THEORETICAL ISSUES IN INTERNATIONAL BORROWING

Jeffrey Sachs

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Cambridge, MA 02138
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ABSTRACT

The current crisis in international lending points up a lesson re-learned several times in the past 150 years: the international loan markets function very differently from the textbook model of competitive lending. This paper discusses various extensions of the basic model. First, we amend the textbook model to show how limitations on a government's taxing authority may greatly affect its optimal borrowing strategy. Second, we explore the implications of a debtor country's option to repudiate debt. Third, we show that efficient lending may require collective actions by bank syndicates, and that a breakdown in collective action can result in serious inefficiencies and even financial panics.

Jeffrey Sachs
Professor of Economics
Harvard University
Cambridge, MA 02138
(617) 495-4112

National Bureau of Economic Research
1050 Massachusetts Avenue
Cambridge, MA 02138
(617) 868-3900
I. Introduction

The current crisis in international lending points up a lesson re-learned several times in the past 150 years: the international loan markets function very differently from the textbook model of competitive lending. In the simple model, borrowers have ready access to loans at a given interest rate; they enter the loan market to finance all investment projects with positive present value at the prevailing interest rate; and they use loans to equate the marginal utility of consumption at various points in time. Actual lending behavior is far from this rosy view. Borrowers are extensively rationed in the international markets; they may be unable to obtain credit at any price, much less the posted market price. Highly profitable investment projects may be left standing for want of foreign capital, or worse, may be abandoned mid-stream after creditors withdraw capital in a sudden loss of confidence. Various institutions, such as the IMF and the BIS, have an acknowledged role in maintaining stability in the loan markets, even though such institutions are superfluous in the simple model.

The large gap between theory and practice has led to a search for new theoretical concepts to explain actual loan behavior. A number of recent models have taken seriously the possibility of debt repudiation (or "sovereign risk") on loans to developing country governments, and have shown that such a risk radically alters the behavior of borrowers and lenders. The presence of sovereign risk can help to explain credit rationing, debt re-scheduling, conditionality, and even the maturity structure of international obligations (see [6], [17]). Other models (e.g., [19]) have shown that credit rationing may arise for other reasons as well, such as when lenders cannot
evaluate the risk categories of potential borrowers. Still other models (e.g., [9]) have explored the interaction of the domestic financial systems in developing countries with international borrowing, to derive more useful policy guidelines for international borrowing decisions.

These new models are helpful not only in restoring the relevance of a central class of economic models but also in shedding light on actual loan market behavior. The theoretical advances can help us to define the proper role of the IMF in the present debt crisis, and of the banks and borrowing countries in a very imperfect market.

This paper discusses some of the elements needed for a richer and more realistic model of international lending to developing-country governments. It draws on recent work and offers some new results as well, aimed at highlighting what is right and wrong with the standard models of international lending. The textbook case is both insightful and a natural starting point for discussion. It is treated in Section II of the paper. We then proceed to three areas of that model, to show how more realistic assumptions can fundamentally alter our views of borrower and lender behavior. The first revision comes in modelling the borrowing country itself. The textbook case treats the borrower as a "representative agent" maximizing utility subject to a budget constraint based on national wealth (described below). Since most international borrowing is by governments or government enterprises, the textbook model implicitly assumes that governments have an unlimited taxing power over national wealth. We follow Kharas [9] in introducing a limit to the government's taxing authority, and show that optimal borrowing strategies may be quite different from standard
prescriptions. In particular, governments should no longer borrow in order to finance all investment projects with a positive present value at world interest rates. Such a strategy almost surely leads to slow growth and a credit squeeze.

The second extension to the textbook model comes in the assumption about loan repayments. Standard models assume that loans are repaid as long as resources are available to repay them. Implicitly, the models assume that the costs to a country of repudiating its debts always outweigh the benefits. There is no doubt that the costs of debt repudiation are high, in both economic and diplomatic terms. Nevertheless, governments have at times preferred unilateral debt repudiation (or at least a unilateral debt moratorium) to the arduous and politically unpopular task of servicing a heavy debt burden, even when the debt servicing was technically feasible.

In the middle third of the paper, the textbook model is extended by giving the borrower the option of debt repudiation. The costs and benefits of debt repudiation are made explicit, and all market participants are assumed to understand these costs and benefits. A number of important implications are then derived. Most obviously, the repudiation risk leads to an upward-sloping supply of funds to borrowing countries, and to credit rationing once high levels of indebtedness are reached. Less intuitively, a number of inefficiencies arise in the dynamic behavior of borrowing countries, since the default risk distorts several of the borrower's incentives. Debtors may be led to: (1) overborrow period to period, relative to a path that would maximize ex ante expected utility; (2) over-invest in risky projects at the expense of safe projects, relative to a choice that would maximize ex ante utility; and (3) over-consume
and under-invest relative to levels that would maximize ex ante expected utility.

The problems arise from the inability of borrowers to pre-commit themselves to certain behavior once a loan is arranged. For example, a borrower might try to convince lenders that it will act prudentially, and avoid overly risky investments, in order to attract large loans at low rates. The problem is that after the loan is made, the borrower can often reduce the expected burden of the debt by going ahead with the risky projects (or by over-borrowing, or over-consuming, etc.). Since creditors anticipate that these actions will in fact be pursued, they charge a risk premium for them in the original loan contract. Not surprisingly, borrowers are best off when they can convincingly foresewre these actions and thereby reduce their initial cost of borrowing. In domestic capital markets, bond covenants are used for that purpose. In the international markets, convincing ways to foresewre such costly behavior have yet to be found.

The third extension to the basic model involves loan supply behavior. The competitive loan model typically ignores the institutional structure of lending. Credit is assumed to be supplied elastically at a given interest rate, and little attention is paid as to whether that lending comes from a bank loan or a bond flotation, a single lender or a syndicate, etc. In fact, the nature of intermediation can be of profound importance. Inherent risk considerations coupled with prudential bank lending rules have resulted in the syndication as the preferred form of lending. A standard loan to a borrowing country may be extended by several hundred participating banks. The problem with the
syndication is that member banks do not necessarily act in their collective best interest when key contingencies arise, as would be the case, for example, if the loan were made by a single bank operating in a competitive environment.

Many actions that a syndicate should take have "public good" features to them. For example, efficient loan packages may require that banks monitor a country's economic performance after a loan is made, but the banks may have no way to share in the finance of that monitoring. Even if the need for monitoring is clear, each bank might try to be a free rider on the monitoring expenditures of other banks. An insufficient amount of monitoring would result. More importantly, an efficient loan package sometimes requires that banks re-finance the debts of a heavily indebted country, at below market rates, in order to keep it from imminent default. Collectively, the need for re-scheduling may be clear, but once again each individual bank might try to withdraw its own credits in order to leave the debt burden to the other banks.

The most dramatic breakdown of loan supply comes in a panic, in which a fundamentally sound economy is forced into default by a shortage of credit. This type of market failure can result from the rational behavior of a large number of small lenders. Each bank rationally bases its loan supply decision regarding a country on the actions of the other banks. Suppose, for example, that a country needs a large loan to tide it over a short-run fall in income. If all banks but one stop lending, only a very large loan from the remaining bank may save the country from default. But such a large loan may be precluded by risk or regulatory considerations. Thus, if no other banks loan to the country, the given bank may also choose to stop loaning; whereas if the other
banks were to continue their loans, the given bank could safely contribute a share of the total credit line. In these situations, two equilibria may be present. In the favorable case, all banks continue to lend; in a panic, all banks stop lending, because the others have stopped lending, and the "healthy" country finds itself forced into default.

There are several ways to overcome the collective action problems of the creditors, though none is costless or foolproof. Syndicated loans typically include loan managers, who provide public-good services, such as monitoring or legal representation, in return for management fees. Also, informal "fair share" rules have emerged in the course of re-schedulings, in which various burdens are divided on the basis of the banks' existing shares in the total loans to a re-scheduling country. Finally, institutions like the IMF (and to a much lesser degree the BIS and leading central banks) assume some of the public-good aspects of international lending, such as monitoring and enforcement costs.

In the next sections, we illustrate these considerations in a series of models of international lending to a developing-country sovereign borrower. The models are kept simple to illustrate the main points as clearly as possible. I make no attempt at generality and little attempt at putting all of the various models together. Each remains a single facet of an evolving general model. Section II introduces the standard borrowing model, and Sections III, IV and V consider the three major extensions of that model: imperfections in the borrowing economy (Section III); risks of debt repudiation (Section IV); and collective-action problems of the creditors (Section V).
II. The Basic Model of International Borrowing

Consider the standard borrowing problem facing a social planner of a small open economy (see [2], [3], [14] for examples). The economy produces a pure traded good in amount $Q_t$ in period $t$, according to a production function $Q_t = F(K_t, L_t)$. The labor supply is exogenous (or perfectly elastic with a fixed wage $w$). The capital stock evolves according to the equation $K_{t+1} = K_t(1-d) + I_t$, where $I$ is gross investment and $d$ is the rate of depreciation. In the closed economy, total spending (the sum of consumption and investment) is equal to output. In the open economy, spending can be augmented by foreign borrowing. It is typical to assume that the economy can engage in one-period international loans at given world interest rates. Let $D_{t+1}$ be the stock of loans undertaken at time $t$ for re-payment at time $t+1$, and let $r$ be the real rate of interest on the loan (we will simplify substantially and assume that $r$ is fixed through time). With national output given by $Q_t$, national income (denoted by $Y_t$) is given by $Q_t$ net of interest payments on debt coming due: $Y_t = Q_t - rD_t$. Total consumption in period $t$ is national income minus investment plus net new borrowing:

$$C_t = Y_t - I_t + (D_{t+1} - D_t).$$

The model is closed by specifying the terms of borrowing. How much can the borrower borrow? The standard assumption here is that the borrower can attract any loan that can feasibly be paid back. The incentive to make the necessary loan repayments is simply assumed. In a finite-horizon version of the model, say $T$ periods, it is also assumed that there is no last-period debt, so that $D_{T+1} < 0$. The borrower faces the set of constraints:
(1) \[ D_2 = (1+r)D_1 + C_1 + I_1 - Q_1 \]
\[ D_3 = (1+r)D_2 + C_2 + I_2 - Q_2 \]
\[ \vdots \]
\[ D_{T+1} = (1+r)D_T + C_T + I_T - Q_T \]
\[ D_{T+1} < 0 \]

By a simple set of manipulations we can re-write (1) as:

(2) \[ \frac{T}{\sum_{i=1}^{T} (1+r)^{-(i-1)}} C_i < \frac{T}{\sum_{i=1}^{T} (1+r)^{-(i-1)}(Q_i - I_i)} - (1+r)D_1 \]

This equation implicitly defines an upper bound to borrowing. Note that \[ \sum_{i=1}^{T} (1+r)^{-(i-1)} \] must be non-negative, since \( C_i \) is non-negative. Therefore, by bringing \((1+r)D_1\) to the left-hand side of the inequality in (2) we find that:

(3) \[ (1+r)D_1 < \max_{i=t}^{T} \frac{T}{\sum_{i=1}^{T} (1+r)^{-(i-1)}(Q_i - I_i)} \]

In any period \( t \), we could perform the same manipulations for the remaining \( T-t \) periods, to find:

(4) \[ (1+r)D_t < \max_{i=t}^{T} \frac{T}{\sum_{i=1}^{T} (1+r)^{-(i-t)}(Q_i - I_i)} \]

Equation (4) states a very important point:

In order for debt re-payment to be feasible, i.e., for \( D_{T+1} < 0 \), indebtedness at any time must be less than national productive
wealth, where the latter is defined as the maximum discounted value of GDP net of investment in the remaining periods.

If the constraint in (4) is ever strictly binding, it implies that consumption is zero along the entire remaining growth path. For that reason it is unlikely that an optimal borrowing program involves borrowing $D_t$ up to its maximum feasible level. It is a simple matter to transform (4) to the appropriate expression for the infinite-horizon case. Feasibility conditions for re-payment in that case are found simply by replacing $T$ with $\infty$ in (4):

\[(5) \quad (1+r)D_t < \max_{i=t} \sum_{i=t}^{\infty} (1+r)^{t-i} (Q_i - I_i) \quad \text{(Infinite-Horizon Case)}\]

Now, let us state the full borrowing problem.

\[(6) \quad \text{The Borrowing Problem in Finite Horizon}\]

maximize $U(C_1, C_2, \ldots, C_T)$

subject to:

\[Q_t = F(K_t, L_t)\]

\[K_{t+1} = K_t(1-d) + I_t\]

\[C_t = (Q_t - rD_t) - I_t + (D_{t+1} - D_t)\]

\[D_t < \max_{i=t} \sum_{i=t}^{T} (1+r)^{t-i} (Q_i - I_i)\]

$K_1, D_1$ are given; $L_t$ is given for all $t$

The infinite-horizon problem is found simply by substituting $\infty$ for $T$ in (6). This problem has been heavily explored, in various guises, in the economics
and planning literature, and in various degrees of sophistication. When stated as in (6), the problem results in the following necessary conditions of optimization.

(7) Optimal Borrowing in the Finite Horizon

The solution to (6) is a set of sequences \( \{C_1, C_2, \ldots, C_T\} \{I_1, \ldots, I_{T-1}\} \), and \( \{D_1, \ldots, D_T\} \) that satisfy the conditions in (6) together with:

(a) \( U_i = \frac{\partial U}{\partial C_i} = \lambda (1+r)^{-i} \)

(b) \( \frac{\partial F}{\partial K_i} = r + d \) for \( i = 2, \ldots, T-1 \)

(c) \( \frac{\partial F}{\partial K_T} = 1 + r \)

\[ \sum_{i=1}^{T} (1+r)^{-(i-1)} C_i = \sum_{i=1}^{T} (1+r)^{-(i-1)} (Q_i - I_i) - (1+r)D_i \]

There are three main conditions here. First (7)(a) states that the international loan market should be used to equate the marginal utility of consumption in each period, \( U_i \), with a discounted marginal utility of wealth \( \lambda (1+r)^{-i} \). Second, (7)(b) states that investments should be undertaken in each period (except the last) in order to equate the marginal product of capital, \( \frac{\partial F}{\partial K_i} \), with the cost of capital, \( r + d \). Finally, (7)(c) holds that the discounted value of total consumption equals the discounted value of total productive wealth net of initial indebtedness. We see from (2) that this condition is equivalent to assuming that \( D_{T+1} = 0 \).

The conditions (7)(a)-(c) are also properties of richer models, e.g., those
that allow for productivity shocks or terms-of-trade fluctuations. When carefully interpreted they describe many of the standard guidelines in the development literature for foreign borrowing. For example, (7)(a) is really a prescription to **smooth** consumption over time relative to income, by borrowing when output is low relative to trend and paying back loans, on net, when output is high. (See [14] and [16] for this interpretation). The country should borrow in order to finance consumption during a **temporary** drop in income, but not during a **permanent** drop in the trend of income. On close analysis, (7)(a) makes explicit the IMF dictum to finance temporary shocks, but "adjust" to permanent shocks.

Equation (7)(b) states the standard cost-benefit condition for investment projects in a small open economy. Regardless of the consumption stream, the planner should invest so as to equate the marginal product of capital, evaluated at world market prices, with the cost of capital also at world market prices. A nearly equivalent condition is that all projects should be undertaken with positive present value at the world market prices and interest rates.

A useful simplification for solving the borrowing model is to write utility as **additively separable**:

\[
U(C_1, C_2, \ldots, C_T) = \sum_{i=1}^{T} u(C_i)(1+\delta)^{-i}
\]

That is, total utility is the sum of sub-utilities based on consumption in each period. These sub-utilities are discounted according to the subjective rate of
time preference, $\delta$. With this formulation, (7)(a) becomes $u_i = \lambda(1+\delta)^i/(1+r)^i$. From this point forward, we will assume additive separability.

A second transformation is also helpful. Let $V(K_t, D_t)$ be the maximum value of utility that a borrower can achieve starting with $K_t, D_t$. It is found by plugging the solution from (6) into the utility function. Then a $T$-period or infinite horizon problem can be reduced to a two-period or three-period problem. Thus, for example, the problem stated in (6) can be re-written as:

\[
(6') \max u(c_1) + U(c_2)/(1+\delta) + V(K_3, D_3)/(1+\delta)^2
\]

subject to:

\[
Q_t = F(K_t, L_t), \quad t = 1, 2
\]

\[
K_{t+1} = K_t(1-d) + I_t
\]

\[
C_t = (Q_t - rD_t) - I_t + (D_{t+1} - D_t)
\]

\[
K_1, D_1 \text{ are given; } L_t \text{ is given for all } t
\]

The first-order conditions are now:

\[
(7') \quad u_i(c_1) = -(1+r)V_{K_2}(K_3, D_3)/(1+\delta)^2
\]

(where $u_i$ signifies $\partial u/\partial c_i$)

\[
F_K(K_2) = r + d
\]

\[
V_K(K_3, D_3) = -V_D(K_3, D_3)
\]

III. **International Borrowing with Domestic Financial Constraints**

We now expand the model to allow for an explicit role for public-sector
financial variables. For simplicity we illustrate our results in the three-
period version of the model just introduced.

Kharas [9], Katz [8], and others have pointed out that the pure
borrowing model should differentiate between the private and public sectors and
take seriously the empirical fact that most international lending to developing
countries is to the public sector, or to the private sector with public-sector
guarantees. In these circumstances, debt-servicing capacity depends not only on
national wealth but on the public sector's ability to tax that wealth.
Moreover, domestic capital markets in the borrowing country tend to be highly
segmented and imperfect so that the public sector must use its borrowing powers
to bring about an efficient level of aggregate investment in the economy.

Thus, let us suppose that the private sector in the developing country
saves a fixed fraction of post-tax income, which is available for private
investment, while the government uses its taxing and borrowing authority to
supplement private investment and/or private consumption (see Arrow and Kurz
[1], Ch. VI, for a similar set-up in a closed economy). Private investors
have no direct access to the international loan market. The government taxes
domestic output at rate \( \tau_t \), which may change over time. This rate must be less
than 1.0, and may be less than zero if the government is making net income
transfers to the private sector.

With domestic output given by \( Q_t \), tax revenues are \( \tau_t Q_t \), and private sector
savings are \( s(1-\tau_t)Q_t \). Private consumption is given by \( C_t = (1-s)(1-\tau_t)Q_t \). In
any period, the government borrows \( D_{t+1} \) and repays \( (1+r)D_t \). Total investment in
the economy is given by:
(9) \[ I_t = s(1-\tau_t)Q_t + \tau_tQ_t + (D_{t+1} - (1+r)D_t) \]
\[ \text{(private savings)} \quad \text{(tax revenue)} \quad \text{(net foreign resource inflow)} \]

As written, it appears that all foreign borrowing is used for investment rather than consumption, but this is true only as an accounting matter. Suppose, for example, that the government wants to raise private consumption while holding investment levels fixed. It merely raises \( D_{t+1} \) while reducing \( \tau_t \) sufficiently to keep \( I_t \) constant; in that case the borrowing finances consumption 100% on the margin.

Now, let us calculate the optimal financial policy of the government, assuming again that it tries to maximize an intertemporal utility function of the form \( u(C_1) + u(C_2)/(1+\delta) + V(K_3, D_3)/(1+\delta)^2 \).

(10) **The Basic Public Finance Problem**

with International Borrowing

\[
\begin{align*}
\text{max} & \quad I_1, I_2, \tau_1, \tau_2 \\
& \quad u(C_1) + u(C_2)/(1+\delta) + V(K_3, D_3)/(1+\delta)^2 \\
\text{subject to} & \quad Q_t = F(K_t, L_t) \\
& \quad K_{t+1} = K_t(1-d) + I_t \\
& \quad C_t = (1-s)(1-\tau_t)Q_t \\
& \quad I_t = s(1-\tau_t)Q_t + \tau_tQ_t + D_{t+1} - (1+r)D_t
\end{align*}
\]
As long as tax rates are completely flexible, the solution to this problem is identical to the social planner's solution of the earlier section, since the dynamic budget constraint facing the government is no different whether it chooses $C_t$ and $I_t$ as before or $\tau_t$ and $I_t$ as here.

To show this formally, simply substitute for $C_1$, $C_2$, $K_2$ and $D_3$ in the utility function in (10), to find:

\[
U = u[(1-s)(1-\tau_1)F(K_1)] + u[(1-s)(1-\tau_2)F(K_1(1-d) + I_1)]/(1+\delta)
\]

\[
+ V(K_1(1-d)^2 + I_1(1-d) + I_2, (1+r)[I_1 - \tau_1 F(K_1) - s(1-\tau_1)F(K_1)] + I_2
\]

\[
+ \tau_2 F(K_1(1-d) + I_1) - s(1-\tau_2)F(K_1(1-d) + I_1))/(1+\delta)^2
\]

Now, the first-order conditions are given by:

(11) (a) $\partial U/\partial \tau_1 = 0 \implies u_1 = -(1+r)\rho D/(1+\delta)^2$

(b) $\partial U/\partial \tau_2 = 0 \implies u_2 = -\rho D/(1+\delta)$

(c) $\partial U/\partial I_1 = 0 \implies (1-s)(1-\tau_2)F_K(K_2) \cdot U_2 + (1-d)\rho V_K/(1+\delta) + (1+r)\rho D/(1+\delta)
\]

\[
+ [\tau_2 - s(1-\tau_2)]F_K(K_2) \cdot \rho V_K/(1+\delta)
\]

(d) $\partial U/\partial I_2 = 0 \implies V_K = -\rho D$

By substituting (11)(a),(b),(d) into (11)(c), we find that $F_K(K_2) = r + d$, as before; the conditions (11)(a),(b) and (d) are exactly as before. Thus, the demonstration is complete.

To find the tax rates implied by (11), note that $C_1 = (1-s)(1-\tau_1)F(K_1)$, so that $\tau_1 = 1 - [C_1/F(K_1)][1/(1-s)]$. A typical optimal growth path for a developing
economy will involve a rising \( \tau \). Low tax rates in the early period allow households to benefit early on from the growth that will be achieved in periods 2 and 3. Higher taxes later on are necessary to service the international debt.

Now let us introduce a simple yet crucial hitch into the model. Suppose that the government can only raise tax rates to a limit \( \bar{\tau} < 1 \), and that the constraint is binding in the sense that the optimal \( \tau_1 \) and/or \( \tau_2 \) exceeds \( \bar{\tau} \). The first effect of the tax ceiling is to tighten significantly the feasibility constraint derived earlier in (4). Debt repayment now depends on taxing authority as well as national wealth. The new constraint is that \( D_t \) must be less than or equal to the maximum level of tax revenues net of government investment. Government investment is \( I_t \) minus private investment, \( s(1-\tau_t)Q_t \). Thus,

\[
(4') \quad D_t (1+r) < \max_{\tau, I} \sum_{i=t}^t (1+r)^{-(i-t)} [\tau_i Q_t - I_t + s(1-\tau_t)Q_t] 
\]

It is more likely that (4') holds as a binding constraint than (4), since (4') does not imply that future consumption must equal zero when the constraint binds. Nonetheless, in the examples that follow, we do not consider the binding case. We focus rather on the binding constraint \( \tau = \bar{\tau} \), assuming \( D_t \) remains below its maximum level.

Since the optimal tax path tends to involve rising \( \tau \), a natural case to consider is one in which the tax constraint does not bind in period 1 while it does bind in period 2. We consider that case first. By the Kuhn-Tucker conditions for optimization we know that \( \partial U/\partial \tau_i > 0 \), with the strict inequality
holding when \( \tau_1 \) is binding at the constraint \( \tau \). Thus, when \( \tau_1 < \tau \) and \( \tau_2 = \tau \), \( \partial U / \partial \tau_1 = 0 \) and \( \partial U / \partial \tau_2 > 0 \). What are the implications of the tax constraint? From (11)(b), \( u_2 < -V_D / (1+\delta) \). The second-period marginal utility of consumption is "too low." The government would like to raise second-period taxes, reduce \( C_2 \) and thereby raise \( u_2 \), but it has already taxed to the limit. Let \( \theta \) be such that \( u_2(1+\theta) = -V_D / (1+\delta) \) (clearly \( \theta \) must be positive). Substituting this relationship and (11)(d) into (11)(c) we see that:

\[
(12) \quad F_K(K_2) = (r + d) \cdot \gamma
\]

\[
\gamma = (1+\theta) / \left[ (1+\theta) - \theta (1-s)(1-\tau) \right] > 1
\]

We have the key result:

Under a regime of constrained tax levies, the marginal product of capital should no longer be equated with the world market cost of capital but rather should be kept higher, to reflect a lower shadow value of second-period output.

The utility value of second-period output may be measured by \( u_2 \). Since this is no longer equated to \( V_D / (1+\delta) \), second-period returns to investment should be given a weight less than \( 1.0 \) in project analysis. By following the standard rule \( F_K(K_2) = r + d \), the country is led to over-borrow, with the result that social welfare is reduced.

Let us consider a graphic case of this issue that follows the analysis in Kharas [9]. Suppose that the government only cares about growth, in the sense that \( u(C_1) \equiv u(C_2) \equiv 0 \), and \( V(K_3, D_3) = F(K_3) - (1+r)D_3 \). The government is
trying to maximize third-period national income (net of international indebtedness). If $\tau_t$ is not constrained, $\tau_1$ and $\tau_2$ should be set at 1.0, with government revenue plus net foreign borrowing used to equate $F_k(K_2)$ with $r + d$, according to the classic policy prescription.

Now suppose that $\tau_1, \tau_2 < \tau < 1$. Since consumption has no weight in utility, it is optimal to set taxes at their maximum rate: $\tau_1 = \tau_2 = \tau$. Then, $D_3$ and $K_3$ are given by:

$$D_3 = (1+r)[I_1 - (s(1-\tau) + \tau)F(K_1)] + [I_2 - (s(1-\tau) + \tau)F(K_1(1-d) + I_1)]$$

$$K_3 = K_1(1-d)^2 + I_1(1-d) + I_2$$

By setting $\partial V/\partial I_1 = \partial V/\partial I_2 = 0$, we find the optimal investment policy. After some algebra, we find:

$$F_k(K_2) = \frac{(r+d)}{s(1-\tau) + \tau} > r + d; F_k(K_3) = (r + d)$$

Once again, the country should not invest enough to equate $F_k(K_2)$ and $r + d$.

To understand (13), consider how a foreign-financed, $A$-change in $I_1$ affects third-period income (that is all that counts to the government!). $A_{I_1}$ raises $K_3$ by $(1-d)A_{I_1}$, and so $\Delta F(K_3)$ equals $F_k(K_3) \cdot (1-d)A_{I_1}$. Since $F_k(K_2)$ is chosen to equal $(1+r)$, $\Delta F(K_2) = (1+r)(1-d)A_{I_1}$. $A_{I_1}$ affects third-period debt in a number of ways: second-period taxes rise by $\tau F_k(K_2)A_{I_1}$; second-period savings rise by $s(1-\tau)F_k(K_2)A_{I_1}$; and second-period debt rises by $(1+r)A_{I_1}$. Thus, third-period debt rises by $(1+r)$ times $(1+r)A_{I_1} - \tau F_k(K_2)A_{I_1} - s(1-\tau)F_k(K_2)A_{I_1}$, or:
\[ \Delta(1+r)D_3 = (1+r)[(1+r)\Delta I_1 - \frac{\Delta F}{K_2}(1-r)F_K(K_2) + \Delta I_1 - s(1-r)F_K(K_2)] \]

At the optimum, \( \Delta F(K_2) \) is equated to \( \Delta(1+r)D_3 \). This condition immediately yields (13).

This model provides a powerful indictment against foreign borrowing, even for productive investment projects, if the domestic fiscal system is not equipped to handle rising debt-service ratios. Figure 1 illustrates how aggregate growth is slowed by excessive borrowing in a tax-constrained regime, for specific parameter values of the model. In the unconstrained regime, optimal borrowing is at \( D_2^* \); in the constrained case, with a low \( r \), the optimum is at \( D_2^{**} < D_2^* \); and in the constrained case with a high \( r \), the optimum is at \( D_2^{***} \).

IV. International Borrowing with Possible Debt Repudiation

So far, the creditors have only considered the feasibility of debt repayment in setting debt ceilings for the borrowing country. In practice, however, a loan must pass two hurdles to make it a reasonable bet: (1) it must be feasible for the loan to paid off; and (2) the borrower must have the incentive to pay off the loan when it comes due. In some cases it may be less onerous for a borrower to repudiate a debt obligation, and accept the penalties that may arise, than to undertake the task of paying off the debt. In this section we study how the creditors and borrowers operate when such a possibility exists.

A word concerning terminology is useful at this point. In technical terms, a default is any failure to respect the terms of a loan agreement. A default
Figure 1. Foreign Borrowing and Growth in a Tax-Constrained Regime
may occur because $D_t$ is mistakenly allowed to rise above productive wealth; or because the government is unable to levy the necessary taxes to service the debt; or, as we shall see in Section V, because the creditors panic and create a liquidity shortage for the borrowing country. In this section we analyze the possibility of defaults caused purely by the choice of the borrowing-country government in a situation in which debt repayment is feasible but perhaps unpleasant. For this type of default we follow Eaton and Gersovitz [6] in reserving the term "debt repudiation."

A. The Basic Model of Debt Repudiation

The key to modelling debt repudiation is an explicit assumption regarding its benefits and costs. The benefits are straightforward: the borrower saves the real value of the outstanding debt, which it no longer services. The costs are far more problematic, to judge from historical experience (see [15]). One aspect of the costs is a partial or complete inability to obtain new loans in the world capital markets, at least for some time after the repudiation occurs. Another aspect of the cost may be a direct seizure of the country’s overseas assets, including bank accounts, direct foreign investments, ships and aircraft. A third, and even more important cost, may be a dramatic decline in the country’s capacity to engage in trade, even if no net new borrowing is involved. Modern trade is built on a sophisticated system of revolving trade credits. Even if a country’s net debt is zero, its gross stocks of trade-related financial assets and liabilities are likely to be large. Because a borrower would have difficulty in arranging trade credits
after a repudiation, the mechanics of trade would be made onerous. Moreover, merchandise at ports ready to be dispatched to the debtor country could be subject to seizure by the creditors.

To introduce these elements, we assume that when a debt is repudiated the creditors retaliate by imposing two costs: in all future periods, the borrower's production is reduced, for given $K$ and $L$, by a fixed fraction $\lambda$; and second, the borrower is excluded from all further borrowing. Importantly, we assume that this retaliation yields neither costs nor benefits to the creditors (or that the costs and benefits cancel).

As an easy start, we begin with a two-period version of the international borrowing model (we simply drop $V(K_3,D_3)$). The tax considerations are ignored, so that we implicitly assume that domestic tax levies are not constrained. Loans are made to the sovereign borrower in period 1. If they are not repaid in period 2, the penalty is enforced and second-period output is reduced by $\lambda Q_2$. The borrower makes the repudiation decision in the second period (there is no way that it can pre-commit itself to a decision before the second period arrives). Since second-period utility is simply $u(C_2)$, the borrower compares consumption levels with and without repudiation. With repudiation, $C_2$ equals $Q_2 - \lambda Q_2 = (1-\lambda)Q_2$. (We denote this level as $C_2^R$.) With no repudiation, $C_2$ equals $Q_2 - (1+r_2)D_2$, which we denote $C_2^N$. The borrower defaults whenever $C_2^N$ exceeds $C_2^R$, and thus whenever $(1+r_2)D_2 < \lambda Q_2$. Note that the interest rate has a time-subscript; we can no longer assume a unique world interest rate for all loans, since creditors will now impose a risk premium to allow for default risk.

There are two choices with respect to the timing of loans. The level of
credit $D_2$ may be extended before or after the investment decision $I_1$ is made. We shall see shortly that it is a great advantage to the country to be able to choose $I_1$ before going to the capital markets, since $I_1$ may then be chosen to make the credit terms on a given loan more favorable, or to increase the total amount that the country can borrow. A more natural assumption, however, is that loans are arranged first and that the government then allocates them to consumption and investment. This is more natural because the government will generally have an incentive to renege on a promised level of $I_1$ once a loan is arranged, even if ex ante it would be better off to fix the $I_1$ initially. Thus, promises concerning $I_1$ will be unconvincing. We term the case in which $I_1$ is set first the "pre-commitment" equilibrium, and regard the other case as the "standard" assumption.

The trick to solving the borrowing problem under certainty is to calculate the loan supply ceiling $\tilde{D}_2$, beyond which the creditors will not make loans. As long as $D_2$ is less than or equal to $\tilde{D}_2$, the country will choose not to default. The loan will be safe, and the interest rate will equal the safe rate of interest, denoted by $\rho$. For $D_2 > \tilde{D}_2$, the country will default for any interest rate greater than or equal to $\rho$. No risk premium can compensate for the certainty of debt repudiation. All lending is cut off at the point $\tilde{D}_2$.

In order to find $\tilde{D}_2$, we first compute the country's investment choice as a function of $D_2$. For each $D_2$, we find the level of utility of the borrowing country for alternative values of $I_1$, and choose the optimal $I_1$ as a function of $D_2$. We thereby derive $I_1 = I_1(D_2)$. The borrowing ceiling $\tilde{D}_2$ is found as the point for which the default penalty, $\lambda F[K_1(1-d) + I_1(D_2)]$, just equals $(1+\rho)D_2$.

In other words:
(14) \[ \beta_2 = \frac{\lambda F[K_1(1-d) + I_1(\beta_2)]}{(1+\rho)} \]

It remains to calculate \( I_1(D_2) \). Note that the borrower defaults if and only if \( \lambda F[K_1(1-d) + I_1] > (1+\rho)D_2 \). Thus, for each \( D_2 \) there is a threshold \( \bar{I}_1 \), for which the country defaults if and only if \( I_1 < \bar{I}_1 \). To find the best investment policy for given \( D_2 \), the country makes two calculations: its best utility if it heads for default (i.e., with \( I_1 < \bar{I}_1 \)), or if it plans to repay its debt (i.e., with \( I_1 > \bar{I}_1 \)). It then picks the strategy which yields the higher utility. Thus:

\[ I_1(D_2) \text{ is given as the solution to } \max_{I_1} (U^R, U^N) \]

where

\[ U^R = \max_{I_1 < \bar{I}_1} U(C_1 + D_2 - I_1) + U[\lambda Q_2]/(1+\delta) \]

\[ U^N = \max_{I_1 > \bar{I}_1} U(C_1 + D_2 - I_1) + U(Q_2 -(1+\rho)D_2)/(1+\delta) \]

and \( Q_2 = F[K_1(1-d) + I_1] \)

Armed with the investment function, (14) may be solved for \( D_2 \).

Once \( D_2 \) is known, the borrower's problem is easily specified.

(16) \textbf{Borrower's Problem with Repudiation Risk}

\[ \max_{C_1, D_2} u(C_1) + u(C_2)/(1+\delta) \]

\[ I_1 = Q_1 - C_1 + D_2 \]

\[ K_2 = K_1(1-d) + I_1 \]

\[ D_2 < \bar{D}_2 \]
$$C_2 = Q_2 - (1-\rho)D_2; Q_2 = F(K_2)$$

The solution to this is readily given, as:

(17) \[
u_1(C_1) = u_2(C_2)(1+\rho+\gamma)/(1+\delta)
\]

$$F_{K_2} = (1+\rho+\gamma)$$

$$\gamma = 0 \text{ for } D < D$$

$$\gamma > 0 \text{ for } D = D$$

The interpretation is as follows. If the credit ceiling is not binding, we are back to the textbook model. Marginal utility of incomes are equated over time, with \(u_1/[u_2/(1+\delta)]\) equal to \((1+\rho)\). Investments are carried out to the point where the marginal product of capital equals the world interest rate. (Note that in the two-period model, this means \(F_{K_2} = 1 + \rho\), rather than \(F_{K_2} = r + d\) as in the three-period model.) When the credit ceiling binds, it is as if the domestic interest rate exceeds the world market rate. Less investments are undertaken, since the marginal product of capital must now equal the higher rate \(1 + \rho + \gamma\). \(u_1(C_1)\) rises relative to \(u_2(C_2)\), meaning that the consumption path is pushed into the future.

As \(\lambda\) (the default penalty) rises, \(D_2\) rises as well, and the credit is more easily obtained. Utility rises, and investment and \(C_1\) increase as well. Thus, in a world of certainty, borrowers prefer higher penalties for debt repudiation. The higher the penalty, the easier are the credit conditions.

Now let us modify the model by allowing the country to pre-commit to \(I_1\) before \(D_1\) is selected. The borrower's problem becomes:
(18) **Borrower’s Problem with Investment Pre-commitment**

\[
\begin{align*}
\max_{I_1, C_1} & \quad u(C_1) + u(C_2)/(1+\delta) \\
& \quad (1+\rho)D_2 < \lambda F[(1-d)K_1 + I_1] \\
Q_1 & = C_1 + I_1 - D_2 \\
C_2 & = Q_2 - (1+\rho)D_2 \\
Q_2 & = F(K_2)
\end{align*}
\]

Borrowers now set \( I_1 \) knowing that their choice **influences the size of loans** that they can hope to arrange, since creditors restrict \((1+\rho)D_2\) to be less than or equal to \(\lambda Q_2\). Implicitly there is a non-linear constraint on \(C_1\) and \(I_1\), such that \((1+\rho)(C_1 + I_1 - Q_1) < \lambda F[(1-d)K_1 + I_1]\). The solution to the pre-commitment problem is:

(19) \( u_1(C_1) = u_2(C_2)/(1+\delta) \cdot \left[ (1+\rho)(1+\theta(1+\delta)/u_2) \right] \)

\( F_K(K_2) = [(u_2+\theta)/u_2 + \lambda\theta](1+\rho) > (1+\rho) \)

\( \theta = 0 \quad \text{for} \quad D_2(1+\rho) > \lambda Q_2 \)

\( \theta > 0 \quad \text{for} \quad D_2(1+\rho) = \lambda Q_2 \)

Once again, when \( \theta = 0 \), we are back to an unconstrained optimum, with \( u_1/u_2 = (1+\rho)/(1+\delta) \) and \( F_K(K_2) = (1+\rho) \). When \( \theta > 0 \), we are again in a situation where \( F_K \) should be held **above** the world cost of capital, as should the ratio of \( u_1 \) to \( u_2/(1+\delta) \). A key point here, relative to the "standard case" in (17), is that \( F_K \) is now set **below** \( u_1/[u_2/(1+\delta)] \). That is because there are now **two** benefits of
investment: higher second-period income, and a relaxation of the borrowing constraint. The first motive for investment (second-period consumption) generally leads the planner to equate $F_K$ with $u_1/[u_2/(1+\delta)]$. The second consideration raises investment, and thus lowers $F_K$ relative to $u_1/[u_2/(1+\delta)]$.

In general, pre-commitment results in a higher level of $I_1$, greater debt, and higher utility. The utility level must be greater than or equal to utility in the "standard" case, since the policy-maker in (18) could choose to pre-commit $I_1$ at the equilibrium level in the standard solution. We see, then, that pre-committing one's country to a high investment profile is a method of enhancing credit-worthiness and raising social welfare. Of course, we should not lose sight of the previous section's conclusion that a weak public finance structure may militate against extensive foreign borrowing for investment purposes.

A linear model offers a vivid illustration of the effects of repudiation risk, and of investment pre-commitment. Let:

$$Q_1 = \delta$$

$$Q_2 = \delta + (1+\gamma)I_1 , \quad I < I$$

$$U = C_1 + C_2/(1+\delta)$$

$$\delta > \gamma > \rho$$

Thus, there we assume a quantity $I$ of investment projects with a rate of return $\gamma$ exceeding the world interest rate $\rho$. The rate of time discount $\delta$ is also assumed to be greater than the world interest rate. In the textbook model, the borrowing equilibrium involves $I_1 = I$ (all investment projects undertaken), with consumption shifted entirely to the first period, and no consumption in the
second (since $\delta > \rho$ with linear utility). In sum:

(21) The Textbook Case:

$$C_1 = \bar{Q} + \left[ \bar{Q} + (1+\gamma)\bar{I} \right]/(1+\rho)$$

$$I_1 = \bar{I}$$

$$C_2 = 0$$

$$D_2 = C_1 + I_1 - \bar{Q}$$

Now, we turn to the "standard" repudiation model. For any given $D_2$, $I_1$ will be chosen to equal zero, since $\delta > \gamma$. Therefore $Q_2 = \bar{Q}$, and the debt ceiling is given by $D_2 = \lambda \bar{Q}/(1+\rho)$. The complete solution is:

(22) The Standard Repudiation Case

$$C_1 = \bar{Q} + \lambda \bar{Q}(1+\rho)$$

$$I_1 = 0$$

$$C_2 = \bar{Q} - \lambda \bar{Q}$$

$$D_2 = \lambda \bar{Q}/(1+\rho)$$

Thus, the risk of repudiation reduces $D_2$, $I_1$, and $C_1$, and raises $C_2$.

Finally, we have the pre-commitment case. In this model, the borrowing country will choose to pre-commit to $I_1 = \bar{I}$ when $\gamma$ is close to $\delta$, and when $\delta$ is much greater than $\rho$. Specifically we find:

(23) The Pre-Commitment Repudiation Case
\[ C_1 = \bar{Q} + D_2 - I_1 \]
\[ I = 0 \text{ for } (\delta - \rho)(1+\gamma) < (\delta - \gamma)(1+\rho) \]
\[ I = \bar{I} \text{ for } (\delta - \rho)(1+\gamma) > (\delta - \gamma)(1+\rho) \]
\[ C_2 = Q_2 - \lambda Q_2 \]
\[ D_2 = \lambda Q_2/(1+\rho) \]

Thus, the pre-commitment case may be the same as the no-pre-commitment case, but might (and generally will) result in an equilibrium somewhere between the textbook model and the standard repudiation model. Pre-commitment allows greater borrowing, greater investment in profitable projects, and higher first-period consumption.

B. The Debt Repudiation Model Under Uncertainty

So far, an actual default never occurs in the model, though the threat of default has a profound effect on economic welfare and the nature of macroeconomic equilibrium. Once uncertainty is introduced into the model, debt repudiations will actually occur as random events. The presence of uncertainty has several effects. First, the loan supply schedule becomes upward sloping, rather than perfectly elastic up to a maximum debt level \( \bar{D} \). Second, and even more important, the incentive structure for macroeconomic management may become perverse, in ways soon to be described. A more complete treatment of debt repudiation under uncertainty may be found in [6] and [17]. Here, we will discuss some simple yet revealing examples.

Consider the linear model just described, but with \( I = 0 \) and \( Q_2 \) a random variable, equal to \( Q \) with probability \( \Pi \) and \( \theta Q \) (\( \theta < 1 \)) with probability \( (1-\Pi) \).
We assume \( \Pi > 0 \), which proves convenient below. Creditors are assumed to be risk-neutral, charging an interest rate \( r_2 \) that yields an expected rate of return equal to \( \rho \) (we relax the assumption of risk neutrality in the next section). Debtors are assumed to repudiate debt whenever \((1+r_2)D_2 > \lambda Q_2\), and to repay debt otherwise.

Let us specify the loan supply schedule. Let \( \alpha \) be the probability of a debt repudiation. The interest rate \( r_2 \) will be set so that \((1+r_2)(1-\alpha) = (1+\rho)\), assuming risk neutral creditors. By using the relations \((1+r_2)(1-\alpha) = (1+\rho)\) and \( \alpha = \Pr[(1+r_2)D_2 > \lambda Q_2] \) we may easily derive the following loan schedule.

\[
D_2 < D_2 = \frac{\lambda Q_2}{(1+\rho)}
\]

with interest rates:

\[
r = \rho \text{ for } D_2 < \theta_2 M_2
\]

\[
r = \frac{\rho + \Pi}{(1-M)} \text{ for } \theta_2 M_2 < D < D_2
\]

The supply schedule is shown in Figure 2, where we see an important point. Though the loan supply is upward sloping (in this case a step function), there is still a point \( D_2 \) above which higher risk-premia do not compensate for repudiation risk. Creditors will not extend loans beyond \( D_2 \), at any interest rate. Thus, even in more general models, there tend to be credit ceilings, rather than ever-higher risk premia, as a property of the loan supply schedule.

Now, suppose that the social welfare function is \( U = C_1 + C_2/(1+\delta) \), and the goal of the government is to maximize expected utility, \( E(U) \). Since we are ignoring investment, the only issue is how much to borrow, with \( C_1 = \bar{Q} + D_2 \), and
C² = \max (Q₂ - (1+r₂)D₂, (1-\lambda)Q₂). A little algebra yields the following rules:

(25) For \delta < \rho, the country lends \ D₂ = -Q₁

For \rho < \delta < [\theta(1-\Pi) + \Pi\rho(1-\theta)]/(\Pi-\theta), the country borrows

D₂ = \lambda\theta\delta/(1+\Pi) at interest rate \Pi, and with zero probability of debt repudiation.

For \rho < [\theta(1-\Pi) + \Pi(1-\theta)] < \delta, the country borrows \ D₂ = \tilde{D}, at interest rate \ r = (\rho+\Pi)/(1-\Pi), and with probability (1-\Pi) of debt repudiation.

Thus, the more "impatient" the country, i.e., the greater is \delta, the higher is the borrowing, which comes at a greater cost and a greater risk of default.

There is a simple, yet important lesson in (25). The probability of debt repudiation does not depend on \lambda but rather on comparisons of \delta and \rho. Higher penalties (\lambda) do not necessarily reduce the frequency of debt repudiation. In a more general model, a rise in \lambda might actually raise that frequency! The reason is that while higher \lambda makes default more costly, it also makes lenders willing to extend more credit. Thus, when \lambda rises both the costs and benefits of debt repudiation increase, and in the example, the probability of debt repudiation remains unchanged.

C. Debt Repudiation and Macroeconomic Incentives

A recent theme of financial economics is that the various claimants on a firm's income stream (e.g., the shareholders, bondholders, workers) have differing interests regarding the firm's policies because alternative policies
Figure 2. Loan Supply Schedule Under Repudiation Risk
affect the relative valuation of the different claims. Thus, the shareholders may urge policies that raise shareholder wealth at the expense of bondholder wealth, as described in [7], [13], and [18]. Or coalitions of the shareholders and banks may engage in policies at the expense of bondholders, especially in the context of bankruptcy actions (see [4]). A notable feature of these examples is that the firm may pursue inefficient policies that reduce the overall value of the firm, because some groups will benefit even though other groups will be hurt more.

A related theme is that all groups are generally left better off, \textit{ex ante}, if the firm can be constrained from pursuing inefficient policies. As an example, consider the case of risky investments. It is well known that shareholders can sometimes devalue the claims of bondholders on the firm by selecting overly risky investment projects. (A bond is an option on the firm's income stream; an increase in variance of an income stream reduces the value of the related option.) Since bondholders know this, \textit{ex ante}, they may charge a high risk premium in anticipation of the investment policy. This high risk premium reduces shareholder wealth while it allows the bondholder to earn the expected market rate of return. After the loan is made and the risky investment selected, the bond claim is reduced in value (relative to the hypothetical value if a safe investment is instead selected), but the initial risk premium has already compensated for that effect. If the shareholders could have somehow committed themselves to choose safe investments, the initial high risk premium could have been avoided, to their own advantage.

Several direct analogies can be made to macroeconomic behavior by the
borrowing country. Like a firm, the country also has various claimants on the income stream, including the government, domestic citizens, and international creditors. And like the firm, the country may be led to select inefficient policies to transfer income from the creditors to the "shareholders" (the government and domestic private sector). Generally, the country would like to forewear these policies _ex ante_ but may find it difficult to do so.

Let us consider an example in the linear stochastic framework, also involving the riskiness of investment projects. Suppose that there are two options for an investment project. Option A yields \( Q_2 = (1 + \gamma^A)I_1 \) with certainty. Option B yields \( Q_2 = (1 + \gamma^B)I_1 \) with probability \( \Pi \), and \( Q_2 = 0 \) with probability \( 1 - \Pi \). The yield on A is \( \gamma^A \), and the expected yield on B is \( \Pi(1 + \gamma^B) - 1 \). We will assume that \( \gamma^A \) is greater than \( \Pi(1 + \gamma^B) - 1 \). The government borrows to finance \( I_1 \), so that \( D_2 = I_1 \). Second-period consumption equals \( Q_2 - (1 + r_2)D_2 \) if there is no default, and \( (1 - \lambda)Q_2 \) if there is a default. Default occurs if and only if \( (1 + r_2)D_2 > \lambda Q_2 \). We also assume that \( (1 + r_2) \) is less than \( \lambda(1 + \gamma^A) \) and \( \lambda(1 + \gamma^B) \).

Now suppose that social welfare is simply the expected value of \( C_2 \), \( E(C_2) \). Which investment project should be selected? If the investment project is selected _before_ the loan is made, it is easy to check that option A is preferred. Under option A, the borrowing rate is simply \( \rho \), since there is zero probability of default. \( E(C_2^A) \) is simply \( (1 + \gamma^A)I_1 - (1 + \rho)I_1 \). Under option B, default occurs with the "bad" outcome, which occurs with probability \( (1 - \Pi) \). The borrowing rate equates \( \Pi(1 + r_2) \) with \( (1 + \rho) \). \( EC_2^B = \Pi(1 + \gamma^B)I_1 - (1 + \rho)I_1 \). By assumption, \( \Pi(1 + \gamma^B) < (1 + \gamma^A) \), so \( EC_2^B < EC_2^A \).

If the loan is made _before_ the investment project is selected, the country
may well choose B instead of A! To see the problem, suppose that the banks lend \( D_2 = I_1 \) at rate \( \rho \) in anticipation that option A will be selected. At rate \( \rho \), \( EC^A_2 = (\gamma^A - \rho)I_1 \), as before, while \( EC^B_2 \) now equals \( \Pi(\gamma^B - \rho)I_1 \). As long as \( 1 + \gamma^A > \Pi(1 + \gamma^B) > (1 + \gamma^A) - (1 + \rho)(1 - \Pi) \), \( EC^B_2 > EC^A_2 \). Since the creditors will recognize the country’s ex post incentive to choose B, the loan will in fact carry the interest rate \( r_2 \) such that \( \Pi(1 + r_2) = 1 + \rho \), and project B will again be preferred.

The problem here is as follows. When the investment project is chosen first, the borrower must consider the effect of his investment choice on the terms of the loan. When the loan is arranged first, the borrower does not consider this effect, since the terms of the loan are already set. The borrower would like to promise the creditor that the safe project will be pursued, but such a promise will look unconvincing given the incentive to renge on it once the loan agreement is set.

There are several other areas of behavior in which timing and default risk interact to produce bad macroeconomic choices. The earlier discussion of investment pre-commitment can be thought of precisely in these terms. From an ex ante point of view it is best for the country to choose a high level of investment, because high investment relaxes credit ceilings. However, once a loan package is arranged, the country prefers to raise first-period consumption at the expense of investment. Since creditors understand this, they will tend to discount initial promises of high investment plans, and indeed they will be right.

A similar phenomenon occurs when countries borrow with long-term debt.
When a country owes long-term debt, each new amount of borrowing tends to reduce the expected value of the original debt by making eventual its repudiation more likely. In many cases, the borrowing country would like to be able to promise a potential long-term creditor that it will not over-borrow once the long-term debt is arranged. Such a promise would reduce the risk premium on the long-term debt. However, there will generally be strong incentives, *ex post*, to do precisely the contrary. The result is, in general, that long-term debt will command a high risk premium and that, as expected, over-borrowing will occur.

Market participants search for ways to reduce these deleterious incentives. It may be the case that countries can establish reputations for maintaining macroeconomic policies in line with announced plans. There is a growing economics literature on establishing a reputation (e.g., [11]) that may well give some insights in this direction. Other specific actions, such as relying on short-term borrowing rather than long-term borrowing, may reduce some of the incentive problems. In domestic capital markets, and to a much smaller extent in international lending, bond covenants can be used to pre-commit the borrower to a future line of action. Smith and Warner [18] provide an excellent survey of such covenants, which indicate how they help to enforce an efficient borrowing and investment plan by corporate borrowers. For example, covenants often directly restrict dividend payments, which may be tantamount to requiring the shareholders to invest rather than "consume" their loans. Other types of provisions include restrictions on new borrowing, maintenance of the firm's existing assets, financial disclosure requirements, and restrictions on merger activity. Such provisions are typically unenforceable with foreign sovereign
borrowers and thus are not part of the most syndicated loan agreements.

V. Collective Action Problems in Syndicated Lending

The problems with the textbook model have so far all involved the borrower (whether in its tax system, or its incentive to repudiate debt). Problems at least as serious can arise on the creditor side, especially when credit market imperfections interact with the problems already identified. We have so far treated the creditor side as a "black box" operation, extending loans that yield the appropriate rate of return. In fact, on a typical loan, the creditor side tends to be composed of a large number of financial intermediaries who join together as a syndicate on an ad hoc basis. While the syndication process helps in risk diversification, it leads to several other problems of great significance.

Our amended model of the supply side posits a very large number of banks, each with an upward sloping schedule of loan supplies to the borrowing country. Let $E_{i}^{L}$ be the expected return on a loan made to the country by bank $i$ (the expectation takes account of risks of debt repudiation, insolvency, etc.). Let $L_{i}$ be the amount of the bank's total lending to the country, and $B_{i}$ be total bank capital. The main hypothesis is that the inverse loan supply schedule takes the form:

\begin{equation}
E_{i}^{L} = \rho + F\left(\frac{L_{i}}{B_{i}}\right) \quad f(0) = 0
f'(\cdot) > 0
\end{equation}

According to (26), banks demand an expected return close to $\rho$ when the country
loans constitute a small fraction of bank capital but demand a higher expected return as the loans constitute a growing fraction of bank capital. There may even be a cut-off point \( \bar{l} \) such that \( L_i/B_i < \bar{l} \). According to American banking law, for example, no bank may allocate more than 10 percent of bank capital to a single borrower. While there are many technical ways around such ceilings, these ceilings seem also to be self-imposed by banks.

The loan supply schedule in (26) provides a powerful incentive for loan diversification among a large number of creditors. If a single bank makes a loan of size \( l = L_i/B_i \), the country pays expected return \( E(r^L_i) = \rho + f(l) \). If the same loan is equally divided among \( N \) banks, the rate is \( E(r^P_i) = \rho + f(l/N) < \rho + f(l) \). Indeed, as \( N \to \infty \), the cost of borrowing approaches \( \rho \), and the loan supply schedule mimics that of the risk-neutral creditors of earlier models.

A loan supply schedule like (26) can be derived from the utility maximization of risk-averse banks, via a CAPM approach. Suppose, for example, that expected utility (to bank managers) of the bank portfolio is given by \( E(r^P_i) - \beta \sigma^2(r^P_i) \), where \( r^P_i \) is the return on the overall bank portfolio. It is simple to show that the expected return on the country loan must satisfy \( E(r^L_i) = \rho = \text{cov}(r^L_i, r^P_i) \). In general, as \( L_i/B_i \) increases, the covariance of \( r^L_i \) and \( r^P_i \) also rises, so that the bank requires a higher risk premium on its loans.

The assumption of risk-averse banks requires some justification. Standard finance theory holds that under certain conditions firms should ignore own-risk in choosing policies, since shareholders can diversify whatever specific risk
the firm undertakes. However, these conditions are extremely restrictive and more general conditions lead to risk-averse behavior. For example, serial costs of bankruptcy mean that firm valuation will depend on own risk. Also with imperfect monitoring of managerial decisions by the shareholders, firm decisions will tend to involve risk-averse behavior. And in the context of commercial banks, bank regulators impose portfolio requirements limiting risk. Such policies are necessary in light of the moral hazards engendered by official deposit insurance programs in the U.S., Western Europe, and Japan.

A. The Possibility of Panics

The same drive towards diversification also gives rise to the possibility of liquidity crisis or panic in international lending. Consider the following example. Suppose that a country has debt obligations due in the first period equal to \((1 + r_1) D_1\); current income \(Q_1\) less than \((1 + r_1) D_1\); and stochastic second-period income given by \(Q_2 = 0\) with probability \(P\) and \(Q_2 = 0\) with probability \(1 - P\). We assume further that the existing debt \(D_1\) is held by a large number of creditors who do not act as a unified group engaged in negotiation with the debtor country (more on this below), and whose individual holdings are too small to give rise to any individual bargaining with the debtor country.

There are a number of possibilities at hand. Consider first the standard assumption that all debt is highly diversified among the creditors, so that \(E(r_i) = \rho\) for all banks \(i\), and for all periods. Since the country cannot feasibly repay \((1 + r_1)D_1\) out of current income, it must borrow \(D_2 > (1 + r_1)D_1 - Q_1\). New loans must be at least at interest rate \(r_2\) such that \(P(1 + r_2) = 1 + \rho\) in order to satisfy the new creditors. Thus, \((1 + r_2) = (1 + \rho)/P\).
Now,

(27) If \( \bar{Q} < (1 + \rho) \left[ (1 + r_1)D_1 - Q_1 \right] / \Pi \)

the country will be forced into default on grounds of negative net
worth (insolvency);

If \( \lambda \bar{Q} < (1 + \rho) \left[ (1 + r_1)D_1 - Q_1 \right] / \Pi \), the country will be forced into
default on the grounds that repudiation risk precludes new loans,
in spite of overall solvency;

If \( \lambda \bar{Q} > (1 + \rho) \left[ (1 + r_1)D_1 - Q_1 \right] / \Pi \), the country can obtain new loans on a
competitive basis.

A liquidity crisis, as distinct from a solvency or repudiation crisis,
occurs when the last condition in (27) is satisfied but the country is
nevertheless unable to obtain the requisite loans. The surprising result is
that such an outcome can occur in competitive equilibrium. Assume that all
banks lend according to (26), and take as given the amount of loans extended by
the other banks. If all banks suddenly expect all other banks to stop
lending to the country, it will be rational for certain parameter values for
each bank to stop lending as well on the basis of that expectation, with the
result that it becomes self-confirming. To see this, consider a single bank
planning its loans under the assumption of no loans from other banks. The bank
knows that unless it loans \( D_2 = (1 + r_1)D_1 - Q_1 \), the country will default.
According to (26), a loan of size \( D_2 \) requires an expected rate of return
\( E(r_2^L) = \rho + f(D_2/B) \). Since the debtor can only hope to be paid off with
probability \( \Pi \) in the second period, the interest rate \( r_2 \) on the loan must at
least be such that \( N(1+r_2) = 1 + \rho + f(D_2/B) \).

A liquidity crisis can arise when the following condition holds:

(28) \[ (1 + \rho + f(D_2/B))D_2/N > \lambda Q > (1+\rho)D_2/N \]

where \( D_2 = (1+r_1)D_1 - Q_1 \)

In this case, it is safe for many banks to lend \( D_2 \), but not safe for any single bank to lend at all. The risk premium required for single-bank lending is enough to push the country to debt repudiation in the next period, with

\[ (1+r_2)D_2 > \lambda Q \].

If all banks believe that all other banks will stop lending, that outcome will in therefore occur.

Note that a panic requires a fairly high level of initial debt. Let \( r_2^L(D_2) \) be the loan supply schedule for a single debt. Then, a panic requires

\[ (1 + r_2^L(D_2))D_2 > \lambda Q \].

Clearly, there exists a \( D_2^{\min} \), for which \( D_2 < D_2^{\min} \) a panic is impossible and \( D_2 > D_2^{\min} \) a panic can occur. Since \( D_2 \) equals \( (1+r_1)D_1 - Q_1 \), there is similarly a condition for \( (1+r_1)D_1 \). Figure 3 shows the individual banks loan supply schedule; for \( D_2 > D_2^{\min} \), \( r_2^L \) is such that

\[ (1+r_2^L)D_2 > Q \].

It is precisely because panics occur only at high levels of debt that they are so difficult to distinguish from other forms of default. In every true liquidity crisis, it will seem to some observers that the problem really lies with the risk of debt repudiation or insolvency, rather than with the supply of credit.

A good historical candidate for a liquidity crisis is cited in [15] (also
see [10]). In mid-1930, the international bond markets "shut down" to the developing countries. While about $411 million of non-Canadian debt was floated from January to July, 1930, only $5 million was floated during the rest of the year. It was six months after the bond market collapsed that the first country (Bolivia) defaulted. Remarkably, much of the panic can be traced to a single week when a military coup in Brazil caused bond prices to plummet. Some representative bond prices for that week are shown in the table below.

<table>
<thead>
<tr>
<th>Country</th>
<th>Closing Price October 3</th>
<th>Closing Price October 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina 6s</td>
<td>95</td>
<td>54 1/8</td>
</tr>
<tr>
<td>Bolivia 8s</td>
<td>76 3/4</td>
<td>66</td>
</tr>
<tr>
<td>Brazil 6 1/2s</td>
<td>73</td>
<td>48 1/2</td>
</tr>
<tr>
<td>Chile 6s</td>
<td>83 1/2</td>
<td>71</td>
</tr>
<tr>
<td>Columbia 6s</td>
<td>66 5/8</td>
<td>58</td>
</tr>
<tr>
<td>Uruguay</td>
<td>101</td>
<td>88</td>
</tr>
</tbody>
</table>


B. Other Collective Action Problems

A financial panic is not the only case in which cooperation among creditors can improve the efficiency of the international loan markets. In general, loan
contracts can be made more efficient if the creditors are able to act collectively under certain contingencies. For example, an efficient loan contract may require that existing creditors re-schedule the debts of the borrower at below-market rates in order to avoid a default. If the creditors are widely dispersed, without an institutional structure to enforce collective action, it may be impossible to arrange the re-scheduling. No creditor alone has incentive to re-schedule at below-market rates, while each creditor has an incentive to free ride (by demanding full repayment of his claims) if the others re-schedule. This seems to be the historical experience with international lending in the bond market.

When creditors are better organized, as in syndicated bank loans, there is at least the possibility that mutually advantageous collective action can be arranged. We will cite evidence below, however, that even in bank syndicates significant free-rider problems remain.

Let us illustrate the role for collective action by considering a simple numerical example in the three-period borrowing model. We will assume that $Q_1$ is fixed at $\bar{Q}$, while $Q_2$ and $Q_3$ may equal $\bar{Q}$ (probability $1/2$) or zero (probability $1/2$). $Q_2$ and $Q_3$ are assumed to be independently distributed. We assume that the country defaults only when it runs out of cash (i.e. it never voluntarily repudiates its debt). The safe interest rate is 0.10, and creditors are risk-neutral.

The borrower is interested in maximizing $C_1$, which equals $Q_1 + D_2$, so that equivalently, he is interested in maximizing first-period loans. Now, we compare two types of loan contracts. In the first case, the loan $D_2$ is made by
a large number of creditors without organizational cohesion. If $Q_2 = 0$, the borrower must try to borrow again on market terms, in order to repay $(1+r_2)D_2$.

If he cannot attract new loans, he defaults. In the second case, a cohesive syndicate is formed. If $Q_2 = 0$, the borrower must first try to borrow again on market terms. If he cannot attract new loans, the syndicate guarantees that the borrower can roll-over the credit at the initial interest rate.

The borrower will be able to borrow more, at better terms, under the syndicate arrangement. First, we calculate the loan supply schedule in the case without syndication. It turns out that for $0 < D_2 < \bar{Q}/2.42$, the lenders can lend at the safe interest rate of 10 percent. If $Q_2$ turns out to equal zero, borrowers will be able to attract the necessary new loans to pay off the existing debt. The new loan $D_3$ will equal $1.1D_2$, with interest rate $r_3 = 1.2 = (1.1/2) - 1$. For $\bar{Q}/2.42 < D_2 < \bar{Q}/2.2$, the interest rate $r_2$ is 1.2. If $Q_2 = 0$, the country will default, since it will not be able to attract new loans at market terms. $\bar{Q}/2.2$ is the loan ceiling.

In the syndicate, loans $0 < D_2 < \bar{Q}/2.42$ will also be made at the 10 percent rate, for the reasons above. For $\bar{Q}/2.42 < D_2 < \bar{Q}/1.85$, the interest rate will equal 0.36. The syndicate will not loan $D_2 > \bar{Q}/1.85$. To find the syndicate interest rate (0.36), note that the syndicated loan is an asset that pays $(1+r_2)$ in the second period with probability $\Pi$, and $(1+r_2)^2$ in the third period with probability $(1-\Pi) \cdot \Pi$ (i.e., with prob $(Q_2 = 0, Q_3 = \bar{Q})$). Thus, $(1+p) = \Pi \cdot (1+r_2) + \Pi \cdot (1-\Pi) \cdot (1+r_2)^2/(1+p)$, with $\Pi = 1/2$ and $p = 0.1$. The solution to this equation is $r_2 = 0.36$. The loan ceiling is found by solving $(1.36)^2D_2 = Q$.

Thus, we find that a syndicate will loan more $(\bar{Q}/1.85$ versus $\bar{Q}/2.2$) at a better interest rate (0.36 versus 1.2). But the syndicate depends on the ability to make a second-period loans at 36 percent when new creditors would
Figure 3. The Loan Supply Schedule and Financial Panic in the Loan Market
offer nothing at that rate. The risk for the syndicate is that an informal bargain among creditors to re-lend may break down in the second period, leading to a default rather than a re-scheduling. Recent experience with commercial bank re-scheduling points up the tensions with re-lending. Banks with small participation in a loan agreement try to escape with their credit intact, relying on the larger banks to forestall default. Figure 4 on the next page shows this vividly with respect to the current Brazilian re-scheduling package, where the U.S. regional banks are contributing new short-term credits at systematically far less than their existing shares in Brazilian debt.

Other types of collective action that a syndicate might engage in include: monitoring existing loans for compliance; enforcement of loan agreements; and retaliatory actions in the event of non-compliance. In each of these cases, a free-rider problem potentially exists, with resulting inefficiencies in loan supply. Cline [5] has reported the difficulty of the commercial bank syndicates to Peru in exercising all three of these functions, and their ultimate resort to the IMF as a way to escape from this problem:

In March 1976 the Bermudez government sought a large balance-of-payments loan from major U.S. banks, without a prior IMF standby agreement. The government felt that agreeing to IMF conditions would be unacceptable politically, although in its discussion with the banks, the government proposed a program very much like that which might have secured IMF support. Partly out of fear of a more leftist coup if Bermudez lost power, the banks eventually agreed, but only after the regime demonstrated willingness to take unpopular stabilization measures....

The program called for an initial $200 million in loans with a second $200 million to follow after several months, contingent on government adherence to the policy purchase. Signed only by the end of 1976, the package soon demonstrated the frailty of such direct intervention by banks; for reasons of data availability, technical capacity, and political sensitivity, it proved impossible for the banks to enforce their lending conditions, and adverse publicity for the intervention (plus its ineffectiveness) caused the leading banks to resolve that they would not become entangled in similar packages in the future but would rely on the
Figure 4

INTERBANK LOANS TO BRAZIL
(in millions $)

Big Banks

Smaller Banks

"Fair Share"  Actual loans as of May 17

200                                               50
150                                               100
100                                               50

Manufacturers  Citibank  Morgan  Bank of  National  Wells  First
Hanover  Guaranty  Bankers  New York  Bank of  Fargo  Interstate
Trust    Trust    Trust  New York  Detroit  Bank  Bancorp

Source: Fortune Magazine, July 11, 1983
IMF as the monitoring authority. (pp. 305-306)

VI. Conclusions and Extensions

This paper has suggested three areas in which the standard model of international borrowing requires major revision. In the typical planning or project-analysis framework, too little attention is generally given to the domestic budgetary implications of foreign borrowing. We have seen that in an economy with limitations on the government's ability to raise revenues, official foreign borrowing is often less attractive than standard calculations might suggest. Since the shadow cost of tax revenues is greater than 1.0, claims on tax revenues (like amortization payments on foreign borrowing) must also given a cost greater than 1.0.

The second area of focus was on the effects of default risk, particularly repudiation risk, on the nature of international loans. We found two phenomena of great importance: the loan supply schedule becomes upward sloping, with eventual credit rationing, where the position of the supply schedule depends on the penalties of default; and various incentives are introduced that lead to inefficient borrowing and investment behavior by the debtor country.

The final area of concern involved the supply side of the credit market. Liquidity crises were shown to result from the risk averse behavior of individual lenders. Thus, we identified situations where the credit markets in the aggregate would be willing to lend but in which each individual bank withholds loans because of the fear that other banks will do so as well. No individual bank will break the credit squeeze, but a coalition of banks acting
cooperatively might well be able to restore the flow of lending. In general, the ability to form binding coalitions among creditors allows for more sophisticated, and ultimately more efficient loan packages. A particularly important example is a loan agreement which guarantees a debt-rescheduling at below-market rates in the event of an output shortfall (or other real income loss) in the borrowing country.

Further research should explore the role of international organizations (such as the IMF and IBRD) in light of the market imperfections we have identified. Without doubt the IMF has an important function to play with respect to each of the three categories of market breakdown. Its standard country consultations already involve the review of the domestic financial structure of borrowing countries, where a constant theme has been the intricate connection of budget financing and foreign borrowing.

The more novel Fund involvement in recent years has come in the second and third categories. To an increasing extent, IMF conditionality involves the application of loan covenants to borrowing packages, for the purposes analyzed in Section IV. In this regard Fund programs might do better to emphasize the distribution of spending between investment and consumption rather than the overall level of spending in the borrowing country.

The most visible role of the Fund in recent months has come in the third category, wherein Fund cajolery has been useful in overcoming a classic panic in the international loan markets. One of the great conceptual weaknesses in that role, however, has come from the inability of analysts to distinguish convincingly among the three forms of default risk discussed in Section V.
(solvency risk, repudiation risk, panic risk). Better, empirically oriented dynamic models of international lending are still needed to identify the middle-term prospects for developing country debts.
References


