Economists have long been interested in explaining the observed instability of economic aggregates. Though several reasons for variation in the pace of production are easily given, such as exogenous variation in tastes or in production possibilities, it is hard to see why there should be large variations in those factors that are synchronized across the entire economy. Instead, it seems more likely to suppose that variations in demand or in production costs in different parts of the economy should be largely independent and hence that a law of large numbers would imply that significant variations in aggregate activity (relative to the typical size of aggregate activity) are not likely to occur.

An alternative approach proposes that economies possess intrinsically unstable dynamics, which even in the absence of external shocks would result in persistent deterministic fluctuations. This type of model implies, however, that aggregate fluctuations should involve motion on a low-dimensional attractor, while analysis of economic time series has not revealed structure of this kind. Another alternative proposes that the economy possesses multiple equilibria and that it can therefore switch between equilibria for arbitrary reasons. This possibility suffers, however, from the difficulty that one must explain how people succeed in coordinating their expectations about the times at which a shift should occur.

Here we explore another type of explanation, which relies on an entirely different mechanism. Our proposal is that the effects of many small independent shocks to different sectors of the economy need not cancel out in the aggregate, due to the presence of significantly nonlinear, strongly localized interactions between different parts of the economy. The type of macroscopic instability that can result has been studied by condensed-matter physicists, under the name of "self-organized criticality" (Per Bak and Kan Chen, 1991).

Physicists have noted, in several contexts, the possibility of a "critical state," in which independent microscopic fluctuations can propagate so as to give rise to instability on a macroscopic scale. This is a state in which chain reactions initiated by local disturbances neither damp out over a short distance (the "subcritical" case) nor propagate explosively so that the system cannot remain in that state (the "supercritical" case), as in the controlled nuclear fission that allows a reactor to generate power without exploding. Often this has seemed to depend upon parameters being carefully "tuned" to exactly their critical values. (In the case of a reactor, an elaborate control mechanism is required to keep it near criticality.) More recently, it has been argued that some systems may spontaneously evolve toward a critical state and return to it even if perturbed by an external shock.

The prototypical example of such "self-organized criticality" is a sandpile. When the slope of the pile is nowhere too steep, dropping on additional grains of sand at randomly chosen sites has no macroscopic effects, as at most small numbers of grains will shift position in each case. However, randomly dropping on additional sand will eventually result in the slope of the pile increasing to a critical slope, at which point large avalanches can occur in response to the dropping of a single additional grain of sand. A sandpile with a slope that is initially greater than the critical slope also evolves toward it, in this case through an immediate...
large avalanche that collapses the pile. Thus while the existence of macroscopic instability without large external shocks depends upon a particular critical slope, the system endogenously evolves toward exactly that state.

In our work with Bak and Chen (Bak et al., 1993), we show the occurrence of a self-organized critical state as a result of the factor-demand linkages between sectors in a large economy. Our model is one in which autonomous shocks to final-goods demand have effects on production that propagate, both through time and between sectors, due to inventory dynamics. Before describing this model, we first discuss why, in order for aggregate fluctuations to persist in the large-economy limit, it is important that interactions between economic units be both local and significantly nonlinear.

I. Laws of Large Numbers for Large Economies

When different decision units affect one another only through their effects upon economy-wide variables, such as market prices, independent shocks to individual units tend to have an effect on the variability of aggregate quantities (moments of the distribution of quantities referring to individual units) that vanishes as the number of units becomes large. For example, in competitive economies, random variations in individual traders’ characteristics have a vanishing effect on equilibrium prices—and hence upon the distribution of quantities referring to individual units—that vanishes as the number of units becomes large. For example, in competitive economies, random variations in individual traders’ characteristics have a vanishing effect on equilibrium prices—and hence upon the distribution of quantities referring to individual units—that vanishes as the number of units becomes large. (Werner Hildenbrand, 1971).

Boyan Jovanovic (1985) obtains a similar result for anonymous noncooperative games. A game is anonymous if the payoff of an individual depends only upon his own action and the distribution of other players’ actions (and not upon which other agents play particular strategies). Suppose that all players have the same set of strategies, and that the utility function of each has for arguments his own strategy and the distribution of strategies chosen by other players.

The independent player-specific shocks are variations in this utility function. One can then define a limit game in which there is a continuum of players with a given distribution of utility functions, and the distribution of possible “sample” games, where \( n \) players’ utility functions are chosen independently from that same distribution. Jovanovic shows that if the limit game possesses a unique Nash equilibrium, then the distribution of actions chosen in “sample” games of size \( n \) converges as \( n \) is made large to the distribution of actions associated with the Nash equilibrium of the limit game.\(^2\)

Additional possibilities arise if we allow for local interaction instead of anonymity. Jovanovic (1985) presents an example of this kind, in which each player’s payoff depends only upon his own action and that of the player who precedes him in the sequence. In the unique Nash equilibrium of this game, each player’s action is a linear function of the preceding player’s action (unless he is first) and of his own shock. Jovanovic shows that the variance of the average individual player’s move does not necessarily go to zero as the number of players becomes large.

Still, we wish to argue that a cascade of this kind in which the interactions between neighboring units are linear is inadequate as a model of aggregate fluctuations. First, the variance of the average action is not an obvious measure of the variability of aggregate activity. For when the actions are nonnegative, then the mean of the average action grows with \( n \) as well. If one instead considers aggregate activity scaled by its mean, rather than by the number of interacting units, one finds that it approaches a constant as \( n \) is made large. A similar result is obtained if one scales aggregate activity by another measure of its average value.

\(^2\)Jovanovic (1985) provides an example that shows that aggregate fluctuations remain possible in the limit if the limiting game does not itself possess a determinate equilibrium; but the example requires “tuning” the value of a parameter to a critical value.
such as the median. By contrast, in the next section we show how a corresponding measure of aggregate activity, even when scaled by its median value, can have a nontrivial distribution in the large-economy limit. Second, the Jovanovic (1985) example depends upon an unbounded set of possible actions for each player, and in particular upon there being no limit to the amount that a player may be induced to change his actions by the actions of others. Our model avoids this as well.

Thus we conclude that in order for aggregate fluctuations to persist in the large-economy limit, we must introduce both local interactions and strong nonlinearities; that is, small changes in a given unit’s action must be able to produce large effects on the actions of its neighbors under certain circumstances, even though they do not have a uniform effect of that size. We turn now to a particular class of models that illustrate this idea.

II. Self-Organized Criticality in a Model of Production and Inventory Dynamics

One source of strongly nonlinear local interactions of the sort called for above, which we believe to be of considerable empirical relevance, results from nonconvexities in the production technology for individual producers. Nonconvex production costs appear to be pervasive, due for example to indivisibilities. In the presence of such nonconvexities, optimization typically requires alternation between discrete levels of production; the optimal policy is an “S-s rule” for inventories, with inventories reset to their upper target level, through a burst of production, whenever they fall to their lower target level. Such nonconvexities are now widely recognized to be important factors in production scheduling and inventory management at the plant level, where the empirical inadequacy of the “production-smoothing” model with convex costs has become evident. Here, we draw attention to the possible consequences of such plant-level nonconvexities for aggregate fluctuations. We wish to emphasize that the presence of nonconvexities at the level of individual units may be crucial for the generation of aggregate fluctuations, even in the absence of any cost advantage to “bunching” aggregate production.

Nonconvexities at the level of the individual plant need have no such effect when the number of plants is made large; it depends upon the nature of the interaction between individual plants’ production decisions. If, on the one hand, each plant produces for a single economy-wide market, total demand in which is the sum of independently fluctuating buyers’ demands, or alternatively if each plant supplies its own independently varying order flow, with no connection between any sector’s production decisions and the orders received by other sectors, then the variability of aggregate production will vanish as the number of sectors becomes large.

In Bak et al. (1993), we obtain a different result by assuming that each productive unit is connected through sales and purchases to only a few “nearby” units. For the sake of concreteness, productive units are located on a cylindrical lattice with $L$ rows and $L$ columns. Each unit (except in the bottom row) purchases inputs from two units in the row below it, and (except in the first row) sells goods to two units in the row above it. We assume a technology in which it is always optimal to produce a batch of two units at a time. Production of two units of output is assumed to require two units of produced inputs, one from each of the unit’s two suppliers. Production is also assumed to require one unit of primary inputs for each unit of output, though this is irrelevant for the production dynamics, since we assume that the primary inputs are always available when needed, and that purchases of them have no effect upon the demand for any of the produced goods. (Because of these primary-goods purchases, value added by each productive unit equals one unit of primary input per unit of output produced at that stage of production.) Finally, we assume inventory holding costs that make it optimal for each unit always to hold in inventory either zero units or one unit of the good that it produces. New production only occurs when an order cannot be filled out of
existing inventory and never results in more than one unit remaining in inventory.

The initial state of the economy in any period is described by the inventory holdings of each productive unit. Each unit's production and end-of-period inventory holdings are both deterministic functions of initial inventories and the unit's current sales. For instance, if a productive unit starts with one unit of inventories and sells two units, it produces a batch of two units and ends up with an inventory of one unit. If on the other hand it sells only one unit, it does not produce and ends up with an inventory of zero. The current orders received by each unit in the second row or lower are equal to half the sum of the quantities produced by its two downstream buyers. Finally, the orders received by the units in the first row are specified as exogenous shocks, determined outside the system. The relations just summarized then completely determine the new state (inventory configuration) as a function of the initial state and the current period's vector of exogenous shocks (final-goods orders).

Given a probability distribution from which the shock vector is independently drawn each period, these dynamics define a Markov chain on the set of possible inventory configurations. We consider in particular the behavior of aggregate demand $N$ for final goods each period, and of aggregate production $Y$, in each case defining the aggregate as a sum over units. (This output measure corresponds to aggregate value added.) We are interested in whether it is possible for significant fluctuations in $Y$ to occur despite an absence of significant exogenous shocks in $N$. We further specify that each final-goods supplier receives one order during the period with probability $p$ and no orders with probability $1-p$, independent of the orders received elsewhere. We consider systems in which the number of sectors $L$ is made very large, while the probability that any sector receives an order goes to zero; specifically we set $p \sim L^{-\gamma}$, with $2/3 < \gamma < 1$. The mean of $N$ is then $p(L)L$, which grows as $L^{1-\gamma}$. The random variable $\tilde{N} = N/L^{1-\gamma}$ then has a mean that does not change with $L$, and the limiting distribution of $\tilde{N}$, if it exists, is accordingly a reasonable indicator of the degree to which there are exogenous aggregate shocks in the large-economy limit. It is easily seen that as $L$ is made large, $\tilde{N}$ converges in distribution to a constant. Thus there exists no aggregate variability in the exogenous flow of final-goods orders in the limit.

We now wish to consider a similar question about the limiting variability of aggregate production. It can be shown that the median value of $Y$ grows asymptotically as $L^{3(1-\gamma)}$. Hence we consider the limiting behavior of the scaled aggregate-production measure $\tilde{Y} = Y/L^{3(1-\gamma)}$.

In Bak et al. (1993), we show that $\tilde{Y}$ converges in distribution as $L$ is made arbitrarily large but that the distribution is not a constant; instead, it is a Pareto-Levy stable law with exponent 1/3. In this sense we argue that aggregate fluctuations in production continue to occur in the large-economy limit, even though aggregate exogenous shocks cease to exist. Note that this is possible despite the facts that we scale aggregate activity by a measure of its average value and that our model is one in which production by each unit in each period is bounded.

The reasoning may be sketched as follows. Each order received by a final-goods producer initiates a chain reaction whose length depends upon the initial inventory configuration. If the unit receiving the order can fill it out of existing inventory, no further orders are generated. If it cannot, it produces and consequently sends orders to two units in the row below, each of which may or may not be required to produce and so send orders to their suppliers in the third row, and so on. The size distribution of the resulting "avalanche" of production depends upon the asymptotic distribution for inventory configurations. One can show that the probability of an avalanche of production of any given size is independent of the...
size of the system \( L \), for all \( L \) large enough, since it depends only upon the local inventory configuration near the final-goods producer receiving the order. The limiting distribution of avalanche sizes has a "fat tail"; the probability of an avalanche of size \( y \) or greater falls off only as \( y^{-1/3} \). Aggregate production \( Y \) is a sum of \( N \) drawings from this distribution, and in the large-economy limit the drawings are independent. Then the Levy central-limit theorem gives the limiting distribution for \( Y/N^3 \), which coincides asymptotically with the distribution of \( Y \). The existence of a well-behaved limiting distribution for individual avalanche sizes with such a "fat tail," just as in the sandpile model referred to in the Introduction, results from the fact that the rate of propagation of the chain reaction initiated by a single final-goods order is neither explosive nor too strongly damped. This "critical state" of the system is a property of the endogenous asymptotic distribution of inventory configurations, rather than a feature of the system determined by a careful "tuning" of parameters.

The fact that very long chain reactions become possible in the case of a large enough economy is obviously crucial to our result, and it will be observed that this results from our assuming that the number of stages of production (rows) grows along with the number of independently disturbed sectors (columns). This may seem undesirable. However, we suspect that the need for it is an artifact of our assumption of a rigid sequence of stages of production; allowing for a more general pattern of supply relationships among units (which would in any event be more realistic), we expect that an increase in the number of units should typically allow for correspondingly longer chain reactions, as arranged here in a relatively contrived manner. The study of more general patterns of connection remains a topic for further research.

REFERENCES


