}} \) greatly exceeds the
physical extent of the disk \( R_d \), its value quickly decreases as the disk optical depth and
accretion rate drop, such that the transition to the irradiated regime occurs at \( R_d = R_{\text{irr}} \)
(Fig. 3.1). If irradiation of the outer disk is blocked during the super-Eddington phase, then
the transition instead coincides with the sub-Eddington transition.

### 3.2.1.4 Photo-Evaporation Phase

The disk begins to evaporate due to photo-ionization heating once it spreads to radii approaching the so-called gravitational radius \( R_g = \frac{GM_{\text{NS}}}{c_{s,g}^2} \approx 400 \text{ AU} \),

\[
R_g = \frac{GM_{\text{NS}}}{c_{s,g}^2} \approx 400 \text{ AU}, \tag{3.14}
\]

at which the escape velocity \( \sim (GM_{\text{NS}}/r)^{1/2} \) equals the sound speed \( c_{s,g} \approx 2 \text{ km s}^{-1} \) of photo-ionized gas with an approximately fixed temperature \( \approx 5 \times 10^3 \text{ K} \) set by the balance between photo-ionization and line cooling (e.g. Melis et al. 2010, for a metal-enriched composition).

Naively, one would expect photo-evaporation to become relevant only once the disk spreads to \( R_d \sim R_g \). However, more detailed estimates consider the exponential profile of the disk atmosphere and the fact that outflowing matter is accelerated further after detaching from the disk, as it is subjected to further external irradiation. Adams et al. (2004) find that the mass evaporation rate from the outer disk edge can be expressed as (for \( R_d < R_g \))

\[
\dot{M}_{\text{evap}} \approx 2\pi R_d \Sigma_d c_{s,g} \left( \frac{R_g}{R_d} \right)^2 e^{-R_g/2R_d}. \tag{3.15}
\]

The sensitive dependence of \( \dot{M}_{\text{evap}} \) on \( R_d/R_g \) shows that, as the disk expands approaching \( R_g \), the mass loss rate rises exponentially. Conversely, the evaporation timescale, \( t_{\text{evap}} \sim M_d/\dot{M}_{\text{evap}} \), decreases exponentially. This elucidates the subsequent evolution — once the disk expands to the critical radius \( R_{\text{evap}} \) at which the evaporation timescale equals the
accretion timescale, the disk evolution stalls.

Up to inessential numerical factors of order unity, the evaporation disk radius \( R_{\text{evap}} \) at which \( t_{\text{evap}} = t_{\text{visc}} \) is determined by the solution of the transcendental equation

\[
x^{-3/2}e^{x/2} = 1/\alpha \theta_d^2 ,
\]

which in the range \( \alpha \theta_d^2 \sim 10^{-10} - 10^{-2} \) is approximately given by

\[
R_g/R_{\text{evap}} \equiv x \simeq -5.024 \log_{10} (\alpha \theta_d^2) + 8.283 .
\]

For typical parameters, this yields values of \( R_{\text{evap}} \sim \) of a few percent of \( R_g \), corresponding to tens of AU.

Since the disk radius \( R_d \approx R_{\text{evap}} \) is regulated to an approximately fixed value in the photo-evaporation phase, the viscous timescale (which depends most sensitively on radius) also remains approximately constant. This breaks the self-similarity of the disk solution, resulting in the exponential evaporation/accretion of the remaining disk mass over the viscous timescale

\[
t_{\text{evap}} \sim \frac{M_d}{M_d|_{R_{\text{evap}}}} \approx \frac{7A\mu m_p \Omega r^2}{9\alpha k_B T} |_{R_{\text{evap}}} \\
\simeq 4 \text{ Myr} \left( \frac{\alpha}{0.01} \right)^{-1} \left( \frac{R_{\text{evap}}}{10 \text{AU}} \right)^{1/2} \left( \frac{T|_{R_{\text{evap}}}}{100 \text{ K}} \right)^{-1}
\]

at the time of the transition to the evaporative phase, where we have taken a characteristic value of \( T \approx 100 \text{ K} \) as an estimate of the temperature at the evaporative transition (see
equation (3.23), and $A \approx 3$ is a constant relating $M_d$ to the local mass at $R_d$ (equation 3.21).

We neglect additional mass sinks due to solid condensation on the disk evolution, as was included, e.g., in Currie & Hansen (2007). Although solid formation is obviously relevant to planet formation, neglecting its effect on the disk evolution should not appreciably alter our results, in part because roughly half of the disk mass is comprised of oxygen and must remain in gaseous form (a significant fraction may also remain trapped in inert CO). There is also some evidence that the solid condensation responsible for forming planets in the PSR B1257+12 system happened quickly once the appropriate conditions were first reached near the outer radius of the disk (§6.5).

3.2.2 Model Description

Following Phinney & Hansen (1993), we model the disk evolution by a patchwork of matched self-similar solutions corresponding to each stage described in §3.2.1. The viscous time at the outer disk radius $R_d(t)$ controls the rate of mass accretion rate $\dot{M}_d(t)$ reaching smaller scales and hence the temporal spreading rate set by angular momentum conservation, as described in Appendix F. Transitions between different regimes occur once either (1) the value of $R_d$ exceeds the critical radii $R_{\text{rad}}$, $R_{\text{irr}}$, or $R_{\text{evap}}$; (2) the disk becomes optically thin ($\tau = \sqrt{2/3}$); (3) the opacity regime at $R_d$ changes (if the disk is in the radiative regime), or (3) the accretion rate becomes sub-Eddington (if the disk is in the irradiated phase).

We assume that the disk achieves a new self-similar solution instantaneously following any regime transition. As such, the complete solution is continuous in all variables, with two exceptions: (a) the transition from the unstable radiation-dominated regime to the gas
pressure-dominated solution at the radiative disk transition. (b) the transition to a low temperature (neutral) solution at onset of the recombination instability. In both cases, the disk discontinuously transitions to a lower $\dot{M}_d$ regime in which the viscous timescale is larger than just prior to the transition. The disk therefore stalls at these transitions for a time $t \sim t_{\text{visc}}$, until the new self-similar evolution proceeds (see also Shen & Matzner 2014).

The similarity solutions described in Appendix F provide the radial scaling of disk variables, which then allows us to extrapolate the solution at any given time from $R_d$ to smaller radii. Using the same criteria discussed above, we determine the critical radii at which the disk transitions between different regimes, and extrapolate from any such transition radii inwards using the radial similarity solutions for the new state.

### 3.3 Results

Starting from the final configuration of WD-NS merger disks calculated by MM16, we follow the remainder of the disk evolution. Our main goal is to investigate the disk conditions at $R_p \sim 0.3 – 0.5$ AU where the PSR B1257+12 planets reside, and close to where they likely formed originally (Hansen et al. 2009). We focus on the mass available for planet formation near this radius as compared to the $\simeq 8M_\oplus$ required to explain the pulsar planets, along with the local disk temperature. Solid condensation and subsequent planet formation (§3.4) begins only once the temperature has decreased to the condensation of gaseous carbon to graphite grains at a critical temperature of $T_c \lesssim 2000$ K (Goeres 1996).

We begin by describing a fiducial model, which overviews the basic stages of the disk evolution. We then move on to a parameter survey that more thoroughly examines the
requisite conditions for planet formation.

3.3.1 Fiducial Model

Our baseline model corresponds to the disruption of a 0.6\(M_\odot\) C/O WD by a NS of mass 1.4\(M_\odot\). We assume that the WD composition is half carbon and half oxygen. For simplicity, we take a constant mean molecular weight of \(\mu \simeq 13\) (corresponding to the neutral atomic C/O phase) while calculating the disk evolution depicted in Figs. 3.1, 3.2. In practice, the mean molecular weight should be smaller, \(\mu \simeq 1\), at earlier times when the gas is fully ionized, and larger, \(\mu \simeq 28\), at late times once a significant fraction of the gas condenses into carbon-monoxide molecules. We adopt a Shakura-Sunyaev viscosity parameter of \(\alpha = 0.1\), and a wind mass-loss exponent of \(p \simeq 0.44\) during the RIAF phase\(^2\). The numerical calculations of [MM16] corresponding to these parameters conclude at \(t \simeq 136\) s, at which point \(R_d \simeq 5.8 \times 10^9\) cm, \(\Sigma_d \simeq 1.5 \times 10^{12}\) g cm\(^{-2}\), and \(\theta_d \simeq 0.43\). A total of 0.34\(M_\odot\) has been lost from the disk by this early time, due to disk winds and accretion onto the NS. Finally, in our fiducial solution we assume that the outer disk is irradiated by the inner accretion flow, even during the super-Eddington phase.

Fig. 3.1 shows the temporal evolution of the outer disk radius (top panel), surface density (middle), and temperature (bottom panel), for this fiducial model. Vertical dashed curves indicate key transitions in the outer disk’s accretion regime, and vertical dotted curves demarcate transitions in the opacity law at \(R_d\). The disk initially evolves in the RIAF phase, until \(t \simeq 4 \times 10^5\) s when the disk transitions to a radiatively cooled regime (\(R_d = R_{\text{rad}}\)). This

\(^2\)In the [MM16] model, the value of \(p\) is determined by the asymptotic velocity of the winds relative to the local escape speed of the disk and the value of the Bernoulli integral to which the midplane is regulated by wind cooling.
transition is discontinuous in temperature and scale height due to onset of the Lightman-Eardley instability, following which the disk collapses to the geometrically-thin, gas pressure-dominated solution. The disk stalls at this transition radius until \( t \) equals the new (larger) viscous timescale, after which the self-similar radiative evolution commences.

At \( t \approx 5 \times 10^8 \) s the temperature decreases to 8000 K and the disk encounters the recombination instability, transitioning discontinuously to the lower temperature solution in the opacity gap. Once again, a brief stalling in the evolution is associated with this sudden transition. Shortly thereafter, \( R_d = R_{\text{irr}} \) and the outer disk becomes irradiation heated. Irradiation is dominant throughout the remainder of the disk evolution, and is only affected by the transition from a super-Eddington to sub-Eddington accretion rate around \( t \sim 2 \times 10^{10} \) s. The evolution finally terminates once \( R_d = R_{\text{evap}} \), following which the disk photo-evaporates on a viscous timescale.

The conditions of the gaseous disk near the radius of presumptive planet formation for this baseline model are illustrated in Fig. 3.2, which shows the local mass and midplane temperature at \( R_p = 0.4 \) AU. Initially, \( R_d < R_p \) and thus no appreciable mass is present at this location. When the outer disk radius first crosses \( R_p \), the mass at this position is \( \simeq 65M_\oplus \), but the temperature is marginally higher than required for solid condensation, \( T = T_c \simeq 2000 \) K. As time elapses (grey arrow), the disk continues to spread, accreting some of its mass, and carrying the remainder to larger radii, so the local mass at \( R_p \) decreases. The temperature at this fixed radius remains constant for some period, as the disk is locally in the super-Eddington irradiation heated regime (equation [3.22]). Only later, once the accretion rate becomes sub-Eddington, does the local temperature decrease to the point to allow solid
condensation.

Evidently, by the time the temperature at $R_p$ decreases to 2000 K, the local disk mass available for planet formation is an order of magnitude too low. At face value, this rules out our fiducial model as the source of planets in the PSR B1257+12 system. However, if we instead consider the variation of the fiducial model described earlier, in which the irradiation phase is delayed until the accretion rate becomes sub-Eddington (dashed line in Fig. 3.2), then planet formation can commence at an earlier phase, when the disk mass is still $\sim 60M_\oplus$. Although $T_p$ again temporarily increases above the condensation temperature once irradiation turns on at $\dot{M} = \dot{M}_{\text{Edd}}$, this may not destroy the solids that have formed because the sublimation and melting temperature of solid carbon is higher than the condensation
temperature.

We now discuss the broader range of disk parameters that allow the mass-temperature constraints needed for planet formation to be satisfied.

### 3.3.2 Analytic Considerations

The two main free parameters in our model are the strength of the viscosity $\alpha$ and the wind mass-loss exponent $p$. Some uncertainty also arises due to our approximate opacity curve, but this has little affect on the results if the disk is in the irradiation phase at the time of solid condensation. The initial conditions are also reasonably well determined: the disk aspect ratio has a fixed value $\theta \sim 0.4$ set by its initial thermal energy determined by energy conservation during the merger. The initial size and surface density of the disk are solely determined by WD mass (MM16), which varies by less than a factor of 2 (e.g. Liebert et al. 2005b).

In order to explore a wider parameter space than the specific models studied by MM16, we consider the initial disk conditions at yet earlier times of minutes-hours, near the time when the accretion rate first peaks following the WD disruption and before wind mass loss has become significant. Specifically, we consider $R_{d,0} = 2 \times 10^9$ cm, $\Sigma_{d,0} = 2 \times 10^{13}$ g cm$^{-2}$, $\theta_{d,0} = 0.4$, for an initial disk mass of $M_{d,0} = 0.6M_\odot$. These values, informed by our previous numerical studies, describe the initial disk well.
3.3.2.1 Analytic Estimates of Mass and Temperature at Planet Formation Radius

As illustrated by our fiducial model, the available mass budget at the planet formation radius \( R_p \) peaks when the outer disk radius first reaches this location (Fig. 3.2). We thus focus on estimating the disk mass at this time using global angular momentum considerations. In doing so we must treat the RIAF and the radiatively cooled regimes separately.

The RIAF phase ends when the disk first becomes radiative \( (R_d = R_{rad}) \). Assuming electron scattering dominates the opacity during this early phase, then \( R_{rad} \propto \tau^2 \propto \Sigma_d^2 \) (eq. 3.11). During the RIAF phase, the disk radius evolves according to equation (3.8). At the radiative transition, the disk mass has therefore decreased to a fraction \( (R_{rad,0}/R_{d,0})^{-(2p+1)/(4p+12)} \) of its initial value, where

\[
R_{rad,0} \equiv \left( \frac{3}{2} \right)^4 \eta_{rad}^2 \alpha^2 \theta_{d,0}^2 \kappa_{es}^2 \Sigma_{d,0}^2 \frac{GM_{NS}}{c^2} \approx 6.7 \times 10^{25} \text{ cm} \left( \frac{\alpha}{0.01} \right)^2 \left( \frac{\Sigma_{d,0}}{2 \times 10^{13} \text{ g cm}^{-2}} \right)^2.
\]

is the initial value of the radiative transition radius, and in the second equality we assume canonical values of \( \eta_{rad} = 0.5, \theta_{d,0} = 0.4 \) and \( M_{NS} = 1.4M_\odot \).

After the radiative transition, the disk evolves without further angular momentum loss, expanding according to equation (3.1), and losing additional mass solely to accretion. The remaining mass once the disk first reaches \( R_p \) is thus

\[
M_d (R_d = R_p) = M_{d,0} \left( \frac{R_p}{R_{d,0}} \right)^{-1/2} \left( \frac{R_{rad,0}}{R_{d,0}} \right)^{-p/(2p+6)}.
\]
This mass is notably significantly smaller than the standard result for a spreading disk under the assumption of zero wind mass loss \((p = 0; \text{e.g. Phinney & Hansen 1993; Currie & Hansen 2007})\). Finally, the total disk mass, \(M_d\), is related to the local mass at the outer radius via

\[
A\pi R_d^2 \Sigma_d = M_d, \tag{3.21}
\]

where \(A \approx 3\) is a constant determined by an analysis of our numerical results (see Appendix A of Metzger et al. 2008b).

The temperature at \(R_p\) depends on local energy balance. Three separate cases require consideration based on the evolutionary phases described in §3.2.1.

For an irradiated disk during the super-Eddington phase, the temperature at the planet formation radius is constant

\[
T_p^{\text{irr,Edd}} \approx 2600 \text{ K} \left( \frac{R_p}{0.4 \text{ AU}} \right)^{-3/7} \left( \frac{\mu}{28} \right)^{-1/7}, \tag{3.22}
\]

where we have used equations (3.12) and (3.9) with \(f(\tau) = 1\). Here we adopt \(\mu = 28\) appropriate if most of the disk mass is locked up in molecular CO, as expected at temperatures \(\lesssim 4000\) K (see §3.4).

Similarly, for an irradiated disk during the sub-Eddington phase, the temperature at
\( R_p \) is given by

\[
T_p^{(\text{irr})} \approx \left[ \frac{\alpha M_p}{R_p^2} \frac{27}{392 \pi \sigma R_{NS}} \left( \frac{k_B}{\mu m_p} \right)^{3/2} \right]^{2/5} \approx 1000 \text{ K} \left( \frac{\alpha}{0.01} \right)^{2/5} \left( \frac{M_p}{10 M_\oplus} \right)^{2/5} \left( \frac{R_p}{0.4 \text{ AU}} \right)^{-4/5} \left( \frac{\mu}{28} \right)^{-3/5},
\]

where \( M_p \equiv \pi R_p^2 \Sigma(R_p) \) is the local mass at radius \( R_p \). In deriving the accretion luminosity used to calculate the irradiation heating from equation (3.12), we have expressed the local accretion rate as \( \dot{M} = 2 \pi r \Sigma |v_r| = 9 \alpha \theta^2 \Omega M_p / 7 \), where the factor 9/7 is specified by the self-similar solution in this regime.

Finally, we consider a third scenario in which the disk first reaches \( R_p \) during the viscously heated phase. Although the temperature in this phase is not easily tractable in general due to the complicated form of \( \tilde{\kappa}(\rho, T) \), we focus on a simple and common case in which the disk is in the opacity-gap region of the opacity curve (Appendix G). In this temperature range of \( 2000 \text{ K} \lesssim T \lesssim 8000 \text{ K} \), where the gas is mainly neutral but not yet cool enough to form dust, we approximate the opacity by a constant low value of \( \tilde{\kappa}_{\text{gap}} = 10^{-2} \text{ cm}^2 \text{ g}^{-1} \).

Using equations (3.5) and (3.9) along with equation (3.10) in the optically thick regime,
we find that the disk temperature at $R_p$ in the viscously heated regime, is given by

$$T_p^{(\text{visc})} \approx \frac{\alpha M_p^2 27 \kappa_{\text{gap}}}{R_p^{11/2} 32 \pi^2 \sigma \mu m_p} \frac{k_B}{\sqrt{GM_{\text{NS}}}}$$

$$\approx 105 \text{K} \left( \frac{\kappa_{\text{gap}}}{10^{-2} \text{cm}^2 \text{g}^{-1}} \right)^{1/3} \left( \frac{\alpha}{0.01} \right)^{2/5} \left( \frac{M_p}{10M_\oplus} \right)^{2/3} \times \left( \frac{R_p}{0.4 \text{AU}} \right)^{-11/6} \left( \frac{\mu}{28} \right)^{-1/3}.$$ (3.24)

Note that this expression is only applicable for temperatures above the dust-condensation temperature $T_c \sim 2000 \text{K}$. Grain opacity becomes dominant once dust formation commences, increasing the value of $\kappa$. The resulting strong dependence of $\kappa$ on temperature regulates the latter to a fixed value $\sim T_c$ across a wide range of densities.

### 3.3.2.2 Parameter Study

We now use our derived expressions for the disk mass $M_d$ and temperature once the disk first reaches the planet formation radius $R_p = 0.4 \text{ AU}$ to constrain the parameter space required for planet formation. Fig. 6.6 shows with black contours the local disk mass at the planet formation radius, $M_p/M_\oplus$, in the space of viscosity $\alpha$ and RIAF wind mass loss parameter $p$. Also shown with red (brown) curves are contours of constant temperature in K for the irradiation (viscously)-heated regimes. The viscous regime temperature curves are calculated assuming a constant opacity of $\kappa = 10^{-2} \text{cm}^2 \text{g}^{-1}$ (equation 3.24).

For large values of $p$ and small values of $\alpha$ (upper left corner of Fig. 6.6), when the disk first reaches $R_p$ it is irradiation-heated and the accretion rate is sub-Eddington, such that equation (3.23) applies. Below the dotted red curve ($\dot{M} = \dot{M}_{\text{Edd}}$) the accretion rate is
Figure 3.3 Mass and temperature conditions when the disk first spreads to the planet formation radius $R_d = R_p = 0.4$ AU in the parameter space of the viscosity ($\alpha$) and wind mass-loss exponent ($p$). Black contours show the local mass at $R_p$, $M_p$ (labeled in units of $M_\oplus$) calculated from equation (3.20) assuming $A = 3$ (see equation 3.21). The local temperature at the same position, $T_p$ is labeled in units of Kelvin and plotted in red (brown) for an irradiation (viscously)-heated disk, as calculated via equations (3.22-3.24). Larger values of $p$ result in strong initial outflows which therefore decrease the available mass at late times, whereas lower values of $\alpha$ decrease the viscous heating and accretion rate, leading to smaller temperatures. The shaded blue regions show the allowed parameter space if a mass of $100 M_\oplus$ when the temperature first reaches 2000 K is required to produce the observed pulsar-planets assuming a formation efficiency of 8% ($\S$ 3.4). Despite this conservative assumption, a reasonably wide parameter-space satisfies these constraints.

super-Eddington at $R_p$. For small values of $p$ and large values of $\alpha$ (bottom right corner of Fig. 6.6), the disk reaches $R_p$ in the viscously-heated radiative phase (equation 3.24). The transition between the viscously-heated and irradiation-heated regimes for the fiducial model is shown with a dotted brown curve, along which $T_p^{(\text{visc})} = T_p^{(\text{irr,Edd})}$ (equations 3.22, 3.24). Thus, between the dotted curves, the disk is irradiation-heated and accreting above the Eddington limit and the temperature is constant, $T = T_p^{(\text{irr,Edd})} \approx 2600$ K (equation 3.22).

In the delayed-irradiation model, the disk instead remains viscously heated until $\dot{M} \lesssim \dot{M}_{\text{Edd}}$, such that the temperature is less than 2600 K in between the two dotted lines.

In reality, the disk temperature in the viscous-heated regime below the condensation temperature $T_c \sim 2000$ K will differ from those depicted in Fig. 6.6 due to the increase in
opacity that accompanies grain formation. Once a supply of small carbonaceous grains becomes available, rapid growth by coagulation onto larger grains should proceed efficiently, rapidly forming larger solids which sink to the disk midplane, while simultaneously depleting and regulating the number of small grains that contribute most to the opacity (e.g., Dullemond & Dominik 2005). Although detailed modeling of this process is beyond the scope of this paper, the net effect of grain formation is to regulate the disk temperature to a value near the condensation temperature for a wide range of densities (i.e. independent of $M_p$). This will extend the parameter space with $T \lesssim 2000$ K to lower values of $\alpha$ and higher values of $p$ than predicted by our constant opacity model.

The shaded blue regions in Fig. 6.6 show the allowed parameter-space to form the PSR B1257+12 planets, under the assumption that a local disk mass of $100M_\oplus$ is required at the time when the temperature first drops below 2000 K (corresponding to a planet formation efficiency of 8%; see §3.4). The dark blue area constrains the fiducial model, whereas the underlying lightly shaded region applies to the irradiation-delayed model. Both are limited from above by the $M_p = 100M_\oplus$ contour, and terminate at the maximal value of $\alpha$ where this mass contour meets the $T_p = 2000$ K curve. The allowed parameter-space includes cases in which the disk first reaches $R_p$ at temperatures $\gtrsim 2000$ K. These models are still viable if enough mass remains near $R_p$ once the disk further expands and cools below the condensation threshold (as heuristically pictured in Fig. 3.2). Since equations (3.22-3.24) do not depend explicitly on $p$, the boundaries of the permitted parameter-space are vertical lines.

Favorable conditions for planet formation within the fiducial model require low values
of $\alpha \lesssim 6 \times 10^{-3}$, due to the lower temperature of the disk when the accretion rate is sub-Eddington. The simplest version of the delayed-irradiation model permits larger values of $\alpha$ since in this case the disk first reaches $R_p$ in the cooler, viscously-heated phase. The parameter-space constraints are further alleviated if the planet-formation efficiency is larger than the value of 8% we have assumed. Even relaxing the efficiency moderately to $\sim 16\%$ (see §3.4), would expand the allowed parameter-space up to the $M_p = 50 M_\oplus$ contour. This would permit viscosities up to $\alpha \approx 10^{-2}$ for the fiducial model and values of $p \sim 0.5$ favored by numerical simulations of RIAFs for both fiducial and delay-irradiation models.

A time-varying value of $\alpha$, as might be expected as the disk ionization state changes, can also increase the allowed parameter-space. The mass contours in Fig. 6.6 depend on $\alpha$ only through the value of $R_{\text{rad},0}$ (equation 3.19), which determines the radiative transition time at early times when the disk is hot and fully ionized. By contrast, the temperature contours of Fig. 6.6 are determined by the value of $\alpha$ at late times, after the disk material has largely recombined. If the lower ionization state of the disk reduces the effective value of $\alpha$, e.g. due to suppression of the MRI by non-ideal MHD effects, then it would become easier to simultaneously satisfy both mass and temperature constraints on planet formation. However, we note that it is unlikely that the disk will become entirely ‘dead’, since even the low ionization levels of trace alkaline elements are sufficient to sustain the MRI (Gammie 1996; Armitage 2010). X-ray irradiation from the inner accretion flow will also maintain a significant ionized column on the disk surface.
3.3.2.3 Application to Supernova Fall-back Disks

Although our focus is on the WD-NS merger scenario, we can apply our estimates to show why the supernova fallback model is disfavored for pulsar planet formation. The expected angular momentum and mass of such disks are at most $J_{d,0} \sim 10^{49} \text{erg s}$ and $M_{d,0} \approx 10^{-3} - 10^{-1} M_\odot$ (Chevalier 1989). The factor of $\lesssim 10^{-2}$ smaller angular momentum in the fallback scenario compared with the WD-NS merger case severely limits the remaining mass reservoir at the planet forming radius.

Even neglecting disk winds, the local mass reaching $R_p$ (for $A = 3$) is only

$$M_p < 17M_\oplus \left( \frac{J_{d,0}}{10^{49} \text{erg s}} \right) \left( \frac{R_p}{0.4 \text{AU}} \right)^{-1/2},$$

in tension with the observed PSR B1257+12 planetary system unless the metallicity of the accreting gas is very high.

In reality, unbound winds during the early RIAF phase are challenging to avoid (e.g. MacFadyen & Woosley 1999b). The actual mass reaching $R_p$ is thus smaller than the above upper-limit by a factor of $(R_{\text{rad},0}/R_{d,0})^{-p/(2p+1)}$ (eq. 3.20). For canonical parameters and a wind mass-loss exponent of $p = 0.5$, this further reduces the gaseous mass by an order of

\footnote{In fact, Perna et al. (2014) find that disk formation is disfavored altogether for single star stellar evolution models which include commonly used prescriptions for interior angular momentum transport due to magnetic torques resulting from the Spruit-Taylor dynamo.}
magnitude,

\[
M_p \approx 1.6M_\oplus \left( \frac{J_{d,0}}{10^{49}\text{ erg s}} \right)^{12/7} \left( \frac{M_{d,0}}{10^{-2}M_\odot} \right)^{-6/7} \times \left( \frac{\alpha}{0.01} \right)^{-1/7} \left( \frac{R_p}{0.4\text{ AU}} \right)^{-1/2} \leq \text{eq. (3.25)},
\]

(3.26)

inconsistent with even an 100% planet-formation efficiency model (note that for any range of parameters, equation \textbf{3.26} must be truncated from above by equation \textbf{3.25}). We conclude that low-angular momentum disks, such as those anticipated in most core collapse events, cannot channel enough mass to radii \( \approx 0.4\text{ AU} \) to explain the PSR B1257+12 planetary system.

The debris disk detected from its infrared excess around the young millisecond pulsar 4U 0142+61 provides possible evidence for the existence of fallback disks (Wang et al. 2006). However, the inferred disk radius \( R_d \approx 0.04\text{ AU} \) and mass \( M_d \sim 10M_\oplus \) in this case imply a modest angular momentum of \( J_d \sim 5 \times 10^{47}\text{ erg s} \), consistent with our estimates for the effects of mass loss during an early RIAF phase as given by equation \textbf{3.26}.

3.4 Planet Formation Scenario

For realistic parameters, we find that the mass in the disk when it reaches \( R_p \) greatly exceeds the combined masses of the observed PSR B1257+12 planets \( \approx 8M_\oplus \). This is especially true if the outer disk is shielded from irradiation by the inner disk during the super-Eddington phase. In the following, we outline a possible formation mechanism for the pulsar planets which differs in several respects from the ‘standard’ planetary formation scenario. An entire
separate and self-contained work could be devoted to this complex issue, yet here we only propose a general scenario for this process and point-out some of the key issues involved.

A unique aspect of planet formation in our scenario is the unusual C/O-dominated composition. The innermost regions of the disk undergo nuclear burning at early times in the RIAF phase, synthesizing intermediate mass and iron group elements (Metzger 2012a; MM16). However, these inner layers are either unbound from the system by winds or accreted by the NS, such that the composition of the outermost disk remains dominated by unburned C/O.

The Z = 1 metallicity of our proto-planetary disk would suggest a very efficient planet-formation process, since an order unity of the available disk mass could condense into solids. Upon closer examination, while carbonaceous dust grains can form from gaseous carbon, predominantly as graphite (there is no hydrogen from which PAHs could form), most of the oxygen is effectively inert because it must combine with other intermediate mass elements to form silicate grains.

Furthermore, a substantial fraction of the carbon mass reservoir will inevitably be trapped in carbon-monoxide (CO), which is stable and forms at a higher temperature than graphite condensation. In particular, using the thermal CO formation/destruction rates of Lazzati & Heger (2016), we find that carbon-monoxide condenses at $T_{\text{CO}} \sim 4000$ K for characteristic densities of $n_{14} = n/10^{14}$ cm$^{-3}$, and which in detail is well fit by the formula $T_{\text{CO}}/K \approx 4124 + 304 \log_{10}(n_{14}) + 21 \log_{10}(n_{14})^2$.

The formation of CO at temperatures higher than solid condensation suggests that

$^4$However, the possibility cannot be excluded that moderate amounts of heavier elements synthesized at small radii reach the upstream, either due to radial turbulent diffusion within the disk or due to the fall-back of wind ejecta launched from the inner disk.
the solid formation efficiency could be much lower than the maximum \( \sim 100\% \) efficiency allowed for unity metallicity. Two ingredients control the amount of carbon that will be trapped in CO versus that available to condense into solids. The first is the initial C/O number ratio of our disk, which is essentially that of the initially disrupted WD. The WD C/O ratio is generally believed to be close to unity, with increasing oxygen abundances with larger WD mass. Still, current stellar evolution model estimates of WD composition are highly uncertain due to underlying uncertainties in the nuclear physics, primarily in the \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reaction rate. Fields et al. (2016) recently applied a Monte Carlo approach to estimate the uncertainties in WD central carbon and oxygen abundances, finding that \( \Delta X_{^{12}\text{C}} \approx \Delta X_{^{16}\text{O}} \approx 0.4 \), indicating that the C/O ratio is essentially unconstrained. Given these uncertainties, it is plausible that \( X_{^{12}\text{C}} \gtrsim X_{^{16}\text{O}} \) yielding C/O ratios of order 1.1 or larger. This would suggest a \( \sim 10\% \) or greater efficiency in the condensation of solids.

Even if the C/O ratio is below unity, carbonaceous dust formation may not be inhibited. UV/X-ray irradiation from the inner disk will photodissociate CO molecules in the upper disk layers, continuously generating a fresh supply of free carbon. The fate of these carbon monomers depends sensitively on the ambient conditions. If the rate of CO formation exceeds the rate of carbonaceous dust condensation, then recently freed carbon will immediately capture onto nearby oxygen before sinking to the midplane. However, the rate of thermal CO formation decreases exponentially with the declining disk temperature (Lazzati & Heger 2016), such that as the disk continues to cool solid formation may eventually come to eclipse CO formation. This could occur even in regions of the disk which remain shielded by X-rays and hence are conducive to the required gas phase chemistry.
Once carbon solids condense out of the gas phase, they will sink to the disk midplane and grow to large sizes by two body collisions. The stage by which larger planetesimals grow beyond this point is uncertain. One possibility is the direct collapse of the solid disk into gravitationally bound entities due to gravitational instability on a dynamical time (Goldreich & Ward [1973]). Vertical shear between the gaseous and solid disks induces turbulence, which can inhibit the gravitational instability (Weidenschilling [1995]). However, Youdin & Shu (2002) have shown that sufficiently large ratios of the solid to gas surface densities inhibit such turbulence. Specifically, the critical mass fraction of solids required for the gravitational instability to act effectively is (within factors of order unity) given by (Sekiya 1998; Youdin & Shu [2002])

\[ X_{\text{solids}} \gtrsim \theta_{\text{gas}} \approx 0.014 \left( \frac{T}{2000 \text{ K}} \right)^{1/2} \left( \frac{R_p}{0.4 \text{ AU}} \right)^{1/2}. \] (3.27)

As discussed above, the C/O-dominated disks envisioned in our current scenario should easily satisfy this criterion.

The most unstable wavelength by which the gravitational instability proceeds is

\[ \lambda \sim \frac{2\pi^2 G \Sigma}{\Omega^2} \bigg|_{\text{solids}} \approx 3 \times 10^8 \text{ cm} \left( \frac{M_{\text{solids}}}{10M_{\oplus}} \right) \left( \frac{R_p}{0.4 \text{ AU}} \right), \] (3.28)

implying a characteristic (maximal) planetesimal mass of

\[ m \sim 2\pi \Sigma R_p \lambda \approx 3 \times 10^{-4} M_{\oplus} \left( \frac{M_{\text{solids}}}{10M_{\oplus}} \right)^2. \] (3.29)

The remaining ‘rubble pile’ must therefore undergo \( \sim 10^4 \) collisions to build up the earth mass planets observed in the PSR B1257+12 system. As pointed out by Miller & Hamilton
these collisions are unlikely to eject bodies from the system, since the escape velocity from such planetesimal embryos’ surface, ≲ 1 km s$^{-1}$ \((\text{equation 3.29})\), is significantly below the system escape speed $\sim 50$ km s$^{-1}$. This suggests that subsequent collisional assembly of the earth mass planets may be quite efficient, necessitating only $M_{\text{solids}} \approx 10M_\oplus$.

### 3.5 Discussion

Compared to previous work modeling the time-dependent disk evolution \cite{Phinney1993, Currie2007} we begin with initial conditions for the WD-NS merger scenario motivated by \cite{MM16} and include for the first time the important effects of wind mass loss during the RIAF phase at early times. We identify a range of parameters which are consistent with necessary conditions for planet-formation around PSR B1257+12, and briefly discuss key aspects of the formation process unique to this scenario. Finally, we show that including the early RIAF phase of mass loss in ‘low angular momentum’ models, such as supernova fall-back disks, significantly reduces the gaseous disk mass, disfavoring these models for the PSR B1257+12 system. This would help explain the striking lack of planetary systems around the vast majority of pulsars, as summarized in Fig. 3.4 adapted from \cite{Kerr2015}. By contrast, the rate of observed pulsar planetary systems agrees well with the (albeit uncertain) rate estimates of WD-NS mergers \cite{OShaughnessy2010, Kim2015, Bobrick2016}.

Though we have considered only C/O WDs, more massive O/Ne WDs can likewise merge with a binary NS companion, with rates that may even greatly exceed the C/O WD - NS merger rate \cite{Bobrick2016}. Our analysis can be applied to this scenario as well,
with the exception that silicate dust (as apposed to graphite) will be the primary means of solid formation. Since the amount of silicates is limited by their \( \sim \)solar trace abundances, the planet-formation efficiency in this scenario drops and \( \sim \) 10 times more mass is required at \( R_p \). This severely constrains the prospects of an O/Ne WD merger as the progenitor of the PSR B1257+12 planetary system.

Miller & Hamilton (2001) argue that the millisecond pulsar PSR B1257+12 cannot have been recycled by accretion from the same disk responsible for forming the planets, due to the detrimental effects of the large and sustained X-ray accretion luminosity which is associated with the spin-up process. In the case of a WD-NS merger, however, most of the angular momentum is deposited on the NS during the earliest phases (first \( \lesssim \) minute) following the disruption, when the Alfvén radius is pushed to the NS surface. This accretion is capable of
spinning up the pulsar to periods of

\[ P_0 \approx \frac{4p + 1}{3} \frac{2\pi I_{NS}}{M_{d,0} \sqrt{GM_{NS}R_{NS}}} \left( \frac{R_{NS}}{R_{d,0}} \right)^p , \]  

(3.30)

as illustrated in Fig 3.5. Here \( I_{NS} \approx 10^{45} \text{ g cm}^2 \), \( R_{NS} \approx 12 \text{ km} \) are the NS moment of inertia and radius (Lattimer & Schutz 2005b). Clearly, rapid post-merger accretion can spin-up the NS to millisecond periods. At the same time, the disk maintains a reservoir of mass large enough to form planets on much longer timescales, over which the additional accreted mass does not appreciably change the pulsar spin. Requiring that PSR B1257+12 is spun-up post-accretion to periods shorter than the currently observed \( \approx 6.2 \text{ ms} \) yields an additional constraint on the mass-loss exponent, \( p \lesssim 0.4 \).

We additionally speculate that the low X-ray efficiency of PSR B1257+12 (Pavlov et al. 2007; Yan et al. 2013) may be an artifact of a dramatic rapid-accretion event, since magnetic fields would be buried by such an event, possibly causing the final field topology to differ from that of standard pulsars.

The typical column densities \( \sim 10^3 \text{ g cm}^{-2} \) of gaseous matter (presumably CO) at onset of solid condensation would shield the disk midplane from impinging X-ray or particle radiation from the NS. Combined with the comparatively short formation timescale implied by the Goldreich-Ward mechanism envisioned in our scenario, it is reasonable that ablation/evaporation of solids can be averted (Miller & Hamilton 2001). At late times after disk dispersal, the planets would remain largely unperturbed by the pulsar’s hazardous radiation due to their implied diamond composition (boiling temperature \( \gtrsim 4000 \text{ K} \); Kuchner & Seager 2005).
Figure 3.5 Pulsar spin period following WD-NS merger accretion phase, $P_0$, versus the RIAF phase mass-loss exponent, $p$ (equation 3.30). More mass reaches the NS surface for low values of $p$, leading to more significant spin-up, until the NS rotates at breakup frequency $\approx \Omega_k(R_{NS})$. The dashed red curve indicates the currently observed PSR B1257+12 period $P_{\text{obs}} \approx 6.2$ ms. If PSR B1257+12 has only spun-down since its presumptive initial rapid accretion event, then $P_0$ must be $\lesssim P_{\text{obs}}$, constraining the mass-loss exponent to $p \lesssim 0.4$.

One of the mysteries of the PSR B1257+12 planetary system is the relatively narrow range of semi-major axes ($\approx 0.2 - 0.5$ AU) and low eccentricities, which N-body calculations of the final rocky planet assembly by Hansen et al. (2009) indicate requires that the original solid condensation be concentrated over a similarly limited radial range. The observed outer truncation of the planetary system would appear to favor a situation in which the condensation temperature is achieved immediately once the disk first reaches $R_p \sim 0.4$ AU. The inner truncation could also be explained if solid formation is so efficient that no condensible matter (i.e. free carbon not locked up in CO) remains in the disk after this initial formation epoch, once the inner regions of the disk cool to the required temperatures. It is thus a nontrivial feature of our model that values for the solid mass at the $R_p$ achieve values close to those required for reasonable values of the disk viscosity $\alpha \sim 10^{-2}$ (Fig. 6.6).

Planets formed from purely carbonaceous grains, as in our scenario, will likely have
very different properties, such as mean densities, from those of silicate-dominated planets in
our own solar system (e.g., Kuchner & Seager 2005; Mashian & Loeb 2016). Unfortunately,
it is challenging to devise of an observation which could confirm the predicted ‘diamond’
composition of the pulsar planets. Spectroscopy of the planetary atmosphere (if one exists)
or surrounding nebular matter seems unfeasible, but could in principle provide such a test.
We would expect a high-metallicity environment dominated by CO and an anomalously
large $^{12}\text{C}/^{13}\text{C}$ isotopic ratio $\gg 100$. The latter might distinguish WD-NS merger from SN
fallback scenarios, which both predict a high metallicity formation scenario.

The merger of WD-NS binaries have also been proposed to explain the so-called Calcium-
rich transients (Perets et al. 2010b; Kasliwal et al. 2012b). These are a class of dim SN-like
optical transients which are observed to occur often in the very outskirts of their host galax-
ies, where very few stars reside. One of the motivations for associating WD-NS mergers
with these events are the birth kicks received by the NS during the SN (Metzger 2012a;
Lyman et al. 2016b; Bobrick et al. 2016), which could cause the gravitational wave-driven
merger to take place well outside of the galactic plane of the host. It is thus worth noting
that the transverse velocity of PSR B1257+12 is $326 \text{ km s}^{-1}$, making it the only known
millisecond pulsar with a velocity clearly exceeding $300 \text{ km s}^{-1}$ (Yan et al. 2013; our model
naturally explains this fact due to the NS spin-up, since such velocities are not atypical of
normal pulsars). If a connection between WD-NS mergers and the PSR B1257+12 system
is confirmed, this in turn links some WD-NS merger systems to the high proper motions
necessary to explain the remote positions of the Ca-rich transients. Though uncertain, the

$^{5}$WD carbon matter is expected to be composed of nearly pure $^{12}\text{C}$, since any $^{13}\text{C}$ traces in the WD
progenitor’s interior would be consumed by rapid $\alpha$-captures during the He burning phase.
rate of Ca-rich transients is indeed also about 1% of the core collapse SN rate. However, the
types of binary WDs (He or ONe) giving rise to these explosions may well be different than
the C/O WDs which appear to be the most conducive to planet formation.

Finally, we note that alternative ‘merger’ models for the pulsar-planets suffer from vari-
ous shortcomings. Although the WD-WD merger scenario has similar angular momentum to
WD-NS mergers, recent work shows that the merger remnant in this case evolves viscously
into a quasi-spherical envelope, can ignite shell carbon burning, and expand to $\sim$ AU scales
[Shen et al. 2012; Schwab et al. 2012]. It is difficult to imagine planet formation proceeding
in this setting. The model is further constrained by the details of accretion-induced collapse
(AIC) which is necessary in forming the NS. If the planets formed prior to AIC, then the
$\sim 0.1M_\odot$ loss in rest mass energy due to collapse would easily induce larger eccentricities in
the planets’ orbits than observed\(^6\). A separate problem (which plagues also the case in which
the planets formed after AIC) is explaining the $\gtrsim 300\, \text{km}\, \text{s}^{-1}$ velocity of PSR B1257+12,
since AIC is not thought to impart a significant birth-kick to the NS. The merger (or colli-
sion) of a NS with a non-degenerate companion is not as well studied, but assuming a disk
of initial mass and angular momentum similar to WD-NS mergers is formed, the expected
$\sim 10$ times lower metalicity of the disk in this scenario seems constraining given our current
analysis.

\(^6\)the energetic ejecta expected from AIC (e.g. $\sim 10^{50}\, \text{erg}$ in $\sim 0.01M_\odot$; Metzger et al. 2009) could ablate
the planets, so they may not even survive in this scenario.
Subpart B

Binary NS Mergers
A canonical mass of neutron stars born in supernova explosions is $M \approx 1.4M_\odot$. The distribution of $M$ around $1.4M_\odot$ might, however, extend above $2M_\odot$, especially if the neutron star is born spinning fast, with a period approaching the minimum (“breakup”) $P_{\text{spin}} \sim 1$ ms. The additional centrifugal support allows a stable hydrostatic configuration with mass $M$ that would be forbidden for non-rotating stars.

Neutron stars in binary systems have additional chances to gain mass through accretion. The second most massive known pulsar J1614-2230 is in a binary system and has $M \approx 2M_\odot$ (Demorest et al. 2010c). Its spin period is 3.15 ms. It is unclear if accretion is capable of spinning up the star to $P_{\text{spin}} \sim 1$ ms. If this happens, the star mass may keep growing and

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remain stable even when it exceeds $2M_\odot$.

Such centrifugally supported “supramassive” neutron stars (SMNS) may also be created in mergers of neutron star binaries. Recent observations of J1614-2230 and J0348+0432 (Demorest et al. 2010c; Antoniadis et al. 2013a) indicate that the equation of state (EOS) of dense nuclear matter is relatively stiff, and therefore some mergers may initially result in a stable object supported by pressure and fast rotation (e.g., Özel et al. 2010). Numerical simulations show that the object will initially rotate differentially (Shibata & Uryū 2000; Rosswog & Davies 2002; Shibata et al. 2005; Oechslin et al. 2007a; Hotokezaka et al. 2011a), but that solid body rotation will be rapidly established following outwards transport of angular momentum via magnetic stresses and gravitational waves. The timescale for differential rotation to be removed could be as short as tens of ms (Shibata & Taniguchi 2006a), and will almost certainly be much shorter than 10 s (e.g., Shapiro 2000). The heat stored in the merger product is also mostly lost to neutrino emission within seconds (e.g., Burrows & Lattimer 1986).

The SMNS is fated to collapse to a black hole. Its lifetime is controlled by the eventual loss of angular momentum (spindown-induced collapse) or excessive mass growth (accretion-induced collapse). The collapse is associated with a huge release of gravitational energy and could produce a bright transient event — a burst of electromagnetic radiation, such as a cosmological gamma-ray burst (GRB).

This GRB trigger is plausible if the equatorial part of the neutron star is not immediately swallowed by the black hole but forms a compact, massive, centrifugally supported disk around it. Jets of hot plasma and radiation are expected to emerge from the debris disk and
power the burst (e.g., Narayan et al. 1992).

In the merger scenario, the SMNS eventually collapses due to its gradual spindown, which removes the rotational support in minutes to hours. The spindown timescale depends on the magnetic field of the merger product, which is likely amplified to $B \sim 10^{15} \text{ G}$ during the merger (Thompson & Duncan 1993a; Giacomazzo et al. 2014). This implies a moderate delay of the collapse-powered burst following the gravitational waves that are emitted during the merger and hopefully detected by Advanced LIGO (Rezzolla & Kumar 2014; Ciolfi & Siegel 2015).

The goal of this Letter is to assess if the key condition for this burst scenario — a massive debris disk after the collapse — can be satisfied. The structure of the SMNS and hence the outcome of its collapse are controlled by the EOS of the dense nuclear matter $P(\rho)$. Available general relativistic simulations of the collapse do not show disk formation (Shibata 2003; Baiotti et al. 2005). These simulations, however, implemented only simplified EOS. In particular, Shibata (2003) used the polytropic $P \propto \rho^{1+1/n}$ with index $n \leq 2$, and found that less than $10^{-3}M_\odot$ remains outside the black hole at the termination of the simulation, comparable to their numerical resolution. They also found that for an extremely soft EOS (with $n = 2.9$ and 3) disks can form, however such EOS are incompatible with observations of neutron stars. The remaining open possibility is that a different form of the EOS could lead to disk formation, e.g. soft at high densities (which gives a compact inner core — the seed for a future black hole) and stiff at lower densities (which gives an extended outer core with a high angular momentum).

In this Letter we explore a wide range of EOS in search for one that could possi-
bly give a debris disk. Instead of carrying out full-fledged and computationally expensive hydrodynamic simulations of SMNS collapse, we employ a simple method. We analyze the equilibrium hydrostatic configuration prior to the collapse and check if it satisfies a necessary condition for formation of a debris disk after the collapse.

### 4.1 Condition for Disk Formation

A stringent criterion on disk formation can be derived by assuming that all but an infinitesimal amount of the SMNS’s mass and angular momentum are inherited by the newly formed Kerr black hole. Matter at the SMNS equator has the largest specific angular momentum, \( j_e \), and hence is the most likely to comprise a disk. The angular momentum is conserved during collapse, as long as magnetic and viscous torques are negligible and the spacetime remains axisymmetric. The centrifugal barrier will stop the equatorial matter from plunging the horizon if \( j_e \) exceeds the specific angular momentum of the inner-most stable circular orbit (ISCO) in the Kerr metric of the nascent black hole,

\[
j_e > j_{\text{isco}}(a) \Rightarrow \text{disk formation is possible.} \tag{4.1}
\]

Note that \( j_{\text{isco}} \) depends on the spin parameter \( a = Jc/GM^2 \) where \( J \) is the angular momentum inherited by the black hole from the SMNS. A similar criterion has been employed previously to the collapse of supermassive gas clouds (Shapiro & Shibata 2002).
4.2 Maximally Rotating Maximal Mass

We construct axisymmetric neutron star models using the rns code (Stergioulas & Friedman 1995a; Nozawa et al. 1998), which calculates relativistic rotating hydrostatic equilibria following the method outlined in Cook et al. (1994b,a). The collapse occurs when the stellar mass exceeds $M_{\text{max}}$ at which the star becomes secularly unstable (Friedman et al. 1988) and no hydrostatic solution is found.

$M_{\text{max}}$ depends on the angular momentum $J$ and the EOS of dense nuclear matter. For a given EOS, we calculate $M_{\text{max}}(J)$ and find $a$ and $j_{e}$ immediately prior to collapse. Disk formation is clearly impossible for a non-rotating star because matter will fall radially into the newly-formed Schwarzschild black hole. As $J$ and hence $j_{e}$ are increased, black hole spin $a$ increases and hence $j_{\text{isco}}$ decreases. Condition (4.1) could thus in principle be satisfied at some point along the maximal mass sequence.

The maximal mass sequence $M_{\text{max}}(J)$ cannot be extended indefinitely as it eventually reaches the mass-shedding limit, beyond which the co-rotating orbital frequency at the SMNS equator exceeds the SMNS rotation frequency. This point defines the maximally rotating maximum mass (MRMM), $M_{\text{max}}(J_{\text{max}})$, which is typically 10-30% higher than $M_{\text{max}}(0)$. The collapsing MRMM has the best chance to form a debris disk but this is not guaranteed. Although $j_{e}$ of the MRMM is just sufficient to orbit the hydrostatic star, the spacetime metric changes after the collapse and the same $j_{e}$ can fail to sustain Keplerian rotation around the nascent black hole. If condition (4.1) is not met for the MRMM, it will not be met for any slower rotating maximal mass models and we may conclude that disk formation...
The input parameters of the rns code are the central energy density and the oblateness of the star. For a given oblateness we find the maximal mass model by varying the central energy density. Then we step along the maximum mass sequence toward MRMM by increasing the oblateness parameter. At the end of the sequence we iterate the oblatness until the mass shed limit is found to within a specified accuracy. At each step we check if the disk formation criterion \(4.1\) is satisfied.

Figure 4.1 illustrates our procedure for three representative EOS, labeled eosA, eosB, and eosC, respectively. For eosA, \(j_e < j_{isco}\) for all black hole spin \(a\), so disk formation is impossible according to criterion \(4.1\). For eosC, \(j_e > j_{isco}\) for \(a \gtrsim 0.5\), indicating that a
disk could form; however, the maximum non-rotating mass for this unrealistically soft EOS is only $0.48M_\odot$. Disk formation is also possible for eosB, but only for a very narrow range of $J$ near the mass-shedding limit.

### 4.3 Survey of the EOS Space

The possibility of disk formation is controlled by the high density EOS, which is poorly known. Therefore, below we conduct a survey over a broad range of EOS. Our goal is to check whether it is possible to simultaneously satisfy the disk formation criterion and current observational constraints on neutron star radii and masses.

We parametrize the EOS at $\rho > \rho_0 = 10^{14.3} \text{ g cm}^{-3}$ as a broken power law. This choice is motivated by previous works (Read et al. 2009a) which show that a piecewise polytrope can reliably reproduce a variety of EOS models. The break is fixed at density $\rho_1 = 10^{14.7} \text{ g cm}^{-3}$. At densities below $\rho_0$ we use the SLy EOS (Douchin & Haensel 2001) with the approximation of Read et al. (2009a), and we fix $P(\rho_0)$ to the SLy value.

With fixed $\rho_1$ we are left with only two free parameters: $P_1 = P(\rho_1)$ and the power-law index at $\rho > \rho_1$, $\Gamma_2 = d \ln P / d \ln \rho$. Two degrees of freedom in the EOS may be insufficient to predict observables to within $\sim 1\%$ accuracy (e.g., as in (Read et al. 2009a)). However, this form of EOS is sufficiently flexible for our purposes, allowing independent variation of the SMNS mass $M$ and radius $R$. These parameters determine the star’s compactness $M/R$, the key factor for disk formation.

The results of our numerical survey of the parameter space $P_1 - \Gamma_2$ are shown in Figure 4.2. For “stiff” EOS above the grey strip even the MRMM configuration fails to meet
Figure 4.2 Regions of allowed and forbidden disk formation in the EOS parameter space. Dashed purple curves show contours of constant maximum mass for non-rotating neutron stars, while dotted black lines indicate constant radius values for a $1.4M_\odot$ non-rotating star. The green region shows the $2\sigma$ allowed parameter space based on observed neutron star masses (Antoniadis et al. 2013a) (bottom boundary) and constraints on neutron star radius (Steiner et al. 2013) (left and right side boundaries).

The criterion (4.1), and thus disk formation is ruled out. The criterion is met by the MRMM below the grey strip (and possibly inside the strip where it is numerically unresolved).

Small $P_1$ or $\Gamma_2$ values are however problematic as they predict low $M_{\text{max}}$ while observations demonstrate the existence of neutron stars with $M \approx 2M_\odot$ (Demorest et al. 2010c; Antoniadis et al. 2013a), even at moderate rotation when centrifugal effects may be neglected (the 39 ms spin period of J0348+043 is slow enough that it can be treated as essentially non-rotating for the purpose of constraining the maximal neutron star mass).

An additional observationaly accessible parameter is the radius of normal neutron stars with moderate rotation and canonical mass $M \approx 1.4M_\odot$. For instance using observations of transiently accreting and bursting neutron stars (Steiner et al. 2013) reported $R_{1.4M_\odot} = 10.42 - 12.89$ km at $2\sigma$. We note that current neutron star radius constraints are subject to uncertainties in both astrophysics and nuclear physics modeling and the radius constraints
are not entirely settled yet (cf. e.g., Suleimanov et al. (2011); Guillot et al. (2013)).

For any candidate EOS one should check its prediction for $M_{\text{max}}(J \approx 0)$ as well as $R_{1.4\text{M}_\odot}$, which can be tested against observations. Figure 4.2 shows the contours of constant $M_{\text{max}}(J = 0)$ and $R_{1.4\text{M}_\odot}$ on the $P_1-\Gamma_2$ plane together with the observational constraints. The condition $M_{\text{max}} > 2\text{M}_\odot$ alone excludes almost the entire region where disk formation is possible. A significant gap appears between this region and the allowed region if following Steiner et al. (2013) we also require $R_{1.4\text{M}_\odot} < 13 \text{ km}$. We also note that even below the grey strip formation of a debris disk requires significant fine-tuning toward the MRMM configuration. Disk formation quickly becomes impossible if $J$ is reduced below $J_{\text{max}}$ (see Figure 4.1 in particular model eosB).

4.4 Discussion and Astrophysical Implications

Our method employs a simple parametrization for the high density EOS as a piecewise polytrope, and hence may not replicate nuances of realistic EOSs (for instance, Read et al. (2009a) shows that it tends to overestimate the sound speed). This parametrization is, however, sufficient to capture the overall mass distribution of the star, which is most important to our analysis. Since our results in Figure 4.2 show a significant gap between the observationally allowed and the disk forming regions, any analysis using a more complex parametrization (as in e.g. Read et al. (2009a); Steiner et al. (2013)) is likely to yield a similar conclusion. This is especially so when considering that much of the formally allowed disk formation region will not produce a disk in practice without fine tuning of the SMNS angular momentum.

In particular, we have applied our method directly to the entire list of parametrized
EOSs given in Read et al. (2009a) (their Table III), and find that none of these support disk formation. Interestingly, the robustness of our main conclusion relies in part on the recent discovery of a $2M_\odot$ neutron star (Demorest et al. 2010c; Antoniadis et al. 2013a) and hence could not have been made with as much confidence prior to 2010, when the largest known mass was $1.74\pm0.04M_\odot$.

Although lower limits on the maximum neutron star mass are well established by dynamical measurements, observational constraints on the neutron star radius are subject to systematic uncertainties (e.g., Miller 2013). It is thus important to note that our conclusion that disk formation is unlikely depends most sensitively on the established maximum mass constraints, and less critically on the neutron star radius.

Our analysis assumed axisymmetric collapse. This is reasonable since non-axisymmetric perturbations will likely be damped out via gravitational waves. Furthermore, if the amount of surviving disk mass is determined by deviations from axisymmetry then producing a disk of an interesting mass $\gtrsim 10^{-3}M_\odot$ translates into a radial perturbation of $\gtrsim 2$ km, an unlikely occurrence.

We have additionally assumed that magnetic or viscous torques do not affect the SMNS matter during the collapse. Numerical hydrodynamical simulations consistently show that the SMNS matter collapses on a dynamical timescale with approximate conservation of angular momentum and negligible dissipation effects on fluid streamlines (Shibata 2003). Magnetic fields could become dynamically important only when they are extremely strong. Such fields could also slightly affect the SMNS structure. Its radius would be increased up to $\sim 16\%$ in the most extreme case of magnetic pressure equal to thermal pressure (e.g.,
Our results have implications for some GRB models. Electromagnetic emission from SMNS formed in neutron star binary mergers has been proposed by many authors (e.g. Metzger et al. 2008d; Bucciantini et al. 2012a; Rowlinson et al. 2013a; Gompertz et al. 2015) to explain long-lived X-ray flares (“extended emission”) and plateaus observed following short duration GRBs (e.g. Norris & Bonnell 2006; Nousek et al. 2006; Zhang et al. 2006), which in some cases have been observed to terminate abruptly in a way suggesting a SMNS that has collapsed to a black hole (Rowlinson et al. 2010). These magnetar models have been criticized because it is not clear how to produce the relativistic jet responsible for the initial GRB itself as the result of baryonic pollution from the young neutron star remnant (e.g., Murguia-Berthier et al. 2014). This has recently led to the suggestion of a “Time Reversal” scenario (Rezzolla & Kumar 2014; Ciolfi & Siegel 2015), whereby black hole formation and the GRB is delayed for tens or hundreds of seconds following the merger, but due to light time travel effects is observed before X-rays from the SMNS remnant cease. A similar physical situation, which posits the collapse of a SMNS to a black hole following the accretion of matter from a binary companion (accretion-induced collapse; e.g., MacFadyen et al. 2005; Giacomazzo & Perna 2012) is also commonly invoked as an alternative to neutron star merger models for short GRBs.

Both these alluring models (accretion-induced collapse and Time Reversal) require a debris disk after the SMNS collapse in order to power the short GRB. Our results show that this assumption contradicts the stiff nuclear EOS inferred from observations of neutron stars.
This does not necessarily mean that SMNS collapse will have no observational electromagnetic signature. For instance [Lehner et al. (2012); Falcke & Rezzolla (2014b)] suggest that if the SMNS is initially magnetized, a significant electromagnetic transient could arise regardless of any surrounding accretion disk. However, such a transient is unlikely to last many dynamical times across the black hole horizon and hence may fail to explain the $0.1 - 1 \, \text{s}$ duration of observed short GRBs.

Our model assumes solid body rotation and a cold EOS and hence does not rule out a disk if the black hole forms shortly following a binary neutron star merger. Disk formation in fact appears to be a robust outcome of general relativistic simulations of the merger process (e.g., Shibata & Taniguchi 2006a). Thermal pressure is only sustained for a few seconds after the merger, until neutrino cooling sets in. More importantly, the merger remnant is primarily supported by differential rotation, such that the collapse is usually initiated by the outwards redistribution of angular momentum, as is expected to occur on a timescale of tens or hundreds of milliseconds due to magnetic or viscous stresses. Since in this case collapse occurs prior to the establishment of solid body rotation throughout the remnant, disk formation is much more likely than in the case of a delayed collapse.

Finally, our results also render untenable proposed scenarios for long duration GRBs which postulate a long delay (exceeding hours or days) between the core collapse of a massive star and the formation of a black hole with a debris accretion disk [Vietri & Stella 1998].
Chapter 5

Constraining the Maximum Mass of Neutron Stars From Multi-Messenger Observations of GW170817

5.1 Introduction

On August 17, 2017, the Advanced LIGO and Virgo network of gravitational wave (GW) observatories discovered the inspiral and coalescence of a binary neutron star (BNS) system (LIGO Scientific Collaboration & Virgo Collaboration 2017), dubbed GW170817. The measured binary chirp mass was $M_c = 1.118^{+0.004}_{-0.002} M_\odot$, with larger uncertainties on the mass of the individual neutron star (NS) components and total mass of $M_1 = 1.36-1.60 M_\odot$,

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\[ M_2 = 1.17 \pm 1.36 M_\odot, \] and \[ M_{\text{tot}} = M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_\odot, \] respectively. These masses are derived under the prior of low dimensionless NS spin (\( \chi \lesssim 0.05 \)), characteristic of Galactic BNS systems.

The electromagnetic follow-up of GW170817 was summarized in LIGO Scientific Collaboration et al. (2017a). The Fermi and INTEGRAL satellites discovered a sub-luminous gamma-ray burst (GRB) with a sky position and temporal coincidence within \( \lesssim 2 \) seconds of the inferred coalescence time of GW170817 (Goldstein et al. 2017; Savchenko et al. 2017; LIGO Scientific Collaboration et al. 2017b). Eleven hours later, an optical counterpart was discovered (Coulter et al. 2017; Allam et al. 2017; Yang et al. 2017; Arcavi et al. 2017; Tanvir & Levan 2017; Lipunov et al. 2017) with a luminosity, thermal spectrum, and rapid temporal decay consistent with those predicted for “kilonova” (KN) emission, powered by the radioactive decay of heavy elements synthesized in the merger ejecta (Li & Paczyński 1998; Metzger et al. 2010a). The presence of both early-time visual (“blue”) emission (Metzger et al. 2010a) which transitioned to near-infrared (“red”) emission (Barnes & Kasen 2013; Tanaka & Hotokezaka 2013) at late times requires at least two distinct ejecta components consisting, respectively, of light and heavy r-process nuclei (e.g. Cowperthwaite et al. 2017; Nicholl et al. 2017b; Chornock et al. 2017; Kasen et al. 2017; Drout et al. 2017; Kasliwal et al. 2017). Rising X-ray (Troja et al. 2017; Margutti et al. 2017a) and radio (Hallinan et al. 2017; Alexander et al. 2017) emission was observed roughly two weeks after the merger, consistent with delayed onset of the synchrotron afterglow of a more powerful relativistic GRB whose emission was initially relativistically beamed away from our line of sight (e.g. van Eerten & MacFadyen 2011).
The strength of the red and blue KN signatures of a BNS merger depends on the compact remnant which forms immediately after the merger; the latter in turn depends on the total mass of the original binary or its remnant, $M_{\text{tot}}$, relative to the maximum NS mass, $M_{\text{max}}$. A massive binary ($M_{\text{tot}} \gtrsim 1.3 - 1.6M_{\text{max}}$) results in a prompt collapse to a BH; in such cases, the polar shock-heated ejecta is negligible and the accretion disk outflows are weakly irradiated by neutrinos, resulting in a primarily red KN powered by the tidal ejecta (left panel). By contrast, a very low mass binary $M_{\text{tot}} \lesssim 1.2M_{\text{max}}$ creates a long-lived SMNS, which imparts its large rotational energy $\gtrsim 10^{52}$ erg to the surrounding ejecta, imparting relativistic expansion speeds to the KN ejecta or producing an abnormally powerful GRB jet (right panel). In the intermediate case, $1.2M_{\text{max}} \lesssim M_{\text{tot}} \lesssim 1.3 - 1.6M_{\text{max}}$ a HMNS or short-lived SMNS forms, which produces both blue and red KN ejecta expanding at mildly relativistic velocities, consistent with observations of GW170817.

The discovery of GW170817 implies a BNS rate of $R_{\text{BNS}} = 1540^{+3200}_{-1220}$ Gpc$^{-3}$ yr$^{-1}$, corresponding to $\approx 6 - 120$ BNS mergers per year once LIGO/Virgo reach design sensitivity (LIGO Scientific Collaboration & Virgo Collaboration 2017). This relatively high rate bodes well for the prospects of several scientific objectives requiring a large population of GW detections, such as “standard siren” measurements of the cosmic expansion history (Holz & Hughes 2005; Nissanke et al. 2010; LIGO Scientific Collaboration et al. 2017c) or as probes of the equation of state (EOS) of NSs (e.g. Read et al. 2009b; Hinderer et al. 2010; Bauswein & Janka 2012).
Uncertainties in the EOS limit our ability to predict key properties of NSs, such as their radii and maximum stable mass (e.g. Özel & Freire 2016). Methods to measure NS radii from GWs include searching for tidal effects on the waveform during the final stages of the BNS inspiral (Hinderer et al. 2010; Damour & Nagar 2010; Damour et al. 2012; Favata 2014; Read et al. 2013; Del Pozzo et al. 2013; Agathos et al. 2015; Lackey & Wade 2015; Chatziioannou et al. 2015) and for quasi-periodic oscillations of the post-merger remnant (e.g. Bauswein & Janka 2012; Bauswein et al. 2012; Clark et al. 2014; Bauswein & Stergioulas 2015; Bauswein et al. 2016). Searches on timescales of tens of ms to $\lesssim 500$ s post-merger revealed no evidence for such quasi-periodic oscillations in the GW170817 (LIGO Scientific Collaboration & Virgo Collaboration 2017).

While the radii of NS are controlled by the properties of the EOS at approximately twice the nuclear saturation density, the maximum stable mass ($M_{\text{max}}$) instead depends on the very high density EOS (around 8 times the saturation density; Özel & Psaltis 2009). Observations of two pulsars with gravitational masses of $1.93 \pm 0.07 M_\odot$ (Demorest et al. 2010a; Özel & Freire 2016) or $2.01 \pm 0.04 M_\odot$ (Antoniadis et al. 2013c) place the best current lower bounds. However, other than the relatively unconstraining limit set by causality, no firm theoretical or observational upper limits exist on $M_{\text{max}}$. Indirect, assumption-dependent limits on $M_{\text{max}}$ exist from observations of short GRBs (e.g. Lasky et al. 2014; Lawrence et al. 2015; Fryer et al. 2015; Piro et al. 2017) and by modeling the mass distribution of NSs (e.g. Antoniadis et al. 2016; Alsing et al. 2017).

Despite the large uncertainties on $M_{\text{max}}$, it remains one of the most important properties affecting the outcome of a BNS merger and its subsequent EM signal (Fig. 7.1). If
the total binary mass $M_{\text{tot}}$ exceeds a critical threshold of $M_{\text{th}} \approx k M_{\text{max}}$, then the merger product undergoes “prompt” dynamical-timescale collapse to a black hole (BH) (e.g. Shibata 2005; Shibata & Taniguchi 2006b; Baiotti et al. 2008; Hotokezaka et al. 2011b), where the proportionality factor $k \approx 1.3 - 1.6$ is greater for smaller values of the NS “compactness”, $C_{\text{max}} = (G M_{\text{max}}/c^2 R_{1.6})$, where $R_{1.6}$ is the radius of a 1.6$M_\odot$ NS (e.g. Bauswein et al. 2013). For slightly less massive binaries with $M_{\text{tot}} \lesssim M_{\text{th}}$, the merger instead produces a hyper-massive neutron star (HMNS), which is supported from collapse by differential rotation (and, potentially, by thermal support). For lower values of $M_{\text{tot}} \lesssim 1.2 M_{\text{max}}$, the merger instead produces a supramassive neutron star (SMNS), which remains stable even once its differential rotation is removed, as is expected to occur $\lesssim 10 - 100$ ms following the merger (Baumgarte et al. 2000; Paschalidis et al. 2012; Kaplan et al. 2014). A SMNS can survive for several seconds, or potentially much longer, until its rigid body angular momentum is removed through comparatively slow processes, such as magnetic spin-down. Finally, for an extremely low binary mass, $M_{\text{tot}} \lesssim M_{\text{max}}$, the BNS merger produces an indefinitely stable NS remnant (e.g. Bucciantini et al. 2012b; Giacomazzo & Perna 2013). Figure 5.2 shows the baryonic mass thresholds of these possible BNS merger outcomes (prompt collapse, HMNS, SMNS, stable) for an example EOS as vertical dashed lines.

The different types of merger outcomes are predicted to create qualitatively different electromagnetic (EM) signals (e.g. Bauswein et al. 2013; Metzger & Fernández 2014). In this Letter, we combine EM constraints on the type of remnant that formed in GW170817 with GW data on the binary mass in order to constrain the radii and maximum mass of NSs.
5.2 Constraints from EM Counterparts

This section reviews what constraints can be placed from EM observations on the energy imparted by a long-lived NS into the non-relativistic KN ejecta (§5.2.1) and into the relativistic ejecta of the GRB jet (§5.2.2). Then in §5.2.3 we describe the implications for the type of remnant formed.

5.2.1 Kilonova (Non-Relativistic Ejecta)

Two sources of neutron-rich ejecta, capable of synthesizing $r$-process nuclei, accompany a BNS merger (Fernández & Metzger 2016). First, matter is ejected on the dynamical timescale, either by tidal forces (e.g. Ruffert et al. 1997, Rosswog et al. 1999, Radice et al.)
or by shock heating at the interface between the merging NSs (e.g. Oechslin et al. 2007b; Bauswein et al. 2013; Hotokezaka et al. 2013). The tidal matter emerges in the binary equatorial plane and has a low electron fraction, $Y_e \lesssim 0.1 - 0.2$. Matter from the shocked interface expands into the polar direction and possesses a higher $Y_e \gtrsim 0.25$ (Wanajo et al. 2014; Sekiguchi et al. 2016).

Outflows from the accretion torus around the central compact object provide a second important source of ejecta (e.g. Metzger et al. 2008c; Dessart et al. 2009; Fernández & Metzger 2013b; Perego et al. 2014; Just et al. 2015; Siegel & Metzger 2017, 2018). The disk outflows typically possess a broad distribution of $Y_e \sim 0.1 - 0.5$, with an average $Y_e$ that increases with the lifetime of the HMNS/SMNS, due to neutrino irradiation of the ejecta by the NS (Metzger & Fernández 2014; Perego et al. 2014; Martin et al. 2015).

The KN following GW170817 showed evidence for two distinct emitting ejecta components (Cowperthwaite et al. 2017; Kasen et al. 2017; Tanaka et al. 2017). The early $\lesssim 2$ day timescale “blue” emission phase requires an ejecta mass of $M_{\text{ej, blue}}^{\text{blue}} \approx 1 \times 10^{-2} M_\odot$ of lanthanide-free ejecta ($Y_e \gtrsim 0.25$) with a mean velocity of $v_{\text{ej, blue}}^{\text{blue}} \approx 0.2 - 0.3c$ (Nicholl et al. 2017b). The comparatively “red” emission seen at later times requires $M_{\text{ej, red}}^{\text{red}} \approx 4 - 5 \times 10^{-2} M_\odot$ of lanthanide-rich ejecta ($Y_e \lesssim 0.25$) with $v_{\text{ej, red}}^{\text{red}} \approx 0.1 - 0.2c$ (Chornock et al. 2017). The total kinetic energy of the ejecta is therefore approximately $E_{\text{KN}} \approx M_{\text{ej, blue}}^{\text{blue}} (v_{\text{ej, blue}}^{\text{blue}})^2 / 2 + M_{\text{ej, red}}^{\text{red}} (v_{\text{ej, red}}^{\text{red}})^2 / 2 \approx 1.0 \times 10^{51}$ erg.
5.2.2 Gamma-Ray Burst (Relativistic Ejecta)

The radiated gamma-ray energy from GW170817, and the kinetic energy of its afterglow if it originates from an on-axis GRB jet, were several orders of magnitude lower than those of cosmological short GRBs (Goldstein et al. 2017; LIGO Scientific Collaboration et al. 2017; Fong et al. 2017). This could indicate that we are observing the GRB jet well outside of its core (e.g. Kathirgamaraju et al. 2017; Lazzati et al. 2017). The delayed rise of synchrotron X-ray and radio emission is consistent with the afterglow from a much more powerful relativistic jet pointed away from our line of sight (Troja et al. 2017; Hallinan et al. 2017; Margutti et al. 2017a; Alexander et al. 2017; Evans et al. 2017; Haggard et al. 2017). However, for observing viewing angles relative to the binary axis inferred from the GW data and host galaxy, \( \theta_{\text{obs}} \approx 11 - 33^\circ \) (LIGO Scientific Collaboration et al. 2017c), the inferred kinetic energy of an off-axis GRB jet is \( E_{\text{GRB}} \lesssim 10^{50} \text{ erg} \) (e.g. Alexander et al. 2017; Margutti et al. 2017a), within the range of inferred properties of normal cosmological SGRB jets (Berger 2014).

The production of a GRB may indicate that a BH formed (e.g. Lawrence et al. 2015; Murguia-Berthier et al. 2014), in which case the GRB’s delay of \( \lesssim 2 \text{ s} \) following the merger implicates a remnant that either underwent prompt collapse to a BH, or formed a short-lived HMNS or SMNS. Late-time X-ray emission observed after many short GRBs has been suggested to indicate the presence of a long-lived magnetar (e.g. Metzger et al. 2008b; Rowlinson et al. 2013b), raising doubt about whether BH formation is a strict requirement to produce a GRB. However, GW170817 showed no evidence for temporally extended high-energy emission (LIGO Scientific Collaboration et al. 2017b).
5.2.3 Constraints on the Merger Remnant in GW170817

The KN emission from GW170817 tightly constrains the type of compact remnant that formed in the merger event (Fig. 7.1). Prompt collapse to a BH \( M_{\text{tot}} \gtrsim M_{\text{th}} \) is disfavored by the quantity of the blue KN ejecta. General relativistic numerical simulations show that mergers with prompt collapses eject only a small quantity \( \lesssim 10^{-4} - 10^{-3} M_\odot \) of matter from the merger interface (e.g. Hotokezaka et al. 2011b), inconsistent with the inferred value \( M_{\text{ej}}^{\text{blue}} \gtrsim 10^{-2} M_\odot \) for GW170817. The accretion disk outflows can also contribute; however, the wind ejecta with \( Y_e \gtrsim 0.25 \) is only a fraction of the initial torus, which is already small \( \lesssim 0.01 - 0.02 M_\odot \) for prompt collapse (Ruffert & Janka 1999, Shibata & Taniguchi 2006b, Oechslin et al. 2007c). Furthermore, the predicted velocities of the disk winds \( \sim 0.03 - 0.1c \) (e.g. Fernández & Metzger 2013b, Just et al. 2015) are lower than the velocities \( \gtrsim 0.2 - 0.3c \) inferred for the blue KN of GW170817 (e.g. Nicholl et al. 2017b).

A HMNS remnant, due to its longer lifetime \( \gtrsim 10 \) ms, produces a greater quantity of dynamical and disk wind ejecta. The expansion rate \( \sim 0.2 - 0.3c \) and ejecta mass of \( M_{\text{ej}}^{\text{blue}} \sim 0.01 - 0.02 M_\odot \) of the blue KN ejecta inferred for GW170817 are consistent with the properties of the high-\( Y_e \) shock-heated dynamical ejecta found by BNS merger simulations (e.g. Sekiguchi et al. 2016), provided that the radius of the NS is relatively small, \( R_{\text{ns}} \lesssim 11 \) km (Nicholl et al. 2017b). The higher quantity and lower velocity of the red KN emission are also broadly consistent with those expected from the outflows of a relatively massive accretion torus \( \approx 0.1 - 0.2 M_\odot \) (e.g. Siegel & Metzger 2017) following the collapse of a relatively short-lived HMNS.

At the other extreme, a long-lived SMNS or indefinitely stable NS remnant is strongly
disfavored by the moderate kinetic energy of the observed KN and GRB afterglow. Even once its differential rotation has been removed, a SMNS possesses an enormous rotational energy, $T \approx 10^{53}$ erg, which is available to be deposited into the post-merger environment. Not all of this energy is “extractable” insofar as, even just prior to spinning down to the threshold for collapse, the NS remnant is still rotating quite rapidly. Margalit et al. (2015) show that the collapse of a SMNS is unlikely to produce a centrifugally-supported accretion disk outside of the innermost stable circular orbit, in which case all of the mass and angular momentum of the star are trapped in the BH.

The extractable rotational energy of a BNS merger remnant is more precisely defined as

$$\Delta T = T_0 - T_\star,$$

(5.1)

where $T_0$ is the energy available immediately after differential rotation has been removed, and $T_\star$ is the rotational energy at the point of gravitational collapse to a BH. We take $T_0$ equal to the rotational energy at the mass-shedding limit, a condition which approximates the state of the remnant immediately after differential rotation is removed. However, the constraints obtained hereafter would be similar if we had instead taken $T_0$ to equal the threshold value $T/|W| \approx 0.14$ for the growth of secular instabilities (e.g. Lai & Shapiro 1995). Fig. 5.2 shows that $\Delta T$ rises sharply from zero at the HMNS-SMNS boundary to $\Delta T = T_0 \approx 10^{53}$ erg for stable remnants.

The most likely mechanism by which $\Delta T$ is removed, enabling the SMNS to collapse, is the extraction of angular momentum via a magnetized outflow or jet. MHD BNS merger simulations find that ultra-strong magnetic fields $\gtrsim 10^{15} - 10^{16}$ G are generated in the merger
remnant (e.g. Kiuchi et al. 2014). A NS of radius $R_{\text{ns}}$, rotation frequency $\Omega = 2\pi/P$, and spin period $P$ loses energy to a magnetic wind at a rate (Spitkovsky 2006)

$$\dot{E}_{\text{mag}} = \frac{\mu^4 \Omega^4}{c^3} (1 + \sin^2 \chi), \quad (5.2)$$

where $B_d, \mu = B_d R_{\text{ns}}^3$, and $\chi$ are the surface magnetic dipole field strength, dipole moment, and angle between the rotation and dipole axes, respectively. Taking $R_{\text{ns}} = 12 \text{ km}$ and $\chi = 0$, the SMNS’s available rotational energy is removed by magnetic torques on a timescale

$$\tau_{\text{sd}} = \frac{\Delta T}{\dot{E}_{\text{mag}}} \approx 24 \text{ s} \left( \frac{\Delta T}{10^{52} \text{ erg}} \right) \left( \frac{B_d}{10^{15} \text{ G}} \right)^{-2} \left( \frac{P}{0.8 \text{ ms}} \right)^4. \quad (5.3)$$

If we demand that BH formation occur on a timescale of $\tau_{\text{sd}} \lesssim 2 \text{ s}$ following the merger in order to explain the observed gamma-ray emission (§5.2.2), then this requires a SMNS remnant with $B_d \gg 10^{15} \text{ G}$ or $\Delta T \ll 10^{53} \text{ erg}$.

A SMNS can in principle also spin down through gravitational wave emission, as may result from the quadrupolar moment of inertia induced by a strong interior magnetic field which is misaligned with the rotation axis (e.g. Stella et al. 2005; Dall’Osso et al. 2009, 2015). Figure 5.3 shows that GW spin-down dominates over magnetic spin-down (Eq. 5.2) only if the interior toroidal magnetic field exceeds the external poloidal one by a factor of $\gtrsim 100$. However, such a strong toroidal to poloidal field configuration would be unstable (Braithwaite 2009; Akg"{u}n et al. 2013; grey shaded region in Fig. 5.3) and would furthermore imply a relatively long collapse time of $\tau_{\text{sd}} \gtrsim 100 \text{ s}$, potentially incompatible with the gamma-ray burst emission observed on a timescale $\lesssim 2 \text{ s}$ (§5.2.2). It would also produce quasi-periodic
Figure 5.3 Parameter-space of external dipole magnetic field $B_d$, responsible for EM spin-down (Eq. 5.2), and internal toroidal field $B_t$, which can deform the NS causing GW-driven spin-down. Contours show the spin-down timescale (blue) and ratio of EM to GW extracted spin-down energy (black) calculated by integrating equations for the spin frequency $\Omega$ and misalignment angle $\chi$ as a function of time (Cutler & Jones 2001; Dall’Osso et al. 2009). The region where GWs could dominate over EM emission falls below the thick black curve, but this region is: (a) susceptible to magnetic instabilities (grey shaded areas Braithwaite 2009; Akgün et al. 2013), (b) implies long spin-down timescales $\gtrsim 100$ s at odds with the detection of a GRB only 2 s after the merger, and (c) would produce a strong GW signal.

GW emission which is not observed in the GW170817 post-merger signal (albeit with only weakly constraining upper limits; Abbott et al. 2017b).

In summary, all signs point to GW170817 having produced a HMNS or very short-lived SMNS remnant. If the merger had instead produced a long-lived SMNS, then a large fraction of its available rotational energy $\gtrsim 10^{52}$ erg should have been deposited into the merger environment, either into a collimated relativistic jet or shared more equitably with the merger ejecta, on a timescale $\sim \tau_{sd}$. Such a large energy input is incompatible with the GRB and KN observations of GW170817.
Table 5.1. EOS Properties and Consistency with EM Observations

<table>
<thead>
<tr>
<th>EOS</th>
<th>$M_{\text{max}}^g$ ($M_\odot$)</th>
<th>$R_{1.3}$ (km)</th>
<th>$M_{\text{smns}}^g$ ($M_\odot$)</th>
<th>$\Delta T_{\text{max}}$ (10$^{53}$erg)</th>
<th>Consistency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS1</td>
<td>2.77</td>
<td>14.9</td>
<td>3.31</td>
<td>1.8</td>
<td>0.0</td>
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<tr>
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<td>1.8</td>
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<td>1.7</td>
<td>0.2</td>
</tr>
<tr>
<td>ENG</td>
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<td>12.0</td>
<td>2.67</td>
<td>1.4</td>
<td>5.2</td>
</tr>
<tr>
<td>WFF2</td>
<td>2.20</td>
<td>11.1</td>
<td>2.63</td>
<td>1.6</td>
<td>10.2</td>
</tr>
<tr>
<td>APR4</td>
<td>2.19</td>
<td>11.3</td>
<td>2.61</td>
<td>1.5</td>
<td>18.4</td>
</tr>
<tr>
<td>SLy</td>
<td>2.05</td>
<td>11.8</td>
<td>2.43</td>
<td>1.2</td>
<td>100.0</td>
</tr>
<tr>
<td>H4</td>
<td>2.02</td>
<td>14.0</td>
<td>2.38</td>
<td>0.8</td>
<td>100.0</td>
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<tr>
<td>ALF2</td>
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<td>12.7</td>
<td>2.41</td>
<td>0.9</td>
<td>100.0</td>
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<td>2.29</td>
<td>0.7</td>
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<tr>
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<td>11.5</td>
<td>2.35</td>
<td>1.0</td>
<td>99.8</td>
</tr>
<tr>
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<td>2.27</td>
<td>1.1</td>
<td>99.4</td>
</tr>
<tr>
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<td>14.3</td>
<td>2.10</td>
<td>0.6</td>
<td>99.9</td>
</tr>
</tbody>
</table>

Note. — all EOS are approximated as piecewise broken polytropes (Read et al. 2009b)

5.3 Constraints on NS Properties

The masses of the binary components inferred for GW170817, combined with evidence from the KN disfavoring a prompt collapse, places a lower limit on the maximum mass $M_{\text{max}}$ of a (slowly rotating) NS. Likewise, upper limits on the rotational energy injected by a long-lived SMNS place an upper limit on $M_{\text{max}}$.

In order to translate GW+EM inferences into constraints on the properties of NSs, we use the RNS code (Stergioulas & Friedman 1995b) to construct general relativistic rotating hydrostationary NS models for a range of nuclear EOS. We use the piecewise polytropic approximations to EOS available in the literature provided in Read et al. (2009b)
(circles in Fig. 5.4, summarized in Table 5.1), supplemented by EOS constructed from the two-parameter piecewise polytropic parameterization of Margalit et al. (2015) (triangles in Fig. 5.4). Our simplified parameterization is limited in its ability to model micro-physically motivated EOS with high accuracy, yet allows us to efficiently survey the EOS parameter space, and we leave it to future work to extend this first analysis with more flexible EOS parameterizations (e.g. Raithel et al. 2016). For each EOS, the uncertainty range of the GW170817-measured binary gravitational mass (LIGO Scientific Collaboration & Virgo Collaboration 2017) translates into a corresponding uncertainty range of baryonic mass, defined by the probability distribution

\[
P(M_{\text{rem}}^b | O, \text{EoS}) = \\
\int dM_1^b \int dM_2^b \delta(M_1^b + M_2^b - M_{\text{ej}} - M_{\text{rem}}^b) \\
\times P(g_{\text{EoS}}(M_1^b), g_{\text{EoS}}(M_2^b) | O) \left| g'_{\text{EoS}}(M_1^b) \right| \left| g'_{\text{EoS}}(M_2^b) \right|.
\]

Here \( P(M_1^b, M_2^b | O) \) is the posterior joint probability distribution function of NS gravitational masses inferred from the BNS waveform \( O \) (LIGO Scientific Collaboration & Virgo Collaboration 2017), \( M_{\text{ej}} = 2 \times 10^{-2} M_\odot \) is a conservative lower limit for the mass loss from the system as inferred from the KN ejecta, and the EOS enters in converting between gravitational and baryonic masses, \( M^g = g_{\text{EoS}}(M^b) \). We approximate the posterior by changing variables to the chirp mass \( M_c \) and mass-ratio \( q = M_1^g / M_2^g \),

\[
P(M_1^g, M_2^g | O) = P(q, M_c) M_c^{-1} q^{6/5} (1 + q)^{-2/5},
\]
assuming independent asymmetric Gaussian distributions for both $P(M_c)$ and $P(q)$, consistent with the median and 90% quoted confidence levels on $M_c$, $M_1^{g}$, $M_2^{g}$, and $M_{\text{tot}}^{g}$. Specifically, we assume $P(q, M_c) \propto \exp \left[ -\left( M_c - \mu_{M,\pm} \right)^2/2\sigma_{M}^2 - \left( q - \mu_q \right)^2/2\sigma_q^2 \right]$ for $q \leq 1$ and $P = 0$ otherwise, with $\mu_q = 1$, $\sigma_q \simeq 0.164$, $\mu_M \simeq 1.188 M_\odot$ and where $\sigma_{M,\pm} \simeq 2.63 \times 10^{-3} M_\odot$ ($2.07 \times 10^{-3} M_\odot$) for $M_c \geq \mu_M$ ($M_c < \mu_M$), respectively.

For each EOS, we then compare the inferred remnant mass to the “allowed” range between the maximum mass to avoid prompt collapse (using the relation $M_{\text{th}}(R_{1.6}, M_{\text{max}})$ of Bauswein et al. 2013), to the minimum baryonic mass which results in a SMNS with an extractable energy $\Delta T$ (Eq. 5.1) less than the upper limits on the kinetic energy of the KN and GRB emission $E_{\text{EM}} = E_{\text{KN}} + E_{\text{GRB}} \lesssim 10^{51}$ erg. Integrating the probability distribution of the remnant mass within this allowed range yields the “consistency” of the given EOS with the GW170817 observations,

$$\text{Consistency} = \int_S P(M_{\text{rem}}^b | O, \text{EoS}) \ dM_{\text{rem}}^b,$$

where $S$ is the domain in which both $\Delta T(M_{\text{rem}}^b) \leq E_{\text{EM}}$ and $M_{\text{rem}}^b \leq M_{\text{th}}$.

One example of this analysis is illustrated in Fig. 5.2. Clearly, $E_{\text{EM}}$ is so much smaller than $\Delta T_{\text{max}}$ that the extractable energy curve intersects $E_{\text{EM}}$ at the very precipice of the SMNS-HMNS transition. We also find for all the EOS we have examined that $M_{\text{smns}}^b \approx 1.18 M_{\text{max}}^b$ largely irrespective of compactness, consistent with previous findings (e.g. Lasota et al. 1996). These two facts allow formulation of an approximate analytic criterion on the
maximal non-rotating NS mass consistent with GW170817,

\[ M_{\text{max}}^b \lesssim M_{\text{rem}}^b / \xi, \quad (5.7) \]

where \( \xi \simeq 1.16 - 1.21 \) and the EOS is only necessary in translating baryonic to gravitational masses.

In addition to several key properties of each EOS, Table 5.1 provides the probability that each EOS is consistent with constraints from GW170817. For instance, the very hard MS1, MPA1 and ENG EOS are disfavored, with consistencies of 0.0%, 0.0% and 5.2%, respectively.
However, the softer EOS’s with $M_{\text{max}}^g \lesssim 2.1 - 2.2M_\odot$ show much higher consistencies. Figure 5.4 shows where each of our EOS lie in this $M_{\text{max}} - R_{1.3}$ parameter space, with the strength of the symbol representing the probability of its consistency with GW170817. Additionally shown are consistency values for polytropic EOSs of the form $p \propto \rho^{1+1/n}$ with indices $n = 0.5, 0.6, 0.7$. These define diagonal curves in the $M_{\text{max}}^g - R_{1.3}$ plane parameterized by the pressure normalization of the polytrope. Regions of large compactness are ruled-out by the requirement of causality, $R_{\text{max}} \geq 2.82GM_{\text{max}}^g/c^2$ (dark shaded region; Koranda et al. 1997) which is conservative since $R_{1.3}$ is generally larger than the radius of a maximum mass NS, $R_{\text{max}}$. A tighter estimate of $R_{1.3} \geq 3.1GM_{\text{max}}^g/c^2$ is therefore also shown (light shaded region). The background grey curve shows the cumulative probability distribution function that the maximum mass $M_{\text{max}}$ is less than a given value. This was calculated by marginalizing over the $R_{1.3}$ axis and treating the consistency values as points in a probability distribution function. We weight EOS with $M_{\text{max}}^g$ below $2.01M_\odot$ by a Gaussian prior accounting for consistency with the maximum measured pulsar mass of $2.01 \pm 0.04M_\odot$ (Antoniadis et al. 2013c). Thus, we find $M_{\text{max}}^g \lesssim 2.17M_\odot$ at 90% confidence.

5.4 Discussion

Several works have explored the potentially exotic EM signals of BNS mergers in cases when a long-lived SMNS or stable neutron star remnant is formed (Metzger et al. 2008b, Bucciantini et al. 2012b, Yu et al. 2013, Metzger & Bower 2014, Metzger & Piro 2014, Gao et al. 2016, Siegel & Ciolfi 2016a,b). However, one of the biggest lessons from GW170817 was the well-behaved nature of its EM emission, under the simplest case of a relatively short-
lived HMNS remnant \cite{ShibataT2006}, with an off-axis afterglow \cite{vEertenM2011} and KN emission \cite{Metzgereta2010a} closely resembling “vanilla” theoretical predictions.

Here we have made explicit the argument that the BNS merger GW170817 formed a HMNS. In combination with the GW-measured binary mass, this inferred outcome places upper and lower limits on the maximum NS mass. The lower limit on $M_{\text{max}}$ is not constraining compared to those from well-measured pulsar masses, though tighter constraints would be possible by the future detection of similar fast-expanding blue KN ejecta (indicating HMNS formation) from a future BNS merger with a similar observing inclination but higher binary mass than GW170817. The lack of a luminous blue KN following the short GRB050509b \cite{Bloometal2006, Metzgereta2010a, Fongetal2017} may implicate a high mass binary and prompt collapse for this event.\footnote{A prompt collapse might also be consistent with the low measured gamma-ray fluence of GRB050509b, because the mass of the remnant accretion torus responsible for powering the GRB jet would also be lower for a prompt collapse than if a HMNS had formed.}

On the other hand, our upper limits on $M_{\text{max}} \lesssim 2.17M_\odot$ (90% confidence limit; Fig. 5.4) are more constraining than the previous weak upper limits from causality, and less model-dependent than other methods \cite{Laskyetal2014, Lawrenceetal2015, Fryeretal2015, Gaoetal2016, Alsingetal2017}. A low value of $M_{\text{max}}$ has also been suggested based on Galactic NS radius measurements \cite{Ozeletal2016, OzelFreire2016} and would be consistent with the relatively small NS radius $\lesssim 11$ km inferred from modeling the blue KN \cite{Nicholletal2017b, Cowperthwaiteetal2017}. Furthermore, the lack of measurable tidal-effects in the inspiral of GW170817 similarly imply a small NS radius and thus a low $M_{\text{max}}$ \cite{LIGOVirgoCollab2017}. Upper limits on $M_{\text{max}}$
will be improved by the future discovery of EM emission from a merger with a lower total mass than GW170817. Conversely, the detection of a substantially brighter afterglow or faster evolving KN emission could instead point to the formation of a long-lived SMNS or stable remnant. The NS masses measured for GW170817 are broadly consistent with being drawn from Galactic NS population, which is well-fit by a Gaussian of mean $\mu = 1.32M_\odot$ and standard deviation $\sigma = 0.11M_\odot$ \cite{kiziltan2013}; this hints that the HMNS formation inferred in GW170817 is likely a common—if not the most frequent—outcome of a BNS merger.

A simple analytic estimate of our result can be obtained from Eq. (5.7), using the approximation $M_b = M_g + 0.075M_g^2$ for the relation between baryonic and gravitational masses \cite{timmes1996}. From this relation, the total baryonic binary mass is constrained, $M^b_{\text{rem}} \lesssim M^b_{\text{tot}} \lesssim 3.06M_\odot$. A typical value of $\xi \approx 1.18$ then implies that

$$M^g_{\text{max}} \lesssim \sqrt{1 + 0.3M^b_{\text{rem}}/\xi} - 1 \lesssim 2.2M_\odot,$$

(5.8)

in agreement with our more elaborately calculated result. We stress that the calculation above is intended only as an approximate analytic estimate, and that we do not use the \textit{Timmes et al.} (1996) relation nor do we assume a universal value for $\xi$ in our complete analysis ($\S$ 5.3).

Our approach differs in several respects from similar works constraining $M_{\text{max}}$ (e.g. \textit{Lawrence et al.} 2015, \textit{Fryer et al.} 2015). These works generally assume (a) that creation of a GRB implies a BH formed, and (b) that BH formation necessarily implies either prompt-collapse or a HMNS post-merger remnant. The central engine and emission mechanisms
of GRBs are still widely debated in the literature, and the validity of the “GRB=BH” assumption remains unclear. Baryon-pollution by neutrino-driven winds launched off a long-lived NS remnant may hinder ultra-relativistic jets (Murguia-Berthier et al. 2014), however the Lorentz factors of short GRB jets and GRB 170817A in particular are poorly constrained, and there remains room for the possibility that short GRBs may be powered by strongly-magnetized rapidly-rotating NSs. NS GRB engines have also been suggested on grounds of the ‘extended’ X-ray emission observed after some short GRBs (e.g. Metzger et al. 2008b; Rowlinson et al. 2013b), emission which is difficult to interpret within the BH engine model. Furthermore, the peculiar properties of GRB 170817A accompanying GW170817, although broadly consistent with a normal GRB viewed off-axis (e.g. Margutti et al. 2017a), may also point at a difference between this event and cosmological short GRBs (e.g. Gottlieb et al. 2018), necessitating further caution in the GRB modeling and interpretation. Secondly, even if a BH did form as a prerequisite to the GRB in this event (i.e. within ~ 2s post-merger), there is nothing a-priori preventing this from occurring through the spin-down induced collapse of a SMNS merger remnant, negating assumption (b) above. Here we have circumvented both assumptions and GRB engine modeling altogether by instead relying on a simple energetic consideration — that a SMNS merger remnant would inevitably release an enormous amount of rotational energy into the surrounding KN ejecta and circum-stellar medium. Therefore, only merger remnants with an extractable rotational energy $\lesssim 10^{51}$ erg are consistent with the energetics inferred from EM observations of GW170817.

Several uncertainties affect our conclusions. Our upper limits on $M_{\text{max}}$ implicitly assume that this is the most important parameter controlling the HMNS-SMNS boundary, and that
the suite of EOS we have taken are sufficiently “representative” in the requisite sense. We cannot obviously exclude the possibility that an alternative EOS could be found with large $M_{\text{max}}$ that would still be consistent with our observational constraints.

Another uncertainty affecting our conclusions is the possibility that in counting the KN and GRB components of the ejecta, we are somehow “missing” substantial additional energy imparted by a putative SMNS remnant to the environment; however, any such hidden ejecta should be at least mildly relativistic and thus tightly constrained by radio synchrotron emission on timescales of months to years following the merger (Metzger & Bower 2014). Yet another uncertainty is the possibility that a SMNS did form, but most of its rotational energy was lost to GW radiation instead of being transferred to the merger ejecta. Though unlikely, we found this would only be possible for remnant lifetimes of $\sim 100$ s (Fig. 5.3). Searches in the GW170817 waveform have revealed no evidence for such signals, although the detectors’ decreasing sensitivity at high-frequencies currently limits these constraints (Abbott et al. 2017b).

Finally, our analysis neglects the effects of thermal pressure on the stability of the SMNS (Kaplan et al. 2014), which can be important on timescales of hundreds of milliseconds to seconds post merger (depending also on the effects of neutrino-driven convection; Roberts et al. 2012). Thermal pressure in the outer layers of the star generally acts to reduce the maximum mass of the SMNS remnant by up to $\lesssim 8\%$ (mainly by reducing the angular velocity at the mass-shedding limit), which would act to weaken our constraints on $M_{\text{max}}$. Future numerical work exploring the transition from the HMNS to SMNS phase, which includes the effects of neutrino cooling and convection self-consistently, is required to better
understand how this would quantitatively affect our conclusions.
Part II

Birth of Compact Objects — Magnetar Models
Chapter 6

The GRB-SLSN Connection:
mis-aligned magnetars, weak jet emergence, and observational signatures

6.1 Introduction

Type I “superluminous supernovae” (SLSNe; Quimby et al. 2011; Gal-Yam 2012) are a rare class of core collapse supernovae (SNe) with hydrogen-poor spectra that reach peak luminosities $\sim 10^{43} - 10^{45}$ erg s$^{-1}$ which are usually too large to be powered by the radioactive

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decay of $^{56}\text{Ni}$. The nickel mass required to power these events would in most cases exceed the total ejecta mass inferred from the timescale of the light curve rise (e.g. Inserra et al. 2013; Nicholl et al. 2013; however see Kozyreva et al. 2017); the UV-rich spectrum of SLSNe also disfavors the ejecta being so rich in iron-group elements (Kasen et al. 2011; Dessart et al. 2012; Yan et al. 2017). These facts argue against most SLSNe being the result of pair instability in extremely massive stars (Barkat et al. 1967; Kasen et al. 2011; Dessart et al. 2013; Kozyreva et al. 2017).

Another potential power source for SLSNe is a shock-mediated collision between the SN ejecta and the external circumstellar medium (Chevalier & Irwin 2011; Ginzburg & Balberg 2012; Moriya et al. 2013; Chatzopoulos et al. 2013). The latter might be produced by pre-explosion stellar mass loss (as in Type IIIn supernovae; Chevalier & Fransson 1994; Moriya et al. 2014; Dessart et al. 2015), pulsational pair instability (e.g. Woosley et al. 2007; Chatzopoulos & Wheeler 2012; Woosley 2016), or a relic proto-stellar disk (Metzger 2010). However, the lack of spectroscopic evidence such as emission lines for shock interaction or the presence of extended circumstellar matter again appears to disfavor interaction models for the majority of Type I SLSNe (Nicholl et al. 2015b). Line-emitting gas could be sufficiently deeply embedded to remain hidden at all times, but this appears to require fine-tuning of the distribution of external matter. A small number of SLSNe-I show clear signatures of interaction through late-time H$\alpha$ emission (Yan et al. 2015, 2017), but these occur at late, $\gtrsim 100$ d, epochs indicating that such interaction does not power the peak luminosity (although see Liu et al. 2017 for an alternative model).

$^1$Pair instability SNe remain a contender for explaining slowly-evolving SLSNe (e.g., Gal-Yam et al. 2009; Lunnan et al. 2016) if the persistent blue colors can somehow be reconciled with the predicted high abundance of Fe-group elements in the ejecta.
The most strongly favored model for SLSNe is sustained energy input from the young compact stellar remnant, such as the electromagnetic dipole spin-down of a strongly magnetized neutron star (“millisecond magnetar”; Kasen & Bildsten 2010; Woosley 2010; Metzger et al. 2014) or an accreting black hole (Dexter & Kasen 2013). The simplest form of the magnetar model — which assumes that 100% of the spin-down power of the magnetar is converted to thermal energy behind the ejecta — predicts a light curve evolution which can be successfully fit to those of most SLSNe (Inserra et al. 2013; Chatzopoulos et al. 2013; Nicholl et al. 2014; Nicholl et al. 2017).

The association between SLSNe and the birth of energetic compact objects is further supported by the growing observational connection between SLSNe and long duration gamma-ray bursts (GRB). GRB originate from the deaths of a rare class of massive stars, a fact well-established by their observed coincidence with hyper-energetic, broad-lined Type Ic SNe (SN Ic-bl; e.g. Galama et al. 1998; Stanek et al. 2003; Hjorth et al. 2003; Woosley & Bloom 2006; Modjaz et al. 2016). As in SLSNe, the relativistic jets responsible for powering GRB could be fed by the rotational energy of a millisecond magnetar (e.g., Usov 1992; Wheeler et al. 2000; Thompson et al. 2004; Metzger et al. 2011) or the accretion energy of a stellar mass black hole (e.g. MacFadyen & Woosley 1999b; MacFadyen et al. 2001). SLSNe and SNe Ic-bl have comparable average spectral features and absorption line velocities at all phases, which are systematically broader than those of normal SNe Ic (Liu & Modjaz 2016), while also sharing remarkably similar nebular spectra (Nicholl et al. 2016b; Jerkstrand et al. 2017). SLSNe share the preference of GRBs for faint, low metallicity galaxies, indicating a

\footnote{although see e.g. Zhang & Dai (2008); Bernardini et al. (2013) for an alternative accretion-powered magnetar model motivated by relativistic jets observed in some accreting NS systems (e.g. Fender et al. 2004)}
possible link between their formation channels (e.g. Lunnan et al. 2014; Stanek et al. 2006; Chen et al. 2015; Perley et al. 2016b; Japelj et al. 2016; Perley et al. 2016a). Yet while the typical hosts of the two are quite similar, SLSN hosts on average seem to have even lower metal content and higher star formation rates (Leloudas et al. 2015; Angus et al. 2016; Schulze et al. 2016). The recently discovered SN2017egm (Nicholl et al. 2017b; Bose et al. 2017; Chen et al. 2017) was localized to a massive (∼10^{10.7} M_\odot) spiral galaxy with ~solar metallicity, indicating that at least some SLSNe can be produced in solar metallicity environments, a result which is also broadly in line with GRBs (Nicholl et al. 2017b). Taking the overall similarity of the host galaxies along with other evidence such as spectroscopic evolution, SN 2011kl, and the general requirement for a central engine in both classes, it appears that the two populations are closely related.

Further indirect evidence for a connection between SLSNe, GRB, and magnetar birth was provided by the recent localization of the repeating fast radio burst (FRB) FRB 121102 (Chatterjee et al. 2017) in a dwarf irregular galaxy (Tendulkar et al. 2017b) with properties of mass, metallicity, and star formation remarkably similar to the hosts of SLSNe and GRB (Tendulkar et al. 2017b; Metzger et al. 2017b; Nicholl et al. 2017a). Prior to this discovery, several works hypothesized that FRBs originate from flaring magnetars (e.g. Lyubarsky 2014; Kulkarni et al. 2014; Lyutikov et al. 2016), thus supporting an association between FRB 121102 and the birth of a millisecond magnetar embedded in a young supernova remnant with an age of decades to a century (Metzger et al. 2017b; Beloborodov 2017; Lyutikov 2017; Waxman 2017).

If both SLSNe and GRB are engine-powered, it is natural to question what distinguishes
them. Their most important distinction is probably the *duration* of the engine’s peak luminosity, which is characteristically minutes or less in GRB, and usually days or longer in the case of SLSNe (e.g. Metzger et al. 2015). A long-lived engine is essential in SLSNe to enhance their luminosity above the minimum level set by radioactive $^{56}$Ni, the latter of which instead powers the luminosity of the Ic-bl SNe accompanying most GRB (e.g. Cano et al. 2016; however, see Wang et al. 2016).

A class of ultra-long GRB (ULGRB) with durations of $\gtrsim 10^3$ s has recently received attention (Gendre et al. 2013; Levan et al. 2014; Zhang et al. 2014; Boër et al. 2015). The ULGRB 111209A was observed in coincidence with a highly luminous $\sim 10^{43}$ erg s$^{-1}$ and short-lived $\sim 15 \text{d}$ supernova (Greiner et al. 2015) with a blue spectrum consistent with those associated with SLSNe (Liu & Modjaz 2016; Kann et al. 2016), providing a potential direct link between GRB and SLSNe (Metzger et al. 2015; Bersten et al. 2016; Gompertz & Fruchter 2017). In retrospect, this connection is unsurprising because a long-lived engine (as required to power an ultra-long GRB) is identical to that needed to maintain a luminosity $\sim 10^{43}$ erg s$^{-1}$ a couple weeks after the explosion, near the super-luminous level (Metzger et al. 2015).

On the other hand, ULGRB 111209A raises the question of how it is possible theoretically for a single engine to power both a successful jet (which escapes the exploding star to power the GRB) while also thermalizing a large fraction of its energy behind the ejecta (to power the SN at later times). If a jet can escape from the ejecta of a SLSNe, then it is also natural to ask what observable signatures such jets would display for the more typical off-axis observer.
The key questions raised above motivate additional theoretical studies of the GRB-SLSN connection and ways to test it observationally. Metzger et al. (2015) recently highlighted the diversity of transients potentially associated with millisecond magnetar birth, providing a common theoretical framework for interpreting diverse phenomena from long-duration GRB (LGRB) and ULGRB to SLSNe and broad-line SN-Ic (see also Kashiya et al. 2016; Ioka et al. 2016).

This work goes beyond these initial steps to describe an explicit mechanism by which magnetars can power both a GRB jet and a SLSN. In particular, previous analytic and numerical calculations either explicitly or implicitly assumed $\hat{\Omega} \cdot \hat{\mu} = 1$. In general, we expect mis-alignment between the magnetic and rotation axes, implying an explicit mechanism for thermalization of the spindown power via reconnection in the equatorial striped magnetar wind (Lyubarsky 2003). For a range of mis-alignment angles we expect either complete thermalization ($\hat{\Omega} \cdot \hat{\mu} = 0$), as assumed in models of SLSNe (Kasen & Bildsten 2010; Woosley 2010), or virtually no thermalization and strong jet production ($\hat{\Omega} \cdot \hat{\mu} = 1$), as in models of GRBs (e.g., Bucciantini et al. 2009).

We develop this model and apply it. For the weak jets that we expect are launched generically by SLSNe, we extend earlier work to address whether or not they can escape the supernova explosion, on what timescale, and with what observational signature, whether the observer is on- or off-axis. Our general model of thermalization and jet production allows us to provide a unified picture of the GRB-SLSN dichotomy and connection.

Setting the details of the magnetar thermalization mechanism we propose aside, our estimates for low-luminosity jet emergence and its observational signature can also be applied
to black hole accretion models.

This paper is organized as follows. In §6.2 we review the magnetar scenario and present our model for partitioning spin-down luminosity between both jetted and thermal components. We then examine whether weak jets can break-out of the confining stellar matter (§6.3). Readers uninterested in the jet-propagation details are encouraged to skip to §6.3.2.2 where we derive our primary results. We continue by exploring the observational signatures such off-axis jets may give rise to (§6.4). Our novel model for powering early optical/UV emission in SLSNe by (post-breakout) jet interaction with the confining SN-ejecta walls is presented in §6.4.2 and applied to the SLSN LSQ14bdq. We discuss implications of our results in §6.5 and summarize the landscape of engine-powered transients in Fig. 6.6. We end with bulleted conclusions (§6.6).

6.2 Magnetar Misalignment: Powering Both Jet and SN

We begin this section with a brief review of the magnetar model, following which we describe an explicit mechanism by which a misaligned magnetar can partition its power into both thermal and magnetically-dominated (jetted) components. The engine luminosity’s time evolution can generally be expressed as \cite{KasenBildsten2010}

\[
L_e = \frac{E_e}{t_e} \left( \frac{\ell - 1}{1 + t/t_e} \right) \ell, \tag{6.1}
\]
where $E_e$ is the total energy of the engine and $t_e$ the engine lifetime, over which the power is approximately constant. At late times $t \gg t_e$ the power decays as $\sim t^{-\ell}$. For the magnetar scenario, $\ell = 2$ and the values of $E_e$ and $t_e$ are related to the magnetar’s surface dipole field $B_d$ — which is presumably amplified during core-collapse by, e.g. a large-scale dynamo (Mösta et al. 2015a) — and initial spin period, $P_0$, by

$$E_e = \frac{1}{2} I_{\text{ns}} \Omega^2 \simeq 2.5 \times 10^{52} \text{erg} \left(\frac{M_{\text{ns}}}{1.4 M_\odot}\right)^{3/2} \left(\frac{P_0}{1 \text{ ms}}\right)^{-2}, \quad (6.2)$$

$$t_e = \frac{E_{\text{mc}}^3}{\mu^2 \Omega^4 (1 + \sin^2 \alpha)} \simeq \frac{147 \text{ s}}{(1 + \sin^2 \alpha)} \left(\frac{M_{\text{ns}}}{1.4 M_\odot}\right)^{3/2} \left(\frac{P_0}{1 \text{ ms}}\right)^2 \left(\frac{B_d}{10^{15} \text{ G}}\right)^{-2}, \quad (6.3)$$

where $I_{\text{ns}} \simeq 1.3 \times 10^{45} \text{ g cm}^2 (M_{\text{ns}}/1.4 M_\odot)^{3/2}$ is an estimate of the neutron-star moment of inertia for a range of plausible nuclear density equations of state (Lattimer & Schutz 2005b), $\mu = B_d R_{\text{ns}}^3$ is the magnetic dipole moment, and the factor $(1 + \sin^2 \alpha)$ accounts for the dependence on the misalignment angle $\alpha$ between magnetic and rotational axes ($\cos \alpha \equiv \hat{\Omega} \cdot \hat{\mu}$, see Fig. 6.1 Spitkovsky 2006). We assume that all of the rotational energy goes into electromagnetic spin-down, instead of gravitational wave radiation (e.g. Moriya & Tauris 2016; Ho 2016).

The notion of simultaneously powering both a collimated jet and an isotropic thermal SN by a single magnetar has previously been discussed, e.g. in the context of relating hyper-
Figure 6.1 Schematic diagram (not to scale) showing how the same millisecond magnetar engine can power both a relativistic GRB jet and a SLSN via isotropic radiative diffusion. A magnetar (grey) with a non-zero misalignment between the rotation and magnetic dipole axes develops a striped-wind configuration in a wedge near the equatorial plane. The fraction of the spin-down energy carried by the striped wind is thermalized when the alternating field undergoes magnetic reconnection near the wind termination-shock, heating the pulsar-wind nebula (PWN; yellow). This thermal energy diffuses through the spherical SN ejecta (blue), powering luminous SN emission. By contrast, the spin-down power at high latitudes is channeled into a bi-polar collimated jet (orange; §6.2). Even once the jet has escaped from the star, a fraction of its power will continue to be thermalized at the interface between the jet and the ejecta walls, driving a hot mildly-relativistic wind of velocity $v_w$. Thermal radiation from this wind may give rise to relatively isotropic optical/UV emission viewable off the jet axis, producing a pre-maximum peak in the light curves of SLSNe (§6.4.2; Fig. 6.4).

energetic broad-lined Ic SNe to GRBs (e.g. Thompson et al. 2004). Such models, though, could not address a fundamental question — how is the magnetar energy partitioned between jet and SN? Later numerical simulations by Bucciantini et al. (2009) found that nearly none of the magnetar spindown power was deposited into the spherical SN component (see also Komissarov & Barkov 2007), raising questions as to the viability of magnetar-driven SNe. Here, we propose a solution to this “thermalization problem” by introducing a new, explicit, model for magnetar thermalization. The idea rests on consideration of the misalignment angle $\alpha$ between the magnetar’s rotation and magnetic axes. In this respect, the
2D axisymmetric simulations mentioned above implicitly assumed $\alpha = 0$, and could not capture the physics of our proposed model.

For $\alpha \neq 0$, the magnetar develops a ‘striped-wind’ configuration where the toroidal magnetic field switches polarity in the equatorial plane [Coroniti 1990, Lyubarsky & Kirk 2001], as illustrated schematically in Fig. 6.1. The consequences of this wind geometry are well-studied in the pulsar community, as they may play an role in solving the so-called “$\sigma$ problem” first identified in the Crab Nebula.

Outside of the light cylinder, in the wind zone, the power pattern assuming a split-monopole field varies with latitude $\theta$ (measured from the rotation axis) as $dL_e/d\Omega \propto \sin^2 \theta$. For a misaligned rotator, a fraction of the magnetic energy at low latitudes (within $\pm \alpha$ from the equator) will be dissipated by forced reconnection of the striped wind in the equatorial wedge near the temination shock which separates the wind from the magnetar nebula. Following Lyubarsky (2003) and Komissarov (2013), the fraction of the wind power remaining in Poynting flux at latitude $\theta$ is given by

$$\chi(\theta; \alpha) = \begin{cases} 
1, & 0 \leq \theta < \pi/2 - \alpha \\
[2\phi(\theta; \alpha)/\pi - 1]^2, & \pi/2 - \alpha \leq \theta < \pi/2 
\end{cases}, \quad (6.4)$$

where $\phi(\theta; \alpha)$ is the stripe wave phase defined by $\cos \phi(\theta; \alpha) \equiv -\cot(\theta) \cot(\alpha)$.

Thus, the total fraction of magnetar power which remains in the ordered magnetic field following reconnection at the termination shock is

$$f_j(\alpha) = \frac{\int (dL_e/d\Omega) \chi d\Omega}{\int (dL_e/d\Omega) d\Omega} = \frac{3}{2} \int_0^{\pi} \frac{2}{\pi} \chi(\theta; \alpha) \sin^3 \theta d\theta. \quad (6.5)$$
Similarly, the thermalized energy fraction is $f_{\text{th}}(\alpha) = 1 - f_j(\alpha)$. We find that $f_{\text{th}}$ is well approximated (to within an accuracy of a couple percent) by

$$f_{\text{th}}(\alpha) \approx \frac{1 + (\pi/2)^{-4} b}{(b + \alpha^4)^{1/4}} = \frac{1.025\alpha}{(0.636 + \alpha^4)^{1/4}},$$  \hspace{1cm} (6.6)$$

where $\alpha$ is in radians and in the second equation $b \simeq 0.636$. The model thus implies that any oblique rotator will partition its spin-down power into both an ordered magnetic and a thermal component, with thermalization increasing for greater misalignment angles $\alpha$.

Note that numerical simulations find $dL_e/d\Omega \propto \sin^n \theta$ with $n \sim 4$ at large inclination angles (e.g. Tchekhovskoy et al. 2013). Our results hold equally in this scenario, with the qualitative difference that $f_{\text{th}}$ rises faster with $\alpha$ for larger $n$. In particular, for $n = 4$ we find that the functional form (6.6) still well fits $f_{\text{th}}(\alpha)$, with $b \simeq 0.268$.

It is therefore natural to interpret $f_j$ — the fraction of the energy remaining in an
ordered toroidal magnetic field — as that which may contribute to a collimated jet component (GRB), while \( f_{\text{th}} \) is the complementary power energizing the SN ejecta which may contribute to powering to the isotropic thermal emission (SN). The mis-alignment angle can thus be observationally inferred by identifying the thermalization fraction with \( f_{\text{th}} \approx E_{\text{SLSN}}/(E_{\text{SLSN}} + E_{\text{GRB}}) \) and inverting equation (6.6), yielding

\[
\alpha \approx 0.893 f_{\text{th}} \left(1.105 - f_{\text{th}}^4\right)^{-1/4} \text{ rad.} \tag{6.7}
\]

The simplified picture outlined above assumes that magnetic energy is only dissipated through reconnection of a striped wind at the termination shock. However, other forms of dissipation related to MHD instabilities (e.g. kink or sausage), may operate on larger scales throughout the nebula as well (e.g. Begelman 1998; Porth et al. 2013; Zrake & Arons 2016), depending in part on how effectively the build-up of toroidal flux is “relieved” by the escape of a successful polar jet. The details of such a thermalization processes are more complex, and we briefly discuss their affect on the jet component in §6.4.

Finally, note that the jet model we develop in the following section can equally be applied to black-hole (accretion-powered) engines. In this case, we expect \( \ell \approx 5/3 \) in equation (6.1), as set by the rate of mass fall-back of marginally-bound stellar debris following the SN\(^4\). The engine energy is related to the total fall-back mass \( M_{fb} \) according to \( E_e = \epsilon_{fb} M_{fb} c^2 \), where \( \epsilon_{fb} < 1 \) is an efficiency factor for producing a relativistic jet or disk wind. The engine timescale \( t_e \) is generally set by the gravitational free-fall timescale of the progenitor star, \( t_e \sim \)

\(^4\)However, see Tchekhovskoy & Giannios (2015), who argue that the jet power may be set by the rate of accumulation of magnetic flux onto the black hole, rather than the accretion rate, in which case the time-dependence of the engine luminosity will be more complicated.
$1/\sqrt{G\bar{\rho}}$, where $\bar{\rho}(r)$ is the average density of the enclosed mass within radius $r$ of the stellar progenitor (e.g. Dexter & Kasen 2013). This timescale is substantially shorter in the case of the compact Wolf-Rayet progenitors responsible for SNe Ic-bl, compared to the more radially-extended outer layers of blue or red supergiants. As in the magnetar case, accretion-powered engines may exhibit a dichotomy between a bipolar relativistic outflow and slower wide-angle wind, which contribute to powering the GRB and isotropic SN, respectively. However, the energetic partitioning between these two components depends in a more complex manner on the details of the accretion model, such as the angular momentum distribution (Dexter & Kasen 2013) and magnetic flux (Tchekhovskoy & Giannios 2015) of the fall-back mass.

### 6.3 Weak Jet Break-out

Jet propagation and break-out from stationary stellar progenitors have been extensively studied in the context of GRB (Aloy et al. 2000; MacFadyen et al. 2001; Zhang et al. 2003; Morsony et al. 2007; López-Cámara et al. 2013; Bromberg & Tchekhovskoy 2016). The emerging picture is of a self-collimated relativistic outflow confined by an isobaric cocoon of hot shocked matter which spills around the jet-head (e.g. Bromberg et al. 2011).

Central-engine powered SNe generally require long-lived engine activity ($\sim 10$ d) compared to those of typical LGRB ($\sim 100$ s). For a fixed engine energy budget, this implies dramatically lower power. Such low-luminosity jets may therefore not manage to burrow their way out of the surrounding stellar progenitor faster than the SN shock-front, resulting in dramatically different ambient conditions for jet propagation. In the following, we extend analytic jet models to low-luminosity jets propagating within an exploding stellar profile.
and expanding SN ejecta. The latter has been previously considered by Quataert & Kasen (2012), but has not received attention beyond that work. Our main finding is that weak jets — of the kind expected to accompany SLSNe if the central engine partitions comparable energy into jetted and thermal components (see §6.2) — may successfully break out of the expanding SN ejecta at late times (cf. equations 6.26, 6.28).

Although the magnetar picture presented in §6.2 dictates a magnetically dominated jet, recent numerical simulations by Bromberg & Tchekhovskoy (2016) illustrate that such jets behave similarly to hydrodynamic jet models during their escape from the star. We therefore follow Bromberg et al. (2011) and adopt a hydrodynamic collimated-jet model. Our results differ from those of Quataert & Kasen (2012) primarily due to the fact that these authors assumed an uncollimated jet model. Readers uninterested in the model details are encouraged to jump to §6.3.2 (in particular 6.3.2.2), where we present our main findings.

6.3.1 Jet Propagation Model

6.3.1.1 Ambient Density Profile

We assume an initial stellar progenitor of mass $M_\star$, radius $R_\star$, described by a power-law density profile $\rho \propto r^{-w}$ with $w < 3$, such that

$$\rho(r, t = 0) = \frac{(3 - w)M_\star}{4\pi R_\star^3} \left(\frac{r}{R_\star}\right)^{-w}. \quad (6.8)$$
A SN explosion of energy $E_{sn} \sim 10^{51}$ erg generates an outgoing shock front which propagates as

$$R_{sh}(t \leq t_\star) = R_\star \left( \frac{t}{t_\star} \right)^{2/(5-w)} ,$$

(6.9)

where

$$t_\star = \sqrt{\frac{2M_\star R_\star^2}{(5-w)^2 E_{sn}}}$$

$$\simeq 173 \text{ s} \left( \frac{R_\star}{10^{11} \text{ cm}} \right) \left( \frac{M_\star}{5M_\odot} \right)^{1/2} \left( \frac{E_{sn}}{10^{51} \text{ erg}} \right)^{-1/2} \quad (6.10)$$

is the time at which the shock front reaches the stellar surface. Up to numerical factors of order unity (e.g. Chevalier 1976) this is the standard Sedov-Taylor solution.

We schematically follow Chevalier & Soker (1989) who focus on the case of a progenitor density power-law of $w = 17/7 \simeq 2.43$. This profile is particularly relevant as it is characteristic of stripped envelope stars which are the possible progenitors of LGRB, Type Ib/c SNe, and Type I SLSNe, and are well modeled by $\rho \sim r^{-2.5}$ (Woosley & Heger 2006). The precise value of $w = 17/7$ is convenient as it considerably simplifies the solution by reducing to the so-called Primakoff blast-wave. In this case, the density increases linearly up to the radius of the outwardly-propagating shock front, and can be expressed as

$$\rho(r < R_{sh}, t < t_\star) = 7\rho(R_{sh}, t = 0) \left( \frac{r}{R_{sh}} \right) = \frac{M_\star}{\pi R_\star^3} \left( \frac{r}{R_\star} \right) \left( \frac{t}{t_\star} \right)^{-8/3} .$$

(6.11)

The unshocked region ahead of the blast-wave ($r > R_{sh}$) remains unperturbed and is still
described by the initial stellar density profile (equation 6.8).

After reaching the progenitor’s surface at \( t = t_\star \), the shock is accelerated in the dilute stellar atmosphere, the details of which we ignore here. Eventually, at late times \( (t \gg t_\star) \) the SN ejecta expands homologously, such that \( v = r/t \) and

\[
\rho(r \leq v_{ej}t, t \gg t_\star) = \zeta_\rho \frac{M_{ej}}{(v_{ej}t)^3} \left( \frac{r}{v_{ej}t} \right)^{-\delta},
\]

with \( \delta = 1 \) a typical value (Chevalier & Soker 1989). The characteristic ejecta velocity is given by

\[
v_{ej} = \zeta_v \sqrt{E_{sn}/M_{ej}},
\]

and the numerical constants \( \zeta_\rho, \zeta_v \) are defined as

\[
\zeta_\rho = \frac{3 - \delta}{4\pi}, \quad \zeta_v = \sqrt{\frac{2(5 - \delta)}{3 - \delta}}.
\]

We compare our analytic model for \( \rho(r, t) \) with 1D numerical simulation results in Fig. 6.3. The numerical profiles were calculated using the 1D implicit hydrodynamic code KEPLER (Weaver et al. 1978b). Starting from a presupernova model of a \( M_\star = 10M_\odot \) carbon-oxygen core from the same code (Sukhbold & Woosley 2014) the explosion of \( E_{sn} = 10^{52} \) erg is calculated using the moving inner boundary method (i.e. ‘piston’ method, Woosley & Weaver 1995). Our analytic model is fully described by specifying \( M_\star, R_\star \) and \( E_{sn} \), which we take directly from the simulation setup. While our simplified analytic prescription cannot reproduce many of the finer features present in the numerical results, the large-scale structure of the density profiles is reasonably approximated by our model.
6.3.1.2 Jet Propagation

The jet dynamics are determined by the dimensionless jet luminosity parameter $\tilde{L} \equiv L_j/A_j \rho c^3$, where $A_j$ is the jet head cross-section as it burrows through the star’s ambient density $\rho$, $L_j = f_j L_e/2$ is the one-sided jet luminosity, and $L_e$ is the total engine luminosity (equation [6.1]). A jet which obeys $\tilde{L} \lesssim \gamma_j^{4/3}$, where $\gamma_j$ is the jet Lorentz factor, remains self-collimated as it propagates through the star. Additionally, a jet with $\tilde{L} \ll 1$ moves through the star at a non-relativistic velocity. In these limits, the dimensionless jet power — as a function of the jet-head position within the ambient medium $z_h$ — is given by (Bromberg et al. 2011)

$$\tilde{L}(z_h) = \zeta L^2 (\rho(z_h) z_h^{-2} c^3)^{2/3} \equiv \tilde{L} \zeta L (\rho(z_h) z_h^{-2} c^3)^{-2/3} \left( \frac{\rho(z_h) z_h^{-2} c^3}{M_* R_*^{-2} t_*^{-3}} \right)^{-2/3}.$$  (6.15)
Here $\zeta_\tilde{L}$ is an order unity numerical factor derived in Appendix H (equation H12), and in the second equation we have defined

$$\tilde{L}_* \equiv \left( \frac{L_j \gamma_j^4}{M_* R_*^2 t_*^3} \right)^{2/3} \simeq 0.89 \times \left( \frac{L_j}{10^{48} \text{ erg s}^{-1}} \right)^{2/3} \left( \frac{\gamma_j}{2} \right)^{8/3} \times \left( \frac{M_*}{5M_\odot} \right)^{1/3} \left( \frac{R_*}{10^{11} \text{ cm}} \right)^{2/3} \left( \frac{E_{\text{sn}}}{10^{51} \text{ erg}} \right)^{-1}$$

(6.16)

as a convenient measure of the jet luminosity which depends only on $L_j$ and the stellar parameters (and not on $z_h$). In §6.3.2 we show that $\tilde{L}_*$ differentiates between various jet break-out regimes.

The jet-head velocity $v_h$ in the self-collimated non-relativistic limit is related to the dimensionless luminosity by (e.g. Matzner 2003)

$$v_h = \frac{c}{1 + \tilde{L}^{-1/2}} \approx c \left[ \tilde{L}(z_h) \right]^{1/2}.$$ 

(6.17)

This is a differential equation for the vertical location of the jet-head $z_h(t)$, which is readily solved for any given ambient density and jet luminosity. For a power-law density profile the solution is derived in Appendix H and acquires the simple form $z_h = \zeta_z v_h t$.

Though subject to many uncertainties, the jet Lorentz factor appearing in equation (6.15) can be constrained from inferred GRB jet-opening angles, since $\gamma_j \sim \theta_j^{-1}$ (Bromberg et al. 2011). Mizuta & Ioka (2013) numerically calibrate this relation to $\gamma_j \approx 1/5\theta_j$, and infer typical GRB Lorentz factors of $\gamma_j \approx 2-3$. We therefore adopt $\gamma_j = 2$ as a fiducial value throughout this work.

Bromberg & Tchekhovskoy (2016) show that magnetically dominated jets are suscepti-
ble to the global kink-instability when the dimensionless parameter $\Lambda \lesssim 1$, where

$$\Lambda \equiv 10\eta \frac{t_{\text{kink}}}{t_{\text{dyn}}} = 10\eta \frac{2\pi \gamma j R_j/v_A}{z_h/(c - v_h)},$$  \hspace{1cm} (6.18)$$

$R_j \approx \sqrt{A_j/\pi}$ is the cylindrical radius of the jet head, $v_A \approx c$ is the Alfvén speed, and $\eta$ is an order unity ignorance factor which Bromberg & Tchekhovskoy (2016) numerically find satisfies $0.5 \lesssim \eta \lesssim 1$. Expressing the jet-head velocity and cross-section as a function of $\tilde{L}$ using the general unapproximated expressions (i.e. relaxing the assumption $\tilde{L} \ll 1$), we find after some algebraic manipulation that

$$\Lambda = 20\sqrt{\pi \eta \gamma_j^{-1} z_h^{-1/2} \tilde{L}^{1/4}}$$  \hspace{1cm} (6.19)$$

### 6.3.2 Jet Breakout Conditions

#### 6.3.2.1 from Stellar Surface ($\tilde{L}_* \gtrsim 1$)

For sufficiently large luminosities, the jet burrows out of the stellar progenitor before the SN blast-wave has any significant effect on the outer layers of the star (i.e. $t_{\text{bo}} \ll t_*$). This is the usual scenario considered in the literature (e.g. Bromberg et al. 2011), where the jet can be treated as propagating within a hydrostatic stellar environment described by equation (6.8). Equating $R_*$ with $\zeta z v_h(t_{\text{bo}})t_{\text{bo}}$ yields an estimate of the jet breakout time in this limit,

$$t_{\text{bo}} = \zeta_\rho \zeta_\varepsilon \zeta_L^{-1/3} \tilde{L}_*^{-1/2} t_* \approx 0.19 \tilde{L}_*^{-1/2} t_*$$  \hspace{1cm} (6.20)$$

$$\simeq 7.6 \times 10^5 \left( \frac{L_j}{10^{50} \text{ erg s}^{-1}} \right)^{-1/3} \left( \frac{\gamma_j}{2} \right)^{-4/3} \left( \frac{M_*}{5M_\odot} \right)^{1/3} \left( \frac{R_*}{10^{11} \text{ cm}} \right)^{2/3}$$

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where $\zeta_\rho, \zeta_z, \zeta_L$ are calculated using equations (6.14, H12, H13) with the understanding that “$\delta = w$ in the considered case (density described by equation 6.8). Equation (6.20) is therefore applicable in the regime $\tilde{L}_* \gg 1$, characteristic of the jet luminosities of normal long duration GRB (equation 6.16).

The jet fails to break out in this regime if the break-out time $t_{bo}$ exceeds the engine duration $t_e$. Combining equations (6.20) and (6.1), successful breakout requires a minimum engine duration of

$$t_e \gtrsim 2.9 \text{s} \left( \frac{f_j E_e}{10^{52} \text{erg}} \right)^{-1/2} \left( \frac{\gamma_j}{2} \right)^{-2} \left( \frac{M_*}{5M_\odot} \right)^{1/2} \left( \frac{R_*}{10^{11} \text{cm}} \right).$$

(6.21)

When $\tilde{L}_* \sim 1$ the jet escape timescale is comparable to the time required for the SN shock to reach the stellar surface, in which case $t_{bo}$ differs from equation (6.20). In the marginal case defining the boundary between jet-breakout from the original stellar surface $R_*$ versus from an expanding SN envelope, i.e. $t_{bo} = t_*$, the jet can be considered as propagating entirely within the density profile (6.11). This marginal case occurs for a critical jet luminosity$^5$

$$\tilde{L}_*(t_{bo} = t_*) = \zeta_\rho^{2/3} \zeta_z^{-2} \zeta_L^{-2/3} \simeq 0.37,$$

(6.22)

or, by virtue of equation (6.16), $L_j(t_{bo} = t_*) \simeq 3 \times 10^{47} \text{erg s}^{-1}$.

Finally, in the weak jet regime ($\tilde{L}_* \ll 1$) relevant to long-lived engines capable of powering SLSNe, the jet propagates predominantly within the expanding SN ejecta described by equation (6.12). We focus on this regime for the remainder of this section.

$^5$Note that the numerical values of $\zeta_\rho$, $\zeta_z$ and $\zeta_L$ differ slightly between equations (6.20) and (6.22). In the latter case, these constants are calculated from equations (6.14, H12, H13) with the understanding that $\delta = -1, \beta = 8/3$, as implied by equation (6.11).
We consider two criteria for the jet to successfully escape from the expanding SN ejecta. First, we require that the jet head overtakes the expanding ejecta \( z_h(t_{bo}) \gtrsim v_{ej} t_{bo} \) at breakout, which is essentially equivalent to the requirement that the head velocity exceed the ejecta velocity \( v_h \gtrsim v_{ej} \) \cite{QuataertKasen2012}. Second, the jet must also satisfy the kink-stability criterion \( \Lambda \gtrsim 1 \) (equation 6.18). Combining these criteria, successful jet break-out occurs if both of the following conditions on its luminosity are satisfied,

\[
v_h \gtrsim \frac{v_{ej}}{\zeta z} \Rightarrow \tilde{L} \gtrsim \tilde{L}_{\text{crit}} = \left( \frac{v_{ej}}{\zeta z c} \right)^2 \sim 10^{-3} \left( \frac{v_{ej}}{10^9 \text{ cm s}^{-1}} \right)^2 \quad (6.23)
\]

\[
\Lambda \gtrsim 1 \Rightarrow \tilde{L} \gtrsim \tilde{L}_{\text{crit}} = \left( \frac{\gamma_j^{1/2} \tilde{L}}{20 \sqrt{\pi \eta}} \right)^4 \sim 10^{-4} \left( \frac{\gamma_j}{2} \right)^4 \eta^{-4}, \quad (6.24)
\]

or, in terms of the physical jet luminosity (equation 6.15),

\[
L_j(t) \gtrsim \zeta^2 \zeta^2 \gamma_j^{-4} c^3 \tilde{L}_{\text{crit}}^3 M_{ej} v_{ej} t, \quad (6.25)
\]

where we have substituted equation (6.12) for the ejecta density, and evaluated \( \rho z^2 \) at the ejecta front \( (z = v_{ej} t) \) where it attains its maximal value, and the breakout condition is hardest to satisfy.

For typical parameters, the more stringent break-out threshold \( \tilde{L}_{\text{crit}} \) is set by the dynamical condition (6.23) on the head velocity instead of kink instability. Also note that our assumption at the beginning of this section that the jet propagates through the star at non-relativistic velocities in the collimated regime is also justified because \( \tilde{L}_{\text{crit}} \ll 1, \gamma_j^{4/3} \) are
satisfied for all reasonable parameters.

In both the magnetar and fall-back scenarios the central engine power (equation 6.1) decays at times \( t \gg t_e \) as \( L_j \sim t^{-\ell} \) with \( \ell > 1 \). In such cases the LHS of equation (6.25) decreases faster with time than the RHS, indicating that successful jet break-out can only occur prior to the engine shut-off time \( t_e \). More precisely, condition (6.25) must be satisfied before the critical time \( t_e/(\ell - 1) \) at which \( \partial \ln L_j/\partial \ln t = -1 \). By substituting equation (6.1) at this critical time in equation (6.25) and expressing the jet energy in terms of the total engine energy, \( E_j = f_j E_e/2 \) (the factor of two accounts for the bipolar nature of the jet), this breakout condition may be recast entirely as a constraint on the total engine energy \( E_e \) as follows

\[
E_e \gtrsim E_{e,\text{min}} = 2f_j^{-1}f_\ell \xi_0 \xi_0^{-1} \gamma_j^{-4} \tilde{L}_{\text{crit}}^{3/2} \frac{M_{ej} c^3}{v_{ej}} \approx 0.195 E_{\text{sn}} \left( \frac{\gamma_j}{2} \right)^{-4} f_j^{-1}, \tag{6.26}
\]

where \( f_\ell \equiv \ell^\ell (\ell - 1)^{(1-\ell)} \approx 4 \) and in the second line we have used the threshold critical luminosity \( \tilde{L}_{\text{crit}} \) from equation (6.23) along with equation (6.13). In magnetar models, this lower limit on the engine energy implies a maximum birth spin-period for successful jet breakout (equation 6.2) of

\[
P_0 \lesssim 11.3 \text{ ms} \left( \frac{E_{\text{sn}}}{10^{51} \text{ erg}} \right)^{-1/2} \left( \frac{\gamma_j}{2} \right)^2 f_j^{1/2}. \tag{6.27}
\]

For engine energies \( E_e > E_{e,\text{min}} \) exceeding this minimum value, equation (6.25) yields a
break-out time of

\[
t_{bo} = \zeta_{\rho} \zeta_{z}^{-1} \zeta_{\nu}^{-3} \zeta_{j}^{-4} E_{\text{sn}} / L_{j}
\]

\[
\simeq 2.4 \times 10^3 \text{s} \left( \frac{\gamma_{j}}{2} \right) ^{-4} \left( \frac{E_{\text{sn}}}{10^{51} \text{erg}} \right) \left( \frac{L_{j}}{10^{46} \text{erg s}^{-1}} \right) ^{-1}.
\]  \hspace{1cm} (6.28)

Equation (6.28) is identical in form to the timescale for shock break-out from spherical pulsar wind nebulae derived by [Chevalier & Fransson (1992)](see also Chevalier 2005), differing only by numerical constants \( \propto \gamma_{j}^{-4} \).

### 6.4 Off-axis Jet Signature

The results of \*§6.3\* illustrate that jets with relatively low luminosities, of the same order of magnitude as the engine luminosities needed to power SLSNe, may successfully escape their SN ejecta on timescales comparable or less than the engine lifetime (equation 6.28). This is consistent with the discovery of GRB 111209A in association with the luminous SN 2011kl [Greiner et al. (2015)](Greiner et al. 2015).

In this section, we turn to the next natural question — what are the observable signatures of such jets? We focus on the most common case in which the jet axis is not aligned with the observer’s line of sight, precluding an associated GRB. In §6.4.1 we explore cocoon break-out emission, which can give rise to short-duration (\( \sim \)hr long) UV transients. We then propose a novel model whereby radiation from a thermally-driven wind launched off the jet-ejecta interface can produce an early (\( \sim \)several day) maxima in SLSNe light-curves (6.4.2). Finally, in 6.4.3 we examine radio afterglow constraints on observed SLSNe.
6.4.1 Cocoon Breakout Emission

We consider first the signature of the hot cocoon which encases the jet after it breaks out of the stellar surface. As discussed by Nakar & Piran (2016), the most promising cocoon signature is that arising from the isotropic non-relativistic shocked-star component of the cocoon (as opposed to the dilute, relativistic shocked-jet component). This component expands with an average bulk velocity of \( v_c \sim \sqrt{P_c/\bar{\rho}} \approx \gamma_j^{-1} v_h \), and in the stationary-stellar model produces emission from the cooling envelope (Nakar & Piran 2016). By contrast, in the case of breakout from an expanding SN-envelope considered here, the jet overtakes the ejecta only once the head velocity reaches \( v_h \sim v_{ej} \) (condition 6.23). This implies that the shocked-star component of the cocoon (which may be more appropriately named shocked SN-ejecta in this case) will be expanding at velocities \( < v_{ej} \) and therefore cannot itself breakout ahead of the expanding ejecta. For this reason, we do not expect a signature from the bulk shocked-star component of the cocoon in the late-breakout scenario, unless potentially in rare cases when the jet axis aligns with the observer’s line of sight.

An alternative isotropic signature may arise if some fraction of the shocked-jet cocoon component attains trans-relativistic (as opposed to exclusively ultra-relativistic) velocities, thereby managing to overtake the surrounding SN-ejecta while also producing emission that is not highly relativistically beamed. Following the notation of Nakar & Piran (2016), we assume that a fraction \( f_{\Gamma,1} \sim 0.1 \) of the cocoon’s energy is deposited into a trans-relativistic component expanding at proper-velocities \( \Gamma \beta \sim 1 \), as appropriate for the case of significant, but not complete mixing between the shocked-star and shocked-jet. The main free parameter of the model is the expansion velocity, \( \beta_{c,j}c \), the value of which we take fiducially to be
\[ \beta_{c,j} = 0.7. \]

The cocoon’s thermal energy at breakout, \( E_c \approx 2 L_j t_{bo} \), implies a trans-relativistic cocoon component mass

\[
M_{c,j} \approx 2 f_{\Gamma_{\beta,1}} L_j t_{bo} / (\beta_{c,j} c)^2,
\]

of order \( \sim 10^{-5} M_\odot \) for typical parameters.

We now consider two cases, depending on the shock break-out time \( t_{bo} \). For relatively short break-out times \( t_{bo} \ll 1 \) d, the optical depth of the cocoon after one initial expansion timescale (over which the cocoon wraps spherically around the SN ejecta) exceeds the value \( \sim \beta_{c,j}^{-1} \). In this case the initial photon diffusion time exceeds the expansion time, and so the bulk of the radiation is initially trapped and the observed luminosity will peak only after further radial expansion, at times \( t \gg t_{bo} \). This situation is well described by the standard cooling envelope emission model (e.g. Nakar & Piran 2016), resulting in emission for a characteristic duration of

\[
t_{pk} = \sqrt{\frac{3 \kappa M_{c,j}}{4 \pi c^2 \beta_{c,j}}} \approx 9.2 \times 10^2 \text{s} \left( \frac{E_{sn}}{10^{51} \text{erg}} \right)^{1/2} \left( \frac{\gamma_j}{2} \right)^{-2} \times \left( \frac{f_{\Gamma_{\beta,1}}}{0.1} \right)^{1/2} \left( \frac{\beta_{c,j}}{0.7} \right)^{-3/2},
\]

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with a peak (bolometric) luminosity of

\[
L_{pk} \approx \frac{E_0 t_{exp,0}}{t_{pk}^2} \simeq 6.5 \times 10^{43} \text{ erg s}^{-1} \left( \frac{L_j}{10^{46} \text{ erg s}^{-1}} \right)^{-1} \times \left( \frac{E_{sn}}{10^{51} \text{ erg}} \right)^{-14/3} \left( \frac{v_{ej}}{10^9 \text{ cm s}^{-1}} \right)^{1/3} \left( \frac{f_{\Gamma\beta,1}}{0.1} \right)^{1/3} \left( \frac{\beta_{c,j}}{0.7} \right)^2.
\]

Here \( E_0 = (V_{c,j}/V_0)^{1/3} E_{c,j} \approx (f_{\Gamma\beta}\gamma_j^{-2}/28)^{1/3} E_{c,j} \) is the thermal energy of the cocoon following one expansion time \( t_{exp,0} = v_{ej} t_{bo}/\beta_{c,j} c \) after breakout, and \( V_{c,j} = f_{\Gamma\beta,1} V_c \) is the volume required by pressure equilibrium inside the cocoon (which also implies that \( \theta_{c,j} = \theta_{c,j} f_{\Gamma\beta,1} \approx \gamma_j^{-1} f_{\Gamma\beta,1} \)).

In the opposite limit of long breakout times (\( t_{bo} \gtrsim 1 \text{ d} \)), the shocked ejecta becomes transparent to thermal radiation prior to significant radial expansion. The standard cooling envelope model does not apply because the thermal energy is radiated before the shocked ejecta has time to expand quasi-spherically around the SN ejecta. In particular, for jet luminosities in the range

\[
L_{\text{min}} < L_j < L_{\text{min}} \times \theta_{c,j}^{-1}(1 - \beta_{c,j})^{-1/2}
\]

\[
\simeq 11.6L_{\text{min}} \left( \frac{\gamma_j}{2} \right)^{1/2} \left( \frac{f_{\Gamma\beta,1}}{0.1} \right)^{-1/2} \left( \frac{1 - \beta_{c,j}}{0.3} \right)^{-1/2}
\]

where

\[
L_{\text{min}} \simeq 6.7 \times 10^{43} \text{ erg s}^{-1} \left( \frac{v_{ej}}{10^9 \text{ cm s}^{-1}} \right) \left( \frac{E_{sn}}{10^{51} \text{ erg}} \right)^{1/2} \times \left( \frac{\gamma_j}{2} \right)^{3} \left( \frac{\kappa}{0.2 \text{ cm}^2 \text{g}^{-1}} \right) \left( \frac{\beta_{c,j} (1 - \beta_{c,j})}{0.21} \right)^{1/2}
\]
the expansion timescale at breakout — while longer than the timescale for radiative diffusion in the azimuthal direction — is still shorter than the initial (pre-expansion) radial diffusion timescale. Condition \( (6.32) \) is equivalent to a range of break-out times \( 0.4 \, d < t_{bo} < 4.2 \, d \) according to equation \( (6.28) \), somewhat longer than those expected for SLSNe engines.

In this timescale hierarchy, radiation leaks azimuthally as the cocoon emerges from the ejecta. The light-curve duration and luminosity are approximately given by

\[
t_{pk} \approx t_{\exp,0} = \frac{v_{ej} t_{bo}}{\beta_{c,j} c} \approx 2 \times 10^3 - 2 \times 10^4 \, \text{s},
\]

\[
L_{pk} \approx \frac{E_{c,j}}{t_{exp,0}} \approx f_{\Gamma,1} \frac{\beta_{c,j} L_j}{v_{ej}} \approx 10^{44} - 2 \times 10^{45} \, \text{erg s}^{-1}.
\]

In summary, the most promising source of isotropic cocoon emission originates from the part of the shocked-jet material that attains transrelativistic velocities following jet break-out. Thermal energy released as this matter expands from the jet head may power a UV flare, which lasts a few hours starting after jet break-out (which itself is typically delayed by hours or more after the explosion) and reaches a peak luminosity of \( \gtrsim 10^{44} \, \text{erg s}^{-1} \).

### 6.4.2 Thermal Wind from Jet-Ejecta Interface

While a potentially promising target for future high cadence surveys like the Zwicky Transient Facility (ZTF; Bellm 2014) and ULTRASAT (Ganot et al. 2016), the cocoon break-out emission described in the previous section cannot explain the early maxima observed in SLSNe light curves (Leloudas et al. 2012; Nicholl et al. 2015a; Nicholl & Smartt 2016; Smith et al. 2016). The latter instead occur over a duration of several days, a timescale which is
suggestively comparable to the lifetime $t_e$ of the engine needed to power the peak of the SLSNe at later times.

One explanation for this coincidence is that the early emission is powered by a spherical shock break-out through the ejecta, driven by the pressure of the pulsar wind nebula inflated by the magnetar wind \cite{Kasen2016, Chen2016, Suzuki2016}. However, \cite{Kasen2016} found that it was challenging to make this emission component stick out above the rising SN light curve unless the thermalization efficiency of the engine power is suppressed at early times. Here we outline a new mechanism for producing early-time emission from an off-axis jet, also acting on a timescale comparable to the engine duration $t_e$.

Even after the jet has breached the stellar surface, and once a steady outflow of relativistic matter has been established, the jet should continue to transfer a small fraction of its kinetic energy and momentum to the confining ejecta walls. Common sense dictates that the process by which a jet escapes the star will not be entirely “clean”, a fact which should have observable consequence for an off-axis observer. The process by which this “friction” occurs is left unspecified, but it could plausibly be related to intrinsic variability of the central engine \cite{Morsony2010}, conical recollimation shocks between the jet and confining (evacuated) funnel \cite{BarniolDuran2016}, Kelvin-Helmholtz instabilities at the jet-ejecta interface (or propagation instabilities in general; e.g. \cite{Lazzati2011}, or some other unspecified process. High resolution numerical hydrodynamical simulations, focusing on this shearing interface, are needed to quantify the efficiency of this energy deposition and the mass entrained by this process.
To make progress, in the following we simply assume the jet deposits heat into the ejecta walls at a rate of \( \dot{E}_w = 2 \epsilon L_j \) (where \( \epsilon \ll 1 \)), driving mass-loss at a rate \( \dot{M}_w \). We find it convenient to express the mass and energy loss rates in terms of the terminal wind velocity (achieved after this heat is reconverted into kinetic energy via adiabatic expansion) according to \( v_w = (2 \dot{E}_w / \dot{M}_w)^{1/2} \) because the jet interface-driven wind scenario is only meaningful across a small range of parameters, \( v_{ej} \lesssim v_w < c \).

The characteristic rise timescale for the light curve produced by this heated wind is again set by radiative diffusion,

\[
t_{pk} = \frac{3 \kappa \dot{M}_w}{4 \pi c v_w} = \frac{3 \kappa \epsilon L_j}{\pi c v_w^3} \approx 1.4 \text{ d} \left( \frac{\kappa}{0.2 \text{ cm}^2 \text{ g}^{-1}} \right) \left( \frac{2 \epsilon L_j}{10^{45} \text{ erg s}^{-1}} \right) \left( \frac{v_w}{3 \times 10^9 \text{ cm s}^{-1}} \right)^{-3}.
\]

After this time, the \textit{bolometric} luminosity will track the engine (jet) power,

\[
L(t_{bo} + t_{pk} \lesssim t) \approx \dot{E}_w(t) = 2 \epsilon L_j(t),
\]

consistent with the standard \textbf{Arnett} (1979) rule.

Across the range of radii \( v_{ej} t \ll r \ll R_{bo} + v_w(t - t_{bo}) \), the wind assumes a secularly-evolving ‘steady-state’ density profile where \( \rho_w(t) \approx \dot{M}_w(t) / 4 \pi \nu_w r^2 \), and the radius of the
photosphere is approximately given by

\[ R_{\text{eff}} = \frac{\kappa_{\text{es}} M_w(t)}{4\pi v_w} = \frac{\kappa_{\text{es}} \epsilon L_j(t)}{\pi v_w^3} \]

\[ \simeq 1.2 \times 10^{15} \text{ cm} \left( \frac{2\epsilon L_j(t)}{10^{45} \text{ erg s}^{-1}} \right) \left( \frac{v_w}{3 \times 10^9 \text{ cm s}^{-1}} \right)^{-3}, \]  

\[ \text{(6.38)} \]

where we have adopted \( \kappa_{\text{es}} = 0.2 \text{ cm}^2 \text{ g}^{-1} \) as the electron scattering opacity. The effective temperature of the emission is given by

\[ T_{\text{eff}} \equiv \left( \frac{L}{4\pi \sigma R_{\text{eff}}^2} \right)^{1/4} = \left( \frac{\pi v_w^6}{2\sigma \kappa^2 \epsilon L_j} \right)^{1/4} \]

\[ \simeq 3.2 \times 10^5 \text{ K} \left( \frac{2\epsilon L_j(t)}{10^{45} \text{ erg s}^{-1}} \right)^{-1/4} \left( \frac{v_w}{3 \times 10^9 \text{ cm s}^{-1}} \right)^{3/2}, \]

\[ \text{(6.39)} \]

where we have expressed our results in terms of \( 2\epsilon L_j(t) \), the peak bolometric wind luminosity (equation 6.37).

The high temperatures implied by equation (6.39) show that the interface wind emission is typically observed on the Rayleigh-Jeans tail. The optical-band luminosity in this regime scales as \( \propto L_{\text{bol}} T_{\text{eff}}^{-3} \propto L_{\text{bol}}^{1/4} R_{\text{eff}}^{3/2} \), and therefore depends sensitively on emission temperature.

As the engine luminosity decays, the radius of the expanding ejecta will overtake the photosphere of the interface wind. Once \( v_{ej} t \gtrsim R_{\text{eff}}(t)/2 \), the optically-thick spherical wind model described above breaks down, and the wind radiates immediately after emerging from the ejecta. The effective emitting region shrinks to the (one-sided) ‘polar cap’, \( R_{\text{eff}} \sim 0.5\theta_j v_{ej} t \), causing an abrupt drop in the optical-band luminosity, primarily due to the rapidly rising temperature of the wind emission.\(^6\)

\(^6\)Note that once the wind becomes optically thin, the emission reaching a typical off-axis observer will
Figure 6.4 Model fit to the r-band light curve (solid black line) of the double peaked SLSN LSQ14bdq (red points). The main SN peak is powered by radiative diffusion through the ejecta, as in the standard engine-powered supernova model. The early maximum is instead thermal emission powered by a hypothesized wind driven from the interface between the off-axis relativistic jet (which has successfully escaped from the star) and the supernova ejecta ($\S$ 6.4.2). The best-fit model parameters are: $E_e = 7.4 \times 10^{52}$ erg, $t_e = 37.2$ day, $M_{ej} = 5.3 M_\odot$, $E_{sn} = 3.3 \times 10^{50}$ erg, $f_j = 0.55$, $\epsilon = 0.14$, and $v_w = 3.2 \times 10^9$ cm s$^{-1}$.

Figure 6.4 shows a fit of our engine-powered SN + interface wind model to the r-band light-curve of the double-peaked SLSN LSQ14bdq [Nicholl et al. 2015a]. The model can reproduce both the early bump and main SN peak within the same framework, for a reasonable set of parameters as marked in the caption. The numerical model from which the fit is derived is calculated by integrating the two-zone SN+wind equations while self-consistently evolving $R(t)$, $E(t)$ for each component, as described in Appendix I. The effective photospheric radius of each component is tracked at each timestep, from which the observed optical-band luminosities are then calculated assuming thermal emission. In particular, the SN photosphere is calculated by integrating the density profile equation (6.12) and assuming originate from only one side of the bi-polar jets; therefore the bolometric luminosity in this regime is a factor of two smaller than implied by equation (6.37).
a constant opacity, resulting in

\[
R_{\text{eff,sn}}(t) = v_{ej} t \left\{ \begin{array}{l}
\exp \left[ -\frac{v_{ej}}{c} \left( \frac{t}{t_{pk,sn}} \right)^2 \right] ; \delta = 1 \\
\left[ 1 + \frac{(\delta-1)v_{ej}}{c} \left( \frac{t}{t_{pk,sn}} \right)^2 \right]^{-1/(\delta-1)} ; \delta \neq 1
\end{array} \right.
\]  

(6.40)

where \( t_{pk,sn} = \sqrt{\zeta \rho \kappa M / v_{ej} c} \). We note that the simple estimate above neglects temperature and ionization effects on the ejecta opacity, which may become increasingly important at late times \( t \gg t_{pk,sn} \).

6.4.3 Orphan Radio Afterglow — the Case of SN2015bn

Our finding that energetic off-axis jets may commonly accompany SLSNe also predicts radio afterglows from these events produced by the interaction between the relativistic jet and the surrounding external medium. Though not as bright at early times as the radio emission for an on-axis observer, off-axis viewers should nevertheless observe radio emission once the jet decelerates to mildly relativistic speeds, allowing the observer to enter the causal emission region (a so-called “orphan afterglow”, because the prompt jet emission was relativistically beamed away from the observer line of sight).

Relatively few constraining radio follow-up observations have been published of SLSNe \cite{Chomiuk_2011}. \cite{Nicholl_2016a} recently obtained deep radio limits on the nearby SN2015bn, which were claimed to rule out a ‘healthy’ jet coincident with this event. Here we re-examine these radio non-detection constraints in greater detail by conducting a parameter survey over the two primary afterglow parameters: the injected jet energy \( f_j E_o \) and the ambient density. The latter is parameterized by \( n_0 \) for the case of a constant density.
interstellar-medium (ISM) surrounding, and the wind mass-loss parameter $A_*$ for the case of a wind environment,

$$\rho_{\text{CSM}}(r) = \begin{cases} 
n_0 m_p & \text{ISM} \\
5 \times 10^{11} \text{g cm}^{-1} A_* r^{-2} & \text{Wind} 
\end{cases} \quad (6.41)$$

where $A_* = 1$ corresponds to a progenitor stellar wind of velocity $10^3 \text{ cm s}^{-1}$ and mass loss rate $10^{-5} \mathcal{M}_\odot \text{ yr}^{-1}$.

We adopt a semi-analytic afterglow model. The blast-wave dynamics are calculated following Oren et al. (2004) and assuming azimuthal expansion of the jet at the local sound speed (see also Huang et al. 2000), while the emitted synchrotron radiation is calculated following Sari et al. (1998b). We neglect self-absorption, which is irrelevant at these late epochs, and adopt conservative fiducial values of $\theta_j = \epsilon_e = \epsilon_B = 0.1$, $p = 2.5$ for the jet half-opening angle and microphysical parameters. Smaller values of $\theta_j$, $\epsilon_e$ or $\epsilon_B$ would only reduce the model-predicted flux and thus weaken the non-detection constraints. Following a procedure similar to Soderberg et al. (2006), we survey the jet-energy – ambient-density parameter space and locate ‘allowed’ regions, where the model-predicted 22 GHz (7.4 GHz) radio flux at $t_{\text{obs}} = 330 \text{ day}$ falls below the $40 \mu\text{Jy} \ (75 \mu\text{Jy})$ upper limit constraints on SN2015bn. In practice, we find the 7.4 GHz upper limit unconstraining in light of the deeper 22 GHz limits (see also Nicholl et al. 2016a).

Fig. 6.5 shows our results for both constant density (dashed purple curves) and wind (solid black curves) environments. Heavy, medium, and lightly weighted curves show constraints for off-axis observers at viewing angles of $90^\circ$, $60^\circ$, and $30^\circ$ respectively. The regions
Figure 6.5 Constraints on jet energy and ambient density for SN2015bn (top) and SN2017egm (bottom), based on late time radio non-detections (Nicholl et al. 2016a; Bose et al. 2017). Solid black (dashed purple) curves separate allowed and forbidden regions of parameter-space for different angle off-axis observers, assuming a wind (ISM) ambient-density profile. Regions to the left (lower-density) of any given curve are permitted. The horizontal red curve shows the approximate condition on late-time jet breakout (equation 6.26). Also shown are best-fit parameters from detailed afterglow modelling of the ULGRB 111209A and GRB 130427A (Stratta et al. 2013; Perley et al. 2014). A reasonably powerful jet accompanying SN2015bn or SN2017egm cannot be ruled out for most off-axis observers if this event occurred in an environment similar to GRB 130427A and GRB 111209A.
to the right of each curve are ruled-out for a given observer. The horizontal red curve shows the jetted outflow’s required energy for successful breakout (equation 6.26). We therefore do not expect successful jets below this limit.

Even with this additional constraint, we find that there is a significant parameter-space where a ‘healthy’ jetted outflow coincident with SN2015bn could have gone undetected. While this would require a low ambient density, such densities are in fact inferred from observations of many LGRB afterglows, as indicated by the points in the plot. For example, best-fit models of the extraordinarily well-observed afterglow of GRB 130427A (Perley et al. 2014) yield $A_* \sim 10^{-3}$. Similarly, afterglow modeling of the ULGRB 11209A associated with SLSN 2011kl implies an ISM density of $n_0 = 0.07 \text{ cm}^{-3}$ (Stratta et al. 2013). We conclude that a relativistic jet accompanying SN2015bn would go undetected by most off-axis observers if it had occurred in a similar environment to these well-studied events.

A low-density external environment for SN2015bn is also consistent with X-ray upper limits recently obtained by Margutti et al. (2017a), which imply $A_* < 10^3$. Tighter constraints of $A_* < 2$ are obtained by these authors for the SLSN PTF12dam. More broadly, for the SLSN-I population as a whole, Margutti et al. (2017a) find that energetic jets are not constrained by current X-ray data for off-axis observers at significant viewing angles $\gtrsim 30^\circ$. We therefore conclude that current upper-limits cannot rule-out a reasonably powerful jet coincident with the SLSN 2015bn, as we might expect would have accompanied it.

Finally, we examine recent radio limits obtained for the nearby SN2017egm (Nicholl et al. 2017b; Bose et al. 2017). As shown in the bottom panel of Fig. 6.5, the 10 GHz upper limits of 23.3 $\mu$Jy at $t_{\text{obs}} \approx 41 \text{ day}$ (Bose et al. 2017) rule out a healthy jet within a
parameter space region similar to SN2015bn. An interesting point regarding SN2017egm is the relatively large magnetar spin period inferred from fitting the light-curve (Nicholl et al. 2017b). This implies a weak engine, $f_{th}E_c \sim 10^{51} \text{ erg}$, and points towards the possibility that a putative jet did not successfully break out of the confining SN ejecta in this case (unless $f_j \gg f_{th}$, c.f. equation 6.26).

6.5 Discussion and Implications

We have explored implications of the growing observational connection between long GRB and SLSNe, and their likely common association with the birth of energetic compact objects. Both phenomena can conveniently be interpreted within a single theoretical framework of engine-powered transients (equation 6.1; Metzger et al. 2015). We have focused explicitly on magnetar engines, for which we have proposed a novel mechanism of driving both jetted and thermal outflows (§6.2), but our subsequent results for weak-jet break-out (§6.3) and associated observational signatures (§6.4) can equally be applied to alternative engine models within this framework (e.g. the black-hole accretion, i.e. ‘fall-back’, scenario).

6.5.1 Landscape of Engine-Powered Transients

Figure 6.6 summarizes the landscape of such engine-powered transients, and illustrates the diversity of potential observational signatures which may be produced by the collapse of rapidly rotating stellar cores, including long GRB, ULGRB, broad-lined SNe-Ic, and SLSNe-I. Furthermore, it demonstrates the result of equation (6.26) — that SLSNe may generically be accompanied by successful jets.
Figure 6.6 Central-engine phase space, plotted for equal thermal and jetted energy fractions ($f_{th} = f_j = 0.5$, equivalent to $\alpha \simeq 0.4$ in the magnetar model; Fig. 6.2): the generic axes of engine timescale $t_e$ and energy $E_e$ are related to magnetar spin-period $P_0$ and dipole magnetic field $B_d$ in the magnetar scenario (equations 6.2,6.3), or average density of the stellar progenitor and fallback mass $E_e/\epsilon_{fb}c^2$ in accretion-powered model. The blue-shaded region bounds the range of plausible magnetar parameters ($E_e \lesssim 10^{55}$ erg, $B_d \lesssim 10^{16}$ G) and light brown shaded regions depict several fallback progenitor model parameters. Black contours show the peak SN luminosity powered by the central engine, while black points show the best-fit parameters of a population of observed Type I SLSNe. The solid red curve separates between parameter-space regions where a self-collimated jet manages or fails to break-out of the entraining stellar matter (equations 6.21,6.26). Above the dotted red curve (equation 6.22), such break-out occurs within the expanding SN ejecta. The figure illustrates the diversity of transients which can arise from the collapse of rapidly rotating stellar cores, and shows that successful jets may commonly accompany SLSNe. Adopted model parameters are $\gamma_j = 2$, $R_* = 10^{11}$ cm, $E_{sn} = 10^{51}$ erg, $M_{ej} = M_* = 5M_\odot$, and for the SN luminosity contours we adopt ejecta opacity $\kappa = 0.2$ cm$^2$ g$^{-1}$ and central engine power-law decay rate $\ell = 2$ (equation 6.1). See text for further details.
The primary axes of Fig. 6.6 are the total energy $E_e$ and characteristic lifetime $t_e$ of the engine. In this example the energy is partitioned equally between the jetted and thermal components, i.e. $f_{th} = 0.5$, corresponding to a dipole inclination angle $\alpha \approx 0.4$ in the magnetar model (Fig. 6.2). In the magnetar model, the values of $E_e$ and $t_e$ are related to the dipole field $B_d$ and initial spin period $P_0$ through equations (6.2,6.3), which we show as additional axes floating above the top of the figure and which cross the diagram as inclined lines. The blue shaded region highlights the rather generous parameter space spanned by realistic constraints on the magnetar engine based on the maximum rotational energy $E_e \lesssim 10^{53}$ erg (corresponding to centrifugal break-up for a massive neutron-star; Metzger et al. 2015) and maximum realistic magnetic field strength $B_d \lesssim 10^{16}$ G. In the black hole accretion model, the engine energy and lifetime are instead related, respectively, to the total accreted mass ($\sim E_e/\epsilon_{fb}c^2$) and mean density of the stellar core, the latter being shown in a separate axis to the right of the figure. Light brown regions denote the space of fall-back accretion models associated with different stellar progenitor types (iron core, Wolf-Rayet outer layers, and Red/Blue Supergiant envelopes, as labeled; Sukhbold & Woosley 2014; Sukhbold et al. 2016), where a span in $\epsilon_{fb} = 10^{-3} - 10^{-1}$ is taken.

Black annotations in Fig. 6.6 relate to the SN emission. Black curves depict contours of constant peak SN luminosity $L_{e, pk}$ with black points showing the engine properties required to explain individual observed SLSNe within the magnetar model (Nicholl et al. 2017). The SLSNe populating this region require a central engine which deposits $\sim 10^{52}$ erg of thermal energy over an engine lifetime of typically several days, corresponding to $P_0 \gtrsim 1$ ms and $B_d \sim 10^{14}$ G in the magnetar scenario. A black-and-red point shows the best fit central-engine

\footnote{We consider only engine-powered emission, neglecting any contribution from $^{56}$Ni.}
model of Metzger et al. (2015) to SN 2011kl and the associated ULGRB 111209A (Greiner et al. 2015), which straddles the region between typical SLSNe and observed ULGRB and thus represents a potential hybrid or transitional event (see also Ioka et al. 2016; Gompertz & Fruchter 2017).

SNe which are instead powered predominantly by $^{56}$Ni occupy the region to the bottom left of the $L_{e,\text{pk}} = 10^{43}$ erg$\,\text{s}^{-1}$ contour. In this region, the engine duration is too short to appreciably contribute to the SN luminosity: the majority of the energy deposited by the engine suffers adiabatic degradation by the time the ejecta becomes transparent around the time of the SN peak. The engine can still enhance the kinetic energy of the ejecta in this case, as long as $f_{\text{th}} E_{e} \gtrsim E_{\text{sn}}$, where $E_{\text{sn}}$ is the initial kinetic energy of the explosion. Standard Ni-powered SNe Ic are therefore divided from their energetic broad-lined counterparts (‘hypernova’) at approximately $E_{e} \sim 10^{52}$ erg.$^{8}$

Thermal energy deposited by the magnetar inflates a hot bubble behind the SN ejecta, analogous to a pulsar wind nebula. Kasen et al. (2016) show that, for a sufficiently energetic engine, this hot bubble drives a shock wave through the outer layers of the SN ejecta, producing an early-time shock-break out signature (see Chen et al. 2016; Suzuki & Maeda 2016 for two-dimensional simulations of this process). The purple curve shows the combination of energy and lifetime above which this shock break-out emission is particularly pronounced, because it occurs while the engine is still near its peak activity (corresponding to quadrants 1 and 2 of Fig. 2 of Kasen et al. 2016). This break-out was proposed by Kasen et al. (2016) as an explanation for the early peak observed in the light curves of SLSNe (Leloudas et al. 2016).

$^{8}$However, note that kinetic energy is a challenging quantity to measure accurately from the supernova spectra if the ejecta are highly asymmetric (e.g., Dessart et al. 2017).
2012; Nicholl et al. 2015a; Nicholl & Smartt 2016; Smith et al. 2016), to which we have offered an alternative explanation (§6.4.2).

Red curves and annotations within Fig. 6.6 relate to the jetted component of the engine, and show the approximate regions of parameter space giving rise to classical GRB, ULGRB, and low-luminosity GRB (ℓℓGRB). A red curve separates the engine energy above which the relativistic jet successfully escapes the SN ejecta instead of being choked (equations 6.21, 6.26). This criterion is a non-trivial function of the engine properties because — in the case of a long engine duration — the ejecta has time to expand appreciably following the explosion (§6.3.2). Even relatively weak jets below a critical luminosity $\simeq 3 \times 10^{47}$ erg s$^{-1}$ (dotted red curve; equation 6.22), which could not escape the (effectively stationary) stellar progenitor in the case of a short-lived engine, are capable of escaping at late times as the ejecta dilutes following its expansion in the SN explosion.

Figure 6.6 therefore summarizes the break-out criterion derived in §6.3.2. Importantly, it illustrates that observed SLSNe inhabit a parameter-space region where they are expected to be accompanied by successful, albeit low-luminosity, jets. Although we have assumed $f_j = 0.5$ in plotting Fig. 6.6, this result is robust and holds for even modest jetted energy fractions of only several percent. We thus conclude that off-axis jets may be a common feature of SLSNe.

6.5.2 Jetted High-Energy Transients

The early break-out times of jets from SLSNe engines (within days of the explosion) implies that their signatures may be missed in most optically-selected events, which are generally
discovered at later times and are viewed off the jet axis.

However, if successful jets are commonly associated with SLSNe, one implication is the existence of a class of high energy transients with extremely long durations $T_{90} \sim 10^5 - 10^6$ s (e.g., Quataert & Kasen 2012, Woosley & Heger 2012) and associated bright optical counterparts (Metzger et al. 2015). Figure 6.7 shows our predictions for the duration and peak luminosity distribution of these SLSNe-associated events, which we have calculated using the engine properties (energy $E_e$, lifetime $t_e$, assuming a jet fraction $f_j = 0.5$) and volumetric rates of the observed sample of SLSNe (Quimby et al. 2013), and assuming a beaming fraction of $f_b = 1/50$, similar to those typical of long GRBs. The density is normalized with respect to the average volumetric event rate of such jetted events within $z \leq 1$ (following a calculation similar to equation 6.43), so that the integral over the distribution equals $\langle R_{jSLSN} \rangle \simeq 2.2 \text{ Gpc}^{-3} \text{yr}^{-1} (f_b^{-1}/50)^{-1}$.

Shown for comparison is the engine duration distribution of the observed long and ultra-long GRBs, adapted from Zhang et al. (2014), and the inferred LGRB luminosity function from Wanderman & Piran (2010) (similarly normalized with respect to the $z \leq 1$ GRB rate, $\langle R_{LGRB} \rangle \simeq 2.3 \text{ Gpc}^{-3} \text{yr}^{-1}$). Our estimates show that the predicted rate of jetted transients from SLSNe is comparable to the long GRB rate, though the distribution is bimodal. This deficit of intermediate luminosity and duration jets may be physical and related to a bimodal population of engine properties, or it may be the result of selection effects against detecting long-lived low-luminosity gamma-ray transients (Gendre et al. 2013, Levan et al. 2014) or of identifying engine-powered supernovae when their engines have shorter durations and they may not be classified as “superluminous”.
Figure 6.7 Distribution of duration $t_e$ (in seconds) and peak isotropic luminosity $L_{\text{iso}}$ (in erg s$^{-1}$) of jetted transients, normalized to their volumetric rates at redshift $z \lesssim 1$. Top Panel: Measured distribution of U/LGRB engine durations (Zhang et al. 2014; solid-black) and jetted-tidal disruption events (blue; scaled up by a factor of $\times 10$) compared to the predicted durations distribution of jets accompanying SLSNe (solid-red; based on the engine duration obtained by fitting the magnetar model to the SLSNe optical light curves). Dashed curves account for an assumed factor of two uncertainty in the event rates. Bottom Panel: Isotropic-equivalent luminosity function of long GRB (as derived by Wanderman & Piran 2010, accounting for detection bias; solid-black) and jetted tidal disruption events (blue; scaled up by a factor of $\times 10$) compared to the predicted distribution for jets accompanying SLSNe (solid-red). Note that luminosity decreases to the right, facilitating an easier comparison with the duration distribution.
The properties of the SLSNe-associated jets we predict are similar to those of the “jetted tidal disruption events” (Bloom et al. 2011; Levan et al. 2011; Burrows et al. 2011; Zauderer 2011), of which there are currently only three examples (Cenko et al. 2012; Brown et al. 2015). A lower limit on their rates can be crudely estimated by considering that Swift has observed 3 jetted-TDEs, which would have been detected up to redshift \( z \sim 1 \), over a \( \sim 12 \) yr baseline. Therefore \( \langle R_{\text{JTDE}} \rangle \sim f_\Omega (3 \text{ events}/12 \text{ yr})/V(z \leq 1) \simeq 0.01 \text{ Gpc}^{-3} \text{ yr}^{-1} \), where \( f_\Omega \simeq 1/7 \) is Swift’s fractional all-sky field of view, and \( V \) the co-moving volume within redshift \( z \). These events are shown for comparison in Fig. 6.7 with blue colored lines.

Although a tidal disruption origin is favored for these high energy transients, in part due to the coincidence of the prototype Swift J1644+57 with the nucleus of its galaxy, this association is not air tight (the possibility of a chance coincidence with the nucleus is high, and the angular size of the host galaxies of the other events are too small to tell), especially considering the lack of evidence for powerful jets in other tidal disruption flares (e.g. Bower et al. 2013; van Velzen et al. 2013; Generozov et al. 2017). This raises the possibility that some or all of these events may in fact be core collapse events (Quataert & Kasen 2012; Levan et al. 2014), possibly of the type described here. Although Swift J1644+57 was highly dust extincted, it did show evidence for a separate component of (possibly thermal) optical/IR emission (Levan et al. 2016), while Swift J2058+05 showed clear evidence for a very luminous \( \sim 10^{45} \text{ erg s}^{-1} \) thermal component (Cenko et al. 2012; Pasham et al. 2015), broadly consistent with the properties of SLSNe. The fact that the estimated rates of these transients is lower than those we predict in association with SLSNe could suggest the engine luminosities of the observed “jetted tidal disruption events” are on the high end of the distribution; that only
a small fraction of SLSNe produce successful jets; or that selection effects against detecting long-lived low-luminosity transients \cite{Gendre2013, Levan2014} again result in larger underestimates of the volumetric rates of these transients.

Selection effects are crucial in understanding why the fact that, as of yet, jetted events have not been directly observed in association with SLSNe, is not in tension with our suggestion that such jets may commonly accompany these SNe. A conservative upper bound on the fluence of such putative jetted-SLSNe is $S \lesssim 0.5E_e/4\pi D_L^2$, where $D_L$ is the luminosity distance to the source. \cite{Levan2014} show that the fluence sensitivity of \textit{Swift} is greatly reduced for long-duration transients. In particular, low-luminosity transients of $\sim$day timescale durations are not detectable within a single exposure, and would have to be integrated over several orbits by the \textit{Swift} BAT transient monitor. Using the BAT’s sensitivity (integrated variance) over 1-16 day periods \cite{Krimm2013}, we find an approximate $5\sigma$ threshold detectable fluence of $S_{\text{lim}} \sim 1.1 \times 10^{-4} \text{ erg cm}^{-2} (T_{90}/10^6 \text{ s})^{0.4}$. This implies that only events with

$$t_e \lesssim 9 \times 10^4 \text{ s} \left( \frac{E_e}{10^{52} \text{ erg}} \right)^{2.5} \left( \frac{D_L}{1 \text{ Gpc}} \right)^{-5}$$

(6.42)

would be observable by \textit{Swift}. Of the 35 SLSNe considered in our sample, only 3 meet this requirement (of which 2 are only marginally consistent with this conservative upper limit), yet if the beaming fraction of SLSNe jets is similar to that inferred for LGRBs, we expect only $\sim 1$ in 50 jetted events to be pointed in our direction. It is thus unsurprising that jets are not observed in either of the 3 SLSNe meeting equation (6.42).
6.5.3 UV Flash from Jet Break-Out

Beyond potential on-axis jet signatures, we have considered other sources of jet-powered emission, which may be visible also for off-axis viewers, i.e. in coincidence with most/all SLSNe. One source is UV emission due to the cocoon break-out ([6.4.1]), which is expected to last for a few hours and will reach peak luminosities of $\sim 10^{44} - 10^{45}$ erg s$^{-1}$, corresponding to an absolute magnitude $M \sim -21.3$ to $-23.8$. Cocoon break-out emission from SLSNe could be a promising source for future wide-field UV survey missions, such as ZTF ([Bellm 2014] the proposed ULTRASAT satellite (e.g., [Sagiv et al. 2014] Ganot et al. 2016).

ULTRASAT[9] will achieve a sensitivity of 21.5 AB magnitude in the 220-280 nm wavelength range for an integration time of 900 seconds. Its 210 deg$^2$ instantaneous field of view covers a fraction $f_\Omega \simeq 5 \times 10^{-3}$ of the sky. Assuming a detection threshold at twice the sensitivity, a source of magnitude $M \sim -21.3$ to $-23.8$ would be visible to $z_{\text{max}} \sim 0.4 - 1.2$, out to which the co-moving volume of the Universe is $V_{\text{max}} \sim 23 - 224$ Gpc$^3$. The rate of SLSNe at $z \approx 0.16$ is estimated to be $R_{\text{SLSN}} \approx 32^{+77}_{-26}$ Gpc$^{-3}$ yr$^{-1}$ ([Quimby et al. 2013]), which if increasing as the star formation rate $\propto (1 + z)^{3.28}$ ([Hopkins & Beacom 2006]) would increase to $R_{\text{SLSN}} \approx 75 - 338$ Gpc$^{-3}$ yr$^{-1}$ by $z_{\text{max}} \approx 0.4 - 1.2$. The number of SLSN jet break-outs detectable by ULTRASAT is therefore very roughly

$$\dot{N} = f_\Omega \int_0^{z_{\text{max}}} \left( \frac{R_{\text{SLSN}}(z)}{1 + z} \frac{dV}{dz} \right) dz \sim f_\Omega \frac{R_{\text{SLSN}}(z_{\text{max}}) V_{\text{max}}}{1 + z_{\text{max}}} \sim 5 - 114 \text{ yr}^{-1}. \quad (6.43)$$

6.6 Summary

This paper investigates the SLSN-GRB connection, exploring the powering-mechanism, break-out conditions, and observational signatures of engine-powered jets. Our main findings are the following.

1. Mis-alignment between rotation and magnetic axes of a millisecond magnetar provides a natural mechanism for dissipating a fraction $f_{\text{th}}$ of the magnetar’s spin-down luminosity and powering an energetic SN (Fig. 6.1). The remaining spin-down luminosity, $f_j = 1 - f_{\text{th}}$, can power an ordered, magnetically dominated jet (Fig. 6.2). Within this model, $f_{\text{th}}$ and $f_j$ are solely functions of the mis-alignment angle $\alpha$ (equation 6.6). Thus, observational measurements of both jet and SN energies for a common event can be used to infer $\alpha$ (equation 6.7).

2. Break-out of weak jets is studied in §6.3, regardless of the underlying mechanism by which they are powered (irrespective of the magnetar model described in §6.2). Jets below a threshold luminosity $\simeq 3 \times 10^{47} \text{erg s}^{-1}$ (equation 6.22) cannot break-out of the stellar progenitor before the SN shock-wave reaches the stellar surface (equation 6.10). This regime, which is of interest in the case of SLSNe engines (Fig. 6.6), requires separate treatment from canonical GRB jet models (§6.3.2.2).

3. We find that jet break-out in this regime is set by the condition $v_h \gtrsim v_{ej}$ (equation 6.23), for which the jet is also kink-stable (equation 6.24). This yields a simple condition on jet energy for successful break-out, $f_j E_c \gtrsim 0.19 E_{\text{sn}}$ (equation 6.26). In the magnetar scenario this is commensurate with a maximum birth spin-period of $\simeq 10 \text{ms}$ (equa-
tion 6.27). Successful jets break typically out of the expanding SN ejecta on timescales of hours following the explosion (equation 6.28).

4. Observational signatures of off-axis jets associated with SLSNe are explored in §6.4. Break-out of a transrelativistic ‘shocked-jet’ cocoon component can produce $\sim$ hr long $\sim 10^{44}$ erg s$^{-1}$ UV flares (§6.4.1). We estimate that $5 - 100$ such events may be detectable by ULTRASAT per year (equation 6.43). In contrast to jet break-out from a stationary stellar progenitor, we find that the Newtonian ‘shocked-star’ cocoon component cannot overtake the expanding SN ejecta, and is therefore unobservable.

5. We have proposed a novel signature of long-duration jets — emission from a thermal wind driven off the jet-ejecta interface (Fig. 6.1; §6.4.2). We hypothesize that some, hitherto unspecified, process dissipates jet-power at this interface, driving such a wind. The dissipation mechanism and resulting wind properties should be constrained by future high-resolution numerical studies. Wind emission lasts for the duration of jet activity ($\sim$ days for SLSNe engines) and can enhance optical SN emission, producing early light-curve maxima consistent with observed SLSNe (Fig. 6.4).

6. Our model generically predicts radio afterglow emission from off-axis SLSNe jets. Although radio follow-up observations have so far yielded only upper limit constraints, we show that these are consistent with expected $\gtrsim 30^\circ$ off-axis observer viewing angles (§6.4.3). We strongly encourage further study of the late-time radio properties of SLSNe, a result with potential implications also for the origin of fast radio bursts (Metzger et al. 2017b; Nicholl et al. 2017a).
7. Finally, we have illustrated the diversity of transients which can arise from the collapse of rapidly rotating stellar cores (Fig. 6.6), and explored statistical properties of the population of jetted transients (Fig. 6.7).
Chapter 7

Unveiling the Engines of Fast Radio Bursts, Super-Luminous Supernovae, and Gamma-Ray Bursts

7.1 Introduction

Neutron stars with exceptionally strong magnetic fields ("magnetars"; Duncan & Thompson 1992) are promising engines for astrophysical transients across a range of timescales and wavelengths. The magnetized relativistic winds from young magnetars, which are born rapidly spinning following core collapse supernovae (SNe) are candidates for powering long duration gamma-ray bursts (GRB; e.g., Usov 1992; Thompson et al. 2004; Metzger et al. 2011; Mösta et al. 2015b; Beniamini et al. 2017) and super-luminous supernovae (SLSNe; e.g., Maeda et al. 2007; Kasen & Bildsten 2010; Woosley 2010; Dessart et al. 2012; Metzger 2012).
Hydrogen-poor SLSNe are a rare subset of the terminal explosions of massive stars stripped of their outer hydrogen envelopes which exhibit peak optical luminosities exceeding those of other SNe by factors of $\gtrsim 10 - 100$ (Quimby et al. 2011; Chomiuk et al. 2011; Gal-Yam 2012; Inserra et al. 2013; Nicholl et al. 2014; Liu et al. 2017; De Cia et al. 2017; Lunnan et al. 2018; Quimby et al. 2018) and which occur preferentially in small and irregular low-metallicity host galaxies, with properties broadly similar to those of long GRB hosts (Lunnan et al. 2015; Chen et al. 2015; Perley et al. 2016c; Japelj et al. 2016; Schulze et al. 2018). The merger of neutron star binaries can also create massive magnetar remnants (e.g. Price & Rosswog 2006; Metzger et al. 2008b; Kiuchi et al. 2015), which are temporarily supported against gravitational collapse by their rapid rotation; such meta-stable objects could help shape the electromagnetic counterparts to these gravitational wave sources (e.g. Metzger et al. 2018b). Later in their evolution, magnetars can evolve to become sources of high energy radiation powered by dissipation of their enormous reservoirs of magnetic energy, which are observed as primarily Galactic sources of transient outbursts and giant flares (Thompson & Duncan 1995; see Kaspi & Beloborodov 2017 for a review).

A new window into magnetized compact objects was opened by the discovery of fast radio bursts (FRBs) — coherent pulses of radio emission lasting a few milliseconds that occur at an all-sky rate of $10^3 - 10^4$ per day above 1 Jy (Lorimer et al. 2007; Keane et al. 2012; Thornton et al. 2013; Spitler et al. 2014; Ravi et al. 2015; Petroff et al. 2016; Champion et al. 2016; Lawrence et al. 2017). FRBs are characterized by large dispersion measures $DM \approx 300 - 2000$ pc cm$^{-3}$, well above the contribution from propagation through the Milky Way or its halo and thus implicating an extragalactic origin. The cosmological distance of at least
one FRB was confirmed by the discovery of a repeating FRB 121102 (Spitler et al. 2014, 2016) and its subsequent localization (Chatterjee et al. 2017) to a dwarf star-forming galaxy at a redshift of $z = 0.1927$ (Tendulkar et al. 2017a). The unusual host galaxy properties are similar to those of long GRBs and SLSNe (Metzger, Berger, & Margalit 2017b), supporting a possible connection between FRBs and young magnetars (Popov & Postnov 2013; Lyubarsky 2014; Kulkarni et al. 2014; Katz 2016; Lu & Kumar 2016; Metzger et al. 2017b; Nicholl et al. 2017; Kumar et al. 2017; Lu & Kumar 2017).

One mechanism by which a young magnetar could power a burst of coherent radio emission is through the synchrotron maser instability in the plasma behind magnetized shocks (Gallant et al. 1992; Lyubarsky 2014; Waxman 2017). Such shocks could be produced by transient ejections from the magnetar which collide with the external medium at ultra-relativistic speeds. This medium could represent the baryon-rich wind of material accumulated from the succession of previous recent flares (Beloborodov 2017) or, on larger scales, with the hot nebula of magnetic fields and particles confined behind the expanding SN ejecta (Lyubarsky 2014). Other FRB emission mechanisms have been proposed that occur closer to the magnetar surface, such as antenna curvature emission within the magnetosphere (e.g. Kumar et al. 2017; Lu & Kumar 2017). Radio bursts from FRB 121102 have now been observed intermittently for over four years, with separations between bursts as short as seconds (Spitler et al. 2016; Michilli et al. 2018). Any magnetar responsible for this behavior must be significantly more active than the Galactic population, which are largely dormant (Kaspi & Beloborodov 2017).

Radio interferometric localization of FRB 1211012 (Marcote et al. 2017a) revealed a
luminous \( (\nu L_\nu \approx 10^{39} \text{ erg s}^{-1}) \) steady radio synchrotron source coincident to within \( \lesssim 0.8 \text{ pc} \) of the FRB location (Tendulkar et al. 2017a). This could be interpreted as a nascent “nebula” surrounding the magnetar, powered by its rotational (Metzger et al. 2017b; Kashiyma & Murase 2017; Omand et al. 2018) or magnetic energy (Beloborodov 2017). A plasma-dense environment surrounding FRB sources is supported also by the observed scattering tails following some FRB pulses (Thornton et al. 2013; Ravi et al. 2015; Luan & Goldreich 2014) and possible evidence for plasma lensing of the bursts by intervening screens of dense ionized material (Pen & Connor 2015; Cordes et al. 2017; though much of the latter could be the ISM of the host galaxy). A subsequent search for similar persistent radio sources discovered 11 candidates, which under the assumption that each source is active for \( \sim 100 \text{ yr} \) implies a birth rate \( \lesssim 500 \text{ Gpc}^{-3} \text{ yr}^{-1} \), consistent with the GRB and SLSNe rates (Ofek 2017).

Constraints can be placed on the age, \( t_{\text{age}} \), of the putative compact remnant responsible for FRB 1211012 (Metzger et al. 2017b). Upper limits on the size of the quiescent radio source relative to predictions for an expanding nebula place a rough upper limit of \( t_{\text{age}} \lesssim 100 \text{ yr} \). On the other hand, a lower limit of \( t_{\text{age}} \gtrsim 20 - 30 \text{ yr} \) follows from the requirement that the supernova ejecta not attenuate the FRB radiation via free-free absorption or overproduce the observed DM or its time derivative (Connor et al. 2016; Piro 2016; Metzger et al. 2017b; Bietenholz & Bartel 2017). The young inferred age may be connected to the repeater’s high activity as compared to Galactic magnetars (Beloborodov 2017), which are typically much older, \( t_{\text{age}} \sim 10^4 \text{ yr} \). Lu & Kumar (2017), Nicholl et al. (2017b), and Law et al. (2017) show that if all FRB sources repeat in a manner similar to FRB 121102, then the birth rate of FRB-producing magnetars is consistent with those of SLSNe and long GRBs, and
thus potentially with the subpopulation of magnetars born with particularly short rotation periods (high rotational energies).

Also supporting the existence of a dense electron-ion plasma surrounding FRB 121102 is its large rotation measure, \( \text{RM} \sim 10^5 \text{ rad m}^{-2} \) (Michilli et al. 2018; see also Masui et al. 2015). This RM value exceeds those of other known astrophysical sources, with the exception of the flaring magnetar SGR J1745-2900 located in the Galactic Center at a projected offset of only 0.1 pc from Sgr A* (Eatough et al. 2013). The magnetic field of the medium responsible for FRB 121102’s RM exceeds \(^1\) \( \sim 1 \) mG (Michilli et al. 2018). Though too high for the ISM of the host galaxy, the large field strength could instead be reasonably attributed to the same quiescent synchrotron nebula which is co-located with the bursting source. The RM was furthermore observed to decline by \( \sim 10\% \) over a 7 month interval (Michilli et al. 2018). This may suggest that a turbulent magnetized environment surrounds the burst, as in the Galactic Center. Alternatively, the decline may implicate secular evolution originating from the source being embedded in an expanding, diluting magnetized medium, either from the supernova shock wave interacting with circumstellar gas (Piro & Gaensler 2018) or the burst-powered synchrotron nebula (see §7.4.1).

Despite the growing circumstantial evidence tying young magnetars to a range of astrophysical transients (GRB, SLSNe, FRB, NS mergers), definitive proof for this connection remains elusive and alternative models remain viable. Long GRBs can be powered by fallback accretion onto a black hole of the ejecta of a massive star following a failed explosion (Woosley 1993; MacFadyen & Woosley 1999b). SLSNe could instead be powered through cir-

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\(^1\)This minimum average magnetic field strength is derived under the conservative assumption that the RM-producing medium also contributes all of the DM, once the Milky Way and intergalactic medium values have been subtracted off.
cumstellar interaction (Chevalier & Irwin 2011; Moriya et al. 2013) or by fall-back accretion from a radially-extended star (Quataert & Kasen 2012; Dexter & Kasen 2013). The high RM of FRB 121102 could indicate the bursting source just happens to be located in a magnetized galactic center environment close to an accreting massive black hole (e.g. Eatough et al. 2013) or that such a location is somehow essential to the emission process (Zhang 2017; Thompson 2017; Zhang 2018), rather than originating from the birth nebula of a young stellar-mass compact object.

One challenge in testing the magnetar model is our inability to directly view the central engine at early times due to the large absorbing column of the supernova or merger ejecta. Metzger et al. (2014) propose to search for the emergence of UV or X-ray radiation from the young magnetar nebula on timescales of years after explosion, once the ejecta become transparent to bound-free absorption (see also Kotera et al. 2013; Murase et al. 2015). Such transparency can occur gradually as the ejecta column density dilutes, or abruptly due to a sudden drop in opacity when a photo-ionization front driven by the nebula radiation reaches the ejecta surface (Metzger et al. 2014). Margutti et al. (2017c) invoked such an “ionization break-out” to explain the unusual UV re-brightening of the highly luminous optical transient ASASSN-15lh (Dong et al. 2016) observed a few months following the explosion; this explanation could in principle apply regardless of whether the event was a true SLSN (e.g. ejecta ionization by a central magnetar engine) or a tidal disruption event (stellar debris ionization by an accreting supermassive black hole; Leloudas et al. 2016; Krühler et al. 2018). While most SLSNe only show upper limits on their X-ray luminosity $L_X$ (Margutti et al. 2017a), SCP06F6 (Barbary et al. 2009) was detected with $L_X \sim 10^{44} - 10^{45}$
erg s$^{-1}$ roughly 70 days after the explosion \cite{Levan2013}. The slowly-evolving SLSN PTF12dam \cite{Nicholl2013} also showed detectable X-ray emission with \(L_X \approx 2 \times 10^{40}\) erg s$^{-1}$ \cite{Margutti2017a}, though this could originate from star formation in the host galaxy.

\cite{Kashiyama2016, Omand2018} proposed to search for the emergence of late-time radio synchrotron emission from the engine-powered nebula, once the ejecta becomes transparent to free-free and synchrotron self-absorption (a similar condition as that needed for FRB emission to escape). Searches for long-lived radio emission from magnetars have already been conducted for short GRBs, leading to non-detections \cite{Metzger2014, Horesh2016, Fong2016}. \cite{Metzger2017b} proposed to search for FRB emission from \(\gtrsim\) decade-old old SLSNe and long GRBs to directly connect these sources to the birth of young magnetars. \cite{Nicholl2017b} proposed the same idea for the magnetar remnants of binary neutron star mergers, a search conducted following the recent LIGO-discovered merger GW170817 \cite{Andreoni2017} which yielded non-detections.

Given the growing sample of FRBs with detections or upper limits on local contributions to their DM and RM (and in the case of repeating sources like FRB 121102, also of their time derivatives), as well as GRBs, SLSNe, and NS mergers with late-time X-ray and radio observations, it is essential to revisit predictions for the time-evolving properties of the supernova/merger ejecta and the observability of the flaring magnetar or magnetar-powered nebula inside. The ionization state of the ejecta, which is controlled by photo-ionization from the UV/X-ray flux of the central nebula \cite{Metzger2014, Metzger2017b} or by passage of the reverse shock as the ejecta interacts with the ISM \cite{Piro2016, Piro2018},
controls the escape of X-ray/radio emission from the central sources and determines the local contribution to DM and $d\text{DM}/dt$. Likewise, the magnetized nebula fed by the accumulation of past magnetar flares, provides both a steady synchrotron source and the dominant source of both RM and $d\text{RM}/dt$.

Despite the importance of the temperature and ionization structure of the ejecta on these observables, previous studies of the ejecta properties have been semi-analytic in approach (Metzger et al. 2014, 2017b). A more accurate treatment must account self-consistently for the ionization-recombination balance for all relevant atomic states for a realistic ejecta composition, including a self-consistent solution for the temperature structure of the ejecta. Here, we perform such a calculation using the photo-ionization code CLOUDY, applied at snapshots in time after the merger, which we then use to infer the evolution of DM, as well as the bound-free and free-free optical depth. Combining the latter with physically- or observationally-motivated models for the intrinsic nebula radiation, we are able to predict the X-ray and radio light curves for individual SLSNe.

This paper is organized as follows. In §7.2 we discuss basic properties of the engine and ejecta and describe our numerical approach. In §7.3 we present our CLOUDY results and describe the ejecta ionization state; properties of X-ray break-out with application to SLSNe, NS mergers and ASASSN-15lh; and the implied radio absorption and DM evolution in the context of FRBs. In §7.4 we discuss the radio properties of the nebula and origin of the high RM associated with FRB 121102. We summarize our results in §7.5.
Figure 7.1 Schematic diagram illustrating the components of the model for engine-powered transients considered in this paper. An expanding cloud of SN (or NS-merger) ejecta material (blue) envelopes a magnetar, whose spin-down and/or magnetically-driven wind shocks the ejecta interior, producing a hot nebula (yellow). UV and X-ray radiation from this nebula photo-ionizes the ejecta. An additional column of ionized material is created by the forward and reverse shocks (green and blue, respectively) generated by the ejecta’s expansion into the surrounding CSM (dotted green). FRB- or steady synchrotron radio emission produced within or interior to the nebula will undergo dispersion and free-free absorption traveling through the ejecta towards an observer, as well as scattering and Faraday rotation within the magnetized nebula.

7.2 Properties of the Engine and Ejecta

7.2.1 Central Engine

A young neutron star possess two reservoirs of energy: rotational and magnetic. Given the short spin-down timescales of magnetars, rotational energy is most important at early times following the supernova explosion or merger and is needed to power an ultra-relativistic jet in GRBs, or to inflate the nebula of radiation responsible for powering SLSNe. However, rotational energy cannot readily explain the large instantaneous power source of the luminous FRB emission itself, at least for FRB 121102 and its implied age of \( \gtrsim \) several decades after magnetar formation (e.g. Lyutikov 2017). Magnetic energy, though generally smaller in magnitude than rotational energy, can emerge from the stellar interior more gradually over
timescales of years to centuries, and thus may be responsible for powering intermittent magnetic flares responsible for FRBs, as well as an ion-electron synchrotron nebula behind the ejecta.

7.2.1.1 Rotational Energy and Ionizing Radiation

A magnetar born with an initial spin period $P_0$ and mass $M_{\text{ns}} = 1.4M_\odot$ possesses a reservoir of rotational energy

$$ E_{\text{rot}} \simeq 2 \times 10^{52} \text{ergs} \left( \frac{P_0}{1 \text{ ms}} \right)^{-2}. \quad (7.1) $$

If the magnetic dipole and rotational axes are aligned, this energy is extracted through a magnetized wind on a characteristic timescale

$$ t_{\text{rot}} \simeq 4.7 \text{d} \left( \frac{B}{10^{14} \text{ G}} \right)^{-2} \left( \frac{P_0}{1 \text{ ms}} \right)^2, \quad (7.2) $$

where $B$ is the surface magnetic dipole field strength. The spin-down luminosity at time $t$ after the explosion is given by

$$ L_{\text{rot}} = \frac{E_{\text{rot}}/t_{\text{rot}}}{(1 + t/t_{\text{rot}})^2} \approx 8 \times 10^{40} \text{ergs s}^{-1} \left( \frac{B}{10^{14} \text{ G}} \right)^{-2} \left( \frac{t}{10 \text{ yr}} \right)^{-2}, \quad (7.3) $$

where the last equality applies at times $t \gg t_{\text{rot}}$.

Millisecond magnetars which are invoked to power cosmological GRBs must possess large magnetic fields $B \gtrsim 10^{15} \text{ G}$ and short spin-down times $t_{\text{sd}} \lesssim 10 - 10^3 \text{ s}$, commensurate with the duration of long GRBs emission (e.g. [Thompson et al. 2004]) or the temporally-extended X-ray emission following some short GRBs (e.g. [Metzger et al. 2008b]). By contrast,
magnetars invoked as the power source of SLSNe must instead possess weaker magnetic fields $B \lesssim 10^{14}$ G and spin-down times of $\gtrsim$ days, comparable to the photon diffusion timescale of optical radiation through the expanding supernova ejecta (e.g. Maeda et al. 2007; Kasen & Bildsten 2010; Woosley 2010; Nicholl et al. 2017b). The rotationally-powered magnetar wind is ultra-relativistic with a low baryon-loading. As in pulsar wind nebulae (PWNe), dissipation of the wind energy is expected to accelerate a power-law distribution of electrons/positrons, powering non-thermal radiation extending from radio to gamma-ray frequencies. The spectral energy distribution of the PWNe-like emission from a young magnetar engine is highly uncertain. It depends on poorly understood details such as the pair multiplicity of the wind and the location of particle acceleration (e.g. close to the wind termination shock, or within regions of magnetic reconnection in the upstream wind zone; Sironi & Spitkovsky 2011). Furthermore, while the rotationally-powered wind may be ultra-relativistic, it may be periodically interrupted or entirely subsumed by transient ejections of mildly-relativistic baryon-rich material which accompany magnetic flares (see §7.2.1.2 below) and may induce significant Faraday rotation.

Given these uncertainties, we make the simplifying assumption in our photo-ionization calculations (§7.3) that a fraction $\epsilon_i \leq 1$ of the spin-down power $L_{\text{rot}}(t)$ is placed into ionizing radiation, $L_e$. We furthermore assume a spectrum, $L_{e,\nu} \propto \nu^{-1}$ which distributes the energy equally per decade in frequency between $h\nu_{\text{min}} = 1$ eV and $h\nu_{\text{max}} = 100$ keV. In other words, we take

$$\nu L_{e,\nu} = \frac{\epsilon_i L_{\text{rot}}}{\log (\nu_{\text{max}}/\nu_{\text{min}})}.$$  

(7.4)

However, note that the difference between the magnetar field strengths capable of powering long GRB jets versus SLSNe can be reduced if the magnetar experiences fall-back accretion because the latter enhances the spin-down luminosity of the magnetar relative to its isolated evolution (Metzger et al. 2018a).
Although this spectrum is somewhat ad hoc, a high value of $\epsilon_i \sim 1$ is motivated by the likelihood that the nebular electrons/positrons will be fast cooling at such a young age.

### 7.2.1.2 Magnetic Energy and Radio Emission

A magnetar formed with a strong interior magnetic field of strength $B_*$ contains a reservoir of magnetic energy given approximately by

$$E_B \approx \frac{B_*^2 R_{\text{ns}}^3}{6} \approx 2 \times 10^{50} \text{ ergs} \left( \frac{B_*}{5 \times 10^{16} \text{ G}} \right)^2,$$

where $R_{\text{ns}} = 12$ km is the neutron star radius. A field strength $B_* \approx 5 \times 10^{16}$ G corresponds to only a few percent of equipartition with the rotational energy (equation 7.1) for $P_0 \sim 1$ ms.

If this magnetic energy emerges from the stellar interior in the form of intermittent flares, this could be responsible for powering FRB emission, for instance through coherent emission in magnetized internal shocks (Lyubarsky 2014; Beloborodov 2017; Waxman 2017). The enhanced activity of FRB 121102, as compared to older Galactic magnetars, could result from more rapid leakage of the magnetic field from the neutron star interior driven by ambipolar diffusion in the core over the first $t_{\text{mag}} \lesssim 10 - 100$ yrs following birth (Beloborodov 2017). Beyond youth alone, the timescale of magnetic field diffusion $t_{\text{mag}} \propto B_*^{-2}$ would be shortened in magnetars for larger $B_*$ (Beloborodov & Li 2016). Stronger interior fields might be expected if the FRB-producing sources are born rotating particularly rapidly (Thompson & Duncan 1993b), as required for the central engines of GRBs and SLSNe. The rate of magnetic flux leakage, and thus potentially of external flaring activity, would also be enhanced if the
neutron star core cools through direct URCA reactions \cite{BeloborodovLi2016}. The latter is activated in the cores of the massive neutron stars formed from the collapse of particularly massive stars \cite{Brown2018}, also implicated as the progenitors of long GRBs and SLSNe, e.g. given their observed locations in the highest star-forming regions of their host galaxies \cite{Lunnan2016,Blanchard2016}.

In analogy with equation (7.3) for the rotation-powered luminosity, we parameterize the time-averaged magnetic luminosity as

\[
L_{\text{mag}} = \frac{E_{\text{mag}}}{t_{\text{mag}}} \left(1 + t/t_{\text{mag}}\right)^{\alpha - 1},
\]

(7.6)

However, the precise value of the decay index \(\alpha\) — and indeed whether a power-law evolution is even appropriate — remains highly uncertain. Determining this evolution with greater confidence will require future modeling of the rate of magnetic flux escape from young magnetars.

In addition to powering FRB emission itself, magnetic energy deposited in a nebula behind the ejecta could be responsible for the quiescent synchrotron radio emission \cite{Beloborodov2017}. The high rotation measure \(RM \sim 10^5 \text{ rad m}^{-2}\) of FRB 121102 implicates an electron-ion environment surrounding the source \cite{Michilli2018}, favoring the ion-loaded composition expected based on Galactic giant magnetar flares \cite{Granot2006}, but disfavoring the relatively baryon-clean environment expected for a rotationally-powered PWN \cite{7.2.1.1}. We estimate the RM contributed by the magnetar nebula in \cite{7.4.1} and use it to place constraints on the required value of \(E_{\text{mag}}\) and baryon-loading of the magnetically-powered ejections.
7.2.2 Density Profile and Composition of the Ejecta

We model the evolution of the supernova ejecta at radius $r$ and time $t$ as one of homologous expansion, with a broken power-law density profile

$$\rho(r, t) = \frac{3M_{ej}}{8(v_{ej}t)^3} \begin{cases} 1 & r \leq v_{ej}t \\ (r/v_{ej}t)^{-6} & r > v_{ej}t, \end{cases}$$  \hspace{1cm} (7.7)

such that the mass in material expanding above a given velocity $v = r/t$ obeys $M(> v) \propto v^{-3}$. This particular choice for the high-velocity tail is motivated by recent numerical multidimensional simulations of the early-time interaction of the magnetar-inflated nebula and the surrounding ejecta (Suzuki & Maeda 2017; see also Chen et al. 2016, Blondin & Chevalier 2017). Here $M_{ej}$ is the total ejecta mass and $v_{ej}$ the characteristic ejecta velocity at which the transition from a flat core to steep envelope occurs,

$$v_{ej} = \sqrt{10E_{ej}/9M_{ej}},$$  \hspace{1cm} (7.8)

and the ejecta energy $E_{ej} = E_{sn} + E_{rot}$ is the sum of the initial explosion energy, which we take as $E_{sn} = 10^{51}$ erg, and the rotational energy $E_{rot}(P_0)$ fed by the magnetar engine (equation 7.1). Hence, the parameters $M_{ej}$ and $P_0$ fully determine the ejecta density distribution at times greater than a few spin-down timescales (typically days for SLSNe). Although the coupling efficiency between the magnetar spin-down luminosity and the ejecta is uncertain, the magnetar parameters we use in this study are based on fits to the photo-
metric light-curves of SLSNe (Nicholl et al. 2017b), which only probe the energy deposited by the engine into the ejecta. It is therefore self-consistent to assume that the entirety of the observationally-inferred rotational energy is deposited within the ejecta, and eventually converted predominantly into kinetic energy (the amount of radiated energy is typically only a small fraction of the total energy).

We consider a few different assumptions about the elemental composition of the ejecta. We first consider a spatially homogeneous composition of exclusively hydrogen, in order to whet our intuition in a simple limit and to explore ejecta ionization in related events like tidal disruption events of solar-metallicity stars. For SLSNe and long GRBs we instead assume spatially homogeneous O-rich hydrogen-poor composition characteristic of energetic broad-lined SNe-Ic (which show similarities with SLSNe-I; e.g. Liu & Modjaz 2016; Quimby et al. 2018). Specifically, we adopt the composition resulting from the explosion of a 16$M_\odot$ He core model for an explosion energy $10^{52}$ erg from Nakamura et al. (2001). The mass fraction of the first several dominant elements are: O (0.65), He (0.16), Fe (0.05), Ne (0.04), Si (0.04), Mg (0.02), and C (0.01).

Finally, we explore a few Fe-dominated ejecta models in order to explore composition approximating the ejecta from a binary neutron star merger. For instance, in GW170817 a large fraction of the merger ejecta was inferred to possess exclusively light $r$-process nuclei (e.g. Nicholl et al. 2017a; Cowperthwaite et al. 2017; Villar et al. 2017), which are expected to possess electron shell structures relatively similar to Fe (Tanaka et al. 2017; Kasen et al. 2017).
### 7.2.3 Numerical Method to Calculate the Ejecta Ionization State

We employ the publicly-available photo-ionization code CLOUDY\(^3\) (version C13.1: Ferland et al. 2013) to calculate the ionization state of the expanding SN ejecta at different snapshots in time. Given an incident radiation field, gas density profile (equation 7.7) and composition (§7.2.2), CLOUDY calculates the ionization-recombination equilibrium solution and self-consistent temperature profile within the ejecta. As discussed in §7.2.1.1, we adopt as the incident radiation field from the central engine the flat spectral energy distribution normalized to a fraction of the spin-down luminosity, \(L_{e,\nu} \propto L_{\text{rot}}(t)\nu^{-1}\) (equation 7.4). Though at most times of interest \(t \gg t_{\text{rot}}\) we are in the \(L_{\text{rot}} \propto t^{-2}\) portion of the decay (equation 7.3), in our analytic discussion we generalize the central ionizing luminosity to an arbitrary power-law decay, \(L_e \propto t^{-\alpha}\) (for instance, if \(t \lesssim t_{\text{rot}}\), or in case the ionizing luminosity instead tracks the release of magnetic energy; equation 7.6).

Since CLOUDY does not treat radiation transfer in regimes where the medium is optically-thick to electron scattering, it cannot be reliably used at very early epochs (\(\lesssim 1\) yr for typical SLSNe ejecta velocities) when the Thomson optical depth through the ejecta exceeds unity. On the other hand, our implicit assumption of ionization-recombination equilibrium is itself valid only at sufficiently early times, when the density is high enough that the recombination timescale is shorter than the heating/cooling or expansion timescales. Assessing precisely when the equilibrium assumption breaks down is non-trivial, as it depends on the self-consistent ionization-state and temperature of the ejecta. However, using the CLOUDY output, we estimate that ionization-recombination equilibrium holds for the domi-

\(^3\)http://www.nublado.org/
nant species of interest to the latest times of interest (∼several decades), when the ejecta becomes transparent to free-free absorption at GHz frequencies (see §7.3.3).

As CLOUDY is configured to calculate time-independent equilibrium solutions, we have implicitly also neglected adiabatic cooling in the heating balance. This assumption holds well throughout the bulk of the ejecta initially but becomes more difficult to satisfy at late times, and is only marginally valid for the inner high-temperature ejecta layers on timescales at which the ejecta becomes transparent to free-free absorption at GHz frequencies. Comparing the adiabatic cooling timescale (∼ time since explosion) $t$ to the radiative cooling timescale within the ejecta, we find that typically $t_{\text{cool}}/t \lesssim 10$ at all radii within the ejecta at the epoch when the ejecta becomes free-free transparent. By contrast, $t_{\text{cool}}/t \gtrsim 1$ only in the very inner parts of the ejecta, $r \lesssim 10^{-2} R_{\text{ej}}$. As this region contains only a small fraction of the total mass, and contributes little to the ejecta DM, free-free or bound-free optical depths, we expect that our neglect of adiabatic cooling is a reasonable approximation. The effects of adiabatic cooling would be to moderately overestimate the temperature of the inner ejecta, and therefore slightly overestimate the DM and underestimate the free-free optical depth.

### 7.3 Photo-ionization Results

#### 7.3.1 Ionization State

The ionization fraction of the ejecta is defined as

$$f_{\text{ion}}(r) \equiv \frac{n_e(r)}{\sum_i Z_i n_i(r)},$$

(7.9)
where $n_e$ is the electron number density and $Z_i$, $n_i$ are the atomic number and density, respectively, of ion $i$. The ionization fraction along with the neutral fraction, $f_n$, of the ejecta, and particularly their density-weighted averages,

$$
\langle f_{\text{ion}} \rangle_{\rho} \equiv \frac{\int f_{\text{ion}} \rho dr}{\int \rho dr}; \quad \langle f_n \rangle_{\rho} \equiv 1 - \langle f_{\text{ion}} \rangle,
$$

are crucial ingredients in determining the X-ray (§7.3.2) and radio (§7.3.3) opacity of the ejecta, as well as the local DM for an embedded FRB source. This section describes the time-dependent evolution of the ionization state determined by photo-ionization, starting with pure hydrogen composition (§7.3.1.1) and building up to the O-rich composition relevant to GRBs and SLSNe (§7.3.1.2) and Fe-like composition relevant to NS mergers (§7.3.1.3). In §7.3.3.1, we compare the DM from central photo-ionization to collisional ionization from the reverse shock traveling back through the ejecta as it begins to interact with the circumstellar medium.

### 7.3.1.1 Pure Hydrogen Ejecta (e.g. TDE)

We start with the case of ejecta with a pure hydrogen composition. This provides an illustrative example of the relevant physical processes with the added benefit of being analytically tractable. Though not applicable to stripped-envelope supernovae, this case is relevant to photo-ionization of the hydrogen-rich stellar ejecta in a tidal disruption event.

Figure 7.2 shows time snapshots of the radial profile of the electron temperature, $T_e(r)$, and the ionization fraction $f_{\text{ion}}(r)$. The high temperature of the inner ejecta is set by a balance between Compton heating and Compton cooling, at an approximately fixed value.
$T_e \sim 10^7$ K which corresponds to the “Compton temperature” of the nebular radiation field. At larger radii, where the radiation energy density is weaker, free-free cooling exceeds Compton, leading to a steep temperature drop, until at sufficiently large radii photo-electric heating by photo-ionization exceeds the Compton heating. Most of the ejecta mass is concentrated at large radii, near the outer edge of the ejecta, around which $T_e$ reaches an approximately constant value $\approx 10^4$ K. The temperature profile described qualitatively above can be estimated more precisely analytically, as described in Appendix J and illustrated for comparison by the dashed grey curves in Figure 7.2.

The inner portions of the ejecta are nearly fully ionized ($f_{\text{ion}} \approx 1$), until reaching the ionization front at which $f_{\text{ion}}$ declines to values $\lesssim 1$ (there is also an associated drop in temperature at this location). The bottom panel of Fig. 7.2 also shows the density-averaged ionization fraction, which evolves only weakly with time. As follows below, this result can be understood semi-quantitatively through a simple Stromgren sphere analysis (Strömgren 1939).

Assuming that a centrally illuminating source fully ionizes a homogeneous cloud of hydrogen up to $r \lesssim R_s$, and that $f_{\text{ion}} \approx 0$ at larger radii. Equating the total production rate of ionizing photons $Q_0 = \int_\nu_0^\infty (L_{e,\nu}/h\nu) d\nu \propto t^{-\alpha}$ to the total recombination rate yields the familiar Stromgren radius,

$$R_s = \left( \frac{3Q_0}{4\pi n^2 \langle \alpha_B \rangle_m} \right)^{1/3} ,$$

(7.11)

where $n$ is the density (assumed for simplicity here to be radially-constant) and $\langle \alpha_B \rangle_m$ is the mass-averaged type-B recombination rate coefficient. The density-averaged ionization
fraction (equation 7.10) for a spherical, homologously expanding cloud of radius $v_{ej}t$ is then

$$\langle f_{\text{ion}} \rangle \rho \sim \frac{R_s(t)}{v_{ej}t} \propto M_{ej}^{-2/3} v_{ej} t^{1-\alpha/3}.$$  \hspace{1cm} (7.12)

For pure hydrogen composition, the temperature in the regions of greatest interest is regulated to an approximately constant value (see Appendix J), and thus \(\langle \alpha_B \rangle_m\) does not add significant temporal dependence. Therefore \(\langle f_{\text{ion}} \rangle \rho \propto t^{1/3}\) for the standard $\alpha = 2$ case where the ionizing luminosity is powered by magnetar spin-down at times $t \gg t_{\text{rot}}$.

Equation (7.12) only applies while $R_s < v_{ej}t$, because once the ionization front reaches the outer ejecta radius the Stromgren analysis predicts \(\langle f_{\text{ion}} \rangle \rho = 1\). However, when calculating the bound-free X-ray opacity we are more interested in the small residual neutral fraction. In this regime $f_n = 1 - f_{\text{ion}} \ll 1$ a local version of ionization-recombination balance yields

$$f_n(r, t) \approx \frac{4\pi \alpha_B n(r, t)r^2}{\sigma_{pe} Q_0(t)},$$  \hspace{1cm} (7.13)

where $\sigma_{pe}$ is the photo-ionization cross section. Therefore the density-averaged neutral fraction evolves as

$$\langle f_n \rangle \rho \approx \frac{4\pi \alpha_B M_{ej}}{3\sigma_{pe} \mu m_p v_{ej} t Q_0(t)} \propto M_{ej} v_{ej}^{-1} t^{\alpha - 1}.$$  \hspace{1cm} (7.14)

### 7.3.1.2 Oxygen-rich Ejecta (SLSNe)

The case of O-rich ejecta relevant to GRB and Type I-SLSNe cannot be simply described by the Stromgren sphere analysis due to the large number of different ionization states. Figure 7.3 shows our CLOUDY calculation of the radial profile of the ionization fraction and electron
temperature of the ejecta at three different epochs. In contrast to the pure hydrogen case, the radial profiles show significant structure representative of the multiple ionization fronts for different species, consistent with the picture outlined in [Metzger et al. (2014, 2017b)]. Qualitatively, the temperature is still set by Compton heating near the inner edge of the ejecta and by photo-electric heating at large radii. However, in the O-rich case line-cooling instead exceeds free-free cooling throughout most of the ejecta volume.

Given this analytically-untractable complexity, it is fortunate that our main interest is in global properties related to the average ionization state and temperature of the ejecta. Empirically, for O-rich ejecta and a $L_e \propto t^{-2}$ decaying ionizing luminosity ($\alpha = 2$) we find that the density-averaged ionization fraction remains nearly constant in time, $\langle f_{\text{ion}} \rangle_\rho \propto t^0$.

\footnote{However, note that one can still define an ‘effective’ Stromgren radius $R_{\text{seff}} \equiv \langle f_{\text{ion}} \rangle_\rho v_{\text{ej}} t$ (see equation \ref{eq:seff}).}
Figure 7.3 Same as Fig. 7.2 but for the fiducial O-rich ejecta model relevant to SLSNe. The temperature and ionization profiles are significantly more complex than for the pure-hydrogen ejecta due to multiplicity of ionization fronts. The density averaged ionization fraction $\langle f_{\text{ion}} \rangle_\rho$ remains roughly constant in time in this scenario.

We also consider the case of a constant luminosity source, $L_e \propto t^0 \ (\alpha = 0)$, for example, describing the early plateau phase of spindown evolution at $t < t_{\text{rot}}$ (equation 7.2). In this case we find the density averaged ionization fraction increases as roughly $\langle f_{\text{ion}} \rangle_\rho \propto t^{0.4}$ before saturating at close to complete ionization. As discussed further in §7.3.2 these results indicate that the normally-considered picture of ionization breakout is only possible for a temporally constant or slowly-decaying ionizing radiation sources. Stated another way, if ionization break-out does not occur by $t \lesssim t_{\text{rot}}$, it will not occur at later times.

### 7.3.1.3 Pure Iron Ejecta (NS Merger)

To explore the photo-ionization of ejecta by a long-lived central remnant in the case of binary NS mergers, we apply our methods to ejecta of mass $M_{\text{ej}} = 0.05M_\odot$ and velocity $v_{\text{ej}} = 0.2 \ c$, motivated by the inferred properties of the kilonova emission accompanying GW170817 (e.g. Cowperthwaite et al. 2017 [Cowperthwaite et al. 2017] Kasen et al. 2017 [Kasen et al. 2017] Villar et al. 2017 [Villar et al. 2017]). Although
Figure 7.4 Same as Fig. 7.2 but for pure Fe composition, meant to approximate the properties of the $r$-process ejecta from binary NS mergers. As in the O-rich ejecta case, the value of $\langle f_{\text{ion}} \rangle_\rho$ is approximate constant in time.

the merger ejecta is expected to be composed of freshly-synthesized $r$-process material, the atomic data for these elements is not currently incorporated into CLOUDY. For this first approach to the problem, we therefore assume an iron-rich composition, which exhibits the closest degree of complexity to at least the light $r$-process nuclei that is achievable within the current limitations. Also note that the implicit assumption within CLOUDY that the Thompson optical depth is small does not always hold for our models at early times. We are therefore likely underestimating the ionization fraction of the merger ejecta due to the inability of CLOUDY to treat backscattering. We continue with the calculations despite these two important caveats, leaving more accurate modeling of merger ejecta photo-ionization to future studies.

Figure 7.4 shows the ionization state for the fiducial merger-ejecta model, with a magnetar of dipole magnetic field $B = 10^{16}$ G and initial spin-period $P_0 = 0.8$ ms (corresponding to the break-up limit for a NS, the relevant scenario given the large orbital angular momentum at merger) as the ionizing radiation source. The snapshots shown are earlier than
in the SN case, due to the faster evolution of the merger ejecta given its lower mass and higher velocity. Models run for lower assumed dipole fields (larger ionizing fluxes at times $t \gg t_{\text{rot}}$) result in nearly complete ionization of the NS merger ejecta at all epochs. We also find, similarly to the O-rich SLSN case, that the density-averaged ionization fraction $\langle f_{\text{ion}} \rangle_\rho$ is roughly constant in time at times $t > t_{\text{rot}}$, when $L_e \propto t^{-2}$. Again, ionization breakout appears not to be effective unless it has already taken place by $t \sim t_{\text{rot}}$ (see §7.3.2 for further discussion).

### 7.3.2 X-ray Light Curves

#### 7.3.2.1 Oxygen-rich Ejecta (SLSNe)

One potential test of the magnetar model (or more, broadly, engine-powered models) for SLSNe is the onset of late-time X-ray emission, produced once ionizing radiation from the rotationally-powered nebula escapes from the expanding ejecta (Metzger et al. 2014; Kotera et al. 2013). At early times, X-rays are attenuated by bound-free absorption in the ejecta. The X-ray optical depth through the ejecta is given by

$$\tau_X = \int \sigma_{\text{bf}} n (1 - f_{\text{ion}}) \, dr = \frac{3 \sigma_{\text{bf}} M_{\text{ej}}}{8 \mu m_p v_0^2 t_2} \langle f_n \rangle_\rho,$$  (7.15)

where $n = \rho / (\mu m_p)$ and $\sigma_{\text{bf}} = \int F_\nu \sigma_\nu(\nu) d\nu / \int F_\nu d\nu$ is the flux averaged bound-free cross section within the observed X-ray frequency band.

There are two ways that $\tau_X$ can decrease below unity, initiating the X-ray light curve to rise to its peak. First, $\tau_X$ can decrease abruptly, driven by changes in the ejecta’s ionization
Figure 7.5 Unattenuated X-ray luminosity from the magnetar nebula (light blue light curves) and transmitted luminosity through the supernova ejecta (dark blue light curves) from our CLOUDY calculations, for different engine and ejecta parameters from the sample of SLSNe modeled by Nicholl et al. (2017b). We assume an efficiency $\epsilon_i = 1$ for converting spin-down luminosity into broad-band ionizing luminosity (equation 7.4). The nebular X-rays are initially attenuated by bound-free absorption, until the ejecta undergoes sufficient dilution for the bound-free optical depth to decrease below unity (equation 7.16), after which time the incident and transmitted light-curves converge. The detection of SCP06F6 (Levan et al. 2013) and PTF12dam (Margutti et al. 2017a) are shown in yellow/black respectively, while red circles show current upper limits (Margutti et al. 2017a). The X-ray light-curve of PTF12dam predicted by our fiducial model is also highlighted in black. Despite exhibiting the strongest light-curve among SLSNe in our sample, it seems difficult to interpret the early X-ray flux from PTF12dam as originating from the central engine. A green curve shows the X-ray light-curve for an artificial model with temporally-constant ionizing luminosity, $L_{rot} = 10^{43}$ erg s$^{-1}$; this model exhibits an ionization breakout whereby X-rays escape due to a decrease in the average neutral fraction $\langle f_n \rangle_\rho$ (Metzger et al. 2014) as opposed to dilution effects.

state (i.e. because $\langle f_{ion} \rangle_\rho \rightarrow 1$), a so-called ‘ionization breakout’ (Metzger et al. 2014). Alternatively, the condition $\tau_X < 1$ can be achieved more gradually, due to the $\propto t^{-2}$ decrease in the ejecta column at fixed $\langle f_{ion} \rangle_\rho$, a process we refer to as ‘expansion-dilution’.

The previous section showed that $\langle f_{ion} \rangle_\rho$ is approximately constant for O-rich ejecta if $L_e \propto t^{-2}$. Thus, in the $t \gtrsim t_{rot}$ portion of the magnetar spin-down evolution (equation 7.3), we conclude that ionization-breakout is irrelevant and X-rays can only escape due to expansion-dilution. Stated another way, if an ionization break-out is not achieved by $t \sim t_{rot}$, then it is unlikely to be achieved at later times $t \gtrsim t_{rot}$.

This behavior is apparent in Figure 7.5, which shows the transmitted X-ray light curves.
extracted from our CLOUDY results using parameters relevant to the sample of SLSNe in Nicholl et al. (2017b) and assuming $\epsilon_i = 1$. The peak luminosities, achieved on timescales of $\sim 3-30$ years, are in all cases low, $L_X \lesssim 10^{39}$ erg s$^{-1}$ and consistent with non-detection upper-limits shown for comparison from Margutti et al. (2017a). Note that the CLOUDY models and resulting light-curves are only calculated starting at $t = 1$ yr because the SLSNe ejecta are typically optically thick to Thompson scattering at earlier times, a regime in which CLOUDY is not designed to treat radiation transfer correctly.

If ionization-breakout were instead responsible for the X-ray escape, then the calculated light-curves would rise to peak more abruptly, when the dominant ionization front corresponding to the X-ray observing band reaches the ejecta surface. Instead, for the ejecta-dilution scenario the light-curves evolve gradually, displaying the expected $L_X \propto (t/t_e)^{-2} \exp[-(t/t_{bf})^{-2}]$ behavior, where $t_{bf}$ is the time at which $\tau_X = 1$. For a temporally-constant $\langle f_n \rangle / \rho \approx 0.5$ the latter can be estimated as (equation 7.15)

$$t_{bf} \approx 130 \text{ yr} \left( \frac{M_{ej}}{10M_\odot} \right)^{1/2} \left( \frac{v_{ej}}{10^4 \text{ km s}^{-1}} \right)^{-1} \left( \frac{Z}{8} \right)^{-3/2} \left( \frac{\langle f_n / \rho \rangle}{0.5} \right)^{1/2}, \quad (7.16)$$

where we have taken $\mu \approx 2Z$ and have approximated the bound-free opacity $\sigma_{bf} \approx 8 \times 10^{-18}$ cm$^2$ Z$^{-2}$ by its value near the ionization threshold frequency for hydrogen-like ion of atomic number $Z$.

To explore a particularly optimistic case in which ionization break-out might occur, we calculate models with a temporally-constant ionizing radiation source. These models, presented also in \`[\text{7.3.1.2}]\` exhibit a temporal increase in the mean ionization fraction which dominates over the $t^{-2}$ expansion-dilution of the ejecta, such that ionization break-out oc-
curs. The transmitted X-ray luminosity of one such model, with an ionizing luminosity of $10^{43}$ erg s$^{-1}$, is depicted by the green curve in Fig. 7.5. The sharp transition at $t \approx 2$ yr due to the rapid increase in the ejecta's ionization state marks the onset of ionization break-out. This behavior differs significantly from the slow-evolving light-curves of $L \propto t^{-2}$ ionizing radiation sources (blue curves in Fig. 7.5), which are characterized instead by ejecta-dilution. The qualitative result is not strongly dependent on the value of the assumed fixed ionizing luminosity source. For example, even lower luminosities of $10^{41}$ erg s$^{-1}$ induced a rapid increase in the ejecta’s ionization state and led to an ionization break-out on timescales of $\sim 5$ yr. We expect this behavior as long as the luminosity is sufficient to drive $\langle f_{\text{ion}} \rangle \rho \rightarrow 1$ faster than the expansion-dilution effect.

Finally, we note that density inhomogeneities in the ejecta, e.g. due to Rayleigh-Taylor instabilities caused by the PWN accelerating into this medium (Chen et al. 2016; Suzuki & Maeda 2017; Blondin & Chevalier 2017), can allow X-rays to escape at earlier times than predicted by our spherical models. Blondin & Chevalier (2017) show that the column density along certain viewpoints can decrease by more than an order of magnitude due to such inhomogeneities. This would allow radiation leakage reaching some observers at $\sim$three times earlier than predicted by the spherical models. Variability of the column density due to turbulent motions expected within this inhomogeneous ejecta may also affect the X-ray light-curves, and we leave study of such effects to further work.
7.3.2.2 ASASSN-15lh

As a test case illustrating the strong dependence of the transmitted X-ray flux on model parameters, we examine the X-ray and UV emission of the very luminous transient ASASSN-15lh (Dong et al. 2016). The nature of ASASSN-15lh has been debated extensively, the two prominent models interpreting the event as either a SLSN or a tidal disruption event (e.g. Metzger et al. 2015; Dong et al. 2016; Leloudas et al. 2016; Sukhbold & Woosley 2016; Margutti et al. 2017c; Krühler et al. 2018). The ASASSN-15lh optical light curve peaked at $\sim 35$ d with luminosity $\sim 2 \times 10^{45}$ erg s$^{-1}$ (Dong et al. 2016), and later showed a re-brightening in UV, reaching $\sim 5 \times 10^{44}$ erg s$^{-1}$ at $t \sim 200$ d (Brown et al. 2016). Coincidental with the re-brightening, X-rays were first detected from the location of the transient at a luminosity of $\sim 6 \times 10^{41}$ erg s$^{-1}$ (Margutti et al. 2017c; there are deeper non-detections at earlier times).

One suggested interpretation of the UV (and possibly also X-ray) brightening is ionization break-out of a central-engine, whether the latter is a millisecond magnetar in the SLSN case or an accreting supermassive black hole in the TDE case (Margutti et al. 2017c). Here we assess whether the detected UV and X-ray luminosities at $t \sim 200$ d can be attributed to an ionizing central radiation source behind a layer of expanding ejecta. We model the engine’s incident radiation field by interpolating between the detected luminosities at UV and X-ray frequencies with a power-law SED. This assumption implies that the model will

$^5$ We performed a similar calculation but instead modeling the incident SED as two black-bodies of temperatures $1.5 \times 10^4$ K and $2 \times 10^6$ K, respectively, with luminosities necessary to match the X-ray and UV detections (Brown et al. 2016; Margutti et al. 2017c). The qualitative results for this spectral model are identical to the power-law SED case, though quantitatively the value of $L_{\text{trans}}/L_{\text{incident}}$ in Fig. 7.6 at saturation is several orders of magnitude lower.
reproduce the observations if the transmitted UV and X-ray luminosities are unattenuated \((L_{\text{trans}} \approx L_{\text{incident}})\). Given the unknown mass of the ejecta \(M_{\text{ej}}\), we explore our results as a function of \(M_{\text{ej}}\).

Figure 7.6 shows the ratio of transmitted to incident X-ray (circles) and UV (squares) luminosities, separately for O-rich (blue) and solar composition (red) ejecta. While the UV flux escapes nearly unattenuated for any ejecta mass we explore (unsurprising given the large measured UV luminosity), the X-ray flux show an abrupt step-function transition between being able to ionize the ejecta at low ejecta mass and instead undergoing strong absorption at high \(M_{\text{ej}}\). The O-rich ejecta exhibits stronger absorption than the Solar composition one due to the larger abundance of bound-free transitions in the X-ray band for this high-metallicity material. The most striking feature of this result is the nearly bimodal nature of X-ray absorption — either the incident radiation manages to ionize its way out and escapes nearly unattenuated, or ionization break-out is unsuccessful and the incident radiation is strongly absorbed within the ejecta. A change of only \(\sim 50\%\) in ejecta mass can result in six orders of magnitude difference in the escaping X-ray flux (see also Fig. 7.5).

The dashed vertical curve in Fig. 7.6 shows the minimum ejecta mass \(M_{\text{ej}} \approx 3M_\odot\) which is consistent with the observed 35 d optical peak of ASASSN-15lh. This is made under the assumption that the peak time is determined by the usual photon diffusion timescale \(t_{\text{pk}} \approx (3\kappa M_{\text{ej}}/4\pi c v_{\text{ej}})^{1/2}\) for an expansion velocity of \(v_{\text{ej}} = 10^4\) km s\(^{-1}\) and we take a conservative upper limit on the opacity of \(\kappa = 0.2\) cm\(^2\) g\(^{-1}\).

Our results confirm in greater quantitative detail the conclusions of Margutti et al. (2017c), namely that (1) the observed UV brightening could be the result of an ionization
break-out from a central engine, regardless of its precise nature (e.g. a magnetar in the SLSNe case or accreting supermassive black hole in the TDE case); (2) if the observed X-ray emission originates from the same central engine (as opposed to an unrelated source like a nuclear star cluster or AGN), then the ejecta is more consistent with being a TDE than a SLSNe. For the TDE case the ejecta is expected to be of solar composition and to possess a relatively low mass $\lesssim M_\odot$ (most TDEs are expected to be of solar or sub-solar mass stars; e.g. Stone & Metzger 2016; Kochanek 2016). By contrast, the large mass $\gtrsim 3M_\odot$ of oxygen-rich ejecta required in the SLSN scenario would be challenging to photo-ionize through on timescales of the observed X-ray emission. On the other hand, if the X-rays are unrelated to the transient, as would be the case if they do not fade away in time, then the interpretation would remain ambiguous. Late-time X-ray observations of ASASSN-15lh to determine if the source has faded would help distinguish these scenarios.

7.3.2.3 Pure Fe Ejecta (Binary NS Merger)

A key question regarding the outcome of binary NS mergers such as GW170817 is whether a central engine, such as a long-lived magnetar or accreting black hole, might contribute to powering or re-energizing the kilonova or afterglow emission (Zhang 2013; Yu et al. 2013; Metzger & Piro 2014; Metzger & Bower 2014; Horesh et al. 2016; Kisaka et al. 2016; Fong et al. 2016; Matsumoto et al. 2018; Li et al. 2018). If present, such an engine could reveal itself through its direct X-ray emission. Binary NS mergers have also been suggested as possible sources of FRBs if the merger produces a stable remnant (Yamasaki et al. 2018). Constraints on non-afterglow contributions to the X-ray emission from GW170817 have
Figure 7.6 The ratio of transmitted to incident flux in the UV (squares) and X-ray (circles) as a function of assumed ejecta mass for O-rich (blue; SLSN case) and solar composition (red; TDE case). The incident luminosity is set to fit the observed UV and X-ray data on ASASSN-15lh at $\sim 200$ days (Brown et al. 2016; Margutti et al. 2017c), with a power-law extrapolation of the SED in between these frequency bands. The incident UV radiation propagates through the ejecta nearly unattenuated for any ejecta mass. By contrast, the X-ray attenuation is extremely sensitive to the assumed ejecta mass, exhibiting a sharp cut-off above a characteristic ejecta mass, which is $\approx 1\,M_\odot$ for the O-rich case and $\approx 3\,M_\odot$ for the solar composition case. The ejecta velocity in both cases is taken to be $v = 10^4\,\text{km}\,\text{s}^{-1}$, consistent with the observed spectrum of ASASSN-15lh. A vertical dashed line shows the approximate ejecta mass inferred from the light curve peak under the assumption of a supernova origin for the emission for an assumed opacity $\kappa = 0.2\,\text{cm}^2\,\text{g}^{-1}$.

been used to argue against the formation of a long-lived NS remnant (Pooley et al. 2017; Margutti et al. 2018; see Margalit & Metzger 2017 for alternative arguments against a long-lived NS remnant in GW170817). However, there have thus far been no detailed calculations of the photo-ionization of the merger ejecta and its affect on attenuating a central X-ray source or FRB.

Figure 7.7 shows X-ray light curves resulting from our CLOUDY calculations of expanding ejecta with pure-Fe composition. Blue curves in the top panel show the transmitted luminosity for a spin-down powered magnetar ionizing source with dipole magnetic fields in the range $B = 10^{14} - 10^{16}\,\text{G}$ and $\epsilon_i = 1$ (except for the bottom curve). Light-grey curves (in some cases overlapping underneath the blue curves) show for comparison the unattenuated (incident) radiation for each model. Even high $B$-field spin-down powered engines
Figure 7.7 X-ray emission from a long-lived magnetar remnant following a binary NS merger. Similar to Fig. 7.5 unattenuated (light blue) and transmitted (dark blue/brown/green) X-ray light-curves for pure-Fe models of binary NS merger ejecta. The blue curves assume a rotationally-powered magnetar ionizing radiation source of dipole magnetic field strengths $B = 10^{14} - 10^{16}$ G, while the brown and green curves are for various engine powered kilonova models of GW170817 (see text). The red circles, squares and triangle denote GW170817 upper limits from NuSTAR, Swift, and Chandra X-ray Observatory, respectively (Evans et al. 2017; Margutti et al. 2017b). Clearly, even large $B$ magnetar remnants are ruled out for GW170817. Meanwhile, a more modest luminosity ionizing source is not constrained by the observations.

successfully ionize their way out of the ejecta, such that the transmitted luminosity is nearly equal to the incident one, for large $\epsilon_i$. A comparison to early-time X-ray upper limits on GW170817 from NuSTAR, Swift, and Chandra (red symbols in Fig. 7.7; Evans et al. 2017; Margutti et al. 2017b) rules out magnetar models for this event.

On the other hand for $B = 10^{16}$ G and a lower radiative efficiency $\epsilon_i = 0.1$, X-ray absorption at early epochs is significantly stronger and the presence of a magnetar would be left unconstrained by the X-ray data alone. This again illustrates the extreme sensitivity of the transmitted flux on model parameters (see also §7.3.2.2) — a reduction of the incident flux from $\epsilon_i = 1$ to $\epsilon_i = 0.1$ results in almost seven orders of magnitude difference in the early-time transmitted luminosity.

We stress that a magnetar remnant in GW170817 is separately ruled out based on energetic grounds alone (Margalit & Metzger 2017) — the large angular momentum at
merger implies that the remnant is born rotating near breakup, with $\sim 10^{53}$ erg of rotational energy. If even a few percent of this energy is released via magnetic-dipole spindown, it would exceed the GW170817 ejecta energy inferred from the kilonova and afterglow observations. The additional constraints we discuss in this paper are based solely on early X-ray non-detections, and thus serve as further independent evidence against a long-lived magnetar remnant in GW170817.

One hypothetical way to bypass these constraints is by invoking a large deformation of the NS so that the bulk of its rotational energy is lost to GWs instead of magnetic-dipole spindown. However, this requires a combination of large internal toroidal magnetic field with a small external dipole field, a configuration which is thought to be magnetically unstable (Braithwaite 2009; Margalit & Metzger 2017).

Nevertheless, Li et al. (2018) have proposed GW spindown dominated magnetar model in which the kilonova emission is powered by the subdominant magnetic-dipole luminosity. Here we show that in addition to the magnetic-stability consideration, such a model would also be constrained by the early X-ray non-detections. The brown curve in bottom panel of Fig. 7.7 shows the transmitted X-ray light curve resulting from our CLOUDY calculations adopting the best-fit model parameters of Li et al. (2018). The emission at $\sim 2-30$ d exceeds the X-ray upper limits for GW170817 (red points), and is thus inconsistent with the data.

The bottom panel of Fig. 7.7 also shows the transmitted light curve (green curves) through NS merger ejecta, this time for a central engine of luminosity $L_e = 6 \times 10^{41} \text{ erg s}^{-1} (t/1\text{d})^{-1.3}$. Such an engine has the expected temporal power-law resulting from radioactive decay of $r$-process material to the valley of stability (Metzger et al. 2010a) and roughly tracks the
bolometric luminosity of the kilonova associated with GW170817 (e.g. Arcavi 2018). This second case was chosen to constrain models in which the GW170817 kilonova was powered by a central engine rather than by radioactivity as proposed by Matsumoto et al. (2018).

Different curves show results for different assumed ejecta masses (labeled for each curve) for fixed ejecta velocity $v_{ej} = 0.2c$. For the highest mass cases $M_{ej} \gtrsim 10^{-2}M_{\odot}$, the ejecta provides a sufficient column density to absorb the X-rays at early times, consistent with GW170817 non-detections. However, lower mass ejecta, such as $M_{ej} \sim 10^{-3}M_{\odot}$ advocated by Li et al. (2018), are ruled out. We stress that for $M_{ej} \gtrsim 10^{-2}M_{\odot}$, the r-process radioactive heating rate becomes comparable to the assumed engine luminosity, essentially bypassing the need for invoking such an engine. These results differ from the analytic estimates of Matsumoto et al. (2018) who found large X-ray optical depths (their eq. 28) because the ejecta was assumed to be fully neutral in their calculation, and the ionizing affect of the incident radiation field (which lowers the bound-free optical depth) was not accounted for.

### 7.3.3 Radio Transparency and Ejecta DM

Having calculated both the ionization and temperature structure of the ejecta for an ensemble of properties motivated by SLSNe, we now determine the local DM contribution to the ejecta and its free-free absorption. The latter controls both when a putative FRB and associated nebular emission produced within such SLSNe ejecta become visible.

The DM is naturally expressed in terms of the density-averaged ejecta ionization fraction

$$DM = \int n_e \, dr = \frac{9M_{ej}}{20\pi \mu_e m_p v_{ej}^2 t^2} \langle f_{ion} \rangle \rho(t),$$

(7.17)
Figure 7.8 Time evolution of the ejecta dispersion measure DM(t) for the SLSNe in our sample (solid grey curves; extrapolated by dotted extensions). Blue points show the time at which the ejecta first becomes transparent to free-free absorption at 1 GHz. Observational constraints for FRB 121102 are shown as dashed black curves, whose intersection with the DM(t) tracks indicate the minimum allowed age of FRB 121102 assuming it originates from each SLSN (purple points). The bottom panel shows the distribution of free-free transparency timescales and minimal age (blue and purple respectively). If FRB 121102 originates from a population of young magnetars with properties similar to those inferred for observed SLSNe, then its age is \( \gtrsim 30 - 100 \) yr.

where \( \mu_e = \rho/(m_p n_e) \approx 2 \) is the mean molecular weight per electron.

Figure 7.8 shows the time evolution of the DM for the population of SLSNe in our sample for \( \epsilon_i = 1 \) (solid grey curves). Since we found that the ionization fraction is nearly constant in time (§7.2.3), the DM evolution closely follows a simple \( \propto t^{-2} \) power-law decay due to the decreasing ejecta column, as shown by the dotted-grey extrapolations to late times after conclusion of the CLOUDY calculations. Various tracks are therefore differentiated almost entirely based on their different normalizations imprinted by the ejecta mass and ionizing spin-down power.

The free-free optical depth can be approximately expressed as

\[
\tau_{ff} = \int \kappa_0 g_{ff}(T_e, \nu) \nu^{-2} Z^2 n_e n_i T_e^{-3/2} dr
\]

\[
\approx \kappa_0 g_0 \nu^{-2.118} Z^{1.882} \int T_e^{-1.323} f_{ion}^2 n^2 dr,
\]

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where the gaunt factor is 

\[ g_{\text{ff}} \approx g_0 (Z \nu)^{-0.118} T^{0.177} \]  

(Draine 2011), and \( g_0 = 13.907 \), \( \kappa_0 = 0.01772 \) in appropriate cgs units. Neglecting temperature changes and for a \( \sim \) constant \( \langle f_{\text{ion}} \rangle \rho \), this implies a temporal scaling of \( \tau_{\text{ff}} \propto t^{-5} \). From the numerical CLOUDY calculations, we find \( d \ln \tau_{\text{ff}} / d \ln t \sim -4.2 \) to \(-4.6 \) up to the 1 GHz transparency timescale \( t_{\text{ff}} \sim 10 - 100 \) yr for the SLSNe in our sample.

### 7.3.3.1 Comparison to the Reverse Shock

The SN/merger ejecta will become collisionally-ionized after being heated by the reverse shock, which propagates back into the ejecta as the ejecta interacts with the circumstellar medium (Piro 2016). This process occurs on the Sedov-Taylor timescale,

\[ t_{\text{ST}} \approx 450 \text{ yr} \left( \frac{E}{10^{52} \text{ ergs}} \right)^{-1/2} \left( \frac{M_{\text{ej}}}{10 M_\odot} \right)^{5/6} \left( \frac{n_{\text{csm}}}{1 \text{ cm}^{-3}} \right)^{-1/3}, \]  

(7.19)

where \( n_{\text{csm}} \) is the number density of the external medium. Here we compare this source of external shock-ionization to that due to photo-ionization from the central engine.

Piro (2016) assume a constant density ejecta and find, using the approximate hydrodynamic solutions of McKee & Truelove (1995), a large ejecta DM contribution at early times \( t \ll t_{\text{ST}} \) from the reverse shock. However, more realistic models for the ejecta structure find steeply-declining outer envelopes outside of the relatively flat, constant-density core (e.g. Chevalier & Soker 1989, Suzuki & Maeda 2017; see equation 7.7), which exhibit dramatically different behavior at early times, \( t \ll t_{\text{ST}} \). Using the approximate solutions of Truelove & McKee (1999) we re-evaluate this contribution to the local DM and ionization-state (see Appendix K).
Figure 7.9 Comparison of the DM contribution from the reverse shock driven into the ejecta by interaction with the circumstellar medium (black curves) with that of the DM contribution from photo-ionization due to a central engine (red curve), as presented previously in Figure 7.3. The dashed black curve shows the DM for the idealized case of homogeneous ejecta (Piro 2016), while the solid black line shows results for the more realistic case of a steeply-declining outer envelope (equation 7.7), which exhibits a much lower DM at early times when the reverse shock is propagating within the low-density envelope. As expected, the two solutions converge once the shock enters the ejecta core (dotted-vertical curve marked by $t_{\text{core}}$) and the dynamics transition to that of the Sedov-Taylor phase (second dotted-vertical line, $t_{\text{ST}}$). In either case, the DM contributed by the forward/reverse shocks is significantly lower than that due to photo-ionization by a central engine on timescales $\lesssim 10^2$ yr.

Figure 7.9 shows our results for the time-dependent DM contribution from the reverse shock for a characteristic ejecta profile (equation 7.7; $n = 6$ in the notation of Truelove & McKee 1999), and compared to the idealized constant-density ejecta distribution ($n = 0$) as well as the fiducial O-rich photo-ionization model presented previously in Figure 7.3. In both models photo-ionization dominates the DM at $t \lesssim 200$ yr, while the reverse shock dominates at later times. Although the case shown assumes typical SLSNe parameters ($E = 10^{52}$ ergs; $M = 10 M_\odot$; and $n_{\text{csm}} = 1 \text{ cm}^{-3}$), the time axis can be scaled trivially with $t_{\text{ST}}$ (equation 7.19). Our DM estimate does not account for details of the post-shock density distribution due to compression or radiative cooling, which however will only act to reduce the ionized column by a factor of $\lesssim 3$ (Appendix K). Furthermore, for $n = 6$ the majority

$^6$We obtain similar results for other values $n \gtrsim 5$, although the limit of $n \to \infty$ reverts back to the case of homogeneous ejecta (Piro 2016).
contribution to the ionized mass and DM comes from the shocked circumstellar medium instead of the shocked ejecta.

The main physical effect leading to the smaller DM when the ejecta has a steep outer envelope compared to the constant density ejecta (Piro 2016) is the larger blast-wave and reverse shock radii at a given time (and thus smaller column for a fixed ejecta mass) in the envelope case, since at early times the forward and reverse shocks are located at velocity coordinates greater than \( v_{\text{ej}} \) (at which the core-envelope transition occurs).

### 7.3.3.2 Application to FRB 121102

Observational constraints on the local contributions of the DM and its time derivative for FRB 121102, as well as the system age, are given by (e.g. Spitler et al. 2016; Piro 2016; Chatterjee et al. 2017)

\[
\text{DM}_{\text{local}} \lesssim 140 \pm 85 \text{ pc cm}^{-3}, \tag{7.20}
\]

\[
|d\text{DM}/dt| \lesssim 2 \text{ pc cm}^{-3} \text{ yr}^{-1}, \tag{7.21}
\]

\[
t_{\text{age}} \gtrsim 6 \text{ yr}, \tag{7.22}
\]

and are depicted as dashed black curves in Fig. 7.8. The ejecta first becomes optically thin to free-free absorption (blue points) at \( t_{\text{ff}} \sim 10 - 100 \text{ yr} \) after explosion (see histogram on the bottom panel), while the minimal age of FRB 121102 (purple points), assuming it originates from a magnetar with properties characteristic of the SLSN population, is \( \sim 30 - 100 \text{ yr} \). Our detailed numerical result is thus consistent with previous analytic estimates of the repeater’s minimal age (Metzger et al. 2017b).
Figure 7.8 illustrates that the constraints on FRB 121102 are consistent with all of the SLSNe in our sample at sufficiently late times. However, this alone says nothing about the probability we are observing the repeater at such a late time. To explore this issue, we assume that the probability distribution for detecting an FRB at time $t$ is given by $P(t|t_{ff}) \propto t^{-\alpha}$, provided that $t > t_{ff}$, the free-free transparency timescale, and $t < t_a$, the engine activity timescale. While $t_{ff}$ is determined by our CLOUDY calculations, the parameter values $\alpha$ and $t_a$ which control the rate of FRB activity are uncertain (though $t_a$ is expected to be less than a few hundred years if the flares result from the diffusion of magnetic flux from the magnetar core; §7.2.1.2). For $\alpha$ we consider two cases: (i) a 'flat' evolution ($\alpha = 0$), motivated by magnetic dissipation powered FRB models (equation 7.6); and (ii) a rapidly decaying evolution $\alpha = 2$ model, e.g. motivated by spin-down powered FRB scenarios (equation 7.3).

Since the DM undergoes a simple time evolution $\propto t^{-2}$, it is easy to invert the problem to calculate the probability of observing an FRB from a specific SLSN with some DM. Summing over the distribution of $t_{ff}$ and the DM at this time for the population of SLSNe then yields the marginalized probability distribution of observing an FRB with some given dispersion measure, $P(DM)$ (see Appendix L for further details). The same procedure can be repeated for the DM derivative, allowing comparison with constraints on FRB 121102.

Figure 7.10 shows the resulting probability distributions for $\alpha = 0$ (solid curves) and $\alpha = 2$ (dotted curves), and different assumptions regarding the engine active lifetime, $t_a$ (different colors). Shown for comparison are the constraints on DM and its derivative for FRB 121102. In practice, as in Fig. 7.8 the upper limit on $dDM/dt$ provides the tightest constraints. In the fiducial, magnetically powered model ($\alpha = 0$), even a relatively
Figure 7.10 Probability distributions of observing an FRB with a given local dispersion measure and DM derivative, assuming FRBs are produced by a population of magnetars (and their enveloping SN ejecta) similar to the observed SLSN population. The probability distribution functions are calculated based on the DM and free-free transparency times found using our CLOUDY models, and assuming an FRB activity lifetime $t_a$ (different colored curves) and an FRB activity / detection metric which evolves with time as $t^{-\alpha}$ (see §7.3.3 and Appendix L for further details). Solid curves show results for $\alpha = 0$ while dotted curves are for $\alpha = 2$. The observational constraints (upper-limits) for FRB 121102 are shown as dashed vertical curves. The probability that the local DM and $d\text{DM}/dt$ of FRB 121102 be consistent with predictions for the population of SLSNe is clearly non-negligible for a wide range of model parameters. The hypothesis that FRB 121102 arises from a young magnetar with parameters $(P_0, B, M_{ej})$ drawn from the observed SLSN population is therefore consistent with the observed dispersion measure and its time derivative.

A short engine lifetime of $t_a = 100\,\text{yr}$ results in a non-negligible probability $P(d\text{DM}/dt < 2\,\text{pc cm}^{-3}\,\text{yr}^{-1}) = 0.34$ of observing FRB 121102 at times consistent with the constraints. A longer assumed engine activity timescale obviously results in a higher probability of detecting the source at sufficiently late times. The rapidly decaying model ($\alpha = 2$) predicts somewhat lower, yet still significant, probabilities of randomly detecting an FRB with DM and $d\text{DM}/dt$ consistent with the repeater. At the low end, for $t_a = 100\,\text{yr}$, we obtain $P(d\text{DM}/dt < 2\,\text{pc cm}^{-3}\,\text{yr}^{-1}) = 0.15$, while even slightly longer timescales approach the asymptotic $t_a \to \infty$ limit of $P(d\text{DM}/dt < 2\,\text{pc cm}^{-3}\,\text{yr}^{-1}) = 0.47$.

In summary, we conclude that, under a wide range of assumptions, the observed DM and $d\text{DM}/dt$ of the repeater are completely “characteristic” of those expected if FRB 121102 originates from an engine embedded within a young SLSN.
7.4 Nebula Rotation Measure and Synchrotron Emission

In the previous section we presented calculations of the time-dependent ionization state of the SN ejecta, in order to predict its DM and the X-ray light curves from the central engine. This section extends this connection to the RM and the radio emission from synchrotron nebulae with relation to the quiescent radio source associated with FRB 121102 (Chatterjee et al. 2017; Marcote et al. 2017b). Given the ejecta’s free-free optical depth calculated in §7.3.3, we also estimate the late-time radio emission from SLSNe and long GRBs, and from magnetar-powered FRB sources more generally.

7.4.1 Rotation Measure

The large RM ≈ 10^5 rad m^{-2} of FRB 121102 (Michilli et al. 2018), and its observed ~ 10% decrease over a baseline of 7 months, if related to the dilution of an expanding nebula, strongly constrain the age and origin of the bursting source.

7.4.1.1 An Electron-Ion Nebula

Since the RM contributions from positrons and electrons cancel one another, the observed large RM value requires an electron-ion plasma rather than the pair-dominated ultra-relativistic wind from a rotationally-powered pulsar wind (Michilli et al. 2018). Though young pulsars produced primarily electron-positron winds, a large ion loading is not necessarily surprising in the context of a bursting magnetar. Observations of the synchrotron radio afterglows of
giant flares from Galactic magnetars indeed find the bulk of the matter ejected from these events to be expanding at mildly- or trans-relativistic speeds (e.g. Granot et al. 2006). This substantial baryon loading is presumably from the neutron star surface after being heated during the fireball phase (following which most electron/positron pairs annihilate; e.g. Beloborodov 2017).

A key, but theoretically uncertain, property of the magnetically-powered outflow is the average ratio, $\xi$, of the number of ejected baryons to the released magnetic energy (Beloborodov 2017)

$$10^2 \text{erg}^{-1} \lesssim \xi \equiv \frac{N}{E} \lesssim 10^4 \text{erg}^{-1}. \quad (7.23)$$

Here the lower limit on $\xi$ follows from an estimate of the minimum number of radio-emitting electrons responsible for the afterglow of the giant flare of SGR 1806-20 (Granot et al. 2006), while the upper limit corresponds to the escape speed of a neutron star,

$$\xi_{\text{max}} \approx \frac{R_{\text{ns}}}{GM_{\text{ns}}m_p} \approx 4 \times 10^3 \text{erg}^{-1}. \quad (7.24)$$

If the kinetic energy of the ejecta flare thermalizes at the termination shock, transferring a fraction $\epsilon_e$ of its energy to the electrons, then the latter enter the nebula with a mean thermal Lorentz factor

$$\bar{\gamma}_e \approx \frac{\epsilon_e}{\xi} \approx 150 \left( \frac{\epsilon_e}{0.5} \right) \left( \frac{\xi}{\xi_{\text{max}}} \right)^{-1}. \quad (7.25)$$

As discussed below, the characteristic value $\bar{\gamma}_e \sim 10^2$ implied for $\xi \lesssim \xi_{\text{max}}$ is consistent with that required to power the quiescent sychrotron source from FRB 121102. The limited frequency range $\nu \sim 1-20$ GHz over which the quiescent source is observed, and the potential
impact of cooling on the spectrum, makes it challenging to determine whether the radiating electron population is a non-thermal power-law (as assumed in previous works), or whether it might be also consistent with a relativistic Maxwellian (or superposition of Maxwellians) with $kT \approx \bar{\gamma}_e m_e c^2$ (equation 7.25).

### 7.4.1.2 Radio-Emitting Electrons, Injected Recently

We first show that FRB 121102’s high RM cannot originate from the same relativistic electrons responsible for powering the quiescent radio emission. Synchrotron emission from electrons with Lorentz factor $\gamma_e = 100 \gamma_{100}$ embedded in a magnetic field $B$ (in Gauss) peaks at a frequency $\nu \approx 5.6 B \gamma_{100}^2$ GHz. The observed spectral luminosity $L_\nu \approx 10^{29} \nu^{-0.2} \text{erg s}^{-1} \text{Hz}^{-1}$ of the source at $\nu < 10$ GHz is related to the number of radiating electrons $N_{\gamma_e} \equiv dN_{\gamma_e}/d\ln \gamma_e$ with $\gamma_e(\nu)$ according to (Beloborodov 2017)

$$L_\nu \approx \frac{3 e^3 B}{m_e c^2} N_{\gamma_e} \Rightarrow N_{\gamma_e} B \approx 2 \times 10^{50} \text{G}. \quad (7.26)$$

For a homogeneous spherical nebula of radius $R_n = 10^{17} R_{17} \text{cm}$, and magnetization parameter $\sigma$ (ratio of magnetic to particle energy), one finds individually that (Beloborodov 2017)

$$B \approx 0.06 \sigma^{2/7} R_{17}^{-6/7} \text{G}; N_{\gamma_e} \approx 3 \times 10^{51} \sigma^{-2/7} R_{17}^{6/7}, \quad (7.27)$$

and thus the Lorentz factor of the emitting particles is

$$\gamma_e \approx 540 \nu_{10}^{1/2} \sigma^{-1/7} R_{17}^{3/7}, \quad (7.28)$$
while their average number density in the nebula is

$$n_e \approx \frac{3N_{\gamma e}}{4\pi R_n^3} \approx 0.7 \text{ cm}^{-3} \sigma^{-2/7} R_{17}^{-15/7}. \quad (7.29)$$

The maximum RM through the nebula, from the same electrons which power the observed synchrotron radiation, is then given by

$$\text{RM}_{\gamma e} = \frac{e^3}{2\pi m_e^2 c^4} \int n_e B_{\|} \frac{\ln \gamma_e}{2\gamma_e^2} ds \lesssim \frac{e^3}{2\pi m_e^2 c^4} n_e B R_n \ln \gamma_e,$$

$$\approx 0.01 \text{ rad m}^{-2} \sigma^{2/7} R_{17}^{-20/7} \quad (7.30)$$

where the $\ln \gamma_e/2\gamma_e^2$ factor accounts for suppression of the RM contributed by relativistically-hot electrons (Quataert & Gruzinov 2000a) and in the second line we have neglected the parameter dependence of the logarithmic terms. Clearly, the value of $\text{RM}_{\gamma e}$ from the radio-emitting electrons is many orders of magnitude too low to explain the observed RM $\sim 10^5$ rad m$^{-2}$.

**7.4.1.3 Cooled Electrons, Injected in the Distant Past**

Recently-injected electrons responsible for the quiescent radio source of FRB 121102 cannot produce its large RM, in part because of the $\propto 1/\gamma_e^2$ suppression for relativistic temperatures. However, prospects are better if a greater number of electrons were ejected when the source was younger, especially since they may by now be sub-relativistic ($\gamma_e \lesssim 2$) due to synchrotron and adiabatic cooling. Such cooling is reasonable if the present source age is $\gtrsim 3 - 10$ times greater than the timescale $t_{\text{mag}}$ around when magnetic activity peaked (equation 7.6) and
presumably when most of the baryons were deposited in the nebula. Adiabatic expansion alone will reduce the energy of relativistic electrons by a factor $\sim t/t_{\text{mag}}$ from their injected ultra-relativistic values, while the synchrotron loss timescale $\propto 1/B_n^2 \propto t^2 \dot{E}_{\text{mag}}$ will also be considerably shorter at early times when the magnetic field inside the nebula is stronger (see equation 7.32 below).

To explore this possibility with a rough estimate, consider that the magnetar has up until now released a magnetic energy $E \sim E_B = 10^{51} E_{51} \text{erg}$, comparable to its total magnetic reservoir $E_B$ (equation 7.5). The total number of electrons in the nebula is therefore (equation 7.23)

$$N_e = \xi E_B \approx 4 \times 10^{54} (\xi/\xi_{\text{max}}) E_{51}$$

(7.31)

where a value $\xi \sim \xi_{\text{max}}$ (equation 7.24) is again motivated by matching the thermal Lorentz factor of the injected electrons (equation 7.25) to those required to explain the frequency of the quiescent radio emission of FRB 121102 (equation 7.28). The average density of electrons in the nebula is then $n_e \approx 3N_e/(4\pi R_n^3)$.

If the magnetic energy of the nebula, $B_n^2 R_n^3/6$, is a fraction $\epsilon_B$ of the energy $\sim L_{\text{mag}} t$ injected in relativistic particles over an expansion time $\sim t$, then the magnetic field strength in the nebula is given by

$$B_n \approx \left( \frac{6\epsilon_B E_B (\alpha - 1)}{R_n^3} \right)^{1/2} \left( \frac{t}{t_{\text{mag}}} \right)^{(1-\alpha)/2},$$

(7.32)

where we have used equation (7.6) for $L_{\text{mag}}(t)$.

Combining results, the maximum contribution to the RM (assuming all the electrons
are mildly relativistic) is given by

\[
RM = \frac{e^3}{2\pi m_e^2 c^4} \int n_e B_\parallel ds \approx \frac{3e^3}{8\pi^2 m_e^2 c^4} \frac{N_e B_n}{R_n^2} \left( \frac{\lambda}{R_n} \right)^{1/2}
\approx 6 \times 10^7 \left[ \epsilon_B (\alpha - 1) \right]^{1/2} E_{51}^{3/2} \left( \frac{\xi}{\xi_{\text{max}}} \right) \times \left( \frac{t}{t_{\text{mag}}} \right)^{(1-\alpha)/2} \left( \frac{\lambda}{R_n} \right)^{1/2} \text{rad m}^{-2}
\]

(7.33)

where \( \lambda \) quantifies the correlation lengthscale of the magnetic field in the nebula. Thus, we see it is possible to obtain RM values \( \sim 10^5 \text{ rad m}^{-2} \) comparable to those measured for FRB121102 for optimistic parameters, e.g. \( \epsilon_B = 0.1, E_{51} \approx 1, t = t_{\text{age}} \sim 10t_{\text{mag}}, \lambda \sim R_n \).

An important prediction of this model is the expected secular decrease in the RM. Assuming that \( R_n \propto t \), the time derivative of the RM is given by

\[
\frac{dRM}{dt} = -\frac{(6 + \alpha)}{2} \frac{RM}{t}
\]

(7.34)

If the observed \( \Delta RM/RM = -0.1 \) change in FRB 121102’s RM over the baseline of \( \Delta t = 0.6 \) yr (Michilli et al. 2018) is entirely due to this secular decline, then this requires a source age of

\[
t = t_{\text{age}} \approx \frac{\alpha + 6}{2} \left( \frac{RM}{\Delta RM} \right) (\Delta t) \approx 5(\alpha + 6) \text{ yr}
\]

(7.35)

Instead treating the observed change as an upper limit on the change in the RM due to secular expansion (e.g. if the observed variability is dominated by some stochastic process, e.g. internal turbulence, at fixed nebular size), then equation (7.35) becomes a lower limit...
on the source age. This RM-inferred age estimate is compatible with the ∼ 30 − 100 yr age independently estimated based on the DM-derivative.

7.4.2 Radio Synchrotron Emission

Though X-rays from the engine appear challenging to detect (Fig. 7.5), prospects may be better at radio frequencies once the ejecta becomes transparent to free-free absorption (Kashiyama et al. 2016). Indeed, such a nebula was proposed as the origin of the quiescent radio source associated with FRB 121102 (Metzger et al. 2017b; Kashiyama & Murase 2017). If FRB 121102 is indeed associated with a young magnetar, its birth heralded by a SLSN or long GRB, then one may invert the problem to ask what radio emission we might expect to detect at late times from other SLSNe or long GRB remnants (Metzger et al. 2017b). In the following, we adopt a phenomenological approach to estimating the late-time quiescent radio flux, using a minimal set of assumptions and scaling whenever possible to the observed properties of the repeater’s quiescent source.

We consider two assumptions about the energy spectrum of electrons injected into the nebula which radiation synchrotron emission. First, we assume that the electrons are injected with a relativistic Maxwellian of constant temperature (or, equivalently, mean Lorentz factor) given by equation (7.25). We also consider a power-law population of electrons accelerated in the magnetar nebula to Lorentz factors γ, ∂N/∂γ ∝ γ−p, which emit at frequencies ν = γ²eB/2πmₑc. We additionally assume that the nebula is observed at early times in the
fast-cooling regime ($\nu_c < \nu$) such that the radio luminosity is

$$\nu L_{e,\nu} \propto \dot{E}_n \gamma^2 (\partial N/\partial \gamma) ,$$  \hspace{1cm} (7.36)

where $\dot{E}_n(t)$ energy injection rate into the nebula from the engine, and the synchrotron spectrum is $L_{e,\nu} \propto \nu^{-p/2}$. Finally, we assume that the nebula magnetic field is in equipartition, such that $B \propto (E_n/R_n^3)^{1/2}$.

We can now use equation (7.36) to rescale the repeater’s observed quiescent flux density to other sources. We assume a power-law energy injection rate to the nebula using similar notation as for the detection-metric defined in the previous section, $\dot{E}_n = \tilde{\dot{E}} t^{-\alpha}$, and that the nebula size is set by the outer ejecta radius so that $R_n = v_{ej} t$. This implies that the predicted flux at frequency $\nu$ and time $t$ is

$$F_\nu(t) \lesssim F_\nu \left( \frac{D}{D_t} \right)^2 \left( \frac{\nu}{\nu_t} \right)^{-p/2} \left( \frac{\tilde{L}}{L_t} \right)^{(p+2)/4} \times \left( \frac{v_{ej} t_r}{R_{VBLI}} \right)^{3(2-p)/4} \left( \frac{t}{t_r} \right)^{1-p/2-\alpha(p+2)/4}$$  \hspace{1cm} (7.37)

$$\approx 188 \, \mu Jy \, D_{\text{Gpc}}^{-2} \nu_{10}^{-1.15} v_{ej,9}^{-0.225} t_{30 \text{ yr}}^{-0.15} t_{r,30 \text{ yr}}^{-0.075}$$

where quantities with subscript $X_t$ refer to the assumed/measured properties of the repeater, and the inequality results from the VLBI constraints on the marginally resolved emitting region, $v_{ej,t,r} \lesssim R_{VBLI} \approx 0.7 \, \text{pc}$ \cite{Marcote2017} and assuming $p > 2$ (otherwise the inequality in equation (7.37) is reversed). The last equality in equation (7.37) adopts an electron power-law index of $p = 2.3$ consistent with the FRB 121102 spectrum above 10 GHz.
Figure 7.11 Quiescent radio emission predicted for the sample of SLSNe as a function of time at 6 GHz (grey curves) and at the current observed age of each SLSN (blue circles; purple triangles illustrate similar predicted radio fluxes at 100GHz). Symbols colored in grey indicate that the SLSN ejecta is not yet transparent to free-free absorption at the given epoch and frequency band. The calculation assumes a naive scaling of the repeater’s observed properties, following equation (7.37). The top panel shows results for a constant nebula energy injection rate ($\alpha = 0$), while the bottom panel is for a decaying injection rate proportional to $t^{-2}$, as appropriate for a spin-down powered model ($\alpha = 2$).

assuming this is above the cooling frequency.

Figure 7.11 shows the predicted quiescent radio flux in the VLA and ALMA bands assuming an age (time since SN) of $t_r = 30$ yr for the repeater (see previous section), and for two energy injection models — $\alpha = 0$ (2) heuristically corresponding to a magnetic-dissipation (spin-down) powered models, respectively. As in equation (7.37), the electron index $p$ is set by the observed spectral slope of the FRB121102 quiescent source, which can be approximate as $F_\nu \sim \nu^{-1.15}$ above $\sim 10$ GHz, implying $p = 2.3$. We model free-free absorption through multiplication by a prefactor $\exp[-(t/t_{ff})^{-4.5}]$, where $t_{ff}$ and the power-law scaling $\tau_{ff} \sim t^{-4.5}$ are obtained numerically from our CLOUDY models (see §7.3.3).

A major underlying assumption in this simplified phenomenological model is that we would observe putative SLSNe quiescent sources within the same spectral region (in this case — the fast-cooling optically thin regime). This assumption is motivated by the interpretation
of the spectral turn-over at $\sim 10$ GHz as the cooling break. If this interpretation is correct, then the fact that FRB 121102 is likely observed later after SN then current SLSNe (see previous section) implies that for such SLSNe, the cooling break should be at even lower frequencies, thus validating the implicit assumption of $\nu_c < \nu$.

Another assumption of this calculation is that the injected electron spectrum is a power-law. As already mentioned, if the nebula is powered by the escape of magnetic energy from the engine in a baryon-loaded wind, then the electrons are heated at the wind termination shock to a thermal energy $kT \sim \gamma_e m_e c^2$ with a mean Lorentz factor $\gamma_e \sim 100$ (equation 7.25) possibly sufficient to explain the observed GHz radio emission of FRB 121102. In this case, if the baryon loading of the wind is fixed $\xi$, then at earlier times when the magnetic field is higher then the Lorentz factors of electrons contributing in the GHz range will be a part of the Rayleigh-Jeans tail and we will expect a $F_\nu \propto \nu^{1/3}$ spectrum (e.g. Giannios & Spitkovsky 2009). The luminosity should scale with magnetic field in this case, $F_\nu \propto B$ such that

$$F_\nu \lesssim F_{\nu_1} \left( \frac{D}{D_r} \right)^{-2} \left( \frac{\nu}{\nu_r} \right)^{1/3} \left( \frac{\tilde{L}}{\tilde{L}_r} \right)^{1/2} \left( \frac{v_{ej} t_r}{R_{VLBI}} \right)^{-3/2} \left( \frac{t}{t_r} \right)^{-(2+\alpha)/2}$$

$$\approx_{\alpha=0} 240 \mu Jy D_{10}^{-2} \nu_{10}^{1/3} v_{ej,9}^{-3/2} t_{30yr}^{-1} t_{30yr}^{-0.5}$$

(7.38)

### 7.5 Conclusions

We have examined the photo-ionization of homologously expanding ejecta by a central ionizing radiation source, with application to GRBs, Type I SLSNe, NS mergers (specifically GW170817), FRBs (focusing on the repeating source FRB 121102), and the very luminous
transient ASASSN-15lh. These diverse phenomena share a commonality — the possibility that their driving power source is a newly-born magnetar, or otherwise similarly-acting central engine like an accreting black hole.

Our investigation of the time-dependent ionization state of the expanding ejecta cloud surrounding such a putative central engine is used to address a multitude of its potential observable signatures. We additionally address the question of whether FRB 121102 is consistent with a young ‘SLSN-type’ magnetar origin, as suggested by e.g. [Metzger et al. (2017b)], and provide simple analytic models for its observed DM, RM and quiescent radio emission.

Our main conclusions are summarized as follows:

1. The (density-averaged) ionization fraction of metal rich (e.g. O-rich, pure-Fe) ejecta remains roughly constant in time for an ionizing luminosity source declining as $L_e \propto t^{-2}$, as would apply to the late-time magnetar spin-down power.

2. X-rays from SLSNe engines are severely attenuated in the first $\sim$ decades post explosion and escape the ejecta due to expansion-dilution rather than classical X-ray break-out [Metzger et al. (2014)]. This is consistent with X-ray non-detections for the majority of SLSNe and indicates that, except in possible extreme cases, or if density inhomogeneities play an important role, X-rays may not provide the easiest means of testing the magnetar hypothesis for SLSNe.

3. The observed X-ray flux of ASASSN-15lh can only be explained as unabsorbed flux from a central engine if the ejecta mass is assumed to be low ($\lesssim 1M_\odot$ or $\lesssim 3M_\odot$ for O-rich or solar composition, respectively). This is in possible tension with the peak
timescale of this event, if this timescale is attributed to the photon-diffusion timescale through the ejecta (though in a TDE, the light curve peak could be set by other effects like the fallback time of the stellar debris). Alternatively, the X-ray source may be unrelated to the optical/UV transient.

4. For canonical parameters, photo-ionization of SLSNe ejecta can induce significantly larger DM on $\lesssim 10^2$ yr timescales than that caused by collisional ionization of shocked matter due to the ejecta-CSM interaction (as advocated by Piro 2016, Piro & Gaensler 2018).

5. A magnetar central engine operational on $\sim 1-100$ d timescales is ruled out for the NS merger GW170817 unless the amount of spin-down power emitted in ionizing radiation is small ($\epsilon_i \ll 1$). Similarly, the hypothesis that the kilonova associated with GW170817 was powered by a central engine (instead of by radioactive decay of freshly synthesized r-process material) is ruled out by early X-ray non-detections, unless the ejecta mass is large $\gtrsim 10^{-2} M_\odot$. However, for such a large ejecta mass, radioactive heating would provide a comparable luminosity to that of the supposed engine, negating the need for the latter.

6. The age of FRB 121102, assuming it originates from a flaring magnetar within a typical Type-I SLSN, is $\gtrsim 30-100$ yr, consistent with previous analytic estimates (Metzger et al. 2017b).

7. The observed DM and upper limits on $|dDM/dt|$ of FRB 121102 are statistically consistent with the assumption that FRB 121102 originates from a magnetar with properties...
\((B, P_0, M_{eq})\) drawn from the inferred parameters for magnetar-powered Type-I SLSNe (Nicholl et al. 2017b).

8. The high RM of FRB 121102 cannot be caused by the \(\gamma \sim 100\) electrons responsible for the associated quiescent radio emission. The RM can be explained by a population of electrons and ions which were injected into the nebula at early times and have since cooled to non-relativistic velocities.

9. Interpreting the observed change in RM as secular within the framework mentioned above results in an estimate for the repeater’s age of \(5(\alpha + 6)\) yr, where \(\dot{E}_n \propto t^{-\alpha}\) is the rate of magnetic energy injection to the nebula. This is again consistent with other age constraints on FRB 121102.

10. The maximal number of ejected baryons per unit energy released by a flaring magnetar \(\xi_{max}\) (as set by the escape speed from the NS surface) corresponds to a characteristic electron Lorentz factor \(\bar{\gamma}_e \sim 10^2\). Remarkably, this value agrees with that required to produce the frequency of the quiescent radio emission coincident with FRB 121102 (Beloborodov 2017), providing additional support for the magnetar model.

Future telescopes like UTMOST (Caleb et al. 2017) and Apertif (Colegate & Clarke 2011; van Leeuwen 2014; Maan & van Leeuwen 2017), will enable a large expansion in the study of FRB properties, including those with good localizations, particularly if all FRBs are accompanied by bright persistent radio sources similar to the quiescent emission of FRB 121102 (Eftekhari et al. 2018). In the absence of luminous radio nebulae, robust FRB host galaxy association requires higher resolution, sub-arcsecond,
localization only accessible to facilities such as VLBA, EVN, VLA, ASKAP, DSA-10 and MeerKAT, which are expensive for this purpose given the large number of observing hours likely required to detect an FRB (Eftekhari & Berger 2017).

In addition to expanding the sample size of well-localized repeating FRBs, further monitoring of FRB 121102 may provide crucial information for testing the magnetar hypothesis. In particular, our models predict a secular decline in both DM and RM of the repeater due to the surrounding SN ejecta’s expansion. Thus, a falsifiable test of our model, at least in its simplest form, is if both rotation and dispersion measures are not found to decrease (averaging over any random fluctuations) over a baseline of ~several decades.

Finally, we point out the importance of further investigation of X-ray break-out from SLSNe, given that such a signature would provide a smoking-gun indication of a magnetar engine. Though our current analysis suggests that X-rays cannot, for typical ‘SLSN-type’ magnetar and ejecta parameters, ionize their way out of the ejecta, our idealized models assume spherical symmetry and neglect inhomogeneities expected due to e.g. Rayleigh-Taylor instabilities from the nebula-ejecta interface (e.g. Blondin & Chevalier 2017). The ‘fractured’ density distribution in this case may allow X-rays to escape at earlier times (and higher luminosities) than predicted by our current spherical models, and we leave investigation of this issue to future work.

We also note that we have focused in this work on SLSNe rather than long-GRB engines because the latter should emit significantly lower luminosity at the late times of interest ($t \gg t_{\text{rot}}$). This is a natural consequence of the shorter engine timescale of
long-GRBs, $\sim 100\, \text{s}$, compared to $\sim \text{days}$ for SLSNe and the fact that $L \propto (t/t_{\text{rot}})^{-2}$ for magnetar spin-down (e.g. Margalit et al. [2017]). Long-GRB engines are therefore expected to have little effect on the ionization state of their surrounding ejecta on timescales of years or later.
Bibliography


Allam, S. et al. 2017, GCN, 21530, 1


Andreoni, I. et al. 2017, PASA, 34, e069


Antoniadis, J. et al. 2013c, Science, 340, 448


Arcavi, I. et al. 2017, Nature

Armitage, P. J. 2010, Astrophysics of Planet Formation, 294

—. 2013, Astrophysics of Planet Formation


Baade, W. & Zwicky, F. 1934a, Proceedings of the National Academy of Science, 20, 259

—. 1934b, Physical Review, 46, 76


Bellm, E. 2014, in The Third Hot-wiring the Transient Universe Workshop, ed. P. R. Wozniak, M. J. Graham, A. A. Mahabal, & R. Seaman, 27–33
Beloborodov, A. M. & Mészáros, P. 2017, SSR, 207, 87

281

Bloom, J. S. et al. 2011, Science, 333, 203


Burbidge, E. M., Burbidge, G. R., Fowler, W. A., & Hoyle, F. 1957, Reviews of Modern Physics, 29, 547

282
Caughlan, G. R. & Fowler, W. A. 1988, Atomic Data and Nuclear Data Tables, 40, 283
Colegate, T. M. & Clarke, N. 2011, PASA, 28, 299
Coulter, D. A. et al. 2017, Science, 1
Damour, T. & Nagar, A. 2010, Phys. Rev. D, 81, 084016
Del Pozzo, W., Li, T. G. F., Agathos, M., Van Den Broeck, C., & Vitale, S. 2013, Physical Review Letters, 111, 071101
—. 2010b, Nature, 467, 1081
—. 2010c, Nature, 467, 1081
Dong, S. et al. 2016, Science, 351, 257

Draine, B. T. 2011, Physics of the Interstellar and Intergalactic Medium


Gal-Yam, A. 2012, Science, 337, 927
—. Observational and Physical Classification of Supernovae, ed. A. W. Alsabti & P. Murdin, 195


Greiner, J. et al. 2015, Nature, 523, 189


Hansen, C. J., Kawaler, S. D., & Trimble, V. 2004, Stellar interiors : physical principles, structure, and evolution


Hinderer, T., Lackey, B. D., Lang, R. N., & Read, J. S. 2010, Phys. Rev. D, 81, 123016


—. 2011b, Phys. Rev. D, 83, 124008
Hotokezaka, K., Piran, T., & Paul, M. 2015, Nature Physics, 11, 1042
Kasen, D., Metzger, B., Barnes, J., Ramirez-Ruiz, & Quataert, E. 2017, Nature
Leloudas, G. et al. 2016, Nature Astronomy, 1, 0002

Levan, A. J. et al. 2011, Science, 333, 199


   —. 2005b, ApJS, 156, 47


LIGO Scientific Collaboration & Virgo Collaboration. 2017, PhRvL, 119, 161101

   —. 2017c, Nature submitted

Lipunov, V. et al. 2017, GCN, 21546, 1


   —. 2017, ArXiv e-prints


Lyutikov, M. 2017, ArXiv e-prints


Marcote, B. et al. 2017b, ArXiv e-prints


Metzger, B. D., Beniamini, P., & Giannios, D. 2018a, ArXiv e-prints


—. 2017b, ArXiv e-prints


Metzger, B. D., Thompson, T. A., & Quataert, E. 2018b, ArXiv e-prints
Miller, M. C. 2013, ArXiv e-prints


Nakar, E. 2007, PhysRep, 442, 166


—. 2016, ArXiv e-prints


—. 2015a, ApJL, 807, L18
—. 2007b, Astron. Astrophys., 467, 395
—. 2007c, A&A, 467, 395
Paschalidis, V., MacLeod, M., Baumgarte, T. W., & Shapiro, S. L. 2009, Phys. Rev. D, 80, 024006


—. 2004, Reviews of Modern Physics, 76, 1143


—. 2009b, Phys. Rev. D, 79, 124032
Rezzolla, L. & Kumar, P. 2014, ArXiv e-prints


Savchenko, A. et al. 2017


Sekiya, M. 1998, Icarus, 133, 298


—. 2005, Physical Review Letters, 94, 201101

Shibata, M. & Taniguchi, K. 2006a, Phys. Rev. D, 73, 064027

—. 2006b, Phys. Rev. D, 73, 064027


306


Thornton, D. et al. 2013, Science, 341, 53


Totani, T. 2013, PASJ, 65, L12


Usov, V. V. 1992, Nature, 357, 472


van Leeuwen, J. 2014, in The Third Hot-wiring the Transient Universe Workshop, ed. P. R. Wozniak, M. J. Graham, A. A. Mahabal, & R. Seaman, 79–79


Wolszczan, A. 1994, Science, 264, 538

—. 2016, ArXiv e-prints
Yamasaki, S., Totani, T., & Kiuchi, K. 2018, PASJ, 70, 39
Yang, S. et al. 2017, GCN, 21531, 1
Yu, Y.-W., Zhang, B., & Gao, H. 2013, ApJL, 776, L40
—. 2018, ArXiv e-prints


Appendices

Appendix

A Initial Conditions

This section provides additional details on the initial radial profile of the WD accretion disk (§2.2). Assuming a radial surface density profile as parameterized in equation (2.4), the normalization factor $N$ is found by requiring that the disk mass equal that of the disrupted WD,

$$\int 2\pi r \Sigma \, dr = M_{\text{WD}} . \quad (A1)$$

This yields

$$N(m,n) \equiv \left( \frac{m + 2}{n - 2} \right)^{m+2} \frac{\Gamma(m + n)}{\Gamma(m + 2)\Gamma(n - 2)} , \quad (A2)$$

where $\Gamma(x)$ is the gamma function.

The characteristic radius of the disk $R_d$ is determined by requiring that the total angular momentum of the torus,

$$\int 2\pi r \Sigma \times \Omega r^2 \, dr = J_{\text{tot}} , \quad (A3)$$

equal that of the binary at the time of disruption,

$$J_{\text{tot}} = M_{\text{WD}}\sqrt{GM_{\text{NS}}R_c} , \quad (A4)$$
where Keplerian rotation, $\Omega = \Omega_k$, is assumed. The proportionality constant which relates $R_d$ to the circularization radius, $R_c$ (equation 2.3), is given by

$$R(m,n) \equiv \frac{R_d}{R_c} = \frac{m+2}{n-2} \left[ \frac{\Gamma(m+2)\Gamma(n-2)}{\Gamma(m+5/2)\Gamma(n-5/2)} \right]^2. \quad (A5)$$

As shown in Fig. A1 the value of this constant is typically $R(m,n) \lesssim 1$, indicating that the disk radius $R_d$ is generally smaller than the circularization radius.

Finally, the function $T(m,n)$, which determines the initial disk aspect ratio through equation (2.5), is found by equating the energy of the binary at disruption, $E_{\text{tot}} = -\frac{GM_{\text{WD}}M_{\text{NS}}}{2(1+q)R_c}$, (A6)

with the combined gravitational, kinetic, and internal energy (which is proportional to $\theta^2$) of the disk, i.e.

$$\left[ 1 - \frac{2\theta^2}{(\gamma - 1)} \right] \int 2\pi r \Sigma \left( -\frac{1}{2} \Omega_k^2 r^2 \right) dr = E_{\text{tot}}. \quad (A7)$$
For our initial density prescription, this yields
\[ T(m, n) \equiv \frac{n - 2}{m + 1} \left[ \frac{\Gamma(m + 2)\Gamma(n - 2)}{\Gamma(m + 5/2)\Gamma(n - 5/2)} \right]^{-2}. \] (A8)

**B Derivation of ‘Precursor’ Outflow**

Here we estimate the fraction of the initial disk mass which is lost to ‘precursor’ winds at very early times within the framework of our outflow model. As described in §2.4.1, these winds occur if the initial aspect ratio of the disk exceeds its steady-state value, \( \theta_{\text{ss}} \) (equation 2.28). On a short timescales \( \sim t_w < t_{\text{dyn}} \ll t_{\text{visc}} \), the disk aspect ratio is regulated by this excess energy loss until \( \theta \approx \theta_{\text{ss}} \).

Since the mass loss timescale is shorter than other timescales in the problem, we assume that wind cooling dominates during the initial transient phase, such that
\[ \frac{\dot{\theta}}{\theta} = \frac{\dot{\theta}}{\theta} = \frac{1}{2} \frac{\dot{\rho}_w/\Sigma}{u} = \frac{\gamma - 1}{2} \left[ \eta_w - \text{Be}_d(\theta) \right] \theta^{-2} \dot{\Sigma}_w, \] (B1)

where we have used the wind cooling prescription (equation 2.22) and the fact that \( \dot{\theta}/\theta = \dot{u}/2u \) for a gamma-law EOS (for which \( \theta \propto \sqrt{u} \)). The short transient timescale also implies that the disk density changes almost entirely due to wind mass losses, such that \( d\Sigma_w \approx -d\Sigma \), and therefore
\[ \frac{\gamma - 1}{\gamma} \Sigma^{-1} d\Sigma = \frac{\theta}{\eta_w + 1/2 - \theta^2/\gamma/(\gamma - 1)} d\theta. \] (B2)

Here we have explicitly used equation (2.20) for the Bernoulli parameter. Integrating from the initial to final density/aspect-ratio, we obtain
\[ \frac{\Sigma_i}{\Sigma_f} = \left( \frac{\chi - \theta^2_i}{\chi - \theta^2_f} \right)^{1/\gamma}, \] (B3)

where we have defined \( \chi \equiv (\eta + 1/2)\gamma(\gamma - 1)^{-1} \).

Finally, we obtain the amount of wind launched off the disk during the transient by
radially integrating the disk surface density. This yields

\[ M_w^{(\text{precursor})} = - \int 2\pi r (\Sigma_i - \Sigma_f) \, dr \]

\[ = M_d \times \left[ 1 - \left( \frac{\chi - \theta_{\text{initial}}^2}{\chi - \theta_{\text{ss}}^2} \right)^{1/\gamma} \right]. \]  

(B4)

Here we have identified the initial mass as \( M_d \), and the initial (final) aspect ratios as \( \theta_{\text{initial}} \) (\( \theta_{\text{ss}} \)) respectively, in accordance with previous notation. Expanding equation (B4) in powers of \( \Delta \theta / \theta_{\text{ss}} \ll 1 \), where \( \Delta \theta \equiv \theta_{\text{initial}} - \theta_{\text{ss}} \), and plugging in our definition for \( \chi \) as well as the explicit solution for \( \theta_{\text{ss}} \) (equation 2.28), we obtain our final result

\[ M_w^{(\text{precursor})} \approx \frac{1 + 2 \text{Be}_{\text{crit}}' \gamma}{\eta_w - \text{Be}_{\text{crit}}'} \left( \frac{\Delta \theta}{\theta_{\text{ss}}} \right) \times M_d. \]  

(B5)

C Solution for Mass Inflow Exponent

Here we derive an explicit analytic expression for the steady-state mass inflow exponent, \( p \), as a function of the model parameters. Along the way we obtain a few additional results of interest.

Using the definition of \( p \) (equation 2.30) and equation (2.10) for the radial accretion velocity, we find by solving the continuity equation (2.8) with \( \partial_t = 0 \), a steady-state wind mass loss rate of

\[ \dot{\Sigma}_{\text{w,ss}} = 3 \alpha \theta_{\text{ss}}^2 p \left( p + \frac{1}{2} \right) \Sigma \Omega_k. \]  

(C1)

The steady-state disk aspect ratio, \( \theta_{\text{ss}} \), can be substituted into this expression using equation (2.28). Note that in contrast to previous expressions for the wind mass loss rate (such as equation 2.21), this result is independent of our adopted wind prescription, being entirely a consequence of mass conservation.

Turn now to energetic considerations. It is straightforward to show that the ratio of advective cooling relative to the viscous heating rate in steady state is a constant value,

\[ \left| \frac{\dot{q}_{\text{adv}}}{\dot{q}_{\text{visc}}}_{\text{ss}} \right| = \frac{4}{3} \left( p + \frac{1}{2} \right) \left( \frac{1}{\gamma - 1 + \frac{1}{2} - p} \right) \theta_{\text{ss}}^2. \]  

(C2)
The specific advective cooling rate in the steady-state regime is defined as $\dot{q}_{\text{adv}}/\Sigma = v_r \partial_r u + c_s^2 v_r \partial_r \ln \Sigma$, and this expression is derived assuming a gamma-law EOS (equation 2.17).

Using for the first time the specifics of our wind parameterization, equation (2.22), the ratio of wind cooling to viscous heating is given by

$$\left| \frac{\dot{q}_w}{\dot{q}_{\text{visc}}}_{\text{ss}} \right| = \frac{4}{9} \alpha^{-2} \theta_{\text{ss}}^2 \frac{\dot{\Sigma}_{\text{ws}}}{\Sigma_k} (\eta_w - \text{Be}'_{\text{crit}})$$

$$= \frac{4}{3} p \left( p + \frac{1}{2} \right) (\eta_w - \text{Be}'_{\text{crit}}),$$

where in the second equality we have substituted $\dot{\Sigma}_{\text{ws}}$ from equation (C1). The only inherent assumption in the wind parameterization of equation (2.22) which we have used in deriving this result, is that the specific energy of the wind scales with the escape velocity $v_k$, and that the disk is regulated to a fixed Bernoulli parameter Be'_{\text{crit}}. Equation (C3) does not depend on the less certain form of $\dot{\Sigma}_w$ given by equation (2.21).

This last result allows us to solve for the mass inflow exponent $p = p(\eta_w, \text{Be}'_{\text{crit}}, \gamma)$, by requiring energy conservation in steady-state, i.e.

$$\left| \frac{\dot{q}_w}{\dot{q}_{\text{visc}}}_{\text{ss}} \right| + \left| \frac{\dot{q}_{\text{adv}}}{\dot{q}_{\text{visc}}}_{\text{ss}} \right| = 1,$$

as implied by equation (2.16) for $\partial_t = 0$ (and neglecting nuclear heating, $\dot{q}_{\text{nuc}}$). Substituting equations (C2) and (C3) into the last expression and using $\theta_{\text{ss}}$ from equation (2.28), we rearrange to find

$$p = p(\eta_w, \text{Be}'_{\text{crit}}, \gamma) =$$

$$= \frac{1}{2} \left[ 1 - 2\text{Be}'_{\text{crit}} + \text{Be}'_{\text{crit}} \gamma - \eta_w \gamma ight. \right.$$}

$$\left. + \left( 12\text{Be}'_{\text{crit}} \gamma - 5 \gamma^2 - 18\text{Be}'_{\text{crit}} \gamma^2 + 10\eta_w \gamma^2 ight. \right.$$}

$$\left. + 9\text{Be}'_{\text{crit}}^2 \gamma^2 + \eta_w^2 \gamma^2 - 6\text{Be}'_{\text{crit}} \eta_w \gamma^2 + 6\gamma \right)^{1/2} \right]$$

$$\left/ \left[ 2\text{Be}'_{\text{crit}} - \gamma - 4\text{Be}'_{\text{crit}} \gamma + 2\eta_w \gamma + 1 \right] \right.$$. 

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D Threshold Accretion Rate for Effective Triple-α Burning

In the following we calculate the accretion rate above which triple-α burning can effectively fuse $^{12}$C seeds, a necessary condition for further α-capture and the limiting factor in helium WD merger nucleosynthesis.

The triple-α nuclear reaction rate is analytically approximated by (Caughlan & Fowler 1988)

$$\dot{X}_{3\alpha \rightarrow ^{12}C} = \rho^2 X_{3\alpha}^2 \left[ 2.79 \times 10^{-8} T_9^{-3} e^{-4.4027/T_9} 
+ 1.35 \times 10^{-8} T_9^{-3/2} e^{-24.811/T_9} \right]$$  \hspace{1cm} (D1)

in cgs units, where $T_9 \equiv T/10^9$ K. The reverse reaction rate is similarly given by

$$\dot{X}_{^{12}C \rightarrow 3\alpha} = 2 \times 10^{20} \dot{X}_{3\alpha \rightarrow ^{12}C} T_9^3 e^{-84.424/T_9}.$$  \hspace{1cm} (D2)

Equating the two rates yields a critical temperature at which they are in equilibrium,

$$T_{\text{lim}} \simeq 1.74 \times 10^9 \text{ K},$$  \hspace{1cm} (D3)

a result which is insensitive to the precise carbon/helium mass fractions due to the strong temperature dependence of equation (D2).

Neglecting mixing, the steady-state version of equation (2.23) shows that significant nuclear burning only occurs in disk regions where the mass inflow (accretion) timescale is comparable to the nuclear reaction rate, i.e. at the disk radius where $v_r/r = \dot{X}$. If the burning radius for $\dot{X}_{3\alpha \rightarrow ^{12}C}$ is larger than the reverse triple-α burning radius, then $^{12}$C can be fused before it is disintegrated at smaller radii. In the opposite case, any carbon in the disk is disintegrated into α-particles faster than the triple-α reaction can fuse new carbon $^{12}$C.

Using equation (2.10) for the accretion velocity, and relating the disk density to its
temperature $\rho = \rho(r, T)$ via the radiation pressure dominated EOS (equation 2.7), we find that the three timescales are equal and

$$\frac{v_r}{r}|_{r_{\text{lim}}, T_{\text{lim}}} = \dot{X}_{\text{3}\alpha \rightarrow ^{12}\text{C}}|_{r_{\text{lim}}, T_{\text{lim}}} = \dot{X}_{^{12}\text{C} \rightarrow 3\alpha}|_{r_{\text{lim}}, T_{\text{lim}}}$$

(D4)

at the critical radius given by

$$r_{\text{lim}} \approx 6.75 \times 10^7 \text{ cm} \left(\frac{\alpha}{0.1}\right)^{2/7} \left(\frac{\theta}{0.4}\right)^{12/7} \left(\frac{M_{\text{NS}}}{1.4M_{\odot}}\right)^{5/7}.$$  

(D5)

The accretion rate at this limiting radius is

$$\dot{M}_{\text{in}}^{\text{(lim)}}(r_{\text{lim}}) = 4.85 \times 10^{-5} \text{ } M_{\odot} \text{ s}^{-1}$$

(D6)

$$\times \left(\frac{\alpha}{0.1}\right)^{12/7} \left(\frac{\theta}{0.4}\right)^{37/7} \left(\frac{M_{\text{NS}}}{1.4M_{\odot}}\right)^{9/7}.$$  

For $\dot{M} > \dot{M}_{\text{in}}^{\text{(lim)}}(r_{\text{lim}})$, the triple-$\alpha$ burning front occurs at large radii as compared with the reverse reaction burning front, where $\dot{X}_{^{12}\text{C} \rightarrow 3\alpha} < \dot{X}_{3\alpha \rightarrow ^{12}\text{C}}$, and therefore $^{12}\text{C}$ is efficiently fused. At smaller local accretion rates, the reverse triple-$\alpha$ reaction immediately disintegrates any carbon into $\alpha$ particles and $^{12}\text{C}$ fusion becomes ineffective.

E Tabulated Outflow Properties

F Self-Similar Disk Solutions

The set of disk evolution equations presented in 3.2 are well known to permit a variety of similarity solutions. This Appendix briefly summarizes, extends, and organizes the large number of such solutions presented in the literature (e.g. Cannizzo et al. 1990, Pringle 1991, Phinney & Hansen 1993, Metzger et al. 2008b, and references therein). We separately describe the RIAF, radiative, and irradiated phases.
<table>
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<th>(^4\text{He}) \times 10^{-3}</th>
<th>(^{12}\text{C}) \times 10^{-1}</th>
<th>(^{16}\text{O}) \times 10^{-1}</th>
<th>(^{20}\text{Ne}) \times 10^{-3}</th>
<th>(^{24}\text{Mg}) \times 10^{-3}</th>
<th>(^{28}\text{Si}) \times 10^{-3}</th>
<th>(^{32}\text{S}) \times 10^{-3}</th>
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Table E1: Ejected mass in various elements at the simulation end time, \(t_{\text{end}}\).
F1 RIAF Phase

During the RIAF phase, the radius and accretion rate of the outer disk evolve in time as

\[ R_d \propto t^{2/3}, \quad (F1) \]
\[ \dot{M}_d \propto t^{-{(2p+4)/3}}. \quad (F2) \]

Assuming that radiation provides the dominant component of the midplane pressure, the surface density, temperature and aspect ratio evolve as

\[ \Sigma \propto r^{p-1/2}t^{-4(p+1)/3}, \quad (F3) \]
\[ T \propto r^{(p-5/2)/4}t^{-(p+1)/3}, \quad (F4) \]
\[ \theta \propto r^0t^0. \quad (F5) \]

F2 Radiative Phase

During the radiative phase, a range of different solutions are permitted, depending on the opacity law and the vertical optical depth of the disk. Assuming that gas pressure dominates and adopting a general opacity law of the form \( \bar{\kappa} \propto \rho l T^{-k} \) (eq. G1), one can manipulate the energy equation to obtain

\[ \theta \propto \Sigma^m r^n \quad (F6) \]

where

\[ m = \frac{\eta(l + 1) + 1}{6 + \eta(2k + l)}; \quad n = \frac{\eta(k + l) + 3/2}{6 + \eta(2k + l)}; \quad (F7) \]

\[ \eta \equiv \frac{\partial \ln f(\tau)}{\partial \ln \tau} = \begin{cases} 1; & \tau \geq \sqrt{2/3} \\ -1; & \tau < \sqrt{2/3}, \end{cases} \quad (F8) \]

and \( f(\tau) \) is our approximation to the flux function given by equation (3.10).
In terms of these shorthand variables, the radius and mass of the disk evolve as

\[ R_d \propto t^{1/(3/2-2n+5m)} , \quad (F9) \]

\[ \dot{M}_d \propto t^{-(2-2n+5m)/(3/2-2n+5m)} , \quad (F10) \]

The solution for the local disk variables then follows

\[ \Sigma \propto r^{-(1/2+2n)/(2m+1)} \dot{M}^{1/(2m+1)} , \quad (F11) \]

\[ T \propto r^{-(3m-2n+1)/(2m+1)} \dot{M}^{2m/(2m+1)} , \quad (F12) \]

\[ \theta \propto r^{(n-m/2)/(2m+1)} \dot{M}^{m/(2m+1)} . \quad (F13) \]

We have intentionally written the local variables \( \Sigma, T, \) and \( \theta \) as a function of the control parameter \( \dot{M} \), and do not explicitly substitute equation (F9). This is because the outer disk evolves independently of the inner disk and at most times will reside in an alternative opacity regime, i.e. the values of \( m, n \) appropriate to the current state of the outer disk can differ from those at some smaller radii where equation (F11) is to be applied.

**F3 Irradiated Phase**

Using the same notation as for radiative phase from equation (F6), we obtain the same class of solutions described by equations (F9,F11), except that in this case

\[ m = \begin{cases} 
0; & \dot{M} \geq \dot{M}_{\text{Edd}} \\
1/5; & \dot{M} < \dot{M}_{\text{Edd}} 
\end{cases} \quad , \quad n = \begin{cases} 
2/7; & \dot{M} \geq \dot{M}_{\text{Edd}} \\
1/2; & \dot{M} < \dot{M}_{\text{Edd}}. 
\end{cases} \quad (F14) \]

The solution in the irradiated regime therefore differs chiefly based on whether the accretion rate is above or below the Eddington limit.
G Opacity Curve

As shown in Fig. G1, we employ a broken power-law approximation to the Planck-averaged mean opacity of the form

\[ \tilde{\kappa} = \tilde{\kappa}_0 \rho T^{-k}, \]  

(G1)

which qualitatively mimics the main features expected from the nearly pure carbon/oxygen composition of the disrupted WD disk. The analytical tractability allowed by this form is convenient for permitting self-similar solutions in the radiative regime (Appendix F). Apart from the recombination “cliff” below \( T \approx 8000 \text{ K} \), the opacity curve transitions continuously between different regimes.

At the highest temperatures, where the gas is at least partially ionized, we employ OPAL\(^7\) opacity tables for an assumed composition of half carbon and half oxygen by mass (Iglesias & Rogers 1996). These are shown as open triangles in Fig. G1 for three representative densities of \( 10^{-6} \text{ g cm}^{-3} \) (orange), \( 10^{-7} \text{ g cm}^{-3} \) (brown), and \( 10^{-8} \text{ g cm}^{-3} \) (red). The opacities converge at high temperatures, \( T \gtrsim 10^6 \text{ K} \), where electron scattering dominates (\( \tilde{\kappa} = 0.2 \text{ cm}^2 \text{ g}^{-1} \)), as well as at the recombination interface at \( T \approx 8000 \text{ K} \). At intermediate temperatures, the opacity increases with density, approximately as \( \rho^{0.8} \). Although our parameterization does not capture some of the more subtle details (such as the “wiggles” around \( T \sim 2 \times 10^4 \text{ K} \)), it provides an reasonable first-order approximation to the OPAL opacities.

Below the recombination threshold and before dust condensation, in what is commonly referred to as the ‘opacity-gap’, the opacity is somewhat uncertain. Atomic and molecular opacity obtained for our composition using the AESOPUS\(^8\) code (Marigo & Aringer 2009) indicates that the minimal opacity at relevant disk densities

\[ \rho_p \approx 3 \times 10^{-8} \text{ g cm}^{-3} \left( \frac{M_p}{100M_{\odot}} \right) \left( \frac{T_p}{2000 \text{ K}} \right)^{-1/2} \times \left( \frac{R_p}{0.4 \text{ AU}} \right)^{-7/2} \left( \frac{\mu}{28} \right)^{1/2} \]  

(G2)

is \( \tilde{\kappa} \sim 10^{-2} \text{ cm}^2 \text{ g}^{-1} \), and roughly scales as \( \sim \rho^{0.5} \).

At temperatures below a few thousand K, solids can condense and grow, and the opacity

\(^7\)http://opalopacity.llnl.gov/opal.html  
\(^8\)http://stev.oapd.inaf.it/cgi-bin/aesopus
Figure G1 Opacity curve for pure C/O matter as a function of temperature. Open triangles denote opacities obtained from the OPAL project for an ionized carbon-oxygen gas (equal by number). These are plotted at three representative densities (different colors). At lower temperatures, the graphite opacity for an MRN grain size distribution is plotted as open blue circles, and for a fixed grain size of $a = 10^{-3}$ µm as purple crosses. The illustrated graphite opacities are for a dust-to-gas ratio of 0.1, and the results scale linearly with this parameter. At intermediate temperatures, atomic/molecular opacity for $\rho = 10^{-8}$ g cm$^{-3}$ obtained using the AESOPUS code is plotted as the red-dotted curve. The black curves depict our adopted power-law approximation to the opacity curve.

becomes dominated by dust. Given the disk composition in our scenario, the dust composition will be dominated by carbonaceous grains (predominantly graphite) because oxygen on its own cannot condense into silicate grains. We adopt opacities in this range based on the Planck-averaged graphite cross sections calculated by Draine & Lee (1984) and Laor & Draine (1993). These are converted to opacities taking a graphite density of $\approx 2.2$ g cm$^{-3}$, as shown with open blue circles and purple crosses in Fig. G1. We assume a fiducial dust-to-gas ratio of 10% (see §3.4 for further discussion), and that the graphite opacity simply scales linearly with this ratio if other values are assumed. The opacity law in this region is well fit by a three-component power-law in temperature (the opacity is essentially independent of density).

The nominal opacities calculated by Draine & Lee (1984) (blue circles in Fig. G1) assume a canonical MRN grain size distribution (Mathis et al. 1977) of

$$\text{dn} \propto a^{-3.5} \text{da} \ . \quad (G3)$$

However, a nearly identical opacity law results by instead assuming a single grain size of $a = a_{\text{min}} = 10^{-3}$ µm (purple crosses in Fig. G1). This occurs because, for grains smaller
than \( \sim 0.03 \, \mu m \), the dipole approximation holds well, and the grain cross-section per unit mass is nearly independent of grain size. Therefore, as long as the grain size distribution decays steeply with \( a \) (such as characterizes the MRN distribution), then the opacity will be dominated by grains with \( a \sim a_{\text{min}} \). However, as long as \( a_{\text{min}} \lesssim 0.03 \, \mu m \), then the exact value of \( a_{\text{min}} \) is inconsequential and hence neither is the precise size distribution.

### H Self-Similar Jet Solutions

Here we extend the results of [Bromberg et al. (2011)](Bromberg et al. 2011) to the case of a non-relativistic collimated jet propagating within a time-dependent power-law density profile

\[
\rho \propto r^{-\delta} t^{-\beta}, \tag{H1}
\]

and a power-law jet luminosity

\[
L_j \propto t^{-\ell}. \tag{H2}
\]

As pointed out in §6.3.2 and shown later in this Appendix, there is no solution for \( \ell \geq 1 \). Therefore, one should keep in mind \( \ell \approx 0 \) as the canonical case, appropriate for the early \((t \lesssim t_e)\) stages of the central-engine evolution, where the injected power is approximately temporally constant.

The, isobaric, cocoon pressure can be expressed as

\[
P_c = \frac{E_c}{3 V_c} \approx \frac{\int_0^t L_j(t')dt'}{3 \int_0^{z_h} \pi \left( \int_0^t v_c(z_h, t')dt' \right)^2 dz'}, \tag{H3}
\]

where we have neglected terms of order \( v_h/c \), and where

\[
v_c(z_h, t) = \sqrt{P_c(t)/\rho \rho(z_h, t)}, \quad \rho = 3/(3 - \delta) \tag{H4}
\]

is the cocoon’s average (lateral) expansion velocity ([Bromberg et al. 2011](Bromberg et al. 2011)).

Assuming a power-law temporal evolution of the cocoon pressure and jet-head position,
$P_c \propto t^a$, $z_h \propto t^b$, the cocoon volume can be integrated to obtain

$$V_c(t) = \frac{\pi}{\rho A^2} \times \frac{P_c(t)z_h(t)t^2}{\rho(z_h(t), t)},$$  \hspace{1cm} (H5)$$

where

$$A = \frac{2 + \beta + a + b\delta}{2}.$$  \hspace{1cm} (H6)

This, via equation (H3), yields the cocoon pressure

$$P_c = \sqrt{\frac{\rho A^2 \zeta_z}{3\pi(1 - \ell)}} \times \sqrt{\frac{L_j(t)\rho(z_h(t), t)}{z_h(t)^2/v_h(t)}}.$$  \hspace{1cm} (H7)

Here we have substituted $z_h(t) = \zeta_z v_h(t)t$, which defines an integration constant $\zeta_z$ to be specified later.

Bromberg et al. (2011) find that the jet cross-section in the collimated regime is $A_j = L_j/4\gamma_j^2cP_c$, which allows us to find the dimensionless jet parameter $\tilde{L} = L_j/A_j\rho c^3$ and hence the jet-head velocity

$$v_h c \approx \tilde{L}^{1/2} = \left(\frac{4\gamma_j^2 P_c}{\rho c^2}\right)^{1/2}.$$  \hspace{1cm} (H8)

Substituting $P_c$ which itself depends on the jet-head velocity, we obtain

$$v_h(t) = \left[\zeta_L \times \frac{L_j(t)\gamma_j^4}{\rho(z_h(t), t) z_h(t)^2}\right]^{1/3}, \hspace{1cm} \zeta_L \equiv \frac{16\rho A^2 \zeta_z}{3\pi(1 - \ell)}.$$  \hspace{1cm} (H9)

Identifying $v_h = dz_h/dt$ and integrating yields the power-law solution for the jet-head propagation

$$z_h(t) \propto t^b, \hspace{1cm} b = \frac{3 + \beta - \ell}{5 - \delta}.$$  \hspace{1cm} (H10)

This along with equation (H7) yields the closure relation

$$a = -\frac{1 + \beta + \ell + b(\delta + 1)}{2} = -\frac{4 + \delta - \delta\ell + 2\ell + 3\beta}{5 - \delta},$$  \hspace{1cm} (H11)
so that

$$\zeta_L = \frac{16(3 + \beta - \ell)}{\pi(3 - \delta)(5 - \delta)(1 - \ell)}, \quad (H12)$$

and $\zeta_z = 1/b$, i.e.

$$\zeta_z = \frac{5 - \delta}{3 + \beta - \ell}. \quad (H13)$$

Two useful cases of equations $\text{(H12, H13)}$ are for: (a) homologous expansion, $\beta = 3 - \delta$, as suitable for the expanding SN ejecta density profile (equation $6.12$) and (b) stationary medium, $\beta = 0$, which describes the initial stellar progenitor profile (equation $6.8$; here “$\delta$” should be identified with $w$). In either case, the jet can only break-out successfully before the engine turn-off time, $t_e$, so that we take $\ell = 0$ (i.e. constant engine power) in equations $(\text{H12, H13})$ as the fiducial case.

I Jet-energized-wind model

The approximate light-curve produced by the jet-energized wind can be found by integrating a simple variation to the standard one-zone SN equations on the wind internal energy $E(t)$, velocity $v(t)$ (= $v_w$ in this scenario), outer radius $R(t)$, and total mass $M(t)$ (e.g. Metzger et al. 2015):

$$\frac{dE}{dt} = -E \frac{v}{R} - E \frac{4\pi cR}{3\kappa M} + \dot{E}_w; \quad (I1)$$
$$\frac{dv}{dt} = EM/R; \quad \frac{dR}{dt} = v; \quad \frac{dM}{dt} = \dot{M}_w. \quad (I2)$$

We assume that the wind velocity is roughly constant $v \approx v_w$ and there is no appreciable initial mass (and hence energy) in the outflow, i.e. $t_{\dot{M}} < t_{\text{exp},0}$, where $t_{\text{exp},0} = R_0/v_0$ is the initial expansion timescale and $t_{\dot{M}} = M_0/\dot{M}_w$ is mass-loss timescale. In this case, the peak emission time (set by equating the diffusion timescale to the expansion timescale) is given by equation $(6.36)$, $t_{pk} = 3\kappa \dot{M}_w/4\pi c v_w$, and the energy equation $(I1)$ can be recast in terms
of the radiated luminosity $L$ (second term on the RHS of equation (I1)),

$$\frac{dL(t)}{dt} = -L(t) \frac{t_{\text{exp,0}} + t_{\text{pk}}}{t_{\text{pk}}(t_{M} + t)} + \frac{\dot{E}_{w}(t)}{t_{\text{pk}}(t_{M} + t)}. \tag{I3}$$

Note that the time $t$ in all equations in this appendix is measured with respect to the onset of the jet-energized wind, i.e. $t = 0$ here is equivalent to $t = t_{\text{bo}}$ in previous sections (where time was measured with respect to the SN explosion epoch). In the limits $t \ll t_{M}, t_{\text{exp,0}} \ll t_{\text{pk}}$, the approximate solution to equation (I3), assuming also $\dot{E}_{w} \sim \text{constant}$ (i.e. $t \ll t_{e}$), is

$$L(t) \approx \frac{\dot{E}_{w}}{t_{\text{pk}}}. \tag{I4}$$

Next, in the limits $t_{M}, t_{\text{exp,0}} \ll t \ll t_{\text{pk}}$, equation (I3) reduces to

$$\frac{dL}{dt} \approx -\frac{L}{t} + \frac{\dot{E}_{w}}{t_{\text{pk}}} \tag{I5}$$

whose solution (again, for constant wind power) is

$$L(t) \approx \dot{E}_{w} \left( \frac{t - t_{\text{exp,0}}}{2t_{\text{pk}}} + \frac{t_{\text{exp,0}}^{2}}{t_{\text{pk}}t} \right). \tag{I6}$$

Finally, in the limit where $t_{M}, t_{\text{exp,0}} \ll t_{\text{pk}} \ll t$, we find

$$L(t) \approx \dot{E}_{w} \left( 1 - \frac{1}{2} e^{-t/t_{\text{pk}}} \right). \tag{I7}$$

Equations (I1,I2) should generally be integrated numerically to produce model light-curves (this is the procedure adopted in creating Fig. 6.4 for example), but we find that a convenient analytic functional form which schematically tracks the bolometric light-curve in the limits described above is

$$L(t) \sim \dot{E}_{w}(t) \times \left[ 1 - e^{-(t+t_{\text{exp,0}})/t_{\text{pk}}} \right]. \tag{I8}$$

This analytic form may be more easily implemented in fitting procedures, and for the most
part produces similar light-curves to the full calculation, but should non-the-less be used with caution.

J Temperature Profile of Hydrogen-Rich Ejecta

For hydrogen-rich ejecta, the radial profile of the electron temperature, $T_e(r, t)$, can be estimated analytically by considering what sources of heating $\Gamma$ and cooling $\Lambda$ balance on different radial scales.

Absent internal sources of heating (e.g. radioactivity) and neglecting the reverse shock (Appendix K), the ejecta heating is determined by the incident radiation field from the central engine. Compton heating due to inelastic electron scattering occurs at a rate (per unit volume) given by

$$\Gamma_{\text{comp}} = f_{\text{ion}} n \int \frac{\sigma_T u_\nu}{m_e c} h \nu d\nu \approx f_{\text{ion}} n \frac{\sigma_T}{4\pi m_e c^2} \nu L_{e,\nu} (\nu_{\text{max}} - \nu_{\text{min}})$$

where $\sigma_T$ the Thomson cross-section and $u_\nu(r)$ is the radiation energy density. We have assumed in the second line a logarithmically flat ionizing spectrum, $u_\nu \propto \nu^{-1}$ between $\nu_{\min}$ and $\nu_{\max}$, and have neglected radial attenuation of the radiation energy density; the latter is a reasonable approximation for the pure-hydrogen nebula since only photons near the ionization threshold $h\nu \sim 13.6\,\text{eV}$ are absorbed.

Photo-ionization (photo-electric) heating occurs at the rate

$$\Gamma_{\text{pe}} = (1 - f_{\text{ion}}) n \int \frac{\sigma_{\text{pe}}(\nu) u_\nu}{h \nu} (h \nu - h \nu_0) d\nu \approx \alpha_B f_{\text{ion}}^2 n^2 \frac{h \nu_0}{3}$$

where $n$ is the ejecta number density. In the second equality we have assumed ionization-recombination equilibrium, where $\alpha_B$ is the case-B recombination coefficient and $\approx h \nu_0/3$ is the mean energy per photo-ionization for a typical dependence $\sigma_{\text{pe}}(\nu) \propto \nu^{-3}$ of the cross-section for $\nu \geq \nu_0$ and $u_\nu \propto \nu^{-1}$.

Compton and radiative-recombination cooling can be expressed similarly to the heating
terms above,

\[ \Lambda_{\text{comp}} = f_{\text{ion}} n \int \frac{\sigma_T u_\nu}{m_e c} 4 k_B T_e \, d\nu \]

\[ \approx f_{\text{ion}} n \frac{\sigma_T \nu L_{\text{e,}\nu}}{4 \pi m_e c^2 r^2} 4 k_B T_e \ln \left( \frac{\nu_{\text{max}}}{\nu_{\text{min}}} \right) \]

and

\[ \Lambda_{\text{rr}} = \alpha_B(T_e) f_{\text{ion}}^2 n^2 \frac{3}{2} + \left( \frac{\partial \ln \alpha_B}{\partial \ln T_e} \right) k_B T_e, \]

\[ (J4) \]

where the term in brackets is the average energy loss per recombination. Finally, free-free cooling occurs at a rate

\[ \Lambda_{\text{ff}} = f_{\text{ion}}^2 n^2 \lambda_{\text{ff}}(T_e) \approx f_{\text{ion}}^2 n^2 \lambda_{\text{ff},0} T_e^{1/2}, \]

\[ (J5) \]

where \( \lambda_{\text{ff}}(T_e) \approx \lambda_{\text{ff},0} T_e^{1/2} \) for temperatures near \( T_e \sim 10^4 \) K and \( \lambda_{\text{ff},0} \approx 1.42 \times 10^{-27} \) in appropriate cgs units. Since we focus here on pure-hydrogen composition, we do not consider line cooling by metals, even though the latter dominates free-free cooling for O-rich ejecta composition.

Balancing various heating and cooling terms (\( \Gamma = \Lambda \)), we distinguish three regimes relevant at increasing radii within the ejecta. At small radii, Compton heating balance Compton cooling (\( \Gamma_{\text{comp}} = \Lambda_{\text{comp}} \)), and the electron temperature equals the “Compton Temperature” of the radiation field,

\[ T_e = \frac{h (\nu_{\text{max}} - \nu_{\text{min}})}{4 k_B \ln (\nu_{\text{max}}/\nu_{\text{min}})} \propto r^0 \nu^0. \]

\[ (J6) \]

The radially- and temporally-constant value of \( T_e \) is just a consequence of our assumption that the shape of the spectral energy distribution of the nebula radiation is fixed. However, the Compton cooling rate decreases with radius as \( r^{-2} \) or steeper, such that at sufficiently large radii \( \Lambda_{\text{ff}} \gg \Lambda_{\text{comp}} \), and the temperature is instead set by the balance \( \Gamma_{\text{comp}} = \Lambda_{\text{ff}} \), giving

\[ T_e \approx \left[ \frac{h (\nu_{\text{max}} - \nu_{\text{min}})}{4 \pi \lambda_0 m_e c^2 f_{\text{ion}} n r^2} \right]^2 \propto f_{\text{ion}}(r, t)^{-2} r^{-4} \nu^2. \]

\[ (J7) \]

The temporal and radial scaling here apply for the case where \( L_{\text{e,}\nu} \propto t^{-2} \) and a radially constant (homologously expanding) density. Note that the temperature in this region drops
dramatically, roughly as $r^{-4}$, from the $\sim 10^7$ K Compton temperature down to $\sim 10^4$ K at which photo-electric heating and both free-free and radiative recombination cooling terms become dominant. Note also that $\Lambda_{\text{comp}} \propto t^{-7} T_e(t)$ while $\Lambda_{\text{ff}} \propto t^{-6} T_e(t)^{1/2}$ at a given radius, so that the transition between the Compton cooled and free-free cooled regions moves to smaller radii as time progresses.

Finally, in the outer layers of the ejecta, photo-electric heating is balanced by both free-free and radiative-recombination cooling, which are comparable to one another for $T_e \sim 10^4$ K. Setting $\Gamma_{\text{pe}} = \Lambda_{\text{ff}}$ results in

$$T_e \approx \frac{h \nu_0}{3 k_B} \left[ \frac{3}{2} + \left( \frac{\partial \ln \alpha_B}{\partial \ln T_e} \right)^{-1} \right] \approx 8.2 \times 10^4 K \propto r^0 t^0,$$

while equating $\Gamma_{\text{pe}} = \Lambda_{\text{ff}}$ results in

$$T_e \sim \left( \frac{h \nu_0/3}{T_e^{\alpha_B}} \right)^{-1} \left[ \frac{3}{2} \left( \frac{\partial \ln \alpha_B}{\partial \ln T_e} \right)^{-1} \right] \approx 5.3 \times 10^4 K \propto r^0 t^0,$$

where we have here written $\alpha_B(T_e) = \alpha_{B,0} T_e^{(\partial \ln \alpha_B/\partial \ln T_e)}$, and assumed for Hydrogen recombination that $\alpha_{B,0} = 4.68 \times 10^{-10}$, and $(\partial \ln \alpha_B/\partial \ln T_e) = -0.8163 - 0.0208 \ln(T_e/10^4 K)$ in the vicinity of $T_e \sim 10^4$ K [Draine 2011; his eq. 14.6].

**K Contribution of the Reverse Shock to Ejecta DM**

Here we present a detailed analytic estimate of the maximum DM contributed by the ejecta which has been shock-heated by its interaction with the ambient circumstellar material of assumed density $\rho_{\text{csm}}$. We focus on the early “ejecta-dominated” phase, relevant at times $t \ll t_{\text{ST}} \sim 10^3$ yr (eq. 7.19). We utilize the solutions described by Truelove & McKee (1999) and adopt the same notation as in that paper, to which we refer the reader for additional details on the dynamics of the blast wave and reverse shocks.

The forward shock radius $R_b(t)$, for an ejecta of mass $M_{\text{ej}}$, total energy $E$, and a density profile characterized by a constant density core and an outer power-law envelope $\rho \propto r^{-n}$,
is given by the following implicit relationship,

\[
\frac{R_b^*}{t^*} = \left( \frac{\alpha}{2} \right)^{-1/2} \ell_{ED} \left[ 1 + \frac{n-3}{3} \left( \frac{\phi_{ED}}{\ell_{ED} f_n} \right)^{1/2} R_b^{3/2} \right]^{-2/(n-3)}.
\] (K1)

Here the dimensionless physical variables demarcated \(X^* \equiv X/X_{ch}\) are normalized by their characteristic values,

\[
M_{ch} = M_{ej}; \quad R_{ch} = M_{ej}^{1/3} \rho_{csm}^{-1/3}; \quad t_{ch} = E^{-1/2} M_{ej}^{5/6} \rho_{csm}^{-1/3},
\] (K2)

where \(\alpha, \phi_{ED}\) and \(\ell_{ED}\) are constants which depend on the power-law index \(n\) (Truelove & McKee 1999). The reverse shock radius \(R_r\) in this ejecta-dominated phase is simply related to the blast-wave radius by the lead factor, i.e. \(R_r = R_b / \ell_{ED}\).

At times \(t^* \ll t_{CN}^*\) the first term in brackets in equation (K1) is the dominant one, and the solution reduces to free expansion,

\[
R_b^*(t \ll t_{CN}^*) \approx \left( \frac{\alpha}{2} \right)^{-1/2} \ell_{ED} t^*.
\] (K3)

where

\[
t_{CN}^* = \left( \frac{n-3}{3} \right)^{-2/3} \left( \frac{\phi_{ED}}{f_n} \right)^{-1/3} \ell_{ED}^{-2/3} \left( \frac{\alpha}{2} \right)^{1/2} \sim 6.47 \times 10^{-5} \left( \frac{w_{core}}{10^{-2}} \right)^2
\] (K4)

is the onset time of the Chevalier (1982), Nadezhin (1985) solution. Here \(f_n\) is another constant defined by Truelove & McKee (1999), \(w_{core} \equiv v_{core} / v_{ej}\), and \(v_{core}\) is the velocity at which the density transitions between the flat core and power-law envelope. Note that the second equality above is given in the limit \(w_{core} \ll 1\).

At late times \(t^* \gtrsim t_{CN}^*\), the second term in brackets of (K1) instead dominates and the
blast wave radius instead evolves as a power-law that depends on the density profile,

\[ R_b^*(t_{CN}^{*} \lesssim t^* \lesssim t_{core}^*) \approx \left( \frac{n - 3}{n} \right)^{-2/n} \left( \frac{\alpha}{2} \right)^{-(n-3)/2n} f_n^{(n-2)/n} \]

\[ \times \left( \frac{\phi_{ED}}{f_n} \right)^{-1/n} t^*^{(n-3)/n}. \]  

(K5)

This persists until the time \( t_{core}^* \) at which the reverse shock reaches the core-envelope transition, which we estimate from equation (K5) to be

\[ t_{core}^* \approx \left( \frac{n - 3}{3} \right)^{-2/3} \phi_{ED}^{-1/3} \ell_{ED}^{-2/3} f_n^{1/3} \left( \frac{\alpha}{2} \right)^{1/2} w_{core}^{-n/3} \approx 0.647, \]  

(K6)

independent of \( w_{core} \).

Given these expressions for the shock dynamics, we now estimate the accumulation of shocked ejecta with time. Using expressions for the ejecta mass above normalized velocity coordinate \( w_r = R^*_b/v^*_e t^* \) and remembering that \( R^*_b(t^*) = R_b^*(t^*)/\ell_{ED} \) in the ejecta-dominated phase applicable at \( t^* \ll t_{CN}^* \), we find

\[ M_{sh,r}^*(t^*) = \frac{1 - w_r(t^*)^{-(n-3)}}{1 - (n/3)w_{core}^{-(n-3)}}. \]

(K7)

Expanding equation (K1) as a Taylor-series in \( 1 - w_r \ll 1 \) (as applicable at \( t^* \ll t_{CN}^* \)), and using the approximate free-expansion solution, we find

\[ 1 - w_r(t^*) \approx \frac{2}{3} \left( \frac{\phi_{ED}}{f_n} \right)^{1/2} \left( \frac{\alpha}{2} \right)^{-3/4} \ell_{ED} t^*^{3/2}, \]  

(K8)

and thus \( M_{sh,r}^*(t^* \ll t_{CN}^*) \propto t^{3/2} \). At later times, \( t_{CN}^* \lesssim t^* \lesssim t_{core}^* \), it is easy to show that \( w_r \propto t^{-(n-3)/n} \), and thus \( M_{sh,r}^*(t_{CN}^* \lesssim t^* \lesssim t_{core}^*) \propto t^{3(n-3)/n} \).

The ionization fraction of the ejecta, as results from heating due to the reverse shock, depends on details such as the ejecta composition and cooling, both radiative and from subsequent adiabatic expansion. Here we estimate the largest possible contribution to the ionized ejecta by making the generous assumption of negligible cooling and complete ionization of shocked matter at all subsequent times. In this case, the mass-averaged ionization
fraction of the ejecta simply becomes

$$\langle f_{\text{ion}} \rangle_m = M_{\text{sh, r}}^* (t^*) \quad (K9)$$

The density-averaged ionization fraction, relevant to calculating the DM, depends on the post-shock density profile, which is not easily described analytically. However, the distribution of shocked ejecta matter between $R_r$ and the contact discontinuity will introduce at most an order unity correction to $\langle f_{\text{ion}} \rangle$. Again taking the most conservative scenario (largest possible $\langle f_{\text{ion}} \rangle$) in which the entire shocked mass is concentrated at the reverse shock, $\rho \sim \delta (r - R_r)$, we find

$$\langle f_{\text{ion}} \rangle \rho \lesssim \frac{n - 1}{3n} \left( \frac{\alpha}{2} \right)^{-1} t_{\text{ED}}^2 w_{\text{core}} M_{\text{sh, r}}^* \left( \frac{R_b}{t^*} \right)^{-2} \quad (K10)$$

$$\propto \begin{cases} t^{*-1/2}, & t^* < t_{\text{CN}}^* \\ t^{*(n-3)/n}, & t_{\text{CN}}^* < t^* < t_{\text{core}}^* \end{cases}.$$ 

The density-averaged ionization fraction, and thus the DM, initially decreases with time in the free expansion phase, before increasing again at $t^* > t_{\text{CN}}^*$ (this result is only applicable to an ejecta with $n \gtrsim 5$). Since the $n > 5$ solution must converge to the $n = 0$ solution at $t^* \gtrsim t_{\text{core}}^*$, this implies that the ratio between the DM predicted by the $n > 5$ ejecta at early times and the constant density ($n = 0$) ejecta is limited to a maximum value,

$$\frac{\text{DM}_{n > 5}}{\text{DM}_{n = 0}} = \frac{\langle f_{\text{ion}} \rangle_{\rho, n > 5}}{\langle f_{\text{ion}} \rangle_{\rho, n = 0}} \lesssim \left( \frac{t_{\text{CN}}^*}{t_{\text{core}}^*} \right)^{(n-3)/n} = w_{\text{core}}^{(n-3)/3}. \quad (K11)$$

Thus, the maximum DM discrepancy between envelope-less and $n > 5$ envelope ejecta models is related only to the ratio between the core-envelope transition velocity and the outer (fastest) ejecta velocity. Given the total ejecta mass/energy budget, $w_{\text{core}}$ can be
expressed in terms of the (uncertain) outer ejecta velocity as

\[
\frac{w_{\text{core}}}{w_{\text{core}} \ll 1} \approx \left[ \frac{10(n-5)E}{3(n-3)M_{\text{ej}}v_{\text{ej}}^2} \right]^{1/2} \ \ \ \ (K12)
\]

\[
\approx 2.5 \times 10^{-2} \left( \frac{E}{10^{52}\text{ergs}} \right)^{1/2} \left( \frac{M_{\text{ej}}}{10M_{\odot}} \right)^{-1/2} \left( \frac{v_{\text{ej}}}{c} \right)^{-1}.
\]

For the reasonable assumption that \(v_{\text{ej}} \ll c\), we find \(w_{\text{core}} \approx 0.1\) as a reasonable estimate, in which case the ratio (K11) is at most a factor of a few for reasonable \(n\).

Finally, note that the swept-up shocked circumstellar material can also be important in contributing to the ionized column density. In the \(n = 0\) case, this is only important after \(t \gtrsim t_{\text{ST}}\), and thus on longer timescales than of typical interest in our scenario. For the case of an \(n > 5\) ejecta, the swept-up mass exceeds (yet remains comparable to) the shocked-ejecta mass already following \(t \gtrsim t_{\text{CN}}\); however the swept-up circumstellar mass only dominates the shocked ejecta \(M_{\text{sh,r}}\) by a significant amount at very late times \(t \gtrsim t_{\text{ST}}\).

### L DM Probability Distributions

The probability distribution of dispersion measures and their derivatives can be computed using our \textsc{Cloudy} photoionization calculations and under the assumption that the magnetar/ejecta parameters inferred for the adopted sample of SLSNe is characteristic of the underlying population. Focusing in particular on the chance of detecting an FRB with given DM and \(d\text{DM}/dt\), we assign an arbitrary probability metric of FRB detectability as a function of time, adopting a power-law parameterization, \(P(t) \propto t^{-\alpha}\) with a detectable activity lifetime \(t_a\). We expect \(\alpha\) and \(t_a\) to be related to some physical measure of burst detectability such as a possible decay in burst luminosity or repetition frequency with time, and adopt toy models with \(\alpha = 0\) and \(\alpha = 2\) and various values for \(t_a\).

The probability density function (PDF) of detecting an FRB with dispersion measure DM, given that the assumed SLSN progenitor has a free-free transparency time \(t_{\text{ff}}\) at radio frequencies and dispersion measure \(\text{DM}_{\text{ff}}\) at this time, is then

\[
P(DM|t_{\text{ff}}, \text{DM}_{\text{ff}}) = P(t|\text{DM}; t_{\text{ff}}, \text{DM}_{\text{ff}}) \times \left| \frac{d\text{DM}}{dt} \right|^{-1}, \ \ \ \ (L1)
\]
where $t(\text{DM})$ is given by inverting the dispersion measure temporal behavior. For O-rich ejecta we have shown in §7.3.3 that the ionization fraction of the ejecta remains approximately constant, and therefore $\text{DM}(t) = \text{DM}_{\text{ff}} (t/t_{\text{ff}})^{-2}$. Using this, in conjunction with the detection PDF at time $t$

$$P(t|\text{ff}) = \begin{cases} t^{-\alpha}, & t_{\text{ff}} < t < t_a \\ 0, & \text{else} \end{cases} \quad (L2)$$

normalized such that the integrated distribution over the detectable time-slot $t_{\text{ff}} < t < t_a$ is unity.

Combining the above equations we arrive at an expression for the PDF of measuring an FRB with dispersion measure $\text{DM}$, for a given set of SLSN parameters $t_{\text{ff}}, \text{DM}_{\text{ff}}$. It is

$$P(\text{DM}|t_{\text{ff}}, \text{DM}_{\text{ff}}) = \frac{1 - \alpha}{2\text{DM}_{\text{ff}}} \left[ \left( \frac{t_a}{t_{\text{ff}}} \right)^{1-\alpha} - 1 \right]^{-1} \left( \frac{\text{DM}}{\text{DM}_{\text{ff}}} \right)^{(\alpha-3)/2} \quad (L3)$$

To complete the calculation, we use the distribution of SLSNe parameters $t_{\text{ff}}, \text{DM}_{\text{ff}}$ as found by our CLOUDY calculations and apply Bayes’ law to obtain

$$P(\text{DM}) = \int dt_{\text{ff}} d\text{DM}_{\text{ff}} P(\text{DM}|t_{\text{ff}}, \text{DM}_{\text{ff}}) P(t_{\text{ff}}, \text{DM}_{\text{ff}}). \quad (L4)$$

where, for our finite sample of SLSNe parameters

$$P(t_{\text{ff}}, \text{DM}_{\text{ff}}) \approx \frac{1}{N} \sum_{i=1}^{N} \delta(t_{\text{ff},i}) \delta(\text{DM}_{\text{ff},i}), \quad (L5)$$
and thus, the final observable FRB DM distribution is

$$P(\text{DM}) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1 - \alpha}{2 \text{DM}_{\text{ff},i}} \left[ \left( \frac{t_a}{t_{\text{ff},i}} \right)^{1-\alpha} - 1 \right]^{-1} \left( \frac{\text{DM}}{\text{DM}_{\text{ff},i}} \right)^{(\alpha - 3)/2} \times \begin{cases} 1, & \frac{\text{DM}}{\text{DM}_{\text{ff},i}} \in \left[ 1, \left( \frac{t_a}{t_{\text{ff},i}} \right)^{-2} \right] \\ 0, & \text{else} \end{cases}$$

(L6)

where the index $i$ enumerates the free-free transparency time $t_{\text{ff},i}$ and the dispersion measure at that time $\text{DM}_{\text{ff},i}$ for the $i$th SLSN in our sample.

Since $d\text{DM}/dt = -2\text{DM}/t$, a similar analysis can be performed for the DM derivative, resulting in

$$P \left( \frac{d\text{DM}}{dt} \right) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{(1 - \alpha)t_{\text{ff},i}}{2 \text{DM}_{\text{ff},i}} \left[ \left( \frac{t_a}{t_{\text{ff},i}} \right)^{1-\alpha} - 1 \right]^{-1} \left( -\frac{t_{\text{ff},i}}{2 \text{DM}_{\text{ff},i}} \frac{d\text{DM}}{dt} \right)^{(\alpha - 4)/3} \times \begin{cases} 1, & \left| \frac{d\text{DM}}{dt} \right| \in \left[ \frac{2 \text{DM}_{\text{ff},i}}{t_{\text{ff},i}}, \left( \frac{t_a}{t_{\text{ff},i}} \right)^{-3} \right] \\ 0, & \text{else} \end{cases}$$

(L7)