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Tax-Collection Costs, Public Welfare
And the Predatory State

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The collection of taxes, in any economic system, clearly requires the use of resources. In modern democratic states tax legislation is almost always controversial, and subject to extensive lobbying. In developing counties the wealthy often successfully avoid payment of taxes and the burden has to be borne by relatively impoverished rural classes, who are themselves not easy to tax directly because of poor record -keeping and difficulty of communications. In earlier times kings and princes often lacked the necessary means of direct taxation and were forced to rely on decentralized institutions such as feudalism.

Despite its importance, however, tax-collection costs have been a relatively neglected issue in the literature of economics. Perhaps it is felt that the costs are still small relative to the revenue that is raised, and that their effects are too obvious to be worth introducing into theoretical models.

To convince the skeptical reader that the issue of tax-collection costs is neither trivial nor obvious, we pose the following question. What is the effect of greater efficiency in tax collection on the welfare of the tax-paying public? If the government is benign, taxing only to defray socially necessary public expenditure, a reduction in the costs of collecting these minimal taxes would clearly be a 'good thing'. What, however, if the state is inherently "predatory" in nature, as argued by Brennan and Buchanan (1980) and a number of others? In this case the state taxes not only to pay for public services but also to raise revenue for its own, possibly nefarious, purposes. Would an increase in the efficiency if tax-collection be undesirable under this alternative scenario?

To answer this question we extend the model of the state presented in Findlay and Wilson (1984) and Findlay and Wellisz (2003) to include the explicit resource cost of revenue extraction, either by an utterly benign Philosopher-King or a purely self-interested Leviathan.

We begin with the now familiar production function

$$Y = A(L_g)F(L_P,K) \qquad (1)$$

where

$$A(0) = 1,\ A'(L_g) > 0,\ \ A''(L_g) < 0$$

and the function F is homogenous of first degree with positive first and negative second derivatives. The function $A(L_g)$ is a public intermediate input, the "infrastructure" that sustains the productive activities of the private sector, which is assumed to operate with perfectly competitive markets.

The novelty of the present paper is that we specify a *Revenue Extraction Function*

$$R = R(Y, L_t) \qquad (2)$$

where

R = government revenue

Y_t = labor employed for tax-collection

We assume that the Revenue Extraction Function has the properties:

$$R = 0 \text{ if } Y = 0 \text{ or } L_t = 0 \qquad (3)$$

and that it is homogenous of the first degree

$$\lambda R = R(\lambda Y, \lambda L_t) \tag{4}$$

i.e. multiplication of Y and L_t by λ also multiplies R by λ.

These assumptions enable us to treat the extraction of revenue exactly like a production function with constant returns to scale. Thus we can picture a family of iso-revenue contours that are convex to the origin in a diagram with Y and L_t on the axes, just like an isoquant map in input space. We also have:

$$\partial R/\partial Y > 0, \qquad \partial R/\partial L_t > 0 \tag{5}$$

$$\partial^2 R/\partial R^2 < 0, \qquad \partial^2 R/\partial L_t^2 < 0 \tag{6}$$

and

$$\partial^2 R/\partial Y \partial L_t = \partial^2 R/\partial L_t \partial Y > 0 \tag{7}$$

Finally we have the labor constraint

$$L_g + L_P + L_t = L \tag{8}$$

Consider Figure 1 with Y and L_t as the axes. The convex functions represent iso-revenue curves showing the level of revenue corresponding to the values of Y and L_t along it. The concave function $Y(L_t)$ shows the maximum Y corresponding to each value of L_t . When L_t is zero the entire labor force L can be allocated optimally between L_P and L_g to maximize Y at Y* by equating the marginal product of labor between the provision of public services and employment in the private sector. As L_t is increased the residual labor force $(L - L_t)$ falls and so the maximized value $Y(L_t)$ declines, even though labor is still being allocated optimally between public services and private sector employment. When L_t is equal to L itself Y must then fall to zero. The concavity of the $Y(L_t)$ function follows from diminishing returns in the $A(L_g)$ and $F(L_P, K)$ functions.

Revenue is clearly maximized in Figure 1 at α, where the $Y(L_t)$ function is tangential to the highest attainable iso-revenue curve. $Y^\#$ and $L_t^\#$ are the values of Y and L_t corresponding to the maximized revenue $R^\#$. Having determined $R^\#$ and $Y^\#$ we can now determine

$$t^\# = R^\#/Y^\# \quad . \tag{9}$$

which is the *maximum effective tax rate* for the economy. Clearly it is possible for a government to *set* any tax-rate it wants, even a hundred per cent, but that does not mean that any tax –rate can actually be implemented. The maximum effective tax rate $t^\#$ is what can be obtained when the tax base $Y^\#$ and the tax-collection effort $L_t^\#$ are such that the resulting revenue $R^\#$ is at the maximum possible level. Note also that an effective tax-rate is defined at each point along the $Y(L_t)$ function, as the ratio of the revenue level of the iso-revenue contour that passes through it and the corresponding value of $Y(L_t)$. The effective tax-rate clearly rises from zero to $t^\#$ as L_t rises from zero to $L_t^\#$.

A rational predatory state, however, would *not* choose the revenue-maximizing solution. The appropriate maximand would be the "surplus", the difference between revenue and the costs incurred in generating that revenue. To obtain the surplus-maximizing solution we first define the Revenue Function $R(L_t)$ and the Expenditure Function $E(L_t)$.

The Revenue Function $R(L_t)$ is readily constructed from the iso-revenue map and the $Y(L_t)$ function in Figure 1. For each value of L_t we simply note the revenue obtained at the corresponding $Y(L_t)$ point. Since Y is *maximized* for each L_t by allocating the remaining labor force $(L - L_t)$ optimally between L_P and L_g, the corresponding revenue is also maximized for each L_t . In Figure 2 the $R(L_t)$ function is a concave function with revenue rising at a diminishing rate from zero at L_t equal zero, to the maximum $R^{\#}$ at $L_t^{\#}$, and falling thereafter.

The Expenditure Function $E(L_t)$ is defined as:

$$E(L_t) = w(L_t)\,[L_t + L_g(L_t)] \qquad (10)$$

where $w(L_t)$ denotes the after-tax real wage.

The Expenditure Function is constructed as follows. For each L_t we have the corresponding maximized $Y(L_t)$ and the values of L_g and L_P that are necessary to achieve it by equating the marginal products of labor in the public and the private sectors as in

$$A'(L_g)F[(L - L_t - L_P),K] = A(L_g)F_L(L_P) \qquad (11)$$

Since we know $Y(L_t)$ and $R(L_t)$ for each L_t we also know the effective tax rate $t(L_t)$ for each L_t . We can therefore determine the after-tax wage $w(L_t)$ as

$$w(L_t) = [1- t(L_t)]A(L_g)\,F_L(L_P) \qquad (12)$$

We can now determine $E(L_t)$ since, for each value of L_t , we have $L_g(L_t)$ and $w(L_t)$. When L_t is zero $(L_P + L_g)$ is equal to L and national income is maximized at Y*. Government expenditure is then equal to wL_g^*, but there is no tax revenue at all. In Figure 2 the distance 0M on the vertical axis indicates expenditure wL_g^* when L_t is equal to zero. As L_t is increased, the productive labor force $(L - L_t)$ shrinks, and so the marginal product of labor in the private sector $A(L_g) F_L$ rises. Both L_P and L_g fall when L_t rises, so total public employment $(L_t + L_g)$ rises along with the real wage $w(L_t)$. The $E(L_t)$ function is therefore convex upwards from the vertical intercept 0M.

We now define the 'surplus' $S(L_t)$ as

$$S(L_t) = R(L_t) – E(L_t) \tag{13}$$

which is maximized when

$$R'(L_t) = E'(L_t) \tag{14}$$

i.e. when the slopes of $R(L_t)$ and $E(L_t)$ are equal, with $\tilde{L}_t$ as the number of tax-collectors necessary to achieve this.

It is instructive to examine $R'(L_t)$ and $E'(L_t)$ in more detail. The $R(L_t)$ function depicted in Figure 2 is the Revenue Extraction Function (2), in which the argument for national income, Y, has been 'maximized out' by the optimal allocation of $(L - L_t)$ between L_g and L_P . The derivative $R'(L_t)$ is

$$R'(L_t) = (\partial R/\partial Y)\,(\partial Y/\partial L_g)\,dL_g/dL_t + \partial R/\partial L_t \tag{15}$$

in which

$\partial R/\partial Y > 0$, $\partial Y/\partial L_g > 0$, $dL_g/dL_t < 0$, $\partial R/\partial L_t > 0$ (16)

so that the first term of (15) is clearly negative. This term indicates the loss of revenue due to the decline in national income resulting from the reduction in productive employment $(L - L_t)$ consequent to the increase in L_t.

The last term of (15), which is positive, is the increase in revenue, holding national income constant, that is brought about by hiring one more tax-collector. For tax-collection to be feasible at all this last term must be greater in absolute value than the first when L_t is initially zero. As L_t increases Y falls, as we see in Figure 2, reflecting the negative sign of the first term in (15). Revenue, however, increases as L_t increases, as we cross higher iso-revenue curves along the $Y(L_t)$ function in Figure 1, until revenue is maximized at $R^{\#}$ at the point α.

Due to diminishing returns in productive employment the first term in (15) is rising as $(L - L_t)$ gets smaller, while diminishing returns to adding more tax collectors causes the second term to keep falling. Eventually the two terms will be equal in absolute value, making $R'(L_t)$ equal to zero and maximizing revenue at $R^{\#}$, at the tangency point α in Figure 1.

Differentiating (10) with respect to L_t we see that:

$$E'(L_t) = w(1 + d L_g/dL_t) + (L_t + L_g)\partial w/\partial L_t \quad (17)$$

Because both L_g and L_P decline when L_t rises, dL_g/dL_t is negative, but smaller than unity in absolute value, so that the first term in (17) is positive. The second term is also positive, because the marginal product of labor in the private sector rises as $(L - L_t)$

falls. Thus both sides of (14) must be positive when “surplus” is maximized at $\tilde{L}_t$, which proves that it must be to the left of $L_t^{\#}$ in Figure 2. We denote the levels of national income, revenue, expenditure and “surplus” corresponding to $\tilde{L}_t$ as $\tilde{Y}$, $\tilde{R}$, $\tilde{E}$ and $\tilde{S}$.

Looking at Figure 1 we note that the maximum national income Y* corresponding to L_t equal to zero is not feasible since revenue at this point is zero, while expenditure is wL_g* as indicated by 0M in Figure 2. We see that L_t must increase to $L^{**}{}_t$ before sufficient revenue can be raised to exactly offset public expenditure of $w(L_t + L_g)$ at the point where the convex function $E(L_t)$ first intersects the concave function $R(L_t)$. National income Y^{**} corresponding to $L^{**}{}_t$ is therefore the maximum *feasible* national income corresponding to the “Philosopher-King” solution of Findlay and Wellisz (2003) in the presence of tax-collection costs.

The following relations between the various solution values are readily apparent

$$L^{**}{}_t < \tilde{L}_t < L^{\#}{}_t \quad (18)$$

$$L^{**}{}_g > \tilde{L}_g > L^{\#}{}_g \quad (!9)$$

$$Y^{**} > \tilde{Y} > Y^{\#} \quad (20)$$

It is interesting to note that the size of government as measured by the correspoñ ding effective tax rate is ordered as

$$t^{**} < \tilde{t} < t^{\#} \quad (21)$$

Total public employment in the case of the surplus-maximizing Leviathan

$(\tilde{L}_t + \tilde{L}_g)$ is *greater* than $(L^{**}_t + L^{**}_g)$ in the case of the Philosopher-King, even though the number of productive public servants $\tilde{L}_g$ is less than L^{**}_g. The reason is that the number of additional tax- collectors hired by the Leviathan exceeds the decline in the number of productive public servants. Because the predatory state hires more tax collectors, it is larger than the optimally productive state, even though the former provides *less* intermediate public goods than the latter. The consumption available to citizens is less not only because national income $\tilde{Y}$ is less than Y^{**}, but because $\tilde{S}$ is positive while S^{**} is zero.

We now examine the consequences of changes in the efficiency of tax collection on both the Philosopher-King and Leviathan regimes. As we have seen the Revenue Extraction Function that we have specified in (2) behaves exactly like a constant returns to scale production function, with the 'output' being the revenue R and the 'inputs' national income Y and the number of tax collectors L_t. We assume that changes in tax-collection efficiency can be represented by 'Hicks-neutral' shifts in the Revenue Extraction Function so that an x per cent increase in efficiency means that revenue R increases by x per cent with Y and L_t constant. The 'marginal productivities' of Y and L_t in raising revenue, $\partial R/\partial Y$ and $\partial/\partial L_t$, will also increase by x per cent in the case of Hicks-neutrality, leaving the ratio of $\partial R/\partial Y$ and $\partial R/\partial L_t$ unchanged at any given ratio Y/Lt of national income to the number of tax collectors.

In Figure 1 a Hicks- neutral shift in the Revenue Extraction Function of x per cent will leave the $Y(L_t)$ function unchanged, since it depends only on the production technology of the economy, while simply renumbering the family of iso-revenue curves by x per cent. Thus at each L_t and $Y(L_t)$ the revenue will be x per cent higher in Figure 1. In Figure 2 the $R(L_t)$ function will also shift upward by x per cent at each value of L_t. To see what happens to the derivative $R'(L_t)$ at each point on the $R(L_t)$ function after the increase in tax efficiency consider equation (15). The Hicks-neutral shift raises both $\partial R/\partial Y$ and $\partial R/\partial L_t$ by x per cent, while leaving $\partial Y/\partial L_g$ and dL_g/dL_t functions unchanged. Each of the two terms on the RHS of (15) therefore increases by x per cent, but because the absolute value of the positive second term is greater than that of the negative first term, $R'(L_t)$ must increase, though by less than x per cent.

To show that the $E(L_t)$ function shifts down we look at (10) and (12). It is readily seen that L_g and L_P are unchanged at any given L_t , and so the marginal product of labor in the private sector, equal to the real wage *before* tax , is also unchanged. Since revenue increases at any given L_t and $Y(L_t)$, the effective tax- rate at each L_t goes up, and therefore the after-tax real wage $w(L_t)$ goes *down* for any given L_t. This proves that the $E(L_t)$ function shifts down at every given L_t. It also follows from (16) that $E'(L_t)$.shifts down as well at each value of L_t .

Putting these results together we see that the 'surplus' of revenue over public expenditure clearly goes up at each value of L_t since revenue rises, while public expenditure falls. What will be the impact on L^{**}_t, the number of tax- collectors hired by the Philosopher-King? Since he now has a positive surplus instead of just breaking even as before the increase in tax- collection efficiency, he will want to reduce his revenue and so will cut down on L_t. This will raise national income since $Y'(L_t)$ is negative, hence L_g and L_P both go up in response to the decline in L_t. Total public expenditure will fall along with revenue, since the reduction in tax- collectors will be greater than the increase in productive public servants. The new value of L^{**}_t, at which the budget is balanced, will therefore clearly raise public welfare since all of the larger national income is still available to the citizens. In the limit the economy will approach the national income Y* corresponding to L_t equal zero and the entire labor force allocated only between L_g and L_P.

Consider now the impact on $\tilde{L}_t$, the case of the Leviathan. Starting at the original value of $\tilde{L}_t$ we see that surplus is increased and that $R'(L_t)$ rises, while $E'(L_t)$ falls. Since they have to be equal for his surplus being maximized, by equation (14), he must increase L_t and therefore reduce L_g and L_P, lowering in the process national income $Y(L_t)$. Since the maximized surplus $\tilde{S}$ goes up, and the corresponding $\tilde{Y}$ goes down the public welfare measured by $(\tilde{Y} - \tilde{S})$ is clearly reduced by the increase in efficiency of tax- collection. In the limit as tax- efficiency is increased the consumption available to the public $(\tilde{Y} - \tilde{S})$ will be driven down to the 'anarchy' level Y_o, as defined in Findlay and Wellisz (2003), the point at which 'civil society' reverts to the 'state of nature'.

Thus we see that as tax- collection costs fall the gap between public welfare under the Philosopher-King and under the Leviathan increases, rising with the former up to an upper bound Y* and falling under the latter to a lower bound of Y_o. Denoting public consumption under the two regimes by C^{**} and $\tilde{C}$ we can define an *Index of Predation*

$$\pi \equiv (C^{**} - \tilde{C})/ C^{**} \qquad (22)$$

for any level of tax-efficiency common to the two regimes. As we have seen

$$\pi_{max} = (C^* - C_o)/C^* \qquad (23)$$

where C* equals Y* and C_o equals Y_o.

An interesting question that remains is what happens to C^{**} and $\tilde{C}$ as tax efficiency declines. By applying the previous reasoning in reverse, we know that the $R(L_t)$ function will fall and the $E(L_t)$ will rise. In the limit the concave $R(L_t)$ function will just be tangential to the convex $E(L_t)$ function. Surplus will be zero as required by the Philosopher-King situation, while $R'(L_t)$ and $E'(L_t)$ will be equal, so that surplus is maximized at zero by the Leviathan. At this point C^{**} and $\tilde{C}$ will therefore coincide so that in the limit we have zero as the minimum value of the Index of Predation. As tax-efficiency increases C^{**} rises toward C* as the upper bound, and $\tilde{C}$ falls toward C_o as the lower bound, so that the Index of Predation π rises from zero to $(C^* - C_o)/C^*$ as stated in (23).

We have shown that improvements in the efficiency of revenue- extraction are a double- edged sword. In the hands of a benign ruler the benefits are passed to the public in the form of higher consumption that they can enjoy because the socially necessary public expenditures on productivity-enhancing activities are less costly to maintain at the optimal levels. In the hands of a Leviathan, however, improvements in the efficiency of revenue-extraction enhance the ‘parasitic’ activities of the predatory state, enabling it to extract more surplus from a shrinking flow of national income to the detriment of the general public.

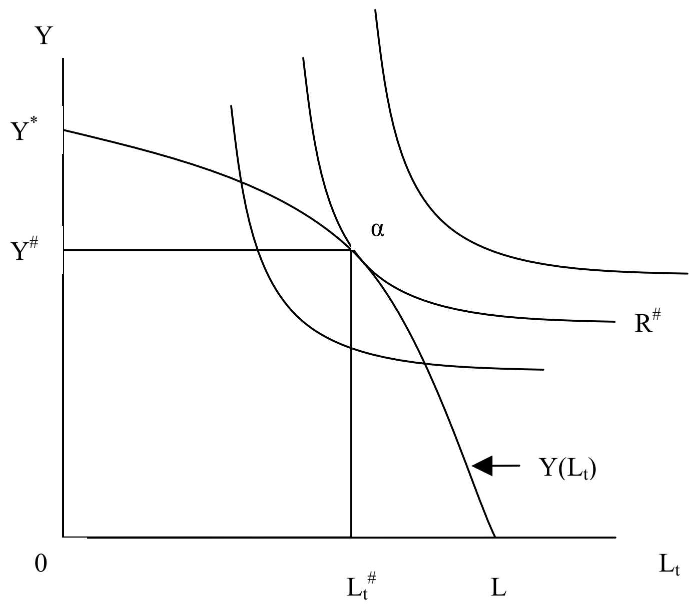

Figure 1

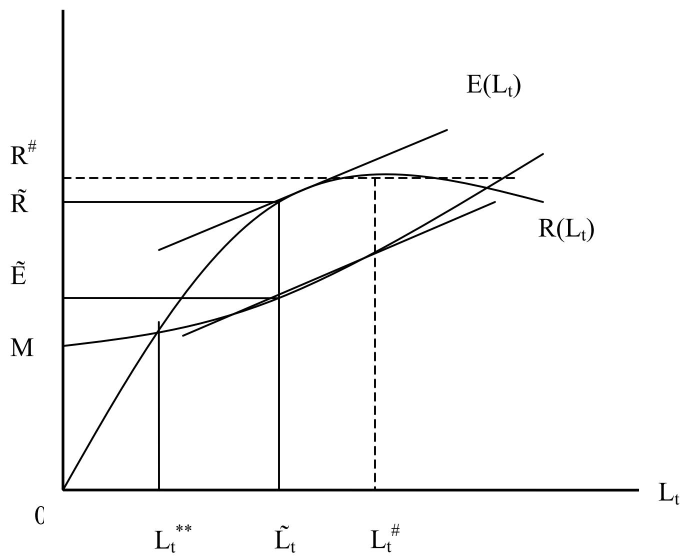

Figure 2

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