Three Essays on Economic Fluctuations

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ABSTRACT

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This dissertation consists of three essays on the sources and desirability of economic fluctuations. Chapter 1 focuses on a source of fluctuations that has long been attached to the history of economic thought on business cycles: sticky prices. I provide a microfounded theory for one of the oldest, but so far informal, explanations of price rigidity: the kinked demand curve theory. Assuming that some customers observe at no cost only the price of the store they happen to be at gives rise to a kink in firms’ demand curves: a price increase above the market price repels more customers than a price decrease attracts. The kink in turn makes a range of prices consistent with equilibrium, but an intuitive criterion—the adaptive rational-expectations criterion—selects a unique equilibrium where prices stay constant for a long time. The kinked-demand theory is consistent with price-setters’ account of price-rigidity as arising from the customer’s—not the firm’s—side, and can be tested against menu-cost models in micro data: it predicts that prices should be more likely to change if they have recently changed, and that prices should be more flexible in markets where customers can more easily compare prices. The kinked-demand theory has novel implications for monetary policy: its Phillips curve is strongly convex but does not contain any (present or past) expectations of inflation; its trade-off between output and inflation persists in the long-run; changes to the distribution of sectoral productivity shift the Phillips curve; and monetary shocks have a much longer-lasting real effect than in a menu-cost model, despite also being a model of state-dependent pricing.

Chapter 2, written with Emi Nakamura and Jón Steinsson, starts from the assumption of nominal rigidities—asymmetric wage rigidity this time—to investigate the welfare costs of business cycles. We document that the dynamics of unemployment fit what Milton Friedman labeled a plucking model: a rise in unemployment is followed by a fall of similar amplitude, but the amplitude of the rise does not depend on the previous fall. We develop a microfounded plucking model of the business cycle to account for these phenomena. The model features downward nominal wage rigidity within an explicit search model of the labor market. Our search framework implies that downward nominal wage rigidity is fully consistent with optimizing behavior and equilibrium. We reassess the costs of business cycle fluctuations through the lens of the plucking model. Contrary to New-Keynesian models where fluctuations are cycles around an average natural rate, the plucking model generates fluctuations that are gaps below potential (as in Old-Keynesian models). In this model, business cycle fluctuations raise not only the volatility but also the average level of unemployment, and stabilization policy can reduce the average level of unemployment and therefore yield sizable welfare benefits.

Chapter 3 is a contribution to a second branch of Keynesian economics, which sees the possibility of
inefficient economic fluctuations not as a consequence of sticky prices, but instead as a more intrinsic property of a system of decentralized production. I ask: how do agents coordinate in a world that they do not fully understand? I consider a dispersed-information coordination game with ambiguity-averse agents who do not trust their models. Because distinguishing models is harder in a noisier economy, the model is one of endogenous ambiguity. Because one agent’s noise is another’s private information, one agent’s reliance on his private information increases how much ambiguity his neighbor faces. I revisit the role of private and public information in this new light. On the positive side, I show that the equilibrium depends less on fundamentals as agents become more ambiguity averse, and not at all in the limit where they become infinitely so. I also show that, because it makes agents trust their model more, the release of public information drives the economy toward fundamentals whenever ambiguity-aversion is high enough, in contrast to the standard result under rational expectations. On the normative side, I show that the equilibrium features too much dependence on fundamentals: agents would rather live in a world that they understand better, even if it means living in a world that is less responsive to changes in fundamentals.
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Chapter 1

A Kinked-Demand Theory of Price Rigidity

Introduction

Why are prices sticky? Most monetary macro models—when they do not assume that firms cannot change their prices for a number of periods—have looked for a rationale for nominal rigidities on the firm’s side. Menu-cost models assume that a firm can calculate its desired price for free, but adjusts its posted price to its desired level at a cost.\(^1\) Models of producers’ inattentiveness assume that a firm can adjust its posted price for free, but uncovers its desired price at a cost.\(^2\) Both classes of models face a theoretical difficulty: if prices do not adjust, quantities must. Menu-cost models need to assume that quantity adjustment-costs (hiring costs) are less important than price adjustment-costs, while models of producers’ inattentiveness need to assume that firms do not notice that they produce and hire more.

But firm-side models of price rigidity also face an empirical challenge: surveys of price-setters repeatedly point instead at the customer’s side as the source of price rigidity.\(^3\) When asked “why they don’t change their prices more often than that”, a majority of price-setters surveyed by Blinder et al. (1998) stress their fear of “antagonizing customers”. The answer is vague\(^4\) But price-setters seem to be telling us they charge a fixed


\(^2\) The renewed interest in models of producers’ inattentiveness was kindled by Mankiw and Reis (2002a)’s model of delayed information and Woodford (2003a)’s model of partial information. The subsequent literature has endogenized the information structure by modeling the information choice, as in Reis (2006), Woodford (2009), Mackowiak and Wiederholt (2009), Alvarez et al. (2011) combine observation and menu costs.


\(^4\) One could interpret price-setters’ answer as simply referring to the price-elasticity of demand: when prices increase, customers are antagonized, so they leave and demand falls. This is the puzzle: standard models of either perfect or imperfect competition very much incorporate elastic demand curves. Yet they predict prices should be flexible.
price not because their desired price is too costly to implement or too difficult to learn about, but because their desired price itself is sticky. They point at customers’ reaction to price changes: demand curves.

A persistent, if less prominent, literature in monetary economics has sought a rationale for price rigidity on the customer’s side, appealing to ideas of implicit contracts and fairness, customer-base markets, or customers’ search. A likely reason for their lesser popularity, customer-side arguments for price rigidity often lack the microfoundations of their firm-side counterparts. As a result, their internal consistency, and the key assumptions that distinguish them from the benchmark models of consumer demand, have remained open questions. This is in particular true of one of the oldest, and at a time leading, theories of price rigidity: the kinked demand curve theory, dating back to Hall and Hitch (1939) and Sweezy (1939).[^5]

I provide microfoundations—optimizing behaviors, standard equilibrium concepts and rational expectations—for the old informal argument for price rigidity of the kinked-demand curve. I show that relaxing a single assumption on customers in a model of imperfect competition gives rise to a kink in firms’ demands: that all customers are equally informed on all prices charged by firms within a market. When instead some customers observe at no cost only the price at the firm they happen to be at, firms’ demands become kinked: a firm loses more customers by increasing its price above the market price than it gains by decreasing it below. The kink in demand in turn justifies prices that stay constant for substantial periods of time, despite firms’ ability to change them at any moment and at not cost. Although imperfect competition alone does not account for price rigidity, it does when coupled with customers’ asymmetric information on prices. It does so though firms have perfect information on all shocks, though no agent is subject to money illusion, and in tune with price-setters’ reported impression that keeping prices fixed is their optimal response even in the absence of any physical or cognitive constraint on changing prices.

To isolate the key assumption responsible for flexible or sticky prices, I proceed by step-by-step departures from the benchmark model of price-setting under imperfect competition: monopolistic competition in face of the consumption aggregator of a representative agent.[^6] There, demand is elastic at the intensive margin only—when prices increase, customers can buy fewer goods. My first departure is to consider instead a switching-cost model where demand is also elastic at the extensive margin—when prices increase, customers can leave to competitors (section 2). I introduce the extensive margin of demand in a way that preserves the tractability of the benchmark model: the only difference is that the elasticity of demand is now the sum of the elasticities at the intensive and extensive margins. The introduction of the extensive margin in itself does not make prices sticky: I show that when customers (and firms) have perfect information, prices are flexible and the classical dichotomy holds.

Yet, the new framework allows me to consider alternative assumptions on the information available to customers. I relax the assumption that all customers know all prices in a market. Instead, I assume that a fraction of them observe at no cost only the price at the firm they happen to be at, and need to first move to a competitor to observe its price (section 3). Customers are not subject to money illusion however: they all

[^5]: On the history of the kinked-demand curve theory, see Reid (1981) and Stigler (1978).
[^6]: This benchmark is spelled out for instance in section 1.1 of chapter 3 of Woodford (2003a).
have perfect information on the distribution of prices in all markets, and therefore on the price level. This single assumption gives rise to a kink in a firm’s demand curve, located at the market price: all customers at the firm notice and react to a price increase, but only a fraction of customers at other firms notice and react to a price decrease. This provides microfoundations for the informal theory of the kinked-demand curve first propounded by [Sweezy (1939)] and [Hall and Hitch (1939)], especially the versions of the argument relating the existence of the kink to information asymmetries ([Scitovsky (1978); Stiglitz (1979); Woglom (1982)]).

As conjectured by [Woglom (1982)] however, the implication of the kink is not price rigidity. Instead, it is price-multiplicity: a whole range of equilibrium prices are possible in a given market, and the indeterminacy translates into an indeterminacy at the macro level. The possible equilibrium prices are all higher or equal to the full-information price. The competitive force that drives prices down under full-information—the incentive to attract new customers with lower prices—is no longer effective.

The multiplicity is a fragile theoretical feature however. The root of multiplicity is the same as in menu-cost models: strategic complementarities. If all storekeepers wake up one morning expecting everyone else to switch to a new price, then it is in every storekeeper’s interest to follow the herd. Such an extent of coordination seems unrealistic. To capture the intuition that firms are reluctant to be the first firm to raise prices in the market—a reluctance price-setters rank very high in surveys as the reason why they do not change prices—I define a new equilibrium selection criterion: I assume price-setters start each period using yesterday’s price as their initial guess for what the market price will be today, deduce all firms’ best response to this price, and iterate until the guessed price is the best-response price. Firms’ mental process is adaptive, but expectations are ultimately always rational: I call such equilibria adaptive rational-expectations equilibria. I show that there exists a unique adaptive rational-expectations equilibrium. In it, a firm’s pricing function features an inaction region: prices are sticky (section 4).

Menu-cost models too predict infrequent price changes. Does the kinked-demand theory make predictions that distinguishes it from menu-cost models? I stress two micro-level predictions that set the theory apart and make it testable (section 5). I restrict to predictions that are robust to the assumption on the process for costs, so I venture away from statistics such as the frequency and size of price changes, which are the joint product of a theory and a process for costs. First, the kinked-demand theory predicts that prices should be more likely to change if they have recently changed: hazard functions should be decreasing at first. Second, it predicts that prices should be more flexible in markets where customers can more easily compare prices. I find evidence of both in the empirical literature.

Turning from the micro to the macro, I show that the kinked-demand theory has novel implications for monetary policy (section 6). First, the Phillips curve of the kinked-demand theory is strongly convex but does not contain any inflation expectations shifters. The strong convexity limits the extent to which inflation can increase output despite the absence of expectation shifters. Yet the absence of expectations shifters implies that disinflation remains costly even when the change in monetary policy is not only credible.
but also widely known. The convexity also explains the flattening of the Phillips cure since the early 1980s, and the missing disinflation during the Great Recession. Second, the kinked-demand theory predicts a long-run trade-off between output and inflation: the long-run Phillips curve is non-vertical. Third, in the kinked-demand theory relative price changes such as oil price shocks shift the Phillips curve up, and monetary policy can undo these inflationary pressures only at the cost of a decrease in output. Fourth, the kinked-demand theory, despite being a model of state-dependent pricing, predicts long-lasting real effects of monetary shocks, if anything even longer-lived than in a comparable Calvo model. It shows that the often short-lived effects of monetary shocks found in menu-cost models are no feature of state-dependent pricing per se: endogenizing the changing response of price-setting to a changing environment needs not write down the extent of monetary non-neutrality.

1.1 Literature review

This paper is not the first one to look for a rationale for price rigidity on the customer’s side. Several strands of the literature have considered reasons why customers may behave differently than the canonical model of demand predicts. Although these papers differ enough to be classified into different subliteratures, they often start from the same desire to reconcile demand theory with the perception of practitioners. As such, they can be seen as various attempts to rationalize a common idea. (Because many of these papers lack microfoundations, whether they tell the exact same story is ultimately subjective.) Since the present paper certainly belongs to the same effort, I contrast the different models with the present rationalization. If the single assumption of customers’ asymmetric information is enough to give a theory of demand that makes prices sticky, what assumptions are not necessary?

Several papers appeal to the ideas of fairness or implicit norms to justify price rigidity. Okun (1981) suggests firms do not change their prices so often because they have “implicit contracts” with their customers not to take advantage of them by raising prices when demand is high. Whether Okun’s contracts are meant to be enforced by reputation concerns or constitute social norms is unclear. Other authors like Kahneman et al. (1986), Rotemberg (2005), and Eyster et al. (2015) make a more explicit appeal to the notion of fairness. I show that a customer-side theory of price-rigidity needs not arise from concerns about fairness. I derive firms’ incentives not to vary prices using standard preferences and equilibrium concepts.

Following Phelps and Winter (1970), customer-base models consider markets where firms and customers interact repeatedly. If customers learn slowly about prices charged by different competitors, Phelps and Winter conjecture, customers should flow only slowly from expensive to cheap stores. Current prices should thus have little impact on demand today, but much on demand tomorrow, providing incentives for firms not to adjust their prices to short-term variations in costs. A main challenge in modeling the intuition behind customer-base markets is to keep track of the dynamics of market shares. Ravn et al. (2006) find an ingenious way to circumvent the aggregation problem by keeping demand elastic at the intensive margin—
not the extensive margin—and basing firms’ market-share concerns on customers’ building habits in their goods—not on the slow diffusion of information. Soderberg (2011) also dispenses with the extensive margin of demand, but returns to an informational interpretation of a customer-base by having the representative agent only occasionally reoptimizing its allocation of consumption. These microfounded models do not produce sticky prices however. An exception is Nakamura and Steinsson (2011) who show in a customer-base model based on habits formation that price rigidity is one (of many) possible equilibrium outcome because it allows firms to commit not to price-gouge their customers. Their model may be seen as a rationalization of the fairness argument.

The model of this paper shares with the customer-base literature the focus of its early papers on the extensive margin of demand and on customers’ imperfect information. In contrast however, I do not emphasize the intertemporal considerations associated to a customer-base: although in the model a customer starts a period attached to one firm and can switch to another only by incurring a cost, I assume the customer is randomly reassigned to a new firm in the next period. Thus, although the model is a switching-cost model (Klemperer (1987); Farrell and Klemperer (2007)), it may well not deserve the name of a customer-base model. The abstraction from the dynamics of firms’ market-shares is intentional: it stresses that the rationale for price rigidity presented in this paper does not rely on firms’ forward-looking considerations on the evolution of their customer-bases.

Another literature the paper connects to is the good-market search literature. The connection is different. The kinked-demand papers of Stiglitz (1979) and Woglom (1982) appealed to customer search to postulate a kink. Yet, the microfounded customer-search literature seldom addressed the question of price rigidity, and focused instead on other issues, first of which the possibility of price dispersion in equilibrium. At any rate, the microfounded papers show no kink in demand. This raises two questions. Is the present kinked-demand model a search model? If so, how come it is the only one to find kinked demand curves?

Both questions are best answered through a concrete search model, so I take on them in a companion paper, Dupraz (2016). Here, I jump to the bottom lines. On the first question: the present model is not an explicit search model, but the kind of markets it seeks to capture can be seen as including markets

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8Kleshchelski and Vincent (2009) argue that switching costs can bring the pass-through between costs and prices below one-for-one. In contrast, in the model of this paper, switching costs alone do not account for price rigidity. Only with information asymmetries—absent in their paper—do they generate sticky-prices, in the strong sense of no pass-through at all for extended periods of time.

9See McMillan and Rothschild (1994) for a survey of the search literature. A small branch of the search literature—the Bayesian search models of Benabou and Gertner (1993); Fishman (1996); Dana (1994)—does address monetary issues. However, the mechanism these models focus on does not go through a kink in demand: the Bayesian search literature considers the inference that a customer can make from one firm’s price on the distribution of prices in the market. Benabou and Gertner (1993) study how inflation can increase customers’ search, and therefore competition in the market. Dana (1994) shows that the price level is less responsive to cost changes than under customers’ full information. Fishman (1996) shows that cost shocks have different short-term and long-run effects on prices. Independently of the Bayesian search literature, and still distinct from the kinked-demand curve, Head et al. (2012) make the case that in a search model with price dispersion, the unique equilibrium distribution of prices can be implemented through several pricing policies by individual firms, some of which consist in keeping prices fixed for many periods.

10Stiglitz (1987) claims to prove the existence of a kink. However, as Stahl (1989) notes: “[S]tiglitz makes the dubious assumption that consumers can ‘see’ deviations by stores before they actually search. This departure from the notion of an NE is crucial to [his] results.” Whether the assumption is considered dubious or not, it rules out in any case the mechanism that drives the kink in the present paper: that some customers do not notice firms’ price decreases. The same remark applies to Braverman (1989) (who is not interested in price rigidity).
with search. The model shows that two departures from the benchmark theory of consumer demand are enough to generate kinked demand curves: an extensive margin of demand, and asymmetrically informed customers. Because a search model fulfills both requirements, the argument for a kink applies to a search model. The companion paper shows this more concretely by re-deriving the argument of the present paper within an explicit search model. So, to the second question: search models did not find a kink in demand because most assumed search costs to be the same for all customers. When search costs are the same for all customers, firms face discontinuous demand curves. If instead search costs are both continuously distributed among customers and with a positive density at zero—as switching costs are in the framework of the present paper—then demand curves are kinked. In other words, the kink in demand in search models has been hidden by the discontinuity in demand arising from the particular assumption of homogeneous search costs.

Some papers have tackled the issue of providing microfoundations for the theory of the kinked-demand curve. However, they microfound different arguments for the kink. Maskin and Tirole (1988) analyze the Markov-perfect equilibrium of a dynamic price-competition model where firms take turns choosing prices. They show that some equilibria are reminiscent of the kinked-demand theory because a price decrease triggers a price war while competitors do not react to a price increase. As such, Maskin and Tirole microfound Hall and Hitch (1939)’s and Sweezy (1939)’s argument for a kink in demand. Besides, they do not address the question of the responsiveness of prices to changing costs: costs do not fluctuate in their model. Heidhues and Koszegi (2008) generate kinked demand curves by assuming that consumers are loss averse relative to some expectation they have formed on the price. In a recent paper, Ilut et al. (2015) propose a theory of price rigidity that arise through a kink in the perceived demand curve of firms: a combination of Knightian uncertainty and non-parametric learning generates a kink in the expected demand of a firm as the firm fears the worst if it changes its price. Yet demand is in reality smooth. In contrast, I show that demand is effectively kinked when some customers are asymmetrically informed on prices. Firms perceive demand to be kinked because it is.

Although the kinked-demand theory receded from monetary macroeconomics in the 1970’s, Kimball (1995) reintroduced a modified version in the 1990’s: quasi-kinked demand curves. Quasi-kinked demand curves—which Kimball postulates—capture the same general intuition as kinked-demand curves: firms lose more customers by raising their prices above the market price than they gain by lowering them below. But the kink of a quasi-kinked demand curve is smoothed-out. This matters: quasi-kinked demand curves do not produce price rigidity. Instead, Kimball shows that they amplify existing sources of price rigidity by increasing strategic complementarities. Guren (2016) finds evidence of quasi-kinked, concave, demand curves in the housing market, and introduces the assumption of concave demand curves within a search model of the housing market to explain house price momentum. Levin and Yun (2008) consider a search model

---

11 Guren’s results can be seen as empirical support for the kinked-demand theory of the present paper too. There is a qualification however: interpreted literally, the model of the present paper does not predict that actual price changes should be met with asymmetric responses of demand. Indeed, kinked demand curves remain an outside-equilibrium phenomenon: because demand curves are kinked if firms deviate, firms do not deviate, and in equilibrium—thus in the data—actual price increases and price decreases are met with the same changes in demand.
that produces quasi-kinked demand curves. The model of this paper produces an unadulterated kink, and therefore a basis for price rigidity.

Finally, other recent papers tackle the issue of price rigidity and consumers’ imperfect information through different mechanisms. [Matejka (2011)] considers the information choice of rationally-inattentive costumers and shows it can make fixed prices appealing to price-setters. The mechanism Matejka investigates relies on customers’ aversion to variations in their consumption of the numeraire. [L’Huillier (2015)] considers a monopolist facing consumers who are uninformed about the level of inflation. He shows that the monopolist can benefit from charging a fixed price in order not to tip off customers about the inflation rate. Because L’Huillier’s model (and Matejka’s) considers a monopoly, it does not address the mechanism that I stress: the nature of competition when customers are unequally informed on all competitors’ prices.

### 1.2 The extensive margin of demand

In this section, I introduce a general-equilibrium set-up that distinguishes between the intensive and extensive margins in a firm’s demand. I present the set-up absent any form of frictions—in particular all households and all firms have perfect information—and show that price flexibility and monetary neutrality hold in this case. This establishes the benchmark to which subsequent sections are compared.

The economy engages in the production of the continuum of goods $i \in [0, 1]$ from an homogeneous labor input, each consumed by a continuum of households $j \in [0, 1]$. Good $i$ is produced in sector $i$, by any of a continuum of firms $k \in [0, 1]$—each firm is designated by the double index $i, k$. Firms are price-setters; the labor market is competitive. Although customers are perfectly indifferent between all goods sold in market $i$ once acquired, competition in market $i$ is hindered by the existence of switching costs: each customer $j$ is initially attached to one firm, knows of one firm he can switch to at a cost, and cannot shop at any other firm.

#### 1.2.1 Households

All households $j$ have the same intertemporal preferences over a continuum of goods $i \in [0, 1]$ at each period, and labor at each period:

$$
E_0 \sum_{t=0}^{\infty} \beta^t (\log(C_{jt}^i) - L_{jt}^i),
$$

where $L_{jt}^i$ is hours worked and $C_{jt}^i$ is total consumption defined as the CES aggregator of the consumption of all goods $i$:

$$
C_{jt}^i = \left( \int_i (C_{jt,i})^{\frac{\theta - 1}{\theta - 1}} di \right)^{\frac{\theta}{\theta - 1}}.
$$


Household $j$ starts a period $t$ with nominal wealth $B_t$ and has access to complete financial markets, where $Q_{t,t+1}$ is the unique nominal stochastic discount factor. On each market $i$ customer $j$ is randomly assigned at the beginning of each period $t$ to one firm $A_{i,j}$, to which he has free access. (Every day, he goes shopping and randomly bumps first into one shop.) In addition, customer $j$ randomly draws a second firm $B_{i,j}$, to which he can switch only by incurring a cost. (He notices another shop, a few blocks down the street.) Finally, customer $j$ has no access to other firms $k$ on market $i$. His shopping decision on market $i$ therefore reduces to whether to stay at $A_{i,j}$, or switch to $B_{i,j}$. The switching cost is expressed as a fraction of total consumption, and is proportional to the size of market $di$: it is $\gamma_i^j di C_{j,t}$, where the proportionality factor $\gamma_i^j \geq 0$ is specific to both the customer and the market.

Once he has decided where to shop on all markets, household $j$ faces a standard consumption problem taking as given prices $(P_{j,t,i})_i$, where on a given market $i$, $P_{j,t,i}$ is either $P_{A_{i,j}}^t$ or $P_{B_{i,j}}^t$ depending on his switching decision. His flow budget constraint is then:

$$\int P_{j,t,i}^t C_{j,t,i}^t di + E_t(Q_{t,t+1}B_{j,t+1}^t) = W_t L_t + \Pi_t + B_t^j = I_t^j,$$  \hspace{1cm} (1.3)

where $W_t$ is the nominal wage, $\Pi_t$ nominal profits coming from the ownership of firms—I assume the ownership of all firms is equally divided between all households—and $I_t^j$ denotes total nominal incomes. (I assume that switching costs, although expressed in consumption equivalents, are incurred in effort and therefore do not show up in the budget constraint). In addition, a terminal constraint forbids household $j$ to enter Ponzi-schemes.

Solve household $j$’s problem backward. After deciding where to shop, $j$ consumes and works (and invests, but I won’t use the optimal portfolio decisions) to maximize utility. The consumption/leisure trade-off sets:

$$C_{j,t}^i = \frac{W_t}{P_t^i},$$ \hspace{1cm} (1.4)

and demand for the individual good $i$ takes the form:

$$C_{j,t,i}^i = \left(\frac{P_{j,t,i}^i}{P_t^i}\right)^{-\theta} C_{j,t}^i,$$ \hspace{1cm} (1.5)

where $P_t^i$ is the subjective price index of customer $j$:

$$P_t^i = \left(\int (P_t^{j,i})^{1-\theta} di\right)^{1/(1-\theta)}.$$ \hspace{1cm} (1.6)

1.2.2 Household’s shopping decisions

Consider now the shopping problem of household $j$ in market $i$. Because he is randomly reassigned to two new firms $A_i$ and $B_i$ next period, $j$ faces only present-period benefits of switching from firm $A_i$ to firm $B_i$. 

If he shops at firm $A_i$, customer $j$ gets indirect utility $C^j(P_i^A)$—total consumption when the price of good $i$ is $P_i^A$; if he shops at firm $B_i$, he gets indirect utility $(1 - \gamma_i^j di)C^j(P_i^B)$—the fraction $1 - \gamma_i^j di$ of total consumption when the price of good $i$ is $P_i^B$. He therefore switches to firm $B_i$ if and only if:

$$
\frac{C^j(P_i^A)}{C^j(P_i^B)} < 1 - \gamma_i^j di
$$

(1.7)

I take a log-linear approximation to indirect utility. Since $C^j = I_j^j P_i^j$, at nominal income and all other prices fixed $d\ln(C^j) = -d\ln(P_i^j) di$, which lets condition (3.20) be approximated as:

$$
\left(\frac{P_i^A}{P_i^B}\right)^{-di} < 1 - \gamma_i^j di.
$$

(1.8)

(The expression is exact in the Cobb-Douglas case $\theta = 1$.) Since a market is infinitesimally small $di \to 0$, this is equivalent to:

$$
\frac{P_i^A}{P_i^B} > e^{\gamma_i^j}.
$$

(1.9)

In words: customer $j$ switches to firm $B_i$ when $A_i$’s price is more than $(e^{\gamma_i^j} - 1)\%$ above $B_i$’s price.

### 1.2.3 Firm’s market share and firm’s demand

Look now at market $i$ from firm $k$’s perspective. A fraction $dk$ of customers have been randomly assigned to firm $k$ as their primary $A$-firm, and another fraction $dk$ have been assigned to firm $k$ as their secondary $B$-firm. I consider symmetric equilibria in market $i$ where all firms charge the same price $P_i$ (so that there will be ultimately no switching in equilibrium). If firm $k$ charges the same price $P_i$ as all other firms, it maintains its initial market share $dk$. By charging a lower price, $k$ can increase its market share up to $2dk$ by attracting the customers who are not at $k$ but can switch to it. In contrast, a higher price triggers the departures of some of $k$’s own customers toward their secondary $B$-firm, and decreases $k$’s market share.

This exact way in which firm $k$’s market share responds to the price it charges is determined by the distribution of switching costs $\gamma_i^j \in \mathbb{R}_+$ across customers, or equivalently the distribution of $e^{\gamma_i^j} \in [1, +\infty)$. I note $F$ the CDF of the distribution of $e^{\gamma_i^j}$ (the same in all subgroups of customers defined by their primary firm), and $p^k = P^k / P_i$ the price of firm $k$ relative to the market price. If $k$ charges a higher price $p^k \geq 1$, then $k$ will retain only its customers with a large enough switching cost $e^{\gamma_i^j} \geq p^k$: a fraction $1 - F(p^k)$. Besides, $k$ will attract no customer from other firms. Instead, if $k$ charges a lower price $p^k$, then $k$ will retain all its customers, and will attract those of other firms’ customers with switching costs small enough $e^{\gamma_i^j} \leq 1/p^k$: a

---

12Some details: $(1 - \gamma di)^{-1} = e^{\ln(1-\gamma di)(-\frac{1}{\gamma di})} \to e^\gamma$.
fraction $F(1/p^k)$. The market share of firm $k$ is therefore $\nu dk$, where $\nu$ is the market share function\(^\text{13}\)
\[
\nu (p^k) \equiv \begin{cases} 
1 - F(p^k) & \text{if } p^k \geq 1, \\
1 + F\left(1/p^k\right) & \text{if } p^k \leq 1.
\end{cases}
\] (1.10)

I assume $F$ is a continuous and differentiable distribution. This guarantees that the market share function $\nu$ is continuous and differentiable everywhere. In particular, it is differentiable at the relative price $p^k = 1$, which is where the elasticity of the market share function matters in a symmetric equilibrium. I note $\alpha$ (minus) this elasticity:
\[
\alpha \equiv -\varepsilon_\nu(1) = F'(1).
\] (1.11)

As Klemperer (1987) notes, the elasticity of the market share is driven by the density of customers with no switching costs: these are the marginal customers, who arbitrage any price discrepancy between firms.

The market-share function measures whether a customer buys or not at firm $k$: the extensive margin of demand. On top of it, a customer who buys at $k$ buys more or less depending on $k$’s price: the intensive margin of demand. This last margin is the standard one incorporated in models of monopolistic competition and is given by equation (1.5): the constant-elasticity individual demand-curve of CES preferences. As will be shown below, all customers will have the same total consumption $C$ and subjective price index $P$ in equilibrium. Firm $k$’s demand is therefore $D^kCdk$, where:
\[
D^k\left(\frac{P^k}{P_i}, P\right) = \nu \left(\frac{P^k}{P_i}\right)^{-\theta}.
\] (1.12)

This demand curve inherits the continuity and differentiability of the market-share curve: the extensive margin of demand does not in itself generate any kink in firms’ demand curves.

With a fully unrestricted distribution little restricts firm $k$’s demand curve, and thus nothing prevents $k$’s profit function to be multi-modal. To avoid this peripheral issue and have the local optimality conditions be global, I restrict to the assumption of a Pareto distribution for $F$ (equivalently an exponential distribution for the switching costs $\gamma_i^j$):
\[
F(p) = 1 - p^{-\alpha},
\] (1.13)

whose single parameter $\alpha > 0$ is the elasticity of the resulting market-share function at $p = 1$. The assumption of a Pareto distribution makes the market-share function constant-elastic over $p \geq 1$, and is thus the analog for the extensive margin of demand of the assumption of a CES aggregator for the intensive margin of demand.

\(^{13}\)I call $\nu$—and later $s$—the market share although it can be greater than one: it lies between 0 and 2. There is no contradiction: what should be rigorously referred to as the market share is $\nu \times dk < 1$—and later on $s \times dk < 1$. I proceed with the slight abuse of language.
demand. Figure 1.1 depicts the market share function in the case of a Pareto distribution.

1.2.4 Firm’s pricing

Let us turn to firm $k$’s pricing decision. Firm $k$, as all other firms in sector $i$, uses the constant returns to scale production function in labor $Y = A_{t,i}L$, where $A_{t,i}$ is the state of technology in sector $i$ at period $t$—I allow for heterogeneity in technology across sectors, but assume homogeneity within sector $i$. Because firm $k$ gets to reset its price every period, and because customers are randomly reassigned every period so that firm $k$’s price today does not affect its market share tomorrow, firm $k$ sets its price $P^k$ to maximize present profits

$$\pi^k = (P^k - \frac{W}{A})D^kCdk,$$

taking the nominal wage $W$ as given.

Because $k$’s demand curve is differentiable, $k$’s optimal price needs to satisfy the first-order condition. From this local optimality condition, the only possible symmetric equilibrium is thus for all firms to charge:

$$P^k = P_i = M(\alpha + \theta)\frac{W}{A_i},$$

where $M(\varepsilon) \equiv \frac{\varepsilon}{\varepsilon - 1}$ is the Lerner markup function,

which is well-defined provided $\varepsilon = \theta + \alpha > 1$. The appendix checks that under the assumption of a Pareto distribution, firms’ profit functions are single-peaked, and thus that this is indeed an equilibrium.
The market stays equally divided between all firms, and there is no switching in equilibrium. The markup is determined by the total elasticity of demand \( \varepsilon = \alpha + \theta \), which is the sum of the elasticities of demand at the intensive and extensive margins. The pricing decision is exactly identical to the pricing decision in the representative-agent benchmark of consumer demand, except the elasticity is now the sum of the elasticities at the intensive and extensive margins.\(^{14}\)

### 1.2.5 Monetary neutrality

Because the pricing decision is the same as in the benchmark, monetary neutrality obtains as in the benchmark. Since on all markets all firms set the same price, all customers face the same prices on all markets and there is no heterogeneity among households in equilibrium, as claimed earlier. The common subjective price level (1.6) is given by:

\[
P = \frac{M(\alpha + \theta)W}{A}, \tag{1.15}
\]

where:

\[
A \equiv \left( \int A_i^{\theta-1} dG(A_i) \right)^{\frac{1}{\theta-1}}, \tag{1.16}
\]

where \( G \) is the cross-sectional distribution of productivity across markets, and \( A \) is a measure of aggregate productivity. Equation (1.15) sets the real wage \( W/P \) to \( A/M(\alpha + \theta) \). The real wage is in turn equal to the marginal rate of substitution between labor and consumption, which is given by equation (1.4) to be equal to consumption. The equilibrium is independent of monetary factors.

**Proposition 1** Under perfect information—on both firms’ and customers’ sides, the equilibrium is unique and money is neutral. Regardless of monetary factors, there exists a unique equilibrium for real allocations, with output equal to:

\[
C = \frac{A}{M(\alpha + \theta)}. \tag{1.17}
\]

The classical dichotomy holds. The introduction of switching costs and of an extensive margin of demand does not challenge the standard result that imperfect competition per se does not provide a rationale for price rigidity.

\(^{14}\)The property of the benchmark of monopolistic competition and CES demand that prices are a constant markup over costs only carries over here under the assumption that firms are homogeneous within a market. The symmetry between firms prevents them to face the bottom part \( P_b < P_i \) of their demand curves in equilibrium. I do not rely on this non constant-elastic component of the demand curve here. Yet, when firms differ in their costs of productions, the model gives further insight into the mechanisms of competition at the extensive margin: while higher-cost firms price a constant markup over their marginal cost, lower-cost firms set their price closer to the one of higher-cost firms.
1.3 Price indeterminacy

I now relax a single assumption of the previous benchmark: that all customers know both the price at the A-firm they are at, and the price at the B-firm they can switch to. Instead, I assume that although all customers in market \( i \) perfectly observe the price \( P^A_i \) at the firm A they are attached to, only a fraction \( 1 - \lambda \) of customers also have full information on the price at their B-firm—I call them informed customers. The remaining fraction \( \lambda \)—that I will refer to simply as uninformed—have asymmetric information on the two prices. An uninformed customer \( j \) observes its B-firm’s price only once he switches to \( B_i \), if he does.

Before observing \( B_i \)’s price and deciding whether to switch to \( B_i \), customer \( j \) forms rational expectations on \( P^B_i \). This requires assumptions on customer \( j \)’s information set. I assume that although \( j \) does not observe \( P^B_i \), he knows the distribution of prices in market \( i \). This assumption captures the idea that although customers do not know the individual price charged at each firm, they have a (rationally expected) sense of the prices they can expect to find if they look for a better bargain—as in the search literature.\(^{15}\)

Because switching may therefore reveal information that may alter the optimal shopping decision, I also need to make assumptions on whether customer \( j \) can return to his A-firm after he switched to his B-firm. I assume that return is possible, at the same cost \( \gamma^j_i \) as the outward trip. Because there will be no switching in equilibrium, the assumption on the cost of return is not essential however.

This form of imperfect information is not an assumption of money illusion on the part of customers. All customers have perfect information on the distribution of prices in all markets, and therefore perfect information on the price level. Their uncertainty only bears on the prices of individual firms within a market. I am moving the information problem from firms to customers, but this does not mean I am simply moving money illusion from firms to customers.

1.3.1 Household’s shopping decision

Once a household has decided where to shop, his consumption, investment and labor-supply decisions are still given by equations (1.4), (1.5), and (1.6). Move to the shopping decision of household \( j \) in market \( i \), under the new assumption on information. If \( j \) is informed in market \( i \), he faces the same problem as before, and his switching decision between his A and B firms is still given by (1.9). In contrast, if household \( j \) is uninformed he observes \( P^A_i \) but not \( P^B_i \), and the best he can do is to rationally expect \( P^B_i \) based on the distribution of prices in equilibrium. I still consider symmetric equilibria where all firms in market \( i \) charge the same price \( P_i \). The distribution of prices in the market is therefore a Dirac at \( P_i \), and household \( j \) expect \( P^B_i \) to be \( P_i \). Thus, household \( j \) switches to \( B \) if:

\[
\frac{P^A_i}{P_i} > e^{\gamma^j_i}.
\]

\(^{15}\)This assumption circumvents the issue of customers’ inference, from their primary firm’s price, of the distribution of prices in the market. For insights into this inference problem, see the Bayesian search literature, for instance Benabou and Gertner (1993), Dana (1994), Fishman (1996).

13
If he did switch to $B_i$ and it turns out that $B_i$’s price is higher than $P_i$—something that was unanticipated given the equilibrium distribution of prices in the market—customer $j$ may still switch back to its primary $A$-firm charging $P_i^A$. This choice is made under full information, and therefore customer $j$ switches back if:

$$\frac{P_i^B}{P_i^A} > e^{\gamma_j}.$$  

(1.19)

### 1.3.2 Firm’s market share and firm’s demand

Consider now how the asymmetry in customers’ information on prices affects firm $k$’s market-share curve. Still note $p^k = P^k / P_i$ the price of firm $k$ relative to the market’s price $P_i$. The fraction $1 - \lambda$ of informed customers—both those assigned to $k$ as their primary $A$-firm and those assigned to $k$ as their secondary $B$-firm—behave in the same way as in the previous section. Firm $k$’s market share coming from informed customers is therefore $(1 - \lambda)\nu(p^k)dk$. What differs is the market share coming from uninformed customers. Nothing changes for customers attached to $k$ as their primary $A$-firm. If $k$ charges a price lower or equal to the market price $p^k \leq 1$, all stay; if $k$ charges a higher price $p^k \geq 1$, all notice and those with a low enough switching cost:

$$p^k \geq e^{\gamma_j}$$  

(1.20)

leave to a competitor. Because all competitors indeed charge $P_i$, none of those who left return.

Things change for customers attached to $k$ as their secondary $B$-firm. Such a customer $j$ does not observe $P^k$ and bases its switching decision not on $P^k$, but on $P_i$, the average price in the market. This is crucial as the market price $P_i$ is something firm $k$ cannot affect. Because customer $j$ therefore expects no price gap between his $A$-firm and his $B$-firm $k$, he does not switch to $k$, regardless of the value of its switching cost. Firm $k$’s price decreases never attract uninformed customers from its competitors, as uninformed customers do not even notice that $k$ decreased its price. Put otherwise, firm $k$’s problem is that it cannot advertise a price decrease to uninformed customers. Its market share coming from uninformed customers is $\lambda s^u(p^k)dk$, where:

$$s^u(p^k) \equiv \begin{cases} 1 - F(p^k) & \text{if } p^k \geq 1, \\ 1 & \text{if } p^k \leq 1. \end{cases}$$  

(1.21)

In words, the market share function among uninformed is the same as among informed for $p^k \geq 1$, but is inelastic for $p^k \leq 1$. Because a price increase is a signal that all uninformed shopping at firm $k$ hear, while there is no uninformed to hear the signal of a price decrease beyond the walls of $k$, price increases above and price decreases below the market price $P_i$ have a sharply asymmetric effect on uninformed customers. Asymmetric information translates into asymmetric customer flows.
Firm $k$’s total market share $s(p^k)dk$ is the sum of its market share among informed and uninformed:

$$s(p^k) \equiv (1-\lambda)\nu(p^k) + \lambda s^u(p^k).$$  \hfill (1.22)

Contrary to the market share among uninformed, the total market share is not inelastic for $p^k \leq 1$: informed customers at other firms do notice $k$’s price decrease and some do switch to $k$. But uninformed customers do mute—if not cancel—the elasticity of the total market share for $p^k \leq 1$. The robust manifestation of the asymmetric customer flows in the market share functions is a kink: provided the density of customers with no switching costs $\alpha = F'(1)$ is positive—provided demand is elastic at the extensive margin—the total market share function is kinked at $p^k = 1$, where $k$ charges the same price as other firms. Figure 1.2 depicts the market-share function among uninformed $s^u$ (left panel) and the total market-share function $s$ (right panel) in the case of a Pareto-distribution for $F$. 

Figure 1.2: Firm $k$’s market share as a function of its relative price $p^k = P^k/P_i$, for a Pareto distribution. Calibration: $\alpha = 6, \lambda = 1/2$. The left panel is the market share among uninformed customers, and the right panel the market share among all customers. In both cases, the market share function is kinked at 1—where $k$ charges the same price as other firms.
The intensive margin of demand still adds to the extensive margin of demand embedded in the market share function. Again, all customers still have the same total consumption $C$ and subjective price index $P$ in equilibrium. Firm $B$’s demand is therefore $D^k C_{dk}$, where:

$$D^k \left( \frac{P^k}{P_i}, \frac{P^k}{P_i} \right) = s \left( \frac{P^k}{P_i} \right) \left( \frac{P^k}{P_i} \right)^{-\theta}.$$ (1.23)

This demand curve inherits the kink of the market-share function. The model microfound old informal arguments for the kinked-demand curve (Sweezy (1939); Hall and Hitch (1939)), especially those relating the existence of the kink to information asymmetries (Scitovsky (1978); Stiglitz (1979); Woglom (1982)).

In his seminal article, Sweezy (1939) introduces the kinked-demand curve by noting that “businessmen frequently explain that they would lose their customers by raising prices but would sell very little more by lowering prices. Economists who are accustomed to thinking in terms of traditional demand-curve analysis are likely to attribute this kind of answer to ignorance or perversity.” We can, instead, attribute it to customers’ asymmetric information:

**Lemma 1** If:

1. The density of customers with no switching costs $\alpha = F’(1)$ is positive,
2. Some customers have asymmetric information on prices,

then firms’ demand curves are kinked.

Note that this demand curve captures what happens if firm $k$ deviates from the market price $P_i$: it needs not correspond to the effective change in demand that occurs in equilibrium in response to an effective change in price. And it will not: as under full information, there will be no switching in equilibrium, and the kinked-demand curve will remain an outside-equilibrium phenomenon. Taken at face value, the model does not predict that effective price decreases—the ones an econometrician would observe in the data—should have a smaller effect on demand than effective price increases. Yet the kinked-demand curve has very real implications for pricing and equilibrium—to which I now move.

### 1.3.3 Firm’s pricing

Apart from the new demand function, firm’s $k$ pricing problem is unchanged: to maximize present-period profits $\pi^k(P^k) = (P^k - \frac{W}{A}) D^k(P^k)C_{dk}$. This profit function inherits the kink of the demand function: it is differentiable everywhere but at $P^k = P_i$, where its left and right derivatives differ. Since I consider symmetric equilibria where firm $k$ ends up charging the market price $P_i$, the kink in profits at $P^k = P_i$ is located precisely at the point where it matters. Formally, with a kink at $P^k = P_i$ the requirement that $P^k = P_i$ maximizes firm $k$’s profits does not imply that the first-derivative of profits cancels at $P^k = P_i$. Instead the necessary first-order condition takes the weaker form that the left-derivative of profits needs to
be weakly positive, and the right-derivative of profits needs weakly negative:
\[
\begin{align*}
\frac{\partial \pi^k}{\partial P^k}(P_i) &\leq 0, \\
\frac{\partial \pi^k}{\partial P^k}(-P_i) &\geq 0.
\end{align*}
\]
(1.24)

This is equivalent to:
\[
\begin{align*}
P_i &\geq \mathcal{M}\left(\varepsilon_+(1) + \theta\right)\frac{W}{A_i}, \\
P_i &\leq \mathcal{M}\left(\varepsilon_-(1) + \theta\right)\frac{W}{A_i},
\end{align*}
\]
(1.25)

where \(\varepsilon_+(1)\) and \(\varepsilon_-(1)\) are (minus) the right and left elasticities of the market-share function [1.22] at \(p^k = 1\), and \(\mathcal{M}\) is still the Lerner markup function. Because the right and left elasticities are equal to:
\[
\begin{align*}
\varepsilon_+(1) &= \alpha, \\
\varepsilon_-(1) &= \alpha(1 - \lambda),
\end{align*}
\]
(1.26)

any price between \(\mathcal{M}(\alpha + \theta)W/A_i\) and \(\mathcal{M}((1 - \lambda)\alpha + \theta)W/A_i\) is a possible equilibrium. The appendix checks that, under the assumption of a Pareto distribution, profit functions are single-peaked so that all these prices are indeed equilibrium prices.

**Lemma 2** When customers have asymmetric information on prices, there is a continuum of equilibrium prices in market \(i\). For a given nominal wage \(W\), any price \(P_i\) between:
\[
P_i \in \left[\mathcal{M}(\alpha + \theta)\frac{W}{A_i}, \mathcal{M}((1 - \lambda)\alpha + \theta)\frac{W}{A_i}\right]
\]
(1.27)
defines a (partial) equilibrium in market \(i\).

As in [Woglom (1982)], there exists a continuum of equilibrium prices. These equilibrium prices are bounded below by the full-information price \(P_i = \mathcal{M}(\alpha + \theta)\frac{W}{A_i}\). All higher prices up to \(P_i = \mathcal{M}(\alpha(1 - \lambda) + \theta)\frac{W}{A_i}\), which were ruled out as equilibrium prices under full information, are now sustainable. The competitive force that incentivized firms to decrease their prices to attract new customers under full information is muted. Customers’ asymmetric information goes in the direction of more (downward) inelastic demands, higher markups, and higher prices.

### 1.3.4 Equilibria

To get to general equilibrium, aggregate across markets. All firms within a market still charge the same price \(P_i\); all households still face the same prices and are therefore identical in equilibrium. They have the same subjective price level [1.6], which given the indeterminacy in sectoral prices, is indeterminate. They
have the same consumption $1.4$, which given the indeterminacy in the price level, is indeterminate. In
other words, the price indeterminacy in partial equilibrium translates into a real indeterminacy in general
equilibrium.

Proposition 2 When customers have asymmetric information on prices, there is a continuum of general
equilibria. Given nominal wages $W$, the price level can take any value between:

$$P \in \left[ \frac{M(\alpha + \theta)}{A}, \frac{M((1 - \lambda)\alpha + \theta)}{A} \right],$$

and output $C$ can take any value between:

$$C \in \left[ \frac{A}{\frac{M((1 - \lambda)\alpha + \theta)}{A}}, \frac{A}{M(\alpha + \theta)} \right].$$

Yet, the qualitative effect of customers’ imperfect information on output is unambiguous: because prices are
necessarily above their full-information level, consumption is necessarily below its full-information level.

1.4 Price rigidity

In the previous section, I proposed a microfoundation for old informal arguments for the existence of a
kink in firms’ demand curves. These informal arguments relied on such a kink to justify price rigidity. So
far however, the microfounded kinked-demand curve of the model provides no rationale for price rigidity:
instead, it is a theory of price indeterminacy. In this section, I discuss the source of the multiplicity, propose
an equilibrium selection criterion—adaptive rational expectations—to select a unique equilibrium, and show
that the selected equilibrium features sticky prices. Last, I discuss the new view the model—including its
equilibrium selection criterion—takes on the widespread absence of indexation.

1.4.1 Price multiplicity vs. price rigidity

Some equilibria of the model do feature price rigidity: since a given price $P_i$ in market $i$ is an equilibrium
price over a range of values for nominal cost:

$$\frac{W}{A_i} \in \left[ \frac{P_i}{M((1 - \lambda)\alpha + \theta)}, \frac{P_i}{M(\alpha + \theta)} \right],$$

an equilibrium where $P_i$ stays constant over several periods is possible as long as fluctuations in $W/A_i$ remain
contained within the interval $1.30$ over the length of time the market price stays constant to $P_i$. Nothing in
the model so far however goes in the direction of selecting such equilibria more than any other, nor of being
more predictive as to what value would such sticky prices $P_i$ take. Full price-flexibility equilibria—equilibria
where the market price $P_i$ changes every period—exist just as well: even more so than under full information
since even for a constant level of nominal costs $W/A_i$ the market could coordinate on different prices in different periods.

The large multiplicity of equilibria that the model predicts can be traced to the few constraints that the model puts on the location of the kink in the demand curve. (Justifying the location of the kink has always been the Achilles’ heel of informal arguments for the kinked-demand curve.) Rationalizing price rigidity—prices that stay constant over several periods despite variations in costs—requires the kink to be located at the value of the price previously charged by the firm. In the present microfounded model, the kink is located at the market price this period. This present-period market price is little constrained in the previous section: it can take any value within the interval $[1.27]$. In particular it is not constrained to bear any resemblance to previous values of the market price.

This does not write off the kinked demand curve theory as a theory of price-rigidity however. One of the most relied-upon theories of price-rigidity—menu-cost models—typically feature multiplicity too, and for quite the same reasons. As first pointed out by Ball and Romer [1991] for menu-cost models, strategic complementarities are to blame: if all price-setters suddenly decide to switch to a new common price, it can well be in every individual price-setter’s interest to follow the herd[16] The extent of coordination necessary to move the market price to a different value each period, even when the previous period’s price does constitute an equilibrium, seems unlikely to occur in practice however. I propose an equilibrium selection criterion that rules out these unintuitive equilibria. The criterion can be used to select an equilibrium in menu-cost models too.

1.4.2 The adaptive rational-expectations criterion

To specify the transition from an equilibrium at period $t – 1$ to the new equilibrium at period $t$, I consider the following criterion. At the beginning of every period—every morning before he opens his store—a storekeeper goes through the following mental process. First, he backward-lookingly assumes that the market-price today is going to be the same as yesterday[17] He then reasons what price he and all his competitors would set in response. If the answer happens to be the conjectured past price, the storekeeper has found a (rational-expectations) equilibrium price and his mental process stops. He keeps his price unchanged. If the best-response price happens to differ from the initial guess, then the storekeeper repeats his reasoning with the new guessed price, and iterates until the process converges to a (rational-expectations) equilibrium price. He posts the price his mental process converged to.

[16]Although strategic complementarities drive multiplicity in the present model too, the appropriate measure of complementarity is no longer the slope of the notional short-run aggregate supply (SRAS). Indeed, the notional SRAS in (this version of) the model has slope 1, which corresponds to no strategic complementarity in the benchmark model of price-setting: under full-information, a firm’s desired price is independent of other firms’ prices—equation (1.14). The kinked demand curve strengthens strategic complementarity in a new way. Kimball [1995] shows how the strategic complementarities created by a quasi-kinked demand curve (which he postulates) can be measured through the slope of the SRAR. Because demand curves are truly kinked here, the new strategic complementarities can less easily be captured by a revised measure of the elasticity of the SRAS. Caballero and Engel [1993] show that the extent of multiplicity in a menu-cost model also depends on the dispersion of prices in the market. In the present model, all firms within a market set the same price: price-setting is synchronized.

[17]The equilibrium-selection criterion needs only describe how price-setters form expectations on the price of competitors in the same market, not on the price level: price-setters’ optimal price is independent of the price level.
Through the choice of the first guess, the convergence process that selects an equilibrium is backward-looking. Yet, expectations are not: the selected equilibrium is a rational-expectations equilibrium, not an adaptive-expectations equilibrium. I call the criterion the adaptive rational-expectations criterion, and the resulting equilibrium the adaptive rational-expectations equilibrium.

Adaptive rational expectations capture firms’ reluctance to make the first step—to be the first firm in the market to adjust its price. In the coordination problem faced by firms within a market, the adaptive rational-expectations criterion will by design select an equilibrium where a firm changes its price only when it has an incentive to do so even if the price change is unilateral—when the incentive to change it does not rely on the expectation that competitors will do the same. Essential in modeling the reluctance to make the first step is the backward-looking initial guess of the adaptive rational-expectations mental algorithm: this first guess captures the role of the status quo as a point of coordination.

The reluctance to make the first step that adaptive rational-expectations capture ranks very high among the reasons why firms declare not changing their prices in survey evidence. In Blinder et al. (1998)’s survey for instance, the idea that “[firms] do not want to be the first ones to raise prices, but, when competing goods rise in price, firms raise their own price promptly” is the single most popular theory among the twelve tested in the survey. If Blinder refers to the reluctance to make the first step as coordination failure, and although this reluctance definitely requires the existence of strategic complementarities, it matters to notice that the question points at a more precise notion than what the concept of strategic complementarities often embraces in the theoretical literature. Strategic complementarities are a very important way through which the effects of price rigidity get amplified in many theories of sticky prices. Yet, in standard models strategic complementarities do not act by creating a reluctance to be the first one to change prices, which is what the Blinder survey—and others—find strong support for. Instead, they act by having all firms make steps—even if they are alone to move—but steps in the direction of others. In contrast, when defined to include the adaptive rational-expectations criterion, the kinked-demand theory is a theory of not wanting to go first.

Adaptive rational expectations can be thought of as modeling the convergence to equilibrium, although it locates convergence not in actual time but in mental (or notional) time. As a mental-time model of the convergence to equilibrium, adaptive rational-expectations connect to models of eductive—as opposed to evolutive—learning, such as proposed by Guesnerie (1992), or, in game-theoretic vocabulary, to rationalizable-expectations equilibria. The difference is that rationalizable expectations enlarge instead of restrict the set of equilibria: there are always more—not fewer—rationalizable-expectations equilibria than rational-expectations equilibria. Adaptive rational-expectations become an equilibrium-selection criterion through the role they assign to the first guess. While rationalizable expectations start from the whole set of possible strategies and iteratively eliminate strategies that are not best responses, adaptive rational-

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18 The ranking applies to the closed-ended questions evaluating how important respondents find each of the twelve theories in explaining why they do not change prices. It is in contrast to the open-ended question that I mentioned in the introduction.

19 The distinction between eductive and evolutive approaches to equilibrium is due to Binmore (1987). Eductive agents get to equilibrium by reasoning—they are theorists—while evolutive agents get to equilibrium by noticing patterns in the past plays of the game—they are empiricists. Rationalizable solutions have been introduced by Bernheim (1984) and Pearce (1984) as a solution concept weaker than Nash equilibrium.
expectations give a central role to the starting value of the iteration process. Adaptive rational expectations (and thus eductive learning) also closely relate to the concepts of calculation equilibrium proposed by Evans and Ramey (1992), and of reflective equilibrium proposed by Garcia-Schmidt and Woodford (2015) to study price-level determination under interest-rate rules. In contrast, calculation equilibria and reflective equilibria consider a finite number of iterations in the mental process of agents, not the limit as the process converges, and thus constitute a departure from rational expectations.

1.4.3 Pricing function

How do firms set their prices in a given market in the adaptive rational-expectations equilibrium? The appendix proves the following characterization of the (partial) equilibrium.

Lemma 3 There exists a unique adaptive rational-expectations equilibrium in market \( i \). In it, firms vary their prices according to the single state variable \( X_{t,i} \equiv \frac{W_t}{A_t,i} P_{t-1,i} \equiv \frac{\text{nominal costs}}{\text{past price}} \). The pricing function \( q \) gives the change in price as a function of the state:

\[
\frac{P_{t,i}}{P_{t-1,i}} = q(X_{t,i}) = \begin{cases} 
M\left(\theta + \alpha(1-\lambda)\right) X_{t,i} & \text{if } X_{t,i} \leq \frac{1}{M(\theta + \alpha(1-\lambda))} \\
1 & \text{if } X_{t,i} \in \left[\frac{1}{M(\alpha(1-\lambda)+\theta)}, \frac{1}{M(\alpha+\theta)}\right] \\
M\left(\theta + \alpha\right) X_{t,i} & \text{if } X_{t,i} \geq \frac{1}{M(\theta+\alpha)}.
\end{cases}
\] (1.31)

The left panel of figure 1.3 gives a graphical illustration of the pricing function \( q \) (and its right panel gives an alternative illustration through the lens of the markup function \( P_{t,i}/A_t = q(X_i)/X_i \)). As in menu-cost models—but not because of any cost to change prices—the pricing function features an inaction region: a firm keeps its price constant (thus varies its markup) as long as variations in costs remain within an interval. As in menu-cost models, prices are sticky:

Proposition 3 The adaptive rational-expectations equilibrium of the model under customers’ asymmetric information features sticky prices: prices can stay constant for several periods despite changes in costs.

The kinked-demand theory thus joins menu-cost models in passing a simple test, which models of producer’s imperfect information fail: prices respond infrequently, not just incompletely, to changes in costs.

The pricing function also tells what happens to prices when they do change. For price increases, firms charge the minimal markup \( M(\theta + \alpha) \) over nominal costs; for price decreases, they charge the maximal markup \( M(\theta + \alpha(1-\lambda)) \). In both cases, prices change by the minimal amount necessary to remain an equilibrium price.

1.4.4 Real vs. nominal rigidity

The adaptive rational-expectations criterion turns a real indeterminacy into a nominal rigidity. How it achieves this is transparent: the selection criterion assigns a role to past prices in determining the equilibrium,
linking periods together and breaking the classical irrelevance of the unit of account. Yet, it begs the question: why past prices? I already argued that past variables are a natural focal point for coordination, and that, when a backward-looking reference point does not conflict with individual rationality or rational expectations, equilibria that are backward-looking are more realistic. But the question has a second component: among all past variables, why the nominal price?

Behind this question looms the issue of indexation: why don’t firms mechanically index their prices on some (likely lagged) price index, keeping the price fixed in terms of this index instead? The problem of indexation is a long discussed issue in theories of price rigidity. As noted by McCallum (1986), most theories of price rigidity have difficulty explaining the absence of indexation, as they often only rationalize a real, not a nominal, rigidity.[20] Responding to the objection, McCallum argues the rigidity applies to nominal prices because it is precisely part of the convenience of a unit of account to be a familiar reference point.

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[20] Menu-cost models taken at face value account for the absence of indexation since the menu cost bears on changing nominal prices. However, unless one commits to a literal interpretation of menu costs, this explanation of the absence of indexation only begs the question: what is so difficult—so costly—in indexing prices?
The present model adds a new dimension to McCallum’s explanation of the absence of indexation: it is not only a lack of convenience, but also a lack of coordination that discourages indexation. A firm does not find it optimal to index its prices if it is the only one to do so. Were all firms to coordinate on a common index, then indexation would be individually optimal. Firms in the U.S. could for instance coordinate on keeping their prices constant in euros, indexing their prices on the euro/dollar exchange rate to have their dollar-denominated prices adjust accordingly. Yet, they instead coordinate on the prices in dollars because, labeled on all their goods, they are the ubiquitous public signals. By essence, the unit of account is the natural coordination device. It defines the status quo on which firms coordinate.

1.5 Microeconomic predictions

Both the kinked-demand theory and menu-cost models predict infrequent price adjustments. Does the kinked-demand theory microfound menu-cost models? Indeed, a common agnostic view on menu costs is to take them as an “as if” assumption. Instead of the literal physical cost of printing a new menu—an interpretation that the ubiquitousness of temporary sales in some sectors makes hard to defend—menu costs should be seen as a reduced-form way to capture the true underlying motive for price rigidity, among which the adverse reaction of customers to price changes ranks high.21

I show that the two theories are not equivalent, and emphasize two predictions that allow to test the kinked-demand theory against a menu-cost model. Prices should be more likely to change if they have recently changed; markets where customers can more easily compare prices should have more flexible prices. I find evidence of both in the empirical literature.

1.5.1 Sectoral productivity shocks and monetary policy

I choose the two predictions to be robust predictions of the kinked-demand theory, independent of the assumed process for costs. For illustration however, I will sometimes rely on simulating data, which requires to commit to a process for costs. Whenever I do, I use the following simple assumptions on sectoral productivity shocks and the demand side of the economy. I restrict sectoral productivity to follow an AR(1) process in logs:

\[
\ln(A_{t,i}) = \rho \ln(A_{t-1,i}) + \varepsilon_{t,i},
\]

(Average technology is normalized to 1.) I assume that the innovations \((\varepsilon_{t,i})_{t,i}\) to the productivity processes are normally distributed with standard deviation \(\sigma_{\varepsilon}^2\).

In addition to the state of technology in the sector, a firm’s nominal cost depends on the aggregate nominal wage. Given equation (1.32), the nominal wage \(W \) is equal to nominal spending \(PC\). I assume that

21[Rotemberg (1987) and Ball and Mankiw (1994) express this view: Rotemberg favors customer-side interpretations; Ball and Mankiw lean toward managers’ inattention.]
nominal spending grows at a constant rate $\mu$:
\[
\ln(W_t) = \ln(W_{t-1}) + \mu.
\] (1.33)

1.5.2 Calibration

Whenever I simulate data, they are based on the following calibration. I set a period to be a month. The calibration boils down to six parameters: the fraction of uninformed customers $\lambda$, the extensive elasticity of demand $\alpha$, the intensive elasticity of demand $\theta$, the growth rate of nominal spending $\mu$, and the parameters for the technology processes $\rho^a$ and $\sigma^a_\epsilon$.22 I assume that half of customers are uninformed. I set the elasticities $\theta$ to 1 and $\alpha$ to 6, so that the lower-bound full-information markup corresponds to an elasticity of demand of 7 as in Golosov and Lucas (2007), and the upper-bound markup corresponds to an elasticity of demand of 4 as in Nakamura and Steinsson (2010). The decomposition of the elasticity between the intensive and extensive margins, which puts most at the extensive margin, is in line with Levin and Yun (2008)’s results. I set the autoregressive coefficient $\rho^a$ of technology to correspond to an annual 0.8, in line with the estimate of Foster et al. (2008). I set the standard deviation of innovations in productivity $\sigma^a_\epsilon$ to get a standard deviation of productivity (meant to capture the volatility of idiosyncratic costs) of 7.5%. The volatility of costs is likely to be very heterogeneous across goods categories, but this value corresponds to what Eichenbaum et al. (2011) reports for the costs of a US retailer. It is also in line with the volatility of costs in Nakamura and Steinsson (2010), although because I assume a more persistent process than they do, I do not assume as volatile innovations as they do. I set the growth in nominal spending $\mu$ to an annual 2%, in line with an average inflation target of 2%. Table 1.1 sums it all up.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>6</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho^a$</td>
<td>0.81/12</td>
</tr>
<tr>
<td>$\sigma^a_\epsilon$</td>
<td>$0.075 \times \sqrt{1 - (\rho^a)^2}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.02/12</td>
</tr>
</tbody>
</table>

Table 1.1: Calibration. In section 6, I consider various values for the growth in nominal spending $\mu$.

1.5.3 The frequency and size of price changes

Figure 1.4 provides a typical path for an individual price for the calibration at hands. The inaction region for price changes in the pricing function (1.31) translates into infrequent price adjustments: a price goes through long spells at some values. As long as the variations in costs are not so large as to make a firm want to unilaterally deviate from the market price, the firm keeps its price constant. When a price is no longer consistent with equilibrium, it adjusts by just the minimal amount necessary to make it an equilibrium price

22The discount factor $\beta$ plays no role.
again.

How infrequent are price adjustments exactly? The illustrative calibration gets the order of magnitude right: the frequency of price changes is 18% per month, when Nakamura and Steinsson (2008) and Klenow and Kryvstov (2008) report a median frequency for regular prices in the CPI data between 10% and 14%, and a mean frequency between 19% and 30%. The frequency of price changes is not a particularly relevant way to evaluate the theory however. First because microdata show a considerable heterogeneity in the frequency of price changes across different product categories (which accounts for the discrepancy between the median and the mean). Second because the frequency of price changes is as much a product of the theory as of the assumed process on costs. Third, because menu-cost models too can produce a variety of frequencies, provided they are fed with the appropriate assumptions on the process for costs (and the appropriate values for menu-costs). I look instead for more essential prediction of the kinked-demand theory, both independent of the assumption on costs, and which distinguish the theory from menu-cost models.

In addition to the frequency of price changes, the empirical literature on microdata has documented a second key statistics: the size of price changes. For the CPI data, Nakamura and Steinsson (2008) and Klenow and Kryvstov (2008) report a median absolute size of regular prices changes of 8.5% and 10%. But Klenow and Kryvstov (2008) also point at the large fraction of small price changes: 44% are smaller than 5%; 12% are smaller than 1%. Subsequent literature has argued that these figures are likely an overestimation. Eichenbaum et al. (2014) point at two measurement problems likely to highball the statistic: in some product categories prices are calculated as the ratio of sales to quantities (Unit Value Indices, or UVI), and certain prices pertain to bundle of goods. Nevertheless, after correcting for these biases small price changes remain quite common: Eichenbaum and his coauthor count 32% of price changes smaller than 5%.

The ordinariness of small price changes is a thorn in the side of menu-cost models. Intuitively, if a firm keeps its price fixed because it incurs a cost when changing it, it should make large price adjustments in order to minimize the number of price changes. Klenow and Kryvstov reject Golosov and Lucas (2007)'s benchmark menu-cost model precisely on the ground that it fails to produce small price changes. The kinked-demand theory in contrast has no problem accommodating small price changes. A firm has no reason to make a large price adjustment in response to a small change in cost. Instead, the firm changes its price by just the necessary amount, knowing that it will always have the possibility to change it again if nominal costs keep moving in the same direction. The illustrative calibration at hands, illustrated in figure 1.4, produces many small price changes.

This calibration actually produces too many small price changes as costs vary gradually by small amounts in-between fixed-price regimes, running into the opposite problem. Just as the frequency of price changes however, the size of price changes is a joint prediction of the theory and the assumed process for costs, and

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23I do not discuss evidence on the size of price changes coming from scanner data, since, as Eichenbaum et al. (2014) argue, the price-as-UVI problem is particularly present in these datasets where all prices are calculated as UVI. In addition, Campbell and Eden (2014) point at a second reason to suspect spurious small price changes in scanner data: weekly averaging. Daily online data “scraped” on online shopping platforms such as the ones collected in the “Billion Prices Project” are immune to these measurement error and do feature fewer small price changes (Cavallo and Rigobon 2011; Cavallo 2016a,b).
thus not a good judge of a theory of price rigidity. Midrigan (2011) show that Golosov and Lucas (2007)’s menu-cost model does not produce as many large price-changes as in the data, but that it can if the process on costs is amended to a process with fatter tails. The argument applies to the kinked-demand theory: larger sudden changes in costs would translate into larger price changes.

Again, because I want to provide tests of the kinked-demand theory alone, I do not try to backward-engineer a process for costs that could match the data. Instead, I point at a prediction the kinked-demand theory makes on the size of price-changes that is independent of the process for costs, and distinguishes it from menu-cost models.

1.5.4 Decreasing hazard functions

The first such prediction I stress is not how frequent price changes are, but how clustered together they are. In the kinked-demand theory, most processes on costs have the price oscillate between long spells at some values, and rapid successions of price changes in-between fixed prices. The appropriate statistics to capture this pattern is the hazard function—the probability of a price change as a function of a price’s age. Because prices that just changed are likely to change again, the pattern maps into decreasing hazard functions at young ages—as illustrated on figure 1.5 for the calibration at hands.

In contrast, menu cost models have much difficulty accommodating a decreasing hazard function. Intuitively, the logic of a menu-cost model goes exactly against clusters of price changes: when a firm resets its price, it resets it in the middle of the inaction region, in order to minimize the probability of having to reset it again soon—and to pay the menu-cost again soon. The intuition maps into increasing hazard functions: the older a price, the more likely it is to be away from its optimal target, and thus to be reset. The intuition needs some qualification: some assumptions on the cost process can make the hazard function of menu-cost model non monotonically increasing. Transitory cost-shocks make quickly-reversed price changes worthwhile and flatten the hazard function. But to get a menu-cost model to produce decreasing hazard functions requires unrealistically large idiosyncratic shocks, as Nakamura and Steinsson (2008) find.

Microdata show evidence of decreasing hazard functions. Nakamura and Steinsson (2008) find them in CPI data, while Campbell and Eden (2014) find them in a weekly scanner dataset, precisely reporting that an individual price “typically goes through several price changes in rapid succession before settling down”. The evidence on decreasing hazard functions is subject to qualifications however. Using the same dataset as Nakamura and Steinsson, but controlling for survivor bias in a different way, Klenow and Kryvstov (2008) report flat hazard functions.

1.5.5 The more customers know, the more flexible prices are

Which are the sticky-price goods? A main insight from the microdata literature is the tremendous heterogeneity in price rigidity across product categories: Klenow and Malin (2011) report that the frequency of price changes varies from less than 3% per month for some services to 91% for gasoline. A natural expla-
nation for the heterogeneity in the flexibility of prices is heterogeneity in the volatility of costs: the price of gasoline at the pump may be quite flexible because the price of a barrel of crude oil is quite volatile.

The kinked-demand theory predicts other determinants to the rigidity of a good’s price. In both menu-cost models and the kinked-demand theory, a price is stickier the wider its inaction region for price changes. In menu-cost models, a main determinant of the width of the inaction region is the size of menu costs. In the kinked-demand theory, the inaction region depends mostly on two parameters: the extensive-margin elasticity of demand $\alpha$, and the fraction of uninformed customers $\lambda$. Both $\alpha$ and $\lambda$ widen the inaction region: a higher fraction of uninformed customers mutes the extensive elasticity of demand more, and there is more to mute the larger the extensive elasticity is.

First thus, goods with stickier prices should be those for which demand is, not necessarily more elastic, but more elastic at the extensive margin. For instance, among goods with the same elasticity of demand, goods that are bought in one-or-zero quantity should tend to have more rigid prices, while goods that customers can buy more or less of should have more flexible prices. Incidentally, canonical examples of flexible-price
goods such as gasoline and fresh fruits are divisible goods the quantity of consumers can adjust continuously.

Second, the kinked-demand theory asserts that prices should be more flexible in markets where consumers are better informed on competitors’ prices, a prediction that may be easier to test. The challenge is nonetheless to identify goods that customers are more informed about. A first indirect test is to consider durable goods. Klenow and Malin (2011) point to the fact that durable goods have more flexible prices—once fresh food and energy, which I discussed above, are excluded. If one takes the view that households make more thorough research on the goods that they are going to keep longer, the kinked-demand theory provides an explanation.

A more direct test is to look at online shopping platforms, where customers can more easily compare competitors’ prices, and to assess whether these online markets feature more flexible prices. This is precisely what Gorodnichenko and Talavera (2016a) and Gorodnichenko and Talavera (2016b) do. They find that online prices are much more flexible than offline prices: the frequency of regular price changes in their data is three to seven times higher than the one reported in Nakamura and Steinsson (2008). A problem in

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Gorodnichenko and Talavera (2016a) use scraped data while Gorodnichenko and Talavera (2016b) use a data set directly provided by an online platform.
interpreting the result as support for the kinked-demand theory is that online markets can also be seen as ones where price-setters face lower menu costs—if one interprets menu cost in a literal sense. Speaking less ambiguously in favor of the kinked-demand theory is Gorodnichenko and Talavera’s result that goods that receive more clicks have more flexible prices, which the authors suggest “points to a greater flexibility for price quotes that matter to consumers”. If more clicks means more research and thus more awareness of all sellers’ prices, the fact fits with the kinked-demand theory.

1.6 Macroeconomic implications

The kinked-demand theory can be distinguished from other theories of price rigidities by its implications at the micro level. Does it have different implications at the macro level? I show it does: the theory has novel implications for monetary policy. I derive the Phillips curve of the kinked-demand theory and stress four of its implications. First, the kinked-demand Phillips is strongly convex, and relies on this convexity instead of inflation expectations shifters to limit the extent to which inflation can increase output. Second, the long-run Phillips curve is essentially similar to the short-run Phillips curve, leading to a permanent trade-off between output and inflation. Third, relative price changes such as oil price shocks shift the Phillips curve up. Fourth, monetary shocks have a long-lasting impact on output, invalidating the idea that a state-dependent pricing model necessarily speaks in favor of little monetary non-neutrality.

1.6.1 The Kinked-demand Phillips curve

To sum up the aggregate-supply side of the model into a Phillips curve, aggregate the price-setting behavior of firms across sectors. Differentiating the price-level (1.6) (still common to all households), inflation appears as a weighted average of sectoral inflations:

$$\Pi_t^{1-\theta} = \int \Pi_t^{1-\theta} \left( \frac{P_{t-1,i}}{P_{t-1}} \right)^{1-\theta} dG(A_{t,i}). \quad (1.34)$$

Lemma 3 precisely expresses the sectoral inflation rate $\Pi_{t,i}$ as the pricing function $q$ of the sectoral state variable $X_{t,i} = W_t/A_{t,i}P_{t-1,i}$—nominal cost in sector $i$ divided by the lagged market price. Using the labor-supply equation (1.4) to rewrite the state $X_{t,i}$ turns equation (1.34) into the following Phillips curve:

**Proposition 4** The Phillips Curve of the adaptive rational-expectations equilibrium is:

$$\Pi_t^{1-\theta} = \int \left( q \left( \frac{C_t}{A_{t,i} \Pi_{t-1} P_{t-1,i}} \right) \right)^{1-\theta} \left( \frac{P_{t-1,i}}{P_{t-1}} \right)^{1-\theta} dG(A_{t,i}). \quad (1.35)$$

The kinked-demand Phillips curve gives—by definition—a relationship between inflation $\Pi_t$ and output $C_t$. Two shifters affect the kinked-demand Phillips curve: the distribution of the sectoral technology shocks $(A_{t,i})_i$, and the past distribution of prices $(P_{t-1,i}/P_{t-1})_i$. To set these distributions, I simulate the aggregate
economy under the following assumptions. I assume a large number of markets—100,000 in the simulation—which sees a market as consisting of very similar goods, in line with the perfect indifference in customers’ preferences between goods produced by different firms in a given market. I assume the productivity processes are identical across sectors, and independent—ruling out any aggregate productivity shocks. Under these assumptions, the large number of markets lets the law of large numbers equalize the cross-sectional distribution of productivity $A_i$ to its ergodic distribution. Finally, I take the distribution of prices $(P_{t-1,i}/P_{t-1})_t$ to be at its steady-state value—I consider the Phillips curve starting from the steady-state. The steady-state consists of the economy under a constant growth of nominal spending (1.33), in which case the law of large numbers again sets the distribution of the sectoral state variable $(A_t, X_t)$ to its ergodic distribution.

Figure 1.6 plots this Phillips curve for the calibration in table 1.1. The calibration is monthly but figure 1.6 considers the inflation rate and average output over the entire year—not just month—to come. The inflation rate can affect output substantially: high inflation rates push output up against its upper bound, while high deflation rates push it up against its lower bound, where the bounds are the ones defined in section 3. From one extreme case to the other, output can change by as much as 11%. The trade-off between output and inflation is also substantial when measured by the slope of the Phillips curve: at a moderate level of inflation: a one-percent increase in output corresponds to an increase in inflation of 0.3 points when inflation is 0%, and 1.15 points when inflation is 2%.

Yet the upper and lower bounds on output set a limit on how much inflation can affect output: the key feature of the kinked-demand Phillips curve is its strong convexity for positive inflation rates, and strong concavity for negative ones—all in all, the long-run Phillips curve is S-shaped. At 10% inflation, a one-percent increase in output generates an increase in inflation of 7.2 percentage points. The convexity allows to combine a rather flat Phillips curve at moderate rates of inflation with a virtually vertical one under hyperinflation (and hyperdeflation). The convexity arises mechanically from the pricing function (1.31): small increases in nominal marginal costs do not trigger much price increases as most firms do not reach the upper end of their inaction regions, but for larger increases in costs many firms are pushed to the end of their inaction regions and increase their prices.

The convexity of the kinked-demand Phillips curve is a constitutive property of the theory. Yet, it does not necessarily set the theory apart. Indeed, on this account what first distinguishes the present model from alternative theories of price rigidity is its tractability, which allows to easily solve for the Phillips curve without relying on approximation methods. The norm in monetary models is to rely, often by necessity, on linearizations methods, ruling out the possibility of investigating the extent of non-linearity in the Phillips curve. Exceptions exist. Ball et al. (1988) consider a model of time-dependent pricing where firms set prices for a deterministic, but endogenously chosen, amount of time. They find that firms choose a higher frequency of price changes when inflation is higher, and that the slope of the Phillips curve increases with the inflation rate.

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25I fix the inflation rate to take the same value every of the 12 months, and solve for the corresponding output each month. Fixing output to take the same value every month and taking the cumulative inflation rate over the 12 months yields almost exactly the same result.
-rate. They present both features as properties of optimizing models of nominal rigidity, which endogenize the extent of nominal rigidity. However, as the authors mention, if the higher frequency of price changes is a robust feature which is for instance also predicted by state-dependent pricing menu-cost models (and, for that matter, the present model), that the slope of the Phillips curve increases with inflation needs not follow. The menu-cost model of Caplin and Spulber (1987), where money is completely neutral, offers a counter-example, while the menu-cost model of Caplin and Leahy (1991, 1997), where monetary shocks impact output differently depending on the level of output, suggests menu-costs models can generate non-linearities in the Phillips curve. How convex a Phillips curve menu-cost models predict appears as an open question however, because of the greater difficulty to solve menu-cost models. Akerlof et al. (2000) generate non-linearities in the Phillips curve by considering near-rational agents who only incorporate inflation in their wage-setting when inflation reaches a threshold of salience. In contrast, the convexity of the kinked-demand Phillips curve does not rely on any departure from rationality.

\footnote{Menu-cost papers do not usually derive the Phillips curves of their models. The exception is Gertler and Leahy (2008). They derive the Phillips curve of their model by relying on an approximation around steady-state however.}
The convexity of the kinked-demand Phillips curve provides simple explanations to some puzzles in the evolution of the U.S. Phillips curve in the past decades. Atkeson and Ohanian (2001) document that the slope of the Phillips curve is lower when estimated on the post early-1980s sample: the Phillips curve has flattened. Some economists, such as Roberts (2006), have interpreted it as a consequence of changes in monetary policy, which would have achieved a better anchoring of expectations. The convex kinked-demand Phillips curve sees the decrease in the slope of the Phillips curve in the 1980s as exactly what one would expect when inflation fell from the double-digit levels it had known during the 1970s.

A related puzzle is the missing disinflation observed during the Great Recession: although output decreased sharply in 2008-2009, deflation did not follow as the estimates of the slope of the Phillips curve would have suggested. As Ball and Mazumder (2011) argue, this can be explained by the convexity of the Phillips curve. In the kinked-demand Phillips curve, not only is the Phillips curve flatter at 2% inflation than at 10%: the relationship becomes even flatter around zero inflation. A slack in output is associated with little decrease in inflation until it reaches, not Great Recession, but Great Depression levels, at which point strong deflation ensues.

1.6.2 The absence of expectations shifters

Many theories of the Phillips curve build on the insights of Friedman (1968) and Phelps (1968) to explain the trade-off between output and inflation: it is because prices (or wages) are set based on imperfect expectations of inflation that inflation can affect output. This assigns a key role to expectations of inflation: because output depends only on the gap between inflation and inflation expectations, past expectations of present inflation are a shifter of the Phillips curve. Models that rely on the Phelps-Friedman rationale include models of producers’ imperfect information, from the New-Classical Phillips curve of Lucas (1972) to the sticky-information Phillips curve of Mankiw and Reis (2002a). Other models of the output-inflation trade-off, such as time-dependent pricing or menu-cost models, do not explicitly rely on any form of nominal mis-perception but, because they make pricing forward-looking, also assign a central role to expectations of inflation—present expectations of future inflation this time. Such models give rise to the New-Keynesian Phillips curve, with present expectations of future inflation as a shifter.

The kinked-demand Phillips curve contrasts with both accounts of the output/inflation trade-off: equation (1.35) does not contain any expectations shifters. This means, first, that the kinked-demand Phillips curve contains no past expectations of present inflation. Firms have full—not outdated—information when setting their prices. Thus, the Phillips curve does not lug past expectation errors. Second, the kinked-demand Phillips curve contains no more present expectations of future inflation. The pricing decision of firms is not forward-looking: to set prices, firms only look at present-day’s variables and need not form any expectations on future variables since they can always reset their prices tomorrow if new conditions call for it. Thus, the

27 Stock and Watson (2009) survey the subsequent literature on Phillips curve inflation forecasts.
Phillips curve is not forward-looking.

An important appeal of augmenting the Phillips curve with inflation expectations is its ability to reconcile the idea of a trade-off between inflation and output with the fact that episodes of hyperinflation are not associated with hyperoutput. Despite the absence of expectations shifters however, the kinked-demand Phillips curve passes the sanity check of hyperinflation. What limits the extent to which inflation can increase output is not expectations shifters, but the strong convexity of the Phillips curve: when output approaches its full-information upper-bound, the Phillips curve becomes asymptotically vertical.

That it is possible to account for episodes of hyperinflation without relying on expectations shifters obviously does not prove wrong the possibility of such expectations shifters. The kinked-demand theory itself could possibly be amended to include some forward-looking behavior—for instance if customers are not randomly assigned every period, so that firms care about long-term market shares in setting their prices. Yet, what the kinked-demand Phillips curve shows is that the trade-off between output and inflation needs not arise from any form of nominal mis-perception nor forward-lookingness in pricing, and thus needs not be shifted by expectations of inflation.

This matters first in order to fit the evidence on pricing at the micro level. The absence of forward-lookingness in price-setting finds support in surveys of price-setters. Fewer than a third of the price-setters surveyed in Blinder et al. (1998) report that they often "take forecasts of economy-wide inflation into account", and more generally 55% report that they rarely or never "raise [their] own price in anticipation when [they] can see cost or wage increases coming" (p.203). This is confirmed by Coibion et al. (2015)'s recent survey of firms, which finds that about 60% of firms report not tracking inflation. These results are not a problem for models based instead on nominal mis-perception, but these last models do not explain why prices stay fully fixed (not just partially adjust).

Second, this matters for the macro-level implications. Relying on the convexity of the Phillips curve to limit the extent to which inflation affects output gives a different account of the costs of disinflation—the sacrifice ratio—than relying on expectations shifters in the Phillips curve does. In models of price-rigidity that rely on expectations shifters, a policy of disinflation needs not be contractionary, provided it succeeds in shifting expectations of inflation. This point was central to the "rational-expectations view" on the Phillips curve, propounded for instance in Sargent (1982). The absence of a sacrifice ratio takes the most drastic form in models whose Phillips curve contains present expectations of future inflation, such as the New-Keynesian Phillips curve. With such Phillips curves, the credibility of the disinflation policy is enough to guarantee it will not lead to a recession. As shown by Ball (1994), a credible disinflation even leads to a boom. Models whose Phillips curve contains past expectations of present inflation, such as models of producers' inattentiveness, avoid this extreme prediction. When price-setters are inattentive, even a credible disinflation policy can be costly in output because many firms remain unaware of the change in policy. Mankiw and

29The absence of expectations in the Phillips curve does not strongly depend on the equilibrium selection criterion however. Only if firms were to use expectations of inflation as their coordination device would expectations appear in the kinked-demand Phillips curve. These are sufficiently disagreed upon to not qualify as ideal public signals.
Reis (2002a) present this property as support for models of producer’s inattention, as opposed to the New-Keynesian Phillips curve. Yet, the common prediction of both types of Phillips curves is that a disinflation policy that is both credible and widely known is not costly. In contrast, the kinked-demand Phillips curve predicts a high sacrifice ratio can apply to such disinflations too. Even if all firms know about, and believe in, the policy change, they still face a coordination problem in deciding to increase prices more slowly.

The kinked-demand theory predicts so while being consistent with the historical experiences that have been presented as evidence against the necessity of a high sacrifice ratio. The theory predicts that disinflating the economy from around 10% inflation to moderate levels—such as during the Volcker disinflation of the early 1980s in the US—is costly. Yet, the convexity of the kinked-demand Phillips curve implies very small costs of disinflating from higher inflation rates to a 10% rate. Most arguments against a high sacrifice ratio come precisely from such historical experiences. Sargent (1982) points at the four disinflations of Austria, Hungary, Poland and Germany in 1922-1924 as evidence against “prohibitively high costs [of] eradicating inflation”, if only monetary policy is credible enough to warrant an immediate adjustment of expectations. However, Sargent does document a substantial rise in unemployment in Austria, Hungary and Poland (data are missing for Germany). His argument is not that disinflation was not costly, but that it was much less costly than a linear extrapolation of the standard costs of disinflation proposed for moderate levels of inflation would infer. But this is exactly what the kinked-demand theory predicts. Another argument against a high sacrifice ratio comes from the study of disinflations in developing countries, as surveyed in Calvo and Vegh (1999). They document an initial boom following exchange-rate-based stabilization plans (exchange-rate pegs), although contraction does materialize in the following years. In these event studies however, inflation rarely falls below double-digit levels, and never in the first years of the stabilization plan. Through the lens of the kinked-demand Phillips curve, the absence of a contraction should not be seen as a puzzle, nor evidence against a large sacrifice ratio when disinflating below 10% inflation. As it turns out, in one of the few cases where disinflation was pursued significantly below 10%—the 1991 Argentinian currency board against the dollar where inflation reached 0% in 1996—unemployment rose from 5.8% to 18.8% between 1991 and 1996.

1.6.3 Long-run monetary non-neutrality

In addition to their different implications for the sacrifice ratio, expectations shifters and convexity also differ in their implications for the long-run trade-off between inflation and output. In the Phelps-Friedman rationale, there exists a trade-off between inflation and output in the short-run as prices are set based on imperfect expectations of inflation, but in the long-run expectations adjust and the trade-off vanishes.\textsuperscript{30} Because the kinked-demand theory is not a theory of money illusion, there is no room for the argument

\textsuperscript{30} Models of time-dependent pricing do not always back the intuition of an exactly vertical long-run Phillips curve. For instance, Calvo pricing gives birth to the New-Keynesian Phillips Curve $\pi_t = \kappa x_t + \beta E_t(\pi_{t+1})$, where $x_t$ is the output gap, $\beta$ the discount factor, and $\kappa$ the slope of the (short-run) Phillips curve. The long-run Phillips curve is thus $\pi = 1/(1-\beta)x$. It is not exactly vertical, but close to it. Besides, in a non-cashless economy, the tax-like distortions created by shoe-leather costs also give room for the growth rate of the money supply to affect steady-state output. In this case however, steady-state output decreases with steady-state inflation.
that mis-perceptions should disappear in the long run, making the long-run Phillips curve vertical. Figure 1.7 plots the long-run Phillips curve of the kinked-demand theory: the relationship between steady-state output and the steady-state inflation rate \( \mu \), capturing the trade-off that monetary policy faces if the change in inflation considered in the short-run Phillips curve is to stay permanent. The long-run Phillips curve is virtually identical to the short-run Phillips curve, implying a long-run trade-off between output and inflation. Again however, this trade-off is limited by the strong convexity of the Phillips curve. In the kinked-demand theory, it is not that the Phillips curve is flat in the short-run and vertical in the long-run. Instead, it is flat at low levels of inflation, and vertical at high levels.\(^{31}\)

![Long-Run and Short-Run Phillips Curve](image)

Figure 1.7: Long-run Phillips curve, superimposed on the short-run Phillips curve. The vertical lines are the bounds on output derived in section 3. The cross corresponds to the \( \mu = 2\% \) steady-state.

A higher inflation target increases output persistently because it persistently affects the position of firms within their inaction regions. The upper panel of figure 1.8 plots, for annual inflation rates of \(-2\%\) to \(2\%\),

\(^{31}\)The model of Akerlof et al. (2000) yields a similar property. In their model with near-rational agents, it is not that agents face money illusion in the short-run and not in the long-run, but instead that agents face money illusion at low inflation rates and not high inflation rates. In contrast to Akerlof et al. (2000), the kinked-demand theory departs altogether from the interpretation of the output/inflation trade-off as a mis-perception between real and nominal variables (and does not rely on any departure from rationality).
the cross-sectional distribution of the $X_i$’s within the inaction region. Higher inflation shifts the distribution of the $X_i$’s to the right. As a consequence, the distribution of markups shifts to the left, as shown in the bottom panel. (The bottom panel plots the distribution of $P_i/W$ instead of the distribution of markups $P_i/WA_i$ because the two masses in $\mathcal{M}(\alpha + \theta)$ and $\mathcal{M}(\alpha(1-\lambda) + \theta)$ in the distribution of markups makes it hard to visualize.) As markups decrease, output increases.

![Steady-state cross-sectional distribution of $X_i$](image1)

![Steady-state cross-sectional distribution of $P_i/W$](image2)

Figure 1.8: Steady-state distributions of the state variable $X_i$ (top panel) and of $P_i/W$ (bottom panel) across sectors for different (annual) values of nominal spending growth. On the left panel, the vertical lines are the limits of the inaction region.

The long-run effect of inflation on output is best understood by remembering that the price rigidity of the kinked-demand theory is a downward rigidity. In the kinked-demand theory, prices have a tendency not to adjust downward to decreasing costs, because price decreases cannot be advertised. When positive productivity shocks lower the production costs of a firm, the firm does not decrease its price, not until the state variable $X_i = W/A_i P_{-1,i}$—nominal costs divided by past price—reaches the lower end of its inaction region. Instead, the markup increases. Trend inflation tempers this tendency by steadily increasing nominal costs, thus steadily eroding markups. Because markups are lower, output is higher.

The mechanism behind the existence of a non-vertical long-run Phillips curve is thus not unlike the
existing argument in favor of a long-run trade-off: downward nominal wage rigidity. The argument, dating back to \textit{Tobin} \cite{72} and given a detailed account in \textit{Akerlof et al.} \cite{96}, is that if for any reason nominal wages have trouble adjusting downward, then positive inflation can increase output by eroding real wages without touching to nominal wages. Although the argument has usually needed to assume disequilibrium in the labor market and thus been subject to the Barro critique, in chapter 2 of this dissertation, written with Emi Nakamura and Jón Steinsson, we show that a labor search framework makes it consistent with equilibrium and robust to the Barro critique.

Here prices, not wages, are the sticky ones. But here too inflation acts by having relative prices adjust in a way they would not without the help of inflation: inflation greases the wheels of the goods market. There is a substantial difference however: the reasons for the lack of adjustment in the absence of inflation is different. The downward rigidity of nominal wages is usually argued to come from psychological barriers to wage cuts combined with money illusion. Here there is neither money illusion nor loss aversion. Instead, the problem is a coordination failure: a firm has no incentive to adjust its price after a shock if it is the only firm to do so. Inflation makes the adjustment for everyone at once.

The existence of a non-vertical long-run Phillips curve has long been a contentious empirical issue. Traditional tests—assessing whether the coefficients on lagged inflations sum to one—fell out of fashion after \textit{Sargent} \cite{71} pointed out that identifying the slope of the long-run Phillips curve in this way requires to assume that inflation expectations load one for one on past inflation, a strong assumption given that inflation had been little serially correlated historically. However, \textit{King and Watson} \cite{94} show that the slope of the long-run Phillips is identified provided inflation has a unit root, and find this slope to be non-vertical when using a structural VAR with a Keynesian identification. Yet, using a higher-dimensional VAR and different identification assumptions, \textit{Benati} \cite{15} cannot reject the hypothesis of a vertical long-run Phillips curve. As \textit{Benati} stresses, the incontrovertible result is the substantial uncertainty surrounding the estimates: at bottom the problem is that there is little information in the existing data to estimate the slope of the long-run Phillips curve, both because of the generic difficulty in estimating the long-run, and because of the need to rely only on the permanent variations in inflation. If the empirical evidence is inconclusive, what the kinked-demand theory shows is that there is no theoretical basis to exclude the possibility of a long-run trade-off as inconsistent with rationality or rational expectations.

The existence of a long-run trade-off between inflation and output summons the normative question: should monetary policy exploit it by targeting a higher inflation target? The benefit of a higher inflation target implied by the kinked-demand theory—eroding markups—adds to the two main arguments in favor of higher inflation targets: greasing the wheels of the labor markets by eroding real wages when nominal wages are downwardly rigid, as already mentioned, and creating a buffer away from the liquidity trap.\footnote{The case for a positive inflation target to avoid falling in the liquidity trap is often attributed to \textit{Summers} \cite{91}.}

Pinning down the optimal inflation target called for by the possibility of eroding markups requires to compare this new benefit to the costs of inflation. The present version of the model does incorporates the one...
cost of inflation usually taken into account in studies of optimal monetary policy (Woodford (2003a)): the welfare loss stemming from possibly more dispersed relative prices. Yet, in the model higher inflation does not increase relative price dispersion within markets. Thus, the optimal inflation rate is trivially infinite. The absence of connection between inflation and price dispersion is confirmed empirically in Nakamura and Steinsson (2016). As they stress, it is thus still unclear what the relevant costs of inflation are and how to quantify them. As a consequence, I do not undertake a formal cost-benefits analysis, and do not attempt to pin down the optimal inflation target.

Instead, I simply make three remarks. First, higher inflation always implies higher output in the model, but only if output is sub-optimally low does it imply higher welfare. It is in this version of the model, since the distortions created by monopolistic competition are not corrected by any tax or subsidy. And it is likely to be in reality too. Yet, I know of no reason why steady-state distortions, which do not fluctuate over the business cycle, should be corrected by monetary policy instead of fiscal policy, if correcting them through monetary policy comes with the additional costs created by inflation.

Second, putting this concern aside, the convexity of the long-run Phillips curve implies that most of the benefits of a higher inflation target are achieved with reasonably low inflation rates. In the calibration at hand, full-information output is about 7% higher than zero-inflation output. But an inflation target of 5% already closes 75% of the gap: much inflation is mostly useless. Thus, the kinked-demand argument for a higher inflation target is likely not to justify a target much higher than the recent 4% proposal by Blanchard et al. (2010) and Ball (2014), which they base on liquidity-trap concerns.

Third, a distinct implication of the convexity of the long-run Phillips curve is actually to qualify the liquidity-trap argument for a higher inflation target. As the argument goes, maintaining a distance in normal times between the nominal interest rate and its lower bound of zero leaves room for monetary policy to cut rates and stimulate demand during recessions. Yet, the present model suggests that stimulating demand becomes harder at high inflation rates. It implies that a higher inflation target would save ammunitions for bad times only at the cost of making these ammunitions less effective.

### 1.6.4 Inflation and relative price changes

If the convex Phillips curve plotted in figure 1.6 gives a simple account of such events as the flattening of the Phillips curve in the 1980s or the missing disinflation of 2008-2009, it does not in itself explain others, such as the stagflation of the 1970s, where high inflation was combined with low activity. It has long been stressed—for instance in Gordon (1982)'s triangle model—that such an event does not invalidate the existence of a Phillips curve: the Phillips curve can shift under the influence of shocks independent of monetary policy. A first obvious shifter of the Phillips curve, in the kinked-demand theory like other models, is an increase in aggregate productivity, which shifts the Phillips curve to the right.\(^{33}\)

\(^{33}\)It also shifts right the unemployment Phillips curve as long as changes in productivity affect the unemployment rate, as it does for instance in a search model with real-wage rigidity (Blanchard and Gali (2010)).
Yet, a much-discussed suspect for the shifts of the Phillips curve in the 1970s is not aggregate productivity but the OPEC oil price shocks—distinct from their effect on aggregate productivity. Although the impact of oil prices on inflation is very present in policy discussions and has, like productivity shocks, long been introduced in the empirical work of the Phillips curve (Gordon (1982)), it is both less discussed and more contentious on the theoretical side. On the one hand, Friedman’s monetarist motto that “inflation is always and everywhere a monetary phenomenon” (Friedman (1970)) intends to rule out such shocks as a source of inflation. In a much-cited article in Newsweek (Friedman (1974)), he asked: “Why should the average level of prices be affected significantly by changes in the price of some things relative to others?” Under this monetarist view, oil price shocks were only an excuse for the lenient monetary policy of the 1970s, which was the only cause of inflation. Stagflation should be understood as monetary policy shifting inflation expectations by trying to exploit the output/inflation trade-off, as Friedman’s accelerationist Phillips curve had predicted.

On the other hand, Mankiw and Ball (1995) show that in a model with sticky prices, there is no fallacy in a link between changes in relative prices and changes in the price level: relative-price changes are shifters of the Phillips curve. Ball and Mankiw show this point in a static menu-cost model which takes initial prices to be optimal. If dynamic menu-cost models are less tractable and make the issue harder to analyze, the analytical tractability of the kinked-demand model makes the argument easier to investigate.

To analyze the effect of an oil-price shock, I consider the following thought experiment. Starting from the symmetric steady-state distribution of sectoral productivity, I assume that 8% of the sectors—the weight of energy in the PCE in 1980—face a decrease in productivity of 2.1% per month over the year—the average increase in the price of energy relative to the CPI from March 1979 to March 1980. In order to isolate the effect of relative-price changes from the effect of changes in aggregate productivity, I assume that the remaining sectors face an increase in productivity so that aggregate productivity is left unchanged. Figure 1.9 gives the effect of this simulated oil price shock. The Phillips curve shifts up: the steady-state level of output at 2% inflation is now associated to an inflation rate of 3.4%. Firms hit by the negative productivity shocks increase their prices, while for most of the other firms the increase in productivity is not sufficiently large to trigger a price decrease. The order of magnitude of the increase in inflation is itself a lower bound since the model abstracts from intermediary inputs, and thus from any spillover to the non-energy sectors.

A less rigorous interpretation of Friedman’s monetarist motto is that, although such things as an increase in the relative price of oil may put inflationary pressures on the economy, it is always in the power of the central bank to undo these shocks, so that the central bank does ultimately have full control over the price level. The model fits this interpretation: all inflation rates are indeed parts of the menu of options that the central bank can choose from on the Phillips curve. Yet, the central bank can only decrease inflation by trading off a decrease in output. Figure 1.9 gives the extent of output loss the central would have to be willing to endure to undo the inflationary shock. Maintaining the pre-shock 2% inflation rate requires a decrease in output of 1.3%. Containing inflation requires to decrease nominal spending enough so that
enough firms reach the bottom end of their inaction region and decrease their prices. Yet, in doing so, many other firms respond to the decrease in nominal spending without changing prices, decreasing production instead.

![Short-Run Phillips Curve](image)

Figure 1.9: Shift in the short-run Phillips curve after an oil-price shock. The vertical lines are the bounds on output derived in section 3. The cross corresponds to the $\mu = 2\%$ steady-state.

### 1.6.5 Dynamics of monetary shocks

Looking at the effect of monetary policy through the Phillips curve abstracts from the dynamics in the output/inflation trade-off. I turn to the dynamics of monetary policy shocks. Because impulse-response functions are more common than Phillips curves as a way to evaluate the effect of monetary policy in theoretical models, this also allows me to compare the extent of monetary non-neutrality in the kinked-demand theory and alternatives theories of price rigidity, in particular menu-cost models.

The literature on menu-cost models has often served as a cautionary tale against the large and long-lasting non-neutrality found in models of time-dependent pricing, such as Calvo (1983)’s model of staggered pricing. With time-dependent pricing, the fraction of firms that adjust their prices is fixed and the selection...
of firms that adjust is random. The more microfounded alternative of state-dependent pricing endogenizes both the fraction and the selection of adjusting firms. When these characteristics are endogenized through a menu-cost model, the real effect of monetary shocks can be dramatically reduced. Caplin and Spulber (1987) exhibit a particular case where money is completely neutral. Golosov and Lucas (2007) show that a more realistic version of the menu-cost model, which adds in particular idiosyncratic shocks, predicts, if no complete neutrality, very transient effects of monetary shocks.

State-dependent pricing is often used as a synonym for menu-cost models. Yet it is not: the kinked demand curve theory is a model of state-dependent pricing too. I show that the kinked-demand theory predicts much longer-lived real effects of monetary shocks than in the benchmark menu-cost model of Golosov and Lucas (2007), and thus that the transient effects of monetary shocks in menu-cost models is not a general feature of state-dependent pricing, but only of the specific way menu-cost models make pricing state-dependent.

I use Golosov and Lucas’s results as a comparison point to contrast the predictions of the kinked-demand theory with the predictions of menu-cost models. To be sure, the model of Golosov and Lucas is only a particular version of the menu-cost framework. In particular, it considers a case with no strategic complementarity in price-setting. Subsequent literature—for instance Gertler and Leahy (2008) and Nakamura and Steinsson (2010)—has shown that strategic complementarities amplify the real effect of monetary shocks in menu-cost models just as it does in models of time-dependent pricing. But the version of the kinked-demand model I consider in this paper shares the same sin as Golosov and Lucas’s model: insofar as strategic complementarities are measured by the slope of the notional short-run aggregate supply (SRAS), there is none. Golosov and Lucas’s model is thus the appropriate benchmark.

To evaluate the extent of short-run monetary non-neutrality, I run the same experiment as the one Golosov and Lucas use to show the transient effects of monetary shocks in their benchmark menu-cost model: starting at the steady-state, I simulate a one-time 1% increase in nominal spending and look at how output responds.34 Figure A.1 plots the quarterly impulse-response function of output when the inflation target $\mu$ is 2%. On impact, most of the increase in nominal spending translates into an increase in output. More critically, the shock is much more persistent than in Golosov and Lucas (2007)’s menu-cost model. There, all effects of the monetary shock disappear in about half-a-quarter; here, the effect of the shock is still about 40% of its initial impact after a year, and 20% after two. The persistence of the shock is much more in line with the persistence predicted by the Calvo model. Figure A.1 superimposes the impulse-response for the Calvo model, absent strategic complementarities.35

What accounts for the difference between the kinked-demand theory and a menu-cost model? Both in

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34 Because behavior is derived under the expectation of no shock, the question arises of whether this is consistent with rational expectations. A general argument to answer yes, which applies to both the menu-cost and the kinked-demand models, is that zero-probability events can happen. An argument specific to the kinked-demand model is that the expectations of price-setters on the process for costs do not matter anyway: price-setting is not forward-looking.

35 Specifically, this is the IRF implied by the log-linearized New-Keynesian Phillips curve $\pi_t = \kappa x_t + \beta E_t(\pi_{t+1})$, where $\kappa = (1 - \xi)(1 - \beta)\xi$, with $\xi$ the probability of not resetting prices calibrated to $\xi = 8/9$—a price duration of 9 months—and $\beta$ the discount factor calibrated to $\beta = 0.99$. 

41
a menu-cost model and in the kinked-demand theory, the firms that are triggered to increase their prices following a positive monetary shock are the ones that are at the upper end of their inaction region. What differs is the connection between the size of cost-changes and the size of price-changes discussed in section 5.3. In a menu-cost model, a firm makes a large price-adjustment regardless of the size of the cost-shock. As a result, in response to the small increase in nominal costs driven by monetary policy, the firms that adjust their prices adjust them by enough to compensate for the firms that do not adjust. In contrast, in the kinked-demand theory the change in price is at most the size of the change in cost. As a result, in response to a monetary shock, no firm adjusts its price by more than the size of the monetary shock. Over time all prices do adjust. But since none overadjusts, the price level adjusts slowly, leaving output durably high.

The example of the kinked-demand theory shows that what is true for menu-cost models needs not be true of all models of state-dependent pricing: deriving the inaction region from the cost of changing prices, as menu-cost models do, is not innocuous. Just as models of time-dependent pricing constrain the frequency of price changes to be little dependent on the economic environment, the assumption in menu-cost models of a cost of adjusting prices constrains the size of a price change to be little dependent on the size of the cost shock.

1.7 Conclusion

When it comes to sticky prices, economists and practitioners tend to disagree. Practitioners perceive price-rigidity as the optimal response to the demand curves they face. We economists rely on models where demand curves predict flexible desired prices, and assimilate price-rigidity to firms’ inability to track these desired prices.

In this paper, I have argued that theory needs not contradict the perception of practitioners. I have shown that the oldest of customer-side theories of price rigidity—the kinked demand-curve theory—is as consistent with optimizing behavior, equilibrium, and rational expectations, as firm-side theories are. I have pointed at ways by which deciding between the theories can be moved from price-setters’ surveys to microdata empirical studies. And I have shown that the question matters: the kinked-demand theory has implications for monetary policy that substantially differ from the ones of firm-side models.

I conclude by pointing at a last implication of the kinked-demand theory I have not touched here. The key assumption that gives the kinked-demand curve in this paper is that not all consumers know about all prices in a market. Information is endogenous in the model: uninformed customers decide whether to move to competitors and learn their prices. Yet, throughout the paper I have maintained the fraction of uninformed customers as exogenous. It can be endogenized: by considering firms’ decision to inform customers, the kinked-demand theory naturally lends itself to a theory of advertising at business-cycle frequency.
Figure 1.10: Impulse-response function of output to a 1% increase in nominal spending, at $\mu = 2\%$ steady-state inflation.
Chapter 2

A Plucking Model of Business Cycles

with Emi Nakamura and Jón Steinsson

Introduction

The unemployment rate in the United States displays a striking asymmetry: much of the time, it hovers around 5%, but occasionally it rises far above this level, peaking each time at a different maximum. Milton Friedman proposed a “plucking model” analogy to describe this behavior of the economy: “In this analogy, ... output is viewed as bumping along the ceiling of maximum feasible output except that every now and then it is plucked down by a cyclical contraction” (Friedman, 1964, 1993). Friedman highlighted one manifestation of these asymmetric dynamics: economic contractions are followed by expansions of a similar amplitude—as if the economy is recovering back to its maximum level—while the amplitude of expansions are not related to the previous contractions—each pluck seems to be a new event.

Workhorse models of the business cycle do not capture this asymmetry in unemployment and output. Instead, they see the business cycle as symmetric ups and downs of unemployment and output around an average level. An important implication of this view is that stabilization policy cannot affect the average level of output or unemployment. At best, stabilization policy can reduce inefficient fluctuations. As a consequence, in these models the welfare gains of stabilization policy are trivial (Lucas, 1987, 2003).

Friedman’s plucking model view of the business cycle potentially has very different implications for the welfare gains from stabilization policy. In this view, economic contractions involve drops below the economy’s full-potential “ceiling,” rather than symmetric cycles around a “natural rate.” Eliminating such drops increases average output and decreases average unemployment, which raises welfare by non-trivial amounts.

We develop this thesis by building a plucking model of the business cycle. The key ingredient for
generating the plucking property in our model is downward nominal wage rigidity. We depart from the
previous literature by introducing downward nominal wage rigidity within an explicit search model of the
labor market. The search framework rationalizes unemployment as an equilibrium phenomenon and, most
importantly, makes the downward rigidity of wages fully consistent with optimizing behavior, and thus
robust to Barro’s (1977) critique that wage rigidity should neither interfere with the efficient formation of
employment matches nor lead to inefficient job separations.

Our plucking model captures the pronounced asymmetry of the distribution of unemployment. Empiri-
cally the distribution of unemployment has a longer right tail than left tail. The unemployment rate spends
much time around 5%. Occasionally, it rises to levels much higher than this. In contrast, it never falls much
below this level. Our plucking model generates this type of asymmetry in sharp contrast with standard
models of unemployment dynamics.

In our plucking model, unemployment always lies above its no-shock steady-state level. Fluctuations
in unemployment are shocks (plucks) away from this steady-state level and subsequent drifts back toward
this level. This property is what allows the model to match the asymmetry in the distribution of the
unemployment rate. This also shows that the natural-rate view of business cycles—and its corollary that
stabilization policy can’t effect mean output and unemployment—is not a necessary implication of imposing
the discipline of optimizing behavior, equilibrium analysis, and rational expectations.

Intuitively, the distribution of unemployment is right-skewed in our model because good shocks mostly
lead to increases in wages, while bad shocks mostly lead to increases in unemployment. The source of this
asymmetry is our assumption of downward nominal wage rigidity. This notion has a long history within
macroeconomics going back at least to Tobin (1972). The main theoretical challenge for this line of thinking
has been how to justify the notion that wages don’t fall in recessions despite obvious incentives of unemployed
workers to bid wages down.

To make downward nominal wage rigidity robust to this critique, we build on the recent insights from
the labor search literature. Hall (2005) pointed out that, once a search and matching model is purged of
its ad hoc assumption of Nash-bargaining, wages are not uniquely pinned down. They are only constrained
to lie within a wage-band, making some amount of wage-rigidity consistent with individual rationality and
equilibrium. Intuitively, because of search frictions, unemployed workers cannot freely meet with firms and
offer to replace employed workers at a lower wage. Instead, unemployed workers and potential employers
must engage in a costly matching process. But after the worker and employer have matched, the worker has
some monopoly power and therefore no longer has any reason to bid the wage down. As a result the wage
has no reason to be driven to market-clearing level.

The plucking nature of our model has important normative implications. Reductions in the volatility
of shocks not only reduces the volatility of the unemployment rate, but also reduces its average level.
Eliminating all shocks in our model reduces the average unemployment rate from 5.8% to 4.2%. The welfare
benefits of stabilization policy are therefore more than an order of magnitude larger in our model than in
standard models in which stabilization policy cannot affect the average level of output and unemployment.

In our model, a modest amount of inflation can “grease the wheels of the labor market” by allowing real wages to fall in response to adverse shocks even though nominal wages are downward rigid. Increasing the average inflation rate from 2% (our baseline calibration) to 4% yields a drop in average unemployment from 5.8% to 4.9%. The benefits of inflation diminish at higher levels of inflation but are quite large at low levels. Reducing the average inflation rate from 2% to 1% increases the average unemployment rate from 5.8% to 7.2%.

Our work is related to several strands of existing literature. Kim and Ruge-Murcia (2009, 2011) and Benigno and Ricci (2011) assume downward nominal wage rigidity in models where employment is restricted to fluctuate at the intensive margin only, i.e., they dispense with unemployed workers altogether. Akerlof et al. (1996) and Schmitt-Grohe and Uribe (2016) close the labor market through some variant of the short-side rule, assuming the labor-market is demand-constrained when wages need to fall, but without explaining explicitly why unemployed workers do not bid down the wage of employed workers.

Our assumption of downward nominal wage rigidity is motivated by the microdata evidence on the existence of asymmetric wage adjustments. Micro-data panel studies of downward nominal wage rigidity, starting with McLaughlin (1994), Kahn (1997) and Card and Hyslop (1997), and more recently Barattieri et al. (2014), point at a spike at zero in the density of nominal wage changes, strongly suggestive of downward nominal rigidity. As emphasized in Pissarides (2009) and Haefke et al. (2013), only the existence of wage-rigidity for new hires has allocative implications. Haefke et al. (2013) argue that wages of new hired are less rigid than those of existing workers. Gertler and Trigari (2009) argue that this result may be mainly due to a compositional effect. Bewley (1999, ch. 9) gives evidence that employers report a constraint to maintain internal equity between similar workers within the firm and therefore to tie the wage of new hires to the wage of older workers in the firm.

Recent work has explored several ways in which Lucas’ (1987, 2003) calculations may underestimate the costs of business cycle fluctuations and therefore the potential benefits of stabilization policy. Fluctuations are more costly when output is difference stationary (Obstfeld 1994) and when shocks have fat tails (Barro 2009). Uninsurable income risk also increases the cost of fluctuations (Krebs, 2007, Krusell et al., 2009). Our work highlights the notion that fluctuations may be more costly than Lucas estimates because they reduce the average level of output.

The paper proceeds as follows. Section 2.1 lays out our plucking model of business cycles. Section 2.2 describes the model’s implications for the distribution of the unemployment rate. Section 2.3 shows that fluctuations increase the average level of unemployment and higher inflation reduces the average level of unemployment. Section 2.4 concludes.

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1Recall that Lucas (2003) shows that the consumption equivalent welfare loss of business cycle fluctuations in consumption over the period 1947-2001 is 0.05% if consumers are assumed to have log-utility and face trend stationary fluctuations in output with normally distributed innovations.
2.1 An Equilibrium Model of Downward Nominal Wage Rigidity

Households supply an exogeneous quantity of labor—which we normalize to 1—to firms, from which firms produce a homogeneous good using the decreasing returns to scale technology:

\[ Y_t = Z_t F(N_t), \]  

(2.1)

where \( Y_t \) is output, \( N_t \) is employment, and \( Z_t \) is an exogenous productivity shifter, meant to capture any (non-modeled) change in labor productivity. Firms sell the good back to households in competitive markets at price \( P_t \). (Because the goods market is competitive, the production side can equivalently be seen as consisting of a representative firm.) We focus the model on the functioning of the labor market. Because of search frictions, not all households are employed: workers are divided between employed \( N_t \) and unemployed \( U_t = 1 - N_t \). Employed workers earn the nominal wage \( W_t \), while unemployed earn nothing. We note \( w_t = W_t / P_t \) the real wage.

Because we abstract from the intensive-margin labor-supply decision of households, households play an essentially passive role: they work, or at least try to, whatever the wage, and for this reason our assumption of no unemployment benefits has no consequences on the determination of employment. Since there is no capital accumulation, households’ consumption/savings decisions do not matter much for the determination of aggregate variables either. Nevertheless, because the firm’s hiring decision will be intertemporal, households do matter to determine the stochastic discount factor. We assume the stochastic discount factor between \( t \) and \( t + s \) takes the standard form 

\[ Q_{t,t+s} = \beta^s u'(C_{t+s}) / u'(C_t), \]

where \( \beta \) is the discount factor and \( C_t \) is aggregate consumption at \( t \). Thus, implicitly, we assume a representative agent with standard intertemporal separable preferences, which can be justified despite the heterogeneity in incomes by the existence of complete markets, or the assumption that households meet at the end of each period to share their incomes.

2.1.1 Labor-Demand

We first derive firms’ demand for labor. Hiring workers is subject to hiring costs that take the form of search costs: with cost \( Z_t c \)—proportional to productivity—a firm can post a vacancy, which will translate into a hire if the job offer matches a job seeker. A match happens with probability \( q_t \), which a firm takes as given and the determination of which is described below. We assume that each firm is big enough so that it can abstract from the randomness in seeking a worker: hiring one worker requires to post \( 1/q_t \) vacancy and has the certain cost \( Z_t c / q_t \). Hiring costs are in terms of the same composite final good as the household consumes, and thus have price \( P_t \). Besides, each period a fraction \( s \in (0, 1) \) of a firm’s workforce leaves the firm for
exogenous reasons. Noting $H_t$ the number of hires at $t$, a firm’s workforce therefore evolves according to:

\[ N_t = (1 - s)N_{t-1} + H_t. \]  

(2.2)

(This assumes that workers hired at $t$ start to work for the firm at $t$.)

A firm’s real profits at $t$ are real revenues $Z_t F(N_t)$, minus real labor costs $w_t N_t$, minus real hiring costs $\frac{Z_c}{q_t} H_t I_{|H_t \geq 0}$. A firm chooses employment and hires in order to maximize intertemporal real profits, discounting them using the representative household’s discount factor, and subject to the flow equation (2.2). If firms hire every period, which we will impose in equilibrium, firms’ labor-demand (equivalently hiring decision) is characterized by the first-order condition:

\[ Z_t F'(N_t) = w_t + \frac{cZ_t}{q_t} - (1 - s) E_t \left( Q_{t, t+1} cZ_{t+1} \right), \]  

(2.3)

which equates the marginal productivity of a worker to its cost to the firm, itself equal to the wage, plus the hiring cost, minus the expected savings of having a worker next period without having to hire him next period. For firms not to be willing to fire workers in equilibrium, it must be that the value of an (already hired) worker is positive. This imposes the following upper-bound on the wage:

\[ w_t \leq Z_t F'(N_t) + (1 - s) E_t \left( Q_{t, t+1} cZ_{t+1} \right), \]  

(2.4)

The probability of filling a vacancy $q_t$ is determined in equilibrium through an exogenous matching function $q(.)$ of the tightness ratio $\theta_t$, the ratio of the number of vacancy posted $H_t$ to the number $S_t$ of job-seekers at the beginning of the period: $\theta_t = H_t/(q_t S_t)$. The probability for an unemployed worker to find a job is equal to the ratio of hires to job-seekers $f(\theta_t) \equiv H_t/S_t = \theta_t q(\theta_t)$. We assume that a worker losing his job between periods $t - 1$ and $t$ gets a chance to find a new job at the beginning of period $t$ and therefore to work in period $t$, spending no period without a job. Thus, the number of job-seekers at $t$ is $S_t = 1 - (1 - s) N_{t-1}$.

The employment flow equation (2.2) can therefore be rewritten using the tightness ratio $\theta_t$ instead of hires $H_t$:

\[ N_t = 1 - (1 - f(\theta_t))[1 - (1 - s)N_{t-1}] \]  

(2.5)

### 2.1.2 Wage-Setting

Because of search frictions, unemployed workers cannot instantly meet with firms to offer to replace employed workers at a lower wage. Instead, an unemployed worker always meets a firm after a match, at which point he no longer has any reason to bid the wage down. As a result the wage has no reason to be driven to

\[ \text{w_t} \leq Z_t F'(N_t) + (1 - s) E_t \left( Q_{t, t+1} cZ_{t+1} \right), \]  

(2.4)

The number $S_t$ of job seekers at $t$, although it can be seen as the number of unemployed at the beginning of the period $t$, is not equal to what we defined as the unemployment rate $U_t$ at $t$, which only counts those job seekers who did not find a job at $t$.\footnote{The number $S_t$ of job seekers at $t$, although it can be seen as the number of unemployed at the beginning of the period $t$, is not equal to what we defined as the unemployment rate $U_t$ at $t$, which only counts those job seekers who did not find a job at $t$.}
market-clearing level, nor to be uniquely pinned down to any level: nothing forces the equilibrium to be at the crossing of the labor-demand curve \( (2.3) \) and labor supply curve \( N_t = 1 \). Instead, there are only upper and lower bounds on an equilibrium wage. The upper-bound is defined by the no-firing condition \( (2.4) \). Since we assume an exogenous, inelastic labor supply, there is no lower bound coming from workers’ unwillingness to work for too low a wage. However, an equilibrium wage must prevent firms from being willing to hire more workers than the supply of them. Using the labor-demand \( (2.3) \), the condition of no excess labor demand \( N_t \leq 1 \) translates into the following lower-bound on the wage:

\[
\begin{align*}
w_t & \geq Z_t \frac{F'(1)}{q_t} + (1 - s)E_t \left( Q_{t,t+1} \frac{cZ_{t+1}}{q_{t+1}} \right) \\
& \geq Z_t \frac{F'(1)}{q_t} + (1 - s)E_t \left( Q_{t,t+1} \frac{cZ_{t+1}}{q_{t+1}} \right) 
\end{align*}
\]

(2.6)

In-between these two bounds, all wages are consistent with individual optimality. This continuum of wages defines an infinity of equilibria, each characterized by an assumption on wage-setting. We consider three wage-setting assumptions: downward nominal wage rigidity, and two benchmarks: flexible wages and symmetric real wage rigidity.

Start with flexible wages. Following Blanchard and Gali (2010) and Michaillat (2012), we specify wage flexibility through the short-cut assumption that real wages follow productivity:

\[
w_t = \bar{w}Z_t,
\]

(2.7)

where \( \bar{w} \) is a constant. The short-cut is justified by the fact that this is a very close approximation to the dynamics of wages under the assumptions of either market-clearing or Nash-bargaining. However, as Shimer (2005) shows, the search framework with flexible wages fails to account for the fluctuations in the unemployment rate: when wages follow productivity, all the effect of shocks goes to prices, leaving quantities unchanged.

Our second benchmark is symmetric real wage rigidity, as considered in previous papers on wage rigidity in search models. Specifically, we assume that real wages adjust slowly to productivity by following Shimer (2010)’s specification of the real wage as a weighted average of the past real wage and present flexible wage:

\[
w_t = \rho w_{t-1} + (1 - \rho)\bar{w}Z_t,
\]

(2.8)

where \( \rho \) is a weight between 0 and 1.

We contrast symmetric real wage rigidity with our main assumption of downward nominal wage rigidity. We assume that the nominal wage is set to the flexible wage, except if this requires the nominal wage to fall below a threshold defined as a fraction of the past nominal wage: \( W_t = \max\{P_t\bar{w}Z_t, \gamma W_{t-1}\} \). The weight on the past wage \( \gamma \in [0, 1] \) characterizes the extent of wage rigidity. Expressed in terms of real wages, and
noting \( \Pi_t \) the inflation rate \( \Pi_t = P_t/P_{t-1} \), the wage-setting equation becomes:

\[
w_t = \max \left\{ \bar{w}Z_t, \gamma \frac{w_{t-1}}{\Pi_t} \right\}.
\]  

(2.9)

Downward nominal wage rigidity adds the lower-bound \( \gamma w_{t-1}/\Pi_t \) on the present real wage.

The three specifications of wage-setting do not explicitly impose that the wage remains within the wage band defined by the no-firing condition (2.4) and no excess demand condition (2.6). However, we will check that they do in all our simulations.

### 2.1.3 Equilibrium

To close the model, we assume that the good market clears: production meets households’ demand for consumption and firms’ demand for hiring services:

\[
Y_t = C_t + \frac{cZ_t}{q(\theta_t)}[N_t - (1 - s)N_{t-1}],
\]  

(2.10)

An equilibrium is then, given an exogenous process for productivity \( (Z_t) \) and an initial condition for employment \( N_0 \), processes for the six endogenous variables \( N_t, C_t, Y_t, \theta_t, w_t, \Pi_t \) that satisfy the production function (2.1), the labor demand (2.3), the employment flow equation (2.5), the no-firing condition (2.4) and no-excess-demand condition (2.6), the good market-clearing condition (2.10), and the downward nominal wage rigidity wage-setting rule (2.9)—or the alternative benchmark (2.8). This only determines an equilibrium once monetary policy is specified. We assume that monetary policy sets the inflation rate to a constant target \( \Pi \) at all periods.

### 2.1.4 Productivity Growth

Productivity growth matters when considering downward nominal wage rigidity. In an economy with high trend growth, episodes where wages need to decrease are short-lived, as trend productivity soon brings the flexible wage back to previous levels. In contrast, in a low-growth economy, such episodes can have much more devastating consequences, as the downward nominal wage rigidity constraint can be binding for much longer. We thus allow for the possibility of trend growth.

We consider a time-trend in productivity: we assume that productivity shocks \( Z_t \) are the sum of a deterministic trend at rate \( g \), and an AR(1) process \( (A_t) \):

\[
\ln(Z_t) = \ln(A_t) + \ln(A_t),
\]  

(2.11)

\[
\ln(A_t) = \rho_a \ln(A_{t-1}) + \sigma_a^2 \varepsilon_t^A,
\]  

(2.12)

where \( \varepsilon_t^A \) has mean zero and variance one. Accordingly, we detrend consumption, output and wages by
defining $\tilde{C}_t \equiv \frac{C_t}{s \sigma}$, $\tilde{Y}_t \equiv \frac{Y_t}{s \sigma}$, and $\tilde{w}_t \equiv \frac{w_t}{s \sigma}$, while $N_t$ and $\theta_t$ are stationary and need no detrending.

### 2.1.5 Solving Method

Given the asymmetries and non-linearities our model is meant to capture, we rely on global methods to numerically solve for the equilibrium. A solution to the model can be described as policy functions for the 5 variables $C, \theta, N, Y,$ and $w$ as a function of a 3-variable state: the exogenous state of productivity $A$, and the endogenous states of lagged employment $N_{-1}$ and lagged wage $w_{-1}$. We form a discrete grid of the state-space, approximate the stochastic processes for the exogenous productivity variable using the Tauchen method, and solve the model by iteration on the policy functions. Details are provided in appendix A.

An issue arises in solving the model: on a grid of the state $(A, N_{-1}, w_{-1})$, some points of the grid necessarily feature high lagged wages and low productivity. As soon as the calibrated value of $\gamma$ is high enough (and inflation low enough) so that the downward-rigidity constraint on wages has some bite, some of these states will have firms willing to fire workers, violating the no-firing condition. However this is not to say that the no-firing constraint is likely to be violated on an equilibrium path: these states are very unlikely to occur—we check ex post that our simulated paths remain well away from these states. Solving the equilibrium in these unlikely extreme states is nevertheless necessary to calculate expectations in states that do occur with reasonable probability on an equilibrium path. We adopt the following approach: in a state where the no-firing condition fails, we assume that firms are forbidden to fire workers and simply do not hire\(^3\).

### 2.1.6 Calibration

We calibrate the model to a monthly frequency. We assume a CRRA utility $u'(C) = C^{-\sigma}$, a Cobb-Douglas production function $F(N) = N^\alpha$, and a Cobb-Douglas matching function $q(\theta) = \mu \theta^{-\eta}$. Start with the parameters that determine the steady-state of the model. For preferences, we calibrate the household’s discount factor $\beta$ to correspond to an annual risk-free interest rate of 4%, and we assume utility to be logarithmic in consumption: $\sigma = 1$. For production, we set decreasing returns to $\alpha = 2/3$ to get the standard labor share. We consider the case of no growth $g = 0$. We set the separation rate to the average $s = 3.4\%$ per month reported by Shimer (2005). We set the elasticity of the matching function to $\eta = 0.5$, in the middle of the range reported in Petrongolo and Pissarides (2001)'s survey. The parameters $\mu$ and $c$ jointly determined hiring costs. One of the two is redundant: only $c \mu^{1-\eta\sigma}$ is identified—details are provided in appendix B. We normalize $\mu$ to 1. We set $c$ so that steady-state hiring costs $c/q$ are 10% of the monthly steady-state wage $\bar{w}$, in line with what Jose and Manuel (2009) report based on the Employer Opportunity Pilot Project survey in the US. We set the last parameter, the steady-state wage $\bar{w}$, to target the average level of unemployment over the period 1964-2009: 5.8%. In the benchmark of symmetric real wage rigidity,

\(^3\)The symmetric problem may occur with the no-excess-demand condition under symmetric real wage rigidity: wages may be so much below productivity that firms are willing to hire more workers than there are. We deal with such cases in the same way: we assume that firms hire all workers but no more, and leave the wage at its value.
Table 2.1: Calibration: steady-state parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1-0.04/12</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2/3</td>
</tr>
<tr>
<td>$g$</td>
<td>2</td>
</tr>
<tr>
<td>$s$</td>
<td>0.034</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1</td>
</tr>
<tr>
<td>$c$</td>
<td>0.150</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>0.674 (DNWR)</td>
</tr>
<tr>
<td></td>
<td>0.678 (SRWR)</td>
</tr>
</tbody>
</table>

Table 2.2: Calibration: shock and wage-rigidity parameters.

<table>
<thead>
<tr>
<th></th>
<th>$\rho_a$</th>
<th>$\sigma^a_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock process</td>
<td>0.98</td>
<td>0.005</td>
</tr>
<tr>
<td>Inflation target</td>
<td>$\bar{\Pi}$</td>
<td>1+0.02/12</td>
</tr>
<tr>
<td>Downward nominal wage rigidity</td>
<td>$\gamma$</td>
<td>1</td>
</tr>
<tr>
<td>Symmetric real wage rigidity</td>
<td>$\rho$</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The steady-state is equal to the average. In the case of downward nominal wage rigidity, targeting the average of 5.8% sets the steady-state level to 4.2%. Table 2.1 sums it all up.

The other parameters only affect the dynamics of the economy: the parameters of the productivity process govern the shocks that hit the economy while the parameters of wage rigidity govern the responsiveness of the economy to these shocks. We assume the innovations $\epsilon^a_t$ are normal and calibrate $\rho_a = 0.98$ and $\sigma^a_e = 0.005$, following [Shimer (2010)]. Finally, we calibrate wage rigidity. We set $\gamma = 1$ (nominal wages cannot fall). We assume the inflation target $\bar{\Pi}$ is 2% annually. Table 2.2 sums it all up.

2.2 An Elastic String Glued Lightly to a Board

In this section, we consider the positive properties of our model. We check that individual incentives are never violated in our simulations, then contrast the properties of unemployment in the model with the natural-rate perspective, as illustrated by the benchmark of symmetric real wage rigidity. First, our model is a plucking model: unemployment always lies above its steady-state level, not around. Second, the model fits the asymmetric distribution of the unemployment rate in the data.

2.2.1 No Violation of Individual Optimality

First, we confirm that downward nominal wage rigidity does not conflict with individual rationality and equilibrium. We check that in our simulations, the no-firing constraint—and the constraint of no excess-demand of labor—is satisfied. Figure 2.1 illustrates this result (here in the case of a time trend) by plotting a simulated sample path of the wage under our main assumption of downward nominal wage rigidity, along
with the upper and lower bound defining the wage band. The wage remains always well below the level at which workers would begin to cost more to firms than they bring in. Firms do not fire workers in downturns (or more exactly do not fire more than the exogeneous destruction rate) and keep hiring, although less.

![Wage Band Graph](image)

Figure 2.1: Simulated path for wages and the wage band under downward nominal wage rigidity.

### 2.2.2 Around the Natural Rate Mean vs. Drops Below the Potential Ceiling

We turn to the properties of unemployment in our model. Figure plots a simulated unemployment rate series in the case of a time trend in productivity. We plot on the same graph the response of the unemployment rate to the same shocks under our main assumption of downward nominal wage rigidity, and in the benchmark of symmetric real wage rigidity. In addition, we superimpose the steady-state rates of unemployment—the rates that would prevailed absent any shock—under both assumptions. The figure illustrates the sharp contrast between two views of the business cycle.

In the benchmark of symmetric real-wage rigidity, unemployment fluctuates symmetrically above and below its steady-state level (set to 5.8%), which corresponds to its average. This steady-state average fits the most standard acceptations of the concept of a natural level of unemployment, which in this case can be equivalently defined as the long-run unemployment rate, the average unemployment rate, and the unemployment rate that would prevail absent any form of wage rigidity. (It is not the Non Accelerating-
Inflation Rate of Unemployment (NAIRU) however—the unemployment rate consistent with a constant inflation rate—which is instead the red plain curve since money neutrality holds under symmetric real wage rigidity.)

In contrast, with downward nominal wage rigidity unemployment always lies above its steady-state level (set to 4.2%). Decreases in productivity increase unemployment above a lower bound of about 4.6%, while increases in productivity never decrease it below because wages adjust easily upward when higher productivity shifts labor demand. The average unemployment rate is no longer the steady-state level of unemployment. Flipping the figure upside-down to go from the unemployment rate to output, the model sees business cycle fluctuations as, in the words of Milton Friedman, “an elastic string glued lightly to a board, and plucked at a number of points chosen more or less at random”. This view of business cycles is also the Old-Keynesian view, which is still present today as a vestige in the terminology of “potential output” and “output gap”—although “potential output” is now used as a synonym of “natural output”, and a “gap” is no longer meant to always be positive. Indeed, Okun (1962) defined potential output as the answer to the question: “how much output can the economy produce under condition of full employment?” Although he qualified that “the full employment goal must be understood as striving for maximum production without inflationary pressure; or, more precisely, as aiming for a point of balance between more output and greater stability, with appropriate regard for the social valuation of these two objectives.”, his use of the concept had little resemblance with either a natural rate or the NAIRU.

2.2.3 Asymmetries

The natural-rate view and the plucking model differ as to where they locate the steady-state level of unemployment—unemployment absent any shock. Simply put, on the graph of the empirical unemployment rate, the natural view would draw the steady-state in the middle, while the plucking model would draw it below. Which is right? Is it possible to distinguish empirically between the plucking model and the natural-rate view? Such a test was the initial focus of Friedman (1964). Friedman pointed at one feature of the data that speaks in favor of the plucking model. Because in the plucking model contractions can be of various sizes but expansions are returns to the potential ceiling, the amplitude of an expansion should depend on the size of the previous contraction, but the amplitude of a contraction should not depend on the amplitude of the previous expansion. In contrast, no such correlation is predicted by the natural-rate model.

Another related distinction between the two models is their predictions for the distribution of the level of the unemployment rate. Again, the plucking model speaks in favor of an asymmetry: the distribution of the unemployment rate is skewed to the right, as the unemployment rate can reach high levels, but is bounded below. In contrast, in the natural-rate view, fluctuations are symmetric and this symmetry translates to the distribution of the unemployment rate: it is not skewed.

Figure 2.3 plots the histograms of the unemployment rate predicted by the plucking model with downward nominal wage rigidity, by the natural-rate model with symmetric real wage rigidity, and the empirical
Figure 2.2: Simulated path for the unemployment rate. The blue curve is for the model under our main assumption of downward nominal wage rigidity; the red curve is for the benchmark of symmetric real wage rigidity. The shocks are the same in both cases.

An empirical histogram of the civil unemployment rate among workers aged 15 to 64 from 1970 to 2015. The empirical distribution is right-skewed, although not as much as in our calibration of the plucking model.

2.3 Costs of Business Cycles and Benefits of Stabilization Policy

We now turn to the normative implications of our model. First, we reassess the costs of business cycle fluctuations through the lens of the plucking model. Second, we consider how monetary policy can achieve the benefits of stabilization implied by the model.

2.3.1 First-Order Effect of Economic Fluctuations

In a thought-provoking exercise, Lucas (1987, 2003) asked whether a reasonable estimate of the benefits of stabilization policies justifies the attention that their design receive. He answered negatively: replacing the stochastic stream of consumption of a representative agent by a constant stream with the same mean would yield extremely small welfare gains, unlikely to compensate for the costs of stabilization.

Subsequent literature has considered whether Lucas’s result is robust to alternative assumptions on
preferences toward risk (Obstfeld (1994), Dolmas (1998), Tallarini (2000)), or to removing the assumption of perfect insurance against idiosyncratic shocks induced by the existence of complete markets (Imrohoroglu (1989), Atkeson and Phelan (1994), Krussel and Smith (1999)). Most of these papers show that such extensions can beef up the costs of business cycles, and thus the benefits of stabilization policy. Yet, because Lucas’s initial estimate is so small, finding bigger estimates does not necessarily overcome the general conclusion that fluctuations don’t matter much: these papers still find small—although not as small as Lucas’s—costs of business cycle fluctuations.

The robustness of Lucas’s result is not necessarily surprising. The contrary intuition that stabilization policies can do much—the intuition that prevailed before Lucas’s at least—relies on the presumption that they can eliminate slumps, and can do so without getting rid of the boom: that they affect not only the volatility, but also the mean level of unemployment and output. Because Lucas assumes away the possibility for policy to change the mean, there is nothing counter-intuitive or paradoxal in his result.

The question then is whether the assumption that stabilization policy can affect the mean is a reasonable assumption to entertain. We have shown that it is fully consistent with a commitment to methodological individualism, and with our current understanding of the reason for wage rigidity. In our model, replacing the process for productivity with a process with the same mean but no volatility—as in Lucas’s experiment—
would decrease the unemployment rate. In our calibration, the decrease would be from an average 5.8% when inflation is targeted to be 2%, to the steady-state level of 4.2%.

2.3.2 Greasing the Wheels of the Labor Market

Lucas’s experiment of eliminating all fluctuations is meant to give an upper-bound of the benefits of stabilization policy, abstracting from the constraints that may exist on what outcomes policy can actually achieve. Our microfounded model permits to consider specific policies, and to not assume but derive their effects. In the rest of the paper, we consider one specific such policy: monetary policy. The reliance on monetary policy to alleviate the inefficiency created by downward nominal wage rigidity is as old as the early emphasis on downward nominal wage rigidity by Tobin (1972).

We consider the effect of a simple policy choice: the inflation target. Figure 2.4 plots the reaction of the unemployment rate to the same shocks, under different values for the inflation target, from 1% to 4%. A higher inflation target decreases the average unemployment rate by facilitating the adjustment of wages. Inflation allows real wages to adjust without touching to nominal wages, and as such alleviates the constraints of downward nominal wage rigidity. Inflation greases the wheels of the labor market.

In our calibration, increasing the inflation target from 2% to 4% decreases average unemployment from 5.8% to 4.9%. Decreasing the inflation target to 1% instead increases average unemployment to 7.2%.

2.4 Conclusion

We build a plucking model of the business cycle that captures the pronounced skewness of the unemployment rate. Unemployment arises from search frictions in the labor market and is skewed due to downward nominal wage rigidity. In contrast to earlier models of downward nominal wage rigidity, our model is fully consistent with optimizing behavior and therefore robust to the Barro (1977) critique.

We show that in our model eliminating business cycles has large welfare benefits since it lowers the average unemployment rate. Our simulations imply that eliminating all fluctuations could lower the average unemployment rate by about 1.5 percentage points. Downward nominal wage rigidity provides one rationale for a positive inflation rate. Our results imply that moving from a 2% inflation target to a 4% inflation target would lower the average unemployment rate by roughly 1 percentage point, while lowering the inflation target to 1% would raise the average unemployment rate by about 1.5 percentage points.
Figure 2.4: Simulated path for the unemployment rate for different levels of the inflation target. The shocks are the same in all cases.
Chapter 3

Coordination under Ambiguity

Introduction

The importance of the dispersion and slow diffusion of information has recently received a renewed interest in macroeconomic models of the business cycles. Dispersed information has important consequences in environments where coordination is essential, be it in monetary models—the strategic complementarity in price-setting (Woodford (2003a), Mankiw and Reis (2002b))—or real models—the strategic complementarity arising from demand spillovers (Angeletos and La’O (2009, 2013); Lorenzoni (2009)). Just as reduced-form models of strategic complementarities (Cooper and John (1980); Cooper (1999), and references therein) had shed light on the mechanisms at play in macroeconomic models of coordination failures popular in the New Keynesian literature, reduced-form models of strategic complementarities under dispersed information (Morris and Shin (2002), Angeletos and Pavan (2007)) have shed light on the mechanisms at play in these new macroeconomic models. In such models, where an agent’s welfare depends partly on how close his action is to others’, introducing imperfect and dispersed information lets higher-order beliefs—what I believe you believe and so on—take central stage. Because public information is a better predictor of others’ actions, public signals become a coordination device, driving the economy away from fundamentals. Some of the general insights that have emerged from these models are as follows. First, the stronger strategic complementarities are—the stronger the motive to do the same thing as others—the less dependent on fundamentals the equilibrium is. Second, releasing public information decreases the equilibrium dependence on fundamentals. Third, the equilibrium reliance on private and public information, and resulting departure from fundamentals, may however be efficient.

While these models have increased our understanding of the role of dispersed information in business cycles, in such complex interactions as pricing and production decisions which depend on the whole macroeconomy, the uncertainty agents face is likely to consist not only of risk, but also of ambiguity—ignorance of the true model of the economy. The assumption of rational expectations on which beauty contests rely
assumes however that agents fully trust their models, ruling out ambiguity as a source of uncertainty. I consider a beauty contest model where agents face both risk and ambiguity, building on the robust control literature (Hansen and Sargent (2008), and references therein) to model the way agents fear model misspecifications. Agents have more trouble uncovering the true model of the economy when the economy features more unforecastable fluctuations—when there is more noise in the regressions they run to learn the model. As a consequence, ambiguity is endogenously determined in equilibrium: each agent’s action depends on the ambiguity he faces; but in turn, ambiguity depends on agents’ actions. Ultimately, the noise one agent faces is another’s private information. Section 1 and 2 detail the set-up and how ambiguity is endogenously determined in equilibrium.

Section 3 analyses the positive properties of the model. The two main results revisit the role of public signals through their interaction with ambiguity-aversion. First, in addition to increasing with strategic complementarities, the dependence on public signals increases with ambiguity aversion. When agents face ambiguity in the actions of others, they are less willing to take the risk to respond to fundamentals and seek to act more like others. To do so, they rely more on public information, driving the economy away from fundamentals. For any degree of fundamental strategic complementarity, at the limit where the concern for model misspecification increases, the equilibrium depends on public signals only, and not at all on fundamentals. Second, releases of public information do not necessarily drive the economy away from fundamentals, as happens under rational expectations, and as some authors have worried could be detrimental to welfare. Whenever ambiguity-aversion is high enough, releasing public information increases agents’ understanding of the world, decreases ambiguity, and makes them more willing to take the risk to respond to fundamentals: the economy tracks fundamentals more closely.

Section 4 considers the normative properties of the equilibrium. I show that, although the equilibrium depends more on public signals than it does under rational expectations, it does not depend on public signals enough. Under rational expectations, the equilibrium of the game I consider is efficient; no longer so when agents fear model misspecifications. In equilibrium, each agent fails to internalize that when he reacts to his private information and idiosyncratic shock, he creates noise for everyone else, decreasing everyone’s understanding of the world, and increasing ambiguity. The equilibrium generally features an overproduction of ambiguity.

Although I stress the interpretation of robust control as a model of ambiguity-aversion, the multiplier preferences I use are consistent with expected utility, leading to an interpretation of the model that involves risk-aversion only. Seen under this light, the model describes how the positive and normative results of coordination games change when one moves away from the quadratic preferences on which much of the reduced-form literature on dispersed information has focused. When the certainty equivalence no longer holds, agents’ aversion to risk moves the equilibrium toward public signals, but not as much as efficiency requires: the equilibrium is too risky.

The paper builds on both the literature on dispersed information in coordination games, and the literature
on decision-theory under ambiguity, especially robust control. Models of dispersed information have wide applications in macroeconomics. Early applications include the possibility of multiple equilibria in currency attacks (Morris and Shin (1998), Angeletos and Werning (2006)) and the persistence of inflation in models of price-setting (Woodford (2003a), Mankiw and Reis (2002b)). More recently, several articles have stressed how these models also apply to a central strategic complementarity in decentralized market economies: demand linkages. Angeletos and La’O (2009, 2013) and Lorenzoni (2009) show how common errors in expectations can drive business cycles, while Benhabib et al. (2015, 2013) show how dispersed information can give rise to multiple equilibria. Schaal and Taschereau-Dumouchel (2015) consider investment decisions and show how an economy can fall into a coordination trap, remaining in a recession for a very long time. The present paper is closest to the reduced-form branch of the literature, such as Morris and Shin (2002) and Angeletos and Pavan (2007).

On the ambiguity-aversion side, this paper explicitly relies on the robustness literature as reviewed in Hansen and Sargent (2008). Many of the early models on robustness are decision-theoretical and not equilibrium models. Closer to the spirit of this model are papers that consider ambiguity-averse agents within an equilibrium model. Ilut and Schneider (2014) consider the effect of ambiguity shocks in a real business cycles model where ambiguity—how large the set of models agents consider is—is exogenous. Backus et al. (2015) also consider a real business cycles model with ambiguity-averse agents, this time using Klibanoff et al. (2009)’s recursive smooth ambiguity to model ambiguity-aversion. Again, ambiguity is exogenous in their model. In contrast, Bidder and Smith (2012) use the multiplier preferences of robust control in a similar real business cycles model, and stress the dependence on risk of the worst-case scenario agents fear. Part of my contribution is to explain why robust control generates such a dependence of ambiguity on risk. In addition, none of these models considers strategic complementarities between heterogenous agents or dispersed information, which is at the heart of this paper.

### 3.1 Set-up

Each agent $i$ in a continuum $i \in [0,1]$ seeks to pick an action $y_i$ so as to minimize a weighted average of the expected distances to two targets. First, an exogenous, idiosyncratic and observed target $a_i$. This fundamental is the sum of an aggregate component $\bar{a}$, with mean $m$ and variance $1/\kappa_a$, and an idiosyncratic component $b_i$, independently and identically distributed with zero mean (so that the average fundamental is $\bar{a}$) and variance $1/\kappa_b$: $a_i = \bar{a} + b_i$. I assume all fundamentals, as well as the signals to be defined, to be jointly gaussian. Second, the average action of others:

$$\bar{y} = \int y_i di.$$  

(3.1)
Given an information set $\omega_i$, agent $i$ seeks to minimize its expectation, with respect to his information set $\omega_i$, of the quadratic loss:

$$L(y_i, \bar{y}, a_i) = (1 - \alpha) \left( \frac{(y_i - a_i)^2}{2} \right) + \alpha \left( \frac{(y_i - \bar{y})^2}{2} \right), \quad \alpha \in (0, 1). \quad (3.2)$$

The weight $\alpha$ on the average action parameterizes the degree of strategic complementarities. I assume it to be positive, and strictly so: for $\alpha = 0$, there is no tradeoff between the two targets and no uncertainty in targeting $a_i$, so that the solution is trivially $y_i = a_i$.

This reduced-form model is emblematic of the literature on beauty contests. One feature worth stressing is that I assume an idiosyncratic and observed fundamental target $a_i$. Agents know their own fundamentals and the only variable they need to expect is the aggregate action $\bar{y}$: uncertainty arises only from the need to coordinate with others. My reason for doing so is that in many applications, this is the central information problem agents face. For instance, in the application to demand linkages developed in Angeletos and La'O (2009, 2013), the fundamental $a_i$ is a firm’s idiosyncratic technology shock which we can think of as being relatively well known to the firm; the firm’s uncertainty bears on aggregate demand $\bar{y}$. Technically, it is also easier to treat the case where agents need to form expectations on a single variable.

The information set $\omega_i$ of agent $i$ consists in private and public information. The idiosyncratic signal $a_i$ is in itself a private signal of $\bar{a}$. I let there be additional private information by considering a second private signal $\tilde{x}_i = \bar{a} + \tilde{\varepsilon}_i^x$, with precision $\kappa_x$, such that $a_i$ and $\varepsilon_i^x$ are independent, and $\varepsilon_i^x$ are independent across agents. Private information can be summed up by the single signal $x_i = \frac{\kappa_x}{\kappa_a} a_i + \frac{\tilde{\kappa}_x}{\kappa_x} \tilde{x}_i$, with precision $\kappa_x \equiv \kappa_x + \kappa_b$. There exists a public signal $z$ which also takes the form of average fundamental plus noise: $z \equiv \bar{a} + \varepsilon^z$, with precision $\kappa_z$, such that $\varepsilon^z$ is independent of $\varepsilon_i^x$ for all $i$. When this prior is not flat ($\kappa_a \neq 0$), the unconditional mean $m$ plays the role of a second public signal. The weighted average $z' = \frac{\bar{\kappa}_a}{\kappa_a + \kappa_z} z + \frac{\kappa_a}{\kappa_a + \kappa_z} m$ of the two public signals $z$ and $m$, with precision $\kappa_{z'} = \kappa_a + \kappa_z$, can be treated as the unique public signal.

### 3.1.1 Rational Expectations non robust solution

As a benchmark, I derive the rational expectations solution of the game.

**Definition 1** A RE equilibrium is strategies for all $i$ such that all agents minimize their expectations of their loss (3.2) where the aggregate action $\bar{y}$ is defined by (3.1) and beliefs are the equilibrium ones.

The unique equilibrium can be derived along the same lines as in Morris and Shin (2002). The objective of agent $i$ is concave, so that the first-order condition selects the unique best response of agent $i$. It is a

---

1The early literature focused on a common and imperfectly observed fundamental (e.g. Morris and Shin (2002); Angeletos and Pavan (2007)). Allowing for idiosyncratic fundamentals has become common. For instance, Bergemann et al. (2015) analyze how the information structure shapes aggregate volatility in a framework that allows for idiosyncratic shocks.

2Demand spillovers were also one of the major application of games of strategic complementarities in the early new keynesian literature. See e.g. Hart (1982); Heller (1986); Cooper (1994); Bryant (1983); Roberts (1987); Kiyotaki (1988); Shleifer (1986); Murphy et al. (1989).
weighted average of its two targets \( a_i \) and \( \bar{y} \):

\[
y_i = (1 - \alpha)a_i + \alpha E_i(\bar{y}).
\] (3.3)

So that the average action is:

\[
\bar{y} = (1 - \alpha)\bar{a} + \alpha \bar{E}(\bar{y}),
\] (3.4)

where \( \bar{E}(.) \equiv \int E_i(.)di \) designates the average expectation. It is practical to rely on the apparatus of formal power series that is standard in time-series. Note \( H \) the average higher-order belief operator; the equation the average action solves can be written:

\[
(I - \alpha H)\bar{y} = (1 - \alpha)\bar{a}.
\] (3.5)

Restricting to solutions such that \( \alpha^k \bar{E}^{(k)}(\bar{y}) \) tends to zero with \( k \), that is excluding that agents believe that agents believe ... that the aggregate action is infinite, the polynomial can be inverted to give:

\[
\bar{y} = (1 - \alpha)\sum_{k=0}^{\infty} \alpha^k \bar{E}^{(k)}(\bar{a}),
\] (3.6)

where \( \bar{E}^{(k)}(.) = H^k \) is the \( k \)th-order average belief operator, defined recursively by \( \bar{E}^{(k)}(.) = \bar{E}(\bar{E}^{(k-1)}(.)) \).

The higher-order beliefs of \( \bar{a} \) are easy to solve for by induction:

\[
\bar{E}^{(k)}(\bar{a}) = \mu^k \bar{a} + (1 - \mu^k)z',
\] (3.7)

where \( \mu = \kappa_x/\kappa \) is the bayesian weight on private information. Beliefs of higher-orders depend more and more on the public signal \( z' \) at the expense of the private signal \( x_i \). Plugging in this expression of higher-order beliefs (3.7) in (3.6) gives the unique solution for \( \bar{y} \). This solution for \( \bar{y} \) can be plugged into (3.3) to give the corresponding unique best-response function.

**Lemma 4** There exists a unique RE equilibrium. The aggregate action follows \( \bar{y} = \Phi^{*}\bar{a} + (1 - \Phi^{*})z' \), with:

\[
\Phi^{*} \equiv \frac{1 - \alpha}{1 - \alpha \mu},
\] (3.8)

Best-responses take the linear form \( y_i = \phi_a^{*}a_i + \phi_x^{*}x_i + \phi_z^{*}z' \), with:

\[
\phi_a^{*} \equiv 1 - \alpha,
\] (3.9)

\[
\phi_x^{*} \equiv \Phi^{*} - (1 - \alpha),
\] (3.10)

\[
\phi_z^{*} \equiv 1 - \Phi^{*}.
\] (3.11)
With no public information $\mu = 1$, the equilibrium aggregate action follows fundamentals $\bar{y} = \bar{a}$, regardless of the degree of strategic complementarities $\alpha$. With any positive amount of public information $\mu < 1$, the dependence on fundamentals $\Phi^*$ is decreasing from 1 to 0 as $\alpha$ increases from 0 to 1: public signals combined with strategic complementarities move the economy away from fundamentals to common errors in expectations. Besides, the dependence on fundamentals $\Phi^*$ is increasing in $\mu$ the weight of private information. Releasing public information thus decreases the weight of private information and decreases the dependence on fundamentals.

Is the equilibrium efficient? Define the efficient allocation as the one that agents would pick if they could commit to an action ex ante, before the realization of their idiosyncratic shocks and signals. Under this veil of ignorance, this is akin to minimizing a utilitarian social loss functions, weighted by the probability of occurrence of the idiosyncratic fundamentals and information. For the welfare criterion to be relevant, and as stressed by Angeletos and Pavan (2007), it must incorporate the same information constraints as in equilibrium: the action $y_i$ of agent $i$ needs to be measurable with respect to his information set $\omega_i = \{x, z\}$; the aggregate action needs to be measurable with respect to the aggregate variables $a$ and $z$. Formally, an efficient allocation solves the program:

$$\min_{(y_i(x, z))} \mathbb{E} \left((1 - \alpha) \frac{(y_i - a_i)^2}{2} + \alpha \mathbb{E}_i \left(\frac{(y_i - \bar{y})^2}{2}\right)\right),$$

$$\text{st.} \bar{y} = \int y_i d_i.$$

As shown in the appendix, the equilibrium is efficient under rational expectations.

**Lemma 5** The rational expectations equilibrium corresponds to the unique efficient allocation.

### 3.1.2 Concerns for model misspecifications and robust RE equilibrium

I now introduce a concern for model misspecifications in agents’ preferences. I follow the literature on robust control (Hansen and Sargent (2008), and references therein) to model how agents value and choose actions in the face of ambiguity. I consider that agents are unsure of their model, not that they worry that others may have a different model (Woodford (2010) is an example of the latter).

Agent $i$ does not know where $\bar{y}$ lies on the real line. Although it is always possible to represent agent $i$’s uncertainty of the location of $\bar{y}$ through a conditional distribution $f(\bar{y}|\omega_i)$ on $\mathbb{R}$, Knight (1921)’s classical distinction between risk and ambiguity stresses that it is completely legitimate to do so only when uncertainty consists of risk, defined as a “quantity susceptible of measurement”\(^3\). A measure is a probability distribution; call it a model.

\(^3\)To be sure, risk and uncertainty in Knight’s own words. The word uncertainty is however commonly used in economics without a precise definition, to refer to several things, including risk. I follow the common practice in the literature and use uncertainty to refer to any form of not knowing, and ambiguity to refer to Knightian uncertainty.
Definition 2 A model is any probability distribution on $\bar{y} \in \mathbb{R}$, conditional on available information $\omega_i$, $f(\bar{y} | \omega_i)$.

Note that the definition of a model does not connote a theoretical model: the definition is agnostic on how the agent came to form his model. It could be from theory—reasoning in a representation of the world in which it is possible to deduce relationships by making assumptions and thinking about their implications for equilibrium—or pure atheoretical econometrics—observing the past predictive power of observables contained in $\omega_i$—or any combination of the two. Risk is then the fluctuations in $\bar{y}$ perceived by a model $f(\bar{y} | \omega_i)$. As far as decision-theory is concerned, it matters little whether the distribution $f(\bar{y} | \omega_i)$ is the objective one $f^*(\bar{y} | \omega_i)$, that is whether it corresponds to the actual distribution of equilibrium fluctuations: the agent can be wrong in his perception of risk, without altering the manner he behaves under uncertainty.

Ambiguity is the uncertainty the agent faces on top of the uncertainty embedded in a model because he is unsure of the model: the inability to reduce the set of possible models $\mathcal{M} = \{ f(. | \omega_i) \}$ to a singleton. Robust control proposes an approach in two steps to describe how an agent makes decisions under ambiguity. First, given a favor model $f_0$ and a degree of ambiguity $A$, the agent restricts the set $\mathcal{M}$ of possible models to the neighborhood of a favored model $f_0$, $B(f_0, A) \equiv \{ f / D(f||f_0) \leq A \}$. The single parameter $A$ parameterizes the lack of confidence in the favored model $f_0$. When $A = 0$, the agent fully trusts his model, while when $A$ tends to infinity, he makes no prior restrictions on the set of possible models. The metric used is relative entropy (the Kullback-Leibler distance) defined as:

$$D(f||f_0) \equiv E^f \left( \log(f) - \log(f_0) \right),$$

for any distribution $f$ that is absolutely continuous with respect to $f_0$; distributions that are not are considered infinitely distant from $f_0$. Relative entropy is the most standard metric between distributions used in information theory. Although there is always some arbitrariness in defining a metric, it is an appealing measure of the distance between models as it considers as close models that would be hard to distinguish empirically with any inference method based on the likelihood function. Indeed, given the unknown model $f$, picking a model $f_0$ so as to minimize the Kullback-Leibler distance $D(f||f_0)$ to the true model $f$ is equivalent to maximizing $E^f \log(f_0)$. This is unobserved as $f$ is unknown, but maximizing the sample analogue with an iid sample boils down to maximizing the loglikelihood. Relative entropy selects the models that ML estimation, but also bayesian methods and other estimation methods that rely on the likelihood, would have difficulty distinguishing from the favored model $f_0$. Although the definition of a model was agnostic about where models come from, restricting the space of possible models through relative entropy highlights the empirical way of coming to a model.

Second, within the restricted class of models $B(f_0, A)$, the agent makes decisions according to the maximin

---

4 On information theory, see e.g. Cover and Thomas (2006). Relative entropy is not a symmetric metric, so that there exist in principle two different measures of relative entropy: $E_f(\ln(f/f_0))$ and $E_{f_0}(\ln(f_0/f))$. The robust control literature considers the former: when entertaining the possibility that the true density is $f$, the agent measures the difference between the two distributions using the distribution $f$. 

---
principle, which has long been offered as a way to model decisions under ambiguity (e.g. \textcite{Wald1945,Bellman1956}). The agent’s desire to make decisions that work well not only under his favored model but also under neighboring models is captured by considering the worst-case model (from the agent’s perspective). It is formally as if the agent was playing a zero-sum game against nature, fearing that nature would systematically react to his action $y_i$ by picking the model that hurts him most.

Instead of a constraint problem that fixes the size of the entropy ball $A$, I consider a multiplier problem: increasing the upper-bound on relative entropy has a constant cost $\lambda > 0$, and the agent chooses the size of the set of models given the exogenous cost $\lambda$. I do so merely for analytical convenience. Concerns for model misspecifications are still parameterized by a single free parameter $\lambda$ representing confidence in the model.

To sum up, the robust program of agent $i$ is:

$$\min_{y_i} \max_{f_i(y|\omega_i)} \left(1 - \alpha\right) \frac{(y_i - a_i)^2}{2} + \alpha E_i^f \left(\frac{(y_i - \bar{y})^2}{2}\right) - \lambda D(f_i||f_0^i),$$

where $E_i^f(.)$ designates the expected value under the neighboring model with density $f_i^i$.

As it is, agent $i$’s program is a function of exogenous beliefs parameterized by the favored model $f_0^i$, in the logic of a temporary equilibrium. The equilibrium concept disciplines beliefs in much the same way as rational expectations do, by imposing that the favored model be the equilibrium one. Just as with rational expectations, an equilibrium can be seen as a fixed point between beliefs and actual outcomes. I therefore still refer to the equilibrium concept as rational expectations, although robust rational expectations.

**Definition 3** A robust RE equilibrium with (common) degree of ambiguity $\lambda$ consists in:

1. Best responses such that each agent $i$, for a favored model $f_0^i$, solves the robust program (3.14).

2. The aggregate action $\bar{y}$ is given by (3.1), and agent $i$’s favored model $f_0^i$ is the equilibrium one.

Rational expectations stricto sensu correspond to the absence of ambiguity $\lambda = \infty$. This highlights the two components of rational expectations: first the agent’s uncertainty consists of risk only—he faces no ambiguity—second he is correct in his assessment of risk. A robust rational expectations equilibrium weakens the requirement on beliefs: an agent’s favored assessment of risk is the equilibrium one, but he may face uncertainty stemming from model ambiguity.$^5$

---

$^5$ In line with what the agent needs to expect, a model is a distribution on $\bar{y}$, and relative entropy bounds the distance between distributions on $\bar{y}$, not distributions on the exogenous $a$. A nice feature of relative entropy is that bounding the distance between distributions on $\bar{a}$ would not affect the relative entropy metric. However, as the agent perceives only $\bar{y}$, it is more natural to phrase the problem this way. Besides, the invariance property is likely to depend on the linear structure of the model, and on the fact that $\bar{y}$ and $\bar{a}$ have the same dimension.

$^6$ Bayesian decision theory dismisses the practical relevance of the distinction between risk and ambiguity, as defended most famously on axiomatic grounds by \textcite{Savage1954}. The argument is that agents should form a distribution on the set of possible models, deduce from it a unique probability on $\bar{y}$, and from there rely on standard decision theory—(subjective) expected utility. However, this way of taking ambiguity into account is ruled out in a rational equilibrium model: when equating $f_i(\bar{y}|\omega_i)$ to the true equilibrium distribution $f^*(\bar{y}|\omega_i)$, rational expectations assume both that the agent perceives the risk correctly, and that the ambiguity he faces is nil. In contrast, robust control allows for an equilibrium concept that imposes that agents have a correct assessment of risk, but do not necessarily face zero ambiguity.
I solve the model using a guess and verify/identify approach. I assume agent $i$ has a favored model $f^0_i(\bar{y}|\omega_i)$ that is Gaussian with variance $V_0$ independent of his information set $\omega_i$:

$$\bar{y}|\omega_i \sim \mathcal{N}\left(E^{f_0}(\bar{y}|\omega_i), V_0\right).$$

($V_0$ is common to all agents). I then check that these beliefs generate gaussian conditional distributions of this form, and identify the function $E^{f_0}(\bar{y}|.)$ and $V_0$ to determine the equilibrium. I make no claim to the uniqueness of equilibrium in a broader class of distributions, although the gaussian-linear structure of the model would make the existence of non gaussian-linear equilibria surprising. Note that in the case of rational expectations studied in the previous section, the uniqueness of equilibrium was proven without any prior restriction to a class of models, and the gaussian-linear nature of the unique equilibrium was proved and not assumed.

3.1.3 Risk interpretation

Although I will mostly focus on the interpretation of the model as one of ambiguity aversion, it is useful to be aware that the multiplier preferences of robust control can equivalently be interpreted as standard (expected-utility) decision under risk, up to increasing the degree of risk aversion embedded in the utility function. The interpretation of the multiplier preferences in (3.14) as “risk-sensitive” (Hansen and Sargent (1995), Tallarini (2000)) is justified by the possibility to rewrite them as:

$$\min_{y_i} \lambda \log\left(E_i\left(e^{\frac{1}{\lambda}L(y_i, \bar{y}, a_i)}\right)\right),$$

where the loss function $L$ is the one defines in (3.2)—a proof is in the appendix. This representation is of the expected utility form, with the more convex Bernouilli utility function $\exp\left(\frac{1}{\lambda}L(.\right))$. The model will shed some light on the reason for this equivalence.

3.2 Equilibrium characterization

Although it is formally possible to solve for the best-response by taking first-order conditions with respect to beliefs and action simultaneously, solving the program sequentially—first deriving worst-case beliefs, plugging them into the program, then maximizing in the action $y_i$—is insightful.

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7 The fixed point problem goes: from beliefs on the aggregate action to the best response; from the best response to the aggregate action; from the aggregate action to beliefs on the aggregate action. Formally, it is possible to start with a guess at any of these three points. Starting at beliefs stresses the view of an equilibrium as a fixed point in beliefs, and stresses that agents do not need to have any knowledge of the underlying structure of the game. He does not even need to be aware that the distribution of $\bar{y}$ is the result of some other people’s actions.
3.2.1 Worst-case beliefs

The maximization in the distribution \( f(\tilde{y}|\omega_i) \) is a concave program, so that a solution is characterized by the first-order conditions. Noting \( \zeta \) the Lagrange multiplier on the \( \int f = 1 \) constraint, the Lagrangian corresponding to agent \( i \)'s robust program (3.14) is:

\[
\min_{y_i} \max_{f(\tilde{y}|\omega_i)} (1 - \alpha) \left( \frac{(y_i - a_i)^2}{2} \right) + \int \left( \frac{\alpha(y_i - \bar{y})^2}{2} - \lambda \left( \log(f(\tilde{y}|\omega_i)) - \log(f_0(\tilde{y}|\omega_i)) \right) - \zeta \right) f(\tilde{y}|\omega_i) d\tilde{y}.
\]

(3.17)

The first-order condition on \( f(\tilde{y}|\omega_i) \) is:

\[
\alpha \left( \frac{y_i - \bar{y}}{2} \right) - \lambda \left( \log(f(\tilde{y}|\omega_i)) - \log(f_0(\tilde{y}|\omega_i)) + 1 \right) - \zeta = 0.
\]

(3.18)

This implies that the worst-case belief is the following normal distribution:

\[
\tilde{y}|\omega_i \sim N \left( \frac{E_{f_0}(\tilde{y}|\omega_i)}{V_0} \frac{\alpha y_i}{\lambda} \frac{1}{\lambda} \frac{1}{V_0} \right),
\]

(3.19)

provided the following condition to guarantee that the variance is positive:

\[
\lambda > \alpha V_0.
\]

(3.20)

When condition (3.20) is not satisfied, the supremum is infinite and worst-case beliefs are not defined. For any action \( y_i \), agent \( i \) fears that any of his actions will be followed by a worst-case scenario that hurts him infinitely. There is no meaningful equilibrium in this case. Condition (3.20) imposes that considering alternative models is costly enough so that this does not happen. To see the effect of a concern for model misspecification under the maximin principle, consider the distortion in the mean:

\[
E_i^f(y|\omega_i) - E_{f_0}^f(\tilde{y}|\omega_i) = - \left( \frac{1}{\lambda V_0 - 1} \right) (y_i - E_0^f(\tilde{y}|\omega_i)).
\]

(3.21)

Worst-case beliefs systematically act to exaggerates the distance of agent \( i \)'s action \( y_i \) to its expectation of the uncertain target \( \tilde{y} \). First, consider the sign of the distortion. If agent \( i \) is at the right of the expectation of the target given by his favored model, he will fear that the true model is one that predicts a target even further on the left. Second, consider the amplitude of the distortion. It is determined by the factor \( \left( \frac{1}{\lambda V_0 - 1} \right) \). Quite expectedly, the factor is decreasing in \( \lambda \)—increasing in the concern for model misspecification. It is increasing in \( \alpha \) as model misspecifications are more of a concern when the agent initially cares more about \( \tilde{y} \).

But—and it is key—the inflating factor is also increasing in \( V_0 \), the conditional variance. This essential
feature tells that in an economy that is more risky—an economy with a higher conditional variance $V_0$ of the endogenous $\bar{y}$—agent $i$ will also be facing more ambiguity—as measured by a higher mean distortion between the favored and worst-case models. In the vocabulary of econometrics, parameter and specification uncertainty (ambiguity) increase with innovation uncertainty (risk). This is because more volatility makes it more difficult to detect statistical regularities, something the relative entropy metrics takes into account. If $\bar{y}$ depends on variables the agent does not observe—noise from his perspectives—not only does the forecasting power of the variables in his information set $(x_i, z)$ diminishes: the greater noise also reduces the confidence he puts in the relationship between $(x_i, z)$ and $\bar{y}$. As an extreme example, consider the case of a nil variance $V_0 = 0$. Regressing $\bar{y}$ on the information set $(x_i, z)$ would yield a $R^2$ of one and nil standard deviations: the relationship is deterministic. In this situation, the agent should fear no ambiguity, regardless of his concern for model misspecifications $\lambda$. Indeed, relative entropy assesses that all distributions are infinitely distant from a dirac distribution. The intuition carries over with a positive variance $V_0$: relative entropy assesses that as $V_0$ increases, the agent should be less and less confident in his model. This parallels how the standard deviations of the coefficients in a regression of $\bar{y}$ on $(x_i, z)$ increase with the variance of noise $V_0$. To see the point more formally, note that the relative entropy between two gaussian distributions is:

$$D(f\|f_0) = \frac{1}{2} \left( \frac{(E_f - E_{f_0})^2}{V_{f_0}} \right) + \frac{1}{2} \left( \frac{V_f}{V_{f_0}} - 1 + \ln \left( \frac{V_{f_0}}{V_f} \right) \right).$$

(3.22)

Normal distributions are characterized by their first two moments. The first term in the relative entropy accounts for the distance between the two distributions due to the difference in their means; it is zero if and only if the two distributions have the same mean. The second term accounts for the distance between the two distributions due to differences in their variances; it is zero if and only if the two distributions have the same variance. What accounts for the issue at hands is that the mean term has the variance $V_f$ at the denominator: two normal distributions with different means will be harder to distinguish if they have a higher variance.

Note that relative entropy focuses on the dependence of the measure of uncertainty in the variance of the unobservables but abstracts from its dependence in the sample size. In contrast, usual measures of the degree of confidence in one’s estimates, such as standard deviations in classical statistics, also decrease as data accumulate. Whether agents who learn the model from data converge to rational expectations is an important question, addressed in the adaptive learning literature (Evans and Honkapohja (2001) and references therein). On the way to the asymptotic rational expectations limit however, standard deviations are non-zero. Whenever standard deviations are positive, agents face some ambiguity. How they react to this amount of ambiguity necessarily requires assumptions on their behaviors. Robust control reduces this assumption to the single parameter $\lambda$ which can be interpreted as a preference parameter, much like risk aversion. Away from the asymptotic limit, the dependence of the degree of confidence in the sample size could matter for practical purposes if standard deviations were known to have decreased in macroeconometrics.
over time. It does not seem that they have however, partly because the possibility of structural breaks pushes macroeconometricians not to rely on data too far in the past, limiting the increase in the size of the sample from which they learn past regularities.

### 3.2.2 Best response

I now turn to how ambiguity—the worst-case scenario—affects the choice of the optimal action $y_i$. Straightforward algebra shows that:

$$E_i^f (y_i - \bar{y})^2 = C(V_0)^2 (y_i - E_i^{f_0} (\bar{y}))^2 + C(V_0) V_0,$$

$$D(f||f_0) = \frac{1}{2} \left[ V_0 \left( \frac{\alpha}{\lambda} C(V_0) \right)^2 (y_i - E_i^{f_0} (\bar{y}))^2 + \left( C(V_0) - 1 - \ln(C(V_0)) \right) \right],$$

where $C(V_0) \equiv \frac{1}{1 - \frac{\alpha V_0}{\lambda}} \geq 1$. (3.25)

Plugging these into the objective, minimizing the loss in $y_i$ is equivalent to minimizing:

$$\min_{y_i} (1 - \alpha) \left( \frac{(y_i - a_i)^2}{2} \right) + \alpha C(V_0) E_i^{f_0} \left( \frac{(y_i - \bar{y})^2}{2} \right) - \frac{\lambda}{2} \left( C(V_0) - 1 - \ln(C(V_0)) \right).$$

As the last term is taken as given by agent $i$, the objective is of the same form as the one with no concern for robustness, but with an inflated weight on the uncertain target $\bar{y}$. It was $\alpha$; it is now $\alpha C(V_0) \geq \alpha$. To understand why, remember that worst-case beliefs systematically act to exaggerate the distance of agent $i$’s action $y_i$ to its expectation of the uncertain target $\bar{y}$, all the more so that his action is far from the expected target. Without any concern for robustness, the agent shoots somewhere in between his two targets $a_i$ and $\bar{y}$. But if he fears model misspecifications, he then considers alternative models that exaggerate the distance of his action to the uncertain target $\bar{y}$. He therefore adjusts his action closer to $\bar{y}$. As a result, he acts as if he put more weight on the uncertain target $\bar{y}$. Note that robustness matters only because of the existence of a trade-off between two targets. Were he to try to reach a unique uncertain target, agent $i$’s concern for robustness would not affect his action: he would still act at the best forecast of his best model.

Because the objective is of the same form as in the rational-expectations benchmark, so does the best-response. It is of the same linear form as in (3.3):

$$y_i = (1 - \alpha'(V_0)) a_i + \alpha'(V_0) E_i^{f_0} (\bar{y}|\omega_i).$$

with a weight:

$$\alpha'(V_0) \equiv \frac{\alpha C(V_0)}{1 - \alpha + \alpha C(V_0)}.$$ (3.28)

where condition (3.20) guarantees $\alpha'(V_0) \in (0, 1)$. The weight $\alpha'$ on the target $\bar{y}$ no longer reduces to strategic
complementarities $\alpha$ alone: it now also depends on ambiguity. Both higher strategic complementarity and higher ambiguity are reasons to put more weight on $\bar{y}$. Ambiguity aversion therefore argues in favor of higher calibration for the parameter of strategic complementarity in models of price-setting and demand spillovers, a calibration which has been shown to be of great importance. The dependence of $\alpha'$ in ambiguity follows the same intuition as the dependence of the mean distortion (3.21): $\alpha'$ decreases in $\lambda$, but also increases in $V_0$.

3.2.3 Aggregate action and identification

The derivation in section 3.1.1 still applies, replacing $\alpha$ by $\alpha'(V_0)$:

$$\bar{y} = \Phi(V_0)\bar{a} + (1 - \Phi(V_0))z',$$

(3.29)

where $\Phi(V_0) \equiv \frac{1 - \alpha'(V_0)}{1 - \alpha'(V_0)\mu} = \frac{1 - \alpha}{1 - \alpha + \alpha(1 - \mu)C(V_0)}$.

(3.30)

As in the case of rational expectations, the coefficient $\Phi$ measures how much $\bar{y}$ depends on fundamental, as opposed to the public signal. The key difference is that $\Phi$ is now a function of $V_0$, because risk determines ambiguity, which in turn determines individual and aggregate behavior. But equilibrium imposes that the favored model of agent $i$ be the one effective in equilibrium. From the expression (3.29) for $\bar{y}$, I verify that the equilibrium distribution of $\bar{y}$ conditional on $\omega_i$ is linear and gaussian:

$$\bar{y}|\omega_i \sim N\left(\Phi\mu x_i + (1 - \Phi \mu)z', \Phi^2/\kappa\right),$$

(3.31)

and coefficients can be identified:

$$E_i^{f_0}(\bar{y}|\omega_i) = \Phi\mu x_i + (1 - \Phi \mu)z',$$

(3.32)

$$V_0 = \frac{\Phi^2}{\kappa}.$$  

(3.33)

The risk $V_0$ in the endogenous variable $\bar{y}$ depends on the risk $\kappa^{-1}$ in the exogenous fundamental, but also in how the equilibrium $\bar{y}$ loads on the fundamentals $\bar{a}$—the parameter $\Phi$. Because $\Phi$ results from agents’ actions, the model is one of endogenous risk: an economy that depends on imperfectly observed fundamentals is more risky than one that depends on observed public signals. But because risk $V_0$ affects how much ambiguity agents face, the model is also one of endogenous ambiguity. One agent’s behavior affects his neighbor’s utility—and ultimately his action—by affecting his neighbor’s understanding of the world. If he responds to private signals that he is the only one to observe, his actions will be pure noise for others, creating a world that is difficult to understand. If instead he responds to public signals, his actions will become predictable, making the world easier to understand.

Equation (B.2) gives $V_0$ as a function of $\Phi$—how the dependence on fundamentals determines risk—when
equation (B.1) gives $\Phi$ as a function of $V_0$—how risk determines the dependence in fundamentals. Jointly, they determine $\Phi$ and $V_0$.

**Lemma 6** An equilibrium is characterized as $(\Phi, V_0)$ that satisfy equations (B.1) and (B.2), with condition (3.20).

The best-response function $y_i = \phi_a a_i + \phi_x x_i + \phi_{z'} z'$ can then be recovered from $\Phi$ and $V_0$:

\[
\phi_a = 1 - \alpha', \quad (3.34)
\]
\[
\phi_x = \Phi - (1 - \alpha'), \quad (3.35)
\]
\[
\phi_{z'} = 1 - \Phi. \quad (3.36)
\]

### 3.3 Public information and ambiguity-aversion

Now that the equilibrium is characterized at the crossing of the two curves in lemma 6—plotted in figure 3.1—I consider how ambiguity-aversion affects agents’ reliance on private and public information, and the dependence of the aggregate action $\bar{y}$ on the average fundamental $\bar{a}$.

#### 3.3.1 Ambiguity-aversion as a driver toward public signals

There are two cases to distinguish: the equilibrium with private information only ($\mu = 1$), and the equilibrium with public information ($\mu < 1$). Because the unconditional mean $m$ plays the role of a public signal, the case of private information corresponds to both $\kappa_z = 0$ and $\kappa_a = 0$: it requires a flat, uninformative prior. Although the case of a flat prior is of little practical relevance, considering the case of private information only is helpful to later interpret the effect of public information. With no public signal $z'$, the aggregate action $\bar{y}$ cannot depend on $z'$ and need be equal to the average fundamental $\bar{a}$: $\Phi = 1$—equation (B.1) is horizontal. The crossing of the two curves merely equates the risk in the endogenous action $\bar{y}$ to the risk in the exogenous fundamentals $\bar{a}$, $1/\kappa_x$. Risk and ambiguity do impact individual action however: the weight $\alpha'$ on the target $E_{t+1}^0(\bar{y})$.

**Proposition 5** With no public information $\kappa_z = \kappa_a = 0$, and for any $\alpha \in (0,1)$, there exists a unique equilibrium whenever $\lambda > \alpha/\kappa_x$, given by $\Phi = 1$, $\phi_{z'} = 0$ and:

\[
\phi_x = \alpha' = \frac{\alpha}{\alpha + (1 - \alpha) \left(1 - \frac{\alpha}{\lambda \kappa_x}\right)}, \quad (3.37)
\]
\[
\phi_a = 1 - \alpha'. \quad (3.38)
\]

The condition $\lambda \kappa_x > \alpha$ is simply condition (3.20): for an equilibrium to exist, agents cannot be too ambiguity-averse.
Figure 3.1: Graphical illustration of the equilibrium. The equilibrium is at the crossing of equation (B.1) and equation (B.2). The figure is for $\alpha = 0.5$. 
With public information (including the case of public information only $\kappa_x = 0$), the existence and uniqueness of equilibrium is always guaranteed: equation (B.1) is decreasing from $\Phi^*$ to 0 as $V_0$ increases from 0 to $\frac{1}{\alpha}$, while equation (B.2) is increasing.

**Proposition 6** With public information $\kappa_z + \kappa_a > 0$, and for any $\alpha \in (0, 1)$, there exists a unique equilibrium for any $\lambda > 0$.

With public information, risk and the dependence on fundamentals become jointly determined in equilibrium in a meaningful way. Just as when there is only private information, higher risk induces higher ambiguity, which leads agents to track the unknown action of others more closely. But now, putting more weight on the action of others means putting more weight on the public signal, which does not get washed out when aggregating individual behaviors: higher risk $V_0$ induces a lower dependence on fundamentals. The decrease in the dependence on fundamentals in turn decreases risk, as $\bar{y}$ depends more on public signals that are known to everyone, but not enough to wholly counteract the effect: the equilibrium always feature less dependence on fundamentals than under rational expectations.

More generally, the more ambiguity-averse agents are—the lower $\lambda$—the less does the equilibrium depend on fundamentals. At the limit where the concern for model misspecifications becomes infinite, the equilibrium depends on public signals only, even when there is little strategic complementarities.

**Proposition 7** With public information $\kappa_z + \kappa_a > 0$,

- $\Phi$ is increasing in $\lambda$, from zero as $\lambda$ tends to zero to $\Phi^*$ as $\lambda$ tends to infinity.
- $\phi_a$ is increasing in $\lambda$, from zero as $\lambda$ tends to zero to $\phi_a^* = 1 - \alpha$ as $\lambda$ tends to infinity.
- $\phi_z'$ is decreasing in $\lambda$, from 1 as $\lambda$ tends zero to $\phi_z'^* = 1$ as $\lambda$ tends to infinity.
- $\phi_x$ can be non-monotonic, since as $\lambda$ increases, agents focus less on the uncertain target, but also more on private relative to public information. Nevertheless, it always tends to zero as $\lambda$ tends to zero, and to its value under no concern for robustness $\phi_x^*$ as $\lambda$ tends to infinity.

Figures 3.2 illustrates the comparative statics results for a value of strategic complementarity $\alpha = 0.5$ and equal precision of public and private information $\kappa_x = \kappa_z'$. It plots the three coefficients $\phi_a$, $\phi_x$, $\phi_z'$ as the concern for model misspecification increases (as $\lambda$ decreases to zero).

### 3.3.2 Releasing public information

Under rational expectations, the equilibrium dependence of $\bar{y}$ on fundamentals $\bar{a}$ depends only on the relative quantity of private vs. public information $\mu$, not on the absolute levels of information $\kappa_x$ and $\kappa_z'$—equation (3.8). When agents fear model misspecifications, this is no longer the case, as the quantity of information

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8 There is no convenient closed-form solution for the equilibrium. Plugging in equation (B.2) into (B.1), the equilibrium $\Phi$ appears as the unique solution to a cubic equation that satisfies condition (3.20). Cubic equations do have closed-form solutions, but these are messy and not very insightful.

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Figure 3.2: Equilibrium best response coefficients as a function of confidence $\lambda$. Dotted horizontal lines are the coefficients under no concern for robustness. The figure is for $\alpha = 0.5$ and $\kappa_a = \kappa_z$. 

75
\(\kappa\) affects the risk, hence the ambiguity, in \(\bar{y}\)—equation (B.2). When private information \(\kappa_x\) increases, both curves shift up in figure 3.1 as under rational expectations, equation (B.1) shifts up because the weight on private information increases; but in addition, equation (B.2) shifts up because risk, hence ambiguity decreases. As a result, the dependence on fundamental unambiguously increases.

Of greater interest are the comparative statics in the precision of the public signal \(\kappa_z\), since it corresponds to the policy experiment of releasing public information on the state of the economy, an issue central bankers, for instance, repeatedly face. Under rational expectations, the sole effect of releasing public information is to decrease \(\mu\)—(B.1) shifts down—therefore decreasing the dependence of \(\bar{y}\) on fundamentals. It is this move away from fundamentals that some authors, the first of whom [Morris and Shin (2002)], have worried could be detrimental to welfare. However, when agents fear model misspecifications, there is a countervailing effect: when information is released—either private or public—risk diminishes. It is then easier to learn the model of the economy, and agents face less ambiguity. As a result, the are less worried to miss the target of the aggregate action of others \(\bar{y}\), and track their individual fundamental \(a_i\) more. Formally, equation (B.2) shifts up. The overall impact of a release of public information is thus ambiguous. The following proposition states that the second effect dominates when ambiguity-aversion is high enough.

**Proposition 8** The dependence on fundamentals \(\Phi\) increases as more public information is released if and only if \(\lambda \kappa_x < \alpha (1 - \alpha)^2\).

Figure 3.3 illustrates these comparative statics in both cases.

### 3.3.3 Risk interpretation

I have focused so far on the standard interpretation of the multiplier preferences of robust control as ambiguity-aversion. As mentioned in the previous section however, these preferences are consistent with expected utility, and thus can alternatively be interpreted as modelling risk-aversion only. Under this interpretation of preferences, the higher weight on the uncertain target \(\bar{y}\) in the objective (3.26) is to be read differently. In arbitrating between reaching his two targets, the agent trades off the certain loss of missing \(a_i\) against the uncertain loss of missing \(\bar{y}\). Therefore, the more risk-averse the agent is—the lower \(\lambda\)—the more he targets the aggregate action \(\bar{y}\). This effect of risk-aversion is however missed in the literature on dispersed information that relies on quadratic preferences, because the certainty equivalence applies: best-response functions are independent of risk. The positive results in this section can be read as the consequences of risk-aversion on the equilibrium reliance on private and public information when going beyond the convenient but ultimately restrictive case of the certainty equivalence: risk-aversion increases the dependence of the economy on public signals, away from fundamentals.

Why exactly do risk-aversion and ambiguity-aversion play the same role in this model? The exact equivalence between the robust control multiplier preferences and expected utility is a bit of a technical coincidence—the equivalence does not apply if one considers a constraint problem that bounds the relative
Figure 3.3: Equilibrium best response coefficients as a function of the precision of public information $\kappa_z$. 
entropy between models instead of a multiplier problem. There is however a deep reason why agents who dislike ambiguity ends up disliking risk, a reason the model sheds light on. The relative entropy metric takes into account that agents are more unsure of the model when models are hard to distinguish empirically; learning from data is in turn harder in an economy with more unpredictable fluctuations—a riskier economy. Whether agents dislike risk per se, or because risk makes it hard to learn the true model, does not matter for the predictions of the model.

3.4 The Overproduction of Ambiguity

Ambiguity drives the equilibrium away from fundamentals relative to the rational expectations equilibrium, as agents react to the ambiguity in \( \bar{y} \) by putting more weight on \( \bar{y} \), and therefore on public information. In doing so, agents jointly reduce the amount of ambiguity in equilibrium. It is however a non-intended consequence of everyone’s actions: no individual agent internalizes his impact on ambiguity. Is the equilibrium amount of ambiguity efficient? I adjust the welfare criterion used under rational expectations to incorporate ambiguity aversion. To make it relevant, the planner cannot directly affect the worst-case scenario that each agent fears, since it is part of their preferences: the planner cannot just tell agents that they have nothing to worry about and hope to be believed on his word—the criterion rules out the confidence fairy. However, the planner can engineer a less risky allocation which will have agents worry less about model misspecifications. Relative to the welfare objective under rational expectations, this is precisely this new concern that I want to focus on. Formally, an efficient allocation is solution to:

\[
\min_{(y_i(x_i,\bar{a},z))} \mathbb{E}^0 \left( \max_{(f_i(\bar{y},\omega_i))} \left\{ (1 - \alpha) \frac{(y_i - a_i)^2}{2} + \alpha \mathbb{E}^f_i \left( \frac{(y_i - \bar{y})^2}{2} \right) - \lambda D(f_i || f^0_i) \right\} \right),
\]

\[\text{st.} \bar{y} = \int y_i d_i.\]

Again, I restrict to solutions where agent \( i \)'s favored model is Gaussian with variance \( V_0 \) independent of his information set \( \omega_i \), as in equation (3.15). Worst-case beliefs of agent \( i \) given \( (y_i, \bar{y}) \) are still given by equation (3.19), provided condition (3.20). Injecting these beliefs in the objective (3.39), the problem of the planner reduces to:

\[
\min_{(y_i(x_i,\bar{a},z))} (1 - \alpha) \mathbb{E}^0 \left( \frac{(y_i - a_i)^2}{2} \right) + \alpha C(V_0) \mathbb{E}^0 \left( \frac{(y_i - \bar{y})^2}{2} \right) - \frac{1}{2} \lambda \left( C(V_0) - 1 - \ln(C(V_0)) \right),
\]

\[\text{st.} \bar{y} = \int y_i d_i.\]
Lemma 7 The efficient best-response of agent \( i \) satisfies:

\[
(1 - \alpha + \alpha C(V_0))y_i = (1 - \alpha)a_i + \alpha C(V_0)E_i(\bar{y}) - \delta E_i(\bar{y} - \bar{E}(\bar{y})),
\]

\[
\delta = C'(V_0)\alpha[E(y_i - \bar{y})^2 - V_0] \geq 0, \text{ and } > 0 \text{ provided } \lambda < \infty.
\]

Agent \( i \)'s efficient best-response corrects the equilibrium best-response $$(3.27)$$ by the term $${-\delta E_i(\bar{y} - \bar{E}(\bar{y}))}$$, which is the marginal social cost of increasing \( y_i \) due to the resulting increase in ambiguity. It is the product of \( \delta \)—the marginal social cost of increasing \( V_0 \)—and \( \bar{y} - \bar{E}(\bar{y}) \)—the marginal increase in \( V_0 \) with \( \bar{y} \), which increases when \( y_i \) increases. Agent \( i \)'s action impacts everybody else’s understanding of the model through his impact on \( V_0 \), which in turn affects everybody else’s welfare because agents are ambiguity-averse. Efficiency commands that he should take into account this understanding externality and attempt to close the gap between \( \bar{y} \) and \( \bar{E}(\bar{y}) \), that is to reduce the average unexpected component of \( \bar{y} \). Because agent \( i \) does not observe the gap \( \bar{y} - \bar{E}(\bar{y}) \), efficiency commands that he should take his best expectation of the term. For instance, if agent \( i \) expects others to hold on average too pessimistic expectations on \( \bar{y} \) (\( \bar{y} > \bar{E}(\bar{y}) \)), he should temper his private incentives to act so as not to widen the gap \( \bar{y} - \bar{E}(\bar{y}) \) further.

The presence of the corrective term in the best-response function hints at the inefficiency of the equilibrium. To prove it, I solve for the efficient allocation implied by the efficient best-response. Averaging $$(3.41)$$ over \( i \), the aggregate action \( \bar{y} \) satisfies:

\[
(1 - \alpha + \alpha C)\bar{y} = (1 - \alpha)\bar{a} + \alpha C\bar{E}(\bar{y}) - \delta \bar{E}(\bar{y} - \bar{E}(\bar{y})).
\]

The efficient aggregate action follows a second-order linear difference equation, as opposed to the first-order difference equation the equilibrium allocation satisfies. Using the higher-order polynomial \( H \), this can be written \( \bar{y} = P(H)\bar{a} \), with:

\[
P(H) = (1 - \alpha)\left( (1 - \alpha + \alpha C)I - (\alpha C - \delta)H - \delta H^2 \right)^{-1}.
\]

Since \( H^k\bar{a} = \mu^k\bar{a} + (1 - \mu^k)z' \), it follows that \( \bar{y} = \Phi\bar{a} + (1 - \Phi)z' \) with:

\[
\Phi = P(\mu) = \frac{1 - \alpha}{1 - \alpha + \alpha C(1 - \mu) + \delta \mu(1 - \mu)}.
\]

Just as for equilibrium, the (common) conditional variance of \( \bar{y} \) needs satisfy $$(3.20)$$ and $$(B.2)$$. An efficient allocation is characterized by equations $$(3.45)$$ and $$(B.2)$$, provided condition $$(3.20)$$. Simply put, the efficient allocation can be seen at the crossing of two curves, just the as equilibrium allocation. A technical difficulty
however is that the new term $\delta$ depends on $E(y_i - \bar{y})^2$, which is itself a function of $\Phi$:

$$E(y_i - \bar{y})^2 = (\Phi^2 - \phi^2_a) \frac{1}{\kappa_x} + \phi^2_a \frac{1}{\kappa_b}. \tag{3.46}$$

$$\phi_a(V_0) = \frac{1 - \alpha}{1 - \alpha + \alpha C(V_0)}. \tag{3.47}$$

Using equation (B.2) to replace $\Phi^2$ by its equilibrium value $V_0\kappa$ in equation (3.46), equation (3.45) expresses $\Phi$ as an explicit function of $V_0$:

$$\Phi = \frac{1 - \alpha}{1 - \alpha + \alpha C(V_0)(1 - \mu) + \alpha C'(V_0)\mu(1 - \mu) \left( \frac{\phi^2_a(V_0)}{\kappa_a} - \frac{1}{\kappa_x} \right) + V_0 \left( \frac{\kappa_a}{\kappa_x} - 1 \right)}. \tag{3.48}$$

It follows that:

**Lemma 8** The efficient allocation is characterized as $(\Phi, V_0)$ that satisfy equations (3.48) and (B.2), with condition (3.20).

Figure 3.4 illustrates the determination of the efficient allocation at the crossing of the two curves. Comparing equation (B.1) and (3.48), the equilibrium appears efficient if and only if $\delta\mu(1 - \mu) = 0$. More generally, the appendix shows that equation (3.48) always lies below equation (B.1), and strictly so except when $\delta\mu(1 - \mu) = 0$.

**Proposition 9** The efficient allocation is unique.

- Under rational expectations $(\lambda = \infty)$, the equilibrium is efficient (lemma 5).
- If there is only private information $(\mu = 1)$, the equilibrium is efficient.
- If there is only public information $(\mu = 0)$, the equilibrium is efficient.
- In all other cases, the equilibrium is inefficient. The equilibrium features too much dependence on fundamentals to the detriment of public information $(\Phi^{eq} > \Phi^{eff})$, and too much conditional volatility $(V_0^{eq} > V_0^{eff})$.

With only private information, there is no way to move the economy toward a better understood world, so the efficiency of the equilibrium is not surprising. But since the prior always acts as a public signal, the absence of public information requires the unlikely assumption of a flat prior $\kappa_a = 0$, which makes the case irrelevant for practical concerns. With only public information, there is no uncertainty whatsoever in this model—all agents face the same fundamental shock $\bar{a}$ that they all observe—so that the efficiency of the equilibrium is not surprising: again, the case is irrelevant for practical concerns.

In all relevant cases, the equilibrium is inefficient when agents fear model misspecifications. Agents do not internalize the effect of their actions on how easy it is to learn the equilibrium. Agents react to their private information too much, creating too much noise for others, creating too much ambiguity on the model...
Figure 3.4: Graphical illustration of the efficient allocation. The efficient allocation is at the crossing of equation (3.48) and equation (B.2). The figure is for $\alpha = 0.5$.

governing $\tilde{y}$. Simply put, there is an overproduction of ambiguity in equilibrium: agents would rather live in a world they understand better, even if it means living in a world that does not track fundamentals as well.

Again, it is possible to interpret the model as one of risk-aversion instead of ambiguity-aversion. The inefficiency is then to be interpreted differently. Agents do not internalize that in reaction to their private signals, they make the economy more risky for everybody else. The efficiency of the equilibrium in the benchmark case of quadratic preferences ($\lambda = \infty$) appears as the tree that hides the forest. Away from the knife-edge case of the certainty equivalence, the equilibrium is generically inefficient. This matters since much of the literature on the efficient use of information, such as Angeletos and Pavan (2007), has focused on the case of quadratic preferences (of a more general form than the ones covered here).

### 3.5 Conclusion

I have looked at a beauty contest with ambiguity-averse agents to reassess the role of public information in coordinating agents’ actions. When agents do not fully understand the world they live in, public information differs from private information along a new dimension. A world in which others respond to signals that I also observe is one that I can easily understand; a world in which others respond to signals only they
can see is one whose functioning I have trouble uncovering. In equilibrium agents fail to internalize the negative externality of their reliance on private information and private fundamentals on everybody else’s understanding of the model: the world is inefficiently ambiguous.

The key feature of the model is that ambiguity is endogenous: it increases with risk, which depends on whether others behave in a predictable way. Alternatively however, the model can be interpreted as one where agents directly care about risk, simply because they are risk-averse. A world where others react to signals I observe is one that is less risky, something I can value per se. The reliance on private information and fundamentals makes for a riskier world, which has positive and normative implications when we move away from the certainty equivalence of the quadratic framework popular in the literature.

The present model is a static one. In a dynamic setting, an increase in risk can be expected to have long-lasting effects on ambiguity: even a short-lived episode of macroeconomic turmoil—volatile GDP and asset prices—could lead agents to become less confident in their understanding of the economy for an extended period of time. A dynamic extension of the present framework may be able to model the often vaguely-defined notion of a “crisis of confidence”. It could do so without considering it as an exogenous disturbance, but as an endogenous and long-lasting increase in ambiguity following a short-lived increase in risk. The endogenous reaction of ambiguity could generate long-lasting effects of risk shocks, something the uncertainty literature—which most often equates uncertainty to risk only—typically has difficulty finding (Bloom (2009)).
Bibliography


Appendix A

Appendix to Chapter 1

A.1 Proof of the existence of the equilibrium under full information

To complete the proof that all firms charging $P_i = \mathcal{M}(\alpha + \theta)W/A_i$ is indeed an equilibrium, I need to check that when all firms charge $P_i$, firm $k$’s profits have a global—not just local—optimum at $P^k = P_i$. Under the assumption of a Pareto distribution for $F$, for $P^k \geq P_i$ firm $k$ faces a constant-elastic demand curve with elasticity $\alpha + \theta$, and it is easy to check that the derivative of profits is negative over $P^k > P_i$. The more involved verification is to check that profits are increasing over $P^k < P_i$. Equivalently, using the relative price $p = P^k/P_i$ and noting $c = W/A_iP_i = \frac{1}{\mathcal{M}(\alpha + \theta)}$, I need to check that:

\[(p - c)(2 - p^\alpha)p^{-\theta}\]  (A.1)

is increasing over $p < 1$. The derivative of (A.1) has the sign of:

\[G(p) = (2 - p^\alpha) + \left(1 - \frac{c}{p}\right)(-\alpha p^\alpha - \theta(2 - p^\alpha)).\]  (A.2)

Both term are strictly positive for $p < c$, but the second term is negative for $p > c$. In this case however, since $1 - \frac{c}{p} < 1 - c = \frac{1}{\alpha + \theta}$:

\[G(p) > (2 - p^\alpha) + \frac{1}{\alpha + \theta}(-\alpha p^\alpha - \theta(2 - p^\alpha)) = 2(1 - p^\alpha)\frac{\alpha}{\alpha + \theta} > 0,\]  (A.3)

which ends the proof.
A.2 Proof of the existence of the equilibrium under asymmetric information

The proof is the same as the one under full information of the previous appendix. For \( P^k \geq P_i \) firm \( k \), faces a constant-elastic demand curve with elasticity \( \alpha + \theta \), and the derivative of profits is negative over \( P^k > P_i \).

To show profits are increasing for \( P^k < P_i \), it is equivalent to show, noting \( p = P^k / P_i \) and \( c = W/A_i P_i \), that:

\[
(p - c) [\lambda + (1 - \lambda)(2 - p^\alpha)] p^{-\theta}
\]

is increasing over \( p < 1 \). The derivative of (A.4) has the sign of:

\[
G(p) = \left(2 - \lambda - (1 - \lambda)p^\alpha\right) + \left(1 - \frac{c}{p}\right) \left(-\alpha(1 - \lambda)p^\alpha - \theta \left[2 - \lambda - (1 - \lambda)p^\alpha\right]\right).
\]

Both term are strictly positive for \( p < c \), but the second term is negative for \( p > c \). In this case however, since \( 1 - \frac{c}{p} < 1 - c \leq 1 - \frac{1}{M[(1 - \lambda)\alpha + \theta]} = \frac{1}{(1 - \lambda)\alpha + \theta} \):

\[
G(p) > \left(2 - \lambda - (1 - \lambda)p^\alpha\right) + \frac{1}{(1 - \lambda)\alpha + \theta} \left(-\alpha(1 - \lambda)p^\alpha - \theta \left[2 - \lambda - (1 - \lambda)p^\alpha\right]\right) = (2 - \lambda)(1 - p^\alpha) \frac{\alpha(1 - \lambda)}{\alpha(1 - \lambda) + \theta} > 0,
\]

which ends the proof.

A.3 Characterization of the adaptive rational-expectations equilibrium

First, I derive firm \( k \)'s best-response to any market price \( P_i \) it expects. (This in effect extends the problem treated in section 3, where \( P_i \) was restricted to be an equilibrium price; now it can be any price.) Firm \( k \) still maximizes present profits \( \pi^k = \left(P^k - \frac{W}{A_i}\right) D^k(P^k) \). There are three cases. First, if the optimal response \( P^k \) is greater than \( P_i \), firm \( k \) faces a demand curve with constant elasticity \( \alpha + \theta \) around \( P^k \), and sets \( P^k = \mathcal{M}(\alpha + \theta) \frac{W}{A_i} \). This occurs when \( P_i \leq \mathcal{M}(\alpha + \theta) \frac{W}{A_i} \). Second is the case \( P^k = P_i \), treated in section 3. As seen there, this occurs when \( P_i \in [\mathcal{M}(\theta + \alpha)W/A_i, \mathcal{M}(\theta + \alpha(1 - \lambda))W/A_i] \), then firm \( k \) sets \( P^k = P_i \). Finally, if \( P^k \) is less than \( P_i \), profits are differentiable at \( P^k \) and the optimal \( P^k \) is determined by the first-order condition again. It solves:

\[
-\left(1 - \frac{W}{A_i P^k}\right) \left[\theta \left(1 - \lambda\right) \left(2 - \left(\frac{P^k}{P_i}\right)^\alpha\right) + \lambda\right] + \alpha(1 - \lambda) \left(\frac{P^k}{P_i}\right)^\alpha + (1 - \lambda) \left(2 - \left(\frac{P^k}{P_i}\right)^\alpha\right) + \lambda = 0.
\]

(A.7)
which implicitly defines the best-response $P^k$ as a function of $P_i$. The function is only implicit, but the contrapositive of the two previous cases guarantees that this last case occurs when $P_i \geq M(\theta + \alpha(1 - \lambda))W/A_i$, completing the partition of mutually exclusive cases. Taken together, the three cases define, for any given value of $W/A_i$, the best-response price $P^k$ to any price $P_i$. It is illustrated in figure A.1.

![Best-response $P^k$ to a market price $P_i$](image)

Figure A.1: Best-response $P^k$ to a market-price $P_i$. Calibration: $\alpha = 6$, $\theta = 1$, and $\lambda = 1/2$.

This best-response function defines the sequence $(P_s)_{s \geq 0}$ of mental iterations, starting from the initial value $P_0 = P_{-1,i}$, the market price in the past period. Consider now the price to which the iterative process converges. Again, distinguish three cases. If $P_0 = P_{-1,i} \leq M(\alpha + \theta)\frac{W}{A_i}$—the case when the past price is too low for equilibrium—firms set $P^k = M(\alpha + \theta)\frac{W}{A_i}$ in the first round of the iteration, which is an equilibrium price: the process stops at round 2 with $P_i = M(\alpha + \theta)\frac{W}{A_i}$: a price increase. If $P_0 = P_{-1,i} \in [M(\theta + \alpha)W/A_i, M(\theta + \alpha(1 - \lambda))W/A_i]$—the case when the past price is still consistent with equilibrium—firms set $P^k = P_{-1,i}$ in the first round of iteration: the process stops at round 1 and firms keep their prices fixed $P_i = P_{-1,i}$. Finally, if $P_{-1,i} \geq M(\theta + \alpha(1 - \lambda))W/A_i$—the case when the past price is too low for equilibrium—then the process defines a strictly decreasing sequence that remains within $[M(\theta + \alpha(1 - \lambda))W/A_i, +\infty)$. By the monotone convergence theorem, the iterative process converges (in an
infinity of rounds) to the unique possible fixed point $P_i = \mathcal{M}(\theta + \alpha(1 - \lambda))W/A_i$: a price decrease.
Appendix B

Appendix to Chapter 2

B.1 Solution Algorithm

We describe the details of our solving method, here in the case of our main assumption of downward nominal wage rigidity—the method is similar for symmetric real wage rigidity. Incorporating the assumptions of CARA preferences and Cobb-Douglas technology, and a monetary policy of constant inflation $\bar{\Pi}$, solving for the equilibrium consists, given the exogenous productivity perturbations $\ln(A_t)$ in solving for the 5 endogeneous variables—some of which have been detrended—$N_t$, $\hat{C}_t$, $\hat{Y}_t$, $\theta_t$, $\ln(\hat{w}_t)$, defined by the five-equation system:

\[ e^{\ln(A_t)} \alpha N_t^{\alpha-1} = e^{\ln(\hat{w}_t)} + \frac{c e^{\ln(A_t)}}{q(\theta_t)} - (1 - s) \beta \hat{C}_t E_t \left( \hat{C}_{t+1} e^{(1-\sigma)g} \frac{c e^{\ln(A_{t+1})}}{q(\theta_{t+1})} \right), \]  
\[ \hat{w}_t = \max \left\{ \bar{w} e^{\ln(A_t)}, \frac{\gamma}{e^{\sigma}} \hat{w}_{t-1} \right\}, \]  
\[ 1 - N_t = (1 - f(\theta_t))[1 - (1 - s)N_{t-1}], \]  
\[ \hat{Y}_t = e^{\ln(A_t)} F(N_t), \]  
\[ \hat{Y}_t = \hat{C}_t + \frac{c e^{\ln(A_t)}}{q(\theta_t)} [N_t - (1 - s)N_{t-1}]. \]

B.1.1 Steady-State

A non-stochastic steady-state equilibrium with $A_t = 1$ is such that $(N, \theta)$ solve the two-equation system:\[\alpha N^{\alpha-1} = \bar{w} + \frac{c}{q(\theta)} [1 - \beta (1 - s) e^{(1-\sigma)g}], \] \[f(\theta) = \frac{N}{1 - (1 - s)N}. \]

\[1\]For the downwardly-rigid nominal-wages equilibrium, we assume $\bar{\Pi} e^{\theta} \geq \gamma$ otherwise there is no steady-state equilibrium.
Once $N$ and $\theta$ are solved for, $\tilde{Y}$ and $\tilde{C}$ are given by:

$$\tilde{Y} = N^\alpha,$$  \hspace{1cm} (B.8)

$$\tilde{C} = \tilde{Y} - \frac{c}{q(\theta)} s N.$$  \hspace{1cm} (B.9)

**B.1.2 Iteration Method**

A solution to the model can be described as policy functions for the 5 variables $\tilde{C}, \theta$ (or equivalently and for convenience, $1/q$), $N$, $\tilde{Y}$, and $\ln(\tilde{w})$ as a function of the 3-variable state: the exogenous state of productivity $\ln(A)$, and the endogeneous states of lagged employment $N_{-1}$ and lagged wage $w_{-1}$. We form a discrete $21 \times 21 \times 21$ grid of the state-space, approximate the stochastic processes for the exogenous productivity variable using the Tauchen method, and solve for the policy functions at each point of the grid by policy function iteration. Specifically, we start from an initial guess on the policy functions $C$ and $1/q$, and use this guess to calculate $C_{t+1}$ and $1/q_{t+1}$, and from there the expectation term in equation (B.1). In calculating the expectation term, we need to evaluate the policy function at points that are not on the grid. We do so through linear interpolation. Given this expectation term, we solve for the equilibrium in this state of the grid—details are provided below—and store the solution for $C$ and $\theta$. Done in all states of the grid, this provides a new guess for the policy functions. We repeat until convergence of the policy functions.

**B.1.3 Solving Within a Loop**

At each iteration of the iterative algorithm, and at each point of the grid, we need to solve for the system (A.1)-(A.5), given the expectation term $E_t^1$. To do so, we define a function of $N$ in the following way:

- Through equation (A.3): $\theta(N)$.
- Through equation (A.4): $Y(N)$.
- Through equation (A.5): $C(N)$.

Equation (A.1) is then an equation in $N$ alone that can be solved for $N$. (We use the bisection method to do so.)

**B.1.4 Dealing with the Constraints**

A solution $N$ to equation (A.1) needs to lie between $(1-s)N_{t-1}$ and 1. Otherwise, the firm would need to fire people and the no-firing condition \(2.4\), or no excess-demand condition \(2.6\) would fail. In the unlikely states—which do not occur on the sample paths in our simulations—where the no-firing constraint fails, we assume that the firm does not hire nor fire workers and thus set $N_t = (1-s)N_{t-1}$. In the unlikely states—which do not occur on the sample paths in our simulations—where the no-excess demand constraint fails, we assume that the firm hires all the available workers and thus set $N_t = 1$. 

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B.2 Normalization of $\mu$

We show that only the calibration of $c \mu^{-\frac{1}{n}}$—not of $\mu$ and $c$ separately—matters. The two parameters $c$ and $\mu$ only show up as $c/q(\theta)$ and $f(\theta)$ in the system characterizing the equilibrium. Because $q = \mu^{-\frac{1}{n}} f^{\frac{n}{n-1}}$, we have that:

$$\frac{c}{q} = \left( c \mu^{-\frac{1}{n}} \right) f^{\frac{n}{n-1}}, \quad \text{(B.10)}$$

so that only $c \mu^{-\frac{1}{n}}$ is identified. We thus normalize $\mu$ to one.
Appendix C

Appendix to Chapter 3

C.1 Proof of lemma \[5\]

The program of the planner is:

$$\min_{(y(x_i,z'),\bar{y}(\bar{a},z'))} \left( \frac{1-\alpha}{2} \int (y_i(x_i,z') - a_i)^2 P(x_i,z')dx_i dz' + \frac{\alpha}{2} \int \int (y_i(x_i,z') - \bar{y}(\bar{a},z'))^2 P(x_i,z',\bar{a})dx_i dz' d\bar{a}, \right.$$ 

$$\text{st.} \forall (\bar{a},z'), \bar{y}(\bar{a},z') = \int y_i(x_i,z') P(x_i|\bar{a},z') dx_i.$$ \hspace{1cm} (C.1)

It is concave. Noting $\beta(\bar{a},z') P(\bar{a},z')$ the Lagrange multiplier on the $(\bar{a},z')$ constraint, the Lagrangian is:

$$\mathcal{L} = \frac{1-\alpha}{2} \int (y_i(x_i,z') - a_i)^2 P(x_i,z')dx_i dz' + \frac{\alpha}{2} \int \int (y_i(x_i,z') - \bar{y}(\bar{a},z'))^2 P(x_i,z',\bar{a})dx_i dz' d\bar{a}$$ 

$$- \int \beta(\bar{a},z') \bar{y}(\bar{a},z') P(\bar{a},z')d\bar{a}dz' + \int \beta(\bar{a},z') y_i(x_i,z') P(x_i,z',\bar{a})dx_i dz' d\bar{a}.$$ \hspace{1cm} (C.2)

The first-order conditions are:

$$/y_i : (1-\alpha)(y_i - a_i) + \alpha(y_i - E_i(\bar{y})) + E_i(\beta) = 0,$$ \hspace{1cm} (C.3)

$$/\bar{y} : \beta = 0.$$ \hspace{1cm} (C.4)

Therefore, $y_i = (1-\alpha)a_i + \alpha E_i(\bar{y})$, which is the same best-response function as the equilibrium one, therefore yielding the same allocation.
C.2 Proof of the EU representation of robust multiplier preferences

For any density \( f^* \),

\[
D(f\| f_0) = D(f\| f^*) + E^f \log \left( \frac{f^*}{f_0} \right). 
\] (C.5)

Pick \( f^* \) defined such that \( \log \left( \frac{f^*(\bar{y})}{f_0(\bar{y})} \right) = \frac{L(\bar{y})}{\lambda} - \log \left( E^{f_0} (e^{\frac{L(\bar{y})}{\lambda}}) \right) \). Then:

\[
E^f(L) - \lambda D(f\| f_0) = -\lambda D(f\| f^*) + \lambda \log \left( E^{f_0} (e^{\frac{L}{\lambda}}) \right). 
\] (C.6)

The robust multiplier preferences maximize this function in \( f \). Note that only the first term of the right-hand side depends on \( f \), and has a maximum of zero reached in \( f = f^* \). Hence:

\[
\max_f E^f(L) - \lambda D(f\| f_0) = \lambda \log \left( E^{f_0} (e^{\frac{L}{\lambda}}) \right). 
\] (C.7)

C.3 Proof of proposition 7

Note that \( \lambda \) is a shifter of (B.1) only, through its negative impact on \( C(V_0) = \frac{1}{1-\alpha^{2}} \). As \( \lambda \) increases from 0 to \( \infty \), \( \Phi \) increases from 0 to \( \Phi^* \), and \( V_0 \) increases. The coefficient \( \phi_z = 1 - \Phi \) decreases, from 1 to \( \phi_z^* \). Given equation (B.1), \( \Phi^* \) increases means that \( C \) decreases. Therefore, \( \phi_a = \frac{1-\alpha}{1-\alpha + \alpha C} \) increases, from 0 to \( \phi_a^* \). The last coefficient \( \phi_x = \Phi - \phi_a \) is a bit more tricky as it can be non-monotonic. Differentiating \( \phi_a = \frac{1-\alpha}{1-\alpha + \alpha C} \) and \( \Phi = \frac{1-\alpha}{1-\alpha + \alpha C(1-\mu)} \), it appears that \( \frac{\partial \phi_x}{\partial \lambda} \) has the sign of \( (1-\alpha)^2 - (\alpha C)^2(1-\mu) \). Since \( C \) is decreasing in \( \lambda \), \( \frac{\partial \phi_x}{\partial \lambda} \) has the sign of \( (\alpha C)^2(1-\mu) - (1-\alpha)^2 \). This last expression is positive as \( \lambda \to 0 \) and decreasing in \( \lambda \). It follows that \( \phi_x \) is non-monotonically increasing in \( \lambda \) if and only if \( \alpha^2(1-\mu) < (1-\alpha^2) \).

C.4 Proof of proposition 8

Differentiating equation (B.1) and (B.2) in \( \Phi \), \( V_0 \) and \( \kappa_z \), and eliminating \( dV_0 \) between the two equations, the implicit function theorem gives:

\[
\frac{\partial \Phi}{\partial \kappa_z} = \frac{\phi_a C}{\kappa^2} \left( \frac{1}{\kappa} (1-\mu) CV_0 \kappa - \kappa_a \right) = \frac{1}{1-\alpha + \alpha (1-\mu) C} + \frac{\alpha^2}{\kappa} (1-\mu) C \frac{2 \Phi^2}{\kappa}. \] (C.8)

It follows that \( \frac{\partial \Phi}{\partial \kappa_z} \) has the sign of \( \alpha V_0 - \mu \lambda \), that is \( \alpha \Phi^2 - \kappa_z \lambda \). Equation (B.1) and (B.2) can be combined to characterize the equilibrium \( \Phi \) as the unique root that lies between 0 and 1 of the polynomial of degree
three:

\[ P(X) = \alpha X^3 - \alpha X^2 - \lambda \kappa \left( \frac{1 - \alpha \mu}{1 - \alpha} \right) X + \lambda \kappa. \]  

(C.9)

Multiplying \( \alpha \Phi^2 - \kappa_x \lambda \) by \((1 - \Phi)\) and using the fact that \( \Phi \) satisfies equation (C.9), we get that \( \frac{\partial \Phi}{\partial \kappa_x} > 0 \) if and only if \( \Phi < 1 - \alpha \). Because \( 1 - \alpha \in (0, 1) \), \( \frac{\partial \Phi}{\partial \kappa_x} > 0 \) if and only if \( P(1 - \alpha) < 0 \), that is if and only if \( \alpha(1 - \alpha)^2 > \lambda \kappa_x \).

### C.5 Proof of lemma 7

Using the fact that posterior variance is constant across agents, it can be written \( V_0 = E^0 E_i^{f_o} (\bar{y} - E_i^{f_o}(\bar{y}))^2 \). The program of the planner is:

\[
\min_{(y(x_i, z'), \bar{y}(\bar{a}, z'))} \frac{1 - \alpha}{2} \int (y_i(x_i, z') - a_i)^2 P(x_i, z') dx_i dz' + \frac{\alpha}{2} C(V_0) \int C(V_0) \int (y_i(x_i, z') - \bar{y}(\bar{a}, z'))^2 P(x_i, z', \bar{a}) dx_i dz' d\bar{a} \\
- \frac{1}{2} \lambda(C(V_0) - 1 - \ln(C(V_0))),
\]

st. \( \forall(\bar{a}, z'), \bar{y}(\bar{a}, z') = \int y_i(x_i, z') P(x_i | \bar{a}, z') d\bar{a} dz' \),

st. \( V_0 = \int (\bar{y}(\bar{a}, z') - E_i(\bar{y}))^2 P(x_i, z', \bar{a}) dx_i dz' d\bar{a}. \)  

(C.10)

It is concave. Noting \( \beta(\bar{a}, z') P(\bar{a}, z') \) the Lagrange multiplier on the first constraints, and \( \delta / 2 \) the constraint on the second constraint, the Lagrangian is:

\[
\mathcal{L} = \frac{1 - \alpha}{2} \int (y_i(x_i, z') - a_i)^2 P(x_i, z') dx_i dz' + \frac{\alpha}{2} C(V_0) \int (y_i(x_i, z') - \bar{y}(\bar{a}, z'))^2 P(x_i, z', \bar{a}) dx_i dz' d\bar{a} \\
- \frac{1}{2} \lambda(C(V_0) - 1 - \ln(C(V_0))) \\
- \int \beta(\bar{a}, z') \bar{y}(\bar{a}, z') P(\bar{a}, z') d\bar{a} dz' \int \beta(\bar{a}, z') y_i(x_i, z') P(x_i, z', \bar{a}) dx_i dz' d\bar{a} \\
- \frac{\delta}{2} \left( V_0 - \int (\bar{y}(\bar{a}, z') - E_i(\bar{y}))^2 P(x_i, z', \bar{a}) dx_i dz' d\bar{a} \right). \]

(C.11)

The first-order conditions are:

\[
/y_i : (1 - \alpha)(y_i - a_i) + \alpha C(V_0)(y_i - E_i(\bar{y})) + E_i(\beta) = 0, \]

(C.12)

\[
/\bar{y} : \beta = \delta(\bar{y} - E(\bar{y})), \]

(C.13)

\[
/V_0 : \delta = \alpha C'(V_0) E(y_i - \bar{y})^2 - \lambda \left( C'(V_0) - \frac{C'(V_0)}{C(V_0)} \right). \]

(C.14)

Injecting equations (C.13) and (C.14) in (C.12) yields equation (3.41).

The expression of \( \delta \) can be simplified to \( \delta = \alpha C'(V_0)[E(y_i - \bar{y})^2 - V_0] \). To show that it is positive,
first notice that $C'(V_0) = \frac{\alpha}{\lambda}C^2(V_0) \geq 0$, with equality if and only if $\lambda = \infty$, i.e. under non-robust rational expectations. Besides, $E(y_i - \bar{y})^2 - V_0 = E(y_i - E_i(\bar{y}))^2 \geq 0$.

### C.6 Proof of proposition 9

An efficient allocation is any crossing of the two curves (3.48) and (B.2) that satisfies condition (3.20). Equation (3.48) is a decreasing function of $V_0$ because $\delta$ is an increasing function of $V_0$. Indeed, write $\delta$ as:

$$\delta = C'(V_0)\alpha \left(\frac{1 - \alpha}{1 - \alpha + \alpha C(V_0)}\right)^2 \left(\frac{1}{\kappa_b} - \frac{1}{\kappa_x}\right) + C''(V_0)\alpha V_0 \left(\frac{\kappa}{\kappa_x} - 1\right). \quad (C.15)$$

It is easy to check that both terms are increasing in $V_0$. Besides, the limit of equation (3.48) as $V_0 \to 0$ is positive, and the limit as $V_0 \to \frac{\lambda}{\alpha}$ is zero. It follows that the efficient allocation exists and is unique. That the equilibrium is efficient when $\mu = 0$, $(1 - \mu) = 0$, or $\delta = 0$ (rational expectations) is immediate. In all other cases, the curve of equation (3.48) is strictly below the curve of equation (B.1), so that $\Phi^{eq} > \Phi^{eff}$ and $V_0^{eq} > V_0^{eff}$. 