Four Essays in International Economics

Qingyuan Du

Submitted in partial fulfillment of the

Requirements for the degree

of Doctor of Philosophy

in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2011

Abstract

Four Essays in International Economics

Qingyuan Du

Developing countries are more likely than developed countries to pursue a fixed exchange rate regime, yet this pattern is not directly predicted by conventional theories of optimum currency area. The first Chapter proposes a new theory of exchange rate regime choice—different from the policy maker's credibility argument in the "fear of floating" theory—that stresses the roles of both stage of economic development and labor market frictions. In general, for a typical developing country with low labor productivity and high labor market frictions, a fixed exchange rate regime would yield a higher level of welfare than a floating regime as the former generates more export revenue. The opposite is true for a country with high labor productivity or a more flexible labor market. We provide empirical evidence that is consistent with the key predictions of the theory.

The second chapter investigates some new hypothesis of the high savings rates and current account surpluses in countries like China. Large savings and current account surpluses by China and other countries are said to be a contributor to the global current account imbalances. In this chapter, we propose a theory of excess savings based on a major transformation in many of these societies, namely, a steady increase in the surplus of men relative to women. We construct an OLG model with two sexes and a desire to marry. We show conditions under which an intensified competition in the marriage market can induce men to raise their savings rate, and produce a rise in both the aggregate savings and current account surplus. This effect is economically significant if the biological desire to have a partner of the opposite sex is strong. A calibration of the model suggests that this factor could generate economically significant current account responses, or between one third and a half of the actual current account imbalances observed in the data.

In the third chapter, we analyze how the social structual change–the rise in the sex ratios–may affect the real exchange rate. We find that a rise in the sex ratio, in theory, can simultaneously generate a decline in the real exchange rate (RER) and a rise in the current account surplus. We demonstrate this logic through both a savings channel and an effective labor supply channel. In this model, a low RER is not a cause of the current account surplus, nor is it a consequence of currency manipulations. Empirically, those economies with a high sex ratio tend to have a low real exchange rate, beyond what can be explained by the Balassa-Samuelson effect, financial underdevelopment, dependence ratio, and exchange rate regime classifications. Once these factors are accounted for, the Chinese real exchange rate is estimated to be undervalued by only a relatively trivial amount.

The last chapter studies the entrepreneurial activities in countries like China who have experienced a severe rise in the pre-marriage age cohort's sex ratio. In this chapter, we present a theoretical model and find that, when the sex ratio is large, a rise in the sex ratio will induce men to take the risk and pursue the high returns, which leads to an increase in the entrepreneurial activities in the economy. In an open economy model with two sectors, a risky sector and a risk free sector, we show that a country with a very skewed sex ratio is more likely to have a comparative advantage in the risky sectors. We provide empirical evidence that is consistent with the theoretical predictions.

Contents

List of Figures

List of Tables

Acknowledgments

This dissertation would not have been possible without the support and help of countless amazing people, whom I would like to thank.

First of all, I was extremely fortunate to have the immense support of advisors. I am truly grateful to Shang-Jin Wei, who provided the perfect mix of letting me develop my own ideas, making sure I would not lose sight of the big picture questions, and pushing me as hard as possible. He worked extremely hard on turning me into a true researcher and introducing me to the world of economists. I am deeply indebted to Ronald Findlay who always gave me invaluable advice in my research. I have also benefitted tremendously from Martin Uribe. Without the support from him, none of the work in Chapter 1 would have been possible. I truly hope that one day I will be able to pay back some of this debt as an advisor to my own students.

My research benefited enormously from comments and discussions at conferences and seminars. These are too many people to name individually, but I am very grateful to seminar/conference participants at the Columbia University, Monash University, University of Hawaii–Manoa, and the 2010 NBER Conference on the Chinese Economy.

The Economics Department at Columbia University provided a wonderful environment for developing and executing research ideas. From the first day on, some of the best teachers I had were my fellow graduate students. In particular I want to thank Jae Bin Ahn, Li An, Ryan Chahrour, Chin-Han Chiang, Yinghua He, Ran Huo, Tao Li and Xing Xia who not only supported me with their sharp economic insights, but also by becoming my closest friends.

I had many productive interactions with a very large number of faculty members during my years at Columbia. No matter how different their own research interests, I always found an open ear and insightful thoughts regarding my ideas. While I received some sort of support from more faculty members than I could possibly name here, particular thanks go to Amit Khandelwal, Serena Ng, Bruce Preston, Stephanie Schmitt-Grohe, Jon Steinsson and Mike Woodford.

This work has also been supported with various resources by the Program for Economic Research at Columbia, and Columbia GSAS. I am, of course, responsible for any mistakes.

To my wife and my parents

Introduction

This dissertation consists of four chapters on international economics. The first chapter studies the role of the development stage of a country and wage rigidity in the determination of the optimal exchange rate regime choices. The second and the third chapters analyze how a major social transformations in countries like China, namely a rise in the surplus of men relative to women, will affect countries' current accounts and real exchange rates. The last chapter studies how the rise in the sex ratios in those economies will also affect the entrepreneurial activities and countries' comparative advantages.

While developing countries are more likely than developed countries to pursue a fixed exchange rate regime, this pattern is not directly predicted by conventional theories of optimal currency area. Calvo and Reinhart (2002), to our knowledge, is the only paper that interprets this pattern. They find that, even in the best of times, when countries retain voluntary access to international capital markets, lack of credibility in developing countries will give rise to "fear of floating." However, the authors consider no direct role of a country's stage of development in choosing the exchange rate regime, and they do not provide a welfare-based model that can be used to analyze the policy choice.

The aim of the first chapter is to fill this important void in the literature. Without assuming any "lack of credibility" problems, we provide a theoretical model to analyze the role of countries' stage of development in the choice of exchange rate regime. We show that, in general, for a typical developing country with low labor productivity and high labor-market frictions, a fixed exchange rate regime yields a higher level of welfare than would a floating regime, as the former generates more export revenue. The opposite is true for a country with high labor productivity or a more flexible labor market.

The related literature on the choice of exchange rate regime traces back to the "optimum currency area" theory (OCA thereafter), originally associated with Mundell (1961), McKinnon (1963) and Kenen (1969). This approach to a fixed-versus-flexible dilemma weighs the trade and welfare gains from a stable exchange rate vis-à-vis the rest of the world (or, more precisely, the country's main trade partners) against the benefits of exchange rate flexibility as a means of adjusting to shocks in the presence of nominal rigidities. The traditional OCA theory delivers four key criteria for a successful currency union: i) labor mobility across the region; ii) openness with capital mobility and price and wage flexibility across the region; iii) a risk-sharing system such as an automatic fiscal transfer mechanism to redistribute money to areas/sectors that have been adversely affected by the first two characteristics; and iv) participant countries with similar business cycles. The theory has been most frequently applied in recent years to the euro. However, a major shortcoming of OCA theory is that it is very hard to find a group of countries in the real world that meet all the criteria. Even in the case of the Eurozone, member countries do not fulfill all of the requirements: First, while capital is quite mobile in the Eurozone, labor mobility is relatively low. Second, wages in the Eurozone are quite rigid. In fact, Babecký et al. (2009) find that the incidence of downward nominal wage rigidity is substantial in Europe. One criticism of the OCA theory is that the only area that has optimal conditions for a single currency is one that already has a single currency.

More recently, Devereux and Engel, in a series of papers, investigate the choice of exchange rate regime—fixed vs. floating—in a dynamic intertemporal general equilibrium framework with price stickiness. In Devereux and Engel (2003), they find that in the presence of local currency pricing (LCP), a fixed exchange rate regime is preferred. Intuitively, when the policy maker chooses an optimal monetary rule under LCP, she does not attempt to use monetary policy to alter the relative price of home to foreign goods because movements in exchange rates do not affect the prices consumers face. The exchange rate is not a part of the optimal monetary policy. Gertler et al. (2001) study fixed and flexible exchange rates in an economy with a financial accelerator. They find that the financial-accelerator effects are much stronger under fixed rates than under flexible rates (with a suitably managed monetary policy). Roughly speaking, an exchange rate peg forces the central bank to adjust the interest rate in a manner that enhances financial distress. Bacchetta and Wincoop (2000) develop a simple general-equilibrium framework to study the effect of the exchange rate regime on trade and welfare. They find that, in general, both trade and welfare can be higher under either exchange rate regime, depending on preferences and the monetary-policy rules. There is no one-to-one relationship between the levels of trade and welfare across exchange rate regimes. Using a similar framework as in Bacchetta and Wincoop (2000), Bergin and Lin (2008) find that currency unions and direct exchange rate pegs can raise trade significantly. They do not analyze the welfare effect of switching from a flexible to a fixed exchange rate regime. However, if trade is important to the country, their work may be interpreted as favoring a fixed exchange rate regime.

These papers reach no consensus on which regime—fixed or flexible—is better; nor do they analyze why country income seems to be so tightly linked to exchange rate regime choice. As mentioned above, Calvo and Reinhart (2002) appears to be the only paper that draws a link, albeit indirect, between country income and the choice of exchange rate regime. They argue that developing countries' central banks usually lack credibility and, therefore, prefer a fixed exchange rate regime since exchange rate stabilization provides the economy with a clear-cut nominal anchor.

Unlike Calvo and Reinhart's (2002) "fear of floating" theory, we stress the roles of both stage of economic development and labor market frictions under a New Keynesian framework. Our model expands on Devereux and Engel's (2003) framework with some important features. First, we assume physical capital in production. Capital is used in both the domestic and export sectors; hence, the capital rental rate (the factor price of capital) will be affected by both domestic and foreign demand shocks. Under a fixed exchange rate regime, domestic demand always moves in the same direction as foreign demand; under a flexible exchange rate regime, however, domestic demand is independent of changes in foreign demand. In response to foreign shocks, a flexible exchange rate regime, by adjusting the nominal exchange rate to stabilize the domestic economy, can generate a less volatile capital market than can a fixed exchange rate regime.

Second, we assume price stickiness and wage rigidity in the model. Monopolistically competitive firms will set prices one period before sales happen. In the presence of local currency pricing, firms in the export sector will set optimal prices by taking the future nominal exchange rate and marginal costs into account. Fluctuations in both the marginal cost and nominal exchange rate will influence the firms' choices. If labor is the main factor in production—i.e., production in the export sector is labor-intensive—under the assumption of wage rigidity, a part of the marginal cost can be predicted. Firms will be concerned primarily with the volatility of the nominal exchange rate when pre-setting optimal prices. However, if capital makes the main contribution to production—i.e., production in the export sector is capital-intensive—both future capital market conditions and nominal exchange rate movements are important to firms' optimal decisions. The nominal exchange rate then plays distinct roles in optimal price setting, depending on the kind of production technology a country uses: If a country has a capital-intensive export sector, firms may care more about the capital rental rate risk, and a floating exchange rate may help to reduce such a risk; if a country has a labor-intensive export sector, firms will be more concerned about nominal exchange rate fluctuations, and a fixed exchange rate may reduce that volatility.

Finally, we assume a CES production function with the substitution elasticity between capital and labor below one, instead of the Cobb-Douglas production function. Antras (2004) estimates the substitution elasticity between capital and labor in a very similar CES production and obtains the result that, allowing for labor-augmenting technology, the elasticity is significantly below one. The lower-than-one elasticity of substitution means that capital and labor are gross complements rather than substitutes: when labor-augmenting productivity is high, firms will input more capital. In developed countries, labor is usually more efficient than in developing countries, and, hence, production in developed countries is more capital-intensive. Taking all three assumptions into account, it is not hard to see that the choice of exchange rate regime may have different implications for developed and developing countries.

Our model can successfully interpret the pattern that developing countries are more likely than developed countries to pursue a fixed exchange rate regime. A fixed exchange rate regime can generate higher exports, as well as higher consumption good prices, for developing countries than can a flexible exchange rate regime. If wages are sufficiently rigid, switching to a fixed exchange rate regime will have a positive net effect on welfare. The result reverses for developed countries. A flexible exchange rate regime creates higher exports and lower consumption good prices than does a fixed exchange rate regime and, therefore, is preferred. Interestingly, our result is opposite to the OCA theory: a country with a higher degree of wage rigidity tends to choose a fixed exchange rate regime, while OCA theory predicts that wage flexibility should be one important criterion for a successful currency union.

The result also differs from Devereux and Engel (2003). The main reason is that we assume capital in production while they do not. Intuitively, if firms use capital intensively in production, under a flexible exchange rate regime, the home country can adjust the nominal exchange rate to reduce the impact of foreign shocks on the capital market and, thus, stabilize the home capital rental rate. The existence of a capital market in the model works to provide an advantage to a flexible exchange rate regime. As the domestic sector becomes more important, the desire for a flexible exchange rate regime is stronger.

Our model is useful in analyzing the optimal exchange rate regime choices for countries in different development stages. For a country with low labor productivity—i.e., at an earlier stage in its development—it is optimal to have a fixed exchange rate regime if wages are sufficiently rigid. As the country reaches a higher development stage (labor productivity rises), it would be optimal to switch from a fixed to a flexible exchange rate regime.

We document empirical evidence supporting the mechanism described in the first chapter. We first examine how *de facto* exchange rate regime choices (Reinhart and Rogoff, 2004) influence the export growth in 24 manufacturing sectors. We find that: i) sectoral export growth tends to be higher in more capital-intensive sectors under a flexible exchange rate regime than that under a fixed exchange rate regime; and ii) sectoral exports tend to grow faster under a fixed exchange rate regime if wages are more rigid. Both results are consistent with our theoretical prediction. In a separate regression, we test how countries' income and wage rigidity affect the choice of exchange rate regime. Consistent with the theory, the result shows that countries with higher initial incomes or lower wage rigidities are more likely to choose more flexible exchange rates. To further examine the validity of our empirical analysis, we run 2SLS regressions to deal with endogeneity issues and use Levy-Yeyati and Sturzenegger (2003) *de facto* exchange rate regime classifications to do robustness checks. Those results are similar to our benchmark regressions.

In summary, the main results in the first chapter show that: for a typical developing country with low labor productivity and high labor market frictions, a fixed exchange rate regime would yield a higher level of welfare than a floating regime as the former generates more export revenue. The opposite is true for a country with high labor productivity or a more flexible labor market.

Chapters 2 and 3 are joint works with Professor Shang-Jin Wei of Columbia University Business School. We focus on the current accounts and real exchange rates in a special group of countries.

High savings rates in excess of domestic investment rates in many Eastern and South-eastern Asian countries have produced a massive current account surplus as a share of GDP, and are said to be a major contributor to the global current account imbalances, to the unusually low long-term interest rates, and possibly to the onset of the 2008-2009 global financial crisis. As to theories of savings behavior, the existing literature has highlighted the roles of life-cycle considerations (Modigliani, 1970), precautionary savings (Kimball, 1990), habit formation (Carroll, Overland, and Weil, 2008), culture (Belton and Uwaifo Oyelere, 2008), and financial under-development (Caballero, Farhi, and Gourinchas, 2008; Ju and Wei, 2006, 2008 and 2010; Mendoza, Quadrini and Rios-Rull, 2007). The aim of Chapter 2 is to propose an alternative theory that gives prominence to a major, albeit insufficiently recognized by macroeconomists, social transformation in many economies, namely an increasing gap in the numbers of men and women in the marriage market. The basic thesis is that as competition intensifies in the marriage market, men or parents with sons raise their savings rates with the hope of improving their relative standing in the marriage market. Because the biological desire to have a partner of the opposite sex is strong, this effect is quantitatively important enough to reveal itself in the aggregate savings rate and the current account balance.

A direct source of the idea for the theory is an empirical paper by Wei and Zhang (2009), which studies household savings behavior in China. They provide both cross-regional and cross-household evidence that is consistent with the notion that a worsening prospect for men in the marriage market has motivated them and their parents to raise their savings rates substantially. They call this the "competitive saving motive." Chinese household savings as a share of disposable income rose from 16% in 1990 to 30% in 2007. Wei and Zhang suggest that the rise in the sex ratio imbalance could account for half the total increase in the savings rate. Because their paper does not have a formal theory, there is a need to construct a model to see if the hypothesis can work in a general equilibrium, and whether a calibration of the model can produce an effect whose magnitude is economically significant.

Chapter 2 aims to fill these important voids. The core part of the chapter is to analyze theoretically whether and how a sex ratio imbalance will influence the economy-wide savings rate and the current account. We construct a simple overlapping generations (OLG) model with two sexes and a desire to marry. To focus on the macroeconomic implications of sex ratio imbalances, we intentionally shut down channels such as the usual precautionary savings motive, habit formation, culture, and financial development. Because it is an OLG model, there are still life-cycle considerations, which, however, do not lead to current account imbalances on their own.

Under reasonable conditions, we show that men respond to a rise in the sex ratio by raising their savings rates. Moreover, the increment in their savings is always enough to offset any decrease in women's savings. As a result, the aggregate savings rises with the sex ratio. We also discuss a number of extensions that aim to allow for additional realism: (a) incorporate parental savings for children, (c) introduce intra-household bargaining, and (c) consider an OLG structure in which each generation lives for 50 periods and makes savings decisions in multiple periods. In each case, under reasonably general conditions, both the aggregate savings rate and current account rise in response to a rise in the sex ratio.

To check if the model can deliver an effect that is economically significant, we go to quantitative calibrations. In a more realistic case when allowing intra-household bargaining, for a small open economy, as the sex ratio rises from 1 to 1.15, the economy-wide savings rate and the current account will rise by more than 6%. We also consider the case of two large economies, whose relative sizes and income levels are calibrated to mimic China and the United States. The synthetic United States is assumed to always have a balanced sex ratio, while the synthetic China experiences a rise in the sex ratio from 1 (balanced) to 1.5 (very unbalanced). The rise in China's sex ratio produces a rise in its current account surplus, and a corresponding rise in the current account deficit for the United States. The magnitudes of the current account imbalances in the simulations (about 6.1% of GDP for China and -2.0% of GDP for the United States) are such that they are around one-half of the actual current account imbalances observed in the data. While the sex ratio imbalance is not the sole reason for the global current account imbalances in recent years, it could be one of the significant, and yet thus far unrecognized, factors.

A desire to enhance one's prospects in the marriage market through a higher level of wealth could be a motive for savings even in countries with a balanced sex ratio. But such a motive is not as easy to detect when the competition is modest. When the sex ratio gets out of balance, obtaining a marriage partner becomes much less assured. A host of behaviors that are motivated by a desire to succeed in the marriage market may become magnified. But sex ratio imbalances so far have not been investigated by macroeconomists. This may be a serious omission. A sex ratio imbalance at birth and in the marriage age cohort is a common demographic feature in many economies, especially in East, South, and Southeast Asia, such as Korea, India, Vietnam, Singapore, Taiwan and Hong Kong, in addition to

China. In many economies, parents have a preference for a son over a daughter. This used to lead to large families, not necessarily an unbalanced sex ratio. However, in the last three decades, as the technology to detect the gender of a fetus (Ultrasound B) has become less expensive and more widely available, many more parents engage in selective abortions in favor of a son, resulting in an increasing relative surplus of men. The spread of technology started in the early 1980s and accelerated quickly afterwards. 1985 was the first year in which half of the county-level hospitals in China had acquired at least one Ultrasound B machine. By early 1990s, all county-level hospitals had at least one such machine (Ebenstein, Li, and Meng, 2010). The strict family planning policy in China, introduced in the early 1980s, has induced Chinese parents to engage in sex-selective abortions more aggressively than their counterparts in other countries. The sex ratio at birth in China rose from 106 boys per hundred girls in 1980 to 122 boys per hundred girls in 1997 (see Wei and Zhang, 2009, for more detail). It may not be a coincidence that the Chinese current account surplus started to garner international attention around 2002 just when the first cohort born after the implementation of the strict family planning policy was entering the marriage market.

Throughout the model, we assume an exogenous sex ratio. While the sex ratio is endogenous in the long-run as parental preference evolves, the assumption of an exogeous sex ratio can be defended on two grounds. First, the technology that enables the rapid rise in the sex ratio has only become inexpensive and widely accessible in developing countries within the last 25 years or so. As a result, it is reasonable to think that the rising sex ratio affects only the relatively young cohorts' savings decisions, but not those who have passed half of their working careers. Second, data suggests that if the preference for son has a mean-reverting property, it must be a very slow-moving process. Almost all countries that have a skewed sex ratio today have exhibited a gradual climb over the last decade or two. Korea is the only economy whose sex ratio appears to have started to revert back from a very skewed level. This suggests that a systematic reversal of the sex ratio is unlikely to happen in most economies in the short run.

To see if the theoretical predicion has any support in the data, we check if a country's private sector savings rate (defined as current account minus government savings, divided by GDP) is systematically linked to its sex ratio. After controlling for the effects on the savings rate from income, the share of working age people in the population (i.e., a proxy for the life cycle theory), the ratio of private bank credit to GDP (a proxy for financial development), and social security expenditure as a share of GDP (a proxy for the precautionary savings motive), we find that a rise in the sex ratio from a balanced level to 1.15 (the current sex ratio for the pre-marital age cohort in China) is associated with a higher current account (excluding government savings) by over 10% of GDP.

In Chapter 3, we explore neglected implications of the sex ratio imbalance for the real exchange rate. Real exchange rate undervaluation due to currency manipulation is a frequent topic in international economic policy discussions. Two commonly used criteria by researchers and international financial institutions for judging undervaluations are deviations from the purchasing power parity (PPP) and large and persistent current account surpluses. The goal of this chapter is to demonstrate that a rise in the sex ratio can generate both phenomena. In other words, a low real exchange rate need not be the cause of a current account surplus. (Given a current account surplus, foreign exchange reserve accumulation could be a passive outcome of a country's capital account controls, rather than exchange rate interventions. In other words, if a country has no capital controls, e.g., Japan, a current account surplus shows up as an addition to its private sector's holding of foreign assets. With capital controls, which typically require compulsory surrender of foreign exchange earnings by firms or households, a current account surplus has to be converted into additional holding of foreign exchange reserves by the official sector.)

We highlight two channels through which a sex ratio imbalance could lead to an appearance of currency undervaluation. The first is a savings channel. If an economy experiences a shock that raises its savings rate, then the real exchange rate often falls. To see this, we recognize that a rise in the savings rate implies a reduction in the demand for both tradable and non-tradable goods. Since the price of the tradable good is tied down by the world market, this translates into a reduction in the relative price of the nontradable good, and hence a decline in the value of the real exchange rate (a departure from the PPP). The effect can be persistent if there are frictions that impede the reallocation of factors between the tradable and nontradable sectors.

The second theoretical channel works through effective labor supply. A rise in the sex ratio can also motivate men to cut down leisure and increase labor supply. This leads to an increase in the economy-wide effective labor supply. If the nontradable sector is more labor intensive than the tradable sector, this generates a Rybzinsky-like effect, leading to an expansion of the nontradable sector at the expense of the tradable sector. The increase in the supply of nontradable good leads to an additional decline in the relative price of nontradable and a further decline in the value of the RER. There is evidence from China that the effective labor supply is indeed larger in regions with a higher sex ratio (Wei and Zhang, 2010).

Putting the two channels together, a rise in the sex ratio generates a real exchange rate that appears too low relative to the purchasing power parity. Of course, if there are structural factors, other than a rise in the sex ratio, that have also triggered an increase in the aggregate savings rate (e.g., an increase in the government savings rate) or an increase in the effective labor supply (e.g., peculiar patterns of the rural-urban migration within a country), they would reinforce the mechanisms discussed in this chapter, causing the real exchange rate to fall further.

There are four bodies of work that are related to the second and third chapter. First, the literature on status goods, positional goods, and social norms (e.g., Cole, Mailath and Postlewaite, 1992, Corneo and Jeanne, 1999, Hopkins and Kornienko, 2004 and 2009) has offered many useful insights. One key point is that when wealth can improve one's social status (including improving one's standing in the marriage market), in addition to affording a greater amount of consumption goods, there is an extra incentive to save. This element is in our model as well. However, all existing theories on status goods feature a balanced sex ratio. Yet, an unbalanced sex ratio presents some non-trivial challenges. In particular, while a rise in the sex ratio is an unfavorable shock to men (or parents with sons), it is a favorable shock to women (or parents with daughters). Could the latter group strategically reduce their savings so as to completely offset whatever increments in savings men or parents with sons may have? In other words, the impact on aggregate savings appears ambiguous. Our model in Chapter 2 will address this question. In any case, the literature on status goods has no discernible impact in policy circles. For example, while there are voluminous documents produced by the International Monetary Fund or speeches by U.S. officials on China's high savings rate and large current account surplus, no single paper or speech thus far has pointed to a possible connection with its high sex ratio imbalance.

Second, the theoretical and empirical literature on the real exchange rate is too voluminous to summarize comprehensively here. Sarno and Taylor (2002) and Chinn (2011) provide recent surveys. A third related literature is the economics of family, which is also too vast to be summarized here comprehensively. One interesting insight from this literature is that a married couple's consumption has a partial public goods feature (Browning, Bourguignon and Chiappori, 1994; Donni, 2006). We make use of this feature in our model as well. None of the papers in this literature explores the general equilibrium implications for exchange rates from a change in the sex ratio. The fourth literature examines empirically the causes of a rise in the sex ratio. The key insight is that the proximate cause for the recent rise in the sex ratio imbalance is sex-selective abortions, which have been made increasingly possible by the spread of Ultrasound B machines. There are two deeper causes for the parental willingness to disproportionately abort female fetuses. The first is the parental preference for sons, which in part has to do with the relatively inferior economic status of women. When the economic status of women improves, sex-selective abortions appear to decline (Qian, 2008). The second is either something that leads parents to voluntarily have a lower fertility rate than earlier generations, or a government policy that limits the number of children a couple can have. In regions of China where the family planning policy is less strictly enforced, there is also less sex ratio imbalance (Wei and Zhang, 2009). Bhaskar (2011) examines parental sex selections and their welfare consequences.

In Chapter 4, we study the same social structural change as in Chapters 2 and 3, but we focus on the entrepreneurship and comparative advantage. A direct source of idea comes from Wei and Zhang (2010), which empirically studies the effect of a rise in the sex ratio on entrepreneurship and economic growth. They find that the imbalance may stimulate economic growth by inducing more entrepreneurship. Motivated by their empirical findings, we provide a theoretical framework to analyze the consequence of a sex ratio imbalance on entrepreneurial activities in this chapter. We first construct an overlapping generations model with two sexes and desire to marry in a closed economy. At the beginning of the first period, men can choose to be entrepreneurs (with a risky return) and workers (with a certain labor income) while all women are workers. They enter the marriage market at the start of the second period and marriages occur. When the sex ratio is close to a balanced level, only entrepreneurs who receive low income failed in the marriage market. As the sex ratio rises, more men will choose to be workers since being workers will obtain higher returns in the marriage market. However, when the sex ratio is large such that some male workers cannot get matched with women, an increase in the sex ratio raises the probability that a male worker will not get married, while it does not alter the expected utility of being an entrepreneur (to a first-order approximation). Then more men will respond to a higher sex ratio by becoming entrepreneurs.

The results may have important implications in an open economy model. Based on the same idea, the sex ratio imbalance can be an important sourse of the comparative advantage[1] in the risky sectors. We show in this chapter that, in an open economy with two sectors in an economy, a risky sector and a risk free sector, a country with a very skewed sex ratio (above some threshold) may have more entrepreneurs in the risky sector, which in turn may lead to a comparative advantage in the risky sector.

We also provide some empirical support to the theoretical predictions. In addition to reviewing the evidence in Wei and Zhang (2010), we run two types of regressions to test our theoretical predictions in this chapter. First, we find that, when the sex ratio exceeds some threshold, a rise in a country's sex ratio tends to lead to higher exports in more volatile

[1] We define the comparative advantage in a sector as the relative sectoral export position in this paper.

sectors. Second, in a nonlinear least squares test, we find that above some threshold, which is close to the biological mean of the cross-country sex ratios, a rise in the sex ratio will lead to an increase in a country's export volatility. Both findings are consistent with our theoretical predictions. Quantitatively, the effect of a rise in the sex ratio on a country's export volatility can be very significant. For instance, consider a country initially with a sex ratio around 1.05 (mean of the sex ratios in the world) and an export volatility 0.11 (mean of the export volatilities across countries), if the sex ratio rises from 1.05 to 1.13 (China's sex ratio in 2006), the export volatility will increase by almost 25%.

In summary, Chapter 4 provides a theoretical framework to analyze the impact of a rise in the sex ratio on entrepreneurial activities. A very skewed sex ratio may induce more entrepreneurship. In an open economy model, the sex ratio imbalance may be an important sourse of comparative advantages in the more risky sectors.

Chapter 1

To Fix or to Float? The Role of Development Stage and Wage Rigidity

While developing countries are more likely than developed countries to pursue a fixed exchange rate regime, this pattern is not directly predicted by conventional theories of optimal currency area. Calvo and Reinhart (2002), to our knowledge, is the only paper that interprets this pattern. They find that, even in the best of times, when countries retain voluntary access to international capital markets, lack of credibility in developing countries will give rise to "fear of floating." However, the authors consider no direct role of a country's stage of development in choosing the exchange rate regime, and they do not provide a welfare-based model that can be used to analyze the policy choice.

Our aim in this paper is to fill this important void in the literature. Without assuming any "lack of credibility" problems, we provide a theoretical model to analyze the role of countries' stage of development in the choice of exchange rate regime. We show that, in general, for a typical developing country with low labor productivity and high labor-market frictions, a fixed exchange rate regime yields a higher level of welfare than would a floating regime, as the former generates more export revenue. The opposite is true for a country with high labor productivity or a more flexible labor market.

The related literature on the choice of exchange rate regime traces back to the "optimum currency area" theory (OCA thereafter), originally associated with Mundell (1961), McKinnon (1963) and Kenen (1969). This approach to a fixed-versus-flexible dilemma weighs the trade and welfare gains from a stable exchange rate vis-à-vis the rest of the world (or, more precisely, the country's main trade partners) against the benefits of exchange rate flexibility as a means of adjusting to shocks in the presence of nominal rigidities. The traditional OCA theory delivers four key criteria for a successful currency union: i) labor mobility across the region; ii) openness with capital mobility and price and wage flexibility across the region; iii) a risk-sharing system such as an automatic fiscal transfer mechanism to redistribute money to areas/sectors that have been adversely affected by the first two characteristics; and iv) participant countries with similar business cycles. The theory has been most frequently applied in recent years to the euro. However, a major shortcoming of OCA theory is that it is very hard to find a group of countries in the real world that meet all the criteria. Even in the case of the Eurozone, member countries do not fulfill all of the requirements: First, while capital is quite mobile in the Eurozone, labor mobility is relatively low. Second, wages in the Eurozone are quite rigid. In fact, Babecký et al. (2009) find that the incidence of downward nominal wage rigidity is substantial in Europe. One criticism of the OCA theory is that the only area that has optimal conditions for a single currency is one that already has a single currency.

More recently, Devereux and Engel, in a series of papers, investigate the choice of exchange rate regime—fixed vs. floating—in a dynamic intertemporal general equilibrium framework with price stickiness. In Devereux and Engel (2003), they find that in the presence of local currency pricing (LCP), a fixed exchange rate regime is preferred. Intuitively, when the policy maker chooses an optimal monetary rule under LCP, she does not attempt to use monetary policy to alter the relative price of home to foreign goods because movements in exchange rates do not affect the prices consumers face. The exchange rate is not a part of the optimal monetary policy. Gertler et al. (2001) study fixed and flexible exchange rates in an economy with a financial accelerator. They find that the financial-accelerator effects are much stronger under fixed rates than under flexible rates (with a suitably managed monetary policy). Roughly speaking, an exchange rate peg forces the central bank to adjust the interest rate in a manner that enhances financial distress. Bacchetta and Wincoop (2000) develop a simple general-equilibrium framework to study the effect of the exchange rate regime on trade and welfare. They find that, in general, both trade and welfare can be higher under either exchange rate regime, depending on preferences and the monetary-policy rules. There is no one-to-one relationship between the levels of trade and welfare across exchange rate regimes. Using a similar framework as in Bacchetta and Wincoop (2000), Bergin and Lin (2008) find that currency unions and direct exchange rate pegs can raise trade significantly. They do not analyze the welfare effect of switching from a flexible to a fixed exchange rate regime. However, if trade is important to the country, their work may be interpreted as favoring a fixed exchange rate regime.

Schmitt-Grohe and Uribe (2011) investigates how costly it is to maintain a currency peg, in terms of unemployment and welfare, for an emerging economy facing large external shocks. Currency pegs will hinder the efficient adjustment of the economy to negative external shocks. The reason is that such shocks produce a contraction in aggregate demand that requires a decrease in the relative price of nontradables, that is, a real depreciation of the domestic currency. In turn, the required real depreciation may come about via a nominal devaluation of the domestic currency or via a fall in nominal prices or both. The currency peg rules out a devaluation. Thus, the only way the necessary real depreciation can occur is through a decline in the nominal price of nontradables. However, if nominal prices, especially factor prices, are downwardly rigid, the real depreciation will take place only slowly, causing recession and unemployment along the way. In a calibrated version of the model, Schmitt-Grohe and Uribe show that a large contraction can lead to a huge welfare loss (the median welfare cost of a currency peg is about 10 percent of lifetime consumption).

These papers reach no consensus on which regime—fixed or flexible—is better; nor do they analyze why country income seems to be so tightly linked to exchange rate regime choice. As mentioned above, Calvo and Reinhart (2002) appears to be the only paper that draws a link, albeit indirect, between country income and the choice of exchange rate regime. They argue that developing countries' central banks usually lack credibility and, therefore, prefer a fixed exchange rate regime since exchange rate stabilization provides the economy with a clear-cut nominal anchor.

Unlike Calvo and Reinhart's (2002) "fear of floating" theory, we stress the roles of both stage of economic development and labor market frictions under a New Keynesian framework. Our model expands on Devereux and Engel's (2003) framework with some important features. First, we assume physical capital in production. Capital is used in both the domestic and export sectors; hence, the capital rental rate (the factor price of capital) will be affected by both domestic and foreign demand shocks. Under a fixed exchange rate regime, domestic demand always moves in the same direction as foreign demand; under a flexible exchange rate regime, however, domestic demand is independent of changes in foreign demand. In response to foreign shocks, a flexible exchange rate regime, by adjusting the nominal exchange rate to stabilize the domestic economy, can generate a less volatile capital market than can a fixed exchange rate regime.

Second, we assume price stickiness and wage rigidity in the model. Monopolistically competitive firms will set prices one period before sales happen. In the presence of local currency pricing, firms in the export sector will set optimal prices by taking the future nominal exchange rate and marginal costs into account. Fluctuations in both the marginal cost and nominal exchange rate will influence the firms' choices. If labor is the main factor in production—i.e., production in the export sector is labor-intensive—under the assumption of wage rigidity, a part of the marginal cost can be predicted. Firms will be concerned primarily with the volatility of the nominal exchange rate when pre-setting optimal prices. However, if capital makes the main contribution to production—i.e., production in the export sector is capital-intensive—both future capital market conditions and nominal exchange rate movements are important to firms' optimal decisions. The nominal exchange rate then plays distinct roles in optimal price setting, depending on the kind of production technology a country uses: If a country has a capital-intensive export sector, firms may care more about the capital rental rate risk, and a floating exchange rate may help to reduce such a risk; if a country has a labor-intensive export sector, firms will be more concerned about nominal exchange rate fluctuations, and a fixed exchange rate may reduce that volatility.

Finally, we assume a CES production function with the substitution elasticity between capital and labor below one, instead of the Cobb-Douglas production function. Antras (2004) estimates the substitution elasticity between capital and labor in a very similar CES production and obtains the result that, allowing for labor-augmenting technology, the elasticity is significantly below one. The lower-than-one elasticity of substitution means that capital and labor are gross complements rather than substitutes: when labor-augmenting productivity is high, firms will input more capital. In developed countries, labor is usually more efficient than in developing countries, and, hence, production in developed countries is more capital-intensive. Taking all three assumptions into account, it is not hard to see that the choice of exchange rate regime may have different implications for developed and developing countries.

Our model can successfully interpret the pattern that developing countries are more likely than developed countries to pursue a fixed exchange rate regime. A fixed exchange rate regime can generate higher exports, as well as higher consumption good prices, for developing countries than can a flexible exchange rate regime. If wages are sufficiently rigid, switching to a fixed exchange rate regime will have a positive net effect on welfare. The result reverses for developed countries. A flexible exchange rate regime creates higher exports and lower consumption good prices than does a fixed exchange rate regime and, therefore, is preferred. Interestingly, our result is opposite to the OCA theory: a country with a higher degree of wage rigidity tends to choose a fixed exchange rate regime, while OCA theory predicts that wage flexibility should be one important criterion for a successful currency union.

The result differs from Devereux and Engel (2003). The main reason is that we assume capital in production while they do not. Intuitively, if firms use capital intensively in production, under a flexible exchange rate regime, the home country can adjust the nominal exchange rate to reduce the impact of foreign shocks on the capital market and, thus, stabilize the home capital rental rate. The existence of a capital market in the model works to provide an advantage to a flexible exchange rate regime. As the domestic sector becomes more important, the desire for a flexible exchange rate regime is stronger.

Our framework also differs from Schmitt-Grohe and Uribe (2011). One important departure of this paper from Schmitt-Grohe and Uribe (2011) is that we focus on the *ex ante* optimal policies while they study the *ex post* exchange rate policy. Schmitt-Grohe and Uribe (2011) focus on analyzing how a small open economy deviates from the steady state after a negative external shock. The steady states under the two exchange rate regimes in their paper are the same and hence both flexible and fixed exchange rate regimes will yield the same the welfare before the shock. The *ex post* best policy will be the optimal policy. However, our paper focuses on the difference in the *ex ante* equilibrium under the two exchange rate regimes. We allow firms choosing their own prices, and since firms have different expections about future under two exchange rate regimes, they will set different prices which in turn leads to different *ex ante* equilibria under the two regimes. The goal of this paper is to compare the expected social welfare under different *ex ante* equilibria and find the optimal exchange rate policy.

Our model is useful in analyzing the optimal exchange rate regime choices for countries in different development stages. For a country with low labor productivity—i.e., at an earlier stage in its development—it is optimal to have a fixed exchange rate regime if wages are sufficiently rigid. As the country reaches a higher development stage (labor productivity rises), it would be optimal to switch from a fixed to a flexible exchange rate regime.

We document empirical evidence supporting the mechanism described in this paper. We first examine how *de facto* exchange rate regime choices (Reinhart and Rogoff, 2004) influence the export growth in 24 manufacturing sectors. We find that: i) sectoral export growth tends to be higher in more capital-intensive sectors under a flexible exchange rate regime than that under a fixed exchange rate regime; and ii) sectoral exports tend to grow faster under a fixed exchange rate regime if wages are more rigid. Both results are consistent with our theoretical prediction. In a separate regression, we test how countries' income and wage rigidity affect the choice of exchange rate regime. Consistent with the theory, the result shows that countries with higher initial incomes or lower wage rigidities are more likely to choose more flexible exchange rates. To further examine the validity of our empirical analysis, we run 2SLS regressions to deal with endogeneity issues and use Levy-Yeyati and Sturzenegger (2003) *de facto* exchange rate regime classifications to do robustness checks. Those results are similar to our benchmark regressions.

The rest of the paper is organized as following: In section 2, we present the model and solve it up to the second order. Section 3 provides empirical facts that support our theory. Section 4 concludes and suggests future research directions.

1.1 Model

1.1.1 Households

We assume that there are two countries in the world: home and foreign. The households in each country have the same preferences. As in Devereux and Engel (2003) and Corsetti and Pesenti (2005), the representative household in the home country will maximize the optimization problem

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \chi \ln \left(\frac{M_t}{P_t} \right) - \kappa L_t \right]$$

where C_t and L_t are the consumption of the final good and supply of labor. M_t/P_t is the real money balance in period t. β is the discount factor. The intertemporal budget constraint for the household is

$$\begin{aligned} & C_t + \frac{P_{inv,t}}{P_t} I_t + \frac{B_t}{P_t} + \frac{S_t B_t^*}{P_t} + \frac{M_t}{P_t} \\ \leq \; & \frac{W_t}{P_t} L_t + r_t K_t + (1 + i_{t-1}) B_{t-1} + (1 + i_{t-1}^*) S_t B_{t-1}^* + \frac{\Pi_t}{P_t} + \frac{M_{t-1} + T_t}{P_t} \end{aligned}$$

where W_t and r_t are the nominal wage rate and the real capital rental rate, respectively, in period t. Π_t is the aggregate profit in period t. P_t and $P_{inv,t}$ are the prices for the final consumption good and the capital good, respectively. K_t is the capital stock held by the households at the beginning of period t. B_t and B_t^* are home-currency denominated and foreign-currency denominated bonds, respectively.

To facilitate an analytical solution, we assume complete capital depreciation. Relaxing this assumption will not change any of the qualitative results.

$$K_{t+1} = I_t$$

The optimal conditions for the representative household in the home country are

$$(1+i_t)^{-1} = E_t\left[Q_{t,t+1}\right] \tag{1.1}$$

$$(1+i_t^*)^{-1} = E_t\left[Q_{t,t+1}\frac{S_{t+1}}{S_t}\right] \tag{1.2}$$

$$\frac{M_t}{P_t} = \frac{\chi C_t}{1-E_t\left[Q_{t,t+1}\right]} \tag{1.3}$$

and

$$E_t\left[\beta\frac{C_t}{C_{t+1}}\frac{P_t}{P_{inv,t}}r_{t+1}\right] = 1 \tag{1.4}$$

where $Q_{t,t+1} = \frac{\beta P_t C_t}{P_{t+1}C_{t+1}}$ is the stochastic discount factor.

Households in the foreign country will have similar optimal conditions. If we assume a complete financial market, then the nominal exchange rate is determined by

$$S_t = \frac{P_t C_t}{P_t^* C_t^*} \tag{1.5}$$

as in the standard literature.

The final consumption good consists of a tradable good and a non-tradable good:

$$C_t = \frac{C_{Tt}^{\gamma} C_{Nt}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

where the tradable good bundle can be written as an index over the home produced tradable good and foreign produced tradable good.

$$C_{Tt} = \left[\omega^{\frac{1}{\psi}} C_{Ht}^{\frac{\psi-1}{\psi}} + (1-\omega)^{\frac{1}{\psi}} C_{Ft}^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}$$

where ψ is the constant elasticity of substitution between the home produced tradable good and foreign produced tradable good. We assume $\psi > 1$. We assume that both countries have the same tradable good basket and parameter ω indicates the expenditure share of the foreign country's goods in the consumption basket of households in either country. C_{Ht} and C_{Ft}, in turn, are indices of home produced tradable good and foreign produced tradable good, respectively.

$$\begin{aligned} C_{Ht} &= \left[\omega^{-\frac{1}{\theta_H}} \int_0^{\omega} C(h,j)^{\frac{\theta_H-1}{\theta_H}} dj\right]^{\frac{\theta_H}{\theta_H-1}} \\ C_{Ft} &= \left[(1-\omega)^{-\frac{1}{\theta_F}} \int_{\omega}^{1} C(f,j)^{\frac{\theta_F-1}{\theta_F}} dj\right]^{\frac{\theta_F}{\theta_F-1}} \end{aligned}$$

The elasticity of substitution between any two home produced tradable goods is θ_H, and the elastiticity of substitution between any two foreign produced tradable goods is θ_F. We assume θ_H and θ_F are both greater than one.

Similarly, for the nontradable good, we have

$$C_{Nt} = \left[\int C(n,j)^{\frac{\theta_N-1}{\theta_N}} dj\right]^{\frac{\theta_N}{\theta_N-1}}$$

where θ_N is the constant elasticity of subsitution between nontradable goods in the home country.

We assume that ω is small (close to zero)—i.e., the home country is a small open economy. As in Obstfeld and Rogoff (1995), for simplicity, we assume that only tradable goods can be transformed into capital goods.[1]

[1] This assumption greatly simplifies the calculation. However, relaxing it will not change the qualitative

1.1.2 Government

The government alters the money supply with direct transfers. The government budget constraint (in per capita terms) is

$$M_t = M_{t-1} + T_t$$

1.1.3 Firms

Firms in each country will produce a tradable good and a non-tradable good. The production of the tradable goods is:

$$y_{Ht} = \left[(1-\alpha_H)^{\frac{1}{\eta_H}} (A_t L_{Ht})^{\frac{\eta_H - 1}{\eta_H}} + \alpha_H^{\frac{1}{\eta_H}} K_{Ht}^{\frac{\eta_H - 1}{\eta_H}} \right]^{\frac{\eta_H}{\eta_H - 1}}$$

where A_t is the labor productivity in the home country. η_H is the constant elasticity of substitution between capital and labor, which is a central parameter in the economic theory. Models investigating the sources of economic growth and the determinants of the aggregate distribution of income have been found to deliver substantially different implications depending on the value of the elasticity of substitution. Antràs (2004) assumes a very similar production function as in our model and obtains the result that this elasticity of subsitution is significantly below one. Therefore, we assume $\eta_H < 1$ in our paper.

The non-tradable good sector has a similar production function

$$y_{Nt} = \left[(1-\alpha_N)^{\frac{1}{\eta_N}} (A_t L_{Nt})^{\frac{\eta_N - 1}{\eta_N}} + \alpha_N^{\frac{1}{\eta_N}} K_{Nt}^{\frac{\eta_N - 1}{\eta_N}} \right]^{\frac{\eta_N}{\eta_N - 1}}$$

where $\eta_N < 1$. Since there is no direct evidence to the contrary, we assume $\eta_H = \eta_N = \eta$. As in Obstfeld and Rogoff (1995), we also assume that $\alpha_N \leq \alpha_H$.

All the markets are monopolistically competitive. We assume price stickiness and local currency pricing—i.e., firms set their own prices one period before the sales happen, and

results.

export prices must be denominated in the foreign currency.

The optimization problem for a representative firm in the non-tradable sector is

$$\max \ E_{t-1}\left[Q_{t-1,t}\left(p_{Nt}-MC_{Nt}\right)y_{Nt}\right]$$

where

$$y_{Nt}=\left(\frac{p_{Nt}}{P_{Nt}}\right)^{-\theta_N}C_{Nt}$$

is the demand function facing each individual firm.

Since all non-tradable good producers will set the same optimal price

$$P_{Nt}=\frac{\theta_N}{\theta_N-1}\frac{E_{t-1}\left[Q_{t-1,t}MC_{Nt}P_tC_t\right]}{E_{t-1}\left[Q_{t-1,t}P_tC_t\right]} \tag{1.6}$$

then

$$P_{Nt}=\frac{\theta_N}{\theta_N-1}E_{t-1}\left[MC_{Nt}\right]$$

where

$$MC_{Nt}=\left[(1-\alpha_N)\left(\frac{W_t}{A_t}\right)^{1-\eta}+\alpha_N\left(r_tP_t\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

1.1.4 Tradable good sector

Due to the assumption of a small open economy (ω is close to zero), almost all tradable goods produced by the home country will be exported abroad. Then the optimization problem for a representative firm in the tradable good sector is approximately as following:

$$\max \ E_{t-1}\left[Q_{t-1,t}\left(S_tp^*_{Ht}-MC_{Ht}\right)y_{Ht}\right]$$

where p^*_{Ht} is the price denominated in foreign currency and MC_{Ht} is the marginal cost in the tradable good sector. And

$$y_{Ht}=\omega\left(\frac{p^*_{Ht}}{P^*_{Ht}}\right)^{-\theta_H}\left(\frac{P^*_{Ht}}{P^*_{Tt}}\right)^{-\psi}\left(C^*_{Tt}+I^*_t\right)$$

is the demand function facing each individual firm.

Due to the symmetry, all firms in the tradable good sector set the same optimal price

$$P_{Ht}^{*}=\frac{\theta_H}{\theta_H-1}\frac{E_{t-1}\left[Q_{t-1,t}MC_H y_{Ht}\right]}{E_{t-1}\left[Q_{t-1,t}S_t y_{Ht}\right]} \tag{1.7}$$

For simplicity, we assume that $\theta_H=\theta_N=\theta$. According to the production function, we have

$$MC_{Ht}=\left[(1-\alpha_H)\left(\frac{W_t}{A_t}\right)^{1-\eta}+\alpha_H\left(r_tP_t\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

In period t, the capital market clears

$$K_t=\alpha_H\left(\frac{r_tP_t}{MC_{Ht}}\right)^{-\eta}\left(\frac{P_{Ht}^{*}}{P_{Tt}^{*}}\right)^{-\psi}\left(C_{Tt}^{*}+I_t^{*}\right)+\alpha_N\left(\frac{r_tP_t}{MC_{Nt}}\right)^{-\eta}\frac{(1-\gamma)P_tC_t}{P_{Nt}} \tag{1.8}$$

and the total employment is

$$L_t=(1-\alpha_H)\left(\frac{W_t/A_t}{MC_H}\right)^{-\eta}\left(\frac{P_{Ht}^{*}}{P_{Tt}^{*}}\right)^{-\theta}\frac{\left(C_{Tt}^{*}+I_t^{*}\right)}{A_t}+(1-\alpha_N)\left(\frac{W_t/A_t}{MC_{Nt}}\right)^{-\eta}\frac{(1-\gamma)P_tC_t}{A_tP_{Nt}} \tag{1.9}$$

Lemma 1. *If the home country is small—i.e., $\omega\to 0$—then*

$$P_{Ht}^{*}=\frac{\theta}{\theta-1}E_{t-1}\left[\frac{MC_{Ht}}{S_t}\right]$$

Proof. (See Appendix A1.1.). □

By Lemma 1, firms in the tradable good sector will set prices taking into account the future marginal cost and the nominal exchange rate. Using this result, the expected value of a firm's future profit and export revenues are

$$E_{t-1}\left[Q_{t-1,t}\left(S_tP_{Ht}^{*}-MC_{Ht}\right)y_{Ht}\right]=\frac{\omega\beta\left(\gamma+d\right)}{\theta}P_{t-1}C_{t-1}\left(\frac{P_{Ht}^{*}}{P_{Tt}^{*}}\right)^{1-\psi} \tag{1.10}$$

and

$$E_{t-1}\left[Q_{t-1,t}S_tP^*_{Ht}y_{Ht}\right] = \omega\beta\left(\gamma + d\right)P_{t-1}C_{t-1}\left(\frac{P^*_{Ht}}{P^*_{Tt}}\right)^{1-\psi} \tag{1.11}$$

respectively, where d is defined in the proof of Lemma 1. Since $\psi > 1$, a lower price will lead to higher expected export revenues and profit. Obviously, if the future marginal cost is predictable or does not fluctuate much, MC_{Ht} is close to a constant. By Jensen's inequality, under a fixed exchange rate regime, firms are able to set more competitive (lower) prices and earn higher export revenues. However, results may differ if MC_{Ht} is volatile. For instance, if capital is the main factor used in production, i.e., the home country uses a capital-intensive technology, firms should also take the capital market conditions into account when setting their optimal prices. By (1.8), the capital rental rate will be affected by both home and foreign shocks. It is also clear from (1.8) that the capital rental rate is not linear in home and foreign demand, which means volatility in the capital rental rate will influence the pricing rule by firms. It is ambiguous as to which regime, fixed or floating, can create a competitive edge for domestic firms in the tradable good sector. It depends on several conditions: i) the convexity of marginal cost in the shocks, and ii) the ability of each regime to reduce the volatility in $\frac{MC_{Ht}}{S_t}$.

1.1.5 Wage rigidity

If there exists no friction in the labor market (flexible wages), from the firms' side $\frac{W^o_t}{P_t} = mrn_t$, where mrn_t is the real value of firms' marginal product of labor. From the consumer-workers' side,

$$\frac{W^o_t}{P_t} = mrs_t = \kappa C_t$$

where mrs_t is consumer-workers' real value of marginal substitution between leisure and consumption. In equilibrium, the nominal wage is $W^o_t = \kappa P_t C_t$.

However, almost all the countries in the world exhibit some degree of wage rigidities in their labor markets. Therefore, we assume in this paper that wages respond sluggishly to labor market conditions, as a result of some (unmodeled) imperfection or friction in labor markets. Specifically, we assume a partial adjustment model:[2] [3]

$$W_t = W_{t-1}^{\lambda} W_t^{o1-\lambda} \tag{1.12}$$

where λ is the degree of the nominal wage rigidity. We view equation (1.12) as an admittedly ad-hoc but parsimonious way of modeling the slow adjustment of wages to labor market conditions, as found in a variety of models of real wage rigidities, without taking a stand on what the "right" model is.

1.1.6 Money supply

In the benchmark model, we consider only the monetary shocks in the economy–i.e., the labor productivity is non-stochastic. Assume that the foreign country's money balance growth rule is

$$\ln M_t^* = \ln \mu + \ln M_{t-1}^* + \varepsilon_t^* \tag{1.13}$$

where ε_t^* is a white noise following a normal distribution $N(0, \sigma^2)$. We assume $\mu > \beta$ in order to make sure that consumption is positive in both countries.

If the home country takes a flexible exchange rate regime and chooses the monetary growth as the monetary policy, we assume that the equilibrium money growth rate would be similar to that of the foreign country.

$$\ln M_t = \ln \mu + \ln M_{t-1} + \varepsilon_t \tag{1.14}$$

[2] Blanchard and Gali (2005) similarly assume real wage rigidity. We can also assume the same real wage rigidity as in their paper: no qualitative results will change.

[3] Similar to Blanchard and Gali (2005), in principle, one would want to guarantee $\frac{W_t}{P_t} \geq mrs_t$ at all times, to prevent workers from working more than desired, given the wage (as would be the case for example in a model where wages set in bargaining vary over time, but always remain above the workers' reservation wage). This would be easily achieved by introducing a (sufficiently large) positive steady-state wage markup, as in

$$W_t = W_{t-1}^{\lambda} \left(\mu_w P_t \cdot mrs_t\right)^{1-\lambda}$$

without altering any of the conclusions below, though at the cost of burdening the notation.

where ε_t is a white noise subject to a normal distribution $N(0, \sigma^2)$. We may think of the shock as a money velocity shock. When the policy maker sets the money supply goal, the nominal money balance in the home country will be the money supply plus the velocity shock.

If the home country chooses a fixed exchange rate regime, $S_t = S_{t-1}$.

1.1.7 Solving the benchmark model

Given these processes, we have that the domestic and foreign interest rates are both equal to a constant, which we denote by i.

As the model cannot be solved analytically, we will solve the model up to the second order (of σ). Our strategy is to first compute the steady state ($\varepsilon_t = \varepsilon_t^* = 0$) and then take approximation around the steady state.

In the steady state, since there is no monetary shock, we have

$$\bar{W} = \kappa \bar{P} \bar{C} \tag{1.15}$$

$$\bar{r}\bar{P} = P_T / \beta \tag{1.16}$$

$$\bar{K} = \alpha_H \left(\frac{\bar{r}\bar{P}}{\overline{\overline{MC_H}}} \right)^{-\eta} \left(\frac{\bar{P}_H^*}{\bar{P}_T^*} \right)^{-\psi} \frac{(\gamma + d)\bar{P}^*\bar{C}^*}{\bar{P}_T^*} + \alpha_N \left(\frac{\bar{r}\bar{P}}{\overline{\overline{MC_N}}} \right)^{-\eta} \frac{(1-\gamma)\,\bar{P}\bar{C}}{\bar{P}_N} \tag{1.17}$$

$$\bar{P}_H^* = \frac{\theta}{\theta - 1} \frac{M_{t-1}^*}{M_{t-1}} \left[\alpha_H \left(\bar{r}\bar{P} \right)^{1-\eta} + (1-\alpha_H) \left(\frac{\bar{W}}{A_t} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{1.18}$$

$$\bar{P}_N = \frac{\theta}{\theta - 1} \left[\alpha_N \left(\bar{r}\bar{P} \right)^{1-\eta} + (1-\alpha_N) \left(\frac{\bar{W}}{A_t} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{1.19}$$

$$P_T = \frac{\theta}{\theta - 1} \frac{M_{t-1}}{M_{t-1}^*} \left[\alpha_H \left(\bar{r}^*\bar{P}^* \right)^{1-\eta} + (1-\alpha_H) \left(\frac{\bar{W}^*}{A_t^*} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{1.20}$$

In period t, all the prices and capital stock are predetermined and the price in the foreign country will not be affected by the home country's production. We log approximate equation (1.8) around $\varepsilon_t = \varepsilon_t^* = 0$ and obtain

$$K_t = \bar{K} \exp \left\{ \begin{array}{c} k_H \left(\varepsilon_t^* - \frac{\eta(1-\alpha_H)\left(\frac{W_{t-1}}{A_t}\right)^{1-\eta}}{\overline{MC_H}^{1-\eta}} \widehat{\frac{r_t P_t}{W_t/A_t}} - \psi \widehat{P_{Ht}^*} \right) \\ +k_N \left(\varepsilon_t - \frac{\eta(1-\alpha_N)\left(\frac{W_{t-1}}{A_t}\right)^{1-\eta}}{\overline{MC_N}^{1-\eta}} \widehat{\frac{r_t P_t}{W_t/A_t}} - \widehat{P_{Nt}} \right) \end{array} \right\}$$

where

$$\begin{aligned} k_H &= \frac{\alpha_H \left(\overline{\frac{r_t P_t}{MC_H}}\right)^{-\eta} \left(\frac{\overline{P}_{Ht}^*}{\overline{P}_{Tt}^*}\right)^{-\psi} \frac{1}{\chi} \frac{i}{1+i} \frac{(\gamma+d)\mu M_{t-1}^*}{P_T^*}}{\bar{K}} \\ k_N &= \frac{\alpha_N \left(\frac{r_t P_t}{MC_{Nt}}\right)^{-\eta} \frac{1}{\chi} \frac{i}{1+i} \frac{(1-\gamma)\mu M_{t-1}}{P_{Nt}}}{\bar{K}} \\ \widehat{P_{Ht}^*} &= \log\left(\frac{P_{Ht}^*}{\bar{P}_H^*}\right) \\ \widehat{P_{Nt}} &= \log\left(\frac{P_{Nt}}{\bar{P}_N}\right) \end{aligned}$$

Using the factor market clearing condition, we can solve for the relative factor price in the home country

$$\widehat{\frac{r_t P_t}{W_t}} = m_H \left(\varepsilon_t^* - \psi \widehat{P_{Ht}^*} \right) + m_N \left(\varepsilon_t - \widehat{P_{Nt}} \right) - (m_H + m_N) \widehat{K_t} \tag{1.21}$$

where

$$\begin{aligned} m_H &= \frac{k_H}{\frac{\eta k_H (1-\alpha_H)\left(\frac{\bar{W}}{A_t}\right)^{1-\eta}}{\overline{MC_H}^{1-\eta}} + \frac{\eta k_N (1-\alpha_N)\left(\frac{\bar{W}}{A_t}\right)^{1-\eta}}{\overline{MC_N}^{1-\eta}}} \\ m_N &= \frac{k_N}{\frac{\eta k_H (1-\alpha_H)\left(\frac{\bar{W}}{A_t}\right)^{1-\eta}}{\overline{MC_H}^{1-\eta}} + \frac{\eta k_N (1-\alpha_N)\left(\frac{\bar{W}}{A_t}\right)^{1-\eta}}{\overline{MC_N}^{1-\eta}}} \end{aligned}$$

The optimal price set in the tradable good sector is

$$P_{Ht}^{*}=\frac{\theta}{\theta-1}E_{t-1}\left[\frac{\left((1-\alpha_H)\left(\frac{W_t}{A_t}\right)^{1-\eta}+\alpha_H\left(r_tP_t\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}M_t^{*}}{M_t}\right]$$

In the standard literature, capital-intensity is commonly defined as the ratio of capital input cost to the total cost. Under the assumption of CES production function, $\frac{\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}$ is the steady state capital-intensity in the tradable good sector. Given all the assumptions and equations above, we can show the following Proposition.

Proposition 1. *Up to the second order, if the share of nontradable goods in the aggregate consumption good basket is large enough such that*

$$\gamma<\frac{\left(\frac{1}{\alpha_H}-\eta\right)\left(\beta\frac{\theta}{\theta-1}\right)^{\psi-1}}{1+\left(\frac{1}{\alpha_H}-\eta\right)\left(\beta\frac{\theta}{\theta-1}\right)^{\psi-1}}$$

then there exists a critical value A_0, which is an increasing function of λ, such that $(P_{Ht}^{*})^{flexible}=(P_{Ht}^{*})^{fixed}$.

(i) For $A_t>A_0$, $(P_{Ht}^{*})^{flexible}<(P_{Ht}^{*})^{fixed}$, as a result, exports in the tradable good sector are higher under a flexible exchange rate regime than that under a fixed exchange rate regime. For $A_t<A_0$, $(P_{Ht}^{*})^{flexible}>(P_{Ht}^{*})^{fixed}$, as a result, exports in the tradable good sector are lower under a flexible exchange rate regime than that under a fixed exchange rate regime.

(ii) For all $A_t>0$, $(P_{Nt})^{flexible}<(P_{Nt})^{fixed}$.

Proof. (See Appendix A1.2.) □

A few remarks are in order. First, the tradable good price is more likely to be lower (higher) in a labor-intensive (capital-intensive) sector under a fixed exchange rate regime. Here is the intuition. Suppose that only labor is used in production. Due to wage rigidity, part of the marginal cost is predetermined. Firms will set prices based mainly on the expectation of the future nominal exchange rate. By Lemma 1, the expectional term is a convex function of the nominal exchange rate which means that higher volatility in the nominal exchange rate will yield a higher premium in the price. Therefore, fixing the exchange rate at a constant may help firms to set more competitive prices and earn higher revenues. In the opposite case, if capital is the main factor input in production, the result will reverse. We can consider $\frac{MC_{Ht}}{S_t}$ as the marginal cost in the foreign currency, which approximately equals $\frac{r_t P_t}{S_t}$. By (1.8), (1.13), (1.14) and Lemma 1, we can show that the capital rental rate denominated in foreign currency is convex in both home and foreign demand shocks. A volatile capital rental rate (in foreign currency) will yield a positive premium in the optimal prices. Suppose that there is an expansion in foreign demand. Under a fixed exchange rate regime, this immediately leads to a double increase in the demand for capital, from both the tradable and the nontradable good sector. As a result, the capital rental rate (in terms of foreign currency) rises drastically. However, under a flexible exchange rate regime, the nontradable good sector in the home country will not be influenced by the foreign shock, the capital rental rate (in foreign currency) will rise only moderately. Then, a flexible exchange rate can generate a less volatile capital rental rate (in foreign currency) and firms in the tradable good sector are able to set more competitive prices. Combining the two cases together, in general, a higher volatility in $\frac{MC_{Ht}}{S_t}$ will yield higher prices, and by (1.10) and (1.11), will generate both lower profits and lower export revenues. In fact, we can understand this result from another aspect. We consider a special case when prices are fully flexible, in which firms set prices after the shocks. By (1.7), a flexible tradable good price equals

$$P_{Ht}^{*o} = \frac{\theta}{\theta - 1} \frac{MC_{Ht}}{S_t}$$

If $\frac{MC_{Ht}}{S_t}$ becomes more volatile, firms are facing higher possibilities that tradable good prices set by (1.7) may deviate significantly from a flexible prices ex post. However, flexible tradable good prices are virtually the best response functions to the shocks and thus are desirable to firms. An increase in the volatility of $\frac{MC_{Ht}}{S_t}$ will then lead to a loss to firms.

Second, the difference between tradable good prices under two exchange rate regimes, $(P^*_{Ht})^{fixed} - (P^*_{Ht})^{flexible}$, is decreasing in λ, which means that as wages become more rigid, firms are more likely to set lower prices under a fixed exchange rate regime. Here is the intuition. Suppose wages become fully rigid—i.e., wages in period t are completely determined by period $t-1$'s information. This will greatly reduce the volatility of marginal cost in period t. Firms probably will be concerned mainly with nominal exchange rate movements. If the home country can fix the nominal exchange rate, firms do not have to worry about the loss caused by exchange rate fluctuations and hence they set lower prices.

Third, if wages are fully flexible, firms will always set lower prices under a flexible exchange rate regime. In this case, nothing can be predetermined and wage is also volatile in each period. Based on similar reasoning in the first remark, factor prices (in foreign currency) are less volatile under a flexible exchange rate regime. Therefore, firms under a flexible exchange rate regime are always more competitive.

Fourth, by Proposition 1, for all positive labor productivities, $(P_{Nt})^{flexible} \leq (P_{Nt})^{fixed}$. Here is why. The nominal exchange rate does not directly influence the price in the nontradable good sector. Firms will be concerned only with the volatility of marginal cost when setting their prices. Once there is a capital input in nontradable good production, firms will be affected by foreign shocks. Since a flexible exchange rate regime works to reduce the impact from foreign shocks, the capital rental rate is less volatile, thus firms in the nontradable good sector are able to set lower prices. However, as we can see from the proof of Proposition 1, when wages become more rigid, the difference between nontradable good prices under the two exchange rate regimes becomes smaller.

Finally, the inequality condition in Proposition 1 is easy to be satisfied under proper parameter settings. Using the result of Lemma 1, we can rewrite the condition as

$$\gamma < \frac{1 - \alpha^*_F \left(\beta \frac{\theta}{\theta - 1}\right)^{1-\psi} \left(\frac{I}{C}\right)^*}{1 + \alpha^*_F \left(\beta \frac{\theta}{\theta - 1}\right)^{1-\psi}}$$

where $\left(\frac{I}{C}\right)^*$ stands for the investment-to-consumption ratio in the foreign country. We can find investment and consumption to GDP data from Penn World Table 6.3 and compute the investment-to-consumption ratio. If we look at this ratio across countries from 2000 to 2004, the mean and median are both smaller than 0.3. For β and θ, β takes a value above 0.98 for quarterly frequency, and θ takes a value such that the markup in the tradable good sector $\frac{1}{\theta-1}$ is about 15 percent. ψ is the elasticity of substitution between home produced tradable good and foreign produced tradable good. As in Corsetti et al. (2007), it takes a value about 1.5 or above. Then the right hand side of the inequality takes a value of 0.32 with an extremely high α_F^* ($\alpha_F^* = 1$), and a value of 0.40 with $\alpha_F^* = 0.8$. In the literature, the share of nontradable good consumption in the aggregate consumption good basket is usually assumed to be around or even greater than 70 percent, which means $\gamma \leq 0.3$ in our model, (Burstein, Neves and Rebelo, 2003). Then, the condition in Proposition 1 is easy to be satisfied under proper parameter settings.

1.1.8 Welfare

In this section, we will compare welfare levels under the two exchange rate regimes. As in Corsetti and Pesenti (2005), we consider the problem faced by a policymaker who seeks to maximize home agents' expected utility.

$$V\left(K_0, B_{-1}, B_{-1}^*\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \chi \ln\left(\frac{M_t}{P_t}\right) - \kappa L_t\right]$$

We use V^{fixed} and $V^{flexible}$ denote the value functions under a fixed exchange rate regime and a flexible exchange rate regime, respectively. Given the information in period $t-1$, the policy maker will choose a fixed exchange rate regime from period $t-1$ to period t if and only if

$$V^{fixed}\left(K_t, B_{t-1}, B_{t-1}^*\right) > V^{flexible}\left(K_t, B_{t-1}, B_{t-1}^*\right)$$

Under a fixed exchange rate regime,

$$V^{fixed}\left(K_t, B_{t-1}, B^*_{t-1}\right) = \max E_{t-1}\left[\ln C_t + \chi \ln\left(\frac{M_t}{P_t}\right) - \kappa L_t + \beta E_t V^{fixed}\left(K_{t+1}, B_t, B^*_t\right)\right]$$

and similarly,

$$V^{flexible}\left(K_t, B_{t-1}, B^*_{t-1}\right) = \max E_{t-1}\left[\ln C_t + \chi \ln\left(\frac{M_t}{P_t}\right) - \kappa L_t + \beta E_t V^{fixed}\left(K_{t+1}, B_t, B^*_t\right)\right]$$

Then

$$\begin{aligned} & V^{fixed}\left(K_t, B^*_{t-1}\right) - V^{flexible}\left(K_t, B^*_{t-1}\right) \\ = \; & E_{t-1}\left[\left(\ln C_t + \chi \ln\left(\frac{M_t}{P_t}\right) - \kappa L_t\right)^{fixed} - \left(\ln C_t + \chi \ln\left(\frac{M_t}{P_t}\right) - \kappa L_t\right)^{flexible}\right] + \beta \Delta V \end{aligned}$$

where

$$\Delta V = E_{t-1} V^{fixed}\left(K^{fixed}_{t+1}, B^{fixed}_t, B^{*fixed}_t\right) - E_{t-1} V^{flexible}\left(K^{flexible}_{t+1}, B^{flexible}_t, B^{*flexible}_t\right)$$

The policy maker will consider two components if the country switches from a flexible exchange rate regime to a fixed exchange rate regime: i) the welfare gain in period t, and ii) the future welfare gain after period t.

Proposition 2. *Under the assumption in Proposition 1,*

(i) If the nominal wage in the home country is rigid enough—i.e., λ is sufficiently large– there exists a critical value of labor productivity, A^w_0, which is an increasing function of λ, such that the welfare levels are the same under the two regimes. For $A_t > A^w_0$, welfare is higher under a flexible exchange rate regime than that under a fixed exchange rate regime. For $A_t < A^w_0$, welfare is lower under a flexible exchange rate regime than that under a fixed exchange rate regime.

(ii) If the nominal wage in the home country is flexible enough—i.e., λ is sufficiently small–the welfare under a flexible exchange rate regime is always higher than that under a

fixed exchange rate regime.

Proof. (See Appendix A1.3.) □

A few remarks are in order. First, the price of the final consumption good in the home country is lower under a flexible exchange rate regime than that under a fixed exchange rate regime. Here is the intuition: i) As shown in Proposition 1, the nontradable good price in the home country is lower under a flexible exchange rate regime than that under a fixed exchange rate regime; ii) since the foreign country is relatively large, the home country's import price will not be affected by changes in the home country. Therefore, the final consumption good price is lower under a flexible exchange rate regime.

Second, by Proposition 1, for any positive degree of wage rigidity, if $\lambda > 0$, there exists some small A_ts, such that the tradable good price is lower under a fixed exchange rate regime than that under a flexible exchange rate regime. As a result, switching from a flexible exchange rate regime to a fixed exchange rate regime, the home country will obtain more export revenues. However, this does not mean that welfare is unambiguously higher under a fixed exchange rate regime. The higher CPI in the first remark may cause a welfare loss to the representative agent in the home country. One necessary condition for a home country with low labor productivity to optimally choose a fixed exchange rate regime is that wage is sufficiently rigid. As λ becomes larger, the gap between a nontradable good price under a fixed versus a flexible exchange rate regime becomes smaller. For sufficiently large λ, the welfare loss resulting from the higher CPI under a fixed exchange rate regime is small. The positive effect from the higher export revenue will dominate, and there is a welfare gain if the home country switches to a fixed exchange rate regime. In short, the advantage of a flexible exchange rate diminishes with λ.

1.1.9 Other shocks

In this section, we will consider shocks other than monetary shocks. To be more specific, we consider labor productivity shocks—i.e., A_t and A_t^* are stochastic. Assume that $\ln A_t$

and $\ln A_t^*$ follow AR(1) processes:

$$\begin{aligned}\ln A_t &= \rho \ln A_{t-1} + (1-\rho) \ln \bar{A} + \varepsilon_{at} \\ \ln A_t^* &= \rho^* \ln A_{t-1}^* + (1-\rho^*) \ln \bar{A}^* + \varepsilon_{at}^*\end{aligned}$$

where $\bar{A}$ and $\bar{A}^*$ are steady state labor productivities in the home and foreign country, respectively. ε_{at} and ε_{at}^* are white noises following normal distributions $N(0, \sigma_a)$ and $N(0, \sigma_a^*)$, respectively. For simplicity, we assume that $\sigma_a = \sigma_a^*$. ρ and ρ^* shows the convergence speed of labor productivity in the home and foreign country, respectively. We assume that $0 < \rho < 1$ and $0 < \rho^* < 1$.

Assume that the central bank in the foreign country adjusts money growth as following:

$$\ln M_t^* = \ln \mu + \ln M_{t-1}^* + \phi_a^* \ln \left(\frac{A_t^*}{\bar{A}^*} \right) + \varepsilon_t^* \tag{1.22}$$

where $\phi_a^* (> 0)$ is the coefficient showing the reaction of money growth to the deviation of productivity shocks in the foreign country.

If the home country follows a fixed exchange rate regime, money growth in the home country will follow the same process as in the foreign country. If home chooses a flexible exchange rate regime, the home policy maker will set monetary policy rule similar to foreign country

$$\ln M_t = \ln \mu + \ln M_{t-1} + \phi_a \ln \left(\frac{A_t}{\bar{A}} \right) + \varepsilon_t \tag{1.23}$$

where $\phi_a (> 0)$ is the response coefficient to the labor productivity shock set by the home country policy maker. We assume that labor productivity shocks and monetary shocks are independent.

In this extended model, interest rates in both countries are no longer constant. In the foreign country, by (1.1) and (1.3), we have

$$i_t^{*-1} = E_t \left[\beta \frac{M_t^*}{M_{t+1}^*} \left(1 + i_{t+1}^{*-1} \right) \right]$$

Iterating forward, we have

$$i_t^{*-1} = \sum_{n=1}^{\infty} E_t \left[\beta^n \frac{M_t^*}{M_{t+n}^*} \right] + \lim_{N\to\infty} E_t \left[\beta^N \frac{M_t^*}{M_{t+N}^*} i_{t+N}^{*-1} \right]$$

Notice that $\mu > \beta$, and i_t is bounded in each period. Then the last term on the right hand side is zero and

$$i_t^{*-1} = \sum_{n=1}^{\infty} E_t \left[\beta^n \frac{M_t^*}{M_{t+n}^*} \right]$$

By (1.22), we have

$$i_t^{*-1} = \sum_{n=1}^{\infty} \left(\frac{\beta}{\mu}\right)^n \left[\exp\left\{ \begin{array}{c} \frac{n\sigma^2}{2} + \frac{n\phi_a^*\sigma_a^2}{2(1-\rho^*)^2} \\ + \frac{\phi_a^* \frac{\rho^2-\rho^{2n+2}}{1-\rho^2}\sigma_a^2}{2(1-\rho^*)^2} - \frac{\phi_a^* \frac{\rho-\rho^{n+1}}{1-\rho}\sigma_a^2}{(1-\rho^*)^2} \end{array} \right\} \right] \left(\frac{A_t^*}{\bar{A}^*}\right)^{\phi_a \frac{\rho-\rho^{n+1}}{1-\rho}}$$

For simplicity, we assume $\rho^* = \rho$, which is not crucial. Let $\widehat{i_t^*}$ denote $\ln\left(\frac{i_t^*}{\bar{\imath}^*}\right)$, where $\bar{\imath}^*$ denotes the steady state interest rate in the foreign country, then,

$$\widehat{i_t^*} = -\phi_a^* \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho-\rho^{n+1}}{1-\rho}\right) \widehat{A_t^*} \tag{1.24}$$

where

$$s_n^* = \frac{\left(\frac{\beta}{\mu}\right)^n \left[\exp\left\{ \frac{n\sigma^2}{2} + \frac{n\phi_a^*\sigma_a^2}{2(1-\rho^*)^2} + \frac{\phi_a^* \frac{\rho^2-\rho^{2n+2}}{1-\rho^2}\sigma_a^2}{2(1-\rho^*)^2} - \frac{\phi_a^* \frac{\rho-\rho^{n+1}}{1-\rho}\sigma_a^2}{(1-\rho^*)^2} \right\} \right]}{\sum_{j=1}^{\infty} \left(\frac{\beta}{\mu}\right)^n \left[\exp\left\{ \frac{n\sigma^2}{2} + \frac{n\phi_a^*\sigma_a^2}{2(1-\rho^*)^2} + \frac{\phi_a^* \frac{\rho^2-\rho^{2n+2}}{1-\rho^2}\sigma_a^2}{2(1-\rho^*)^2} - \frac{\phi_a^* \frac{\rho-\rho^{n+1}}{1-\rho}\sigma_a^2}{(1-\rho^*)^2} \right\} \right]}$$

We assume that σ^2 and σ_a^2 are small enough, then s_n^* is increasing in n.

Similarly, if the home country is under a flexible exchange rate regime, we have

$$\widehat{i_t} = -\phi_a \sum_{n=1}^{\infty} s_n \left(\frac{\rho-\rho^{n+1}}{1-\rho}\right) \widehat{A_t} \tag{1.25}$$

where

$$s_n = \frac{\left(\frac{\beta}{\mu}\right)^n \left[\exp\left\{\frac{n\sigma^2}{2} + \frac{n\phi_a \sigma_a^2}{2(1-\rho^*)^2} + \frac{\phi_a \frac{\rho^2-\rho^{2n+2}}{1-\rho^2}\sigma_a^2}{2(1-\rho^*)^2} - \frac{\phi_a \frac{\rho-\rho^{n+1}}{1-\rho}\sigma_a^2}{(1-\rho^*)^2}\right\}\right]}{\sum_{j=1}^{\infty} \left(\frac{\beta}{\mu}\right)^n \left[\exp\left\{\frac{n\sigma^2}{2} + \frac{n\phi_a \sigma_a^2}{2(1-\rho^*)^2} + \frac{\phi_a \frac{\rho^2-\rho^{2n+2}}{1-\rho^2}\sigma_a^2}{2(1-\rho^*)^2} - \frac{\phi_a \frac{\rho-\rho^{n+1}}{1-\rho}\sigma_a^2}{(1-\rho^*)^2}\right\}\right]}$$

Notice that this interest rate implication is not inconsistent with Taylor rule, which implies that the policy maker will raise the interest rate if there is an excess inflation and excess output gap. Suppose that labor productivity improves in the home country, there will be two direct results: firms will be able to set lower prices and expand output. If inflation has a very strong effect on the interest rate, the nominal interest rate will go down, which is consistent with (1.25) by assuming $\phi_a > 0$. This is also consistent with Corsetti and Pesenti (2005). In their paper, if there is a positive productivity shock, the home country will increase its expenditures on consumption goods. In our paper, combining (1.23) and (1.25), we find this to be true as well.

Assume that in period $t-1$, both countries stay at the steady states, then we can show the following proposition.

Proposition 3. *Up to the second order, if $\phi_a > 0$ and under the same assumption as in Proposition 1, there exists a critical value A_{a0} for the steady state labor productivity such that $(P_{Ht}^*)^{flexible} = (P_{Ht}^*)^{fixed}$.*

(i) For $\bar{A} > A_{a0}$, $(P_{Ht}^*)^{flexible} < (P_{Ht}^*)^{fixed}$, as a result, export in the tradable good sector is higher under a flexible exchange rate regime than that under a fixed exchange rate regime. For $\bar{A} < A_{a0}$, $(P_{Ht}^*)^{flexible} > (P_{Ht}^*)^{fixed}$, as a result, export in the tradable good sector is lower under a flexible exchange rate regime than that under a fixed exchange rate regime.

(ii) For all $\bar{A} > 0$, $(P_{Nt})^{flexible} < (P_{Nt})^{fixed}$.

Proof. (See Appendix A1.4.) □

Proposition 3 and Proposition 1 are almost the same. One remark on the price of the nontradable good is that, as we can find, $(P_{Nt})^{flexible}$ is increasing in ϕ_a, which means that if the money supply responds more aggressively to the labor productivity shock, the price of the nontradable good sector will be lower.

As for welfare, we can show the following proposition:

Proposition 4. *(i) If the nominal wage in the home country is rigid enough—i.e., λ is sufficiently large–there exists a critical value of labor productivity, A^w_{a0}, such that welfares are the same under a flexible and a fixed exchange rate regime. For $\bar{A} > A^w_{a0}$, welfare is higher under a flexible exchange rate regime than that under a fixed exchange rate regime. For $A_t < A^w_{a0}$, welfare is lower under a flexible exchange rate regime than that under a fixed exchange rate regime.*

(ii) If the nominal wage in the home country is flexible enough—i.e., λ is sufficiently small—welfare under a flexible exchange rate regime is always higher than that under a fixed exchange rate regime.

Proof. (See Appendix A1.5.) □

Proposition 4 and Proposition 2 are similar. As stated in Proposition 3, when ϕ_a becomes larger, firms are more likely to set lower prices in the nontradable good sector which in turn may potentially reduce the consumption good price. However, the effect on welfare of raising ϕ_a is ambiguous. Suppose that there is an adverse labor productivity shock in the home country. As we analyzed above, higher ϕ_a can result in a lower nontradable good price; however, at the same time, this also reduces the aggregate demand greatly, which has a negative effect on welfare. If the second effect on the aggregate demand dominates the first effect, the home country will result in a welfare loss by raising ϕ_a.

We can also extend the model, as in Devereux and Engel (2003), so that each country can adjust the money supply by responsing to the foreign productivity shock. Then the foreign country will follow a monetary policy rule:

$$\ln M^*_t = \ln \mu + \ln M^*_{t-1} + \phi^*_a \ln\left(\frac{A^*_t}{\bar{A}^*}\right) + \delta^*_a \ln\left(\frac{A_t}{\bar{A}}\right) + \varepsilon^*_t$$

and if the home country chooses a flexible exchange rate regime, it will follow a similar monetary policy rule

$$\ln M_t = \ln \mu + \ln M_{t-1} + \phi_a \ln \left(\frac{A_t^*}{\bar{A}^*} \right) + \delta_a \ln \left(\frac{A_t^*}{\bar{A}^*} \right) + \varepsilon_t$$

where δ and δ^* are positive as in Devereux and Engel (2003).

Similar to the analysis above, the interest rate in the foreign country is

$$\widehat{i}_t^* = -\sum_{n=1}^{\infty} \left(\phi_a^* s_n^* \widehat{A_t^*} + \delta_a^* s_n \widehat{A_t} \right) \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \tag{1.26}$$

and the interest rate in the home country is

$$\widehat{i}_t = -\sum_{n=1}^{\infty} \left(\phi_a s_n \widehat{A}_t + \delta_a s_n^* \widehat{A_t^*} \right) \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \tag{1.27}$$

(1.26) and (1.27) are very similar to (1.25) and (1.24) except for the additional terms resulted from responding to the external productivity shocks. Those additional terms work only as constants and will not change any of the theoretical results in Propositions 3 and 4.

1.1.10 Producer pricing in the foreign country

In this section, we consider an extension that firms in the foreign country, a large country, will set their export prices in the domestic currency (producer currency pricing). This change will affect only the import price for the home country and, hence, will not affect the export and nontradable good sector. Proposition 1 still holds.

In the welfare analysis, the import price in the home country now becomes $S_t P_{Ft}^*$ where P_{Ft}^* is the price for the foreign produced tradable good denominated in foreign currency. Since the foreign country is much bigger than the home country, P_{Ft}^* cannot be affected by the demand shocks in the home country. In other words, the home country takes P_{Ft}^* as given. The final consumption good price index in the home country is

$$P_t = P_{Nt}^{1-\gamma} \left(S_t P_{Ft}^{*}\right)^{\gamma} \tag{1.28}$$

Since $\gamma < 1$, and $(P_{Nt})^{flexible} < (P_{Nt})^{fixed}$, the expectation of the final consumption good price index is lower under a flexible exchange rate regime than that under a fixed exchange rate regime.

For developed countries, we can obtain the same result: They prefer flexible exchange rate regimes since i) as analyzed in Proposition 1, they obtain higher export revenues under a flexible exchange rate regime; and ii) a flexible exchange rate regime can generate a lower consumption good price level than a fixed exchange rate regime. Both effects are welfare-enhancing.

Developing countries face a tradeoff similar to that in the benchmark model. If they switch from a flexible exchange rate regime to a fixed exchange rate regime, their exports increase according to Proposition 1. However, this also leads to a higher consumption good price, which potentially yields welfare loss. The net effect is ambiguous. Substituting (1.28) into the proof of Proposition 2, we can obtain a very similar result: If wages are rigid enough in developing countries, a fixed exchange rate regime is preferred since the first effect (higher export revenues when switching to a fixed exchange rate regime) dominates the second effect (higher consumption good price).

1.2 Empirics

1.2.1 Benchmark regressions

In this section, we examine whether the data supports our theoretical predictions. Two separate experiments are done in this section.

We first estimate how sectoral export growth varies under different exchange rate regimes. Our industry-level data can be obtained from the Centre D'Etudes Prospectives et D'Informations

Internationales (CEPII) database that covers sectoral data in 42 countries from 1980 to 2004.[4] There are 24 sectors in the sample, defined using 3-digit International Standard Industrial Classification (ISIC system), Revision 2. For the exchange rate regimes, we use the *de facto* regime classifications by Reinhart and Rogoff (2004) (henceforth RR). As in Aghion et al. (2006), RR can take values from set $\{1, 2, 3, 4\}$, in which 1, 2, 3 and 4 represent a peg, a crawling peg, a managed floating and a free floating, respectively.[5] As the number becomes larger, the exchange rate is more flexible. Macro data such as real GDP per capita can be found in Penn World Table 6.2.

As in the literature, we construct a panel data set by transforming our time series data into five-year averages. There are five non-overlapping periods in this sample: 1980-1984, 1985-1989, 1990-1994, 1995-1999 and 2000-2004.

We test the predictions by Proposition 1 (or Proposition 3). We look at how sectoral export growth varies in response to the change in the choice of exchange rate regime and wage rigidities in all the sectors. We use sectoral export revenue growth as the dependent variable and regress on the *de facto* exchange rate regime index, the interaction between exchange rate regime index and sectoral capital-intensity, the interaction between exchange rate regime index and wage rigidity index, and other control variables including country's log initial income, wage rigidity index, capital-intensity and country, time and sectoral dummies. The regression equation is as following:

$$\begin{aligned} y_{ijt} &= \alpha + \beta_1 \cdot RR_{jt} + \beta_2 \cdot (RR_{jt} \cdot \text{capital-intensity}_{it}) \qquad (1.29) \\ &\quad + \beta_3 \cdot \left(RR_{jt} \cdot \text{wage rigidity}_{jt}\right) + \gamma \cdot Z_t + \varepsilon_{ijt} \end{aligned}$$

where i stands for sector. If we use the sectoral export growth as the dependent variable, according to theory, we expect to have positive β_2 and negative β_3, which means that as the exchange rate becomes more flexible, the marginal effect of exchange rate regime on sectoral

[4] We drop four large countries in the regressions: United States, Japan, Germany and China.

[5] We drop the "free falling" from this classification.

export growth is increasing in the capital-intensity and decreasing in the wage rigidity.

The capital-intensity is computed as one minus labor compensation in value added. There is no direct measure for the nominal wage rigidities for most countries in the world. But Babecký et al. (2009) find that firing cost is positively associated with DNWR; we then use firing cost as one proxy for the nominal wage rigidity in this paper. We also use the index of labor union power as another proxy for wage rigidity. Both data can be obtained from Doing Business Report, World Bank database. We don't think the two indices are perfect measures for the nominal wage rigidities. In fact, we will use another measure in this paper to do the sensitivity checks. As in the macroeconomics literature, we can compute the nominal wage rigidity level by estimating the wage Phillips curve. As in Gali (2010), the coefficient on the predicted unemployment rate (Column (8) in Table 1, Gali, 2010) is a linear function of nominal wage rigidity implied by the model. Since we don't assume any cross-country differences in the utility function, that parameter can be used directly as the measure for nominal wage rigidity. In this paper, we estimate the wage Phillips curve for 38 countries using the quarterly data of wage income, unemployment rate and inflation from 1990 to 2010, which can be obtained from the ILO database. However, the parameter we will use for the nominal wage rigidity measure is an estimator from the wage Phillips curve estimation; there will be a measurement error problem in the sectoral regression, which yields biases to the coefficients. In this paper, we only use the wage rigidity parameter from the wage Phillips curve in the sensitivity checks to show how different our result would be by changing the wage rigidity index. We don't aim to find any proper instruments to solve the measurement error problem.

Table 1.3 shows the regression results. We find that, in all regressions, most coefficients on the interaction term between exchange rate regime and capital-intensity are significant with the right signs, which means in more capital-intensive sectors, sectoral exports tend to grow faster under a flexible exchange rate regime. The opposite results hold in more labor-intensive sectors. We also find that most coefficients on the interaction term between exchange rate regime and wage rigidity are negative and significant, which means that in countries with a higher wage rigidity level, flexible exchange rates tend to impede the sectoral export growth. All these results are consistent with the theoretical predictions.

One concern in (1.29) is that switching from a more fixed to a more flexible exchange rate regime might have nonlinear effects on sectoral export growth. To check this, we run the regression by using exchange rate dummies. In the first experiment, we broadly classify the exchange rates into two regimes: fixed (a peg or a crawling peg) and flexible (a managed floating or a free floating). The regression equation is

$$\begin{aligned} y_{ijt} &= \alpha + \beta_1 \cdot flexible_{jt} + \beta_2 \cdot (flexible_{jt} \cdot \text{capital-intensity}_{it}) \\ &\quad + \beta_3 \cdot \left(flexible_{jt} \cdot \text{wage rigidity}_{jt}\right) + \gamma \cdot Z_t + \varepsilon_{ijt} \end{aligned} \tag{1.30}$$

We then use the four exchange rate regime dummies in Reinhart and Rogoff (2004) (a peg, a crawling peg, a managed floating and a free floating) and run the regression equation:

$$\begin{aligned} y_{ijt} &= \alpha + \sum_{k=2}^{4} \beta_{1k} \cdot RR_{k,jt} + \sum_{k=2}^{4} \beta_{2k} \cdot (RR_{k,jt} \cdot \text{capital-intensity}_{it}) \\ &\quad + \sum_{k=2}^{4} \beta_{3k} \cdot \left(RR_{k,jt} \cdot \text{wage rigidity}_{jt}\right) + \gamma \cdot Z_t + \varepsilon_{ijt} \end{aligned} \tag{1.31}$$

Table 1.4 show the regression results. Compared to a fixed exchange rate regime, under a flexible exchange rate regime, sectoral exports tend to grow faster i) in more capital-intensive sectors and ii) in countries with lower wage rigidity levels. Both results are consistent with our theory.

We also test whether there exists empirical support for Proposition 2 (or Proposition 4). We first examine the capital-intensities in different countries. If the model is correct, then one reason why rich countries are willing to choose a flexible exchange rate regime is that they use more capital-intensive technology in production. Gollin (2002) measures the labor share in GDP. Using his data, we calculate the capital share by using one minus the labor share obtained from Gollin (2002)[6]. Figure 1.1 shows the scatter plot, from which we

[6] We use the first labor share measure in Gollin (2002).

can see a clear positive correlation between capital share in GDP and a country's income. There exist some caveats in this test: i) the sample is very small (only 24 countries); and ii) the labor share obtained in Gollin (2002) may be overstated since he put all operating surplus into the labor income. Therefore, we don't claim this is the perfect evidence to support our theory.

Instead of seeking further cross-country evidence on the capital-intensities, we run an direct ordered Probit using the *de facto* exchange rate regime index as the dependent variable, and country's income and wage rigidity index as the independent variables. The specification equation is as following:

$$RR_{jt} = \alpha + \beta \cdot \ln\left(\text{GDP per capita}_{jt}\right) + \gamma \cdot \text{wage rigidity}_{jt} + \delta \cdot Z_{jt} + \varepsilon_{jt} \tag{1.32}$$

where Z_{jt} is the set of other control variables that may affect the countries' exchange rate regime choices. Levy-Yeyati, Sturzenegger and Reggio (2004) find that exchange rate regime choice may depend on several determinants such as OCA theory, financial openness and policy crutch. Aghion et al. (2006) find that financial development may also be a key factor in choosing the optimal exchange rate regimes. We will include all such variables in our regression by adding trade openness, capital openness, financial development index, years in office and vetopoints to the regressors. Trade openness data can be obtained from the World Bank dataset. Capital openness index can be obtained from the IFS dataset. We use private credit to GDP ratio as the measure of financial development index, which can be obtained from the World Bank dataset. Years in office measures the years the incumbent administration has been in office which can be obtained from the Database of Political Institutions 2000. Vetopoints refer to the extent of institutionalized constraints on the decision-making powers of chief executives, whether individuals or collectivities. This data is from the Polcon 2002 Database.

Tables 1.5 and 1.6 show the regression results when using different wage rigity measures. In all columns in Table 1.5, the coefficients on ln(GDP per capita) are positive and significant, which means that developed countries (usually with high productivities) are more likely to choose flexible exchange rate regimes, while developing countries (usually with low productivities) are more likely to choose fixed exchange rate regimes. The coefficients on firing cost are negative and significant, which means that, for countries with more rigid wage settings, they may be more likely to choose fixed exchange rate regimes. Both results are consistent with our theoretical predictions. In Table 1.6, we again find negative and significant coefficients on the wage rigidity measure obtained from estimations of the wage Phillips curves. However, the coefficients on ln(GDP per capita) are no longer significant though they are with the correct signs. One possible reason is that the sample shrinks significantly when we use the wage Phillips curve parameter to measure the wage rigidity.

1.2.2 Endogeneity issues

Using exchange rate regime as the regressor in the estimation will inevitably lead to discussions on the endogeneity issues. We will adopt the similar methodology in Levy-Yeyati, Sturzenegger, and Reggio (2004) to solve this problem. They find that there are the three main competing approaches to explaining the choice of exchange rate regimes: i) the optimal currency area (OCA) theory pioneered by Mundell (1961), which relates the choice of regime to the country's trade links, size, and openness; ii) the financial view, which highlights the consequences of international financial integration; and iii) and the political view, which regards the use of a peg (or, more generally, an exchange rate anchor) as a "policy crutch" for governments lacking (nominal and institutional) credibility. Based on their work, we will run 2SLS regressions to deal with the endogeneity problems.

In the first stage, we run a multinomial Probit regression by using the flexible dummy or the four *de facto* exchange rate regime index as the dependent variable, and the same set of regressors as in Tables 1.5 and 1.6. We can obtain the predicted values (the likelihoods of the exchange rate regimes) from the first-stage regression. Then in the second stage, we directly use those predicted values obtained from the first stage regression to replace the orginal exchange rate regime dummies in (1.30) or (1.31). For instance, in the period

2000-2004, by the first stage regression, we obtain the likelihood 0.23 that Brazil takes the crawling peg regime and likelihood 0.44 that it will take the managed floating regime. Then we will use the likelihood 0.23 and the likelihood 0.44 to replace the crawling peg dummy and managed floating dummy respectively for Brazil in the second stage regression.

Table 1.7 show the regression results by running 2SLS regressions to (1.30) and (1.31). We find that most results are similar to those in Table 1.3. Exports grow faster under a flexible exchange rate regime than that under a fixed exchange rate regime in capital-intensive sectors or in countries with low wage rigidities. The opposite results hold if capital-intensity is low or wage rigidity is high.

1.2.3 Alternative exchange rate regime classifications

It is useful to examine our results with another exchange rate regime index. In this section, we redo all the regressions by using the *de facto* exchange rate regime classification defined by Levy-Yeyati and Sturzenegger (2003).[7] The original LYS index takes values from 1 to 5, which represent exchange rate regimes inconclusive, float, dirty float, crawling peg and fixed, respectively. We modify the LYS index by i) using 6 minus the LYS index and ii) dropping the observations with "inconclusive". The objective is to have a similar *de facto* exchange rate regime index to the RR index that, as the number becomes larger in the new LYS index, the exchange rate regime is more flexible. Tables 1.8, 1.9 and 1.10 show the regression results when we use the LYS index to replace the RR index and redo (1.29), (1.30), (1.31) and (1.32). In the sectoral export growth regressions, we find very similar results to the case when using RR exchange rate regime classifications. In the regression (1.32), some coefficients on the log country's initial income, and wage rigidity index are not significant. However, all the coefficients have the correct signs that are consistent with our theoretical predictions. We think that the robustness checks still provide empirical support to our theory.

---

[7] We use the 5-way classification in this paper.

1.3 Conclusion

We investigate how a country's stage of development and labor market frictions will influence its choice of exchange rate regime in this paper. Our conclusion is that whether to fix or to float the currency depends on the country's labor productivity and wage rigidity level. For countries that have high labor productivities (namely developed countries), or flexible wages, a flexible exchange rate regime is preferred because under such a regime: i) exporting firms will earn more revenues, and ii) the domestic consumption good price level is lower. Both of these effects are welfare-enhancing. For countries that have low labor productivities (namely developing countries), though domestic consumption good prices are higher under a fixed exchange rate regime, a stable nominal exchange rate will result in more exporting income. If wages are sufficiently rigid, the latter effect (higher exports) will dominate the former (higher consumption good prices), and there is a welfare improvement when switching to a fixed exchange rate regime.

The choice of either a flexible or a fixed exchange rate regime may not always remain the optimal policy. The optimal choice should depend on a country's development stage. At a low level of development, the country might deem a fixed exchange rate regime as optimal if wages are rigid enough. However, as the country develops, an eventual switch from a fixed to a flexible exchange rate regime will be optimal.

To test the theoretical predictions, we use the *de facto* exchange rate regime index from Reinhart and Rogoff (2004) to examine the relationships among sectoral export growth, capital-intensity, exchange rate regime choice and wage rigidity. We find that, consistent with the theory, a flexible exchange rate regime will enhance sectoral export growth in more capital-intensive sectors or in countries with low wage rigidities. Then, we test how a country's initial income and wage rigidity affect the choice of the exchange rate regime. We find that, as the theory predicts, countries with high incomes or low wage rigidities are more likely to choose flexible exchange rate regimes.

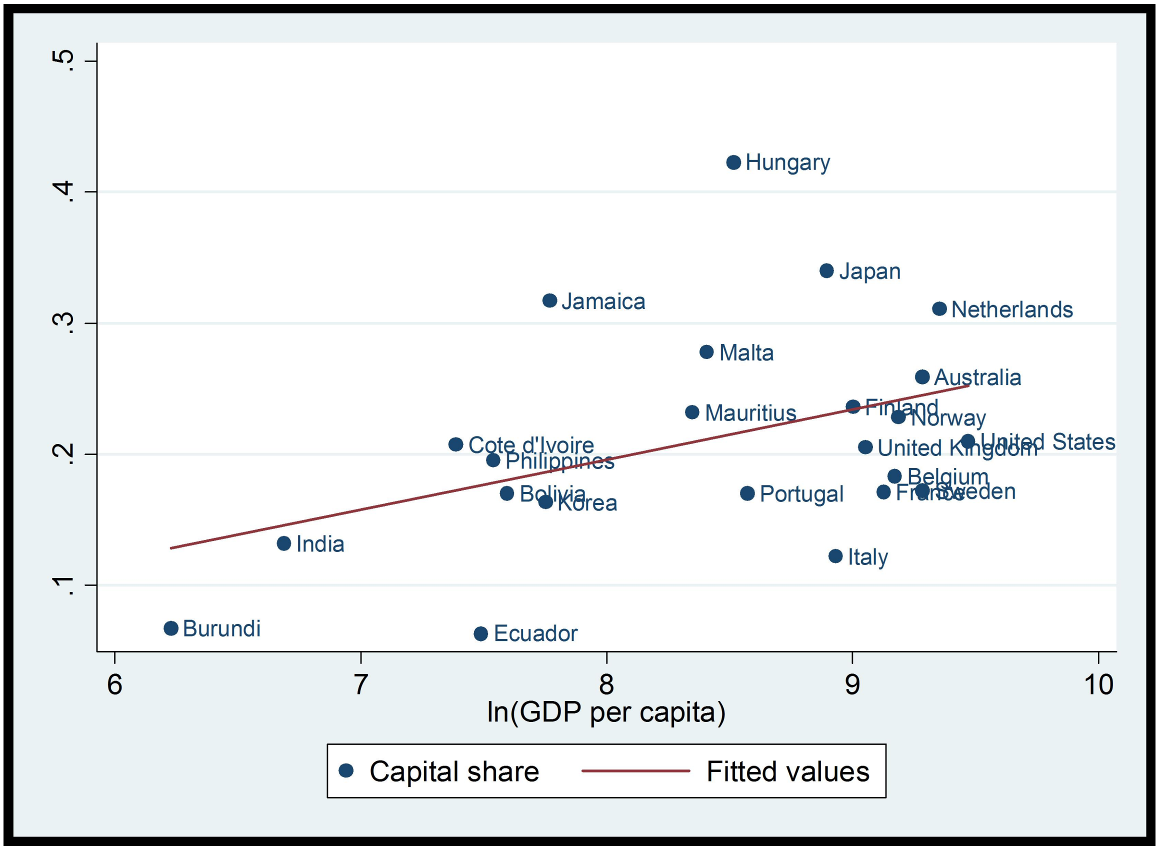

Figure 1.1: Capital share vs ln(GDP per capita)

Notes: a. Capital share is calculated as one minus the labor share. We use the first adjustment in Gollin (2002) to calculate the labor share. b. Since labor shares in Botswana and Congo fall drastically from 1980 to 1990 (more than 20 percent), we exclude the two countries from the sample.

Table 1.1: A country's exchange rate regime and its income

	(1)		(2)		(3)		(4)	
	Obs	Fixed	Obs	Fixed	Obs	Fixed	Obs	Fixed
Low income countries	94	0.74	94	0.74	94	0.74	81	0.71
High income countries	34	0.65	23	0.44	22	0.45	14	0.29
t-test		0.09		0.30**		0.29**		0.45***
		(0.10)		(0.12)		(0.12)		(0.09)

Notes: a. We report the share of countries that take a fixed exchange rate regime in Table 1. A fixed exchange rate regime is defined as a peg or a crawling peg using Reinhart and Rogoff (2004) index. A flexible exchange rate regime is defined as a managed floating or a free floating. b. We divide the countries into two groups, low income countries and high income countries, using 20,000 dollars in year 2004 as the cutoff value of GDP per capita. High income countries include: Australia, Austria, Bahrain, Barbados, Belgium, Canada, Cyprus, Denmark, Finland, France, Germany, Greece, Hong Kong China, Iceland, Ireland, Israel, Italy, Japan, Korea, Kuwait, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Puerto Rico, Qatar, Saudi Arabia, Singapore, Spain, Sweden, Switzerland, United Kingdom, and United States. All other countries in the sample are low income countries. c. In column (2), we treat countries that use Euro as the currency as one big country. d. In column (3) and (4), we drop countries that use Euro as the currency. In column (4), we also drop city economies (Hong Kong and Singapore) and countries with population smaller than 1,000,000.

Table 1.2: Summary statistics of key variables

	Obs	Mean	Median	Std Dev	Min	Max
RR	487	1.56	2	1.31	1	4
LYS	603	1.95	1.5	1.06	1	4
GDP per capita	617	9059	5384	9886	258	65480
Labor union	67	0.75	1	0.44	0	1
Firing cost	128	0.31	0.30	0.23	0	1
Wage Phillips	38	0.032	0.046	0.018	0.002	0.191
Sectoral export growth	4354	0.23	0.22	0.48	-2.96	3.18
Capital-intensity	6911	0.60	0.61	0.18	0.08	0.91

Table 1.3: Sectoral export growth vs exchange rate regime index (RR), capital-intensity and wage rigidity

	(1)	(2)	(3)	(4)	(5)
RR	-0.111***	0.029	-0.038*	-0.096**	-0.124***
	(0.025)	(0.029)	(0.021)	(0.042)	(0.048)
RR*K-intensity	0.227***			0.228***	0.175**
	(0.034)			(0.054)	(0.088)
RR*labor union		-0.067**		-0.070**	
		(0.027)		(0.028)	
RR*firing cost		-0.110*		-0.119*	
		(0.063)		(0.072)	
RR*wage Phillips			-1.76**		-2.08***
			(0.764)		(0.797)
ln(initial income)	-0.474***	-0.146***	-0.175***	-0.127***	-0.175***
	(0.070)	(0.018)	(0.038)	(0.022)	(0.044)
K-intensity	0.182			-0.327***	-0.207
	(0.125)			(0.161)	(0.239)
Labor union		0.087		0.090	
		(0.058)		(0.059)	
Firing cost		0.164		0.168	
		(0.164)		(0.164)	
Wage Phillips			5.15**		5.71**
			(2.46)		(2.52)
Observations	3,489	867	1,088	867	1,088
R-squared	0.23	0.20	0.24	0.22	0.24

Notes: Standard errors in parentheses, *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 1.4: Sectoral export growth vs exchange rate regime dummy (RR), capital-intensity and wage rigidity

	(1)	(2)	(3)	(4)	(5)
Flexible	-0.074	0.062	-0.104**	-0.204*	-0.160
	(0.065)	(0.066)	(0.050)	(0.104)	(0.150)
Flexible*K-intensity	0.403***			0.588***	0.317
	(0.096)			(0.170)	(0.304)
Flexible*labor union		-0.158**		-0.201**	
		(0.062)		(0.064)	
Flexible*firing cost		-0.136		-0.290**	
		(0.143)		(0.145)	
Flexible*wage Phillips			-2.48**		-2.62*
			(1.31)		(1.51)
ln(initial income)	-0.549***	-0.137***	-0.174***	-0.117***	-0.162***
	(0.070)	(0.019)	(0.038)	(0.023)	(0.044)
K-intensity	0.556			-0.115	0.045
	(0.107)			(0.148)	(0.219)
Labor union		0.016		0.027	
		(0.035)		(0.035)	
Firing cost		0.014		0.064	
		(0.089)		(0.091)	
Wage Phillips			1.86		2.03**
			(1.42)		(1.44)
Observations	3,489	867	1,088	867	1,088
R-squared	0.22	0.20	0.24	0.21	0.24

Notes: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 1.5: Exchange rate regimes (RR) vs ln(GDP per capita), wage rigidity (using labor union index and firing cost)

	(1)	(2)	(3)	(4)	(5)
ln(initial GDP per capita)	0.374*	0.386*	0.446	0.384*	0.637*
	(0.223)	(0.244)	(0.336)	(0.233)	(0.363)
Labor union	-0.269	-0.215	-0.376	-0.256	-0.251
	(0.313)	(0.336)	(0.403)	(0.373)	(0.429)
Firing cost	-1.46**	-1.39**	-1.44**	-1.51**	-2.09**
	(0.68)	(0.69)	(0.69)	(0.727)	(0.791)
Size (country's GDP/US GDP)		7.46*			66.9**
		(4.41)			(30.3)
Geographical area		0.129			0.36
(10^6 square kilometers)		(0.130)			(0.26)
Island dummy		0.645			0.342
		(0.426)			(0.495)
Trade openness		-0.003*			-0.006**
		(0.002)			(0.003)
Financial openness			-0.141		-0.126
			(0.126)		(0.130)
Financial development			0.003		0.001
			(0.005)		(0.006)
Reserve to money base ratio			1.04		2.22*
			(0.92)		(1.32)
Years in office				-0.018	-0.016
				(0.025)	(0.026)
Veto points				-0.871	4.22**
				(1.13)	(1.89)
Continent dummies	Y	Y	Y	Y	Y
Observations	59	59	57	59	57
Pseudo R-squared	0.18	0.21	0.20	0.19	0.24

Notes: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 1.6: Exchange rate regimes (RR) vs ln(GDP per capita), wage rigidity (using wage Phillips curve index)

	(1)	(2)	(3)	(4)	(5)
ln(initial GDP per capita)	0.267	0.249	1.79**	0.487	1.29
	(0.237)	(0.272)	(0.746)	(0.366)	(0.759)
Wage Phillips	-12.3**	-10.86*	-15.38**	-11.09**	-13.49
	(5.64)	(5.91)	(8.09)	(5.88)	(8.57)
Size (country's GDP/US GDP)		3.49			6.93
		(3.68)			(5.35)
Geographical area		0.357			0.346
(10^6 square kilometers)		(0.302)			(0.362)
Island dummy		0.240			0.372
		(0.654)			(0.485)
Trade openness		-0.002			-0.002
		(0.002)			(0.003)
Financial openness			-0.209		-0.354
			(0.237)		(0.330)
Financial development			0.002		0.000
			(0.007)		(0.006)
Reserve to money base ratio			3.47		3.52
			(1.83)		(2.32)
Years in office				-0.018	-0.026
				(0.025)	(0.029)
Veto points				-0.871	3.12
				(1.13)	(2.19)
Continent dummies	Y	Y	Y	Y	Y
Observations	38	38	37	36	36
Pseudo R-squared	0.27	0.28	0.27	0.28	0.30

Notes: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 1.7: Sectoral export growth vs exchange rate regime dummy (RR), capital-intensity and wage rigidty, 2SLS

	(1)	(2)	(3)	(4)	(5)
IV Flexible	-1.28***	-0.175	-0.210	-0.202	-1.19
	(0.383)	(0.791)	(0.154)	(0.791)	(1.40)
IV Flexible*K-intensity	1.69***			1.97*	2.35
	(0.536)			(1.12)	(1.91)
IV Flexible*labor union		-0.154**		-0.117**	
		(0.045)		(0.045)	
IV Flexible*firing cost		-1.13**		-1.14**	
		(0.453)		(0.451)	
IV Flexible*wage Phillips			-12.4***		-2.62*
			(3.85)		(1.51)
ln(initial income)	-0.549*	-0.163**	-0.124	-0.147**	-0.172*
	(0.234)	(0.069)	(0.078)	(0.063)	(0.094)
K-intensity	0.456*			-0.105	0.049
	(0.247)			(0.218)	(0.359)
Labor union		0.023		0.032	
		(0.057)		(0.043)	
Firing cost		0.019		0.084	
		(0.097)		(0.105)	
Wage Phillips			2.86		1.56**
			(4.42)		(4.84)
Observations	2,934	742	914	742	914
R-squared	0.21	0.17	0.28	0.22	0.32

Notes: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 1.8: Sectoral export growth vs exchange rate regime index (LYS), capital-intensity and wage rigidity

	(1)	(2)	(3)	(4)	(5)
LYS	-0.148***	0.026	-0.007	-0.037	-0.050
	(0.024)	(0.021)	(0.015)	(0.032)	(0.036)
LYS*K-intensity	0.206***			0.111***	0.098
	(0.032)			(0.042)	(0.071)
LYS*labor union		-0.045**		-0.039*	
		(0.020)		(0.021)	
LYS*firing cost		-0.045		-0.079*	
		(0.044)		(0.045)	
LYS*wage Phillips			-0.946*		-1.03*
			(0.523)		(0.594)
ln(initial income)	-0.356***	-0.144***	-0.183***	-0.138***	-0.194***
	(0.072)	(0.018)	(0.042)	(0.023)	(0.047)
K-intensity	0.179			-0.180***	-0.163
	(0.125)			(0.157)	(0.235)
Labor union		0.045		0.038	
		(0.045)		(0.046)	
Firing cost		0.067		0.081	
		(0.120)		(0.122)	
Wage Phillips			3.11		3.13
			(2.29)		(2.35)
Observations	3,746	804	1,046	804	1,046
R-squared	0.19	0.19	0.21	0.20	0.22

Notes: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 1.9: Sectoral export growth vs exchange rate regime dummy (LYS), capital-intensity and wage rigidity

	(1)	(2)	(3)	(4)	(5)
Float	-0.458***	0.083	-0.017	-0.020	-0.081
	(0.081)	(0.064)	(0.025)	(0.107)	(0.114)
Float*K-intensity	0.651***			0.229*	0.133
	(0.130)			(0.132)	(0.216)
Float*labor union		-0.134**		-0.132**	
		(0.061)		(0.063)	
Float*firing cost		-0.122		-0.191	
		(0.128)		(0.135)	
Float*wage Phillips			-3.48**		-3.75*
			(1.85)		(1.90)
ln(initial income)	-0.389***	-0.138***	-0.182***	-0.132***	-0.180***
	(0.072)	(0.019)	(0.040)	(0.023)	(0.042)
K-intensity	0.388***			-0.007	0.045
	(0.114)			(0.145)	(0.219)
Labor union		-0.000		0.001	
		(0.030)		(0.031)	
Firing cost		0.020		0.030	
		(0.081)		(0.084)	
Wage Phillips			2.83		2.94**
			(1.89)		(1.90)
Observations	3,746	804	1,046	804	1,046
R-squared	0.18	0.19	0.21	0.20	0.22

Notes: Standard errors in parentheses, *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 1.10: Exchange rate regimes (LYS) vs ln(GDP per capita), wage rigidity (using labor union index and firing cost)

	(1)	(2)	(3)	(4)	(5)
ln(initial GDP per capita)	0.246	0.466*	1.08**	0.279	1.70**
	(0.237)	(0.264)	(0.481)	(0.262)	(0.73)
Labor union	-0.319	-0.310	-0.050	-0.298*	0.052
	(0.427)	(0.441)	(0.483)	(0.161)	(0.502)
Firing cost	-0.860	-1.05*	-0.881**	-0.569	-1.68**
	(0.715)	(0.610)	(0.742)	(0.784)	(0.791)
Size (country's GDP/US GDP)		2.69*			16.7
		(4.48)			(40.3)
Geographical area		0.756**			1.41
(10^6 square kilometers)		(0.329)			(0.94)
Island dummy		0.594			-0.934
		(0.497)			(1.01)
Trade openness		-0.004			-0.001
		(0.003)			(0.003)
Financial openness			-0.279*		-0.292
			(0.172)		(0.223)
Financial development			-0.012*		-0.025**
			(0.007)		(0.011)
Reserve to money base ratio			5.34***		7.32***
			(1.69)		(2.65)
Years in office				-0.018	0.065*
				(0.025)	(0.035)
Veto points				-0.871	-2.58
				(1.13)	(2.18)
Continent dummies	Y	Y	Y	Y	Y
Observations	56	56	53	56	53
Pseudo R-squared	0.10	0.14	0.12	0.10	0.18

Notes: Standard errors in parentheses, *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 1.11: Exchange rate regimes (LYS) vs ln(GDP per capita), wage rigidity (using wage Phillips curve index)

	(1)	(2)	(3)	(4)	(5)
ln(initial GDP per capita)	0.400	0.336	0.351	0.085	0.702
	(0.321)	(0.362)	(0.489)	(0.407)	(0.722)
Wage Phillips	-2.45	-0.457	-2.21	-0.074	-1.67
	(6.46)	(6.71)	(7.17)	(6.53)	(7.86)
Size (country's GDP/US GDP)		9.23			14.2
		(8.24)			(10.35)
Geographical area		0.254			0.346
(10^6 square kilometers)		(0.267)			(0.662)
Island dummy		0.620			0.472
		(0.824)			(0.985)
Trade openness		-0.008			-0.004
		(0.008)			(0.009)
Financial openness			-0.827		-0.354
			(0.656)		(0.730)
Financial development			0.037		0.004
			(0.025)		(0.036)
Reserve to money base ratio			56.7*		43.5
			(22.1)		(25.3)
Years in office				-0.013	-0.017
				(0.027)	(0.031)
Veto points				-0.732	-0.982
				(1.23)	(2.19)
Continent dummies	Y	Y	Y	Y	Y
Observations	37	37	36	36	36
Pseudo R-squared	0.23	0.28	0.22	0.29	0.36

Notes: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 1.12: Definitions and Sources of Variables Used in Regression Analysis

Variable	Sourse
RR	De facto exchange rate regime index, using the classification in Reinhart and Rogoff (2004).
	1–peg, 2–crawling peg, 3–managed floating, 4–free floating
Flexible	A dummy variable that takes value 1 if the de facto exchange rate regime is a managed floating
	or a free floating, using RR exchange rate regime index.
LYS	De facto exchange rate regime index. Author's calcualtion by modifying the classification in
	Levy-Yeyati and Sturzenegger (2003). 1–fixed, 2–crawling peg, 3–dirty float, 4–float
Float	A dummy variable that takes value 1 if the de facto exchange rate regime is a managed floating
	or a free floating, using the modified LYS index
Labor union	Data from Doing Business Report, Worldbank
Firing cost	An index shows the cost of firing a worker. Data from Doing Business Report, Worldbank
Wage Phillips	The parameter in the wage Phillips curve that governs the wage rigidity levels.
	Author's calculation, estimating the wage Phillips curves for 38 countries using the method in Gali (2010).
Sectoral export growth	Export growth in 24 manufacturing sectors. Data from CEPII database.
K-intensity	Capital-intensity, defined as the share of total capital input to the total cost.
Size	Country's GDP to US GDP
Geographical area	Data can be obtained from CEPII database.
Island dummy	Data can be obtained from CEPII database.
Trade openness	(import+export)/GDP, data from Penn World Table 6.3
Financial openness	Measure of capital openness provided by Chinn and Ito (2007)
Financial development	Private credit/GDP, data from Worldbank database.
Reserve to money base ratio	Data can be obtained from World bank database.
Years in office	Years the incumbent administration has been in office. Database of Political Institutions 2006
Veto points	An index between 0 and 1, referred to the extent of institutionalized constraints on the
	decision-making powers of chief executives, whether individuals or collectivities. Polcon 2005 Database.

Chapter 2

Sex Ratios, Savings Rates and Current Account Imbalances

with Shang-Jin Wei

High savings rates in excess of domestic investment rates in many countries in East and Southeast Asia have produced a massive current account surplus as a share of GDP, and are said to be a major contributor to the global current account imbalances, to the unusually low long-term interest rates, and possibly to the onset of the 2008-2009 global financial crisis. As to theories of savings behavior, the existing literature has highlighted the roles of life-cycle considerations (Modigliani, 1970), precautionary savings (Kimball, 1990), habit formation (Carroll, Overland, and Weil, 2008), culture (Belton and Uwaifo Oyelere, 2008), and financial under-development (Caballero, Farhi, and Gourinchas, 2008; Ju and Wei, 2006, 2008 and 2010; Mendoza, Quadrini and Rios-Rull, 2007). The aim of the current paper is to propose an alternative theory that gives prominence to a major, albeit insufficiently recognized by macroeconomists, social transformation in many economies, namely an increasing gap in the numbers of men and women in the marriage market. The basic thesis is that as competition intensifies in the marriage market, men or parents with sons raise their savings rates with the hope of improving their relative standing in the marriage market. Because the biological desire to have a partner of the opposite sex is strong, this effect is quantitatively important enough to reveal itself in the aggregate savings rate and the current account balance.

A direct source of the idea for the theory is an empirical paper by Wei and Zhang (2009), which studies household savings behavior in China. They provide both cross-regional and cross-household evidence that is consistent with the notion that a worsening prospect for men in the marriage market has motivated them and their parents to raise their savings rates substantially. They call this the "competitive saving motive." Chinese household savings as a share of disposable income rose from 16% in 1990 to 30% in 2007. Wei and Zhang suggest that the rise in the sex ratio imbalance could account for half the total increase in the savings rate. Because their paper does not have a formal theory, there is a need to construct a model to see if the hypothesis can work in a general equilibrium, and whether a calibration of the model can produce an effect whose magnitude is economically significant.

In this paper, we aim to fill these important voids. The core part of the paper is to analyze theoretically whether and how a sex ratio imbalance will influence the economy-wide savings rate and the current account. We construct a simple overlapping generations (OLG) model with two sexes and a desire to marry. To focus on the macroeconomic implications of sex ratio imbalances, we intentionally shut down channels such as the usual precautionary savings motive, habit formation, culture, and financial development. Because it is an OLG model, there are still life-cycle considerations, which, however, do not lead to current account imbalances on their own.

Under reasonable conditions, we show that men respond to a rise in the sex ratio by raising their savings rates. Moreover, the increment in their savings is always enough to offset any decrease in women's savings. As a result, the aggregate savings rises with the sex ratio. We also discuss a number of extensions that aim to allow for additional realism: (a) incorporate parental savings for children, (c) introduce intra-household bargaining, and (c) consider an OLG structure in which each generation lives for 50 periods and makes savings decisions in multiple periods. In each case, under reasonably general conditions, both the aggregate savings rate and current account rise in response to a rise in the sex ratio.

To check if the model can deliver an effect that is economically significant, we go to quantitative calibrations. In the benchmark case, for a small open economy, as the sex ratio rises from 1 to 1.15, the economy-wide savings rate and the current account will rise by more than 10%. We also consider the case of two large economies, whose relative sizes and income levels are calibrated to mimic China and the United States. The synthetic United States is assumed to always have a balanced sex ratio, while the synthetic China experiences a rise in the sex ratio from 1 (balanced) to 1.5 (very unbalanced). The rise in China's sex ratio produces a rise in its current account surplus, and a corresponding rise in the current account deficit for the United States. The magnitudes of the current account imbalances in the simulations (about 7.7% of GDP for China and -2.6% of GDP for the United States) are such that they are more than one-half of the actual current account imbalances observed in the data. While the sex ratio imbalance is not the sole reason for the global current account imbalances in recent years, it could be one of the significant, and yet thus far unrecognized, factors.

A desire to enhance one's prospects in the marriage market through a higher level of wealth could be a motive for savings even in countries with a balanced sex ratio. But such a motive is not as easy to detect when the competition is modest. When the sex ratio gets out of balance, obtaining a marriage partner becomes much less assured. A host of behaviors that are motivated by a desire to succeed in the marriage market may become magnified. But sex ratio imbalances so far have not been investigated by macroeconomists. This may be a serious omission. A sex ratio imbalance at birth and in the marriage age cohort is a common demographic feature in many economies, especially in East, South, and Southeast Asia, such as Korea, India, Vietnam, Singapore, Taiwan and Hong Kong, in addition to China. In many economies, parents have a preference for a son over a daughter. This used to lead to large families, not necessarily an unbalanced sex ratio. However, in the last three decades, as the technology to detect the gender of a fetus (Ultrasound B) has become less expensive and more widely available, many more parents engage in selective abortions in favor of a son, resulting in an increasing relative surplus of men. The spread of technology started in the early 1980s and accelerated quickly afterwards. 1985 was the first year in which half of the county-level hospitals in China had acquired at least one Ultrasound B machine. By early 1990s, all county-level hospitals had at least one such machine (Ebenstein, Li, and Meng, 2010). The strict family planning policy in China, introduced in the early 1980s, has induced Chinese parents to engage in sex-selective abortions more aggressively than their counterparts in other countries. The sex ratio at birth in China rose from 106 boys per hundred girls in 1980 to 122 boys per hundred girls in 1997 (see Wei and Zhang, 2009, for more detail). It may not be a coincidence that the Chinese current account surplus started to garner international attention around 2002 just when the first cohort born after the implementation of the strict family planning policy was entering the marriage market.

Throughout the model, we assume an exogenous sex ratio. While the sex ratio is endogenous in the long-run as parental preference evolves, the assumption of an exogeous sex ratio can be defended on two grounds. First, the technology that enables the rapid rise in the sex ratio has only become inexpensive and widely accessible in developing countries within the last 25 years or so. As a result, it is reasonable to think that the rising sex ratio affects only the relatively young cohorts' savings decisions, but not those who have passed half of their working careers. Second, data suggests that if the preference for son has a mean-reverting property, it must be a very slow-moving process. Almost all countries that have a skewed sex ratio today have exhibited a gradual climb over the last decade or two. Korea is the only economy whose sex ratio appears to have started to revert back from a very skewed level. This suggests that a systematic reversal of the sex ratio is unlikely to happen in most economies in the short run.

To see if the theoretical predicion has any support in the data, we check if a country's private sector savings rate (defined as current account minus government savings, divided by GDP) is systematically linked to its sex ratio. After controlling for the effects on the savings rate from income, the share of working age people in the population (i.e., a proxy for the life cycle theory), the ratio of private bank credit to GDP (a proxy for financial development), and social security expenditure as a share of GDP (a proxy for the precautionary savings motive), we find that a rise in the sex ratio from a balanced level to 1.15 (the current sex ratio for the pre-marital age cohort in China) is associated with a higher current account (excluding government savings) by over 10% of GDP.

There are three bodies of work that are related to the current paper. First, the literature on status goods, positional goods, and social norms (e.g., Cole, Mailath and Postlewaite, 1992, Corneo and Jeanne, 1999, Hopkins and Kornienko, 2004 and 2009) has offered many useful insights. One key point is that when wealth can improve one's social status (including improving one's standing in the marriage market), in addition to affording a greater amount of consumption goods, there is an extra incentive to save. This element is in our model as well. However, all existing theories on status goods feature a balanced sex ratio. Yet, an unbalanced sex ratio presents some non-trivial challenges. In particular, while a rise in the sex ratio is an unfavorable shock to men (or parents with sons), it is a favorable shock to women (or parents with daughters). Could the latter group strategically reduce their savings so as to completely offset whatever increments in savings men or parents with sons may have? In other words, the impact on aggregate savings appears ambiguous. Our model will address this question. In any case, the literature on status goods has no discernible impact in policy circles. For example, while there are voluminous documents produced by the International Monetary Fund or speeches by U.S. officials on China's high savings rate and large current account surplus, no single paper or speech thus far has pointed to a possible connection with its high sex ratio imbalance.

A second related literature is the economics of family, which is too vast to be summarized here comprehensively. One interesting insight of this literature is that a married couple's consumption has a partial public goods feature (Browning, Bourguignon and Chiappori, 1994; Donni, 2006). We make use of this feature in our model as well. None of the papers in this literature explores the general equilibrium implications for aggregate savings from a change in the sex ratio.

The third literature examines empirically the causes of a rise in the sex ratio. The key insight is that the proximate cause responsible for a majority of the recent rise in the sex ratio imbalance is sex-selective abortions, which have been made increasingly possible by the spread of Ultrasound B machines. There are two deeper causes for parental willingness to disproportionately abort female fetuses. The first is the parental preference for sons, which in part has to do with the relatively inferior economic status of women. When the economic status of women improves, sex-selective abortions appear to decline (Qian, 2008). The second is either something that leads parents to voluntarily choose to have fewer children than the earlier generations, or a government policy that limits the number of children a couple can have. In regions of China where the family planning policy is less strictly enforced, there is also less sex ratio imbalance (Wei and Zhang, 2009). Bhaskar (2011) examines parental sex selections and their welfare consequences.

The rest of the paper is organized as follows: in Section 2, we present a benchmark model with no intra-household bargaining. In Section 3, we consider a number of extensions. One such extension allows for intra-household bargaining, and shows that the key propositions still hold. In Section 4, we calibrate the model to see if the sex ratio imbalance can produce changes in the aggregate savings rate and current account whose magnitudes are economically significant. In Section 5, we provide some empirical evidence that the sex ratio may have a significant impact on a country's current account. Finally, in section 6, we offer concluding remarks and discuss possible future research.

2.1 The Benchmark Model

We construct an overlapping generations model with two sexes. Both men and women live two periods: young and old. An individual (of either sex) receives an exogenous endowment in the first period and nothing in the second period. She or he consumes a part of the endowment in the first period and saves the rest for the second period.

A marriage can only take place between a man and a women in the same generation and at the beginning of their second period. Once married, the husband and the wife pool their first-period savings together and consume an identical amount in the second period. The second period consumption within a marriage has a partial public good feature. In other words, the husband and the wife can each consume more than half of their combined second period income - the exact proportion is an exogenous parameter to be explained below. Everyone is endowed with an ability to give his/her spouse some emotional utility (or "love" or "happiness"). This emotional utility is a random variable in the first period with a common and known distribution across all members of the same sex, and its value is realized and becomes public information when the individual enters the marriage market.

Each generation is characterized by an exogenous ratio of men to women $\phi(>= 1)$. All men are identical *ex ante*, and all women are identical *ex ante*. Men and women are symmetric in all aspects except that the sex ratio may be unbalanced.

We describe the equilibrium in this economy in six steps. First, we start with a representative woman's optimization, followed by a representative man's optimization problem. Second, we describe how the marriage market works. Third, we perform comparative statics, in particular, on how the savings rates change in response to a rise in the sex ratio. Fourth, we consider a small open economy with production and discuss the current account response to a change in the sex ratio. Fifth, we solve for a two-country model in which the global interest rate is endogenous. Sixth, we use numerical calibrations to see if the model can deliver current account responses that are economically significant.

2.1.1 A Representative Woman's Optimization Problem

A representative woman makes her consumption/saving decisions in her first period, taking as given the choices made by men and all other women. If she is not married, her second-period consumption is

$$c_{2w,n} = Rs^w y^w$$

where R, y^w and s^w are the gross interest rate, her endowment, and savings rate, respectively.

If she is married (at the beginning of the second period), her second-period consumption is

$$c_{2w} = \kappa \left(Rs^w y^w + Rs^m y^m \right)$$

where y^m and s^m are her husband's endowment and savings rate, respectively. κ ($\frac{1}{2} =< \kappa =< 1$) represents the notion that consumption within a marriage is a public good with congestion. As an example, if a couple buys a car, both spouses can use it. When $\kappa = \frac{1}{2}$, the husband and the wife only consume private goods. In contrast, when $\kappa = 1$, all the consumption is a public good with no congestion[1].

She chooses her savings rate to maximize the following objective function:

$$V^w = \max_{s^w} u(c_{1w}) + \beta E \left[u(c_{2w}) + \eta^m \right]$$

subject to the budget constraints that

$$c_{1w} = (1 - s^w) y^w \tag{2.1}$$

$$c_{2w} = \begin{cases} \kappa \left(Rs^w y^w + Rs^m y^m \right) & \text{if married} \\ Rs^w y^w & \text{otherwise} \end{cases} \tag{2.2}$$

where V^w is her value function, and E is the expectation operator. η^m is the emotional utility (or "love") she obtains from her husband, which is a random variable with a distribution function F^m. Utility function $u(\cdot)$ satisfies the standard properties that $u' > 0$, and $u'' < 0$. The exact value of emotional utility is revealed at the beginning of the second period and becomes a common knowledge at that time. Bhaskar (2009) also introduces a

[1] By assuming the same κ for the wife and the husband, we abstract from a discussion of bargaining within a household. In an extension later in the paper, we allow κ to be gender specifc, and to be a function of the sex ratio and the relative wealth levels of the two spouses, along the lines of Chiappori (1988 and 1992) and Browning and Chiappori (1998). This tends to make the response of the aggregate savings stronger to a given rise in the sex ratio.

similar "love" variable.

2.1.2 A Representative Man's Optimization Problem

A representative man has a similar optimization problem as the representative woman. In particular, if he is not married, his second-period consumption is

$$c_{2m,n} = Rs^m y^m$$

If he is married, his second-period consumption is

$$c_{2m} = \kappa \left(Rs^w y^w + Rs^m y^m \right)$$

He chooses his savings rate to maximize the following value function:

$$V^m = \max_{s^m} u(c_{1m}) + \beta E \left[u(c_{2m}) + \eta^w \right]$$

subject to the budget constraints that

$$c_{1m} = (1 - s^m) y^m \tag{2.3}$$

$$c_{2m} = \begin{cases} \kappa \left(Rs^w y^w + Rs^m y^m \right) & \text{if married} \\ Rs^m y^m & \text{otherwise} \end{cases} \tag{2.4}$$

where V^m is his value function. η^w is the emotional utility he obtains from his wife, which is drawn from a distribution function F^w. We assume η^w and η^m are independent.

2.1.3 The Marriage Market

In the marriage market, every woman (or man) ranks all members of the opposite sex by a combination of two criteria: (1) the level of wealth (which is determined solely by the first-period savings), and (2) the size of "love" he/she can obtain from his/her spouse. The weights on the two criteria are implied by the utility functions specified earlier. More precisely, woman i prefers a higher ranked man to a lower ranked one, where the rank on man j is given by $u(c_{2w,i,j}) + \eta_j^m$. Symmetrically, man j assigns a rank to woman i based on the utility he can obtain from her $u(c_{2m,j,i}) + \eta_i^w$. (To ensure that the preference is strict for men and women, when there is a tie in terms of the above criteria, we break the tie by assuming that a woman prefers j if $j < j'$ and a man does the same.) Note that "love" is not in the eyes of the beholder in the sense that every woman (man) has the same ranking over men (women).

The marriage market is assumed to follow the Gale-Shapley algorithm, which produces a unique and stable equilibrium of matching (Gale and Shapley, 1962; and Roth and Sotomayor, 1990). The algorithm specifies the following: (1) Each man proposes in the first round to his most preferred choice of woman. Each woman holds the proposal from her most preferred suitor and rejects the rest. (2) Any man who is rejected in round k-1 makes a new proposal in round k to his most preferred woman among those who have not yet rejected him. Each available women in round k "holds" the proposal from her most preferred man and rejects the rest. (3) The procedure repeats itself until no further proposals are made.[2]

With many women and men in the marriage market, all women (and all men) approximately form a continuum and each individual has a measure close to zero. Let I^w and I^m denote the continuum formed by women and men, respectively. We normalize I^w and let $I^w = (0, 1)$. Since the sex ratio is ϕ, the set of men $I^m = (0, \phi)$. Men and women are ordered in such a way that a higher value means a higher ranking by members of the opposite sex.

In equilibrium, there exists a unique mapping (π^w) for women in the marriage market.

$$\pi^w : I^w \to I^m$$

That is, woman i ($i \in I^w$) is mapped to man j ($j \in I^m$), given all the initial wealth and

[2] If only women can propose and men respond with deferred acceptance, the same matching outcomes will emerge. What we have to rule out is that both men and women can propose, in which case, one cannot prove that the matching is unique.

emotional utility draws. This implies a mapping from a combination $(s_i^w,\, \eta_i^w)$ to another combination $(s_j^m,\, \eta_j^m)$. In other words, for woman i, given all her rivals' $(s_{-i}^w,\, \eta_{-i}^w)$ and all men's $(s^m,\, \eta^m)$, the type of husband j she can marry depends on her $(s_i^w,\, \eta_i^w)$. Before she enters the marriage market, she knows only the distribution of her own type but not the exact value. As a result, the type of her future husband $(s_j^m,\, \eta_j^m)$ is also a random variable. Woman i's second period expected utility is

$$\begin{aligned} & \int \max \left[\; u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(\, i | s_i^w, \eta_i^w, s_{-i}^w, \eta_{-i}^w, s^m, \eta^m\right)}, \quad u\left(R s_i^w y_i^w\right) \; \right] dF^w\left(\eta_i^w\right) \\ = \; & \int_{\bar{\pi}_i^w} \left[u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(\, i | s_i^w, \eta_i^w, s_{-i}^w, \eta_{-i}^w, s^m, \eta^m\right)} \right] dF^w\left(\eta_i^w\right) + \int^{\bar{\pi}_i^w} u\left(R s_i^w y_i^w\right) dF^w\left(\eta_i^w\right) \end{aligned}$$

where $\bar{\pi}_i^w$ is her threshold ranking on men such that she is indifferent between marriage or not. Any lower-ranked man, or any man with $\pi_i^w < \bar{\pi}_i^w$, won't be chosen by her.

Since we assume there are (weakly) fewer women than men, we expand the set I^w to $\widetilde{I}^w$ so that $\widetilde{I}^w = (0, \phi)$. In the expanded set, women in the marriage market start from value $\phi - 1$ to ϕ. The measure for women in the marriage market remains one. In equilibrium, there exists an unique mapping for men in the marriage market:

$$\pi^m : I^m \to \widetilde{I}^w$$

where π^m maps man j $(j \in I^m)$ to woman i $(i \in I^w)$. Those men who are matched with a low value $i < \phi - 1$ in set $\widetilde{I}^w$ will not be married. In that case, $\eta^w_{\pi^m(j)} = 0$ and $c_{2m,j,i} = R s_j^m y_j^m$. In general, man j's second period expected utility is

$$\begin{aligned} & \int \max \left[\; u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(\, j | s_j^m, \eta_j^m, s_{-j}^m, \eta_{-j}^m, s^w, \eta^w\right)}, \quad u\left(R s_j^m y_j^m\right) \; \right] dF^m\left(\eta_j^m\right) \\ = \; & \int_{\bar{\pi}_j^m} \left[u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(\, j | s_j^m, \eta_j^m, s_{-j}^m, \eta_{-j}^m, s^w, \eta^w\right)} \right] dF^m\left(\eta_j^m\right) + \int^{\bar{\pi}_j^m} u\left(R s_j^m y_j^m\right) dF^m\left(\eta_j^m\right) \end{aligned}$$

where $\bar{\pi}_j^m$ is his threshold ranking on all women. Any woman with a poorer rank, $\pi_j^m < \bar{\pi}_j^m$, will not be chosen by him.

We assume that the density functions of η^m and η^w are continuously differentiable. Since all men (women) in the marriage market have identical problems, they make the same savings decisions. In equilibrium, a *positive assortative matching* emerges for those men and women who are matched. In other words, there exists a mapping M from η^w to η^m such that

$$\begin{aligned} 1 - F^w(\eta^w) &= \phi\left(1 - F^m\left(M(\eta^w)\right)\right) \\ &\Leftrightarrow \\ M(\eta^w) &= \left(F^m\right)^{-1}\left(\frac{F^w(\eta^w)}{\phi} + \frac{\phi - 1}{\phi}\right) \end{aligned}$$

For simplicity, we assume that η^w and η^m are drawn from the same distribution, $F^w = F^m = F$. The lowest possible value of emotional utility $\eta^{\min}$ is assumed to be sufficiently small (and can be negative) such that any person with a low realized value of emotional utility may not succeed in getting married. Define $\bar{\eta}^w$ and $\bar{\eta}^m$ as the threshold values of emotional utility for women and men, respectively, such that only those with emotional utilities higher than the threshold value will get married. In other words,

$$\bar{\eta}^w = \max\left\{u_{2m,n} - u_{2m}, M^{-1}\left(\bar{\eta}^m\right)\right\} \text{ and } \bar{\eta}^m = \max\left\{u_{2w,n} - u_{2w}, M(\bar{\eta}^w)\right\} \tag{2.5}$$

For woman i, given all her rivals' and men's savings decisions and η^w, her second period utility is

$$\delta_i^w u(\kappa\left(Rs_i^w y^w + Rs^m y^m\right)) + \left(1 - \delta_i^w\right) u(Rs^w y^w) + \int_{\tilde{\eta}_i^w \geq \bar{\eta}^w} M(\tilde{\eta}_i^w) dF(\eta_i^w)$$

where $\tilde{\eta}_i^w = u(\kappa\left(Rs_i^w y^w + Rs^m y^m\right)) - u(\kappa\left(Rs^w y^w + Rs^m y^m\right)) + \eta_i^w$. δ_i^w is the probability that she will get married,

$$\begin{aligned} \delta_i^w &= \Pr\left(u(\kappa\left(Rs_i^w y^w + Rs^m y^m\right)) - u(\kappa\left(Rs^w y^w + Rs^m y^m\right)) + \eta_i^w \geq \bar{\eta}^w \middle| Rs^w y^w, Rs^m y^m\right) \\ &= 1 - F\left(\bar{\eta}^w - u(\kappa\left(Rs_i^w y^w + Rs^m y^m\right)) + u(\kappa\left(Rs^w y^w + Rs^m y^m\right))\right) \end{aligned} \tag{2.6}$$

Due to symmetry (i.e., all women are identical ex ante), we drop sub-index i for women in subsequent discussion. Given men's savings decisions, the first order condition for her optimization problem is

$$-u'_{1w}y^w + \beta \left[\begin{array}{c} \delta^w u'_{2w} \frac{\partial c_{2w}}{\partial s^w} + (1-\delta^w)\, u'_{2w,n} R y^w + \frac{\partial \int_{\tilde{\eta}^w \geq \bar{\eta}^w} M(\tilde{\eta}^w) dF(\eta^w)}{\partial s^w} \\ + \frac{\partial \delta^w}{\partial s^w} \left(u_{2w} - u_{2w,n}\right) \end{array} \right] = 0 \qquad (2.7)$$

where

$$\begin{aligned} \frac{\partial \int_{\tilde{\eta}^w \geq \bar{\eta}^w} M(\tilde{\eta}^w) dF(\eta^w)}{\partial s^w} &= \kappa u'_{2w} R y^w \left[\int_{\bar{\eta}^w} M'(\eta^w) dF(\eta^w) + M(\bar{\eta}^w) f(\bar{\eta}^w) \right] \\ \frac{\partial \delta^w}{\partial s^w} &= f\left(\bar{\eta}^w\right) \kappa u'_{2w} R y^w \end{aligned}$$

Similarly, a representative man's second-period utility, given his rivals' and all women's savings decisions, is

$$\delta_j^m u\left(\kappa\left(Rs^w y^w + Rs_j^m y^m\right)\right) + (1-\delta_j^m) u(Rs_j^m y^m) + \int_{\tilde{\eta}_j^m \geq \bar{\eta}^m} M^{-1}\left(\tilde{\eta}_j^m\right) dF(\eta_j^m)$$

where $\tilde{\eta}_j^m = u\left(\kappa\left(Rs^w y^w + Rs_j^m y^m\right)\right) - u(\kappa\left(Rs^w y^w + Rs^m y^m\right)) + \eta_j^m$ and δ_j^m is his probability of marriage.

$$\begin{aligned} \delta_j^m &= \Pr\left(u\left(\kappa\left(Rs^w y^w + Rs_j^m y^m\right)\right) - u(\kappa\left(Rs^w y^w + Rs^m y^m\right) + \eta_j^m \geq \bar{\eta}^m \middle| Rs^w y^w, Rs^m y^m \right) \\ &= 1 - F\left(\bar{\eta}^m - u\left(\kappa\left(Rs^w y^w + Rs_j^m y^m\right)\right)\right) + u(\kappa\left(Rs^w y^w + Rs^m y^m\right)) \qquad (2.8) \end{aligned}$$

The first order condition for his optimization problem is

$$-u'_{1m}y^m + \beta \left[\begin{array}{c} \delta^m u'_{2m} \frac{\partial c_{2m}}{\partial s^m} + \frac{\partial \int_{\tilde{\eta}^m \geq \bar{\eta}^m} M^{-1}(\tilde{\eta}^m) dF(\eta^m)}{\partial s^m} + (1-\delta^m)\, u'_{2m,n} R y^m \\ + \frac{\partial \delta^m}{\partial s^m} \left(u_{2m} - u_{2m,n}\right) \end{array} \right] = 0 \quad (2.9)$$

where

$$\begin{aligned}
\frac{\partial \int_{\tilde{\eta}^m \geq \bar{\eta}^w} M^{-1}(\tilde{\eta}^m) dF(\eta^m)}{\partial s^m} &= \kappa u'_{2m} R y^m \left[\int_{\bar{\eta}^m} \frac{\partial M^{-1}(\eta^m)}{\partial \eta^m} dF(\eta^m) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m) \right] \\
\frac{\partial \delta^m}{\partial s^m} &= f\left(\bar{\eta}^m\right) \kappa u'_{2m} R y^m
\end{aligned}$$

In the rest of the paper, we assume that the average value of emotional utility $E\eta$ is sufficiently high such that a representative man, ex ante, always prefers marriage to being single. For simplicity, we also assume $\beta R = 1$ througout the paper.

2.1.4 Equilibrium Savings Rates

In the benchmark, we assume that all women and men automatically enter the marriage market (We will later consider an extension in which agents decide whether or not to enter the marriage market). An equilibrium is defined as a collecton of savings rates by men and women that solve their respective optimalization problems, taking all other men and women's decisions as given.

Definition 1. *An equilibrium is* $\{s^w, s^m | y^w, y^m, F^w, F^m\}$ *that satisfies the following conditions:*

$$s_i^w = \arg\max \left(V_i^w \middle| s_{-i}^w, s^m, y^w, y^m, F^w, F^m \right)$$

and

$$s_j^m = \arg\max \left(V_j^m \middle| s^w, s_{-j}^m, y^w, y^m, F^w, F^m \right)$$

where i *and* j *stand for a representative woman and man, respectively, and* $-i$ *and* $-j$ *represent all women other than* i *and all men other than* j*, respectively.* $s^w = \left(s_i^w, s_{-i}^w\right)$ *and* $s^m = \left(s_j^m, s_{-j}^m\right)$ *are the sets of women's and men's savings rates respectively.*

To simplify the discussion, we assume that the population growth rate is zero, and women and men receive the same first period income ($y^w = y^m = y$). Before period t, the economy has a balanced sex ratio. In this case, $s^w = s^m = s$, and s can be obtained from solving the set of first order conditions (3.14) or (3.16):

$$-u'_{1w} + 2\left(1 - F\left(\bar{\eta}\right)\right)\kappa u'_2 + F\left(\bar{\eta}\right)u'_{2n} = 0 \tag{2.10}$$

and

$$\bar{\eta} = u_2 - u_{2n}$$

where we use the fact that at $\phi = 1$, $M(\eta) = \eta$.

The first key proposition concerns the effect of a rise in the sex ratio on the aggregate savings rate. The thought experiment assumes that people in the old cohort have made their savings decision when the sex ratio is balanced. When the sex ratio rises, any change in the aggregate savings is driven by a change in the savings by the young cohort. This simplifying assumption is motivated by the reality: A rise in the sex ratio in almost all economies is a recent phenomenon, since large-scale sex-selective abortions are a recent phenomenon. More precisely, while the diagnostic sonography used for prenatal checkups was available in the 1960s, the procedure became gradually more affordable to people in countries that have a high sex ratio only since the 1980s. (The strict version of the Chinese family planning policy, another contributor to the spread of sex-selective abortions, was also put in place in the early 1980s.) For this reason, the savings pattern for the currently old was largely decided when there was no severe sex ratio imbalance.

In what follows, whenever we say a man (or woman), we mean a young man (or woman), unless otherwise specified. We first state the proposition formally, and then explain the intuition behind the key parts of the proposition. A detailed proof is provided in Appendix A.

Proposition 5. *Assume emotional utility η^w and η^m are drawn from an independent and identical uniform distribution $[\eta^{\min}, \eta^{\max}]$, and*

$$E(\eta) \geq \frac{R\kappa u'_2}{2}\sqrt{\frac{\max\left(0, \kappa u'_2\left(u'_{2w,n} + u'_{2m,n}\right) - u'_{2w,n}u'_{2m,n}\right)}{u''_{1m}u''_{1w}}}$$

Then, as the sex ratio rises, (1) the savings rate of the representative man goes up, but the change in the savings rate by the representative woman is ambiguous; (2) however, the economy-wide savings rate increases unambiguously.

Proof. See Appendix A2.1. □

Three remarks are in order. First, the inquality condition

$$E(\eta) \geq \frac{R\kappa u_2'}{2}\sqrt{\frac{\max\left(0, \kappa u_2'\left(u_{2w,n}' + u_{2m,n}'\right) - u_{2w,n}' u_{2m,n}'\right)}{u_{1m}'' u_{1w}''}}$$

basically states that the expected value of the emotioinal utility one gets from his/her spouse is not too small (so marriage is valuable). This is not a demanding condition. If the period utility function takes the log form, $u(c) = \ln c$, then the right-hand-side of the inequality can be shown to be zero. That is,

$$\kappa u_2'\left(u_{2w,n}' + u_{2m,n}'\right) - u_{2w,n}' u_{2m,n}' = R^2 y^2 \left(\frac{1}{s^m + s^w}\left(\frac{1}{s^w} + \frac{1}{s^m}\right) - \frac{1}{s^m s^w}\right) = 0$$

This means that the condition becomes $E\eta \geq 0$.

Second, it is perhaps not surprising that the representative man raises his savings rate in response to a rise in the sex ratio since the need to compete in the marriage market becomes greater. Why is the impact of a higher sex ratio on a representative woman's savings rate ambiguous? The answer is that a higher sex ratio produces two offsetting effects for her. On the one hand, as she anticipates more savings from her future husband, she can free-ride and does not need to sacrifice her first-period consumption as much as she otherwise would have to. On the other hand, precisely because men have increased their savings rate in the first period in response to a higher sex ratio, they will be more reluctant to share their wealth with a woman with both a low savings rate and a low emotional utility. The last point raises the probability that low-savings women may not get married. Since the representative women also prefers marriage than spinsterhood, she may raise her savings rate to improve her chance in the marriage market. Because the two effects go in the opposite directions, the net effect of a higher sex ratio on a representative woman's savings is ambiguous.

Third, why does the aggregate savings rate rise in response to a higher sex ratio even if women reduce their savings? Put it differently, why is the increment in men's savings greater than the decline in women's savings? To see this intuitively, one has to recognize two separate motivations for a representative man to raise his savings rate. In addition to improving his relative standing in the marriage market, he has to raise his savings rate to make up for the lower savings rate by his future wife. The more his future wife is expected to cut down her savings, the more he would have to raise his own savings to compensate. Heuristically, this ensures that his incremental savings is more than enough to offset any reduction in his future wife's savings. In addition, since men save more, the rising share of men in the population as a result of a higher sex ratio would also raise the aggregate savings rate. While both channels contribute to a rise in the aggregate savings rate, it is easy to verify that the first channel (the incremental competitive savings by any given man) is more important than the second effect (a change in the composition of the population with different savings propensities).

2.1.5 Mixed-strategy equilibrium

In this section, we extend our benchmark model by allowing men and women to choose to enter and exit the marriage market. Formally, this is a mixed-strategy game in which the representative woman chooses the probability of entering the marriage market ρ^w, a savings rate if she decides to enter, and a separate savings rate if decides to abstain from the marriage market.

Conditioning on deciding to enter the marriage market, she has the same optimization problem as in the previous section. However, she can also choose to be single, and conditional on such a choice, her life-time utility is

$$V_n^w = \max_{s_n^w} u(c_{1w,n}) + \beta u(c_{2w,n})$$

where V_n^w denotes the value function of a representative woman who is single throughout her life.

Her overall optimization problem when she is young is

$$\max_{\rho^w, s^w, s_n^w} \rho^w V^w + (1 - \rho^w) V_n^w$$

Obviously, she would choose $\rho^w = 1$ if and only if $V^w > V_n^w$.

Similarly, a representative man chooses the probability of entering the marriage market ρ^m as well as two potentially separate savings rates. His overall optimization problem is

$$\max_{\rho^m, s^m, s_n^m} \rho^m V^m + (1 - \rho^m) V_n^m$$

where V_n^m denotes the value function of a representative man who is single throughout his life. Obviously, the representative man decides to enter the marriage market with probability one if and only if the expected utility of doing so is greater than otherwise, or $V^m > V_n^m$.

Now we can re-define the equilibrium as following:

Definition 2. *An equilibrium is* $\{s^w, s^m, s_n^w, s_n^m, \rho^w, \rho^m | \, y^w, y^m, F^w, F^m\}$ *that satisfies the following conditions:*

$$\left(s_i^w, s_{n,i}^w, \rho_i^w\right) = \arg\max \left(\rho_i^w V_i^w + (1 - \rho_i^w) V_{n,i}^w \middle| \, s_{-i}^w, s^m, s_{n,-i}^w, s_n^m, \rho_{-i}^w, \rho^m, y^w, y^m, F^w, F^m\right)$$

and

$$\left(s_j^m, s_{n,j}^m, \rho_j^m\right) = \arg\max \left(\rho_j^m V_j^m + (1 - \rho_j^m) V_{n,j}^m \middle| \, s^w, s_{-j}^m, s_n^w, s_{n,-j}^m, \rho^w, \rho_{-j}^m, y^w, y^m, F^w, F^m\right)$$

where i and j stand for a representative woman and man, respectively, and $-i$ and $-j$ represent all women other than i and all men other than j, respectively. $s^w = \left(s_i^w, s_{-i}^w, s_{n,i}^w, s_{n,-i}^w\right)$ *and* $s^m = \left(s_j^m, s_{-j}^m, s_{n,j}^m, s_{n,-j}^m\right)$ *are the sets of women's and men's savings rates respectively.* $\rho^w = \left(\rho_i^w, \rho_{-i}^w\right)$ *and* $\rho^m = \left(\rho_j^m, \rho_{-j}^m\right)$ *are the sets of women's and men's probabilities of* *entering the marriage market respectively.*

Now we can show a more general proposition:

Proposition 6. *Assume emotional utility η^w and η^m are drawn from an independent and identical uniform distribution $\left[\eta^{\min}, \eta^{\max}\right]$ with the mean $E\eta \geq 0$, and $u(c) = \ln c$, then there exists a threshold value $\phi_1 > 1$ that satisfies $V^m = V_n^m$.*

(i) For $\phi < \phi_1$, both women and men choose to enter the marriage market with probability one. In addition, as the sex ratio rises, a representative man increases his savings rate while the change in the savings rate of a representative woman is ambiguous. However, the economy-wide savings rate increases unambiguously.

(ii) For $\phi \geq \phi_1$, as the sex ratio rises, a representative man chooses a positive probability of being single while a representative woman still chooses to enter the marriage market with probability one. The effect on the aggregate savings rate is ambiguous.

Proof. **See Appendix A2.2.** □

Three remarks are in order. First, for $\phi < \phi_1$, as the sex ratio rises, men endure a welfare loss while the effect on women's welfare is ambiguous. Men lose because (i) they face a lower probability of marriage, and (ii) the reductions in their first-period consumption do not in the end alter their probability of marriage. In comparison, women face two opposing effects. On the one hand, they may gain both from an ability to free ride on their future husbands' higher savings rates and from an improved chance to marry a man with a higher level of emotional utility. On the other hand, precisely because men have raised their savings, they become more choosy in their choice of a mate as sharing their higher savings rate with a low-type woman may be worse than being single. As a result, women ex ante may face a rising risk of not getting married. The net effect of a higher sex ratio on women's welfare is ambiguous.

Second, after reaching the threshold ϕ_1, with an savings rate already very high, some men would find it better to skip the marriage market (or equivalently, the representative man would assign a positive probability for not entering the marriage market). Otherwise, they would have to share their high savings rate with a low-type woman, resulting in a lower level of welfare than being single. From women's point of view, however, as long the mean level of emotional utility is high enough, they always achieve a higher level of welfare by choosing to enter the marriage market. In this case, the sex ratio in the marriage market is always equal to ϕ_1. Both men and women who choose to enter the marriage market will keep their savings rates constant. The rest of men choose another constant savings rate to maximize their utilities, but it is ambiguous whether the life-time bachelors' savings rate is lower than womens' savings rate or not. Therefore, the effect of a rise in the sex ratio on the aggregate savings rate is ambiguous.

Third, the log utility assumption greatly simplifies the proof. More general utility function forms may also yield the same results if the mean of the emotional utility is sufficiently large such that (i) the condition in Proposition 5 holds

$$E(\eta) \geq \frac{R\kappa u_2'}{2}\sqrt{\frac{\max\left(0, \kappa u_2'\left(u_{2w,n}' + u_{2m,n}'\right) - u_{2w,n}'u_{2m,n}'\right)}{u_{1m}''u_{1w}''}}$$

and (ii), at the balanced sex ratio, all women and men enter the marriage market.

2.1.6 A Production Economy

To analyze how the sex ratio imbalance affects a country's current account imbalance, we need to compare economy-wide savings with investments. In this subsection, we introduce a production sector. We assume that both the final good market and the factor markets are perfectly competitive. The production function is Cobb-Douglas:

$$Q_t = \zeta K_t^{\alpha} L_t^{1-\alpha} \tag{2.11}$$

where K_t is the capital stock and L_t is the labor input. α is the share of capital input to total output and ξ is the total factor productivity (TFP). Everyone in the economy inelastically supplies one unit of labor and earns the same income[3].

A representative firm maximizes the profit

$$\max_{K_t,L_t} Q_t - R_t K_t - W_t L_t$$

The capital return and the wage rate are determined by

$$R_t = \frac{\partial Q_t}{\partial K_t} = \alpha\zeta \left(\frac{1}{K_t}\right)^{1-\alpha} \quad (2.12)$$

$$W_t = \frac{\partial Q_t}{\partial L_t} = (1-\alpha)\,\zeta K_t^{\alpha} \quad (2.13)$$

where we normalize the aggregate labor supply in the economy to be 1, i.e., $L_t = 1$.

For simplicity, we assume no tax or government expenditure; then $y_t = W_t$ where y_t is the corresponding first period disposable income in the endowment economy. We also assume complete depreciation in each period. The aggregate capital supply in period $t+1$ is predetermined by the aggregate savings in period t

$$K^s_{t+1} = \frac{\phi}{1+\phi} s^m_t W_t + \frac{1}{1+\phi} s^w_t W_t \quad (2.14)$$

2.1.7 Current Account in a Small Open Economy

In a small open economy, we assume that capital can flow freely among countries and the gross interest rate R is exogenously determined by the rest of the world. By (4.2) and (4.3), the wage rate is also a constant, and the aggregate investment in the economy is

$$K^d_t = \frac{\alpha W_t}{(1-\alpha) R_t} \quad (2.15)$$

---

[3] Allowing men and women to earn different wages (with a fixed proportional gap) would not change our results.

Substituting (4.2) and (2.15) into the production function, we have

$$Q_t = \frac{W_t}{1-\alpha}$$

The current account in period t equals the increase in net foreign assets,

$$\Delta NFA_t = Q_t + (R-1) \cdot NFA_{t-1} - C_{1t} - C_{2t} - K_{t+1}^d$$

where $(R-1) \cdot NFA_{t-1}$ is the factor income from abroad. C_{1t} and C_{2t} represent the aggregate consumptions by young and old people respectively. Then

$$\Delta NFA_t = \frac{\phi}{1+\phi} s_t^m W_t + \frac{1}{1+\phi} s_t^w W_t - NFA_{t-1} - K_{t+1}^d$$

We define the economy-wide savings rate as the aggregate private savings to GDP ratio; then

$$s_t^P = \frac{Q_t + (R-1) \cdot NFA_{t-1} - C_{1t} - C_{2t}}{Q_t} \tag{2.16}$$

We assume that the country has a balanced sex ratio in period $t-1$, and the sex ratio in the young cohort in period t, rises from one to $\phi(>1)$. Then the ratio of the current account to GDP is

$$\begin{aligned} ca_t &= \frac{Q_t + (R-1) \cdot NFA_{t-1} - C_{1t} - C_{2t} - K_{t+1}^d}{Q_t} \\ &= (1-\alpha)\left(\frac{\phi}{1+\phi} s_t^m + \frac{1}{1+\phi} s_t^w - s_{t-1}\right) \end{aligned} \tag{2.17}$$

where the second equality holds because[4]

$$NFA_{t-1} = s_{t-1} W_{t-1} - K_t^d$$

[4] In overlapping generations models, net foreign asset is equal to the difference between the savings by the young cohort and the domestic investment demand.

where s_{t-1} is the savings rate by the cohort born in period $t-1$. Since the sex ratio is balanced at that time, both the women and the men will have the same savings rate.

Since the wage rate is constant in the small open economy, we can show that a country's current account rises as its sex ratio rises (up to a point).

Proposition 7. *In a small open economy with production, both the economy-wide savings rate and the current account would rise in response to a rise in the sex ratio.*

Proof. See Appendix A2.3. □

The assumption of an exogenous interest rate holds only for a small open economy. But some of the countries that motivate this study are large. An increase in the savings rate in such economies could lower the world interest rate, which could alter investment and savings decisions in all countries. We examine the large country case in the next subsection.

2.1.8 Two Large Countries

Consider a world consisting of only two countries. The two countries are identical in every respect except for their sex ratios in period t (they both have balanced sex ratios in period $t-1$). Country 1's sex ratio ϕ^1 is smaller than Country 2's sex ratio ϕ^2. There are no barriers to either goods trade or capital flows (although labor is not mobile internationally). We can show the following result:

Proposition 8. *Country 1 (with a more balanced sex ratio) runs a current account deficit while Country 2 runs a current account surplus.*

Proof. See Appendix A2.4. □

To see the intuition, let us fix $\phi^1 = 1$ (i.e., Country 1 has a balanced sex ratio). If Country 2 were to have a balanced sex ratio, the current account must be zero for both countries since they are identical in every respect. In other words, within each country, the investment must be equal to the aggregate savings. However, the sex ratio imbalance in Country 2 causes it to have a higher aggregate savings for a given world interest rate. This depresses the world interest rate. The lower interest rate raises the investment level in both economies (and reduces the savings rate a little bit). This must imply that the desired investment level in Country 1 is now greater than its desired savings rate. As a result, capital flows from Country 2 to Country 1. That is, Country 1 runs a current account deficit, and Country 2 a surplus.

2.2 Calibrations and extensions

Are the actual sex ratios observed in the data capable of generating a current account response whose magnitude is economically significant? We answer this question in this section by quantitative calibrations of the model. We start with a small open economy and allow endogenous entry/exit to the marriage market. Then we move on to two cases of a large economy. We also consider two extensions that would add some more realism to the model. First, we discuss potential intra-family bargaining between husband and wife, with their relative bargaining power depending in part on their relative savings rate. Second, we extend the benchmark two-period model to a multi-period model.

2.2.1 The Small Open Economy

Assume that the utility function is of the log form

$$u(c) = \ln(c)$$

In the calibrations for a small open economy, we fix $R = \beta^{-1}$. (In the large country case, the interest rate is endogenously determined.)

The emotional utility η needs to follow a continuously differential distribution. In the benchmark calibration, we assume a normal distribution which might be more realistic than the uniform distribution used in the analytical model (although the uniform distribution assumption is more convenient in the analytical proof). We choose the mean and the standard deviation of emotional utility/love by matching with some empirical patterns reported in Blachflower and Oswald (2004). To be precise, here is what we do to calibrate the mean value. Within the model, holding all other factors constant, we compute the income compensation to a life-time bachelor that makes him indifferent between being married and being single.

$$u\left(\frac{1}{1+\beta}(1+m)y\right) = u\left(\frac{1}{1+\beta}y\right) + E(\eta)$$

where $m \cdot y$ is the compensation paid to a life-time bachelor for being single and $\frac{1}{1+\beta}(1+m)y$ is his second period consumption. Regressing a measure of subjective well-being on income and marital status (and other determinants of happiness) in the United States during 1972-1998, Blanchflower and Oswald (2004) estimate that, on average, a lasting marriage is equivalent to augmenting one's income by \$100,000 (in 1990 dollars) per year every year. Since the average income per working person was about \$48,000 during that period, a sustained marriage is worth twice the average income. We therefore choose $m = 2$ as the benchmark. This implies that the mean value of emotional utility/love is:

$$E(\eta) = u\left(\frac{3y}{1+\beta}\right) - u\left(\frac{y}{1+\beta}\right)$$

We will vary the value of m in the robustness checks.

Since the t-statistic for the marriage status dummy in the happiness regression reported in Blanchflower and Osward (2004) is around 20, we pin down the standard deviation of emotional utility, σ, by

$$\frac{E(\eta)}{\sigma} = 20 \Longrightarrow \sigma = \frac{E(\eta)}{20} = \frac{\ln\left(\frac{3y}{1+\beta}\right) - \ln\left(\frac{y}{1+\beta}\right)}{20} \simeq 0.05$$

As a robustness check, we also consider $\sigma = 0.1$.

For other parameters, whenever possible, we assign values that are consistent with the standard literature.

Choice of Parameter Values

Parameters	Benchmark	Source and robustness checks
Discount factor	$\beta = 0.45$	Prescott (1986) suggests that the discount factor takes a value of 0.96 on annual frequency. As we take 20 years as one period, we set $\beta = 0.96^{20} \simeq 0.45$
Share of capital input	$\alpha = 1/3$	Chari, Kehoe and McGrattan (2001)
Congestion index	$\kappa = 0.8$	$\kappa = 0.7, 0.9$ in the robustness checks.
Love, standard deviation	$\sigma = 0.05$	$\sigma = 0.1$ in the robustness checks
Love, mean	$m = 2$	$m = 0.5$ in the robustness checks

Tables 2.1, 2.2 and 2.3 report the calibration results (when the sex ratio changes from 1 to 1.5). Figure 2.1 plots the aggregate savings rate as a function of the sex ratio (which changes from 1 to 1.5). In our benchmark case, we assume that the share that each spouse can consume out of the combined second-period income is $\kappa = 0.8$, the mean of emotional utility $m = 2$, and its dispersion is $\sigma = 0.05$. In this case, when the sex ratio goes up from 1 to 1.15, the savings rate would go up by 5.8 percentage points. As the sex ratio continues rising, the savings rate may decrease. This is because the sex ratio has exceeded the threshold ϕ_1 in Proposition 6, some men quit the marriage market and choose a lower savings rate, which drives down the economy-wide savings rate.

For sensitivity analyses, we consider different combinations involving $\kappa = 0.7, 0.8$ and 0.9, $m = 2$ and 0.5, and $\sigma = 0.05$, and 0.1. There are a few noteworthy patterns. First, the economy-wide savings rate always rises in response to a rise in the sex ratio. Second, as κ becomes larger, the economy-wide savings rate and the current account respond more strongly to a given rise in the sex ratio. Intuitively, as κ becomes larger, consumption within a marriage acquires more public goods features. Consequently, the desire to marry (and the need to compete in the marriage market) also increases. However, the response of the aggregate savings is not very sensitive to small perturbations of this parameter.

Third, when the mean value of emotional utility becomes higher (e.g., comparing $m = 2$ to $m = 0.5$), both the economy-wide savings rate and the current account respond more strongly to a given rise in the sex ratio. This is intuitive since men have a stronger desire to compete for a marriage partner.

Fourth, as the dispersion for emotional utility becomes smaller, the economy-wide savings rate and the current account respond more strongly to a rise in the sex ratio. This is because, since all men are more similar in terms of the amount of "love" they can offer to women, the need to compete on the basis of wealth also rises.

2.2.2 Two Large Countries

We now consider a two-country model. The interest rate R and the wage rate W are now endogenously determined. All other parameter values are the same as in the small open economy case. We discuss two cases.

In the first case, we assume that the two countries are identical in every respect except for their sex ratios. While Country 1 always has a balanced sex ratio ($\phi^1 = 1$), we vary the sex ratio in Country 2 from 1 to 1.5. Table 2.4 reports the calibration results. Figure 2.2 traces out the current account responses in both countries as Country 2's sex ratio increases. The most important result is that a rise in Country 2's sex ratio first triggers a rise in its current account surplus and a rise in Country 1's current account deficit. After Country 2's sex ratio exceeds threshold ϕ_1, a further rise in Country 2's sex ratio induces a decline in its current account surplus and a rise in Country 1's deficit.

We have also done robustness checks by varying the values of κ, m, and σ. Based on the same reasoning as in the small open economy case, for larger κ, m, or smaller σ, a given increase in Country 2's sex ratio results in a greater current account imbalance in the two countries.

In the second case, we attempt to let Countries 1 and 2 mimic the United States and China, respectively. In particular, we assume that $L_1 = 1/5 \cdot L_2$ to match the fact that the U.S. population is around 1/5 that of China. In addition, we choose the TFP parameter in Country 1, ζ_1, to match the fact that the U.S. per capita GDP was about 15 times the Chinese level around 2000 when the sex ratio in China for the marriage age cohort was not yet seriously out of balance. The remaining parameters are set to be the same as before. We let the sex ratio in the United States be always balanced, and vary the Chinese sex ratio from 1 to 1.5.

Tables 2.5, 2.6 and 2.7 report the benchmark result and robustness checks. Figure 2.3 plots the calibration results. Qualitatively, they look similar to the first large-country experiment. Quantitatively, Country 2's (China) current account response (as a share of GDP) becomes stronger. With China's sex ratio at 1.15 (and $\kappa = 0.8$), it runs a current account surplus on the order of 4.5% of its GDP, and at the same time, the United States runs a current account deficit of 1.5% of GDP. This resembles one third to a half of the real world pattern in which the U.S. deficit is about 4-6% of GDP, whereas the Chinese surplus is on the order of 7-10% of GDP in recent years. In other words, a rise in the Chinese sex ratio does not provide a complete explanation for the observed current account patterns, but could be a potentially significant contributor to the current account imbalances. If the sex ratio rises to threshold ϕ_1, the Chinese surplus may begin to decline.

To summarize, the calibrations suggest that a rise in the sex ratio (when the sex ratio takes some reasonable values) could produce an economically significant increase in the aggregate savings rate that results in a current account surplus. If the country is large enough, this could induce other countries to run a current account deficit even if they have a balanced sex ratio.

2.2.3 Welfare

There are two sources of market failure in the model economy. On one hand, a part of the savings in the competitive equilibrium is motivated by a desire to out-save one's competitors in the marriage market. The increment in the savings, while individually rational, is not useful in the aggregate, since when everyone raises the savings rate by the same amount, the ultimate marriage market outcome is not affected by the increase in the savings. In this sense, the competitive equilibrium produces too much savings. On the other hand, because the savings contribute to a pubic good in a marriage (an individual's savings raises the utility of his/her partner), but an individual in the first period does not take this into account, he/she may under-save relative to the social optimum. Note that these sources of market failure exist even with a balanced sex ratio. They also have opposite effects on the aggregate savings rate. (In the calibrations that will be reported later, these two effects cancel each other out when the sex ratio is balanced.) As the sex ratio rises, the importance of the over-saving effect also increases, which gives rise to our first proposition.

We now consider what a welfare-maximizing central planner would do. The central planner gives equal weight to each man and women. He assigns the marriage matching outcomes and chooses women's and men's savings rates to maximize the following social welfare function,

$$\max\ U = \frac{1}{1+\phi}U^w + \frac{\phi}{1+\phi}U^m$$

The first order conditions are

$$- u'_{1w} + \left[1 - F(\bar{\eta}^w) + \phi\left(1 - F(M\left(\bar{\eta}^w\right))\right)\right]\kappa u'_{2w} + F(\bar{\eta}^w)u'_{2w,n} = 0 \quad (2.18)$$

$$-u'_{1m} + \left[1 - F(M\left(\bar{\eta}^w\right)) + \frac{1}{\phi}\left(1 - F(\bar{\eta}^w\right)\right]\kappa u'_{2m} + F(M\left(\bar{\eta}^w\right)u'_{2m,n} = 0 \quad (2.19)$$

Comparing (2.18), (2.19) to (4.40) and (4.41), in general, it is not obvious whether women or men will save at a higher rate in a decentralized equilibrium than that under central planning due to the two opposing sources of market failure. However, when $\phi = 1$, since women and men have the same optimal conditions, by (2.10), women and men will save the same in the competitive equilibrium as in the central planner economy.

As a thought experiment, one may also consider what the central planner would do if she can choose the sex ratio (in addition to the savings rates) to maximize the social welfare. The first order condition with respect to ϕ is

$$\frac{U^m - U^w}{\left(1+\phi\right)^2} = 0$$

The only sex ratio that satisfies condition above is $\phi = 1$. In other words, the central planner would have chosen a balanced sex ratio. Deviations from a balanced sex ratio represent welfare losses.

In calibrations with a log utility function, we show that men's welfare under a decentralized equilibrium relative to the central planner's economy declines as the sex ratio increases. In comparision, women's relative welfare increases as the sex ratio goes up. The social welfare (the sum of all men's and women's welfare) goes down as the sex ratio rises.[5] Figures 2.4, 2.5 and 2.6 trace out the savings rates for men (the upper left panel), women (the upper right panel), the economy as a whole (the lower left panel) and the welfare (the lower right panel). With a log-utility function, the optimal savings rates chosen for men and women by the planner do not depend on the sex ratio and intra-household bargaining powers.[6] When the sex ratio is balanced, the savings rates by women, men and the economy as a whole are the same as those under the planner's economy. With unbalanced sex ratios, men's (decentralized) savings rates overshoot the socially optimally level, and the extent of excessive savings rises with the sex ratio. Women's savings rates follow an opposite pattern. The economy-wide savings rate follows a pattern that is qualitatively similar to the men's savings rate. In particular, the economy in a decentralized equilibrium tends to save too much relative to the social optimum, and the excess savings rises with the sex ratio. In the lower right panel, we can see that welfare levels for both men and the economy as a whole decline as the sex ratio increases, while the welfare for women rises with the sex ratio.

2.2.4 Endogenous Intra-household Bargaining

One problem in the benchmark calibration is that, as the sex ratio rises, women's savings rates decline very quickly. In this extension, we incorporate intra-household bargaining between wives and husbands into the model. To goal is to show that, when allowing intra-household bargaining, women's savings rate declines much more slowly.

---

[5] The results are similar if we change the utility function to a CRRA form.

[6] This feature does not hold when we use the CRRA utility function.

We assume that everyone consumes two goods in the second period, a public good (e.g., a house) and a private good. The aggregate second period consumption index is

$$c_{2i} = \frac{z_i^{\gamma} h^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \quad i = w, m$$

where z_w and z_m are private goods consumption by women and men, respectively, and h is the public good consumption. γ is the share of private expenditure in the second period consumption index.

A representative household maximizes the weighted sum of the utilities of the husband and the wife. Let μ denote the weight on the wife's utility, which represents her bargaining power in the family. Then the household's optimization problem is

$$\max_{h, z_w, z_m} \; \mu u(c_{2w}) + (1-\mu) u(c_{2m})$$

with the resource constraint

$$z_w + z_m + h = R s^w y^w + R s^m y^m \tag{2.20}$$

If we assume $u(c) = \ln c$, solving the household's maximization problem, we have

$$\begin{aligned} c_{2w} &= \mu^{\gamma} \left(R s^w y^w + R s^m y^m \right) \\ c_{2m} &= (1-\mu)^{\gamma} \left(R s^w y^w + R s^m y^m \right) \end{aligned}$$

If $\mu = \frac{1}{2}$, this is the case in our benchmark model and $\frac{1}{2} < \kappa = 2^{-\gamma} < 1$.

More generally, similar to Browning et al. (1994), μ is a function of the sex ratio ϕ, the relative wealth and other characteristics of the household member. For simplicity, we assume that the intra-household bargaining power depends only on the relative wealth of household members. In particular, we assume that the wife's bargaining power within a family is

$$\mu = \frac{(s^w)^\varepsilon}{(s^w)^\varepsilon + (s^m)^\varepsilon}$$

and the husband's bargaining power is $1-\mu$. ε is the parameter that governs the sensitivity of bargaining power to relative wealth. A larger ε means that household bargaining power will respond to the relative wealth more strongly.

We take the same values for other parameters as in the benchmark. Table 2.8 reports the calibration results, and Figures **??** and 2.8 plot the saving rates. Relative to the case of no intra-household bargaining, women now reduce their savings rates more slowly as the sex ratio rises. Since there is no big change in men's response to the rise in the sex ratio, the economy-wide savings rate responds more strongly to a rise in the sex ratio than the benchmark case. For $\sigma = 0.05$, $\varepsilon = 0$ and $\gamma = 0.5$, as the sex ratio rises from 1 to 1.15, the current account to GDP ratio rises by 5.3%. For $\sigma = 0.05$, $\varepsilon = 0.5$ and $\gamma = 0.5$, as the sex ratio rises from 1 to 1.15, the current account to GDP ratio rises by 8.1%. As the sex ratio keeps rising (and exceeds the threshold ϕ_1, which is around 1.25 in this case), some men quit the marriage market and aggregate savings rate declines.

We re-calibrate the case of two otherwise identical countries except for the sex ratio. Figures 2.9 and 2.10 plot the current account responses and the welfare changes in the corresponding cases. The results are similar to our benchmark calibration's. In both cases, the only difference relative to the benchmark model is the allowance for the endogenous bargaining power with a family. The qualitative results on the aggregate savings and the current account are similar to before. However, the savings response by women becomes more realistic. For $\sigma = 0.05$, $\varepsilon = 0$ and $\gamma = 0.5$, we can find in that as the sex ratio in China rises from 1 to 1.15, this can generate an 2.7% current account suplus in China and a 0.9% deficit in the U.S. For $\sigma = 0.05$, $\varepsilon = 0.5$ and $\gamma = 0.5$, as the sex ratio in China rises from 1 to 1.15, this can generate an 6.1% current account suplus in China and a 2.0% deficit in the U.S, which resembles more than a half of the real word pattern. As the sex ratio in China becomes very large, the Chinese current account surplus will decline.

In the right panel in both Figures 2.9 and 2.10, we trace out the economy-wide welfare in a decentralized equilibrium for a given sex ratio relative to the welfare in a decentralized equilibrium but with a balanced sex ratio. The country that experiences a rise in the sex ratio (e.g., China) clearly suffers from an ever-deteriorating welfare. Interestingly, the country with a balanced sex ratio (e.g., the United States) could enjoy a small welfare gain as China's sex ratio starts to become imbalanced. Intuitively, a rise in the Chinese sex ratio depresses the global interest rate, but this produces two effects of opposite signs on the United States. On one hand, the lower cost of capital boosts the real wage in the United States, which is positive for the Americans. On the other hand, the lower interest rate also implies a lower interest income for a given amount of savings, which is negative for the Americans. For a moderately unbalanced sex ratio in China, the positive effect for the United States dominates. As the Chinese sex ratio becomes seriously out of balance, the welfare levels in both countries could both go down.

We note, however, that the quantitative effect of a rise in the Chinese sex ratio on the U.S. welfare is small. The Chinese lose the most from a rise in the sex ratio. As an illustration, based on the right panel of Figure 2.10, if the Chinese sex ratio reaches 1.15, the Americans have a utility gain that is equivalent to an increment in consumption by 0.7%. In contrast, the Chinese suffer a welfare loss that is equivalent to a decline in consumption by 18.7%.

2.2.5 Multi-period model calibrations

We now extend our benchmark model to a setting in which every cohort lives for 50 periods. Everyone works in the first 30 periods, and retires in the remaining 20 periods. If one gets married, the marriage take place in the τth period. We have not been able to solve the problem that allows for parental savings for their child in the 50-period setup. Instead, we study a case in which men and women save for themselves. However, as we recognize the quantitative importance of parental savings in the data, we choose $\tau = 10$ as our benchmark case so the timing of the marriage is somewhere between the typical number of working years by parents when their child gets married and the typical number of working years by children themselves when they get married. Generally speaking, the greater the value of τ, the stronger is the aggregate savings response to a given rise in the sex ratio.

A representative woman's optimization problem is

$$\max \sum_{t=1}^{\tau-1} \beta^{t-1} u(c_t^w) + E_1 \left[\sum_{t=\tau}^{50} \beta^{t-1} \left(u(c_t^w) + \eta^m \right) \right]$$

For $t < \tau$, when the woman is still single, the intertemporal budget constraint is

$$A_{t+1} = R\left(A_t + y_t^w - c_t^w\right)$$

where A_t is the her wealth level at the beginning of period t. After marriage ($t >= \tau$), her family budget constaint becomes

$$A_{t+1}^H = \begin{cases} R\left(A_t^H + y_t^w - c_t\right) & \text{if } t \leq 30 \\ R\left(A_t^H - c_t^w\right) & \text{if } t > 30 \end{cases}$$

where A_t^H is the level of family wealth (held by wife and husband) at the beginning of period t. c_t is the public good consumption by wife and husband, which takes the same form as in the two period OLG model. The optimization problem for a represenative man is similar.

We adjust some parameters in the 50-period OLG calibrations. As in the standard literature, we will take $R = 1.04$ as the annual gross interest rate. The subjective discount factor now takes the value of $\beta = 1/R$. We also assume an annual capital depreciation rate equal to 0.1 as in the standard literature. Besides the base case of $\tau = 10$, we also examine the case of $\tau = 20$ as a sensitivity check. All other parameters are the same as in the benchmark 2-period OLG model.

The calibration results are shown in Figures 2.11 and 2.12. In the benchmark calibration, when $\sigma = 0.05$, if the marriage takes place in the 10th period, as sex ratio rises from 1 to 1.1, the economy-wide savings rate and current account rise by about 4.1% of GDP (Figure

2.11). As a robustness check, if the marriage takes place in the 20th period, then as the sex ratio rises from 1 to 1.1, the economy-wide savings rate and current account can rise by about 5.2% of GDP (Figure 2.12). On the other hand, if the marriage takes place in the 5th period, as the sex ratio rises from 1 to 1.1, the economy-wide savings rate and current account can rise by around 3% of GDP (the figure not reported to save space).

We also calibrate the multi-period model by incorporating the endogenous intra-household bargaining. The responses of the savings rate and current account to the rise in the sex ratio are stronger. When $\varepsilon = \gamma = 0.5$, and $\sigma = 0.05$, if the marriage takes place in the 10th period, as sex ratio rises from 1 to 1.1, the economy-wide savings rate and current account rise by almost 5% of GDP (Figure 2.13). As a robustness check, if the marriage takes place in the 20th period, then as the sex ratio rises from 1 to 1.1, the economy-wide savings rate and current account can rise by about 6.3% of GDP (Figure 2.14).

2.3 Some Empirical Evidence

We discuss two types of empirical approaches that allow us to check for plausibility and empirical importance of the theory. First,, we provide some cross-country evidence on the relationship between a country's sex ratio and its non-government part of the current account. Second, we review household-level evidence from China on the association between sex ratios and savings rates.

2.3.1 Cross country data patterns

We define a country's non-governmental part of the current account as its current account balance minus its government savings (or government revenue minus expenditure), divided by its GDP. We exclude government savings because our theory is about private sector savings.

We run a multivariate regression of the ratio of non-governmental current account to GDP on sex ratio and other control variables. To be precise, the specification equation is the following:

$$cagdp_i = \beta_0 + \beta_1 \cdot \text{sex ratio}_i + \beta_2 Z_i + \varepsilon_i$$

where $cagdp_i$ is the ratio of current account minus government savings to country i's GDP. Sex ratio is defined as the male to female ratio for the age group 0-15. Our choice of the control variables is guided by the life-cycle theory, precautionary saving theory, and financial development theory. We therefore include variables in Z_i log per capita GDP, the share of working age people in the population (a proxy for life-cycle theory), social security expenditure as a share of GDP (a proxy for the precautionary saving theory), private credit to GDP ratio (a proxy for financial development), and continental dummies (a proxy for possible cultural factors). (We only conduct a cross-sectional regression as we are not able to obtain a panel data set on the sex ratios.)

Current account, GDP, the share of working age in the population and private credit to GDP ratio can be obtained from the World Bank's WDI database. The sex ratio data is obtained from the World Factbook. Social security expenditure as a share of GDP data is obtained from the International Labor Organization (ILO) database. A series of regression results are reported in Table 2.9, where the set of control variables is progressively enlarged. In each regression, we have a positive and statistically significant coefficient on the sex ratio: as the sex ratio becomes more unbalanced, the current account balance tends to go up. This result still holds after we exclude Kuwait, which is a potential outlier with a very large current account surplus in 2006. From Table 2.9, as the sex ratio rises from 1.05 (normal biological level without sex selection) to 1.15 (China's sex ratio), based on the last column, current account will rise by around 12.5 percent.

In Table 2.9, we can find that the financial development index has significant negative signs as the theory predicts. This means that a country with a well developed financial market tends to have a current account deficit. The age profile of populations and the social security expenditure index produce some puzzling patterns. In contrast to the life-cycle theory and precautionary saving theory, the share of working age population has a negative coefficient and social security expenditure as a share of GDP has a positive coefficient. The coefficients mean that old-age households and households with children save more than do households in between, and people save more even though they may get more social security benefits. However, those coefficients are not significant in several regressions which means that life-cycle theory and precautionary saving theory are not capable of explaining the global current account imbalances.

The intertemporal theory predicts that a country's current account should be sensitive to temporary shocks. To minimize the influence of year-to-year fluctuations in the current account due to temporary shocks, we also conduct a robustness check whereby the dependent variable is the average ratio of non-governmental current account to GDP over a five-year period (2004-2008). We report the results in Table 2.10. Again, the positive relationship between the local sex ratio and the local non-governmental current account is robustly positively.

We also study the relationship between a country's sex ratio and its savings rate (% of GDP). Table 2.11 and 2.12 provide the regression results. The coefficients on the sex ratio in all the regressions are positive and significant which means that as a country's sex ratio rises, the aggregate savings rate will increase.

There are many caveats with the empirical patterns. First, in spite of our best efforts, there may still be potential control variables that are missing from our list. Second, because the sex ratio data is not available for most countries in the earlier years, we are not able to conduct a panel regression. In any case, the sex ratio data are likely to be strongly serially correlated, which would have required a long time series to successfully identify the parameters. Third, the sex ratio can be endogenous and/or measured with errors. This would normally call for an instrumental variable approach. At this point, we are not able to come up with convincing instrumental variables in a cross-country context. For these reasons, it is important to review some micro-evidence from within China.

2.3.2 Cross-household and cross-region evidence from China

The sex ratio at birth in China increased from being slightly unbalanced in 1990 to about 120 boys per 100 girls in 2007. Its household savings rate (out of disposable income) almost doubled from 16% to 30% during the same period. While China is not the only economy with a high sex ratio (and a high savings rate), it is the one with the most extreme sex ratio imbalance at the moment, and, because of its size, its savings rate and current account attract the most international attention. For this reason, it is useful to highlight a few empirical patterns documented in Wei and Zhang (2009) that are most relevant for the current paper.

First, let us look at Chinese households' self-reported reasons for savings. A survey of rural households (Chinese household income project in 2002) asked households why they save. There were seven possible categories for savings in the questionnaire: (1) children's wedding, (2) children's education, (3) bequest to children, (4) building a house, (5) (own) retirement, (6) medical expenses, and (7) others. The first three reaons could be grouped under the heading of "savings directly for children." If we just focus on families with an unmarried child, one sees a stunning difference between families with a son versus those with a daughter. 29.8% of families with a son list savings for their child's wedding as either the most or the second most important reason for savings, versus 18.3% of families with a daughter who do the same. Overall, 92.2% of son-families list one of the top three reasons as their primary reasons for savings, which is 5.8 percentage points higher than the percent of the daughter families who say the same. In comparison, 45.5% of daugher families and 37.3% of son-families say their most or the second most important reason for savings is their own retirement. (Note that the sum of the percentage of households that list various reasons as the most or the second most important reason for savings can be more than 100% since a given household could list one category as the most important reason for savings, and another category as the second most important reason for savings.)

Second, we now look at the relationship between household savings rates (out of disposable income) and local sex ratios (at the county or city level), holding constant other determinants of savings rate (household income, household head's age, gender, ethnicity, and educational level, and children's age, and whether there is a family member that has a major illness). What is most revealing for our theory is not just a direct comparison in the savings rates between son-families and daughter families, but the effect of an interaction term between having a son and living in a region with a high local sex ratio. This exercise is interesting in China because the migration rate for the purpose of marriage is low (about 92% of marriages take place between a man and a woman from the same county). When focusing on families with a son in rural areas, Wei and Zhang report that these families' savings rate tends to be higher in regions with a more skewed sex ratio. In comparison, the savings rate by families with a daughter appears to be uncorrelated with the local sex ratio. Across Chinese cities, the savings rates by both son-families and daughter families tend to rise with the local sex ratio. These patterns are consistent with our model that allows for intra-family bargaining. When women (or their parents) are concerned with erosion of bargaining power within a family, they may not reduce their savings rate in response to a higher sex ratio. When the effect of intra-family bargaining dominates, the savings rate by daughter-families could rise in response to a rise in the sex ratio.

Third, across Chinese provinces, Wei and Zhang report a strong positive correlation between local savings rates and local sex ratios (for the age cohort of 7-21 years old), controlling for the age structure of local population, per capita income, the share of employment in state-owned firms in the local labor force, and the share of local labor force enrolled in social security). To go from correlation to causality, Wei and Zhang employ variations in the local enforcement of family planning policy (including monetary penalties for violating birth quotas) as instruments for the sex ratio. The 2SLS estimation confirms the basic finding: regions with a higher sex ratio are also likely to have a higher household savings rate. Based on the 2SLS estimates, 40-60% of the rise in the household savings rate from 1990 to 2007 can be attributed to the observed rise in the sex ratio for the pre-marital age cohort during the period.

Overall, the evidence from within China is consistent with the theoretical predictions.

2.4 Concluding Remarks and Future Research

This paper builds a theoretical model to analyze whether and how a rise in the sex ratio may trigger a competitive race in the savings rate by men (or households with sons). Generally speaking, men raise their savings rate in order to improve their relative standing in the marriage market. If we don't consider intra-household bargaining, women may respond by reducing their savings rate because they may free ride on the increased savings from their husbands. If we consider intra-household bargaining, then the women's response becomes ambiguous because they also have an incentive to raise their savings rate in order to protect their bargaining power within a family. In any case, the aggregate savings always rises unambiguously in response to a rise in the sex ratio, as long as the sex ratio is below some threshold. We argue conceptually and through calibrations that the sex ratios in real economies are unlikely to exceed the threshold.

When the country with an unbalanced sex ratio is large, this could have global ramifications. In particular, when the sex ratio rises, the world interest rate becomes lower. Other countries with a balanced sex ratio could be induced to run a current account deficit. Calibration results suggest that the sex ratio effect could potentially explain more than half of China's current account surplus and the U.S. current account deficit. In other words, the effect is economically significant.

The theory can be extended in a number of directions. First, the sex ratio could endogenously respond to the economic burden of raising a son (as in Bhaskar, 2009). As a result, there may be forces that will eventually induce a correction in the trajectory of a country's sex ratio. It will be a useful extension to endogenize the sex ratio in the model. This will help us understand better the future trajectories of global current account imbalances. Second, while the model focuses on the responses of savings and current account to a rise in the sex ratio, one may extend it to study entrepreneurship and growth effects. These will be useful topics for future research.

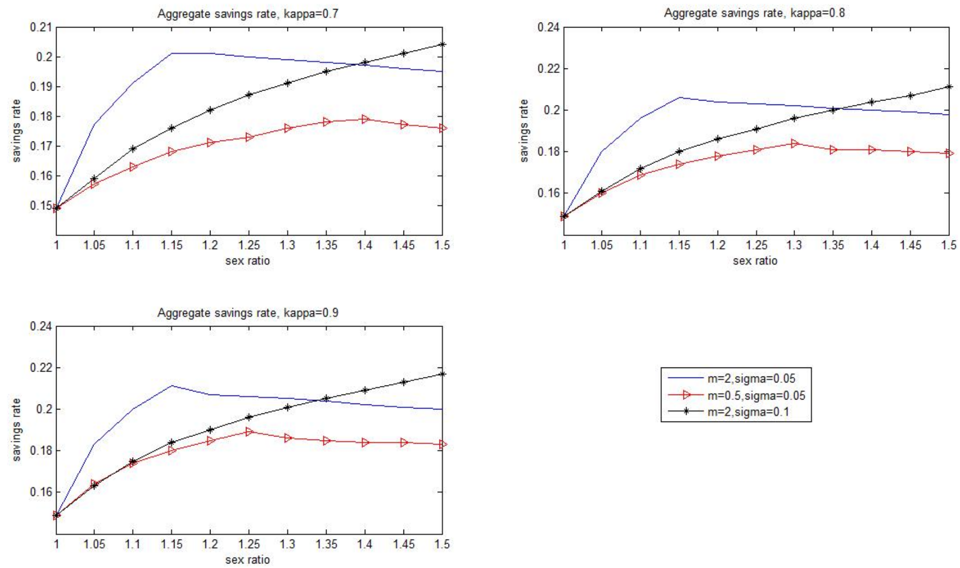

Figure 2.1: Economy-wide savings rate vs sex ratio

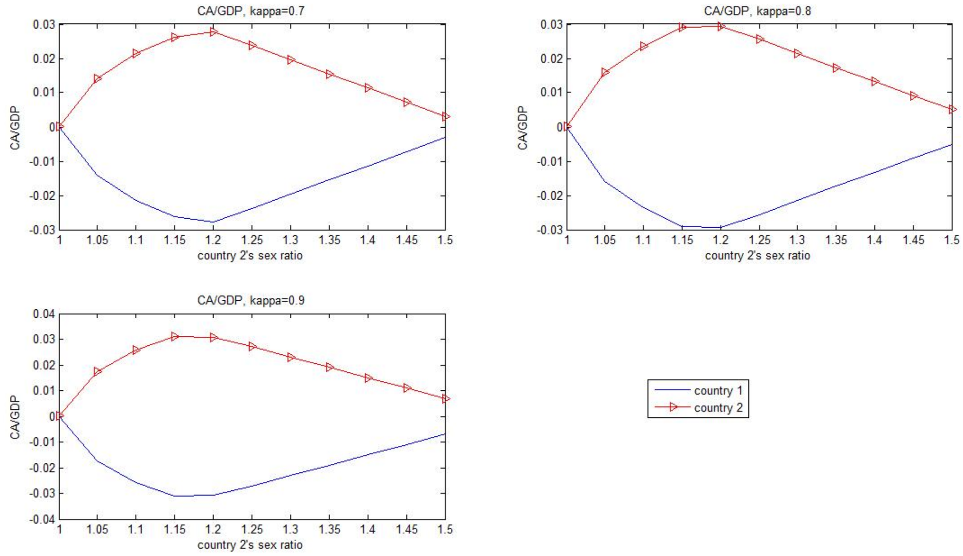

Figure 2.2: Two large countries, differing only in the sex ratios, sigma=0.05

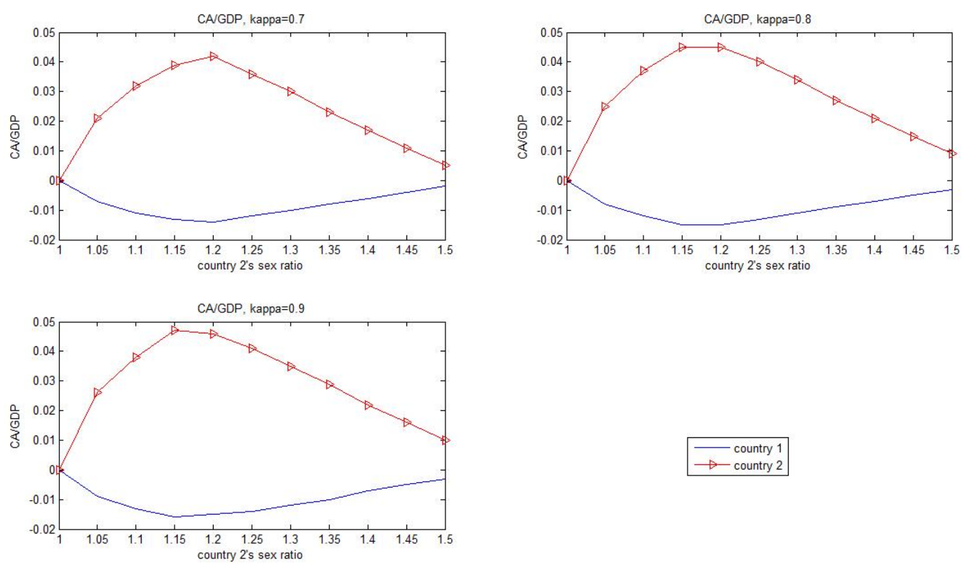

Figure 2.3: Two large countries, (GDP per capita)1=15*(GDP per capita)2, sigma=0.05

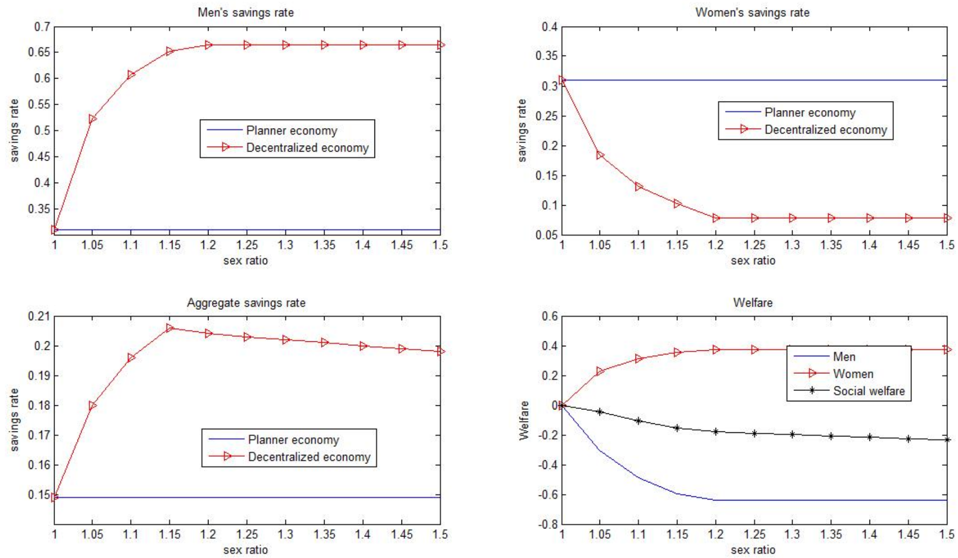

Figure 2.4: The planner's economy vs the decentralized economy, kappa=0.8, sigma=0.05

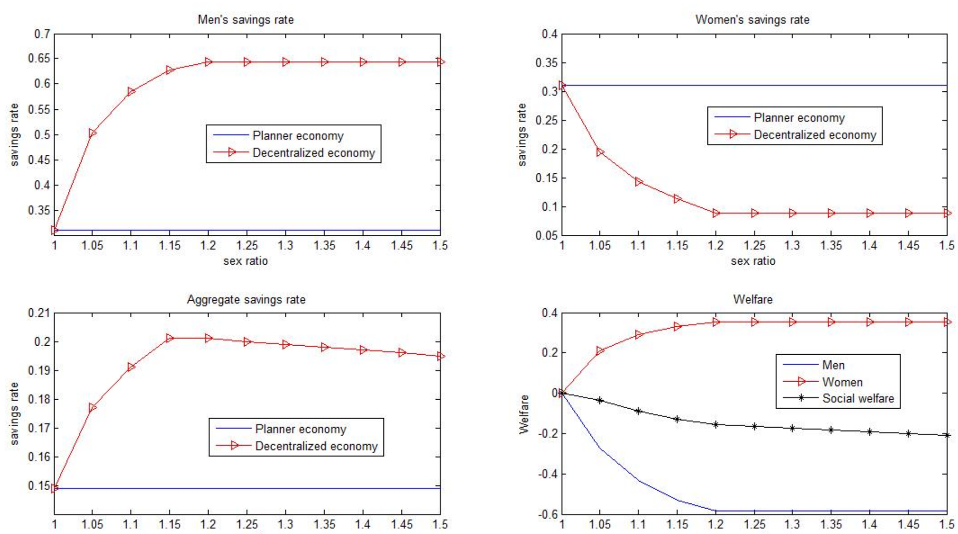

Figure 2.5: The planner's economy vs the decentralized economy, kappa=0.7, sigma=0.05

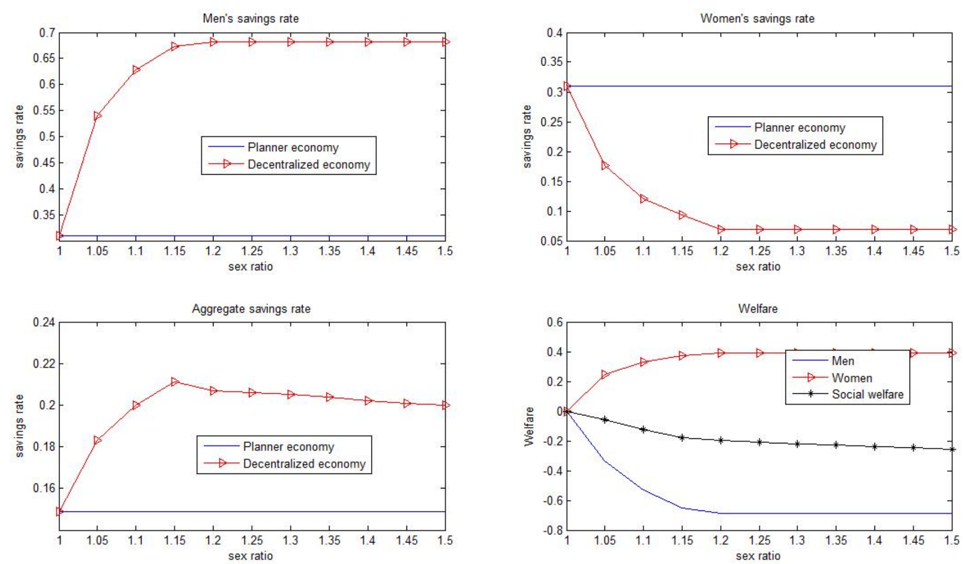

Figure 2.6: The planner's economy vs the decentralized economy, kappa=0.9, sigma=0.05

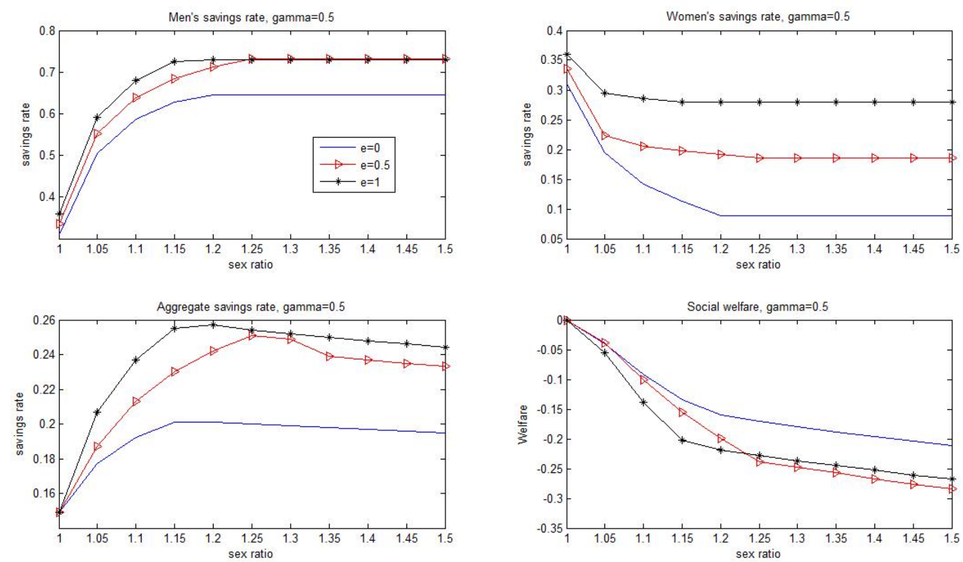

Figure 2.7: Savings rates vs sex ratios, endogenous intra-household bargaining, sigma=0.05

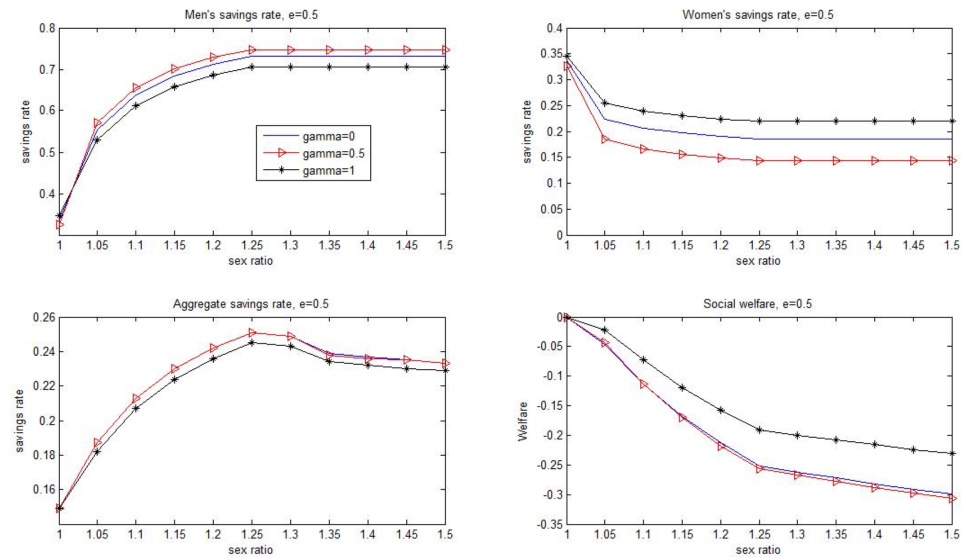

Figure 2.8: Savings rates vs sex ratios, endogenous intra-household bargaining, sigma=0.05

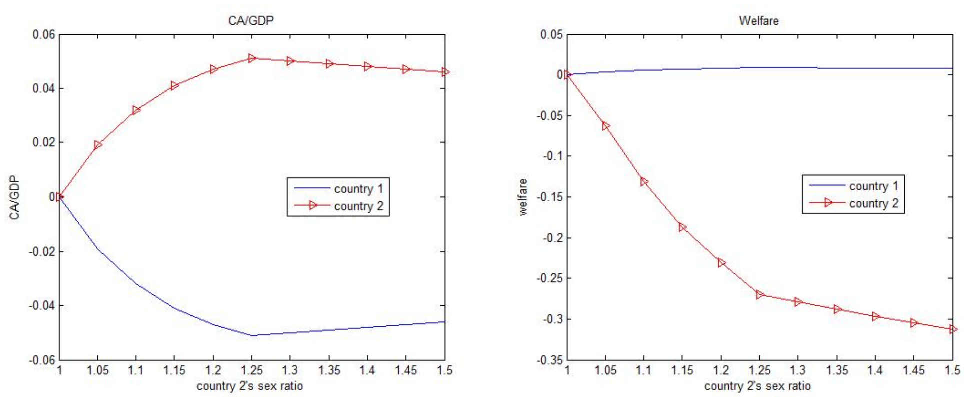

Figure 2.9: Two large countries, differing in the sex ratios, endogenous bargaining power, welfare loss in units of consumption goods relative to the case of phi=1

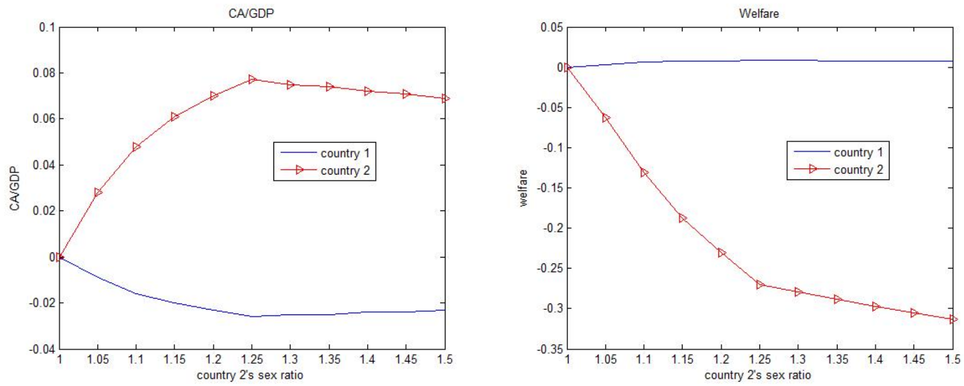

Figure 2.10: Two large countries, endogenous bargaining power, welfare loss in units of consumption goods relative to the case of phi=1, (GDP per capita)1=15*(GDP per capita)2

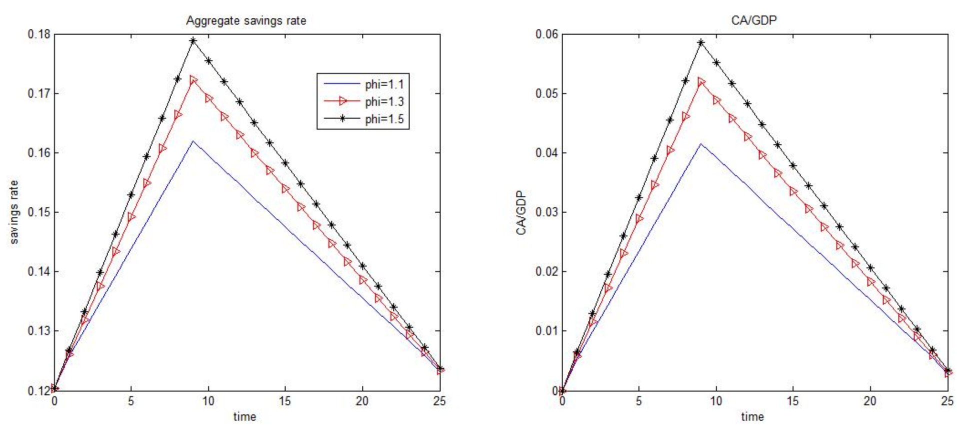

Figure 2.11: 50-period calibrations, tao=10

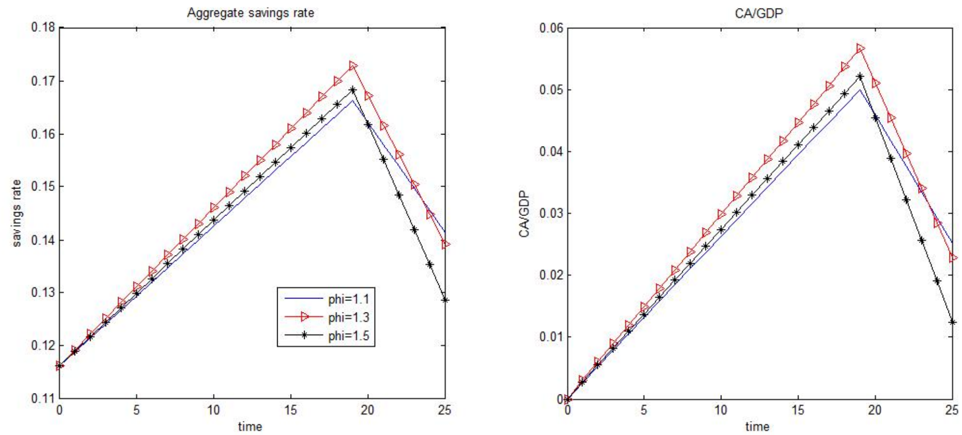

Figure 2.12: 50-period calibrations, tao=20

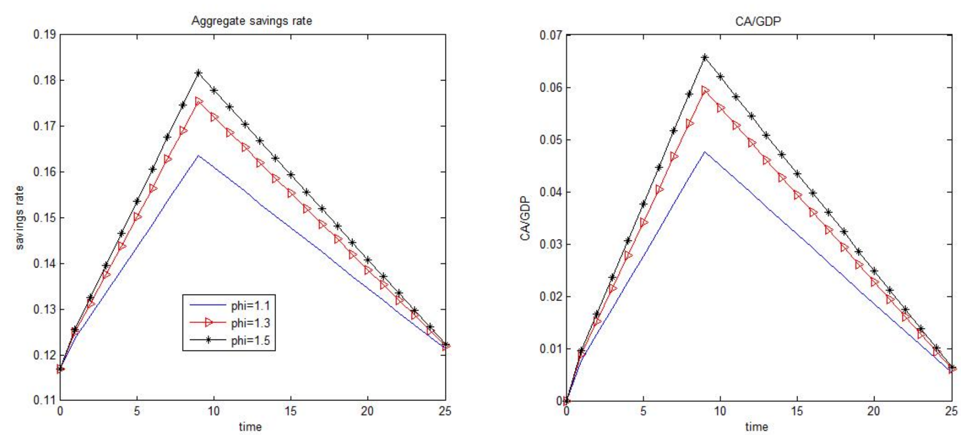

Figure 2.13: 50-period calibrations, tao=10, endogenous intra-household bargaining

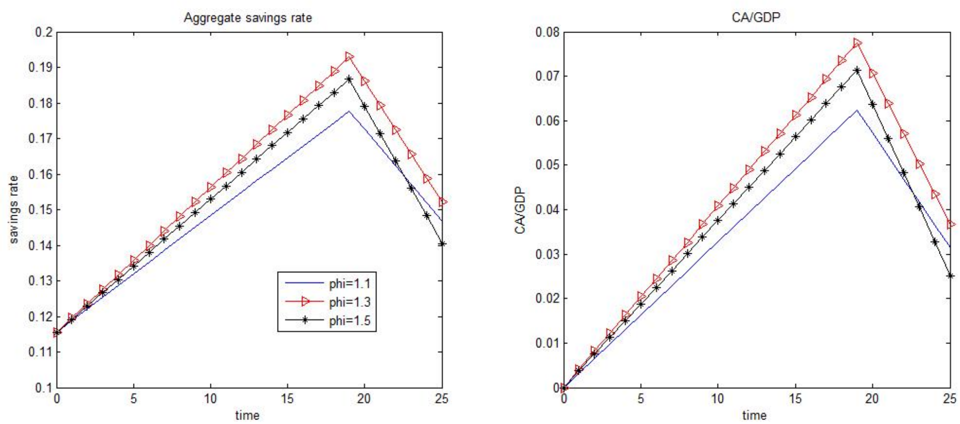

Figure 2.14: 50-period calibrations, tao=20, endogenous intra-household bargaining

Table 2.1: Savings rates vs sex ratios, small country, k=0.8

κ=0.8	m=2, σ=0.05				m=0.5, σ=0.05				m=2, σ=0.1			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.00	0.310	0.310	0.149	0	0.310	0.310	0.149	0	0.310	0.310	0.149	0
1.05	0.522	0.184	0.18	0.032	0.408	0.244	0.16	0.012	0.412	0.242	0.161	0.012
1.10	0.608	0.131	0.196	0.047	0.464	0.204	0.169	0.020	0.484	0.194	0.172	0.024
1.15	0.653	0.102	0.206	0.058	0.493	0.182	0.174	0.025	0.524	0.166	0.18	0.032
1.20	0.664	0.078	0.204	0.056	0.511	0.166	0.178	0.029	0.550	0.146	0.186	0.038
1.25	0.664	0.078	0.203	0.055	0.523	0.155	0.181	0.033	0.568	0.132	0.191	0.043
1.30	0.664	0.078	0.202	0.053	0.532	0.146	0.184	0.036	0.582	0.121	0.196	0.048
1.35	0.664	0.078	0.201	0.052	0.529	0.138	0.181	0.033	0.591	0.112	0.200	0.052
1.40	0.664	0.078	0.200	0.051	0.529	0.138	0.181	0.032	0.598	0.106	0.204	0.055
1.45	0.664	0.078	0.199	0.05	0.529	0.138	0.18	0.032	0.604	0.100	0.207	0.059
1.50	0.664	0.078	0.198	0.049	0.529	0.138	0.179	0.031	0.608	0.095	0.211	0.062

Notes: sm-men's savings rate, sw-women's savings rate, st-economy-wide savings rate, ca-current account to GDP ratio

Table 2.2: Savings rates vs sex ratios, small country, k=0.7

κ=0.7	m=2, σ=0.05				m=0.5, σ=0.05				m=2, σ=0.1			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.00	0.310	0.310	0.149	0	0.310	0.310	0.149	0	0.310	0.310	0.149	0
1.05	0.502	0.195	0.177	0.028	0.387	0.255	0.157	0.008	0.399	0.248	0.159	0.010
1.10	0.584	0.143	0.191	0.043	0.435	0.219	0.163	0.015	0.467	0.203	0.169	0.021
1.15	0.627	0.114	0.201	0.052	0.461	0.199	0.168	0.019	0.505	0.175	0.176	0.028
1.20	0.643	0.088	0.201	0.052	0.476	0.185	0.171	0.022	0.53	0.156	0.182	0.033
1.25	0.643	0.088	0.200	0.051	0.486	0.174	0.173	0.025	0.547	0.142	0.187	0.038
1.30	0.643	0.088	0.199	0.05	0.493	0.166	0.176	0.027	0.560	0.132	0.191	0.042
1.35	0.643	0.088	0.198	0.049	0.497	0.160	0.178	0.029	0.569	0.123	0.195	0.046
1.40	0.643	0.088	0.197	0.048	0.500	0.155	0.179	0.031	0.576	0.117	0.198	0.050
1.45	0.643	0.088	0.196	0.047	0.497	0.149	0.177	0.028	0.581	0.111	0.201	0.053
1.50	0.643	0.088	0.195	0.046	0.497	0.149	0.176	0.027	0.585	0.106	0.204	0.056

Notes: sm-men's savings rate, sw-women's savings rate, st-economy-wide savings rate, ca-current account to GDP ratio

Table 2.3: Savings rates vs sex ratios, small country, k=0.9

κ=0.9	m=2, σ=0.05				m=0.5, σ=0.05				m=2, σ=0.1			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.00	0.31	0.31	0.149	0	0.31	0.31	0.149	0	0.31	0.31	0.149	0
1.05	0.54	0.176	0.183	0.035	0.429	0.233	0.164	0.015	0.424	0.235	0.163	0.015
1.10	0.628	0.121	0.2	0.051	0.491	0.19	0.174	0.025	0.5	0.186	0.175	0.027
1.15	0.674	0.093	0.211	0.062	0.524	0.165	0.18	0.032	0.542	0.156	0.184	0.035
1.20	0.682	0.069	0.207	0.059	0.545	0.149	0.185	0.037	0.569	0.136	0.19	0.042
1.25	0.682	0.069	0.206	0.057	0.559	0.136	0.189	0.041	0.588	0.122	0.196	0.047
1.30	0.682	0.069	0.205	0.056	0.558	0.127	0.186	0.037	0.602	0.111	0.201	0.052
1.35	0.682	0.069	0.204	0.055	0.558	0.127	0.185	0.037	0.612	0.102	0.205	0.057
1.40	0.682	0.069	0.202	0.054	0.558	0.127	0.184	0.036	0.619	0.096	0.209	0.061
1.45	0.682	0.069	0.201	0.053	0.558	0.127	0.184	0.035	0.625	0.09	0.213	0.065
1.50	0.682	0.069	0.2	0.052	0.558	0.127	0.183	0.034	0.63	0.086	0.217	0.068

Notes: sm-men's savings rate, sw-women's savings rate, st-economy-wide savings rate, ca-current account to GDP ratio

Table 2.4: CA/GDP vs country 2's sex ratio, two large countries, differing in the sex ratios

κ=0.8	m=2, σ=0.05				m=0.5, σ=0.05				m=2, σ=0.1			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.00	0.208	0.208	0	0	0.208	0.208	0	0	0.208	0.208	0	0
1.05	0.208	0.24	-0.016	0.016	0.208	0.22	-0.006	0.006	0.208	0.22	-0.006	0.006
1.10	0.208	0.255	-0.024	0.024	0.208	0.228	-0.01	0.01	0.208	0.232	-0.012	0.012
1.15	0.208	0.266	-0.029	0.029	0.208	0.233	-0.013	0.013	0.208	0.239	-0.016	0.016
1.20	0.208	0.267	-0.029	0.029	0.208	0.237	-0.015	0.015	0.208	0.246	-0.019	0.019
1.25	0.208	0.259	-0.026	0.026	0.208	0.241	-0.016	0.016	0.208	0.251	-0.021	0.021
1.30	0.208	0.251	-0.021	0.021	0.208	0.244	-0.018	0.018	0.208	0.255	-0.024	0.024
1.35	0.208	0.243	-0.017	0.017	0.208	0.238	-0.015	0.015	0.208	0.26	-0.026	0.026
1.40	0.208	0.234	-0.013	0.013	0.208	0.228	-0.010	0.010	0.208	0.263	-0.028	0.028
1.45	0.208	0.226	-0.009	0.009	0.208	0.219	-0.006	0.006	0.208	0.267	-0.029	0.029
1.50	0.208	0.218	-0.005	0.005	0.208	0.21	-0.001	0.001	0.208	0.27	-0.031	0.031

Notes: s1-country 1's economy-wide savings rate, s2-country 2's economy-wide savings rate, ca1-country 1's current account to GDP ratio, ca2-country 2's current account to GDP ratio

Table 2.5: CA/GDP vs country 2's sex ratio, k=0.8, two large countries: US and China

κ=0.8	m=2, σ=0.05				m=0.5, σ=0.05				m=2, σ=0.1			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.00	0.208	0.208	0	0	0.208	0.208	0	0	0.208	0.208	0	0
1.05	0.208	0.241	-0.008	0.025	0.208	0.221	-0.003	0.01	0.208	0.221	-0.003	0.01
1.10	0.208	0.257	-0.012	0.037	0.208	0.231	-0.006	0.017	0.208	0.233	-0.006	0.019
1.15	0.208	0.268	-0.015	0.045	0.208	0.236	-0.007	0.021	0.208	0.241	-0.008	0.025
1.20	0.208	0.268	-0.015	0.045	0.208	0.241	-0.008	0.025	0.208	0.248	-0.01	0.03
1.25	0.208	0.261	-0.013	0.04	0.208	0.245	-0.009	0.028	0.208	0.253	-0.011	0.034
1.30	0.208	0.253	-0.011	0.034	0.208	0.244	-0.009	0.027	0.208	0.258	-0.013	0.038
1.35	0.208	0.244	-0.009	0.027	0.208	0.235	-0.007	0.02	0.208	0.262	-0.014	0.041
1.40	0.208	0.236	-0.007	0.021	0.208	0.226	-0.004	0.013	0.208	0.266	-0.015	0.044
1.45	0.208	0.228	-0.005	0.015	0.208	0.217	-0.002	0.007	0.208	0.27	-0.015	0.046
1.50	0.208	0.22	-0.003	0.009	0.208	0.208	0	0	0.208	0.273	-0.016	0.049

Notes: s1-country 1's economy-wide savings rate, s2-country 2's economy-wide savings rate, ca1-country 1's current account to GDP ratio, ca2-country 2's current account to GDP ratio

Table 2.6: CA/GDP vs country 2's sex ratio, k=0.7, two large countries: US and China

κ=0.7	m=2, σ=0.05				m=0.5, σ=0.05				m=2, σ=0.1			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.00	0.208	0.208	0	0	0.208	0.208	0	0	0.208	0.208	0	0
1.05	0.208	0.236	-0.007	0.021	0.208	0.216	-0.002	0.006	0.208	0.218	-0.003	0.008
1.10	0.208	0.251	-0.011	0.032	0.208	0.223	-0.004	0.011	0.208	0.229	-0.005	0.016
1.15	0.208	0.26	-0.013	0.039	0.208	0.227	-0.005	0.014	0.208	0.236	-0.007	0.021
1.20	0.208	0.263	-0.014	0.042	0.208	0.23	-0.006	0.017	0.208	0.241	-0.008	0.025
1.25	0.208	0.256	-0.012	0.036	0.208	0.233	-0.006	0.019	0.208	0.246	-0.01	0.029
1.30	0.208	0.247	-0.01	0.03	0.208	0.235	-0.007	0.02	0.208	0.25	-0.011	0.032
1.35	0.208	0.239	-0.008	0.023	0.208	0.237	-0.007	0.022	0.208	0.254	-0.012	0.035
1.40	0.208	0.231	-0.006	0.017	0.208	0.239	-0.008	0.023	0.208	0.258	-0.012	0.037
1.45	0.208	0.222	-0.004	0.011	0.208	0.232	-0.006	0.018	0.208	0.261	-0.013	0.04
1.50	0.208	0.214	-0.002	0.005	0.208	0.223	-0.004	0.011	0.208	0.264	-0.014	0.042

Notes: s1-country 1's economy-wide savings rate, s2-country 2's economy-wide savings rate, ca1-country 1's current account to GDP ratio, ca2-country 2's current account to GDP ratio

Table 2.7: CA/GDP vs country 2's sex ratio, k=0.9, two large countries: US and China

κ=0.9	m=2, σ=0.05				m=0.5, σ=0.05				m=2, σ=0.1			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.00	0.208	0.208	0	0	0.208	0.208	0	0	0.208	0.208	0	0
1.05	0.208	0.243	-0.009	0.026	0.208	0.223	-0.004	0.011	0.208	0.222	-0.004	0.011
1.10	0.208	0.259	-0.013	0.038	0.208	0.233	-0.006	0.019	0.208	0.235	-0.007	0.02
1.15	0.208	0.27	-0.016	0.047	0.208	0.239	-0.008	0.024	0.208	0.243	-0.009	0.026
1.20	0.208	0.27	-0.015	0.046	0.208	0.244	-0.009	0.027	0.208	0.25	-0.01	0.031
1.25	0.208	0.262	-0.014	0.041	0.208	0.249	-0.01	0.031	0.208	0.255	-0.012	0.035
1.30	0.208	0.254	-0.012	0.035	0.208	0.242	-0.009	0.026	0.208	0.26	-0.013	0.039
1.35	0.208	0.246	-0.01	0.029	0.208	0.233	-0.006	0.019	0.208	0.265	-0.014	0.043
1.40	0.208	0.238	-0.007	0.022	0.208	0.224	-0.004	0.012	0.208	0.269	-0.015	0.046
1.45	0.208	0.23	-0.005	0.016	0.208	0.215	-0.002	0.006	0.208	0.273	-0.016	0.048
1.50	0.208	0.222	-0.003	0.01	0.208	0.207	0	-0.001	0.208	0.276	-0.017	0.051

Notes: s1-country 1's economy-wide savings rate, s2-country 2's economy-wide savings rate, ca1-country 1's current account to GDP ratio, ca2-country 2's current account to GDP ratio

Table 2.8: Savings rates vs sex ratios, endogenous intra-household bargaining, small country, sigma=0.05

γ=0.5	ε=0				ε=0.5				ε =1			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.00	0.31	0.31	0.149	0	0.336	0.336	0.149	0	0.36	0.36	0.149	0
1.05	0.504	0.194	0.177	0.028	0.553	0.224	0.187	0.038	0.591	0.295	0.207	0.058
1.10	0.586	0.142	0.192	0.043	0.638	0.206	0.213	0.065	0.68	0.286	0.237	0.089
1.15	0.629	0.113	0.201	0.053	0.684	0.197	0.23	0.081	0.725	0.28	0.255	0.106
1.20	0.645	0.088	0.201	0.053	0.712	0.191	0.242	0.093	0.73	0.279	0.257	0.108
1.25	0.645	0.088	0.2	0.052	0.732	0.186	0.251	0.102	0.73	0.279	0.254	0.105
1.30	0.645	0.088	0.199	0.05	0.732	0.186	0.249	0.1	0.73	0.279	0.252	0.103
1.35	0.645	0.088	0.198	0.049	0.732	0.186	0.239	0.09	0.73	0.279	0.25	0.101
1.40	0.645	0.088	0.197	0.048	0.732	0.186	0.237	0.088	0.73	0.279	0.248	0.099
1.45	0.645	0.088	0.196	0.047	0.732	0.186	0.235	0.086	0.73	0.279	0.246	0.097
1.50	0.645	0.088	0.195	0.046	0.732	0.186	0.233	0.084	0.73	0.279	0.244	0.095
γ=0.3	ε=0				ε=0.5				ε =1			
1.00	0.31	0.31	0.149	0	0.326	0.326	0.149	0	0.341	0.341	0.149	0
1.05	0.525	0.183	0.181	0.032	0.571	0.186	0.187	0.038	0.597	0.246	0.205	0.057
1.10	0.611	0.13	0.196	0.048	0.656	0.166	0.213	0.065	0.686	0.233	0.235	0.086
1.15	0.656	0.101	0.207	0.059	0.701	0.155	0.23	0.081	0.731	0.225	0.252	0.104
1.20	0.667	0.077	0.205	0.056	0.729	0.148	0.242	0.093	0.735	0.224	0.254	0.105
1.25	0.667	0.077	0.204	0.055	0.747	0.144	0.251	0.102	0.735	0.224	0.252	0.103
1.30	0.667	0.077	0.202	0.054	0.747	0.144	0.249	0.1	0.735	0.224	0.25	0.101
1.35	0.667	0.077	0.201	0.053	0.747	0.144	0.238	0.089	0.735	0.224	0.248	0.099
1.40	0.667	0.077	0.2	0.052	0.747	0.144	0.236	0.087	0.735	0.224	0.246	0.097
1.45	0.667	0.077	0.199	0.051	0.747	0.144	0.235	0.086	0.735	0.224	0.244	0.095
1.50	0.667	0.077	0.198	0.05	0.747	0.144	0.233	0.084	0.735	0.224	0.242	0.093
γ=0.7	ε=0				ε=0.5				ε =1			
1.00	0.31	0.31	0.149	0	0.346	0.346	0.149	0	0.378	0.378	0.149	0
1.05	0.482	0.205	0.173	0.024	0.53	0.255	0.182	0.033	0.579	0.333	0.203	0.054
1.10	0.559	0.155	0.186	0.038	0.611	0.239	0.207	0.059	0.666	0.326	0.233	0.084
1.15	0.6	0.127	0.195	0.047	0.657	0.23	0.224	0.075	0.713	0.321	0.251	0.102
1.20	0.621	0.099	0.197	0.049	0.686	0.224	0.236	0.087	0.719	0.323	0.254	0.105
1.25	0.621	0.099	0.196	0.048	0.706	0.22	0.245	0.096	0.719	0.323	0.252	0.103
1.30	0.621	0.099	0.195	0.047	0.706	0.22	0.243	0.094	0.719	0.323	0.25	0.101
1.35	0.621	0.099	0.194	0.046	0.706	0.22	0.234	0.085	0.719	0.323	0.248	0.099
1.40	0.621	0.099	0.193	0.045	0.706	0.22	0.232	0.083	0.719	0.323	0.246	0.097
1.45	0.621	0.099	0.192	0.044	0.706	0.22	0.23	0.081	0.719	0.323	0.244	0.095
1.50	0.621	0.099	0.191	0.043	0.706	0.22	0.229	0.08	0.719	0.323	0.242	0.093

Notes: sm-men's savings rate, sw-women's savings rate, st-economy-wide savings rate, ca-current account to GDP ratio

Table 2.9: Sex ratios and current accounts, average 2004-2008

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
sex ratio	125.5***	126.6***	134.3**	166.3***	171.6***	131.1**	125.1**
	(44.940)	(44.310)	(54.600)	(55.710)	(55.140)	(55.410)	(53.270)
ln(real GDP per capita)	0.554	-12.40*	0.269	0.603	2.307	3.280**	-20.58*
	(0.664)	(6.846)	(1.088)	(1.075)	(1.418)	(1.437)	(10.590)
social security expenditure/GDP			0.044	0.122	0.109	0.479**	0.434*
			(0.179)	(0.179)	(0.177)	(0.229)	(0.221)
working age population				-0.545*	-0.596**	-0.405	-0.103
				(0.277)	(0.276)	(0.273)	(0.294)
private credit/GDP					-0.0478*	-0.0639**	-0.0836***
					(0.026)	(0.025)	(0.026)
Africa						17.89***	17.32***
						(6.121)	(5.881)
Asia						16.98***	15.58***
						(5.484)	(5.301)
Europe						5.546	4.645
						(5.276)	(5.081)
North America						11.03*	11.48**
						(5.819)	(5.591)
Oceania						16.74**	18.09***
						(6.565)	(6.330)
ln(real GDP per capita) square		0.793*					1.444**
		(0.417)					(0.636)
Observations	93	93	62	62	61	60	60
R-squared	0.12	0.15	0.11	0.17	0.21	0.41	0.47

Notes: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 2.10: Sex ratios and current accounts, year 2006

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
sex ratio	110.0***	112.0***	107.2**	142.3***	144.8***	109.4**	107.5**
	(38.730)	(37.820)	(44.370)	(45.070)	(45.350)	(46.110)	(43.940)
ln(real GDP per capita)	0.817	-12.59**	0.507	0.955	1.85	2.667**	-17.66**
	(0.558)	(5.514)	(0.891)	(0.876)	(1.170)	(1.203)	(8.135)
social security expenditure/GDP			-0.01	0.071	0.064	0.355*	0.320*
			(0.151)	(0.149)	(0.150)	(0.192)	(0.183)
working age population				-0.542**	-0.576**	-0.454**	-0.144
				(0.223)	(0.227)	(0.226)	(0.247)
private credit/GDP					-0.026	-0.0404*	-0.0594***
					(0.022)	(0.021)	(0.021)
Africa						13.71***	13.02***
						(5.084)	(4.848)
Asia						14.55***	13.21***
						(4.631)	(4.440)
Europe						5.076	4.161
						(4.452)	(4.254)
North America						8.337*	8.925*
						(4.941)	(4.710)
Oceania						14.00**	15.35***
						(5.566)	(5.326)
ln(real GDP per capita) square		0.832**					1.241**
		(0.341)					(0.492)
Observations	104	104	65	65	64	63	63
R-squared	0.14	0.19	0.12	0.2	0.21	0.39	0.46

Notes: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 2.11: Savings rates versus sex ratios, year 2006

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
sex ratio	144.1***	102.9**	182.7***	169.5***	165.9***	113.5*	121.8**
	(50.570)	(50.220)	(54.720)	(57.690)	(56.310)	(56.630)	(57.410)
ln(real GDP per capita)	0.551	19.67***	-0.13	-0.303	-0.839	-0.56	-11.51
	(0.688)	(5.850)	(1.085)	(1.113)	(1.590)	(1.634)	(11.920)
social security expenditure/GDP			-0.0855	-0.111	-0.103	0.221	0.153
			(0.190)	(0.193)	(0.193)	(0.259)	(0.270)
working age population				0.209	0.281	0.443	0.641*
				(0.278)	(0.278)	(0.284)	(0.356)
private credit/GDP					0.00303	-0.00091	-0.0133
					(0.028)	(0.027)	(0.030)
Africa						12.00*	11.75*
						(6.063)	(6.078)
Asia						16.05***	15.10***
						(5.326)	(5.431)
Europe						6.424	6.243
						(5.217)	(5.228)
North America						6.944	6.259
						(6.267)	(6.320)
Oceania						18.17***	18.88***
						(6.316)	(6.372)
ln(real GDP per capita) square		-1.219***					0.683
		(0.371)					(0.736)
Observations	124	124	60	60	59	58	58
R-squared	0.1	0.18	0.17	0.18	0.21	0.39	0.4

Notes: Standard errors in parentheses, *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 2.12: Savings rates versus sex ratios, average 2004-2008

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
sex ratio	127.8***	102.5**	180.5***	159.3***	157.5***	118.6**	124.6**
	(43.180)	(42.940)	(47.470)	(49.210)	(48.070)	(49.740)	(50.250)
ln(real GDP per capita)	0.713	14.72***	0.12	-0.111	-0.63	0.248	-8.574
	(0.589)	(4.898)	(0.939)	(0.944)	(1.338)	(1.439)	(9.788)
social security expenditure/GDP			-0.104	-0.16	-0.153	0.0083	-0.0419
			(0.163)	(0.166)	(0.165)	(0.218)	(0.225)
working age population				0.335	0.390*	0.412*	0.553*
				(0.229)	(0.227)	(0.235)	(0.281)
private credit/GDP					0.00469	-0.0051	-0.0163
					(0.024)	(0.024)	(0.027)
Africa						10.07*	9.926*
						(5.253)	(5.264)
Asia						13.42***	12.65**
						(4.827)	(4.908)
Europe						6.535	6.365
						(4.624)	(4.635)
North America						6.154	6.224
						(5.291)	(5.300)
Oceania						12.73**	13.19**
						(5.709)	(5.741)
ln(real GDP per capita) square		-0.895***					0.55
		(0.311)					(0.603)
Observations	134	134	63	63	62	61	61
R-squared	0.12	0.17	0.21	0.24	0.26	0.39	0.4

Notes: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Chapter 3

Sex Ratios and Exchange Rates

with Shang-Jin Wei

The Chinese real exchange rate (RER) is widely believed to be substantially undervalued. This in turn has created enormous tension in the global monetary system. The standard narrative goes as follows. The Chinese exchange rate is undervalued largely through deliberate and massive government interventions in the currency market. The rapid accumulation of the country's foreign exchange reserve is the prima facie evidence that the Chinese authorities have engaged in a massive currency market intervention. The undervalued currency has in turn created both a growing currency account surplus and an increasing departure from the purchasing power parity.

However, this narrative is not an inevitable way to piece together the value of the real exchange rate, the current account and the foreign exchange reserve. In this paper, we explore an alternative narrative. It starts from some technology and policy shocks, unrelated to currency market interventions, that cause simultaneously a rise in the country's savings rate and an expansion in the country's effective labor supply. These developments in turn lead to a simultaneous decline in the value of the real exchange rate and a rise in the current account balance (even though the exchange rate decline is not the cause of the current account surplus). Once the current account is put into a surplus gear, the foreign exchange reserve accumulation can happen passively as a result of the country's capital control regime - put in place long before the exchange rate became an issue - which, as capital control regimes in many other countries, requires mandatory surrender of foreign exchange earnings by firms and households.

The initial technology shock in the new narrative was the spread of ultrasound B machine in China in the 1980s that allowed expectant parents to easily detect the gender of the fetus. 1985 was the first year in which half of the county level hospitables acquired at least one such machine (Li and Zheng, 2009). The initial policy shock was the implementation of a strict version of the family planning policy (popularly known as the "one-child policy") that severely restricts many couples' legally permissable number of children to a level below their desire. By interacting with a long-existing parental preference for sons, the combination of the two shocks started to produce an unnaturally high ratio of boys to girls at birth from early 1980s, and the sex ratio at birth became worse progressively as the use of ultrasound machines became more widespread, and the enforcement of the family planning policy tightened over time. Around 2003, the first cohort born with an excess number of males was entering the marriage market. The competition for marriage partner by young men becomes progressively more fierce. In 2007, the sex ratio for the pre-marital age cohort (5-20) is about 115 young men per 100 young women. This implies that about one out every nine young men cannot get married mathematically speaking.

How would a rise in the sex ratio imbalance trigger a significant increase in the savings rate? The key is that family wealth is a key status variable in the marriage market (other things equal). As the competition for brides intensifies, young men and their parents raise their savings rate in order to improve their relative standing in the marriage market. If the biological desire to have a female partner is strong, the response of the savings rate to a rise in the sex ratio can also be quantitatively large. Of course, any complete story has to investigate why the behavior by women or their parents does not undo the competitive savings story.

The empirical motivation for the savings channel comes from Wei and Zhang (2011).

They provide evidence from China at both the household level and regional level. First, across rural households with a son, they document that the savings rate tends to be higher in regions with a higher sex ratio imbalance (holding constant family income, age, gender, and educational level of the household head and other household characteristics). In comparison, for rural households with a daughter, their savings rate appears to be uncorrelated with the local sex ratio. Across cities, both households with a son and households with a daughter tend to have a higher savings rate in regions with a more skewed sex ratio, although the elasticity of the savings rate with respect to the sex ratio tends to be bigger for son families. Second, across Chinese provinces, they find a strong positive correlation between the local savings rate and the local sex ratio, after controlling for the age structure of the local population, income level, inequality, recent growth rate, local enrollment rate in the social safety net, and other factors. Third, to go from correlation to causality, they explore regional variations in the enforcement of the family planning policy as instruments for the local sex ratio, and confirm the findings in the OLS regressions. The sex ratio effect is both economically and statistically significant. While the Chinese household savings rate approximately doubled from 16% (of disposable income) in 1990 to 31% in 2007, Wei and Zhang (2011a) estimate that the rise in the sex ratio could explain about half of the increase in the household savings rate.

When the economy-wide savings rate rises, the real exchange rate often falls. To see this, we recognize that a rise in the savings rate implies a reduction in the demand for both tradable and non-tradable goods. Since the price of the tradable good is tied down by the world market, this translates into a reduction in the relative price of the nontradable good, and hence a decline in the value of the real exchange rate (a departure from the PPP). The effect would be persistent if there are frictions that impede the reallocation of factors between the tradable and nontradable sectors.

The second channel for the sex ratio imbalance to affect the real exchange rate works through effective labor supply. A rise in the sex ratio can also motivate men to cut down leisure and increase labor supply. This leads to an increase in the economy-wide effective labor supply. If the nontradable sector is more labor intensive than the tradable sector, this generates a Rybzinsky-like effect, leading to an expansion of the nontradable sector at the expense of the tradable sector. The increase in the supply of nontradable good leads to an additional decline in the relative price of nontradable and a further decline in the value of the RER.

Putting the two channels together, a rise in the sex ratio generates a real exchange rate that appears too low relative to the purchasing power parity (or relative to the standard approaches used by the IMF to assess equilibrium exchange rate that include additional terms beyond a departure from PPP but do not include the sex ratio, savings rate, and effective labor supply). Because the effect of a skewed sex ratio on the real exchange rate comes from competition for sex partners, this is fundamentally a Darwinian perspective on the exchange rate.

Of course, other structural factors may also have contributed to an increase in the aggregate savings rate (e.g., an increase in the government savings or an increase in the private-sector precautionary savings) or an increase in the effective labor supply (e.g., gradual relaxation of restrictions on rural-urban migration). These other factors would reinforce the Darwinian mechanism discussed in this paper, causing the real exchange rate to fall further.

A desire to enhance one's prospect in the marriage market through a higher level of wealth could be a motive for savings even in countries with a balanced sex ratio. But such a motive is not as easy to detect when the competition is modest. When the sex ratio gets out of balance, obtaining a marriage partner becomes much less assured. A host of behaviors that are motivated by a desire to succeed in the marriage market may become magnified. But sex ratio imbalances so far have not been investigated by macroeconomists. This may be a serious omission.

A sex ratio imbalance is a common demographic feature in many economies, especially in East, South, and Southeast Asia, such as Korea, India, Vietnam, Singapore, Taiwan and Hong Kong, in addition to China. It is quite possible that the sex ratio effect plays an important in the real exchange rate of these economies. To be clear, most countries in the world do not have a severe sex ratio imbalance. Correspondingly, it cannot be a significant determinant of the real exchange rate for them. However, if one only considers the standard determinants of the real exchange rate and ignore the sex ratio effect, one could mistakenly conclude that countries with a severe sex ratio imbalance to have a severely undervalued currency. This set of countries happens to include China - the world's second largest economy and the largest exporter. Given the enormous effort by international financial institutions and many national governments to pass judgment on its exchange rate, getting it right has global importance.

There are four bodies of work that are related to the current paper. First, the theoretical and empirical literature on the real exchange rate is too voluminous to summarize comprehensively here. Sarno and Taylor (2002) and Chinn (2011) provide recent surveys. Second, the literature on status goods, positional goods, and social norms (e.g., Cole, Mailath and Postlewaite, 1992, Corneo and Jeanne, 1999, Hopkins and Kornienko, 2004 and 2009) has offered many useful insights. One key point is that when wealth can improve one's social status (including improving one's standing in the marriage market), in addition to affording a greater amount of consumption goods, there is an extra incentive to save. This element is in our model as well. However, all existing theories on status goods feature a balanced sex ratio. Yet, an unbalanced sex ratio presents some non-trivial challenges. In particular, while a rise in the sex ratio is an unfavorable shock to men, it is a favorable shock to women. Could the women strategically reduce their savings so as to completely offset whatever increments in savings men may have? In other words, the impact on aggregate savings from a rise in the sex ratio appears ambiguous. Our model will address this question. In any case, the literature on status goods has no discernible impact in macroeconomic policy circles. For example, while there are voluminous documents produced by the International Monetary Fund or speeches by US officials on China's high savings rate and large current account surplus, no single paper or speech thus far has pointed to a possible connection with its high sex ratio imbalance.

A third related literature is the economics of family, which is also too vast to be summarized here comprehensively. One interesting insight from this literature is that a married couple's consumption has a partial public goods feature (Browning, Bourguignon and Chiappori, 1994; Donni, 2006). We make use of this feature in our model as well. None of the papers in this literature explores the general equilibrium implications for exchange rates from a change in the sex ratio. The fourth literature examines empirically the causes of a rise in the sex ratio. The key insight is that the proximate cause for the recent rise in the sex ratio imbalance is sex-selective abortions, which have been made increasingly possible by the spread of Ultrasound B machines. There are two deeper causes for the parental willingness to disproportionately abort female fetuses. The first is the parental preference for sons, which in part has to do with the relatively inferior economic status of women. When the economic status of women improves, sex-selective abortions appear to decline (Qian, 2008). The second is either something that leads parents to voluntarily have a lower fertility rate than earlier generations, or a government policy that limits the number of children a couple can have. In regions of China where the family planning policy is less strictly enforced, there is also less sex ratio imbalance (Wei and Zhang, 2011). Bhaskar (2011) examines parental sex selections and their welfare consequences.

The rest of the paper is organized as follows. In Section 2, we construct a simple overlapping generations (OLG) model with only one gender, and show that structural shocks, by raising the savings rate, can simultaneously produce a real exchange rate depreciation and a current account surplus. In Section 3, we present an OLG model with two genders, and demonstrate that a rise in the sex ratio could lead to a rise in both the aggregate savings rate and the current account, and a fall in the value of the real exchange rate. In Section 4, we calibrate the model to see if the sex ratio imbalance can produce changes in the real exchange rate and current account whose magnitudes are economically significant. In section 5, we provide some empirical evidence on the connection between the sex ratio and the real exchange rate. Section 6 offers concluding remarks and discusses possible future research.

3.1 A benchmark model with one gender

We start with a simple benchmark model with one gender. This allows us to see the savings channel in a transparent way. The setup is standard, and the discussion will pave the way for a model in the next section that features two genders and an unbalanced sex ratio.

There are two types of agents: consumers and producers. Consumers consume and make the saving decisions to maximize their intertemporal utilities. Producers choose capital and labor input to maximize the profits.

3.1.1 Consumers

Consumers live for two periods: young and old. They receive labor income in the first period and nothing in the second period after retiring. In the first period, consumers consume a part of the labor income in the first period and save the rest for the second period.

The final good C_t consumed by consumers consists of two parts: a tradable good C_{Tt} and a nontradable good C_{Nt}.

$$C_t = \frac{C_{Nt}^{\gamma} C_{Tt}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

We normalize the price of the tradable good to be one, and let P_{Nt} denote the relative price of the nontradable good. The consumer price index is $P_t = P_{Nt}^{\gamma}$.

Consumers earn labor income when they are young and retire when they are old. The optimization problem for a representative consumer is

$$\max \ u(C_{1t}) + \beta u(C_{2,t+1})$$

with the intertemporal budget constraint

$$P_t C_{1t} = (1 - s_t) y_t \text{ and } P_{t+1} C_{2,t+1} = R s_t y_t$$

where y_t is the disposable income and s_t is the savings rate of the young cohort. R is the gross interest rate in terms of the tradable good.

The optimal condition for the representative consumer's problem is

$$\frac{u'_{1t}}{P_t} = \beta R \frac{u'_{2,t+1}}{P_{t+1}} \tag{3.1}$$

We start with the case of a small open economy, and assume that the law of one price for the tradable good holds. The price of the tradable good is determined by the world market, and is set to be one in each period. The interest rate R in units of the tradable good is also a constant.

3.1.2 Producers

There are two sectors in the economy: a tradable good sector and a non-tradable good sector. Both markets are perfectly competitive. For simplicity, we make the same assumption as in Obstfeld and Rogoff (1996) that only the tradable good can be transformed into capital used in production.[1]

Tradable good producers

For simplicity, we assume a complete depreciation of capital at the end of every period. Tradable producers maximize

$$\max \ E_t \sum_{\tau=0}^{\infty} (R)^{-\tau} \left[Q_{T,t+\tau} - w_{t+\tau} L_{T,t+\tau} - K_{T,t+\tau+1}\right]$$

where the production function is

$$Q_{Tt} = \frac{A_{Tt} K_{Tt}^{\alpha_T} L_{Tt}^{1-\alpha_T}}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}}$$

[1] Relaxing this assumption will not change any of our results qualitatively.

Without any unanticipated shocks, the factor demand functions are, respectively,

$$R = \frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\alpha_T A_{Tt}\left(\frac{L_{Tt}}{K_{Tt}}\right)^{1-\alpha_T} \tag{3.2}$$

$$w_t = \frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}(1-\alpha_T) A_{Tt}\left(\frac{K_{Tt}}{L_{Tt}}\right)^{\alpha_T} \tag{3.3}$$

It is useful to note that when there is an unanticipated shock in period t, (3.2) does not hold since K_{Tt} is a predetermined variable.

Nontradable good producers

Nontradable good producers maximize the following objective function:

$$\max\ E_t \sum_{\tau=0}^{\infty} (R)^{-\tau}\left[P_{N,t+\tau}Q_{N,t+\tau} - w_{t+\tau}L_{N,t+\tau} - K_{N,t+\tau+1}\right]$$

with the production function given by

$$Q_{Nt} = \frac{A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}$$

Without unanticipated shocks, we have

$$R = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}\alpha_N A_{Nt}\left(\frac{L_{Nt}}{K_{Nt}}\right)^{1-\alpha_N} \tag{3.4}$$

$$w_t = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}(1-\alpha_N) A_{Nt}\left(\frac{K_{Nt}}{L_{Nt}}\right)^{\alpha_N} \tag{3.5}$$

If there is an unanticipated shock in period t, (3.4) does not hold.

In equilibrium, the market clearing condition for the nontradable good pins down the price of the nontradable good,

$$Q_{Nt} = \frac{\gamma P_t\left(C_{2t}+C_{1t}\right)}{P_{Nt}} \tag{3.6}$$

The labor market clearing condition is given by

$$L_{Tt} + L_{Nt} = 1 \tag{3.7}$$

Assuming no labor income tax (for simplicity), $y_t = w_t$.

Definition 3. *An equilibrium in the small open economy is a set*

$$\{s_t, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$$

that satisfies the following conditions:

(i) The households' savings rates, $s_t = \{s_{it}, s_{-i,t}\}$, maximize the household's welfare

$$s_t = \arg\max \{ V_t |\, s_{-i,t}, K_{Tt+1}, K_{Nt+t}, L_{Tt}, L_{Nt}, P_{Nt}\}$$

(ii) The allocation of capital stock and labor, and the output of the non-tradable good clear the factor and the output markets, and maximize the firms' profit. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ solves (3.2), (3.3), (3.4), (3.5), (3.6) and (3.7).

3.1.3 A shock to the savings rate and the effect on the exchange rate

To illustrate the idea that a shock that raises the savings rate could lower the value of the real exchange rate, we now consider an unanticipated increase in the discount factor β that makes the young cohort more patient. In period t, (3.3) and (3.5) hold, but (3.2) and (3.4) fail.

The market clearing condition for the nontradable good can be re-written as

$$\frac{P_{Nt} A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} = \gamma \left(R s_{t-1}^{young} w_{t-1} + (1 - s_t^{young}) w_t \right)$$

We can solve (3.1), (3.6), (3.3) and (3.5) to obtain the equilibrium in period t. Let $R = \frac{RP_t}{P_{t+1}}$ denote the real interest rate. We assume that the utility function is of the CRRA form, i.e., $u(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$. Following Obstfeld and Rogoff (1996) and assuming that the nontradable good sector is relatively more labor-intensive, i.e., $\alpha_N < \alpha_T$, we can obtain the following proposition.

Proposition 9. *With an increase in the discount factor β of the young cohort, the aggregate savings rate rises, and the price of the nontradable good falls. As a result, the real exchange rate depreciates and the current account increases.*

Proof. See Appendix A3.1. □

In the period in which the shock occurs, as a representative consumer becomes more patient, he would save more and consume less. The reduction in aggregate consumption leads to a decrease in the relative price of nontradable good (and a depreciation of the real exchange rate). As the rise in savings is not accompanied by a corresponding rise in investment, the country's current account increases. In summary, without currency manipulations, real factors that lead to a rise in a country's savings rate can simultaneously produce a fall in the real exchange rate and a rise in the current account. The low value of the real exchange rate is not the cause of the current account surplus.

Note that the effect on the RER and the current account last for one period. In period $t+1$, since the shock has been observed and taken into account by consumers and firms, (3.2) and (3.4) hold in equilibrium. By solving (3.2), (3.3), (3.4) and (3.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \text{ and } P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

In other words, the price of the nontradable good and the consumer price index go back to their initial levels. Later in the paper, we will demonstrate how frictions in the factor market can produce longer-lasting effects on the real exchange rate and the current account.

3.2 Unbalanced sex ratios and real exchange rates

In this section, we extend our benchmark model to a two-sex overlapping generations model. Within each cohort, there are women and men. A marriage can take place at the beginning of a cohort's second period, but only between a man and a women in the same cohort. Once married, the husband and the wife pool their first-period savings together and consume an identical amount in the second period. The second period consumption within a marriage has a partial public good feature. In other words, the husband and the wife can each consume more than half of their combined second period income. Everyone is endowed with an ability to give his/her spouse some additional emotional utility (or "love"). This emotional utility is a random variable in the first period with a common and known distribution across all members of the same sex, and its value is realized and becomes public information when an individual enters the marriage market. There are no divorces.

Each generation is characterized by an exogenous ratio of men to women $\phi(\geq 1)$. All men are identical *ex ante*, and all women are identical *ex ante*. Men and women are symmetric in all aspects - in particularly, men do not have an intrinsic tendency to save more - except that the sex ratio may be unbalanced.

Throughout the model, we maintain the assumption of an exogenous sex ratio. While it is surely endogenous in the long-run as parental preference should evolve, the assumption of an exogenous sex ratio can be defended on two grounds. First, the technology that enables the rapid rise in the sex ratio has only become inexpensive and widely accessible in developing countries within the last 25 years or so. As a result, it is reasonable to think that the rising sex ratio affects only the relatively young cohort's savings decisions, but not those who have passed half of their working careers. Second, in terms of cross country experience, most countries with a skewed sex ratio have not shown a sign of reversal. This suggests that, if the sex ratio follows a mean reversion process, the speed of reversion is very low.

3.2.1 A small open economy

We start from a small open economy. As in the benchmark model, the price of the tradable good is always one and the interest rate in units of the tradable good is a constant R. As in Obstfeld and Rogoff (1995), we assume that only tradable goods can be converted into capital used in production.

A Representative Woman's Optimization Problem

A representative woman makes her consumption/saving decisions in her first period, taking into account the choices by men and all other women, and the likelihood that she will be married. If she fails to get married, her second-period consumption is $P_{t+1}C^{w,n}_{2,t+1} = Rs^w_t y^w_t$, where R, y^w_t and s^w_t are the gross interest rate of an international bond, her endowment, and her savings rate, respectively, all in units of the tradable good. If she is married, her second-period consumption is $P_{t+1}C^w_{2,t+1} = \kappa\,(Rs^w_t y^w_t + Rs^m_t y^m_t)$, where y^m_t and s^m_t are her husband's first period endowment and savings rate, respectively. κ ($\frac{1}{2} \leq \kappa \leq 1$) represents the notion that consumption within a marriage is a public good with congestion. As an example, if two spouses buy a car, both can use it. In contrast, were they single, they would have to buy two cars. When $\kappa = \frac{1}{2}$, the husband and the wife only consume private goods. When $\kappa = 1$, then all the consumption is a public good with no congestion[2].

The optimal savings rate is chosen to maximize the following objective function:

$$V^w_t = \max_{s^w_t} u(C^w_{1t}) + \beta E_t\left[u(C^w_{2,t+1}) + \eta^m\right]$$

[2]By assuming the same κ for the wife and the husband, we abstract from a discussion of bargaining within a household. In an extension later in the paper, we allow κ to be gender specifc, and to be a function of both the sex ratio and the relative wealth levels of the two spouses, along the lines of Chiappori (1988 and 1992) and Browning and Chiappori (1998). This tends to make the response of the aggregate savings stronger to a given rise in the sex ratio.

subject to the budget constraints that

$$P_t C_{1t}^w = (1 - s_t^w) y_t^w \tag{3.8}$$

$$P_{t+1} C_{2,t+1}^w = \begin{cases} \kappa \left(R s_t^w y_t^w + R s_t^m y_t^m\right) & \text{if married} \\ R s_t^w y_t^w & \text{otherwise} \end{cases} \tag{3.9}$$

where E_t is the conditional expectation operator. η^m is the emotional utility (or "love") she obtains from her husband, which is a random variable with a distribution function F^m. Bhaskar (2011) also introduces a similar "love" variable.

A Representative Man's Optimization Problem

A representative man's problem is symmetric to a women's problem. In particular, if he fails to get married, his second period consumption is $P_{t+1} C_{2,t+1}^{m,n} = R s_t^m y^m$. If he is married, his second period consumption is $P_{t+1} C_{2,t+1}^m = \kappa \left(R s_t^w y_t^w + R s_t^m y_t^m\right)$. He will choose his savings rate to maximize the following value function

$$V_t^m = \max_{s_t^m} u(C_{1t}^m) + \beta E_t \left[u(C_{2,t+1}^m) + \eta^w\right]$$

subject to the budget constraints that

$$P_t C_{1t}^m = (1 - s_t^m) y_t^m \tag{3.10}$$

$$P_{t+1} C_{2,t+1}^m = \begin{cases} \kappa \left(R s_t^w y_t^w + R s_t^m y_t^m\right) & \text{if married} \\ R s_t^m y_t^m & \text{otherwise} \end{cases} \tag{3.11}$$

where V^m is his value function. η^w is the emotional utility he obtains from his wife, which is drawn from a distribution function F^w.

The Marriage Market[3]

In the marriage market, every woman (or man) ranks all members of the opposite sex by a combination of two criteria: (1) the level of wealth (which is determined solely by the first-period savings), and (2) the size of "love" she/he can obtain from her/his spouse. The weights on the two criteria are implied by the utility functions specified earlier. More precisely, woman i prefers a higher ranked man to a lower ranked one, where the rank on man j is given by $u(c_{2w,i,j}) + \eta_j^m$. Symmetrically, man j assigns a rank to woman i based on the utility he can obtain from her $u(c_{2m,j,i}) + \eta_i^w$. To ensure that the preference is strict for both men and women, whenever there is a tie in terms of the above criteria, we break the tie by assuming that a woman prefers j if $j < j'$ and a man does the same. Note that "love" is not in the eyes of a beholder in the sense that every woman (man) has the same ranking over men (women).

The marriage market is assumed to follow the Gale-Shapley algorithm, which produces a unique and stable equilibrium of matching (Gale and Shapley, 1962; and Roth and Sotomayor, 1990). The algorithm specifies the following: (1) Each man proposes in the first round to his most preferred choice of woman. Each woman holds the proposal from her most preferred suitor and rejects the rest. (2) Any man who is rejected in round k-1 makes a new proposal in round k to his most preferred woman among those who have not have rejected him. Each available women in round k holds the proposal from her most preferred man and rejects the rest. (3) The procedure repeats itself until no further proposals are made, and the women accept the most attractive proposals.[4]

With many women and men in the marriage market, all women (and all men) approximately form a continuum and each individual has a measure close to zero. Let I^w and I^m denote the continuum formed by women and men respectively. We normalize I^w and

[3]We use the word "market" informally here. The pairing of husbands and wives is not done through prices.

[4]If only women can propose and men respond with deferred acceptance, the same matching outcomes will emerge. What we have to rule out is that both men and women can propose, in which case, one cannot prove that the matching is unique.

let $I^w = (0,1)$. Since the sex ratio is ϕ, the set of men $I^m = (0,\phi)$. Men and women are ordered in such a way that a higher value in the set means a higher ranking by members of the opposite sex.

In equilibrium, there exists a unique mapping (π^w) for women in the marriage market, $\pi^w : I^w \to I^m$. That is, woman i ($i \in I^w$) is mapped to man j ($j \in I^m$), given all the savings rates and emotional utility draws. This implies a mapping from a combination (s_i^w, η_i^w) to another combination (s_j^m, η_j^m). Before she enters the marriage market, she knows only the distribution of her own type but not the exact value. As a result, the type of her future husband (s_j^m, η_j^m) is also a random variable. Woman i's second period expected utility is

$$\begin{aligned} & \int \max \left[\; u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(i|s_i^w,\eta_i^w,s_{-i}^w,\eta_{-i}^w,s^m,\eta^m\right)}, \quad u\left(R s_i^w y_i^w\right) \; \right] dF^w\left(\eta_i^w\right) \\ = & \int_{\bar{\pi}_i^w} \left[u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(i|s_i^w,\eta_i^w,s_{-i}^w,\eta_{-i}^w,s^m,\eta^m\right)} \right] dF^w\left(\eta_i^w\right) + \int^{\bar{\pi}_i^w} u\left(R s_i^w y_i^w\right) dF^w\left(\eta_i^w\right) \end{aligned}$$

where $\bar{\pi}_i^w$ is her threshold ranking on men such that she is indifferent between marriage or not. Any lower-ranked man, or any man with $\pi_i^w < \bar{\pi}_i^w$, won't be chosen by her.

Since we assume there are (weakly) fewer women than men, we expand the set I^w to $\widetilde{I}^w$ so that $\widetilde{I}^w = (0,\phi)$. In the expanded set, women in the marriage market start from value $\phi - 1$ to ϕ. The measure for women in the marriage market remains one. In equilibrium, there exists an unique mapping for men in the marriage market: $\pi^m : I^m \to \widetilde{I}^w$, where π^m maps man j ($j \in I^m$) to woman i ($i \in I^w$). Those men with a low value $i < \phi - 1$ in set $\widetilde{I}^w$ will not be married. In that case, $\eta^w_{\pi^m(j)} = 0$ and $c_{2m,j,i} = R s_j^m y_j^m$. In general, man j's second period expected utility is

$$\begin{aligned} & \int \max \left[\; u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(j|s_j^m,\eta_j^m,s_{-j}^m,\eta_{-j}^m,s^w,\eta^w\right)}, \quad u\left(R s_j^m y_j^m\right) \; \right] dF^m\left(\eta_j^m\right) \\ = & \int_{\bar{\pi}_j^m} \left[u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(j|s_j^m,\eta_j^m,s_{-j}^m,\eta_{-j}^m,s^w,\eta^w\right)} \right] dF^m\left(\eta_j^m\right) + \int^{\bar{\pi}_j^m} u\left(R s_j^m y_j^m\right) dF^m\left(\eta_j^m\right) \end{aligned}$$

where $\bar{\pi}_j^m$ is his threshold ranking on all women. Any woman with a poorer rank, $\pi_j^m <$

$\bar{\pi}_j^m$, will not be chosen by him.

We assume that the density functions of η^m and η^w are continuously differentiable. Since all men (women) in the marriage market have identical problems, they make the same savings decisions. In equilibrium, a *positive assortative matching* emerges for those men and women who are married. In other words, there exists a mapping M from η^w to η^m such that

$$\begin{aligned} 1 - F^w(\eta^w) &= \phi\left(1 - F^m\left(M(\eta^w)\right)\right) \\ &\Leftrightarrow \\ M(\eta^w) &= (F^m)^{-1}\left(\frac{F^w(\eta^w)}{\phi} + \frac{\phi - 1}{\phi}\right) \end{aligned}$$

For simplicity, we assume that η^w and η^m are drawn from the same distribution, $F^w = F^m = F$. The lowest possible value of emotional utility $\eta^{\min}$ is sufficiently small (which can be negative) so that some women and so men may not get married. Let $\bar{\eta}^w$ and $\bar{\eta}^m$ denote the threshold values for women's and men's emotional utilities in equilibrium, respectively. Only women (men) with emotional utilities higher than the threshold value $\bar{\eta}^w$ ($\bar{\eta}^m$) will get married. In other words,

$$\bar{\eta}^w = \max\left\{u_{2m,n} - u_{2m}, M^{-1}\left(\bar{\eta}^m\right)\right\} \text{ and } \bar{\eta}^m = \max\left\{u_{2w,n} - u_{2w}, M(\bar{\eta}^w)\right\} \tag{3.12}$$

For woman i, given all her rivals' and men's savings decisions and η^w, her second period utility is

$$\delta_i^w u\left(\frac{\kappa\left(Rs_i^w y^w + Rs^m y^m\right)}{P_{t+1}}\right) + \left(1 - \delta_i^w\right) u\left(\frac{Rs^w y^w}{P_{t+1}}\right) + \int_{\tilde{\eta}_i^w \geq \bar{\eta}^w} M(\tilde{\eta}_i^w) dF(\eta_i^w)$$

where $\tilde{\eta}_i^w = u\left(\frac{\kappa\left(Rs_i^w y^w + Rs^m y^m\right)}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs^w y^w + Rs^m y^m)}{P_{t+1}}\right) + \eta_i^w$. δ_i^w is the probability that woman i will get married,

$$\begin{aligned}\delta_i^w &= \Pr\left(\left[\begin{array}{c} u\left(\frac{\kappa\left(Rs_i^w y^w + Rs^m y^m\right)}{P_{t+1}}\right) \\ -u\left(\frac{\kappa(Rs^w y^w + Rs^m y^m)}{P_{t+1}}\right) + \eta_i^w \end{array}\right] \geq \bar{\eta}^w \middle| Rs^w y^w, Rs^m y^m\right) \\ &= 1 - F\left(\bar{\eta}^w - u\left(\frac{\kappa\left(Rs_i^w y^w + Rs^m y^m\right)}{P_{t+1}}\right) + u\left(\frac{\kappa\left(Rs^w y^w + Rs^m y^m\right)}{P_{t+1}}\right)\right) \quad (3.13)\end{aligned}$$

Due to symmetry, we drop the sub-index i for women. Given men's savings decisions, the first order condition for her optimization problem is

$$-u_{1w}' y^w + \beta\left[\begin{array}{c} \delta^w u_{2w}' \frac{\partial c_{2w}}{\partial s^w} + (1-\delta^w)\, u_{2w,n}' \frac{RP_t}{P_{t+1}} y^w + \frac{\partial \int_{\tilde{\eta}^w \geq \bar{\eta}^w} M(\tilde{\eta}^w) dF(\eta^w)}{\partial s^w} \\ + \frac{\partial \delta^w}{\partial s^w}\left(u_{2w} - u_{2w,n}\right) \end{array}\right] = 0 \quad (3.14)$$

where

$$\begin{aligned}\frac{\partial \int_{\tilde{\eta}^w \geq \bar{\eta}^w} M(\tilde{\eta}^w) dF(\eta^w)}{\partial s^w} &= \kappa u_{2w}' \frac{RP_t}{P_{t+1}} y^w \left[\int_{\bar{\eta}^w} M'(\eta^w) dF(\eta^w) + M(\bar{\eta}^w) f(\bar{\eta}^w)\right] \\ \frac{\partial \delta^w}{\partial s^w} &= f\left(\bar{\eta}^w\right) \kappa u_{2w}' \frac{RP_t}{P_{t+1}} y^w\end{aligned}$$

Similarly, a representative man's second-period utility, given his rivals' and all women's savings decisions, is

$$\delta_j^m u\left(\frac{\kappa\left(Rs^w y^w + Rs_j^m y^m\right)}{P_{t+1}}\right) + (1-\delta_j^m) u\left(\frac{Rs_j^m y^m}{P_{t+1}}\right) + \int_{\tilde{\eta}_j^m \geq \bar{\eta}^m} M^{-1}\left(\tilde{\eta}_j^m\right) dF(\eta_j^m)$$

where $\tilde{\eta}_j^m = u\left(\frac{\kappa\left(Rs^w y^w + Rs_j^m y^m\right)}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs^w y^w + Rs^m y^m)}{P_{t+1}}\right) + \eta_j^m$ and δ_j^m is the probability he gets married

$$\delta_j^m = \Pr\left(\left[\begin{array}{c} u\left(\frac{\kappa\left(Rs^w y^w + Rs_j^m y^m\right)}{P_{t+1}}\right) \\ -u\left(\frac{\kappa(Rs^w y^w + Rs^m y^m)}{P_{t+1}}\right) + \eta_j^m \end{array}\right] \geq \bar{\eta}^m \middle| Rs^w y^w, Rs^m y^m\right)$$
$$= 1 - F\left(\bar{\eta}^m - u\left(\frac{\kappa\left(Rs^w y^w + Rs_j^m y^m\right)}{P_{t+1}}\right) + u\left(\frac{\kappa\left(Rs^w y^w + Rs^m y^m\right)}{P_{t+1}}\right)\right) \quad (3.15)$$

The first order condition for a representative man's optimization problem is

$$-u'_{1m} y^m + \beta \left[\begin{array}{c} \delta^m u'_{2m} \frac{\partial c_{2m}}{\partial s^m} + \frac{\partial \int_{\tilde{\eta}^m \geq \bar{\eta}^m} M^{-1}(\tilde{\eta}^m) dF(\eta^m)}{\partial s^m} + (1-\delta^m) u'_{2m,n} \frac{RP_t}{P_{t+1}} y^m \\ + \frac{\partial \delta^m}{\partial s^m} (u_{2m} - u_{2m,n}) \end{array}\right] = 0 \quad (3.16)$$

where

$$\frac{\partial \int_{\tilde{\eta}^m \geq \bar{\eta}^w} M^{-1}(\tilde{\eta}^m) dF(\eta^m)}{\partial s^m} = \kappa u'_{2m} \frac{RP_t}{P_{t+1}} y^m \left[\int_{\bar{\eta}^m} \frac{\partial M^{-1}(\eta^m)}{\partial \eta^m} dF(\eta^m) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m)\right]$$
$$\frac{\partial \delta^m}{\partial s^m} = f(\bar{\eta}^m) \kappa u'_{2m} \frac{RP_t}{P_{t+1}} y^m$$

For simplicity, we assume that women and men will earn the same first period labor income and that there is no tax, i.e., $y_t^w = y_t^m = w_t$. We now define an equilibrium in this economy.

Definition 4. *An equilibrium is a set of savings rates, capital and labor allocation by sector, and the relative price of nontradable good* $\{s_t^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *that satisfies the following conditions:*

(i) The savings rates by the representative woman and the representative man, conditional on other women and men's savings rates, $s_t^w = \left\{s_{it}^w, s_{-i,t}^w\right\}$ *and* $s_t^m = \left\{s_{jt}^w, s_{-j,t}^m\right\}$*, maximize their respective utilities*

$$s_{it}^w = \arg\max\left\{V_t^w |\, s_{-i,t}^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\}$$
$$s_{jt}^w = \arg\max\left\{V_t^m |\, s_t^w, s_{-j,t}^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\}$$

(ii) The markets for capital, labor, and tradable and nontradable goods clear, and firms maximize their profits. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *solves (3.2), (3.3), (3.4), (3.5), (3.6) and (3.7).*

Shocks to the sex ratio We now consider an unanticipated shock to the young cohort's sex ratio, i.e., the sex ratio rises from one to $\phi(>1)$ from period t onwards. The nature of the shock is motivated by the facts about the sex ratio imbalance in China. Since a severe sex ratio imbalance for the pre-marital age cohort is a relatively recent phenomenon, the older generations' savings decisions were largely made when there was no severe sex ratio imbalance. As the shock is unanticipated, (3.2) and (3.4) do not hold in period t.

As in the benchmark model, the market clearing condition for the nontradable good can be re-written as

$$\frac{P_{Nt}A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} = \gamma\left(Rs_{t-1}w_{t-1} + (1-s_t)w_t\right) \tag{3.17}$$

where $s_t = \frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$ is the aggregate savings rate by the young cohort in period t.

By (3.3) and (3.5), we have

$$w_t = \frac{(1-\alpha_T)A_{Tt}}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\left(\frac{K_{Tt}}{1-L_{Nt}}\right)^{\alpha_T} = \frac{P_{Nt}(1-\alpha_N)A_{Nt}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}\left(\frac{K_{Nt}}{L_{Nt}}\right)^{\alpha_N} \tag{3.18}$$

We can solve (3.14), (3.16), (3.17) and (3.18) to obtain the equilibrium in period t. With some restrictions on the utility function and the distribution of emotional utility, we have the following proposition.

Proposition 10. *Assume that the utility function is of log form,* $u(C) = \ln C$, *for all men and women, and that* η *is drawn from a uniform distribution, then, as the sex ratio in the young cohort rises, a representative man in the cohort increases his savings rate while the savings response by a representative woman is ambiguous. However, the economy-wide savings rate increases unambiguously. The real exchange rate depreciates and the current account rises.*

Proof. See Appendix A3.2. □

A few remarks are in order. First, it is perhaps not surprising that the representative man raises his savings rate in response to a rise in the sex ratio because the need to compete in the marriage market becomes greater. Why does the representative woman reduce her savings rate? Because she anticipates a higher savings rate from her future husband, she does not need to sacrifice her first-period consumption as much as she otherwise would have to.

Second, why does the aggregate savings rate rise in response to a rise in the sex ratio? In other words, why is the increment in men's savings greater than the decline in women's savings? Intuitively, a representative man raises his savings rate for two reasons: in addition to improving his relative standing in the marriage market, he raises his savings rate to make up for the lower savings rate by his future wife. The more his future wife is expected to cut down her savings, the more he would have to raise his own savings to compensate. This ensures that his incremental savings is more than enough to offset any reduction in his future wife's savings. In addition, since men save more, the rising share of men in the population would also raise the aggregate savings rate. While both channels contribute to a rise in the aggregate savings rate, it is easy to verify that the first channel (the incremental competitive savings by any given man) is more important than the second effect (a change in the composition of the population with different saving propensities).

Third, once we obtain an increase in the aggregate savings rate, the logic from the previous one-gender benchmark model applies. In particular, the relative price of the non-tradable good declines (and hence the real exchange rate depreciates), and the current account rises.

Similar to the benchmark model with a single gender, once the shock is observed and taken into account in period $t+1$, (3.2) and (3.4) hold in equilibrium. By solving (3.2), (3.3), (3.4) and (3.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \text{ and } P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

This means that the real exchange rate and the current account will return to the previous values after one period.

3.2.2 Mixed-strategy equilibrium

In this section, we extend our benchmark model by considering an endogenous choice of entering/exiting the marriage market. Formally, we consider a mixed-strategy game in which (a) a representative woman will choose the probability of entering the marriage market ρ^w, a savings rate if she decides to enter, and a separate savings rate if decides to abstain from the marriage market; and (b) a representative man has similar choices.

The representative woman will have the same optimization problem as in the previous section if she enters the marriage market. She can also choose to be single, and if she does so, her life-time utility is

$$V_n^w = \max_{s_n^w} u(c_{1w,n}) + \beta u(c_{2w,n})$$

where V_n^w denotes the value function of a woman who is single throughout her life.

Her overall optimization problem in the mixed-strategy game is

$$\max_{\rho^w, s^w, s_n^w} \rho^w V^w + (1 - \rho^w) V_n^w$$

Obviously, she will choose $\rho^w = 1$ if and only if $V^w > V_n^w$.

Similarly, a representative man's overall optimization problem is

$$\max_{\rho^m, s^m, s_n^m} \rho^m V^m + (1 - \rho^m) V_n^m$$

where V_n^m denotes the value function of a representative man who is single throughout his life, and ρ^m is hi probability of entering the marriage market. He would decide to enter the marriage market with probability one if and only if the expected utility of doing so is greater than otherwise, or $V^m > V_n^m$.

Now we can show a more general proposition in the following:

Proposition 11. *Assume that the utility function is of log form and that emotional utility is drawn from an independent and identical uniform distribution, then there exists a threshold value* $\phi_1 > 1$ *that satisfies* $V^m = V_n^m$*.*

(i) For $\phi < \phi_1$*, both women and men choose to enter the marriage market with probability one. In addition, as the sex ratio rises, the savings rate of a representative man increases while the change in the savings rate of a representative woman is ambiguous. However, the economy-wide savings rate increases unambiguously, and the real exchange rate declines.*

(ii) For $\phi \geq \phi_1$*, as the sex ratio rises, a representative man chooses a positive probability of being single while a representative woman still chooses to enter the marriage market with probability one. The changes in the aggregate savings rate and the real exchange rate are ambiguous.*

Proof. See Appendix A.3.3. □

Two remarks are in order. First, the proposition states that as the sex ratio rises, up to a threshold ϕ_1, a representative man always chooses to enter the marriage market and raises his savings rate in response to a higher sex ratio. A representative woman also always chooses to enter the marriage market but reduces her savings rate in response to a higher sex ratio. This part is similar to Proposition 6. However, once the sex ratio exceeds the threshold ϕ_1, the representative man would respond to an additional increase in the sex ratio by choosing a progressively greater probability of not entering the marriage market. He does so because his savings rate is already high enough such that sharing his savings with a low-type spouse could yield him a lower utility. For the representative woman, entering the marriage market is still a dominant strategy even after the threshold.

Second, as the sex ratio rises, the representative man suffers a welfare loss from two sources. A higher sex ratio reduces his chance of marriage. In addition, while he has to increase his savings in order not to lose out to his competitors in the marriage market, the increased savings in the end does not alter his probability of marriage. Interestingly, the effect of a higher sex ratio on a presentative woman is ambiguous. On the one hand, she could gain both from her future husband's higher savings rates and from the improved probability to be matched with a man with a higher level of emotiional utility. On the other hand, precisely because men have rasied their savings rate, they become more reluctant to share their high savings rate with a low-type woman. As a result, a representative woman's chance of getting married declines. These two opposing forces produce an ambiguous net effect on the representative woman. It is useful to note that, while a representative woman could lose from a higher sex ratio, her utility level is always higher than that of the representative man.

3.2.3 Capital adjustment costs

Without additional frictions, a shock to the sex ratio can only affect the real exchange rate for one period. If there are capital adjustment costs in each sector, the effect on the real exchange rate can be prolonged. We assume that the capital accumulation in each sector is as following:

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$

where δ is the depreciation rate and I_t is investment. $\frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$ represents the adjustment cost as in Chari, Kehoe and McGrattan (2002).

Then (3.2) and (3.4) become, respectively,

$$\begin{aligned} R &= 1-\delta+\frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\alpha_T A_{Tt}\left(\frac{L_{Tt}}{K_{Tt}}\right)^{1-\alpha_T} \\ &\quad -bR\left(\frac{I_{Tt}}{K_{Tt}}-\delta\right)-\frac{b}{2}\left(\left(\frac{I_{Tt}}{K_{Tt}}\right)^2-\delta^2\right) \qquad (3.19) \\ R &= 1-\delta+\frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}\alpha_N A_{Nt}\left(\frac{L_{Nt}}{K_{Nt}}\right)^{1-\alpha_T} \\ &\quad -bR\left(\frac{I_{Nt}}{K_{Nt}}-\delta\right)-\frac{b}{2}\left(\left(\frac{I_{Nt}}{K_{Nt}}\right)^2-\delta^2\right) \qquad (3.20) \end{aligned}$$

Without capital adjustment cost, i.e., $b = 0$, the price of the nontradable good will go back to its equilibrium level in period $t+1$. If $b > 0$, then

$$P_{Nt} = \frac{\frac{\alpha_T A_{Tt+1}\left(\frac{L_{Tt+1}}{K_{Tt+1}}\right)^{1-\alpha_T}}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}} - bR\left(\frac{I_{Tt+1}}{K_{Tt+1}} - \frac{I_{Nt+1}}{K_{Nt+1}}\right) - \frac{b}{2}\left(\left(\frac{I_{Tt+1}}{K_{Tt+1}}\right)^2 - \left(\frac{I_{Nt+1}}{K_{Nt+1}}\right)^2\right)}{\frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}\alpha_N A_{Nt+1}\left(\frac{L_{Nt+1}}{K_{Nt+1}}\right)^{1-\alpha_T}}$$

P_{Nt} is now a function of $\frac{I_{Tt+1}}{K_{Tt+1}}$ and $\frac{I_{Nt+1}}{K_{Nt+1}}$. If $\frac{I_{Tt+1}}{K_{Tt+1}} \neq \frac{I_{Nt+1}}{K_{Nt+1}}$, P_{Nt} is not a constant. This means that, with capital adjustment costs, the price of the nontradable good does not return immediately to its long-run equilibrium level. As a result, a rise in the sex ratio can have a long-lasting and depressing effect on the real exchange rate.

3.2.4 Two large countries

We now turn to a world with two large countries: Home and Foreign. Assume that they are identical in every respect except for their sex ratios. Specifically, in period t, the sex ratio of the young cohort in Home rises from one to ϕ ($\phi > 1$), while Foreign always has a balanced sex ratio. Households in each country consume a tradable good and a nontradable good.

$$C_t = \frac{C_{Nt}^{\gamma} C_{Tt}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \text{ and } C_t^* = \frac{(C_{Nt}^*)^{\gamma}(C_{Tt}^*)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

where C_t and C_t^* represent home and foreign consumption indexes, respectively. Since we choose the tradable good as the numeraire, the consumer price index is $P_t = P_{Nt}^{\gamma}$, where P_{Nt} is the price of the home produced nontradable good. Similarly, the consumer price index in Foreign is $P_t^* = (P_{Nt}^*)^{\gamma}$.

The rise in Home's sex ratio in period t is assumed to be unanticipated. As a result, (3.2) and (3.4) fail in both Home and Foreign. By the same reasoning, Home experiences a real exchange rate depreciation in period t, but a real appreciation in period $t+1$. We can write the current account in Home and Foreign as follows:

$$CA_t = s_t w_t - s_{t-1}w_{t-1} + K_t - K_{t+1} \text{ and } CA_t^* = s_t^* w_t^* - s_{t-1}^* w_{t-1}^* + K_t^* - K_{t+1}^*$$

Before the shock, we had

$$s_{t-1} = s^*_{t-1},\ w_{t-1} = w^*_{t-1} \text{ and } K_t = K^*_t$$

In period $t+1$, we have

$$P_{Nt} = P_{Nt},\ w_{t+1} = w^*_{t+1}, \text{ and } P_{t+1} = P^*_{t+1}$$

and the demand for the nontradable good is

$$Q_{N,t+1} = \frac{\gamma w_{t+1}\left(\left(R-1\right)s_t + 1\right)}{P_{Nt}} \text{ and } Q^*_{N,t+1} = \frac{\gamma w^*_{t+1}\left(\left(R-1\right)s^*_t + 1\right)}{P_{Nt}}$$

Since Home now has a higher sex ratio than Foreign, we have $s_t > s^*_t$, and therefore

$$Q_{N,t+1} > Q^*_{N,t+1}$$

We assume that the nontradable sector is more labor-intensive, i.e., $\alpha_N < \alpha_T$. Given the same technologies and the same labor endowments in the two countries, we have

$$K_{t+1} < K^*_{t+1}$$

In period t, since nothing changes in Foreign, it must be the case that $s^*_t w^*_t = s_{t-1}w_{t-1}$. Following the same steps as in the case of a small open economy, we can show that $s_t w_t > s_{t-1}w_{t-1} = s^*_t w^*_t$. Then it is easy to show that $CA_t > 0 > CA^*_t$. In other words, Home exhibits a current account surplus while Foreign experiences a current account deficit.

3.2.5 Endogenous labor supply

We turn to the case of endogenous labor supply. Just as a male raises his savings rate to gain a competitive advantage in the marriage market, he may choose to increase his supply of labor for the same reason in response to a rise in the sex ratio. This can translate into an increase in the effective aggregate labor supply if women do not decrease their labor supply too much. If the production of the nontradable good is more labor-intensive, the increase in the effective labor supply can reduce the relative price of the non-tradable good (and the value of the real exchange rate). Therefore, endogenous labor supply could reinforce the savings channel from the sex ratio shock, leading to an additional reduction in the real exchange rate.

We allow each person to endogenously choose the first period labor supply and the utility function of the first period is $u(C)+v(1-L)$, where L is the labor supply and $v(1-L)$ is the utility function of leisure. As in the standard literature, we assume that $v' > 0$ and $v'' < 0$. Again, for simplicity, we assume no tax on the labor income. The utility function governing the leisure-labor choice is the same for men and women. In other words, by assumption, men and women are intrinsically symmetric except for their ratio in the society.

We can rewrite the optimization problem for a representative woman as following:

$$\max \ u(C_{1t}^w) + v(1 - L_t^w) + \beta E_t \left[u(C_{2,t+1}^w) + \eta^m \right]$$

with the budget constraint

$$\begin{aligned} P_t C_{1t}^w &= (1 - s_t^w) w_t L_t^w \\ P_{t+1} C_{2,t+1}^w &= \begin{cases} \kappa \left(R s_t^w L_t^w + R s_t^m L_t^m \right) w_t & \text{if married} \\ R s_t^w w_t L_t^w & \text{otherwise} \end{cases} \end{aligned}$$

The first order condition with respect to her labor supply is

$$u'_{1w} \frac{(1 - s_t^w) w_t}{P_t} + \beta \left[\begin{array}{c} \delta^w u'_{2w} \frac{\partial c_{2w}}{\partial s^w} + (1 - \delta^w)\, u'_{2w,n} \frac{R P_t}{P_{t+1}} y^w + \frac{\partial \int_{\tilde{\eta}^w \geq \bar{\eta}^w} M(\tilde{\eta}^w) dF(\eta^w)}{\partial s^w} \\ + \frac{\partial \delta^w}{\partial s^w} \left(u_{2w} - u_{2w,n} \right) \end{array} \right] - v'_w = 0$$

Notice that $\frac{\partial C_{2,t+1}^w}{\partial L_t^w} = \frac{\partial C_{2,t+1}^w}{\partial s_t^w} \frac{s_t^w}{L_t^w}$ and $\frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}^w(\tilde{\eta}^w)}{\partial L_t^w} = \frac{\partial \int M(\tilde{\eta}^w) d\tilde{F}^w(\tilde{\eta}^w)}{\partial s_t^w} \frac{s_t^w}{L_t^w}$. Combining the equation above with (3.14), we have

$$\frac{w_t}{P_t} = \frac{v'_w}{u'_{1w}} \tag{3.21}$$

The optimization problem for a representative man is similar:

$$\max \ u(C^m_{1t}) + v(1 - L^m_t) + \beta E_t \left[u(C^m_{2,t+1}) + \eta^w \right]$$

with the budget constraint

$$\begin{aligned} P_t C^m_{1t} &= (1 - s^m_t) w_t L^m_t \\ P_{t+1} C^m_{2,t+1} &= \begin{cases} \kappa \left(R s^m_t L^m_t + R s^m_t L^m_t \right) w_t & \text{if married} \\ R s^m_t w_t L^m_t & \text{otherwise} \end{cases} \end{aligned}$$

The optimization condition for his labor supply is

$$\frac{w_t}{P_t} = \frac{v'_m}{u'_{1m}} \tag{3.22}$$

On the supply side, all equilibrium conditions remain the same except for the labor market clearing condition, which now becomes

$$L_{Tt} + L_{Nt} = \frac{1}{1+\phi} L^w_t + \frac{\phi}{1+\phi} L^m_t \tag{3.23}$$

We now define an equilibrium for such an economy.

Definition 5. *An equilibrium is a set* $\{(s^w_t, L^w_t), (s^m_t, L^m_t), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *that satisfies the following conditions:*

(i) The savings and labor supply decisions by women and men

$$(s^w_t, L^w_t) = \left\{ s^w_{it}, s^w_{-i,t}, L^w_{it}, L^w_{-i,t} \right\}$$

and

$$\left(s_t^m, L_t^m\right) = \left\{s_{it}^m, s_{-i,t}^m, L_{it}^m, L_{-i,t}^m\right\}$$

maximize their utilities, respectively,

$$\begin{aligned}
\left(s_{it}^w, L_{it}^w\right) &= \arg\max \left\{ V_t^w | \left(s_{-i,t}^w, L_{-i,t}^w\right), \left(s_t^m, L_t^m\right), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt} \right\} \\
\left(s_{jt}^m, L_{jt}^m\right) &= \arg\max \left\{ V_t^m | \left(s_t^w, L_t^w\right), \left(s_{-j,t}^m, L_{-j,t}^m\right), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt} \right\}
\end{aligned}$$

(ii) The markets for both goods and factors clear, and firms' profits are maximized. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *solves (3.2), (3.3), (3.4), (3.5), (3.6) and (3.23).*

As before, we assume that $u(C) = \ln C$. We let L_t denote the aggregate labor supply in period t, and assume that $\frac{v''L}{v'}$ is non-decreasing in L.

Proposition 12. *Under the same assumptions as in Proposition 10, as the sex ratio (in the young cohort) rises, a representative man increases both his labor supply and his savings rate, while the responses in a representative woman's savings rate and labor supply are ambiguous. However, the economy-wide labor supply and savings rate both increase unambiguously. The real exchange rate depreciates, and the current account rises.*

Proof. See Appendix A3.4. □

In response to a rise in the sex ratio, for the same reason that men may cut their consumption and increase their savings rate, they may cut down their leisure and increase their labor supply. Similarly, for women, for the same reason that induce them to reduce their savings, they may reduce their labor supply (and increase leisure). In the aggregate, for the same reason that the increase in savings by men is more than enough to offset the decrease in savings by women, the increase in labor supply by men is also larger than the decrease in labor supply by women. Therefore, the aggregate labor supply rises in response to a rise in the sex ratio.

With a fixed labor supply, it is worth remembering that the nontradable sector shrinks after a rise in the sex ratio. The reason is that a decline in the relative price of the nontradable goods (due to the savings channel) makes it less attractive for labor and capital to stay in the nontradable sector. Now, with an endogenous labor supply, the total effective labor supply increases after a rise in the sex ratio according to Proposition 11. By a logic similar to the Rybzinksy theorem, this by itself has a tendency to induce an expansion of the nontradable sector if the production of the nontradable good is more labor intensive. Relative to the case of a fixed labor supply, adding the effect of endogenous labor supply leads to either an expansion of the nontradable sector, or at least a smaller reduction in the size of the nontradable sector. The exact scenario depends on parameter values. However, regardless of what happens to the size of the nontradable sector, the price of the nontradable good (and the value of the real exchange rate) must fall by a greater amount when the endogenous labor supply effect is added to the savings effect.

3.3 Numerical Examples

We start from a simple OLG model allowing mixed strategies in which every cohort lives two periods and there are no capital adjustment costs. We then add some more realisms by (1) assuming a 50-period life and (2) introducing capital adjustment costs.

3.3.1 Parameters

In the benchmark, the nontradable sector has a lower capital intensity $\alpha_N = 0.3$. We take all other parameters the same as in Chapter 2.

Choice of Parameter Values

Parameters	Benchmark	Source and robustness checks
Discount factor	$\beta = 0.45$	Prescott (1986) suggests that the discount factor takes a value of 0.96 on annual frequency. As we take 20 years as one period, we set $\beta = 0.96^{20} \simeq 0.45$
Share of capital input	$\alpha = 1/3$	Chari, Kehoe and McGrattan (2001)
Congestion index	$\kappa = 0.8$	$\kappa = 0.7, 0.9$ in the robustness checks.
Love, standard deviation	$\sigma = 0.05$	$\sigma = 0.1$ in the robustness checks
Love, mean	$m = 2$	$m = 0.5$ in the robustness checks

3.3.2 Results for the 2-period OLG model

In Figure 3.1, we set parameter κ equal to 0.8. We set $m = 2$ and $\sigma = 0.05$ as a benchmark case. With an unbalanced sex ratio ($\phi > 1$), the real exchange rate depreciates. As the sex ratio rises from 1 to 1.15, the real exchange rate depreciates by 6.4%, while aggregate savings rate and current account both rise by around 2.9%. As the sex ratio continues to rise, the real exchange rate begins to appreciate. The turning point for the real exchange rate corresponds to when the sex ratio crosses the threshold ϕ_1 in Proposition 10.

As a first set of robustness checks, we experiment with different combinations of m and σ by setting $m = 0.5$ or 2, $\sigma = 0.05$ or 0.1. The results are also reported in Figure 3.1, and generally do not deviate from the benchmark by much.

We also set κ to be 0.7 or 0.9, respectively, and experiment with different combinations of other parameters. The results are reported in Figures 3.2 and 3.3. Generally speaking, the real exchange rate always depreciates more with a higher sex ratio. Both the savings rate and the current account (as a share of GDP) rise in response to a rise in the sex ratio.

We now consider endogenous labor supply in Figure 3.4. With $\kappa = 0.8$, $m = 2$ and $\sigma = 0.05$, we obtain a much stronger exchange rate depreciation. As the sex ratio rises from 1 to 1.15, the extent of the real exchange rate depreciation also rises from 0% to about 25%. The aggregate savings rate rises from 17% to 25%, while the current account surplus rises from 0% first to close to 9% of GDP. As the sex ratio continues to rise, it would cross the threshold value ϕ_1, at which point, the real exchange rate begins to appreciate. Correspondingly, the aggregate savings rate and current account both decline.

Robustness checks with other combinations of the parameters are reported in Figures 3.5 and 3.6. The results are broadly in line with the benchmark calibration. In particular, with an endogenous labor supply, a given rise in the sex ratio leads to a greater response in both the real exchange rate and the current account.

3.3.3 An OLG model in which a cohort lives 50 periods

We now extend our benchmark model by assuming that every cohort lives 50 periods. Everyone works in the first 30 periods, and retires in the remaining 20 periods. If one gets married, the marriage take place in the τth period. While differences in the savings rates by parents with a son versus parents with a daughter are an important feature of the data (Wei and Zhang, 2009), we are not able to solve the problem that features simultaneously parental savings for children and a nontradable sector. Instead, we study a case in which men and women save for themselves. However, as we recognize the quantitative importance of parental savings in the data, we choose $\tau = 20$ as our benchmark case so the timing of the marriage is somewhere between the typical number of working years by parents when their child gets married and the typical number of working years by a young person when he/she gets married. Generally speaking, the greater the value of τ, the stronger is the aggregate savings response to a given rise in the sex ratio.

A representative woman's optimization problem is

$$\max \sum_{t=1}^{\tau-1} \beta^{t-1} u(c_t^w) + E_1 \left[\sum_{t=\tau}^{50} \beta^{t-1} \left(u(c_t^w) + \eta^m \right) \right]$$

For $t < \tau$, when the woman is still single, the intertemporal budget constraint is

$$A_{t+1} = R \left(A_t + y_t^w - P_t c_t^w \right)$$

where A_t is the wealth held by the woman at the beginning of period t. $y_t^w = w_t L_t^w$ is her labor income at the age t. After marriage ($t \geq \tau$), her family budget constraint becomes

$$A_{t+1}^H = \begin{cases} R\left(A_t^H + w_t L_t^w - P_t c_t\right) & \text{if } t \leq 30 \\ R\left(A_t^H - c_t^w\right) & \text{if } t > 30 \end{cases}$$

where A_t^H is the level of family wealth (held by wife and husband) at the beginning of period t. c_t is the public good consumption by wife and husband, which takes the same form as in the two period OLG model. The optimization problem for a representative man is similar. To simplify the calculation and generate interesting results, we assume that there is a lower bound of labor supply $\bar{L}$, $L_t^i \geq \bar{L}$ $(i = w, m)$.

As before, we take $R = 1.04$ as the annual gross interest rate. The subjective discount factor now takes the value of $\beta = 1/R$. We assume capital accumulation evolves in the following way:

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$

where $\frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$ represents the quadratic capital adjustment cost. Following Chari, Kehoe and McGrattan (2002), we assume $\delta = 0.1$ and $b = 3$.

To think of the sex ratio shock, we use demographic changes in China over the last two decades as a guide. As the data exhibits a steady increase in the sex ratio in the pre-marital age cohort, we let the sex ratio at birth in the model rise continuously and smoothly until it reaches 1.2 in period 20. The sex ratio then stays at that level in all subsequent periods. For technical reason, we set $\sigma = 0.1$. Under such a standard deviation, ϕ_1 does not appear in this experiment.

The simulation results are shown in Figures 3.7. As the sex ratio rises from 1 in period 0 to 1.2 in period 20, the real exchange rate depreciates by around 9 percent. The economy-wide savings rate and the current account rise by around 3.5 percent of GDP. As a robustness check, if capital adjusts more slowly, i.e., with a higher cost of capital adjustment, the real exchange rate depreciates by almost 10 percent. The converse is true when the adjustment cost is lower.

3.4 Some empirics

Since the sex ratio effect is novel, it is useful to present and discuss some empirical evidence. We recall first the evidence in Wei and Zhang (2009) that a higher sex ratio has led to a rise in the household savings rate in China. Chinese households with a son in both rural and urban areas tend to save more in regions with a more skewed sex ratio. The savings rate by urban households with a daughter also tend to rise with the local sex ratio, although the savings rate by rural households with a daughter appears to be insensitive to the local sex ratio. The savings behavior by daughter-households is consistent with the notion that intra-household bargaining is sufficiently important that they do not cut down savings rate in response to a higher sex ratio (The model of Du and Wei,, 2010, formalizes this intuition). Using regional variations in the eneforcement of the family planning policy as instruments for the local sex ratio, Wei and Zhang (2009) suggest that the positive correlation reflects a causal effect from a higher sex ratio to a higher savings rate. Based on the IV regressions, they estimate that the rise in the sex ratio may explain about half of the observed rise in the household savings rate in the last two decades.

Some evidence that a higher sex ratio has increased effective labor supply is provided in Wei and Zhang (2011). In particular, the number of days a rural migrant worker chooses to work away from home tends to rise with the local sex ratio, especially if the migrant worker has a son at home. Similarly, migrant workers with a son from a region with a more skewed sex ratio are also more willing to work in a job that are more dangerous and less pleasant, such as in minining or construction, or with exposure to extreme heat, cold or hazardous material, presumably for a better wage.

We now provide some suggestive cross-country evidence on how the sex ratio imbalance may affect the real exchange rate. We first run regressions based on the following specification:

$$\ln RER_i = \alpha + \beta \cdot \text{sex ratio} + \gamma \cdot Z + \varepsilon_i$$

where RER_i is the real exchange rate for country i. Z is the set of control variables. We consider a sequentially expanding list of control variables including log GDP per capita, financial development index, government fiscal deficit, dependence ratio, and *de facto* exchange rate regime classifications.

The data for the real exchange rate and real GDP per capita are obtained from Penn World Table 6.3. The "price level of GDP" in the Penn World Table is equivalent to the inverse of the real exchange rate in the model: A higher value of the "price level of GDP" means a lower value of the real exchange rate. The sex ratio data is obtained from the World Factbook. As we are not able to find the sex ratio for the age cohort 10-25 for a large number of countries, we use age group 0-15 instead to maximize the country coverage.

We use two proxies for financial development. The first is the ratio of private credit to GDP, from the World Bank's WDI dataset. This is perhaps the most commonly used proxy in the standard literature. There is a clear outlier with this proxy: China has a very high level of bank credit, exceeding 100% of GDP. However, 80% of the bank loans go to state-owned firms, which are potentially less efficient than private firms (see Allen, Qian, and Qian, 2004). To deal with this problem, we modify the index by multiplying the credit to GDP ratio for China by 0.2. Because this measure is far from being perfect, we also use a second measure, which is the level of financial system sophistication as perceived by a survey of business executives reported in the Global Competitiveness Report (GCR).

For exchange rate regimes, we use two de facto classifications. The first comes from Reinhart and Rogoff (2004), who classify all regimes into four groups: peg, crawling peg, managed floating and free floating. The second classification comes from Levy-Yeyati and Sturzenegger (2005), who use three groups: fix, intermediate and free float.

For the dependent variable, logRER, and most regressors where appropriate, we use their average values over the period 2004-2008. The averaging process is meant to smooth out business cycle fluctuations and other noises. The period 2004-2008 is chosen because it is relatively recent, and the data are available for a large number of countries. (We have also examined a single year, 2006, and obtained similar results).

Table 3.1 provides summary statistics for the key variables. The log RER ranges from -2.22 to 0.41 in the sample, with a mean of -0.74 and a standard deviation of 0.59. The value of log RER for China indicates a substantial undervaluation on the order of 45% when compared to the simple criterion of purchasing power parity.

For the sex ratio for the age cohort 0-15, both the mean and the median across countries are 1.04, and the standard deviation is 0.02. For this age cohort, all countries in the sample have a sex ratio that is at least 1. The sex ratio for most of the countries is between 1 and 1.07. The following economies have a sex ratio that is 1.07 or higher: China (1.13), Macao (1.11), Korea (1.11), Singapore (1.09), Switzerland (1.08), Hong Kong (1.08), Vietnam (1.08), Jordan (1.07), Portugal (1.07) and India (1.07). They represent the most skewed sex ratios in the sample. China, by far, has the most unbalanced sex ratio in the world. If the same sex ratio persists into the marriage market, then at least one out of every eight young men cannot get married. As wives are typically a few years younger than their husbands, the actual probability of not being able to marry is likely to be modestly better in a country with a growing population (for which later cohorts are slightly larger). Nonetheless, the relative tightness of the marriage market for men across countries should still be highly correlated with this sex ratio measure. In addition, unlike most other countries, China exhibits a progressively smaller age cohort over time as a result of its strict family planning policy. As a result, the relative tightness of the marriage market for Chinese men when compared to their counterparts in other countries is likely to be worse than what is represented by this sex ratio. Furthermore, the Chinese sex ratios at birth in 1990 and 2005 are estimated to be 1.15 and 1.20, respectively (see Wei and Zhang, 2009). This implies that the sex ratio for the pre-marital age cohort will likely worsen in the foreseeable future.

We present a series of regressions in Table 3.2. The first column shows that the real exchange rate tends to be lower in poorer countries. This is commonly interpreted as confirmation of the Balassa-Samuelson effect. In Column 2, we add a proxy for financial development by the ratio of private sector credit to GDP. The positive coefficient on the new regressor indicates that countries with a poorer financial system tend to have a lower RER. In Column 3, we add the sex ratio. The coefficient on the sex ratio is negative and statistically significant, indicating that countries with a higher sex ratio tend to have a lower RER. Since oil exporting countries have a current income that is likely to be substantially higher than their permanent income (until they run out of the oil reserve), their current account and RER patterns may be different from other economies. In Column 4, we exclude major oil exporters and re-do the regression. This turns out to have little effect on the reult. In particuar, countries with a higher sex ratio continue to exhibit a lower RER.

In Column 5 of Table 3.2, we add several additional control variables: government fiscal deficit, terms of trade, capital account openness, and dependency ratio. Due to missing values for some of these variables, the sample size is dramatically smaller (a decline from 123 in Column 4 to 75 Column 5). Of these variables, the dependence ratio is the only significant variable. The positive coefficient on the dependence ratio (0.0093) means that countries with a low dependency ratio (fewer children and retirees as a share of the population) tend to have a low RER. By the logic of the life-cycle hypothesis, a lower dependency ratio produces a higher savings rate. By the model in Section 2, this could lead to a reduction in the value of the real exchange rate. It is noteworthy, however, even with these additional controls and in a smaller sample, the sex ratio effect is still statistically significant, although its point estimate is slightly smaller.

In Column 6 of Table 3.2, we take into account exchange rate regimes using the Reinhart-Rogoff (2004) de facto regime classifications. Relative to the countries on a fixed exchange rate regime (the left out group), those on a crawling peg appear to have a lower RER. Countries on other currency regimes do not appear to have a systematically different RER. With these controls, the negative effect of the sex ratio on the RER is still robust. In Column 7, we measure exchange rate regimes by the de facto classifications proposed by Levy Yeyati and Sturzenneger (2003). It turns out this does not affect the relationship between the sex ratio and the real exchange rate.

In Table 3.3, we re-do the regressions in Table 3.2 except that we now measure a country's financial development by the financial system system sophistication index from the Global Competitiveness Report. The results are broadly similar to Table 3.2. In particular, the coefficients on the sex ratio are negative in all five cases, and are significant in four of the five cases. The sex ratio coefficient is (marginally) not significant in Column 6 of Table 3.3, where the Reinhart-Rogoff exchange rate classifications are used as controls. We note, however, that this regression also has far fewer observations (35 only), which also reduces the power of the test. In any case, when the LYS exchange rate classifications are used instead (reported in Column 7), the sex ratio coefficient becomes significant again.

In Tables 3.4 and 3.5, we examine the relationship between the sex ratio and the (private-sector) current account. Because our theory does not discuss government savings behavior, we choose to define the dependent variable as a country's current account account (as a share of GDP) minus the government savings (as a share of GDP). Otherwise, the regression specifications are similar to those in Tables 3.2 and 3.3. The sex ratio has a positive coefficient which is statistically significant in almost all cases except when the sample size becomes very small.

In sum, we find that the sex ratio has a significant impact on the real exchange rate and current account in a way consistent with our theory: as the sex ratio rises, a country tends to have a real exchange rate depreciation and a current account surplus. (An important caveat is that we do not have a clever idea to instrument for the sex ratio in the cross country context; future research will have to investigate the causality more thoroughly.)

To be clear, as the sex ratio imbalance is a severe problem only in a subset of countries, it is not a key fundamental for the real exchange rate in most countries. Nonetheless, for those countries with a severe sex ratio imbalance, including China, one might not have an accurate view on the equilibrium exchange rate unless one takes it into account. To illustrate the quantitative significance of the empirical relations, we compute the extent of the Chinese real exchange rate undervaluation (or the value of the RER relative to what can be predicted based on the fundamentals) by taking the point estimates in Columns 1-2 and 5 of Tables 3.2-3.5, respectively, at their face value. The results are tabulated in Table 3.6. As noted earlier, relative to the simple-minded PPP, the Chinese exchange rate is undervalued by about 45%. Once we adjust for the Balassa-Samuelson effect, the extent of the undervaluation becomes 55% (column 1 of Table 3.6) - apparently the Chinese RER is even lower than other countries at the comparable income level. If we additionally consider financial underdevelopment (proxied by the ratio of private sector loans to GDP), the Chinese RER undervaluation is reduced to 43% (column 2, row 1 of Table 3.6), which is still economically significant. If we also take into account government deficit, terms of trade, and capital account openness, the extent of the RER undervaluation is 35% (column 3, row 1). If we further take into account the dependency ratio, the extent of undervaluation drops to 18% (column 4, row 1). Finally, if we add the sex ratio effect, the extent of undervaluation becomes 8% (column 5, row 1 of Table 3.6). The last number represents a relatively trivial amount of undervaluation since major exchange rates (e.g., the euro/dollar rate or the yen/dollar rate) could easily fluctuate by more than 8% in a year. If we proxy financial development by the rating of financial system sophistication, and also take into account the sex ratio effect and other structural variables, the extent of the Chinese RER undervaluation becomes 2% (column 5, row 2 of Table 3.6), an even smaller amount.

We can do similar calculations for the Chinese (private sector) current account (as a share of GDP) in excess of the fundamentals. If we only take into the regularity that poorer countries tend to have a lower current account balance, the Chinese excess CA is on the order of 14%. If we take into account the sex ratio effect as well as financial underdevelopment, the dependency ratio and other variables in the regressions, the excess amount of current account becomes somewhere between 0.3% and 2.0%, depending on which proxy for financial development is used. These numbers illustrate that the sex ratio is a quantitatively important structural factor, though it is not the only one. In particular, the dependency ratio is also a very important factor. In any case, if these structural factors are not taken into account, one might mistakenly exaggerate the role of currency manipulation in affecting both the RER and the current account.

3.5 Conclusion

A low value of the real exchange rate (i.e., deviations from the PPP from below), a large current account surplus, and accumulation of foreign exchange reserve are the commonly used criteria for judging currency undervaluation or manipulation. We argue that none of them is a logically sound criterion. Instead, a dramatic rise in the sex ratio for the premarital age cohort in China since 2003, could generate both a depreciation of the real exchange rate and a rise in the current account surplus. With capital controls (including mandatory surrender of foreign exchange earnings), a persistent current account surplus can mechanically be converted into a rise in a country's foreign exchange reserve.

The usual narrative about the Chinese external economy connects the three variables in the following way: The authorities intervene aggressively in the currency market in order to generate an artificial undervaluation of its currency. This generates a rise in the foreign exchange reserve holdings and a fall in the real exchange rate. As a result of the currency undervaluation, the country manages to produce a current account surplus. The model and the evidence in this paper encourage the reader to consider an alternative way to connect the three variables: structural factors, such as a rise in the sex ratio, simultaneously generate a rise in the current account (through a rise in the savings rate) and a fall in the real value of the exchange rate. The low real exchange rate is not the cause of the current account surplus. With mandatory surrender of foreign exchange earnings required of by the country's capital control regime, the current account surplus is converted passively into an increase in the central bank's foreign exchange reserve holdings.

If other factors, in addition to a rise in the sex ratio, have also contributed to a rise in the Chinese savings rate, such as a reduction in the dependency ratio, or a rise in the corporate and government savings rates, they can complement the sex ratio effect and reinforce an appearance of an undervalued currency even when there is no manipulation. To be clear, this paper is not saying that no manipulations have occured. Instead, it illustrates potential pitfalls in assessing the equilibrium exchange rate when important structural factors are not accounted for.

Empirically, countries with a high sex ratio do appear to have a low value of the real exchange rate and a current account surplus. If we take the econometric point estimates at face value, it appears that the Chinese real exchange rate has only a relatively small amount of undervaluation (2-8%) once we take into account the sex ratio effect and other structural factors.

In future research, the model could be extended to allow for endogenous adjustment of the sex ratio. This will help us to assess the speed of the reversal of the sex ratio and the unwinding of the current account surplus and currency "undervaluation."

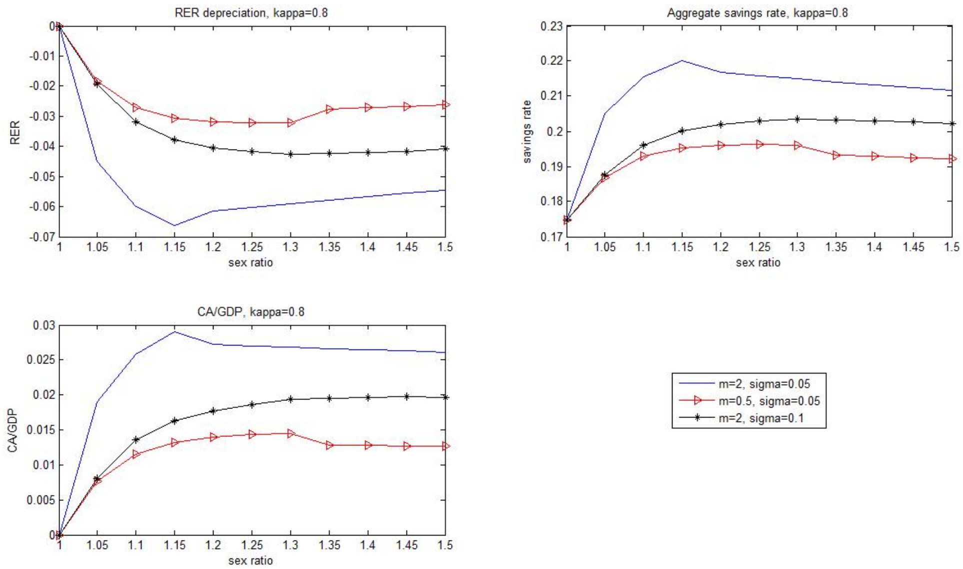

Figure 3.1: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.8

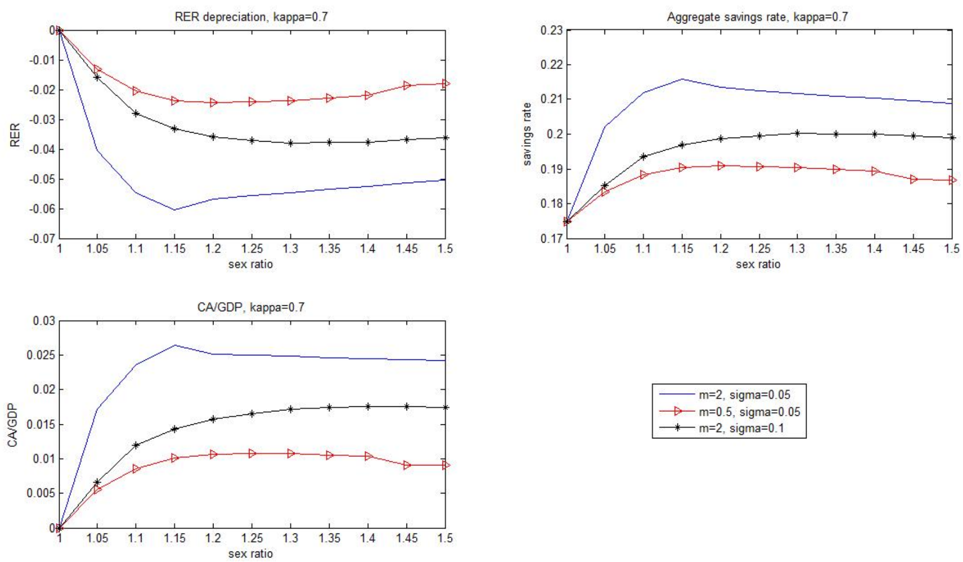

Figure 3.2: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.7

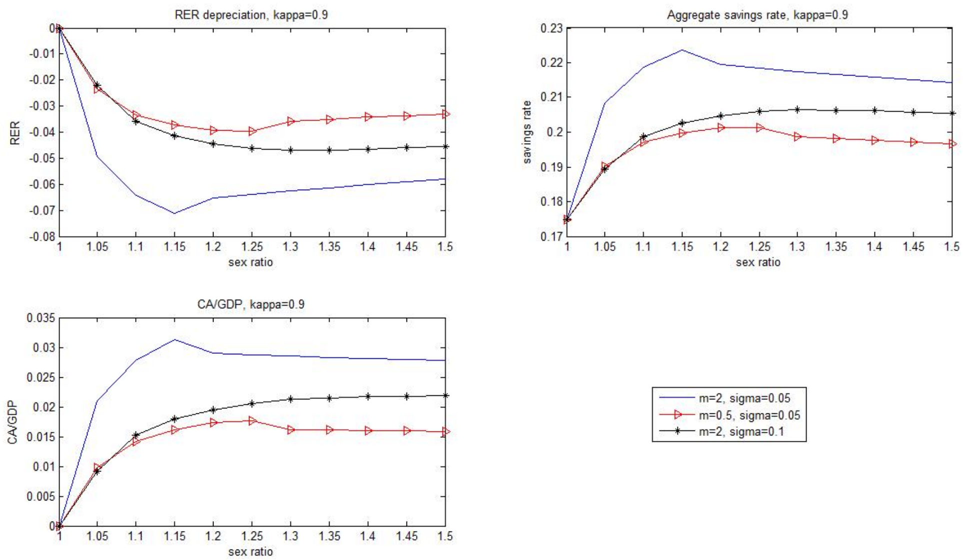

Figure 3.3: RER, aggregate savings rate, CA/GDP vs sex ratio, no labor supply effect, kappa=0.9

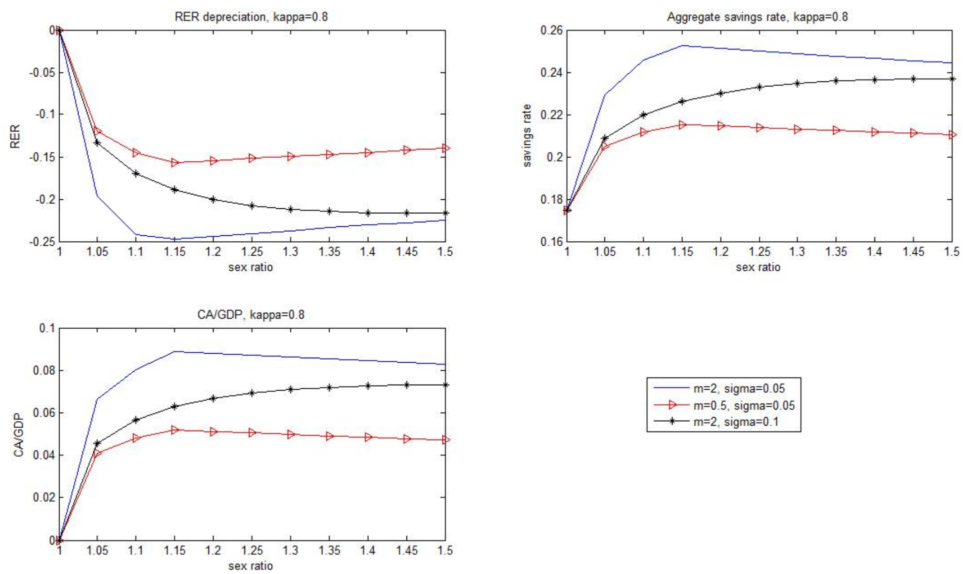

Figure 3.4: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.8

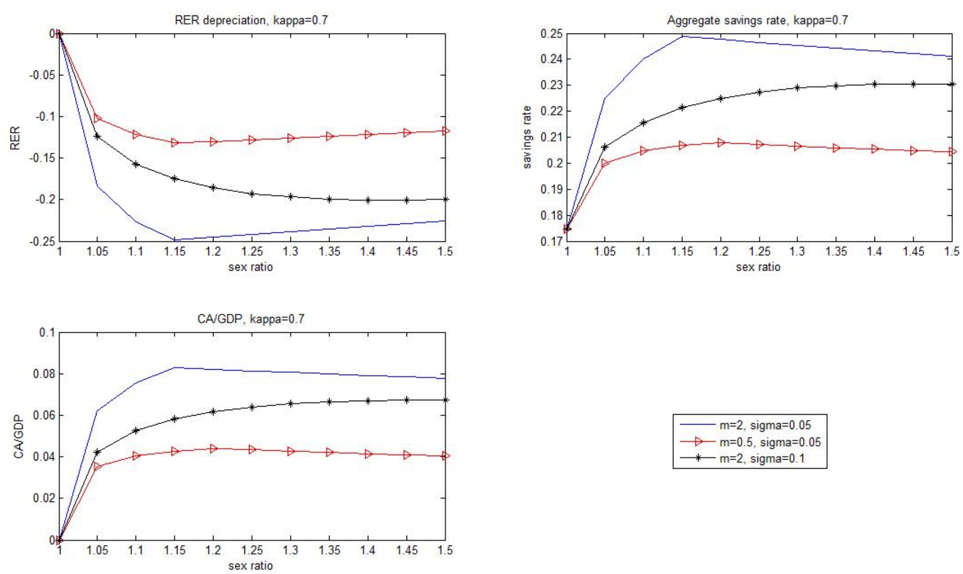

Figure 3.5: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.7

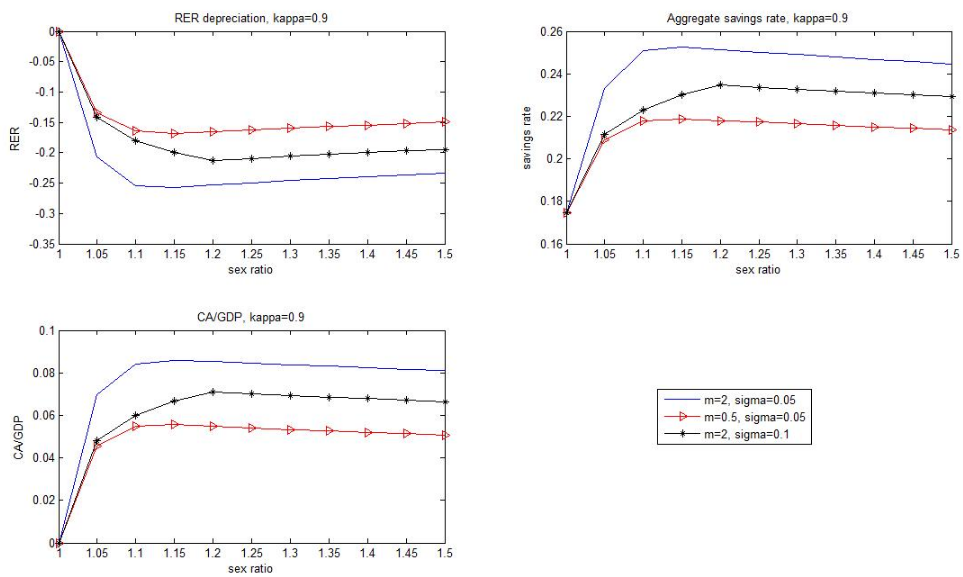

Figure 3.6: RER, aggregate savings rate, CA/GDP vs sex ratio, with labor supply effect, kappa=0.9

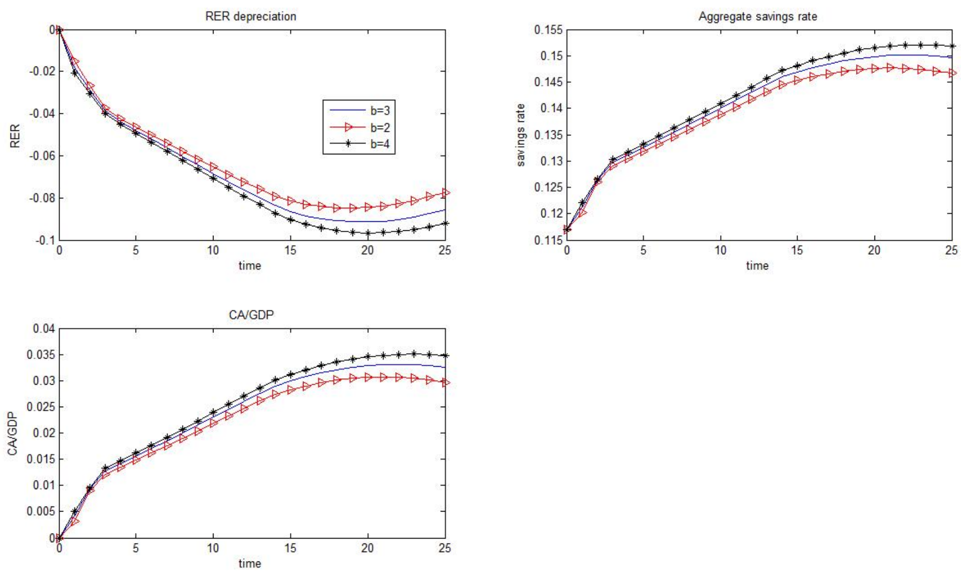

Figure 3.7: Impulse responses of RER, aggregate savings rate and CA/GDP, tao=20

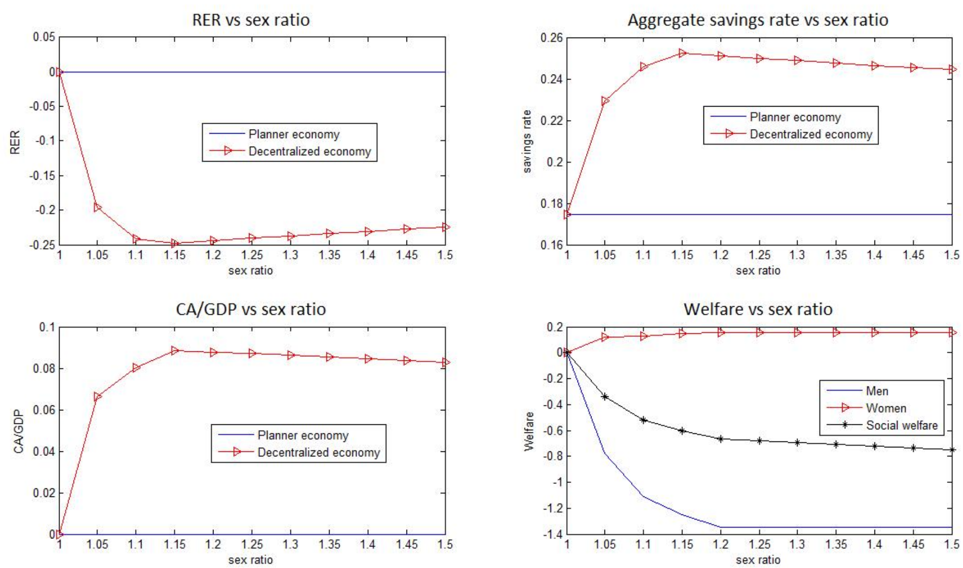

Figure 3.8: Planner's economy vs Decentralized economy, with labor supply effect, kappa=0.8, m=2, sigma=0.05

Table 3.1: Summary statistics, 2004-2008 average

Variable	Mean	Median	Standard deviation	Min value	Max value
Ln(RER)	-0.74	-0.8	0.59	-2.22	0.41
(Private Sector) Current account	-3.63	-2.93	9.32	-31.51	26.91
Real GDP per capita (US$)	12986	7747	13733	367	77057
Private credit (% of GDP)	56.63	38.7	52.26	2.08	319.72
Financial system sophistication	3.78	3.66	0.79	2.52	5.28
Sex ratio	1.04	1.04	0.02	1	1.13
Fiscal deficit (% of GDP)	-1.47	-0.37	5.98	-25.98	11.38
Terms of trade	113	102	33.8	70	205.8
Capital account openness	0.53	0.118	1.64	-1.83	2.5
Dependency ratio	60.75	54.84	17.64	28.47	107.6

Notes: a. The real exchange rate data is obtained from Penn World Tables 6.3. The variable p (called price level of GDP) in the Penn World Tables is equivalent to the real exchange rate relative to the US dollar: A lower value of p means a depreciation in the real exchange rate. b. Private Sector Current account is equal to current account to GDP ratio minus the government savings to GDP ratio.

Table 3.2: Ln(real exchange rate) and the sex ratio, using private credit to GDP ratio as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			-4.290**	-4.012**	-3.193*	-3.408**	-3.500**
			(1.667)	(1.713)	(1.797)	(1.568)	(1.754)
Ln(GDP per capita)	0.318**	0.190**	0.236**	0.233**	0.360**	0.402**	0.359**
	(0.030)	(0.038)	(0.041)	(0.044)	(0.073)	(0.063)	(0.073)
Private credit (% of GDP)		0.004**	0.004**	0.004**	0.003**	0.002**	0.002**
		(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Fiscal deficit					-0.007	0.002	-0.005
					(0.009)	(0.008)	(0.009)
Terms of trade					0.0002	-0.001	0.0003
					(0.001)	(0.001)	(0.001)
Capital account openness					0.060**	0.029	0.058**
					(0.027)	(0.024)	(0.027)
Dependency ratio					0.009**	0.010**	0.008*
					(0.004)	(0.004)	(0.004)
Crawling peg (RR)						-0.397**	
						(0.075)	
Managed floating (RR)						-0.036	
						(0.077)	
Free floating (RR)						-0.081	
						(0.119)	
Intermediate (LYS)							-0.078
							(0.092)
Float (LYS)							-0.145*
							(0.085)
Observations	142	132	132	123	92	89	92
R-squared	0.444	0.542	0.564	0.579	0.706	0.801	0.716

Notes: Dependent variable = log(RER). Standard errors are in parentheses, ** $p<0.05$, * $p<0.1$

Table 3.3: Ln(real exchange rate) and the sex ratio, using financial system sophistication as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			-6.192**	-6.255**	-5.051*	-4.664*	-4.43
			(1.964)	(1.995)	(2.500)	(2.802)	(2.908)
Ln(GDP per capita)	0.318**	0.480**	0.443**	0.447**	0.529**	0.526**	0.531**
	(0.030)	(0.082)	(0.077)	(0.088)	(0.123)	(0.119)	(0.127)
Financial system sophistication		0.170*	0.252**	0.245**	0.099	0.034	0.086
		(0.089)	(0.086)	(0.099)	(0.110)	(0.121)	(0.116)
Fiscal deficit					-0.022	-0.014	-0.025
					(0.015)	(0.015)	(0.017)
Terms of trade					-0.004	-0.006**	-0.005
					(0.003)	(0.003)	(0.003)
Capital account openness					0.063	0.058	0.073
					(0.042)	(0.047)	(0.047)
Dependency ratio					0.014**	0.017**	0.017*
					(0.007)	(0.007)	(0.008)
Crawling peg (RR)						-0.285*	
						(0.147)	
Managed floating (RR)						0.045	
						(0.102)	
Free floating (RR)						0.053	
						(0.173)	
Intermediate (LYS)							-0.052
							(0.137)
Float (LYS)							0.044
							(0.125)
Observations	142	54	54	49	43	42	43
R-squared	0.444	0.748	0.791	0.797	0.844	0.866	0.845

Notes: Dependent variable = log(RER). Standard errors are in parentheses, ** $p<0.05$, * $p<0.1$

Table 3.4: Non-governmental CA/GDP vs sex ratio, using private credit to GDP ratio as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			66.43*	78.43**	134.7**	111.6**	94.24
			(37.090)	(36.650)	(37.520)	(56.430)	(56.510)
Ln(GDP per capita)	2.025**	3.683**	2.964**	2.050**	4.941**	4.035	3.834
	(0.639)	(0.876)	(0.957)	(0.975)	(1.529)	(3.415)	(3.115)
Private credit (% of GDP)		-0.048**	-0.046**	-0.030*	-0.054**	-0.053**	-0.051**
		(0.018)	(0.018)	(0.018)	(0.018)	(0.025)	(0.024)
Fiscal deficit					0.079	-0.031	0.101
					(0.187)	(0.379)	(0.345)
Terms of trade					0.021	0.127	0.131*
					(0.029)	(0.076)	(0.076)
Capital account openness					-0.315	-0.017	-0.081
					(0.563)	(1.508)	(1.353)
Dependency ratio					0.175*	0.209	0.439
					(0.089)	(0.745)	(0.720)
Share of working age						0.163	0.797
						(1.884)	(1.885)
Social security expenditure						0.137	0.115
						(0.250)	(0.233)
RR regime dummy	N	N	N	N	N	Y	N
LYS regime dummy	N	N	N	N	N	N	Y
Continent dummies	N	N	N	N	N	Y	Y
Observations	130	127	127	120	91	47	48
R-squared	0.073	0.125	0.147	0.121	0.275	0.543	0.532

Notes: Dependent variable = Non-governmental CA/GDP. Standard errors are in parentheses, ** p<0.05, * p<0.1

Table 3.5: Non-governmental CA/GDP vs sex ratio, using financial system sophistication as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			103.3**	77.57**	129.6**	51.98	13.34
			(43.290)	(37.420)	(52.890)	(96.330)	(80.390)
Ln(GDP per capita)	2.025**	1.715	2.327	-0.47	2.009	3.752	-0.194
	(0.639)	(1.744)	(1.688)	(1.641)	(2.613)	(5.031)	(3.845)
Financial system sophistication		-0.925	-2.29	0.876	-0.477	4.029	3.48
		(1.889)	(1.896)	(1.861)	(2.337)	(3.336)	(2.951)
Fiscal deficit					-0.185	0.081	0.385
					(0.313)	(0.533)	(0.512)
Terms of trade					0.039	-0.07	0.0772
					(0.055)	(0.118)	(0.105)
Capital account openness					-0.103	-1.3	-0.569
					(0.879)	(2.619)	(1.765)
Dependency ratio					0.251	0.167	-1.807
					(0.150)	(5.527)	(2.375)
Share of working age						0.125	-3.446
						(12.230)	(5.400)
Social security expenditure						0.056	-0.092
						(0.302)	(0.257)
RR regime dummy	N	N	N	N	N	Y	N
LYS regime dummy	N	N	N	N	N	N	Y
Continent dummies	N	N	N	N	N	Y	Y
Observations	130	54	54	49	43	32	33
R-squared	0.073	0.023	0.123	0.118	0.2	0.478	0.505

Notes: Dependent variable = Non-governmental CA/GDP. Standard errors are in parentheses, ** $p<0.05$, * $p<0.1$

Table 3.6: Real exchange rate undervaluation and excess current account: The Case of China

	% of RER undervaluation					Excess (non-governmental) current account				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
	Only BS	FD+BS	Add GD +TT+KA	Add DR	Add SR	Only BS	FD+BS	Add GD +TT+KA	Add DR	Add SR
Financial development index										
Private credit (% of GDP)										
Financial system sophistication	55.26	46.38	31.31	16.78	2.24	13.52	10.26	10.11	7.97	0.37

Notes: a. Excess RER undervaluation is equal to model prediction minus actual log RER. A positive number describes the percentage of undervaluation b. Excess current account is equal to private sector current account, i.e., current account net of government savings, minus model prediction c.The five columns include progressively more regresors: (i) The only regressor (other than the intercept) is log income, a proxy for the Balassa-Samuelson (BS) effect; (ii) Add financial development (FD) to the list of regressors; (iii) Add government fiscal deficit (GD), terms of trade (TT), and capital account openness (KA); (iv) Add the dependence ratio (DR); (v) Add the sex ratio (SR) d. The last two rows correspond to estimates when two different proxies for financial development are used. The first row uses the ratio of credit to the private sector to GDP, and the second row uses an index of local financial system sophistication from the Global Competitiveness Report.

Chapter 4

Sex Ratios, Entrepreneurship and Comparative Advantage

Many Asian countries, including China, Korea, India, Vietnam, Singapore, Taiwan, and Hong Kong, have experienced a rise in the sex ratio in the pre-marriage age cohort. In many of such economies, parents have a strong preference for a son over a daughter. This used to lead to large families, not necessarily an unbalanced sex ratio. However, in the last three decades, as the technology to detect the gender of a fetus (Ultrasound B) has become less expensive and more widely available, more parents engage in selective abortions in favor of a son, resulting in an increasing relative surplus of men. The strict family planning policy in China, introduced in the early 1980s, has induced Chinese parents to engage in sex-selective abortions more aggressively than their counterparts in other countries. The sex ratio at birth in China used to be 106 boys per hundred girls in 1980, and it rose to 122 boys per hundred girls in 1997 (see Wei and Zhang, 2009, for more detail).

The existing literature has identified several consequences on the economy of a serious sex ratio imbalance. First, the sex ratio imbalance may cause crimes. Edlund, Li, Yi, and Zhang (2007) use Chinese data to estimate the effect of a rise in the sex ratio in crimes. They find that every one basis point increase in the sex ratio (e.g., from 1.10 to 1.11 boys per girl) raises violent and property crime rates by 3%, and the rise in the sex ratio imbalance may account for up to one-seventh of the overall rise in crime in China. Second, the imbalance may also trigger competitive savings among 2 households – men and households with sons forego current consumption to accumulate wealth in order to improve a young man's standing in the marriage market relative to other men. This increase in the savings rate is inefficient since it does not alter the number of unmarried men in the aggregate. Wei and Zhang (2009) estimate that about half the increase in the household savings rate in China during 1990-2005 can be attributed to the rise in the sex ratio. Du and Wei (2010) provide a theoretical framework to show this effect. In Du and Wei (2010), they also find that, as the sex ratio rises, social welfare will decline. Third, the sex ratio imbalance can generate a decline in the real exchange rate. They highlight two channels through which a sex ratio imbalance could lead to an appearance of currency undervaluation. The first is a savings channel. A rise in the savings rate implies a reduction in the demand for both tradable and non-tradable goods. Since the price of the tradable good is tied down by the world market, this translates into a reduction in the relative price of the nontradable good, and hence a decline in the value of the real exchange rate. The second theoretical channel works through effective labor supply. A rise in the sex ratio can also motivate men to cut down leisure and increase labor supply. This leads to an increase in the economy-wide effective labor supply. If the nontradable sector is more labor intensive than the tradable sector, this generates a Rybzinsky-like effect, leading to an expansion of the nontradable sector at the expense of the tradable sector. The increase in the supply of nontradable good leads to an additional decline in the relative price of nontradable and a further decline in the value of the RER.

However, not many papers have focused on the consequence of a sex ratio imbalance on economic growth and entrepreneurship. To our knowledge, Wei and Zhang (2010) is the first paper that empirically studies the effect of a rise in the sex ratio on entrepreneurship and economic growth. They find that the imbalance may stimulate economic growth by inducing more entrepreneurship. Motivated by their empirical findings, we provide a theoretical framework to analyze the consequence of a sex ratio imbalance on entrepreneurial activities in this paper. We first construct an overlapping generations model with two sexes and desire to marry in a closed economy. At the beginning of the first period, men can choose to be entrepreneurs (with a risky return) and workers (with a certain labor income) while all women are workers. They enter the marriage market at the start of the second period and marriages occur. When the sex ratio is close to a balanced level, only entrepreneurs who receive low income failed in the marriage market. As the sex ratio rises, more men will choose to be workers since being workers will obtain higher returns in the marriage market. However, when the sex ratio is large such that some male workers cannot get matched with women, an increase in the sex ratio raises the probability that a male worker will not get married, while it does not alter the expected utility of being an entrepreneur (to a first-order approximation). Then more men will respond to a higher sex ratio by becoming entrepreneurs.

The results may have important implications in an open economy model. Based on the same idea, the sex ratio imbalance can be an important sourse of the comparative advantage[1] in the risky sectors. We show in this paper that, in an open economy with two sectors in an economy, a risky sector and a risk free sector, a country with a very skewed sex ratio (above some threshold) may have more entrepreneurs in the risky sector, which in turn may lead to a comparative advantage in the risky sector.

We also provide some empirical support to the theoretical predictions. In addition to reviewing the evidence in Wei and Zhang (2010), we run two types of regressions to test our theoretical predictions in this paper. First, we find that, when the sex ratio exceeds some threshold, a rise in a country's sex ratio tends to lead to higher exports in more volatile sectors. Second, in a nonlinear least squares test, we find that above some threshold, which is close to the biological mean of the cross-country sex ratios, a rise in the sex ratio will lead to an increase in a country's export volatility. Both findings are consistent with our theoretical predictions. Quantitatively, the effect of a rise in the sex ratio on a country's export volatility can be very significant. For instance, consider a country initially with a sex

[1] We define the comparative advantage in a sector as the relative sectoral export position in this paper.

ratio around 1.05 (mean of the sex ratios in the world) and an export volatility 0.11 (mean of the export volatilities across countries), if the sex ratio rises from 1.05 to 1.13 (China's sex ratio in 2006), the export volatility will increase by almost 25%.

The rest of the paper is organized as following. In Section 2, we present a model and derive the main results of the paper. Section 3 is the empirical part in which we provide some empirical support to the theoretical predictions. In Section 4, we conclude and discuss about the future research.

4.1 Model

We construct an overlapping generations model with two sexes. Both men and women live two periods: young and old. At the beginning of the first period, men decide whether to become an entrepreneur or a worker while all women are workers. Every entrepreneur will run a firm by renting capitals and hiring workers. The return on the entrepreneurial activity is uncertain. All workers earn the same wage income. After receiving the first period income, men and women will consume part of the income and save the rest for the second period.

A marriage can only take place between a man and a women in the same generation and at the beginning of their second period. Once married, the husband and the wife pool their first-period savings together and consume an identical amount in the second period. The second period consumption within a marriage has a partial public good feature. In other words, the husband and the wife can each consume more than half of their combined second period income - the exact proportion is an exogenous parameter to be explained below. Everyone is endowed with an ability to give his/her spouse some emotional utility (or "love" or "happiness"). This emotional utility is a random variable in the first period with a common and known distribution across all members of the same sex, and its value is realized and becomes public information when the individual enters the marriage market.

We describe the equilibrium in three steps: first, we introduce the optimization problems for men and women. Second, we discuss the matching in the marriage market. Finally we derive the equilibrium in this economy.

4.1.1 Optimization problems for men and women

We denote the wage income by W_t and let ϕ and n_t^m denote the sex ratio in the young cohort and the fraction of young men who choose to be entrepreneurs. Then the total number of entrepreneurs in the economy is $\frac{\phi}{1+\phi}n_t^m$ if we normalize the measure of young people to be one.

For each entrepreneur, he runs a firm and produces a differentiated good with production function

$$y_{it} = \frac{z_{it}K_{it}^{\alpha}L_{it}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$$

where K_{it} and L_{it} are the capital and labor input, respectively. z_{it} is the productivity shock. For simplicity, we assume that the productivity z is drawn from a binomial distribution: $z = z^H$ with probability π and $z = z^L$ with probability $1 - \pi$.

The final consumption good is a index over all the varieties:

$$Y_t = \left[\left(\frac{\phi n_t^m}{1+\phi} \right)^{-\frac{1}{\theta}} \int_0^{\frac{\phi n_t^m}{1+\phi}} y_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

As in Dixit and Stiglitz (1977), parameter θ denotes the elasticity of intratemproal substitution between different varieties, with $\theta > 1$.

The individual demand function is then

$$y_{it} = \left(\frac{\phi n_t^m}{1+\phi} \right)^{-1} \left(\frac{p_{it}}{P_t} \right)^{-\theta} Y_t$$

where P_t is the aggregate price

$$P_t = \left[\left(\frac{\phi n_t^m}{1+\phi} \right)^{-1} \int_0^{\frac{\phi}{1+\phi}n_t^m} p_{it}^{1-\theta} di \right]^{\frac{1}{1-\theta}}$$

For simplicity, we assume complete capital depreciation in this paper. Then for an individual entrepreneur, he will maximize the profit

$$\max_{p_{it},K_{it},L_{it}} \{p_{it}y_{it} - W_t L_{it} - R_t K_{it}\}$$

the optimal price chosen by the entrepreneur is

$$p_{it} = \frac{\theta}{\theta - 1} MC_{it} \tag{4.1}$$

and factor demands are determined by

$$R_t = \frac{z_{it} MC_{it}}{\alpha^{\alpha-1}(1-\alpha)^{1-\alpha}} \left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} \tag{4.2}$$

$$W_t = \frac{z_{it} MC_{it}}{\alpha^{\alpha}(1-\alpha)^{-\alpha}} \left(\frac{K_{it}}{L_{it}}\right)^{\alpha} \tag{4.3}$$

where MC_{it} is the marginal cost which equals $R_t^{\alpha} W_t^{1-\alpha}$. By (4.2) and (4.3), we have

$$\frac{W_t}{R_t} = \frac{1-\alpha}{\alpha} \frac{K_{it}}{L_{it}} \tag{4.4}$$

Given the equilibrium interest rate and wage rate, each firm will have the same capital to labor ratio which should be equal to the economy-wide capital to labor endownment in period t.

For a representative entrepreneur with z^H and a reprensentative entrepreneur with z^L, under the demand structure in Dixit and Stiglitz (1977), the ratio of their revenues is

$$\frac{p^H y^H}{p^L y^L} = \left(\frac{p^H}{p^L}\right)^{1-\theta} = \left(\frac{z^L}{z^H}\right)^{1-\theta}$$

By (4.1), (4.2) and (4.3), we can obtain

$$\frac{p^H y^H}{p^L y^L} = \frac{\frac{\theta}{\theta-1} \frac{W_t L_t^H}{(1-\alpha) z^H}}{\frac{\theta}{\theta-1} \frac{W_t L_t^L}{(1-\alpha) z^L}} = \frac{L_t^H}{L_t^L} \frac{z^L}{z^H}$$

where L_t^H and L_t^L are the labor hired by the representative entrepreneur with z^H and the representative entrepreneur with z^L, respectively. Then

$$\left(\frac{z^L}{z^H}\right)^{1-\theta} = \frac{L_t^H}{L_t^L}\frac{z^L}{z^H} \Rightarrow \frac{L_t^H}{L_t^L} = \left(\frac{z^L}{z^H}\right)^{-\theta}$$

If we use L_t denote the total labor supply in the economy, which equals

$$L_t = \frac{\phi n_t^m}{1+\phi}\left(\pi L_t^H + (1-\pi) L_t^L\right)$$

then

$$L_t^L = \frac{\left(z^L\right)^\theta}{\pi\left(z^L\right)^\theta + (1-\pi)\left(z^H\right)^\theta}\frac{L_t}{\frac{\phi n_t^m}{1+\phi}} \text{ and } L_t^H = \frac{\left(z^H\right)^\theta}{\pi\left(z^L\right)^\theta + (1-\pi)\left(z^H\right)^\theta}\frac{L_t}{\frac{\phi n_t^m}{1+\phi}}$$

In this model, P_tY_t is the aggregate sales revenue by all the firms. Given the demand structure in Dixit and Stiglitz (1977), each individual firm has the same markup which equals $\frac{\theta}{\theta-1}$, which means that the total revenue equals the product of the total cost and $\frac{\theta}{\theta-1}$ given the economy-wide wage rate and rental rate. Then by the law of large number, we have

$$\begin{aligned}
P_tY_t &= \frac{\theta\int_i^{\frac{\phi n_t^m}{1+\phi}} MC_{it}y_{it}di}{\theta-1} = \frac{\theta\int_i^{\frac{\phi n_t^m}{1+\phi}} \frac{1}{z_{it}}\frac{W_tL_{it}}{1-\alpha}di}{\theta-1} \\
&= \frac{\theta\frac{\phi n_t^m}{1+\phi}W_t}{(1-\alpha)(\theta-1)}\left(\frac{\pi L_t^H}{z^H} + \frac{(1-\pi)L_t^L}{z^L}\right) \\
&= \frac{\theta}{(1-\alpha)(\theta-1)}\left(\begin{array}{c} \frac{\pi}{z^H}\frac{\left(z^H\right)^\theta}{\pi\left(z^L\right)^\theta+(1-\pi)\left(z^H\right)^\theta} \\ +\frac{1-\pi}{z^L}\frac{\left(z^L\right)^\theta}{\pi\left(z^L\right)^\theta+(1-\pi)\left(z^H\right)^\theta}\end{array}\right)W_tL_t
\end{aligned}$$

Using this result, the profit of entrepreneur i is

$$\begin{aligned}\Pi_{it}^{e} &= (p_{it} - MC_{it}) \left(\frac{\phi n_t^m}{1+\phi}\right)^{-1} \left(\frac{p_{it}}{P_t}\right)^{-\theta} Y_t \\ &= \frac{\left(\frac{\phi n_t^m}{1+\phi}\right)^{-1} \left(\frac{p_{it}}{P_t}\right)^{1-\theta}}{(1-\alpha)(\theta-1)} \left(\begin{array}{c} \frac{\pi}{z^H} \frac{\left(z^H\right)^\theta}{\pi (z^L)^\theta + (1-\pi)(z^H)^\theta} \\ + \frac{1-\pi}{z^L} \frac{\left(z^L\right)^\theta}{\pi (z^L)^\theta + (1-\pi)(z^H)^\theta} \end{array} \right) W_t L_t\end{aligned}$$

Since the total labor supply is

$$L_t = \frac{\phi(1-n_t^m)+1}{1+\phi}$$

the profit of entrepreneur i is

$$\Pi_{it}^{e} = \frac{\frac{\pi}{z^H} \frac{\left(z^H\right)^\theta}{\pi (z^L)^\theta + (1-\pi)(z^H)^\theta} + \frac{1-\pi}{z^L} \frac{\left(z^L\right)^\theta}{\pi (z^L)^\theta + (1-\pi)(z^H)^\theta}}{(1-\alpha)(\theta-1)} \frac{z_{it}^{\theta-1}\left(\frac{1+\phi}{\phi n_t^m} - 1\right) W_t}{\pi \left(z^H\right)^{\theta-1} + (1-\pi)\left(z^L\right)^{\theta-1}} \tag{4.5}$$

By (4.5), one key factor that determines the profit is the productivity. A larger productivity will yield a higher profit. In the rest of the paper, we call entrepreneurs with z^H "successful entrepreneurs", and entrepreneurs with z^L "failed entrepreneurs". We will solve the optimization problems for men by backward induction: we first solve the optimization savings decisions for successful entrepreneurs, male workers and failed entrepreneurs, respectivley, take the number of entrepreneurs in the economy as given. Then by comparing the *ex post* utilities, we derive the number of entrepreneurs in equilibrium.

Given the sex ratio ϕ and the fraction of men who choose to be entrepreneurs n_t^m, if a man choose to be an entrepreneur and he succeeds, his optimization problem is

$$\max \; u(c_{1m,t}^e) + \beta E\left[u(c_{2m,t+1}^e) + \eta^w\right]$$

with budget constraint

$$P_t c_{1m,t}^e = (1 - s_t^e)\Pi_t^H$$

$$P_{t+1}c^e_{2m,t+1} = \begin{cases} \kappa(R_t s^e_t \Pi^H_t + R_t s^w_t I^w_t) & \text{if married} \\ R_t s^e_t \Pi^H_t & \text{otherwise} \end{cases}$$

where s^e_t and Π^H_t are the savings rate and the profit of the man, respectively. s^w_t and I^w_t are the savings rate and first period income of his wife, respectively. η^w is the emotional utility obtained from his wife, which is drawn from a distribution $F^w(\eta)$. If the man fails, his optimization problem is

$$\max \; u(c^{e,L}_{1m,t}) + \beta E\left[u(c^{e,L}_{2m,t+1}) + \eta^w\right]$$

with budget constraint

$$\begin{aligned} P_t c^{e,L}_{1m,t} &= (1 - s^{e,L}_t)\Pi^L_t \\ P_{t+1}c^{e,L}_{2m,t+1} &= \begin{cases} \kappa(R_t s^{e,L}_t \Pi^L_t + R_t s^w_t I^w_t) & \text{if married} \\ R_t s^{e,L}_t \Pi^L_t & \text{otherwise} \end{cases} \end{aligned}$$

where $s^{e,L}_t$ and Π^L_t are the savings rate and the profit of the man, respectively.

The optimization problem for a representative male worker is

$$\max \; u(c^L_{1m,t}) + \beta E\left[u(c^L_{2m,t+1}) + \eta^w\right]$$

with budget constaint

$$\begin{aligned} P_t c^L_{1m,t} &= (1 - s^{m,L}_t)W_t \\ P_{t+1}c^L_{2m,t+1} &= \begin{cases} \kappa R_t(s^w_t + s^{m,L}_t)W_t & \text{if married} \\ R_t s^m_t W_t & \text{otherwise} \end{cases} \end{aligned}$$

where $s^{m,L}_t$ is the savings rate of the representative male worker.

For a representative female worker, the optimization problem is

$$\max\ u(c_{1w,t}) + \beta E\left[u(c_{2w,t+1}) + \eta^m\right]$$

with budget constaint

$$\begin{aligned} P_t c_{1w,t} &= (1-s_t^w)W_t \\ P_{t+1}c_{2w,t+1} &= \begin{cases} \kappa R_t(s_t^w W_t + s_t^m I_t^m) & \text{if married} \\ R_t s_t^m W_t & \text{otherwise} \end{cases} \end{aligned}$$

where s_t^m and I_t^m denotes the savings rate and first period income of her future husband, respectively.

For simplicity, we assume log utility function throughout the paper.

4.1.2 Marriage market

We assume that η^w and η^m are drawn from the same uniform distribution with a lower bound $\eta^{\min}$ and an upper bound $\eta^{\max}$. In the benchmark model, we assume that everybody enters the marriage market.

In the marriage market, every woman (or man) ranks all members of the opposite sex by a combination of two criteria: (1) the level of wealth (which is determined solely by the first-period savings), and (2) the size of "love" he/she can obtain from his/her spouse. The weights on the two criteria are implied by the utility functions specified earlier. More precisely, woman i prefers a higher ranked man to a lower ranked one, where the rank on man j is given by $u(c_{2w,i,j}) + \eta_j^m$. Symmetrically, man j assigns a rank to woman i based on the utility he can obtain from her $u(c_{2m,j,i}) + \eta_i^w$. (To ensure that the preference is strict for men and women, when there is a tie in terms of the above criteria, we break the tie by assuming that a woman prefers j if $j < j'$ and a man does the same. Note that "love" is not in the eyes of the beholder in the sense that every woman (man) has the same ranking over men (women).

The marriage market is assumed to follow the Gale-Shapley algorithm, which produces a unique and stable equilibrium of matching (Gale and Shapley, 1962; and Roth and Sotomayor, 1990). The algorithm specifies the following: (1) Each man proposes in the first round to his most preferred choice of woman. Each woman holds the proposal from her most preferred suitor and rejects the rest. (2) Any man who is rejected in round k-1 makes a new proposal in round k to his most preferred woman among those who have not yet rejected him. Each available women in round k "holds" the proposal from her most preferred man and rejects the rest. (3) The procedure repeats itself until no further proposals are made.[2]

4.1.3 Equilibrium

To derive the general equilibrium, we will first solve the optimization problems for successful entrepreneurs, failed entrepreneurs, male workers and female workers. Since there are three types of men that receive different first period incomes in the economy, men will not make the same savings decisions. Hopkins (2010) analyzes a more general two-sided matching game. One implication by his paper is that men's choice decision (savings in this paper) is an increasing function of their initial wealth. In other words, successful entrepreneur will have the highest wealth in the marriage market while failed entrepreneurs have the lowest wealth.

For simplicity, we assume that the dispersion of the emotional utility distribution is very small, $\eta^{\min}$ is very close to $\eta^{\max}$, such that the equilibrium matching between men and women is: (i) successful entrepreneurs get matched with the best typed women; (ii) male workers get matched with mid typed women; and (iii) failed entrepreneurs get matched with the worst typed women.

Let $\phi(\geq 1)$ denote the sex ratio in the economy. We assume ϕ is exogenous throughout the paper. If ϕ is small (close to one), then in equilibrium, only failed entrepreneurs cannot get married. In this case, the equilibrium matching between successful entrepeneurs and

[2] If only women can propose and men respond with deferred acceptance, the same matching outcomes will emerge. What we have to rule out is that both men and women can propose, in which case, one cannot prove that the matching is unique.

female workers is

$$\begin{aligned} 1-F\left(\eta^{w}\right) &= \frac{\pi\phi n_t^m}{1+\phi}\left[1-F\left(M^1\left(\eta^w\right)\right)\right] \\ &\Rightarrow M^1\left(\eta^w\right)=F^{-1}\left(1-\frac{1+\phi}{\pi\phi n_t^m}\left[1-F\left(\eta^w\right)\right]\right) \end{aligned}$$

The equilibrium matching between male workers and female workers is

$$M^2\left(\eta^w\right)=F^{-1}\left(1-\frac{1+\phi}{\phi\left(1-n_t^m\right)}\left[F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)-F\left(\eta^w\right)\right]\right)$$

and the equilibrium matching between failed entrepeneurs and female workers is

$$M^3\left(\eta^w\right)=F^{-1}\left(1-\frac{1+\phi}{\left(1-\pi\right)\phi n_t^m}\left[F\left(\left(M^2\right)^{-1}\left(\eta^{\min}\right)\right)-F\left(\eta^w\right)\right]\right)$$

For a representative successful entrepreneur i, given all his rivals choices, his second period utility is

$$\begin{aligned} &\delta_i^e \ln\left(\kappa R_t\left(s_{i,t}^e\Pi_t^H+s_t^w W_t\right)\right)+\left(1-\delta_i^e\right)\ln\left(R_t s_{i,t}^e\Pi_t^H\right)+\int_{\bar{\eta}_{1i}^m}\left(M^1\right)^{-1}\left(\tilde{\eta}_{1i}^m\right)dF\left(\eta_i^m\right) \\ &+\int_{\bar{\eta}_{2i}^m}^{\bar{\eta}_{1i}^m}\left(M^2\right)^{-1}\left(\tilde{\eta}_{2i}^m\right)dF\left(\eta_i^m\right)+\int_{\bar{\eta}_{3i}^m}^{\bar{\eta}_{2i}^m}\left(M^3\right)^{-1}\left(\tilde{\eta}_{3i}^m\right)dF\left(\eta_i^m\right) \end{aligned}$$

where

$$\begin{aligned} \tilde{\eta}_{1i}^m &= \eta_i^m+\ln\left(\kappa R_t\left(s_{it}^e\Pi_t^H+s_t^w W_t\right)\right)-\ln\left(\kappa R_t\left(s_t^e\Pi_t^H+s_t^w W_t\right)\right) \\ \tilde{\eta}_{2i}^m &= \eta_i^m+\ln\left(\kappa R_t\left(s_{it}^e\Pi_t^H+s_t^w W_t\right)\right)-\ln\left(\kappa R_t\left(s_t^{m,L}W_t+s_t^w W_t\right)\right) \\ \tilde{\eta}_{3i}^m &= \eta_i^m+\ln\left(\kappa R_t\left(s_{it}^e\Pi_t^H+s_t^w W_t\right)\right)-\ln\left(\kappa R_t\left(s_t^{e,L}\Pi_t^L+s_t^w W_t\right)\right) \end{aligned}$$

$$\begin{aligned} \bar{\eta}_{1i}^m &= \eta^{\max}+\ln\left(\kappa R_t\left(s_t^{m,L}W_t+s_t^w W_t\right)\right)-\ln\left(\kappa R_t\left(s_{it}^e\Pi_t^H+s_t^w W_t\right)\right) \\ \bar{\eta}_{2i}^m &= \eta^{\max}+\ln\left(\kappa R_t\left(s_t^{e,L}\Pi_t^L+s_t^w W_t\right)\right)-\ln\left(\kappa R_t\left(s_{it}^e\Pi_t^H+s_t^w W_t\right)\right) \\ \bar{\eta}_{3i}^m &= M^3\left(\eta^{\min}\right)+\ln\left(\kappa R_t\left(s_t^{e,L}\Pi_t^L+s_t^w W_t\right)\right)-\ln\left(\kappa R_t\left(s_{it}^e\Pi_t^H+s_t^w W_t\right)\right) \end{aligned}$$

and

$$\delta^e = \Pr\left(\left[\begin{array}{c}\eta_i^m + \ln\left(\kappa R_t\left(s_{it}^e \Pi_t^H + s_t^w W_t\right)\right) \\ -\ln\left(\kappa R_t\left(s_t^{e,L}\Pi_t^L + s_t^w W_t\right)\right)\end{array}\right] \geq \eta^{\min} \middle| s_t^e, s_t^{m,L}, s_t^{e,L}, s_t^w, W_t, R_t\right)$$

is the probability that successful entrepreneur i gets married. Due to the symmetry, we drop index i and obtain the first order condition for a successful entrepreneur as

$$-\frac{1}{1-s_t^e} + \beta\frac{1}{s_{i,t}^e + s_t^w W_t/\Pi_t^H}\left(1 + \frac{\pi\phi n_t^m}{1+\phi}\right) = 0 \tag{4.6}$$

For a representative male worker, if the sex ratio is small (close to one), then his first order condition is

$$-\frac{1}{1-s_t^{m,L}} + \beta\frac{1}{s_t^{m,L} + s_t^w}\left(1 + \frac{\phi\left(1-n_t^m\right)}{1+\phi}\right) = 0 \tag{4.7}$$

If the sex ratio is large such that some male workers cannot get married, then his first order condition is

$$-\frac{1}{1-s_t^{m,L}} + \frac{\beta}{s_t^{m,L}+s_t^w}\left[\begin{array}{c}\left(1-F\left(M^2\left(\eta^{\min}\right)\right)\right)\left(1+\frac{\phi(1-n_t^m)}{1+\phi}\right) \\ +F\left(M^2\left(\eta^{\min}\right)\right)\frac{s_t^{m,L}+s_t^w}{s_t^{m,L}} \\ +f\left(\eta^{\min}\right)\left(\ln\left(\kappa\left(s_t^{m,L}+s_t^w\right)\right)+\eta^{\min}-\ln\left(s_t^w\right)\right)\end{array}\right] = 0 \tag{4.8}$$

For a representative failed entrepreneur, if the sex ratio is small (close to one), with a positive possibility he can get married, then the first order condition is

$$-\frac{1}{1-s_t^{e,L}} + \beta\frac{\left[\begin{array}{c}\left(1-F\left(M^3\left(\eta^{\min}\right)\right)\right)\left(1+\frac{\phi(1-\pi)n_t^m}{1+\phi}\right)+F\left(M^3\left(\eta^{\min}\right)\right)\frac{s_t^{e,L}+s_t^w W_t/\Pi_t^L}{s_t^{e,L}} \\ +f\left(\eta^{\min}\right)\left(\ln\left(\kappa\left(s_t^{e,L}+s_t^w W_t/\Pi_t^L\right)\right)+\eta^{\min}-\ln\left(s_t^{e,L}\right)\right)\end{array}\right]}{s_t^{e,L}+s_t^w W_t/\Pi_t^L} = 0 \tag{4.9}$$

If the sex ratio becomes large such that no failed entrepreneurs can get matched with some women, then the first order condition is

$$-\frac{1}{1-s_t^{e,L}}+\beta\frac{1}{s_t^{e,L}}=0 \tag{4.10}$$

For a representative woman, her expected second period utility is

$$\begin{aligned}
&\delta^{w,e}\ln\left(\kappa R_t\left(s_t^e\Pi_t^H+s_t^wW_t\right)\right)+\delta^{w,L}\ln\left(\kappa R_t\left(s_t^{m,L}W_t+s_t^wW_t\right)\right)\\
&+\left(1-\delta^{w,e}-\delta^{w,L}\right)\ln\left(\kappa R_t\left(s_t^{e,L}\Pi_t^L+s_t^wW_t\right)\right)\\
&+\int_{(M^1)^{-1}(\eta^{\min})}M^1\left(\eta^w\right)dF\left(\eta^w\right)+\int_{(M^2)^{-1}(\eta^{\min})}^{\left(M^1\right)^{-1}\left(\eta^{\min}\right)}M^2\left(\eta^w\right)dF\left(\eta^w\right)\\
&+\int_{\eta^{\min}}^{\left(M^2\right)^{-1}\left(\eta^{\min}\right)}M^3\left(\eta^w\right)dF\left(\eta^w\right)
\end{aligned}$$

If the sex ratio is close to one, $M^3\left(\eta^w\right)\neq 0$, then her first order condition is

$$-\frac{1}{1-s_t^w}+\beta\left[\begin{array}{c}\frac{F\left(\left(M^2\right)^{-1}\left(\eta^{\min}\right)\right)\left(1+\frac{1+\phi}{(1-\pi)\phi n_t^m}\right)}{s_t^{e,L}\Pi_t^L/W_t+s_t^w}+\frac{\left(1+\frac{1+\phi}{\pi\phi n_t^m}\right)\left(1-F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)\right)}{s_t^e\Pi_t^H/W_t+s_t^w}\\+\frac{\left(1+\frac{1+\phi}{\phi\left(1-n_t^m\right)}\right)\left(F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)-F\left(\left(M^2\right)^{-1}\left(\eta^{\min}\right)\right)\right)}{s_t^{m,L}+s_t^w}\end{array}\right]=0 \tag{4.11}$$

If the sex ratio becomes sufficiently large such that no failed entrepreneur can get married, then the representative woman's first order condition is

$$-\frac{1}{1-s_t^w}+\beta\left[\begin{array}{c}\frac{\left(1+\frac{1+\phi}{\pi\phi n_t^m}\right)\left(1-F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)\right)}{s_t^e\Pi_t^H/W_t+s_t^w}\\+\frac{\left(1+\frac{1+\phi}{\phi\left(1-n_t^m\right)}\right)F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)}{s_t^{m,L}+s_t^w}\end{array}\right]=0 \tag{4.12}$$

Since men can freely choose to be entrepreneurs and workers *ex ante*, in equilibrium, men will feel indifferent between being an entrepreneur and a worker, which means n_t^m will solve the equation in the following

$$\pi V_t^{m,e}+(1-\pi)V_t^{m,eL}=V_t^{m,L} \tag{4.13}$$

Labor market and capital market must clear in equilibrium, therefore, the two equations in the following hold, respectively.

$$\frac{\phi(1-n_t^m)+1}{1+\phi} = \frac{1-\alpha}{\alpha}\frac{R_t}{W_t}K_t \tag{4.14}$$

and

$$R_tK_t = \frac{\alpha(\theta-1)}{\theta}\left\{\begin{array}{c} \pi\frac{\phi_{t-1}n_{t-1}^m}{1+\phi_{t-1}}s_{t-1}^{e,H}R_t\Pi_{1,t-1}^{e,H} + (1-\pi)\frac{\phi_{t-1}n_{t-1}^m}{1+\phi_{t-1}}s_{t-1}^{e,L}R_t\Pi_{1,t-1}^{e,L} \\ +\frac{\phi_{t-1}\left(1-n_{1t-1}^m\right)}{1+\phi_{t-1}}s_{t-1}^{m,L}R_tW_{t-1} + \frac{1}{1+\phi_{t-1}}s_{t-1}^{w}R_tW_{t-1} \\ +\pi\frac{\phi n_t^m}{1+\phi}\Pi_t^{e,H} + (1-\pi)\frac{\phi n_t^m}{1+\phi}\Pi_t^{e,L} + \left(1-\frac{\phi n_t^m}{1+\phi}\right)W_t \end{array}\right\} \tag{4.15}$$

where the term $\pi\frac{\phi_{t-1}n_{t-1}^m}{1+\phi_{t-1}}s_{t-1}^{e,H}R_t\Pi_{1,t-1}^{e,H}+(1-\pi)\frac{\phi_{t-1}n_{t-1}^m}{1+\phi_{t-1}}s_{t-1}^{e,L}R_t\Pi_{1,t-1}^{e,L}+\frac{\phi_{t-1}\left(1-n_{1t-1}^m\right)}{1+\phi_{t-1}}s_{t-1}^{m,L}R_tW_{t-1}+\frac{1}{1+\phi_{t-1}}s_{t-1}^{w}R_tW_{t-1}$ represents the aggregate consumption of the old cohort in period t.

We consider how a structual shock, an unexpected rise in the young cohort's sex ratio in period t, may affect the entrepreneurial activities in this paper. We will assume that $\eta^{\min}$ is large enough such that marriage is appealing for everyone in the economy. Now we can show the following proposition.

Proposition 13. *If π is small enough,*

(i) When the sex ratio is small (close to one), as the sex ratio rises, a smaller number of men choose to be entrepreneurs.

(ii) When the sex ratio becomes sufficiently unbalanced such that no failed entrepreneurs can get married, as the sex ratio rises, a larger fraction of men choose to be entrepreneurs.

Proof. See Appendix A4.1. □

Three remarks are in order. First, there always exist a positive fraction of men who choose to be entrepreneurs. Here is the reason, for a representative man, if he observes that all his rivals are workers, he will optimally choose to be entrepreneurs since, no matter what level of productivity he gets, he will earn infinity profit by (4.5). Therefore, $n_t^m = 0$ cannot be the equilibrium in this economy.

Second, the number of entrepreneurs in the economy depends on the expected welfare of being an entrepreneur and the welfare of being a male worker. When the sex ratio is close to one, every male worker will get married, if the possibility of getting a low productivity draw is high, a rise in the sex ratio harms the failed entrepreneurs most. Men will switch their choices to workers since being a worker can always get married with some woman.

Third, if the sex ratio is very unbalanced such that some male workers cannot get married, as the sex ratio rises, more men choose to be entrepreneurs. Here is the intuition. Suppose there is an increase in the number of men in one period, for a representative "new" man in the economy, he can choose either to be an entrepreneur or a worker. If he choose to be a worker, the competition for a wife among workers become even more severe, male workers will experience a welfare loss. However, if he choose to be an entrepreneur, there are two outcomes: (i) he gets a high productivity draw z^H, and he can marry a woman in the second period; and (ii) he gets a low productivity draw z^L, he can never get married so he optimally choose his savings as a life-time bachelor. Notice that, if the lower bound of emotional utility is sufficiently high, even a rise in the number of successful entrepreneurs may leads to a welfare loss, but it is smaller than the welfare loss of male workers who are facing a bigger possibility of being single. On the other hand, failed entrepreneurs can never get married, a rise in the number of failed entrepreneurs will not lead to a big welfare loss to them. Therefore, a representative "new" man will optimally choose to become an entrepeneur.

4.1.4 A mixed-strategy equilibrium

In this section, we extend our benchmark model by considering the choice of entering/exiting the marriage market. In a mixed-strategy game, both women and men can choose the probability of entering the marriage market as well as their saving decisions.

A representative woman will have the same optimization problem as in the previous section if she enters the marriage market. She can also choose to be single and if she does so, her life-time utility is

$$V_n^w = \max_{s_n^w} u(c_{1w,n}) + \beta u(c_{2w,n})$$

where V_n^w denotes the value function of a representative woman who is single throughout her life.

In a mixed-strategy game, the representative woman will choose the probability of entering the marriage market ρ^w, a savings rate if she decides to enter, and a separate savings rate if decides to abstain from the marriage market. The optimization problem is

$$\max_{\rho^w, s^w, s_n^w} \rho^w V^w + (1-\rho^w) V_n^w$$

where V^w is the value function when she chooses to enter the marriage market. Obviously, she will choose $\rho^w = 1$ if and only if $V^w > V_n^w$. All men will have the similar optimization problem given their first period income.

We assume $E\eta$ is sufficiently large in this paper such that entering the marriage market is a dominant strategy for people who face an *ex pos*t probability one to get married. Then, when the sex ratio is small (close to one), everyone enters the marriage market with probability one. As the sex ratio keeps rising and exceeds some threshold, some failed entrepreneurs will quit the marriage market. Here is the reason. As in Proposition 13, there always exists a sex ratio at which failed entrepreneurs get married with probability zero *ex post*. For instance, let ϕ' denote such a sex ratio. If all failed entrepreneurs still enter the marriage market with probability one at ϕ', the value function of a representative failed entrepreneur is $V_t^{e,L} = \ln\left(\left(1 - s_t^{e,L}\right)\Pi_t^L\right) + \beta \ln\left(R_t s_t^{e,L} \Pi_t^L\right)$. Notice that if a failed entrepreneur deviate by choosing being single, he will maximize his life-time utility $\ln\left(\left(1-s_t^n\right)\Pi_t^L\right) + \beta \ln\left(R_t s_t^n \Pi_t^L\right)$ by selecting s_t^n. The first order condition for this failed entrepreneur who chooses to be single is

$$-\frac{1}{1-s_t^n} + \beta \frac{1}{s_t^n} = 0$$

which is different from (4.9). This means that, at s_t^n, $\ln\left(\left(1-s_t^n\right)\Pi_t^L\right)+\beta\ln\left(R_t s_t^n \Pi_t^L\right)$ takes its maximum, any deviation in the savings rate from s_t^n will yield a welfare loss. Therefore, being single will lead to a higher *ex post* utility than entering the marriage market in this case. Entering the marriage market with probability one cannot be the optimal strategy in equilibrium at ϕ' and some failed entrepreneurs will optimally choose to be single. Due to the continuity of all the variables, there exists a threshold ϕ^0 at which all failed entrepreneurs still enter the marriage market but they obtain the same *ex post* utility as being single.

$$V_t^{e,L} = V_{n,t}^{e,L} \tag{4.16}$$

As the sex ratio keeps rising, failed entrepreneurs will choose a positive probability of being single.

We total differentiate the equations (4.6), (4.7), (4.9), (4.11), (4.13), (4.14), (4.15) and (4.16) at ϕ^0 to see how a further rise in the sex ratio may influence the number of entrepreneurs in the economy. Similar to the proof of Proposition 13, we find that the effect is ambiguous. Here is the intuition. As the sex ratio keeps rising, we consider those "new" men in the economy. They will have a tradeoff in choosing to be entrepreneurs or workers. If all of them choose to be workers, this will raise the competition for better wives among male workers which potentially leads to a welfare loss. However, if some of them choose to become entrepreneurs, once they fail, they will face a more severe competition for getting married which again yield an expected welfare loss to entrepreneurs. When the sex ratio is small and the mean of the emotional utility is large, the second effect dominates the first one and the number of total entrepreneurs in the economy will decrease. This is because, once an entrepreneur fails, it may face a positive possibility of being single, the marginal loss is proportional to the mean of the emotional utility which can be potentially very large. Then, to be safe, men are more likely to be workers since male workers can always get married if the sex ratio is very low. However, when the sex ratio becomes large ($\geq \phi^0$), failed entrepreneurs are indifferent between entering the marriage market and being single, the net effect of a rise in the sex ratio on the relative welfare of being an entrepeneur compared to being a worker is ambiguous. The reason is that, the marginal welfare loss to the failed entrepreneurs in this case may not be large since failed entrepreneurs can choose to be single if they foresee the bad situations in the marriage market. In other words, there exists a quasi lower bound for the welfare of failed entrepreneurs at which failed entrepreneurs' welfare does not vary much no matter how sex ratio changes. In this case, it is possible that the first effect dominates the second one and the total number of entrepreneurs may increase as the sex ratio rises.

As the sex ratio continues rising, when no failed entrepreneurs can get married, similar to the analysis above, there exists another threshold ϕ^1 above which male workers will consider a positive possibility of being single. As in Du and Wei (2010, 2011), such a threshold may be very large if the mean of the emotional utility is very large. Some numerical examples show that in the real world, no single country has a sex ratio beyond the threshold. Therefore, for simplicity, we do not consider the case that male workers choose a positive probability of being single in this paper.

4.1.5 An open economy model with two sectors

In this extension, we assume there are two production sectors, 1 and 2, in the economy. The final good is a composite of the two goods

$$Y = \frac{Y_1^{\gamma} Y_2^{1-\gamma}}{\gamma^{\gamma} \left(1-\gamma\right)^{1-\gamma}}$$

where γ and $1-\gamma$ are the shares on the good 1 and the good 2, respectively. Y_1 and Y_2 are the aggregate index of differentiated goods in sector 1 and sector 2, respectively. Young men can choose to be entrepreneurs in both sectors and factor can freely flow between sectors. We assume that sector 1 is a risky sector and sector 2 is a risk free sector. The production function for a representative firm in sector j is

$$y_j = \frac{z_j K_j^{\alpha_j} L_j^{1-\alpha_j}}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}}$$

where z_j takes value z^H with probability π and z^L with probability $1-\pi$ in sector 1. And z_2 is a constant in sector 2.

Let n_{1t}^m and n_{2t}^m denote the fractions of young men who choose to be entrepreneurs in sector 1 and sector 2, respectively. Similar to the benchmark analysis, the profit for entrepreneur i in sector 1 is

$$\begin{aligned}
\Pi_{it}^{1e} &= \left(p_{it}^1 - MC_{it}^1\right)\left(\frac{\phi n_{1t}^m}{1+\phi}\right)^{-1}\left(\frac{p_{it}^1}{P_t^1}\right)^{-\theta} Y_{1t} \\
&= \gamma \frac{p_{it}^1 - MC_{it}^1}{p_{it}^1}\left(\frac{\phi n_{1t}^m}{1+\phi}\right)^{-1}\left(\frac{p_{it}^1}{P_t^1}\right)^{1-\theta} P_t Y_t \\
&= \frac{\gamma\left(\frac{\phi n_{1t}^m}{1+\phi}\right)^{-1}}{\theta-1} \frac{z_{it}^{\theta-1}\left(\frac{L_{1t}}{1-\alpha_1}\left(\begin{array}{c}\frac{\pi}{z^H}\frac{\left(z^H\right)^\theta}{\pi\left(z^L\right)^\theta+(1-\pi)\left(z^H\right)^\theta} \\ +\frac{1-\pi}{z^L}\frac{\left(z^L\right)^\theta}{\pi\left(z^L\right)^\theta+(1-\pi)\left(z^H\right)^\theta}\end{array}\right)+\frac{L_{2t}}{z_2(1-\alpha_2)}\right)}{\pi\left(z^H\right)^{\theta-1}+(1-\pi)\left(z^L\right)^{\theta-1}} W_t
\end{aligned}$$

and the profit for entrepreneur k in sector 2 is

$$\begin{aligned}
\Pi_{kt}^{2e} &= (1-\gamma)\left(p_{kt}^2 - MC_{kt}^2\right)\left(\frac{\phi n_{2t}^m}{1+\phi}\right)^{-1}\left(\frac{p_{kt}^2}{P_t^2}\right)^{-\theta} Y_t \\
&= \frac{(1-\gamma)\left(\frac{\phi n_{2t}^m}{1+\phi}\right)^{-1}\left(\frac{L_{1t}}{1-\alpha_1}\left(\begin{array}{c}\frac{\pi}{z^H}\frac{\left(z^H\right)^\theta}{\pi\left(z^L\right)^\theta+(1-\pi)\left(z^H\right)^\theta} \\ +\frac{1-\pi}{z^L}\frac{\left(z^L\right)^\theta}{\pi\left(z^L\right)^\theta+(1-\pi)\left(z^H\right)^\theta}\end{array}\right)+\frac{L_{2t}}{z_2(1-\alpha_2)}\right)}{\theta-1} W_t
\end{aligned}$$

In equilibrium, given the composition of the final consumption good, the ratio of the share of good 1 in the total consumption basket to the share of good 2 in the total consumption basket is

$$\frac{P_{1t}Y_{1t}}{P_{2t}Y_{2t}} = \frac{\gamma}{1-\gamma}$$

where we can obtain the aggregate revenue income in sectors 1 and 2 similar to the benchmark analysis,

$$
\begin{aligned}
P_{1t}Y_{1t} &= \frac{\theta \frac{L_{1t}}{1-\alpha_1}\left(\frac{\pi}{z^H}\frac{\left(z^H\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}+\frac{1-\pi}{z^L}\frac{\left(z^L\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}\right)}{\theta-1}W_t \\
P_{2t}Y_{2t} &= \frac{\theta}{\theta-1}\frac{L_{2t}}{z_2\left(1-\alpha_2\right)}W_t
\end{aligned}
$$

Then

$$
\frac{\frac{L_{1t}}{1-\alpha_1}\left(\frac{\pi}{z^H}\frac{\left(z^H\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}+\frac{1-\pi}{z^L}\frac{\left(z^L\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}\right)}{\frac{L_{2t}}{z_2(1-\alpha_2)}}=\frac{\gamma}{1-\gamma}
$$

Again, if we denote the aggregate labor supply in the economy by L_t, where $L_t = L_{1t} + L_{2t}$, we can solve L_{1t} and L_{2t}, respectively.

$$
L_{1t} = \frac{\frac{\gamma}{1-\gamma}\frac{1}{z_2(1-\alpha_2)}}{\frac{\gamma}{1-\gamma}\frac{1}{z_2(1-\alpha_2)}+\frac{1}{1-\alpha_1}\left(\frac{\pi}{z^H}\frac{\left(z^H\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}+\frac{1-\pi}{z^L}\frac{\left(z^L\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}\right)}L_t \quad (4.17)
$$

$$
L_{2t} = \frac{\frac{1}{1-\alpha_1}\left(\frac{\pi}{z^H}\frac{\left(z^H\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}+\frac{1-\pi}{z^L}\frac{\left(z^L\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}\right)}{\frac{\gamma}{1-\gamma}\frac{1}{z_2(1-\alpha_2)}+\frac{1}{1-\alpha_1}\left(\frac{\pi}{z^H}\frac{\left(z^H\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}+\frac{1-\pi}{z^L}\frac{\left(z^L\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}\right)}L_t \quad (4.18)
$$

We can then rewrite the profit function of entrepreneur i in sector 1 as

$$
\Pi_{it}^{1e} = B_1\frac{\gamma\left(\frac{\phi n_{1t}^m}{1+\phi}\right)^{-1}}{\theta-1}\frac{z_{it}^{\theta-1}}{\pi\left(z^H\right)^{\theta-1}+(1-\pi)\left(z^L\right)^{\theta-1}}W_tL_t
$$

where

$$
B_1 = \frac{\frac{1}{1-\gamma}\left(\frac{\pi}{z^H}\frac{\left(z^H\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}+\frac{1-\pi}{z^L}\frac{\left(z^L\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}\right)}{\frac{\gamma}{1-\gamma}\left(1-\alpha_1\right)+z_2\left(1-\alpha_2\right)\left(\frac{\pi}{z^H}\frac{\left(z^H\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}+\frac{1-\pi}{z^L}\frac{\left(z^L\right)^\theta}{\pi(z^L)^\theta+(1-\pi)(z^H)^\theta}\right)}
$$

Since sector 2 is risk free, in equilibrium, we have

$$
\Pi_t^{2e} = W_t
$$

By (4.17) and (4.18), we have

$$B_1 L_t = \frac{\phi n_{2t}^m}{1+\phi}\frac{\theta-1}{1-\gamma} \tag{4.19}$$

In equilibrium, both labor market and capital market will clear,

$$\frac{\phi(1-n_{1t}^m-n_{2t}^m)+1}{1+\phi} = L_t \tag{4.20}$$

and

$$R_t K_t = \frac{\alpha(\theta-1)}{\theta}\left\{\begin{array}{c} \pi\frac{\phi_{t-1}n_{1t-1}^m}{1+\phi_{t-1}}s_{t-1}^{e,H}R_t\Pi_{1,t-1}^{e,H} + (1-\pi)\frac{\phi_{t-1}n_{1t-1}^m}{1+\phi_{t-1}}s_{t-1}^{e,L}R_t\Pi_{1,t-1}^{e,L} \\ +\frac{\phi_{t-1}\left(1-n_{1t-1}^m\right)}{1+\phi_{t-1}}s_{t-1}^{m,L}R_tW_{t-1} + \frac{1}{1+\phi_{t-1}}s_{t-1}^{w}R_tW_{t-1} \\ +\pi\frac{\phi n_{1t}^m}{1+\phi}\Pi_{1t}^{e,H} + (1-\pi)\frac{\phi n_{1t}^m}{1+\phi}\Pi_{1t}^{e,L} + \left(1-\frac{\phi n_{1t}^m}{1+\phi}\right)W_t \end{array}\right\} \tag{4.21}$$

By (4.19) and (4.20), we can further pin down the number of entrepreneurs in sector 2,

$$\frac{\phi n_{2t}^m}{1+\phi} = \frac{B_1\left(1-\frac{\phi n_{1t}^m}{1+\phi}\right)}{B_1+\frac{\theta-1}{1-\gamma}} \tag{4.22}$$

If there are more entrepreneurs flowing into sector 1, i.e., $\frac{\phi n_{1t}^m}{1+\phi}$ increases, the number of entrepreneurs in sector 2 will decrease. Substitute (4.22) into the profit function of firms in sector 1, we have

$$\Pi_{it}^{1e} = \frac{B_1}{B_1+\frac{\theta-1}{1-\gamma}}\frac{z_{it}^{\theta-1}}{\pi\left(z^H\right)^{\theta-1}+(1-\pi)\left(z^L\right)^{\theta-1}}\left(\frac{1+\phi}{\phi n_{1t}^m}-1\right)W_t \tag{4.23}$$

Now we can show the following proposition.

Proposition 14. *If π is small enough,*

(i) When the sex ratio is small (close to one), as the sex ratio rises, a smaller number of men will choose to become entrepreneurs in sector 1 while a larger number of men choose to become entrepreneurs in sector 2;

(ii) When the sex ratio becomes sufficiently unbalanced such that no failed entrepreneurs can get married, as the sex ratio rises, a larger fraction of men choose to be entrepreneurs

in sector 1 while a smaller number of men choose to become entrepreneurs in sector 2.

Proof. See Appendix A4.2. □

The intuition behind Proposition 14 is similar to that behind Proposition 13. A rise in the sex ratio is an adverse shock on men in the marriage market, which will stimulate men to choose more risky but potentially higher return choices. Therefore, the risky sector (sector 1) will expand relative to the risk free sector (sector 2).

Now we consider an open economy case. Consider a small open economy. We evaluate the comparative advantage of the country by comparing its relative sectoral export positions among different sectors. If there are two sectors in the country, let x_i and x_j denote the shares of sector i's export and sector j's export in the home country's total export, respectively. If

$$x_i > x_j, i \neq j$$

the country has a comparative advantage in sector i. We call the small open economy "the home country", then we can show the following proposition.

Proposition 15. *Under the same assumption in Proposition 14, (i) when the home country's sex ratio is small (close to one), as the sex ratio rises, the home country is more likely to have a comparative advantage in sector 2; (ii) when the home country's sex ratio becomes sufficiently unbalanced such that no failed entrepreneurs in sector 1 can get married, as the sex ratio rises, the home country is more likely to have a comparative advantage in sector 1.*

Proof. See Appendix A4.3. □

Proposition 15 points out that some structural factors may influence the trade pattern significantly. For instance, countries such as China have very unbalanced sex ratios, men in those countries are more likely to take the risky choices. If the turning points of the sex ratios as we discussed in Propositions 13, 14 and 15 are small (close to the balanced level), then countries with unbalanced sex ratios are very likely to have a larger risky sectors than countries with moderate sex ratios. This leads to a comparative advantage towards the risky sectors in countries with skewed sex ratios.

4.2 Empirics

We discuss two types of empirical approaches that allow us to check for plausibility and empirical importance of the theory. First, we review the evidence from China on the association between sex ratios and entrepreneurial activities. Second, we provide some cross-country evidence on the relationship between a country's sex ratio and its structure.

4.2.1 Cross-section evidence in China

The sex ratio at birth in China increased from being slightly unbalanced in 1990 to about 120 boys per 100 girls in 2007. While China is not the only economy with a high sex ratio (and a high savings rate), it is the one with the most extreme sex ratio imbalance at the moment. For this reason, it is useful to highlight two empirical patterns documented in Wei and Zhang (2010) that are most relevant for the current paper.

First, using data from two censuses of industrial firms in 1995 and 2004, Wei and Zhang (2010) find that the local sex ratio is a significant predictor of which regions are more likely to have new domestic private firms (beyond other determinants of the birth of new firms). They also compute economic impact. Using the most conservative estimate in Wei and Zhang (2010), an increase in the sex ratio by 3 basis points (e.g., from 1.08 to 1.11), which is equal to the increase in the average sex ratio from 1995 to 2004, generates an increase in the natural log number of private firms by 0.39. The actual increase in log number of firms in this period is 0.83, the rise in the sex ratio can potentially explain 47% (=0.39/0.83) of the actual increase in the number of private firms in China during this period. In other words, the economic impact of the rise in sex ratio in promoting entrepreneurial activities in rural China is potentially very big.

Second, Wei and Zhang (2010) look at the China Population 1% Survey in 2005 to find the information on firm owners, and then run a Probit test to see how the sex ratio imbalance may influence the likelihood for parents to be entrepreneurs. Across households, they find that a combination of having a son and living in a region with a skewed sex ratio significantly raises the likelihood for parents to be business owners or self-employed.

In summary, Wei and Zhang (2010) provide strong empirical evidence in China that a larger sex ratio may induce more entrepreneurial activities, which is very relevant for our current paper.

4.2.2 Cross-country evidence

We run two types of regressions in this section. First, we directly test how sex ratios and sectoral volatilities will influence the sectoral export. Second, we construct a country export volatility index by using export data and sectoral volatility data. Then we examine how the changes in countries' sex ratios will affect the country export volatilities.

Data

Koren and Tenreyro (2005) have shown that increasing levels of economic development across countries are associated with a pattern of comparative advantage towards less volatile sectors–where this volatility is measured as the aggregate sector volatility of output per worker. Assuming that the rankings on industrial volatilities do not vary much across countries, we compute a similar measure of aggregate productivity volatility from the NBER-CES Manufacturing Productivity database. The NBER-CES Manufacturing Productivity database covers annual sectoral data from 1958 to 2005. We compute the volatility of sector-level output per worker (VOL 1) by taking the standard deviation of its annual growth rate.

As a robustness check, we replicate the sectoral volatility measure as in Cunat and Melitz (2009). We use a reference country, the US, to measure the industrial volatility index. We measure differences in firm-level volatility across sectors using COMPUSTAT data from Standard & Poor's. This data covers all publicly traded firms in the US, and contains yearly sales and employment data since 1980 (the past 26 years). We use the standard deviation of the annual growth rate of firm sales (measured as year-differenced log sales) as the second industrial volatility index. Similar to Cunat and Melitz (2009), we include in our analysis all firms with at least 5 years of data (using all the data going back to 1980) and all sectors with at least 10 firms. We do not exclude the observations where the absolute value of the growth rate is above 300%. We compute the sector-level volatility as the average of the firm-level volatility measures, weighted by the firm's average employment over time.

Other sectoral characteristics such as factor intensity data in manufacturing are available over time from the NBER-CES Manufacturing Industry Database at the 4-digit US SIC level. For each sector, we measure capital intensity as capital per worker and skill intensity as the ratio of non-production wages to total wages. Again, we use the most recent data available, but also average out the data across the latest five years, in order to smooth out any small yearly fluctuations (especially for very small sectors). All measures are also aggregated to the 3-digit SIC level.

The cross-country sex ratio data can be obtained from World Fact dataset. We use the sex ratio in the group from age 0 to age 15 in the regression. We obtain the labor flexibility data from Cunat and Melitz (2009). All other country-level data can be obtained from Penn World Table 6.3. We measure capital abundance as the physical capital stock per capita. Capital stock is taken from Caselli (2005) and is constructed from the investment data reported in PWT 6.3 (based on the perpetual inventory method). Human skill abundance is calculated as the average years of schooling in the total population from Barro and Lee (2000). We use the ratio of private credit to GDP as the financial development index. In order to control the effect that major commodity exporters may have different volatilities in their exports, we construct an dummy variable–commodity exporters–in the regression.[3] Cashin et al. (2003) list a group of non-oil commodity exporters. We will include those countries and major oil exporters in this paper.

[3]We consider a country as a commodity exporter if the share of commodity exports in the country is greater than 30% of the total exports.

The country-sector exports data can be obtained from WITS database. We compute the average export between year 2004-2008 for each sector in all countries.

Estimation

We run two estimations in this section: (i) we test how sex ratios and volatilities together can influence the sectoral export, and (ii) we estimate the correlation between countries' export weighted volatilities and their sex ratios.

In the first type of estimation, we run the estimation to see how countries' sex ratios and sectoral volatilities affect the sectoral exports. We propose a nonlinear regression equation as in the following:

$$\ln(\exp_{ki}) = \begin{cases} \beta_1 \cdot sex.ratio_i \cdot VOL_k + \gamma \cdot Z_{ki} + f_i + f_k + error_{ki} & \text{if } sex.ratio_i < \phi^* \\ \beta_2 \cdot sex.ratio_i \cdot VOL_k + \gamma \cdot Z_{ki} + f_i + f_k + error_{ki} & \text{otherwise} \end{cases}$$

where the dependent variable is the sector k's log export in country i. Z_{ki} is a set of control variables used in Cunat and Melitz (2009) (see Table 3). f_i and f_k are the country- and sector-fixed effects, respectively.

In practice, the estimation is done in sequence. The value of ϕ^* is determined by a grid search. Estimation of this model can be done via maximum likelihood or sequential conditional least squares. Procedurally, we perform a grid search over possible values of ϕ^*. Starting with an initial value of ϕ^* at 1.0, the search adds 0.001 in each successive round until $\phi^* = 1.15$ (which is an upper bound of the sex ratios in our sample). Given ϕ^* in each round, we then estimate a linear equation

$$\begin{aligned} \ln(\exp_{ki}) = \; & \beta_1 \cdot sex.ratio_i \cdot VOL_k \cdot \mathbf{I}[sex.ratio_i < \phi^*] \\ & + \beta_2 \cdot sex.ratio_i \cdot VOL_k \cdot (1 - \mathbf{I}[sex.ratio_i < \phi^*]) \\ & + \gamma \cdot Z_{ki} + f_i \cdot \mathbf{I}[sex.ratio_i < \phi^*] + f_k \cdot \mathbf{I}[sex.ratio_i < \phi^*] \\ & + f_i' \cdot (1 - \mathbf{I}[sex.ratio_i < \phi^*]) + f_k' \cdot (1 - \mathbf{I}[sex.ratio_i < \phi^*]) + error_{ki} \end{aligned}$$

and collect the residuals. We then compare the sum of squared residuals for all ϕ^*s and pick the ϕ^* at which we obtain the smallest sum of squared residuals.

Table 3 shows the regression results. We find that, when a country's sex ratio is small (below the threshold ϕ^*), a rise in the sex ratio does not lead to an increase in the export in more volatile sectors (β_1s are positive or insignificant in all regressions). However, if country's sex ratio is large (above the threshold ϕ^*), a rise in the sex ratio can generate higher export in more volatile sectors (β_2s are significant in most regressions), which is consistent with our theoretical predictions. Interestingly, results in Table 3 also support the predictions from standard trade theories: the coefficient on the interaction term between log capital abundance and log sectoral capital-intensity is positive and significant, which means that capital abundant countries will export more capital-intensive goods. The coefficient on the interaction term between log skill abundance and log sectoral skill-intensity is also positive and significant, which means that skill abundant countries will export more skill-intensive goods.

Next, we construct a country export volatility index and examine how changes in the sex ratio may influence a country's export volatility. By Proposition 15, as the sex ratio in a country rises, the country may have a larger export towards the more volatile sectors or less volatile sectors, which depends on the level of the sex ratio. In other words, a country's sex ratio may nonlinearly influence the country's average export volatility. We propose a nonlinear estimation to test our theory.

$$VOL_i = \begin{cases} \alpha_1 + \beta_1 \cdot sex.ratio_i + \gamma \cdot Z_i + error_i & \text{if } sex.ratio_i < \phi^* \\ \alpha_2 + \beta_2 \cdot sex.ratio_i + \gamma \cdot Z_i + error_i & \text{otherwise} \end{cases}$$

The dependent variable, country i's export weighted volatility is computed as

$$VOL_i = \sum_k \frac{X_{ki}}{X_i} VOL_k$$

where X_{ki} and X_i denote the sector k's export and total export in country i, respectively.

VOL_k is the sectoral volatility index. We include the sex ratio (*sex.ratio*), and all control variables used in Cunat and Melitz (2009) such as labor market flexibility, log(capital abundance), skill abundance, log(real GDP per capita) and etc. (which is denoted by set Z) as the regressors. ϕ^* is the threshold of the sex ratio in Proposition 15. We will estimate parameters α, β, γ and ϕ^* by using a nonlinear least squares method.

Figures 1 and 2 show the scatter plot and the LOWESS smoother of countries' export weighted volatilities and their sex ratios, using two different measures of industrial volatilities. The patterns showed in those figures are consistent with our theoretical predictions: starting at the balanced sex ratio, as the sex ratio rises, a country's export volatility will first decrease. Once the sex ratio passes some threshold, export volatility will increase.

Table 4 shows the estimation results. In the first experiment of each regression, we include the sex ratio, log country's income and commodity exporters dummy variable as the regressors. The reason for adding commodity exporters dummy is that by looking at the sectoral volatilities, the commodity production sectors seem to have relatively higher volatility levels on average. In the second experiment, we include all other control variables in Cunat and Melitz (2009) as the regressors. Finally, we exclude the commodity exporters from our sample in the last experiment. We find that β_2s, the marginal effect on the export volatility of a rise in the sex ratio when the sex ratio is large, are all positive and half of them are significant. This means that countries like China with very unbalanced sex ratios will export goods in more volatile sectors than a country with a sex ratio around ϕ^*. This effect is also economically significant. Consider a country with both export volatility and its sex ratio around the mean level in the world, i.e., the country has an export volatility around 0.11 and a sex ratio around 1.05. If the sex ratio in the country rises to 1.13 (China's sex ratio in 2005), the export volatility of the country will increase by $0.3 \cdot (1.13 - 1.05) \simeq 0.025$, where 0.3 comes from the empirical results in Table 3 that β_2 is about 0.3 on average. This means that as the sex ratio in the country rises to an very unbalanced level, 1.13 in this case, the export volatility will increase by almost $0.025/0.11 \cdot 100\% \simeq 24\%$, which is economically significant.

We also find in Table 4 that most β_1s are negative and some of them are significant, which means that a small rise in the sex ratio from the balanced level may possibly induce countries to export more in less volatile sectors, but the effect can be very small. All these results are consistent with our theoretical predictions.

There are many caveats with the empirical patterns. First, in spite of our best efforts, there may still be potential control variables that are missing from our list. Second, the sex ratio (and some other variables) can be endogenous and/or measured with errors. This would normally call for an instrumental variable approach. At this point, we are not able to come up with convincing instrumental variables in a cross-country context. We leave this to the future research.

4.3 Conclusion

We construct a theoretical model in this paper to analyze how a major social structure changes in countries like China, namely a rise in the surplus of men relative to women, will affect the entrepreneurial activities. We find that, when the sex ratio is close to one, as the sex ratio rises, fewer men choose to become entrepreneurs which in turn leads to a reduction in the aggregate entrepreneurial activities. However, when the sex ratio becomes very unbalanced, a rise in the sex ratio will induce more men to take the risk and pursue the returns, which leads to an increase in the entrepreneurial activities in the economy. In an open economy model with two sectors, a risky sector and a risk free sector, we show that, when the home country's sex ratio is sufficiently large, it is more likely to have a comparative advantage in the risky sector.

There exists some empirical evidence that supports our theory. Wei and Zhang (2010) find that a rise in the sex ratio in China leads to more entrepreneurship. In addition to their work, we also test the relationship between exports and sex ratios. We find that (i) if the sex ratio becomes very skewed, the exports in more volatile sectors tend to rise, and (ii) a country with a very unbalanced sex ratio tends to have a large aggregate export volatility.

Both of the results are consistent with our theoretical findings.

The paper can be extended in a number of directions. First, since entrepreneurship is an important source of economic growth, we can analyze how sex ratio can influence the growth in countries like China. Second, we can investigate the welfare implications of the rise in the sex ratio. This could be very interesting because, a rise in the sex ratio is an adverse shock on men which may result in a worse situation to men. Social welfare may fall as the sex ratio rises (Du and Wei (2010)). But at the same time, a rise in the sex ratio may stimulate the economic growth by inducing more men to be entrepreneurs. This means that high welfare may not be equivalent to fast economic growth, which is very useful to discuss the real optimal policies. We leave these topics for future research.

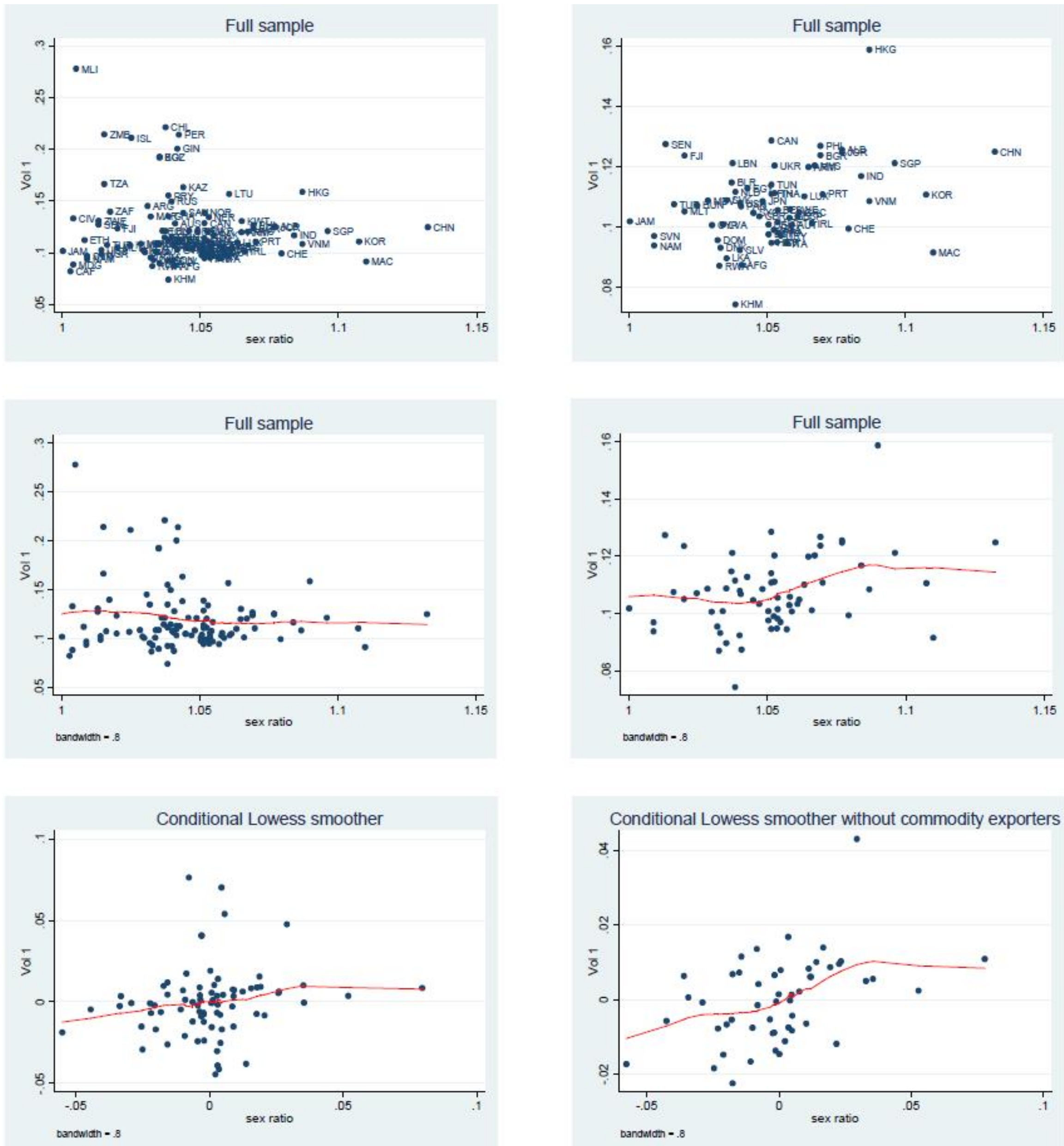

Figure 4.1: Volatility 1 (output per worker growth) vs Sex ratio, scatter plot and Lowess smoother

Notes: a. The fifth graph shows the Lowess smoother conditional on variables ln(country's income), labor market flexibility and commodity exporter dummy. We first regress both Vol 1 and sex ratios on those three variables and collect the residuals. Then we trace out the lowess smoother between the residuals. b. The sixth graph shows the Lowess smoother conditional on variables ln(country's income) and labor market flexibility after excluding commodity exporters. We first regress both Vol 1 and sex ratios on the two variables and collect the residuals. Then we trace out the lowess smoother between the residuals

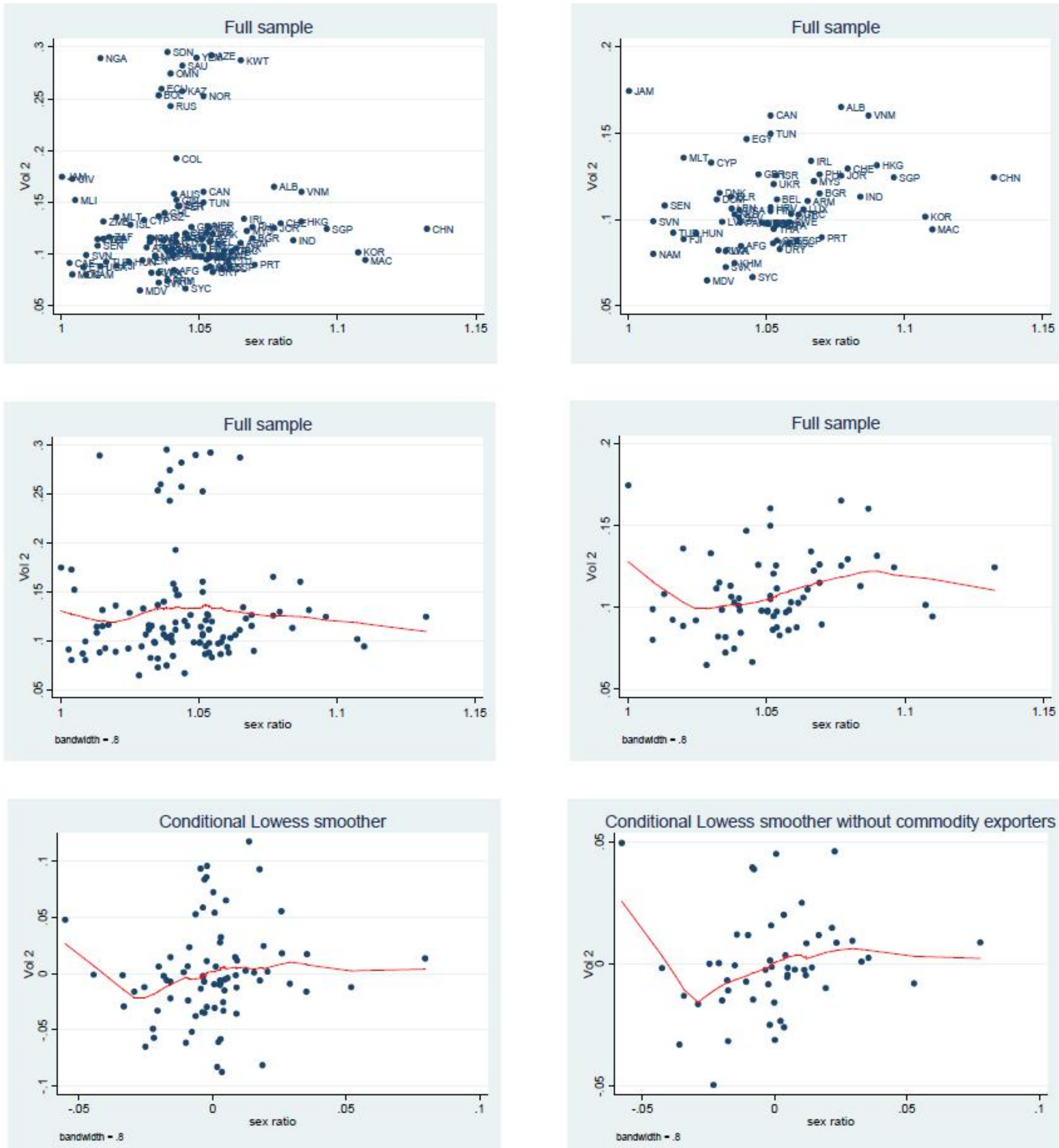

Figure 4.2: Volatility 2 (annual sales growth) vs Sex ratio, scatter plot and Lowess smoother

Notes: a. The fifth graph shows the Lowess smoother conditional on variables ln(country's income), labor market flexibility and commodity exporter dummy. We first regress both Vol 2 and sex ratios on those three variables and collect the residuals. Then we trace out the lowess smoother between the residuals. b. The sixth graph shows the Lowess smoother conditional on variables ln(country's income) and labor market flexibility after excluding commodity exporters. We first regress both Vol 2 and sex ratios on the two variables and collect the residuals. Then we trace out the lowess smoother between the residuals

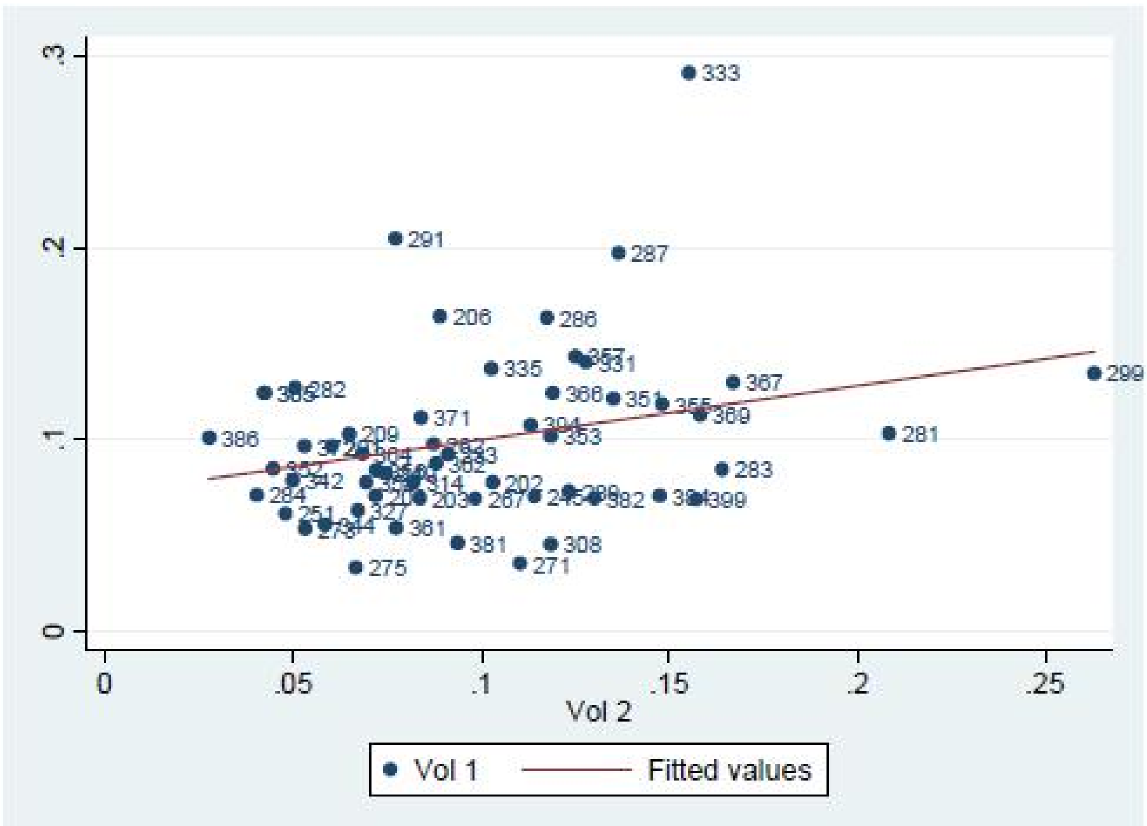

Figure 4.3: Scatter plot of two sectoral volatility measures

Table 4.1: The Ten Least and Most Volatile Sectors at the 3-Digit SIC Level, Vol1

SIC	Vol 1	# of firms	Description
275	0.033	-	Commercial Printing, Lithographic and Gravure
271	0.036	-	Newspapers: Publishing, or Publishing and Printing
308	0.046	-	Plastics Products
381	0.046	-	Detection, Navigation, and Nautical Systems and Instruments
205	0.048	-	Bread and Other Bakery Products
279	0.05	-	Typesetting, Platemaking and Related Services
359	0.051	-	Industrial and Commercial Machinery and Equipment
276	0.051	-	Manifold Business Forms
323	0.051	-	Glass Products, Made of Purchased Glass
328	0.053	-	Cut Stone and Stone Products

SIC	Vol 1	# of firms	Description
333	0.291	-	Primary Smelting and Refining of Copper, Aluminum
261	0.266	-	Pulp Mills
214	0.232	-	Secondary Smelting and Refining of Nonferrous Metals
334	0.219	-	Animal and Marine Fats and Oils, oil mills
207	0.215	-	Fertilizers, Pesticides and Agricultural Chemicals
291	0.205	-	Leather Gloves and Mittens
287	0.197	-	Cigars
315	0.193	-	Leather Gloves and Mittens
206	0.164	-	Sugar, candy and gum
286	0.163	-	Industrial Organic Chemicals

Notes: NBER-CES Manufacturing Productivity database does not report the number of firms in the industries.

Table 4.2: The Ten Least and Most Volatile Sectors at the 3-Digit SIC Level, Vol2

SIC	Vol 2	# of firms	Description
386	0.028	14	Photographic Equipment and Supplies
284	0.04	38	Perfumes,soap and Other Toilet Preparations
365	0.042	26	Household Audio and Video Equipment
352	0.045	11	Farm Machinery and Equipment
251	0.048	18	Household Furniture
342	0.05	10	Cutlery, Handtools, & Hardware
282	0.051	21	Plastics Materials, Synthetic Resins
372	0.053	33	Aircraft
273	0.053	13	Books: Publishing, or Publishing and Printing
371	0.054	80	Motor Vehicles and Passenger Car Bodies

SIC	Vol 2	# of firms	Description
131	0.296	207	Crude Petroleum and Natural Gas
299	0.262	15	Lubricating Oils and Greases
281	0.208	30	Alkalies and Chlorine & Industrial Chemicals
122	0.201	25	Bituminous Coal and Lignite Surface Mining
367	0.172	260	Electronic Components & Accessories
283	0.164	447	Medicinal Chemicals and Botanical Products
369	0.158	44	Electrical Machinery, Equipment
399	0.157	22	Linoleum, and Other Hard Surface Floor Coverings
333	0.155	13	Primary Smelting and Refining of Copper, Aluminum
355	0.148	59	Textile and woodworking machinery, Printing Machinery

Table 4.3: List of Commodity Exporters

Algeria	Angola	Argentina	Australia	Azerbaijan	Bolivia
Brazil	Burundi	Cameroon	Central African Rep	Chile	Colombia
Costa Rica	Cote d'Ivoire	Dominica	Ecuador	Ethiopia	Guatemala
Honduras	Iceland	Indonesia	Iran	Iraq	Kazakhstan
Kenya	Kuwait	Kyrgyzstan	Madagascar	Libya	Malawi
Mali	Mauritania	Mexico	Myanmar	New Zealand	Nicaragua
Niger	Nigeria	Norway	Oman	Papua New Guinea	Paraguay
Peru	Russia	Saudi Arabia	South Africa	St. Vincent & G	Sudan
Suriname	Syrian Arab Rep	Tanzania	Togo	Uganda	United Arab Emirates
Uruguay	Venezuela	Zambia	Zimbabwe		

Notes: Non-oil commodity exporters data can be obtained from Cashin et al (2004). Major oil exporters data can be obtained from US Energy Information Administration.

Table 4.4: Exports vs Volatilities, Sex Ratios

Dependent variable = log(export in sector i country k)								
Variables	Using Vol 1 measure				Using Vol 2 measure			
Vol*sex ratio (if sex ratio<ϕ*)	16.07	37.83	136.2**	36.15	108.3***	-10.82	73.56	-9.17
	(30.890)	(51.080)	(64.520)	(51.150)	(32.440)	(92.100)	(49.230)	(92.580)
Vol*sex ratio (if sex ratio>=ϕ*)	94.95*	149.82**	151.3**	143.13	417.9***	82.20*	67.63	89.16
	(55.420)	(62.940)	(68.190)	(94.120)	(140.200)	(48.210)	(48.540)	(55.370)
Vol*commodity exporter dummy	6.15***	5.01***	-	5.06***	8.08***	2.31	-	2.25
	(0.925)	(1.300)		(1.310)	(1.140)	(1.710)		(1.760)
Vol*ln(financial development)		1.22	0.732	1.22		2.41*	1.89	2.37*
		(1.140)	(1.870)	(1.140)		(1.410)	(2.190)	(1.430)
Vol*ln(capital abundance)		-3.38***	-2.63*	-3.16***		-2.19*	-2.06	-2.13
		(1.020)	(1.410)	(1.060)		(1.310)	(2.040)	(1.340)
Vol*ln(skill abundance)		19.81***	12.66	19.75***		0.507	3.11	0.576
		(4.550)	(5.830)	(4.750)		(5.870)	(7.750)	(5.920)
Vol*ln(labor flexibility)		-4.68**	-0.201	-4.88**		1.31	0.484	1.17
		(2.070)	(2.770)	(2.060)		(2.580)	(3.310)	(2.590)
ln(capital abundance)		0.161***	0.184***	0.14***		0.051	0.146***	0.028
*ln(sectoral capital intensity)		(0.026)	(0.032)	(0.028)		(0.039)	(0.045)	(0.041)
ln(skill abundance)		3.12***	3.38***	3.11***		3.68***	3.91***	3.17
*ln(sectoral skill intensity)		(0.308)	(0.376)	(0.311)		(0.516)	(0.612)	(0.521)
ln(labor flexibility)		-0.041	-0.027	-0.01		-0.068	-0.049	-0.031
* ln(sectoral capital intensity)		(0.082)	(0.011)	(0.083)		(0.117)	(0.139)	(0.119)
ϕ*	1.062	1.064	1.024	1.065	1.082	1.049	1.048	1.047
Country fixed effects	Y	Y	Y	Y	Y	Y	Y	Y
Sector fixed effects	Y	Y	Y	Y	Y	Y	Y	Y
Observations	13,356	6,647	4,165	6,510	5,808	2,706	1,693	2,651
R-squared	0.77	0.78	0.79	0.78	0.76	0.8	0.81	0.8

Notes: a. Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. b. The third column in each experiment shows the regression results after excluding the commodity exporters. c. The fourth column in each experiment shows the regression results after excluding China from the sample.

Table 4.5: Volatilities vs Sex Ratios

Dependent variable = country's export volatility								
Variables	Vol 1				Vol 2			
ϕ*	1.05***	1.05***	1.05***	1.05***	1.002***	1.002***	1.02***	1.02***
	(0.015)	(0.011)	(0.031)	(0.025)	(0.001)	(0.001)	(0.011)	(0.009)
sex ratio (if < ϕ*)	-0.24	-0.666	-0.059	-0.072	-31.88***	-32.46	-3.60***	-4.13
	(0.282)	(0.441)	(0.457)	(0.484)	(0.066)	(0.055)	(0.098)	(0.087)
sex ratio (if >=ϕ*)	0.324	0.46	0.325	0.434	0.420*	0.336*	0.262	0.298
	(0.236)	(0.324)	(0.270)	(0.323)	(0.239)	(0.188)	(0.217)	(0.259)
ln(real GDP per capita)	0.003	-0.001	-0.01	-0.015	0.005	0.001	-0.011	-0.013
	(0.003)	(0.003)	(0.015)	(0.016)	(0.005)	(0.012)	(0.021)	(0.020)
Commodity exporters	0.039***	0.038***	0.034***	0.035***	0.064***	0.038***	0.038***	0.042***
	(0.006)	(0.010)	(0.011)	(0.011)	(0.011)	(0.010)	(0.013)	(0.011)
ln(capital abundance)		0.003	0	0.002		0.003	-0.001	0.003
		(0.010)	(0.012)	(0.013)		(0.010)	(0.016)	(0.016)
Skill abundance		0.006	0.013	0.012		0.006	-0.004	-0.004
		(0.014)	(0.013)	(0.014)		(0.014)	(0.019)	(0.017)
labor flexibility			0	0			0.001	0
			(0.000)	(0.000)			(0.001)	(0.000)
Financial development			0	0			0	0
			(0.001)	(0.000)			(0.001)	(0.000)
Observations	108	67	50	49	108	67	50	49
R-squared	0.29	0.27	0.37	0.37	0.26	0.28	0.29	0.36

Notes: a. Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. b. The fourth column in each experiment shows the regression results after excluding China from the sample.

Table 4.6: Top ten export sectors, China

SIC 3	Description	share in total export	Vol 1	Vol 2
357	Electronic Computers	0.105	0.145	0.125
366	Communications Equipment	0.062	0.124	0.119
367	Electronic Capacitors	0.05	0.13	0.172
331	Steel Works, Electrometallurgical Products	0.049	0.14	0.128
365	Household Audio and Video Equipment	0.041	0.124	0.042
342	Cutlery, Hand and Edge Tools	0.023	0.079	0.05
232	Men's and Boys' Clothing	0.022	0.091	.
394	Games, Toys, Dolls, and Children's Vehicles	0.021	0.108	0.113
233	Women's, Misses', and Juniors' Suits, Skirts, and Coats	0.019	0.092	0.091
333	Primary Smelting and Refining of Copper, Aluminum	0.018	0.291	0.155

Table 4.7: Top ten export sectors, Macau

SIC 3	Description	share in total export	Vol 1	Vol 2
232	Men's and Boys' Clothing	0.265	0.091	.
236	Girls', Children's, and Infants' Dresses, Blouses, and Shirts	0.208	0.102	.
231	Men's and Boys' Suits, Coats, and Overcoats	0.208	0.087	.
233	Women's, Misses', and Juniors' Suits, Skirts, and Coats	0.104	0.092	0.091
234	Household Audio and Video Equipment	0.052	0.076	.
211	Cutlery, Hand and Edge Tools	0.024	0.107	.
367	Electronic Capacitors	0.019	0.13	0.172
222	Broadwoven Fabric Mills, Manmade Fiber and Silk	0.014	0.076	.
365	Household Audio and Video Equipment	0.013	0.124	0.042
283	Medicinal Chemicals and Botanical Products	0.009	0.085	0.164

Table 4.8: Top ten export sectors, Korea

SIC 3	Description	share in total export	Vol 1	Vol 2
373	Ship and Boat Building and Repairing	0.092	0.088	.
132	Natural Gas Liquids	0.083	.	.
371	Motor Vehicles	0.079	0.111	0.054
366	Communications Equipment	0.079	0.124	0.119
367	Electronic Capacitors	0.077	0.13	0.172
331	Steel Works, Electrometallurgical Products	0.054	0.14	0.128
282	Plastics Materials, Synthetic Resins	0.043	0.127	0.051
342	Cutlery, Hand and Edge Tools	0.043	0.079	0.05
352	Farm Machinery and Equipment	0.029	0.085	0.045
353	Construction Machinery and Equipment	0.027	0.102	0.118

Table 4.9: Top ten export sectors, Singapore

SIC 3	Description	share in total export	Vol 1	Vol 2
367	Electronic Capacitors	0.225	0.13	0.172
132	Natural Gas Liquids	0.176	.	.
357	Electronic Computers	0.136	0.145	0.125
355	Textile and woodworking machinery, Printing Machinery	0.049	0.118	0.148
366	Communications Equipment	0.029	0.124	0.119
353	Construction Machinery and Equipment	0.027	0.102	0.118
282	Plastics Materials, Synthetic Resins	0.021	0.127	0.051
286	Industrial Organic Chemicals	0.02	0.163	0.117
369	Electrical Machinery, Equipment	0.018	0.113	0.158
365	Household Audio and Video Equipment	0.017	0.124	0.042

Table 4.10: Top ten export sectors, Vietnam

SIC 3	Description	share in total export	Vol 1	Vol 2
131	Crude Petroleum and Natural Gas	0.155	.	0.296
204	Flour and Other Grain Mill Products	0.05	0.116	.
314	Footwear	0.045	0.077	0.082
232	Men's and Boys' Clothing	0.043	0.091	.
233	Women's, Misses', and Juniors' Suits, Skirts, and Coats	0.039	0.092	0.091
259	Furniture and Fixtures	0.037	0.07	.
231	Men's and Boys' Suits, Coats, and Overcoats	0.031	0.087	.
302	Rubber and Plastics Footwear	0.028	0.093	.
206	Sugar, candy and gum	0.023	0.164	0.089
123	Anthracite Mining	0.022	.	.

Table 4.11: Top ten export sectors, Hong Kong

SIC 3	Description	share in total export	Vol 1	Vol 2
333	Primary Smelting and Refining of Copper, Aluminum	0.302	0.291	0.155
231	Men's and Boys' Suits, Coats, and Overcoats	0.068	0.087	.
391	Silverware, Plated Ware, and Stainless Steel Ware	0.068	0.079	.
366	Communications Equipment	0.063	0.124	0.119
283	Medicinal Chemicals and Botanical Products	0.034	0.085	0.164
367	Electronic Capacitors	0.028	0.13	0.172
232	Men's and Boys' Clothing	0.018	0.091	.
236	Girls', Children's, and Infants' Dresses, Blouses, and Shirts	0.012	0.102	.
282	Plastics Materials, Synthetic Resins	0.012	0.127	0.051
233	Women's, Misses', and Juniors' Suits, Skirts, and Coats	0.012	0.092	0.091

Bibliography

[1] Aghion, Philippe, Bacchetta, Philippe, Ranciere, Romain and Rogoff, Kenneth (2008). Exchange Rate Volatility and Productivity Growth: The Role of Financial Development. Journal of Monetary Economics, vol. 56(4), pages 494-513.

[2] Allen, Franklin, Jun Qian, and Meijun Qian, 2005, "Law, Finance, and Economic Growth in China," *Journal of Financial Economics*, 77(1): 57-116.

[3] Antràs, Pol (2004). Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution. Harvard University, mimeo.

[4] Alesina, Alberto, and Robert J. Barro (2002). Currency Unions. Quarterly Journal of Economics, 117 (2), May, pp. 409-436.

[5] Babecký, Jan, Philip Du Caju, Kosma, Theodora, Lawless, Martina, Messina, Julián and Rõõm, Tairi, (2009), Downward Nominal and Real Wage Rigidity: Survey Evidence from European Firms. European Central Bank working paper

[6] Bacchetta, Philippe and Eric van Wincoop, (2000). Does Exchange-Rate Stability Increase Trade and Welfare? American Economic Review 90, 1093-1109.

[7] Becker, G. S. (1973). "A Theory of Marriage: Part I," *Journal of Political Economy* 81(4): 813-46.

[8] Becker, G. S. (1974). "A Theory of Marriage: Part II," *Journal of Political Economy* 82(2, Part II): S11-S26.

[9] Becker Gary S., 1991, A Treatise on the Family. Enlarged Edition. Cambridge, Mass. Harvard University Press.

[10] Benigno, Pierpaolo (2004). Optimal Monetary Policy in a Currency Area.? Journal of International Economics 63 (2), pp. 293-320

[11] Bergin, Paul R. and Lin, Ching-Yi, (2008). Exchange Rate Regimes and the Extensive Margin of Trade. NBER working paper W14126

[12] Betts, Caroline and Michael B. Devereux, (1996). The Exchange Rate in a Model of Pricing-to-market, European Economic Review 40, 1007-1021.

[13] Betts, Caroline and Michael B. Devereux, (2000). Exchange-rate Dynamics in a Model of Pricing-to-market, Journal of International Economics 50, 215-244.

[14] Bhaskar, V, 2011, "Parental Sex Selection and Gender Balance," American Economic Journal: Microeconomics, 3(1): 214–44.

[15] Blanchard, Oliver and Gali, Jordi, (2005). Real Wage Rigidities and the New Keynesian Model, Massachusetts Institute of Technology Department of Economics Working Paper Series

[16] Blanchard, Olivier J. and Francesco Giavazzi, 2005. "Rebalancing Growth in China: A Three-Handed Approach." MIT Department of Economics Working Paper No. 05-32.

[17] Blanchflower, D.G., Oswald, A.J., 2004. "Well-being over time in Britain and the USA." Journal of Public Economics 88 (7–8), 1359–1386.

[18] Browning, M., F. Bourguignon, P.-A. Chiappori, and V. Lechene (1994): "Income and Outcomes: A Structural Model of Intrahousehold Allocation," *Journal of Political Economy*, 102, 1067–1096.

[19] Browning, M. and P.-A. Chiappori (1998): "Efficient Intra-household Allocations: A General Characterization and Empirical Tests," *Econometrica*, 66, 1241–1278.

[20] Browning, M., P.-A. Chiappori, and V. Lechene (2006): "Collective and Unitary Models: A Clarification," *Review of Economics of the Household*, 4, 5–14.

[21] Burstein, Ariel, Martin Eichenbaum and Sergio Rebelo, 2005, "Large Devaluations and the Real Exchange Rate," *Journal of Political Economy*, 113: 742-784.

[22] Burstein, Ariel, Joao Neves, and Sergio Rebelo, 2003, "Distribution Costs and Real Exchange-Rate Dynamics During Exchange-Rate-Based Stabilizations," *Journal of Monetary Economics*, 50: 1189—1214.

[23] Caballero, Ricardo, Emmanuel Farhi, and Pierre-Olivier Gourinchas, 2008, "An Equilibrium Model of 'Global Imbalances' and Low Interest Rates," American Economic Review, 98(1): 358-393.

[24] Carroll, Christopher D., Overland, Jody R. and Weil, David N., 2008, "Saving and Growth with Habit Formation," Working Paper No 95-42.

[25] Campbell, C. M. and Kamlani, K. S. (1997). The Reasons for Wage Rigidity: Evidence from a Survey of Firms, Quarterly Journal of Economics, 112(3), pp. 759–89.

[26] Calvo, Guillermo A. and Carmen M. Reinhart, (2002). Fear Of Floating, Quarterly Journal of Economics, v107(2,May), 379-408

[27] Chamon, Marcos, and Eswar Prasad, 2008, "Why Are Saving Rates of Urban Households in China Rising?" *American Economic Journal – Macroeconomics*, forthcoming.

[28] Chari, V.V.; Patrick J. Kehoe; and, Ellen McGrattan, (1997). Monetary Policy and the Real Exchange Rate in Sticky Price Models of the International Business Cycle, National Bureau of Economic Research, working paper no. 5876.

[29] Chari, V.V., Patrick J. Kehoe, and Ellen McGrattan, 2002, "Can Sticky Prices Generate Volatile and Persistent Real Exchange Rates?," *Review of Economic Studies* 69, 633-63

[30] Chinn, Daniel M., and Preston J. Miller, (1998). Fixed vs. floating rates: A Dynamic General Equilibrium Analysis, European Economic Review 42, 1221-1249.

[31] Chinn, Daniel M., and Wei, S. J, (2008). A Faith-based Initiative: Does a Flexible Exchange Rate Regime Really Facilitate Current Account Adjustment?, NBER working paper, W14420

[32] Clarida, Richard, Jordi Gali and Mark Gertler, (2001). Optimal Monetary Policy in Open vs. Closed Economies: An Integrated Approach. American Economic Review Papers and Proceedings, 91 (2), May, pp. 248-252.

[33] Richard Clarida, Jordi Gali, and Mark Gertler, (2002). A Simple Framework for International Monetary Policy Analysis. Journal of Monetary Economics 49: 879-904.

[34] G. Corneo and Olivier Jeanne, 1999, "Social Organization in an Endogenous Growth Model", *International Economic Review* 40, pp. 711-25

[35] Cole, Harold L., George J. Mailath, and Andrew Postlewaite, 1992, "Social Norms, Savings Behavior, and Growth," Journal of Political Economy, 100(6): 1092-1125, December.

[36] Corsetti, Giancarlo, and Paolo Pesenti (2001). Welfare and Macroeconomic Interdependence. Quarterly Journal of Economics, 116 (2), May, pp. 421- 446.

[37] Corsetti, Giancarlo, and Paolo Pesenti (2005). International Dimensions of Optimal Monetary Policy. Journal of Monetary Economics.

[38] Corsetti, Giancarlo, Dedola, Luca and Leduc, Sylvain, (2008). DSGE models of High exchange rate volatility and low pass-through. Journal of Monetary Economics.

[39] Cunat, Alejandro, and Melitz, Marc, 2009, "Volatility, Labor Market Flexibility and Comparative Advantage", *Journal of the European Economics Association*

[40] Dellas, Harris and Tavlas, George, (2005). Wage Rigidity and Monetary Union. Economic Journal.

[41] Devereux, Michael B., and Charles Engel (2003). Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility? Review of Economic Studies 70, pp. 765-783.

[42] Devereux, Michael B.; Charles Engel; and Peter E. Stoorgard (2004). Endogenous Exchange Rate Pass-Through when Nominal Prices are Set in Advance, Journal of International Economics 63, pp. 263-291.

[43] Dickens, W. T. and Goette, L. (2006). Estimating Wage Rigidity for the International Wage Flexibility Project. Mimeo, Brookings Institution.

[44] Donni, Olivier, 2006, "The Intrahousehold Allocation of Private and Public Consumption: Theory and Evidence from U.S. Data," *IZA DP no.* 2137

[45] Du Caju, P., Fuss, C. and Wintr, L. (2009). Understanding Sectoral Differences in Downward Real Wage Rigidity: Workforce Composition, Competition, Technology and Institutions. ECB Working Paper, No. 1006.

[46] Du Caju P., Fuss, C. and Wintr, L. (2007). Downward Wage Rigidity for Different Workers and Firms: An Evaluation for Belgium Using the IWFP Methodology. ECB Working Paper, No. 840.

[47] Ebenstein, Avraham, Hongbin Li, and Lingsheng Meng, 2010, "The Impact of Ultrasound Technology on the Status of Women in China," working paper, Hebrew University and Tsinghua University.

[48] Eeckhout, Jan, 2000, "On the Uniqueness of Stable Marriage Matchings," Economic Letters, Vol. 69, No. 1, 1-8.

[49] Edlund, Lena, Hongbin Li, Junjian Yi, and Junsen Zhang, 2008, "More Men, More Crime: Evidence from China's One-Child Policy," *IZA DP no.* 3214.

[50] Erceg Christopher, Dale Henderson, and Andrew Levin, (2000). Optimal Monetary Policy with Staggered Wage and Price Contracts. Journal of Monetary Economics, 46(2), 281-313.

[51] Frankel, Jeffrey A., and Andrew K. Rose (1998). The Endogeneity of the Optimum Currency Area Criteria. Economic Journal 108 (449), July, pp. 1009-25.

[52] Frankel, Jeffrey A., and Wei S. J., (2007). Assessing China's Exchange Rate Regime. NBER working paper W13100

[53] Friedman, Milton (1953). The Case for Flexible Exchange Rates,? reprinted in Essays in Positive Economics, Chicago, IL: University ofChicago Press, pp. 157-203.

[54] Gale, D., and L. S. Shapley, 1962, "College Admissions and the Stability of Marriage", American Mathematical Monthly, 69: 9-14.

[55] Gali, Jordi, (2010). The Return of the Wage Phillips Curve. CREI working paper.

[56] Hopkins, Ed, 2009, "Job Market Signaling of Relative Position, or Becker Married to Spence," working paper, University of Edinburgh.

[57] Hopkins, Ed, and Tatiana Kornienko, 2004, "Running to Keep in the Same Place: Consumer Choice as a Game of Status," *American Economic Review* 94(4): 1085-1107.

[58] Ju, Jiandong, and Shang-Jin Wei, 2006, "A Solution to Two Paradoxes of International Capital Flows," NBER working paper 12668.

[59] Ju, Jiandong, and Shang-Jin Wei, 2008, "When is the Quality of Financial Institutions a Source of Comparative Advantage," NBER working paper 13984, and forthcoming, *Journal of International Economics.*

[60] Ju, Jiandong, and Shang-Jin Wei, 2010, "Domestic Institutions and the Bypass Effect of Financial Globalization," *American Economic Journal: Economic Policy*, 2(4): 173–204.

[61] Kenen, Peter (1969). The Theory of Optimum Currency Areas: An Eclectic View, reprinted in Exchange Rates and the Monetary System: Selected Essays of Peter B. Kenen, Aldershot: Elgar, 1994, pp. 3-22.

[62] Kimball, Miles S, 1990. "Precautionary Saving in the Small and in the Large," *Econometrica*, 58(1): 53-73.

[63] Levy-Yeyati, E. and Sturzenegger, F., (2003). To Float or to Fix: Evidence on the Impact of Exchange Rate Regimes on Growth. American Economic Review 93, 1173-1193.

[64] Levy-Yeyati, E., Sturzenegger, F. and Reggio, I., (2004), On the Endogeneity of Exchange Rate Regimes. mimeo.

[65] Lucas, Robert E. (1969). Labor-Capital Substitution in U.S. Manufacturing, in *The Taxation of Income from Capital*, edited by Arnold Harberger and Martin J. Bailey, Washington, DC: The Brookings Institution, pp. 223-274.

[66] Maddala, G. S. (1965). Differential Industry Effects and Differential Factor Effects of Technological Change, memo 36, Research Center in Economic Growth, Stanford University.

[67] McKinnon, Ronald (1963). Optimum Currency Areas. American Economic Review 53, September, pp. 717-724.

[68] Mendoza, Enrique G., Vincenzo Quadrini, Jose-Victor Rios-Rull, 2007, "Financial Integration, Financial Deepness and Global Imbalances," NBER Working Papers 12909.

[69] Modigliani, Franco, 1970, "The Life Cycle Hypothesis of Saving and Intercountry Differences in the Saving Ratio," in W.A. Elits, M.F. Scott, and J.N. Wolfe, eds., *Growth and Trade, Essays in Honor of Sir Roy Harrod,* Oxford University Press.

[70] Modigliani, Franco, and Shi Larry Cao, 2004, "The Chinese Saving Puzzle and the Life Cycle Hypothesis," *Journal of Economic Literature*, 42: 145-170.

[71] Mundell, Robert (1961). A Theory of Optimum Currency Areas. American Economic Review 51, September, pp. 657-664.

[72] Obstfeld, Maurice, and Kenneth Rogoff (1996). Foundations of International Macroeconomics. Cambridge, MA: MIT Press.

[73] Obstfeld, Maurice, and Kenneth Rogoff (2000). New Directions for Stochastic Open Economy Models. Journal of International Economics 50 (1), February, pp. 117-153.

[74] Peters, M., and A. Siow, 2002, "Competing Pre-marital Investments," Journal of Political Economy 113:1163-1216.

[75] Qian, Nancy, 2008, “Missing Women and the Price of Tea in China: The Effect of Sex-specific Earnings on Sex Imbalance,” *Quarterly Journal of Economics*, 123(3): 1251-1285.

[76] Razin, Ofair, and Susan M. Collins, 1997, "Real Exchange Rate Misalignments and Growth," Georgetown University.

[77] Reinhart, C. M. and Rogoff, K. S., (2004). The Modern History of Exchange Rate Arrangements: A Reinterpretation. Quarterly Journal of Economics 119(1):1-48, February.

[78] Rodrik, Dani, 2008, "The Real Exchange Rate and Economic Growth," *Brookings Papers on Economic Activity*, 2008:2.

[79] Rose, Andrew, and Engel, Charles (2000). Currency Unions and International Integration. National Bureau of Economic Research Working Paper No. 7872, September.

[80] Roth, A. E.,1984, "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory," *Journal of Political Economy*, 92: 991–1016.

[81] Roth, A. E., and Sotomayor, M. A. O., 1990, *Two-sided Matching: A Study in Game-theoretic Modeling and Analysis,* Cambridge University Press.

[82] Wei, Shang-Jin, and Xiaobo Zhang, 2009, “The Competitive Saving Motive: Evidence from Rising Sex Ratios and Savings Rates in China,” *NBER working paper* 15093, June, and forthcoming, *Journal of Political Economy.*

[83] Wei, Shang-Jin, and Xiaobo Zhang, 2011, "Sex Ratios, Entrepreneurship, and Economic Growth in the People’s Republic of China", NBER working paper #16800.

[84] Woodford, Michael (2003). Interest and Prices. Foundations of a Theory of Monetary Policy, Princeton University Press

Appendices

Appendices to Chapter 1

A1.1. Proof of Lemma 1

Proof. Under the assumption that ω is close to zero, we can rewrite the optimal prices as

$$P_{Ht}^{*} = \frac{\theta}{\theta - 1} \frac{E_{t-1}\left[Q_{t-1,t} MC_{Ht}\left(C_{Tt}^{*} + I_{t}^{*}\right)\right]}{E_{t-1}\left[Q_{t-1,t} S_{t}\left(C_{Tt}^{*} + I_{t}^{*}\right)\right]} \tag{4.24}$$

In period t, capital market clears in the foreign country, then

$$I_{t-1}^{*} = \alpha_{F}^{*}\left(\frac{W_{t}^{*}}{r_{t}^{*} P_{t}^{*}}\right)^{1-\alpha_{F}^{*}} \frac{\eta P_{T,t}^{*}\left(C_{Tt}^{*} + I_{t}^{*}\right)}{P_{Ft}^{*}} + \alpha_{N}^{*}\left(\frac{W_{t}^{*}}{r_{t}^{*} P_{t}^{*}}\right)^{1-\alpha_{N}} \frac{\gamma P_{t}^{*} C_{t}^{*}}{P_{Nt}^{*}}$$

We multiply each side of the equation above by $Q_{t-1,t}^{*} r_{t}^{*} P_{t}^{*}$ and we can obtain

$$E_{t-1}\left[Q_{t-1,t}^{*} r_{t}^{*} P_{t}^{*}\right] I_{t-1}^{*} = \frac{\theta - 1}{\theta} \alpha_{F}^{*} \eta E_{t-1}\left[Q_{t-1,t}^{*} P_{T,t}^{*}\left(C_{Tt}^{*} + I_{t}^{*}\right)\right] + \alpha_{N}^{*} E_{t-1}\left[Q_{t-1,t}^{*} P_{t}^{*} C_{t}^{*}\right]$$

By (1.4), we have

$$\frac{P_{T,t-1}^{*} I_{t-1}^{*}}{P_{t-1}^{*} C_{t-1}^{*}} = \frac{\theta - 1}{\theta} \alpha_{F}^{*} \eta E_{t-1}\left[\gamma + \frac{P_{T,t}^{*} I_{t}^{*}}{P_{t}^{*} C_{t}^{*}}\right] + \alpha_{N}^{*}$$

Iterate forward, we obtain

$$\frac{P_{T,t-1}^{*} I_{t-1}^{*}}{P_{t-1}^{*} C_{t-1}^{*}} = \gamma \sum_{k=1}^{\infty}\left[\frac{\theta - 1}{\theta} \alpha_{F}^{*} \eta\right]^{k} + \alpha_{N}^{*} + \lim_{k \to \infty}\left[\frac{\theta - 1}{\theta} \alpha_{F}^{*} \eta\right]^{k+1} E_{t-1}\left[\frac{P_{T,t+k}^{*} I_{t+k}^{*}}{P_{t+k}^{*} C_{t+k}^{*}}\right]$$

In this model, $\frac{P^*_{T,t+k}I^*_{t+k}}{P^*_{t+k}C^*_{t+k}}$ is bounded, and notice that $0 < \frac{\theta-1}{\theta}\alpha^*_F\eta < 1$, then as k goes to infinity, the second term on the right hand side will converge to zero. As a result, $\frac{P^*_{T,t}I^*_t}{P^*_tC^*_t}$ is equal to a constant, $\frac{\gamma\frac{\theta-1}{\theta}\alpha^*_F\eta}{1-\frac{\theta-1}{\theta}\alpha^*_F\eta}$.

Using this result in (4.24), we have

$$P^*_{Ht} = \frac{\theta}{\theta-1}\frac{E_{t-1}\left[Q_{t-1,t}MC_{Ht}P^*_tC^*_t\right]}{E_{t-1}\left[Q_{t-1,t}S_tP^*_tC^*_t\right]}$$

By (1.5), we can further rewrite the optimal prices as

$$\begin{aligned} P^*_{Ht} &= \frac{\theta}{\theta-1}E_{t-1}\left[MC_{Ht}\frac{P^*_tC^*_t}{P_tC_t}\right] \\ &= \frac{\theta}{\theta-1}E_{t-1}\left[\frac{MC_{Ht}}{S_t}\right] \end{aligned}$$

which completes the proof. □

A1.2. Proof of Proposition 1

Proof. (i) We approximate the optimal price in the home produced tradable good sector around $\varepsilon_t = \varepsilon^*_t = 0$, and we obtain

$$\begin{aligned} P^*_{Ht} &= \bar{P}^*_H E_{t-1}\left[\exp\left\{\varepsilon^*_t - \lambda\varepsilon_t + \frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}\frac{\widehat{r_tP_t}}{W_t}\right\}\right] \\ &= \bar{P}^*_H E_{t-1}\left[\exp\left\{\varepsilon^*_t - \lambda\varepsilon_t + \frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}\begin{pmatrix} m_H\left(\varepsilon^*_t - \psi\widehat{P^*_{Ht}}\right) \\ +m_N\left(\varepsilon_t - \widehat{P_{Nt}}\right) \\ -\left(m_H+m_N\right)\widehat{K_t} \end{pmatrix}\right\}\right] \end{aligned}$$

We can rewrite the equation above as

$$\begin{aligned}
&\left(1+\psi\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)\widehat{P_{Ht}^*}+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N\widehat{P_{Nt}}\\
=&\left[\left(1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)^2+\left(\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\right)^2\right]\frac{\sigma^2}{2}\\
&-\left(m_H+m_N\right)\widehat{K_t}
\end{aligned}$$

Similarly, for the optimal price set in the nontradable good sector, we have

$$P_{Nt}=\frac{\theta}{\theta-1}E_{t-1}\left[\left[(1-\alpha_N)\left(\frac{W_t}{A_t}\right)^{1-\eta}+\alpha_N\left(r_tP_t\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}\right]$$

and

$$\begin{aligned}
&\psi\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\widehat{P_{Ht}^*}+\left(1+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)\widehat{P_{Nt}}\\
=&\left[\left(\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)^2+\left(1-\lambda+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)^2\right]\frac{\sigma^2}{2}\\
&-\left(m_H+m_N\right)\widehat{K_t}
\end{aligned}$$

Under a flexible exchange rate regime,

$$\begin{aligned}
\left(\widehat{P_{Ht}^*}\right)^{flexible}=&\frac{\sigma^2}{2}\frac{\left[\begin{array}{c}\left(1+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)\left[\begin{array}{c}\left(1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)^2\\+\left(\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\right)^2\end{array}\right]\\-\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N\left[\begin{array}{c}\left(\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)^2\\+\left(1-\lambda+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)^2\end{array}\right]\end{array}\right]}{\det\left(\Omega\right)}\\
&-\frac{\sigma^2}{2}\det\left(\Omega\right)^{-1}\left(m_H+m_N\right)\widehat{K_t}
\end{aligned}$$

and

$$\left(\widehat{P_{Nt}}\right)^{flexible} = \frac{\sigma^2}{2}\frac{\left[\begin{array}{c}\left(1+\psi\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)\left[\begin{array}{c}\left(\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)^2\\+\left(1-\lambda+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)^2\end{array}\right]\\-\psi\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\left[\begin{array}{c}\left(1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)^2\\+\left(\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\right)^2\end{array}\right]\end{array}\right]}{\det(\Omega)} - \frac{\sigma^2}{2}\det(\Omega)^{-1}(m_H+m_N)\widehat{K_t}$$

where

$$\det(\Omega) = 1+\psi\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N>0$$

Similarly, under a fixed exchange rate regime, we have

$$\left(\widehat{P^*_{Ht}}\right)^{fixed} = \frac{\sigma^2}{2}\frac{\left[\begin{array}{c}\left(1+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)\left(\begin{array}{c}1-\lambda+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\\+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N\end{array}\right)^2\\-\left(\begin{array}{c}1-\lambda+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\\+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\end{array}\right)^2\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N\end{array}\right]}{\det(\Omega)} - \frac{\sigma^2}{2}\det(\Omega)^{-1}(m_H+m_N)\widehat{K_t}$$

and

$$\left(\widehat{P_{Nt}}\right)^{fixed} = \frac{\sigma^2}{2}\frac{\left[\begin{array}{c}\left(1+\psi\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)\left(\begin{array}{c}1-\lambda+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\\+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\end{array}\right)^2\\-\left(\begin{array}{c}1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\\+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\end{array}\right)^2\psi\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\end{array}\right]}{\det(\Omega)} - \frac{\sigma^2}{2}\det(\Omega)^{-1}(m_H+m_N)\widehat{K_t}$$

By the budget constraint of the households, we have

$$\begin{aligned} & C_t + \frac{P_{inv,t}}{P_t} I_t + \frac{B_t}{P_t} + \frac{S_t B_t^*}{P_t} \\ = \; & \frac{S_t P_{Ht}^* Y_{Ht}^*}{P_t} + \frac{P_{Nt} C_{Nt}}{P_t} + \frac{(1+i_{t-1})B_{t-1} + (1+i_{t-1}^*) S_t B_{t-1}^*}{P_t} \end{aligned}$$

If we assume in period $t-1$, $B_{t-1} = B_{t-1}^* = 0$, then

$$\begin{aligned} & \frac{M_t}{\chi} \frac{i}{1+i} + P_{inv,t} I_t + S_t B_t^* + B_t \\ = \; & \frac{M_t}{\chi} \frac{i}{1+i} \left[\left(\frac{P_{Ht}^*}{P_{Tt}^*} \right)^{-\psi} \frac{(\gamma + d)}{P_{Tt}^*} + (1-\gamma) \right] \end{aligned}$$

then

$$K_{t+1} = \frac{\frac{M_t}{\chi} \frac{i}{1+i} \left(\left(\frac{P_{Ht}^*}{P_{Tt}^*} \right)^{-\psi} \frac{(\gamma+d)}{P_{Tt}^*} - \gamma \right) - B_t - S_t B_t^*}{P_{Tt}}$$

The similar condition holds for the foreign capital stocks.

By the market clearing condition in the bonds market, $\int B_t = \int B_t^* = 0$, we have

$$\begin{aligned} \omega \frac{P_{Tt-1} K_t}{S_{t-1}} + (1-\omega) P_{Tt-1}^* K_t^* \; = \; & \omega \frac{M_{t-1}^*}{\chi} \frac{i}{1+i} (\gamma + d) \left(\frac{P_{Ht-1}^*}{P_{Tt-1}^*} \right)^{-\psi} \\ & + (1-\omega)(\gamma + d) \frac{M_{t-1}^*}{\chi} \frac{i}{1+i} \end{aligned}$$

then

$$K_t = \frac{1}{\chi} \frac{i}{1+i} \frac{M_{t-1}}{P_{Tt-1}} \left[(\gamma + d) \left(\frac{P_{Ht-1}^*}{P_{Tt-1}^*} \right)^{-\psi} + \frac{1-\omega}{\omega} \gamma \right]$$

The capital stock in period t does not depend on the choice of exchange rate regime from period $t-1$ to period t. Therefore, $\widehat{K_t}$ takes the same value under two exchange rate regimes and we can obtain

$$\begin{aligned}
& \left(\widehat{P_{Ht}^{*}}\right)^{fixed}-\left(\widehat{P_{Ht}^{*}}\right)^{flexible} \\
= \quad & \sigma^{2}\frac{\left[\begin{array}{c}\left(1+\frac{\eta k_{N}\alpha_{N}\overline{r_{t}P_{t}}^{1-\eta}}{\overline{MC_{N}}^{1-\eta}}m_{N}\right)\left[\begin{array}{c}\left(1+\frac{\eta k_{H}\alpha_{H}\overline{r_{t}P_{t}}^{1-\eta}}{\overline{MC_{H}}^{1-\eta}}m_{H}\right)\\ \cdot\left(\frac{\eta k_{H}\alpha_{H}\overline{r_{t}P_{t}}^{1-\eta}}{\overline{MC_{H}}^{1-\eta}}m_{N}-\lambda\right)\end{array}\right]\\ -\left[\begin{array}{c}\left(\frac{\eta k_{N}\alpha_{N}\overline{r_{t}P_{t}}^{1-\eta}}{\overline{MC_{N}}^{1-\eta}}m_{H}\right)\\ \cdot\left(1-\lambda+\frac{\eta k_{N}\alpha_{N}\overline{r_{t}P_{t}}^{1-\eta}}{\overline{MC_{N}}^{1-\eta}}m_{N}\right)\end{array}\right]\frac{\eta k_{H}\alpha_{H}\overline{r_{t}P_{t}}^{1-\eta}}{\overline{MC_{H}}^{1-\eta}}m_{N}\end{array}\right]}{\det\left(\Omega\right)}
\end{aligned} \tag{4.25}$$

By (1.15) and (1.16), we have

$$\overline{\frac{W}{rP}}=\frac{\kappa}{\kappa^{*}}\overline{\frac{W^{*}}{r^{*}P^{*}}}$$

For very small A_t, for instance, $A_t \to 0$, $\overline{\frac{W_t/A_t}{r_tP_t}} \to \infty$, the capital-intensity

$$\frac{\alpha_{H}\overline{r_{t}P_{t}}^{1-\eta}}{\overline{MC_{H}}^{1-\eta}}=\frac{\alpha_{H}}{\alpha_{H}+\left(1-\alpha_{H}\right)\left(\frac{\overline{W_{t-1}/A_{t}}}{r_{t}P_{t}}\right)^{1-\eta}}$$

is close to zero and

$$\left(\widehat{P_{Ht}^{*}}\right)^{fixed}-\left(\widehat{P_{Ht}^{*}}\right)^{flexible}\to-\lambda\sigma^{2}<0$$

For very large A_t, for instance $A_t \to \infty$, then $\overline{\frac{W_t/A_t}{r_tP_t}} \to 0$, the capital-intensity

$$\frac{\alpha_{H}\overline{r_{t}P_{t}}^{1-\eta}}{\overline{MC_{H}}^{1-\eta}}=\frac{\alpha_{H}}{\alpha_{H}+\left(1-\alpha_{H}\right)\left(\frac{\overline{W_{t-1}/A_{t}}}{r_{t}P_{t}}\right)^{1-\eta}}$$

is close to one. We can rewrite the expressions of $\bar{K}$ as

$$\begin{aligned}
\bar{K} \quad = \quad & \frac{1}{\chi}\frac{i}{1+i}\alpha_{H}\left(\bar{r}\bar{P}\right)^{-\eta}\left(\frac{\theta}{\theta-1}\right)^{-\theta}\frac{\left(\gamma+d\right)\mu M_{t-1}^{*}}{P_{T}^{*1-\theta}}\left[\begin{array}{c}\left(1-\alpha_{H}\right)\left(\frac{\bar{W}}{A_{t}}\right)^{1-\eta}\\ +\alpha_{H}\left(\bar{r}\bar{P}\right)^{1-\eta}\end{array}\right]^{\frac{\eta-\theta}{1-\eta}} \\
& +\frac{1}{\chi}\frac{i}{1+i}\alpha_{N}\left(\bar{r}\bar{P}\right)^{-\eta}\left(\frac{\theta}{\theta-1}\right)^{-1}\left[\begin{array}{c}\left(1-\alpha_{N}\right)\left(\frac{\bar{W}}{A_{t}}\right)^{1-\eta}\\ +\alpha_{N}\left(\bar{r}\bar{P}\right)^{1-\eta}\end{array}\right]^{-1}\left(1-\gamma\right)\mu M_{t-1}
\end{aligned}$$

Notice that M_{t-1}, M^*_{t-1} and $\bar{r}\bar{P}$ are not functions of A_t, then

$$0 < \frac{k_H}{k_N} \simeq \left(\bar{r}\bar{P}\right)^{1-\theta}\left(\frac{\theta}{\theta-1}\right)^{1-\theta}\frac{\gamma+d}{(1-\gamma)P_T^{*1-\theta}}\frac{M^*_{t-1}}{M_{t-1}} < \infty$$

which means that as A_t goes to infinity, neigher k_H or k_N will converge to zero. Then m_H and m_N converge to infinity as A_t approaches infinity, and

$$\begin{aligned} & \left(\widehat{P^*_{Ht}}\right)^{fixed} - \left(\widehat{P^*_{Ht}}\right)^{flexible} \\ = \ & \sigma^2 \det\left(\Omega\right)^{-1}\left[\begin{array}{c} \eta^2 k_H k_N m_N^2 + \eta^2 k_H^2 m_N m_H + \eta k_H m_N \\ -\eta^2 k_H k_N m_N m_H - \lambda\left(\eta k_N m_N + \eta k_H m_H\right) - \lambda \end{array}\right] \end{aligned}$$

It is easy to show that one of the following inequalities must hold

$$k_H k_N m_N^2 > k_H k_N m_N m_H$$

or

$$k_H^2 m_N m_H > k_H k_N m_N m_H$$

As a result,

$$\begin{aligned} & \left(\widehat{P^*_{Ht}}\right)^{fixed} - \left(\widehat{P^*_{Ht}}\right)^{flexible} \\ > \ & \frac{\sigma^2\left[\min\left(\eta^2 k_H k_N m_N^2, \eta^2 k_H^2 m_N m_H\right) + \eta k_H m_N - \lambda\left(\eta k_N m_N + \eta k_H m_H\right) - \lambda\right]}{\det(\Omega)} \end{aligned}$$

As both m_H and m_N converge to infinity,

$$\min\left(\eta^2 k_H k_N m_N^2, \eta k_H m_N m_H\right) + \eta k_H m_N - \lambda\left(\eta k_N m_N + \eta k_H m_H\right) - \lambda > 0$$

Then

$$\left(\widehat{P^*_{Ht}}\right)^{fixed} - \left(\widehat{P^*_{Ht}}\right)^{flexible} > 0$$

Now we show that $\left(\widehat{P_{Ht}^{*}}\right)^{fixed}$ and $\left(\widehat{P_{Ht}^{*}}\right)^{flexible}$ is increasing in A_t.

We can rewrite the difference between $\left(\widehat{P_{Ht}^{*}}\right)^{fixed}$ and $\left(\widehat{P_{Ht}^{*}}\right)^{flexible}$ as

$$\left(\widehat{P_{Ht}^{*}}\right)^{fixed} - \left(\widehat{P_{Ht}^{*}}\right)^{flexible}$$

$$= \frac{\left(\frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}\right) m_N}{\psi} + \frac{\left[\begin{array}{c} \frac{\eta k_N \alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}} m_N \left(\left(1-\frac{1}{\psi}\right)\frac{k_N}{k_H} - 1\right) \\ -\lambda\left(\left(\frac{k_N}{k_H}\right)^2 \frac{\alpha_N}{\alpha_H}\frac{\overline{MC_H}^{1-\eta}}{\overline{MC_N}^{1-\eta}} + 1\right) \\ -\frac{\lambda}{\frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}\frac{k_H}{k_N}} \end{array}\right]}{\psi + \left(\frac{k_N}{k_H}\right)^2 \frac{\alpha_N}{\alpha_H}\frac{\overline{MC_H}^{1-\eta}}{\overline{MC_N}^{1-\eta}} + \frac{1}{\frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}\frac{k_H}{k_N}}} \tag{4.26}$$

The first term on the right hand side is

$$\left(\frac{\eta k_H \alpha_H \overline{rP}^{1-\eta}}{\overline{MC_H}^{1-\eta}}\right) m_N$$

$$= \frac{\alpha_H \overline{rP}^{1-\eta}}{(1-\alpha_H)\left(\frac{W_{t-1}}{A_t}\right)^{1-\eta}\left(1+\frac{k_H}{k_N}\right) + \frac{(1-\alpha_N)\overline{MC_H}^{1-\eta}\left(\frac{W_{t-1}}{A_t}\right)^{1-\eta}}{\overline{MC_N}^{1-\eta}}\left(1+\frac{k_N}{k_H}\right)}$$

where

$$\frac{k_H}{k_N} = \frac{\alpha_H\left(\frac{\theta}{\theta-1}\right)^{1-\psi}(\gamma+d)\, M_{t-1}^{*}\overline{MC}_N^{1-\eta}}{\alpha_N(1-\gamma)\, P_T^{*1-\psi} M_{t-1}\overline{MC}_H^{\psi-\eta}}$$

The derivative of $\frac{k_H}{k_N}$ with respect to A_t is

$$\frac{d\left(\frac{k_H}{k_N}\right)}{dA_t} = -\frac{1-\eta}{A_t}\frac{k_H}{k_N}\left(\frac{\bar{W}}{A_t}\right)^{1-\eta}\left(\frac{1-\alpha_N}{\overline{MC}_N^{1-\eta}} - \frac{\psi-\eta}{1-\eta}\frac{1-\alpha_H}{\overline{MC}_H^{1-\eta}}\right)$$

By the expressions of $\frac{k_H}{k_N}$ and $\frac{d\left(\frac{k_H}{k_N}\right)}{dA_t}$, we can show that the derivative of $\left(\frac{\eta k_H \alpha_H \overline{rP}^{1-\eta}}{\overline{MC_H}^{1-\eta}}\right) m_N$ with respect to A_t is:

$$\frac{d\left(\frac{\eta k_H \alpha_H \overline{rP}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_N\right)}{dA_t} = \text{positive}\,.terms \cdot \left[\begin{array}{c} (1-\alpha_H)\frac{k_H}{k_N}\left(1+b_N-\frac{\psi-\eta}{1-\eta}b_H\right) \\ +\frac{(1-\alpha_N)\overline{MC_H}^{1-\eta}}{\overline{MC_N}^{1-\eta}}\frac{k_N}{k_H}\left(1-b_N+\frac{\psi-\eta}{1-\eta}b_H\right) \end{array}\right]$$

where

$$b_H = \frac{(1-\alpha_H)\left(\frac{\bar{W}}{A_t}\right)^{1-\eta}}{\overline{MC_H}^{1-\eta}} \text{ and } b_N = \frac{(1-\alpha_N)\left(\frac{\bar{W}}{A_t}\right)^{1-\eta}}{\overline{MC_N}^{1-\eta}}$$

Notice that both b_H and b_N are less than one, if

$$\frac{\psi-\eta}{1-\eta} < \frac{\alpha_H(1-\alpha_N)}{\alpha_N(1-\alpha_H)}$$

$b_N - \frac{\psi-\eta}{1-\eta} b_H$ is always negative, then

$$\frac{d\left(\frac{\eta k_H \alpha_H \overline{rP}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_N\right)}{dA_t} > 0$$

If

$$\frac{\psi-\eta}{1-\eta} \geq \frac{\alpha_H(1-\alpha_N)}{\alpha_N(1-\alpha_H)}$$

we have

$$\frac{d\left(\frac{k_H}{k_N}\right)}{dA_t} > 0$$

then the ratio $\frac{k_H}{k_N}$ takes its maximum at $A_t \to \infty$. When A_t approaches ∞, $\frac{k_H}{k_N}$ is close to $\frac{\alpha_H \beta^{1-\psi}\left(\frac{\theta}{\theta-1}\right)^{1-\psi}(\gamma+d)}{1-\gamma}$. Then

$$\frac{k_H}{k_N} < \frac{\alpha_H \beta^{1-\psi}\left(\frac{\theta}{\theta-1}\right)^{1-\psi}(\gamma+d)}{1-\gamma}$$

Under the assumption of γ and by Lemma 1 and use $d = \frac{\gamma \frac{\theta-1}{\theta}\alpha_H \eta}{1-\frac{\theta-1}{\theta}\alpha_H \eta}$,

$$\frac{k_H}{k_N} < 1 < \frac{k_N}{k_H}$$

then

$$\frac{(1-\alpha_N)\overline{MC_H}^{1-\eta}}{\overline{MC_N}^{1-\eta}} \frac{k_N}{k_H} > (1-\alpha_H)\frac{k_H}{k_N}$$

We still get

$$\frac{d\left(\frac{\eta k_H \alpha_H \overline{rP}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_N\right)}{dA_t} > 0$$

For the second term in (4.26), it is easy to show that $\frac{\overline{MC_H}^{1-\eta}}{\overline{MC_N}^{1-\eta}}$ is increasing in A_t if $\alpha_H > \alpha_N$. Then by plugging the expression of $\frac{k_H}{k_N}$ into (4.26), we can easily show that, the second term is also increasing in A_t. Therefore,

$$\frac{\partial\left(\left(\widehat{P_{Ht}^*}\right)^{fixed} - \left(\widehat{P_{Ht}^*}\right)^{flexible}\right)}{\partial A_t} > 0$$

Since all functions in the model are continuous, there must exist a critical value of the labor productivity, A_0 such that

$$\left(\widehat{P_{Ht}^*}\right)^{fixed} - \left(\widehat{P_{Ht}^*}\right)^{flexible}\Bigg|_{A_t = A_0} = 0$$

For $A_t > A_0$, $\left(\widehat{P_{Ht}^*}\right)^{flexible} < \left(\widehat{P_{Ht}^*}\right)^{fixed}$. As a result,

$$(y_{Ht})^{flexible} = \omega\left(\frac{(P_{Ht}^*)^{flexible}}{P_{Tt}^*}\right)^{-\psi} Y_{Tt}^* > \omega\left(\frac{(P_{Ht}^*)^{fixed}}{P_{Tt}^*}\right)^{-\psi} Y_{Tt}^* = (y_{Ht})^{fixed}$$

Since $\psi > 1$,

$$(P_{Ht}^* Y_{Ht})^{flexible} > (P_{Ht}^* Y_{Ht})^{fixed}$$

the export revenue in the tradable good sector is higher under a flexible exchange rate regime than that under a fixed exchange rate regime.

For $A_t < A_0$, $\left(\widehat{P_{Ht}^*}\right)^{flexible} > \left(\widehat{P_{Ht}^*}\right)^{fixed}$. As a result,

$$(y_{Ht})^{flexible} = \omega\left(\frac{(P_{Ht}^*)^{flexible}}{P_{Tt}^*}\right)^{-\psi} Y_{Tt}^* < \omega\left(\frac{(P_{Ht}^*)^{fixed}}{P_{Tt}^*}\right)^{-\psi} Y_{Tt}^* = (y_{Ht})^{fixed}$$

and similarly due to $\psi > 1$,

$$(P^*_{Ht} Y_{Ht})^{flexible} < (P^*_{Ht} Y_{Ht})^{fixed}$$

the export revenue in the tradable good sector is lower under a flexible exchange rate regime than that under a fixed exchange rate regime.

(ii) By (4.26), it is easy to show that

$$\frac{\partial \left(\left(\widehat{P^*_{Ht}} \right)^{fixed} - \left(\widehat{P^*_{Ht}} \right)^{flexible} \right)}{\partial \lambda} < 0 \text{ and } \frac{\partial^2 \left(\left(\widehat{P^*_{Ht}} \right)^{fixed} - \left(\widehat{P^*_{Ht}} \right)^{flexible} \right)}{\partial \lambda \partial A_t} < 0$$

and since

$$\frac{\partial \left(\left(\widehat{P^*_{Ht}} \right)^{fixed} - \left(\widehat{P^*_{Ht}} \right)^{flexible} \right)}{\partial \left(\frac{\alpha_H \overline{rP}^{1-\eta}}{\overline{MC_H}^{1-\eta}} \right)} = \frac{\partial \left(\left(\widehat{P^*_{Ht}} \right)^{fixed} - \left(\widehat{P^*_{Ht}} \right)^{flexible} \right)}{\partial A_t} \frac{\partial A_t}{\partial \left(\frac{\alpha_H \overline{rP}^{1-\eta}}{\overline{MC_H}^{1-\eta}} \right)}$$

Under the assumption $\eta < 1$, $\frac{\partial A_t}{\partial \left(\frac{\alpha_H \overline{rP}^{1-\eta}}{\overline{MC_H}^{1-\eta}} \right)} > 0$. Therefore,

$$\frac{\partial \left(\left(\widehat{P^*_{Ht}} \right)^{fixed} - \left(\widehat{P^*_{Ht}} \right)^{flexible} \right)}{\partial \left(\frac{\alpha_H \overline{rP}^{1-\eta}}{\overline{MC_H}^{1-\eta}} \right)} > 0$$

(iii) As for the optimal prices in the nontradable good sector, we have

$$\begin{aligned} & \left(\widehat{P_{Nt}} \right)^{fixed} - \left(\widehat{P_{Nt}} \right)^{flexible} \\ = \; & \sigma^2 \frac{\left[\begin{array}{c} \left(1 + \psi \frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_H \right) \left(\frac{\eta k_N \alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}} m_H \right) \left(1 - \lambda + \frac{\eta k_N \alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}} m_N \right) \\ - \left(1 + \frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_H \right) \left(\frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_N - \lambda \right) \psi \frac{\eta k_N \alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}} m_H \end{array} \right]}{\det (\Omega)} \end{aligned}$$

For $A_t \to 0$, $\frac{\alpha_H \overline{r_t P_t}^{1-\eta}}{MC_H^{1-\eta}}$, $\frac{\alpha_N \overline{r_t P_t}^{1-\eta}}{MC_N^{1-\eta}}$, m_H and m_N are all close to zero, and then

$$\left(\widehat{P_{Nt}}\right)^{fixed} - \left(\widehat{P_{Nt}}\right)^{flexible} \to 0$$

Similar to (i), we can show that

$$\frac{\partial\left(\left(\widehat{P_{Nt}}\right)^{fixed} - \left(\widehat{P_{Nt}}\right)^{flexible}\right)}{\partial A_t} > 0$$

then

$$\left(\widehat{P_{Nt}}\right)^{flexible} < \left(\widehat{P_{Nt}}\right)^{fixed}$$

for all $A_t > 0$. □

A1.3. Proof of Proposition 2

Proof. We first compare the welfare gain in period t if the home country switches from a flexible exchange rate regime to a fixed exchange rate regime.

$$E_{t-1} \ln C_t = \ln \mu + \ln M_{t-1} - \ln P_t + \ln\left(\frac{1}{\chi}\frac{i}{1+i}\right)$$

and

$$E_{t-1} L_t = \frac{1}{\chi}\frac{i}{1+i}\left(\begin{array}{c} (1-\alpha_H)\left(\frac{W_t/A_t}{MC_{Nt}}\right)^{-\eta}\left(\frac{P^*_{1Ht}}{P^*_{1t}}\right)^{-\psi}\frac{\eta(\gamma+d)}{A_t P^*_{1t}}E_{t-1}M^*_t \\ +(1-\alpha_N)\left(\frac{W_t/A_t}{MC_{Nt}}\right)^{-\eta}\frac{(1-\gamma)E_{t-1}M_t}{A_t P_{Nt}} \end{array}\right)$$

The consumption price index P_t is

$$P_t = P_{Tt}^{\gamma} P_{Nt}^{1-\gamma}$$

where P_{Tt} is approximately the price of imported good for very small ω,

$$P_{Tt} \simeq \frac{\theta}{\theta-1} E_{t-1}\left[MC^*_{Ft} S_t\right]$$

Under a flexible exchange rate regime, we have

$$\begin{aligned} P_{Ft}^{flexible} &= \bar{P}_F E_{t-1}\left[\exp\left\{\varepsilon_t - \lambda\varepsilon_t^* + \frac{\eta k_H \alpha_H \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_H}^{1-\eta}}\frac{\widehat{r_t^* P_t^*}}{W_t^*}\right\}\right] \\ &= \bar{P}_F \exp\left\{\frac{\sigma^2}{2} + \left(\frac{\eta k_F^* \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^*\right)^2 \frac{\sigma^2}{2}\right\} \end{aligned}$$

where

$$\bar{P}_F = \frac{\theta}{\theta-1}\left[\alpha_H\left(\overline{r}^*\bar{P}^*\right)^{1-\eta} + (1-\alpha_H)\left(\frac{\overline{W}^*}{A_t^*}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

Then

$$\begin{aligned} (\ln P_t)^{flexible} - \ln\bar{P} &= \frac{\gamma\sigma^2}{2}\left[1 + \left(\frac{\eta k_F^* \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^*\right)^2\right] \\ &\quad +(1-\gamma)\left[(\ln P_{Nt})^{flexible} - \ln\bar{P}_N\right] \end{aligned}$$

Similarly, under a fixed exchange rate regime,

$$P_{Ft}^{fixed} = \bar{P}_F \exp\left\{\left(1 + \frac{\eta k_F^* \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^*\right)^2 \frac{\sigma^2}{2}\right\}$$

and

$$\begin{aligned} (\ln P_t)^{fixed} - \ln\bar{P} &= \frac{\gamma\sigma^2}{2}\left(1 + \frac{\eta k_F^* \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^*\right)^2 \\ &\quad +(1-\gamma)\left[(\ln P_{Nt})^{fixed} - \ln\bar{P}_N\right] \end{aligned}$$

Then

$$\begin{aligned} &[E_{t-1}\ln C_t]^{fixed} - [E_{t-1}\ln C_t]^{flexible} \\ =\ & -\gamma\sigma^2\left(\frac{\eta k_F^* \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^*\right) - (1-\gamma)\left[\left(\widehat{P_{Nt}}\right)^{fixed} - \left(\widehat{P_{Nt}}\right)^{flexible}\right] \end{aligned}$$

As for the disutility from labor supply, up to the second order of σ, we approximately have

$$\begin{aligned}
\left(E_{t-1}L_t\right)^{flexible} &= \bar{L} + \bar{L}E_{t-1}\left[\begin{array}{c} l_H\left(\varepsilon_t^* + \frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}\widehat{\frac{r_tP_t}{W_t/A_t}} - \theta\widehat{P_{Ht}^*}\right) \\ +l_N\left(\varepsilon_t + \frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}\widehat{\frac{r_tP_t}{W_t/A_t}} - \widehat{P_{Nt}}\right)\end{array}\right] \\
&\quad +t.i.regime \\
&= \bar{L}\left[\begin{array}{c} 1-\left(\begin{array}{c} l_H\left(\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+1\right) \\ +l_N\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\end{array}\right)\psi\left(\widehat{P_{Ht}^*}\right)^{flexible} \\ -\left(\begin{array}{c} l_H\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N \\ +l_N\left(\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N+1\right)\end{array}\right)\left(\widehat{P_{Nt}}\right)^{flexible}\end{array}\right] \\
&\quad +t.i.regime
\end{aligned}$$

where

$$\begin{aligned}
\bar{L} &= \frac{1}{\chi}\frac{i}{1+i}\left[\begin{array}{c} \left(1-\alpha_H\right)\left(\frac{\overline{W_t/A_t}}{\overline{MC_{Ht}}}\right)^{-\eta}\left(\frac{\overline{P_{Ht}^*}}{\overline{P_{Tt}^*}}\right)^{-\psi}\frac{(\gamma+d)\mu M_{t-1}^*}{A_tP_{Tt}^*} \\ +\left(1-\alpha_N\right)\left(\frac{\overline{W_t/A_t}}{\overline{MC_{Nt}}}\right)^{-\eta}\frac{(1-\gamma)\mu M_{t-1}^*}{A_tP_{Nt}}\end{array}\right] \\
l_H &= \frac{\frac{1}{\chi}\frac{i}{1+i}\left(1-\alpha_H\right)\left(\frac{\overline{W_t/A_t}}{\overline{MC_{Ht}}}\right)^{-\eta}\left(\frac{\overline{P_{Ht}^*}}{\overline{P_{Tt}^*}}\right)^{-\psi}\frac{(\gamma+d)\mu M_{t-1}^*}{A_tP_{Tt}^*}}{\bar{L}} \\
l_N &= \frac{\frac{1}{\chi}\frac{i}{1+i}\left(1-\alpha_N\right)\left(\frac{\overline{W_t/A_t}}{\overline{MC_{Nt}}}\right)^{-\eta}\frac{(1-\gamma)\mu M_{t-1}^*}{A_tP_{Nt}}}{\bar{L}}
\end{aligned}$$

and $t.i.regime$ means the term independent of the exchange rate regime choices.

Similarly, under a fixed exchange rate regime, we have

$$\begin{aligned}
\left(E_{t-1}L_t\right)^{fixed} &= \bar{L}\left[\begin{array}{c} 1-\left(l_H\left(\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+1\right)+l_N\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)\psi\left(\widehat{P_{Ht}^*}\right)^{fixed} \\ -\left(l_H\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N+l_N\left(\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N+1\right)\right)\left(\widehat{P_{Nt}}\right)^{fixed}\end{array}\right] \\
&\quad +t.i.regime
\end{aligned}$$

As we have shown in the previous section, $\left(\widehat{P_{Ht}^*}\right)^{fixed} - \left(\widehat{P_{Ht}^*}\right)^{flexible}$ and $\left(\widehat{P_{Nt}}\right)^{fixed} - \left(\widehat{P_{Nt}}\right)^{flexible}$ are of the same order of σ^2. Then up to the second order of σ, the welfare gain of switching to a fixed exchange rate regime in period t is

$$
\begin{aligned}
& E_{t-1}\left[\ln C_t+\chi\ln\left(\frac{M_t}{P_t}\right)-\kappa L_t\right]^{fixed}-E_{t-1}\left[\ln C_t+\chi\ln\left(\frac{M_t}{P_t}\right)-\kappa L_t\right]^{flexible} \\
= & -\gamma(1+\chi)\sigma^2\left(\frac{\eta k_F^*\alpha_F^*\overline{r_t^*P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}-\lambda^*\right) \\
& -(1+\chi)(1-\gamma)\left(\left(\widehat{P_{Nt}}\right)^{fixed}-\left(\widehat{P_{Nt}}\right)^{flexible}\right) \\
& -\bar{L}\left[\begin{array}{c}
\psi\left(l_H\left(\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+1\right)+l_N\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right) \\
\cdot\left(\left(\widehat{P_{Ht}^*}\right)^{fixed}-\left(\widehat{P_{Ht}^*}\right)^{flexible}\right) \\
+\left(l_H\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N+l_N\left(\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N+1\right)\right) \\
\cdot\left(\left(\widehat{P_{Nt}}\right)^{fixed}-\left(\widehat{P_{Nt}}\right)^{flexible}\right)
\end{array}\right]
\end{aligned}
$$

Similar to the proof of Proposition 1, the capital stock in period $t+1$ is

$$
K_{t+1}=\frac{1}{\chi}\frac{i}{1+i}\frac{M_t}{P_{Tt}}\left[(\gamma+d)\left(\frac{P_{Ht}^*}{P_{Tt}^*}\right)^{-\psi}+\frac{1-\omega}{\omega}\gamma\right]
$$

If we add up the budget constraint of two countries, then we have

$$
\begin{aligned}
& \omega\left(P_tC_t+P_{inv,t}I_t\right)+(1-\omega)S_t\left(P_t^*C_t^*+P_{inv,t}^*I_t^*\right) \\
\leq\ & \omega\left(W_tL_t+r_tP_tK_t+\Pi_t\right)+(1-\omega)S_t\left(P_{Tt}^*Y_{Tt}^*+\frac{P_{Tt}}{S_t}Y_{Tt}+P_{Nt}^*Y_{Nt}^*\right)
\end{aligned}
$$

we rewrite the inequality above and get

$$
\begin{aligned}
C_t+\frac{P_{inv,t}}{P_t}I_t & \leq \frac{W_t}{P_t}L_t+r_tK_t+\frac{\Pi_t}{P_t} \\
& \quad +\frac{1-\omega}{\omega}\frac{S_t}{P_t}\left(P_{Tt}^*Y_{Tt}^*+\frac{P_{Tt}}{S_t}Y_{Tt}+P_{Nt}^*Y_{Nt}^*-P_t^*C_t^*-P_{inv,t}^*I_t^*\right) \\
(1-\gamma)C_t & \leq \frac{W_t}{P_t}L_t+r_tK_t+\frac{\Pi_t}{P_t}+(1-\omega)\gamma\frac{1}{\chi}\frac{i}{1+i}\frac{M_t}{P_t}
\end{aligned}
$$

The value function can be written as a function of the only predetermined variable K_t based on this new budget constraint.

For small σ^2, we can approximately obtain

$$V_{t+1} = \bar{V} + \overline{\frac{\partial V}{\partial K}} \bar{K} \ln \left(\frac{K_{t+1}}{\bar{K}} \right)$$

where by B.S formula,

$$\overline{\frac{\partial V_t}{\partial K_t}} = \frac{1}{1-\gamma} \overline{\frac{r}{C}} = \frac{\kappa}{1-\gamma} \overline{\frac{rP}{W}}$$

then

$$V_{t+1}^{fixed} - V_{t+1}^{flexible} = \kappa \overline{\frac{rP}{W}} \bar{K} \left(\ln \left(\frac{K_{t+1}}{\bar{K}} \right)^{fixed} - \ln \left(\frac{K_{t+1}}{\bar{K}} \right)^{flexible} \right)$$

$$\begin{aligned}
& \ln \left(\frac{K_{t+1}}{\bar{K}} \right)^{fixed} - \ln \left(\frac{K_{t+1}}{\bar{K}} \right)^{flexible} \\
= \; & \ln \left(\frac{M_t^*}{P_{Tt}} \left[(\gamma + d) \left(\left(\frac{P_{Ht}^*}{P_{Tt}^*} \right)^{fixed} \right)^{-\psi} + \frac{1-\omega}{\omega} \gamma \right] \right) \\
& - \ln \left(\frac{M_t}{P_{Tt}} \left[(\gamma + d) \left(\left(\frac{P_{Ht}^*}{P_{Tt}^*} \right)^{flexible} \right)^{-\psi} + \frac{1-\omega}{\omega} \gamma \right] \right) \\
= \; & \left(\varepsilon_t^* - \psi q \left(\widehat{P_{Ht}^*} \right)^{fixed} - \left(\widehat{P_{Tt}} \right)^{fixed} \right) \\
& - \left(\varepsilon_t - \psi q \left(\widehat{P_{Ht}^*} \right)^{flexible} - \left(\widehat{P_{Tt}} \right)^{flexible} \right)
\end{aligned}$$

where

$$q = \frac{(\gamma + d) \left(\left(\frac{P_{Ht}^*}{P_{Tt}^*} \right)^{fixed} \right)^{-\psi}}{(\gamma + d) \left(\left(\frac{P_{Ht}^*}{P_{Tt}^*} \right)^{fixed} \right)^{-\psi} + \frac{1-\omega}{\omega} \gamma}$$

Then

$$E_{t-1} \left[V_{t+1}^{fixed} - V_{t+1}^{flexible} \right] = -\kappa \overline{\frac{rP}{W}} \bar{K} \left[\begin{array}{c} \psi q \left(\left(\widehat{P_{Ht}^*} \right)^{fixed} - \left(\widehat{P_{Ht}^*} \right)^{flexible} \right) \\ + \left(\left(\widehat{P_{Tt}} \right)^{fixed} - \left(\widehat{P_{Tt}} \right)^{flexible} \right) \end{array} \right]$$

The expected welfare difference between a fixed and a flexible exchange rate regime is

$$\begin{aligned}
& V_t^{fixed} - V_t^{flexible} \\
= & -\gamma\left(1+\chi-\beta\kappa\frac{\overline{rP}}{W}\bar{K}\right)\left(\frac{\eta k_F^*\alpha_F^*\overline{r_t^*P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}-\lambda^*\right)\sigma^2 \\
& -\left[(1-\gamma)(1+\chi)+\begin{pmatrix} l_H\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N \\ +l_N\left(\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N+1\right)\end{pmatrix}\bar{L}\right]\left(\left(\widehat{P_{Nt}}\right)^{fixed}-\left(\widehat{P_{Nt}}\right)^{flexible}\right) \\
& -\psi\left[\beta\kappa\frac{\overline{rP}}{W}\bar{K}q+\begin{pmatrix} l_H\left(\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+1\right) \\ +l_N\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\end{pmatrix}\bar{L}\right]\left(\left(\widehat{P_{Ht}^*}\right)^{fixed}-\left(\widehat{P_{Ht}^*}\right)^{flexible}\right) \quad (4.27)
\end{aligned}$$

The first term on the right hand side of (4.27) is independent of the home production and has an ambiguous sign. The second term is negative. And the last term has an ambiguous sign according to Proposition 1.

For very large A_t, for instance $A_t \to \infty$, the last term on the right hand side of (4.27) is negative. According to Proposition 1, as $A_t \to \infty$, $\left(\widehat{P_{Ht}^*}\right)^{fixed} - \left(\widehat{P_{Ht}^*}\right)^{flexible}$ converges to infinity, which will dominate the first term on the right hand side of (4.27) even if the first term is positive. Then we have

$$V_t^{flexible} > V_t^{fixed}$$

There is a welfare loss if switching from a flexible exchange rate regime to a fixed exchange rate regime.

For very small A_t, for instance $A_t \to 0$, the third term disappears and

$$\begin{aligned}
& V_t^{fixed} - V_t^{flexible} \\
= & -\gamma\left(1-\beta\kappa\frac{\overline{rP}}{W}\bar{K}\right)\left(\frac{\eta k_F^*\alpha_F^*\overline{r_t^*P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}-\lambda^*\right)\sigma^2 \\
& +\theta\left(\beta\kappa\frac{\overline{rP}}{W}\bar{K}q+l_H\bar{L}\right)\lambda\sigma^2
\end{aligned}$$

For large enough λ such that

$$\lambda > \frac{\gamma\left(1+\chi-\beta\kappa\frac{\overline{rP}}{W}\bar{K}\right)\left(\frac{\eta k_F^*\alpha_F^*\overline{r_t^*P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}-\lambda^*\right)}{\psi\left(\beta\kappa\frac{\overline{rP}}{W}\bar{K}q+l_H\bar{L}\right)}$$

we have

$$E_{t-1}V_t^{flexible} < E_{t-1}V_t^{fixed}$$

There is a welfare gain if switching from a flexible exchange rate regime to a fixed exchange rate regime.

Now we calculate the derivative of

$$\frac{\partial\left(E_{t-1}V_t^{fixed}-E_{t-1}V_t^{flexible}\right)}{\partial A_t}$$

As in Proposition 1, we have $\frac{\alpha_H(r_tP_t)^{1-\eta}}{\overline{MC_H}^{1-\eta}}$, $\frac{\alpha_N(r_tP_t)^{1-\eta}}{\overline{MC_H}^{1-\eta}}$ and $\bar{K}$ are increasing in A_t.

Plugging the expressions of l_H and l_N into the derivative, we can obtain

$$\begin{aligned}
&\left(l_H\left(\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+1\right)+l_N\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)\bar{L} \\
= \quad & \frac{\left(\bar{r}\bar{P}\right)^{\eta}}{\bar{W}}\left(\frac{\bar{W}}{A_t}\right)^{1-\eta}\bar{K}\left[\begin{array}{c}\frac{1-\alpha_H}{\alpha_H}\left(\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+k_H\right) \\ +\frac{1-\alpha_N}{\alpha_N}\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\end{array}\right]
\end{aligned}$$

Let Ω denote the term $\left(l_H\left(\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+1\right)+l_N\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)\bar{L}$. We take the derivative of Ω with respect to A_t and get

$$\begin{aligned}
\frac{d\Omega}{dA_t} \quad = \quad & \text{positive}\,.terms \\
& +\text{positive}\,.terms\cdot\frac{1-\alpha_N}{\alpha_N}\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\frac{k_N}{k_H}\left(1+\frac{\psi-\eta}{1-\eta}b_H-b_N\right)
\end{aligned}$$

As we analyzed in previous section, $b_N < 1$, then

$$\frac{d\Omega}{dA_t} > 0$$

Similarly, we can show that $\left(l_H \frac{\eta\alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_N + l_N \left(\frac{\eta\alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}} m_N + 1\right)\right) \bar{L}$ is also increasing in A_t. By Proposition 1,

$$\frac{\partial\left(\left(\widehat{P^*_{Ht}}\right)^{fixed} - \left(\widehat{P^*_{Ht}}\right)^{flexible}\right)}{\partial A_t} > 0$$

and

$$\frac{\partial\left(\left(\widehat{P_{Nt}}\right)^{fixed} - \left(\widehat{P_{Nt}}\right)^{flexible}\right)}{\partial A_t} > 0$$

Therefore,

$$\frac{\partial\left(E_{t-1}V_t^{fixed} - E_{t-1}V_t^{flexible}\right)}{\partial A_t} < 0$$

There exists a critical value there exists a critical value of the labor productivity, A_0^w, such that it is indifferent in choosing a flexible and a fixed exchange rate regime. For $A_t > A_0^w$, there is a welfare loss if switching from a flexible exchange rate regime to a fixed exchange rate regime. For $A_t < A_0^w$, the result reverses. Welfare improves is switching to a fixed exchange rate regime. □

A1.4. Proof of Proposition 3

Proof. In period $t-1$, both countries are at the steady state, then we have

$$K_t = \bar{K}\exp\left\{\begin{array}{c} k_H \left(\begin{array}{c} \varepsilon_t^* + \left(1 - \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a^*\varepsilon_{at}^* \\ -\frac{\eta(1-\alpha_H)\left(\frac{W_{t-1}}{A_t}\right)^{1-\eta}}{\overline{MC_H}^{1-\eta}}\widehat{\frac{r_t P_t}{W_t/A_t}} - \psi\widehat{P^*_{Ht}} \end{array}\right) \\ +k_N \left(\begin{array}{c} \varepsilon_t + \left(1 - \sum_{n=1}^{\infty} s_n \left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a\varepsilon_{at} \\ -\frac{\eta(1-\alpha_N)\left(\frac{W_{t-1}}{A_t}\right)^{1-\eta}}{\overline{MC_N}^{1-\eta}}\widehat{\frac{r_t P_t}{W_t/A_t}} - \widehat{P_{Nt}} \end{array}\right) \end{array}\right\}$$

and

$$\begin{aligned}
\frac{\widehat{r_t P_t}}{W_t} &= m_H \left(\varepsilon_t^* + \left(1 - \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \phi_a^* \varepsilon_{at}^* - \psi \widehat{P_{Ht}^*} \right) \\
&\quad + m_N \left(\varepsilon_t + \left(1 - \sum_{n=1}^{\infty} s_n \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \phi_a \varepsilon_{at} - \widehat{P_{Nt}} \right) \\
&\quad - (m_H + m_N) \widehat{K_t}
\end{aligned}$$

Based on the same reasoning as in Proposition 1, the home country will have the same K_t under two exchange rate regimes. Since we are interested in the difference between $\left(\widehat{P_{Ht}^*}\right)^{fixed}$ and $\left(\widehat{P_{Ht}^*}\right)^{flexible}$, we can simply drop this term in the following calculations.

The optimal price in the tradable good sector is now

$$P_{Ht}^* = \frac{\theta}{\theta - 1} E_{t-1} \left[\frac{\left((1-\alpha_H) \left(\frac{W_t}{A_t} \right)^{1-\eta} + \alpha_H (r_t P_t)^{1-\eta} \right)^{\frac{1}{1-\eta}} \frac{i_t^*}{1+i_t^*} M_t^*}{\frac{i_t}{1+i_t} M_t} \right]$$

We can approximately write the optimal price in the tradable good sector as following:

$$P_{Ht}^* = \bar{P}_H^* E_{t-1} \left[\exp \left\{ \begin{array}{c} \left(1 + \frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_H \right) \\ \cdot \left(\varepsilon_t^* + \left(1 - \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \phi_a^* \varepsilon_{at}^* \right) \\ + \left(\frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_N - \lambda \right) \\ \cdot \left(\varepsilon_t + \left(1 - \sum_{n=1}^{\infty} s_n \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \phi_a \varepsilon_{at} \right) \\ -\lambda \varepsilon_{at} - \frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} \left(m_H \psi \widehat{P_{Ht}^*} + m_N k_N \widehat{P_{Nt}} \right) \end{array} \right\} \right]$$

Similarly, the price in the nontradable good price is

$$
\begin{aligned}
P_{Nt} &= \frac{\theta}{\theta-1}E_{t-1}\left[\left[(1-\alpha_N)\left(\frac{W_t}{A_t}\right)^{1-\eta}+\alpha_N\left(r_tP_t\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}\right] \\
&= \bar{P}_N E_{t-1}\left[\exp\left\{\begin{array}{c}\left(1-\lambda+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N\right) \\ \cdot\left(\varepsilon_t+\left(1-\sum_{n=1}^{\infty}s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a\varepsilon_{at}\right) \\ +\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\left(\varepsilon_t^*+\left(1-\sum_{n=1}^{\infty}s_n^*\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a^*\varepsilon_{at}^*\right) \\ -\lambda\varepsilon_{at}-\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}\left(m_H\psi\widehat{P_{Ht}^*}+m_Nk_N\widehat{P_{Nt}}\right)\end{array}\right\}\right]
\end{aligned}
$$

Under a flexible exchange rate regime, up to the second order of σ and σ_a, we have

$$
\begin{aligned}
\left(\widehat{P_{Ht}^*}\right)^{flexible} &= \frac{\sigma^2}{2}\frac{\left[\begin{array}{c}\left(1+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)\left[\begin{array}{c}\left(1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)^2 \\ +\left(\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\right)^2\end{array}\right] \\ -\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N\left[\begin{array}{c}\left(\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)^2 \\ +\left(1-\lambda+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)^2\end{array}\right]\end{array}\right]}{\det\left(\Omega\right)} \\
&+\frac{\sigma_a^2}{2}\frac{\left[\begin{array}{c}\left(1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)^2\left(1-\sum_{n=1}^{\infty}s_n^*\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)^2\phi_a^* \\ +\left(\left(\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\right)\left(1-\sum_{n=1}^{\infty}s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a-\lambda\right)^2\end{array}\right]}{\det\left(\Omega\right)^{-1}}
\end{aligned}
$$

and

$$\left(\widehat{P_{Nt}}\right)^{flexible} = \frac{\sigma^2}{2}\frac{\left[\begin{array}{c}\left(1+\theta\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)\left[\begin{array}{c}\left(\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)^2\\+\left(1-\lambda+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)^2\end{array}\right]\\-\theta\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\left[\begin{array}{c}\left(1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)^2\\+\left(\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\right)^2\end{array}\right]\end{array}\right]}{\det(\Omega)}$$
$$+\frac{\sigma_a^2}{2}\frac{\left[\begin{array}{c}\left(\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)^2\left(1-\sum_{n=1}^{\infty}s_n^*\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)^2\phi_a\\+\left(\begin{array}{c}\left(1-\lambda+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N\right)\\\cdot\left(1-\sum_{n=1}^{\infty}s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a-\lambda\end{array}\right)^2\end{array}\right]}{\det(\Omega)}$$

Similarly, under a fixed exchange rate regime, up to the second order of σ and σ_a, we have

$$\left(\widehat{P_{Ht}^*}\right)^{fixed} = \frac{\sigma^2}{2}\frac{\left[\begin{array}{c}\left(1+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)\left(1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\right)^2\\-\left(1-\lambda+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)^2\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N\end{array}\right]}{\det(\Omega)}$$
$$+\frac{\sigma_a^2}{2}\frac{\left[\begin{array}{c}\left(1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)\left(1-\sum_{n=1}^{\infty}s_n^*\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a^*\\+\left(\left(\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\right)\left(1-\sum_{n=1}^{\infty}s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a^*-\lambda\right)\end{array}\right]^2}{\det(\Omega)}$$

and

$$\left(\widehat{P_{Nt}}\right)^{fixed} = \frac{\sigma^2}{2}\frac{\left[\begin{array}{c}\left(1+\theta\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)\left(\begin{array}{c}1-\lambda+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\\+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\end{array}\right)^2\\-\left(\begin{array}{c}1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\\+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\end{array}\right)^2\psi\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\end{array}\right]}{\det(\Omega)}$$

$$+\frac{\sigma_a^2}{2}\frac{\left[\begin{array}{c}\left(\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)\left(1-\sum_{n=1}^{\infty}s_n^*\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a^*\\+\left(\begin{array}{c}\left(1-\lambda+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N\right)\\\cdot\left(1-\sum_{n=1}^{\infty}s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a^*-\lambda\end{array}\right)\end{array}\right]^2}{\det(\Omega)}$$

Then we calculate the difference between optimal prices under the two regimes. For the tradable good price, we have

$$\begin{aligned}&\left(\widehat{P_{Ht}^*}\right)^{fixed}-\left(\widehat{P_{Ht}^*}\right)^{flexible}\\&=\sigma^2\frac{\left[\begin{array}{c}\left(1+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)\left[\left(1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)\left(\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\right)\right]\\-\left[\left(\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)\left(1-\lambda+\frac{\eta k_N\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N\right)\right]\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N\end{array}\right]}{\det(\Omega)}\\&\quad+\sigma_a^2\det(\Omega)^{-1}\phi_a^*\left(1+\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H\right)\left(1-\sum_{n=1}^{\infty}s_n^*\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\\&\quad\cdot\left(\left(\frac{\eta k_H\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N-\lambda\right)\left(1-\sum_{n=1}^{\infty}s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a-\lambda\right)\end{aligned}$$

As $\bar{A}\to 0$, i.e., steady state labor productivity in the home country is extremely low, then

$$\left(\widehat{P_{Ht}^*}\right)^{fixed}-\left(\widehat{P_{Ht}^*}\right)^{flexible}\to-\lambda\sigma^2\left[\begin{array}{c}1+\frac{\sigma_a^2}{\sigma^2}\left(1-\sum_{n=1}^{\infty}s_n^*\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\\\cdot\left(\left(1-\sum_{n=1}^{\infty}s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a+1\right)\end{array}\right]$$

Since $|\rho| < 1$, $\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)$ is increasing in n. Due to the fact $\sum_{n=1}^{\infty} s_n = 1$ and s_n is decreasing in n, it is easy to show that

$$\sum_{n=1}^{\infty} s_n \left(\frac{\rho - \rho^{n+1}}{1-\rho}\right) < 1$$

Then, as $\bar{A} \to 0$,

$$\left(\widehat{P^*_{Ht}}\right)^{fixed} < \left(\widehat{P^*_{Ht}}\right)^{flexible}$$

As $\bar{A} \to \infty$, i.e., steady state labor productivity in the home country is extremely high, if $\phi_a > 0$, similar to the proof of Proposition 1, $\frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_H$ and $\frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_N$ will converge to infinity. Then

$$\left(\widehat{P^*_{Ht}}\right)^{fixed} > \left(\widehat{P^*_{Ht}}\right)^{flexible}$$

and $\left(\widehat{P^*_{Ht}}\right)^{fixed} - \left(\widehat{P^*_{Ht}}\right)^{flexible}$ will also converge to infinity.

To calculate the derivative,

$$\frac{\partial\left(\left(\widehat{P^*_{Ht}}\right)^{fixed} - \left(\widehat{P^*_{Ht}}\right)^{flexible}\right)}{\partial \bar{A}}$$

we only need to calculate the derivative of the additional term with respect to $\bar{A}$ resulted from the labor productivity shocks. Notice that the term can be written as following

$$\phi_a \phi_a \sigma_a^2 \left(1 - \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho - \rho^{n+1}}{1-\rho}\right)\right)^2 \left[\begin{array}{c} \frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_N \\ -\frac{\lambda\left(1+\frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_H\right) + \frac{\eta k_N \alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}} m_N}{1+\psi \frac{\eta k_H \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_H + \frac{\eta k_N \alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}} m_N} \end{array}\right]$$

Under the assumption in Proposition 1, we can similarly show that this term is increasing in A_t. Then

$$\frac{\partial\left(\left(\widehat{P^*_{Ht}}\right)^{fixed} - \left(\widehat{P^*_{Ht}}\right)^{flexible}\right)}{\partial \bar{A}} > 0$$

Following the same proof of Proposition 1, we can show (ii) and (iii) hold. □

A1.5. Proof of Proposition 4

Proof. We first compare the welfare gain in period t if the home country switches from a flexible exchange rate regime to a fixed exchange rate regime.

$$E_{t-1}\ln C_t = \ln\mu + \ln M_{t-1} - \ln P_t + \ln\left(\frac{1}{\chi}\frac{i_t}{1+i_t}\right)$$

and

$$E_{t-1}L_t = \begin{array}{c} (1-\alpha_H)\left(\frac{P^*_{1Ht}}{P^*_{1t}}\right)^{-\psi} E_{t-1}\left[\left(\frac{W_t/A_t}{MC_{Nt}}\right)^{-\eta}\frac{\eta(\gamma+d)}{A_tP^*_{1t}}\frac{1}{\chi}\frac{i^*_t}{1+i^*_t}M^*_t\right] \\ +(1-\alpha_N)\,E_{t-1}\left[\left(\frac{W_t/A_t}{MC_{Nt}}\right)^{-\eta}\frac{1}{\chi}\frac{i_t}{1+i_t}\frac{(1-\gamma)M_t}{A_tP_{Nt}}\right]\end{array}$$

By (1.3), we have

$$\ln\left(\frac{M_t}{P_t}\right) = \ln C_t - \ln\left(\frac{i_t}{1+i_t}\right)$$

The consumption price index P_t is

$$P_t = P^{\gamma}_{Tt}P^{1-\gamma}_{Nt}$$

where P_{Tt} is set in period $t-1$ by foreign producers in home currency (local currency pricing).

$$P_{Ft} = \frac{\theta}{\theta-1}E_{t-1}\left[MC^*_{Ft}S_t\right]$$

Under a flexible exchange rate regime, we have

$$P^{flexible}_{Ft} = \bar{P}_F E_{t-1}\left[\exp\left\{\begin{array}{c}\left(\frac{\eta k^*_F\alpha^*_F\overline{r^*_tP^*_t}^{1-\eta}}{\overline{MC^*_F}^{1-\eta}}-\lambda^*\right) \\ \cdot\left(\varepsilon^*_t+\left(1-\sum_{n=1}^{\infty}s^*_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi^*_a\varepsilon^*_{at}\right) \\ +\left(\varepsilon_t+\left(1-\sum_{n=1}^{\infty}s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a\varepsilon_{at}\right) \\ -\lambda\varepsilon^*_{at}-\frac{\eta k^*_F\alpha^*_F\overline{r_tP_t}^{1-\eta}}{\overline{MC^*_F}^{1-\eta}}\left(m^*_F\psi\widehat{P^*_{Ft}}+m^*_N\widehat{P^*_{Nt}}\right)\end{array}\right\}\right]$$

$$= \frac{\sigma^2}{2}\det(\Omega)^{-1}\left[1+\left(\frac{\eta k_F \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}-\lambda^*\right)^2\right]$$

$$+\frac{\sigma_a^2}{2}\frac{\left[\begin{array}{c}\left(1-\sum_{n=1}^{\infty} s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)^2\phi_a^* \\ +\left(\begin{array}{c}\left(\frac{\eta k_F^* \alpha_F^* \overline{r_t P_t}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}-\lambda^*\right) \\ \cdot\left(1-\sum_{n=1}^{\infty} s_n^*\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a-\lambda^*\end{array}\right)^2\end{array}\right]}{\det(\Omega)}$$

where

$$\bar{P}_F = \frac{\theta}{\theta-1}\left[\alpha_H\left(\overline{r}^* \bar{P}^*\right)^{1-\eta}+(1-\alpha_H)\left(\frac{\overline{W}^*}{\bar{A}^*}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

Similarly, under a fixed exchange rate regime,

$$\left(\widehat{P_{Ft}}\right)^{fixed} = \frac{\sigma^2}{2}\frac{\left[\begin{array}{c}\left(1+\frac{\eta k_N^* \alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N^*}^{1-\eta}}m_N^*\right)\left(\begin{array}{c}1-\lambda^* \\ +\frac{\eta k_F^* \alpha_F^* \overline{r_t P_t}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}m_F^* \frac{\eta k_F \alpha_F^* \overline{r_t P_t}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}m_N^*\end{array}\right)^2 \\ -\frac{\eta k_F \alpha_F^* \overline{r_t P_t}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}m_N^*\left(\begin{array}{c}1-\lambda^*+\frac{\eta k_N^* \alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N^*}^{1-\eta}}m_N^* \\ +\frac{\eta k_F^* \alpha_F^* \overline{r_t P_t}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}m_F^*\end{array}\right)^2\end{array}\right]}{\det(\Omega)}$$

$$+\frac{\sigma_a^2}{2}\frac{\left[\begin{array}{c}\left(1-\sum_{n=1}^{\infty} s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a \\ +\left(\begin{array}{c}\left(\frac{\eta k_F^* \alpha_F^* \overline{r_t P_t}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}}-\lambda^*\right) \\ \cdot\left(1-\sum_{n=1}^{\infty} s_n^*\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a-\lambda^*\end{array}\right)\end{array}\right]^2}{\det(\Omega)}$$

Then

$$
\begin{aligned}
& (\ln P_t)^{fixed} - (\ln P_t)^{flexible} \\
= \; & \gamma\sigma^2 \left(\frac{\eta k_F^* \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^* \right) \\
& +(1-\gamma) \left[\left(\widehat{P_{Nt}} \right)^{fixed} - \left(\widehat{P_{Nt}} \right)^{flexible} \right] \\
& +\phi_a^* \sigma_a^2 \left(1 - \sum_{n=1}^{\infty} s_n \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \\
& \cdot \left(\left(\frac{\eta k_F \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^* \right) \left(1 - \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \phi_a - \lambda^* \right)
\end{aligned}
$$

The gain in the consumption-led utility of switching to a fixed exchange rate regime is

$$
\begin{aligned}
& [E_{t-1} \ln C_t]^{fixed} - [E_{t-1} \ln C_t]^{flexible} \\
= \; & -\gamma\sigma^2 \left(\frac{\eta k_F^* \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^* \right) \\
& -(1-\gamma) \left[\left(\widehat{P_{Nt}} \right)^{fixed} - \left(\widehat{P_{Nt}} \right)^{flexible} \right] \\
& -\phi_a^* \sigma_a^2 \left(1 - \sum_{n=1}^{\infty} s_n \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \\
& \cdot \left(\left(\frac{\eta k_F \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^* \right) \left(1 - \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \phi_a - \lambda^* \right)
\end{aligned}
$$

As for the disutility from labor supply, up to the second order of σ, we approximately have

$$
\begin{aligned}
(E_{t-1} L_t)^{flexible} \; = \; & \bar{L} + \bar{L} E_{t-1} \left[\begin{array}{c} l_H \left(\begin{array}{c} \varepsilon_t^* + \left(1 - \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \phi_a \varepsilon_{at}^* \\ + \frac{\eta \alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} \widehat{\frac{r_t P_t}{W_t / A_t}} - \theta \widehat{P_{Ht}^*} \end{array} \right) \\ +l_N \left(\begin{array}{c} \varepsilon_t + \left(1 - \sum_{n=1}^{\infty} s_n \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \phi_a \varepsilon_{at} \\ + \frac{\eta \alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}} \widehat{\frac{r_t P_t}{W_t / A_t}} - \widehat{P_{Nt}} \end{array} \right) - \varepsilon_{at} \end{array} \right] \\
& + t.i.regime
\end{aligned}
$$

$$= \bar{L}\left[\begin{array}{c} 1-\left(l_H\left(\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+1\right)+l_N\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)\psi\left(\widehat{P_{Ht}^*}\right)^{flexible} \\ -\left(l_H\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N+l_N\left(\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N+1\right)\right)\left(\widehat{P_{Nt}}\right)^{flexible}\end{array}\right] + t.i.regime$$

where

$$\begin{aligned}
\bar{L} &= \frac{1}{\chi}\frac{i}{1+i}\left[\begin{array}{c}(1-\alpha_H)\left(\frac{\overline{W_t/A_t}}{\overline{MC_{Ht}}}\right)^{-\eta}\left(\frac{\overline{P_{Ht}^*}}{\overline{P_{Tt}^*}}\right)^{-\psi}\frac{(\gamma+d)\mu M_{t-1}^*}{A_tP_{Tt}^*} \\ +(1-\alpha_N)\left(\frac{\overline{W_t/A_t}}{\overline{MC_{Nt}}}\right)^{-\eta}\frac{(1-\gamma)\mu M_{t-1}^*}{A_tP_{Nt}}\end{array}\right] \\
l_H &= \frac{\frac{1}{\chi}\frac{i}{1+i}(1-\alpha_H)\left(\frac{\overline{W_t/A_t}}{\overline{MC_{Ht}}}\right)^{-\eta}\left(\frac{\overline{P_{Ht}^*}}{\overline{P_{Tt}^*}}\right)^{-\psi}\frac{(\gamma+d)\mu M_{t-1}^*}{A_tP_{Tt}^*}}{\bar{L}} \\
l_N &= \frac{\frac{1}{\chi}\frac{i}{1+i}(1-\alpha_N)\left(\frac{\overline{W_t/A_t}}{\overline{MC_{Nt}}}\right)^{-\eta}\frac{(1-\gamma)\mu M_{t-1}^*}{A_tP_{Nt}}}{\bar{L}}
\end{aligned}$$

and $t.i.regime$ means the term independent of the exchange rate regime choices.

Similarly, under a fixed exchange rate regime, we have

$$(E_{t-1}L_t)^{fixed} = \bar{L}\left[\begin{array}{c} 1-\left(l_H\left(\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+1\right)+l_N\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\right)\psi\left(\widehat{P_{Ht}^*}\right)^{fixed} \\ -\left(l_H\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N+l_N\left(\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N+1\right)\right)\left(\widehat{P_{Nt}}\right)^{fixed}\end{array}\right] + t.i.regime$$

As we have shown before, $\left(\widehat{P_{Ht}^*}\right)^{fixed}-\left(\widehat{P_{Ht}^*}\right)^{flexible}$ and $\left(\widehat{P_{Nt}}\right)^{fixed}-\left(\widehat{P_{Nt}}\right)^{flexible}$ are of the same order of σ^2. Then up to the second order of σ, the welfare gain in period t of switching to a fixed exchange rate regime is

$$
\begin{aligned}
& E_{t-1}\left[\ln C_t + \chi \ln\left(\frac{M_t}{P_t}\right) - \kappa L_t\right]^{fixed} - E_{t-1}\left[\ln C_t + \chi \ln\left(\frac{M_t}{P_t}\right) - \kappa L_t\right]^{flexible} \\
= \; & -\gamma(1+\chi)\sigma^2 \left(\frac{\eta k_F \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^*\right) \\
& -(1+\chi)(1-\gamma)\left(\left(\widehat{P_{Nt}}\right)^{fixed} - \left(\widehat{P_{Nt}}\right)^{flexible}\right) \\
& -\phi_a^* \sigma_a^2 \left(1 - \sum_{n=1}^{\infty} s_n \left(\frac{\rho - \rho^{n+1}}{1-\rho}\right)\right) \\
& \cdot \left(\left(\frac{\eta k_F \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^*\right)\left(1 - \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho - \rho^{n+1}}{1-\rho}\right)\right)\phi_a - \lambda^*\right) \\
& -\bar{L}\left[\begin{array}{c}
\psi\left(l_H\left(\frac{\eta\alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_H + 1\right) + l_N \frac{\eta\alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}} m_H\right) \\
\cdot\left(\left(\widehat{P_{Ht}^*}\right)^{fixed} - \left(\widehat{P_{Ht}^*}\right)^{flexible}\right) \\
+\left(l_H \frac{\eta\alpha_H \overline{r_t P_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}} m_N + l_N\left(\frac{\eta\alpha_N \overline{r_t P_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}} m_N + 1\right)\right) \\
\cdot\left(\left(\widehat{P_{Nt}}\right)^{fixed} - \left(\widehat{P_{Nt}}\right)^{flexible}\right)
\end{array}\right]
\end{aligned}
$$

Similarly to the proof of Proposition 2,

$$
K_{t+1} = \frac{1}{\chi}\frac{i}{1+i}\frac{M_t}{P_{Tt}}\left[(\gamma + d)\left(\frac{P_{Ht}^*}{P_{Tt}^*}\right)^{-\psi} + \frac{1-\omega}{\omega}\gamma\right]
$$

and

$$
V_{t+1}^{fixed} - V_{t+1}^{flexible} = \kappa \frac{\overline{rP}}{W}\bar{K}\left(\ln\left(\frac{K_{t+1}}{\bar{K}}\right)^{fixed} - \ln\left(\frac{K_{t+1}}{\bar{K}}\right)^{flexible}\right)
$$

$$
\begin{aligned}
& \ln\left(\frac{K_{t+1}}{\bar{K}}\right)^{fixed} - \ln\left(\frac{K_{t+1}}{\bar{K}}\right)^{flexible} \\
= \; & \ln\left(\frac{M_t^*}{P_{Tt}}\left[(\gamma+d)\left(\left(\frac{P_{Ht}^*}{P_{Tt}^*}\right)^{fixed}\right)^{-\psi} + \frac{1-\omega}{\omega}\gamma\right]\right) \\
& -\ln\left(\frac{M_t}{P_{Tt}}\left[(\gamma+d)\left(\left(\frac{P_{Ht}^*}{P_{Tt}^*}\right)^{flexible}\right)^{-\psi} + \frac{1-\omega}{\omega}\gamma\right]\right) + \widehat{i}_t^* - \widehat{i}_t \\
= \; & \left(\varepsilon_t^* - \phi_a^*\varepsilon_{at}^* - \psi q\left(\widehat{P_{Ht}^*}\right)^{fixed} - \left(\widehat{P_{Tt}}\right)^{fixed}\right) \\
& -\left(\varepsilon_t - \phi_a\varepsilon_{at} - \psi q\left(\widehat{P_{Ht}^*}\right)^{flexible} - \left(\widehat{P_{Tt}}\right)^{flexible}\right)
\end{aligned}
$$

Then the expected welfare difference between a fixed and a flexible exchange rate regime is

$$
\begin{aligned}
& V_t^{fixed} - V_t^{flexible} \\
= \; & -\gamma\left(1+\chi-\beta\kappa\frac{\overline{rP}}{W}\bar{K}\right)\left(\frac{\eta k_F\alpha_F^*\overline{r_t^*P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^*\right)\sigma^2 \\
& -\beta\kappa\frac{\overline{rP}}{W}\bar{K}\frac{\phi_a^2+\phi_a^{*2}}{2}\sigma_a^2 \\
& -\phi_a^*\sigma_a^2\left(1-\sum_{n=1}^{\infty}s_n\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right) \\
& \cdot\left(\left(\frac{\eta k_F\alpha_F^*\overline{r_t^*P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^*\right)\left(1-\sum_{n=1}^{\infty}s_n^*\left(\frac{\rho-\rho^{n+1}}{1-\rho}\right)\right)\phi_a - \lambda^*\right) \\
& -\left[\begin{array}{c} (1-\gamma)(1+\chi) \\ +\left(\begin{array}{c} l_H\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_N \\ +l_N\left(\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_N+1\right)\end{array}\right)\bar{L}\end{array}\right]\left(\left(\widehat{P_{Nt}}\right)^{fixed} - \left(\widehat{P_{Nt}}\right)^{flexible}\right) \\
& -\psi\left[\beta\kappa\frac{\overline{rP}}{W}\bar{K}q + \left(\begin{array}{c} l_H\left(\frac{\eta\alpha_H\overline{r_tP_t}^{1-\eta}}{\overline{MC_H}^{1-\eta}}m_H+1\right) \\ +l_N\frac{\eta\alpha_N\overline{r_tP_t}^{1-\eta}}{\overline{MC_N}^{1-\eta}}m_H\end{array}\right)\bar{L}\right]\left(\left(\widehat{P_{Ht}^*}\right)^{fixed} - \left(\widehat{P_{Ht}^*}\right)^{flexible}\right)
\end{aligned}
$$

The first three terms on the right hand side of (4.27) are independent of the home production and have ambiguous signs. The fourth term is negative and the last term has an ambiguous sign according to Proposition 3.

For very large $\bar{A}$, for instance $\bar{A} \to \infty$, the last term on the right hand side of (4.27) is negative. According to Proposition **??**, as $\bar{A} \to \infty$, $\left(\widehat{P^*_{Ht}}\right)^{fixed} - \left(\widehat{P^*_{Ht}}\right)^{flexible}$ converges to infinity, which will dominate the first three terms on the right hand side of (4.27). Then we have

$$V_t^{flexible} > V_t^{fixed}$$

It is optimal for the policy maker to choose a flexible exchange rate regime.

For very small $\bar{A}$, for instance $\bar{A} \to 0$, the fourth term disappears and

$$\begin{aligned}
& V_t^{fixed} - V_t^{flexible} \\
= \quad & -\gamma \left(1 - \beta\kappa \overline{\frac{rP}{W}} \bar{K}\right) \sigma^2 \left(\frac{\eta k_F \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^* \right) \\
& -\beta\kappa \overline{\frac{rP}{W}} \bar{K} \frac{\phi_a^2 + \phi_a^{*2}}{2} \sigma_a^2 + \psi \left(\beta\kappa \overline{\frac{rP}{W}} \bar{K} q + l_H \bar{L} \right) \lambda \sigma^2 \\
& -\phi_a^* \sigma_a^2 \left(1 - \sum_{n=1}^{\infty} s_n \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \\
& \cdot \left(\left(\frac{\eta k_F^* \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^* \right) \left(1 - \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \phi_a - \lambda^* \right)
\end{aligned}$$

If

$$\lambda > \frac{\left[\begin{array}{c} \phi_a^* \frac{\sigma_a^2}{\sigma^2} \left(1 - \sum_{n=1}^{\infty} s_n \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \\ \cdot \left(\left(\frac{\eta k_F^* \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^* \right) \left(1 - \sum_{n=1}^{\infty} s_n^* \left(\frac{\rho - \rho^{n+1}}{1-\rho} \right) \right) \phi_a - \lambda^* \right) \\ +\gamma \left(1 + \chi - \beta\kappa \overline{\frac{rP}{W}} \bar{K} \right) \left(\frac{\eta k_F \alpha_F^* \overline{r_t^* P_t^*}^{1-\eta}}{\overline{MC_F^*}^{1-\eta}} - \lambda^* \right) \\ +\beta\kappa \overline{\frac{rP}{W}} \bar{K} \frac{\phi_a^2 + \phi_a^{*2}}{2} \frac{\sigma_a^2}{\sigma^2} \end{array} \right]}{\psi \left(\beta\kappa \overline{\frac{rP}{W}} \bar{K} q + l_H \bar{L} \right)}$$

then we have

$$V_t^{flexible} < V_t^{fixed}$$

which means, for countries with very low labor productivities, it is optimal for the policy maker to choose a fixed exchange rate regime.

Notice that, allowing for labor productivity shocks, the additional term compared to the benchmark case does not depend on $\bar{A}$, therefore, we have

$$\frac{\partial\left[V_t^{fixed}-V_t^{flexible}\right]}{\partial\bar{A}}<0$$

and the same results hold as in Proposition 2. $\square$

Appendices to Chapter 2

A2.1. Proof of Proposition 5

Proof. The first order conditions for a woman and a man, respectively, are:

$$-u_{1w}'+\left[\begin{array}{c}\kappa u_{2w}'\left(\delta^w+\left[\frac{1}{\phi}\left(1-F\left(\bar{\eta}^w\right)\right)+\bar{\eta}^m f(\bar{\eta}^w)\right]\right)\\+\left(1-\delta^w\right)u_{2w,n}'+f\left(\bar{\eta}^w\right)\kappa u_{2w}'\left(u_{2w}-u_{2w,n}\right)\end{array}\right]=0$$

$$-u_{1m}'+\left[\begin{array}{c}\kappa u_{2m}'\left(\delta^m+\left[\phi\left(1-F\left(\bar{\eta}^m\right)\right)+\bar{\eta}^w f(\bar{\eta}^m)\right]\right)\\+\left(1-\delta^m\right)u_{2m,n}'+f\left(\bar{\eta}^m\right)\kappa u_{2m}'\left(u_{2m}-u_{2m,n}\right)\end{array}\right]=0$$

We show by contradiction that $\bar{\eta}^w=u_{2m,n}-u_{2m}$ and $\bar{\eta}^m=M(\bar{\eta}^w)$ hold for $\phi\geq 1$. Suppose not, then

$$\bar{\eta}^m>M(\bar{\eta}^w)\geq\bar{\eta}^w$$

where the second inequality holds because $\phi\geq 1$. Then we have

$$\bar{\eta}^m=u\left(Rs^w y\right)-u\left(\kappa(Rs^w y+Rs^m y)\right)>\bar{\eta}^w\geq u\left(Rs^m y\right)-u\left(\kappa(Rs^w y+Rs^m y)\right)$$

and hence, $s^w>s^m$.

Then

$$\begin{aligned}u_{1w}' &= \delta^w\kappa u_{2w}'\left(1+\left[\frac{1}{\phi}\left(1-F\left(\bar{\eta}^w\right)\right)+M(\bar{\eta}^w)f(\bar{\eta}^w)\right]\right)\\&\quad+\left(1-\delta^w\right)u_{2w,n}'+f\left(\bar{\eta}^w\right)\kappa u_{2w}'\left(u_{2w}-u_{2w,n}\right)\end{aligned}$$

$$
\begin{aligned}
&< \delta^m \kappa u'_{2w}\left(1+\left[\frac{1}{\phi}\left(1-F\left(\bar{\eta}^w\right)\right)+M(\bar{\eta}^w)f(\bar{\eta}^w)\right]\right) \\
&\quad +\left(1-\delta^m\right)u'_{2w,n}+f\left(\bar{\eta}^w\right)\kappa u'_{2w}\left(u_{2w}-u_{2w,n}\right) \\
&< \delta^m \kappa u'_{2m}\left(1+\left[\phi\left(1-F\left(\bar{\eta}^m\right)\right)+M^{-1}(\bar{\eta}^m)f(\bar{\eta}^m)\right]\right) \\
&\quad +\left(1-\delta^m\right)u'_{2m,n}+f\left(\bar{\eta}^m\right)\kappa u'_{2m}\left(u_{2m}-u_{2m,n}\right) \\
&= u'_{1m}
\end{aligned}
$$

[4]which means that

$$s^m > s^w$$

Contradiction. Therefore, we have $\bar{\eta}^m = M(\bar{\eta}^w)$ and $s^m \geq s^w$[5] for $\phi \geq 1$.

Since $\frac{1}{2} \leq \kappa \leq 1$, at $\phi = 1$

$$\kappa(Rs^m y + Rs^w y) \geq \max\left(Rs^w y,\, Rs^m y\right)$$

Then, in the neighbourhood of $\phi = 1$, we have $\kappa u'_{2m} < u'_{2m,n}$.[6]

We proceed in two steps. In the first step, we assume that inequality $\kappa u'_{2m} < u'_{2m,n}$ holds for all values of ϕ, and prove that a higher sex ratio leads to a higher savings rate. In the second step, we prove by contradiction that the inequality indeed holds for all values of ϕ.

Substitute the expression of $\bar{\eta}^w$ and $\bar{\eta}^m$ into (4.40) and (4.41), totally differentiating the

---

[4]The second inequality holds because (i)

$$\frac{1}{\phi}\left(1-F\left(\bar{\eta}^w\right)\right)+M(\bar{\eta}^w)f(\bar{\eta}^w) = \phi\left(1-F(\bar{\eta}^m)\right)+M^{-1}(\bar{\eta}^m)f(\bar{\eta}^m)$$

by using the uniform distribution assumption; and (ii),

$$u_{2m}-u_{2m,n} > u_{2w}-u_{2w,n}$$

[5]We can $s^m \geq s^w$ for $\phi \geq 1$ by contradiction. Suppose not, then $s^w > s^m$, following the same steps as showing $\bar{\eta}^m = M(\bar{\eta}^w)$, we find a contradiction.

[6]The conditions for the equation $\kappa u'_{2m} = u'_{2m,n}$ are $\kappa(Rs^m y + Rs^w y) \geq \max\left(Rs^w y,\, Rs^m y\right)$ and $\kappa = 1$, which cannot hold at the same time.

system and re-arrange the matrix, we obtain

$$\Omega \cdot \mathbf{ds} = \mathbf{dz} \tag{4.28}$$

where

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}, \ \mathbf{ds} = \begin{pmatrix} ds^w \\ ds^m \end{pmatrix} \text{ and } \mathbf{dz} = \begin{pmatrix} 0 \\ A \end{pmatrix}$$

$$\begin{aligned}
\Omega_{11} &= u''_{1w}y + Ry \begin{bmatrix} \kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right) + \left(1-\delta^w\right) u''_{2w,n} \\ +f\left(\bar{\eta}^w\right)\kappa^2 u''_{2w}\left(u_{2w} + M(\bar{\eta}^w) - u_{2w,n}\right) \\ +2f\left(\bar{\eta}^w\right)\kappa u'_{2w}\left(\kappa u'_{2w} - u'_{2w,n}\right) \end{bmatrix} \\
\Omega_{12} &= Ry \begin{bmatrix} \kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right) \\ +f\left(\bar{\eta}^w\right)\kappa^2 u''_{2w}\left(u_{2w} + M(\bar{\eta}^w) - u_{2w,n}\right) \\ +f\left(\bar{\eta}^w\right)\kappa^2 u'^2_{2w} + f(\bar{\eta}^w)\left(u'_{2m,n} - \kappa u'_{2m}\right)\left(u'_{2w,n} - \kappa u'_{2w}\right) \end{bmatrix} \\
\Omega_{21} &= Ry \begin{bmatrix} \kappa^2 u''_{2m}\left(\left(1+\phi\right)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right) \\ +\frac{f(\bar{\eta}^w)\kappa u'_{2m}}{\phi}\left(\kappa u'_{2m} - u'_{2m,n}\right) + f\left(\bar{\eta}^w\right)\left(\kappa u'_{2m}\right)^2 \end{bmatrix} \\
\Omega_{22} &= u''_{1m}y + Ry \begin{bmatrix} \kappa^2 u''_{2m}\left(\left(1+\phi\right)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right) + \left(1-\delta^m\right) u''_{2m,n} \\ -\frac{1}{\phi} f\left(\bar{\eta}^w\right)\left(\kappa u'_{2m} - u'_{2m,n}\right)^2 \\ +f\left(\bar{\eta}^w\right)\left(\kappa u'_{2m} - u'_{2m,n}\right)\kappa u'_{2m} \end{bmatrix}
\end{aligned}$$

and

$$A = \frac{1}{\phi^2}\left[1 - F(\bar{\eta}^w)\right]\left(\kappa u'_{2m} - u'_{2m,n}\right) < 0$$

It is easy to show that

$$\begin{aligned}
\det\left(\Omega\right) &= positve.terms - \frac{Ryf\left(\bar{\eta}^w\right)\left(\kappa u'_{2m} - u'_{2m,n}\right)^2 \Omega_{11}}{\phi} + u''_{1m}u''_{1w}y^2 \\
&\quad + \frac{\left(\kappa u'_2\right)^2 R^2 y^2 \left[u'_{2w,n}u'_{2m,n} - \kappa u'_2\left(u'_{2w,n} + u'_{2m,n}\right)\right]}{\left(\eta^{\max} - \eta^{\min}\right)^2}
\end{aligned}$$

$$> \frac{R^2 y^2 \frac{(\kappa u_2')^2}{\phi} \left(u_{2m,n}'^2 - u_{2m,n}' \kappa u_2' - (\kappa u_2')^2 \right)}{(\eta^{\max} - \eta^{\min})^2} + u_{1m}'' u_{1w}'' y^2$$

Under the assumption

$$E\eta \geq \frac{R\kappa u_2'}{2} \sqrt{\frac{\max\left(0, \kappa u_2' \left(u_{2w,n}' + u_{2m,n}' \right) - u_{2w,n}' u_{2m,n}' \right)}{u_{1m}'' u_{1w}''}}$$

we can obtain

$$u_{1w}'' u_{1m}'' y^2 + \frac{(\kappa u_2')^2 R^2 y^2 \left[u_{2w,n}' u_{2m,n}' - \kappa u_2' \left(u_{2w,n}' + u_{2m,n}' \right) \right]}{(\eta^{\max} - \eta^{\min})^2} \geq 0$$

and hence $\det(\Omega) > 0$.

The derivative of savings rate with respect to the sex ratio are as following:

$$\frac{ds^m}{d\phi} = \frac{A\Omega_{11}}{\det(\Omega)} > 0 \text{ and } \frac{ds^w}{d\phi} = -\frac{A\Omega_{12}}{\det(\Omega)}$$

The sign of $\frac{ds^w}{d\phi}$ is ambiguous. As we showed, for $\phi > 1$, $s^w \leq s^m$, which means that even if $\frac{ds^w}{d\phi} \geq 0$, we will have $\frac{ds^w}{d\phi} \leq \frac{ds^m}{d\phi}$.

The response of the aggregate savings rate in the young cohort to the rise in the sex ratio is as following:

$$\begin{aligned}
\frac{ds^y}{d\phi} &= \frac{s^m - s^w}{(1+\phi)^2} + \frac{\phi - 1}{1+\phi}\frac{ds^m}{d\phi} + \frac{1}{1+\phi}\left(\frac{ds^m}{d\phi} + \frac{ds^w}{d\phi} \right) \\
&= \frac{s^m - s^w}{(1+\phi)^2} + \frac{\phi - 1}{1+\phi}\frac{ds^m}{d\phi} \\
&\quad + \frac{1}{1+\phi} \frac{A \left(u_{1w}'' y + Ry \left[\begin{array}{c} (1-\delta^w) u_{2w,n}'' \\ -f(\bar{\eta}^w) \left(\begin{array}{c} \kappa u_2' \left(u_{2w,n}' - u_{2m,n}' \right) \\ + u_{2w,n}' u_{2m,n}' \end{array} \right) \end{array} \right] \right)}{\det(\Omega)}
\end{aligned}$$

Obviously, the second term on the right hand side are positive. When $\phi = 1$, both women and men have the same savings rates. As the sex ratio rises, we have shown that men raise their savings while women reduce their savings. Then $s^m > s^w$, which implies that the first and the third terms on the right hand side are positive. Therefore, the aggregate savings rate of the young cohort increases as the sex ratio rises. Since the (dis-)savings rate of the old cohort is fixed, an increase in the savings rate by the young cohort translates into an increase in the economy-wide savings rate.

We now show by contradiction that $\kappa u'_{2m} < u'_{2m,n}$ must hold for all ϕs. Suppose not, then $\kappa u'_{2m} > u'_{2m,n}$ may fail sometime. Due to continuity of E, there exists a level of sex ratio ϕ_0 at which $\kappa u'_{2m} = u'_{2m,n}$, which implies that $A = 0$. Then

$$\frac{ds^m}{d\phi} = \frac{ds^w}{d\phi} = 0$$

We calculate the derivative of A with respect to ϕ at $\phi = \phi_0$ and obtain

$$\left.\frac{dA}{d\phi}\right|_{\phi=\phi_0} = \frac{Ry}{\phi_0^2}\left[1 - F(\bar{\eta}^w)\right]\left(\kappa u''_{2m}\left(\frac{ds^m}{d\phi} + \frac{ds^w}{d\phi}\right) - u'_{2m,n}\frac{ds^m}{d\phi}\right) = 0$$

Then we have

$$\begin{aligned}
\left.\frac{d^2 s^w}{d\phi^2}\right|_{\phi=\phi_0} &= \frac{\Omega_{12}}{\det(\Omega)}\left.\frac{dA}{d\phi}\right|_{\phi=\phi_0} = 0 \\
\left.\frac{d^2 s^m}{d\phi^2}\right|_{\phi=\phi_0} &= -\frac{\Omega_{11}}{\det(\Omega)}\left.\frac{dA}{d\phi}\right|_{\phi=\phi_0} = 0
\end{aligned}$$

Using the result that $\left.\frac{d^2 s^w}{d\phi^2}\right|_{\phi=\phi_0} = \left.\frac{d^2 s^m}{d\phi^2}\right|_{\phi=\phi_0} = 0$, we calculate the second order derivative of A with respect to ϕ at $\phi = \phi_0$,

$$\left.\frac{d^2 A}{d\phi^2}\right|_{\phi=\phi_0} = \frac{Ry}{\phi_0^2}\left[1 - F(\bar{\eta}^w)\right]\left(\kappa u''_{2m}\left(\frac{d^2 s^m}{d\phi^2} + \frac{d^2 s^w}{d\phi^2}\right) - u'_{2m,n}\frac{d^2 s^m}{d\phi^2}\right) = 0$$

Then

$$\left.\frac{d^3 s^w}{d\phi^3}\right|_{\phi=\phi_0} = \frac{\Omega_{12}}{\det(\Omega)}\left.\frac{d^2 A}{d\phi^2}\right|_{\phi=\phi_0} = 0$$
$$\left.\frac{d^3 s^m}{d\phi^3}\right|_{\phi=\phi_0} = -\frac{\Omega_{11}}{\det(\Omega)}\left.\frac{d^2 A}{d\phi^2}\right|_{\phi=\phi_0} = 0$$

By iterating this process forward, we obtain that

$$A|_{\phi=\phi_0} = 0 \text{ and } \left.\frac{d^k A}{d\phi^k}\right|_{\phi=\phi_0} = 0 \text{ for any } k > 0$$

This means that A equals zero for all ϕs, which contradicts with the result at the beginning of the proof that $A \neq 0$ when $\phi = 1$. In other words, there exists no ϕ_0 such that $A = 0$ holds. Therefore, inequality $\kappa u'_{2m} < u'_{2m,n}$ must hold for all ϕs.

Remarks: In this proof, we have assumed a uniform distribution for emotional utility η^i ($i = w, m$). We note that many other distributions can give us the same results as long as they satisfy three sufficient conditions:[7]

$$\frac{\partial \int f(M(\eta^w))d\eta^w}{\partial \phi} \geq 0, \; \frac{\partial \left[\frac{1}{\phi}\int \frac{f(\eta^w)}{f(M(\eta^w))}dF(\eta^w) + M(\bar{\eta}^w)f(\bar{\eta}^w)\right]}{\partial \phi} \leq 0$$

and $f(\bar{\eta}^w)$ is small enough.

The first sufficient condition is equivalent to

$$\int \frac{f'(M(\eta^w))}{f(M(\eta^w))}\frac{1-F(\eta^w)}{\phi^2}d\eta^w \geq 0$$

and the second one is equivalent to

$$\frac{1}{\phi^2}\int \left[\frac{f(\eta^w)}{f(M(\eta^w))} - \frac{f(\bar{\eta}^w)}{f(M(\bar{\eta}^w))}\right]dF(\eta^w) + \frac{1}{\phi}\int \frac{f(\eta^w)f'(M(\eta^w))}{f^3(M(\eta^w))}\frac{1-F(\eta^w)}{\phi^2}d\eta^w \geq 0$$

□

[7] We use normal distributions in the calibration section, which gives the same qualitative result.

A2.2. Proof of Proposition 6

Proof. Under the log utility assumption, at the balanced sex ratio, (2.10) becomes

$$-\frac{1}{1-s}+\frac{\beta}{s}=0 \tag{4.29}$$

where we use the fact that men and women are symmetric when $\phi = 1$ and hence they choose the same savings rate. Notice that (4.29) is the same first order condition for a life-time bachelor, then even if there are some men or women choosing to be single, they choose the same savings rate s. For a representative woman, at the balanced sex ratio, if she chooses to enter the marriage market, with probability $F(\bar{\eta})$ she can get married and receive welfare

$$\begin{aligned} V^w &= \ln((1-s)y)+\beta F(\bar{\eta})\ln(\kappa R(2s)y) \\ &\quad +\beta(1-F(\bar{\eta}))\ln(Rsy)+E[\eta|\,\eta^w\geq\bar{\eta}] \\ &\geq \ln((1-s)y)+\beta\ln(Rsy)=V_n^w \end{aligned}$$

where the inequality holds because $\kappa > 1/2$ and $E[\eta|\,\eta^w \geq \bar{\eta}] \geq 0$. Therefore, entering the marriage market is a dominant strategy for all women. Since men and women are symmetric when $\phi = 1$, all men and all women will enter the marriage market with probability one at the balanced sex ratio.

As we have showed in Proposition 5,

$$\frac{ds^m}{d\phi}>0 \text{ and } \frac{ds^m}{d\phi}+\frac{ds^w}{d\phi}>0$$

we can show that

$$\begin{aligned} \frac{\partial V^m}{\partial\phi} &= y\left(-u'_{1m}+\kappa\delta^m u'_{2m}+(1-\delta^m)u'_{2m,n}\right)\frac{ds^m}{d\phi} \\ &\quad +\delta^m y\kappa u'_{2w}\frac{ds^w}{d\phi}-\beta\int_{M(\bar{\eta}^w)}[1-F(\eta)]\,d\eta \end{aligned} \tag{4.30}$$

$$< -\beta \int_{M(\bar{\eta}^w)} [1 - F(\eta)]\, d\eta - (\phi - 1)\, \delta^m y \kappa u'_{2w} \frac{ds^m}{d\phi} < 0$$

where the first equality in (4.30) holds because

$$\begin{aligned} \frac{\partial \delta^m}{\partial \phi} &= -\frac{1 - F(\bar{\eta}^w)}{\phi^2} (u_{2m} - u_{2m,n}) \\ &\quad -\frac{Ryf(\bar{\eta}^w)}{\phi} \left[u'_{2m,n} \frac{ds^m}{d\phi} - \kappa u'_{2m} \left(\frac{ds^m}{d\phi} + \frac{ds^w}{d\phi} \right) \right] (u_{2m} - u_{2m,n}) \end{aligned}$$

$$\begin{aligned} \frac{\partial \left(\int_{M(\bar{\eta}^w)} M^{-1}(\eta^m) dF(\eta^m) \right)}{\partial \phi} &= -\int_{M(\bar{\eta}^w)} [1 - F(\eta)]\, d\eta - \frac{\bar{\eta}^w (1 - F(\bar{\eta}^w))}{\phi^2} \\ &\quad -\frac{\bar{\eta}^w f(\bar{\eta}^w)}{\phi} \left[u'_{2m,n} \frac{ds^m}{d\phi} - \kappa u'_{2m} \left(\frac{ds^m}{d\phi} + \frac{ds^w}{d\phi} \right) \right] \end{aligned}$$

and the first inequality in (4.30) holds because

$$\delta^m < 1 < \phi (1 - F(\bar{\eta}^m)) + \bar{\eta}^m f(\bar{\eta}^m) \text{ and } \frac{ds^w}{d\phi} \leq \frac{ds^m}{d\phi}$$

Men lose as the sex ratio rises while the effect on women's welfare is ambiguous.

Now consider women's welfare. Given the equilibrium s^m and s^w under a sex ratio of ϕ, if a woman deviates from the equilibrium choice s^w, for instance, by choosing a savings rate $s^{w\prime} = s^m$, then she would receive a lower life-time utility $V^{w\prime}$ ($\leq V^w$). Since $s^{w\prime} = s^m \geq s^w$, this woman will have a better situation than all other women in the marriage market, i.e., she is more likely to get married and also more likely to marry a better man. Then

$$\begin{aligned} V^{w\prime} &= u_{1w'} + \beta \left[\delta' u_{2w'} + \left(1 - \delta'\right) u_{2w',n} + \int_{\bar{\eta}^w} M\left(\eta^w + u_{2w'} - u_{2w}\right) dF\left(\eta^w\right) \right] \\ &\geq u_{1w'} + \beta \left[(1 - F(\bar{\eta}^w))\, u_{2w'} + F(\bar{\eta}^w)\, u_{2w',n} + \int_{\bar{\eta}^w} M(\eta^w)\, dF(\eta^w) \right] \\ &= u_{1m} + \beta \left[(1 - F(\bar{\eta}^w))\, u_{2m} + F(\bar{\eta}^w)\, u_{2m,n} + \int_{\bar{\eta}^w} M(\eta^w)\, dF(\eta^w) \right] \end{aligned}$$

$$
\begin{aligned}
&\geq \quad u_{1m} + \beta \left[\begin{array}{c} \left(1 - F(M\left(\bar{\eta}^{w}\right))\right) u_{2m} + F\left(M\left(\bar{\eta}^{w}\right)\right) u_{2m,n} \\ + \int_{M\left(\bar{\eta}^{w}\right)} M^{-1}\left(\eta^{m}\right) dF\left(\eta^{m}\right) \end{array} \right] \\
&= \quad V^{m}
\end{aligned}
$$

where $u_{1w'}$, $u_{2w'}$ and $u_{2w',n}$ denote the first period consumption-led utility, the second period consumption-led utilty when she gets married, and the second period utility when she fails to get matched with any man, respectively. u_{2w} is the second period consumption-led utility for all other women who get married. The first inequality holds because the woman faces a greater possibility of getting married and also she will receive a higher expected emotional utility from her husband. The second inequality holds because, women are more likely than men to get married and also women are expecting to receive higher emotional utilities from their spouses than men.

Therefore, for $\phi \geq 1$, we can show that $V^{w} \geq V^{m}$, women always achieve higher welfare than men.

For a representative man in the marriage market, given his rivals' choices, if he choose to stay in the marriage market, he will follow the first order condition (4.41) and achieves an approximate life time utility $u_{1m} + \beta u_{2m,n}$. If he chooses to be single, he maximizes the life time utility $u_1 + \beta u_2$. The first order condition in this case is

$$
-u_{1m}' + u_{2m}' = 0
$$

The two savings decisions, in the marriage market and being single, will be different since the representative man would follow different first order conditions. Then

$$
V_{n}^{m} = \max u_1 + \beta u_2 > u_{1m} + \beta u_{2m,n} \rightarrow V^{m}
$$

when $\phi \rightarrow \infty$. The representative man will then choose to be single which violates the assumption that, for all ϕs, entering the marriage market is the dominant strategy for all men. Therefore, there much be a threshold ϕ_1 such that for $\phi \geq \phi_1$, $V_{n}^{m} = V^{m}$.

For $\phi \geq \phi_1$, with probability $\frac{\phi_1}{\phi}$, a representative man will choose to enter the marriage market, and with probability $1 - \frac{\phi_1}{\phi}$, he remains single. For a representative woman, since she earns the same first period income as a representative man, we can show that

$$V_n^w = V_n^m = V^m < V^w$$

Therefore, the representative woman would choose to enter the marriage market with probability one.

As for the aggregate savings rate in the young cohort, we have showed in Proposition 5 that for $\phi < \phi_1$, as the sex ratio rises, the aggregate savings rate in the young cohort will rise. For $\phi \geq \phi_1$, as the sex ratio rises, some men begin quitting the marriage market and choose a different savings rate according to (4.43). Compare (4.41) with (4.43), it is ambiguous whether $s^m > s_n^m$ or not, then the effect on the aggregate savings rate is ambiguous. □

A2.3. Proof of Proposition 7

Proof. We rewrite the the economy-wide savings rate as following

$$s_t^P = (1-\alpha)\left(\frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w - s^{young}\right) + \frac{\alpha}{R}$$

As we have shown in Proposition 5, $\frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$ strictly increases in ϕ since men will save at a higher rate than women. s^P then is an increasing function of ϕ. By the expression of the current account to GDP ratio, this is also the condition that the current account is an increasing function of the sex ratio. Therefore, the economy-wide savings rate and the current account rise as the sex ratio becomes more unbalanced. □

A2.4. Proof of Proposition 8

Proof. Since capital can flow freely internationally, the interest rates are equal in both countries. By (4.2) and (4.3), the wage rates are also equal in the two countries.

Given the same wage rates, the households in the two countries have the same first period income. By Proposition 5, Country 2 will have a higher savings rate than Country 1. On the other hand, in equilibrium, given a constant R, the investments in both countries are the same, and the world capital market always clears. Therefore, Country 2 runs a current account surplus and Country 1 runs a current account deficit. $\square$

A2.5. Parental savings and endogenous sex ratios

In this appendix we make the sex ratio for any cohort to be an endogenous choice of their parents. We introdue parental savings for children, which is a part of the economy-wide household savings. To incorporate these features, we consider an OLG model in which every cohort lives two periods (young and old). Everyone works and earns labor income in the first period. If one gets married, the marriage takes place at the beginning of the second period, and the couple produces a single child right away. They derive direct emotional utility from having a child, and the value of this emotional utility could depend on the gender of the child. Parents are altrustic toward their child and can save for their child (and transfer the savings to the child to augument his/her income).

As noted in Wei and Zhang (2011), widespread sex selective abortions are a relatively recent phenomenon because the inexpensive technology (especially Ultrasound B machines) used to detect the gender of a fetus became available only within the last three decades. For example, 1985 was the first year in which half of the county-level hospitables in acquired at least one ultrasound B machine (Li and Zheng, 2009). Therefore, the first cohort born with a severe sex ratio imbalance was entering the marriage market around 2003. In the model, we assume sex-selective abortions are not technologically feasible in periods before t_0 so that the sex ratio is always balanced. Starting from period t_0 that parents can directly choose a sex ratio ϕ_t for the next cohort cohort. As a result, parents in period t have a son with probability of $\frac{\phi_t}{1+\phi_t}$, and a daughter with probability of $\frac{1}{1+\phi_t}$.

Parents can save for their child, and that savings potentially depends on the gender of their child. Let T^i_{t+1}[8] be the amount of the parental savings for their child, where $i = w$ (a daughter) or m (a son). A young person's first-period income is the sum of the labor income y and the transfer from her/his parents:

$$y^i_{t+1} = y + T^i_{t+1}$$

With this setup, the optimization problem for a representative young woman who enters the marriage market is

$$V^w_t = \max_{S^W_t} \left\{ u(c^w_{1t}) + \beta E \left[u(c^w_{2,t+1}) + \eta^m \right] + \beta E \left[\frac{\phi \left(\theta V^m_{t+1} + \eta^s \right)}{1+\phi} + \frac{\theta V^w_{t+1} + \eta^d}{1+\phi} \right] \right\}$$

where V^m_{t+1} (V^w_{t+1}) is the life-time utility of the woman's son (daughter). η^s and η^d are the emotional utility each parent obtains from having a son and a daughter, respectively. θ is the parameter representing the degree of parental altruism toward their child which is assumed to be independent of the child's gender. We assume $\theta \leq 1/2$.[9]

Let c^w_{1t} and S^w_t denote the representative woman's first-period consumption and savings, respectively. Naturally,

$$c^w_{1t} = y^w_t - S^w_t$$

If she fails to get married, her second-period consumption is

$$c^{w,n}_{2,t+1} = RS^w_t$$

If she gets married, her second-period consumption is

$$c^{w,i}_{2,t+1} = \kappa \left(RS^w_t + RS^m_t - T^i_{t+1} \right)$$

where S^m_t is the first period savings by a representative man and i (=w or m) stands for

[8] T^i can be negative, which means young people make a transfer to their parents.

[9] This assumption is made to ensure the existence of a steady state in the long run equilibrium.

the child's gender.

As in the benchmark model, we assume a uniform distribution for η^i. The optimization condition for the representative woman is[10]

$$u'_{1w,t} = \left[\begin{array}{c} \kappa \left(\left(1 + \frac{1}{\phi_t}\right)(1 - F(\bar{\eta}^w_t)) + M(\bar{\eta}^w_t) f(\bar{\eta}^w_t) \right) \left(\frac{\phi_{t+1} u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}} \right) \\ + f(\bar{\eta}^w_t)\, \kappa \left(\frac{\phi_{t+1} u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}} \right) \left(\begin{array}{c} \frac{\phi_{t+1}\left(u^{w,m}_{2,t+1} + \theta V^m_{t+1} + \eta^s\right)}{1+\phi_{t+1}} + \frac{u^{w,w}_{2,t+1} + \theta V^w_{t+1} + \eta^d}{1+\phi_{t+1}} \\ - u_{2w,n,t+1} \end{array} \right) \\ + (1 - \delta^w_t) u'_{2w,n,t+1} \end{array} \right] \tag{4.31}$$

where $u'_{2,m,t+1}$ and $u'_{2,w,t+1}$ stand for her marginal utilities when she has a son and a daughter in the second period, respectively. $u^{w,m}_{2,t+1}$ and $u^{w,w}_{2,t+1}$ stand for the utilities obtained from consumption when the representative woman has a son and a daughter, respectively.

Similarly, for a representative man, the optimal condition is

$$u'_{1m,t} = \left[\begin{array}{c} \kappa \left((1 + \phi_t)(1 - F(\bar{\eta}^m_t)) + M^{-1}(\bar{\eta}^m_t) f(\bar{\eta}^m_t) \right) \left(\frac{\phi_{t+1} u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}} \right) \\ + f(\bar{\eta}^m_t)\, \kappa \left(\frac{\phi_{t+1} u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}} \right) \left(\begin{array}{c} \frac{\phi_{t+1}\left(u^{w,m}_{2,t+1} + \theta V^m_{t+1} + \eta^s\right)}{1+\phi_{t+1}} + \frac{u^{w,w}_{2,t+1} + \theta V^w_{t+1} + \eta^d}{1+\phi_{t+1}} \\ - u_{2m,n,t+1} \end{array} \right) \\ + (1 - \delta^m_t)\, u'_{2m,n,t+1} \end{array} \right] \tag{4.32}$$

Parents optimally choose how much to save for (and transfer to) their children. For parents with a daughter, their optimization problem is

$$\max_{T^w_t} u(c_{2,w,t}) + \theta V^w_t$$

---

[10] In this extension,

$$\bar{\eta}^w = \max\left(M^{-1}(\bar{\eta}^m), u_{2m,n} - \left(\frac{\phi\left(u_{2w,m} + \theta\left(V^m_{t+1} + \eta^s\right)\right)}{1+\phi} + \frac{u_{2w,w} + \theta\left(V^w_{t+1} + \eta^d\right)}{1+\phi} \right) \right)$$

and

$$\bar{\eta}^m = \max\left(M(\bar{\eta}^w), u_{2w,n} - \left(\frac{\phi\left(u_{2w,m} + \theta\left(V^m_{t+1} + \eta^s\right)\right)}{1+\phi} + \frac{u_{2w,w} + \theta\left(V^w_{t+1} + \eta^d\right)}{1+\phi} \right) \right)$$

The first order condition with respect to T_t^w is

$$-\kappa u'_{2,w,t} + \theta u'_{1w,t} = 0 \tag{4.33}$$

Similarly, the first order condition for parents with a son is

$$-\kappa u'_{2,m,t} + \theta u'_{1m,t} = 0 \tag{4.34}$$

Parents also optimally choose the sex ratio (although they don't directly choose the gender of the child). The first order condition on the sex ratio chosen by parents in period t is

$$(u_{2,m,t} + \theta V_t^m + \eta^s) - \left(u_{2,w,t} + \theta V_t^w + \eta^d\right) = 0 \tag{4.35}$$

Since this paper focuses on countries with a preference for son, we assume $\eta^s \geq \eta^d$. We also make the Darwinian assumption that $E\eta^m$ and $E\eta^w$ are sufficiently large so that marriage is strongly attractive. Totally differentiating (4.31), (4.32), (4.33) and (4.34), we have the following proposition:

Proposition 16. *Assume emotional utilities are drawn from an independent and identical uniform distribution* $(\eta^{\min}, \eta^{\max})$*, with the mean of emotional utility sufficiently large such that* $\frac{f(\eta)}{1-F(\bar{\eta})}$ *is close to zero, assume also* $u(c) = \ln c$*, and assume further that the sex ratio becomes a choice variable from period* t_0 *onwards, then there is a unique old steady state before period* t_0*, and*

(i) $\phi_t \geq 1$ $(t \geq t_0)$*;*

(ii) In the long run new steady state, equilibrium sex ratio is greater than one. Compared to the old steady state, both young men and parents with a son in the new steady state have a higher savings rate, while the relative savings rates for young women and parents with a daughter in the two steady states are ambiguous. However, both the total savings rate for the young cohort and the parental savings rate for children in the new steady state are higher than in the old stead state.

(iii) In period t_0, both young men and parents with a son have a higher savings rates relative to their counterparts in the earlier periods, but the changes in the savings rates by young women and parents with a daughter are ambiguous. However, the aggregate savings rate is higher and the country runs a current account surplus in period t_0.

Proof. Similar to the proof of Proposition 5, we first show that $\bar{\eta}_t^m = M(\bar{\eta}_t^w)$ if parents choose a sex ratio $\phi_t \geq 1$. Suppose not, then at least in one period, k,

$$\bar{\eta}_k^m > M(\bar{\eta}_k^w) \geq \bar{\eta}_k^w$$

where the second inequality holds because $\phi_k \geq 1$. Then we have

$$\begin{aligned} & u_{2,k+1}^{w,n} - \left(\frac{\phi_{k+1} \left(u_{2,,k+1}^{w,m} + \theta V_{k+1}^m + \eta^s \right)}{1 + \phi_{k+1}} + \frac{u_{2,k+1}^{w,w} + \theta V_{k+1}^w + \eta^d}{1 + \phi_{k+1}} \right) \\ > \quad & u_{2,k+1}^{m,n} - \left(\frac{\phi_{k+1} \left(u_{2,k+1}^{w,m} + \theta V_{k+1}^m + \eta^s \right)}{1 + \phi_{k+1}} + \frac{u_{2,k+1}^{w,w} + \theta V_{k+1}^w + \eta^d}{1 + \phi_{k+1}} \right) \end{aligned}$$

and hence,

$$S_k^w > S_k^m \tag{4.36}$$

.

Then, similar to the proof of Proposition 5, we have

$$\begin{aligned} u'_{1w,k} \quad = \quad & \kappa \begin{pmatrix} \left(1 + \frac{1}{\phi_k}\right) \left(1 - F\left(\bar{\eta}_k^w\right)\right) \\ + M(\bar{\eta}_k^w) f(\bar{\eta}_k^w) \end{pmatrix} \begin{pmatrix} \frac{\phi_{k+1} u'_{2,m,k+1}}{1+\phi_{k+1}} \\ + \frac{u'_{2,w,k+1}}{1+\phi_{k+1}} \end{pmatrix} + F\left(\bar{\eta}_k^w\right) u'_{2w,n,k+1} \\ & + f\left(\bar{\eta}_k^w\right) \kappa \left(\frac{\phi_{k+1} u'_{2,m,k+1}}{1 + \phi_{k+1}} + \frac{u'_{2,w,k+1}}{1 + \phi_{k+1}} \right) \begin{pmatrix} \frac{\phi_{k+1}\left(u_{2,k+1}^{w,m} + \theta V_{k+1}^m + \eta^s\right)}{1+\phi_{k+1}} \\ + \frac{u_{2,k+1}^{w,w} + \theta V_{k+1}^w + \eta^d}{1+\phi_{k+1}} \\ - u_{2w,n,k+1} \end{pmatrix} \end{aligned}$$

$$
\begin{aligned}
< \;& \kappa\left(\left(1+\phi_k\right)\left(1-F\left(\bar{\eta}_k^m\right)\right)+M^{-1}(\bar{\eta}_k^m)f(\bar{\eta}_k^m)\right)\begin{pmatrix} \frac{\phi_{k+1}u'_{2,m,k+1}}{1+\phi_{k+1}} \\ +\frac{u'_{2,w,k+1}}{1+\phi_{k+1}} \end{pmatrix} \\
& +F\left(\bar{\eta}_k^m\right)u'_{2m,n,k+1} \\
& +f\left(\bar{\eta}_k^m\right)\kappa\left(\frac{\phi_{k+1}u'_{2,m,k+1}}{1+\phi_{k+1}}+\frac{u'_{2,w,k+1}}{1+\phi_{k+1}}\right)\begin{pmatrix} \frac{\phi_{k+1}\left(u^{w,m}_{2,k+1}+\theta V^m_{k+1}+\eta^s\right)}{1+\phi_{k+1}} \\ +\frac{u^{w,w}_{2,k+1}+\theta V^w_{k+1}+\eta^d}{1+\phi_{k+1}} \\ -u_{2m,n,k+1} \end{pmatrix} \\
= \;& u'_{1m,k}
\end{aligned}
$$

which means that

$$S_k^m > S_k^w \tag{4.37}$$

which is a contradiction! Therefore, we must have $\bar{\eta}_t^m = M(\bar{\eta}_t^w)$, (and $S_t^m \geq S_t^w$ also holds) if $\phi_t \geq 1$. By (4.33) and (4.34), we can also show by contradiction that $T_t^m \geq T_t^w$. Suppose not, we have $T_t^m < T_t^w$, then

$$u'_{1m,t} = -\frac{\kappa}{\theta}u'_{2,m,t} > -\frac{\kappa}{\theta}u'_{2,w,t} = u'_{1w,t}$$

Since $S_t^m \geq S_t^w$, $T_t^m > T_t^w$ must hold if the above inequality holds. Contradiction again with the assumption. Thus we much have $T_t^m \geq T_t^w$.

Now we show that $\phi_t \geq 1$. Suppose not, then there exists one period k that $\phi_k < 1$. By (4.35), we have

$$u_{2,w,k} + \theta V_k^w - u_{2,m,k} - \theta V_k^m = \eta^s - \eta^d \geq 0$$

Similar to the previous analysis, we have

$$\bar{\eta}_k^w = M^{-1}\left(\bar{\eta}_k^m\right) > \bar{\eta}_k^m,\ S_k^w > S_k^m \text{ and } T_k^w > T_k^m$$

Then

$$u_{2,w,k} < u_{2,m,k}$$

Similar to the proof of Proposition 5, if $E\eta$ is large enough such that

$$E\eta \geq M\left(\bar{\eta}^{w}\right) + u_{2w} - u_{2w,n}$$

we can show that

$$\begin{aligned}
V_k^w &= u(c_{1k}^w) + \beta\left(1-\delta_k^w\right) u\left(c_{2,k+1}^{w,n}\right) + E\eta^m \\
&\quad + \beta\delta_k^w\left[\frac{\phi_{k+1}\left(u_{2,m,k+1} + \theta V_{k+1}^m + \eta^s\right)}{1+\phi_{k+1}} + \frac{u_{2,w,k+1} + \theta V_{k+1}^w + \eta^d}{1+\phi_{k+1}}\right] \\
&< u(c_{1k}^m) + \beta\left(1-\delta_k^m\right) u\left(c_{2,k+1}^{m,n}\right) + E\eta^w \\
&\quad + \beta\delta_k^m\left[\frac{\phi_{k+1}\left(u_{2,m,k+1} + \theta V_{k+1}^m + \eta^s\right)}{1+\phi_{k+1}} + \frac{u_{2,w,k+1} + \theta V_{k+1}^w + \eta^d}{1+\phi_{k+1}}\right] \\
&= V_k^m
\end{aligned}$$

This would have to imply that

$$u_{2,w,k} + \theta V_k^w - u_{2,m,k} - \theta V_k^m < 0$$

which is a contradiction with the initial assumption! Therefore, $\phi_t \geq 1$ much hold in each period ($t \geq t_0$). We can check that if $\eta^s = \eta^d$, due to the symmetry in men and women's optimization problems, we have $\phi_t = 1$.

By the assumption that sex ratio only becomes a choice variable from $t = t_0$ onwards, parents in all previous periods take as given sex ratio is balanced. That is, $\phi_t = 1$ for $t <= t_0$. They make optimal decisions on savings for themselves and savings for children by solving the first order conditions, (4.31), (4.32), (4.33) and (4.34). In the initial equilibrium when parents are not able to choose the probability of having a son, equation (4.35) becomes

$$\left(u_{2,w,t} + \theta V_t^w + \eta^d\right) - \left(u_{2,m,t} + \theta V_t^m + \eta^s\right) = \Delta_{t_0-1}$$

where Δ_{t_0-1} is obtained by solving (4.31), (4.32), (4.33) and (4.34), which represents the difference in the parental welfare between having a daughter and having a son. In the initial equilibrium, young men and young women, parents with a son and parents with a daughter have a symmetric optimization problem, and hence $s_{t_0-1}^m = s_{t_0-1}^w$, $T_{t_0-1}^m = T_{t_0-1}^w$. Since by assumption $\eta^s \geq \eta^d$, we have $\Delta_{t_0-1} \leq 0$. To see the savings responses to the shock in period t_0 that permits an endogenous choice of the sex ratio, it is equivalent to the savings responses to an permanent increase in Δ (from a negative value to zero).

For any level of Δ, by combining the equations (4.31), (4.32), (4.33), (4.34), (4.35) and women and men's value functions, and totally differentiating them, we can obtain

$$\Omega_t^P \cdot dx_t + \Lambda_{t+1}^P \cdot dx_{t+1} = dz \tag{4.38}$$

where

$$\Omega_t^P = \begin{pmatrix} \Omega_{11,t}^P & \Omega_{12,t}^P & \Omega_{13,t}^P & \Omega_{14,t}^P & \Omega_{15,t}^P & \Omega_{16,t}^P & \Omega_{17,t}^P \\ \Omega_{21,t}^P & \Omega_{22,t}^P & \Omega_{23,t}^P & \Omega_{24,t}^P & \Omega_{25,t}^P & \Omega_{26,t}^P & \Omega_{27,t}^P \\ \Omega_{31,t}^P & \Omega_{32,t}^P & \Omega_{33,t}^P & \Omega_{34,t}^P & \Omega_{35,t}^P & \Omega_{36,t}^P & \Omega_{37,t}^P \\ \Omega_{41,t}^P & \Omega_{42,t}^P & \Omega_{43,t}^P & \Omega_{44,t}^P & \Omega_{45,t}^P & \Omega_{46,t}^P & \Omega_{47,t}^P \\ \Omega_{51,t}^P & \Omega_{52,t}^P & \Omega_{53,t}^P & \Omega_{54,t}^P & \Omega_{55,t}^P & \Omega_{56,t}^P & \Omega_{57,t}^P \\ \Omega_{61,t}^P & \Omega_{62,t}^P & \Omega_{63,t}^P & \Omega_{64,t}^P & \Omega_{65,t}^P & \Omega_{66,t}^P & \Omega_{67,t}^P \\ \Omega_{71,t}^P & \Omega_{72,t}^P & \Omega_{73,t}^P & \Omega_{74,t}^P & \Omega_{75,t}^P & \Omega_{76,t}^P & \Omega_{77,t}^P \end{pmatrix}$$

$$\Lambda_t^P = \begin{pmatrix} \Lambda_{11,t}^P & \Lambda_{12,t}^P & \Lambda_{13,t}^P & \Lambda_{14,t}^P & \Lambda_{15,t}^P & \Lambda_{16,t}^P & \Lambda_{17,t}^P \\ \Lambda_{21,t}^P & \Lambda_{22,t}^P & \Lambda_{23,t}^P & \Lambda_{24,t}^P & \Lambda_{25,t}^P & \Lambda_{26,t}^P & \Lambda_{27,t}^P \\ \Lambda_{31,t}^P & \Lambda_{32,t}^P & \Lambda_{33,t}^P & \Lambda_{34,t}^P & \Lambda_{35,t}^P & \Lambda_{36,t}^P & \Lambda_{37,t}^P \\ \Lambda_{41,t}^P & \Lambda_{42,t}^P & \Lambda_{43,t}^P & \Lambda_{44,t}^P & \Lambda_{45,t}^P & \Lambda_{46,t}^P & \Lambda_{47,t}^P \\ \Lambda_{51,t}^P & \Lambda_{52,t}^P & \Lambda_{53,t}^P & \Lambda_{54,t}^P & \Lambda_{55,t}^P & \Lambda_{56,t}^P & \Lambda_{57,t}^P \\ \Lambda_{61,t}^P & \Lambda_{62,t}^P & \Lambda_{63,t}^P & \Lambda_{64,t}^P & \Lambda_{65,t}^P & \Lambda_{66,t}^P & \Lambda_{67,t}^P \\ \Lambda_{71,t}^P & \Lambda_{72,t}^P & \Lambda_{73,t}^P & \Lambda_{74,t}^P & \Lambda_{75,t}^P & \Lambda_{76,t}^P & \Lambda_{77,t}^P \end{pmatrix}$$

$$dx_t = \begin{pmatrix} dS_t^w \\ dS_t^m \\ dT_t^w \\ dT_t^m \\ d\phi_t \\ dV_t^w \\ dV_t^m \end{pmatrix} \quad \text{and} \quad dz = \begin{pmatrix} -f\left(\bar{\eta}_t^w\right)\kappa\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{d\Delta}{1+\phi_{t+1}} \\ 0 \\ 0 \\ 0 \\ d\Delta \\ \beta\delta_k^w \frac{d\Delta}{1+\phi_{t+1}} \\ \beta\delta_k^m \frac{d\Delta}{1+\phi_{t+1}} \end{pmatrix}$$

where

$$\Omega_{11,t}^P = u''_{1w,t} + R \begin{bmatrix} \kappa^2 \begin{pmatrix} \frac{\phi_{t+1}u''_{2,m,t+1}}{1+\phi_{t+1}} \\ +\frac{u''_{2,w,t+1}}{1+\phi_{t+1}} \end{pmatrix} \left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}_t^w\right)\right)\right) + F\left(\bar{\eta}_t^w\right)u''_{2w,n,t+1} \\ +f\left(\bar{\eta}_t^w\right)\kappa^2 \begin{pmatrix} \frac{\phi_{t+1}u''_{2,m,t+1}}{1+\phi_{t+1}} \\ +\frac{u''_{2,w,t+1}}{1+\phi_{t+1}} \end{pmatrix} \begin{pmatrix} u_{2,t+1}^{m,n} - \bar{\eta}_t^w + M(\bar{\eta}_t^w) \\ -u_{2,t+1}^{w,n} + \frac{d\Delta}{1+\phi_{t+1}} \end{pmatrix} \\ +2f\left(\bar{\eta}_t^w\right)\kappa \begin{pmatrix} \frac{\phi_{t+1}u''_{2,m,t+1}}{1+\phi_{t+1}} \\ +\frac{u''_{2,w,t+1}}{1+\phi_{t+1}} \end{pmatrix} \begin{pmatrix} \kappa\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right) \\ -u'_{2w,n,t+1} \end{pmatrix} \\ +f\left(\bar{\eta}_t^w\right)\kappa \begin{pmatrix} \frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}} \\ +\frac{u'_{2,w,t+1}}{1+\phi_{t+1}} \end{pmatrix} \begin{pmatrix} \kappa\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right) \\ -u'_{2,w,n,t+1} \end{pmatrix} \end{bmatrix}$$

$$\Omega^P_{12,t} = R\left[\begin{array}{c} \kappa^2\left(\frac{\phi_{t+1}u''_{2,m,t+1}}{1+\phi_{t+1}}+\frac{u''_{2,w,t+1}}{1+\phi_{t+1}}\right)\left(\left(1+\frac{1}{\phi_t}\right)\left(1-F\left(\bar{\eta}^w_t\right)\right)\right) \\ +f\left(\bar{\eta}^w_t\right)\kappa^2\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right)^2 \\ +f(\bar{\eta}^w_t)\left(u'_{2m,n}-\kappa\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right)\right) \\ \cdot\left(u'_{2w,n}-\kappa\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right)\right) \\ +f\left(\bar{\eta}^w_t\right)\kappa^2\left(\frac{\phi_{t+1}u''_{2,m,t+1}}{1+\phi_{t+1}}+\frac{u''_{2,w,t+1}}{1+\phi_{t+1}}\right) \\ \cdot\left(u^{m,n}_{2,t+1}+\bar{\eta}^w_t+M(\bar{\eta}^w_t)-u^{w,n}_{2,t+1}+\frac{d\Delta}{1+\phi_{t+1}}\right) \\ +f\left(\bar{\eta}^w_t\right)\kappa^2\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)^2 \end{array}\right]$$

$$\Omega^P_{13,t} = -u''_{1w,t},\ \Omega^P_{14,t}=0,\ \Omega^P_{15,t}=0,\ \Omega^P_{16,t}=0,\ \Omega^P_{17,t}=0$$

$$\Omega^P_{21,t} = R\left[\begin{array}{cc} \multicolumn{2}{c}{\kappa^2\left(\frac{\phi_{t+1}u''_{2,m,t+1}}{1+\phi_{t+1}}+\frac{u''_{2,w,t+1}}{1+\phi_{t+1}}\right)\left(\left(1+\phi_t\right)\left(1-F\left(M\left(\bar{\eta}^w_t\right)\right)\right)\right)} \\ +f\left(\bar{\eta}^w_t\right)\kappa\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right) & \left(\begin{array}{c}\left(1+\frac{1}{\phi_t}\right)\kappa\left(\begin{array}{c}\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}} \\ +\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\end{array}\right) \\ -\frac{1}{\phi_t}u'_{2m,n,t+1}\end{array}\right) \end{array}\right]$$

$$\Omega^P_{22,t} = u''_{1m}+R\left[\begin{array}{c} \kappa^2\left(\frac{\phi_{t+1}u''_{2,m,t+1}}{1+\phi_{t+1}}+\frac{u''_{2,w,t+1}}{1+\phi_{t+1}}\right)\left(\left(1+\phi_t\right)\left(1-F\left(M\left(\bar{\eta}^w_t\right)\right)\right)\right) \\ +\left(1-\delta^m_t\right)u''_{2m,n,t+1} \\ +f\left(\bar{\eta}^w_t\right)\left(\kappa\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right)-u'_{2m,n,t+1}\right) \\ \cdot\left(\left(1+\frac{1}{\phi_t}\right)\kappa\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right)-\frac{1}{\phi}u'_{2m,n,t+1}\right) \end{array}\right]$$

$$\Omega^P_{23,t} = 0,\ \Omega^P_{24,t}=-u''_{1m,t}$$

$$\Omega^P_{25} = \frac{1-F(\bar{\eta}^w_t)}{\phi^2_t}\left(u'_{2m,n,t+1}-\kappa\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right)\right)$$

$$\Omega^P_{26,t} = 0,\ \Omega^P_{27,t}=0$$

$$\Omega^P_{31,t} = -u''_{1w,t},\ \Omega^P_{32,t}=0,\ \Omega^P_{33,t}=\frac{1}{\theta}\kappa^2u''_{2,w,t}+u''_{1w,t}$$

$$\Omega^P_{34,t} = 0,\ \Omega^P_{35,t}=0,\ \Omega^P_{36,t}=0,\ \Omega^P_{37,t}=0$$

$$\begin{aligned}
\Omega^P_{41,t} &= 0,\ \Omega^P_{42,t} = -u''_{1m,t},\ \Omega^P_{43,t} = 0\\
\Omega^P_{44,t} &= \frac{1}{\theta}\kappa^2 u''_{2,m,t} + u''_{1m,t},\ \Omega^P_{45,t} = 0,\ \Omega^P_{46,t} = 0,\ \Omega^P_{47,t} = 0
\end{aligned}$$

$$\begin{aligned}
\Omega^P_{51,t} &= 0,\ \Omega^P_{52} = 0,\ \Omega^P_{53} = -\kappa u'_{2,w,t}\\
\Omega^P_{54} &= \kappa u'_{2,m,t},\ \Omega^P_{55} = 0,\ \Omega^P_{56,t} = \theta,\ \Omega^P_{57,t} = -\theta
\end{aligned}$$

$$\begin{aligned}
\Omega^P_{61,t} &= F(\bar{\eta}^w_t)u'_{2,w,n,t+1} + M(\bar{\eta}^w_t)f(\bar{\eta}^w_t)\kappa\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right) - u'_{1w,t}\\
\Omega^P_{62,t} &= \left(1 - F(\bar{\eta}^w_t)\right)u'_{2,m,n,t+1}\\
\Omega^P_{63,t} &= u'_{1w,t},\ \Omega^P_{64,t} = 0,\ \Omega^P_{65,t} = \frac{E\left[\eta^w \middle| \eta^w > \bar{\eta}^w_t\right]}{\phi^2_t}\\
\Omega^P_{66,t} &= -1,\ \Omega^P_{67,t} = 0
\end{aligned}$$

$$\begin{aligned}
\Omega^P_{71,t} &= \left(1 - F(M(\bar{\eta}^w_t))\right)u'_{2,m,n,t+1}\\
\Omega^P_{72,t} &= F(M(\bar{\eta}^w_t)))u'_{2,m,n,t+1} - u'_{1m,t}\\
&\quad + \frac{\bar{\eta}^w_t f(\bar{\eta}^w_t)}{\phi}\kappa\left(\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right) - u'_{2,m,n,t+1}\right)\\
\Omega^P_{73,t} &= 0,\ \Omega^P_{74,t} = u'_{1m,t},\ \Omega^P_{75,t} = -E\left[\eta^m \middle| \eta^m > M(\bar{\eta}^w_t)\right]\\
\Omega^P_{76,t} &= 0,\ \Omega^P_{77,t} = -1
\end{aligned}$$

and

$$\begin{aligned}
\Lambda^P_{11,t+1} &= 0,\ \ \Lambda^P_{12,t+1} = 0\\
\Lambda^P_{13,t+1} &= -\left[\begin{array}{c}
\kappa^2\left(\left(1+\frac{1}{\phi_t}\right)\left(1-F\left(\bar{\eta}^w_t\right)\right) + M(\bar{\eta}^w_t)f(\bar{\eta}^w_t)\right)\frac{u''_{2,w,t+1}}{1+\phi_{t+1}}\\
+f\left(\bar{\eta}^w_t\right)\kappa^2\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\\
+f\left(\bar{\eta}^w_t\right)\kappa^2\frac{u''_{2,w,t+1}}{1+\phi_{t+1}}\left(u^{m,n}_{2,t+1} - u^{w,n}_{2,t+1} - \bar{\eta}^w_t + \frac{\Delta}{1+\phi}\right)
\end{array}\right]
\end{aligned}$$

$$
\begin{aligned}
\Lambda^P_{14,t+1} &= \begin{array}{c} -\kappa^2\left(\left(1+\frac{1}{\phi_t}\right)(1-F(\bar{\eta}^w_t))+M(\bar{\eta}^w_t)f(\bar{\eta}^w_t)\right)\frac{\phi_{t+1}u''_{2,m,t+1}}{1+\phi_{t+1}} \\ -f(\bar{\eta}^w_t)\,\kappa^2\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}} \\ -f(\bar{\eta}^w_t)\,\kappa^2\frac{\phi_{t+1}u''_{2,m,t+1}}{1+\phi_{t+1}}\left(u^{m,n}_{2,t+1}-u^{w,n}_{2,t+1}-\bar{\eta}^w_t+\frac{\Delta}{1+\phi}\right) \end{array} \\
\Lambda^P_{15,t+1} &= \frac{\kappa\left(u'_{2w,m,t+1}-u'_{2w,w,t+1}\right)}{\left(1+\phi_{t+1}\right)^2}\left(\frac{1-F(\bar{\eta}^w_t)}{\phi_t}+f(\bar{\eta}^w)\left(\begin{array}{c} u^{w,n}_{2,t+1}-u^{w,n}_{2,t+1} \\ +M(\bar{\eta}^w_t)-\bar{\eta}^w_t \end{array}\right)\right) \\
\Lambda^P_{16,t+1} &= \Lambda^P_{17,t+1}=0
\end{aligned}
$$

$$
\begin{aligned}
\Lambda^P_{21,t+1} &= 0,\ \ \Lambda^P_{22,t+1}=0 \\
\Lambda^P_{23,t+1} &= -\left[\begin{array}{c} \kappa^2\left((1+\phi_t)\left(1-F\left(M\left(\bar{\eta}^w_t\right)\right)\right)\right)\frac{u''_{2,w,t+1}}{1+\phi_{t+1}} \\ +f(\bar{\eta}^w_t)\,\kappa^2\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{u'_{2,w,t+1}}{1+\phi_{t+1}} \end{array}\right] \\
\Lambda^P_{24,t+1} &= -\left[\begin{array}{c} \kappa^2\left((1+\phi_t)\left(1-F\left(M\left(\bar{\eta}^w_t\right)\right)\right)\right)\frac{\phi_{t+1}u''_{2,m,t+1}}{1+\phi_{t+1}} \\ +f(\bar{\eta}^w_t)\,\kappa^2\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}} \end{array}\right] \\
\Lambda^P_{25,t+1} &= \frac{\kappa}{(1+\phi_{t+1})^2}\left(\phi_t\left(1-F\left(M(\bar{\eta}^w_t)\right)\right)\right)\left(u'_{2m,m,t+1}-u'_{2m,w,t+1}\right) \\
\Lambda^P_{26,t+1} &= 0,\ \Lambda^P_{27,t+1}=0
\end{aligned}
$$

$$
\begin{aligned}
\Lambda^P_{31,t+1} &= \Lambda^P_{32,t+1}=\Lambda^P_{33,t+1}=\Lambda^P_{34,t+1}=\Lambda^P_{35,t+1}=\Lambda^P_{36,t+1}=\Lambda^P_{37,t+1}=0 \\
\Lambda^P_{41,t+1} &= \Lambda^P_{42,t+1}=\Lambda^P_{43,t+1}=\Lambda^P_{44,t+1}=\Lambda^P_{45,t+1}=\Lambda^P_{46,t+1}=\Lambda^P_{47,t+1}=0 \\
\Lambda^P_{51,t+1} &= \Lambda^P_{52,t+1}=\Lambda^P_{53,t+1}=\Lambda^P_{54,t+1}=\Lambda^P_{55,t+1}=\Lambda^P_{56,t+1}=\Lambda^P_{57,t+1}=0 \\
\Lambda^P_{61,t+1} &= \Lambda^P_{62,t+1}=\Lambda^P_{63,t+1}=\Lambda^P_{64,t+1}=\Lambda^P_{65,t+1}=\Lambda^P_{66,t+1}=\Lambda^P_{67,t+1}=0 \\
\Lambda^P_{71,t+1} &= \Lambda^P_{72,t+1}=\Lambda^P_{73,t+1}=\Lambda^P_{74,t+1}=\Lambda^P_{75,t+1}=\Lambda^P_{76,t+1}=\Lambda^P_{77,t+1}=0
\end{aligned}
$$

By (4.32), (4.33), (4.34) and the assumption that $\theta \leq 1/2$, we can show that

$$\begin{aligned} & u'_{2m,n,t+1} - \kappa\left(\frac{\phi_{t+1} u'_{2w,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right) \\ = \quad & \frac{\frac{1}{\theta}\kappa u'_{2,m,t+1} - \kappa\left(\left(1 - F\left(\bar{\eta}^w_t\right)\right) + 1\right)\left(\frac{\phi_{t+1} u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)}{F(M(\bar{\eta}^w_t))} \\ > \quad & \frac{F\left(\bar{\eta}^w_t\right)\kappa u'_{2,m,t+1}}{F(M(\bar{\eta}^w_t))} > 0 \end{aligned}$$

Then $\Omega^P_{25,t} > 0$.

In the rest of the proof, we assume $E\eta$ is suffciently large such that $f\left(\bar{\eta}^w\right)$ is very small compared to $1 - F\left(\bar{\eta}^w\right)$. In the long run equilibrium where $S^w_{t-1} = S^w_t$, $S^m_{t-1} = S^m_t$, $T^w_{t-1} = T^w_t$, and $T^m_{t-1} = T^m_t$, we have

$$\left(\Omega^P + \Lambda^P\right) \cdot dx = dz$$

We can show that the determinant of matrix $\Omega^P + \Lambda^P$ is

$$\begin{aligned} & \theta\left(\Omega^P_{65} - \Omega^P_{75}\right)\left[\begin{array}{c} \frac{\kappa^4 u''_{2w} u''_{2m}}{\theta^2}\left(\begin{array}{c} \left(\Omega^P_{13} + \Lambda^P_{13}\right)\left(\Omega^P_{24} + \Lambda^P_{24}\right) \\ -\left(\Omega^P_{23} + \Lambda^P_{23}\right)\left(\Omega^P_{14} + \Lambda^P_{14}\right) \end{array}\right) \\ +(1-\delta^m) u''_{2m,n}\Omega^P_{44}\left(\begin{array}{c} \left(1-\delta^w\right) u''_{2w,n}\Omega^P_{33} \\ -\frac{\kappa^2 u''_{2w}\left(\Omega^P_{13}+\Lambda^P_{13}\right)}{\theta} \end{array}\right) \end{array}\right] \\ & -\theta\left(\Omega^P_{25} + \Lambda^P_{25}\right)\left[\begin{array}{c} \left(\Omega^P_{62} - \Omega^P_{72}\right)\Omega^P_{44}\left(\begin{array}{c} \left(1-\delta^w\right) u''_{2w,n}\Omega^P_{33} \\ -\frac{\kappa^2 u''_{2w}\left(\Omega^P_{13}+\Lambda^P_{13}\right)}{\theta} \end{array}\right) \\ +\left(\Omega^P_{61} - \Omega^P_{71}\right)\Omega^P_{33}\frac{\kappa^2 u''_{2m}\left(\Omega^P_{14}+\Lambda^P_{14}\right)}{\theta} \end{array}\right] \\ & -\theta\Lambda^P_{15}\left[\begin{array}{c} \left(\Omega^P_{61} - \Omega^P_{71}\right)\Omega^P_{33}\left(\begin{array}{c} (1-\delta^m) u''_{2m,n}\Omega^P_{44} \\ -\frac{\kappa^2 u''_{2m}\left(\Omega^P_{24}+\Lambda^P_{24}\right)}{\theta} \end{array}\right) \\ +\left(\Omega^P_{62} - \Omega^P_{72}\right)\Omega^P_{44}\frac{\kappa^2 u''_{2w}\left(\Omega^P_{23}+\Lambda^P_{23}\right)}{\theta} \end{array}\right] \end{aligned}$$

Rearrage the terms on the right hand side, we can show that

$$\det\left(\Omega^P+\Lambda^P\right) = \theta\left(\Omega^P_{65}-\Omega^P_{75}\right)\left[\begin{array}{c}\frac{\kappa^4 u''_{2w}u''_{2m}}{\theta^2}\left(\begin{array}{c}\left(\Omega^P_{13}+\Lambda^P_{13}\right)\left(\Omega^P_{24}+\Lambda^P_{24}\right)\\ -\left(\Omega^P_{23}+\Lambda^P_{23}\right)\left(\Omega^P_{14}+\Lambda^P_{14}\right)\end{array}\right)\\ +(1-\delta^m)u''_{2m,n}\Omega^P_{44}\left(\begin{array}{c}\left(1-\delta^w\right)u''_{2w,n}\Omega^P_{33}\\ -\frac{\kappa^2 u''_{2w}\left(\Omega^P_{13}+\Lambda^P_{13}\right)}{\theta}\end{array}\right)\end{array}\right]$$

$$-\theta\Lambda^P_{15}\left[\begin{array}{c}\left(\Omega^P_{62}-\Omega^P_{72}\right)\Omega^P_{44}\left(\begin{array}{c}\left(1-\delta^w\right)u''_{2w,n}\Omega^P_{33}\\ -\frac{\kappa^2 u''_{2w}\left(\Omega^P_{13}+\Lambda^P_{13}\right)}{\theta}\\ +\frac{\kappa^2 u''_{2w}\left(\Omega^P_{23}+\Lambda^P_{23}\right)}{\theta}\end{array}\right)\\ +\left(\Omega^P_{61}-\Omega^P_{71}\right)\Omega^P_{33}\left(\begin{array}{c}(1-\delta^m)u''_{2m,n}\Omega^P_{44}\\ -\frac{\kappa^2 u''_{2m}\left(\Omega^P_{24}+\Lambda^P_{24}\right)}{\theta}\\ +\frac{\kappa^2 u''_{2w}\left(\Omega^P_{23}+\Lambda^P_{23}\right)}{\theta}\end{array}\right)\end{array}\right]$$

$$+\,\text{positive}\,.terms$$

Notice that

$$\begin{aligned}\Omega^P_{61}-\Omega^P_{71} &> -\kappa\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right)\\ &\quad\cdot\left[\frac{1-F(\bar\eta^w)}{\phi}+f(\bar\eta^w)\left(\begin{array}{c}\frac{\phi_{t+1}\left(u^{w,m}_{2,t+1}+\theta V^m_{t+1}+\eta^s\right)}{1+\phi_{t+1}}\\ +\frac{u^{w,w}_{2,t+1}+\theta V^w_{t+1}+\eta^d}{1+\phi_{t+1}}-u_{2w,n,t+1}\end{array}\right)\right]\\ \Omega^P_{62}-\Omega^P_{72} &= u'_{1m,t}+\left(1-F(\bar\eta^w_t)\right)u'_{2,m,n,t+1}-F(M(\bar\eta^w_t)))u'_{2,m,n,t+1}\\ &\quad-\frac{\bar\eta^w_t f(\bar\eta^w_t)}{\phi}\kappa\left(\left(\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\right)-u'_{2,m,n,t+1}\right)\\ &> \left(1-F(\bar\eta^w_t)\right)u'_{2,m,n,t+1}+\left(1+\frac{1}{\phi}\right)\left(1-F(\bar\eta^w)\right)\kappa\left(\begin{array}{c}\frac{\phi_{t+1}u'_{2w,m,t+1}}{1+\phi_{t+1}}\\ +\frac{u'_{2w,w,t+1}}{1+\phi_{t+1}}\end{array}\right)\end{aligned}$$

Under the same assumption in Proposition **??**,

$$\left(\Omega^P_{62}-\Omega^P_{72}\right)\Omega^P_{44}\left(1-\delta^w\right)u''_{2w,n}\Omega^P_{33}+\left(\Omega^P_{61}-\Omega^P_{71}\right)\Omega^P_{33}(1-\delta^m)u''_{2m,n}\Omega^P_{44}<0$$

and

$$\frac{\kappa^2 u''_{2w}\left(\Omega^P_{62}-\Omega^P_{72}\right)\Omega^P_{44}}{\theta}\left(\left(\Omega^P_{23}+\Lambda^P_{23}\right)-\left(\Omega^P_{13}+\Lambda^P_{13}\right)\right)$$
$$< \quad -\frac{\kappa^2 u''_{2m}\left(\Omega^P_{61}-\Omega^P_{71}\right)\Omega^P_{33}}{\theta}\left(\left(\Omega^P_{14}+\Lambda^P_{24}\right)-\left(\Omega^P_{24}+\Lambda^P_{24}\right)\right)$$

Then,

$$\det\left(\Omega^P+\Lambda^P\right)>0$$

In the long run equilibrium,

$$\frac{dT^m}{d\Delta} = \frac{\Omega^P_{42}\begin{bmatrix}\left(\Omega^P_{25}+\Lambda^P_{25}\right)\left(\Omega^P_{11}\Omega^P_{33}-\Omega^P_{31}\left(\Omega^P_{13}+\Lambda^P_{13}\right)\right)\\ -\Lambda^P_{15}\left(\Omega^P_{21}\Omega^P_{33}-\left(\Omega^P_{23}+\Lambda^P_{23}\right)\Omega^P_{31}\right)\end{bmatrix}}{\det\left(\Omega^P+\Lambda^P\right)}$$
$$+\frac{f\left(\bar{\eta}^w_t\right)\kappa\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{\theta d\Delta}{1+\phi_{t+1}}\begin{bmatrix}\left(\Omega^P_{25}+\Lambda^P_{25}\right)\Omega^P_{42}\\ \cdot\left(\Omega^P_{11}\Omega^P_{33}-\left(\Omega^P_{13}+\Lambda^P_{13}\right)\Omega^P_{31}\right)\\ -\Lambda^P_{15}\Omega^P_{42}\left(\Omega^P_{21}\Omega^P_{33}-\Lambda^P_{23}\Omega^P_{31}\right)\end{bmatrix}}{\det\left(\Omega^P+\Lambda^P\right)}$$

$$\frac{dT^w}{d\Delta} = \frac{\Omega^P_{31}\begin{bmatrix}\left(\Omega^P_{25}+\Lambda^P_{25}\right)\left(\Omega^P_{12}\Omega^P_{44}-\left(\Omega^P_{14}+\Lambda^P_{14}\right)\Omega^P_{42}\right)\\ -\Lambda^P_{15}\left(\Omega^P_{22}\Omega^P_{44}-\left(\Omega^P_{24}+\Lambda^P_{24}\right)\Omega^P_{42}\right)\end{bmatrix}}{\det\left(\Omega^P+\Lambda^P\right)}$$
$$+\frac{f\left(\bar{\eta}^w_t\right)\kappa\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}}+\frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{\theta d\Delta}{1+\phi_{t+1}}\begin{bmatrix}\left(\Omega^P_{25}+\Lambda^P_{25}\right)\\ \cdot\Omega^P_{31}\left(\Omega^P_{12}\Omega^P_{44}-\Lambda^P_{14}\Omega^P_{42}\right)\\ -\Lambda^P_{15}\Omega^P_{31}\begin{pmatrix}\Omega^P_{22}\Omega^P_{44}\\ -\left(\Omega^P_{24}+\Lambda^P_{24}\right)\Omega^P_{42}\end{pmatrix}\end{bmatrix}}{\det\left(\Omega^P+\Lambda^P\right)}$$

It is easy to show that

$$\begin{aligned}
\left(\Omega_{25}^P+\Lambda_{25}^P\right)\left(\Omega_{11}^P\Omega_{33}^P-\Omega_{31}^P\left(\Omega_{13}^P+\Lambda_{13}^P\right)\right)-\Lambda_{15}^P\left(\Omega_{21}^P\Omega_{33}^P-\left(\Omega_{23}^P+\Lambda_{23}^P\right)\Omega_{31}^P\right) &> 0 \\
\left(\Omega_{25}^P+\Lambda_{25}^P\right)\Omega_{42}^P\left(\Omega_{11}^P\Omega_{33}^P-\left(\Omega_{13}^P+\Lambda_{13}^P\right)\Omega_{31}^P\right)-\Lambda_{15}^P\Omega_{42}^P\left(\Omega_{21}^P\Omega_{33}^P-\Lambda_{23}^P\Omega_{31}^P\right) &> 0
\end{aligned}$$

then

$$\frac{dT^m}{d\Delta}>0$$

The sign of $\frac{dT^w}{d\Delta}$ is ambiguous. However,

$$\begin{aligned}
\frac{dT^m}{d\Delta}+\frac{dT^w}{d\Delta} &> \text{positive}\,.terms+\frac{\Lambda_{15}^P\left[\begin{array}{c}\Omega_{42}^P\left(\Omega_{11}^P\Omega_{33}^P-\Omega_{31}^P\left(\Omega_{13}^P+\Lambda_{13}^P\right)\right)\\ -\Omega_{31}^P\left(\Omega_{22}^P\Omega_{44}^P-\left(\Omega_{24}^P+\Lambda_{24}^P\right)\Omega_{42}^P\right)\end{array}\right]}{\det\left(\Omega^P+\Lambda^P\right)} \\
&\quad+\frac{\left[\begin{array}{c}\Omega_{31}^P\left(\Omega_{25}^P+\Lambda_{25}^P\right)\left(\Omega_{12}^P\Omega_{44}^P-\left(\Omega_{14}^P+\Lambda_{14}^P\right)\Omega_{42}^P\right)\\ -\Lambda_{15}^P\Omega_{42}^P\left(\Omega_{21}^P\Omega_{33}^P-\left(\Omega_{23}^P+\Lambda_{23}^P\right)\Omega_{31}^P\right)\end{array}\right]}{\det\left(\Omega^P+\Lambda^P\right)}
\end{aligned}$$

we can show that both terms on the right hand side are positive. Therefore,

$$\frac{dT^m}{d\Delta}+\frac{dT^w}{d\Delta}>0$$

Under the log utility assumption,

$$\frac{dS^m}{d\Delta}>0 \text{ and } \frac{dS^m}{d\Delta}+\frac{dS^w}{d\Delta}>0$$

Now we analyze the dynamic transition after the shock. First, we can show that,

$$\det\left(\Omega_t^P\right) = \theta\left(\Omega_{65,t}^P - \Omega_{75,t}^P\right)\left[\begin{array}{c} \Omega_{44,t}^P\Omega_{33,t}^P\left(\Omega_{11,t}^P\Omega_{22,t}^P - \Omega_{12,t}^P\Omega_{21,t}^P\right) \\ -\Omega_{44,t}^P\Omega_{31,t}^P\Omega_{13,t}^P\Omega_{22,t}^P - \Omega_{42,t}^P\Omega_{24,t}^P\left(\begin{array}{c}\Omega_{11,t}^P\Omega_{33,t}^P \\ -\Omega_{31,t}^P\Omega_{13,t}^P\end{array}\right)\end{array}\right] - \theta\Omega_{25,t}^P\left[\begin{array}{c}\left(\Omega_{62,t}^P - \Omega_{72,t}^P\right)\Omega_{44,t}^P\left(\Omega_{11,t}^P\Omega_{33,t}^P - \Omega_{31,t}^P\Omega_{13,t}^P\right) \\ -\left(\Omega_{61,t}^P - \Omega_{71,t}^P\right)\Omega_{33,t}^P\Omega_{22,t}^P\Omega_{44,t}^P\end{array}\right]$$

Both terms on the right hand side are positive. Then $\det\left(\Omega_t^P\right) > 0$. By (4.38),

$$\frac{dT_t^m}{d\Delta} = \frac{\theta\left(\Omega_{65,t}^P - \Omega_{75,t}^P\right)\Omega_{42,t}^P\left[\begin{array}{c}\left(\Omega_{11,t}^P\Omega_{33,t}^P - \Omega_{31,t}^P\Omega_{13,t}^P\right)\left(\begin{array}{c}\Lambda_{24,t+1}^P\frac{dT_{t+1}^m}{d\Delta} \\ +\Lambda_{23,t+1}^P\frac{dT_{t+1}^w}{d\Delta} \\ +\Lambda_{25,t+1}^P\frac{d\phi_{t+1}}{d\Delta}\end{array}\right) \\ +\Omega_{21,t}^P\Omega_{33,t}^P\left(\begin{array}{c}\Lambda_{14,t+1}^P\frac{dT_{t+1}^m}{d\Delta} \\ +\Lambda_{13,t+1}^P\frac{dT_{t+1}^w}{d\Delta} \\ +\Lambda_{15,t+1}^P\frac{d\phi_{t+1}}{d\Delta}\end{array}\right)\end{array}\right]}{\det\left(\Omega_t^P\right)} + \frac{\theta\Omega_{42,t}^P\left[\begin{array}{c}\left(\Omega_{25,t}^P + \Lambda_{25,t}^P\right)\left(\Omega_{11,t}^P\Omega_{33,t}^P - \Omega_{31,t}^P\Omega_{13,t}^P\right) \\ -f\left(\bar{\eta}_t^w\right)\kappa\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{\left(\Omega_{65,t}^P - \Omega_{75,t}^P\right)\Omega_{21,t}^P\Omega_{33,t}^P}{1+\phi_{t+1}}\end{array}\right]}{\det\left(\Omega_t^P\right)}$$

$$\frac{dT_t^w}{d\Delta} = \frac{\theta\left(\Omega_{65,t}^P - \Omega_{75,t}^P\right)\Omega_{31,t}^P \left[\begin{array}{c} \left(\Omega_{22,t}^P\Omega_{44,t}^P - \Omega_{42,t}^P\Omega_{24,t}^P\right)\left(\begin{array}{c} \Lambda_{14,t+1}^P \frac{dT_{t+1}^m}{d\Delta} \\ +\Lambda_{13,t+1}^P \frac{dT_{t+1}^w}{d\Delta} \\ +\Lambda_{15,t+1}^P \frac{d\phi_{t+1}}{d\Delta} \end{array}\right) \\ +\Omega_{12,t}^P\Omega_{44,t}^P \left(\begin{array}{c} \Lambda_{24,t+1}^P \frac{dT_{t+1}^m}{d\Delta} \\ +\Lambda_{23,t+1}^P \frac{dT_{t+1}^w}{d\Delta} \\ +\Lambda_{25,t+1}^P \frac{d\phi_{t+1}}{d\Delta} \end{array}\right) \end{array}\right]}{\det\left(\Omega_t^P\right)} + \frac{\theta\Omega_{31,t}^P \left[\begin{array}{c} \left(\Omega_{25,t}^P + \Lambda_{25,t}^P\right)\left(\Omega_{22,t}^P\Omega_{44,t}^P - \Omega_{42,t}^P\Omega_{24,t}^P\right) \\ -f\left(\bar{\eta}_t^w\right)\kappa\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{\left(\Omega_{65,t}^P - \Omega_{75,t}^P\right)\Omega_{12,t}^P\Omega_{44,t}^P}{1+\phi_{t+1}} \end{array}\right]}{\det\left(\Omega_t^P\right)}$$

Rewrite the equations above, we have

$$\left(\begin{array}{c} \frac{dT_t^m}{d\Delta} \\ \frac{dT_t^w}{d\Delta} \\ \frac{d\phi_t}{d\Delta} \end{array}\right) = F_t + G_{t+1}\left(\begin{array}{c} \frac{dT_{t+1}^m}{d\Delta} \\ \frac{dT_{t+1}^w}{d\Delta} \\ \frac{d\phi_{t+1}}{d\Delta} \end{array}\right)$$

where

$$F_t = \left(\begin{array}{c} \frac{\theta\Omega_{42,t}^P \left[\begin{array}{c} \left(\Omega_{25,t}^P + \Lambda_{25,t}^P\right)\left(\Omega_{11,t}^P\Omega_{33,t}^P - \Omega_{31,t}^P\Omega_{13,t}^P\right) \\ -f\left(\bar{\eta}_t^w\right)\kappa\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{\left(\Omega_{65,t}^P - \Omega_{75,t}^P\right)\Omega_{21,t}^P\Omega_{33,t}^P}{1+\phi_{t+1}} \end{array}\right]}{\det\left(\Omega_t^P\right)} \\ \frac{\theta\Omega_{31,t}^P \left[\begin{array}{c} \left(\Omega_{25,t}^P + \Lambda_{25,t}^P\right)\left(\Omega_{22,t}^P\Omega_{44,t}^P - \Omega_{42,t}^P\Omega_{24,t}^P\right) \\ -f\left(\bar{\eta}_t^w\right)\kappa\left(\frac{\phi_{t+1}u'_{2,m,t+1}}{1+\phi_{t+1}} + \frac{u'_{2,w,t+1}}{1+\phi_{t+1}}\right)\frac{\left(\Omega_{65,t}^P - \Omega_{75,t}^P\right)\Omega_{12,t}^P\Omega_{44,t}^P}{1+\phi_{t+1}} \end{array}\right]}{\det\left(\Omega_t^P\right)} \\ \frac{\left(1+\theta\left(\Omega_{65,t}^P - \Omega_{75,t}^P\right)\right)\left[\begin{array}{c} \Omega_{44,t}^P\Omega_{33,t}^P\left(\Omega_{11,t}^P\Omega_{22,t}^P - \Omega_{12,t}^P\Omega_{21,t}^P\right) \\ -\Omega_{42,t}^P\Omega_{24,t}^P\left(\Omega_{11,t}^P\Omega_{33,t}^P - \Omega_{31,t}^P\Omega_{13,t}^P\right) \end{array}\right]}{\det\left(\Omega_t^P\right)} \end{array}\right)$$

and

$$G_{t+1} = \begin{pmatrix} G_{11,t+1} & G_{12,t+1} & G_{13,t+1} \\ G_{21,t+1} & G_{22,t+1} & G_{23,t+1} \\ G_{31,t+1} & G_{32,t+1} & G_{33,t+1} \end{pmatrix}$$

where

$$G_{11,t+1} = \frac{\theta\left(\Omega^P_{65,t} - \Omega^P_{75,t}\right)\Omega^P_{31,t}\left[\begin{array}{c}\left(\Omega^P_{22,t}\Omega^P_{44,t} - \Omega^P_{42,t}\Omega^P_{24,t}\right)\Lambda^P_{14,t+1} \\ +\Omega^P_{12,t}\Omega^P_{44,t}\Lambda^P_{24,t+1}\end{array}\right]}{\det\left(\Omega^P_t\right)}$$

$$G_{12,t+1} = \frac{\theta\left(\Omega^P_{65,t} - \Omega^P_{75,t}\right)\Omega^P_{31,t}\left[\begin{array}{c}\left(\Omega^P_{22,t}\Omega^P_{44,t} - \Omega^P_{42,t}\Omega^P_{24,t}\right)\Lambda^P_{13,t+1} \\ +\Omega^P_{12,t}\Omega^P_{44,t}\Lambda^P_{23,t+1}\end{array}\right]}{\det\left(\Omega^P_t\right)}$$

$$G_{13,t+1} = \frac{\theta\left(\Omega^P_{65,t} - \Omega^P_{75,t}\right)\Omega^P_{31,t}\left[\begin{array}{c}\left(\Omega^P_{22,t}\Omega^P_{44,t} - \Omega^P_{42,t}\Omega^P_{24,t}\right)\Lambda^P_{15,t+1} \\ +\Omega^P_{12,t}\Omega^P_{44,t}\Lambda^P_{25,t+1}\end{array}\right]}{\det\left(\Omega^P_t\right)}$$

$$G_{21,t+1} = \frac{\theta\left(\Omega^P_{65,t} - \Omega^P_{75,t}\right)\Omega^P_{31,t}\left[\begin{array}{c}\left(\Omega^P_{22,t}\Omega^P_{44,t} - \Omega^P_{42,t}\Omega^P_{24,t}\right)\Lambda^P_{14,t+1} \\ +\Omega^P_{12,t}\Omega^P_{44,t}\Lambda^P_{24,t+1}\end{array}\right]}{\det\left(\Omega^P_t\right)}$$

$$G_{22,t+1} = \frac{\theta\left(\Omega^P_{65,t} - \Omega^P_{75,t}\right)\Omega^P_{31,t}\left[\begin{array}{c}\left(\Omega^P_{22,t}\Omega^P_{44,t} - \Omega^P_{42,t}\Omega^P_{24,t}\right)\Lambda^P_{13,t+1} \\ +\Omega^P_{12,t}\Omega^P_{44,t}\Lambda^P_{23,t+1}\end{array}\right]}{\det\left(\Omega^P_t\right)}$$

$$G_{23,t+1} = \frac{\theta\left(\Omega^P_{65,t} - \Omega^P_{75,t}\right)\Omega^P_{31,t}\left[\begin{array}{c}\left(\Omega^P_{22,t}\Omega^P_{44,t} - \Omega^P_{42,t}\Omega^P_{24,t}\right)\Lambda^P_{15,t+1} \\ +\Omega^P_{12,t}\Omega^P_{44,t}\Lambda^P_{25,t+1}\end{array}\right]}{\det\left(\Omega^P_t\right)}$$

$$G_{31,t+1} = \frac{\theta\left\{\begin{array}{c} \Lambda^P_{14,t+1}\left[\begin{array}{c}\left(\Omega^P_{62,t}-\Omega^P_{72,t}\right)\Omega^P_{44,t}\left(\Omega^P_{21,t}\Omega^P_{33,t}-\Omega^P_{31,t}\Omega^P_{23,t}\right)\\ -\left(\Omega^P_{61,t}-\Omega^P_{71,t}\right)\Omega^P_{33,t}\Omega^P_{22,t}\Omega^P_{44,t}\end{array}\right] \\ -\Lambda^P_{24,t+1}\left[\begin{array}{c}\left(\Omega^P_{62,t}-\Omega^P_{72,t}\right)\Omega^P_{44,t}\left(\Omega^P_{11,t}\Omega^P_{33,t}-\Omega^P_{31,t}\Omega^P_{13,t}\right)\\ -\left(\Omega^P_{61,t}-\Omega^P_{71,t}\right)\Omega^P_{33,t}\Omega^P_{22,t}\Omega^P_{44,t}\end{array}\right]\end{array}\right\}}{\det\left(\Omega^P_t\right)}$$

$$G_{32,t+1} = \frac{\theta\left\{\begin{array}{c} \Lambda^P_{13,t+1}\left[\begin{array}{c}\left(\Omega^P_{62,t}-\Omega^P_{72,t}\right)\Omega^P_{44,t}\left(\Omega^P_{21,t}\Omega^P_{33,t}-\Omega^P_{31,t}\Omega^P_{23,t}\right)\\ -\left(\Omega^P_{61,t}-\Omega^P_{71,t}\right)\Omega^P_{33,t}\Omega^P_{22,t}\Omega^P_{44,t}\end{array}\right] \\ -\Lambda^P_{23,t+1}\left[\begin{array}{c}\left(\Omega^P_{62,t}-\Omega^P_{72,t}\right)\Omega^P_{44,t}\left(\Omega^P_{11,t}\Omega^P_{33,t}-\Omega^P_{31,t}\Omega^P_{13,t}\right)\\ -\left(\Omega^P_{61,t}-\Omega^P_{71,t}\right)\Omega^P_{33,t}\Omega^P_{22,t}\Omega^P_{44,t}\end{array}\right]\end{array}\right\}}{\det\left(\Omega^P_t\right)}$$

$$G_{33,t+1} = \frac{\theta\left\{\begin{array}{c} \Lambda^P_{15,t+1}\left[\begin{array}{c}\left(\Omega^P_{62,t}-\Omega^P_{72,t}\right)\Omega^P_{44,t}\left(\Omega^P_{21,t}\Omega^P_{33,t}-\Omega^P_{31,t}\Omega^P_{23,t}\right)\\ -\left(\Omega^P_{61,t}-\Omega^P_{71,t}\right)\Omega^P_{33,t}\Omega^P_{22,t}\Omega^P_{44,t}\end{array}\right] \\ -\Lambda^P_{25,t+1}\left[\begin{array}{c}\left(\Omega^P_{62,t}-\Omega^P_{72,t}\right)\Omega^P_{44,t}\left(\Omega^P_{11,t}\Omega^P_{33,t}-\Omega^P_{31,t}\Omega^P_{13,t}\right)\\ -\left(\Omega^P_{61,t}-\Omega^P_{71,t}\right)\Omega^P_{33,t}\Omega^P_{22,t}\Omega^P_{44,t}\end{array}\right]\end{array}\right\}}{\det\left(\Omega^P_t\right)}$$

Iterate forward, we can obtain

$$\begin{pmatrix} \frac{dT^m_t}{d\Delta} \\ \frac{dT^w_t}{d\Delta} \\ \frac{d\phi_t}{d\Delta} \end{pmatrix} = \sum_{k=0}^{\infty} G_{t+k}F_{t+k} + \lim_{\tau\to\infty}\prod_{k=0}^{\infty} G_{t+k}\begin{pmatrix} \frac{dT^m}{d\Delta} \\ \frac{dT^w}{d\Delta} \\ \frac{d\phi}{d\eta^s} \end{pmatrix}$$

where we define G_t as the identity matrix, and it is easy to show that

$$G_{11,t+1} > 0,\ G_{12,t+1} > 0,\ G_{13,t+1} > 0,\ G_{21,t+1} > 0,\ G_{22,t+1} > 0 \text{ and } G_{23,t+1} > 0$$

Now we show by contradiction that

$$\frac{dT^m_t}{d\Delta} > 0 \text{ and } \frac{dT^m_t}{d\Delta} + \frac{dT^w_t}{d\Delta} > 0$$

in each period. Suppose not, there exists a $k < \infty$ that, in period $k+1$, both inequalities hold but in period k, at least one of the inequalities fails.

By (4.38), we have

$$\frac{dT_k^m}{d\Delta} = \frac{\theta\Omega_{42,k}^P \left[\begin{array}{c} \left(\Omega_{25,k}^P + \Lambda_{25,k}^P\right)\left(\Omega_{11,k}^P\Omega_{33,k}^P - \Omega_{31,k}^P\Omega_{13,k}^P\right) \\ -f\left(\bar{\eta}_k^w\right)\kappa\left(\frac{\phi_{k+1}u'_{2,m,k+1}}{1+\phi_{k+1}} + \frac{u'_{2,w,k+1}}{1+\phi_{k+1}}\right)\frac{\left(\Omega_{65,k}^P - \Omega_{75,k}^P\right)\Omega_{21,k}^P\Omega_{33,k}^P}{1+\phi_{k+1}} \end{array} \right]}{\det\left(\Omega_k^P\right)} + \begin{pmatrix} G_{11,k+1} & G_{12,k+1} & G_{13,k+1} \end{pmatrix} \begin{pmatrix} \frac{dT_{k+1}^m}{d\Delta} \\ \frac{dT_{k+1}^w}{d\Delta} \\ \frac{d\phi_{k+1}}{d\Delta} \end{pmatrix}$$

It is easy to show

$$F_k > 0$$

Plug in the expressions for $G_{11,k+1}$ and $G_{12,k+1}$,

$$G_{11,k+1}\frac{dT_{k+1}^m}{d\Delta} + G_{12,k+1}\frac{dT_{k+1}^w}{d\Delta} + G_{13,k+1}\frac{d\phi_{k+1}}{d\Delta} > 0$$

since $\phi_{k+1} > 1$, $S_k^m > S_k^w$, and $\frac{dT_{k+1}^m}{d\Delta} + \frac{dT_{k+1}^w}{d\Delta} > 0$. Then $\frac{dT_k^m}{d\Delta} > 0$ and

$$\begin{aligned}\frac{dT_k^m}{d\Delta} + \frac{dT_k^w}{d\Delta} &= \frac{\theta\Omega_{42,k}^P \left[\begin{array}{c} \left(\Omega_{25,k}^P + \Lambda_{25,k}^P\right)\left(\Omega_{11,k}^P\Omega_{33,k}^P - \Omega_{31,k}^P\Omega_{13,k}^P\right) \\ -f\left(\bar{\eta}_k^w\right)\kappa\left(\frac{\phi_{k+1}u'_{2,m,k+1}}{1+\phi_{k+1}} + \frac{u'_{2,w,k+1}}{1+\phi_{k+1}}\right)\frac{\left(\Omega_{65,k}^P - \Omega_{75,k}^P\right)\Omega_{21,k}^P\Omega_{33,k}^P}{1+\phi_{k+1}}\end{array}\right]}{\det\left(\Omega_k^P\right)} \\ &+ \frac{\theta\Omega_{31,k}^P \left[\begin{array}{c} \left(\Omega_{25,k}^P + \Lambda_{25,k}^P\right)\left(\Omega_{22,k}^P\Omega_{44,k}^P - \Omega_{42,k}^P\Omega_{24,k}^P\right) \\ -f\left(\bar{\eta}_k^w\right)\kappa\left(\frac{\phi_{k+1}u'_{2,m,k+1}}{1+\phi_{k+1}} + \frac{u'_{2,w,k+1}}{1+\phi_{k+1}}\right)\frac{\left(\Omega_{65,k}^P - \Omega_{75,k}^P\right)\Omega_{12,k}^P\Omega_{44,k}^P}{1+\phi_{k+1}}\end{array}\right]}{\det\left(\Omega_k^P\right)} \\ &+ \begin{pmatrix} G_{11,k+1} & G_{12,k+1} & G_{13,k+1} \end{pmatrix}\begin{pmatrix} \frac{dT_{k+1}^m}{d\Delta} \\ \frac{dT_{k+1}^w}{d\Delta} \\ \frac{d\phi_{k+1}}{d\Delta} \end{pmatrix} \\ &+ \begin{pmatrix} G_{21,k+1} & G_{22,k+1} & G_{23,k+1} \end{pmatrix}\begin{pmatrix} \frac{dT_{k+1}^m}{d\Delta} \\ \frac{dT_{k+1}^w}{d\Delta} \\ \frac{d\phi_{k+1}}{d\Delta} \end{pmatrix}\end{aligned}$$

Since

$$\theta\Omega_{31,k}^P \left[\begin{array}{c} \left(\Omega_{25,k}^P + \Lambda_{25,k}^P\right)\left(\Omega_{22,k}^P\Omega_{44,k}^P - \Omega_{42,k}^P\Omega_{24,k}^P\right) \\ -f\left(\bar{\eta}_k^w\right)\kappa\left(\frac{\phi_{k+1}u'_{2,m,k+1}}{1+\phi_{k+1}} + \frac{u'_{2,w,k+1}}{1+\phi_{k+1}}\right)\frac{\left(\Omega_{65,k}^P - \Omega_{75,k}^P\right)\Omega_{12,k}^P\Omega_{44,k}^P}{1+\phi_{k+1}}\end{array}\right] > 0$$

and

$$G_{21,k+1}\frac{dT_{k+1}^m}{d\Delta} + G_{22,k+1}\frac{dT_{k+1}^w}{d\Delta} + G_{23,k+1}\frac{d\phi_{k+1}}{d\Delta} > 0$$

$$\frac{dT_k^m}{d\Delta} + \frac{dT_k^w}{d\Delta} > 0$$

Contradiction! Therefore, in each period after the shock,

$$\frac{dT_t^m}{d\Delta} > 0 \text{ and } \frac{dT_t^m}{d\Delta} + \frac{dT_t^w}{d\Delta} > 0$$

but the sign of $\frac{dT_t^w}{d\Delta}$ is ambiguous. In period t_0 when the shock occurs, the aggregate consumption in the old cohort will fall since

$$\begin{aligned}
\frac{dc_{t_0}^{old}}{d\Delta} &= \delta_{t_0-1}\left(\begin{array}{c} \frac{\phi_{t_0}-1}{1+\phi_{t_0}}\frac{dc_{2,m,t_0}}{d\Delta}+\frac{1}{1+\phi_{t_0}}\left(\frac{dc_{2,m,t_0}}{d\Delta}+\frac{dc_{2,w,t_0}}{d\Delta}\right) \\ +\frac{c_{2,m,t_0}-c_{2,w,t_0}}{\left(1+\phi_{t_0}\right)^2} \end{array}\right) \\
&= -\kappa\delta_{t_0-1}\left(\begin{array}{c} \frac{\phi_{t_0}-1}{1+\phi_{t_0}}\frac{dT_{t_0}^m}{d\Delta}+\frac{1}{1+\phi_{t_0}}\left(\frac{dT_{t_0}^m}{d\delta}+\frac{dT_{t_0}^w}{d\Delta}\right) \\ +\frac{T_{t_0}^m-T_{t_0}^w}{\left(1+\phi_{t_0}\right)^2} \end{array}\right)<0
\end{aligned}$$

Under the log utility assumption, by (4.33) and (4.34), in period t_0 when the shock occurs, we have

$$c_{1m,t_0}=\theta\left(2RS_{t_0-1}-T_{t_0}^m\right) \text{ and } c_{1w,t_0}=\theta\left(2RS_{t_0-1}-T_{t_0}^w\right)$$

then

$$\begin{aligned}
\frac{dc_{t_0}^{young}}{d\Delta} &= \frac{\phi_{t_0}-1}{1+\phi_{t_0}}\frac{dc_{1m,t_0}}{d\Delta}+\frac{1}{1+\phi_{t_0}}\left(\frac{dc_{1w,t_0}}{d\Delta}+\frac{dc_{1m,t_0}}{d\Delta}\right) \\
&\quad +\frac{c_{1m,t_0}-c_{1w,t_0}}{\left(1+\phi_{t_0}\right)^2} \\
&= -\theta\left[\begin{array}{c} \frac{\phi_{t_0}-1}{1+\phi_{t_0}}\frac{dT_{t_0}^m}{d\Delta}+\frac{1}{1+\phi_{t_0}}\left(\frac{dT_{t_0}^w}{d\Delta}+\frac{dT_{t_0}^m}{d\Delta}\right) \\ +\frac{T_{t_0}^m-T_{t_0}^w}{\left(1+\phi_{t_0}\right)^2} \end{array}\right]<0
\end{aligned}$$

Therefore,

$$\begin{aligned}
T_{t_0}^m\big|_{\Delta=0}-T_{t_0}^m\big|_{\Delta=\Delta_{t_0-1}} &= \int_{\Delta_{t_0-1}}^0 \frac{dT_{t_0}^m}{d\Delta}d\Delta>0 \\
c_{t_0}^{old}\Big|_{\Delta=0}-c_{t_0}^{old}\Big|_{\Delta=\Delta_{t_0-1}} &= \int_{\Delta_{t_0-1}}^0 \frac{dc_{t_0}^{old}}{d\Delta}d\Delta<0 \\
c_{t_0}^{young}\big|_{\Delta=0}-c_{t_0}^{young}\big|_{\Delta=\Delta_{t_0-1}} &= \int_{\Delta_{t_0-1}}^0 \frac{dc_{t_0}^{young}}{d\Delta}d\Delta<0
\end{aligned}$$

In a small open economy similar to the benchmark model, the aggregate savings rate and current account to GDP ratio are

$$\begin{aligned} s^P_{t_0} &= \frac{Q_t + (R-1)\cdot NFA_{t-1} - c^{young}_{t_0} - c^{old}_{t_0}}{Q_t} \\ &= 1 + (1-\alpha)\frac{(R-1)\cdot NFA_{t-1} - c^{young}_{t_0} - c^{old}_{t_0}}{W} \end{aligned}$$

and

$$\begin{aligned} ca_{t_0} &= \frac{Q_t + (R-1)\cdot NFA_{t-1} - c^{young}_{t_0} - c^{old}_{t_0}}{Q_t} - \frac{K^d_{t_0+1}}{Q_t} \\ &= 1 + (1-\alpha)\frac{(R-1)\cdot NFA_{t-1} - c^{young}_{t_0} - c^{old}_{t_0}}{W} - \frac{\alpha}{R}\frac{1-F(\bar{\eta}^w_{t_0})}{1+\phi_{t_0}} \end{aligned}$$

where NFA_{t-1} is a predetermined variable and W is constant in a small open economy. In period t_0, as we showed above, both the young and the old will reduce their consumptions in response to the new technology that allows for endogenous determination of the sex ratio. The aggregate savings rate rises. If $E\eta$ is large enough such that $\frac{f(\bar{\eta}^w)}{1-F(\bar{\eta}^w_t)}$ is sufficiently small, $\frac{1-F(\bar{\eta}^w_t)}{1+\phi_t}$ decreases after the shock and therefore, current account also rises. □

A few remarks are in order. First, in the new steady state[11], sex ratio is greater than one. To see the intuition, we demonstrate that choosing a balanced sex ratio cannot be optimal when a son brings greater intrinsic utility to parents. Suppose parents did choose to have a balanced sex ratio, then both young men and young women would face a symmetric optimization problem. Then parents would make the same transfer to child regardless of child's gender, and young men and young women would also make the same savings decisions. Since $\eta^s > \eta^d$, at the balanced sex ratio, having a son will yield a greater utility to parents. Parents will optimally deviate by choosing a higher sex ratio.

Second, it is intuitive that parents with a son will make greater savings in period t_0 in response to the technology shock since they expect their son to face more severe competitiion in the marriage market, but why don't parents with a daughter necessarily reduce their

[11]In the longer working paper version, we show that there exists a steady state both before and after the shock.

savings in period t_0? On the one hand, as they expect their daughter to face a more favorable marriage market, they have an incentive to reduce their transfer to her. On the other hand, a higher sex ratio also implies that their daughter has a greater probability to have a son, and she and the son-in-law will likely to have to make a greater sacrifice in their consumption and make a greater transfer to their son. Since parents with a daughter care about the utility of their daughter, they would want to share the burden of their daughter and make a greater transfer to her. Given the two conflicting incentives, the net effect of a higher sex ratio on parental transfer is ambiguous. [Notice that, for parents with a son, a higher sex ratio also raises the probability that their son may have a grandson in the future.]

Third, in the longer working paper version, we show that the sex ratio is higher in the new steady state, and the sex ratio during the transition rises monotonically from the initial to the new steady state. To see this, we note that parents with a son in period t_0 would make a greater transfer to their son based on the logic of the previous remark. The aggregate transfer made by all parents to their children also rises. This raises the initial wealth of the young cohort in period $t_0 + 1$. As this cohort now are more capable of their own son, if they have one, with his marriage, they are more likely to choose to have a higher sex ratio. This process continues until the economy reaches the new long run equilibrium when the utility loss associated with an additional transfer to their child just exceeds the utility gain associated with a further increase in the sex ratio.

Appendices to Chapter 3

A3.1. Proof of Proposition 9

Proof. We totally differentiate the system and have

$$\Omega \cdot \begin{pmatrix} ds_t \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

where

$$\begin{aligned}
\Omega_{11} &= \left(u_1'' + \beta R u_2''\right) \frac{w_t}{P_t}, \; \Omega_{12} = \Omega_{13} = \Omega_{14} = 0 \\
\Omega_{21} &= \gamma w_t, \Omega_{22} = -\gamma(1 - s_t) \\
\Omega_{23} &= \frac{A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1-\alpha_N}}, \; \Omega_{24} = \frac{P_{Nt}(1 - \alpha_N) A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N} (1 - \alpha_N)^{1-\alpha_N}}
\end{aligned}$$

$$\begin{aligned}
\Omega_{31} &= \Omega_{33} = 0, \; \Omega_{32} = -1 \\
\Omega_{34} &= \left(\frac{\alpha_T}{1 - \alpha_T}\right)^{1-\alpha_T} (1 - \alpha_T) A_{Tt} K_{Tt}^{\alpha_T} (1 - L_{Nt})^{-\alpha_T - 1}
\end{aligned}$$

$$\begin{aligned}
\Omega_{41} &= 0, \; \Omega_{42} = -1, \; \Omega_{43} = \frac{w_t}{P_{Nt}} \\
\Omega_{44} &= -\left(\frac{\alpha_T}{1 - \alpha_T}\right)^{1-\alpha_T} (1 - \alpha_T) A_{Tt} K_{Tt}^{\alpha_T} (1 - L_{Nt})^{-\alpha_N - 1}
\end{aligned}$$

and

$$z_1 = -R u_2', \; z_2 = z_3 = z_4 = 0$$

The determinant of matrix Ω is

$$\det(\Omega) = \Omega_{11} \cdot \det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix}$$

and

$$\begin{aligned} \det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} &= \text{negative}\,.terms + \gamma(1-s_t)\left(\frac{w_t}{P_{Nt}}\right)^2 \frac{1-\alpha_T}{L_T} \\ &\quad - \left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) C_{Nt} \end{aligned}$$

Since the consumption on the nontradable goods by the young cohort must be less than the aggregate nontradable good consumption, it follows that $\gamma(1-s_t)w_t < P_{Nt}C_{Nt}$. Therefore,

$$\det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} < 0$$

and $\det(\Omega) > 0$

Then it is easy to show that

$$\frac{ds_t}{d\beta} = \frac{\det \begin{pmatrix} z_1 & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ z_2 & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ z_3 & \Omega_{32} & \Omega_{33} & \Omega_{34} \\ z_4 & \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix}}{\det(\Omega)} = \frac{z_1}{\Omega_{11}} > 0$$

and the price of the nontradable good

$$\frac{dP_{Nt}}{d\beta} = \frac{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} & z_1 & \Omega_{14} \\ \Omega_{21} & \Omega_{22} & z_2 & \Omega_{24} \\ \Omega_{31} & \Omega_{32} & z_3 & \Omega_{34} \\ \Omega_{41} & \Omega_{42} & z_4 & \Omega_{44} \end{pmatrix}}{\det(\Omega)} = \frac{z_1\Omega_{21}\Omega_{32}(\Omega_{44} - \Omega_{34})}{\det(\Omega)} < 0$$

The labor input in the nontradable sector

$$\frac{dL_{Nt}}{d\beta} = \frac{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & z_1 \\ \Omega_{21} & \Omega_{22} & \Omega_{23} & z_2 \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & z_3 \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & z_4 \end{pmatrix}}{\det(\Omega)} = -\frac{z_1\Omega_{21}\Omega_{32}\Omega_{34}}{\det(\Omega)} < 0$$

In period $t+1$, the shock has been observed, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \quad \text{and} \quad P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

which means that after one period the shock occurs, the price of the nontradable good and the consumer price index will go back to their initial levels. As for the current account,

$$CA_t = P_{Nt}Q_{Nt} + Q_{Tt} + (R-1) \cdot NFA_{t-1} - P_tC_t - K_{t+1}$$

where NFA_{t-1} is the net foreign asset holdings in period $t-1$ and K_{t+1} is the sum of capital input in both the nontradable sector and the tradable sector in period $t+1$. Since

$$s_{t-1}w_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = s_t w_t - s_{t-1} w_{t-1} - \Delta K_{t+1}$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w \left((R-1)\, s_t + 1 \right)}{P_N}$$

where we drop the time subindex because wage rate and the relative price of the nontradable good will go back to their initial levels. It is easy to see that since $s_t > s_{t-1}$, $Q_{N,t+1} > Q_{N,t-1}$.

As $\alpha_N < \alpha_T$, the nontradable sector has a lower capital-intensity than the tradable sector. Then, in period $t+1$, $K_{t+1} < K_{t-1}$.

In period $t+1$,

$$A_{Nt} K_{N,t+1}^{\alpha_N} L_{N,t+1}^{1-\alpha_N} = \frac{\gamma w \left((R-1)\, s_t + 1 \right)}{P_{N,t+1}}$$

In the equilibrium, all markets clear and we can obtain

$$K_{t+1} = \frac{\alpha_T - \gamma(\alpha_T - \alpha_N)\left[(R-1)s_t + 1\right]}{(1-\alpha_T)R} w$$

and then

$$CA_t = s_t w_t - s_{t-1} w + \frac{(\alpha_T - \alpha_N)(R-1)\left(s_t - s_{t-1}\right)}{(1-\alpha_T)R} w$$

To show $\frac{dCA_t}{d\beta} > 0$, we only need to show $\frac{d(s_t w_t - s_{t-1} w_{t-1})}{d\beta} > 0$. One sufficient condition for the inequality is

$$s_t P_{Nt} > s_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$s_t \frac{dP_{Nt}}{d\beta} + P_{Nt} \frac{ds_t}{d\beta} > 0$$

which means

$$\frac{dP_{Nt}/d\beta}{ds_t/d\beta} + \frac{P_{Nt}}{s_t} > 0$$

Plugging the expressions of $\frac{dP_{Nt}}{d\beta}$ and $\frac{ds_t}{d\beta}$, we have

$$\begin{aligned}\frac{dP_{Nt}}{ds_t} + \frac{P_{Nt}}{s_t} &= \frac{P_{Nt}C_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) - \gamma(1-s_t)w_tC_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive}.terms} \\ &\quad + \text{positive}.term \\ &= \frac{(P_{Nt}C_{Nt} - \gamma(1-s_t)w_t)\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive}.terms} + \text{positive}.term\end{aligned}$$

As shown above, $P_{Nt}C_{Nt} - \gamma(1-s_t)w_t > 0$, then $\frac{dCA_t}{d\beta} > 0$, in period t, the country will experience a current account surplus. □

A3.2 Proof of Proposition 10

Proof. At $\phi = 1$, all women and men are symmetric and they make the same savings decisions. Since $\frac{1}{2} \leq \kappa \leq 1$,

$$\kappa(Rs_t^m w_t + Rs_t^w w_t) \geq \max\left(Rs_t^w w_t,\, Rs_t^m w_t\right) \tag{4.39}$$

Then, in the neighbourhood of $\phi = 1$, we have $\kappa u'_{2m} < u'_{2m,n}$.[12]

We proceed in two steps. In the first step, we assume that inequality $\kappa u'_{2m} < u'_{2m,n}$ holds for all values of ϕ, and prove that a higher sex ratio leads to a higher savings rate. In the second step, we prove by contradiction that the inequality indeed holds for all values of ϕ.

Assume that inequality $\kappa u'_{2m} < u'_{2m,n}$ holds for all values of $\phi \geq 1$, the first order

---

[12] The condition for the equality $\kappa u'_{2m} = u'_{2m,n}$ is $\kappa(Rs^m y + Rs^w y) \geq \max\left(Rs^w y,\, Rs^m y\right)$ and $\kappa = 1$, which is not possible in the model.

conditions for a woman and a man, respectively, are:

$$-u'_{1w} + \beta R \frac{P_t}{P_{t+1}} \left[\begin{array}{c} \kappa u'_{2w} \left(\delta^w + \left[\frac{1}{\phi} \left(1 - F\left(\bar{\eta}^w\right)\right) + M(\bar{\eta}^w) f(\bar{\eta}^w) \right] \right) \\ + \left(1 - \delta^w\right) u'_{2w,n} + f\left(\bar{\eta}^w\right) \kappa u'_{2w} \left(u_{2w} - u_{2w,n}\right) \end{array} \right] = 0 \tag{4.40}$$

$$-u'_{1m} + \beta R \frac{P_t}{P_{t+1}} \left[\begin{array}{c} \kappa u'_{2m} \left(\delta^m + \left[\phi \left(1 - F\left(\bar{\eta}^m\right)\right) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m) \right] \right) \\ + \left(1 - \delta^m\right) u'_{2m,n} + f\left(\bar{\eta}^m\right) \kappa u'_{2m} \left(u_{2m} - u_{2m,n}\right) \end{array} \right] = 0 \tag{4.41}$$

We show by contradiction that $\bar{\eta}^w = u_{2m,n} - u_{2m}$ and $\bar{\eta}^m = M(\bar{\eta}^w)$ hold for $\phi \geq 1$. Suppose not, then

$$\bar{\eta}^m > M(\bar{\eta}^w) \geq \bar{\eta}^w$$

where the second inequality holds because $\phi \geq 1$. Then we have

$$u\left(\frac{Rs_t^w w_t}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs_t^w w_t + Rs_t^m w_t)}{P_{t+1}}\right) > u\left(\frac{Rs_t^m w_t}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs_t^w w_t + Rs_t^m w_t)}{P_{t+1}}\right)$$

and hence, $s_t^w > s_t^m$.

Then

$$\begin{aligned} u'_{1w} &= \delta^w \kappa u'_{2w} \left(1 + \left[\frac{1}{\phi}\left(1 - F\left(\bar{\eta}^w\right)\right) + M(\bar{\eta}^w) f(\bar{\eta}^w)\right]\right) \\ &\quad + \left(1 - \delta^w\right) u'_{2w,n} + f\left(\bar{\eta}^w\right) \kappa u'_{2w} \left(u_{2w} - u_{2w,n}\right) \\ &< \delta^m \kappa u'_{2w} \left(1 + \left[\frac{1}{\phi}\left(1 - F\left(\bar{\eta}^w\right)\right) + M(\bar{\eta}^w) f(\bar{\eta}^w)\right]\right) \\ &\quad + \left(1 - \delta^m\right) u'_{2w,n} + f\left(\bar{\eta}^w\right) \kappa u'_{2w} \left(u_{2w} - u_{2w,n}\right) \end{aligned}$$

$$\begin{aligned} &< \delta^m \kappa u'_{2m} \left(1 + \left[\phi\left(1 - F\left(\bar{\eta}^m\right)\right) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m)\right]\right) \\ &\quad + \left(1 - \delta^m\right) u'_{2m,n} + f\left(\bar{\eta}^m\right) \kappa u'_{2m} \left(u_{2m} - u_{2m,n}\right) \\ &= u'_{1m} \end{aligned}$$

[13]which means that

$$s_t^m > s_t^w$$

Contradiction! Therefore, we have $\bar{\eta}^m = M(\bar{\eta}^w)$ and $s^m \geq s^w$ for $\phi \geq 1$.

We totally differentiate the system and have

$$\Omega \cdot \begin{pmatrix} ds_t^w \\ ds_t^m \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix}$$

where Ω is a 5×5 matrix with elements

$$\begin{aligned} \Omega_{11} &= u''_{1w} w_t + \beta \left(R \frac{P_t}{P_{t+1}} \right)^2 w_t \begin{bmatrix} \kappa^2 u''_{2w} \left(\left(1 + \frac{1}{\phi}\right)(1 - F(\bar{\eta}^w)) \right) \\ + f(\bar{\eta}^w) \kappa^2 u''_{2w} (u_{2w} + M(\bar{\eta}^w) - u_{2w,n}) \\ + 2 f(\bar{\eta}^w) \kappa u'_{2w} \left(\kappa u'_{2w} - u'_{2w,n} \right) + (1 - \delta^w) u''_{2w,n} \end{bmatrix} \\ \Omega_{12} &= \beta \left(R \frac{P_t}{P_{t+1}} \right)^2 w_t \begin{bmatrix} \kappa^2 u''_{2w} \left(\left(1 + \frac{1}{\phi}\right)(1 - F(\bar{\eta}^w)) \right) \\ + f(\bar{\eta}^w) \kappa^2 u''_{2w} (u_{2w} + M(\bar{\eta}^w) - u_{2w,n}) \\ + f(\bar{\eta}^w) \kappa^2 u'^2_{2w} + f(\bar{\eta}^w) \left(u'_{2m,n} - \kappa u'_{2m} \right) \left(u'_{2w,n} - \kappa u'_{2w} \right) \end{bmatrix} \\ \Omega_{13} &= \Omega_{14} = \Omega_{15} = 0 \end{aligned}$$

---

[13]The second inequality holds because (i)

$$\frac{1}{\phi}(1 - F(\bar{\eta}^w)) + M(\bar{\eta}^w) f(\bar{\eta}^w) = \phi (1 - F(\bar{\eta}^m)) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m)$$

by using the uniform distribution assumption; and (ii),

$$u_{2m} - u_{2m,n} > u_{2w} - u_{2w,n}$$

$$\begin{aligned}
\Omega_{21} &= \beta\left(R\frac{P_t}{P_{t+1}}\right)^2 w_t \left[\begin{array}{c} \kappa^2 u''_{2m}\left(\left(1+\phi\right)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right) \\ +f\left(\bar{\eta}^w\right)\kappa u'_{2m}\left(\left(1+\frac{1}{\phi}\right)\kappa u'_{2m}-\frac{1}{\phi}u'_{2m,n}\right) \end{array}\right] \\
\Omega_{22} &= u''_{1m}w_t+\beta\left(R\frac{P_t}{P_{t+1}}\right)^2 w_t \left[\begin{array}{c} \kappa^2 u''_{2m}\left(\left(1+\phi\right)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right) \\ +\left(1-\delta^m\right)u''_{2m,n} \\ +f\left(\bar{\eta}^w\right)\left(\kappa u'_{2m}-u'_{2m,n}\right)\left(\begin{array}{c} \left(1+\frac{1}{\phi}\right)\kappa u'_{2m} \\ -\frac{1}{\phi}u'_{2m,n} \end{array}\right) \end{array}\right] \\
\Omega_{23} &= \Omega_{24}=\Omega_{25}=0
\end{aligned}$$

$$\begin{aligned}
\Omega_{31} &= \frac{\gamma w_t}{1+\phi},\ \Omega_{32}=\frac{\gamma\phi w_t}{1+\phi},\ \Omega_{33}=-\gamma(1-s_t) \\
\Omega_{34} &= \frac{A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}},\Omega_{35}=\frac{P_{Nt}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} \\
\Omega_{41} &= \Omega_{42}=0,\ \Omega_{43}=-1,\ \Omega_{44}=0 \\
\Omega_{45} &= \left(\frac{\alpha_T}{1-\alpha_T}\right)^{1-\alpha_T}(1-\alpha_T)A_{Tt}K_{Tt}^{\alpha_T}(1-L_{Nt})^{-\alpha_T-1} \\
\Omega_{51} &= \Omega_{52}=0,\ \Omega_{53}=-1,\ \Omega_{54}=\frac{w_t}{P_{Nt}} \\
\Omega_{55} &= -\left(\frac{\alpha_N}{1-\alpha_N}\right)^{1-\alpha_T}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}{}^{-\alpha_N-1}
\end{aligned}$$

and

$$\begin{aligned}
z_1 &= 0,\ z_2=\frac{1}{\phi^2}\left[1-F(\bar{\eta}^w)\right]\left(\kappa u'_{2m}-u'_{2m,n}\right) \\
z_3 &= -\frac{\gamma w_t\left(s_t^m-s_t^w\right)}{1+\phi},\ z_4=z_5=0
\end{aligned}$$

The determinant of matrix Ω is

$$\det(\Omega)=\det\begin{pmatrix}\Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22}\end{pmatrix}\cdot\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix}$$

It is easy to show that

$$\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix} > 0$$

and

$$\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix} = \text{negative}.terms + \gamma(1-s_t)\left(\frac{w_t}{P_{Nt}}\right)^2\frac{1-\alpha_T}{L_T} - \left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)C_{Nt}$$

Notice that the consumption of the nontradable good by the young cohort must be less than the aggregate nontradable good consumption, then $\gamma(1-s_t)w_t < P_{Nt}C_{Nt}$. Therefore,

$$\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix} < 0$$

and $\det(\Omega) < 0$.

Then

$$\frac{ds_t^m}{d\phi} = \frac{z_2\Omega_{11}}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}} > 0 \quad\text{and}\quad \frac{ds_t^w}{d\phi} = \frac{z_2\Omega_{12}}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}$$

The sign of $\frac{ds_t^w}{d\phi}$ is ambiguous. However, the aggregate savings rate by the young cohort,

$s_t^{young} = \frac{\phi}{1+\phi} s_t^m + \frac{1}{1+\phi} s_t^w$, rises as the sex ratio becoms more unbalanced.

$$\frac{ds_t^{young}}{d\phi} = \frac{s_t^m - s_t^w}{(1+\phi)^2} + \frac{\phi - 1}{1+\phi}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\frac{z_2\left(u''_{1w}y + Ry\begin{bmatrix} (1-\delta^w)\, u''_{2w,n} \\ -f(\bar{\eta}^w)\left(\kappa u'_2\left(u'_{2w,n} - u'_{2m,n}\right) + u'_{2w,n}u'_{2m,n}\right)\end{bmatrix}\right)}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22}\end{pmatrix}}$$

where all the terms on the right hand side are positive and hence, $\frac{ds_t^{young}}{d\phi} > 0$.

As for the price of the nontradable good,

$$\frac{dP_{Nt}}{d\phi} = -\frac{z_3 \frac{w_t^2}{P_{Nt}} \frac{\alpha_T}{L_{Tt}}}{\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix}} + \frac{z_2\left(\Omega_{11}\Omega_{32} - \Omega_{12}\Omega_{31}\right)\frac{w_t}{P_{Nt}}\frac{\alpha_T}{L_{Tt}}}{\det(\Omega)}$$

It is easy to show that $\Omega_{11}\Omega_{32} - \Omega_{12}\Omega_{31} < 0$, and since $z_2 < 0$, $\frac{dP_{Nt}}{d\phi} < 0$, which results in a fall in the consumption price index and therefore a real exchange rate depreciation in period t.

As for the current account,

$$CA_t = P_{Nt}Q_{Nt} + Q_{Tt} + (R-1)\cdot NFA_{t-1} - P_tC_t - K_{t+1}$$

where NFA_{t-1} is the net foreign asset holdings in period $t-1$ and K_{t+1} is the sum of capital input in both the nontradable sector and the tradable sector in period $t+1$.

Notice that

$$s_{t-1}w_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = s_t w_t - s_{t-1} w_{t-1} - \Delta K_{t+1}$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. By Obstfeld and Rogoff (1995), if the sex ratio remains constant ϕ after period t, the price of the nontradable good will go back to its initial level, which means that real exchange rate will appreciate in period $t+1$. In this perfect foresight setup, when firms make their optimal decisions, equations (2.2) and (2.4) hold. If we assume log utility function, the aggregate savings rate by the young cohort will remain the same after period t.

The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w \left((R-1) s_t + 1 \right)}{P_{N,t+1}}$$

where we drop the time subscript because both the wage rate and the relative price of the nontradable good would go back to their initial levels. It is easy to see that since $s_t > s_{t-1}$, $Q_{N,t+1} > Q_{N,t-1}$.

As in Obstfeld and Rogoff (1995), we assume that $\alpha_N < \alpha_T$, the nontradable sector has a lower capital-intensity than the tradable sector. Then, in period $t+1$, $K_{t+1} < K_{t-1}$.

In period $t+1$,

$$A_{Nt} K_{N,t+1}^{\alpha_N} L_{N,t+1}^{1-\alpha_N} = \frac{\gamma w \left((R-1) s_t + 1 \right)}{P_{N,t+1}}$$

In the equilibrium, all markets clear and we can obtain

$$K_{t+1} = \left[\frac{\alpha_T}{(1-\alpha_T)} - \left(\frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N} \right) \gamma \left((R-1)s + 1 \right) \right] \frac{w}{R}$$

and then

$$\begin{aligned} \Delta K_{t+1} &= \gamma \left(\frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N} \right) (R-1) \left(s_t - s_{t-1} \right) \frac{w}{R} \\ CA_t &= s_t w_t - s_{t-1} w + \gamma \left(\frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N} \right) (R-1) \left(s_t - s_{t-1} \right) \frac{w}{R} \end{aligned}$$

To show $\frac{dCA_t}{d\phi} > 0$, we only need to show $\frac{d(s_t w_t - s_{t-1} w_{t-1})}{d\phi} > 0$. By (3.9), one sufficient condition is for the inequality is

$$s_t P_{Nt} > s_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$s_t \frac{dP_{Nt}}{d\phi} + P_{Nt} \frac{ds_t}{d\phi} > 0$$

which means

$$\frac{dP_{Nt}/d\phi}{ds_t/d\phi} + \frac{P_{Nt}}{s_t} > 0$$

Plugging the expressions of $\frac{dP_{Nt}}{d\phi}$ and $\frac{ds_t}{d\phi}$, we have

$$\begin{aligned} \frac{dP_{Nt}}{ds_t} + \frac{P_{Nt}}{s_t} &= \frac{P_{Nt} C_{Nt} \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) - \gamma(1-s_t) w_t C_{Nt} \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive}\,.terms} \\ &\quad + \text{positive}\,.term \\ &= \frac{(P_{Nt} C_{Nt} - \gamma(1-s_t) w_t) \left(\frac{w_t}{P_{Nt}}\right) \left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive}\,.terms} + \text{positive}\,.term \end{aligned}$$

As shown above, $P_{Nt} C_{Nt} - \gamma(1-s_t) w_t > 0$, then $\frac{dCA_t}{d\phi} > 0$, in period t, the country will experience a current account surplus.

We now show by contradiction that $\kappa u'_{2m} < u'_{2m,n}$ must hold for all ϕs. Suppose not, then $\kappa u'_{2m} > u'_{2m,n}$ may fail sometime. Due to continuity of E, there exists a level of sex ratio ϕ_0 at which $\kappa u'_{2m} = u'_{2m,n}$, which implies that $z_2 = 0$. Then

$$\frac{ds_t^m}{d\phi} = \frac{ds_t^w}{d\phi} = 0$$

We calculate the derivative of z_2 with respect to ϕ at $\phi = \phi_0$ and obtain

$$\left.\frac{dz_2}{d\phi}\right|_{\phi=\phi_0} = \frac{Ry}{\phi_0^2} \left[1 - F(\bar{\eta}^w)\right] \left(\kappa u''_{2m} \left(\frac{ds_t^m}{d\phi} + \frac{ds_t^w}{d\phi}\right) - u'_{2m,n} \frac{ds_t^m}{d\phi}\right) = 0$$

Then we have

$$\left.\frac{d^2 s_t^w}{d\phi^2}\right|_{\phi=\phi_0} = \frac{\Omega_{12}}{\det(\Omega)} \left.\frac{dz_2}{d\phi}\right|_{\phi=\phi_0} = 0$$

$$\left.\frac{d^2 s_t^m}{d\phi^2}\right|_{\phi=\phi_0} = -\frac{\Omega_{11}}{\det(\Omega)} \left.\frac{dz_2}{d\phi}\right|_{\phi=\phi_0} = 0$$

Using the result that $\left.\frac{d^2 s_t^w}{d\phi^2}\right|_{\phi=\phi_0} = \left.\frac{d^2 s_t^m}{d\phi^2}\right|_{\phi=\phi_0} = 0$, we calculate the second order derivative of A with respect to ϕ at $\phi = \phi_0$,

$$\left.\frac{d^2 z_2}{d\phi^2}\right|_{\phi=\phi_0} = \frac{Ry}{\phi_0^2}\left[1 - F(\bar{\eta}^w)\right]\left(\kappa u''_{2m}\left(\frac{d^2 s_t^m}{d\phi^2} + \frac{d^2 s_t^w}{d\phi^2}\right) - u'_{2m,n}\frac{d^2 s_t^m}{d\phi^2}\right) = 0$$

Then

$$\left.\frac{d^3 s_t^w}{d\phi^3}\right|_{\phi=\phi_0} = \frac{\Omega_{12}}{\det(\Omega)} \left.\frac{d^2 z_2}{d\phi^2}\right|_{\phi=\phi_0} = 0$$

$$\left.\frac{d^3 s_t^m}{d\phi^3}\right|_{\phi=\phi_0} = -\frac{\Omega_{11}}{\det(\Omega)} \left.\frac{d^2 z_2}{d\phi^2}\right|_{\phi=\phi_0} = 0$$

By iterating this process forward, we obtain that

$$z_2|_{\phi=\phi_0} = 0 \text{ and } \left.\frac{d^k z_2}{d\phi^k}\right|_{\phi=\phi_0} = 0 \text{ for any } k > 0$$

This means that z_2 equals zero for all ϕs, which contradicts with the result at the beginning of the proof that $z_2 \neq 0$ when $\phi = 1$. In other words, there exists no ϕ_0 such that $z_2 = 0$ holds. Therefore, inequality $\kappa u'_{2m} < u'_{2m,n}$ must hold for all ϕs. □

A3.3. Proof of Proposition 11

Proof. If $u(c) = \ln c$, solving the first order condition under a balanced sex ratio for both men and women in the marriage market, we can obtain

$$-\frac{1}{1-s_t} + \beta R \frac{P_t}{P_{t+1}} \frac{1}{s_t} = 0$$

which is the same optimal condition when a man or a woman chooses to be single. For a representative woman, at the balanced sex ratio, if she chooses to enter the marriage market, with probability $F(\bar{\eta})$ she can get married and receive welfare

$$\begin{aligned} V_t^w &= \ln\left(\frac{(1-s_t)w_t}{P_t}\right) + \beta F(\bar{\eta}) \ln\left(\frac{\kappa R(2s_t)w_t}{P_{t+1}}\right) \\ &\quad + \beta(1-F(\bar{\eta}))\ln\left(\frac{Rs_t w_t}{P_{t+1}}\right) + E\left[\eta \mid \eta^w \geq \bar{\eta}\right] \\ &\geq \ln\left(\frac{(1-s_t)w_t}{P_t}\right) + \beta \ln\left(\frac{Rs_t w_t}{P_{t+1}}\right) = V_{n,t}^w \end{aligned}$$

where the inequality holds because $\kappa > 1/2$ and $E[\eta \mid \eta^w \geq \bar{\eta}] \geq 0$. Therefore, entering the marriage market is a dominant strategy for all women. Since men and women are symmetric when $\phi = 1$, all men and all women will enter the marriage market with probability one at the balanced sex ratio.

As we have showed in Proposition 2,

$$\frac{ds_t^m}{d\phi} > 0 \text{ and } \frac{ds_t^m}{d\phi} + \frac{ds_t^w}{d\phi} > 0$$

we can show that

$$\begin{aligned} \frac{\partial V_t^m}{\partial \phi} &= y\left(-u'_{1m} + \beta R \frac{P_t}{P_{t+1}}\left(\kappa \delta^m u'_{2m} + (1-\delta^m)u'_{2m,n}\right)\right)\frac{ds_t^m}{d\phi} \\ &\quad + \beta R \frac{P_t}{P_{t+1}} \delta^m y \kappa u'_{2w} \frac{ds_t^w}{d\phi} - \beta \int_{M(\bar{\eta}^w)} [1-F(\eta)]\, d\eta \\ &< -\beta \int_{M(\bar{\eta}^w)} [1-F(\eta)]\, d\eta - \beta R \frac{P_t}{P_{t+1}} (\phi - 1)\, \delta^m y \kappa u'_{2w} \frac{ds_t^m}{d\phi} < 0 \end{aligned} \tag{4.42}$$

where the first equality in (4.42) holds because

$$\begin{aligned}\frac{\partial\delta^m}{\partial\phi} &= -\frac{1-F(\bar{\eta}^w)}{\phi^2}(u_{2m}-u_{2m,n}) \\ &\quad -\frac{Ryf(\bar{\eta}^w)}{\phi}\left[u'_{2m,n}\frac{ds_t^m}{d\phi}-\kappa u'_{2m}\left(\frac{ds_t^m}{d\phi}+\frac{ds_t^w}{d\phi}\right)\right](u_{2m}-u_{2m,n})\end{aligned}$$

$$\begin{aligned}\frac{\partial\left(\int_{M(\bar{\eta}^w)}M^{-1}(\eta^m)dF(\eta^m)\right)}{\partial\phi} &= -\int_{M(\bar{\eta}^w)}[1-F(\eta)]\,d\eta-\frac{\bar{\eta}^w(1-F(\bar{\eta}^w))}{\phi^2} \\ &\quad -\frac{\bar{\eta}^w f(\bar{\eta}^w)}{\phi}\left[u'_{2m,n}\frac{ds_t^m}{d\phi}-\kappa u'_{2m}\left(\frac{ds_t^m}{d\phi}+\frac{ds_t^w}{d\phi}\right)\right]\end{aligned}$$

and the first inequality in (4.42) holds because

$$\delta^m < 1 < \phi(1-F(\bar{\eta}^m))+\bar{\eta}^m f(\bar{\eta}^m) \text{ and } \frac{ds_t^w}{d\phi}\leq\frac{ds_t^m}{d\phi}$$

Men lose as the sex ratio rises while the effect on women's welfare is ambiguous.

Now consider women's welfare. Given the equilibrium s_t^m and s_t^w under a sex ratio ϕ, if one woman deviates from the equilibrium choice s_t^w, for instance, by choosing a savings rate $s_t^{w\prime} = s_t^m$, she would receive a lower life-time utility $V_t^{w\prime}$ ($\leq V_t^w$). Since $s_t^{w\prime} = s_t^m \geq s_t^w$, this woman will have a better situation than all other women in the marriage market, i.e., she is more likely to get married and also more likely to marry a better man. Then

$$\begin{aligned}V_t^{w\prime} &= u_{1w'}+\beta\left[\delta' u_{2w'}+(1-\delta')\,u_{2w',n}+\int_{\bar{\eta}^w}M(\eta^w+u_{2w'}-u_{2w})\,dF(\eta^w)\right] \\ &\geq u_{1w'}+\beta\left[(1-F(\bar{\eta}^w))\,u_{2w'}+F(\bar{\eta}^w)\,u_{2w',n}+\int_{\bar{\eta}^w}M(\eta^w)\,dF(\eta^w)\right] \\ &= u_{1m}+\beta\left[(1-F(\bar{\eta}^w))\,u_{2m}+F(\bar{\eta}^w)\,u_{2m,n}+\int_{\bar{\eta}^w}M(\eta^w)\,dF(\eta^w)\right] \\ &\geq u_{1m}+\beta\left[(1-F(M(\bar{\eta}^w)))\,u_{2m}+F(M(\bar{\eta}^w))\,u_{2m,n}+\int_{M(\bar{\eta}^w)}M^{-1}(\eta^m)\,dF(\eta^m)\right] \\ &= V_t^m\end{aligned}$$

where $u_{1w'}$, $u_{2w'}$ and $u_{2w',n}$ denote the first period consumption-led utility, the second period consumption-led utilty when she gets married, and the second period utility when she fails to get matched with any man, respectively. u_{2w} is the second period consumption-led utility for all other women who get married. The first inequality holds because the woman faces a greater possibility of getting married and also she will receive a higher expected emotional utility from her husband. The second inequality holds because, women are more likely than men to get married and also women are expecting to receive higher emotional utilities from their spouses than men.

Therefore, for $\phi \geq 1$, we can show that $V_t^w \geq V_t^{w\prime} \geq V_t^m$, women always achieve higher welfare than men.

For a representative man in the marriage market, given his rivals' choices, if he choose to stay in the marriage market, he will follow the first order condition (4.41) and achieves an approximate life time utility $u_{1m} + \beta u_{2m,n}$. If he chooses to be single, he maximizes the life time utility $u_1 + \beta u_2$. The first order condition in this case is

$$-u'_{1m} + u'_{2m} = 0 \tag{4.43}$$

The two savings decisions, in the marriage market and being single, will be different since the man will follow different first order conditions. Then

$$V_n^m = \max u_1 + \beta u_2 > u_{1m} + \beta u_{2m,n} \rightarrow V^m$$

when $\phi \rightarrow \infty$. The representative man will then choose to be single which violates the assumption that, for all ϕs, entering the marriage market is the dominant strategy for all men. Therefore, a threshold as ϕ_1 exists and at $\phi \geq \phi_1$, $V_n^m = V^m$.

For $\phi \geq \phi_1$, with probability $\frac{\phi_1}{\phi}$, a representative man will choose to enter the marriage market, and with probability $1 - \frac{\phi_1}{\phi}$, he remains single. For a representative woman, since she earns the same first period income as a representative man, we can show that

$$V_n^w = V_n^m = V^m < V^w$$

the representative woman will enter the marriage market with probability one.

As for the aggregate savings rate in the young cohort, we have showed in Proposition 1 that for $\phi < \phi_1$, as the sex ratio rises, the aggregate savings rate in the young cohort will rise. For $\phi \geq \phi_1$, as the sex ratio rises, some men begin quitting the marriage market and choose a different savings rate according to (4.43). Compare (4.41) with (4.43), it is ambiguous whether $s^m > s_n^m$ or not, then the effect on the aggregate savings rate is ambiguous. □

A3.4. Proof of Proposition 4

Proof. If $u(C) = \ln C$, for $\phi < \phi_1$, by the optimal labor supply condition, we have

$$0 < \frac{dL_t^i}{ds_t^i} = \frac{1}{1 - s_t^i} \frac{v_i' L_t^i}{v_i' - v_i'' L_t^i} \tag{4.44}$$

where $i = w, m$.

Similar to the proof of Proposition 2, we can show that $\bar{\eta}^m = M(\bar{\eta}^w)$ and $s^m L^m \geq s^w L^w$ for $\phi \geq 1$. Since at $\phi = 1$, women and men are symmetric, and hence $s^m = s^w$ and $L^m = L^w$. For $\phi \geq 1$, by (4.44), $s^m L^m \geq s^w L^w$ means $s^m \geq s^w$ and $L^m \geq L^w$.

$$\Omega \cdot \begin{pmatrix} ds_t^w \\ ds_t^m \\ dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix}$$

where

$$
\begin{aligned}
\Omega_{11} &= u''_{1w}\left(1-(1-s_t^w)\frac{dL_t^w}{ds_t^w}\right)\frac{w_t}{P_t} \\
&+\beta\left(R\frac{P_t}{P_{t+1}}\right)^2\left[\begin{array}{c}\kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)(1-F(\bar{\eta}^w))\right)\\ +(1-\delta^w)\,u''_{2w,n}\\ +f(\bar{\eta}^w)\,\kappa^2 u''_{2w}\left(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}\right)\\ +2f(\bar{\eta}^w)\,\kappa u'_{2w}\left(\kappa u'_{2w}-u'_{2w,n}\right)\end{array}\right]\left(L_t^w+s_t^w\frac{dL_t^w}{ds_t^w}\right)\frac{w_t}{P_t} \\
\Omega_{12} &= \beta\left(R\frac{P_t}{P_{t+1}}\right)^2\left[\begin{array}{c}\kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)(1-F(\bar{\eta}^w))\right)\\ +(1-\delta^w)\,u''_{2w,n}\\ +f(\bar{\eta}^w)\,\kappa^2 u''_{2w}\left(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}\right)\\ +2f(\bar{\eta}^w)\,\kappa u'_{2w}\left(\kappa u'_{2w}-u'_{2w,n}\right)\end{array}\right]\left(L_t^m+s_t^m\frac{dL_t^m}{ds_t^m}\right)\frac{w_t}{P_t} \\
\Omega_{13} &= \Omega_{14}=\Omega_{15}=0
\end{aligned}
$$

$$
\begin{aligned}
\Omega_{21} &= \beta\left(R\frac{P_t}{P_{t+1}}\right)^2\left[\begin{array}{c}\kappa^2 u''_{2m}\left((1+\phi)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right)\\ +f(\bar{\eta}^w)\,\kappa u'_{2m}\left(\left(1+\frac{1}{\phi}\right)\kappa u'_{2m}-\frac{1}{\phi}u'_{2m,n}\right)\end{array}\right]\left(L_t^w+s_t^w\frac{dL_t^w}{ds_t^w}\right)\frac{w_t}{P_t} \\
\Omega_{22} &= u''_{1m}\left(1-(1-s_t^w)\frac{dL_t^m}{ds_t^m}\right)\frac{w_t}{P_t} \\
&+\beta\left(R\frac{P_t}{P_{t+1}}\right)^2\left[\begin{array}{c}\kappa^2 u''_{2m}\left((1+\phi)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right)\\ +(1-\delta^m)\,u''_{2m,n}\\ +f(\bar{\eta}^w)\left(\kappa u'_{2m}-u'_{2m,n}\right)\left(\begin{array}{c}\left(1+\frac{1}{\phi}\right)\kappa u'_{2m}\\ -\frac{1}{\phi}u'_{2m,n}\end{array}\right)\end{array}\right]\left(L_t^m+s_t^m\frac{dL_t^m}{ds_t^m}\right)\frac{w_t}{P_t} \\
\Omega_{23} &= \Omega_{24}=\Omega_{25}=0
\end{aligned}
$$

$$
\begin{aligned}
\Omega_{31} &= \frac{\gamma w_t}{1+\phi}\left(L_t^w+s_t^w\frac{dL_t^w}{ds_t^w}\right),\ \Omega_{32}=\frac{\gamma\phi w_t}{1+\phi}\left(L_t^m+s_t^m\frac{dL_t^m}{ds_t^m}\right) \\
\Omega_{33} &= -\gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi}+\frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right] \\
\Omega_{34} &= \frac{A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}},\ \Omega_{35}=\frac{P_{Nt}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}
\end{aligned}
$$

$$\begin{aligned}
\Omega_{41} &= -\frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}} \frac{1}{1+\phi} \frac{dL_t^w}{ds_t^w} \\
\Omega_{42} &= -\frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}} \frac{\phi}{1+\phi} \frac{dL_t^m}{ds_t^m} \\
\Omega_{43} &= -1,\ \Omega_{44} = 0 \\
\Omega_{45} &= \left(\frac{\alpha_T}{1-\alpha_T}\right)^{1-\alpha_T} (1-\alpha_T) A_{Tt} K_{Tt}^{\alpha_T} \left(\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}\right)^{-\alpha_T - 1} \\
\Omega_{51} &= \Omega_{52} = 0,\ \Omega_{53} = -1,\ \Omega_{54} = \frac{w_t}{P_{Nt}} \\
\Omega_{55} &= -\left(\frac{\alpha_N}{1-\alpha_N}\right)^{1-\alpha_N} (1-\alpha_N) A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}{}^{-\alpha_N - 1}
\end{aligned}$$

and

$$\begin{aligned}
z_1 &= 0,\ z_2 = \frac{1}{\phi^2}\left[1 - F(\bar{\eta}^w)\right]\left(\kappa u'_{2m} - u'_{2m,n}\right),\ z_3 = -\frac{\gamma w_t \left(s_t^m L_t^m - s_t^w L_t^w\right)}{1+\phi} \\
z_4 &= \frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}} \frac{L_t^m - L_t^w}{(1+\phi)^2},\ z_5 = 0
\end{aligned}$$

The determinant of matrix Ω is

$$\det(\Omega) = \det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \cdot \det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}$$

Under the assumption that $E\eta$ is sufficiently large, it is easy to show that

$$\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} > 0$$

and

$$\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix} = \text{negative}\,.terms$$
$$+\gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi}+\frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right]\left(\frac{w_t}{P_{Nt}}\right)^2\frac{1-\alpha_T}{L_T}$$
$$-\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}}+\frac{1-\alpha_N}{L_{Nt}}\right)C_{Nt}$$

Notice that the consumption on the nontradable goods by the young cohort must be less than the aggregate nontradable good consumption, then $\gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi}+\frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right]w_t < P_{Nt}C_{Nt}$. Therefore,

$$\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix} < 0$$

and $\det(\Omega) < 0$

Then

$$\frac{ds_t^m}{d\phi} = -\frac{z_2\Omega_{11}}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}} > 0 \text{ and } \frac{ds_t^w}{d\phi} = \frac{z_2\Omega_{12}}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}$$

The sign of $\frac{ds_t^w}{d\phi}$ is ambiguous. By (4.44), we have

$$\frac{dL_t^m}{d\phi} > 0$$

and the sign of $\frac{dL_t^w}{d\phi}$ is ambiguous.

The aggregate savings rate by the young cohort $s_t^{young} = \frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$,

$$\frac{ds_t^{young}}{d\phi} = \frac{\phi}{1+\phi}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\frac{ds_t^w}{d\phi} + \frac{s_t^m - s_t^w}{(1+\phi)^2} > 0$$

The aggregate labor supply in period t

$$\frac{dL_t}{d\phi} = \frac{\phi}{1+\phi}\frac{dL_t^m}{ds_t^m}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\frac{dL_t^w}{ds_t^w}\frac{ds_t^w}{d\phi} + \frac{L_t^m - L_t^w}{(1+\phi)^2}$$

Under the assumption $\frac{v''L}{v'}$ is non-decreasing in L, by (4.44), $\frac{dL_t^m}{ds_t^m} > \frac{dL_t^w}{ds_t^w}$, then we have $\frac{dL_t}{d\phi} > 0$, which means the aggregate labor supply is increasing in the sex ratio.

As for the price of the nontradable good,

$$\frac{dP_{Nt}}{d\phi} = -\frac{z_3\left(\frac{w_t^2}{P_{Nt}}\right)\frac{\alpha_T}{L_{Tt}} + z_4\left(\Omega_{34}\Omega_{55} - \Omega_{35}\Omega_{54}\right)}{\det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix}} + \frac{z_2\left(\Omega_{11}\Omega_{32} - \Omega_{12}\Omega_{31}\right)\frac{w_t^2}{P_{Nt}}\frac{\alpha_T}{L_{Tt}}}{\det(\Omega)}$$

It is easy to show that $\Omega_{34}\Omega_{55} - \Omega_{35}\Omega_{54} < 0$, then $\frac{dP_{Nt}}{d\phi} < 0$, which results in a fall in the consumption price index and therefore a real exchange rate depreciation in period t.

As for the current account,

$$CA_t = P_{Nt}Q_{Nt} + Q_{Tt} + (R-1)\cdot NFA_{t-1} - P_tC_t - K_{t+1}$$

where NFA_{t-1} is the net foreign asset holdings in period $t-1$ and K_{t+1} is the sum of capital input in both the nontradable sector and the tradable sector in period $t+1$.

Notice that

$$s_{t-1}w_{t-1}L_{t-1} = NFA_{t-1} + K_t$$

Then

$$CA_t = \left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right)w_t - s_{t-1}w_{t-1}L_{t-1} - \Delta K_{t+1}$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. Following Obstfeld and Rogoff (1995), if the sex ratio remains constant at ϕ after period t, the price of the nontradable good will go back to its initial level, which means that the real exchange rate will appreciate in period $t+1$. In this perfect foresight setup, when firms make their optimal decisions, equations (2.2) and (2.4) hold. If we take the log utility function, the aggregate savings rate by the young cohort will remain the same after period t.

The demand for the nontradable good is now

$$Q_{N,t+1} = \frac{\gamma w \left((R-1) \left(\frac{s^w_{t+1}}{1+\phi} + \frac{\phi s^m_{t+1}}{1+\phi} \right) + 1 \right)}{P_{N,t+1}}$$

where we drop the time subindex because wage rate and the relative price of the nontradable good will go back to their initial levels. It is easy to see that since $s_t > s_{t-1}$, $Q_{N,t+1} > Q_{N,t-1}$.

In period $t+1$,

$$A_{Nt} K^{\alpha_N}_{N,t+1} L^{1-\alpha_N}_{N,t+1} = \frac{\gamma w \left((R-1) \left(\frac{s^w_t L^w_t}{1+\phi} + \frac{\phi s^m_t L^m_t}{1+\phi} \right) + 1 \right)}{P_{N,t+1}}$$

In equilibrium, all markets clear and we can obtain

$$K_{t+1} = \frac{\alpha_T - \gamma(\alpha_T - \alpha_N) \left[(R-1) \left(\frac{s^w_t L^w_t}{1+\phi} + \frac{\phi s^m_t L^m_t}{1+\phi} \right) + 1 \right]}{(1-\alpha_T) R} w$$

and then

$$\begin{aligned} CA_t &= \left(\frac{s^w_t L^w_t}{1+\phi} + \frac{\phi s^m_t L^m_t}{1+\phi} \right) w_t - s_{t-1} w L_{t-1} \\ &\quad + \frac{(\alpha_T - \alpha_N)(R-1) \left(\left(\frac{s^w_t L^w_t}{1+\phi} + \frac{\phi s^m_t L^m_t}{1+\phi} \right) - s_{t-1} L_{t-1} \right)}{(1-\alpha_T) R} w \end{aligned}$$

To show $\frac{dCA_t}{d\phi} > 0$, we only need to show $\frac{d\left(\left(\frac{s^w_t L^w_t}{1+\phi} + \frac{\phi s^m_t L^m_t}{1+\phi} \right) w_t - s_{t-1} w L_{t-1} \right)}{d\phi} > 0$. By (3.9), one sufficient condition is for the inequality is

$$\left(\frac{s^w_t L^w_t}{1+\phi} + \frac{\phi s^m_t L^m_t}{1+\phi} \right) P_{Nt} > s_{t-1} L_{t-1} P_{Nt}$$

To show this inequality, we just need to show

$$\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right)\frac{dP_{Nt}}{d\phi} + P_{Nt}\frac{d\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right)}{d\phi} > 0$$

which means

$$\frac{dP_{Nt}/d\phi}{d\left(\frac{s_t^w L_t^w}{1+\phi} + \frac{\phi s_t^m L_t^m}{1+\phi}\right)/d\phi} + \frac{P_{Nt}}{s_t} > 0$$

Plug the expressions of $\frac{dP_{Nt}}{d\phi}$ and $\frac{ds_t}{d\phi}$, we have

$$\begin{aligned}\frac{dP_{Nt}}{ds_t} + \frac{P_{Nt}}{s_t} &= \frac{P_{Nt}C_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right) - \gamma(1-s_t)w_t C_{Nt}\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive}\,.terms} \\ &\quad + \text{positive}\,.term \\ &= \frac{\left(P_{Nt}C_{Nt} - \gamma(1-s_t)w_t\right)\left(\frac{w_t}{P_{Nt}}\right)\left(\frac{1-\alpha_T}{L_{Tt}} + \frac{1-\alpha_N}{L_{Nt}}\right)}{s_t \cdot \text{positive}\,.terms} + \text{positive}\,.term\end{aligned}$$

As shown above, $P_{Nt}C_{Nt} - \gamma(1-s_t)w_t > 0$, then $\frac{dCA_t}{d\phi} > 0$, in period t, the country will experience a current account surplus. □

A3.5. Welfare analysis and discussions of policy interventions

We conduct a simple welfare analysis and use it as a basis for evaluating policy interventions aimed at reducing current account imbalances. Consider a benevolent central planner who cares about the overall welfare of men and women when utility is transferable. The central planner can do anything, including cutting down the sex ratio. We first compute the welfare loss of a rise in the sex ratio. Then we compare the welfare consequences of two different ways to reduce the current account surplus: (i) taxing the tradable good and (ii), reducing the sex ratio.

There are two sources of market failures that the central planner would avoid: (a) men save competitively to improve their relative standing in the marriage market; and (b) both men and women may under-save as they do not take into account the benefits of their own savings for the well-being of their future spouses. The central planner assigns the marriage market matching outcome and optimally chooses women's and men's savings rates to maximize the social welfare function,

$$\max\ U = \frac{1}{1+\phi}U^w + \frac{\phi}{1+\phi}U^m$$

The first order conditions are

$$-u'_{1w} + \beta R \frac{P_t}{P_{t+1}}\left[2(1-F(\bar{\eta}^w))\kappa u'_{2w} + F(\bar{\eta}^w)u'_{2w,n}\right] = 0 \quad (4.45)$$

$$-u'_{1m} + \beta R \frac{P_t}{P_{t+1}}\left[2(1-F(M(\bar{\eta}^w)))\kappa u'_{2m} + F(M(\bar{\eta}^w))u'_{2m,n}\right] = 0 \quad (4.46)$$

Comparing (4.45), (4.46) to (3.14) and (3.16), in general, it is not obvious whether women or men will save at a higher rate in a decentralized equilibrium than that under central planning. However, when $\phi = 1$, we can show that the two sets of first order conditions are identical, and therefore, women and men will save the same rates under a central planning economy as in a decentralized economy.

There are two opposing effects. On one hand, a part of the savings in the competitive equilibrium is motivated by a desire to out-save one's competitors in the marriage market. The increment in the savings, while individually rational, is not useful in the aggregate, since when everyone raises the savings rate by the same amount, the ultimate marriage market outcome is not affected by the increase in the savings. In this sense, the competitive equilibrium produces too much savings. On the other hand, because the savings contribute to a pubic good in a marriage (an individual's savings raises the utility of his/her partner), but an individual in the first period does not take this into account, he/she may under-save relative to the social optimum. These two effects offset each other. Therefore, when $\phi = 1$, the final savings rate in the decentralized equilibrium could be the same as the social optimum.

In calibrations with a log utility function, we show that men's welfare under a decentralized equilibrium relative to the central planner's economy declines as the sex ratio increases. In comparison, women's relative welfare increases as the sex ratio goes up. The social welfare (a weighted average of men's and women's welfare) goes down as the sex ratio rises.

As a thought experiment, one may also consider what the central planner would do if she can choose the sex ratio (in addition to the savings rates) to maximize the social welfare. The new first order condition with respect to ϕ is

$$\frac{U^m - U^w}{(1+\phi)^2} = 0 \tag{4.47}$$

The only sex ratio that satisfies (4.47) is $\phi = 1$. In other words, the central planner would have chosen a balanced sex ratio. Deviations from a balanced sex ratio represent welfare losses.

We now consider the welfare effect of two policy interventions aimed at reducing the current account imbalance: i) taxing the tradable good and ii), reducing the sex ratio.

We first consider the case of taxing the tradable good. Suppose the home country will impose a tax τ on the tradable good in period t and fully rebate this tax revenue to consumers, then the price taken by the tradable good producers will be $1-\tau$. In period $t+1$, when the current account goes back to zero, home will reduce the tax to zero. During the period in which the shock occurs, (3.18) becomes

$$w_t^\tau = \frac{(1-\tau)(1-\alpha_T)A_{Tt}}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}} \left(\frac{K_{Tt}^\tau}{1-L_{Nt}^\tau}\right)^{\alpha_T} = \frac{P_{Nt}(1-\alpha_N)A_{Nt}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} \left(\frac{K_{Nt}^\tau}{L_{Nt}^\tau}\right)^{\alpha_N}$$

where variable Z^τ denotes the variable when there is a tax on the tradable good.

As we have shown in the proof of Proposition 10,

$$CA_t = s_t y_t - s_{t-1} w + \gamma \left(\frac{\alpha_T}{1-\alpha_T} - \frac{\alpha_N}{1-\alpha_N}\right)(R-1)(s_t - s_{t-1})\frac{w}{R} \tag{4.48}$$

where y_t is the first period income of the young cohort. We assume that a fraction a $(0 \leq a \leq 1)$ of the tax revenue will distributed to the young cohort in period t while the rest will refund to the old cohort. Then the nontradable good market clearing condition can be re-written as

$$\frac{P_{Nt}A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} = \gamma\left(Rs_{t-1}w_{t-1} + (1-a)\tau Q_{Tt} + (1-s_t)\left(w_t + aQ_{Tt}\right)\right) \tag{4.49}$$

and the wage parity is

$$w_t = \frac{(1-\tau)(1-\alpha_T)A_{Tt}}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\left(\frac{K_{Tt}}{1-L_{Nt}}\right)^{\alpha_T} = \frac{P_{Nt}(1-\alpha_N)A_{Nt}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}\left(\frac{K_{Nt}}{L_{Nt}}\right)^{\alpha_N} \tag{4.50}$$

Given K_{Tt} and K_{Nt} are predetermined, we can show the following proposition:

Proposition 17. *If the tax revenue from the tradable good will only refund to the working people,*

(i) If

$$\alpha_T\left(Rs_{t-1}\frac{w_{t-1}}{w_t} + 1 - s_t\right) + (1-s_t)(w_t - P_{Nt})(\alpha_T + L_{Nt} - 1) \geq 0$$

taxing the tradable good cannot reduce the current account surplus.

(ii) If

$$\alpha_T\left(Rs_{t-1}\frac{w_{t-1}}{w_t} + 1 - s_t\right) + (1-s_t)(w_t - P_{Nt})(\alpha_T + L_{Nt} - 1) < 0$$

taxing the tradable good can reduce the current account surplus. However, everyone in Home will experience a welfare loss (on top of the welfare loss associated with an unbalanced sex ratio).

Proof. As we have shown in Proposition 10, if the utility function is of log form, then savings rates will not depend on the first period income. We then can take the savings rates as given. We totally differentiate the system which consists of (4.49) and (4.50) and obtain

$$\Omega \cdot \begin{pmatrix} dP_{Nt} \\ dw_t \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} d\tau$$

where

$$\begin{aligned}
\Omega_{11} &= C_{Nt},\, \Omega_{12} = -\gamma(1-s_t) \\
\Omega_{13} &= \gamma\left(1-a+a\left(1-s_t\right)\right)\frac{(1-\alpha_T)Q_{Tt}}{1-L_{Nt}} \\
\Omega_{21} &= 0,\, \Omega_{22} = 1,\, \Omega_{23} = -\alpha_T\frac{w_t}{1-L_{Nt}} \\
\Omega_{31} &= \frac{w_t}{P_{Nt}},\, \Omega_{32} = 1,\, \Omega_{33} = \alpha_N\frac{w_t}{L_{Nt}}
\end{aligned}$$

and

$$\begin{aligned}
z_1 &= \gamma\left(1-a+a\left(1-s_t\right)\right)Q_{Tt} \\
z_2 &= -\frac{w_t}{1-\tau} \\
z_3 &= 0
\end{aligned}$$

The determinant of matrix Ω is

$$\begin{aligned}
\det(\Omega) &= wC_N\left(\frac{\alpha_N}{L_N}+\frac{\alpha_T}{1-L_N}\right)+\frac{w}{P_N}\begin{pmatrix} \frac{\gamma(1-s_t)\alpha_T w}{1-L_N} \\ -\frac{\gamma(1-a+a(1-s_t))(1-\alpha_T)Q_{Tt}}{1-L_N} \end{pmatrix} \\
&= \text{positive}\,.terms+\frac{w}{P_N}\begin{pmatrix} P_N C_N\left(\frac{\alpha_N}{L_N}+\frac{\alpha_T}{1-L_N}\right) \\ -\gamma(1-a+a(1-s_t))w \end{pmatrix} \\
&> \frac{w}{P_N}\left(P_N C_N-\gamma(1-a+a(1-s_t))w\right)
\end{aligned}$$

The last inequality holds because we use the fact that

$$\frac{\alpha_N}{L_N} + \frac{\alpha_T}{1 - L_N} \geq \left(\sqrt{\alpha_N} + \sqrt{\alpha_T}\right)^2$$

where the equality holds when $L_N = \left(1 + \sqrt{\frac{\alpha_T}{\alpha_N}}\right)^{-1}$. In the standard literature, both α_N and α_T take value greater than 0.25, then $\frac{\alpha_N}{L_N} + \frac{\alpha_T}{1-L_N} > 1$.

Notice that $\gamma(1 - a + a(1 - s_t))w$ is only part of the demand for the nontradable good, which must be smaller than $P_N C_N$, therefore, $\det(\Omega) > 0$.

Then we can calculate

$$\begin{aligned}
\left.\frac{dy_t}{d\tau}\right|_{\tau=0} &= \frac{dw_t}{d\tau} + Q_{Tt} \\
&= \frac{w_t}{\det(\Omega)}\left[\frac{\alpha_T C_N}{1 - \alpha_T} + \gamma(1 - s_t)\left(\frac{\alpha_T}{1 - L_N} - 1\right)\left(\frac{w_t}{P_{Nt}} - 1\right) Q_{Tt}\right] \\
&= \frac{\gamma w_t^2}{P_{N,t+1}(1 - \alpha_T)\det(\Omega)}\left[\begin{array}{c} \alpha_T\left(R s_{t-1}\frac{w_{t-1}}{w_t} + 1 - s_t\right) \\ +(1 - s_t)(w_t - P_{Nt})(\alpha_T + L_{Nt} - 1) \end{array}\right]
\end{aligned}$$

In period $t - 1$, $w_{t-1} = R^{-\frac{\alpha_T}{1-\alpha_T}} < R^{\frac{\alpha_N - \alpha_T}{\alpha_T}} = P_{Nt}$. In period t, when shock occurs, as we have shown in Proposition 10, $\frac{w_t}{P_{Nt}}$ increases. However, it is unclear whether it exceeds one. Therefore, the sign of $\frac{dy_t}{d\tau}$ at $\tau = 0$ is ambiguous.

If $\alpha_T\left(R s_{t-1}\frac{w_{t-1}}{w_t} + 1 - s_t\right) + (1 - s_t)(w_t - P_{Nt})(\alpha_T + L_{Nt} - 1) \geq 0$, then $\left.\frac{dy_t}{d\tau}\right|_{\tau=0} \geq 0$. By (4.48), taxing the tradable good cannot reduce the current account surplus caused by the unbalanced sex ratio. On the other hand, if

$$\alpha_T\left(R s_{t-1}\frac{w_{t-1}}{w_t} + 1 - s_t\right) + (1 - s_t)(w_t - P_{Nt})(\alpha_T + L_{Nt} - 1) < 0$$

then $\left.\frac{dy_t}{d\tau}\right|_{\tau=0} < 0$. Taxing the tradable good can achieve the goal of cutting down the current account surplus. However, this also reduces the first period income by the young cohort. The welfare of young women and young men will be worse off.

And

$$\begin{aligned}\left.\frac{dC_{2t}}{d\tau}\right|_{\tau=0} &= -\frac{Rs_{t-1}w}{P_{Nt}}\frac{dP_{Nt}}{d\tau} \\ &= -\frac{\gamma Rs_{t-1}w}{P_{Nt}}(1-s_t)\left(\frac{\alpha_N}{L_N}+\frac{2\alpha_T}{1-L_N}-1\right)<0\end{aligned}$$

then the old cohort in period t also suffers from the tax on the tradable good sector.

Therefore, if

$$\frac{\alpha_T\left(Rs_{t-1}\frac{w_{t-1}}{w_t}+1-s_t\right)}{1-\alpha_T}+(1-s_t)\left(w_t-P_{Nt}\right)\frac{\alpha_T+L_{Nt}-1}{1-\alpha_T}<0$$

taxing the tradable good will cut down the current account surplus, however, at the same time, it will reduce the economy-wide welfare. □

When Home taxes the tradable good sector, the wage rate in that sector decreases immediately, which induces a migration of labor from the tradable sector to the nontradable good sector. The tradable good sector shrinks. Since the young people also get all the tax refund, whether this tax refund can offset the decrease in wage rate is ambiguous. Since the total tax refund equals the tax on per unit tradable good multiplied by the quantity of tradable output, a shrinkage of the tradable good sector implies less tax revenue from the tradable sector and a smaller transfer to consumers. However, consumers only bear a part of the tax burden through a lower wage. Firms bear the other part of tax burden by receiving a lower return to capital. Since the entire tax revenue is transferred to consumers, there is an indirect transfer from firms in the tradable sector to consumers. The net effect on the first period income of the young cohort is ambiguous.

If the central planner can reduce the sex ratio, then as shown Proposition 10, a reduction in the sex ratio will yield a fall in the current account. Correspondingly, there will be a welfare gain for young men but a welfare loss for young women. The aggregate social welfare will improve.

Appendices to Chapter 4

A1.1. Proof of Proposition 13

Proof. If the sex ratio is close to one, all male workers get married. Let $N_t^m = \frac{\phi n_t^m}{1+\phi}$ denote the number of entrepreneurs in the economy. We total differentiate the equations (4.6), (4.7), (4.9), (4.11), (4.13), (4.14) and (4.15), we can obtain

$$\Omega^0 \cdot dx = dz$$

where Ω^0 is a 7×7 matrix with elements

$$\begin{aligned}
\Omega_{11}^0 &= -\frac{1}{(1-s_t^e)^2} - \beta \frac{1}{\left(s_t^e + s_t^w W_t/\Pi_t^H\right)^2}\left(1 + \frac{\pi\phi n_t^m}{1+\phi}\right) \\
\Omega_{12}^0 &= \Omega_{13}^0 = 0,\ \Omega_{14}^0 = -\beta \frac{W_t/\Pi_t^H}{\left(s_t^e + s_t^w W_t/\Pi_t^H\right)^2}\left(1 + \frac{\pi\phi n_t^m}{1+\phi}\right) \\
\Omega_{15}^0 &= \beta \frac{1 + \frac{s_t^w W_t/\Pi_t^H}{s_t^e + s_t^w W_t/\Pi_t^H}\frac{1}{(N_t^m)^2}}{s_t^e + s_t^w W_t/\Pi_t^H},\ \Omega_{16}^0 = \Omega_{17}^0 = 0
\end{aligned}$$

$$\begin{aligned}
\Omega_{21}^0 &= 0,\ \Omega_{22}^0 = -\frac{1}{\left(1 - s_t^{m,L}\right)^2} - \beta \frac{1}{\left(s_t^{m,L} + s_t^w\right)^2}\left(1 + \frac{\phi\left(1 - n_t^m\right)}{1+\phi}\right) \\
\Omega_{23}^0 &= 0,\ \Omega_{24}^0 = -\beta \frac{1}{\left(s_t^{m,L} + s_t^w\right)^2}\left(1 + \frac{\phi\left(1 - n_t^m\right)}{1+\phi}\right) \\
\Omega_{25}^0 &= -\beta \frac{1}{s_t^{m,L} + s_t^w},\ \Omega_{26}^0 = \Omega_{27}^0 = 0,\ \Omega_{31}^0 = \Omega_{32}^0 = 0
\end{aligned}$$

$$\Omega_{33}^0 = -\frac{1}{\left(1 - s_t^{e,L}\right)^2} - \frac{\beta\left[\begin{array}{c} \left(1 - F\left(M^3\left(\eta^{\min}\right)\right)\right)\left(1 + \frac{\phi(1-\pi)n_t^m}{1+\phi}\right) \\ + F\left(M^3\left(\eta^{\min}\right)\right)\frac{\left(s_t^{e,L} + s_t^w W_t/\Pi_t^L\right)^2}{\left(s_t^{e,L}\right)^2} \\ + f\left(\eta^{\min}\right)\left(\ln\left(\kappa\left(\frac{s_t^{e,L} + s_t^w W_t/\Pi_t^L}{s_t^{e,L}}\right)\right) + \eta^{\min} + \frac{s_t^w W_t/\Pi_t^L}{s_t^{e,L}}\right) \end{array}\right]}{\left(s_t^{e,L} + s_t^w W_t/\Pi_t^L\right)^2}$$

$$\Omega_{34}^{0}=-\frac{\beta W_t/\Pi_t^L\left[\begin{array}{c}\left(1-F\left(M^3\left(\eta^{\min}\right)\right)\right)\left(1+\frac{\phi(1-\pi)n_t^m}{1+\phi}\right)\\ +F\left(M^3\left(\eta^{\min}\right)\right)\frac{s_t^{e,L}+s_t^w W_t/\Pi_t^L}{s_t^{e,L}}\\ +f\left(\eta^{\min}\right)\left(\ln\left(\kappa\left(s_t^{e,L}+s_t^w W_t/\Pi_t^L\right)\right)+\eta^{\min}-\ln\left(s_t^{e,L}\right)\right)\end{array}\right]}{\left(s_t^{e,L}+s_t^w W_t/\Pi_t^L\right)^2}$$

$$\begin{aligned}\Omega_{35}^{0} &= \beta\frac{s_t^w W_t/\Pi_t^L}{\left(s_t^{e,L}+s_t^w W_t/\Pi_t^L\right)^2}\frac{1+\phi}{\phi\left(n_t^m\right)^2}\left[\begin{array}{c}\left(1-F\left(M^3\left(\eta^{\min}\right)\right)\right)\left(1+\frac{\phi(1-\pi)n_t^m}{1+\phi}\right)\\ +F\left(M^3\left(\eta^{\min}\right)\right)\frac{s_t^{e,L}+s_t^w W_t/\Pi_t^L}{s_t^{e,L}}\\ +f\left(\eta^{\min}\right)\left(\begin{array}{c}1+\ln\left(\kappa\left(\frac{s_t^{e,L}+s_t^w W_t/\Pi_t^L}{s_t^{e,L}}\right)\right)\\ +\eta^{\min}\end{array}\right)\end{array}\right]\\ \Omega_{36}^{0} &= \Omega_{37}^{0}=0\end{aligned}$$

$$\begin{aligned}\Omega_{41}^{0} &= -\beta\frac{\left(1+\frac{1+\phi}{\pi\phi n_t^m}\right)\left(1-F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)\right)}{\left(s_t^e\Pi_t^H/W_t+s_t^w\right)^2}\frac{\Pi_t^H}{W_t}\\ \Omega_{42}^{0} &= -\beta\frac{\left(1+\frac{1+\phi}{\phi(1-n_t^m)}\right)\left(F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)-F\left(\left(M^2\right)^{-1}\left(\eta^{\min}\right)\right)\right)}{\left(s_t^{m,L}+s_t^w\right)^2}\\ \Omega_{43}^{0} &= -\beta\frac{F\left(\left(M^2\right)^{-1}\left(\eta^{\min}\right)\right)}{\left(s_t^{e,L}\Pi_t^L/W_t+s_t^w\right)^2}\frac{\Pi_t^L}{W_t}\end{aligned}$$

$$\begin{aligned}\Omega_{44}^{0} &= -\frac{1}{\left(1-s_t^w\right)^2}-\beta\left[\begin{array}{c}\frac{F\left(\left(M^2\right)^{-1}\left(\eta^{\min}\right)\right)}{\left(s_t^{e,L}\Pi_t^L/W_t+s_t^w\right)^2}+\frac{\left(1+\frac{1+\phi}{\pi\phi n_t^m}\right)\left(1-F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)\right)}{\left(s_t^e\Pi_t^H/W_t+s_t^w\right)^2}\\ +\frac{\left(1+\frac{1+\phi}{\phi(1-n_t^m)}\right)\left(F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)-F\left(\left(M^2\right)^{-1}\left(\eta^{\min}\right)\right)\right)}{\left(s_t^{m,L}+s_t^w\right)^2}\end{array}\right]\\ \Omega_{45}^{0} &= -\frac{\beta\left[\frac{F\left(\left(M^2\right)^{-1}\left(\eta^{\min}\right)\right)\left(1+\frac{1+\phi}{(1-\pi)\phi n_t^m}\right)\frac{s_t^{e,L}\Pi_t^L}{W_t}}{\left(s_t^{e,L}\Pi_t^L/W_t+s_t^w\right)^2}+\frac{\left(1+\frac{1+\phi}{\pi\phi n_t^m}\right)\left(1-F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)\right)\frac{s_t^e\Pi_t^H}{W_t}}{\left(s_t^e\Pi_t^H/W_t+s_t^w\right)^2}\right]}{\left(N_t^m\right)^2}\\ \Omega_{46}^{0} &= \Omega_{47}^{0}=0\end{aligned}$$

$$
\begin{aligned}
\Omega_{51}^0 &= \pi\left(-\frac{1}{1-s_t^e}+\beta\frac{1}{s_t^e+s_t^w W_t/\Pi_t^H}\right),\ \Omega_{52}^0=\frac{1}{1-s_t^{m,L}}-\beta\frac{1}{s_t^{m,L}+s_t^w} \\
\Omega_{53}^0 &= (1-\pi)\left(-\frac{1}{1-s_t^{e,L}}+\beta\frac{1-F\left(M^3\left(\eta^{\min}\right)\right)}{s_t^{e,L}+s_t^w W_t/\Pi_t^L}+\beta\frac{F\left(M^3\left(\eta^{\min}\right)\right)}{s_t^{e,L}}\right) \\
\Omega_{54}^0 &= \frac{\pi\beta}{s_t^e\Pi_t^H/W_t+s_t^w}+\frac{(1-\pi)\,\beta\left(1-F\left(M^3\left(\eta^{\min}\right)\right)\right)}{s_t^{e,L}\Pi_t^L/W_t+s_t^w}-\beta\frac{1}{s_t^{m,L}+s_t^w} \\
\Omega_{55}^0 &= \pi\beta\left(\frac{s_t^w W_t/\Pi_t^H}{s_t^e+s_t^w W_t/\Pi_t^H}\frac{1}{\left(N_t^m\right)^2}+\pi\int_{\eta^{\min}}\left[1-F\left(\eta^m\right)\right]d\eta^m\right) \\
&\quad+(1-\pi)\,\beta\left(\begin{array}{c}\frac{s_t^w W_t/\Pi_t^L}{s_t^e+s_t^w W_t/\Pi_t^L}\frac{1}{\left(N_t^m\right)^2}+(1-\pi)\int_{M^3\left(\eta^{\min}\right)}\left[1-F\left(\eta^m\right)\right]d\eta^m \\ -(1-\pi)\left[1-F\left(M^3\left(\eta^{\min}\right)\right)\right]M^3\left(\eta^{\min}\right)\end{array}\right) \\
&\quad+\beta\int_{\eta^{\min}}\left[1-F\left(\eta^m\right)\right]d\eta^m \\
\Omega_{56}^0 &= \Omega_{57}^0=0
\end{aligned}
$$

$$
\begin{aligned}
\Omega_{61}^0 &= \Omega_{62}^0=\Omega_{63}^0=\Omega_{64}^0=0,\ \Omega_{65}^0=-1 \\
\Omega_{66}^0 &= \frac{1-\alpha}{\alpha}\frac{R_t}{\left(W_t\right)^2}K_t,\ \Omega_{67}^0=-\frac{1-\alpha}{\alpha}\frac{K_t}{W_t} \\
\Omega_{71}^0 &= \Omega_{72}^0=\Omega_{73}^0=\Omega_{74}^0=0
\end{aligned}
$$

$$
\Omega_{75}^0=-\frac{\alpha(\theta-1)}{\theta}\left(\begin{array}{c}\dfrac{\pi\frac{W_t^2}{\Pi_t^H}\frac{1}{\left(N_t^m\right)^2}}{\left(\frac{W_t}{\Pi_t^H}+\alpha\theta\frac{\pi\left(z^H\right)^{\theta-1}+(1-\pi)\left(z^L\right)^{\theta-1}}{\left(z^H\right)^{\theta-1}}\right)^2} \\ +\dfrac{(1-\pi)\frac{W_t^2}{\Pi_t^L}\frac{1}{\left(N_t^m\right)^2}}{\left(\frac{W_t}{\Pi_t^L}+\alpha\theta\frac{\pi\left(z^H\right)^{\theta-1}+(1-\pi)\left(z^L\right)^{\theta-1}}{\left(z^L\right)^{\theta-1}}\right)^2}+\frac{\phi W_t}{1+\phi}\end{array}\right)
$$

$$
\Omega_{76}^0=\frac{\alpha(\theta-1)}{\theta}\left\{\pi N_t^m\frac{\Pi_t^{e,H}}{W_t}+(1-\pi)\,N_t^m\frac{\Pi_t^{e,L}}{W_t}+\left(1-N_t^m\right)\right\},\ \Omega_{77}^0=-K_t
$$

and

$$dx = \begin{pmatrix} ds_t^e \\ ds_t^{m,L} \\ ds_t^{e,L} \\ ds_t^w \\ dN_t^m \\ dW_t \\ dR_t \end{pmatrix}, \; dz^0 = \begin{pmatrix} z_1^0 \\ z_2^0 \\ z_3^0 \\ z_4^0 \\ z_5^0 \\ z_6^0 \\ z_7^0 \end{pmatrix}$$

where

$$\begin{aligned}
z_1^0 &= 0, \; z_2^0 = \beta \frac{1}{s_t^{m,L} + s_t^w} \frac{1}{(1+\phi)^2} > 0 \\
z_3^0 &= -\beta \frac{s_t^w W_t / \Pi_t^L}{\phi^2 n_t^m \left(s_t^{e,L} + s_t^w W_t / \Pi_t^L \right)^2} \begin{bmatrix} \left(1 - F\left(M^3\left(\eta^{\min}\right)\right)\right)\left(1 + \frac{\phi(1-\pi)n_t^m}{1+\phi}\right) \\ +F\left(M^3\left(\eta^{\min}\right)\right) \frac{s_t^{e,L} + s_t^w W_t/\Pi_t^L}{s_t^{e,L}} \\ +f\left(\eta^{\min}\right)\left(1 + \ln\left(\kappa\left(\frac{s_t^{e,L} + s_t^w W_t/\Pi_t^L}{s_t^{e,L}}\right)\right) + \eta^{\min}\right) \end{bmatrix} \\
z_4^0 &= 0, \\
z_5^0 &= -\frac{\beta(1-\pi)^2}{(1+\phi)^2} \begin{pmatrix} \int_{M^3(\eta^{\min})} \left[1 - F\left(\eta^m\right)\right] d\eta^m \\ -\left[1 - F\left(M^3\left(\eta^{\min}\right)\right)\right] M^3\left(\eta^{\min}\right) \end{pmatrix} \\
&\quad - \frac{\beta}{(1+\phi)^2} \int_{\eta^{\min}} \left[1 - F\left(\eta^m\right)\right] d\eta^m \\
z_6^0 &= z_7^0 = 0
\end{aligned}$$

The determinant of matrix Ω^0 is

$$\det\left(\Omega^0\right) = \det\begin{pmatrix} \Omega^0_{66} & \Omega^0_{67} \\ \Omega^0_{76} & \Omega^0_{77} \end{pmatrix} \cdot \left\{ \begin{array}{c}
\Omega^0_{55}\left[\begin{array}{c} \Omega^0_{11}\left(\begin{array}{c} \Omega^0_{22}\left(\begin{array}{c} \Omega^0_{33}\Omega^0_{44} \\ -\Omega^0_{34}\Omega^0_{43} \end{array}\right) \\ -\Omega^0_{42}\Omega^0_{33}\Omega^0_{24} \end{array}\right) \\ -\Omega^0_{41}\Omega^0_{22}\Omega^0_{14}\Omega^0_{33} \end{array}\right] \\
-\Omega^0_{45}\left[\begin{array}{c} \Omega^0_{11}\left(\begin{array}{c} \Omega^0_{22}\left(\begin{array}{c} \Omega^0_{33}\Omega^0_{54} \\ -\Omega^0_{34}\Omega^0_{53} \end{array}\right) \\ -\Omega^0_{52}\Omega^0_{33}\Omega^0_{24} \end{array}\right) \\ -\Omega^0_{51}\Omega^0_{22}\Omega^0_{14}\Omega^0_{33} \end{array}\right] \\
+\Omega^0_{35}\left[\begin{array}{c} \Omega^0_{11}\left(\begin{array}{c} \Omega^0_{22}\left(\begin{array}{c} \Omega^0_{43}\Omega^0_{54} \\ -\Omega^0_{44}\Omega^0_{53} \end{array}\right) \\ +\left(\begin{array}{c} \Omega^0_{42}\Omega^0_{53} \\ -\Omega^0_{52}\Omega^0_{43} \end{array}\right)\Omega^0_{24} \end{array}\right) \\ +\Omega^0_{22}\Omega^0_{14}\left(\Omega^0_{41}\Omega^0_{53} - \Omega^0_{51}\Omega^0_{43}\right) \end{array}\right] \\
-\Omega^0_{25}\left[\begin{array}{c} \Omega^0_{11}\left(\begin{array}{c} \Omega^0_{34}\left(\begin{array}{c} \Omega^0_{42}\Omega^0_{53} \\ -\Omega^0_{43}\Omega^0_{52} \end{array}\right) \\ -\Omega^0_{33}\left(\begin{array}{c} \Omega^0_{42}\Omega^0_{54} \\ -\Omega^0_{44}\Omega^0_{52} \end{array}\right) \end{array}\right) \\ -\Omega^0_{33}\Omega^0_{14}\left(\Omega^0_{41}\Omega^0_{52} - \Omega^0_{51}\Omega^0_{42}\right) \end{array}\right] \\
-\Omega^0_{15}\left[\begin{array}{c} \Omega^0_{22}\left(\begin{array}{c} \Omega^0_{51}\left(\begin{array}{c} \Omega^0_{33}\Omega^0_{44} \\ -\Omega^0_{34}\Omega^0_{43} \end{array}\right) \\ -\Omega^0_{41}\left(\begin{array}{c} \Omega^0_{22}\Omega^0_{44} \\ -\Omega^0_{43}\Omega^0_{24} \end{array}\right) \end{array}\right) \\ +\Omega^0_{24}\Omega^0_{33}\left(\Omega^0_{41}\Omega^0_{52} - \Omega^0_{51}\Omega^0_{42}\right) \end{array}\right]
\end{array} \right\}$$

$$= \det \begin{pmatrix} \Omega^0_{66} & \Omega^0_{67} \\ \Omega^0_{76} & \Omega^0_{77} \end{pmatrix} \cdot \left\{ \begin{array}{c} \Omega^0_{22}\left(\Omega^0_{55}\Omega^0_{11} - \Omega^0_{15}\Omega^0_{51}\right)\left(\Omega^0_{33}\Omega^0_{44} - \Omega^0_{34}\Omega^0_{43}\right) \\ -\Omega^0_{55}\left(\Omega^0_{11}\Omega^0_{42}\Omega^0_{33}\Omega^0_{24} + \Omega^0_{41}\Omega^0_{22}\Omega^0_{14}\Omega^0_{33}\right) \\ +\Omega^0_{33}\left(\Omega^0_{14}\Omega^0_{25} - \Omega^0_{15}\Omega^0_{24}\right)\left(\Omega^0_{41}\Omega^0_{52} - \Omega^0_{51}\Omega^0_{42}\right) \\ +\Omega^0_{11}\left(\Omega^0_{42}\Omega^0_{53} - \Omega^0_{43}\Omega^0_{52}\right)\left(\Omega^0_{35}\Omega^0_{24} - \Omega^0_{25}\Omega^0_{34}\right) \\ +\Omega^0_{54}\Omega^0_{11}\Omega^0_{22}\left(\Omega^0_{35}\Omega^0_{43} - \Omega^0_{33}\Omega^0_{45}\right) \\ +\Omega^0_{53}\Omega^0_{11}\Omega^0_{22}\left(\Omega^0_{34}\Omega^0_{45} - \Omega^0_{35}\Omega^0_{44}\right) \\ +\Omega^0_{35}\Omega^0_{22}\Omega^0_{14}\left(\Omega^0_{41}\Omega^0_{53} - \Omega^0_{51}\Omega^0_{43}\right) \\ +\Omega^0_{15}\Omega^0_{22}\Omega^0_{41}\left(\Omega^0_{22}\Omega^0_{44} - \Omega^0_{43}\Omega^0_{24}\right) \\ +\Omega^0_{45}\left(\Omega^0_{11}\Omega^0_{52}\Omega^0_{33}\Omega^0_{24} + \Omega^0_{51}\Omega^0_{22}\Omega^0_{14}\Omega^0_{33}\right) \\ +\Omega^0_{25}\Omega^0_{11}\Omega^0_{33}\left(\Omega^0_{42}\Omega^0_{54} - \Omega^0_{44}\Omega^0_{52}\right) \end{array} \right\}$$

It is easy to show that

$$\det \begin{pmatrix} \Omega^0_{66} & \Omega^0_{67} \\ \Omega^0_{76} & \Omega^0_{77} \end{pmatrix} < 0$$

Since the utility is log form, by (4.6), (4.7), (4.9) and (4.11), we can show that

$$\begin{aligned} \Omega^0_{22}\left(\Omega^0_{55}\Omega^0_{11} - \Omega^0_{15}\Omega^0_{51}\right)\left(\Omega^0_{33}\Omega^0_{44} - \Omega^0_{34}\Omega^0_{43}\right) &> 0 \\ \Omega^0_{33}\left(\Omega^0_{14}\Omega^0_{25} - \Omega^0_{15}\Omega^0_{24}\right)\left(\Omega^0_{41}\Omega^0_{52} - \Omega^0_{51}\Omega^0_{42}\right) &> 0 \\ \Omega^0_{11}\left(\Omega^0_{42}\Omega^0_{53} - \Omega^0_{43}\Omega^0_{52}\right)\left(\Omega^0_{35}\Omega^0_{24} - \Omega^0_{25}\Omega^0_{34}\right) &> 0 \\ \Omega^0_{54}\Omega^0_{11}\Omega^0_{22}\left(\Omega^0_{35}\Omega^0_{43} - \Omega^0_{33}\Omega^0_{45}\right) &> 0 \\ \Omega^0_{15}\Omega^0_{22}\Omega^0_{41}\left(\Omega^0_{22}\Omega^0_{44} - \Omega^0_{43}\Omega^0_{24}\right) &> 0 \end{aligned}$$

If π is very small, i.e., with a large possibility an entrepreneur will fail, then we can show that

$$\begin{aligned} \Omega^0_{22}\left(\Omega^0_{55}\Omega^0_{11} - \Omega^0_{15}\Omega^0_{51}\right)\left(\Omega^0_{33}\Omega^0_{44} - \Omega^0_{34}\Omega^0_{43}\right) &> \Omega^0_{55}\left(\Omega^0_{11}\Omega^0_{42}\Omega^0_{33}\Omega^0_{24} - \Omega^0_{41}\Omega^0_{22}\Omega^0_{14}\Omega^0_{33}\right) \\ \Omega^0_{45}\left(\Omega^0_{11}\Omega^0_{52}\Omega^0_{33}\Omega^0_{24} + \Omega^0_{51}\Omega^0_{22}\Omega^0_{14}\Omega^0_{33}\right) &> 0 \end{aligned}$$

and if ϕ is small, $F\left(M^3\left(\eta^{\min}\right)\right)$ is close to one. By Lemma 1, $s^{e,L}_t \Pi^L_t < s^{m,L}_t W_t$, Ω^0_{54} can be either positive or very close to zero when the sex ratio is small. Then,

$$\begin{aligned} & \Omega_{45}^0\left(\Omega_{11}^0\Omega_{52}^0\Omega_{33}^0\Omega_{24}^0+\Omega_{51}^0\Omega_{22}^0\Omega_{14}^0\Omega_{33}^0\right)+\Omega_{25}^0\Omega_{11}^0\Omega_{33}^0\left(\Omega_{42}^0\Omega_{54}^0-\Omega_{44}^0\Omega_{52}^0\right) \\ \geq\ & \Omega_{11}^0\Omega_{52}^0\Omega_{33}^0\left(\Omega_{45}^0\Omega_{24}^0-\Omega_{25}^0\Omega_{44}^0\right) \end{aligned}$$

Again, if π is small, n_t^m can be very small and hence, $\Omega_{11}^0\Omega_{52}^0\Omega_{33}^0\left(\Omega_{45}^0\Omega_{24}^0-\Omega_{25}^0\Omega_{44}^0\right)\geq 0$.[14]

Therefore, $\det\left(\Omega^0\right)<0$.

Then,

$$\frac{dN_t^m}{d\phi}=\frac{\det\begin{pmatrix}\Omega_{66}^0 & \Omega_{67}^0\\ \Omega_{76}^0 & \Omega_{77}^0\end{pmatrix}\cdot\left\{\begin{array}{c}\Omega_{22}^0\left(z_5^0\Omega_{11}^0-\Omega_{15}^0\Omega_{51}^0\right)\left(\Omega_{33}^0\Omega_{44}^0-\Omega_{34}^0\Omega_{43}^0\right)\\ -z_5^0\left(\Omega_{11}^0\Omega_{42}^0\Omega_{33}^0\Omega_{24}^0+\Omega_{41}^0\Omega_{22}^0\Omega_{14}^0\Omega_{33}^0\right)\\ +\Omega_{33}^0\left(\Omega_{14}^0z_2^0-z_1^0\Omega_{24}^0\right)\left(\Omega_{41}^0\Omega_{52}^0-\Omega_{51}^0\Omega_{42}^0\right)\\ +\Omega_{11}^0\left(\Omega_{42}^0\Omega_{53}^0-\Omega_{43}^0\Omega_{52}^0\right)\left(z_3^0\Omega_{24}^0-z_2^0\Omega_{34}^0\right)\\ +\Omega_{54}^0\Omega_{11}^0\Omega_{22}^0\left(z_3^0\Omega_{43}^0-\Omega_{33}^0z_4^0\right)\\ +\Omega_{53}^0\Omega_{11}^0\Omega_{22}^0\left(\Omega_{34}^0z_4^0-z_3^0\Omega_{44}^0\right)\\ +z_3^0\Omega_{22}^0\Omega_{14}^0\left(\Omega_{41}^0\Omega_{53}^0-\Omega_{51}^0\Omega_{43}^0\right)\\ +z_1^0\Omega_{22}^0\Omega_{41}^0\left(\Omega_{22}^0\Omega_{44}^0-\Omega_{43}^0\Omega_{24}^0\right)\\ +z_4^0\left(\Omega_{11}^0\Omega_{52}^0\Omega_{33}^0\Omega_{24}^0+\Omega_{51}^0\Omega_{22}^0\Omega_{14}^0\Omega_{33}^0\right)\\ +z_2^0\Omega_{11}^0\Omega_{33}^0\left(\Omega_{42}^0\Omega_{54}^0-\Omega_{44}^0\Omega_{52}^0\right)\end{array}\right\}}{\det\left(\Omega^0\right)}$$

[14] When $\pi\to 0$, $n_t^m\to 0$. The inequality $\Omega_{11}^0\Omega_{52}^0\Omega_{33}^0\left(\Omega_{45}^0\Omega_{24}^0-\Omega_{25}^0\Omega_{44}^0\right)\geq 0$ holds.

Similar to the previous analysis, if π and ϕ are small,

$$\left\{\begin{array}{c} \Omega_{22}^0\left(z_5^0\Omega_{11}^0-\Omega_{15}^0\Omega_{51}^0\right)\left(\Omega_{33}^0\Omega_{44}^0-\Omega_{34}^0\Omega_{43}^0\right) \\ -z_5^0\left(\Omega_{11}^0\Omega_{42}^0\Omega_{33}^0\Omega_{24}^0+\Omega_{41}^0\Omega_{22}^0\Omega_{14}^0\Omega_{33}^0\right) \\ +\Omega_{33}^0\left(\Omega_{14}^0z_2^0-z_1^0\Omega_{24}^0\right)\left(\Omega_{41}^0\Omega_{52}^0-\Omega_{51}^0\Omega_{42}^0\right) \\ +\Omega_{11}^0\left(\Omega_{42}^0\Omega_{53}^0-\Omega_{43}^0\Omega_{52}^0\right)\left(z_3^0\Omega_{24}^0-z_2^0\Omega_{34}^0\right) \\ +\Omega_{54}^0\Omega_{11}^0\Omega_{22}^0\left(z_3^0\Omega_{43}^0-\Omega_{33}^0z_4^0\right) \\ +\Omega_{53}^0\Omega_{11}^0\Omega_{22}^0\left(\Omega_{34}^0z_4^0-z_3^0\Omega_{44}^0\right) \\ +z_3^0\Omega_{22}^0\Omega_{14}^0\left(\Omega_{41}^0\Omega_{53}^0-\Omega_{51}^0\Omega_{43}^0\right) \\ +z_1^0\Omega_{22}^0\Omega_{41}^0\left(\Omega_{22}^0\Omega_{44}^0-\Omega_{43}^0\Omega_{24}^0\right) \\ +z_4^0\left(\Omega_{11}^0\Omega_{52}^0\Omega_{33}^0\Omega_{24}^0+\Omega_{51}^0\Omega_{22}^0\Omega_{14}^0\Omega_{33}^0\right) \\ +z_2^0\Omega_{11}^0\Omega_{33}^0\left(\Omega_{42}^0\Omega_{54}^0-\Omega_{44}^0\Omega_{52}^0\right) \end{array}\right\}<0$$

and therefore,

$$\frac{dN_t^m}{d\phi}<0$$

As the sex ratio rises, a smaller fraction of men will choose to be entrepreneurs.

As the sex ratio keeps rising, the sign of $\frac{dN_t^m}{d\phi}$ becomes ambiguous.

In equilibrium, we must always have $\frac{\Pi_t^H}{W_t}>1$. Otherwise, some entrepreneurs will switch to become workers. By (4.5), we have

$$\frac{\phi\left(1-n_t^m\right)}{1+\phi}>\frac{\phi}{1+\phi}-\frac{1}{1+\alpha\theta\left[\pi+(1-\pi)\left(\frac{z^L}{z^H}\right)^{\theta-1}\right]}$$

As ϕ becomes sufficiently large, $\frac{\phi(1-n_t^m)}{1+\phi}>\frac{1}{1+\phi}$ will hold, which means some male workers cannot get married. In this case, failed entrepreneurs will never get matched with any woman. They optimally choose their savings rate equal to $\frac{\beta}{1+\beta}$ by (4.10). We total differentiate the equations (4.6), (4.8), (4.12), (4.13), (4.14) and (4.15), we can obtain

$$\Omega\cdot dx=dz$$

where Ω is a 6×6 matrix with elements

$$
\begin{aligned}
\Omega_{11} &= -\frac{1}{(1-s_t^e)^2} - \beta \frac{1}{\left(s_t^e + s_t^w W_t/\Pi_t^H\right)^2}\left(1+\frac{\pi\phi n_t^m}{1+\phi}\right) \\
\Omega_{12} &= 0,\ \Omega_{13} = -\beta \frac{W_t/\Pi_t^H}{\left(s_t^e + s_t^w W_t/\Pi_t^H\right)^2}\left(1+\frac{\pi\phi n_t^m}{1+\phi}\right) \\
\Omega_{14} &= \beta \frac{\frac{\phi}{1+\phi} + \frac{s_t^w W_t/\Pi_t^H}{s_t^e + s_t^w W_t/\Pi_t^H}\frac{1+\phi}{\phi (n_t^m)^2}}{s_t^e + s_t^w W_t/\Pi_t^H} \\
\Omega_{15} &= \Omega_{16} = 0,\ \Omega_{21} = 0
\end{aligned}
$$

$$
\Omega_{22} = -\frac{1}{\left(1-s_t^{m,L}\right)^2} - \frac{\beta}{\left(s_t^{m,L}+s_t^w\right)^2}\left[\begin{array}{c} \left(1-F\left(M^2\left(\eta^{\min}\right)\right)\right)\left(1+\frac{\phi(1-n_t^m)}{1+\phi}\right) \\ +F\left(M^2\left(\eta^{\min}\right)\right)\frac{s_t^{m,L}+s_t^w}{\left(s_t^{m,L}\right)^2} \\ +f\left(\eta^{\min}\right)\left(\begin{array}{c} \ln\left(\kappa\left(\frac{s_t^{m,L}+s_t^w}{s_t^w}\right)\right)+\eta^{\min} \\ -\frac{s_t^{m,L}}{s_t^w}\end{array}\right)\end{array}\right]
$$

$$
\Omega_{23} = -\frac{\beta}{\left(s_t^{m,L}+s_t^w\right)^2}\left[\begin{array}{c} \left(1-F\left(M^2\left(\eta^{\min}\right)\right)\right)\left(1+\frac{\phi(1-n_t^m)}{1+\phi}\right) \\ +f\left(\eta^{\min}\right)\left(\ln\left(\kappa\left(\frac{s_t^{m,L}+s_t^w}{s_t^w}\right)\right)+\eta^{\min}-\frac{s_t^{m,L}+s_t^w}{s_t^w}\right)\end{array}\right]
$$

$$
\frac{\beta}{s_t^{m,L}+s_t^w}\frac{s_t^w}{s_t^{m,L}}\frac{1+\phi}{\phi\left(1-n_t^m\right)^2}F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right),\ \Omega_{25}=\Omega_{26}=0
$$

$$
\begin{aligned}
\Omega_{31} &= -\beta \frac{\left(1+\frac{1+\phi}{\pi\phi n_t^m}\right)\left(1-F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)\right)}{\left(s_t^e \Pi_t^H/W_t + s_t^w\right)^2}\frac{\Pi_t^H}{W_t} \\
\Omega_{32} &= -\beta \frac{\left(1+\frac{1+\phi}{\phi(1-n_t^m)}\right)F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)}{\left(s_t^{m,L}+s_t^w\right)^2} \\
\Omega_{33} &= -\frac{1}{(1-s_t^w)^2} - \beta\left[\begin{array}{c} \frac{\left(1+\frac{1+\phi}{\pi\phi n_t^m}\right)\left(1-F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)\right)}{\left(s_t^e\Pi_t^H/W_t+s_t^w\right)^2} \\ +\frac{\left(1+\frac{1+\phi}{\phi(1-n_t^m)}\right)F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right)}{\left(s_t^{m,L}+s_t^w\right)^2}\end{array}\right] \\
\Omega_{34} &= \beta\left(\frac{1}{s_t^{m,L}+s_t^w} - \frac{1}{s_t^e\Pi_t^H/W_t+s_t^w}\right)\frac{\pi\phi}{1+\phi},\ \Omega_{35}=\Omega_{36}=0
\end{aligned}
$$

$$
\begin{aligned}
\Omega_{41} &= \pi\left(-\frac{1}{1-s_t^e}+\beta\frac{1}{s_t^e+s_t^w W_t/\Pi_t^H}\right)\\
\Omega_{42} &= \frac{1}{1-s_t^{m,L}}-\beta\frac{1}{s_t^{m,L}+s_t^w}\\
\Omega_{43} &= \frac{\pi\beta}{s_t^e\Pi_t^H/W_t+s_t^w}-\beta\frac{1}{s_t^{m,L}+s_t^w}\\
\Omega_{44} &= \pi\beta\left(\frac{s_t^w W_t/\Pi_t^H}{s_t^e+s_t^w W_t/\Pi_t^H}\frac{1+\phi}{\phi\left(n_t^m\right)^2}+\frac{\pi\phi}{1+\phi}\int_{\eta^{\min}}\left[1-F\left(\eta^m\right)\right]d\eta^m\right)\\
&\quad +\beta\frac{\phi}{1+\phi}\int_{\eta^{\min}}\left[1-F\left(\eta^m\right)\right]d\eta^m\\
\Omega_{45} &= \Omega_{46}=0
\end{aligned}
$$

$$
\begin{aligned}
\Omega_{51} &= \Omega_{52}=\Omega_{53}=0,\ \Omega_{54}=-\frac{\phi}{1+\phi}\\
\Omega_{55} &= \frac{1-\alpha}{\alpha}\frac{R_t}{\left(W_t\right)^2}K_t,\ \Omega_{56}=-\frac{1-\alpha}{\alpha}\frac{K_t}{W_t}\\
\Omega_{61} &= \Omega_{62}=\Omega_{63}=0\\
\Omega_{64} &= -\frac{\alpha\left(\theta-1\right)}{\theta}\left(\begin{array}{c}\frac{\pi\frac{W_t^2}{\Pi_t^H}\frac{1+\phi}{\phi\left(n_t^m\right)^2}}{\left(\frac{W_t}{\Pi_t^H}+\alpha\theta\frac{\pi\left(z^H\right)^{\theta-1}+(1-\pi)\left(z^L\right)^{\theta-1}}{\left(z^H\right)^{\theta-1}}\right)^2}\\
+\frac{(1-\pi)\frac{W_t^2}{\Pi_t^L}\frac{1+\phi}{\phi\left(n_t^m\right)^2}}{\left(\frac{W_t}{\Pi_t^L}+\alpha\theta\frac{\pi\left(z^H\right)^{\theta-1}+(1-\pi)\left(z^L\right)^{\theta-1}}{\left(z^L\right)^{\theta-1}}\right)^2}+\frac{\phi W_t}{1+\phi}\end{array}\right)\\
\Omega_{65} &= \frac{\alpha\left(\theta-1\right)}{\theta}\left\{\begin{array}{c}\pi\frac{\phi n_t^m}{1+\phi}\frac{\Pi_t^{e,H}}{W_t}+\left(1-\pi\right)\frac{\phi n_t^m}{1+\phi}\frac{\Pi_t^{e,L}}{W_t}\\
+\left(1-\frac{\phi n_t^m}{1+\phi}\right)\end{array}\right\}\\
\Omega_{66} &= -K_t
\end{aligned}
$$

and

$$
dx=\begin{pmatrix} ds_t^e\\ ds_t^{m,L}\\ ds_t^w\\ dn_t^m\\ dW_t\\ dR_t\end{pmatrix},\ dz^0=\begin{pmatrix} z_1\\ z_2\\ z_4\\ z_5\\ z_6\\ z_7\end{pmatrix}
$$

where

$$
\begin{aligned}
z_1 &= -\beta \frac{\frac{n_t^m}{(1+\phi)^2} + \frac{s_t^w W_t/\Pi_t^H}{s_t^e + s_t^w W_t/\Pi_t^H} \frac{1}{\phi^2 n_t^m}}{s_t^e + s_t^w W_t/\Pi_t^H} < 0 \\
z_2 &= -\frac{\beta}{s_t^{m,L} + s_t^w} \frac{s_t^w}{s_t^{m,L}} \frac{1}{\phi^2 (1-n_t^m)} F\left(\left(M^1\right)^{-1}\left(\eta^{\min}\right)\right) < 0 \\
z_3 &= -\beta \left(\frac{1}{s_t^{m,L} + s_t^w} - \frac{1}{s_t^e \Pi_t^H / W_t + s_t^w} \right) \frac{n_t^m \pi}{(1+\phi)^2} < 0 \\
z_4 &= -\pi\beta \left(\frac{s_t^w W_t/\Pi_t^H}{s_t^e + s_t^w W_t/\Pi_t^H} \frac{1}{\phi^2 n_t^m} + \frac{\pi n_t^m}{(1+\phi)^2} \int_{\eta^{\min}} \left[1 - F\left(\eta^m\right)\right] d\eta^m \right) \\
&\quad -\beta \frac{1}{(1+\phi)^2} \int_{\eta^{\min}} \left[1 - F\left(\eta^m\right)\right] d\eta^m \\
z_5 &= \frac{n_t^m}{(1+\phi)^2} \\
z_6 &= \frac{\alpha(\theta-1)}{\theta} \left(\begin{array}{c} \frac{\pi \frac{W_t^2}{\Pi_t^H} \frac{1}{\phi^2 n_t^m}}{\left(\frac{W_t}{\Pi_t^H} + \alpha\theta \frac{\pi (z^H)^{\theta-1} + (1-\pi)(z^L)^{\theta-1}}{(z^H)^{\theta-1}} \right)^2} \\ + \frac{(1-\pi) \frac{W_t^2}{\Pi_t^L} \frac{1}{\phi^2 n_t^m}}{\left(\frac{W_t}{\Pi_t^L} + \alpha\theta \frac{\pi (z^H)^{\theta-1} + (1-\pi)(z^L)^{\theta-1}}{(z^L)^{\theta-1}} \right)^2} + \frac{W_t}{(1+\phi)^2} \end{array} \right)
\end{aligned}
$$

The determinant of matrix Ω is

$$
\begin{aligned}
\det(\Omega) &= \det \begin{pmatrix} \Omega_{55} & \Omega_{56} \\ \Omega_{65} & \Omega_{66} \end{pmatrix} \cdot \left\{ \begin{array}{c} \Omega_{44}\left[\Omega_{11}\left(\Omega_{22}\Omega_{33} - \Omega_{32}\Omega_{23}\right) - \Omega_{31}\Omega_{13}\Omega_{22}\right] \\ -\Omega_{34}\left[\Omega_{11}\left(\Omega_{22}\Omega_{43} - \Omega_{42}\Omega_{23}\right) - \Omega_{41}\Omega_{13}\Omega_{22}\right] \\ +\Omega_{24} \left[\begin{array}{c} \Omega_{11}\left(\Omega_{32}\Omega_{43} - \Omega_{42}\Omega_{33}\right) \\ +\Omega_{13}\left(\Omega_{31}\Omega_{42} - \Omega_{41}\Omega_{32}\right) \end{array} \right] \\ -\Omega_{14} \left[\begin{array}{c} \Omega_{23}\left(\Omega_{31}\Omega_{42} - \Omega_{41}\Omega_{32}\right) \\ -\Omega_{22}\left(\Omega_{31}\Omega_{43} - \Omega_{41}\Omega_{33}\right) \end{array} \right] \end{array} \right\} \\
&= \det \begin{pmatrix} \Omega_{55} & \Omega_{56} \\ \Omega_{65} & \Omega_{66} \end{pmatrix} \cdot \left\{ \begin{array}{c} \Omega_{11}\Omega_{22}\left(\Omega_{33}\Omega_{44} - \Omega_{34}\Omega_{43}\right) \\ +\Omega_{34}\left(\Omega_{11}\Omega_{23}\Omega_{42} + \Omega_{41}\Omega_{13}\Omega_{22}\right) \\ +\left(\Omega_{13}\Omega_{24} - \Omega_{14}\Omega_{23}\right)\left(\Omega_{31}\Omega_{42} - \Omega_{41}\Omega_{32}\right) \\ -\Omega_{11}\Omega_{32}\left(\Omega_{23}\Omega_{44} - \Omega_{43}\Omega_{24}\right) \\ -\Omega_{24}\Omega_{11}\Omega_{42}\Omega_{33} + \Omega_{14}\Omega_{22}\left(\Omega_{31}\Omega_{43} - \Omega_{41}\Omega_{33}\right) \end{array} \right\}
\end{aligned}
$$

It is easy to show that

$$\det\begin{pmatrix}\Omega_{55} & \Omega_{56}\\ \Omega_{65} & \Omega_{66}\end{pmatrix}<0$$

and

$$\begin{aligned}\Omega_{11}\Omega_{22}\left(\Omega_{33}\Omega_{44}-\Omega_{34}\Omega_{43}\right) &> 0\\ \Omega_{11}\Omega_{32}\left(\Omega_{23}\Omega_{44}-\Omega_{43}\Omega_{24}\right) &< 0\\ \left(\Omega_{13}\Omega_{24}-\Omega_{14}\Omega_{23}\right)\left(\Omega_{31}\Omega_{42}-\Omega_{41}\Omega_{32}\right) &> 0\end{aligned}$$

Re-arrange those terms, if π is small, by plugging in equation (4.5), we can further show that

$$\begin{aligned}\Omega_{11}\Omega_{22}\left(\Omega_{33}\Omega_{44}-\Omega_{34}\Omega_{43}\right)+\Omega_{14}\Omega_{22}\left(\Omega_{31}\Omega_{43}-\Omega_{41}\Omega_{33}\right) &> 0\\ \Omega_{34}\left(\Omega_{11}\Omega_{23}\Omega_{42}+\Omega_{41}\Omega_{13}\Omega_{22}\right) &> 0\end{aligned}$$

and hence

$$\det\left(\Omega\right)<0$$

Then

$$\frac{dn_t^m}{d\phi}=\frac{\det\begin{pmatrix}\Omega_{55} & \Omega_{56}\\ \Omega_{65} & \Omega_{66}\end{pmatrix}\cdot\left\{\begin{array}{c}\Omega_{11}\Omega_{22}\left(\Omega_{33}z_4-z_3\Omega_{43}\right)\\ +z_3\left(\Omega_{11}\Omega_{23}\Omega_{42}+\Omega_{41}\Omega_{13}\Omega_{22}\right)\\ +\left(\Omega_{13}z_2-z_1\Omega_{23}\right)\left(\Omega_{31}\Omega_{42}-\Omega_{41}\Omega_{32}\right)\\ -\Omega_{11}\Omega_{32}\left(\Omega_{23}z_4-\Omega_{43}z_2\right)\\ -z_2\Omega_{11}\Omega_{42}\Omega_{33}\\ +z_1\Omega_{22}\left(\Omega_{31}\Omega_{43}-\Omega_{41}\Omega_{33}\right)\end{array}\right\}}{\det\left(\Omega\right)}$$

Similarly, we can show that, if π is small,

$$\begin{aligned}
\Omega_{11}\Omega_{22}\left(\Omega_{33}z_4 - z_3\Omega_{43}\right) &> 0 \\
\Omega_{11}\Omega_{32}\left(\Omega_{23}z_4 - \Omega_{43}z_2\right) &< 0 \\
\left(\Omega_{13}z_2 - z_1\Omega_{23}\right)\left(\Omega_{31}\Omega_{42} - \Omega_{41}\Omega_{32}\right) &> 0
\end{aligned}$$

and

$$\begin{aligned}
\Omega_{11}\Omega_{22}\left(\Omega_{33}z_4 - z_3\Omega_{43}\right) + z_1\Omega_{22}\left(\Omega_{31}\Omega_{43} - \Omega_{41}\Omega_{33}\right) &> 0 \\
z_3\left(\Omega_{11}\Omega_{23}\Omega_{42} + \Omega_{41}\Omega_{13}\Omega_{22}\right) &> 0
\end{aligned}$$

and therefore,

$$\frac{dn_t^m}{d\phi} > 0$$

As the sex ratio rises, a larger fraction of men choose to be entrepreneurs. □

A4.2. Proof of Proposition 14

Proof. By (4.22),

$$\frac{\phi n_{2t}^m}{1+\phi} = \frac{B_1\left(1 - \frac{\phi n_{1t}^m}{1+\phi}\right)}{B_1 + \frac{\theta-1}{1-\gamma}}$$

the number of entrepreneurs in sector 2 negatively depends on the number of entrepreneurs in sector 1.

By (4.19) and (4.20), we can rewrite the labor market clearing condition as

$$\left(1 + B_1\frac{1-\gamma}{\theta-1}\right)L_t = 1 - \frac{\phi n_{1t}^m}{1+\phi} \tag{4.51}$$

We total differentiate the equations (4.6), (4.7), (4.9), (4.11), (4.13), (4.51) and (4.21), or the equations (4.6), (4.8), (4.12), (4.13), (4.51) and (4.21) to analyze the effect of a rise in the sex ratio on the choices of being an entrepreneur in sector 1 or a worker. Notice that, (4.5) and (4.23), (4.14) and (4.51), only differ in constant terms which do not affect any derivative properties. Then, the two systems (when sex ratio is small and when sex ratio is large) are the same as in Proposition 13. Therefore, all the results in Proposition 13 hold if we only take entrepreneurs in sector 1 as the entrepreneurs in Proposition 113.

In sum, If π is small enough, (i) When the sex ratio is small (close to one), as the sex ratio rises, a smaller fraction of men will choose to become entrepreneurs in both sector 1 and sector 2; (ii) When the sex ratio becomes sufficiently unbalanced such that no failed entrepreneurs can get married, as the sex ratio rises, a larger fraction of men choose to be entrepreneurs in sector 1 while a smaller fraction of men will choose to be entrepreneurs in sector 2. □

A4.3. Proof of Proposition 15

Proof. Let n_{jt}^m and N_{jt} denote the fraction of men who choose to be entrepreneurs in sector j in the home country and the total number of sector j's entrepreneurs in the world, respectively. Since we assume the home country is small, it will take the interest rate which is determined by the world capital market as given. This in turn can pin down the wage rate in the home country by (4.2) and (4.3).

For a representative entrepreneur with productivity z_{it} in the home country, the sales revenue is

$$p_{1t}(i)\, y_{1t}(i) = \gamma N_{1t}^{-1} \left(\frac{p_{1t}(i)}{P_{1t}^w} \right)^{1-\theta} P_t^w Y_t^w$$

where P_{1t}^w and P_{2t}^w the aggregate price indices in sector 1 and sector 2, respectively. $P_t^w Y_t^w$ is the world GDP, which is exogenous to the home country. Then

$$p_{1t}(i)\, y_{1t}(i) = \gamma N_{1t}^{-1} \frac{z_{it}^{\theta-1}}{\pi \left(z^H\right)^{\theta-1} + (1-\pi)\left(z^L\right)^{\theta-1}} \left(\frac{P_{1t}}{P_{1t}^w} \right)^{1-\theta} P_t^w Y_t^w$$

The aggregate sales revenue in sector 1 in the home country is

$$TR_{1t} = \int^{\frac{\phi n_{1t}^m}{1+\phi}} p_{1t}(i)\, y_{1t}(i)\, di = \gamma \frac{\frac{\phi n_{1t}^m}{1+\phi}}{N_{1t}} \left(\frac{P_{1t}}{P_{1t}^w} \right)^{1-\theta} P_t^w Y_t^w \tag{4.52}$$

Similary, the aggregate sales revenue in sector 2 in the home country is

$$TR_{2t} = \int^{\frac{\phi n_{2t}^m}{1+\phi}} p_{2t}(i)\, y_{2t}(i)\, di = (1-\gamma)\frac{\frac{\phi n_{2t}^m}{1+\phi}}{N_{2t}}\left(\frac{P_{2t}}{P_{2t}^w}\right)^{1-\theta} P_t^w Y_t^w \tag{4.53}$$

Assume that the aggregate demand for the final good in the home country is Y_t^D. Then the total expenditures on good 1 and good 2 produced by home firms are

$$TE_{1t} = \gamma\frac{\frac{\phi n_{1t}^m}{1+\phi}}{N_{1t}}\left(\frac{P_{1t}}{P_{1t}^w}\right)^{1-\theta} P_t^w Y_t^D$$

and

$$TE_{2t} = (1-\gamma)\frac{\frac{\phi n_{2t}^m}{1+\phi}}{N_{2t}}\left(\frac{P_{2t}}{P_{2t}^w}\right)^{1-\theta} P_t^w Y_t^D$$

respectively. Then, exports in sector 1 and sector 2 are

$$X_{1t} = TR_{1t} - TE_{1t} = \gamma\frac{\frac{\phi n_{1t}^m}{1+\phi}}{N_{1t}}\left(\frac{P_{1t}}{P_{1t}^w}\right)^{1-\theta} P_t^w\left(Y_t^w - Y_t^D\right)$$

and

$$X_{2t} = TR_{2t} - TE_{2t} = (1-\gamma)\frac{\frac{\phi n_{2t}^m}{1+\phi}}{N_{2t}}\left(\frac{P_{2t}}{P_{2t}^w}\right)^{1-\theta} P_t^w\left(Y_t^w - Y_t^D\right)$$

respectively.

When the sex ratio is small, as the sex ratio rises, $\frac{\phi n_{1t}^m}{1+\phi}$ decreases while $\frac{\phi n_{2t}^m}{1+\phi}$ increases. Then $\frac{X_{1t}}{X_{2t}}$ decreases. With a larger possibility, the inequality

$$\frac{x_{1t}}{x_{2t}} = \frac{X_{1t}/Total.Export}{X_{2t}/Total.Export} < 1$$

may hold, where x_{1t} and x_{2t} are the shares of sector 1's export and sector 2's export, respectively. Then the home country is more likely to have a comparative advantage in sector 2. When the sex ratio becomes sufficiently large, as in Proposition 14, if the sex ratio exceeds the threshold value, a rise in the sex ratio will lead to an increase in $\frac{\phi n_{1t}^m}{1+\phi}$ but a decrease in $\frac{\phi n_{2t}^m}{1+\phi}$. $\frac{X_{1t}}{X_{2t}}$ will increase. Then, with a larger possibility, the inequality

$$\frac{x_{1t}}{x_{2t}} = \frac{X_{1t}/Total.Export}{X_{2t}/Total.Export} > 1$$

may hold. Then the home country is more likely to have a comparative advantage in sector 1. $\square$