Modes of Acquisition of Shanghai Mathematics Teachers’ Pedagogical Content Knowledge
within Communities of Practice

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ABSTRACT

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The purpose of this study was to determine the modes of acquisition of Shanghai elementary mathematics teachers’ pedagogical content knowledge within their communities of practice. This study uses the qualitative multiple-case study with a survey research approach with two teachers in two public elementary schools, one each from an urban and a suburban district of Shanghai. In total, forty-four teachers, four teaching research coordinators in the two districts and city, one university professor, and four school administrators were involved in the study. The study shows that Shanghai elementary mathematics teachers acquire and develop their pedagogical content knowledge through positive mentorship; active participation in Teaching Research Group activities in the schools, districts, and city; and informal and formal communications with their colleagues in their school communities. The teaching research coordinators help teachers to better understand the elementary mathematics curriculum, topics, and teaching materials, and students’ learning of mathematics. School policies encourage, support, and ensure that teachers’ professional learning and development occur through their participation within teacher-supported communities of practice. This study has implications for the teachers’ communities of practice, in that policy makers and school administrators should enable teachers to share their teaching practices to improve their mathematics pedagogical content knowledge, and therefore improve students’ learning of mathematics.
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CHAPTER I

INTRODUCTION

The People’s Republic of China is a largely monocultural society that has been governed by the Communist Party for more than 68 years. The central government’s Ministry of Education dictates the school curriculum for Grades 1-12 in most parts of mainland China except Shanghai. Education is compulsory from Grade 1 through 9 throughout China. Students usually spend six years in elementary school, and three years in middle school, or five years in elementary school and four years in middle school. After graduating from middle school, students can attend a vocational, technical school or high school for pursuing college level education in the future. Higher education has grown very rapidly since 1977 when the first National College Entrance Examination was administered after the Cultural Revolution. In 1977, 5.7 million students took the National College Entrance Examination (Gao Kao) with an admission rate of 4.8%, while 9.42 million students took the National College Entrance Examination in 2015 with an admission rate of 75% (Chinese Xinhua News Agency, 2015).

Shanghai is the largest city by population in China. It is one of the four province-level municipalities of the People’s Republic of China, with a total population of over 24 million in 2015. About 9.8 million of them are migrants (Shanghai Bureau of Statistics, 2016). Shanghai produces one fourth of China’s GDP. According to the 2010 census, about 22 percent of residents in Shanghai had college degrees, 21 percent had high school degrees, 36.5 percent middle school, and 1.35 percent primary school education; 2.74 percent of residents age 15 and older were illiterate (Shanghai Bureau of Statistics, 2011).

Need for the Study

Since the 1990s, the high quality of mathematics education in Asian countries, especially
in China, has received international attention (Cai, 1995; Correa, Perry, Sims, Miller, & Fang, 2008; Stevenson, Lee, & Stigler, 1986; Stevenson, et al., 1990). Chinese students’ mathematics performance surprised the world in the International Assessment of Educational Progress (IAEP) (Lapointe & Others, 1992), and the Third International Mathematics and Science Study (TIMSS) (Mullis, Martin, Gonzalez, & Chrostowski, 2004). In particular, Shanghai’s 15-year old students scored, on average, 600 on the mathematics scale in 2009, and 613 in the 2012 PISA, leading the world (OECD, 2010, 2014). In the 2009 PISA assessment, 5,115 Shanghai’s 15-year-olds, including migrant children, from 152 schools took the exam. The schools included middle schools (grades 6-9), middle-high schools (grades 6-12), and vocational schools, which are 89.9 percent public and 10.1 percent private. Of these schools, 39.5 percent are located in urban and 60.5 percent are in rural school districts in Shanghai (Lu, 2009). Some authorities credit the success of mathematics education in China to factors such as Chinese traditional cultural values (Hess, Chang, & McDevitt, 1987; Leung 1995, 2001; Li, 2004; Wong 2004); a coherent curriculum (Newton, 2007; Schmidt, Houang, & Cogan, 2002) that emphasizes computation and problem solving skills (Ni, Li, Li, & Zhang, 2011); rigorous assessment in school education, or the demands of difficult college entrance exams (Tu, 2009; Zhang, Li & Tang, 2004); mathematics teachers’ strong content knowledge (Cai, 2005; Ma, 1999; Stevenson & Stigler 1992); and a pattern of teaching with conceptual variation and procedural variation (Gu, Huang & Marton, 2004; Lopez-Real, Mok, Leung, & Marton, 2004).

More recently, reports uncover that portions of Shanghai’s success are based on its impressive education system, particularly its modes of development and management of teachers. A deep commitment to teacher training, peer learning, and professional development contributes to Shanghai’s successful education (Liang, Kidwai, & Zhang, 2016; Zhang, Ding, &
Xu, 2016). Pre-service elementary teachers in China are required to master strong mathematics content courses and method courses (Li, Zhao, Huang & Ma, 2008; Sun, 2000). In Shanghai, the pre-service elementary teacher education program prepares students who will specialize in teaching mathematics. They take many advanced mathematics courses such as Calculus I and II, Linear Algebra, Analytic Geometry, etc., for the bachelor’s degree in teacher education.

Mathematics content courses for pre-service elementary mathematics teachers are similar to those for pre-service middle school mathematics teachers in the U.S. (Yuan & Han, 2009). Research shows that teachers’ mathematics content and pedagogical content knowledge are related to student achievement (Ball, Thames & Phelps, 2008; Hill, Rowan & Ball, 2008; Ingvarson et al., 2013). The teachers’ knowledge of fundamental mathematics concepts, and their ability to communicate those concepts to their students, especially at the elementary level, is vital to students’ success then and later because mathematics relies so heavily on cumulative knowledge (Greenberg & Walsh, 2008). Ma’s research (1999) found not only that Shanghai elementary teachers demonstrate a profound understanding of fundamental mathematics, but that their depth of understanding is further developed during their careers. Other research (Wang, 2009; Zhang, 2009; Zhang & Zhu, 2009) also shows that Chinese teachers continue to develop professionally within their communities of practice. Chinese teachers’ professional learning activities are organized systematically through Teaching Research Groups (jiaoyanzu, 教研组) (Paine, 1990; Paine & Ma, 1993; Tsui & Wang, 2009; Zhang, Ding, & Xu, 2016).

However, there is little specific discussion about the mathematics preparation for in-service elementary teachers in China, especially during their professional development activities such as Teaching Research Group activities, and much less discussion about how Chinese in-service elementary mathematics teachers acquire and develop their pedagogical content
knowledge within their communities of practice. This also raises the important research problem of the role played by professional development for in-service elementary mathematics teachers’ acquisition of pedagogical content knowledge within their communities of practice in Shanghai.

**Purpose of the Study**

The purpose of this study is to define the acquisition modes of pedagogical content knowledge for Shanghai in-service elementary mathematics teachers within their communities of practice (Lave & Wenger, 1991; Wenger, 1998, 2000; Wenger, McDermott & Snyder, 2002). In this research, pedagogical content knowledge is defined as the knowledge of content and curriculum, knowledge of content and students, and knowledge of teaching (Ball, Thames & Phelps, 2008; Hill, Ball & Schilling, 2008; Shulman, 1986, 1987).

The first goal of this research study is to provide a general picture and in-depth examination of Shanghai in-service elementary mathematics teachers’ acquisition of pedagogical content knowledge within their communities of practice, especially within their Teaching Research Group activities. The second goal is to explore the professional assistance available for Shanghai mathematics teachers’ acquisition of their pedagogical content knowledge within Teaching Research Group activities in and out of the schools and districts. The third goal is to investigate school policies that enable Shanghai mathematics teachers’ acquisition of their pedagogical content knowledge within the communities of practice.

To address these goals, the three research questions for this study are as follows:

1. How do Shanghai in-service elementary mathematics teachers acquire pedagogical content knowledge within their communities of practice?

2. What do teaching research coordinators from the district and the city do to improve teachers’ mathematics pedagogical content knowledge within Teaching Research Group
activities?

3. How does the school administration’s support enable teachers’ acquisition of mathematics pedagogical content knowledge within those communities of practice?

**Procedure of the Study**

After obtaining IRB approval from Teachers College, Columbia University, I explained my study to the Shanghai city-level elementary mathematics teaching research coordinator and two district-level elementary mathematics teaching research coordinators. They strongly supported my research. They introduced me to two elementary schools, Rainbow Elementary School and Blue Sky Elementary School (pseudonyms), in their school districts. I described my study and sent a consent form (see Appendix A) to the two schools. Both of the schools welcomed my research and showed their enthusiasm and support for the study. Because of my schedule and Teaching Research Group activity schedules at the Rainbow and Blue Sky elementary schools, I visited Shanghai in early June 2015, late December 2015, and late December 2016 to collect the research data.

The study was conducted by using mixed methods (Creswell, 2009; Creswell & Plano Clark, 2007; Mertens, 2013; Yin, 2014). It was designed as a qualitative two-case study with a survey component (Stake, 2006; Yin, 2014). The purpose of the quantitative approach is to look for the general description of Shanghai teachers’ acquisition modes of their pedagogical content knowledge within their communities of practice. A questionnaire survey is a source for quantitative data for this study. A qualitative approach with case study methods is this study’s primary research method to seek the in-depth understanding the phenomena of Shanghai elementary mathematics teachers’ acquisition of pedagogical content knowledge within their communities of practice, the professional support, and school policies enabling the acquisition of
their pedagogical content knowledge. The qualitative data include the observations of the two classroom teaching and two Teaching Research Group activities; semi-structured interviews of teachers, mentors, teaching research coordinators in the district and city, and school administrators; teachers’ lesson plans; teachers’ reflection reports; school documents; and my field notes. First, I delivered the questionnaire survey to the Rainbow Elementary School. All mathematics teachers in the school participated and answered all questions in the survey questionnaire. I observed one teacher’s classroom teaching and one follow-up Teaching Research Group activity at Rainbow Elementary School. After the observations, I interviewed teachers, teaching research coordinators, and school administrators. Meanwhile, I collected the teacher’s lesson plan and reflection report. In addition, I took field notes during the observations and interviews. Second, in order to seek the findings with replication logic, I did the same research procedure to the Blue Sky Elementary School as I did to the Rainbow Elementary School.

To answer the first research question, the data from the questionnaire survey, interviews with two teachers and one mentor of one of the two teachers, observations of their teaching and the Teaching Research Group activities, teachers’ lesson plans and their reflection reports, and my field notes were analyzed. To answer the second research question, the data from the interviews with the mathematics teaching research coordinators, the observations of the Teaching Research Group activities, and my field notes explain and describe the professional help. To answer the third research question, the data from the interview with school administrators, the school documents, and my field notes explain the school policies which enable the teachers’ acquisition and development of pedagogical content knowledge within their communities of practice.
CHAPTER II

LITERATURE REVIEW

In this chapter, the discussion includes knowledge that mathematics teachers need (Brown & Borko, 1992; Fennema & Franke, 1992; Shulman, 1986); the concept of pedagogical content knowledge (Ball et al., 2008; Hill et al., 2008; Shulman, 1986; 1987), the correlation between teachers’ pedagogical content knowledge and their students’ learning and achievement (Baumert et al., 2010; Carpenter, Fennema, Peterson, & Carey, 1988; Hill, Rowan, & Ball, 2005; Leinhardt & Smith, 1984; Rowan, Chiang, & Miller, 1997), and in-service teacher training in China (Gu, Zhong, & Zhao, 2008; Paine, 1990; Paine & Fang, 2007; Paine & Ma, 1993; Zhang et al., 2016); I present the theoretical framework of communities of practice to guide the development of this study.

Knowledge for Teaching Mathematics

Shulman (1986) developed a coherent theoretical framework of teacher knowledge through a research on the development of secondary novice teachers in English, biology, mathematics, and social studies. He suggested that teacher knowledge grows in teaching, and “subject matter content knowledge, pedagogical content knowledge, and curricular knowledge” (Shulman, 1986, p. 9) are three categories of teacher content knowledge. Subject matter content knowledge refers to “the amount and organization of knowledge per se in the mind of the teacher” (Shulman, 1986, p. 9). Shulman defined pedagogical content knowledge as the subject matter knowledge for teaching. It represents “the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (Shulman, 1987, p. 8). It includes teachers’ methods of representing and formulating the content that makes
students understand; the teacher’s understanding of why some topics are easy or difficult for students to learn, in part because a diverse group of students has a diverse number of conceptions and preconceptions (Shulman, 1986). Curricula knowledge includes teacher knowledge of topics and instructional materials they teach and alternative curriculum materials of the subject their students learn. For an experienced teacher, the curricula knowledge also includes teacher knowledge of the connections of the curriculum they teach in the preceding and later years, and the knowledge related to their students learn in other subjects at the same time (Shulman, 1986).

Besides the teacher content knowledge, Shulman (1987) proposed that teacher knowledge also includes general pedagogical knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational ends, purposes, and values, and their philosophical and historical grounds from the general perspectives. General pedagogical knowledge especially refers to “broad principles and strategies of classroom management and organization that appear to transcend subject matter” (p. 8). According to Shulman, the range of knowledge of educational contexts is from “the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures” (p. 8).

Knowledge of teaching that mathematics teachers need is the combination of generic and subject matter specific knowledge (Brown & Borko, 1992). Fennema and Franke (1992) modified Shulman’s framework of teacher knowledge. They proposed that mathematics teachers’ knowledge develops in the context of classrooms. It includes four components: the content knowledge of mathematics, knowledge of pedagogy, knowledge of students’ cognitions, and teachers’ beliefs (Fennema & Franke, 1992). Each component is connected to each other. According to Fennema and Franke, the content of mathematics component includes “teacher
knowledge of the concepts, procedures, and problem-solving processes within the domain in which they teach, as well as in related content domains. It includes knowledge of the concepts underlying the procedures, the interrelatedness of these concepts, and how these concepts and procedures are used in various types of problem-solving” (Fennema & Franke, 1992, p. 162).

Pedagogical knowledge includes “teachers’ knowledge of teaching procedures such as effective strategies for planning, classroom routines, behavior management techniques, classroom organizational procedures, and motivational techniques” (Fennema & Franke, 1992, p. 162).

Learners’ cognitions includes “knowledge of how students think and learn and, in particular, how this occurs within specific mathematics content. This includes knowledge of how students acquire the knowledge of the mathematics content being addressed, as well as understanding the processes the students will use and the difficulties and successes that are likely to occur” (p. 162).

Fennema and Franke’s model of teachers’ knowledge shows that nature of teacher knowledge is interactive and dynamic. Teachers develop their knowledge for teaching through interactions with mathematics, with their students, and through their professional experiences (Fennema & Franke, 1992).

**Pedagogical Content Knowledge for Teaching Mathematics**

Based on the research on the difference between subject matter content knowledge and pedagogical content knowledge by Shulman (1986), Ball and her colleagues (2008) spent a year doing a qualitative analysis of the third grade in a public school mathematics classroom to more clearly clarify the distinction between these two pieces of knowledge. They refine Shulman’s categories of the mathematical knowledge for teaching as subject matter knowledge and pedagogical content knowledge (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008) in Fig. 2.1. The domain of subject matter knowledge is Common Content Knowledge (CCK),
Specialized Content Knowledge (SCK), and knowledge at the mathematical horizon. The domain of pedagogical content knowledge includes Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Curriculum (Ball et al., 2008, p.403; Hill et al., 2008, p. 377).

![Diagram showing Subject Matter Knowledge and Pedagogical Content Knowledge]

Fig. 2. 1 Mathematical knowledge for teaching (Hill et al., 2008, p.377)

Ball and her colleagues point out that teachers need to know the material they teach. They define CCK as “the mathematical knowledge and skill used in settings other than teaching” (Ball, Thames, & Phelps, 2008, p.399). For teaching, teachers also should know SCK which is defined as “the mathematical knowledge and skill unique to teaching” (p. 400). The knowledge at the mathematical horizon is the knowledge that teachers know about the mathematics topics and their relations to other mathematics topics students will learn in the future mathematics curriculum (Ball, 1993; Ball et al., 2008).

**The domain of pedagogical content knowledge.** Shulman and his colleagues (1986) first address the Pedagogical Content Knowledge (PCK) as one kind of content knowledge teachers need for teaching. According to Shulman (1986), the category of PCK include:
The most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others. (Shulman, 1986, p. 9)

An understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (Shulman, 1986, p.9)

Based on Shulman’s framework of teacher knowledge for teaching (Shulman, 1986), Ball argues that Shulman’s concept of pedagogical content knowledge is a lack of clarifying the difference between the pedagogical content knowledge and content knowledge, the pedagogical content knowledge and general pedagogical skills. They identify mathematics knowledge for teaching by their studies of mathematics teaching practice. They suggest Shulman’s curricular knowledge be one of the categories of pedagogical content knowledge. Furthermore, Ball and her colleagues state that Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) are two practically noticeable subdomains within PCK. Finally, Ball and her colleagues combine KCS and KCT with Shulman’s curricular knowledge as Knowledge of Content and Curriculum (KCC) to form the domain of PCK (Ball et al., 2008).

KCS is defined as the teacher’s “knowledge that combines knowing about students and knowing about mathematics” (Ball et al., 2008, p. 401.) or “the content knowledge intertwined with knowledge of how students think about, know, or learn this particular content” (Hill, Ball, & Schilling, 2008, p.375). Teachers must anticipate what students are likely to think and what they will find confusing. Teachers should be familiar their students’ common errors. Teachers
need to predict what students will find interesting and motivating; to know students’ difficulties; and to hear and interpret students’ incomplete thinking (Ball et al., 2008).

The concept of KCT combines “knowing about teaching and mathematics” (Ball et al., 2008, p.401). Mathematics teachers must have a thorough mathematical knowledge for the design of an effective lesson. They should understand the logic and the sequence of the mathematics content in the curriculum and choose appropriate examples to guide their students understand the concepts from basic to the complex situation. Teachers also should know how to evaluate and identify the different teaching methods and procedures, and know what each provides instructional (Ball et al., 2008).

KCC, called curricular knowledge by Shulman (1986), includes the knowledge of teaching topics, curriculum materials, and alternative curriculum materials of the subject for the instruction. Shulman suggests that curriculum knowledge combines lateral curriculum knowledge such as the knowledge of curriculum materials that students learn in other subjects at the same time, and vertical curriculum knowledge such as the knowledge of topics and their presentation in the previous and subsequent years in schools.

**Teachers’ pedagogical content knowledge and their students’ achievement.** Research on the relationship between teachers’ content knowledge and pedagogical content knowledge and their students’ achievement is back to 1980s. Leinhardt and Smith (1984) conducted a qualitative research in an elementary school to examine fourth-grade novices and expert teachers’ content knowledge of fractions. The research found that expert teachers had a deeper understanding and better explanation of the fraction problems than novices did. Moreover, some expert teachers had solid conceptual knowledge of fractions while others depended on algorithmic knowledge. According to Leinhardt and Smith, teachers’ strong conceptual knowledge and the connection
between these knowledge and lesson presentations can improve students’ reasoning and understanding concepts rather than simple skill development, then improve students’ mathematics competency. In the late 90s, the result from a quantitative study based on the data from the National Education Longitudinal study of 1988 shows that mathematics teachers’ subject matter knowledge has a causal relationship to their students’ achievement in mathematics (Rowan, Chiang, & Miller, 1997).

However, how do mathematics teachers practice their subject matter knowledge to teach is as important as what do they know the content knowledge. This special knowledge, namely pedagogical content knowledge, links mathematics content and pedagogy. Teachers’ pedagogical content knowledge is rooted in their content knowledge. Ball (2000) suggests that teachers should have the knowledge of “what is typically difficult for students, of representations that are most useful for teaching a specific idea or procedure, and of ways to develop a particular idea” (p. 245). Hill and her colleagues at the University of Michigan conducted a quantitative research that suggests a strong positive correlation between teachers’ mathematical content knowledge for teaching and their students’ mathematics achievement at first and third grades levels. They suggest that teachers’ mathematics explanations, representations, and interactions with students might affect students’ mathematics learning (Hill et al., 2005). German researchers (Baumert et al., 2010) conducted a one-year study to investigate the subject specific knowledge of secondary mathematics teachers. The result showed that teachers’ content knowledge and pedagogical content knowledge is highly related to the quality of teaching and students’ achievement. Teachers’ pedagogical content knowledge could not be substituted by content knowledge even though the pedagogical content knowledge reply on content knowledge. Moreover, researchers at the University of Wisconsin state that teachers’ knowledge of Children’s mathematical
knowledge and thinking help them make instructional decisions (Carpenter, Fennema, & Franke, 1996; Fennema, Franke, Carpenter, & Carey, 1993). Teachers’ pedagogical content knowledge of elementary students’ problem solving is significantly correlated with student achievement (Carpenter, Fennema, Peterson, & Carey, 1988).

In-Service Elementary Teacher Training in Shanghai

In Shanghai, universities, colleges, district teacher training college, and school-based training offer in-service teacher training program (Zhang et al., 2016). In-service teacher training program is based on teachers’ degree improvement and professional development. Teachers are encouraged to study at universities and colleges to meet the degree requirement or achieve higher degrees in the evenings, weekends, or summer and winter breaks. Meanwhile, they are also required to participate in professional development activities in their schools and district teacher training colleges. Sometimes universities and colleges offer in-service teachers professional development programs.

Degree improvement. Compared with their counterparts in the U.S., Chinese elementary teachers have lower degrees. Shanghai has led the country in raising the degree standards of elementary teacher education since 1985. Shanghai pre-service elementary school teachers did not have an associate’s degree until 1989 and a bachelor’s degree until 2003 (Hui, 2000; 2006). Gu and his colleagues (Gu, Zhong, & Zhao, 2008) conducted a quantitative research study of more than three thousand elementary teachers from 60 schools of six districts in Shanghai. The research shows that 69 percent of Shanghai elementary teachers who graduated from secondary normal schools which are four-year teacher education programs admitting middle school graduates, 18.9 percent of them had associate degrees, 11 percent of them had bachelor degrees, and 0.1 percent of them had master degrees when they started their teaching career (Gu, et al.,
However, Paine (1990) pointed out that the Chinese teacher education program shows a strong orientation toward subject-matter knowledge even though their degrees are low.

According to the teacher’s law in the People’s Republic of China issued in 1993, in-service teachers can study to improve their degrees in either teacher colleges, normal universities, or comprehensive universities. Many teachers achieved their degrees without majoring in education. Since 1981, Shanghai in-service teachers have been studying to improve their degrees to meet the government requirement. After continued education during their careers, 48.4 percent of them achieved to associate degrees and 43.6 percent of them bachelor degrees while 0.4 percent of them received master degrees. About 80 percent of Shanghai elementary teachers acquire their highest degrees during their careers (Gu et al., 2008). In Shanghai, the bachelor degree in elementary education for in-service teachers was first offered in 2000. Many in-service elementary teachers pursued other majors in order to acquire their bachelor degrees. Therefore, there is a gap between the curriculum that in-service teachers learn in the colleges and universities and teaching practice in schools they teach (Yang, Li, Gao, & Xu, 2009). However, Ma’s research (1999) shows that Shanghai elementary teachers have stronger mathematics content and pedagogical content knowledge than their counterparts in the U.S.

Shanghai teachers’ depth of understanding of these knowledge is developed during their careers (Ma, 1999; Paine, 1990; Paine & Ma, 1993; Tsui & Wang, 2009; Zhang et al., 2016).

Professional development. Shanghai elementary teachers continue to learn and improve their knowledge of subject matter and pedagogy during the professional development activities (Ma, 1999; Paine, 1990; Paine & Fang, 2007; Paine & Ma, 1993; Yang et al., 2009; Zhang et al., 2016). All Shanghai teachers have been required to participate in in-service training during their first five-year career since 1989. In 2011, all novice teachers are required to participate in an in-
service training program for at least 360 hours in their first five-year career. Moreover, all teachers are required to participate in professional development activities (Zhang et al., 2016). Each novice teacher has a mentor. Novice teachers and their mentors observe each other’s classes. Mentors help their mentees to prepare lessons and give them suggestions of teaching skills (Paine, 1990; Paine & Ma, 1993; Zhang et al., 2016).

Besides learning educational theories and practices in training programs, teachers learn a lot during the Teaching Research Group activities (TRGA) (jiaoyanzu huodong 教研组活动), a specific Chinese version of professional development activities, among the Teaching Research Groups (TRG) (jiaoyanzu 教研组) (Ma, 1999; Paine, 1990; Paine & Ma, 1993; Tsui & Wong, 2009; Yang et al., 2009). Paine and Ma (1993) describe that “the jiaoyanzu is a formal organizational unit within the school or, in the case of a small school, an organizational structure that connects teachers across schools” (p. 677). The TRG has lasted more than sixty years in China. In 1952, the Chinese Ministry of Education issued *High school Interim Regulations* (draft). The document shows that all high schools should set up TRG for each subject. Teachers discuss teaching schedule, content, and methods to improve the quality of teaching in the TRGs. Each TRG has a coordinator called teaching research officer (jiao yan yuan 教研员). In this study, I refer to teaching research officers as teaching research coordinators. TRG activities include discussion of curriculum and lesson plans, observation, and evaluation of colleagues’ classes (Paine, 1990; Paine & Ma, 1993; Yang et al., 2009).

Even though the TRG was initiated in high schools, it was also adapted in elementary schools. In the quantitative research study (Gu et al., 2008) of more than three thousand Shanghai elementary teachers, almost half of them thought the best way to improve their teaching was to observe their counterparts’ teaching outside of their schools. Less than a quarter
of the teachers agreed that teaching research coordinators and the district’s teacher training program helped them to improve their teaching skills; less than 20 percent of them believed that the communication with their colleagues; 16 percent of them thought that the school TRG activities contributed positively to the development of their teaching. However, the study did not single out the mathematics teachers’ opinions.

**Teaching Research Organization.** Yang and his colleagues (2009) pointed out that TRG activities are supported by multi-level teaching research organizations in mainland China. In Shanghai, teaching research organization consists of city teaching research office (shi jiao yan shi 市教研室), district or county teaching research office (qu jiao yan shi 区/县教研室), a school Teaching Research Group (TRG), and school Lesson Preparation Group (LPG) (beikezu 备课组). City teaching research office belongs to the Shanghai Municipal Education Commission while district or county teaching research office belongs to the district’s teacher training college. The majority of teaching research activities happen in the school TRGs and LPGs. They also take courses at their district’s teacher training college. Instructors at the district’s teacher training college or teaching research officers (coordinators) also visit schools to attend school-level teaching research activities and coach school teachers (Zhang et al., 2016). The teaching research coordinators in districts and city play an important role in in-service teachers’ professional development. They were usually experienced teachers before they were appointed as the district or city teaching research coordinators. They are communicators between the government and schools, between theories and teaching practices.

**THEORETICAL FRAMEWORK**

After examining the literature of pedagogical content knowledge for teaching mathematics and its relationship with students’ achievement, Shanghai teachers’ strong
pedagogical content knowledge, and Shanghai teachers’ professional development, I decide to use the theoretical framework of communities of practice to support my research of the modes of acquisition of Shanghai elementary mathematics teachers’ pedagogical content knowledge.

**Communities of Practice**

A social theory of learning focuses on learning as a social participation. Vygotsky (1978) presented a sociocultural perspective view of learning and stated that full cognitive development requires social interaction. He suggested that learning occurs when people interact with each other through discussion, collaboration, and feedback. Lave and Wenger (1991) argued that learning is viewed as a situated activity and learning is “an integral and inseparable aspect of social practice” (p.31). Lave and Wenger observed earlier that a group of professionals learn from each other through the process of sharing information and experiences in the group. Learning occurs through participation (Lave & Wenger, 1991) and knowing develops from the act of participation in complex social learning systems (Wenger, 2000). Wenger and his colleagues stated that “knowledge lives in the human act of knowing (Wenger, McDermott, & Snyder, 2002, p. 27).” Knowledge is dynamic rather than static, tacit and explicit, social and individual. People need to learn from others having a similar situation to develop their expertise within their communities of practice where they communicate and interact each other to share and develop their knowledge (Wenger et al., 2002). Wenger (2000) proposed that communities of practice, boundaries among communities, and identities are the three structuring elements of social learning systems.

After the observations of different apprenticeships from different cultural and historical traditions, Lave and Wenger (1991) proposed that people learn by participating in communities of practitioners. Newcomers acquire knowledge and skills through “legitimate peripheral
participation” (p.29) in a community of practice. Learning through apprenticeship is a form of legitimate peripheral participation. Apprentices acquire and develop their professional skills by learning from their peers and mentors (Wenger & Snyder, 2000). Over time they progress toward full engagement in the community’s social, cultural, and professional activities, learning and increasingly sharing skills and knowledge that they are mastering with their colleagues and mentors (Lave & Wenger, 1991). According to Lave and Wenger, the legitimate peripheral participation is an analytical viewpoint on learning and a way of understanding learning rather than a way of intended teaching.

Communities of practice are “groups of people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an ongoing basis” (Wenger et al., 2002, p.4). These people spend the time to learn together and help each other. A community of practice is a unique combination of three essential components: “a domain of knowledge, which defines a set of issues; a community of people who care about this domain; and the shared practices that they are developing to be effective in their domain” (p. 27).

Practice. Practice means a social practice. It is both explicit and tacit. It includes “the language, tools, documents, images, symbols, well-defined roles, specified criteria, codified procedures, regulations, and contracts that various practices make explicit for a variety of purposes” (Wenger, 1998, p.47). However, it also includes “implicit relations, tacit conventions, subtle cues, untold rules of thumb, recognizable intuitions, specific perceptions, well-tuned sensitivities, embodied understandings, underlying assumptions, and shared worldviews” (p.47). According to Wenger, practice is about meaning as an experience of everyday life. Wenger (1998) argued that meaning exists in a process which he defines as the negotiation of meaning. It
relates the interaction of two-part processes of “participation and reification” (p.52).

Participation is “not just to local events of engagement in certain activities with certain people, but to a more encompassing process of being active participants in the practices of social communities and constructing identities in relation to these communities” (Wenger, 1998, p.4). Wenger (1998) argued that participation has different implications for learning by individuals, communities, and organizations. Wenger stated,

For individuals, it means that learning is an issue of engaging in and contributing to the practices of their communities. For communities, it means that learning is an issue of refining their practice and ensuring new generations of members. For organizations, it means that learning is an issue of sustaining the interconnected communities of practice through which an organization knows what it knows and thus becomes effective and valuable as an organization. (pp.7-8)

Reification means the process or result of reifying. According to the Webster dictionary, reify refers to regard (something abstract) as a material or concrete thing. It is a work that attempts to provide a bridge between what is abstract and what is real. Wenger (1998)’s concept of reification is broader than what the dictionary has. He claimed that the process of reification is central to every practice. Reification covers “a wide range of processes that include making, designing, representing, naming, encoding, and describing, as well as perceiving, interpreting, using, reusing, decoding, and recasting” (Wenger, 1998, p.59). Wenger (1998) explained that reification means a process and its product. Much of the reification related to work practices are from outside a community. Any community and its members have to reify appropriately what they have from the outside into a local process to make it useful.

Participation and reification are considered as a pair. They are distinct and
complementary. Participation and reification interact and imply each other. They transform their relations and describe and interplay. Like the figure (Fig 2.2) shows below, participation and reification are a duality, not opposites (Wenger, 1998).

![Diagram of participation and reification]

Fig. 2.2 The duality of participation and reification (Wenger, 1998, p.63)

**Community.** Wenger (1998) stated that community of practice is a unit. Practice is the source of coherence of a community. There are three dimensions such as “mutual engagement, a joint enterprise, and a shared repertoire” (p.73) of communities of practice. These three elements are dynamic. They interact and interplay to makes for a healthy community. According to Wenger, practice exists based on the mutual engagement of participants. They interact each other and negotiate with one another. The membership, as a matter of mutual engagement, in a community of practice defines the community. Diversity and partiality make mutual engagement happen in a community. Members of a community are different from each other. They could work together, communicate and exchange information and ideas each other. Meanwhile, they also could disagree, challenge, and compete with each other in a community. All these shared practices make a community diverse and complex. To keep a community of practice together,
Wenger (1998) pointed out three points about the enterprise:

“1) It is the result of a collective process of negotiation that reflects the full complexity of mutual engagement.

2) It is defined by the participants in the very process of pursuing it. It is their negotiated response to their situation and thus belongs to them in a profound sense, in spite of all the forces and influences that are beyond their control.

3) It is not just a stated goal, but creates among participants’ relations of mutual accountability that become an integral part of the practice.” (pp.77-78)

The development of a shared repertoire is the third characteristic of practice as a source of community coherence. “The repertoire of a community of practice includes routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions, or concepts that the community has produced or adopted in the course of its existence, and which have become part of its practice. The repertoire combines both reificative and participative aspects” (P.83).

**Teacher development in the community of practice.** Learning to teach takes place within communities (Cochran-Smith & Lytle, 1999; Grossman, Darling-Hammond & Bransford, 2005), and teachers’ knowledge of best practices develops within a professional community of inquiring teachers, as a result of participating in complex “social learning systems” (Wenger, 2000; Darling-Hammond & Bransford, 2005; Cochran-Smith & Lytle, 1999). Interaction and self-assessment by the individual teacher affect the formation of their teaching behaviors.

Cochran-Smith and Lytle (1999) stated that knowledge for practice, knowledge in practice, and knowledge of practice are three conceptions of teacher knowledge and learning. The knowledge for practice means the knowledge that teachers need to develop their teaching practice; the knowledge in practice relies on “the assumption that the knowledge teachers need to...
teach well is embedded in the exemplary practice of experienced teachers” (p.263). This is the knowledge that experienced teachers know as it is expressed in their practice, their reflections, and their narratives (Hammernessa, et al., 2005); the knowledge of practice emphasizes that “the knowledge teachers need to teach well emanates from systematic inquiries about teaching, learners and learning, subject matter and curriculum, and schools and schooling. This knowledge is constructed collectively within local and broader communities” (p.274). It means that teachers are members of a professional community where they learn and develop their knowledge with their colleagues.
CHAPTER III

METHODOLOGY

The purpose of this study is to address 1) a general picture and in-depth exploration of acquisition modes of Shanghai elementary mathematics teachers’ pedagogical content knowledge within their communities of practice; 2) teaching research coordinators’ help to develop and improve mathematics teachers’ pedagogical content knowledge; and 3) school policies to enable the acquisition of teachers’ pedagogical content knowledge. A mixed methods design (Creswell, 2009; Creswell & Plano Clark, 2007, Mertens, 2013; Yin, 2014) was used in the study to answer the research questions. The reason for using this approach is to integrate and validate the results from both quantitative and qualitative data (Creswell, 2009; Creswell & Plano Clark, 2007). A qualitative approach with case study methods is this study’s primary research method. This chapter describes in detail the methodology of the study, including research design, field setting, sample selection, research instruments, data collection, data analysis, issues of ethics and trustworthiness.

Multiple-Case Study Research Design

Multiple-case study (Yin, 2014) with a mixed methods research design is used in this study. The case study is “an intensive description and analysis of a phenomenon or social unit such as an individual group, institution, or community” (Merriam, 2002, p.8). The case study method is a better research method to explore, explain, or describe the social phenomenon and how or why it works. It is an appropriate research method to answer the research questions that “require an extensive and in-depth description of some social phenomenon” and to “understand complex social phenomena” (Yin, 2014, p.4). Stake (1995) argues that case studies are a useful and valuable tool for searching for an explanatory regulation pattern and for better and deeply
understanding human behavior. This study sought to understand the phenomena of Shanghai elementary mathematics teachers having strong pedagogical content knowledge; their acquisition of pedagogical content knowledge within their communities of practice; and the professional support and school policies enabling the acquisition of their pedagogical content knowledge. Therefore a case study is an appropriate method for this study.

Compared to the single-case study, the multiple-case study examines a phenomenon in diverse settings and how the phenomenon performs in a different environment space (Stake, 2006). The replication logic is the use of multi-case study (Stake, 2006; Yin, 2014). This research study is conducted in two schools from two districts, with different school cultures, and different student demographics. Thus, the phenomenon of two different groups of school mathematics teachers acquiring their pedagogical content knowledge in their different communities can provide a more compelling and strong interpretation of how Shanghai elementary mathematics teachers acquire and develop pedagogical content knowledge in their communities of practice (Merriam, 2002; Stake, 2006; Yin, 2014).

Therefore, mixing case study research with other methods enables researchers to address broader and more complicated research questions than case studies alone (Yin, 2014).

**Research Questions**

Once more, the research questions for this study are as follows:

1. How do Shanghai in-service elementary mathematics teachers acquire pedagogical content knowledge within their communities of practice?

2. What do teaching research coordinators from the school district and the city do to improve teachers’ mathematics pedagogical content knowledge within Teaching Research Group activities?
3. How does the school administration’s support enable teachers’ acquisition of mathematics pedagogical content knowledge within those communities of practice?

**Field Setting and Participants**

This study took place in two public schools from two districts in Shanghai. One is located in the north urban area of Shanghai, and another is located in a southwest suburb of Shanghai. For the sake of confidentiality, the sites are referred to as Rainbow Elementary School (Rainbow) from the school District A and Blue Sky Elementary School (Blue Sky) from the school District B. Rainbow was founded as a modern elementary school in 2007 while Blue Sky was founded as a rural elementary school in 1997. Over the last ten years, the rural place where Rainbow is located has been developed to a suburb area. These two districts were selected because their population approximates the average population for all districts in Shanghai. These two public schools were selected because 1) both of schools have more than one thousand students and are among the top performing schools in their districts; 2) there are regional differences, school culture differences, different student demographics, and collaborations between these two schools. Moreover, before I came to the U.S, as a mathematics educator in Shanghai, I worked with both the city-level and district-level teaching research coordinators and one principal in one of the schools. The relationship between me and the participants had already been established. It was convenient for me to communicate with them and conduct this study, which reinforced a very good trustworthy relationship.

All participants were voluntary, which followed the guidelines for research with human subjects as specified by Teachers College’s Institutional Review Board. They were told that the goal of the research study is to find out how Shanghai elementary teachers acquire their mathematics teaching knowledge during their careers. They were informed that they could quit
the involvement of the research study anytime without any penalty, and their data would be deleted. They were also informed that they would neither be compensated directly nor indirectly for their participation in the study.

**Teachers.** All mathematics teachers from Rainbow and Blue Sky are involved in the study. Twenty teachers from Rainbow and twenty-four teachers from Blue Sky responded to the printed Chinese version questionnaire survey and completed all the survey questions.

Among these teachers, three teachers were interviewed; one teacher from Rainbow and one teacher from Blue Sky were interviewed after the observations of their classes and post-observations of Teaching Research Group activities after their teaching, and one mentor from Rainbow was interviewed. About forty mathematics teachers from these two schools attended these two Teaching Research Group activities. Table 3.1 provides a summary profile of the teachers and one mentor who was interviewed in terms of their educational and professional background. For the sake of confidentiality, all names are pseudonyms.

Table 3.1

*Summary of Interviewed Teachers’ and Mentors’ Backgrounds*

<table>
<thead>
<tr>
<th>Name of Participant</th>
<th>Gender</th>
<th>Teacher/Mentor</th>
<th>Name of School</th>
<th>Degree</th>
<th>Major</th>
<th>Years of Teaching Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yu</td>
<td>M</td>
<td>Teacher</td>
<td>Rainbow</td>
<td>Bachelor</td>
<td>Education</td>
<td>3</td>
</tr>
<tr>
<td>Jing</td>
<td>F</td>
<td>Mentor</td>
<td>Rainbow</td>
<td>Bachelor</td>
<td>Education</td>
<td>23</td>
</tr>
<tr>
<td>Hua</td>
<td>F</td>
<td>Teacher</td>
<td>Blue Sky</td>
<td>Bachelor</td>
<td>Education</td>
<td>12</td>
</tr>
</tbody>
</table>

**Teaching research coordinators.** Three district-level mathematics teaching research coordinators from the District A and District B, and one city-level mathematics teaching research coordinator were involved in the two Teaching Research Group activity observations and
interviews. Table 3.2 provides a summary profile of the teaching research coordinators who were interviewed in terms of their educational and professional background.

Table 3. 2

*Summary of Interviewed Teaching Research Coordinators’ Backgrounds*

<table>
<thead>
<tr>
<th>Name of Participant</th>
<th>Gender</th>
<th>Name of Institution</th>
<th>Degree</th>
<th>Major</th>
<th>Experience in the Field</th>
<th>Years of Being Coordinator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qiang</td>
<td>M</td>
<td>City Municipal Education Commission</td>
<td>Bachelor</td>
<td>Education</td>
<td>13 years of teaching mathematics</td>
<td>8 years in the district; 15 years in the city</td>
</tr>
<tr>
<td>Ming</td>
<td>M</td>
<td>District A</td>
<td>Bachelor</td>
<td>Education</td>
<td>26 years of teaching mathematics</td>
<td>4 years in the district</td>
</tr>
<tr>
<td>Ding</td>
<td>F</td>
<td>District B</td>
<td>Bachelor</td>
<td>Education</td>
<td>14 years of teaching mathematics</td>
<td>11 years in the district</td>
</tr>
<tr>
<td>Fu</td>
<td>M</td>
<td>District B</td>
<td>Master</td>
<td>Math Education</td>
<td>10 years of writing elementary mathematics textbook</td>
<td>2 years in the district</td>
</tr>
</tbody>
</table>

*School administrators.* One principal and one curriculum teaching coordinator at Rainbow, one school teaching research coordinator, and one curriculum teaching coordinator at Blue Sky were interviewed and involved in the two Teaching Research Group activity observations. Table 3.3 provides the school administrators who were interviewed in terms of their educational and professional background.
Table 3.3

Summary of Interviewed School Administrators’ Backgrounds

<table>
<thead>
<tr>
<th>Name of Participant</th>
<th>Gender</th>
<th>Title</th>
<th>Name of School</th>
<th>Degree</th>
<th>Major</th>
<th>Years of Teaching Mathematically</th>
<th>Years of Administrator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wei</td>
<td>F</td>
<td>Principal</td>
<td>Rainbow</td>
<td>Bachelor</td>
<td>Education</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>Li</td>
<td>F</td>
<td>Curriculum teaching coordinator</td>
<td>Rainbow</td>
<td>Master</td>
<td>Math Education</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>Fang</td>
<td>F</td>
<td>Curriculum teaching coordinator</td>
<td>Blue Sky</td>
<td>Bachelor</td>
<td>Education</td>
<td>29</td>
<td>10</td>
</tr>
<tr>
<td>Lan</td>
<td>F</td>
<td>School teaching research coordinator</td>
<td>Blue Sky</td>
<td>Bachelor</td>
<td>Education</td>
<td>17</td>
<td>3</td>
</tr>
</tbody>
</table>

University professor. One university professor was involved in one of two Teaching Research Group activity observation.

Data Collection Methods

Data were collected through questionnaire survey; semi-structured in-depth interviews with teachers, mentor, teaching research coordinators, school principal, and curriculum teaching coordinators; observations of classroom teaching and Teaching Research Group activities; teachers’ lesson plans; teachers’ reflection reports, school documents, and my field notes taken during the interviews, the two class observations, and two Teaching Research Group activities. Table 3.4 summarizes the methods applied to address each research question.
Table 3.4

*Summary of Data Collection Methods*

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How do Shanghai in-service elementary mathematics teachers acquire pedagogical</td>
<td>QS, TI, MI, CO,</td>
</tr>
<tr>
<td>content knowledge within their communities of practice?</td>
<td>TRGAO, TLP, TRR,</td>
</tr>
<tr>
<td></td>
<td>FN</td>
</tr>
<tr>
<td>2. What do teaching research coordinators from the school district and the city</td>
<td>DTRCI, CTRCI,</td>
</tr>
<tr>
<td>do to improve teachers’ mathematics pedagogical content knowledge within</td>
<td>TRGAO, FN</td>
</tr>
<tr>
<td>Teaching Research Group activities?</td>
<td></td>
</tr>
<tr>
<td>3. How does school administration’s support enable teachers’ acquisition of</td>
<td>PI, CTCI, STRCI,</td>
</tr>
<tr>
<td>mathematics pedagogical content knowledge within those communities of practice?</td>
<td>SD, FN</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* QS = questionnaire survey; TI = teacher interview; MI = mentor interview; DTRCI = district teaching research coordinator interview; CTRCI = city teaching research coordinator interview; STRCI = school teaching research coordinator interview; PI = principal interview; CTCI = school curriculum teaching coordinator interview; CO = class observation; TRGAO = teaching research group activity observation; TLP = teachers’ lesson plans; TRR = teachers’ reflection reports; SD = school documents, FN = field notes

*Questionnaire survey.* To answer the first research question, the questionnaire survey was designed to provide a quantitative description of how mathematics teachers in the Rainbow and Blue Sky schools acquire and enhance their pedagogical content knowledge within their communities of practice. All forty-four mathematics teachers in both schools were invited to complete the anonymous survey. All of them, twenty mathematics teachers from Rainbow and twenty-four teachers from Blue Sky, responded to the printed Chinese version questionnaire survey and completed all the survey questions.

The questionnaire consists of 28 items. I, a Chinese native speaker, translated the questionnaire survey into Chinese from the English version (Appendix B). The questionnaire was divided into three parts: teachers’ background, mentorship, and participation in professional development activities. The design of the questionnaire items is based on a literature review of
Chinese teachers’ professional development, mathematics pedagogical content knowledge, and communities of practice. Some items are adapted from some items of the survey used to collect data for a Ph.D. dissertation, “The Development of Teachers’ Pedagogical Knowledge: An Investigation of Mathematics Teacher in Three High-Performing High Schools” (Fan, 1998). Items 1 to 5 are designed to provide the educational and professional background information of the teachers. Items 6 to 14 are designed to collect data about the apprenticeship of novice teachers to master teachers (Lave & Wenger, 1991; Paine & Ma, 1993; Wenger, 1998). Items 15 to 19 focus on the teachers’ participation in the Teaching Research Group activities within and outside of the school communities (Paine & Fang, 2007; Paine & Ma, 1993). Items 20, 24, 25, 26, and 27 are designed to provide the description of how teachers communicate, interact, and negotiate with their colleagues, teaching research coordinators, and university professors in the Teaching Research Group activities (Paine & Fang, 2007; Paine & Ma, 1993; Wenger, 1998; 2000; Wenger et al., 2002). Items 21, 22, 23, and 28 are designed to provide the teachers’ view of the degree of use and help in enhancing their mathematics pedagogical content knowledge and the sources for their mathematics pedagogical content knowledge.

**Observations.** There are two kinds of observations conducted in the study to answer research questions 1 and 2. One is classroom teaching observation and another is Teaching Research Group activity observation. One teacher at Rainbow and one teacher at Blue Sky were observed. After each class observation, the post-conference, one of the types of Teaching Research Group activities with mathematics teachers, city-level and district-level teaching research coordinators, school teaching research coordinator, school curriculum teaching coordinators, one university professor, and one principal were observed as well.
In a qualitative study, “the theoretical framework, the problem, and the questions of interest determine what is to be observed” (Merriam, 2009, p.119). The observations of classroom teaching were sought to provide the in-depth explanation of how teachers prepare and teach the lessons by applying their mathematics pedagogical content knowledge. The observations of Teaching Research Group activities were sought for the deep explanation of how two teachers’ colleagues, teaching research coordinators, and university professor commented on, discussed, and suggested improvements to their classroom teaching. The observations of Teaching Research Group activities also showed how the two observed teachers participated and engaged in the activities and how they positioned themselves and were positioned by the community members.

As a non-participant observer, I observed two 40-minute classes and two hour-long Teaching Research Group activities. The Teaching Research Group activities were documented with digital audio-recordings. I saved the digital files to a folder. I also took field notes during the four observations.

**Interviews.** The semi-structured interviews were conducted using suggestions from researchers (Creswell, 2013; Merriam, 2009; Yin, 2014). In total, eleven semi-structured interviews were conducted to answer the research questions 1, 2, and 3. The interview protocols for teachers, mentors, teaching research coordinators, and school administrators can be found in Appendices C, D, E, and F. Each interview lasted about 40 to 60 minutes. Interviews were conducted in person, by telephone, or by video call through Wechat, which is similar to talking through Skype. Requests were sent to the interviewees after each observation of classroom teaching and Teaching Research Group activity. After exchanges of contact information, the interviews were scheduled, conducted, and recorded in digital audio format. The digital audio
files were saved to a folder. Only the researcher can access these data. After the research study is done, the files will be deleted from the accounts in accordance with the confidentiality of participants’ information.

One teacher and his mentor at Rainbow and one teacher at Blue Sky were involved in the follow-up interviews after the class observations and Teaching Research Group activity observations. These three interviews provided data on the in-depth explanation of how these teachers acquired and enhanced their pedagogical content knowledge from the perspectives of the relationship between teachers and their mentors; the relationship between teachers and their colleagues; the concrete pedagogical content knowledge regarding the two lessons that the teachers applied to facilitate the lessons; and teachers’ reflections of the lesson they taught. These data were collected to enhance the answer to the research question 1, namely, how do Shanghai in-service elementary mathematics teachers acquire pedagogical content knowledge within their communities of practice?

One city-level teaching research coordinator and three district-level teaching research coordinators were interviewed after the observations of two classroom teaching and two Teaching Research Group activities. These four interviews were focused on how the teaching research coordinators organize the Teaching Research Group activities in the city and districts; what themes are in the activities and how these themes were decided; how often they visit Rainbow and Blue Sky; from what perspectives do they observe and evaluate lessons and what do they think about the lessons they observed; and how important is it for teachers to have pedagogical content knowledge and how they help to improve teachers’ pedagogical content knowledge. These data were collected to answer research question 2, namely, what do teaching research coordinators from the school district and the city do to improve teachers’ mathematics
pedagogical content knowledge within Teaching Research Group activities?

Four interviews with school administrators were conducted on the role of school policies and regulations in cultivating novice teachers, peer observations, arrangement and establishment of Teaching Research Group activities. These helped answer research question 3, namely, how does school administration’s support enable teachers’ acquisition of mathematics pedagogical content knowledge within those communities of practice?

**Lesson plans, reflection reports, and school document.** One teacher at Rainbow and one teacher at Blue Sky were observed when they taught their classes. Each of them wrote a lesson plan before they taught. They also wrote reports after the post-observation conferences which were Teaching Research Group activities, reflecting on their teaching based on the comments and suggestions from their colleagues, city-level and district-level teaching research coordinators, and the university professor. These lesson plans and reflection reports were additional sources of data that were analyzed to answer research question 1. The reflection reports could enhance the explanation of how these teachers digested the comments and suggestions they received from the Teaching Research Group activities; what kind of pedagogical content knowledge they learned; how they would improve their teaching strategies or improve their lesson plans. Moreover, school documents were collected to discover school history, culture, student demographics, policies, and details about the Teaching Research Group activities. These data added depth to the answer of how school administration enables and supports the acquisition of teachers’ pedagogical content knowledge.

**Data Analysis**

I analyzed data for each of the two schools to explain the phenomena of acquisition of Shanghai elementary teachers’ pedagogical content knowledge (Stake, 2006). The quantitative
data from the questionnaire survey were analyzed by using the Microsoft Excel 2016 software. The qualitative data from the observations, interviews, teachers’ lesson plans and their reflection reports, school documents, and researcher field notes were analyzed by applying the grounded theory coding method (Charmaz, 2014; Merriam, 2009). The theoretical framework of communities of practice guided the analysis process. During the process of analyzing data, I wrote analytical memos to catch my thoughts, tackle the comparisons and connections that I made, and solidified questions and directions for the further analysis (Charmaz, 2014). After I analyzed the data from each case, I wrote a summary of data analysis for each school. Then I conducted cross-case data analysis to find the similarities and differences between the two schools. Finally, I wrote a cross-case report for the study. In this section, I provide a detailed description of the data analysis methods that I used to organize and analyze the data collected from the survey questionnaire, observations, interviews, lesson plans and reflection reports, school documents, and field notes.

**Survey analysis.** For the survey, I numbered the questionnaires from each school and input all data to an excel file. I analyzed the data by Microsoft Excel 2016 to summarize the quantitative description of how mathematics teachers in the Rainbow and Blue Sky acquire and enhance their pedagogical content knowledge within their communities of practice. I compared the summary constantly with the data and codes from the interviews and observations to construct and confirm emergent categories to answer the first research question.

**Qualitative data analysis.** I listened to the audio recordings from the observations and interviews several times. After that, I transcribed and analyzed textual data in Chinese. Then I translated the codes from observations, interviews, lesson plans and reflection reports, and school documents into English.
In order to make sense out of the data and find emergent categories, I analyzed data through the initial and focused grounded theory coding (Charmaz, 2014). This coding method encouraged me to interact with my data (Charmaz, 2014; Merriam, 2009) and “learn what forms these data and in which theoretical directions I can take them” (Charmaz, 2014, p.161). To begin, I read and reread the transcripts, field notes, teachers’ lesson plans and their reflection reports, and school documents. After that, I started to do line-by-line initial coding with pieces by naming each line of the written data. Initial coding reminded me to think of the meaning, indication, and suggestions of the data. It helped me reflect on my data collection and the lack of need. Therefore, I can compare the data from observations and interviews with other participants in the same school and all data from the other school. Initial coding promoted me to look for patterns and think more analytically (Charmaz, 2014). I conducted focused coding by assessing initial codes. I compared these initial codes with data, compared these initial codes with codes, and remained the initial codes that have greater analytic power (Charmaz, 2014). I grouped these initial codes as the focused codes. With these focused codes, I generalized the information and ideas for emerging the tentative scheme of categories first. After that, I compared these tentative categories with the focused codes and the data to sort and refine the tentative scheme of categories for emerging the conceptual categories and generating the subcategories (Charmaz, 2014; Merriam, 2009). Meanwhile, I also wrote analytic memos to record the process of these categories and their corresponding codes. These memos can record my thoughts, the comparisons and connections I made, the questions I raised, and the directions I continued to the ongoing analysis.

**Within-case analysis.** I separately organized and managed the data for each case. Each school was treated as a comprehensive single case. To answer the research questions, I analyzed
the quantitative data from each school by using Excel 2016 and analyzed the qualitative data from each school through the process of initial and focused coding to emerge the conceptual categories. After I completed the analysis for each school, I wrote the within-case summary for each school to describe and analyze the findings and discussion from each school (Stake, 2006; Yin, 2014).

**Cross-case analysis.** Following the within-case summaries, I started to conduct the data analysis across the two cases-Rainbow and Blue Sky to find general assertions that bind the two cases and identify important findings that emerged from the case (Stake, 2006). I compared the findings from both schools to look for the similarities and differences. I emerged the findings into clusters. I placed the similar findings in topics together and dissimilar findings in topics further part (Stake, 2006). Following a replication logic, I wrote the cross-case report as the result of the study (Stake, 2006; Yin, 2014).

**Ethics and Trustworthiness**

To evaluate the trustworthiness of this two-case study with a mixed methods research design, I discuss the standards of rigor of the data collection and analysis in the perspectives of the internal validity, reliability, and external validity (Merriam, 2002). Before that, I begin with the ethics concerns (Merriam, 2002).

**Ethics concerns.** Protection of the human rights is very important when the research’s subjects are human beings. The trustworthiness of a qualitative study depends on the ethics of the researcher (Merriam, 2002). Before I started to collect data, I sent the informed consent (Appendix A) proved by IRB to all of the participants to show the protection of their rights and confidentiality and very low risk of involvement at all stages of the study.
I used a purposive sampling method to choose my participants because I wanted to have two different sites to address the phenomena of Shanghai elementary mathematics teachers’ acquisition of pedagogical content knowledge within their communities of practice. As a mathematics educator, I have taught mathematics content and method courses to pre-service and in-service elementary teachers for many years, visited Shanghai elementary schools for both supervising student teaching practice and attending Teaching Research Group activities in elementary schools, and worked with teaching research coordinators in the school districts and city before I came to the U.S. All of these experiences influenced the data collection and analysis process for this study. Because I am very familiar with the elementary school’s system in Shanghai and familiar with some of the participants in my study, the participants trusted me and felt relaxed and open to involving in the study.

In addition, during the process of the data collection and analysis, I had ongoing reflections by writing memos, communicating with my former colleagues and had member checks with the participants to help me confront any possible biases.

**Internal validity.** This approach provided a richly detailed holistic interpretation of the process of Shanghai elementary mathematics teachers’ development of their pedagogical content knowledge within their communities of practice from the teachers’, teaching research coordinators, and school administrators perspectives. To confirm emerging findings, this two-case study combined quantitative data collection and analysis from all mathematics teachers at two representative schools in Shanghai with a thick description of qualitative data collection and analysis from interviews, observations, teachers’ lesson plans and reflection reports, school documents, and researcher’s field notes. This triangulation strategy supported the internal validity of the study (Merriam, 2002). During the interviews, I had member checks with my
interviewees. After I transcribed the interviews and observations, I also asked my interviewee and my former colleague to review and make comments on my interpretation of the data. Member checks and peer review are two of the common strategies for ensuring validity in qualitative research (Merriam, 2002)

**Reliability.** This study took place in two public schools from two districts in Shanghai. One is located in the north central urban area of Shanghai and another is located in a southwest suburb of Shanghai. These two districts were selected because their population approximates the average population for all districts in Shanghai. These two public schools were selected because both of schools have more than one thousand students in the schools in their districts while one was developed from a rural elementary school. The findings emerged from the two schools followed the replication logic. This two-case study was conducted by utilizing multiple data-collection methods. The replicated findings and the strategies of triangulation ensure consistency and reliability of the study (Merriam, 2002).

**External validity.** Merriam states that “providing a rich and thick description, multisite designs, or maximizing variation in the purposely select sample are major strategies to ensure for external validity or generalizability in the qualitative sense” (Merriam, 2002, p.29). This two-case study with the survey was conducted in two public elementary schools from different districts in Shanghai. The findings from the two-case study can represent the phenomena of elementary mathematics teachers acquire and improve their pedagogical content knowledge within their communities of practice in high-performing urban and suburb schools in Shanghai. The limitations of this approach are the necessarily small size of the sample compared to the number of mathematics teachers in all elementary schools in Shanghai.
CHAPTER IV

FINDINGS

THE CASE OF RAINBOW ELEMENTARY SCHOOL

Rainbow Elementary School (Rainbow), located in district A in the north central urban area of Shanghai, is a modern public school founded in 2007. Currently, there are 41 classes and more than 1,300 students, including more than 100 foreign students in the school. The goals of running the school are internationality, art, and individuality. In total, 115 teachers and staff work at Rainbow, which is one of the top public schools in the district. It is also one of the base schools with which Shanghai Teaching Research Office collaborates to implement the educational reforms of curriculum, teaching strategies, and assessment. In addition, it is one of the public schools where college preservice elementary teachers practice teaching during their senior year. Recently, Rainbow was designated as one of the public elementary schools that collaborates with a normal university in Shanghai to cultivate future excellent elementary teachers.

There are 20 mathematics teachers at Rainbow. From the quantitative data analysis, most of them are young. Fifty percent of them are aged below thirty while 35 percent are aged between thirty and forty. Eighty percent of mathematics teachers hold bachelor’s degrees in education while 10 percent hold a master’s degree in elementary education specialized in teaching mathematics. The rest of the teachers have bachelor’s degrees in other fields such as business and economics. Among these 20 mathematics teachers, 25 percent are novice teachers; 30 percent have been teaching mathematics less than five years; 40 percent have been teaching
mathematics for eight years or more. As at other elementary schools in Shanghai, the overwhelming majority 80 percent of mathematics teachers at Rainbow are female.

**How do mathematics teachers acquire their Pedagogical Content Knowledge?**

To answer the first research question of how mathematics teachers acquire their pedagogical content knowledge, I analyzed the quantitative data from the questionnaire survey of all mathematics teachers at Rainbow and qualitative data, including observations of the classroom teaching and Teaching Research Group Activity (TRGA); interviews with the teacher who was observed and his mentor; the teacher’s lesson plan and his reflection report; and my field notes.

**Part 1 Findings from the Quantitative Analysis**

In this part, I discuss the quantitative description of the general picture of how mathematics teachers acquire and develop their pedagogical content knowledge at Rainbow within their communities of practice: their mentorships with master teachers and in relation to other novice and experienced teachers; their participation in the TRGA; the level of impact on their mathematics teaching after their participation in TRGA; and the communication during these activities.

**Mentorship**

Ninety percent of the teachers at Rainbow reported that they had mentors (master teachers) to help them when they were novice teachers, while 40 percent reported that they mentored novice teachers and junior teachers. Mentors and mentees usually spend an average of six hours a week together for an average of two years. One teacher reported that she used to work with her mentee for 20 hours every week. The longest mentorship among these mathematics teachers lasted four years. Seventy percent of mentees teach the same grade as their
mentors do and half of the mentees share the same office with their mentors. Mentees feel free to ask questions and learn lots from their mentors. When they have difficulties in teaching mathematics, the data showed that 70 percent of mathematics teachers at Rainbow first would first like to communicate with their mentors, followed by experienced teachers and then their fellow novice teachers.

Both mentors and mentees had positive views on their mentorship. On the one side, some mentees reported that they challenged their mentors’ position. On the other side, all mentors reported that they learned something from their mentees, especially educational technology skills, creative learning activity design, and the courage to try new things. The data from the questionnaire also showed that mentors decided on a reasonable compromise after discussing a problem with their mentees. Table 4.1 below shows the positive views of mentors and mentees.

Table 4. 1

<table>
<thead>
<tr>
<th>Mentorship</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>If you are a mentee, do you ask questions if you do not understand your mentor’s explanation about teaching ideas and thoughts?</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>If you are a mentee, do you challenge arguments that you disagree with your mentor?</td>
<td>61%</td>
<td>39%</td>
</tr>
<tr>
<td>If you are a mentor, do you learn anything regarding mathematics teaching from your mentee?</td>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Participation

All mathematics teachers at Rainbow are active participants in the TRGAs at their school, their district and in the city. Ninety-five percent of the mathematics teachers at Rainbow reported that they attended the school TRGAs more than eight times every semester, which means about
once every two weeks. Eighty-five percent of Rainbow mathematics teachers participated in district TRGAs between four and six times every semester, more than once a month. In addition, some of them also participated in TRGAs outside of the district and the city every year. Eighty percent of Rainbow mathematics teachers reported that they joined online TRGAs with other elementary mathematics teachers in the city. However, 35 percent of Rainbow mathematics teachers never joined TRGAs with their counterparts outside of the district and 60 percent of the mathematics teachers never attended TRGA outside of the city. When the teachers were asked the reason that they attended the TRGAs, 80 percent of them reported that the school requires them to participate in TRGAs inside and outside of the school; 60 percent of the teachers said that the district requires them to participate in the district TRGAs. When they were asked to comment on the different types of TRGAs, they enjoyed the school TRGAs most and the online least. Table 4.2 and figure 4.1 show the Rainbow mathematics teachers’ responses to the degree of usefulness of TRGAs in the school, district, and city (online).

Table 4.2

<table>
<thead>
<tr>
<th></th>
<th>Very useful</th>
<th>Useful</th>
<th>Neutral</th>
<th>Not very useful</th>
<th>Not useful</th>
<th>Do not apply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activities in your school</td>
<td>80</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Activities in your district</td>
<td>55</td>
<td>40</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Online activities in the city</td>
<td>20</td>
<td>25</td>
<td>15</td>
<td>20</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Activities in other districts</td>
<td>35</td>
<td>20</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>Activities outside the city</td>
<td>25</td>
<td>15</td>
<td>20</td>
<td>0</td>
<td>10</td>
<td>30</td>
</tr>
</tbody>
</table>
Fig. 4. Distributions in percent of teachers’ evaluations of TRGAs

**TRGA Impact**

The majority of teachers reported that they usually attend the TRGA for designing a lesson plan with their colleagues; observing their colleagues’ classes and attending the post-conference; and discussing mathematics topics, the students’ difficulties, and teaching strategies. The data show that the average times that teachers designed lesson plans with their colleagues is 13 times during an academic year. One of them reported that he designed the lesson plans with his colleagues about 40 times over the last year. Teachers observe their colleagues’ teaching between five and 50 times per year with an average of 17. Table 4.3 shows overwhelming majority mathematics teachers at Rainbow find different TRGAs very useful.
Table 4. 3  
*Distributions in percent of teachers’ evaluations of the TRGA impact on their knowledge of mathematics teaching*

<table>
<thead>
<tr>
<th>Activity</th>
<th>Very useful</th>
<th>useful</th>
<th>Neutral</th>
<th>Not very useful</th>
<th>Not useful</th>
<th>Do not apply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designing a lesson plan with colleagues</td>
<td>90</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Observing colleagues’ teaching</td>
<td>70</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Attending the conference after teaching</td>
<td>70</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Studying mathematics topics and their relation to the curriculum in the textbooks</td>
<td>75</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Discussing mathematics topics and the students’ difficulties learning them</td>
<td>80</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Discussing mathematics topics and the strategies for teaching them</td>
<td>75</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Taking courses about mathematics knowledge</td>
<td>65</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Taking courses about teaching knowledge</td>
<td>60</td>
<td>35</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Taking courses about mathematics teaching knowledge</td>
<td>65</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

In addition, Table 4.4 shows that teachers find that their colleagues and teaching research coordinators are very helpful during the TRGAs.
Table 4.4

*Distributions in percent of teachers’ evaluations of the people who help them improve their knowledge of mathematics teaching*

<table>
<thead>
<tr>
<th></th>
<th>Very helpful</th>
<th>helpful</th>
<th>Neutral</th>
<th>Not very helpful</th>
<th>Do not helpful</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colleagues in your school</td>
<td>95</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Counterparts in your district</td>
<td>45</td>
<td>35</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Counterparts in other districts</td>
<td>40</td>
<td>30</td>
<td>15</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>Teaching research coordinators</td>
<td>75</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>University professors</td>
<td>25</td>
<td>45</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

**Communication**

In Table 4.3, 70 percent of the teachers think that observing colleagues’ teaching and attending the conference after the observation are very useful to improve their knowledge of teaching mathematics. At the post-observation conference, the communication between observees and observers usually goes well. All observers attend the post-observation conference to comment on the class and give suggestions. The data from the questionnaire survey show that 95 percent of mathematics teachers at Rainbow feel very comfortable asking their colleagues questions, and more than half of them would challenge their colleagues’ arguments. However, only 30 percent of mathematics teachers at Rainbow would challenge the arguments of their teaching research coordinators or university professors during the TRGAs. Table 4.5 shows the communication between teachers and their colleagues, the teaching research coordinators and university professors during the post-observation conference and TRGAs.
Table 4. 5

Communication in post-observation conferences and TRGA

<table>
<thead>
<tr>
<th>Communication</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>In a post-observation conference, will you ask questions if you do not understand your colleagues’ mathematics teaching ideas and thoughts?</td>
<td>95%</td>
<td>5%</td>
</tr>
<tr>
<td>In a post-observation conference, will you challenge arguments that you disagree with your colleagues?</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td>In the TRGA, will you ask questions if you do not understand the teaching research coordinators or university professor’s explanation of a mathematics teaching idea and thoughts?</td>
<td>65%</td>
<td>35%</td>
</tr>
<tr>
<td>In the TRGA, will you challenge arguments that you disagree with teaching research coordinators and professors?</td>
<td>30%</td>
<td>70%</td>
</tr>
</tbody>
</table>

Part 2. Findings from the Qualitative Analysis

In this part, I provide the qualitative in-depth explanation of how Rainbow Elementary School mathematics teachers acquire and develop their pedagogical content knowledge within their community of practice. That is, to acquire and improve their pedagogical content knowledge, what do mathematics teachers learn from their mentors, colleagues, the teaching research coordinator, and the university professor through mentorship and TRGAs? The qualitative data are from non-participant observations of Mr. Yu’s class and Teaching Research Group activity after the class; interviews of Yu and his mentor; Yu’s lesson plan and reflection report; and my field notes emerged the following categories: curriculum dialogue, student learning task analysis, and teaching strategy design.

Mr. Yu (pseudonym) is a mathematics teacher with three years of teaching experience at Rainbow ES. He taught a new topic, Volume, to his fourth-grade students. In the class, Yu began
with a video of a fable. The fable shows that a smart crow drinks water by putting stones into a
bottle in order to raise the level of the water. The students were all excited by the video. Then he
asked students what would happen if they put different sizes of stones into the water. After that
he showed a basketball, a volleyball, and a ping pong ball to his students, and asked them which
is the biggest, and which is the smallest. By showing his students that objects occupy space, and
different sizes of objects occupy different spaces, he introduced the concept of volume: the
volume of an object is a measure of how much space the object occupies. In the class, Yu also
asked his students to compare the volumes of two dictionaries, two sticks, and two glasses of
water. Moreover, he asked his students to manipulate a piece of plasticine into different shapes
and observe if the volume of the plasticine would be changed. In the last ten minutes of the class,
he let his students explore how to measure the volume of a Rubik’s pyramid.

Besides myself, Yu’s five colleagues, a teaching research coordinator in the city, and a
university professor observed his class and took notes. After the observation, all observers
attended the TRGA to share their comments, thoughts, and ideas about Yu’s class.

Curriculum Dialogue

The mathematics teachers spent a lot of time analyzing the mathematics textbook and
discussing the mathematics curriculum that they teach. They analyzed the teaching materials in
the textbook; connections of different topics within the textbook and the same topics in different
textbooks; writing supplementary teaching material; and the alternative teaching materials that
could be used to teach the concept of Volume.

Ms. Jing, Yu’s mentor, has a bachelor’s degree in education. She has taught mathematics
for 23 years and mentored Yu for more than three years. Both of them were teaching fourth
grade mathematics. Since Yu’s first year of teaching, Jing had been helping Yu in many different
ways. One of them was to better understand the teaching material in the textbook. Jing went over the mathematics topics in the textbook with Yu when he started to teach mathematics in his first year of career. Mentor Jing explained her help as a mentor:

Jing: When Mr. Yu was a novice teacher, I analyzed the textbook for him by telling him what the important points and difficult points of the teaching material are. After two months, Yu was asked to write a lesson plan by himself and I modified his lesson plan. I observed his teaching and commented on his class after the observation.

Jing continued to help her mentee Yu to prepare for better lesson plans when he had observers in his class. Jing helped Yu to prepare the lesson on Volume. They discussed the important and difficult points of the teaching material before Yu taught the lesson. Three years later, Yu was a lead teacher of the fifth-grade mathematics teachers. He has been responsible for writing fifth-grade mathematics exam papers, and helped his colleagues to prepare lesson plans. Jing has kept helping him and commented on and modified his exam papers.

Besides his mentor’s help, Mr. Yu usually learned from his colleagues through informal exchanges in their offices and through formal exchanges in their TRGAs. During the TRGA following Yu’s class, his colleagues were very willing to comment and share their ideas and suggestions about the lesson. The university professor and the teaching research coordinator also made comments and gave suggestions on how to improve the lesson. Yu was very quiet and took notes when his colleagues, the teaching research coordinator, and the university professor spoke. The following questions regarding mathematics concepts presented in the textbook and the teaching materials were discussed most during the meeting:

1) How does the textbook present the concepts of volume and capacity of an object?
2) Should the teacher teach the concepts of volume and capacity in one lesson or two?
3) What difference would it make if Yu brought rice, sand, or water to measure the volume of the Rubik’s pyramid in the class?

Some examples of the teachers (T), the teaching research coordinator (TRC), and the university professor (P)’s comments and suggestions follow.

T1: Mr. Yu, one of your students made a hollow dumpling by using the plasticine. He was confused about the concepts of volume and capacity. When I taught this lesson before, I brought a plastic bag to the class. I asked my students to observe the size of an empty plastic bag first. Then the bag is full of air. I asked my students to observe if the volume of the plastic bag had changed. I wanted my students to know that everything has its volume including air. But I did not go further because students felt it very hard to understand. I suggest that Mr. Yu could assign a project to ask his students to do some research at home.

P: Actually, students learned that gas has volume in science class when they were third graders. These students are fourth graders. It should not be hard for students to understand the volume of gas in the plastic bag.

T2: I also taught this lesson before. I feel it is a difficult lesson. When Mr. Yu prepared this lesson, I discussed with him the concepts of volume and capacity of an object. What are the similarities and differences between them? How does one refer to another? How does one transfer to another? Then Yu decided to teach only the concept of volume. I think he did a good job.

TRC: I observed one class in another district where the teacher combined the concepts of volume and capacity of an object in one lesson. Students are easily confused about these two concepts. But the teacher conducted the lesson successfully and his students
understood the concepts very well. The textbook only presents concepts and examples. In order to make students better understand concepts, teachers can work together to integrate and create new teaching materials to teach.

Mr. Yu modified the examples in the textbook. In the textbook, students were asked to compare the volumes of two leaves based on the picture of two leaves which are not obviously different in size, and the volumes of two dictionaries without any measurement of their length and width. Mr. Yu presented the pictures below (Fig 4.2) on a PowerPoint slide. The two dictionaries were labeled with measurements of the length and width of their covers. Yu wanted his students to notice that the two dictionaries have the same length and width but different heights. This example was praised by the university professor. He commented that Mr. Yu presented rigorous mathematics to his students. He said:

You gave the students the lengths and widths of the two dictionaries to show that the areas of the two covers are the same. Then to compare their volumes. On the textbook, there is no measurement of length and width (of the two dictionaries), which is not rigor. Middle school teachers always complain that elementary teachers do not teach rigor mathematics, especially geometry. The students will face trouble when they learn geometry in the middle and high schools. They have difficulties in reasoning.

![Fig. 4. 2 From Mr. Yu’s lesson plan](image)

However, the professor criticized another example in the textbook that Yu modified. In the textbook, there is a piece of crushed plasticine. This example shows that the volume of a piece of
plasticine will not change even though its shape is changed. In the class, Mr. Yu let his students manipulate a piece of plasticine to observe if the volume of the plasticine changes. One of his students made a hollow dumpling. This led the students to confuse the concepts of volume and capacity of an object.

More discussion occurred regarding mathematics topics in elementary textbooks and middle school textbooks. The group spoke about making a list of the topics in different grades and then making connections between the same topics in elementary and middle schools. The thought was that teachers should prepare elementary students for the study of mathematics in middle schools. Therefore, the elementary teacher should know not only the mathematics curriculum in each grade of elementary school but in each grade in middle school.

**Student Learning Task Analysis**

In the interview, Yu mentioned that analyzing and designing students’ learning tasks is one of the very important teaching skills he learns from his mentor and colleagues, informally in their offices or formally in the TRG activities. During the TRGAs at Rainbow after Yu’s teaching, his colleagues, the teaching research coordinator, the professor, and the principal also commented on Mr. Yu’s class based on the students’ learning tasks. They argued that it is not hard for teachers to teach students explicit knowledge from the textbook, but it is difficult for teachers to find implicit knowledge and implicit learning tasks for students. They said that all the topics that students need to learn in elementary schools could be listed on one page. Students will learn all of them in five years. However, there is something more important than just teaching all the mathematics knowledge in the textbook.

They all agreed that it is not hard to teach students to complete their explicit learning task but it is difficult to teach students to complete the implicit learning task. Teachers not only teach
their student's mathematics knowledge but also wisdom. It is very important for teachers to know the students’ difficulties in learning this lesson. In this lesson, the explicit learning task is to understand the concept of volume of an object and to compare the volumes of different objects directly. The implicit learning task for students is to know how to measure the volume of an object by using an intermediate tool. The challenge for the students is understanding the concept of conservation of volume. Several teachers agreed that it was too difficult and abstract for children to understand the concept of space. The learning task could be for students to use water, sand, or rice as intermediate tools to explore the volume of a Rubik’s pyramid. However, in the class, Yu used water to demonstrate the experiment. The professor commented,

What is the learning task for today’s class? Mr. Yu, can you offer students more materials like sand, rice, and water to let them manipulate, and decide which they should use to measure the Rubik’s pyramid? Learning mathematics is by doing. Doing mathematics is very important. After they manipulate themselves, they may figure out what is a reasonable way to measure the volume.

The teaching research coordinator commented that teachers should predict their students’ difficulties and responses. Explanation of students’ learning is very important for a teacher. He said:

When teachers design a classroom activity, they should be able to predict their students’ reaction, and know how to respond to their reaction, how to facilitate students’ discussion and even verification.

When one of Yu’s students made a hollow dumpling and wondered about the change in the volume of the plasticine, Yu either did not see or ignored his student’s work. Yu missed the chance to resolve the student’s difficulty in understanding the concepts of the volume and
capacity of an object, which was not covered in the lesson. When Yu prepared his lesson, he did not expect a student to make a hollow dumpling or another object that would confuse understanding of the volume of the plasticine with the plasticine’s capacity to contain or enclose a space.

In the follow-up interviews, both Yu and his mentor Jing admitted that they did not expect the student’s hollow dumpling and his confusion. This was a very open learning task that students were asked to do. Yu and Jing appreciated all the comments and suggestions from their colleagues, the TRC, and professor in the TRGA after the class. In Yu’s reflection report, he realized that he would think deeply about students’ potential learning difficulties when he designs learning tasks.

**Teaching Strategy Design**

Mr. Yu was helped not only by his mentor Ms. Jing and the same grade teachers in preparing this lesson, but he also received many comments and suggestions from the teaching research coordinator, the university professor, and his colleagues, who were teaching different grade but had taught the lesson of volume before. Meanwhile, the group of teachers also learned from each other, the teaching research coordinator, and the university professor regarding teaching strategies during the Teaching Research Group activity. The discussion was focused on clear and logical conceptual teaching, accurate mathematics language, hands-on activities, encouragement of students’ thinking, group work, and presentation of their ideas.

**Logical conceptual teaching.** The teaching research coordinator and Mr. Yu’s colleagues commented that he taught the concept very clearly and logically. They praised the introduction of the concept and the classroom activity design. Instead of telling his students the concept of volume by reading a one-sentence definition, Mr. Yu gradually introduced it by different stages. He began
with a video of a fable to inspire his students’ curiosity about the relationship between the level of water and the stones. Then he encouraged the students to think if the level of water would be changed by putting the different sized stones into the water. After that he showed a basketball, a volleyball, and a ping pong ball to his students, and asked them which is the biggest, and which is the smallest. He showed his students that objects occupy space, and different sized objects occupy different spaces. Finally, he introduced the concept of volume by its definition.

**Accurate Mathematical language.** Yu’s colleagues also praised his accurate mathematical language. Mr. Yu explained clearly about the concept of volume of an object and asked his students to express the correct mathematics language of volume when they answered questions in class. For example, when he asked his students to compare the volumes of two books, he corrected them by adding the condition that the covers have the same area. His students then presented their answers:

In the case of the same size cover, the greater the thickness of the book, the greater its volume. Another example, in the same length of a stick, the thicker the stick, the bigger the volume.

By the end of the class, students presented their work using an accurate mathematical language. They expressed the concept of volume as the volume of an object measured by how much space the object occupies. One of his colleagues commented,

Mr. Chen cultured his students to express the mathematics language very well. At the end of the class, his students had no problem in expressing the volume by using the mathematical language correctly.

**Hands-on activities.** Mr. Yu used more demonstrations of experiments than hands-on activities in his class. He said it was hard to have more hands-on activities because of the size of
the class and the limited teaching time. There were 45 students in the classroom and it was a 40-minute class. However, his colleagues, the teaching research coordinator, and the university professors all agreed that Yu should have more hands-on activities. They argued that students should experience the real world to understand the concept of volume of an object, the measurement of it, and the comparison of the volumes of two objects. The professor emphasized that children learn mathematics by doing:

Mr. Yu, you had lots of demonstrations in the class. But why didn’t you let your students do more hands-on experiments? The textbook does not encourage students to experience learning activity. But children learn mathematics by doing. Doing math is very important. The concept of volume is not abstract. Students can feel the volume of an object… For example, you can let your students do an experiment to measure the volume of a Rubik’s pyramid, not just watch you do the experiment…, you can even let students choose materials such as sands, rice, or water to do the experiment for measuring the volume of a pyramid Rubik’s cube by themselves.

One of Yu’s colleagues suggested that Yu could assign a project of measuring the volume of a Rubik’s pyramid at home by doing experiments:

It was very difficult for fourth graders to understand the concept of volume even though Mr. Yu asked students to observe the objects he brought to the class. I think that it is more important for students to experience hands-on activity to understand the meaning of the size of a space. I suggest each student experience the experiment of measuring the volume of a pyramid Rubik’s cube. They can do experiments for several times, make a table, fill out the data, and write a conclusion of the experiment. This is an after-school project worth doing.
**Encouragement of students’ thinking, group work, and presenting.** Several students were called on to answer the questions even though many students raised their hands and were willing to respond. Mr. Yu called on no more than ten students to give their answers. Sometimes the questions probably were too hard for most of the students. He repeatedly called on the same students to answer his questions. Some of his colleagues suggested that Yu should give his students more time to discuss in groups. They suggested that Yu might set up a scaffolding for his students when the students did not know how to solve the problem. Some examples of his colleagues’ comments follow:

T1: Some problems were very difficult. Students cannot respond to them immediately. I suggest that Mr. Yu should add some group discussion for the problem-solving…. Most of the students who were around me were quiet. They were listeners. A few students expressed their mathematical thinking and ideas. I suggest that students should have enough time to discuss, then Yu can ask someone to present the result as the group representative.

T2: Different students have different ideas. Group working works very well. Good students would help the ones who have difficulties….

After the long discussion, they ended with solutions for how to teach effectively the concepts of measurement in elementary schools. They wanted to list all possible teaching strategies to teach measurement in future school TRGAs. The teaching research coordinator suggested:

We can list the teaching strategies such as manipulation strategy, transfer strategy, constructivism strategy, etc. Moreover, it’s better to attach a case study to each strategy. Then teachers share these with their colleagues. Based on this, every teacher can modify
and create their own teaching strategies.... Manipulation strategy includes students’ learning through practice; transfer strategy includes direct and indirect transformation.…

During the TRGA, Yu took notes on all the comments and suggestions. He was quiet and did not argue with his colleagues, the teaching research coordinator, or the professor about their comments and suggestions. In the interview after the TRGA, Mr. Yu told me that he appreciated these comments and suggestions. He thought that the TRGA is very important for the refinement and development of his knowledge of teaching mathematics. But he mentioned that he would argue with his mentor and his colleagues in the office if he did not agree with them. In the interview, Ms. Jing, his mentor, allowed her mentee to challenge her. She said she sometimes learned new things from mentee Yu. For example, he knows more than her about teaching mathematics with technology.

This is the second time that Yu has taught the lesson Volume. He changed his lesson plan. In the interview, he said:

I taught Volume before, at about the second year of my teaching. At that time, I just directly lectured the content of volume by definitions to my students. But this time, I prepared differently....

In his reflection report, he wrote,

I finished teaching the lesson Volume. This is my second time teaching it. I am glad that the fable at the beginning of the class really worked well. A good start is half of the success. I caught my students’ attention to learn a new topic, volume. It not only directly affected my students’ learning interests and their engagement, but also made the mathematics problem concrete and visual….I did not directly tell my students the definition of the volume of an object and the conservation of volume. I wanted my
students to transfer their knowledge to classroom activity and communication. However, it seemed that the students had difficulties in applying the conservation of volume of liquid to the conservation of volume in solids.

Mr. Yu concluded that he needs to pay more attention to predicting or anticipating students’ difficulties in learning new topics in mathematics. He will modify his lesson plan based on the comments and suggestions in the TRGA and teach it again next year.

**What do the teaching research coordinators do to improve the mathematics teachers’ pedagogical content knowledge during the Teaching Research Group Activities?**

To answer the second research question of what the district and city teaching research coordinators do to help these mathematics teachers to acquire and improve their pedagogical content knowledge during their TRGAs, I analyzed the qualitative data from the interviews of the district teaching research coordinator Ming and the city teaching research coordinator Qiang; the observation of the TRGA, which was a post-observation conference at Rainbow after Yu’s teaching; and my field notes.

Qiang has been working as a city elementary mathematics teaching research coordinator in the Shanghai Municipal Education Commission for more than fifteen years. Before he became a city elementary mathematics teaching research coordinator, he had been a district elementary mathematics teaching research coordinator for about eight years after he was an elementary mathematics teacher for more than thirteen years. After teaching mathematics more than twenty-six years, Ming has been a mathematics teaching research coordinator in the district A for more than four years. Both of them were expert specialist elementary mathematics teachers before they became teaching research coordinators. They are very familiar with the mathematics curriculum, teaching, and learning in Shanghai elementary schools. Usually both Ming and Qiang visit
Rainbow once a month. Sometimes they meet at Rainbow but sometimes visit Rainbow separately. Both of them are familiar with these mathematics teachers at Rainbow. Besides they participate in the TRGAs to help mathematics teachers’ teaching at Rainbow, both of them meet some of these teachers during the district and city TRGAs. Ming organizes the district TRGAs about eight times every semester while Qiang organizes the city TRGAs twice every semester. In addition, Qiang organizes the online TRGAs every semester. At Rainbow, according to the result of the quantitative analysis, the overwhelming majority of mathematics teachers participate in the district TRGAs at least once a month for each semester. Moreover, 10 percent of them participate in all district TRGAs during the semester. Not many teachers are able to participate in city TRGAs because of limits seats for the activities. Thirty-five percent of them reported that they attended the city TRGAs once over the last 12 months, 15 percent of them twice, and 15 percent of them three times over the last 12 months.

The themes of the city TRGAs are determined under the guidance of the working plans in the Central Ministry of Education, Shanghai Municipal Education Commission, and the teaching research office in the Shanghai Municipal Education Commission every year. The themes of the district TRGAs are determined based on the themes of the TRGAs organized by the city teaching research coordinator. Sometimes, the themes could be adjusted according to the needs of the mathematics teachers at Rainbow. In general, the main topics that teachers study in the district and city TRGAs include but not limit analysis of teaching materials inside and outside of textbooks, students’ learning situations, teaching objectives, and the key points and difficult points for teaching. Recently, the themes of the district TRGAs also contain the concerns about students’ cognitive process and their mathematics thinking. At Rainbow, Ming also involved the school project-based teacher training programs such as help teachers to design their digital
homework and develop the school mathematics curriculum. Because the schedule conflicted, Ming did not observe Yu’s classroom teaching and the TRGA afterwards. But Qiang attended the observation and the TRGA at Rainbow.

Based on the analysis of the data from the in-depth semi-structured interviews of Ming and Qiang, the observation of the TRGA at Rainbow after Yu’s teaching, and my field notes, the following emergent categories explain that the district and city teaching research coordinators’ help contribute to Rainbow elementary mathematics teachers’ acquisition and development of their pedagogical content knowledge during their TRGAs in the school, district, and city.

**Interpretation of Teaching Material**

Both Qiang and Ming suggest that studying textbooks is very important for teachers to acquire their mathematics content knowledge and the elementary mathematics curriculum so that they can teach well. Ming usually has been facilitating the TRGA to interpret the teaching materials in the textbooks at the beginning of each semester in the district since 2006 when the new elementary mathematics textbooks were implemented. The series of Shanghai elementary mathematics textbooks match the new Shanghai Primary and Secondary Mathematics Curriculum Standard (SPSMCS) issued in 2004. Some new topics such as statistics and probability are added to the new elementary mathematics textbooks. In order to implement the new curriculum and the textbook, Ming and Qiang spend time in interpreting teaching materials in the new textbooks. Sometimes Ming invites textbook writers to interpret the topics in the textbook to the district mathematics teachers. Ming points out that it is extremely important for the novice teachers and junior teachers who first time teach the new topics to participate in the TRGAs related to interpreting textbooks. There are special activities for these teachers. For example, novice teachers are required to attend the district TRGAs to study the textbook. In
addition, Qiang often visits Rainbow to join the school TRGAs to help teachers better understand the specific topics in the textbooks. Qiang emphasizes to analyze textbooks based on the SPSMCS. He thinks the SPSMCS is an umbrella. Teachers need to better understand the curriculum standard first before they analyze the teaching materials in textbooks. They need to know the ins and outs of all the mathematics knowledge they will teach to their students. He helps teachers to better understand the SPSMCS first, then all the mathematics topics in the curriculum, then these topics in each grade, each semester, each unit, and each lesson. To each lesson, in order to help teachers to better understand the teaching objectives, he explains how each lesson’s teaching objective connects to the teaching objectives in each unit, and then to those in the curriculum standard.

**Analysis of Students’ Learning Conditions**

According to Ming, in his district, the teaching research coordinators spend lots of time in helping mathematics teachers research their students’ learning. In the district TRGAs, Ming focuses on developing teachers’ knowledge of students’ mathematical cognitive learning, students’ experience of mathematics learning, and students’ development of mathematical thoughts and ideas.

Ming thinks it is very important for teachers to know students’ mathematics learning condition before they teach. At Rainbow, many students take afterschool classes to learn more mathematics ahead of their school year. They already have known the content that their teachers are covering in the classroom. They do not have the curiosity to learn and feel bored in the class. Therefore, mathematics teachers need to prepare appropriate teaching material to fit these advanced students. The district teaching research coordinators invited experienced teachers to help junior teachers predict their students’ cognition base. When the experienced teachers lecture
the teaching materials in the textbook with junior teachers, they recall their previous teaching experiences to help junior teachers understand students’ cognition base in learning these mathematics topics. Ming also helps the teachers make pre-test for their students to know their previous mathematics knowledge. Ming said during the interview:

We design an assessment questionnaire to diagnose students’ existing mathematics knowledge. Fifty percent of the students in the class will be selected to do the questionnaire. Usually 10 to 20 percent of students are diagnosed to know the content that their teacher is going to cover in the class. These students will be assigned to help their peers in the class. However, some of these students actually do not understand well about the mathematics concepts because they cannot explain the content to their peers. These students will continue to learn when they realize their difficulties.

Moreover, Ming wants teachers to know the process of students’ mathematics learning. He states that teachers should develop their students’ sense of mathematization and encourage their students to raise mathematics questions in their daily life. He reminds teachers that students’ process of mathematics learning is in the process of understanding mathematics concepts and thinking mathematics questions. Based on this analysis of students’ learning conditions, Ming encourages teachers at Rainbow to design more hands-on activities in the classroom. Ming said in the interview:

Let students experience mathematics learning by participating in hands-on activities that make them learn from the intuitive experiences.

After the observation, Qiang suggested that Yu should have more hands-on activities in his class rather than Yu’s demonstration. Qiang emphasized students’ learning process.
This class’s topic is practicing geometry, but I did not see much practice in the class. Instead, you demonstrated a lot of phenomena. I suggest that you let students practice and communicate with each other after the practice. After that, let them experience the experiment again, then communicate again, and do the experiment again if necessary. Let your students experience the process of learning the concept of volume. Otherwise they do not deeply understand the concept.

Furthermore, during the district and school TRGAs, Ming emphasizes that teachers should develop their students’ mathematical thinking when they teach mathematics. He helps teacher transfer static knowledge in the textbook to dynamic knowledge to their students in the classroom. For example, in the TRGAs, teachers learn different mathematics thoughts and methods such as function thoughts, reasoning thoughts, reductionist methods, etc. These thoughts and methods are the implicit knowledge that teachers can facilitate to their students.

**Observations of Teachers’ teaching and Comments on the Teaching**

From the findings of the quantitative data analysis, mathematics teachers at Rainbow have a very positive view of the district and city teaching research coordinators’ help. Seventy percent of them think the teaching research coordinators are very helpful. Ming and Qiang are invited to visit Rainbow to observe and comment teachers’ classroom teaching periodically. Ming is concerned more about the students’ learning process and logic of the teaching flow. He said that he could observe if teachers’ teaching clearly encourages their students to experience the process of mathematics learning. He wants to observe if teachers’ questions stimulate their students’ mathematical thinking and development of their mathematical thinking. Ming suggests:

If students can immediately answer their teachers’ questions in the class, I would calculate the number of these questions during the observation. If there are many of these
kinds of easy questions in one class, I would conclude that it is not a good lesson. The good questions or problems posed by teachers should lead their students’ thinking. Even though sometimes the students’ thinking results in a great disagreement, the controversial questions are valuable.

Besides the questions and problems that teachers pose in the class, Ming also observed the conversations between teachers and their students. He suggests that teachers respect their students’ expression of their mathematics thoughts and ideas even though they are wrong. He discourages teachers from interrupting their students when they are presenting their thinking and solutions. Based on these concerns, he comments and gives suggestions to teachers during the post-observation discussion.

According to Qiang, he is concerned more about teaching objectives, the content of teaching, the process of learning, and the effectiveness of learning. Qiang also emphasizes his observation of the students’ learning. He suggests that students’ learning reflects their teacher’s teaching. He likes to observe students’ reactions to teachers’ questions and their understanding of the concepts. Qiang realizes the best way to develop teachers’ pedagogy is to observe their classroom teaching and comment on their teaching immediately after the observation:

It is usually not very effective to separately train teachers’ knowledge of mathematics content, knowledge of curriculum and teaching material of textbook, knowledge of students’ learning, and knowledge of teaching strategies. The most efficient and effective way is to observe their class and comment on their classroom teaching. This help is concrete, detailed, and direct to the teachers.
How does school administration’s support enable teachers’ acquisition of mathematics pedagogical content knowledge within those communities of practice?

After analyzing the qualitative data from the interviews with the principal, Ms. Wei, and the school teaching research coordinator Jing, at Rainbow Elementary School, I know that Rainbow mathematics teachers’ acquisition of their PCK within their communities of practice would not happen without the principal and her administration’s support. The qualitative analysis emerged from the categories of recruiting policy, mentorship policy, teaching research group and lesson preparation group activity policies, and provisions for professional development.

**Recruiting Policy**

New teachers are usually recruited among student teachers at Rainbow. According to Wei, they begin to develop teachers’ PCK even before they hire them. Rainbow is one of the top public elementary schools in the city where pre-service teachers practice their teaching for six weeks during their senior year. Each of them has an experienced mathematics teacher as a mentor. Student teachers observe their mentors’ teaching for a week. After that, they prepare lessons by themselves, teach the class, and grade students’ homework. Mentors observe and comment on all of their mentees’ classes. After the six-week practice teaching period, mentors recommend their mentees to the principal as potential recruits if they think their mentees possibly could meet the qualifications of a new teacher whom Rainbow needs or they are worthy of training to be a qualified new teacher. If the principal agrees that these student teachers are potential candidates for hiring, they are invited to return to the school for another two weeks of practice teaching. Wei says that the purpose of the additional practice teaching is the opportunity for both sides to know more about each other.
To the student teachers, they can have more time to experience the working environment at Rainbow. To us, we have more time to observe these student teachers if they are the right candidates we want to hire.

During the two-week unpaid internship, mentors continue to communicate with their mentees to know these student teachers’ understanding of teaching and learning of mathematics. They ask their mentees to write homework problems and lesson plans to see if they have the basic teaching literacy. The candidates experience the teacher’s life at Rainbow to see if they really want to teach there after graduation. Meanwhile, the school observes the student teachers’ development of their practice teaching and decides if they are qualified to be elementary mathematics teachers at Rainbow. Once the school administrator decides and these student teachers are satisfied with each other, these student teachers will apply for the jobs in the school district by the usual procedure. They will take an examination and be interviewed by educational experts in the district. If they pass the examination and the interview, they are hired as elementary mathematics teachers in the district, then transfer to Rainbow.

Yu majored in special education but not specializing in teaching elementary mathematics. Rainbow’s recruiting policy enables Yu’s successful career in teaching elementary mathematics. The school teaching research coordinator Jing took Yu as an example to explain their recruiting policy at Rainbow:

When Yu was a student teacher at Rainbow, his mentor was an excellent mathematics teacher who was later promoted to a teaching research coordinator in our district. His mentor helped him a lot during his six-week practice teaching period. When we realized that he was a potential candidate to be a member of our school, he was invited to come back to experience more of the Rainbow mathematics teacher’s life for another two
weeks after his practice teaching. His previous mentor continued to help him. With the help, Yu better understood the teaching and learning of mathematics at our school. I have been his mentor since he was hired, it is easy for me because he already knew about teaching and experienced mathematics teaching at our school.

**Mentorship Policy**

New teachers attend the new faculty orientation for another three and half days during the summer break. The formal mentorship begins before the new hires begin teaching in their first semester. A novice teacher is assigned a formal mentor who could have been the mentor during their practice teaching or another experienced mathematics teacher. Besides the lectures by the heads of the human resources, student affairs, and information and technology offices, the most important training for the novice teachers are the training programs by the curriculum office. The coordinator of the mathematics curriculum at Rainbow facilitates workshops to help the mathematics novice teachers know and better understand the SPSMCS. Studying the mathematics curriculum for the entire elementary school years first, teachers are more focused on the curriculum in the grade stages they will teach. Teachers who teach lower grades are in one group and ones who teach upper grades are in another. They discuss the curriculum and ask questions that the coordinator answers. After understanding the mathematics curriculum in the grades that they will teach and the all elementary grades, one-to-one communication between mentor and mentee starts to help novice teachers to understand the mathematics topics they will teach in the coming semester. The school mathematics teaching research coordinator also assists mentors to analyze the textbook for novice teachers. They help novice teachers clarify the important points and difficult points in each mathematics topic they will teach. They also
connect the mathematics topics in the previous and following grades textbook to the textbook they will teach.

Moreover, the school mathematics teaching research coordinator and mentors help novice teachers know about how to prepare and teach lessons, how to assign homework, how to help their students after classes, and how to grade homework. Novice teachers also learn about classroom culture of criticism and encouragement and how to implement these culture from the school teaching research coordinator and their mentees. They usually show some videos of successful teaching by their school mathematics teachers to the novice teachers and analyze the lesson. They discuss the objectives and learning tasks of the lesson, the reason behind of the teaching strategies, and what the implicit knowledge and mathematics thoughts of the lesson.

When the semester begins, the mentorship works periodically. The school regulation shows that mentors are required to observe their mentees’ classes at least twice a week while the mentees are required to observe their mentors’ classes at least once a week. The post conference must be held. According to the quantitative analysis in part 1, 72 percent of mentees teach the same grade mathematics as their mentors. Therefore, mentees can get detailed guidance regarding teaching from their mentors. At Rainbow, all lower grade mathematics teachers share one office while higher grade mathematics teachers share another. Fifty percent of mentors do not share the same office with their mentees because some mentors teach different grade mathematics, and some of them have administrator positions. However, phone calls, emails, and social media such as Wechat are other means of communication between mentors and mentees.

TRG and LPG Activity Policies

According to the school’s regulation, all mathematics teachers do not teach on Thursday mornings so that they can participate in their grade Lesson Preparation Group (LPG), grade
TRG, or school TRG activities. Every Thursday morning in the odd weeks of a month, teachers attend the grade TRGA. The grade lead teacher facilitates the grade TRGAs. The same grade teachers usually discuss the teaching material in the textbook and share their concerns about teaching and learning with their colleagues. If someone is to be observed, the grade teachers help their colleague prepare the lesson, observe and comment on the teaching before their colleague is observed by other mathematics teachers at Rainbow, the district and city teaching research coordinators, or the university professor. Sometimes, they invite other grades teachers to join their grade TRGA to learn more about the coherence and consistency of the curriculum. The school teaching research coordinator visits different grade TRGAs to learn what teachers discuss and are concerned about. These issues may be brought up at the school TRGAs as a theme of the curriculum dialogue.

Every Thursday morning in even weeks of a month, all school mathematics teachers attend the school TRGAs. The school teaching research coordinator hosts the school TRGAs. Sometimes, the school invites a university professor, the district and city teaching research coordinators to join the school TRGAs to help mathematics teachers develop their mathematics pedagogical content knowledge and improve their teaching. They usually observe classes with other mathematics teachers at Rainbow and comment on the class after the observation. All mathematics teachers are required to comment their colleague’s teaching. Even though they teach different grade mathematics in the current semester, some of them taught the lesson before while some of them involved in the lesson preparation. After all the mathematics teachers speak up, the district and city teaching research coordinators comment on the lesson and give their suggestions. They also respond the teachers’ comments. Rainbow mathematics teachers are encouraged to exchange their ideas and learn from each other, the district and city teaching
research coordinators during the formal school TRGAs. In addition, some of the mathematics teachers participate in the district TRGAs on Thursday afternoon. Their colleagues substitute their teaching when they attend the district TRGAs. Teachers are also encouraged to participate in the city TRGAs. They do not teach on that day.

At Rainbow, there are five rules in terms of Lesson Preparation Group Activities (LPGA). For each LPGA, it needs to set up a keynote speaker, a theme, time, location, and a host. Novice and junior teachers are strongly encouraged to be active in the LPGAs. With their mentors’ help, they are hosts and keynote speakers in the LPGAs. Same grade teachers help and learn from each other. They prepare lessons together, observe their peers’ lessons, and comment on their colleagues’ teaching. At Rainbow, teachers are required to observe their colleagues’ classes and be observed by their colleagues. Wei explained,

Every semester, each teacher must open his/her classroom twice to allow other teachers who teach the same grade to observe. If necessary, they can open their classrooms to other grades’ teachers, and even to the all the school teachers.

**Competition Policy**

Every Friday afternoon is the school professional development time. The activities include the lectures by experts and school teaching competitions. The school periodically organizes some competitions for the best lesson plan, homework problem development, and homework project design. Principal Wei points out that these competitions offer young teachers’ opportunities to engage themselves and show their professional development to the school community. It inspires more and more young teachers to learn from each other and grow together professionally so as to develop the school community.
Summary of the Case Analysis

Eighty-five percent of the 20 mathematics teachers at Rainbow are aged below 40. Eight out of ten of Rainbow mathematics teachers hold bachelor’s degrees in education and 10 percent hold master’s degrees in elementary education specializing in teaching mathematics. Rainbow mathematics teachers acquire and develop their pedagogical content knowledge through their mentorships; participation in the variety of TRGAs inside and outside of the school and district; informal and formal communication with their colleagues and counterparts, district and city teaching research coordinators, the university professor; and their own reflections. The case study of Mr. Yu shows that Rainbow teachers learn and improve their pedagogical content knowledge through dialogue about the curriculum, analyzing student learning task, and designing teaching strategies.

Almost all mathematics teachers at Rainbow believe that their mentors’ help and TRGA participation are very important to the acquisition of their knowledge of teaching mathematics. Eighteen out of 20 teachers report that they have or had mentors. Both mentors and mentees have positive views on their mentorships. Novice teachers observe and imitate their mentors’ classes when they start teaching in their first semester. After two months, they prepare lessons independently with their mentors’ help. Their mentors observe and comment on their teaching. Junior teachers can still get help from their mentors. In addition, all mentors admit that they learn from their mentees while six out of ten mentees are willing to challenge their mentors if they disagree with them. Rainbow mathematics teachers say their school TRGAs are more helpful than the district and city ones. The great majority of Rainbow mathematics teachers enjoy, in the order of preference, the TRGAs for designing lesson plans, discussing mathematics topics and their relation to the curriculum, analyzing students’ difficulties, discussing teaching strategies,
and observing colleagues’ teaching and attending the post-observation conference. Many of them argue and negotiate with their colleagues if necessary during the TRGAs but not with the district and city teaching research coordinators and the university professor.

All teachers at Rainbow appreciate the district and city teaching research coordinators’ help. Three out of four consider them very helpful to improve their knowledge of teaching mathematics. The analysis of the interviews with the city teaching research coordinator Qiang and the district teaching research coordinator Ming show that they periodically organize the city and district TRGAs. Ming organizes the district TRGAs about eight times per semester while Qiang organizes the city TRGAs twice a semester and one online TRGA as well. Besides helping teachers better understand SPSMCS, they analyze teaching materials inside and outside of textbooks; teaching objectives; the key points and difficult points of the mathematics topics for teaching; students’ learning situations; cognition process; and their mathematical thinking. Meanwhile, they visit Rainbow to participate in the school TRGA and guide the teachers to conduct the school research projects. They observe and comment on Rainbow teachers’ classroom teaching, interpret teaching materials, and analyze students’ learning conditions with them in detailed examples to support Rainbow mathematics teachers’ acquisition of their pedagogical content knowledge.

At Rainbow, under the administration led by Principal Wei who had 15 years teaching experiences and was an expert elementary mathematics teacher, Rainbow formulates the policies for recruiting, mentorship, TRG and LPG activities, and competitions to enable the teachers’ acquisition and development of their pedagogical content knowledge. These policies provide opportunities, space, time, and professional help from the district, city, and university to help Rainbow mathematics teachers, mostly young mathematics teachers, learn from their mentors,
colleagues, counterparts and teaching research coordinators inside and outside of the school community.
CHAPTER V
FININDINGS

THE CASE OF BLUE SKY ELEMENTARY SCHOOL

Compared to Rainbow, Blue Sky is ten years older. It was founded in 1997 as a public rural school in District B, located in a southwest suburb of Shanghai. Blue Sky started with six classes of more than 100 students, many of whom were migrant children. With more people moving to suburban districts from urban districts over the last twenty years, Blue Sky has been expanded to two campuses with 61 classes of more than 2,700 students. Currently, the ratio of students from Shanghai-registered residences to the migrant students is ten to one. Blue Sky has been honored recently as one of the top elementary schools in its district for several consecutive years. It was also recognized as one of the ten best local public schools in Shanghai in 2013. Moreover, like Rainbow, Blue Sky currently is one of the academic cooperation schools with Shanghai Teaching Research Office and one of the public elementary schools where pre-service teachers practice their teaching during their senior year.

Of the 24 mathematics teachers at Blue Sky, 46 percent of them are aged below thirty; 25 percent are aged between thirty and forty; while 21 percent are aged between forty and fifty. Sixty-three percent of mathematics teachers hold a bachelor’s degree and 33 percent hold a master’s degree. At Blue Sky, one in four is a novice teacher. Fewer than 50 percent of the teachers have been teaching mathematics for eight years or more. Like Rainbow, the overwhelming majority, 83 percent of mathematics teachers are female. About 54 percent of all mathematics teachers majored in education, including 33 percent of them have their master’s degrees. Some of them concentrated in teaching mathematics but some did not. Because more people are moving to District B, there is a shortage of elementary teachers and elementary
teachers specializing in mathematics, 46 percent of teachers majored in other fields such as applied psychology, computer science, business, and economic management currently are teaching mathematics at Blue Sky. This raises even more interesting questions. How do Blue Sky mathematics teachers, including those who do not have a background in the educational field or specialist elementary mathematics teachers before their teaching careers, acquire and develop their mathematics pedagogical content knowledge? How do teaching research coordinators help these teachers acquire and improve their pedagogical content knowledge? Finally, how do the school’s policies enable these teachers’ development of their pedagogical content knowledge?

**How do mathematics teachers acquire their Pedagogical Content Knowledge?**

To answer this question, I analyzed the quantitative data from the survey questionnaire that all mathematics teachers participated and completed. I also analyzed the qualitative data including the observations of a class demonstrated by Hua and a Teaching Research Group Activity after the classroom teaching; interviews of Hua; Hua’s lesson plan and her reflection reports; and my field notes.

**Part 1. Findings from the Quantitative Analysis**

Here, I discuss the quantitative description of the general picture of how mathematics teachers acquire and develop their pedagogical content knowledge at Blue Sky. All 24 mathematics teachers responded to the questionnaire survey. The data were analyzed regarding the teachers’ mentorship by their master teachers and in relation to other novice and experienced teachers; frequencies and activities of the TRG inside and outside of the school, and inside and outside of the district and the city; the level of impact on their mathematics teaching by their participation in the TRGAs; and the communication between colleagues and teaching research coordinators during the TRGA.
Mentorship

Seventy-five percent of the teachers at Blue Sky reported that they had mentors when they were novice teachers and 45 percent of the teachers reported that they mentored novice teachers and junior teachers. They usually spend average three hours a week together for an average of two years. One teacher stated that she worked with her mentor ten hours each week during the mentorship. Another teacher claimed that the mentorship between her and her mentor lasted formally and informally for seven years. Ninety-four percent of Blue Sky teachers who had mentors taught the same grade mathematics as their mentors do. But 61 percent of mentees shared the same offices with their mentors.

Both mentors and mentees had positive views of their relationship. When they have difficulties in teaching, 71 percent of mathematics teachers at Blue Sky first would like to communicate with their mentors, followed by experienced teachers and their peers. Teachers feel comfortable asking their mentors questions. Not many of them challenge their mentors’ positions. Table 5.1 below shows the data from the mentors and mentees at Blue Sky.

Table 5.1

<table>
<thead>
<tr>
<th>Mentorship</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>If you are a mentee, do you ask questions if you do not understand mentor’s explanation about teaching ideas and thoughts?</td>
<td>94%</td>
<td>6%</td>
</tr>
<tr>
<td>If you are a mentee, do you challenge arguments that you disagree with your mentor?</td>
<td>44%</td>
<td>56%</td>
</tr>
<tr>
<td>If you are a mentor, do you learn anything regarding mathematics teaching from your mentee?</td>
<td>82%</td>
<td>18%</td>
</tr>
</tbody>
</table>
The data from the survey also showed that all mentors usually decided on a reasonable compromise after discussing a problem with their mentees. Some mentors explained that they learned from their mentees about information technology skills when they prepared the lesson plans together. These skills enhance the lesson plan and teaching strategies to stimulate their students’ curiosity of learning mathematics and improve their understanding of mathematics. Other mentors appreciated their mentees’ dynamic ideas and open thoughts. In addition, two mentors especially praised their mentees’ strong mathematics content knowledge and educational theories that they acquired during their master’s degree study. They were inspired by their mentees with this knowledge to deeply understand mathematics problem-solving methods.

**Participation**

All mathematics teachers at Blue Sky are strongly encouraged to participate in the TRGAs at their school, in the district, and city-wide online TRGAs every semester. Eighty-three percent of mathematics teachers attend school TRGAs twice a month; about 80 percent of mathematics teachers attend a district TRGA between six and eight times every semester; less than half of mathematics teachers joined online TRGAs with their counterparts in the city once to three times every semester. In terms of the opportunities to participate in TRGAs outside of the district and city, about half of the mathematics teachers never attended TRGAs in other districts while 77 percent of mathematics teachers at Blue Sky never had a chance to participate in a TRGA outside of the city. In the questionnaire survey, teachers were asked to evaluate the TRGAs in their school, their district, outside of district and city, and online in enhancing their knowledge of teaching mathematics. Most of them agreed that school and district TRGAs help them more than outside district TRGAs, followed by online and outside of city TRGAs. The detailed data are shown in table 5.2 and figure 5.1 below.
Table 5. 2

*Distributions in percent of teachers’ evaluations of different types of TRGA*

<table>
<thead>
<tr>
<th>Activities in your school</th>
<th>Very useful</th>
<th>Useful</th>
<th>Neutral</th>
<th>Not very useful</th>
<th>Not useful</th>
<th>Do not apply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>79</td>
<td>17</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Activities in your district</td>
<td>75</td>
<td>17</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Online activities in the city</td>
<td>31</td>
<td>45</td>
<td>20</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Activities in other districts</td>
<td>50</td>
<td>17</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>Activities outside the city</td>
<td>33</td>
<td>21</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>42</td>
</tr>
</tbody>
</table>

Fig. 5. 1 Distributions in percent of teachers’ evaluations of TRGAs

**TRGA Impact**

Blue Sky mathematics teachers reported that they usually attend the TRGAs to design a lesson plan with their colleagues, observe their colleagues’ classes, attend the post-observation conference, and discuss mathematics topics and their students’ difficulties in learning. The data
show that, over the past academic year, teachers designed a lesson plan with their colleagues 11 times on average. The average number of times of observing their colleagues’ teaching is 25. As a teacher, the average frequency of observation during an academic year is four times. One novice teacher reported that she observed 50 classes during the year, which means at least twice a week. Also, she was observed by her colleagues five times over the past academic year.

Furthermore, over 90 percent of them also want to discuss the topics in mathematics, their relations to the curriculum in the textbooks, and the strategies for teaching them with their colleagues during the TRGAs. When they were asked the degree of usefulness of the activities that they participated in at the TRGA to improve their knowledge of mathematics teaching, they responded that observing colleagues’ teaching and discussing the mathematics topics and teaching strategies were most useful, followed by designing lesson plans with their colleagues, attending post-observation conferences, and discussing students’ difficulties. Tables 5.3 provides the detailed distribution of the teachers’ responses.
Table 5.3

*Distributions in percent of teachers’ evaluations of the TRGA impact on their knowledge of mathematics teaching*

<table>
<thead>
<tr>
<th>Activity</th>
<th>Very useful</th>
<th>useful</th>
<th>Neutral</th>
<th>Not very useful</th>
<th>Not useful</th>
<th>Do not apply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designing a lesson plan with colleagues</td>
<td>83</td>
<td>13</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Observing colleagues’ teaching</td>
<td>88</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Attending the conference after teaching</td>
<td>83</td>
<td>13</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Studying mathematics topics and their relation to the curriculum in the textbooks</td>
<td>58</td>
<td>33</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Discussing mathematics topics and the students’ difficulties learning them</td>
<td>83</td>
<td>13</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Discussing mathematics topics and the strategies for teaching them</td>
<td>88</td>
<td>8</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Taking courses about mathematics knowledge</td>
<td>54</td>
<td>29</td>
<td>8</td>
<td>4</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Taking courses about teaching knowledge</td>
<td>63</td>
<td>21</td>
<td>8</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Taking courses about mathematics teaching knowledge</td>
<td>67</td>
<td>17</td>
<td>4</td>
<td>8</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

Moreover, when they were asked the degree of helpfulness from their counterparts in the district and city, the teaching research coordinators, and university professors, they responded that the teaching research coordinators and their colleagues were most helpful. Table 5.4 has more information.
Table 5.4

*Distributions in percent of teachers’ evaluations of the people who help them improve their knowledge of mathematics teaching*

<table>
<thead>
<tr>
<th></th>
<th>Very helpful</th>
<th>helpful</th>
<th>Neutral</th>
<th>Not very helpful</th>
<th>Do not helpful</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colleagues in your school</td>
<td>79</td>
<td>21</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Counterparts in your district</td>
<td>38</td>
<td>50</td>
<td>8</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Counterparts in other districts</td>
<td>30</td>
<td>48</td>
<td>9</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>Teaching research coordinators</td>
<td>88</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>University professors</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

**Communication**

During the post-observation conferences, most of the Blue Sky mathematics teachers felt very comfortable asking their colleagues questions if they did not understand their explanations regarding mathematics teaching ideas and thoughts. Some of them would challenge their colleagues’ arguments, but very few would challenge the district and city teaching research coordinators and university professors if they disagreed with their positions, even though two thirds of teachers would ask questions. Table 5.5 shows the communication between teachers and their colleagues, teachers and teaching research coordinators and university professors.
Table 5.5

*Communication in post-observation conferences and TRGA*

<table>
<thead>
<tr>
<th></th>
<th>Communication</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>In a post-observation conference, will you ask questions if you</td>
<td>do not understand your colleagues’ mathematics teaching idea and thoughts?</td>
<td>83%</td>
<td>17%</td>
</tr>
<tr>
<td>In a post-observation conference, will you challenge arguments</td>
<td>that you disagree with your colleagues?</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>In the TRGA, will you ask questions if you do not understand the</td>
<td>teaching research coordinators or university professor’s explanation of a</td>
<td>67%</td>
<td>33%</td>
</tr>
<tr>
<td>teaching research coordinators or university professor’s</td>
<td>mathematics teaching idea and thoughts?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In the TRGA, will you challenge arguments that you disagree with</td>
<td>teaching research coordinators and professors?</td>
<td>8%</td>
<td>92%</td>
</tr>
</tbody>
</table>

**Part 2. Findings from the Qualitative Analysis**

Here, I provide the qualitative in-depth explanation of how Blue Sky mathematics teachers acquire and develop their pedagogical content knowledge within their school communities of practice. What in detail do these teachers learn from their mentors, colleagues, and the district and city teaching research coordinators? The qualitative data from non-participant observations of Hua’s classroom teachings and two TRGAs after the two classes respectively; interviews of Hua, Hua’s lesson plan and the reflection report; and my field notes were analyzed. This qualitative analysis emerged to the categories of supplementing teaching materials, understanding students’ thinking, and teaching mathematical thoughts, which contribute to the further explanation of the Blue Sky mathematics teachers’ acquisition of their pedagogical content knowledge.

Hua has an elementary education bachelor’s degree specialized in mathematics, and 12 years of teaching experience. She had a master teacher as her mentor when she was a novice
teacher. Currently, she is not only a mentor to many novices and junior teachers in her school but in other schools in her district. Involved in a collaborative project of the education departments in Shanghai and England two years ago, Hua was selected to teach mathematics in an elementary school in England for one month. During my time in Shanghai, I observed Hua’s class as a part of the scheduled TRGA at Blue Sky, along with all mathematics teachers, school administrators, city and district teaching research coordinators. She taught a lesson about a tree planting problem to third graders. I also attended another part of the TRGA where all of her colleagues and the district and city teaching research coordinators commented on her teaching after the class.

Hua began by reviewing the video called a micro e-lesson video. It is used as part of students’ homework to prepare for a new lesson. In the video, someone cuts a ribbon into two parts, and cuts another ribbon into three pieces. Hua assigned it the day before the class and asked her students to watch the video, cut ribbons into different pieces as shown in the video, and look for the relationship between the number of times and the number of pieces that the ribbon was cut. Since her students did the homework at home, many of them responded to the answer very well when Hua asked about the relationship between the number of times and the number of pieces that the ribbon was cut in the video. Hua praised her students and asked the rest of the class to discover the pattern in this relationship. Her students all quickly responded that the number of pieces the ribbon turned into is one more than the number of times the ribbon is cut. After that, Hua posed a problem on the smart board: how do we draw trees on a 12-centimeter line segment such that the trees are two centimeters apart? She asked her students to draw trees on their worksheets and she walked around the classroom. She encouraged her students to look for multiple methods. Every student used his or her iPad to take pictures of their solutions and then uploaded them to the smart board immediately. Hua chose different solutions from her
students on the smart board and let her students comment on these solutions. Several students were called upon to share their thinking. They said the solutions represented three different situations: 1) No trees are drawn on both endpoints of the line segment; 2) one tree is drawn on each endpoint of the line segment; 3) one tree is drawn on either endpoint of the line segment.

After that, Hua expanded the problem to how to draw trees on a 12-centimeter line segment such that the trees are three centimeters, four centimeters, or six centimeters apart, respectively. She encouraged her students to think about and look for the relationship between the number of trees drawn and the number of parts separated by trees on the line segment in three different situations. During the last eight minutes of the class, Hua asked her students to solve the three-word problems: 1) how many Christmas gifts could be hung on a 16-meter colored rope such that gifts are four meters apart and there are gifts on both ends of the rope? 2) A gardener wants to plant trees on a 27-meter long road in Shanghai Disney Park. How many trees can he plant if the trees are three meters apart and one tree can be planted at one side of the road? 3) To celebrate the New Year, our school wants to decorate the corridor with some plants. How many plants could be put in a 20-meter corridor if the plants are two meters apart and the ends of the corridor are walls? The students read these problems on their iPads, drew their own line segments to solve the problems, filled out their answers in their iPads, and submitted them to the teacher’s computer that is connected to the smartboard. Before the class was over, Hua played another micro e-lesson video to show two more challenging problems. Hua assigned her students to watch the micro e-lesson video again at home and solve these two problems.

Hua controlled the class during the entire period. She effectively engaged her students in the 40-minute class. She successfully transferred her students’ knowledge of the relationship between the number of times and the number of pieces that the ribbon was cut to the relationship
between the number of trees drawn and the number of parts separated by the trees drawn on the line segment with no trees on both endpoints of the line segment. She encouraged her students to find the pattern of the relationship between the number of trees drawn and the number of parts separated by trees drawn on the line segment in a different situation, then construct a mathematical model and use this model to solve more similar problems in real life.

**Supplementing Teaching Materials**

When they prepare each lesson, Blue Sky mathematics teachers study teaching materials in textbooks. In addition, they also make their own teaching materials for better teaching and learning of mathematics to supplement the examples and exercises in textbooks. They study and work together with their colleagues informally in their offices, or formally during their school TRGAs before and after teaching. Over the past year, Blue Sky teachers have started to make micro e-lessons (wei ke) for their students to preview and review the lesson taught in the class as supplementary of teaching materials.

**Creating micro e-lessons.** Hua made two micro e-lessons for teaching the lesson of tree-planting problems. On the day before the class, she sent one of the micro e-lessons to her students’ parents via the Chinese social media Wechat, which is similar to Facebook. Assigning the e-lesson as homework, Hua asked her students to watch the short video of the ribbon cutting and cut their own six ribbons once, twice, three times, four times, five times, and six times respectively. They needed to observe the numbers of pieces of ribbons created when each ribbon was cut a different number of times. Students were asked to fill out the results in table 5.6 and find the relationship between the number of times a ribbon was cut and the number of pieces of ribbons created after the ribbon was cut.
Table 5. 6

*Exploring Relationships*

<table>
<thead>
<tr>
<th>Number of times cutting a ribbon</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pieces of the ribbon after cutting</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Students all did their homework and responded to Hua very well at the beginning of the class. Hua used this e-lesson to transfer her students’ knowledge of the relationship between the number of ribbon cutting times and the number of the ribbons left after the cutting to solve the problem of drawing trees on a 12-centimeter line segment. Hua used another micro e-lesson she made to help students solve more complicated tree-planting problems at home. In this e-lesson, there is a short video lecture followed by two problems: How many roses can be planted on a circular flowerbed whose circumference is 15 meters if the roses are three meters apart? How many bonsais can be planted on the side of a fenced square whose perimeter is 16 meters if the bonsais are two meters apart? Students can access the e-lesson from their parents’ Wechat accounts and do their homework at home.

During the interview, Hua explained that the idea of making micro e-lessons was inspired by the flipped classroom pedagogical model. Compared with this model, micro e-lessons prepare students for learning new content in the class and digesting what they learned in the class when the students are at home. For students who have difficulties learning the content, they can review the topics by watching videos and solving more problems at home. For students who can look into the content in greater depth, they can solve more complicated problems posed in the micro e-lesson. Hua spent much time making these micro e-lessons. She needed to be very familiar with the topics in the textbook. She thinks these micro e-lessons make her teaching more efficient and effective. More importantly, they can help students’ learning of mathematics topics:
Thanks to most of the parents’ cooperation, the students watched the e-lesson and solved the problems at home. It was not very difficult for them to transfer the ribbon cutting problems to the problems of drawing trees on a line segment with no trees on both ends of the line segment. They have enough time to cutting ribbons at home. It is difficult to do hands-on activities and learn so much content in a 40-minute class.

Most of Hua’s colleagues and the teaching research coordinators all praised Hua’s micro e-lessons during the TRGA after her teaching. They agreed that it was very good for students to study micro e-lessons at home. Some examples of teachers and teaching research coordinators’ comments follow:

T1: Solving tree-planting problems are difficult for our students. At home, the students can watch the micro e-lesson, cutting the ribbons, fill out the form with their data, and look for a pattern. Because of the limited time in the class, students do not have enough time to do a hands-on activity and think the solutions to the problems. They only have time to watch the short video in the class.

T2: Students can actively learn when the problems are related to their real life. From the micro e-lesson, students find the relationship between the number of times of cutting a ribbon and the number of the pieces of the ribbon left after the cutting. This prepares the students well for learning the new lesson of solving tree-planting or drawing problems in the class.

TRC Fu: Micro e-lessons are special and important parts of the whole lesson of tree-planting problems.

However, one teacher and another teaching research coordinator did not appreciate the idea of the micro e-lesson of cutting ribbons. They argued that using the ribbon-cutting problem
to transfer to drawing trees on a line segment restricted students’ thinking. They thought Hua forced her students to assume that no trees were drawn at both edges of the line segment which was similar to cutting ribbons. Teaching research coordinator Ding argued:

“What happens if some students start to draw a tree at the left edge of the line segment? Or some of the students want to draw trees on both ends of the line segment first? These are not the same situations as the cutting of ribbons.”

Ding commented that it would be better not to set up the learning path for the students. Students should have their rights to choose the approach they feel comfortable with to analyze and solve the problems. However, Ding liked the after class micro e-lesson clips Hua played at the end of the class.

**Writing variation problems.** Besides making e-lessons, Blue Sky mathematics teachers also write their own homework problems. During the TRGA, Hua’s colleagues suggested that Hua could ask her students to solve more variation homework problems. In this lesson, Hua wrote and assigned some challenging problems as her students’ homework. For example, how many trees can be planted on the sides of a rectangular yard? One of Hua’s colleagues observed that all of the problems Hua posed in the class and students’ homework were problems of how many trees could be drawn or could be planted. She commented that

In real life, sometimes, we know the number of trees for planting, but we need to find out the distance between each tree.

Another colleague suggested that Hua could let her students solve other word problems at homes such as wood-cutting and ladder-climbing problems. These problems are similar to the tree-planting problems.
Understanding Students’ Thinking

Using the micro e-lesson of cutting ribbons, Hua wanted her students to solve a problem of how many trees can be drawn on a 12-centimeter line segment by transferring their knowledge of the relationship between the number of cutting times and the number of ribbon pieces left after the cutting. During the TRGA after the class, the following questions regarding drawing dots on the given line segment and drawing or planting trees under different situations were discussed at most:

1) Is it necessary for the students to draw dots on a given line segment before they find out the number of trees drawn or planted?

2) Does the order of the three different situations for solving the tree-drawing or planting problems matter?

Hua’s colleagues and the district teaching research coordinators suggest that Hua needs to think more about students’ original learning points and respect that every student thinks mathematics differently.

Students’ original learning point. Hua prepared the lesson with some of her colleagues before teaching it. They discussed how to choose the teaching materials and what the teaching strategies would be used in the lesson. Most of her colleagues who observed her class had positive views on her teaching strategy of requiring her students to draw dots representing trees on a line segment to solve tree-drawing problems and then tree-planting problems. They claimed that this was the best part of teaching this lesson. They explained that the rationale behind the drawing dots on the line segment is to help her students simplify word problems, look for the pattern of the number of dots drawn in terms of the number of parts separated by the dots on the
line segment to build a mathematical model, and then solve more tree-planting problems by applying the model. Teachers’ comments are as follows:

T1: It is difficult for the students to understand the relationship between the number of trees drawn or planted and the number of the parts separated by the trees drawn on the given line segment. Actually, this difficulty could be simplified to the numbers of dots and numbers of parts separated by dots on a given line segment…. Students learned how to draw a line segment when they were first graders. They also used line segments to solve addition, subtraction, multiplication, and division problems when they were second graders. Now they are third graders, it is fine for them to draw line segments to solve tree-planting problems…. 

T2: I think the key point of the lesson is how to find the relationship between the number of dots drawn on the given line and the number of the line segments separated by these dots, and how to find the number of line segments separated by the dots.

However, teaching research coordinator Ding did not think it was a good teaching strategy to ask students to look for the relationship between the number of dots drawn and the number of parts separated by dots on the line segment. She thought at least some of the students would feel confused. She argued that students might have their own approaches to solve the problem and teacher Hua should know her students’ original learning point and respect her students’ own thinking. If drawing dots on the line segment, the students still need to find out how many dots they have to draw. This means that students need to calculate the number of the dots by applying measurement division. They can divide the length of the line segment by the length of each part separated by the dots on the line segment. When they find the answer, they can know the final solutions by analyzing the three different situations. She argued the necessity
of asking students to draw dots on the line segment and look for the pattern of the number of dots drawn in terms of the number of parts separated by the dots on the line segment to solve the problem. Ding commented further:

It is difficult and confusing for the students to solve the tree-planting problems. What were actually students thinking when they tried to solve the problems? Is the quantitative relationship or the nature of the problem? We need to know their thinking. If it was the nature of the problem, we need to understand our students’ basic thinking path. Why did not let students experience the connections and differences between tree-planting problems and application problems for the division? Did they measure two centimeters on the line then draw one tree, measure another two centimeters to draw another tree, …, Or did they calculate that the given line segment would be separated into six parts if trees are two centimeters apart? What is the nature of their thinking?

According to Ding, it is actually a general measurement division problem. In this case, after they solve the problem by applying the measurement division method, the students need to go back to consider the context of the tree problems to find the final solutions. Ding continued to argue that the problem was abstracted to the relation between the line segments and the points when the students were solving the problem. Either the number of the points and line segments are equal, or the number is one in difference. If the number is one in difference, they only need to know which one is more or less.

Hua responded that some of her students drew the dots by using a ruler to measure every two centimeters on the line segment. These students did not calculate the number of the dots on the line segment by division. Therefore, she thought this still might be a good strategy for these students who did not know how to solve the problem by applying division. In the class, Hua
hinted that her students could draw each dot on the line segments by using a ruler. With the different view from Ding, another teaching research coordinator, Fu, gave a positive comment on the method of drawing dots on the line segment:

It’s meaningful to explore how to draw the dots on the line segment. It is helpful for third graders to simplify the complicated word problems. The method of drawing dots on the line to solve the problem is an appropriate way to look for a pattern and then to explore more tree-planting problems. I think the characteristics of this method are general, intuitive, and operational.

At the end, they all agree that teachers should think about their students’ learning points. For some students who did know how to solve the problem by division, they should be encouraged to do so. For some students who did not solve the problem by applying division, they can draw dots by a ruler to measure each tree two centimeters apart to find the pattern of the number of trees drawn in terms of the number of parts separated by trees on the line segment. After that they can solve tree-planting problems by using the pattern. Ding continued commented:

Let our students solve the problem in their own ways. However, what happens if the line segment is very long? Even though students know the pattern, they still need to find the number of parts separated by the trees drawn on the line segment. Some of them may try to draw the dots first, but they cannot draw on the paper if the line segment is very long such as 12 meters. Therefore, they need to calculate the number of parts separated by the trees drawn on the line segment. Some of them apply the division method to find the answer. They can communicate with their peers. They might explain their thoughts and ideas with their peers by drawing a line segment. But they will have to draw some key
points on the graph. We don’t know if our students will have the one-to-one mapping mathematical idea by applying division. Each part of the line segment matches one point. Can we let our students choose the methods they like to solve the problem and then share their thoughts with their peers?

**Different thinking for individual students.** Ding continued to argue that Hua did not ask her students to measure each part first, then draw the dots. If she did not ask them to do it the way she wanted, her students might have different approaches. Some of them would measure the part first while some of them would prefer to calculate first. Once they calculate first, they would know the given line would be separated into six parts by trees. This is a general division problem. It is easy for them to have six trees as their answer. Then they can check their answer with their peers. Why are some of their peers’ answers five or seven? This could generate new questions for the students. It would make them think more. Ding suggested that Hua should let her students have the freedom to think. Different students have different mathematical thinking and methods to solve the same problem.

At the beginning of the class, students responded very well to the questions proposed on the micro e-lesson. They watched the video and cut the ribbons at home. When they solved the first problem Hua proposed in the class, Hua decided that they should start to think the situation of drawing trees on the line segment without trees on both endpoints. Hua wanted them to do so because she wanted her students to transfer the knowledge they knew about cutting ribbons to drawing trees on the line segment. That was her purpose of making the micro e-lesson to prepare her students for the new lesson of solving tree-planting problems. However, some of her colleagues and teaching research coordinators asked why her students had to begin to solve the problem in the situation of no trees on both endpoints. They argued that every student had
different mathematical thinking so that they had different approaches to solve the problem. This is an open-ended problem that asks the students how to draw trees on a 12-centimeter line segment such that trees are two centimeters apart. They thought that Hua forced her students to think the way Hua wanted. Some of her students might start to solve the problem under the situation of one tree on either of the endpoint while some of her students might start to solve the problem under the situation of one tree on each endpoint of the line segment. They argued that it did not matter to solve the problem regarding which situation considered first. But in order to match the micro e-lesson at the beginning of the class, it is fine to ask the students to think about the problem under the situation of no trees on both endpoints, but it is not necessary. We should respect our students’ own thinking.

**Teaching Mathematical Thoughts**

Based on her lesson plan approved by her lesson-preparing team, Hua encouraged her students to look for the relation between the number of the dots they drew on the line segment and the number of parts separated by these dots, and then construct mathematical models in the three situations of drawing trees on a 12-centimeter line segment such that the trees are two centimeters, three centimeters, four centimeters, and six centimeters apart. After that, Hua wanted her students to solve other problems by applying these three models: if no trees are on both endpoints, the number of the trees drawn equals the number of segments separated by trees minus one; if one tree is on either side of the endpoint, the number of the tree drawn equals the number of segments separated by trees; if one tree on each side of the endpoint, the number of trees drawn equals the number of parts separated by trees plus one. Hua tried to teach her students the thoughts of mathematical modeling. Her colleagues all had positive comments on this teaching idea. One of them said:
The best part of this lesson is simplifying the tree-planting problem as the problem of how to draw dots on the line segment.... This is a reductionist thinking method.

Teaching research coordinator Fu also commented:
Following up reviewing the micro e-lesson of cutting ribbons and checking answers of the problems in the e-lesson with her students at the beginning of the class, Hua started the tree-drawing problem with the situation of no trees on both endpoints, then trees on both endpoints and one tree on either side of the endpoints. This implies the reductionist thinking method.

Furthermore, he suggested that the lesson could imply to one-to-one mapping thoughts as well. Teaching research coordinator Ding agreed with this implication. Some of their comments are as follows.

Fu: Alternatively, students can use the one-to-one mapping method to solve the problem. One line segment is followed by one tree, one line segment is followed by one tree, … to the last piece of the line segment. This is the situation of no trees on both endpoints of the given line, which is equivalent to the ribbon-cutting problem. If there is one tree on either side of the endpoints, one line segment is followed by one tree, one line segment is followed by one tree, … and so on until the last tree on the end of the line segment. Alternatively, one tree is followed by one line, one tree is followed by one line, …, and so on until the last piece of the line segment. If one tree is on each side of the endpoint, then one tree is followed by one line segment, one tree is followed by one line segment, … and so on until the last tree at the end of the line segment.

Ding: I agree with Fu. Tree-planting problems are very confusing. Students are easily confused that sometimes trees are on both side of endpoints, sometimes one side of the
endpoint, sometimes no trees on both sides. But what I was thinking is that students might know how to use division to solve the problems. They do not need to draw any dots on the given line segment. They just divide the length of the line segment by the length of the part separated by trees. This answer gives them the number of the trees drawn on the given line segment under the situation of one tree on either side of the endpoint. This implies one-to-one mapping thinking. One tree to one part of the line segment, then another tree to another part of the line segment, … until the last pair of the tree and the rest of the line segment. Or start the first part of the line segment and one tree, then another part to another tree, … until the last part of the line segment and the last tree on the right endpoint.

At Blue Sky, how to make their students experience the process of problem solving and mathematical thinking is one of the themes that mathematics teachers often discuss during their TRGAs about their teaching practices. Hua's lesson is one example. During the interview, Hua explained that one of the important parts of this lesson is to penetrate the reductionist mathematical thought in her students. She wanted her students to discover patterns and construct mathematical models when they solved problems in class. Then they can solve more real-life problems by applying the model. Hua reflected that she overestimated her students, did not give them enough time to think in class, and did not take advantage of the problems that her students generated in the classroom. She wrote in her reflection report:

When I prepared for the lesson, I was reading a lot and learning several teaching strategies. I tried to consider the methods of solving the problems from the students’ perspectives. However, it seemed that I found some regrets after my teaching. I did not immediately respond to my students who had some difficulties. In order to improve my
teaching and the students’ learning, I need to continue to study my students more to improve their mathematical thinking and how to instill mathematics thinking methods into the teaching.

**What do teaching research coordinators do to improve teachers' pedagogical content knowledge?**

There are 65 elementary schools in District B. Over the last ten years, Blue Sky, starting as a rural elementary school, has developed to be one of the top five public schools in the district. Blue Sky could not succeed without its excellent teachers. There is no doubt that mathematics teachers' success could not happen without their district and city research coordinators. The district teaching research coordinators Ding and Fu usually visit Blue Sky twice a month while the city teaching research coordinator Qiang usually visits Blue Sky once a semester. They visit the school and observe teachers' teaching, comment and give suggestions during the TRGAs after the teachers' teaching. Sometimes, they help young teachers prepare their lessons for demonstrating their teaching at other schools inside and outside of the district. Meanwhile, mathematics teachers at Blue Sky usually attend TRGAs on Wednesdays biweekly in their district. All mathematical teachers in the district do not teach on Wednesday afternoons. They attend TRGAs either in their school or district. The district TRGAs include analysis of textbooks and results of district test results, case studies, homework problem design, and assessments such as exam paper writing. In addition, city teaching research coordinator Qiang visits Blue Sky at the request of Blue Sky’s principal. He was asked to not only help all young mathematics teachers but the two lead teachers as well. In this part, I will analyze the finding of how the district and city teaching research coordinators do to help Blue Sky mathematics teachers acquire and develop their pedagogical content knowledge from the qualitative data from the interviews.
with Ding, Fu, and Qiang. The analysis is based on the following categories of detailed textbook analysis, interpretation of the curriculum standards and teaching objectives, and concerns about students’ thinking.

**Detailed Textbook Analysis**

Like Rainbow, Blue Sky mathematics teachers heavily rely on textbooks when they prepare lessons. To conduct a good lesson, it is very important for mathematics teachers to understand the mathematics content and the teaching material presented in the textbook. Like mathematics teachers at Rainbow, Blue Sky mathematics teachers usually discuss mathematics content and textbooks informally with their colleagues in their offices or formally during school TRGAs. They also attend district TRGAs with their counterparts from other schools at the district TRGAs. Teaching research coordinators usually spend much time helping district teachers to understand the mathematics content in textbooks in the beginning of the semester in the district’s Institute of Teacher Continued Education and several times when they visit the individual elementary school during the semester.

At the beginning of each semester, the teaching research coordinators organize the district’s mathematics teachers to study the mathematics content in textbooks for preparing lessons for the whole semester. They lecture on important topics and their connections in the textbooks that they are currently teaching, and how these topics relate to other topics in the previous semester, the next semester, or even all grades in the whole elementary curriculum.

Over the last ten years, the district has hired many novice teachers because of the fast-growing residents in the district. Due to the lack of elementary teachers, some suburb area elementary schools, including Blue Sky, hire some teachers without an education degree or elementary education degree specializing in mathematics. Therefore, teaching research coordinators have to
emphasize the textbook analysis more during the district TRGAs to prepare these novice teachers well. The district offers all novice teachers lectures once every two months. These lectures are very helpful for the novice teachers who have degrees other than education or elementary education specializing in mathematics.

Excerpts from Fu and Ding’s explanations of training teachers’ mathematics content and curriculum knowledge are as follows.

Fu: I always emphasize different mathematics concepts and the connections of these concepts in the textbooks. I think it is very important. When teachers teach, they cannot separate the mathematics concepts that are related to each other. For example, teachers need to know the concepts of perimeter and the area of a rectangle, and the relationship between them. When students learn the concept of perimeter, they have already learned the concept of the area. But they are usually confused by these two concepts when they learn the concept of perimeter. When we analyze these concepts in the textbook with our teachers, especially novice teachers, we need to advise them how to help their students to avoid the confusion.

Ding: During the TRGAs in the district, we analyze the textbook and also criticize the textbook. I think curriculum knowledge is the basic knowledge for teachers. Teachers cannot teach well without mathematics curriculum knowledge. We analyze textbooks on the horizontal and vertical perspectives based on the elementary curriculum standard. We conduct different ways of studying teaching materials in the textbooks. The teaching research coordinator lectures on the difficult and important mathematics concepts in the textbooks. We also invited some teacher experts and textbook writers to give lectures and
share their teaching experiences and explain the arrangement of different mathematics topics in the textbooks.

When Ding mentioned horizontal and vertical perspectives, she was referring to the analysis of mathematics topics in the textbooks teachers teach during the semester and how these topics relate to other or the same topics in different textbooks.

Moreover, Ding and Fu analyze mathematics topics in the textbooks when they visit Blue Sky to help young mathematics teachers prepare specific lesson plans and teaching demonstrations. Teachers need to know the key points of the content that students should learn and the difficult points of the content that students might face. During the process, they also criticize the textbook. If they find the teaching materials are not well presented, they make their own to replace the examples or the homework problems in the textbook. For example, Hua wrote the problems and made the micro e-lessons for her students to supplement the teaching materials in the textbook. Both the Blue Sky teachers and the teaching research coordinators agree that the combination of the textbook analysis, teaching preparation, lesson demonstration, and comments on the demonstrations is the most efficient and effective way to improve teachers’ knowledge of mathematics, curriculum, and teaching. These teachers better understand the mathematics topics and teaching materials in the textbook, learn how to transfer the knowledge of the curriculum to their teaching, and receive immediate feedback on their teaching.

**Interpretation of the Curriculum Standards and Teaching Objectives**

The city teaching research coordinator Qiang also agrees that teachers need to know the mathematics topics and teaching materials in the textbooks, apply that knowledge to write lesson plans, and demonstrate the lessons. He points out, however, they first need to know the big picture that is the SPSMCS and know the ins and outs of all the mathematics curriculum
knowledge they will teach to their students. At the city level, he emphasizes analysis of
textbooks based on the SPSMCS. The SPSMCS is an umbrella of the curriculum, teaching, and
learning of elementary mathematics. He helps teachers to better understand the SPSMS first,
then all the mathematics topics in the curriculum, then these topics in each grade, each semester,
each unit, and each lesson. For each lesson, he helps teachers understand the teaching objectives
better and how these teaching objectives connect to teaching objectives in each unit and
curriculum standard. Qiang explained:

We have been doing this kind of analysis for more than one year. Teachers are gradually
adapting this type of learning to acquire and improve their knowledge of teaching. When
they prepare a lesson, they are required to think about the objectives of the curriculum
first; second, analyze what content is presented in the textbook; third, what students have
known and what difficulties students might have to learn this new content. Based on
these two points, teachers decide the teaching objectives of each lesson, difficult points
and key points of the lesson.

Qiang suggested that teachers couldn’t prepare lessons well if they only prepare each
individual lesson without thinking the whole unit and whole curriculum. The district and city
teaching research coordinators organize teachers from different schools to study together during
their district and city TRGAs. Besides these TRGAs, Qiang, Ding, and Fu visit Blue Sky to help
mathematics teachers once or twice each semester.

Concerns about Students’ Thinking

Since the new Shanghai mathematics curriculum reform was widely implemented in
2004, the Shanghai government and education department devoted substantial funding and
resources to help teachers to better understand the new curriculum and textbooks. According to
Qiang, many teachers are familiar with the contents and teaching materials in the new textbooks except new teachers. However, the more important problem in class is the teachers’ lack of attention to their students. This is probably the result of the Chinese tradition of teacher-centered instruction. Qiang said:

“Teachers’ teaching approaches do not match their students’ learning approaches. Teachers do not know their students’ thinking. Experienced teachers understand their students’ cognition and mathematical thinking. They can conduct a student-centered classroom. They teach based on students’ learning. However, young teachers do not know their students’ thinking paths.

To help these young teachers to better understand their students’ cognition and mathematical thinking, Ding, Fu, and Qiang suggest that the best solution is to observe, comment on teachers’ teaching, and give their suggestions. Sometimes they become involved in the teachers’ lesson preparations before they observe their teaching. They would ask teachers to analyze not only the content in the textbook but to predict their students’ learning situation and their responses to their questions in the class. They encourage teachers to predict different responses from different students and prepare possible teaching strategies to respond to their students’ thinking and responses. When they observe teachers’ teaching, they all carefully observe students’ activities in the classrooms. Qiang pays more attention to students’ responses to their teachers’ questions. He wants to understand students’ thinking from their responses and if teachers can react to their students appropriately. After the observation, he comments on this aspect of the classroom and gives the teachers’ suggestions for improvement.

Ding has further concerns about students’ thinking. Her concerns stand out during the TRGA after Hua’s teaching. She mentioned several times the teachers’ respect for their students’
original learning paths. In the interview, Ding emphasized that teachers’ teaching design should be based on the students’ development of their thinking. She always encourages teachers to let their students play the main roles in the classroom. The teacher should not control students’ thinking. Instead, they should pose open questions to their students. She gave an example to explain her idea about how to develop students’ own thinking rather than having teachers teach them how to think. The example is about how to find the surface area of a rectangular prism, and how the surface area is increased. Ding said,

In the 5th grade, when teachers teach the concept of surface area of a rectangular prism, they usually directly tell their students the concept of the surface area of a rectangular prism and give the formula, then give an example of finding the surface area of a rectangular prism if the length, width, and height are given. They probably give another example of how the surface area of this rectangular prism changes if you cut it into two rectangular prisms. However, I suggested that teachers teach in a different way. I want students to solve the problem before they are told the formula. I would like to pose the problem to the students: “here is a rectangular prism, you can cut it into two rectangular prisms, so how do you cut?” It will stimulate students’ learning curiosity. Let them do a hands-on activity. Different students will have different ways to cut. There are three ways to cut a rectangular prism into two rectangular prisms. After they cut the rectangular prism, I pose the second problem of, if the surface areas of the rectangular prism are increased by 12 square centimeters, 16 square centimeters, and 18 centimeters respectively, how much is the surface area of the original rectangular prism?

According to Ding, when she told the teachers her ideas about teaching the surface area of a rectangular prism, the teachers were shocked. They asked if it is too difficult for the
students. Ding responded that she did not think so. The point is to let students think and do the problem solving, not just substitute numbers into the formula to find the answer. Ding told me that she asked one teacher to call several fourth graders to the teacher’s office to solve the problems Ding posed. They discussed if the different cut would affect the surface area, and why the surface area increases when the rectangular prism is cut. To some of the students, before they cut the rectangular prism, they thought the area should be decreased because of cutting something. Ding said: "This is what I wanted. Let our students think and discuss the problems with their peers!"

**How does the school administration’s support enable teachers’ acquisition of mathematics pedagogical content knowledge within those communities of practice?**

Blue Sky Elementary School has been developed from a public rural school. A little more than 100 students enrolled in Blue Sky when it opened in 1997. Currently, it has two campuses with 61 classes of more than 2,700 students. According to the district and city teaching research coordinators Ding, Fu, and Qiang, Blue Sky is one of the top five public elementary schools in its district. As detailed at the beginning of this chapter, many of the teachers are young, inexperienced, or not trained in teaching elementary mathematics. According to the analysis of the qualitative data from the interviews of school curriculum and teaching coordinator Fang, school teaching research coordinator Lan, school documents, and my field notes, the following school policies on mentorship, classroom teaching observation, lesson preparation group and teaching research group, and competition contribute to the answer of the question.

**Mentorship Policy**

Each novice teacher at Blue Sky is officially assigned a mentor for one year. At the beginning of the first semester, the school teaching research group conducts an online survey for
novice teachers. From the novice teachers’ responses, the school learns about their new teachers’ knowledge of teaching mathematics, practice teaching experiences, and preparation of teaching mathematics at Blue Sky. Based on the survey responses from each novice teacher, especially about his or her weaknesses, the school administrators including the school curriculum teaching coordinator match an experienced teacher with the novice to guide and help his or her teaching of mathematics at Blue Sky. The curriculum and teaching coordinator Fang explained:

We analyze the survey questionnaire and diagnose our novice teachers’ knowledge of teaching and learning mathematics in elementary schools so that we can know their weaknesses and help them improve their knowledge of mathematics teaching and learning.

Just like their students have different difficulties in learning mathematics, different teachers have different difficulties in teaching elementary mathematics. For example, some of the novice teachers lack knowledge of elementary students’ learning; some of them lack knowledge of teaching strategies; some of them might not be very familiar with the elementary mathematics curriculum. With the survey data, each of them can have an appropriate mentor to help in different ways. During the first year of teaching, novice teachers share the same offices with their mentors, except their mentors hold administrator positions as well. They also teach the same grade mathematics as their mentors. Every week, mentors and their mentees are required to observe each other’s classes once or twice a week. After mentors observe their mentees’ class, they hold an observation conference with their mentees to comment on their classroom teaching and give suggestions to improve their teaching. Most of the novice teachers continue to receive informal help from their mentors for several more years.
Besides the mentorship inside Blue Sky, novice teachers also get help from mentors at other schools in their school district for one year. There is a platform for training novice teachers in their school district. Before the fall semester begins, all novice teachers in the school district are required to attend a five-day new faculty orientation at the district’s Institute of Education. The secretary and deputy secretary of the Institute welcome all novice teachers to the district. They encourage new teachers to adapt to the new era of the requirements for new teachers and assimilate into their school communities to be qualified teachers. They also encourage these novice teachers to take advantage of all resources in the district and develop their profession on the stage of the district wide. During the orientation, the novice teachers learn about the district’s probationary teacher’s standardized training program. Good elementary schools are assigned as lead schools and their lead teachers are assigned as mentors to novice teachers. All of these mentors are experienced mathematics or excellent mathematics teachers in the district. Blue Sky is one of them and Ding is one of the district mentors helping train novice elementary mathematics teachers. Blue Sky novice teachers are required to attend the training program. In order to participate in the one-day training program, they do not teach at Blue Sky that day. Instead, they visit their district mentors at the other schools, and observe their mentors’ classes and their whole day routine. Sometimes they help their mentors to grade students’ homework to better understand students’ learning.

In general, Blue Sky novice teachers have mentors inside and outside the school during their first year of teaching. Before the semester begins, they have several days training with other new teachers in other schools from the district. During the first year, they visit the district Institute of Education to attend the lectures by district teaching research coordinators or expert educators once every two months. Every week, they teach four days at Blue Sky and study one
day at other schools from their district mentors or at their own schools from their school mentors who are also district mentors. They experience and learn from experts and district lead mathematics teachers in different learning communities.

**Classroom Teaching Observation Policy**

Compared to other schools, Blue Sky has a unique classroom observation policy. There are four types of observations: drop-in class; invited class; reporting class; and research discussion class. Every teacher must be observed in at least two types of classroom teaching each semester. At the beginning of the semester, the school TRG coordinator, LPG coordinator, and curriculum and teaching coordinator can drop in teachers’ classrooms to observe their teaching without notification. This is called drop-in class observation. Curriculum and teaching coordinator Fang explained:

> We want to observe our teachers’ normal classroom teaching to diagnose their teaching problems so that we can find the issue to focus on the research of classroom teaching improvement. So, we drop into their classrooms without advanced notice.

The invited class observation policy has been implemented for four years at Blue Sky. It usually happens in the middle of the semester. Teachers can invite their colleagues to observe their teaching during the semester. Because they learn from and help each other, mathematics teachers at Blue Sky feel very relaxed when their colleagues observe their teaching. They treat an invited classroom teaching observation as a normal teaching lesson. They do not feel any pressure. They usually invite their colleagues who teach the same grade as they do to observe their class. These mathematics teachers share the same teaching content, teaching objectives, and same grade students so that they can learn from and help each other closely. More important, they are in the same lesson preparation group. They prepare lessons together, share teaching
strategies, observe each other’s classes, comment and give suggestions after the observation, and then modify the lesson plan to teach in their own classes.

All these discussions, observations, and dialogues eventually emerge in a research discussion class teaching. Then all same grade mathematics teachers and other grades mathematics teachers observe the research discussion class teaching. At the end of the semester, the school holds the reporting class observation event to summarize the effectiveness of mathematics teaching practice for the semester. Novice teachers are required to teach reporting classes at the end of the semester. Before they teach the reporting class, they discuss their lesson plans with their mentors and colleagues in the lesson preparation group. Their mentors, school teaching research group coordinator, and colleagues in the lesson preparation group observe the novice teachers’ reporting class teaching. This is one of the methods to evaluate novice teachers’ growth.

A conference usually is held immediately after the class observation. All observers attend to comment on the class. Sometimes the principal and the curriculum and teaching coordinator attend as well to join the discussion. Teachers who present lessons need to write reflection reports based on their colleagues’ comments, suggestions, and discussions. The teachers who present the research discussion classes not only write the reflection reports, but also work with their lesson preparation groups to write case study reports.

Blue Sky teachers not only are required to observe their colleagues’ classes, but they are also strongly encouraged to observe their counterparts’ teaching in other schools inside and outside of their school district. The school policy ensures that teachers have time to participate in TRGAs inside and outside of their school district. They do not need to teach at Blue Sky that day if they have an opportunity to observe classes and attend the TRGA outside the school. However,
they are encouraged to share what they learned outside of the school with their colleagues when they return to Blue Sky. They usually take notes when they observe the classes. After the observation, they write a detailed observation report and their reflection report. The school TRG encourages these teachers to transfer the lessons they observed outside the school, adopt and reconstruct them to their own classrooms.

**LPG and TRG Activity Policies**

At Blue Sky, all teachers join Lesson Preparation Groups (LPG) and school Teaching Research Group (TRG). The teachers who teach the same grade mathematics are in the grade LPG. There are five LPGs in the school. All teachers are members of the school LPGs. The grade Lesson Preparation Group activity (LPGA) is held once every two weeks while the school TRGA is held once a month. Meanwhile, teachers attend the district TRGA once every two weeks and the city TRGA twice per semester. The principal ensures that teachers have time to attend all these group activities. They do not teach any mathematics classes during the period of group activities. Teachers have one day off school if they attend the district and city TRGAs. Furthermore, Blue Sky and Rainbow co-sponsor cross-district TRGA once a year. Mathematics teachers at Blue Sky and Rainbow visit each other and teach classes at their respective sister schools. Other mathematics teachers in the two schools observe classes, comment on them, and share their mathematics teaching thoughts and ideas in the post-observation conferences. Fang commented:

Cross-district teaching research group activities improve young teachers’ professional development in both schools. Teachers not only learn from each other but make friends. The effective interaction among teachers can promote the high quality practice of mathematics teaching in both communities.
The grade LPGA focuses on the discussion of the five links of teaching which include preparing lesson plans, teaching classes, helping students, assigning homework, and assessing students’ learning. Every two weeks, teachers who teach the same grade of mathematics have a meeting to share their thoughts and ideas on these five links, share their difficulties and questions, and discuss and even argue with their colleagues. This is especially helpful for the novice teachers. The themes of the school TRGAs are based on the problems raised by different grade LPGs. Fang explained:

We want our young mathematics teachers to learn from their problems, to practice with their thoughts, to reconstruct the teaching with their reflections... Peer learning, cooperation and mutual assistance, and the combination of research and teaching are the main modes for the school teaching research group activities.

Based on the discussion and problems emerging from the LPG, the group leaders suggest themes to the school TRG coordinator. These themes are the topics that mathematics teachers discuss and research during the school teaching research group activities. The principal invites the district and city teaching research coordinators to participate in these activities. These teaching research coordinators guide and help the mathematics teachers to improve and develop their knowledge of mathematics teaching. They invite the district and city teaching research coordinators to guide the effective direction of the research as well. The school teaching research coordinator Fang said:

The district and city teaching research coordinators can widen our teachers’ vision of mathematics teaching and learning. They have theoretical and practical knowledge of mathematical teaching. Their help makes our objectives of teaching much clearer.
According to the report of the school TRGAs in spring and fall 2016 semesters, 15 teachers presented their research discussion classes based on eight different themes during eight months. All of them are required to write case-study reports to reflect their lesson plans, teaching practices, colleagues and teaching research coordinators’ comments and suggestions, and the reconstruction of the lesson.

**Competition Policy**

At Blue Sky, mathematics teachers, especially the ones who have less than three years of teaching are encouraged to participate in teaching basic skills competitions. There are different types of competitions that usually are held in the middle of each semester. In order to help young teachers to be qualified mathematics teachers, they are strongly encouraged to attend the basic teaching skill competition to improve their mathematics problem solving abilities, their blackboard writing, and efficient and effective use of the blackboard. They are also invited to participate in the competitions of the knowledge of SPSMCS and how to evaluate classroom teaching. In addition, all teachers are invited to competitions on the knowledge of popular topics or new policies in education and mathematics education; micro e-lesson design competition; the young teacher teaching competition; homework problem writing competition.

The youth teaching competition is the most popular competition of all. According to the school teaching research coordinator Lan, participants do not know their assigned teaching content until two days ahead of their teaching. The order of the competition is by lottery. Teachers only have two days to prepare their lessons. They cannot have anyone’s help during their lesson preparation. The youth teaching competition committee members include school lead teachers, the school teaching research coordinator, and the curriculum and teaching coordinator. The winner will be directly helped by the district and city teaching research coordinators to
further their professional development. Taking an example, Lan referred to a young teacher with three-year teaching experience was the winner of the youth teaching competition in the last year. After this teacher won the competition, she also successfully taught a lesson at the sister school Rainbow for the cross-district TRGA with her school LPG members’ help. Furthermore, with the continues help of her colleagues and special help from the city and district teaching research coordinators, she finally won the excellent teaching award in the 2016 national teaching competition.

**Summary of the Case Analysis**

Blue Sky was found as a public rural elementary school. Of the 24 mathematics teachers, 71 percent of them are aged below 40; half of them have less than six years of teaching experience; and one in four of them is a novice teacher, meaning they have less than three years of teaching experience. Sixty-three percent of Blue Sky mathematics teachers hold bachelor’s degrees; and one of three of them hold master’s degrees. However, only a little more than half of the mathematics teachers majored in education, including one third of the teachers have their master’s degrees. Some of them concentrate in teaching elementary mathematics but some do not.

Generally, Blue Sky mathematics teachers acquire and develop their pedagogical content knowledge through the positive mentorships, active participation, and communication with their colleagues and counterparts in different types of TRGAs inside and outside of the school, and the district and city teaching research coordinators’ great help during the TRGAs. All mathematics teachers at Blue Sky consider their mentors, and all but one considered the TRGA, are very important sources for their knowledge of teaching mathematics. Nine out of ten mathematics teachers agree that their own teaching practice and their reflections are similarly important.
Moreover, almost all of them agree on the importance of informal exchanges with their colleagues in contributing to their acquisition and development of mathematics teaching knowledge. Furthermore, the case study of Hua shows that Blue Sky teachers learn and develop their pedagogical content knowledge through supplementing teaching materials, including creating micro e-lessons and writing variation problems; understanding students’ thinking such as their original learning point and individual student’s different thinking; and teaching mathematics thoughts.

Blue Sky mathematics teachers appreciate their mentors’ help. About seven out of ten mathematics teachers report that they have or had mentors. Blue Sky novice teachers not only have their mentors at school but have mentors out of school. They visit and learn from their district mentor one day every week. More than 90 percent of them feel comfortable asking their mentors’ questions, but less than 45 percent of them are willing to challenge their mentors if they disagree with them. On the other hand, over 80 percent of mentors admit that they learn something regarding mathematics teaching from their mentees. Mathematics teachers at Blue Sky think the school TRGAs and the district TRGAs are equivalently helpful to them. The great majority of Blue Sky mathematics teachers enjoy, in order of preference, the TRGAs for observing colleagues’ teaching, discussing teaching strategies, designing lesson plans, attending post-observation conferences, analyzing students’ difficulties, taking courses about mathematics teaching, and studying mathematics topics and their relation to the curriculum. The order of preference also shows that many teachers who did not major in education or specializing in teaching mathematics acquire the knowledge of mathematics teaching through taking method courses besides learning by teaching practices. Many teachers are willing to ask questions to their colleagues in the post-observation conference and teaching research coordinators during the
TRGAs. However, only half of them are willing to challenge their colleagues’ positions and only eight percent are willing to challenge the teaching research coordinators if they disagree with them.

Almost 21 out of 24 Blue Sky mathematics teachers say the district and city teaching research coordinators are very helpful while about 19 out of 24 reported that their colleagues are very helpful in improving their knowledge of teaching. They even think that district teaching research coordinators give them more help than their colleagues. Ding and Fu are the teaching research coordinators in their district. Besides the regular district TRGAs, Ding and Fu visit Blue Sky twice a month to participate in the school TRGAs to help the mathematics teachers’ acquisition and improvement of their knowledge of teaching mathematics. The city teaching research coordinator Qiang visits Blue Sky once a semester. He not only helps young teachers but two lead mathematics teachers as well. They interpret the SPSMCS and teaching objectives during the school, district, and city TRGAs. They observe and comment on Blue Sky teachers’ teaching. During the post-observation conference, they analyze in detail the content in the textbook and state their concerns about students’ thinking to help mathematics teachers to improve their pedagogical content knowledge.

The Blue Sky administration formulates policies for mentorships, classroom teaching observation, LPG and TRG activities, and competitions to ensure that teachers have the opportunities, time, space, and professional support to develop their teaching knowledge and profession. Every grade has its own LPG. There are more grade LPGAs than school TRGAs. Blue Sky has its unique classroom observation policy. There are four types of classroom observation. Every teacher is observed in at least two types. Under these policies, more and more junior mathematics teachers stand out. By the fall of 2017, five Blue Sky mathematics teachers
had participated in the Shanghai and England Mathematics Teacher Exchange program (Boylan, et al., 2016) for three years. They each taught mathematics in English elementary schools for one month. One of them participated in translating Shanghai elementary mathematics textbooks into English. In January 2018, some elementary schools in England started to use Shanghai elementary mathematics textbooks (Qin, 2017)
CHAPTER VI
FININGS

CROSS-CASE REPORT

Rainbow is located in the north central part of Shanghai, and Blue Sky is located in a southwest suburb of the city. Both Rainbow and Blue Sky are among the top performing elementary schools in their districts. Rainbow was founded in 2007 as an international school and currently has 1,300 students, including about 100 foreign students; Blue Sky was founded in 1997 as a rural school and currently has 2,700 students, including nearly 270 migrant students. In both schools, eight out of ten teachers are female and one out of four are novice teachers. The great majority of mathematics teachers at Rainbow and Blue Sky are young. Sixty percent of mathematics teachers at Blue Sky while 70 percent of mathematics teachers at Rainbow have less than eight years of teaching experience. In terms of educational background, more teachers at Blue Sky hold a master’s degree in elementary education concentrated in teaching mathematics than those at Rainbow. However, 90 percent of Rainbow teachers majored in education compared to only 54 percent of Blue Sky.

In analyzing and comparing the quantitative and qualitative data from the two cases of Rainbow and Blue Sky elementary schools, I find more similarities than differences between these schools. In this part, I report on the similarities and dissimilarities of the findings from Rainbow and Blue Sky to answer the following research questions: 1) How do mathematics teachers acquire their pedagogical content knowledge? 2) What do teaching research coordinators do to improve teachers’ pedagogical content knowledge? 3) How does the school
administration’s support enable teachers’ acquisition of mathematics pedagogical content knowledge within those communities of practice?

**How Do Mathematics Teachers Acquire their Pedagogical Content Knowledge?**

**Part 1. Synthesizing the Findings from the Quantitative Analysis**

The questionnaire survey sought to provide a quantitative description of how mathematics teachers in the Rainbow and Blue Sky acquire and enhance their pedagogical content knowledge within their communities of practice. Comparing with the quantitative data from the questionnaire survey from both schools, I found that similarities and differences exist in the two schools regarding mentorship, participation in TRGAs, and communication with their colleagues and teaching research coordinators.

**Mentorship**

The great majority of mathematics teachers in Rainbow and Blue Sky reported that they had mentors. About 40 and 45 percent of mathematics teachers in Rainbow and Blue Sky, respectively, had mentees. Many mentees in both schools shared the same office with their mentors and taught same grade mathematics as their mentors. However, the two schools differ in how much time that mentors and their mentees work together every week. The time that mentors and mentees at Rainbow work together is twice that at Blue Sky. Novice teachers usually spend time with their mentors in preparing lessons and discussing teaching practice. In the findings of research question 3 for the case of Blue Sky, the LPG policy ensures that all mathematics teachers teach the same grade mathematics to meet once a week in their LPG. This explains that mentors and mentees at Blue Sky spend more time in working together with other colleagues every week. Its mentorship policy also shows that novice teachers at Blue Sky have mentors in their school district. Table 6.1 shows the data of mentorship in both schools.
Meanwhile, the relationship between mentors and their mentees is mutual. All mentors at Rainbow and 82% of mentors at Blue Sky said that they had learned something from their mentees. Most of the mentors in two schools reported that they learned about information technology knowledge from their mentees. In particular, several mentors at Blue Sky recalled that they learned dynamic ideas and open thoughts, strong mathematics content knowledge, and educational theories from their mentees because some of their mentees had higher degrees than they did. Thirty-three percent of mathematics teachers at Blue Sky had masters’ degree in elementary education concentrating in teaching mathematics even though only 54 percent of teachers majored in education.

Table 6.1

*Mentorship in both schools*

<table>
<thead>
<tr>
<th>Mentorship</th>
<th>Rainbow</th>
<th>Blue Sky</th>
</tr>
</thead>
<tbody>
<tr>
<td>Office sharing</td>
<td>50% of mentees share office with their mentors.</td>
<td>61% of mentees share office with their mentors.</td>
</tr>
<tr>
<td>Teaching same grade mathematics</td>
<td>72% of mentees teach the same grade mathematics as their mentors.</td>
<td>90% of mentees teach the same grade mathematics as their mentors.</td>
</tr>
<tr>
<td>Time together</td>
<td>6 hours per week at school for 2 years.</td>
<td>3 hours per week at school for 2 years; one day per week with district mentor for 1 year.</td>
</tr>
</tbody>
</table>

**Participation and TRG Impact**

The great majority of mathematics teachers at Rainbow and Blue Sky actively participate in school and district TRGAs. Table 6.2 shows the distributions in percent of teachers’ participation in school, district, and city online TRGAs. Almost all of the mathematics teachers in both schools evaluated the school and district TRGAs as helpful. However, 76 percent of Blue
Sky teachers and only 45 percent of Rainbow teachers rated online TRGAs very useful or useful. Among all the activities in TRGAs, the overwhelming majority of teachers in both schools agreed that designing lesson plans, observing classes and the post-observation conference, studying curriculum and textbooks, and discussing students’ difficulties and teaching strategies are very useful for enhancing their knowledge of teaching mathematics.

Table 6.2

<table>
<thead>
<tr>
<th>Participation</th>
<th>Rainbow</th>
<th>Blue Sky</th>
</tr>
</thead>
<tbody>
<tr>
<td>School TRGA</td>
<td>95% of teachers attend once every two weeks.</td>
<td>83% of teachers attend once every two weeks.</td>
</tr>
<tr>
<td>District TRGA</td>
<td>85% of teachers attend more than once a month.</td>
<td>80% of teachers attend more than once a month.</td>
</tr>
<tr>
<td>City online TRGA</td>
<td>80% of teachers attend 1-3 times a semester.</td>
<td>Less than 50% of teachers attend 1-3 times a semester.</td>
</tr>
</tbody>
</table>

Communication

In both schools, when they had difficulties in teaching mathematics, the great majority of novice teachers listed their master teachers (mentors), experienced teachers, and novice teachers in that order among all their colleagues who they wanted to communicate with. Almost all teachers in both schools stated that informal exchanges with their colleagues are very helpful. All teachers in both schools agreed that their school colleagues are more useful than their counterparts in the school districts for improving their knowledge of teaching mathematics. Moreover, all teachers in both schools agreed that teaching research coordinators are helpful. However, they differed in the degree of helpfulness. More teachers at Blue Sky appreciated their teaching research coordinators’ help than their counterparts in Rainbow did. Both school
teachers rated the TRGA as one of the very important sources for contributing to their knowledge of teaching mathematics.

Nevertheless, two school teachers differ in challenging teaching research coordinators or professors’ positions when they disagree with them. The majority of teachers felt more comfortable challenging their school colleagues’ positions when they disagree with them. Moreover, the two schools also differ in mentees’ willingness to challenge their mentors’ positions. Table 6.3 shows the teachers’ communication with their mentors, colleagues, and teaching research coordinators (TRCs) in both schools.

Table 6.3

*Teachers’ communication with mentors, colleagues, and TRCs in both schools*

<table>
<thead>
<tr>
<th>Communication</th>
<th>Rainbow</th>
<th>Blue Sky</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mentors</td>
<td>70% of mentees agreed that mentors are the first they ask for help.</td>
<td>Same as Rainbow.</td>
</tr>
<tr>
<td>Colleagues</td>
<td>All teachers agreed that school colleagues are more helpful than counterparts in districts.</td>
<td>Same as Rainbow.</td>
</tr>
<tr>
<td>TRCs</td>
<td>75% of teachers very much appreciate TRCs’ help.</td>
<td>88% of teachers very much appreciate TRC’s help.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Negotiation</th>
<th>61% of mentees would challenge their mentors’ positions; all mentors said that they had learned from their mentees</th>
<th>44% of mentees would challenge their mentors’ positions; 82% of mentors said that they had learned from their mentees.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>60% of teachers would challenge their colleagues’ positions.</td>
<td>50% of teachers would challenge their colleagues.</td>
</tr>
<tr>
<td></td>
<td>30% of teachers would challenge TRCs’ positions.</td>
<td>8% of teachers would challenge TRCs’ positions.</td>
</tr>
</tbody>
</table>
Part 2. Synthesizing the Findings from the Qualitative Analysis

Triangulating both schools' qualitative data from observations, interviews, lesson plans, teachers' reflection reports, and my field notes, I compared the qualitative findings from Rainbow and Blue Sky, and also found more similarities than dissimilarities in the findings of how Shanghai mathematics teachers acquire their pedagogical content knowledge within their communities of practice.

Mathematics teachers from both schools acquire and improve their pedagogical content knowledge by creating school curriculum; studying students' learning of mathematics; designing lesson plans with their colleagues, especially with mentors for novice teachers; observing and attending post-observation conferences; and writing reflection reports. They comment on their colleagues’ classes, share their ideas and thoughts, discuss teaching challenges with their colleagues, and learn from district and city teaching research coordinators or university professors in the TRGA that they actively participate in.

Creating School Mathematics Curriculum

Analyzing and criticizing mathematics textbooks and discussing the mathematics curriculum are one of the very important parts when teachers prepare lessons. The two schools share a similarity in this matter. Mentors in both schools analyze the mathematics curriculum and interpret each topic in the textbooks for their mentees’ better understanding of the mathematics contents they will teach. Furthermore, novice teachers discuss more about the mathematics topics with their colleagues of the same and different grades in the LPG and TRG activities. Both Rainbow and Blue Sky teachers spend lots of time in analyzing the mathematics topics in the textbooks, the connections between different topics within the textbooks, and the same topics in different textbooks. If they do not think the teaching materials presented in the textbook are
appropriate, they write their own examples and homework practice problems as an alternative or supplementary teaching material to create the school-based curriculum. During this process, teachers spend time in deep learning and researching the curriculum and mathematics topics not only in the grade, but in the entire elementary school. This results in their acquisition and improvement of their knowledge of mathematics content and the curriculum of elementary mathematics.

At Rainbow, Mr. Yu taught a lesson on Volume. Before he taught, he and his mentor analyzed, discussed, and critiqued the teaching materials in the textbook. In order to present the rigorous mathematics concept of volume to his students, Yu modified two examples in the textbook. In the textbook, students were asked to compare the volumes of two dictionaries without any measurement of the length and width labeled. Yu showed his students the two dictionaries labeled with the same lengths and widths in his PowerPoint slide. He wanted his students to know that they can only compare the volumes of two dictionaries when the areas of the covers of the two dictionaries are equal but not their heights, or the same heights but not the length and width. Yu also modified another example in the textbook but was criticized in the TRG meeting after his teaching. In the textbook, the example of a piece of crushed plasticine shows that the volume of a piece of plasticine will not change even though its shape is changed. Yu took this example to let his students manipulate a piece of plasticine and observe if the volume of the plasticine will change. However, this hands-on activity unexpectedly brought up another concept of capacity. His students were confused and he did not clear up the confusion in the class. This stimulated a big discussion among his colleagues, city teaching research coordinators, and university professors in the TRG meeting after the class. They discussed how the textbook presents the concepts of volume and capacity of an object and if teachers should
teach the two concepts in one lesson or two. Before Yu taught, he analyzed the concept of volume and teaching materials in the textbook. After his teaching, Yu enhanced his knowledge of volume and the volume-related concept of capacity in the elementary mathematics curriculum. Moreover, his colleagues and teaching research coordinators also discussed related mathematics topics in the elementary school curriculum, including the connections to the middle school. They decided to research and list all of them for further analysis these topics later. His colleagues also improved their knowledge of content and curriculum during the discussion in the TRG meeting. They acquire knowledge by creating the school-based curriculum.

At Blue Sky, teachers create their own school curriculum by making micro e-lesson videos as supplementary teaching materials to help their students better understand mathematics content. Like their counterparts at Rainbow, mathematics teachers at Blue Sky also analyze textbooks and discuss the curriculum with their colleagues either in informal office meetings, or in the LPG and TRG formal meetings and activities. Ms. Hua began and finished her class with micro e-lesson videos. These videos were used as part of students’ homework to prepare them for the new lesson before the class, and to consolidate or expand the knowledge that they learned in the class. Hua and her colleagues in the LPG discussed and designed the lesson plan. She made the e-lesson videos with her colleagues’ suggestion and help. During the TRGA after the class observation, her other colleagues who taught in other grades, and the district and city teaching research coordinators commented on the videos. They commented, discussed, and argued the necessity of the e-lessons, their relevance to the lesson in the class, and the coherence of problems posted in the videos and the problems posted in the class. In addition, Hua’s colleagues and teaching research coordinators suggested ways to improve the e-lessons in terms of the varieties of problem solving and the variation problems of tree planting, such as climbing
ladder problems, for their students to practice more and gain a deeper understanding of the applications of the concept of division. After the TRG meeting, Hua reviewed her notes and wrote the reflection report. She would modify the lesson plan including the problems in the e-lessons. Meanwhile, the e-lessons are available for other teachers who are willing use e-lessons to teaching this topic. This is the school curriculum that Blue Sky teachers build, apply, modify, and apply. Teachers develop their knowledge of mathematics content and curriculum when they create and develop their micro e-lessons.

**Studying Students' Learning of Mathematics**

Both Rainbow and Blue Sky teachers focus on their students’ learning of mathematics. During the post-observation conferences, all observers including Yu and Hua’s colleagues, city and district teaching research coordinators, and the university professor commented on their classes in terms of their students’ learning. They discussed students’ difficulties in learning the mathematics topics, how Yu and Hua dealt with students’ difficulties, and what the teaching materials or teaching strategies they might use to help their students overcome the difficulties. Besides Yu and Hua, mathematics teachers in both schools learn and improve their knowledge of mathematics and their students after the observation of classes and discussion after the observation. In both Rainbow and Blue Sky schools, students’ hands-on activity was emphasized as very important for students’ learning. Yu received criticism that he did not allow his students to spend more time on experiments in the class. He responded that time was limited in the class. His colleagues suggested that he could assign a project and let his students do at home. However, Hua made her e-lessons to allow her students to do hands-on activities at home before the class, which was praised by her colleagues and teaching research coordinators. Both Yu and Hua’s classes lacked group work and student presentations.
However, the two schools’ groups discussed the students’ learning of mathematics for different perspectives. At Rainbow, teachers, school and district teaching research coordinators, and the university professor focused on students’ learning tasks while at Blue Sky, teachers and district teaching research coordinators discussed more their students’ original learning points and different thinking of mathematics.

The data from the interviews with Yu, his mentor, and principal show that Rainbow teachers had researched students’ learning tasks as their school research project for a year. They invited the university professor to help them do the research project. They were analyzing students’ learning tasks before they teach mathematics topics to better understand mathematics teaching and learning. In Yu’s case, his colleagues, the teaching research coordinator, the university professor, and even the principal commented on Yu’s class based on his students’ learning tasks. During the post-observation conference, they discussed the explicit and implicit learning tasks for the students. They agreed that the explicit learning task is to understand the concept of volume and know how to compare the volumes of different objects directly. The implicit learning task for this lesson is to know how to measure the volume of an object by using an intermediate tool. The students found it difficult to understand the conservation of volume. From the discussion, teachers learned the importance of knowing students’ learning tasks and predicting students’ difficulties and responses.

In the case of Hua at Blue Sky, teachers learned and improved their knowledge of mathematics and their students from the perspective of understanding students’ thinking. During the post-observation conference, Hua’s colleagues and the teaching research coordinators discussed the students’ original thinking points and how different students think differently. They argued the necessity of the students drawing dots on a given line segment before they find
out the number of trees drawn or planted; and if the order of the three different situations for solving the tree-drawing or planting problems matters. Teachers preferred to ask students to draw dots on a given line segment while teaching research coordinator Ding did not think it was a good idea. Ding argued that they should respect their students’ own thinking and learning point because they had learned the measurement division. This problem was an application of the measurement division. Before students drew the dots, according to Ding, students still needed to calculate how many dots they would draw by division. Hua responded that some of her students did not do the division and they drew the dots by measuring two centimeters by two centimeters. Hua argued that it helped some of her students who did not know division well. The group also discussed how much the e-lesson at the beginning of the class related to the first problem of drawing trees on a 12-centimeter line segment such that the trees are two centimeters apart; should teachers respect that different student think and solve mathematics differently or force their students to think and solve the problem based on the teachers’ thinking?

**Designing and Practicing Teaching Strategies**

Mathematics teachers in Rainbow and Blue Sky acquire and improve their knowledge of mathematics and teaching by designing and practicing teaching strategies. After they analyze the mathematics curriculum, teaching materials, and their students’ learning situations with their mentors, grade mathematics teachers, teachers usually discuss their lesson plans and decide the teaching strategies with their mentors and colleagues in grade LPG or grade TRG activities. Before their teaching, both Yu and Hua discussed their lesson plans with the respective mentors and their colleagues informally in their shared offices and formally in the grade TRG or grade LPG meetings. When they teach, they and their students were observed by their colleagues, teaching research coordinators, or a university professor. After the teaching, they also
immediately received comments and suggestions from their colleagues, the district and city teaching research coordinators, and the university professor during the post-observation conference as part of TRGA. In Yu’s case, the positive comments on his teaching strategies was his clear and logical conceptual teaching and his emphases on accurate mathematics language. In the post-observation conference, Yu’s colleagues, the city teaching research coordinators, and the university professor also suggested that he should facilitate more hands-on activities in the class and encourage his students’ thinking, group work, and presentation more. In the case of Hua in Blue Sky, Hua planed her lesson with the grade LPG members. She created micro e-lessons and taught the lesson by penetrating into mathematics thoughts such as mathematical modeling and reductionist thinking method. In the post-observation conference, her colleagues had positive comments on her teaching strategies, but not the district teaching research coordinators Fu and Ding. They suggested the alternative teaching strategies could be one-to-one mapping thoughts. Furthermore, Ding proposed that it was not necessary to ask students draw any dots on the given line segment before they solving tree-planting problems because they may solve the problem by applying the division.

Meanwhile, Yu, Hua, and their colleagues shared and learned thoughts and ideas of mathematics teaching strategies from each other, teaching research coordinators, or the university professor. They also can teach the same or different topics by applying the teaching strategies that Yu and Hua applied and which were discussed during the TRGAs.

**Writing Reflection Reports**

During the post-observation conference, Yu and Hua listened and took notes when their colleagues, teaching research coordinators, and the university professor commented on and gave suggestions to their teaching. After the post-observation conference, they need to reflect their
lesson plans and teaching based on all the comments and suggestions they get. Then they write reflection reports, modify the lesson plans, and plan to teach the revised lesson plans next time.

Yu’s lesson was to teach a new concept of *volume* to his fourth graders. His lesson plan emphasized conceptual teaching and learning. He taught the lesson logically and systematically. He arranged examples from easy to difficult. He also helped his students use mathematics language to describe the concept of volume. However, he failed to notice when one of his students confused the concepts of volume and capacity. He reflected that he would study more about his students’ difficulties, predict their responses to his teaching, and prepare better teaching strategies. In Hua’s lesson, the topic is about the application of divisions. Her lesson plan was focused on the process of problem solving and mathematical modeling. Her teaching strategies included penetrating the reductionist mathematical thought to her students. Her students were encouraged to discover the relation of the number of trees drawn and the number of parts separated by trees on the line segment, and to construct three mathematical models. Hua reflected that she overestimated her students, did not give them enough time to think in class, and did not take advantage of the problems that her students generated in the classroom. In the future, she would study her students more to improve their mathematical thinking and embed mathematical thoughts into her teaching. For all the teachers who observed Yu and Hua’s classes and participated in the post-observation conferences, they learned from each other, from the district and city teaching research coordinators, or the university professor. They may apply the modified lesson plans to their own classroom.
What do the teaching research coordinators do to improve the mathematics teachers’ pedagogical content knowledge during the TRGAs?

Based on the analysis of the findings from the Rainbow and Blue Sky to answer this second research question, I found most similarities with few dissimilarities in what the teaching research coordinators from the two districts and the city do to improve the mathematics teachers’ pedagogical content knowledge. The teaching research coordinators from both districts and the city organize TRGAs in the city and districts regularly. The three teaching research coordinators Ming, Ding, and Fu from the two districts usually facilitate the TRGA in their district eight times per semester. Qiang, a city teaching research coordinator, usually facilitates TRGA twice a semester besides organizing online TRGAs. In addition, they all periodically visit Rainbow and Blue Sky to help teachers further. Ding and Fu usually visit Blue Sky twice a month while Ming usually visits Rainbow once a month. As a city teaching research coordinator, Qiang also visits Rainbow once a month while he visits Blue Sky once a semester. In order to help mathematics teachers to acquire and develop their pedagogical content knowledge, they usually facilitate TRGAs to interpret and analyze curriculum, mathematics topics, and teaching materials in textbooks in their district and the city. They also participate in school TRGAs to observe classes and comment on teachers’ teaching afterwards.

Facilitating the City and District TRGA for General Training.

At the beginning of each semester, the city teaching research coordinator Qiang usually organizes TRGAs to interpret the SPSMCS to elementary mathematics teacher delegates from different districts. During the semester, he facilitates online TRGAs for mathematics teachers across the city to observe excellent teachers’ teaching demonstration and comment on the teaching via the internet. For the district teaching research coordinators Ming, Fu, and Ding, at
the beginning of the semester, they usually help their mathematics teachers to study mathematics topics and analyze and criticize teaching materials in the textbooks. They offer more TRGAs for novice teachers in the two districts to learn the mathematics curriculum, topics, and teaching materials in the textbooks to acquire and develop their mathematics pedagogical content knowledge.

**Interpreting mathematics curriculum.** The city teaching research coordinator Qiang believes that elementary mathematics teachers need to acquire the knowledge of the curriculum, namely, SPSMCS, before they start to prepare lessons and teach. Therefore, he usually organizes mathematics teachers from different districts to help them learn more about the mathematics knowledge in the curriculum. He suggests that teachers should not only know the mathematics curriculum and topics they teach in the previous, current, and next semesters, but also their connections in the all primary grades. In the city TRGAs, Qiang focuses on helping mathematics teachers better understand the relationship between the objectives of the curriculum and teaching, and the relationship among the mathematics topics in each lesson, each unit, each semester, each grade, and whole elementary curriculum.

**Analyzing mathematics topics and teaching materials in the textbooks.** All the teaching research coordinators in the two districts facilitate district TRGAs to analyze mathematics topics and teaching materials in the textbooks to help their teachers study and better understand what they will teach for the semester. They focus on analyzing the teaching objectives of mathematics topics and their corresponding teaching materials presented in the textbooks as well as mathematics topics that they teach and their relations to other topics in the same grade or the same topics in different grades. They discuss the key points and difficult points of mathematics topics in order to prepare good lesson plans. The teaching research also
coordinators discuss the teaching materials with their mathematics teachers and criticize them when necessary. Sometimes Ding, Fu, and Ming invite some teacher experts to share their teaching experiences with their district mathematics teachers. They also invite the elementary mathematics textbook writers to explain the arrangement of the mathematics topics in the textbooks to help teachers better understand the connections among topics they will teach in the textbook and in other textbooks. In both districts, teaching research coordinators require novice teachers to attend more district TRGAs for the training of their knowledge of the mathematics curriculum and teaching.

**Analyzing students’ situation.** In both school districts, Ming, Ding, and Fu emphasize developing the teachers’ knowledge of their students. It is very important, according to them, that teachers must know their students well in order to prepare and teach lessons well. However, their approaches and focuses are slightly different.

In District A, Ming focuses on developing teachers’ knowledge of their students’ mathematics learning conditions such as mathematics cognitive learning, students’ experience of mathematics learning, and students’ development of mathematics thoughts and ideas. He invites experienced teachers to help junior and novice teachers predict their students’ cognition base. He helps teachers design assessment questionnaires to assess students’ existing mathematics knowledge. Furthermore, Ming encourages teachers to know the process of students’ learning and develop their students’ sense of mathematization, and to design more hands-on activities in the classroom.

In District B, Ding and Fu focus on developing teachers’ knowledge of students’ mathematics thinking. Ding always encourages her district mathematics teachers to design their lesson plans based on their students’ development of their thinking. She sometimes demonstrates
her own lesson plan and experiment of teaching to the teachers to explain her ideas about how to develop students’ own thinking rather than having teacher teach their students how to think.

With the slightly different perspectives of students’ learning situation, nevertheless, Ming, Ding, and Fu in the two districts all suggest that teachers should develop their students’ mathematical thoughts and methods such as reasoning and reductionism when they learn mathematics. According to Ming, these mathematical thoughts and methods are the implicit knowledge that teachers can facilitate to their students.

**Participating in School TRGAs for Individual Help.**

Besides organizing teaching research group activities in the city and districts, the city and districts teaching research coordinators periodically visit Rainbow and Blue Sky to participate in their school TRGAs to help the mathematics teachers develop and improve their knowledge of mathematics content and curriculum, knowledge of mathematics content and teaching, and knowledge of mathematics content and their students. Qiang, Ming, Ding, and Fu involve themselves in school research projects, observe classroom teaching, and offer detailed comments and suggestions afterwards.

**Involvement in School Research Projects.** Qiang, Ming, Ding, and Fu are invited regularly to visit Rainbow and Blue Sky to help according to the schools’ needs. At Rainbow, Qiang and Ming participated in a school project based on teacher training programs, such as helping teachers to design digital homework assignments and developing the school mathematics curriculum. At Blue Sky, Ding and Fu helped their mathematics teachers conduct several school projects, such as micro e-lesson design.

**Observing and Commenting on Classroom Teaching.** From the findings of the first research question, we already knew that the district and city teaching research coordinators
visited the two schools to observe teaching’s teaching demonstrations, and gave the detailed comments and suggestions to improve their content pedagogical knowledge, which in turn helps improve their teaching. As a supplementary form of helping mathematics teachers in the district and city TRGAs, the district and city teaching research coordinators’ participation in the school TRGAs can give directly detailed guidance and help for these mathematics teachers based on their individuals’ needs. Meanwhile, their guidance and suggestions can also help other mathematics teachers in the TRG.

**How does school administration’s support enable teachers’ acquisition of mathematics pedagogical content knowledge within those communities of practice?**

Both Rainbow and Blue Sky have school policies of mentorship, Lesson Preparation Group and Teaching Research Group activities, classroom teaching observation, and competitions to enable their mathematics teachers to acquire and develop their pedagogical content knowledge within their communities of practice. However, some of these policies are regulated slightly differently in the two schools. In addition, Rainbow has its unique recruiting policy.

**Mentorship Policy**

Rainbow and Blue Sky differ in the way that they assign mentors to novice teachers’. Nevertheless, they share the similar observation rules between mentors and mentees.

**Different mentor assignment rule.** At Rainbow, many novice teachers are assigned the same mentors that they had for the six-week practice teaching period during their senior year in the normal university. Mentors and mentees are familiar with each other. Mentors usually start to help them before the novice teachers are hired formally. Their mentors continue to advise them after hiring on the detailed issues of mathematics curriculum and teaching, homework.
assignment and assessment, and the implicit knowledge and mathematics thinking of the lessons that the novice teachers will teach. At Blue Sky, every novice teacher not only has a mentor in his or her school but a mentor from another school in their district. These district mentors are the lead teachers in the lead schools of the district. Each week, novice teachers spend one day learning from their district mentors by observing their teaching and other school work. Sometimes novice teachers help their district mentors to grade their students’ homework in order to understand their learning process. Blue Sky matches mentors to their mentees based on the result of an online survey that the school gives its novice teachers to diagnose their knowledge of the mathematics curriculum, teaching, and learning. According to each novice teacher’s weaknesses and difficulties, the school curriculum teaching coordinator assigns an experienced teachers as a mentor to guide, encourage, and help the novice teacher’s professional growth. Novice teachers at Blue Sky can learn from their mentors inside and outside of their school community while their counterparts in Rainbow learn from their mentors only in their own school community.

**Similar observation rule.** Both Rainbow and Blue Sky require their mentors and mentees observe each other’s classes. Mentors are required to observe their mentees at least once to twice a week and have a post-observation conference meet with them after each observation. Meanwhile, mentees are required to observe their mentors at least once a week. Although the mentorship is officially assigned for one year, it can last longer informally.

**LPG and TRG Activity Policies**

Both Rainbow and Blue Sky have LPG and TRG policies. Both school administrations ensure that their teachers have time to attend activities organized by grade LPGs, grade and school TRGs, and district and city TRGs. In addition, mathematics teachers at Rainbow and Blue
Sky join together in the cross-school TRG once a year to observe and comment on their counterparts’ classroom teaching. The two schools periodically invite district and city teaching research coordinators to join their TRG activities to guide and help their teachers’ teaching and research. Rainbow also invites a university professor to join their school TRG activities periodically and support their mathematics teachers’ teaching and research.

**Different LPG and TRG distributions.** Rainbow and Blue Sky have different ways of grouping their teachers to the teams for lesson preparation and teaching research discussions. At Rainbow, all low-grade teachers are in one LPG while all high-grade teachers are in another. At Blue Sky, each grade’s teachers are in one LPG. Therefore, there are two LPGs in Rainbow but five in Blue Sky. On the other hand, there are five grade TRGs in Rainbow, with all teachers teaching the same grade of mathematics assigned to one TRG. There is no grade TRG at Blue Sky. All mathematics teachers in both schools are members of the school TRG.

**Similar LPG and TRG activities.** Both Rainbow and Blue Sky mathematics teachers who teach the same grade mathematics meet once every two weeks. Each grade’s teachers at Rainbow join their grade TRG and each grade’s mathematics teachers at Blue Sky join their grade LPG to discuss the teaching materials, prepare lesson plans, learn about students’ difficulties, and share their concerns and thoughts about teaching. All the questions and difficulties that teachers raise in the five grade LPGs in Blue Sky and five grade TRGs in Rainbow will be brought up to the school TRG activity that is held in both schools. TRG activities in both schools also include observing and commenting on peers’ classroom teaching.

In addition, among all the LPG and TRG activities in the two schools, Blue Sky has a unique requirement of four types of classroom teaching observation (drop-in class, invited class,
research discussion class, and report class) for LPG and TRG activities, and Rainbow has a special rule of five keys for LPG activities: a keynote speaker, a theme, a host, time, location.

**Competition Policy**

Both schools organize competitions each semester. They encourage their mathematics teachers, especially the junior ones, to participate in the competitions for best lesson plan design and homework problem development. Since Blue Sky has many teachers who did not major in either elementary education or mathematics education, the school organizes a basic teaching skills competition and a knowledge of mathematics curriculum competition to improve their elementary mathematics teaching.

**Recruiting Policy**

Rainbow has a unique recruiting policy. Rainbow usually starts searching for potential candidates among its student teachers. Therefore, the great majority of mathematics teachers at Rainbow hold bachelor’s degrees in education. In the case of Blue Sky, it hires teachers not only in education but in other fields as well because of the school’s expansion of the lack of mathematics teachers. However, Blue Sky has hired more elementary teachers with master’s degrees specialized in teaching mathematics than Rainbow.

**Summary of Cross-Case Report**

Both Rainbow and Blue Sky, located in the north central urban part and southwestern suburb of the city respectively, are top performing public elementary schools in their districts. Rainbow was founded as an international elementary school ten years after Blue Sky was founded as a rural elementary school in 1997. Currently, the ratio of Shanghai residents to international students in Rainbow and the ratio of Shanghai residents to migrant students in Blue Sky are 13 to 1 and 10 to 1 respectively. The ratio of teachers to students is 1 to 65 in Rainbow.
and 1 to 113 in Blue Sky. The great majority of mathematics teachers in both schools are young women. At least 60 percent of mathematics teachers in both schools have less than eight years of teaching experience. In terms of educational background, 90 percent of Rainbow mathematics teachers majored in education compared to only little more than half of those in Blue Sky. However, one third of mathematics teachers at Blue Sky hold master’s degrees in elementary education concentrating in teaching mathematics compared to one tenth of those at Rainbow.

The quantitative and qualitative data analyses show that mathematics teachers in both schools acquire and develop their pedagogical content knowledge in ways more similar than different. The acquisition and development of their pedagogical content knowledge occur through positive mentorships; active participation in different types of TRGAs in the schools, districts, and city; and informal and formal communications with their colleagues and counterparts outside of the schools. The cases of Yu in Rainbow and Hua in Blue Sky provide an in-depth description of Shanghai elementary mathematics teachers’ acquisition and development of their pedagogical content knowledge by creating a school-based mathematics curriculum, studying students’ learning of mathematics, designing and practicing teaching strategies, and writing reflection reports.

As novice teachers in both schools, they learn from their mentors in the first two years of teaching. At least half of them share offices with their mentors and seven out of ten novice teachers in both schools teach the same grade mathematics as their mentors do. Mentors and mentees at Rainbow work together twice as much as their counterparts at Blue Sky every week. But in the mentorship policy in Blue Sky, every novice teacher has another mentor in another school in her or his district. Blue Sky novice teachers learn not only from their school mentors, but they spend one day each week learning from their district mentors as well. Most teachers in
both schools actively participate in school and district TRGAs and they evaluate the school and district TRGAs as more helpful than the city’s online TRGAs. The overwhelming majority of teachers in both schools enjoy the TRG activities of designing lesson plans, observing classes and attending post-observation conferences, studying the curriculum and textbooks, and discussing students’ difficulties and teaching strategies. All are very helpful in enhancing their pedagogical content knowledge. However, Blue Sky teachers also favor taking courses to learn how to teach mathematics. More teachers at Blue Sky attend the district TRGAs than those at Rainbow. These findings reflect the fact that almost half of mathematics teachers at Blue Sky do not major in education, so that they want to learn how to teach mathematics not only through teaching practice but through taking method courses in the district’s teacher training college as well. In addition, all teachers in both schools enjoy communicating with their mentors, experienced teachers, and other novice teachers when they have difficulties. They agreed that informal exchanges with their mentors and colleagues, as well as formal communication with them and the district and city teaching research coordinators, are very helpful and important sources for their knowledge of teaching mathematics. The majority of teachers in both schools are comfortable challenging their colleagues’ positions if they disagree with them. Almost all novice teachers in both schools would ask questions of their mentors. Less than half of novice teachers would challenge their mentors’ positions in Blue Sky while six out of ten novice teachers would do so in Rainbow. However, the great majority of teachers in both schools would not challenge the positions of the district and city teaching research coordinators who they respect as experts.

Before they became city and district teaching research coordinators, Qiang, Ming, and Ding were all expert elementary mathematics teachers while Fu was one of the authors of the
Shanghai elementary mathematics textbooks. All of them are very familiar with the elementary mathematics curriculum, teaching, and learning. Mathematics teachers in both schools respect them as subject experts and leaders. They facilitate the city and district TRGAs for training mathematics teachers. They focus on interpreting the Shanghai mathematics curriculum standards to teachers before they start preparing and teaching lessons; analyzing mathematics topics and teaching materials in the textbooks; and analyzing students’ learning situations. Furthermore, they visit the two schools periodically and participate in the school TRGAs to offer individual assistance. They help the teachers conduct the school research projects and observe and comment on teachers’ classroom teaching. Sometimes, they share teaching practices from other schools and districts with mathematics teachers in Rainbow and Blue Sky.

Both Rainbow and Blue Sky have policies for mentorship, LPG and TRG activities, and competition even though some of the rules may be different. For example, in the LPG and TRG activity policy, there is a unique requirement for four types of class observation at Blue Sky and a special rule of five keys for LPG activities at Rainbow. In addition, Rainbow has its unique recruiting policy of preferring to hire student teachers while Blue Sky has its mentorship policy of having an extra mentor from the district for each novice teacher. All of these policies ensure their mathematics teachers the time, space, opportunities, and professional help to enable their acquisition and development of pedagogical content knowledge.
CHAPTER VII
SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The purpose of this study was to address 1) the general description and in-depth exploration of acquisition modes of Shanghai elementary mathematics teachers’ pedagogical content knowledge within their communities of practice; 2) the district and city teaching research coordinators’ help in developing and improving these teachers’ pedagogical content knowledge; and 3) the school policies to enable the acquisition and development of teachers’ pedagogical content knowledge. The study used the qualitative two-case study with a quantitative survey research approach. In total, forty-four teachers, four teaching research coordinators in the two districts and city, one university professor, and four school administrators were involved. Data were collected through the questionnaire survey; observations of two lessons and two post-observation conferences; semi-structured in-depth interviews with teachers, mentor, teaching research coordinators, school principal, and curriculum teaching coordinators; teachers’ lesson plans and their reflection reports; school documents; and my field notes taken during the observations and interviews. A triangulation of mixed methods was used in the study to answer the research questions.

I found that the acquisition modes of mathematics teachers’ pedagogical content knowledge in both Rainbow and Blue Sky occur through positive mentorship; active participation in many TRG activities in the schools, districts, and city; and informal and formal communications with their colleagues in the schools and counterparts outside of schools. The cases of Yu in Rainbow and Hua in Blue Sky show the in-depth exploration of the acquisition and development of Shanghai elementary mathematics teachers’ pedagogical content knowledge.
with their communities of practice by participating their school TRG activities. In the school
TRGAs, they opened their classrooms, demonstrated the lessons, and attended the post-
observation conferences to have detailed comments and suggestions from their colleagues, the
district and city teaching research coordinators, or the university professor. Through this learning
process, Yu and Hua acquire and improve their pedagogical content knowledge by creating a
school-based mathematics curriculum, studying students’ learning of mathematics, designing and
practicing teaching strategies, and writing reflection reports.

I also found that the district and city teaching research coordinators are either expert
elementary mathematics teachers, very experienced elementary mathematics teachers, or one of
the authors of Shanghai elementary mathematics textbooks. They are very familiar with the
elementary mathematics curriculum, teaching, and learning. They help school teachers through
facilitating the district and city TRG activities and visiting schools to participate in school TRG
activities. They emphasize interpreting the Shanghai mathematics curriculum standards,
analyzing mathematics topics and teaching materials in the textbooks, and understanding
students’ learning situations.

To enable teachers’ acquisition and development of their pedagogical content knowledge,
both principals and their administrations in Rainbow and Blue Sky formulate school policies of
mentorship, LPG and TRG activities, and competition, even though some of the rules may be
different, to offer their teachers encouragement and support. Moreover, Rainbow has its unique
recruiting policy and special rule for LPG activities while Blue Sky has its unique requirements
for class observations and an additional mentor from the district for each novice teacher. All
these policies and regulations enable and ensure the acquisition and development of the
mathematics teachers’ pedagogical content knowledge within their communities of practice.
Conclusions

The two schools’ policies and regulations offer space, time, opportunities, and professional support to encourage and ensure that mathematics teachers can communicate, share, and discuss their teaching practice while interacting and cooperating within their communities. The schools construct communities of practice where their novice teachers can apprentice and progress from peripheral to full participation (Lave & Wenger, 1991). Novice teachers acquire and develop their pedagogical content knowledge through full engagement in the community’s social, cultural, and professional activities with their mentors and peers. They learn their professional skills from their mentors, experienced teachers, and peers (Wenger & Snyder, 2000). In their communities, novice, junior, and experienced teachers in Rainbow and Blue Sky share their concerns, ideas, and knowledge of mathematics teaching and learning; interact and negotiate with each other; and acquire, develop, and deepen their pedagogical content knowledge. These teachers not only learn from their colleagues but from their counterparts and subject experts outside of their schools. The figure below shows how Rainbow and Blue Sky teachers can learn from different people in an expanding array of communities.
Fig. 7. 1 Shanghai teachers learn from different people in different communities

Through the mentorships, novice teachers observe their mentors’ classes, and their mentors observe and comment on their classes to begin their teaching practice. In Rainbow and Blue Sky, mathematics teachers learn through informal and formal communication with their office mates who are either same grade mathematics teachers or same grade-level mathematics teachers. They prepare lessons together, share their teaching practices, ideas, and concerns, and communicate and negotiate with their colleagues when they participate in the grade LPG or TRG activities. Moreover, they can also discuss these with other mathematics teachers when they participate in the school TRG activities. Working together in their school communities, all
mathematics teachers discuss their concerns about the mathematics topics and issues in the curriculum and textbooks, their students’ difficulties learning these topics, and teaching strategies for these topics. They interact, negotiate with, and learn from each other in their school communities of practice to acquire and develop their mathematics pedagogical content knowledge. In addition, Rainbow and Blue Sky’s mathematics teachers acquire and develop this knowledge through participation in the district and city TRG activities. They can learn from the district and city teaching research coordinators and their counterparts outside of their schools. Sometimes the district and city teaching research coordinators visit the schools to contribute to their school TRG activities. However, there are not many exchanges or negotiations between teachers and the district and city teaching research coordinators.

Through participation in their school LPG and TRG activities, Rainbow and Blue Sky mathematics teachers share their ideas and practices and discuss how to find and connect the relationships among the topics and issues in the textbooks and curriculum; how to select alternative teaching materials; and how to create the school-based mathematics curriculum to assist students’ learning. According to Shulman (1986), this is the knowledge of content and curriculum that teachers develop through teaching. However, in Rainbow and Blue Sky, teachers not only acquire and develop this knowledge through teaching but also within their communities of practice (Cochran-Smith & Lytle, 1999; Grossman, Darling-Hammond & Bransford, 2005).

The study shows that mathematics teachers in both schools are required to learn the mathematics content and curriculum first. They acquire and improve their mathematical content knowledge and curriculum by participating the grade LPG or grade TRG, the school, district, and city TRG activities. Besides participating in these activities, novice teachers learn more from their mentors and the district training. Once teachers acquire and develop their content
knowledge and curriculum within their communities, they can then improve their knowledge of content and their students and in turn to develop their knowledge of content and teaching. Both Yu in Rainbow and Hua in Blue Sky discussed the mathematics curriculum, teaching topics, and teaching strategies with their mentor and colleagues when they prepared their lessons within either the grade TRG or grade LPG. Then, in the school TRG activities, they opened their classes and demonstrated the lessons. Their colleagues, the district and city teaching research coordinators, and the university professor observed and commented on their lessons in the post-observation conferences.

In Rainbow, after observing Yu’s class, his colleagues, the university professor, and the city teaching and research coordinator all commented on the student’s confusion of the concepts of volume and capacity. They discussed and shared suggestions on how the textbook presents the concepts of volume and capacity; whether the teacher should teach the two concepts in one lesson or two; how to clarify the two concepts to the students; and how teachers communicate the concepts well to reduce or avoid the students’ confusion. They concluded with the idea of listing all mathematics topics in the elementary mathematics curriculum and finding their connections and presentations in the mathematics textbooks in the entire elementary mathematics curriculum, even including the beginning of the middle school curriculum.

In Blue Sky, after observing Hua’s class, her colleagues and the district and city teaching research coordinators commented. They shared their ideas and suggestions regarding the supplementary teaching materials: micro e-lessons that Hua made and presented to her students at the beginning and end of the class to preview and review the lesson. The discussion was focused on whether the ribbon-cutting problem in the micro e-lesson transferred to drawing trees on a line segment restricted the students’ thinking. Then the discussion extended to how to
understand students’ thinking. The questions regarding drawing dots on the given line segment and drawing or planting trees under different situations were discussed the most: Is it necessary for the students to draw dots on a given line segment before they find out the number of trees drawn or planted? Does the order of the three different situations for solving the tree-drawing or planting problems matter? After the discussion of how to understand different students’ thinking, the issue of teaching mathematical thoughts emerged from the group discussion based on Hua’s class.

The study also shows that Shanghai elementary teachers are more willing to argue with and challenge their peers than with their mentors. Not many teachers are willing to argue with the district and city teaching research coordinators. Some mentors hold administrative positions, which might make their novices reluctant to challenge their mentors’ positions. Also, in Chinese culture, people always respect guests and admire experts. This could contribute to the reason that teachers are not willing to argue with the district and city teaching research coordinators. During the post-observation conferences, Yu and Hua listened to the comments and suggestions and took notes. Yu did not argue with his colleagues, the city teaching research coordinator, or the university professor while Hua only responded twice to the district teaching research coordinator Ding when she tried to defend herself. After the post-observation conferences, they wrote reflection reports and modified their lesson plans for the future class.

Shanghai elementary mathematics teachers acquire and develop their pedagogical content knowledge within their communities of practice through participation and reification. However, there are more negotiations between peers than between novice teachers and mentors. Very few negotiations occur between teachers and teaching research coordinators. According to Wenger’s (1998) concept of practice as meaning, practice includes both the explicit and the tacit
and highlights the social and negotiated character of both. Wenger (1998) claims that “negotiation of meaning, participation, and reification” (p.49) are three basic concepts in the theory of communities of practice. My study shows that Shanghai elementary mathematics teachers acquire and develop their pedagogical content knowledge through participation and reification within their communities of practice, but not totally through the negotiation of meaning. Teachers share their practices, discuss, interact, and negotiate with their peers. But Shanghai teachers usually do not challenge subject experts such as district and city teaching research coordinators or university professors. Therefore, there are few negotiations between teachers and the district and city teaching research coordinators; or between teachers and the university professor. The figure 7.2 below shows the Shanghai teachers’ process of practice.

Fig. 7. 2 Shanghai teachers’ process of practice

This study also shows that novice teachers, experienced teachers, master teachers, university professors, the district and city teaching and research coordinators cross the community of boundaries and engage in the joint enterprise of teaching and learning (Tsui &
As researchers have found elsewhere, teachers improve through immersion in a pedagogical culture or community of best practices regarding teaching techniques, tools, and plans (Darling-Hammond & Bransford, 2005). The difference is that in Shanghai teachers have a strong background in mathematics content knowledge, focusing on conceptual teaching and learning. Their strong mathematics content knowledge helps their acquisition of mathematics pedagogical content knowledge within their communities of practice, which in turn improves their teaching conceptually rather than procedurally. The in-depth description of the study extends the literatures of Chinese teachers learn in their communities of practice (Wang, 2009; Zhang, 2009; Zhang & Zhu, 2009) and in their professional learning activities through Teaching Research Groups (Paine, 1990; Paine & Ma, 1993; Tsui & Wang, 2009; Zhang, Ding, & Xu, 2016) to how Chinese specific subject teachers acquire and develop pedagogical content knowledge within their communities of practice. This study therefore provides a template for larger studies of teachers’ communities of practice in China.

Many studies suggest that teachers can learn efficiently by participating in high-quality professional development (Bell, Wilson, Higgins, & McCoach, 2010; Koellner, Jacobs, & Borko, 2011) and they can learn how to teach within their communities (Cochran-Smith & Lytle, 1999; Grossman, Darling-Hammond, & Bransfor, 2005; Smagorinsky & Valencia, 1999; Wenger, 2000). Just as their students are learners, teachers are learners too, and just as their students can learn mathematics within their learning communities (Walker, 2006; Walker, 2012), mathematics teachers can learn to improve their mathematics pedagogical content knowledge within their communities. The results of my study are consistent with a study showing that elementary mathematics teachers can improve their mathematics content and pedagogical
content knowledge by participating in sustained and ongoing high-quality professional development (Walker, 2007).

My study reinforces the evidence that sustained job-embedded professional development (Croft, Coggshall, Dolan, Powers, & Killion, 2010) and school-based teacher learning can improve teachers’ teaching practice, which in turn can effectively improve students’ learning (Biancarosa, Bryk, & Bexter, 2010; Gallimore, Ermeling, Saunders, & Goldenberg, 2009). During sustained school-based professional development activities, teachers can share their practices of and concerns about teaching, discuss, interact, and learn from each other. Teachers should open and demonstrate their classes to their grade-level peers and engage in lesson studies (Fernandez, 2002; Stigler & Hiebert, 1999). They can observe and comment on each other’s classes; discuss the mathematics curriculum, students’ learning difficulties, and teaching strategies; and reflect on their colleague’s suggestions and their own teaching practices. Thus, teachers can gain and develop pedagogical content knowledge within their communities of practice (Darling-Hammond & Richardson, 2009).

Finally, my study suggests that it is important to build learning communities to ensure space, time, and professional support for teachers especially elementary teachers, so that they can participate in sustained job-embedded professional development. Research shows that teachers’ collaborations and interactions outside of the classroom can improve their teaching practices (Goddard, Goddard, & Tschannen-Moran, 2007; Little, 2003). However, this is difficult to accomplish if they cannot share office space and if they have substantial teaching loads in teaching multiple subjects. Other studies also observe the effectiveness of mentoring programs for novice teachers and recommend them for improving teachers’ confidence and teaching practices (Darling-Hammond, 2010), but outside China/Shanghai this would require a significant
restructuring of teaching schedules, administration, and budgets. Nonetheless, by showing how Shanghai elementary mathematics teachers develop their pedagogical content knowledge and improve their mathematics teaching practices within their communities, this pioneering study offers inspiration and opportunities for teachers, administrators, and policy makers to seek better ways to improve elementary teachers’ mathematics pedagogical content knowledge, thus improving their teaching and their students’ learning of mathematics (Hill et al., 2005).

**Recommendations**

**Limitations**

There are some limitations to this study. The first is the necessarily small size of the survey sample compared to the number of elementary mathematics teachers in Shanghai. The second is that the two public schools in the case studies are among the best in their districts; this limits the understanding of mathematics teachers’ acquisition and development of their pedagogical content knowledge in average and low-performing elementary schools in Shanghai. The third is that neither of the two teachers featured in the case studies are novices. Mr. Yu at Rainbow had three years’ teaching experiences and Ms. Hua at Blue Sky had twelve years’ teaching experience. These teachers were already scheduled to demonstrate their lessons for the TRGA during the time that I was able to visit. In the case of Blue Sky, the teacher Ms. Hua is an experienced teacher with a degree in education. She does not represent the majority of mathematics teachers who have less than eight years’ experience or the 46 percent who teach mathematics without an education degree.

**Future research**

Although the present study provides a model for analyzing acquisition of pedagogical content knowledge within communities of practice, we still do not have a full view of how all
Shanghai schools’ mathematics teachers acquire and develop their pedagogical content knowledge within their communities of practice. In the future, I want to extend my research in three directions. The first is to examine communities of practice for mathematics teachers in average and low-performing elementary schools in Shanghai. In these studies, I want to focus on novice elementary mathematics teachers and teachers with non-education backgrounds. The result will provide us with a fuller view of how Shanghai develops its elementary mathematics teachers’ pedagogical content knowledge within their communities of practice.

Moreover, the Shanghai government encourages schools to conduct action research based on the problems that they face (Zhang et al., 2016). The data from the interviews with the school administrators and the school documents also show that both Rainbow and Blue Sky ask the district and city teaching research coordinators to help them conduct their school research projects, such as research on students’ learning tasks at Rainbow and teaching design for travel problems at Blue Sky. My research questions are how elementary mathematics teachers conduct these projects and how they acquire and develop their content and pedagogical content knowledge through the process of conducting research within their school communities of practice.

Furthermore, Shanghai high school mathematics teachers and principals not only need to do action research but they are also encouraged to publish their work (Jesen, Downing, & Clark, 2017; Zhang et al., 2016). The third direction of research is to investigate how Shanghai high school mathematics teachers conduct their research projects and how they improve their content and pedagogical content knowledge through the process of conducting these projects within their communities of practice. These future research projects can help lay the foundations for a fuller understanding of how Shanghai teachers learn to improve their content and pedagogical content
knowledge, and in turn how their students, and the city’s future citizens become well educated in mathematics.
References


INFORMED CONSENT

Researcher: Hong Yuan                           Faculty Sponsor: Dr. Erica Walker

I, Hong Yuan, invite you to participate in a research study whose goal is to find out “how Shanghai elementary teachers acquire their mathematics teaching knowledge during their careers.” The study is part of my doctoral thesis at Teachers College, Columbia University, USA. The results of the study will be used in my dissertation. This study collects data from questionnaires, observations, and interviews. If you decide to take part, you will be asked to answer a questionnaire. You might be asked to take part in an observation of teaching research group activity, and a follow-up individual interview. Observations will be audio recorded. Only three participants of the observation will be asked to be interviewed. The anticipated time to answer the questionnaire is about 25 minutes. If you are asked to take part in an interview after the observation, it is anticipated to last no more than 45 minutes.

Before you decide whether or not to participate, I kindly ask you to read the following information. If you decide to participate, which I hope you do, you can quit at any time without any penalty. Should you decide to do so; all your data will be deleted. By signing this document, you give your consent for participation.

The risk of taking part in this research study is probably boredom. At any point you can refuse to answer or terminate your participation if you feel bored. You are not likely to benefit directly from your participation. However, by participating you will contribute to the understanding of how in-service elementary mathematics teachers acquire their mathematics teaching knowledge in their careers in Shanghai, China. You will neither be compensated directly nor indirectly for your participation in the research study.

Only Hong Yuan and her sponsor will use the results of this research study in her doctoral thesis and writings of scientific nature. All responses will be kept strictly confidential. Neither schools nor teachers will be identified for the survey, observation, and interviews. To ensure confidentiality, your answer will be coded with a special number and only I, Hong Yuan, will have access to the file that connects that number to your name. Once participants for the interview have been selected that file will be deleted. Your name will never appear in any writings or discussions regarding this research study. Once the study is complete, all data will be deleted.
PARTICIPANT'S RIGHTS

Principal Investigator: Hong Yuan

Research Title: Modes of Acquisition of Shanghai Mathematics Teachers’ Pedagogical Content Knowledge within Communities of Practice

- I have read and discussed the Research Description with the researcher. I have had the opportunity to ask questions about the purposes and procedures regarding this study.
- My participation in research is voluntary. I may refuse to participate or withdraw from participation at any time without jeopardy to future medical care, employment, student status or other entitlements.
- The researcher may withdraw me from the research at her professional discretion.
- If, during the course of the study, significant new information that has been developed becomes available which may relate to my willingness to continue to participate, the investigator will provide this information to me.
- Any information derived from the research project that personally identifies me will not be voluntarily released or disclosed without my separate consent, except as specifically required by law.
- If at any time I have any questions regarding the research or my participation, I can contact the investigator, who will answer my questions. The investigator’s phone number is 1-212-220-8000.
- If at any time I have comments, or concerns regarding the conduct of the research or questions about my rights as a research subject, I should contact the Teachers College, Columbia University Institutional Review Board/IRB. The phone number for the IRB is 001-212-678-4105. Or I can write to the IRB at Teachers College Box 151, Columbia University, 525 West 120th Street, New York, NY, 10027.
- I should receive a copy of the Research Description and this Participant's Rights document.
- When audio recording is part of this research, I consent ( ) to being audio recorded. I do NOT consent ( ) to being audio recorded. The written and audio recorded materials will be viewed and heard only by the principal investigator.
- Written or audio recorded materials may be ( ) viewed or heard in an educational setting outside the research; Written or audio recorded materials may NOT be ( ) viewed or heard in an educational setting outside the research.
- My signature means that I agree to participate in this study.

Participant's signature: ________________________________ Date: _____/____/____
Name: ________________________________
Investigator’s Verification of Explanation

I certify that I have carefully explained the purpose and nature of this research to ______________________ (participant’s name) in age-appropriate language. She/He has had the opportunity to discuss it with me in detail. I have answered all her/his questions and she/he provided the affirmative agreement (i.e. assent) to participate in this research.

Investigator’s Signature: ________________________________

Date: _____________________
APPENDIX B

Teacher Questionnaire

Acquisition of Pedagogical Content Knowledge within Communities of Practice

This is part of a study into how elementary teachers acquire their mathematical pedagogical content knowledge within communities of practice. All responses will be kept strictly confidential; neither schools nor teachers will be identified.

Your response is very important to the researcher. Please respond to all questions as best as you can. The researcher sincerely appreciates your cooperation.

Your background information:

1. Gender: a) Male  b) Female

2. Age:  a) 20 to 29  b) 30 to 39  c) 40 to 49  d) 50 and above

3. Degree earned:

   Degree                               Major                        Year when graduated
   Associate’s                         _______________           _____________
   Bachelor’s                          _______________           _________________
   Master’s                            _______________           _________________

4. How many years have you taught mathematics?
   a) 1 to 2   b) 3 to 5   c) 6 to 8   d) 8 and above

5. What is your title as a professional teacher?
   a) First-class teacher  b) Master teacher  c) Expert teacher

Mentorship

6. When you were/are a novice, did/do you have a mentor to help to prepare lessons?
   a) Yes  b) No

   If yes, please answer the questions in the following order. Otherwise please start to answer question 12.
7. How many years did the mentor help? ______ years
   About how many hours do you communicate every week? ______ hours

8. Do you teach the same grade as your mentor?
   a) Yes b) No

9. Do you share the same office with your mentor?
   a) Yes b) No

10. Will you ask questions if you do not understand your mentor’s explanation about a
    mathematics teaching idea or his/her thoughts?
    a) Yes b) No

11. Will you challenge your mentor’s arguments that you do not agree with?
    a) Yes b) No

12. Did/do you have a mentee?
    a) Yes b) No

   If yes, please answer the questions in the following order. Otherwise please start to answer
   question 15.

13. When your apprentice has different mathematical teaching ideas, usually you will
    a) Accept your apprentice’s suggestions
    b) Persuade your apprentice to accept your ideas
    c) Decide on a reasonable compromise after discussing it with your apprentice

14. Do you learn anything from your apprentice regarding mathematics teaching?
    a) Yes b) No

    If yes, please write down the details of what you learned.

_________________________________________________________________________________________
Participation

15. How often do you usually attend the following activities every semester?

<table>
<thead>
<tr>
<th>Activity</th>
<th>8 times and above</th>
<th>6 to 7 times</th>
<th>4 to 5 times</th>
<th>1-3 times</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching Research Group in the school</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Teaching Research Group in the district</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Teaching Research Group online</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

16. How often do you usually attend the following activities every year?

<table>
<thead>
<tr>
<th>Activity</th>
<th>More than 3 times</th>
<th>3 times</th>
<th>Twice</th>
<th>Once</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching Research Group outside the district</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Teaching Research Group outside the city</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

17. For the activities you attended, why did you attend? Please check all that apply.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Required by district</th>
<th>Required by school</th>
<th>My own choice</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching Research Group in the school</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teaching Research Group in the district</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teaching Research Group in the city</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teaching Research Group outside the district</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teaching Research Group outside the city</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

18. Which of the following activities do you attend in the teaching research group? Your answer may be more than one choice.
   a) Designing a lesson plan with colleagues;
   b) Observing colleagues’ classes and attending the post-conference;
   c) Studying mathematics topics and their relation to the curriculum in the textbooks;
   d) Discussing mathematics topics and the students’ difficulties learning them;
   e) Discussing mathematics topics and the strategies for teaching them;
   f) Taking courses about mathematics knowledge;
   g) Taking courses about teaching knowledge;
   h) Taking courses about mathematics teaching knowledge;

19. In the last twelve months, about how many times did you do the following?

   Lesson plan design with colleagues _________ time(s)
   Classroom observation as an observer _________ time(s)
   Classroom observation as an observee _________ time(s)
20. When you were/are a novice teacher, who did/do you like to communicate with if you had difficulties in teaching mathematics? Place in order of preference.
   a) Novice teachers – experienced teachers – master teachers;
   b) Novice teachers – master teachers – experienced teachers;
   c) Experienced teachers – master teachers – novice teachers;
   d) Experienced teachers – novice teachers – master teachers;
   e) Master teachers – novice teachers – experienced teachers;

21. How useful were the Teaching Research Group activities in enhancing your knowledge of how to teach mathematics?

<table>
<thead>
<tr>
<th>Activities</th>
<th>Very useful</th>
<th>useful</th>
<th>Neutral</th>
<th>Not very useful</th>
<th>Not useful</th>
<th>Not apply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activities in your school</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Activities in your district</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Online activities in the city</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Activities in other districts</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Activities outside the city</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

22. How useful were the following activities in enhancing your knowledge of how to teach mathematics?

<table>
<thead>
<tr>
<th>Activities</th>
<th>Very useful</th>
<th>useful</th>
<th>Neutral</th>
<th>Not very useful</th>
<th>Not useful</th>
<th>Not apply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designing a lesson plan with colleagues</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Observing colleagues’ teaching</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Attending the conference after teaching</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Studying mathematics topics and their relation to the curriculum in the textbooks</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Discussing mathematics topics and the students’ difficulties learning them</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Discussing mathematics topics and the strategies for teaching them</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Taking courses about mathematics knowledge</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Taking courses about teaching knowledge</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Taking courses about mathematics teaching knowledge</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
23. How helpful do the following people in enhancing your knowledge of how to teach mathematics?

<table>
<thead>
<tr>
<th>People</th>
<th>Very helpful</th>
<th>helpful</th>
<th>Neutral</th>
<th>Not very helpful</th>
<th>Not helpful</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colleagues in your school</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Counterparts in your district</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Counterparts in other districts</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Teaching research coordinators</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>University professors</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

24. In the post-observation conference, will you ask questions if you do not understand your colleagues’ explanations about mathematics teaching ideas?
   a) Yes       b) No

25. In the post-observation conference, will you challenge your colleagues’ arguments that you do not agree with?
   a) Yes       b) No

26. In the Teaching Research Group activities, will you ask questions if you do not understand teaching research coordinators or university professors’ explanation about a mathematics teaching idea or his/her thoughts?
   a) Yes       b) No

27. In the Teaching Research Group activities, will you challenge the arguments that you do not agree with research coordinators or university professors?
   a) Yes       b) No

28. Considering the mathematics instructional knowledge, you are using for your teaching, which of the following sources contribute to your knowledge? You may select more than one choice.

   a) Courses are taken in pre-service teacher education program;
   b) Courses are taken in in-service teacher education program;
   c) Learning from my mentor;
   d) Organized professional activities such as teaching research group activities;
   e) Informal exchanges with colleagues;
   f) Reading professional journals or books;
   g) Your own teaching practice and reflection on them.
   h) Others________________________________________________
APPENDIX C

Interview Protocol for Teachers

Interviewee Name: __________________________
Date of Interview: __________________________

1. Would you please describe your background regarding the degree, major, year of graduation, and year of teaching at this school?

2. Did your mentor help you to prepare this lesson? If yes, how?

3. Did you teach this topic before? If yes, how different was from the previous one?

4. Usually how does your mentor help you?

5. How important is your mentor’s help to you?

6. Will you challenge your mentor’s arguments that you disagree with? If so, how?

7. How do you get help from your colleagues regarding teaching mathematics? (Where, how, and what do you communicate?)

8. How important do you think to attend Teaching Research Group activities?

9. How do you learn from the Teaching Research Group activities in your school?

10. How do you learn from the Teaching Research Group activities outside of your school? For example, the Teaching Research Group activities cross your school and Blue Sky Elementary School (or Rainbow Elementary School)?
APPENDIX D

Interview Protocol for Mentors

Interviewee Name: __________________________

Date of Interview: __________________________

1. Would you please describe your background regarding the degree, major, year of graduation, and year of teaching at this school?

2. How do you usually help your mentee to improve his/her mathematics teaching knowledge?

3. Did you help your mentee to prepare this lesson? If yes, how?

4. How do you think his / her class which was observed?

5. Did he/she achieve the goal that you expected? If not, what parts of teaching will he/she need to improve?

6. What will you do if your mentee disagrees with your suggestion to her teaching?

7. Do you think you learn from your mentee sometimes? If yes, what did you learn from him/her?

8. How important do you think the Teaching Research Group activities to novice and junior teachers’ mathematics teaching knowledge?
APPENDIX E

Interview Protocol for Teaching Research Coordinators

Interviewee Name: __________________________

Date of Interview: __________________________

1. Would you please describe your background regarding the number of years as of teaching mathematics and the position of teaching research coordinator at the district/city?

2. How often do you visit this school? How well do you know these mathematics teachers?

3. What kinds of Teaching Research activities have you attended in the Rainbow Elementary School / Blue Sky Elementary School? What is your role in these activities?

4. What do teachers usually do in the Teaching Research Group activities in the district /city? How often each semester?

5. How are the themes for each Teaching Research Group activity selected and facilitated in the district/city?

6. How often there are Teaching Research Group activities cross the district A and B? Why do teachers attend cross-district Teaching Research Group activities?

7. How important do you think that mathematics teachers’ pedagogical content knowledge to their teaching and students’ learning?

8. From what perspective do you usually observe and evaluate the teachers’ classes?
APPENDIX F

Interview Protocol for School Administrators

Interviewee Name: __________________________
Date of Interview: __________________________

1. Would you please describe your background regarding the number of years as of teaching mathematics and the position of a principal and teaching research coordinator at the school?

2. What is your school’s process for developing a novice teacher?

3. Is every teacher required to observe other teachers’ teaching? Why?

4. Is every teacher required to be observed by his/her colleagues? Why?

5. How often is a teacher observed by his/her colleagues?

6. What do teachers usually do in the Teaching Research Group activities in the school?

7. What kinds of activities do mathematics teachers participate in improving their pedagogical content knowledge?

8. How are these activities selected and arranged?

9. How do you encourage teachers to participate these activities?