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Two-Pillar Monetary Policy Strategy?**

*Michael Woodford*

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*Department of Economics  
Columbia University  
New York, NY 10027*

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# Does a “Two-Pillar Phillips Curve” Justify a Two-Pillar Monetary Policy Strategy?\*

Michael Woodford  
Columbia University

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## Abstract

Arguments for a prominent role for attention to the growth rate of monetary aggregates in the conduct of monetary policy are often based on references to low-frequency reduced-form relationships between money growth and inflation. The “two-pillar Phillips curve” proposed by Gerlach (2004) has recently attracted a great deal of interest in the euro area, where it is sometimes supposed to provide empirical support for the wisdom of a “two-pillar strategy” that uses distinct analytical frameworks to assess shorter-run and longer-run risks to price stability. I show, however, that regression coefficients of the kind reported by Assenmacher-Wesche and Gerlach (2006a) among others are quite consistent with a “new Keynesian” model of inflation determination, in which the quantity of money plays no role in inflation determination, at either high or low frequencies. I also show that empirical results of this kind do not in themselves establish that money growth must be useful in forecasting inflation, either in the short run or over a longer run. Hence they provide little support for the ECB’s monetary “pillar.”

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A distinctive feature of the monetary policy strategy of the European Central Bank is the prominent role assigned to the monitoring of measures of the money supply. In what the ECB calls its “two-pillar strategy,” one pillar is “economic analysis,” which “assesses the short-to-medium-term determinants of price developments.” According to the ECB, this analysis “takes account of the fact that price developments over those horizons are influenced largely by the interplay of supply and demand in the goods, services and factor markets.” But in addition, a second pillar, “monetary analysis”, assesses the medium-to-long-term outlook for inflation, “exploiting the long-run link between money and prices.” The two alternative frameworks for assessing risks to price stability are intended to provide “cross-checks” for one another (ECB, 2004, p. 55).

The most important justification for using two quite distinct analytical frameworks in parallel — rather than a single, integrated conceptual framework (within which one might, of course, obtain information from a large number of different indicators) — seems to be the view that different factors determine longer-run trends in inflation than those responsible for shorter-run fluctuations, and that distinct models are accordingly necessary in order to monitor and respond to developments of the two types. In effect, it is supposed that inflation should be viewed as a superposition of two distinct phenomena that each deserve to be separately modeled.

The case for separate treatment of short-run and long-run determinants of inflation is sometimes argued on theoretical grounds. For example, it is sometimes asserted that models of wage and price adjustment in response to the balance of supply and demand in product and factor markets — that provide the basis for the ECB’s “economic analysis” of the inflation outlook — are by their nature unable to determine the long-run trend rate of inflation, even if they correctly describe short-run departures from the inflation trend.<sup>1</sup>

I have shown elsewhere that this is a misunderstanding of the structure of conventional “new Keynesian” models (Woodford, 2007, sec. 2.2).<sup>2</sup> While the long-run inflation trend in such models most assuredly depends on *monetary policy* — it cannot be explained by factors relating to factor markets and product markets *alone*, and indeed, the central bank can ensure any long-run average inflation rate that it wishes (within certain limits) through an appropriate choice of policy, independently

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<sup>1</sup>Comments of this kind can be found, for example, in Nelson (2003, sec. 2.2), Lucas (2006, p. 137), and Reynard (2006, pp. 2-3).

<sup>2</sup>McCallum (2001) also provides an insightful discussion of the nature of inflation determination in models of the same general kind.

of those structural factors — this does not mean that one and the same model cannot simultaneously explain the determination of the inflation trend and of short-run departures from it.<sup>3</sup> In fact, the specification of the monetary policy of the central bank is essential to the explanation of *both* aspects of inflation — without an equation specifying monetary policy,<sup>4</sup> the model would also fail to determine the short-run departures of inflation from trend. At the same time, all changes in the general level of prices, both in the short run and in the long run, are explained as resulting from the optimizing decisions of price-setters, who respond at all times to the same sorts of perceived changes in production costs and demand conditions. Hence there is no fundamental difference in the framework required to understand inflation determination over different time scales.

But probably the most commonly cited arguments for the need for a separate monetary “pillar” are purely empirical ones. The association of money growth with inflation is argued, as an empirical matter, to be highly robust, confirmed by data from different centuries, from different countries, and from economies with different financial institutions and different monetary and fiscal policies. Empirical work in the monetarist tradition often emphasizes simple correlations (and sometimes lead-lag relationships) rather than structural estimation; but it may be argued that the relations thus uncovered represent more certain knowledge, because they are independent of any maintained assumption of the correctness of a particular structural model. Monetarists argue that the causal relation between money growth and inflation is as a consequence one that can more safely be relied upon in designing a policy aimed at controlling inflation than the relations (such as the Phillips curve) that make up a structural macroeconomic model.<sup>5</sup>

The empirical evidence that is relied upon in such arguments relates to primarily to long-run or low-frequency correlations between money growth and inflation. While early advocacy of money-growth targets was often based on analyses of the correlation between money growth and real activity at business-cycle frequencies, these correlations have broken down in many countries since the 1980s,<sup>6</sup> and the more recent

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<sup>3</sup>A simple example of the kind of model that simultaneously explains both is given in section 2.

<sup>4</sup>As discussed in the paper just cited, such a specification of monetary policy need make no reference to the quantity of money.

<sup>5</sup>See Woodford (2007, sec. 3.1) for further discussion both of the kind of evidence that is cited and of its implications for the design of a robust approach to the control of inflation.

<sup>6</sup>See, for example, Friedman and Kuttner (1992) and Walsh (2003, Fig. 1.3) for changes over time

monetarist literature has instead emphasized the wide range of evidence that exists for a long-run relationship between money growth and inflation. This relationship is argued to be more robust, and to suffice as a justification for controlling money growth given a central bank's proper concern with the character of long-run inflation trends. But to the extent that the relationship is asserted only to hold at low frequencies, the possibility is left open that higher-frequency (or shorter-run) inflation developments must be understood in other terms. This is what is asserted in explanations of the ECB's "two-pillar" strategy.

A branch of the empirical literature on the relation between money growth and inflation at low frequencies that has been especially influential among defenders of the ECB's strategy is the one that estimates "two-pillar Phillips curves" of the kind proposed by Gerlach (2004). Because studies of this kind purport to show that different factors explain inflation movements at different frequencies, they may appear to provide an especially straightforward justification for the strategy of the ECB. It is therefore desirable to give particular attention to what exactly can be concluded about the nature of inflation from such studies, and what they imply about the role of measures of money growth in the assessment of risks to price stability.

## 1 "Two-Pillar" Phillips Curves

A recently popular approach to using money growth to forecast longer-run inflation trends has been the estimation of "money-augmented" or "two-pillar Phillips curves," pioneered by Stefan Gerlach (2004).<sup>7</sup> These are forecasting models in which both an output gap measure and a measure of money growth are used to forecast inflation, with the two sources of information argued to each be relevant to forecasting a different frequency component of inflation. The argument about the differing determinants of inflation at different frequencies is made most clearly in the work of

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in the relation between money and national income in the U.S. data. For a recent discussion of the stability of M3 demand in the euro area, and its implications for the usefulness of "excess liquidity" measures based on cumulative M3 growth, see Bordes *et al.* (2007). Fischer *et al.* (2006) document the reduced reliance on estimated money-demand relations in recent years in the monetary analysis of the ECB.

<sup>7</sup>Other examples of work of this kind include Neumann (2003), Neumann and Greiber (2004), Assenmacher-Wesche and Gerlach (2006a, 2006b), and Hofmann (2006). Assenmacher-Wesche and Gerlach (2006b) provide a useful review of related literature.

Assenmacher-Wesche and Gerlach (2006a). In their work, inflation  $\pi_t$  is decomposed into low-frequency and high-frequency components,

$$\pi_t = \pi_t^{LF} + \pi_t^{HF},$$

using linear band-pass filters. The high-frequency component is modeled as forecastable using a relation of the form

$$\pi_t^{HF} = \alpha_g g_{t-1} + \epsilon_t^{HF}, \quad (1.1)$$

where  $g_t$  is the output gap (defined as the log of output, minus its low-frequency component). The low-frequency component is instead modeled by a relation motivated by the quantity theory of money,

$$\pi_t^{LF} = \alpha_\mu \mu_t^{LF} + \alpha_y \gamma_t^{LF} + \alpha_\rho \rho_t^{LF} + \epsilon_t^{LF}. \quad (1.2)$$

Here  $\mu_t$  is the rate of money growth,  $\gamma_t$  the rate of output growth, and  $\rho_t$  the change in a long-term real interest rate (included as a determinant of changes of velocity), and in the case of each of these variables the superscript  $LF$  indicates the low-frequency component of the series in question.

A relation of the form (1.2) is expected to hold at sufficiently low frequencies because of the existence of a relatively stable money-demand relation of the form

$$\log(M_t/P_t) = \eta_y \log Y_t - \eta_i i_t + \epsilon_t^m, \quad (1.3)$$

in which  $M_t$  is the (nominal) money supply in period  $t$ ,  $Y_t$  is an index of aggregate real output,  $i_t$  is a short-term nominal interest rate, the positive coefficients  $\eta_y$  and  $\eta_i$  are the income elasticity and interest-rate semielasticity of money demand respectively, and  $\epsilon_t^m$  is an exogenous disturbance to money demand. First-differencing (1.3) then yields a relation of the form

$$\mu_t - \pi_t = \eta_y \gamma_t - \eta_i \Delta i_t + \Delta \epsilon_t^m. \quad (1.4)$$

Relation (1.2) is expected to hold because of (1.4); thus on theoretical grounds,  $\alpha_\mu$  should equal 1. In the case that the income elasticity of money demand  $\eta_y$  is equal to 1, as long-run estimates often find, one would also predict that  $\alpha_y$  should equal -1 in (1.2).

Table 1: Inflation equations of Assenmacher-Wesche and Gerlach (2006a) for alternative frequency bands. (Note: dependent variable is euro-area inflation; standard errors are given in parentheses below each regression coefficient; \*\* indicates significance at the 1 percent level.)

Freq. range Period (yrs)	HF 0.5-8	LF 8-∞
Money Growth	-0.02 (0.30)	0.96** (0.19)
Output Growth	-0.03 (0.07)	-0.98 (0.97)
RR Change	1.10 (0.46)	3.01 (6.92)
Output Gap	0.12** (0.05)	—

Combining the two models of the separate components of inflation, one obtains a complete forecasting model for inflation of the form

$$\pi_t = \alpha_\mu \mu_t^{LF} + \alpha_y \gamma_t^{LF} + \alpha_\rho \rho_t^{LF} + \alpha_g g_{t-1} + \epsilon_t, \quad (1.5)$$

where the different “causal” variables are expected to have explanatory power at different frequencies. Assenmacher-Wesche and Gerlach argue that this is the case, by using band-spectral regression to estimate an inflation equation of the form

$$\pi_t = \alpha_\mu \mu_t + \alpha_y \gamma_t + \alpha_\rho \rho_t + \alpha_g g_{t-1} + \epsilon_t, \quad (1.6)$$

allowing the coefficients to vary across frequency ranges of the data. Their results (a representative sample of which are reported in Table 1) support the hypothesis sketched above about the difference between high- and low-frequency inflation dynamics. In their regression for the lowest-frequency band (fluctuations with periods of 8 years or longer), the only strongly significant variable is money growth, with a coefficient  $\alpha_\mu$  not significantly different from 1; the point estimates for the coefficients associated with the other “quantity-theoretic” variables, while not significant, have

the signs predicted by the quantity equation,<sup>8</sup> while  $\alpha_g$  is zero at these frequencies (by construction). In their regression for the high-frequency band (periods 0.5 to 8 years), instead, the coefficient  $\alpha_\mu$  is found to be near zero, while  $\alpha_g$  is significantly positive (at the 1% level), and is the only forecasting variable that enters so significantly at this frequency.<sup>9</sup>

Assenmacher-Wesche and Gerlach call equation (1.5) a “two-pillar Phillips curve,” arguing that it provides support for the view (offered as a primary rationale for the ECB’s “two-pillar” strategy) that separate sources of information must be consulted in order to judge the nearer-term and longer-term outlooks for inflation respectively. They argue furthermore that since inflation is the sum of *both* components (technically, a sum of components corresponding to all frequencies), the predictors that are relevant for *either* component are relevant for forecasting inflation. In particular, “the fact that money growth is important only at low frequencies does not mean that it can be disregarded when analyzing current price pressures” (Assenmacher-Wesche and Gerlach, 2006b, p. 25).

The argument that money should not be disregarded does not, of course, imply that there is a need for a separate “monetary pillar.” In fact, Gerlach (2004) explicitly argues against a separate pillar, concluding instead that forecasting equations like (1.6) show how the information contained in monetary aggregates can be used along with real indicators such as the output gap in a single, integrated framework for assessing risks to price stability. But I shall argue that, not only do such regressions provide no evidence for a need for separate, incompatible approaches to modeling inflation dynamics at different frequencies, but they do not in themselves provide any reason to believe that money growth provides any useful information at all in assessing risks to price stability.

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<sup>8</sup>The coefficient  $\alpha_y$  is very close to the theoretical prediction of -1, and  $\alpha_\rho$  is estimated to be positive, as predicted if higher interest rates lower money demand, as in the specification (1.3).

<sup>9</sup>In a subsequent extension of this work, Assenmacher-Wesche and Gerlach (2006b) find that certain “cost-push variables” (notably, import prices) are also significant predictors of inflation, especially at frequencies even higher than those at which the output gap is most important.



## 2 Do Such Relations Imply a Causal Role for Money in Inflation Determination?

Findings such as those reported by Assenmacher-Wesche and Gerlach do not imply a need for two separate models of inflation determination, depending on the time horizon (or frequency range) with which one is concerned. A single model of inflation determination is capable of explaining why inflation would be more closely related to different sets of variables at high and low frequencies. Perhaps more surprisingly, the importance of money growth in their low-frequency inflation equation is perfectly consistent with a model of inflation determination in which money is not among the causal factors that account for inflation variations, and in which observations of the growth rate of money are not of value in forecasting inflation, either at longer horizons or at shorter ones.

### 2.1 A Simple “New Keynesian” Model

To illustrate this, it is useful to recall the structure of a fairly basic “new Keynesian” model.<sup>10</sup> The model presented here is a simplified version of the account of inflation determination given by empirical models such as the euro-zone model of Smets and Wouters (2003), stripped down to a structure that can be solved explicitly.

The log-linearized model consists of three equations. The first is an aggregate supply relation,

$$\pi_t - \bar{\pi}_t = \kappa \log(Y_t/Y_t^n) + \beta E_t[\pi_{t+1} - \bar{\pi}_{t+1}] + u_t, \quad (2.1)$$

where  $\pi_t$  again represents the rate of inflation between periods  $t$  and  $t + 1$ ,  $\bar{\pi}_t$  is the perceived rate of “trend inflation” at date  $t$ ,  $Y_t$  is aggregate output,  $Y_t^n$  is the “natural rate of output” (a function of exogenous real factors, including both technology and household preferences),  $u_t$  is a possible additional exogenous “cost-push” disturbance, and the coefficients satisfy  $\kappa > 0, 0 < \beta < 1$ . This equation represents a log-linear approximation to the dynamics of aggregate inflation in a model of staggered price-setting; in the variant of the model presented here, in periods when firms do not re-optimize their prices, they automatically increase their prices at the trend inflation

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<sup>10</sup>The foundations of models of the type presented here are treated in greater detail in Woodford (2003).

Table 2: Numerical parameter values.

$\beta$	0.99
$\kappa$	0.0238
$\sigma$	6.25
$\phi_\pi$	1.5
$\phi_y$	0.125
$\eta_y$	1.0
$\eta_i$	12.0

rate  $\bar{\pi}_t$ . This assumption of automatic indexation was first used in the empirical model of Smets and Wouters (2003), who assume indexation to the current inflation target of the central bank, part of the specification of monetary policy below.

The second equation is a log-linear approximation to an Euler equation for the timing of aggregate expenditure,

$$\log(Y_t/Y_t^n) = E_t[\log(Y_{t+1}/Y_{t+1}^n)] - \sigma[i_t - E_t\pi_{t+1} - r_t^n], \quad (2.2)$$

sometimes called an “intertemporal IS relation,” by analogy to the role of the IS curve in Hicks’ exposition of the basic Keynesian model. Here  $i_t$  is a short-term nominal interest rate (a riskless “one-period rate” in the theoretical model, earned on money-market instruments held between periods  $t$  and  $t+1$ ) and  $r_t^n$  is the Wicksellian “natural rate of interest” (a function of exogenous real factors, like the natural rate of output). Euler equations of this sort for the optimal timing of expenditure are at the heart of the monetary transmission mechanism in models like that of Smets and Wouters (2003), though they separately model the timing of consumer expenditure and investment spending.

The remaining equation required to close the system is a specification of monetary policy. For purposes of illustration, I shall specify policy by a rule of the kind proposed by Taylor (1993) for the central bank’s operating target for the short-term nominal interest rate,

$$i_t = r_t^* + \bar{\pi}_t + \phi_\pi(\pi_t - \bar{\pi}_t) + \phi_y \log(Y_t/Y_t^n). \quad (2.3)$$

Here  $\bar{\pi}_t$  is the central bank’s inflation target at any point in time, and  $r_t^*$  represents the central bank’s view of the economy’s equilibrium (or natural) real rate of interest,

and hence its estimate of where the intercept needs to be in order for this policy rule to be consistent with the inflation target;  $\phi_\pi$  and  $\phi_y$  are positive coefficients indicating the degree to which the central bank responds to observed departures of inflation from the target rate or of output from the natural rate respectively. I shall assume that both  $\bar{\pi}_t$  and  $r_t^*$  are exogenous processes, the evolution of which represent shifts in attitudes within the central bank taken to be independent of what is happening to the evolution of inflation or real activity.

This is a simplified version (because the relation is purely contemporaneous) of the empirical central-bank reaction function used to specify monetary policy in the empirical model of Smets and Wouters (2003). Like Smets and Wouters, I shall assume that the inflation target follows a random walk,

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \nu_t^\pi, \quad (2.4)$$

where  $\nu_t^\pi$  is an i.i.d. shock with mean zero, while  $r_t^*$  is stationary (or, if the natural rate of interest has a unit root,  $r_t^* - r_t^n$  is stationary). This completes a system of three equations per period to determine the evolution of the three endogenous variables  $\{\pi_t, Y_t, i_t\}$ .

Using (2.3) to substitute for  $i_t$  in (2.2), the pair of equations (2.1) – (2.2) can be written in the form

$$z_t = A E_t z_{t+1} + a (r_t^n - r_t^*), \quad (2.5)$$

where

$$z_t \equiv \begin{bmatrix} \pi_t - \bar{\pi}_t \\ \log(Y_t/Y_t^n) \end{bmatrix}$$

$A$  is a  $2 \times 2$  matrix of coefficients and  $a$  is a 2-vector of coefficients. One can show that the system (2.5) has a unique non-explosive solution (a solution in which both elements of  $z_t$  are stationary processes, under the maintained assumption that the exogenous process  $r_t^n - r_t^*$  is stationary) as long as

$$\phi_\pi + \frac{1 - \beta}{\kappa} \phi_y > 1. \quad (2.6)$$

If this condition holds (as it does for many empirical Taylor rules), the unique non-explosive solution is given by

$$z_t = \sum_{j=0}^{\infty} A^j a E_t [r_{t+j}^n - r_{t+j}^*]. \quad (2.7)$$

Table 3: Parameterization of the disturbance processes.

$\rho^{y^n}$	0.95
$\rho^{r^n}, \rho^{r^d}$	0.8
$\rho^u$	0.6
$\gamma_{11}, \gamma_{22}, \gamma_{33}$	1
$\gamma_{12}, \gamma_{13}, \gamma_{23}$	0.5
$\sigma(\nu^\pi), \sigma(\epsilon^u)$	.0001
$\sigma(\epsilon^{y^n})$	.0003
$\sigma(\epsilon^{r^n})$	0.001
$\sigma(\epsilon^m)$	0.003
$\sigma(\epsilon^{r^d})$	0.004

This implies, in particular, a solution for equilibrium inflation of the form

$$\pi_t = \bar{\pi}_t + \sum_{j=0}^{\infty} \psi_j E_t[r_{t+j}^n - r_{t+j}^*], \quad (2.8)$$

where

$$\psi_j \equiv [1 \ 0] A^j a$$

for each  $j$ .<sup>11</sup> This shows how inflation is determined by the inflation target of the central bank, and by current and expected future discrepancies between the natural rate of interest and the intercept adjustment made to central bank's reaction function. Note that one can solve for the equilibrium path of the inflation rate without any reference at all to the evolution of money.

One can, however, easily enough solve for the evolution of the money supply that should be associated with an equilibrium of the kind just described. Let us suppose that money demand is given by a relation of the form (1.3).<sup>12</sup> The addition of this equation to the system does not change the predicted solution for the equilibrium evolution of inflation, output, and interest rates; but given that solution, equation

<sup>11</sup>For plots of these coefficients in some numerical examples, see Woodford (2003, Figs. 4.5, 4.6). The coefficients are denoted  $\psi_j^\pi$  in the figures.

<sup>12</sup>Such a relation is perfectly consistent with the microeconomic foundations underlying the structural relations expounded earlier in this section; see Woodford (2003, chap. 4, sec. 3).

(1.3) can be solved for the equilibrium evolution of the money supply as well. The model can then be used to make predictions about the co-movement of money and inflation.

It can easily explain the kind of long-run or low-frequency relations between money growth and inflation emphasized in the monetarist literature. One popular approach has been to compare the low-frequency movements in money growth and in inflation through bandpass filtering of the respective time series; essentially, this means taking long moving averages of the data, so as to average out high-frequency fluctuations. For example, Benati (2005) compares the low-frequency variations in money growth and inflation in both the U.K. and the U.S., using various measures of money and prices, and data from the 1870s to the present, and finds that the timing and magnitude of the shifts in the low-frequency trend are similar for both money growth and inflation.<sup>13</sup> Another popular approach to studying the long-run relationship between money growth and inflation in a single country is cointegration analysis. Assenmacher-Wesche and Gerlach (2006a), for example, find that in the euro area, broad money growth and inflation are each non-stationary series (stationary only in their first differences), but that the two series are cointegrated, so that they have a common (Beveridge-Nelson) “stochastic trend”: changes in the predicted long-run path of one series are perfectly correlated with changes in the predicted long-run path of the other series. Moreover, one cannot reject the hypothesis that the linear combination of the two series that is stationary is their difference (*i.e.*, real money growth), so that a one percent upward shift in the predicted long-run growth rate of broad money is associated with precisely a one percent upward shift in the predicted long-run rate of inflation, in accordance with the quantity theory of money.<sup>14</sup>

Results of these kinds are perfectly consistent with the kind of model described above — in which there *is* assumed to exist a stable money-demand relation, but money does *not* play any causal role in inflation determination. As a simple example, let us suppose that in the above model,  $\eta_y = 1$ , that the fluctuations in  $\log Y_t^n$ ,  $r_t^n$ , and  $\epsilon_t^m$  are at least difference-stationary (so that the growth rate  $\gamma_t^n \equiv \log(Y_t^n/Y_{t-1}^n)$  of the

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<sup>13</sup>Similar results are obtained (albeit with shorter time series requiring averaging over a somewhat shorter window) for euro-area data on money growth and inflation by Jaeger (2003) and Assenmacher-Wesche and Gerlach (2006a).

<sup>14</sup>Cointegration analysis is similarly used to establish a long-run relationship between euro-area money growth and inflation by Bruggeman *et al.* (2003) and Kugler and Kaufmann (2005).

natural rate of output is stationary), and that the Taylor-rule intercept  $r_t^*$  tracks the natural rate of interest well enough (at least in the long run) so that the discrepancy  $r_t^n - r_t^*$  is stationary, and moreover has a long-run average value of zero.<sup>15</sup> Then in the case of Taylor-rule coefficients satisfying the inequality (2.6), the result (2.7) implies that the unique non-explosive solution will be one in which both elements of  $z_t$  are stationary mean-zero processes. This in turn implies that inflation will be equal to the stochastic trend  $\bar{\pi}_t$  plus a stationary process, so that  $E_t\pi_{t+1}$  will also equal the stochastic trend  $\bar{\pi}_t$  plus a stationary process. Moreover,  $\log Y_t$  will equal the difference-stationary variable  $\log Y_t^n$  plus a stationary process, so that the growth rate  $\gamma_t$  will be stationary. Equation (2.2) then implies that the nominal interest rate  $i_t$  will equal the difference-stationary variable  $r_t^n$  plus the difference-stationary variable  $E_t\pi_{t+1}$  plus a stationary process, and thus will be difference stationary.

It follows that all terms in (1.4) will be stationary except  $\mu_t$  and  $\pi_t$ . Hence  $\mu_t$  and  $\pi_t$  will both be integrated series of order 1, with the common stochastic trend  $\bar{\pi}_t$ . Thus these two non-stationary series will be cointegrated, with cointegrating vector  $[1 \ -1]$ . Hence the cointegration results of Assenmacher-Wesche and Gerlach for the euro zone are in no way inconsistent with such a model. Moreover, since both inflation and money growth are equal to the integrated series  $\bar{\pi}_t$  plus a stationary series, a bandpass filter that retains only sufficiently low-frequency components of the two series will average out the stationary components, yielding filtered series that are nearly the same in each case. Hence comparisons of the low-frequency movements in money growth and inflation, of the kind presented by Benati (2005) as well as by Assenmacher-Wesche and Gerlach, are consistent with this kind of model as well.

## 2.2 An Explanation for a “Two-Pillar Phillips Curve”

The kind of model just presented is equally consistent with estimates of “two-pillar Phillips curves.” The appearance of money growth in the low-frequency bandpass regression, with a coefficient near 1, simply indicates that inflation and money growth are cointegrated, with a cointegrating vector close to the vector  $[1 \ -1]$  predicted by a money demand relation of the form (1.3). The other variables that appear in the low-frequency regression are similarly consistent with a model in which one of

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<sup>15</sup>This assumption about the central bank’s reaction function is necessary in order for the policy rule to imply that on average inflation will be neither higher nor lower than the target  $\bar{\pi}_t$ .

the structural equations is (1.3), and in which the disturbance term  $\epsilon_t^m$  exhibits little low-frequency variation. The non-appearance of the “output gap” measure  $g_{t-1}$  in the low-frequency regression tells nothing about inflation determination, as this variable exhibits no low-frequency variation *by construction*.

At the same time, the appearance of the output gap as a significant predictor of high-frequency inflation variations is consistent with the existence of *another* structural relation which relates short-run variations in inflation and output to one another (*i.e.*, an aggregate-supply or Phillips-curve relation), in conjunction with substantial high-frequency variation in  $\epsilon_t^m$  (or in inflation expectations), so that there need not be a substantial correlation between inflation and the quantity-equation variables at high frequencies. Under this interpretation of the findings, different *equations* of the structural model play a greater role in determining the coefficients of the (reduced-form) inflation equation in the case of different frequency ranges, but a single (internally coherent) *model* is consistent with both sets of findings.

Here I illustrate this through simulation of a numerical version of the simple model presented above. The model structural equations consist of (1.3), (2.1), (2.2), and (2.3), with numerical parameter values given in Table 1. Here the numerical values of  $\beta$ ,  $\kappa$ , and  $\sigma$  are those estimated by Rotemberg and Woodford (1997) for the U.S. economy,<sup>16</sup> the assumed values of  $\phi_\pi$  and  $\phi_y$  are the coefficients of the celebrated “Taylor rule” (Taylor, 1993),<sup>17</sup> and the coefficients  $\eta_y$  and  $\eta_i$  are those indicated by the “low-frequency” regression of Assenmacher-Wesche and Gerlach (shown in Table 1), if this regression is interpreted as estimating the relation (1.4).<sup>18</sup>

To complete the model, we must specify stochastic processes for the six exogenous disturbances  $\{\bar{\pi}_t, r_t^*, r_t^n, Y_t^n, u_t, \epsilon_t^m\}$ . As above (and in Smets and Wouters, 2003),  $\{\bar{\pi}_t\}$  is assumed to be a random walk (2.4), with white-noise innovation process  $\{\nu_t^\pi\}$ . I assume a VAR(1) specification for the natural rate of interest, the central bank’s estimate of the natural rate (*i.e.*, the intercept of the central-bank reaction function),

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<sup>16</sup>The empirical model of Rotemberg and Woodford is not identical to the simple model assumed here, but has a similar basic structure.

<sup>17</sup>The coefficient  $\phi_y$  here is only 1/4 the size of Taylor’s value (0.5), because I measure the nominal interest rate  $i_t$  in terms of a quarterly rate rather than the annualized rate used in Taylor’s paper.

<sup>18</sup>Here the value  $\eta_i = 12$  refers to the semi-elasticity with respect to the quarterly interest rate  $i_t$ ; this corresponds to a semi-elasticity of 3 with respect to an annualized interest rate (the units in which this parameter is often reported in empirical money-demand studies).

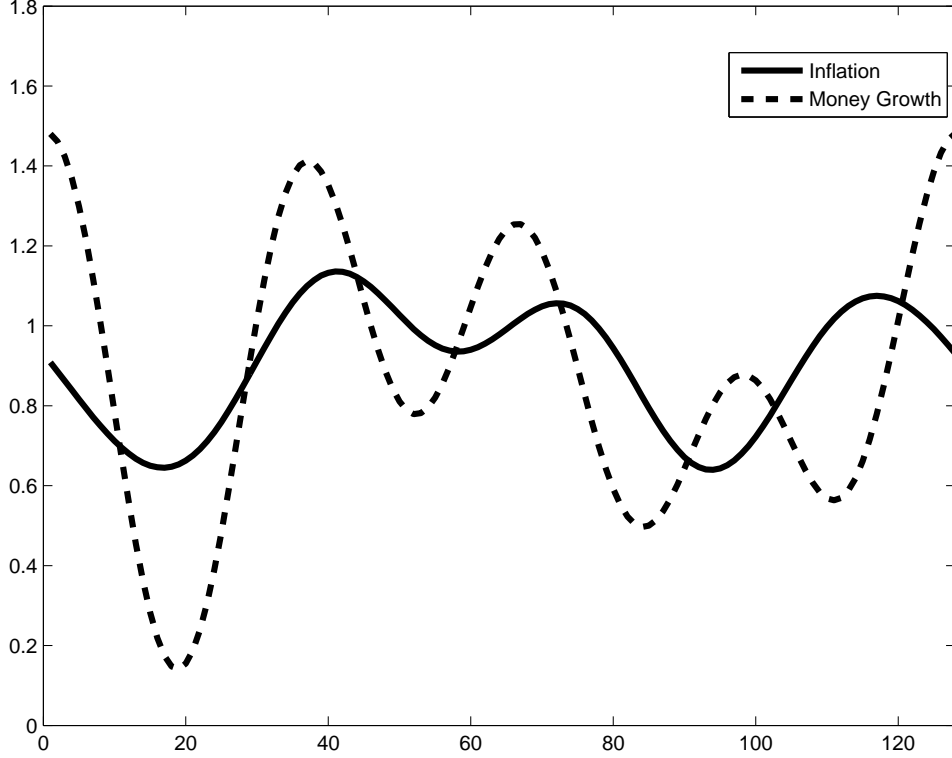


Figure 1: Low-frequency components of inflation and money growth, in a simulation of the new Keynesian model.

and the growth rate of the natural rate of output,

$$\begin{bmatrix} r_t^* - r_t^n \\ r_t^n - \bar{r}^n \\ \Delta \log(Y_t^n) \end{bmatrix} = \begin{bmatrix} \rho_{r^d} & 0 & 0 \\ 0 & \rho_{r^n} & 0 \\ 0 & 0 & \rho_{y^n} \end{bmatrix} \begin{bmatrix} r_{t-1}^* - r_{t-1}^n \\ r_{t-1}^n - \bar{r}^n \\ \Delta \log(Y_{t-1}^n) \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ 0 & \gamma_{22} & \gamma_{23} \\ 0 & 0 & \gamma_{33} \end{bmatrix} \begin{bmatrix} \epsilon_t^{r^d} \\ \epsilon_t^{r^n} \\ \epsilon_t^{y^n} \end{bmatrix} \quad (2.9)$$

where the steady-state natural rate of interest  $\bar{r}^n$  is equal to  $\beta^{-1} - 1$  and the  $\{\epsilon_t^{r^d}, \epsilon_t^{r^n}, \epsilon_t^{y^n}\}$  are each mean-zero white noise processes.<sup>19</sup> The cost-push shock is assumed to follow

<sup>19</sup>The off-diagonal elements in the  $\gamma_{ij}$  matrix allow changes in the growth rate of natural output to affect the natural rate of interest, and allow changes in either the natural growth rate or the natural rate of interest to have transitory effects on the discrepancy between the current natural rate of interest and the central bank's estimate. However, the natural growth rate is assumed to evolve independently of the other factors that affect the natural rate of interest, and other sources of shifts in the intercept of the central bank's reaction function are assumed to have no effect on the evolution of either natural output or the natural rate of interest.



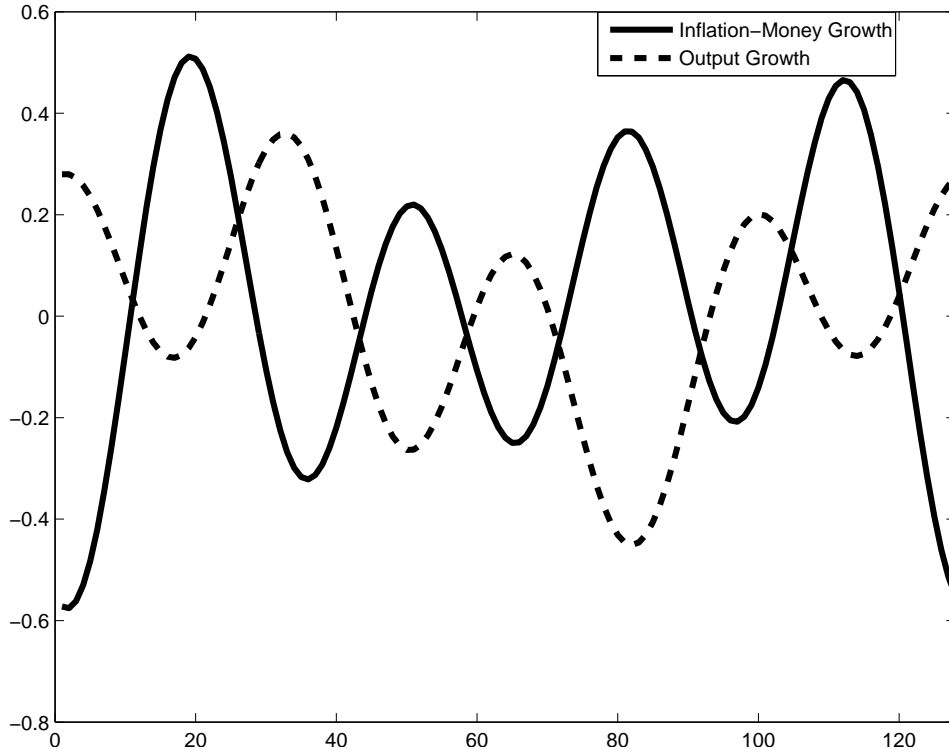


Figure 2: The low-frequency components of inflation minus money growth compared with the low-frequency component of output growth.

an independent AR(1) process

$$u_t = \rho_u u_{t-1} + \epsilon_t^u, \quad (2.10)$$

while the money-demand shock  $\{\epsilon_t^m\}$  is assumed to be a white-noise process. The six white-noise processes  $\{\nu_t^\pi, \epsilon_t^{r^d}, \epsilon_t^{r^n}, \epsilon_t^{y^n}, \epsilon_t^u, \epsilon_t^m\}$  are all assumed to be distributed independently of one another (as well as i.i.d. over time), and each is assumed to be normally distributed with mean zero. The numerical parameter values for these processes assumed in the illustrative simulations here are given in Table 3.<sup>20</sup>

Following Assenmacher-Wesche and Gerlach (2006a), the inflation rate, the various interest rates, and the output growth rate are quoted as quarterly rates of change;

<sup>20</sup>No attempt is made here to estimate parameter values that can be said to be empirically realistic for the euro area, as the model is in any event overly simplistic. The point of the exercise is simply to show that regression coefficients of the kind obtained by Assenmacher-Wesche and Gerlach, among others, are perfectly consistent with a model in which money growth plays no causal role in inflation determination.

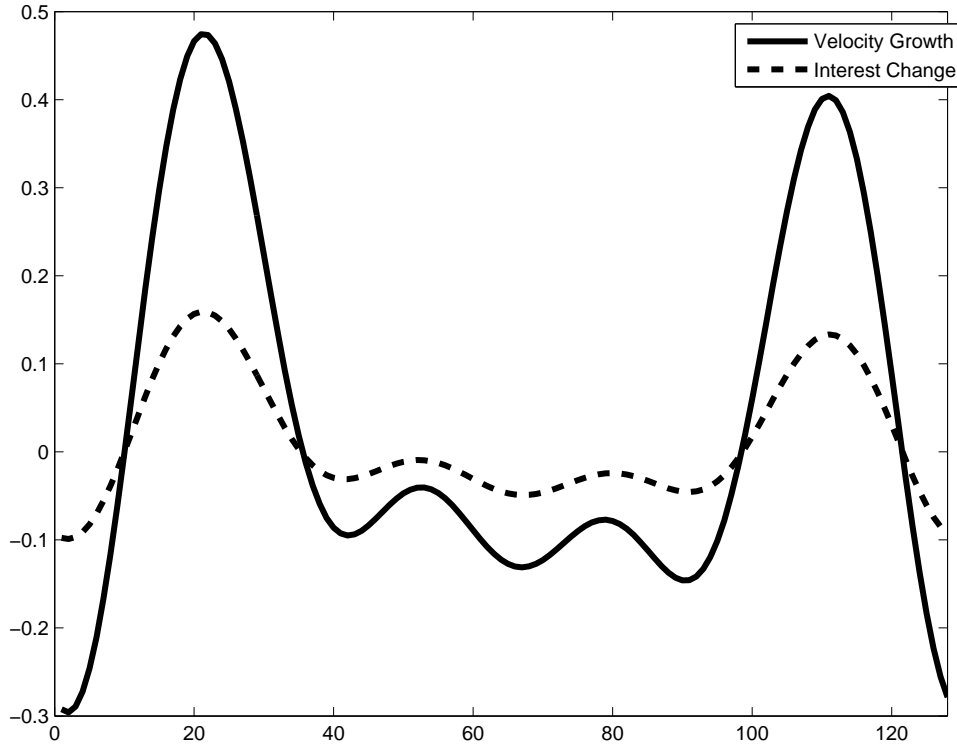


Figure 3: Low-frequency components of velocity growth and of quarterly changes in the nominal interest rate.

thus, for example, the value  $\bar{r}^n = .01$  means one percent per quarter, or a 4 percent annual rate. The initial value assumed for the inflation target in the simulations is .01 (one percent per quarter, or a 4 percent annual rate); this has a permanent effect on the inflation rate in the simulations, since the target is assumed to follow a random walk.<sup>21</sup> Similarly, the value  $\sigma(\nu^\pi) = .0001$  means that a one-standard-deviation shock to the inflation target increases the quarterly target inflation rate by one basis point, or the annualized target inflation rate by 4 basis points.

Given these disturbance processes, the model is solved in the manner indicated above, and then simulated using a random number generator to generate shocks with the indicated standard deviations. To study the kind of regression results that one would expect to obtain in a study like that of Assenmacher-Wesche and Gerlach, if

<sup>21</sup>Note, however, that the absolute level of the inflation rate has no consequences for any of the issues of interest to us here (since we ignore the zero lower bound on interest rates). As it happens, the nominal interest rate is always positive in the simulations reported below.

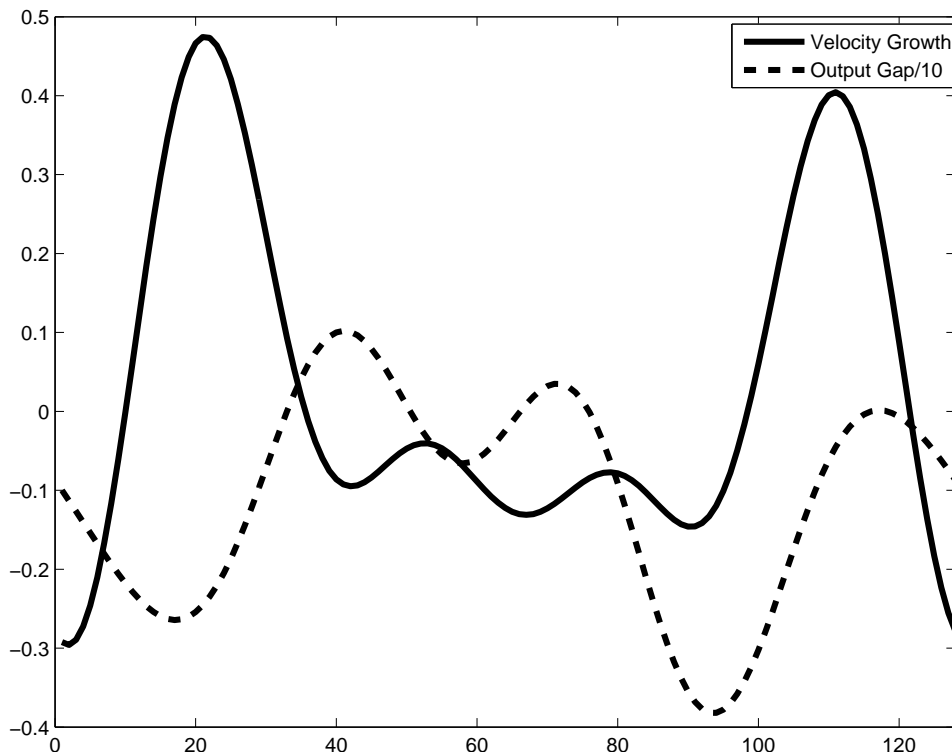


Figure 4: Low-frequency components of velocity growth and of the output gap.

the data generating process were the one specified here, I generate 1001 simulated time series,<sup>22</sup> each 128 quarters (32 years) in length,<sup>23</sup> starting from the same initial conditions (in particular, the same initial inflation target) in each case, but drawing a different series of shocks in each case.

Figures 1-7 illustrate some of the frequency-domain properties of a typical simulation. The simulation chosen for presentation is the one in which the standard deviation of inflation happens to have exactly the median value across all 1001 simulations. Figure 1 compares the low-frequency components of money growth and inflation in the simulated data, after the same bandpass filter is applied to each se-

<sup>22</sup>An odd number of series is generated so that I can report the median values of the statistics in Table 4.

<sup>23</sup>Simulations of this length are chosen as this is approximately the length of the time series studied by Assenmacher-Wesche and Gerlach. It is convenient, when computing the Fourier transforms of the series, to have a number of observations that is a power of 2; this accounts for the choice of exactly 128 quarters as the length of each simulation.

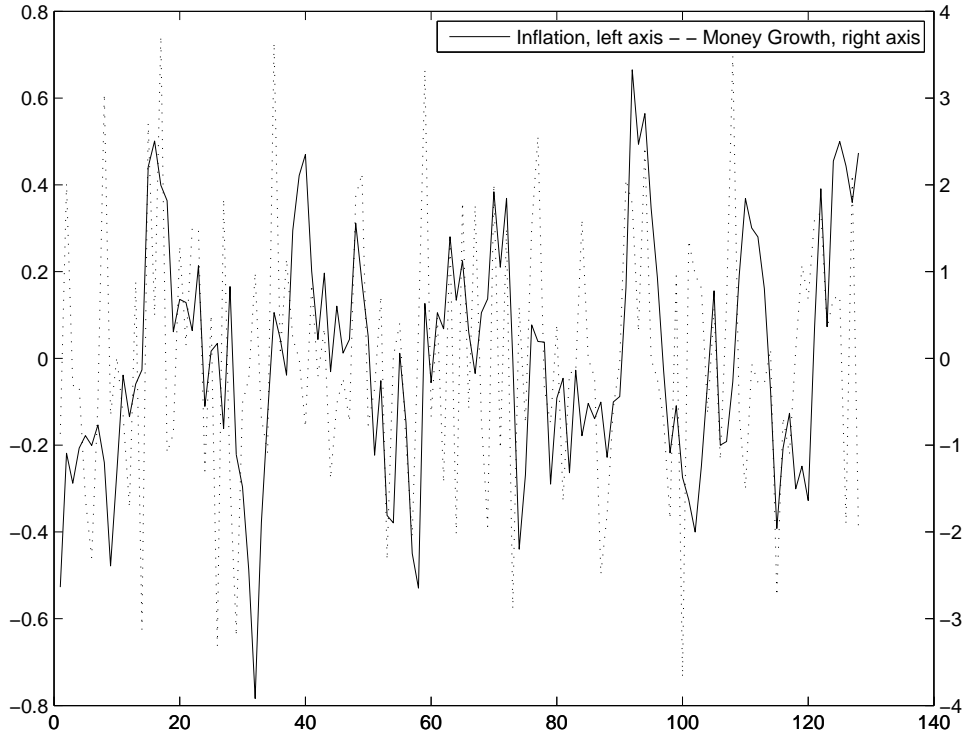


Figure 5: High-frequency components of inflation and money growth, in the same simulation as in Figure 1.

ries. (Here, as in Table 1, the “low-frequency” components are taken to be those with periods longer than 8 years.) One observes, as in the corresponding figure presented by Assenmacher-Wesche and Gerlach for euro-area data, that the low-frequency components of the two series are quite similar.<sup>24</sup>

Next we may consider what accounts for the modest discrepancy between the two series that remains even at low frequencies. Figure 2 compares the low frequency components of inflation minus money growth (the solid line) and of output growth (the dashed line). These are nearly inverses of one another, as one would expect on the basis of a simple quantity-theoretic model of inflation in which money demand is proportional to national income. Finally, Figure 3 compares the low-frequency

<sup>24</sup>Not only are the two series highly correlated, as in the actual euro-area data, but the turning points of the filtered money growth series appear to “lead” the corresponding turning points of the filtered inflation series. But in our simulation model, it is clear that this does not imply *causality* from money growth to inflation!

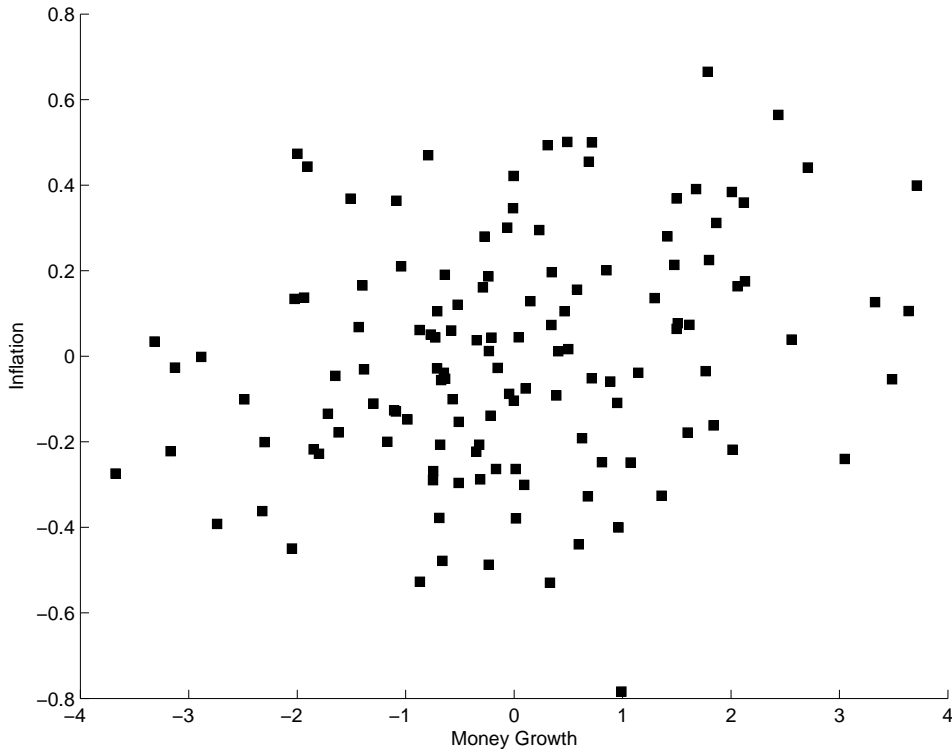


Figure 6: Scatter plot of the high-frequency components of inflation and money growth shown in Figure 5.

component of velocity growth (inflation minus money growth plus output growth) to the low-frequency component of the quarterly change in the nominal interest rate.<sup>25</sup> There is clearly a positive correlation in the low-frequency fluctuations in the two series, though the amplitude of the fluctuations in velocity is larger, consistent with a money-demand equation in which the interest-rate semi-elasticity is well above one. In Figure 4, low-frequency velocity growth is instead compared with the low-frequency component of the output gap,  $\log(Y_t/Y_t^n)$ .<sup>26</sup> While there is substantial low-frequency variation in the output gap in this simulation, given the definition

<sup>25</sup>For comparability with the results of Assenmacher-Wesche and Gerlach, my interest-rate change variable is the change from one quarter to the next in an *annualized* interest rate. However, for simplicity, I plot the (filtered) changes in the short-term (three-month) interest rate, rather than changes in a long-term interest rate. This is also the interest-rate change variable used in the regressions reported in Table 4.

<sup>26</sup>In the figure, the low-frequency component of the output gap is divided by 10, so that the range of variation in the two series plotted in the figure is similar.

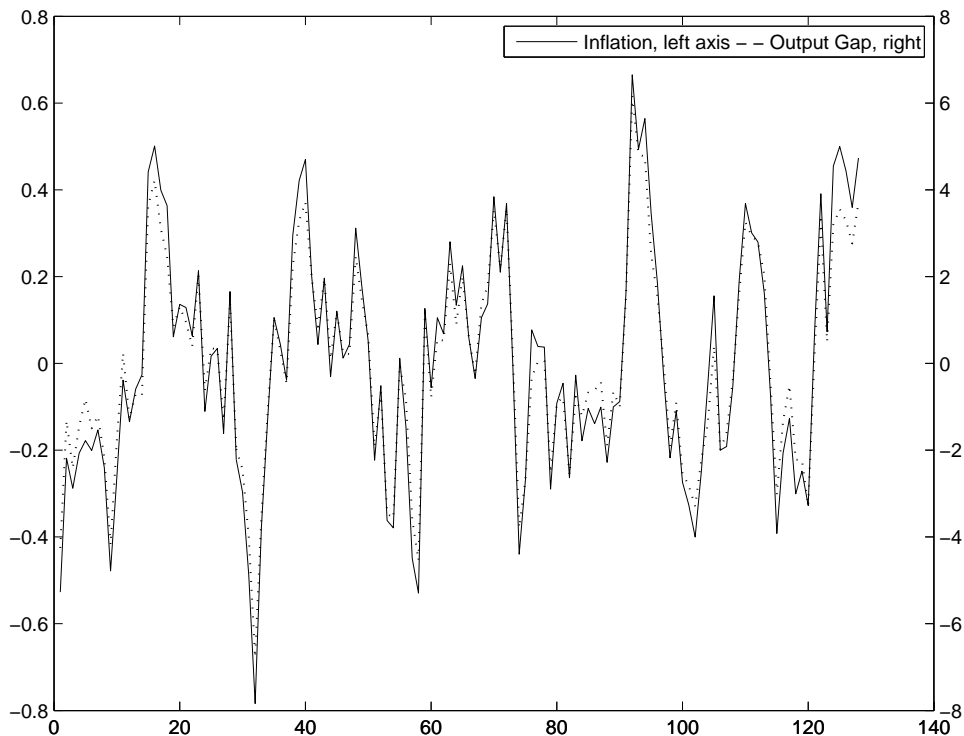


Figure 7: High-frequency components of inflation and of the output gap.

that I am using of the “low frequency” component of the series,<sup>27</sup> the low-frequency variation in the output gap is not nearly as closely related as the other variables to low-frequency variations in inflation.

The correlations that are observed at high frequencies are quite different, as can again be illustrated using plots of bandpass-filtered data from the same simulation. Figure 5 compares the high-frequency components of money growth and inflation (defined as the complements of the low-frequency components plotted in Figure 1). Here a much less close relation between the two series is visible, and a scatter plot of the two series (Figure 6) shows that indeed they are very weakly correlated. Figure 7 instead compares the high-frequency component of inflation with the high-frequency

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<sup>27</sup>Because of the relatively short time series, it is necessary to retain medium-frequency components in the “low-frequency” filtered series, just as in the work of Assemacher-Wesche and Gerlach. It should also be noted that in this model, with the degree of persistence assumed for the disturbance processes, fluctuations in the output gap are fairly persistent, though stationary (and hence of negligible variance at very low frequencies).

component of the output gap. At high frequencies (here corresponding to periodicities between 0.5 and 8 years), inflation and the output gap are very highly correlated. This makes it hardly surprising that the output gap should be the main variable of any significance in a reduced-form inflation equation that is estimated using only high-frequency data.

In fact, if one estimates a “two-pillar Phillips curve” using simulated data from the simple new Keynesian model, one obtains results very similar to those reported by Assenmacher-Wesche and Gerlach (2006a), and reproduced in Table 1. Table 4 presents the results of the Monte Carlo simulation described above. Three regressions are estimated for each of the 1001 simulated time series. The first is a bandpass regression of inflation on a constant,<sup>28</sup> money growth, output growth, and the interest-rate change,<sup>29</sup> using the method of Engle (1974), where the frequency components of the series that are used are those with periodicities longer than 8 years.<sup>30</sup> The second is an alternative low-frequency regression, in which it is assumed to be known that inflation and money growth are cointegrated, with a cointegration vector  $[1 \ -1]$ . Here I regress the stationary variable, inflation minus money growth, on the other (stationary) regressors, again using low-frequency bandpass regression.<sup>31</sup> The third is a high-frequency bandpass regression of inflation on money growth, output growth, the interest-rate change, and the output gap (lagged one quarter).<sup>32</sup> Follow-

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<sup>28</sup>The estimated coefficients on the constant in the low-frequency regressions are omitted in Table 4, as Assenmacher-Wesche and Gerlach do not report the values that they obtain, though they report that constants were included among their regressors. In each of the low-frequency regressions in Table 4, the estimated coefficient on the constant has a median absolute value less than 0.01 and a median standard error less than 0.01 as well. This is not surprising, given that our theoretical model allows for no velocity trend; but of course a velocity trend could be added to the model without changing any of the conclusions reached here.

<sup>29</sup>The interest-rate change variable used in these regressions is the quarterly change in the short-term nominal interest rate, because this is the variable that appears in the theoretical relation (1.4), even though the variable used by Assenmacher-Wesche and Gerlach (2006a) is the change in a real interest rate.

<sup>30</sup>This is not really a correct procedure, because inflation and money growth are non-stationary series; but the results are reported to show that this naive procedure would recover a coefficient close to one for money growth.

<sup>31</sup>This is the kind of low-frequency regression reported by Assenmacher-Wesche and Gerlach, though they estimate the cointegrating relation in a first stage, whereas I treat it as known.

<sup>32</sup>The constant is omitted in the high-frequency regression, as there is no high-frequency variation in a constant. The output gap regressor is lagged one quarter for comparison with the results of

Table 4: Inflation equations for alternative frequency bands, estimated using the data from simulations described in the text.

Freq. range Period (yrs)	HF 0.5-8	LF 8-∞	LF 8-∞
Money Growth	0.02 (0.46) [-.54, .70]	1.00** (0.02) [.95, 1.04]	1 — —
Output Growth	0.10 (0.41) [-.50, .56]	-1.00** (0.03) [-1.07, -.92]	-1.00** (0.02) [-1.05, -.94]
Int. Change	0.05 (1.08) [-1.21, 1.70]	3.00** (0.11) [2.75, 3.24]	3.01** (0.08) [2.82, 3.20]
Output Gap	0.11* (0.05) [.03, .17]	— — —	— — —

ing Assenmacher-Wesche and Gerlach (2006a), the high-frequency regression uses a frequency-domain instrumental-variables approach proposed by Corbae *et al.* (1994) to deal with possible simultaneity bias that might be created by the cointegration relation between inflation and money growth.<sup>33</sup>

In the case of each regressor, Table 4 reports the median value of the regression coefficient over the 1001 simulations, the median standard error for this coefficient (in parentheses under the coefficient), and the range of values for the regression coefficient (in brackets) corresponding to the 5th through 95th percentiles of the distribution of coefficients obtained. Double asterisks again identify the regression coefficients that would be judged to be significant at the 1 percent level (when the t-statistic takes its median value), while a single asterisk indicates significance at the 5 percent level.

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Assenmacher-Wesche and Gerlach.

<sup>33</sup>Lagged money growth is used as an instrument for money growth. No instruments are used for the other regressors, as they are not cointegrated with inflation.



The results are quite similar to those obtained by Assenmacher-Wesche and Gerlach (2006a) for euro-area data, reported in Table 1. In the case of the low-frequency inflation regressions (the last two columns of Table 4), the coefficient on money growth is highly significant and very close to the value of 1 implied by a quantity-theoretic model of inflation determination. The coefficient on output growth is also highly significant and very close to the value of -1 implied by the quantity theory (if one assumes a conventional unit-elastic money demand function), while the coefficient on the interest-rate change is significantly positive, as would also be implied by an interpretation of the low-frequency regression as estimation of the relation (1.4). The values obtained for all three coefficients in the long-run regressions are essentially the same as those obtained by Assenmacher-Wesche and Gerlach; the only notable difference is that in the case of the simulated data, it is possible to estimate the coefficients on output growth and the interest-rate change much more precisely.

The results of the high-frequency inflation regression (the first column of Table 4) are also very similar to those obtained by Assenmacher-Wesche and Gerlach. None of the three “quantity-theoretic” variables appear with coefficients significantly different from zero, and the (median) point estimates of these coefficients are also quite small. Instead, the lagged output gap appears with a significantly positive coefficient,<sup>34</sup> and the (median) numerical value of this coefficient is very close to the value reported by Assenmacher-Wesche and Gerlach. The reported range for the coefficient estimates indicates that a positive coefficient on the output gap is obtained more than 95 percent of the time.<sup>35</sup>

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<sup>34</sup>The median t-statistic, across the 1001 simulations, is 2.14. This is somewhat smaller than the one obtained by Assenmacher-Wesche and Gerlach (2006a) for their (10 percent longer) historical sample, though the median standard error of the coefficient estimate that I obtain is, to the number of decimal places that they report, the same as theirs.

<sup>35</sup>In fact, the coefficient is negative in only 3.6% of the simulations. The theoretical explanation for a positive high-frequency regression coefficient given at the beginning of this section suggests that the high-frequency relation between inflation and the output gap should be *contemporaneous* rather than with a one-quarter lag. Indeed, if the high-frequency regression is run with the current-quarter output gap as the regressor rather than the lagged output gap, the results are the same as those given in Table 4, to the degree of precision reported in the table.

## 2.3 Implications for the Role of Money in Inflation Control

The results presented above show that a finding of different numerical coefficients in a reduced-form inflation equation when it is estimated using different frequency ranges of the data is perfectly consistent with a *single* model of the causal factors responsible for variations in the rate of inflation. The fact that different coefficients values are obtained at different frequencies is simply a sign that the regression equation is *misspecified*, and so cannot be viewed as estimation of a structural relationship. But a coherent structural model may well exist that can simultaneously account for the low-frequency and high-frequency regression coefficients; the model presented above is one simple example. For purposes of predicting the path of inflation under contemplated policy interventions, one should seek to determine an empirically realistic *structural* model, rather than expecting to be able to conduct policy on the basis of equations that are assuredly *not* invariant across alternative monetary policies. But once a structural model is available that simultaneously accounts for both low-frequency and high-frequency relationships, there will be no need for *separate* modeling efforts, intended to capture shorter-run and longer-run inflation dynamics respectively.

Moreover, the type of model needed to account for estimates of “two-pillar Phillips curves” need not be one that assigns any intrinsic role to money, either in the specification of monetary policy or in the monetary transmission mechanism. It could well be a model like the one sketched here (or like the Smets-Wouters model), in which monetary policy is specified by an interest-rate equation like (2.3) that makes no reference to monetary aggregates, and in which the effect of monetary policy on aggregate demand depends solely on the path of interest rates. A system of equations that make no reference to money might suffice to completely determine the evolution of inflation, as in the model presented above, and yet — as long as the central bank’s inflation target  $\bar{\pi}_t$  is non-stationary, and the associated evolution of the money supply is determined by an equation like (1.3) — it will nonetheless be the case that at sufficiently low frequencies inflation will satisfy an equation like (1.2). Thus the findings of Assenmacher-Wesche and Gerlach do not imply any empirical inadequacy of “cashless” models, in either their low-frequency or their high-frequency implications.

Nor do their findings necessarily imply that money growth contains any useful information for forecasting inflation.<sup>36</sup> As a simple example, consider the new Key-

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<sup>36</sup>Here I refer specifically to the finding that money growth is the most important explanatory

nesian model presented above, in the special case in which the interest-rate gap  $r_t^g \equiv r_t^n - r_t^*$  is a white-noise process. (This could be true either because both  $r_t^n$  and  $r_t^*$  are white-noise processes, or because the central bank adjusts  $r_t^*$  to track the changes in the natural rate of interest that are forecastable a period in advance, setting  $r_t^* = E_{t-1}r_t^n$ .) In this case, the solution (2.7) is of the form

$$\begin{aligned}\pi_t &= \bar{\pi}_t + ar_t^g, \\ \log Y_t &= \log Y_t^n + br_t^g,\end{aligned}$$

for certain coefficients  $a, b$ . If inflation evolves in this way, the optimal forecast of future inflation at any horizon  $j \geq 1$  is given by

$$E_t\pi_{t+j} = \bar{\pi}_t = \pi_t + (a/b) \log(Y_t/Y_t^n). \quad (2.11)$$

Thus if one uses the current inflation rate and the current output gap to forecast future inflation, one cannot improve upon the forecast using information from any other variables observed at time  $t$ .

Forecasting future inflation using the output gap *alone* would not be accurate, since inflation has a stochastic trend while the output gap is stationary; one needs to include among the regressors some variable with a similar stochastic trend to that of inflation. In the specification (1.6), the only regressor with that property is money growth. But *inflation itself* is also a variable with the right stochastic trend, and using current inflation to forecast future inflation means that one need not include any other regressors that track the stochastic trend. What one needs as additional regressors are *stationary* variables that are highly correlated with the current *departure* of inflation from its stochastic trend, *i.e.*, the Beveridge-Nelson “cyclical component” of inflation. In the simple example presented above, the output gap is one example of a stationary variable with that property. More generally, the thing that matters is which variables are most useful for tracking relatively high-frequency (or cyclical) variations in inflation, and *not* which variables best track long-run inflation. Hence results like those of Assenmacher-Wesche and Gerlach provide

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variable in a regression such as (1.6) at low frequencies, as shown in Table 1. Other results, beyond the scope of the present discussion, do suggest that money contains information useful for forecasting inflation; for example, Gerlach (2004) shows that forecasts using money growth are superior to ones based on past inflation alone. But just how useful money growth is as an indicator variable depends on what other variables are also available as regressors.

no basis for assuming that money growth should be valuable for forecasting inflation, regardless of the horizon with which one is concerned.

The conditions that lead to an optimal forecast as simple as (2.11) are rather special, but the conclusion reached about the kind of variables that should be most useful for forecasting inflation is a good deal more general. Consider, for example, the somewhat more general specification of the disturbance processes in (2.9). In this case, (2.7) implies an equilibrium of the form

$$\begin{aligned}\pi_t &= \bar{\pi}_t + a'v_t, \\ \log Y_t &= \log Y_t^n + b'v_t, \\ i_t &= \bar{r}^n + \bar{\pi}_t + c'v_t,\end{aligned}$$

where

$$v_t \equiv \begin{bmatrix} r_t^g \\ \hat{r}_t^n \\ \gamma_t^n \end{bmatrix}$$

is the vector of variables relevant to forecasting the evolution of the interest-rate gap, and  $a, b, c$  are now vectors of coefficients. The existence of a solution of this kind implies that the optimal inflation forecast will be of the form

$$E_t\pi_{t+j} = \bar{\pi}_t + d_j'v_t \tag{2.12}$$

for any horizon  $j \geq 1$ , where  $d_j$  is another vector of coefficients.

It furthermore implies that

$$q_t = F v_t, \tag{2.13}$$

where

$$q_t \equiv \begin{bmatrix} x_t \\ i_t - \pi_t \\ \gamma_t - \Delta x_t \end{bmatrix}, \quad F \equiv \begin{bmatrix} b' \\ c' - a' \\ e' \end{bmatrix}$$

using the notation  $e' \equiv [001]$ . The relation (2.13) implies that one should be able to infer the state vector  $v_t$  by observing the elements of  $q_t$ . The optimal inflation forecast can then alternatively be written

$$E_t\pi_{t+j} = \pi_t + (d_j' - a') F^{-1} q_t \tag{2.14}$$

in terms of observables. It is a linear function of the current inflation rate, the current nominal interest rate, and the current and lagged values of the output gap.

Once again, there is no need for information that is uniquely associated with money growth; and if one were to add money growth to the vector of indicators  $q_t$ , the weight on this indicator in the optimal inflation forecast would be zero, given that some of the variation in money growth would be due to disturbances (the money demand shocks) that are independent of the fluctuations in the state vector  $v_t$ .

Of course, the best set of indicators to use in inferring the state vector  $v_t$  in practice might not correspond to the vector  $q_t$  above; it will depend which indicators are available to the central bank with relative precision and in a timely way. But there is no obvious reason to suppose that money growth would be especially useful for this purpose, whatever the defects of other economic statistics may be. One needs to find indicators useful for estimating the current value and forecasting the future evolution of the interest-rate gap, and *not* additional indicators of the inflation trend. Thus the relation that may be found to exist between money growth (or smoothed money growth) and the inflation trend is no reason to expect money growth to be useful for this purpose. Moreover, the state vector  $v_t$  consists entirely of real variables, so it should not be surprising if the most useful indicators are real variables as well — not variables that depend on either the inflation trend or the absolute level of prices.

Nor do reduced-form inflation equations of the kind presented by Assenmacher-Wesche and Gerlach (2006a) provide any basis for supposing that an optimal inflation-stabilization policy should make the central bank's interest-rate operating target a function of the observed rate of money growth. Beck and Wieland (2006) derive an optimal policy rule in which the interest rate depends on money growth, if an estimated “two-pillar Phillips curve” is treated as one of the structural relations of a model of the monetary transmission mechanism (replacing the aggregate-supply relation (2.1)). But in the case discussed here, that would be an incorrect inference. In the correct structural model, the evolution of inflation is fully described by equations (2.1) – (2.4), which do not involve money growth at all. Hence an optimal rule for choosing the central bank's interest-rate operating target — assuming a policy objective that can also be expressed purely in terms of the variables appearing in those equations, and assuming observability of the variables appearing in the equations — can also be formulated with no reference to money growth.

Of course, it remains possible that monetary statistics may have some use as indicator variables. In general, central banks use measures of a wide range of indicators in assessing the state of the economy and the likely effects of alternative policy decisions,

and it is right for them to do so. There is no *a priori* reason to exclude monetary variables from the set of indicators that are taken into account. But the mere fact that a long literature has established a fairly robust long-run relationship between money growth and inflation does not, in itself, imply that monetary statistics must be important sources of information when assessing the risks to price stability. Nor does that relationship provide the basis for an analysis of the soundness of policy that can be formulated without reference to any structural model of inflation determination, and that can consequently be used as a “cross-check” against more model-dependent analyses. To the extent that money growth is useful as an indicator variable (as Beck and Wieland also propose), its interpretation will surely be dependent on a particular modeling framework, that identifies the structural significance of the state variables that the rate of money growth helps to identify (the natural rate of output and the natural rate of interest, in their example). Thus a fruitful use of information revealed by monetary statistics is more likely to occur in the context of a model-based “economic analysis” of the inflationary consequences of contemplated policies than in some wholly distinct form of “monetary analysis.”

### **3 Reflections on the Monetary Policy Strategy of the ECB**

The European Central Bank has already achieved a considerable degree of credibility for its commitment to price stability, and succeeded in stabilizing inflation expectations to a remarkable extent. The achievement is all the more impressive when one considers what a novel kind of institution it was, and how little basis the public had, as a result, for judging what kind of policy to expect from it. It is hardly surprising, then, that the ECB would be proud of the credibility that it has won, and concerned to maintain it. To what extent has its “two-pillar strategy” for monetary policy — and more especially, the prominent role for monetary analysis within that strategy — been a key element in that success?

One obvious advantage of the two-pillar strategy was that the emphasis placed on monetary analysis served as a sign of the new institution’s fidelity to principles stressed earlier by the Bundesbank, which had in turn played a critical role as the anchor of the previous European Monetary System. This was doubtless an important

source of reassurance as to the new institution's degree of commitment to price stability. But however prudent such a choice may have been when the new institution's strategy was first announced, in 1998, it hardly follows that it should never be possible to dispense with pious references to monetary aggregates. At some point, the institution should have earned its own credibility and no longer need to borrow this from an association with past policies of another institution. Of course, it will remain important that the ECB not appear to change its strategy abruptly or capriciously, if its own past successes are to count as a basis for confidence in the institution in the future. But evolution of the details of its strategy should be possible without risking the credibility of the Bank's core commitment to price stability, especially when this evolution can be explained as a result of improved understanding of the means that best serve that unchanging end.

Are there advantages of the two-pillar strategy besides the continuity that it maintains with the past? Two other merits of the strategy are worthy of mention. One is that the existence of the two pillars, acting as cross-checks on one another, underlines the fact that the Bank's preeminent goal is price stability, rather than any particular "intermediate target" or recipe for *reaching* that goal. Rather than drawing attention to any particular quantitative guideline for policy — whether a monetary target like that of the Bundesbank in the past, or the kind of mechanical rules for setting interest rates on the basis of an inflation projection for a specific horizon sometimes offered as an account of inflation-forecast targeting at other central banks — the ECB has instead emphasized its goal of price stability, and shown a willingness to be pragmatic in determining the policy needed to achieve it. There are important advantages to such a "high-level" policy commitment (in the terminology of Svensson and Woodford, 2005). On the one hand, the commitment that is made is closer to what the Bank actually cares about, avoiding the problem of sometimes being forced to take actions that are known *not* to serve the ultimate goal simply because they are prescribed by a guideline that is often but not always congruent with that goal. And on the other, the public's attention is focused on the variable about which it is most useful for them to have well-anchored expectations; for it is inflation expectations (rather than expectations about either money growth or overnight interest rates) that most directly affect the degree to which the Bank can achieve its stabilization objectives.

Yet there are other ways in which a central bank can emphasize the outcome that

it is promising to deliver rather than the particular means that it uses to judge the required policy action. Inflation-targeting central banks all give much more prominence, in their communication with the public, to their quantitative inflation targets (that play essentially the same role for these banks as the ECB's definition of price stability) than to the nature of the decision framework that the use to set interest rates. At the same time, they provide a great deal of information about their decision framework as well — more, in fact, than the ECB does — but in a part of their communication that is addressed to a more specialized audience of financial professionals. The *Inflation Reports* of banks like the Bank of England, the Riksbank, or the Norges Bank provide detailed information about the justification of individual policy decisions — providing a considerable basis for the prediction of future policy, in the case of those in their audience capable of making use of such information — without the banks being tied to a rigid decision framework by their commitment to providing such explanations. And this approach has the important advantage — relative to the strategic ambiguity that is inherent in a “multiple pillar” approach — of requiring a greater degree of coherence in the bank's explanations of its policy. Such discipline should ultimately better serve the bank's interest in allowing verification of its commitment to its putative target and in improving public understanding of how policy is likely to be conducted in the future.

Another notable advantage of the ECB's strategy — over some of the common interpretations of inflation targeting at the time that the Governing Council first had to announce that strategy — is that it is not purely forward-looking. As I have discussed elsewhere<sup>37</sup> the computation of a measure of “excess liquidity” on the basis of a “reference value” for money growth introduces an element of error-correction into the decision process that is not present if a central bank is solely concerned with whether projections of inflation some years in the future conform to its (time-invariant) target. But as I have also explained, the desirable consequences of a commitment to error-correction can be obtained more directly and more reliably through an explicit commitment to adjust policy in response to past target misses; and this only requires monitoring of inflation outcomes, not of monetary aggregates. A commitment instead to correct past excesses or insufficiencies of money growth can only create undesirable uncertainty about the extent to which this may or may not imply stability of the general level of prices at the horizons that are most relevant for

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<sup>37</sup>See the discussion in Woodford (2007, section 4).



economic decisions.

In short, while the general goals of the ECB's strategy are highly praiseworthy — as is the institution's willingness to openly discuss the means that it uses to determine the specific policy actions that serve those goals — there would appear to be room for further refinement of the intellectual framework used as a basis for policy deliberations. And I believe that a serious examination of the reasons given thus far for assigning a prominent role to monetary aggregates in those deliberations provides little support for a continued emphasis on those aggregates.

This is not because a simple formula for sound monetary policy has been discovered and can be shown not to involve money. The quest for a robust decision-making framework for policy is an important one, and there is no reason to regard the procedures currently used by any of the inflation-targeting central banks as the final word on the matter. It makes sense to seek to refine those methods, and to try to find ways to reduce the chance of especially bad outcomes owing to errors in one's model of the monetary transmission mechanism. But there is at present little reason for the quest for such a robust framework to devote much attention to questions such as the construction of improved measures of the money supply or improved econometric models of money demand. For there is little intelligible connection between those questions and the kinds of uncertainty about the effects of monetary policy that are the actual obstacles to the development of more effective, more reliable, and more transparent ways of conducting policy.

## References

- [1] Assenmacher-Wesche, Katrin, and Stefan Gerlach, “Interpreting Euro Area Inflation at High and Low Frequencies,” CEPR discussion paper no. 5632, April 2006a.
- [2] Assenmacher-Wesche, Katrin, and Stefan Gerlach, “Understanding the Link Between Money Growth and Inflation in the Euro Area,” CEPR discussion paper no. 5683, May 2006b.
- [3] Beck, Guenter W., and Volker Wieland, “Money in Monetary Policy Design under Uncertainty: The Two-Pillar Phillips Curve versus ECB-Style Cross-Checking,” working paper, Goethe University Frankfurt, September 2006.
- [4] Benati, Luca, “Long-Run Evidence on Money Growth and Inflation,” *Bank of England Quarterly Bulletin*, Autumn 2005, pp. 349-355.
- [5] Bordes, Christian, Laurent Clerc and Velayoudom Marimoutou, “Is there a Structural Break in Equilibrium Velocity in the Euro Area?” Banque de France working paper no. 165, February 2007.
- [6] Bruggeman, Annick, Paola Donati, and Anders Warne, “Is the Demand for Euro Area M3 Stable?” in Otmar Issing et al, eds., *Background Studies for the ECB’s Evaluation of its Monetary Policy Strategy*, Frankfurt: European Central Bank, 2003.
- [7] Corbae, Dean, Sam Ouliaris, and Peter C. B. Phillips , “A Reexamination of the Consumption Function Using Frequency Domain Regressions,” *Empirical Economics* 19: 595-609 (1994).
- [8] Engle, Robert F. , “Band Spectrum Regression,” *International Economic Review* 15: 1-11 (1974).
- [9] European Central Bank, *The Monetary Policy of the ECB*, Frankfurt: European Central Bank, 2004.
- [10] Fischer, Bjorn, Michele Lenza, Huw Pill, and Lucrezia Reichlin, “Money and Monetary Policy: The ECB Experience 1999-2006,” unpublished, European Central Bank, November 2006.

- [11] Friedman, Benjamin M., and Kenneth N. Kuttner, "Money, Income, Prices and Interest Rates," *American Economic Review* 82: 472-492 (1992).
- [12] Gerlach, Stefan, "The Pillars of the ECB," *Economic Policy* 40: 389-439 (2004).
- [13] Hofmann, Boris, "Do Monetary Indicators (Still) Predict Euro Area Inflation?" Deutsche Bundesbank discussion paper no. 18/2006, May.
- [14] Jaeger, A., "The ECB's Money Pillar: An Assessment," IMF working paper no. 03/82, 2003.
- [15] Kugler, Peter, and Sylvia Kauffman, "Does Money Matter for Inflation in the Euro Area?" Oesterreichische Nationalbank, working paper no. 103, 2005.
- [16] Lucas, Robert E., Jr., "Panel Discussion: Colloquium in Honor of Otmar Issing," remarks presented at the ECB colloquium "Monetary Policy: A Journey from Theory to Practice," Frankfurt, March 2006.
- [17] McCallum, Bennett T., "Monetary Policy Analysis in Models without Money," *Federal Reserve Bank of St. Louis Review* 83: 145-160 (2001).
- [18] Nelson, Edward, "The Future of Monetary Aggregates in Monetary Policy Analysis," *Journal of Monetary Economics* 50: 1029-59 (2003).
- [19] Neumann, Manfred J.M., "The European Central Bank's First Pillar Re-assessed," working paper, University of Bonn, March 2003.
- [20] Neumann, Manfred J.M., and Claus Greiber, "Inflation and Core Money Growth in the Euro Area," Deutsche Bundesbank discussion paper no. 36/2004, November.
- [21] Reynard, Samuel, "Maintaining Low Inflation: Money, Interest Rates, and Policy Stance," working paper, Swiss National Bank, October 2006.
- [22] Rotemberg, Julio J., and Michael Woodford, "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," *NBER Macroeconomics Annual* 12: 297-346 (1997).

- [23] Smets, Frank R., and Raf Wouters, “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area,” *Journal of the European Economic Association* 1: 1123-1175 (2003).
- [24] Svensson, Lars E.O., and Michael Woodford, “Implementing Optimal Monetary Policy through Inflation-Forecast Targeting,” in B.S. Bernanke and M. Woodford, eds., *The Inflation Targeting Debate*, Chicago: University of Chicago Press, 2005.
- [25] Taylor, John B., “Discretion versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy* 39: 195-214 (1993).
- [26] Walsh, Carl E., *Monetary Theory and Policy*, Cambridge, MA: MIT Press, 2d ed., 2003.
- [27] Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press, 2003.
- [28] Woodford, Michael, “How Important is Money in the Conduct of Monetary Policy?” CEPR discussion paper no. 6211, March 2007.