On the Columnar Model for Higher-Order Vagueness

Borderline, clear borderline and borderline clear

by

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I. The Paradox of Higher-Order Vagueness

No clear definition of the predicates ‘mountain’ and ‘hill’ can truly encompass every single case of elevation of the earth’s surface. There will always exist what we call ‘borderline cases’ that are neither clearly hills nor clearly mountains. The existence of those borderline cases is what makes both predicates vague. Vagueness applies at the first-order when we define a predicate that would encompass those borderline cases. For instance, an elevation is a ‘mount’ if it is not clearly a mountain nor clearly a hill. However, an application of vagueness to the first order does not allow us to solve our initial problem – i.e. to clearly define mountain and hills – but enlarges the vagueness we are dealing with. By creating a third predicate, we now have two boundaries (the hill-mount boundary and the mount-mountain boundary) that are both vague. In other words, there are elevations that are neither clearly hills nor clearly mounts and objects that are neither clearly mounts not clearly mountains. This second-order vagueness can in turn be extended to a third order of vagueness, and so on, ad infinitum. We have therefore failed to clearly define any of the above predicates, only delaying the problem of borderlineness to the higher order. Similar arguments lead to unsatisfying results when considering predicates like ‘red’ or ‘bald’ and their respective negation. If we introduce an ‘indeterminate’ value to describe first-order borderline cases, we run into the same issue. It becomes possible for us to apply the ‘indeterminate’ modifier for every higher order of indeterminacy, and infinitely many times. We have thus defined a vague predicate to be a predicate that allows borderline, or indeterminate, cases.

There is an additional way to define vague predicate, advanced by Wright, that relies on a property that they share, the tolerance intuition (Wright 2009): vague predicates are such that they admit small changes along a relevant scale that would make no difference on whether the predicate applies or fails to apply. This property of vague predicates is the consequence of our inability to define a sharp boundary between two predicates. Wright's method in defining vague predicate still relies on the existence of borderline cases. If we were to create a sharp boundary, eliminating borderline cases, then the negligible difference along the scale would lead to a difference in the application of the predicate. Only when the lack of sharp boundaries and borderline cases is a seen as a unsound can Wright's definition be valid. In other words, if there were no mounts at all but only hills and mountains, an elevation that is an inch taller than the tallest hill is a mountain (and an elevation that is an inch shorter than the lowest mountain is a hill) which disproves the tolerance intuition that states that an inch is negligible when considering hills and mountains. For this reason, vague predicates will be defined as those which admit borderline cases and a discussion of borderlineness (§II) will be necessary before discussing vagueness (§III-IV).

Not all predicates lack sharp boundaries (e.g. 'less than exactly 3,000 meters tall' is such a predicate), but for predicates that do, the consequences of such property can be clearly understood when considering a Sorites series. A Sorites series is a series of objects ordered according to a certain dimension (height, redness, etc.) such that the difference between an object and the next one is too small to lead to any change concerning the application of the predicate that relates to the dimension according to which they are ordered. For example, if we list all elevations (existent or non-existent[1]) by their heights, going from an object that is clearly a hill to an object that is clearly a mountain, with a difference of an inch between each, we can create such a series. The lack of

[1] There might not be an elevation that is exactly 532 meters tall but when considering a Sorites series we might want to include it

sharp boundary between hills and mountains allows us to claim that there is a no elevation that is a hill while the next one along the series is a mountain. It is interesting to look at such a series since it will allow us to describe what is called the Sorites paradox. The paradox is a proof by induction along a Sorites series of objects, X_1 to X_n, such that X_1 is a clear case of a predicate P and X_n is a clear case of Q (here, P and Q are predicates describing opposite states of a same dimension). For all X_i and X_{i+1}, the difference in the relevant dimension is negligible, as described above.

1. X_1 is P.
2. By the tolerance principle, for all X_i, if X_i is P then X_{i+1} is also P.
3. Therefore, X_n is P.

Not only does (3) contradict our initial claim, but this proof shows that either all X's are P or all X's are Q. The consequences of the tolerance principle are such that any predicate defined by a certain dimension must be applied to all objects along that dimension. For example, since an inch does not make the difference between a hill and a mountain, we can incrementally add a single inch to a hill and at no point can we stop and say, 'This elevation is now a mountain.' All elevations are proved to be hills through the inductive proof, but we know that there are such objects as mountains. Inversely, if we start with mountains and remove an inch from it incrementally, we can prove that, all elevations are mountains, rather than hills which is also an unsatisfactory result. If at any point we did stop and differentiate between a hill and a mountain because of an inch difference then we would have created a sharp boundary between the two predicates. However, there seems to be no height for which we could set this boundary to be.

In order to solve the above paradox, Susan Bobzien (2015) has argued that columnar higher-order vagueness, as opposed to hierarchical higher-order vagueness, is a more appropriate approach. She claims that her model is able to conserve the existence of higher-order vagueness while rejecting the paradoxes that are associated with it. At the basis of her view is the idea that vagueness is higher-order vagueness which, pictorially, can be seen as a column extending upwards between clear cases of a certain predicate P and clear cases of Q. In order to understand her view, she creates a distinction between two kinds of borderlineness: classificatory and epistemic. By understanding borderlineness, we can explain the two theories of higher-order vagueness, hierarchical and columnar. Rosanna Keefe (2015) has objected to the columnar approach to higher-order vagueness claiming that both clear borderline cases and borderline clear cases are absent from the model proposed by Bobzien. She claims that those two classes of objects are necessary for any view of higher-order vagueness to be coherent. I will reject Keefe's claim by showing that borderline clear cases and clear borderline cases are not necessary in the columnar model advanced by Bobzein and are therefore not necessary for a clear and complete theory of vagueness. This will allow us to accept columnar higher-order vagueness as a more coherent alternative to the traditional hierarchical model.

II. Borderlineness

Before starting our discussion on higher-order vagueness and since a vague predicate is defined by its allowing borderline cases, we will describe two ways to define borderlineness that Bobzien is careful to clearly distinguish (2013) in order to avoid confusion while setting up her model of higher-order vagueness. An object can be considered borderline in two cases. First, if we can establish that an object does not fall in any clear category, we are able to *classify* it as borderline in which case we name it borderline in the classificatory sense. Second, if we cannot establish that an object falls in any clear category, the borderlineness is coming from an *epistemic failure* and is therefore called epistemic borderlineness. Each view has different consequences on how we treat vagueness and on the problem of higher-order vagueness, that will be discussed after we define each. Moreover, we will examine how each understanding of borderlineness treats **borderline clear** cases and **clear borderline** cases since those will be at the center of our discussion of vagueness. For the rest of this paper, suppose that we have two predicates Φ and Ψ such that there exists objects that are borderline Φ/Ψ. Here, Φ and Ψ are predicates that could refer to opposite states of a same dimension (for example, 'long' and 'short') or to a predicate and its negation (e.g. 'red' and 'not red'). I refrain from using F and ¬F, in order to encompass both cases but also since using a predicate and its negation can lead to confusions, especially when negating the later: for example, while the negation of ¬F is equivalent to F, negating Ψ is not equivalent to Φ. This notation will be used throughout the paper. We will note B(Φ/Ψ)α if α is one of the objects for which it is borderline whether Φα or Ψα. The decision to use to B operator as such rather than simply noting 'Bα' will be expanded on later in this paper. Otherwise, if it clear that Φα or that Ψα then we will note CΦα or CΨα respectively. Additionally, let's denote 'possible' with ◊ and 'to identify that' with 'id.'

Classificatory (or in-between) Borderlineness

The first definition of borderlineness is a claim about our ability to clearly identify a case α as existing outside the categories already defined by our existing predicates. By doing so, we are creating a new class of objects, B(Φ/Ψ), such that:

- $B(\Phi/\Psi)\alpha \rightarrow \diamond\, id(\neg\Phi\alpha \wedge \neg\Psi\alpha)$

This formula (Bobzien 2013, 9) represents the main idea of classificatory borderlineness. We call an object borderline only if it is possible to identify it as being both not Φ and not Ψ. For example, by taking a borderline elevation E, we can identify it as not being a hill for being too tall and we can also identify it as not being a mountain to being too small. In this regards, we identify E as being a borderline hill/mountain. By doing so, we define three distinct mutually exclusive categories: Φ, Ψ and B(Φ/Ψ). It is therefore impossible for an object α to be both B(Φ/Ψ) and Φ or Ψ since an object being B(Φ/Ψ) requires that is it clearly not Φ and not Ψ. Moreover, there cannot be sharp boundaries between Φ and Ψ since the boundary is at least as large as the class of objects defined as borderline. Every object α such that B(Φ/Ψ)α exists in the boundary between Φ and Ψ.

Borderline Clear cases and Clear Borderline cases: When considering borderlineness in the classificatory sense, each degree of certainty adds another layer of borderlineness. Boundaries between two clear classes are what Fara calls "gaps" (2003) and an object falls into this gap when we can identify it as falling into neither of the clear cases around it. Moreover, we can clearly identify objects as existing in the gap. This is possible since our definition of borderlineness is a claim of clarity (i.e. a possibility in identifying that a certain object does not fall under any existing category). In this sense, **clear borderline cases** can exist without leading to any contradiction: if it is clear that an object is a clear case of not Φ and a clear case of not Ψ, then it is clear that this object is borderline Φ/Ψ. By creating more clear categories (e.g. clear that is clear, clear that it is borderline, etc.), we create more gaps to fill. Therefore, when considering **borderline clear cases**, we can argue that they merely are borderline cases: an object falls into a gap between Cα and Cβ when I can identify it as neither Cα nor Cβ with α and β complex formulas such as CBCCΦ. The only borderline case that is not a borderline clear case is at the first order.[2] Issues stemming from the existence of borderline clear cases and clear borderline cases will be the center of our discussion on hierarchical higher-order vagueness.

Epistemic Borderlineness

The second definition of borderlineness is a claim about our inability to clearly identify a case α as existing inside the categories already defined by our existing predicates. By doing so, we are creating a class of objects, B(Φ/Ψ), such that:

- $B(\Phi/\Psi)\alpha \rightarrow (\neg\diamond id\Phi\alpha) \wedge (\neg\diamond id\Psi\alpha)$

This formula (Bobzien 2013, 9) represents the main idea of epistemic borderlineness. We call an object borderline only if it is neither possible to identify an object as Φ nor possible to identify it as Ψ. For example, by looking at a borderline elevation E, if it not possible for a rational agent (who knows the definition of both a hill and a mountain) to identify it as a hill, since it is taller than what they take a hill to be, nor is it possible for them to identify it as a mountain, since it is smaller than what they take a mountain to be. In this sense, borderlineness is the result of an epistemic inaccessibility. As a consequence, there could be a sharp boundary between Φ and Ψ that we do not have epistemic access to. Further, it is possible for an object α to be both B(Φ/Ψ) and Φ or Ψ. For example, even when E is a borderline hill/mountain from the point of view of a rational agent, it could still be a hill from the point of view of an omniscient being.

[2] At the first order, $B(\Phi/\Psi)\alpha \rightarrow \diamond id(\neg\Phi\alpha \wedge \neg\Psi\alpha)$ but for any higher order of vagueness, Ψ and Φ are replaced by clear cases, i.e. formulas introduced by a C operator.

Borderline Clear cases and Clear Borderline cases: When considering borderlineness in the epistemic sense, it is more difficult to claim that **clear borderline cases** exist. We would have to claim that it is possible for us to identify that it is not possible for us to identity that Φα which seems to be an incoherent acknowledgement of our epistemic failure. However, **borderline clear cases** are easier to define: α is borderline clear when it is not possible to identify that it is clear that Φα. However, those cases do not need to exist in a standard view of vagueness as we will see later on. The epistemic view of borderlineness and the consequences of adapting such a view will be considered later in this paper as we discuss columnar higher-order vagueness.

III. Hierarchical Higher-Order Vagueness

The hierarchical model for higher-order vagueness, as described by Bobzien, is adapted from Sainsbury 1991. Starting with a vague predicate, since we cannot create a sharp boundary between the cases in which it applies and the cases in which it does not, we have to accept a class of borderlines. This creates two more divisions, between the clear cases and the borderline class and between the borderline class and the negative ones. For the same reasons that our first division led to a borderline class, those two divisions must too. We now have five classes of objects. In general, by repeating the process n times, we draw 2^n boundaries and divide the space in 2n+1 classes of objects. This is why this model of higher-order vagueness is called hierarchical: we are creating a hierarchy between what is most clear and what is most borderline by applying our C and B operators (which are defined in the next paragraph) on a specific predicate n times.

Hierarchical higher-order vagueness can be represented by the following figure that allow us to visualize its hierarchical nature. Bobzien provides us with the following model (2013, 3), from which I will only represent the first two orders of vagueness:

$$\mathrm{CF} \qquad \mathrm{BF} \qquad \mathrm{C}\neg\mathrm{F}$$

$$\mathrm{C}^2\mathrm{F} \qquad \boxed{\mathrm{B}^2\mathrm{F}} \qquad \mathrm{CBF} \qquad \boxed{\mathrm{B}^2\mathrm{F}} \qquad \mathrm{C}^2\neg\mathrm{F}$$

At the first order, Bobzein notes that, for any x,

- $\mathrm{BF}x \leftrightarrow \neg\mathrm{CF}x \wedge \neg\mathrm{C}\neg\mathrm{F}x$,

leaving the two extreme cases of F and ¬F as clear ones. In other words, at the first order, CF and C¬F do not encompasses all cases in a Sorites series but there is a borderline class that is understood, similarly to our epistemic approach above, as a clearly defined class that is independent of both CF and C¬F. However, at the second order, this becomes less obvious. As we move up to the second order, the three classes that we have defined at the first order (CF, BF and C¬F) gain an order of clarity (CCF, CBF and CC¬F) resulting in second-order clear cases and clear borderline cases. As for the second-order borderline cases, both classes that represent all x such that $\neg\mathrm{C}(\mathrm{CF}x) \wedge \neg\mathrm{C}(\mathrm{BF}x)$ and all x such that $\neg\mathrm{C}(\mathrm{BF}x) \wedge \neg\mathrm{C}(\mathrm{C}\neg\mathrm{F}x)$ are represented with $\mathrm{B}^2\mathrm{F}$ by Bobzein even if, intuitively, we might think of them as different. This is where Bobzien places second-order borderline cases in the model, omitting any ‘borderline clear’ case. Indeed, there seems to be misunderstanding around the nature of such cases.

Keefe claims that the standard conception of higher-order vagueness must contain borderline clear cases (Keefe 2015, 95) and must distinguish between those on the side of the clear positive cases of F and those on the side of the clear negative cases of F. Coinciding the two classes, in her view, is ‘counterintuitive’ (2015, 96) since ‘we can typically distinguish between borderline clear Fs and the borderline clear not-Fs.’ Instead, Bobzien considers the second-order borderline cases between C^2F and CBF and those between CBF and $C^2\neg F$ to be one and the same even in the hierarchical model that she presents. She claims that she is only reconstructing Sainbury’s model (1991, 168-9); however, Sainbury describes the five classes resulting from the division at the second order as follow:

> *‘the definite positive cases for the predicate, the definite borderlines and the definite negative cases, together with the cases which are borderline between being definite positive cases and definite borderlines, and those borderline between being definite borderlines and definite negative cases.’*

There does there seem to be a distinction made between the two classes labeled both as B^2F in Bobzien’s model, specifically between those on the clear positive side of F and those on the clear negative side of F. Bobzein is therefore bringing a feature of her columnar model (the absence of borderline clear cases and their overlapping nature) into Sainbury’s description of higher-order vagueness and uses her understanding of borderlineness in the epistemic sense when it seems that Sainsbury refers to borderlineness in its classificatory sense (also called in-between borderlineness).

Sainbury describe borderline cases as existing ‘between’ definitive cases. We can therefore clarify Bobzein’s reconstruction of hierarchical higher-order vagueness in the following way: other than replacing the negation of the predicate in question with another predicate, for clarity, the main difference here is that borderlineness exists in *between* two clear classes. This is made explicit with the use of the B operator as a binary one:

$$\mathrm{C}\Phi \qquad \mathrm{B}(\Phi/\Psi) \qquad \mathrm{C}\Psi$$

$$\mathrm{C}^2\Phi \qquad \mathrm{B}(\mathrm{C}\Phi/\mathrm{B}(\Phi/\Psi)) \qquad \mathrm{CB}(\Phi/\Psi) \qquad \mathrm{B}((\mathrm{B}\Phi/\Psi)/\mathrm{C}\Psi) \qquad \mathrm{C}^2\Psi$$

At the first order, the borderline cases are not completely represented by BF, as Bobzien puts it. It is important to encompass in our borderline class the in-betweenness that Sainsbury is describing: rather than saying it is borderline that Φ or that it is borderline that Ψ, we are saying that it is borderline *whether* it is Φ or Ψ. When it did not seem necessary to encompass both predicates when using F and ¬F, we see the necessity when switching to Φ and Ψ: we can say that 'Mt. Fuji is borderline tall' but does that mean that it is borderline whether it is tall or not tall? Tall or enormous? The encompassing of bordering classes in our expression of borderlineness is crucial. Similarly, at the second order, borderline clear cases and second-order borderline cases cannot by themselves refer to the gap between second-order clear cases and clear borderline cases. It is the conjunction of the two that allows us to express true borderlineness in this case. Still the adequate reconstruction leaves out borderline clear cases, that Keefe believes are an important part of higher-order vagueness, to hold no position even in the hierarchical model.

Even if the hierarchical model that Bobzien describes (in order to reject to reject it) is incomplete, Keefe's reply about the necessity of borderline clear cases does not address the issue with it. Bobzein coincides all second-order borderline cases and omits borderline clear cases in her reconstruction of Sainbury's model which clearly does the opposite. However, the re-adapted and accurate model that I have presented allows us see that borderline clear are unnecessary in the standard hierarchical conception of higher-order vagueness. No α can be such that $BC\alpha$. Rather, treating the B operator as a binary one, to clearly express the classificatory or in-betweenness of borderlineness, limits us to cases such as $B(C\Phi/B(\Phi/\Psi))\alpha$. As a consequence, the existence of 'borderline clear' cases are not ground on which to reject this view. However, clear borderline cases are what allow us to create more and more boundaries with each higher order, and are what lead to the hierarchical nature of the model. They cannot be dismissed at all in the hierarchical model of higher-order vagueness and are a direct consequence of our classificatory understanding of borderlineness. This is why, as we move to describe a columnar approach to higher-order vagueness, it will not be necessary for us to account for any type of borderline clear cases. However, clear borderline cases must be addressed and eventually rejected from our model in order to solve the paradoxes that they lead to as described earlier.

IV. Columnar Higher-Order Vagueness

Columnar Higher-Order Vagueness, as described by Bobzien (2015, 63), provides us with a model in which both clear borderline cases and borderline clear cases are absent. At the basis of the model is the belief that all borderline cases at the first order are also borderline at all higher orders and that clear cases at the first order are also clear at all higher orders. This idea can be represented by the following two formulas (Bobzien 2015, 68):

- $C\Phi \leftrightarrow C^n\Phi$ for any n (COLUMNAR C)
- $B(\Phi/\Psi) \leftrightarrow B^n(\Phi/\Psi)$ for any n (COLUMNAR B)

This allows us to divide any Sorites series into three classes, and only three classes. Borderlineness cannot extend to the higher-order between classes that can become clearer since we cannot clarify a borderline class and are therefore not creating more boundaries other than the Φ/Ψ boundary.

We can reject the existence of **clear borderline cases** as follow. For any α, if it is borderline that α then it is borderline that it is so. If it is borderline that it is borderline, then, following our definition of borderlineness (in the epistemic sense), it is unclear that it is borderline α. Therefore, we cannot have any cases of clear borderline cases of alpha. Formally:

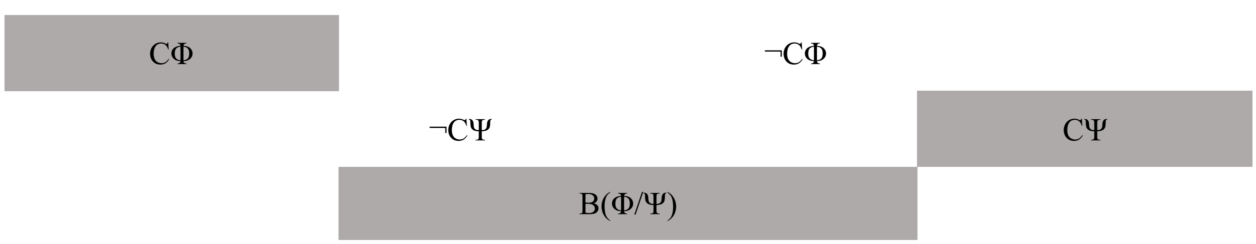
1. $\forall \alpha\ B(\Phi/\Psi)\alpha \rightarrow B^2(\Phi/\Psi)\alpha$
2. $\forall \alpha\ B^2(\Phi/\Psi)\alpha \rightarrow \neg C(B(\Phi/\Psi)\alpha) \wedge \neg C(\neg B(\Phi/\Psi)\alpha)$
3. And therefore, $\forall \alpha\ B(\Phi/\Psi)\alpha \rightarrow \neg C(B(\Phi/\Psi)\alpha)$

When we defined borderlineness in the epistemic sense, we intuitively showed that it was incoherent to have clear borderline cases. The above formally proves this intuition and further prevents higher-order vagueness from extending in a hierarchical way. By limiting our epistemic ability to clearly identify cases of Φ and Ψ to $C^n\Phi$ and $C^n\Psi$ (for any n), we are redefining our borderline classes in a way that can prevent the paradoxes that the hierarchical model led to as follow: an epistemic understanding allows us to that one cannot fail to identify that an elevation is a mountain unless there is an object that is a mountain; in other words, one cannot fail to identify a case of Φ unless there is an *x* that such that Φ*x*. Since the only class of objects that can be clearly identified are $C^n\Phi$ and $C^n\Psi$ (for any n), one can *only* fail to identify such instances. Further, since borderlineness is a failure to identify a certain object as an instance of the predicates, we conclude that borderlineness can only exist in a columnar way between $C^n\Phi$ and $C^n\Psi$ (for an any n). Since $C^n\Phi$ and $C^n\Psi$ coincide with CΦ and CΨ, we conclude that there is only one gap in which the columnar extension of borderline cases can fall.

In the hierarchical model, the existence of clear borderline cases divided our Sorites series into two sides, creating borderline clear cases that are closer to CΦ and borderline cases that are closer to CΨ. However, with the columnar modal, the absence of clear borderline cases and Bobzein's 'mirror axiom' (2016, 67), prevents this kind of division. The axiom states that,

- $B\Phi \leftrightarrow B\Psi$

which confirms our earlier claim that borderlineness can only be understood as a binary operation between the two classes that form the vague boundary: if it is BΦ then it is BΨ, in other words, it is B(Φ/Ψ). Therefore, the cases that, in the hierarchical model, seemed to be divided as closer to CΦ or closer to CΨ now coincide. And so, with the understanding that borderlineness *is* higher-order borderlineness i.e. $B(\Phi/\Psi) \rightarrow B^n(\Phi/\Psi)$ for any n, we observe the extension of borderline cases in a column between clear cases of Φ and clear cases of Ψ. There is an overlap of borderlineness vertically (through the mirror axiom) and horizontally (through the 'Columnar B' formula) leading us to the following model (Bobzein 2013, 13):



The B(Φ/Ψ) column above represents every single case that neither can be identified as Φ nor can be identified as Ψ i.e. the intersection of ¬CΦ and ¬CΨ. Comparing this to the model presented in paragraph III, where B(Φ/Ψ) is defined as the intersection of C(¬Φ) and C(¬Ψ), the difference is that, at the higher order, borderlineness will coincide with the middle column that we see at the first order and so will clarity. A representation of the higher orders will be presented later in this section.

This framework is compatible with an epistemic understanding of borderlineness. A classificatory view necessitates that $B(\Phi/\Psi)$ exists as an exclusive and independent class outside of $C\Phi$ and $C\Psi$, i.e. things that are Φ are bordering things that are $B(\Phi/\Psi)$ which in turn border things that are Ψ. The epistemic understanding of borderlineness allows things that are Φ to border things that are Ψ making claims of borderlineness claims about our epistemic inability to classify an object rather than a claim about the object and the category it falls under (Bobzein 2016, 78). In Part V, I will argue that the columnar model of higher-order vagueness is not only compatible but a direct consequence of our re-evaluating the way we define borderlineness.

First objection: CLEAR BORDERLINE cases. Bobzein defends her rejection of clear borderline cases by maintaining that any proof of their existence uses the two different conceptions of borderlineness defined above interchangeably (2015, 80). Keefe objects to this defense by attempting to show that even if we hold one view throughout, we can show that there are clear borderline case. However, this does not seem to be possible. First, considering that borderlineness is classificatory, existing clearly between two classes, and taking an arbitrary α that is clearly borderline, we get:

1. $CB(\Phi/\Psi)\alpha$
2. $(CC\neg\Phi\alpha \wedge CC\neg\Phi\alpha)$ by the classificatory definition of borderlineness
3. $(C\neg\Phi\alpha \wedge C\neg\Psi\alpha)$ by 'Columnar C'
4. $B(\Phi/\Psi\alpha)$ by the classificatory definition of borderlineness

We can show that any clear borderline case is merely borderline cases. By defining clarity as higher-order clarity, our conception of borderlineness which depends on clearly identify an α as neither Φ nor Ψ cannot be clarified further since clarifying a clear claim does not add any meaning to it. Second, if we consider borderlineness as epistemic, we are led to a contradiction as we proved and already discussed above. The proof goes as follow, taking an arbitrary borderline α:

1. $B(\Phi/\Psi)\alpha$
2. $B^2(\Phi/\Psi)\alpha$ by 'Columnar B'
3. $(\neg C(B(\Phi/\Psi)\alpha \wedge \neg C(\neg B(\Phi/\Psi)\alpha))$ by the epistemic definition of borderlineness
4. $\neg C(B(\Phi/\Psi)\alpha)$ first conjunct of the above formula

It seems that what Keefe has in mind is a definition of clear borderline cases in the following way: for an arbitrary item α,

- $CB(\Phi/\Psi)\alpha \rightarrow C(\neg C\Phi\alpha \wedge \neg C\Psi\alpha)$

Keefe is led to this formula that contradicts our conclusion above. If there are clear borderline cases at the second order, then those cases must be borderline cases at the first order (clear borderline cases are not going to stem from clear cases of Φ or Ψ). However, we have showed above that any borderline case at the first order cannot be a clear borderline case at the second order. Therefore, we have showed that there are no clear borderline cases.

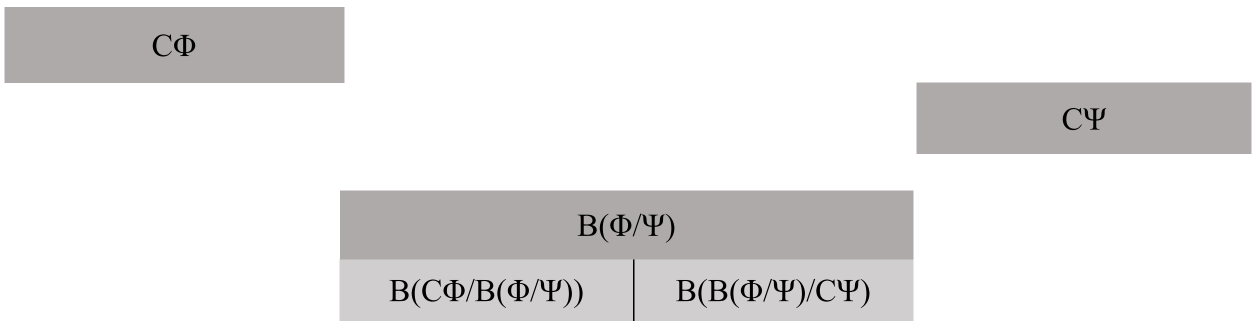
I do not think that Keefe is guilty of switching views between classificatory and epistemic borderlineness in her objection, as Bobzein claim. A switch would yield more incoherent and problematic claims that then ones she presents to us. For any arbitrary α,

- $CB(\Phi/\Psi)\alpha \rightarrow C(\neg C\Phi\alpha \wedge \neg C\Psi\alpha)$ by epistemic borderlineness
- $C(\neg C\Phi\alpha \wedge \neg C\Psi\alpha) \rightarrow BC(\Phi/\Psi)$ by classificatory borderlineness

Clear borderline cases become borderline clear cases if we first use the definition of epistemic borderlineness to understand clear borderline cases and then use the classificatory definition to simplify the resulting formula. Neither Keefe nor Bobzein seem to advocate for a view that would coincide those two classes. Other ways in which the two use of borderlineness can be merged are detailed in Bobzein 2013, 27-29.

Second objection: BORDERLINE CLEAR cases. Keefe's second objection is about borderline clear cases. However, the existence of those cases has already been rejected. By using our B operator only as a binary one, there seems to be no value in describing an object as a 'borderline clear' case since borderlineness must always encompass the two classes around which the vague boundary exist and those are always clear classes. Nevertheless, one aspect of her objection seems more interesting and puzzling. Another reason that Keefe presents to reject Bobzein's model is that we must be able to distinguish between the borderline cases that are more Φ than Ψ and vice-versa (2015, 80). Bobzein has rejected this distinction as we have seen earlier (§III). However, rejecting this distinction is not as plausible and is not necessary to the soundness of her model.

Keefe claims that 'we can typically distinguish between borderline clear Fs [Φ] and borderline clear not-Fs [Ψ]' (2015, 96). Bobzein's model does not seem to allow for such a distinction and even some time uses those two classes as one in order to prove further claims on columnar higher-order vagueness as we have seen in the preceding paragraph. I want to hold, however, that those two classes could still be separated through higher-order borderline cases rather than borderline clear cases. I have already claimed that borderline clear cases are incoherent in the columnar approach to vagueness since borderlineness can only exist between two clear cases and the only clear cases that we have are $C\Phi$ and $C\Psi$. In order to address Keefe's object and preserve our understanding of borderlineness, we can allow for the following continuation of our columnar model at the higher-order:



This does not contradict our rejection of clear borderline cases since the definition of a second-order borderline case does not depend on the existence of clear borderline cases but rather the rejection of such cases. We can further show why $B(\Phi/\Psi)$ coincides with the second-order borderline cases, taking as an example $B(C\Phi/B(\Phi/\Psi))\alpha$. By definition, we have:

- $B(C\Phi/B(\Phi/\Psi))\alpha \rightarrow \neg CC\Phi\alpha \wedge \neg CB(\Phi/\Psi)\alpha$

As we have seen, any α that is first-order borderline is not clearly so rendering the second part of our conjunction trivial. Additionally, saying that α is not clearly clearly Φ is equivalent to saying that it is not clearly Φ, meaning that the first part of our conjunction is also a consequence of α being borderline. Therefore,

- $B(\Phi/\Psi)\alpha \rightarrow \neg CC\alpha$
- $B(\Phi/\Psi)\alpha \rightarrow \neg CB(\Phi/\Psi)\alpha$
- And therefore, $B(\Phi/\Psi)\alpha \rightarrow B(C\Phi/B(\Phi/\Psi))\alpha$

Every case of $B(\Phi/\Psi)$ must be represented in each of the second-order borderline classes (that we have divided above into two classes with a line separating them) so higher-order vagueness cannot exceed the boundaries of first-order vagueness. A better representation would be to put those two classes on top of each other. However, it does seem that each represent something different than the other. It would be counter-intuitive (but more accurate) to see a representation like the one below:

$B(\Phi/\Psi)$
$B(C\Phi/B(\Phi/\Psi))$
$B(B(\Phi/\Psi)/C\Psi)$

However, while those two formulas seem to coincide exactly in the objects that they pick out, there is an expressive power in each that can allow us to be more specific in terms of where the object falls on that spectrum of borderline cases. I believe that the division as it was represented earlier, while inaccurate in terms of our formal and logical view of columnar higher-order vagueness, expresses this distinction.

The refinement of the borderline column is also possible in our clear cases. Clear cases extend in a columnar way with each higher-order of clarity but the expressive power of each formula of the form $C^n\Phi$ can exceed the expressive power of $C\Phi$ with every higher n. The cases that are closer to the extreme in our Sorites series can be described as *more clear* than those closer to the middle. While the logic of columnar higher-order vagueness does not allow for that hierarchy, we can understand this separately in our language and intuition.

V. Reevaluating the paradox

Beyond its simplicity, this model does address paradoxes of higher-order vagueness that result from understanding borderlineness in the classificatory sense, leading to the extension of vagueness to the higher-order in the hierarchical way. The Sorites paradox is a consequence of our lack of sharp boundaries as well as the tolerance principle. However, combining our columnar approach to vagueness and our epistemic understanding of borderlineness does allow for sharp boundaries to exist therefore putting into question the inductive step. Since cases of Φ can border cases of Ψ, it is possible for an α to be both $B(\Phi/\Psi)$ and Φ for example allowing a sharp boundary to exist between Φ and Ψ.

Our definition of borderlineness has important consequences on the way we understand vagueness. A classificatory approach which allowed us to clearly identify borderline cases could lead to problematic results: we do not need to have mountains and hills in order to identify mounts since mounts exist as an exclusive and independent class of objects. The existence of mounts depends only on our ability to identify elevations that can be mounts. However, through the epistemic approach, we have rejected that mounts can exist without the existence of hills and mountains: the existence of mounts depends on our failure to identify objects such as hills and mountains and so without such objects there are no mounts. This should be enough to restrict us to an epistemic understanding of borderlineness. However, the consequences of such a restriction will make it even more appealing.

First, the core of the hierarchical model, the two formulas noted as Columnar B and Columnar C, allow us to understand that vagueness and clarity at the first order coincide with their respective classes at all higher orders. This is a direct consequence of our understanding of borderline cases as resulting from an epistemic failure to clearly identify that an object falls in either existent clear categories. Through this understanding, it is impossible for clear borderline cases to divide our borderline class into multiple sides and therefore preventing the creation of additional boundaries, limiting borderlineness to a single column.

Second, borderline clear cases are also dismissed because of our epistemic approach to borderlineness. While borderline cases do not exist 'between' two classes, they necessitate the existence of two predicates that describe opposite states along a single dimension (which is how we have described Φ and Ψ). This requirement leads us to define the B operator as a binary one that must encompass the two classes within which our object failed to be identify. Borderline clear cases thus become obsolete.

I have thus showed how the hierarchical model advanced by Bobzein is a necessary condition of our adopting an epistemic definition of borderlineness: the lack of clear borderline cases and borderline clear cases, the extension of vagueness to the higher order in a columnar way and the coinciding of first-order clear cases with higher-order clear cases and first-order borderline cases with higher-order borderline cases all stem from the way we understanding borderlineness in the first place.

VI. The intuitive approach to vagueness

Our intuitive approach to vague predicates does not allow us to solve the paradoxes that result from our understanding of borderlineness. If we reject the existence of sharp boundaries between two predicates that refer to opposite states of a same dimension (like 'long' and 'short' or 'red' and 'not red' – in general, Φ and Ψ), we are bound to accept that all cases that fall in the boundary between the two predicates, which can be understood as a gap in truth values, create their own class of borderline cases which is distinct from cases of Φ and cases of Ψ. When we do so, create two more boundaries, starting an infinite regress. This view depends on our accepting of clear borderline cases which serve as a sufficient condition for the existence of additional boundaries at higher orders: by defining some of our borderline cases as being clearly so, we are bound to accept additional borders between the clear Φ and the clear borderlines on one hand and the clear borderline and the clear Ψ on the other. Some objects in those two borderline classes can, in turn, be defined as clearly so, repeating the process of creating more boundaries. In that sense, all borderline cases emerge as an independent class between two clear ones meaning that we are able to clearly distinguish those cases that are borderline from those that are clear. This view of vagueness is therefore dependent on our understanding of borderlineness as 'classificatory' (or 'in-between'): an object is classificatory borderline if we can clearly identify it as being neither Φ nor Ψ. As such, we can only solve the paradoxes of higher-order vagueness by redefining borderlineness in a way that does not lead to additional clear classes, additional boundaries and a hierarchical incremental expansion of higher-order vagueness.

The view presented by Susan Bobzein rejects the necessity of such an expansion by going back to the roots of it: the understanding of borderlineness that leads to the existence of clear borderline cases. Without a clear borderline class, every case that is borderline, at any order, falls into the same 'column' expanding between the clear Φ and the clear Ψ. The core of the view is the idea that borderlineness is higher-order borderlineness i.e. that if an object is borderline Φ/Ψ, it is borderline (not clear) that it is so. Similarly, clear cases extend in a columnar manner, meaning that any clear case is clearly so at any higher order. Those core ideas depend on an epistemic approach in defining borderlineness. A borderline case is so because we cannot identify it as Φ and we cannot identify it as Ψ. Since this definition is a claim on our inability to clearly define any borderline case as so, we cannot have any cases that are clearly borderline. By this virtue, borderlineness does not exist 'between' two clear cases but even if it did, there are only two clear cases for it to expand between: clear cases of Φ and clear cases of Ψ which coincide with higher-order clear cases of Φ and Ψ respectively. The absence of clear borderline cases, which is a consequence of our redefinition of borderlineness, is sufficient to prevent the expansion of vagueness in a hierarchical manner.

To bring this back to our example, it is when we are unable to say that an elevation is a mountain and when we are unable to say that it is a hill that we would call it a mount. There are no clear mounts since we cannot be confident in our classifying anything as a mount: our classifying it as such is the result of uncertainty in the first place. Another example that shows how this view of borderlineness does not commit us to an expansion of vagueness to the higher order would be looking at a professor grading papers: a professor can classify some papers as clearly passing, placing them in a pile on their right, some as clearly failing, placing them in a pile on their left, and some as needing a second look, in a pile in the middle. In the pile that need a second look, there are some that are better than others; however, they are all in the same middle pile since the professor was not able to clearly give them a passing grade nor give them a failing grade. Looking at them a second time, the professor might pass some and fail others but they are doing so out of necessity which does not strip away from the borderlineness of those papers initially in the middle pile. After a second reading, they might place some in a pile that needs a third reading, repeatedly, until every paper is either in the right pile or the left pile. This example allows us to understand a couple of aspects of our columnar modal. First, that a paper can be borderline pass/fail while also passing or while also failing which means that failing papers are bordering passing papers (the paper that is slightly better than the best failing paper is a passing paper and the paper that is slightly worse than the worst passing paper is a failing paper). The borderline papers are not standing as an independent and exclusive class. Second, there are no papers that should clearly be in the borderline pass/fail pass since the mere placing of them in that pile is the result of an epistemic shortcoming.

While our intuitive approach to vagueness led us to a view that commits us to complex results and at sometimes incoherent results, our reframing of our understanding of borderlineness allows us for a simpler and more coherent view of vagueness. This generalization applies to both our definition of borderlineness and our accepting of clear borderline cases in a Sorites series. The model advanced by Bobzein is therefore the first step towards an understanding of vagueness that can conserve its expansion to the higher order without committing us to complex and incoherent conclusions and paradoxes.

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