Bubbles in Japan's Stock Market:  
A Macroeconomic Analysis

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ABSTRACT

Since the 1950s, Japan’s stock market has gone into bubbles every ten years or so (early 50s, early 60s, and early 70s). Then, the 1980s saw a strong bubble that went on for several years (late 1982 to the end of 1989). What caused this bubble to be so strong? Our analysis of the fundamental equation reveals that the key factor was the nominal interest rate which continued to decline until the late 1980s owing to the extremely relaxed monetary policy pursued by the Bank of Japan. Investors’ stock price expectations added to the effect of low interest rates. We show that investors tend to forecast fluctuations on the real side two to three quarters ahead of time. The bubble crashed when investors’ expectations collapsed. Since then, investors have remained bearish and the stock market has remained in the doldrums.
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I. Introduction

In Japan's remarkable growth process from the end of World War II, financial development has been even more spectacular than real growth. Corporate stocks stand out among financial assets because their prices maintained double-digit growth, even in the slow growth periods that have occurred since the mid-1970s.

Stock prices were exceptionally vigorous throughout the 1980s. The Nikkei Stock Price Average, Japan's Dow Jones, started from a low of ¥7042 in August 1982 and continued to rise almost without interruption¹ to a high of ¥38150 in December 1989, a more than fivefold increase. Following the stock market crash, the Nikkei Average bottomed out at ¥15790 in August 1992. The subsequent recovery has been sluggish, and the Nikkei Average hovered at around ¥20000 throughout 1994. Thus, there was a very robust bubble through most of the 1980s and a subsequent collapse.

What was responsible for the stock market bubble and its demise? Was it, as is commonly alleged, investors' speculative zeal? Or were there more mundane factors such as mismanaged monetary policy or business cycles in the corporate sector? The common allegation is apparently based on a first impression that is not necessarily correct. This problem requires a rigorous analysis of statistics. In this paper, we shall show that the basic cause of the bubble was not investors' optimism but more objective factors.

In Section II, we review the ownership structure of Japan's corporate stocks. Section III presents a simple stock market model. We derive the familiar "fundamental equation." Our point of departure from the popular literature is to incorporate expectations formation explicitly. Section IV explains the data we employ. Section V interprets the data in terms of the fundamental equation. Section VI examines the causality governing our basic variables, and Section VII decomposes the bubble into a few components. Section VIII examines the properties of stock-price appreciation expectations. Section IX concludes the paper.²
II. The structure of Japan's corporate finance

How Japanese firms finance their financial needs has an important bearing on the functioning of Japan's stock market. Therefore, we begin by reviewing the structure of Japan's corporate finance.

Funds originate in the household sector. In Japan, the largest part of funds moves through financial intermediaries. Banks receive deposits from households and, based on those funds, supply loans to client firms. Although direct financing through the new equity and bond markets expanded in the late 1980s, it declined again in the latest recession. Bank loan rates and industrial bond yields move together. Bank deposit rates are linked to the central bank's discount rate, which is adjusted as the interest rate structure changes. Thus, the deposit rate moves with the loan rate and bond yield, but at a lower level.

Another route of corporate finance is the issuing of corporate stocks, which then enter into the stock circulation market. Here, a significant feature is that only one quarter of stocks are owned by individuals. In 1992, 23.1% of stocks were owned by individuals, 41.3% by financial institutions, and 24.4% by non-financial corporations. This means that in Japan, businesses own one another. The proportion of cross-share ownership among firms is estimated to be about 40% through 1981-91, with little fluctuation around the mean. Furthermore, Japanese firms have a high propensity to retain earnings. The dividend : earning ratio averaged 32% in 1981-91, much lower than the above 50% ratio for U.S. firms. The low level of stock ownership by individuals and the low dividend : earning ratio implies that the corporate sector lives its own life, largely independently of the household sector, as regards asset management and earnings distribution. It also means that stock prices are formed under the strong influence of corporations themselves. To understand the performance of Japan's stock market, this feature must be kept in mind.

III. A stock market model

(A) Assumptions.

As we are interested in a macroeconomic analysis of the stock market, we assume that all stocks are homogeneous. Also, the stock market is treated as a spot market. \( P_E \) is the market-determined price of the representative stock and \( R_E \) is its net earnings per unit of time (year). The
price: earnings ratio (PER) is $P_E/R_E$. The financial asset that is the closest substitute to stocks is assumed to be corporate bonds.

(B) **Investors' stock-price appreciation expectations.**

Let us assume, for the time being, that investors are principally interested in making short-term capital gains by buying or selling stocks. Those who believe that stock prices will rise more want to add more to their stock holdings. Those who believe that the stock prices will rise less want to reduce their holdings. Let $g(P_E)^e$ be the expected rate of stock-price appreciation. Then each investor has his or her own demand schedule, rising from negative to positive with $g(P_E)^e$. We assume that investors can sell up to what they initially hold and buy up to what financial resources they can mobilize. The crossover point from negative to positive differs among investors. For any given value of $g(P_E)^e$, add up positive demands into $\tilde{D}(E)$ and negative demand (in absolute value) into $\tilde{S}(E)$, where $E$ is the number of stocks to be traded. $\tilde{D}$ is downward sloped and $\tilde{S}$ upward sloped (see Figure 1). The equilibrium $(g(P_E)^e^*, E^*)$ holds at the point of intersection of these two curves. The equilibrium is stable.
(C) **The demand-and-supply equilibrium.**

We restate the equilibrium process in terms of the conventional demand-and-supply framework. Let \( g(P_E)_D \) and \( g(P_E)_S \) be values of \( g(P_E) \) held by a buyer and a seller respectively.

A buyer compares the cost of buying the stock to the returns he or she expects to get when the stock is resold at the end of the unit time. The former is the annual cost of fund, covering both interest cost and "risk premium." We denote their sum by \( i \). The latter is the sum of net earnings, which are wholly distributed to the share holder, and expected capital gains. The investor decides to buy the stock if and only if:

\[
P_E i < R_E + P_E g(P_E)_D^e.
\]

The demand price of the stock is the price that makes this inequality an equality, i.e.,

\[
P_E^D = \frac{R_E}{i - g(P_E)_D^e}.
\]

Noting that \( g(P_E)_D^e = g(E) \), (1) gives the downward-sloped demand function.

Likewise, a seller decides to sell if and only if

\[
P_E i > R_E + P_E g(P_E)_S^e.
\]

Then the supply price of the stock is given by

\[
P_E^S = \frac{R_E}{i - g(P_E)_S^e}.
\]

We have \( g(P_E)_S^e = g(E) \), which is the upward-sloped supply curve.

(1) and (2) yield the familiar demand-and-supply diagram between \( E \) and \( P_E \) (Figure 2). At equilibrium, we have

\[
P_E^* = \frac{R_E}{i - g(P_E)^*}.
\]
is the "fundamental equation" of the stock market. There is an existence condition, i.e., both $g(P_E)^*\text{ and } g(P_e)^*\text{ must be less than } i$. Therefore, $g(P_e)^*\text{ must also be less than } i$.

(D) The fundamental equation.

In the above model, we have assumed that investors are interested in quick resale of the stock. As an alternative model, assume that investors are interested in holding the stock as a permanent asset in their wealth portfolio. Then $P_E$ should be equal to the present discounted value of future income streams. Assume that $R_E$ is expected to grow at a constant rate, $g(R_E)^e\text{, and that the discount variable } i\text{ is expected to remain stationary.}^{13}$ Then we have

\begin{equation}
P_E = \frac{R_E}{i - g(R_E)^e}.
\end{equation}

(4) is the earnings-discount model.$^{14}$ In this case, the expectations variable is not $g(P_e)^e\text{ but } g(R_e)^e\text{.}$

In general, the present discounted value of the stock depends on $g(R_e)^e\text{, } g(P_e)^e\text{, and } T\text{, which is the date at which the stock is expected to be resold. Since } T\text{ is unknown, there is no unique formula for the fundamental equation (see Appendix Note). However, } T\text{ becomes irrelevant when } g(P_e)^e = g(R_e)^e\text{. We assume that this condition is approximately satisfied.}

In what follows, we denote the expectation variable by $g(P_e)^e\text{ rather than } g(R_e)^e\text{.}$ Additionally, as risk premium is statistically unobservable, $i$ is limited to the market interest rate while risk premium is subsumed into $g(P_e)^e\text{.}$

(E) Displacement of the equilibrium.

The fundamental equation, thus modified, shows that the equilibrium solution $(P_E^*, E^*)\text{ is subject to three separate influences--the earnings rate (} R_E\text{, the interest rate (} i\text{, and stock-price appreciation expectations (} g(P_e)^*\text{).}$

The fundamental equation is useful, as we shall show below, in dissecting bubbles into basic factors. The most interesting among them is $g(P_e)^*\text{. Unlike the other two, how this variable is generated is not immediately apparent. As our model has indicated, this variable is
based, behind the scene, on how individual investors' expectations (including risk premium) are distributed. If most investors become bullish, their expectations functions shift to the right. As a result, the demand curve shifts upward more than the supply curve does. $P_E^*$ rises more than $E^*$ does. If most investors become bearish, the opposite takes place, and $P_E^*$ falls more than $E^*$ does.\footnote{15}

Since $P_E^*$, $R_E$, and $i$ are directly measurable, we can obtain $g(P_E)^e$ as the residual of the fundamental equation. The rest of this paper analyzes how this residual behaved in the case of Japanese stock prices.

IV. The data

Japan's stock exchanges suspended normal operations when World War II ended. They resumed operations on 16 May 1949. Initially, there were three stock exchanges (Tokyo, Osaka, and Nagoya). This later expanded to eight exchanges. However, the Tokyo Stock Exchange (TSE) maintained the lion's share of stock tradings.\footnote{16} The TSE is composed of two sections. The First Section is by far the more important, in terms of both market size and quality of firms listed.\footnote{17} The stock price index, called TOPIX, is prepared for stocks traded in the First Section. Our $P_E$ is this index.\footnote{18} The PER given for the First Section is our $P_E/R_E$.\footnote{19} From this, we derive $R_E$. It is important to remember that, in computing PER, the TSE takes earnings recorded for the preceding accounting period. Thus, there is a built-in time lag of, say, one quarter in the statistics of $R_E$.

There are several interest rates. The rate closest to corporate stocks is industrial (corporate) bonds. Hence, we take industrial bond yields for our $i$.\footnote{20}

While these variables are available in monthly series, we take their quarterly averages for our analysis. $i$, $R_E$, and $g(P_E)^e$ are expressed in annual rates.

The sample period is 1981IQ-1994IQ. This is because the monthly series of the PER began only in 1981. Before that date, the series were only available annually. In any case, Japan's financial system was strictly regulated and underdeveloped until around 1973. This underdevelopment is why the statistics on industrial bond yields started in 1974, and then only annually. Though the financial markets were considerably deregulated by the late 1970s, the data
for the 1970s reveal that the fundamental equation of the stock market did not perform well during that decade.

V. The fundamental equation: basic variables

(a) The stock-price index, \( P_E \).

Figure 3 shows the movement of \( P_E \) from 1976IIIQ-1994IIIQ. As \( g(P_E) \), shown in Figure 4, indicates, the curve contains considerable zigzags. However, trends are unmistakable. There was a mini-bubble around 1981. \( P_E \) then went on a sharp upward path from late 1982 to mid-1987. After a dip corresponding to Black Monday, \( P_E \) resumed its upward path, albeit at a reduced speed, until the last quarter of 1989. Then the bubble collapsed. \( P_E \) seems to have bottomed by late 1992.

(b) The earnings rate, \( R_E \).

Figure 6(C) shows the movement of \( R_E \) from 1979IQ-1994IQ. While the curve shows many zigzags, it can be divided into four phases: (i) a rise from 1979IQ-1981IIIQ; (ii) a fall in 1981IIIQ-1983IVQ; (iii) a long upswing from 1983IVQ-1991IVQ; and (iv) a sharp descent in 1991IVQ-1994IQ. Note that (i) and (ii) correspond to the 1980-1983 mini-bubble. It is also interesting that \( R_E \) continued to rise in 1990-91 while \( P_E \) started to fall.

(c) The interest rate, \( i \), and the earnings: price ratio, \( R_E/P_E \).

Figure 5 shows \( i \) and \( R_E/P_E \). It is unmistakable that the two variables are interconnected. This is definitely what the fundamental equation tells us, provided that \( g(P_E)^* \) does not fully offset the interest rate.

(d) Stock-price appreciation expectations, \( g(P_E)^* \).

In the fundamental equation, all variables are measurable except \( g(P_E)^* \). It can be measured as the residual, \(^{21}\) i.e.,

\[
g(P_E)^* = i - R_E/P_E.
\]
Figure 3. The average stock price index, $P_E$

Figure 4. $g(P_E) = (P_E - P_{E,-1})/P_{E,-1}$, in %
Figure 5. \( i \) and \( \frac{R_E}{R_P} \) in %
When this derived statistic is inspected, the first thing that comes to our attention is that it is positively related to $\Delta i$. This is not a mere statistical artifact. There is a reason. In our model, when $P_E$ is determined, investors may not respond fully to the current value of $i$ because of imperfect information. Let $i$ be their estimate of $i$. Then the fundamental equation must be modified to

$$\hat{i} = R_E/P_E + g(P_E)^{**}.$$  

Taking the simplest version of $\hat{i}$,

$$\hat{i} = a i + (1 - a) i_t, \quad 1 > a > 0.$$  

Then we have

$$i - R_E/P_E = (1 - a) \Delta i + g(P_E)^{**}.$$  

We estimate $a$ by regressing $(i - R_E/P_E)$ on $\Delta i$ and a few other instrumental variables.\(^{22}\) This exercise yields robust estimates of $a$ which range between 0.50 and 0.55 (s.e. = 0.25).\(^{23}\) Thus, we assume that

$$\hat{i} = \frac{1}{2} (i + i_t).$$  

An important related finding is that $(i - R_E/P_E)$ is not related to $i_{t+1}$. This supports our assumption that interest expectations are not forward-looking.

Given $\hat{i}$, we estimate $g(P_E)^{**}$ by

$$g(P_E)^{**} = \hat{i} - R_E/P_E.$$  

See Figure 6(A).
Figure 6. Cyclical correspondence

(A) $g(P_E)^{**}$

(B) $Q/Q$

(C) $R_E$
VI. Causality governing cyclical variations

(A) Cyclical variations of variables.

(a) i. The interest rate is a procyclical, lagging economic indicator. In the early 1980s, however, i was on a strong downward path throughout two business cycles (No. 9, which peaked in February 1980 and No. 10, which peaked in July 1985). From mid-1985 on, i behaved more or less as conventionally expected.

(b) $R_E/P_E$. This variable moved very much in parallel with i until 1989. Then the two parted company, especially in 1990-91.

(c) $g(P_E)^e*$. As shown in Figure 6(A), this variable remained positive throughout the period and fluctuated around the mean of 4.0%.

(B) Causal relationships with outside markets.

The fundamental equation says that $P_E$ is determined by three variables. Of these three, i is linked to the financial market in which the government's monetary policy plays an influential role. As for $R_E$, it depends on corporations' total earnings and outstanding volume of stocks. Total corporate earnings depend on the growth and fluctuations of business activities. The growth reflects changes in the business sector's aggregate production function. Fluctuations depend on fluctuations in aggregate demand. The best indicator of the latter turns out to be the degree of capacity utilization in manufacturing, $Q/Q$.24

Figure 6 compares (A) $g(P_E)^e*$, (B) $Q/Q$, and (C) $R_E$. We identify turning points of the three curves which seem to be interrelated. We see that in these identified turning points, $g(P_E)^e*$ generally leads $R_E$ and lags $Q/Q$. The lead and lag times are reported in Table 1.
The table shows that $R_E$ tends to lag behind $Q/Q$. The lag must be due to, among other things, the accounting practice of taking earnings in earlier months. More interesting is the lead time of $g(P_E)^*$. While lead time is considerably variable, the most likely length of the lead is three quarters.

(C) **Regression results.**

We make our impressionistic comparison more quantitative by estimating regression equations.

First, we regress $R_E$ on $Q/Q$. However, $R_E$ must be detrended. Its trend growth rate is estimated to be 4.0% per annum. $\tilde{R}_E$ is the detrended value according to

$$\ln \tilde{R}_E = \ln R_E - 0.010 (t - 1987IIIQ).$$

The regression equation estimated is

(R.1) \[ \ln \tilde{R}_E = 2.29 \ln (Q/Q)_{-1} + 3.057, \text{ R}^2 = 0.393, \text{ D.W.} = 0.21. \]

\[ (0.41) \quad (0.0025) \]

As residuals are highly auto-correlated, we add $\ln \tilde{R}_{E,-1}$ as an explanatory variable. This yields

(R.2) \[ \ln \tilde{R}_E = -1.13 \ln (Q/Q)_{-1} + 1.26 \ln \tilde{R}_{E,-1} - 0.791, \text{ R}^2 = 0.830, \text{ D.W.} = 1.75. \]

\[ (0.37) \quad (0.11) \quad (0.0051) \]
In (R.2), the immediate effect of \((Q/Q)_i\) is negative, but this is due to the overestimation of the autoregressive part. In a steady state, (R.2) yields

\[
\ln \tilde{R}_E = 4.41 \ln (Q/Q) + 3.077.
\]

We conjecture that the true short-run elasticity of \(R_E\) with respect to \((Q/Q)_i\) is about 3.

The autoregressiveness of \(R_E\) must reflect the way business profits are generated. The trend part of \(R_E\) must be due to shifts in the aggregate production function.

We next turn to the relationship between \(g(P^*_E)\) and \(Q/Q\). Suppose that the lead time is on average three quarters. Then we compare \(g(P^*_E)\) with \(Q/Q\) three quarters hence. When this is done, it is found that residuals follow very long cycles. To account for them, a dummy variable \(D\) is introduced. \(D\) takes the value of 1 in 1981IIIQ-1986IVQ and in 1990IIIQ-1994IQ, and takes the value of 0 otherwise. The regression is performed over 1981IQ-1993IIQ. We have

\[
\begin{align*}
\text{(R.3)} \quad \ln Q/Q_{i-3} &= 0.655 \cdot g(P^*_E) - 0.614 \cdot D - 0.030, \quad R^2 = 0.603, \quad D.W. = 1.00 \\
& (0.097) \quad (0.121) \quad (0.010)
\end{align*}
\]

As D.W. is low and \(R^2\) is relatively low, the regression may be improved by adding further explanatory variables. However, (R.3) suffices to indicate that \(g(P^*_E)\) is a leading cyclical indicator, turning some three quarters ahead of cyclical turning points in the real sector. We note that, for \(Q/Q = 1\), \(g(P^*_E)\) takes the value of 4.5% when \(D = 0\) and 5.5% when \(D = 1\). This means that the full capacity value of \(g(P^*_E)\) was lower than otherwise during 1987IQ-1990IIQ, i.e., at the height of the latest bubble and for a while after the collapse of the bubble.

VII. The fundamental equation: the bubble mechanism

(A) Variations of the three elements.

The fundamental equation tells how \(\Delta P_E\) is decomposed into the changes of the three basic variables, namely \(\Delta R_E\), \(\Delta i\), and \(\Delta g(P^*_E)\). Since each can be + or -, we have eight possible sign combinations for the three variables. We classify the total of 53 quarters (1981IQ-1994IQ) into
these eight cases (see Appendix Table). In addition, we divide the whole period into two sub-periods—bubble and off-bubble. \( g(P_E) \) rose above 5% in 1982IVQ and fell below that level in 1989IVQ. Thus, we take the 29 quarters 1982IVQ-1989IVQ for the bubble period,\(^{25} \) and the remaining 24 quarters for the off-bubble period.

Table 2 shows the number of quarters according to the sign mix in the bubble and off-bubble periods (see the Appendix Table). If \( \Delta R_E \) is held constant, the best mix for \( \Delta P_E > 0 \) is \( (\Delta \hat{i} < 0, \Delta R_E > 0) \). This \((-,-)\) mix is Case #1 and #2 in Table 2. There are 14 such quarters. Case #1, in which \( \Delta R_E > 0 \), definitely makes \( \Delta P_E \) positive. It occurred three times in the bubble and one time in the off-bubble. Case #2, in which \( \Delta R_E < 0 \), occurred six times in the bubble and four times off-bubble. At the other extreme is \((+,-)\), which makes for negative \( \Delta P_E \). This is Case #7 and #8. There were only three such quarters. The worst case, #8, occurred in 1990IVQ. The \((-,-)\) combination was most common, occurring 23 times, equally divided between bubble and off-bubble. The \((+,+)\) mix also occurred frequently, roughly divided between bubble and off-bubble.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \Delta R_E )</th>
<th>( \Delta \hat{i} )</th>
<th>( \Delta g(P_E)^* )</th>
<th>bubble</th>
<th>off-bubble</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>6</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>9</td>
<td>8</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>4</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>29</strong></td>
<td><strong>24</strong></td>
<td><strong>53</strong></td>
</tr>
</tbody>
</table>

SOURCE: Appendix Table.

This examination of the sign mix reveals that it is rather rare for all three variables to move simultaneously in the directions required for the strongest bubble condition. At the same time, it is also rare for all the three to move concurrently in the directions required to bring about the worst-case scenario. In the majority of quarters, the three tend to offset one another. This means
that the relative contributions of these three variables must be carefully compared in evaluating
changes in $P_E$.

(B) **The decomposition of the bubble.**

To make such a comparison in a proper fashion, we decompose the stock-price change
into the three elements by means of the fundamental equation. By taking the log-changes of the
fundamental equation, we have

\[
\Delta P_E = \Delta \ln R_E - \Delta \ln (i - g(P_E)^*) \\
= \ln R_E - P_E/R_E \hat{i} + P_R/R_E g(P_E)^* + \text{approx. error.}
\]

Considering directions of change in both $P_E$ and $g(P_E)^*$, the period under study is divided
into several segments. See Table 3 for the seven sub-periods. The bubble is divided into four
phases plus one intermission.

Table 3 shows that none of the three effects assumed the same sign except in the post-
bubble, indicating the severity of the collapse of the bubble. Over the bubble period as a whole,
the interest effect was the strongest of the three, while the expectations effect was the weakest.
Looking at sub-periods of the bubble, we find that the interest and expectations effects offset each
other except in sub-period I. The expectations effect was overwhelmed by the sum of the
earnings and interest effects except in sub-periods I and IV. Overall, quantitatively speaking, the
expectations effect was not a prima donna of the stock market.
Table 3. The decomposition of the bubble

<table>
<thead>
<tr>
<th>Bubble phase:</th>
<th>pre-bubble</th>
<th>whole bubble</th>
<th>I</th>
<th>II</th>
<th>intermission</th>
<th>III</th>
<th>IV</th>
<th>post-bubble</th>
</tr>
</thead>
<tbody>
<tr>
<td>From To</td>
<td></td>
<td></td>
<td>82 IVQ</td>
<td>84 IQ</td>
<td>87 IIQ</td>
<td>88 IQ</td>
<td>88 IVQ</td>
<td>89 IVQ</td>
</tr>
<tr>
<td>Stock-price change</td>
<td>$\Delta \ln P_e$</td>
<td>0.020</td>
<td>1.585</td>
<td>0.327</td>
<td>0.998</td>
<td>-0.072</td>
<td>0.115</td>
<td>0.217</td>
</tr>
<tr>
<td>Earnings effect</td>
<td>$\Delta \ln R_e$</td>
<td>-0.162</td>
<td>0.579</td>
<td>-0.085</td>
<td>0.524</td>
<td>-0.160</td>
<td>0.394</td>
<td>-0.094</td>
</tr>
<tr>
<td></td>
<td>$\Delta \ln (\hat{\bar{\gamma}} - g(P_e)^{*})$</td>
<td>0.182</td>
<td>1.006</td>
<td>0.412</td>
<td>0.474</td>
<td>0.088</td>
<td>-0.279</td>
<td>0.311</td>
</tr>
<tr>
<td>Interest effect</td>
<td>due to $-\Delta \hat{i}$</td>
<td>0.023</td>
<td>0.871</td>
<td>0.211</td>
<td>1.015</td>
<td>-0.444</td>
<td>0.216</td>
<td>-0.127</td>
</tr>
<tr>
<td>Expectations effect</td>
<td>due to $\Delta g(P_e)^{*}$</td>
<td>0.096</td>
<td>0.218</td>
<td>0.269</td>
<td>-0.539</td>
<td>0.536</td>
<td>-0.359</td>
<td>0.311</td>
</tr>
<tr>
<td>Error term</td>
<td></td>
<td>0.063</td>
<td>-0.074</td>
<td>-0.068</td>
<td>-0.002</td>
<td>-0.004</td>
<td>-0.136</td>
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VIII. The nature of stock-price appreciation expectations

(A) $g(P_E)^*$ as a cyclical forecaster.

When considered as a cyclical variable, $g(P_E)$ tends to peak soon after a trough. This property follows from the fundamental equation. Since $R_E$ lags $Q/Q$ while $g(P_E)^*$ leads $Q/Q$, the two together do not impart any clear impact on $g(P_E)$. Then the main variable which affects $P_E$ cyclically is the interest rate, which is known to be a lagging, procyclical indicator. Obviously, $P_E$ responds inversely to $i$. This means that $P_E$ and/or $g(P_E)$ cannot be a leading cyclical indicator.  

However, that is not the case with $g(P_E)^*$. As is clear from Figure 6, $g(P_E)^*$ correctly predicted all of the turning points of $Q/Q$, though there were a few false alarms. This means that, taken with due caution, $g(P_E)^*$ is an excellent indicator of business cycles. The only serious problem is that the lead time is not uniform.

The superiority of $g(P_E)^*$ as a business cycle forecaster becomes apparent when its actual performance in that capacity is compared with that of other well-known business forecasts. There are a few varieties in these forecasts.

One group is forecasts derived from macroeconometric models. Most forecasts prepared by government and private research institutes or by commercial organizations such as banks belong to this category. Japan has a legion of such forecasts. But as reviewed in my earlier study (Sato, 1986), their performances are not very good.

The second group is those based on economic indicators, i.e., the diffusion index constructed from a set of leading, coincident, and lagging indicators. It is prepared by the Economic Planning Agency (EPA) and provides the statistical basis for defining business cycle datings (see Sato (1994)).

The third group is quarterly surveys of business firms. These surveys are not only quantitative but also qualitative, e.g., asking whether business leaders expect the future economic climate to be better or worse. The BOJ has two such surveys, one for large firms and another for all firms. The EPA conducts a similar survey called the "Business Survey."

To see the performance of $g(P_E)^*$ as a business forecaster, we compare it with the two most authoritative surveys of the third group, namely the BOJ's Principal Enterprise Survey and the EPA's Business Survey Index (BSI). Among many items reported in these surveys, we find
the items closest to actual business cycles to be respondents' outlooks on business conditions in
general and within their own industries. Since both surveys produce identical findings, we will
refer to these results as the Business Survey. From the BOJ Survey we include (i) outlooks on
sales (all industries) and (ii) outlooks on operating profits (on a semiannual basis) as separate
items for comparison.

These surveys report percentage differences between better and worse responses. When
the difference is positive, respondents as a whole believe that the economy and the particular
industry will improve over the next quarter. When the difference is negative, the economy and the
industry are believed to be deteriorating.

Figure 7 compares \( g(P_E) \) with the two surveys. Contraction phases in business cycles
and in Q/Q are shown for a basis of comparison. Declining phases of \( g(P_E) \) are marked, as are
negative differences for the two surveys.

From Figure 7, we see that Q/Q was more volatile than business cycles. \( g(P_E) \) predicted
all turning points of Q/Q in the period under study. The EPA Survey captured four out of five
downswings in Q/Q, but only predicted one of the eight turning points of Q/Q before the event.
Of the two, \( g(P_E) \) is clearly the superior predictor. The BOJ Survey is not much better as a
predictor than the EPA Survey. Hence, we conclude that \( g(P_E) \) has been a business forecast par
excellence.

A forecast is effective only if it is timely. The data needed to construct \( g(P_E) \) is taken
from readily available statistical information. Thus, \( g(P_E) \) is an accurate and useful forecasting
indicator that can be constructed at little cost.

(B) Risk premium.

\( g(P_E) \) as measured here consists of two parts: price-change expectations and risk
premium, i.e.,

\[
(12) \quad g(P_E) = \frac{E(g(P_E)) - \delta}{\delta}.
\]

\( E(g(P_E)) \) is some measure of the expected rate of growth of \( P_E \); \( \delta \) is risk premium, which must be
non-negative.
Figure 7. Cyclical indicators (contraction phases), 1990IQ-1994IIQ

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**Business Cycles**

**g(P_e)**

**Q/Q**

**R_t**

**Business Survey Index**

**BOJ**

**BOJ**

**SOURCES:**
- Economic Planning Agency - Economic Planning Agency, Hojin Kigyo Doko Chosa, outlook on the economy and the industry
- Figure 6(A) - Bank of Japan, Short-Term Economic Survey of Principal Enterprises
- Figure 6(B) - BOJ, outlook on sales (all industries), quarterly
- Figure 6(C) - BOJ, Outlook on business profits (all industries), semiannual
The relative contributions of these components to the level and fluctuations of $g(P_E)^{**}$ must be determined. For the level, take the means of (12):

\begin{equation}
(13) \quad \text{mean } g(P_E)^{**} = \text{mean } E(g(P_E)) - \text{mean } \delta.
\end{equation}

We consider the entire bubble period, 1982IVQ-1989IVQ. Over that period, mean $g(P_E)^{**}$ was 4.0%. We expect that mean $E(g(P_E))$ to be some sort of weighted average of the trend values of $g(P_E)$ and $g(R_E)$. We also consider the possibility that mean $E(g(P_E))$ is $g(PY/TE)$, taking PY as either nominal GDP or gross value added, and TE as the total number of stocks outstanding in the TSE First Section (see Table 4).

Since $g(P_E)$ leads to too high a risk premium and $g(PY/TE)$ leads to a negative risk premium, we reject both as estimators for mean $E(g(P_E))$ in favor of the more plausible mean $g(R_E)$.

But $g(R_E)$ is some ten times as volatile as $g(P_E)^{**}$; fluctuations in the latter are much more subdued. Nonetheless, they are persistent, rising or falling over several quarters, apparently reflecting the general bullish or bearish mood of investors. Risk premium presumably remains constant.

Table 4. Estimates of risk premium, 1982IVQ-1989IVQ, percent

<table>
<thead>
<tr>
<th>estimator</th>
<th>mean $E(g(P_E))$</th>
<th>mean $\delta$</th>
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<tbody>
<tr>
<td>$g(P_E)^{**}$</td>
<td>25.4</td>
<td>21.4</td>
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<tr>
<td>$g(R_E)^{a}$</td>
<td>8.7</td>
<td>4.7</td>
</tr>
<tr>
<td>$g(PY/TE)^{b}$</td>
<td>1.2</td>
<td>-2.8</td>
</tr>
<tr>
<td>$g(PY/TE)^{c}$</td>
<td>2.1</td>
<td>-1.9</td>
</tr>
</tbody>
</table>

Notes: mean $g(P_E)^{**} = 4.0%$; $g(TE) = 4.55%$

$^{a}$ $g(P_E)$ and $g(R_E)$ are the trend growth rates over the period
$^{b}$ PY = nominal GDP, 1982IQ-1989IVQ
$^{c}$ PY = gross value added of nonfinancial corporations, 1982-89 (Ministry of Finance, Corporate Enterprise Statistics).

(C) What made Japan's stock prices too high?

It is well known that Japan's stock prices were too high. In 1982-89, the PER was low
and stable in other countries. The mean PER was 12.0 in the U.S., 12.3 in the U.K., 13.6 in Germany, and 12.0 in France. In Japan, the PER rose steadily from 23 in 1982 to 62 in 1989 (Omura and Kawakita (1992), Table 4-1). Why were Japan's stock prices so high? Omura and Kawakita (1992), p. 103, cite four reasons: (i) low interest rates, (ii) differences in accounting practices (e.g., in depreciation methods), (iii) higher growth potential, and (iv) cross-share holdings. We have rejected (iv) as a valid reason. What of the others? To answer the question, we apply the fundamental equation to Japan and to the U.S., which we take as a representative country.

In the U.S., \( g(P_E)^* \) was negative in most years of the 1970s, but expectations were revived and \( g(P_E)^* \) turned positive since 1982. This revival must owe much to the long boom of the 1980s, which lasted 108 months from November 1982 to July 1990. For the U.S., we take annual data from 1982-89. Table 5 reports relevant rates.

We note that, according to \( R_E/P_E \), mean Japanese stock prices were three times higher than mean American stock prices, with the difference widening through the period under study. It is seen that nearly 80% of the U.S.-Japan difference is attributable to the difference in the interest rate, and only 20% attributable to the difference in \( g(P_E)^* \).
Table 5. Comparison of rates, Japan and U.S., percent

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<tbody>
<tr>
<td>mean $R_E/P_E$</td>
<td>2.4</td>
<td>8.1</td>
<td>5.7</td>
</tr>
<tr>
<td>mean $i$</td>
<td>6.4</td>
<td>10.9</td>
<td>4.5</td>
</tr>
<tr>
<td>mean $-g(P_E)^*$</td>
<td>-4.0</td>
<td>-2.8</td>
<td>1.2</td>
</tr>
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</table>

if $-E(g(P_E)) =$

| mean $-g(P_E)$       | -25.4                    | -15.2           | 10.2         |
| mean $\delta$        | 21.4                     | 12.5            | -9.0         |

if $-E(g(P_E)) =$

| mean $-g(R_E)$       | -8.7                     | -8.7            | 0.0          |
| mean $\delta$        | 4.7                      | 5.9             | 1.2          |

| mean $g(P_C)$        | 1.3                      | 4.2             | 2.9          |
| mean $(i - g(P_C))$  | 5.1                      | 6.7             | 1.6          |


$R_E/P_E$ Earnings : price ratio, S&P Composite index

$P_E$ S&P Composite stock price index

$i$ Corporate bond yields, AAA

$P_C$ Consumer expenditure deflator

We have seen that $E(g(P_E))$ is poorly approximated by mean $g(P_E)$. If we suppose the $E(g(P_E))$ is approximated by mean $g(R_E)$, then we find that mean $g(R_E)$ was identical between the two countries. This means that (ii) is not reflected in mean $g(R_E)$. Then, the difference in mean $-g(P_E)^*$ must be solely attributable to the difference in risk premium. Risk premium turns out to be slightly higher in the U.S. than in Japan. Presumably, this difference is due to (iii). More rapid growth in Japan lowered its risk premium.

The nominal interest rate may be decomposed into the rate of inflation and the real interest rate. Measuring the former in terms of the consumption expenditure deflator, we observe that inflation was far more moderate in Japan than in the U.S.

All in all, the most important single factor for raising Japan's stock prices was Japan's low interest rate. It can be noted that loose monetary policy allowed the money supply to grow at a double-digit rate in Japan through most of the 1980s. Since the expanding money supply was absorbed into households' asset portfolios, it did not aggravate inflation. But this expanding money balance was coupled with the land-market bubble that took place in the later 1980s. Thus,
while goods inflation was averted, asset inflation occurred (see Sato (1994)).

In short, the behavior of the nominal interest rate in the decade of the 1980s owed much to mismanaged monetary policy. We can say that the stock-market and land-market bubbles in the 1980s were the products of monetary policy rather than investors' speculative activities as such.

**IX. Concluding remarks**

In this paper, we have examined stock-price formation in Japan in the 1980s and the early 1990s. We have applied the fundamental equation to decompose the stock-price change into changes in the earnings rate, the interest rate, and stock price appreciation expectations.

We have empirically shown that, contrary to common belief, the expectations factor contributed less to the bubble than the interest factor. In fact, we have shown that under ordinary circumstances, the expectations factor and the interest factor offset each other. The expectations factor only seems to have played the leading role in the beginning and end of the bubble period, as well as in the post-bubble period. During the bubble period, more blame must be placed on the interest factor.

Loose monetary policy was pursued during most of the 1980s, causing the continual lowering of the interest rate. We have therefore attributed the stock market bubble of the 1980s to this monetary policy.\(^{35}\) We have also argued that the interest rate, much lower in Japan than in the U.S. and European nations, caused Japan's stock prices to be too high. The high rate of land-price inflation in the late 1980s was also attributed to high money growth (Sato (1994)).

In the 1980s, the Japanese economy faced a macroeconomic contradiction—a low rate of goods inflation and a high rate of asset inflation. Our analysis has resolved this apparent contradiction. The double-digit rate of money growth did not lend itself to goods inflation. Instead, it gave rise to asset inflation, both in stock prices and in land price.
1. Except for a major drop towards the end of 1987. On Black Monday (19 October 1987), the Nikkei Average fell by 14.9% in one day. All other world stock markets fell by similar amounts.

2. Asset inflation in the late 1980s and asset deflation in the early 1990s led to an outpouring of analysis and policy debates on stock pricing. For analyses, see, e.g., Asako et al. (1992), Iwata (1992, chap. 5), Miwa (1990), Tachi et al. (1993), and Ueda (1990). On the role of economic policies, see, e.g., Iwata (1993, chap. 8), and Noguchi (1994).

3. The share was 69% in 1949, 47% in 1960, 40% in 1970, 29% in 1980, and 23% in 1990-92.

4. Other owners in 1992 were: government (0.6%), investment trusts (3.2%), securities firms (1.2%), and foreigners (5.5%). See Bank of Japan (BOJ), Economic Statistics Annual, 1993, Table 90.

5. See Asako et al. (1994), Table 3.

6. In terms of stock tradings, the share of individuals declined from a little over 40% around 1980 to 23% in 1990. The share of securities firms, acting on behalf of their clients, remained at around 25% through the 1980s. The share of banks increased from around 3% in 1980 to the level of 20% or so by the end of the 1980s. See Omura and Kawakita (1992), Table 3-3.

7. Stocks not qualified to be traded in stock exchanges are excluded.

8. The futures and option markets came into being in Japan in 1988.

9. When investors are allowed margin tradings, these minimum and maximum limits are affected.

10. The micro demand schedule need not be continuous. It may be a step-wise function, being negative or positive only at low or high values of \( g(P_e) \). In between, the schedule may be at the zero level. This zero stretch becomes longer when the stock market is depressed, as in the early 1990s.

11. New stock issues can be included in the \( \bar{S} \) curve.

12. Transactions cost, included capital gains tax, are neglected.

13. This means that the elasticity of expectations is 1 as regards \( i \). The empirical validity of this assumption will be seen later in this paper.

14. The dividend-discount model obtains \( P_e \) as the present discounted value of future dividends. If the dividend \( D_e \) is expected to grow at a constant rate \( g(D_e)^e \), we have

\[
(4') \quad P_e = \frac{D_e}{i - g(D_e)^e}.
\]
If the dividend : earning ratio is to be constant, we have $g(D) = g(R)$. Then, apparently, (4') yields a lower value than (4) does. Since (4) and (4') refer to the same firm, (4') must be wrong, obviously because it excludes the contribution of retained earnings to the stock price. Amazingly, (4') is very frequently employed in the investment literature.

Stock market bubbles appear in both $P_E$ and $E_*$. The movement of $E_*$ is analyzed but not reported here to save space.

In 1989, the year stock tradings peaked, the market shares were Tokyo (86.1%), Osaka (10.8%), Nagoya (2.7%), Kyoto (0.1%), and the other four (Sapporo, Niigata, Hiroshima, and Fukuoka) (0.3%). See BOJ, *Economic Statistics Annual*, 1991, Table 104.

In 1990, the First Section covered 1197 brands (1091 firms) and the Second Section 437 brands (436 firms). In terms of the market value of corporations listed, the First Section accounted for 96.7% (December 1989).

The base date of the TOPIX is 4 January 1968. The weights used are market values of brands at that date. Before that date, the TSE stock price index was the unweighted arithmetic average of the stock prices of 225 brands. The latter index was carried into the better known Nikkei Stock-Price Average. The Nikkei Average and the TOPIX move in close parallel.

PER is also available for the Second Section.

One statistical problem with the PER is that $R_E$ tends to be overstated on account of dividends paid to stockholder firms. Let $s =$ cross-firm share ownership and $d =$ dividend : earnings ratio. Then the true measure of earnings is $R_E (1 - sd)$. As noted earlier, $s = 0.41$ and $d = 0.32$. Then $R_E$ must be reduced by 13%.

The commonly accepted procedure then adjusts $P_E$ for cross-share holdings. Thus, the modified PER is $(P_E/R_E)(1 - s)/(1 - sd)$, or 68% of the reported PER in the Japanese case (Omura and Kawakita (1992), p. 105; Asako et al. (1994), Figure 3). This formula, however, is wrong because it does not consider the fact that real assets which are represented by stocks remain unchanged even after cross-held stocks are eliminated. When a merger of involved firms eliminates cross-share holdings, the stock price will rise. Therefore, the PER ought to be raised by 13%, not reduced by 32%.

So long as $s$ and $d$ are stable over time, revisions will affect the absolute level but not the relative changes in the PER.

The monthly series of industrial bond yields (prepared by the Japan Securities Association) started on May 1975. Most studies of the fundamental equation take government bond yields.

The investment literature calls this residual the *spread*. 
22. They include the degree of manufacturing capacity utilization $Q/Q$, the GNP gap $\ln Y/Y$, and $(i - R_E/P_E)_t$.

23. Since $i$ appears on both sides of the regression equation, the estimate of $a$ is biased upward. Small differences in $a$ do not affect our subsequent empirical analysis.

24. Manufacturing is the major activity of corporations listed in the TSE (about two-thirds). Also, in the early 1980s, the GNP gap did not fluctuate as much as $Q/Q$.

25. In some quarters in the bubble period, $g(P_E)$ was below 5%; sometimes even becoming negative.

26. Until its revision in the mid-1970s, the TOPIX was one of the leading indicators for the diffusion index.

27. At the last count, 59 such forecasts are listed (Toyo Keizai, Keizai Tokei 1994, pp. 16-21).

28. However, the best among them, prepared by the Nomura Institute, had a batting average of 0.800, which Professor Lawrence Klein informs me to be a highly satisfactory record.

29. They are called the Short-Term Economic Survey of Principal Enterprises and the Short-Term Economic Survey of Enterprises. Both are conducted in February, May, August, and November, covering the current and subsequent quarters. The first covers about 500 major firms, accounting for 80% of all non-financial corporations in aggregate size. The latter covers some 5500 corporations.

30. The official name is Hojin Kigyo Doko Chosa (Business Operators' Outlook). It is conducted four times a year, also in February, May, August, and November. It covers about 1900 firms.

31. In announcing the end of the latest recession on 6 September 1994, the BOJ based its judgment on the movement of this indicator (called DI) from May to August. The EPA later dated the end of the recession at November 1993.

32. There is the possibility of a political business cycle. Of the seven peaks in $g(P_E)$*, five fell in years of House-of-Representative elections (6/80, 12/83, 7/86, 2/90, 7/93). A general election may help to revive people's confidence, thereby giving a stimulus to the economy. Recall that the mass media predicted that the general election of July 1993 would revive the Japanese economy. But the prediction turned false for 1993.

33. Asako et al. (1994) assume that $E(g(P_E))$ is equal to the 5-year centered moving average of $g(PY)$, where $PY$ is nominal GDP. Taking the average for 1977-86, they derive mean $\delta = 3.86\%$. By assuming that $\delta$ remained at the same level throughout, they compute the "theoretical" value of $P_E$ according to

$$\bar{P}_E = R_E/(i - \delta - g(PY))$$
for January 1981 - December 1993. $i$ is government bond yields. They find that $(P_E - \bar{P}_E)$ was positive throughout the entire period except in 1987. They then conclude that the bubble lasted through the entire period, including during the latest recession. This conclusion is completely contrary to intuition. Since there is no theoretical justification that $E(g(P_E))$ can be approximated by $g(PY)$, we reject this finding.

Ueda (1990), in a more elaborate model, derives the following formula for the "theoretical" value of the PER:

$$\frac{P_E}{R_E} = \frac{1}{(i_C - g(P_Y) + \delta) - g(P_K) + s g(Y)}$$

where $i_C$ is the call rate, $g(P_Y)$ and $g(Y)$ are respectively the growth rates of the implicit GNP deflator and real GNP, $g(P_K)$ is the rate of inflation of real assets, and $s$ is cross-share ownership. $(i_C - g(P_Y) + \delta)$ is the required rate of return, which is the sum of the real interest rate and risk premium. He derives risk premium as the difference of the ex post rate of return on stocks ($R_E/P_E$) and the call rate ($i_C$), averaged for 1956-87. The former was 21.5% and the latter 7.5%. Thus, he estimates $\delta$ to be 14.0%. The derived theoretical value of the PER is about 50% of the measured value in 1956-82 and 15% of the measured value in 1983-88. From this, he concludes that Japanese stock prices were too high, especially in the 1980s. We criticize Ueda's conclusion along the same lines as we did Asako et al.


35. In late 1994, the outgoing Chancellor of the BOJ admitted publicly the BOJ's failure in monetary policy was mostly responsible for the bubbles in the late 1980s.
REFERENCES


Asako, K. et al., “Kabuka no Baburu” (Stock Prices and Bubble), in Nishimuro and Miwa, eds. (1990), pp. 37-86.


Miwa, Y., “Kabuka ‘Moderu’ to Nihon no Kabuka” (Stock-price Model and Japan’s Stock Prices), chap. 1 in Nishimura and Miwa (1990), pp. 3-26.


Appendix Note: The Fundamental Equation

Suppose that the buyer of a stock intends to resell the stock after $T$ years. Then the present value equation is

$$P_E = \frac{R_E}{1 + i} + \frac{R_E (1 + g(R_E)^e)}{(1 + i)^2} + \cdots + \frac{R_E (1 + g(R_E)^e)^{T-1}}{(1 + i)^T} + \frac{R_E (1 + g(R_E)^e)^T}{(1 + i)^T}$$

$$= X \frac{R_E}{i - g(R_E)^e}$$

where

$$X = \frac{1 - \left(\frac{1 + g(R_E)^e}{1 + i}\right)^T}{1 - \left(\frac{1 + g(P_E)^e}{1 + i}\right)^T}$$

We note that

$$\frac{1}{i - g(P_E)^e} < X < \frac{1}{i - g(R_E)^e}$$

according as $g(P_E)^e > g(R_E)^e$.

We immediately find that

$$P_E = \frac{R_E}{i - g(P_E)^e}$$

holds only if $T = 1$ or if $T > 1$ and $g(P_E)^e = g(R_E)^e$.

$X$ is approximated by

$$X = 1 - \frac{g(R_E)^e - g(P_E)^e}{i - g(P_E)^e}.$$

This equation holds exactly for $T = 1$.

Hence, as $g(R_E)^e$ approaches $g(P_E)^e$, $X$ approaches 1.
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<th>Year I</th>
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<th>$\Delta g(P_E)^*$</th>
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