HEFT measurement of the hard X-ray size of the Crab Nebula and the hard X-ray optics of the Nuclear Spectroscopic Telescope Array (NuSTAR)

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ABSTRACT

HEFT measurement of the hard X-ray size of the Crab Nebula and the hard X-ray optics of the Nuclear Spectroscopic Telescope Array (NuSTAR)

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In this thesis, I discuss two topics: The High Energy Focusing Telescope (HEFT) and the Nuclear Spectroscopic Telescope Array (NuSTAR).

HEFT is the first experiment done with imaging telescopes in the hard X-ray energy band (\(\sim\)20-70 keV). I briefly describe the instrument and the balloon campaign. The in-flight calibration of the Point Spread Function (PSF) is done with a point source observation (\(\sim\)50 minutes of Cyg X-1 observation). With the PSF calibrated, I attempt to measuring the size of the Crab Nebula in this energy band. Analysis for aspect reconstruction, optical axis determination and the size measurement are described in detail. The size of the Crab Nebula is energy dependent due to synchrotron burn-off. The measurement of the size at different energies can provide us with important parameters for the pulsar wind nebula (PWN) model such as the magnetization parameter. With \(\sim\)60 minutes of observation of the Crab Nebula with HEFT, I measure the size of the Crab Nebula at energies of 25-58 keV. The analysis technique I used for the size measurement here can be used for measuring the size of astrophysical objects whose sizes are comparable to the width of the PSF.

NuSTAR is a satellite version of the HEFT experiment although the spatial and spectral resolution of the optics are improved significantly. And thus, the fabrication technique for the HEFT optics needed to be modified. I describe the fabrication technique for the NuSTAR optics, focusing on the epoxy selection and process development and the metrology systems for characterizing the figure of the glass surfaces.
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Part I

Introduction
I have studied experimental high energy astrophysics focusing on two goals. The first goal is to evaluate the performance of novel hard X-ray telescopes by observing astrophysical sources from balloons. The second is to further the state-of-the-art in developing hard X-ray telescopes for satellites.

The astrophysics part describes the High Energy Focusing Telescope (HEFT) flight and its observation of Cyg X-1 and the Crab Nebula. HEFT employs innovative technologies for the optics and the detector. The optic is the first one with the capability of focusing hard X-rays. It is a conic approximation to the Wolter-I geometry. The detector is a CdZnTe semiconductor detector, which has a high quantum efficiency. HEFT flew on a balloon payload in May, 2005 and observed the sky for 24 hours. The main interest was the Crab Nebula. The Crab Nebula is believed to be asymmetric in the hard X-ray energy band. The challenge is to reconstruct the image and the spectrum from the noisy balloon data. This work requires a detailed understanding of the aspect control system and the instrument response. First, the image reconstruction procedure is developed and applied to a known point source (Cyg X-1) to make sure that my approach is working properly. Then I apply the reconstruction procedure to the Crab Nebula observation to determine what the implications for the physics are.

The NuSTAR optic will have better resolving power and a larger photon collecting area. The NuSTAR optic shares the same concepts as the HEFT optic, but a lot of detailed studies were conducted to improve the angular resolving power and the photon collecting area. I worked on several key aspects of the optics build, including epoxy characterization and the crucial metrology of the mirror substrates. This work will be discussed in the second section.
Part II

The High Energy Focusing Telescope
X-ray observations of astrophysical objects have been a good probe for understanding the universe since the 1960s. But still, observations above $\sim 10 \text{ keV}$ are crude mostly because of the lack of focusing capability. There are technical difficulties in building focusing telescopes. The grazing incidence optic, which is required above $\sim 0.02 \text{ keV}$ (e.g., $\omega_p \sim 10^{16}/s$, $\bar{h} = 6.6 \times 10^{-16} \text{ eV} \cdot \text{s}$ for gold), has necessarily a small collecting area, and thus has limited sensitivity. Also, the lack of detectors with high quantum efficiency and good spatial resolution at the hard X-ray band is another factor that limits the sensitivity.

Recent developments of thermally slumped X-ray mirrors [Hailey and et al., 1997], error-correcting monolithic assembly and alignment (EMAAL) [Hailey and et al., 2003] and depth-graded multilayer coatings [Joensen and et al., 1995] enabled us to improve the sensitivity and the angular resolution of hard X-ray optics. At the same time, development of high-Z solid state pixel detectors [Harrison and et al., 2003] made true focusing possible at hard X-ray energies. These new technological advances, all employed together, improved the sensitivity and the angular resolution to a level not achievable with the current collimator and coded-aperture instruments.

The High-Energy Focusing Telescope (HEFT) was the first experiment that employed all these technologies for astrophysical observations in the hard X-ray energy band [Harrison and et al., 2005]. The HEFT collaboration developed and optimized high-reflectance depth graded multilayers, low-mass segmented X-ray optics with glass substrates, the mounting techniques, and high spectral resolution Cadmium-Zinc-Telluride pixel detectors to build
hard X-ray focusing telescopes.

Three telescopes were built for HEFT, and the balloon was launched at Fort Summer, New Mexico on May 18, 2005. This was the product of many years of hard work by the collaboration of the California Institute of Technology, Columbia University, Danish Technical University (DTU) and Lawrence Livermore National Laboratory. The balloon reached 40 km in altitude in about 3 hours and stayed there for about 21 hours. With the advanced technologies of the focusing optic, multilayer coating and the CdZnTe detector together with an aspect control system, HEFT successfully detected Cyg X-1 and the Crab Nebula during the flight.

In the next two chapters, I will discuss the overall mission briefly and the physics we learned from the HEFT experiment.
Chapter 2

Instrument overview

The instruments were installed on a gondola frame as shown in figure 2.1. We can categorize the instrument into two parts - the telescope and the aspect control system. The telescope is composed of three optics and three detectors at the focal plane of the optics. The three optics are co-aligned in a triangular configuration and mounted as shown in the figure (‘1’ in the figure). At the focal plane of the optics, three detectors are located in such way that each optic has its own detector. Detectors are stored in a science pressure vessel. It (‘2’ in the figure) is made of two Kevlar/Carbon half-spheres, and has a temperature and pressure controller for the best performance of detectors.

For the aspect control, many sensors were installed, some of which are indicated in the figure. There are two star trackers (‘3’ in the figure). Of the two, at least one can see the sky even when the other is blocked by the balloon because they are separated by 30°. The GPS antennae (‘4’), the elevation drive (‘5’), the azimuthal reaction wheel (‘6’), the cross coupling reaction wheel (‘7’) to prevent pendulation, and the flight computer housing vessel (‘8’) are shown in the figure too. However, other important components such as the gyro and magnetometer, are not shown in the figure.

In this chapter, I will describe each component of the telescope and the aspect control system. Many details are already described elsewhere ([Chonko, 2006], [Madsen, 2007], [Chen, 2008]), therefore my description will be a rather short review.
Figure 2.1: HEFT gondola side view. 1) Optics. 2) Science pressure vessel. 3) Star trackers and baffles. 4) GPS antennae. 5) Elevation drive. 6) Azimuthal reaction wheel. 7) Cross coupling reaction wheel preventing pendulation. 8) Pressurized flight computer vessel.
2.1 Detectors

The HEFT detector is a semiconductor detector made from CdZnTe. When an X-ray comes in, electron-hole pairs are produced in the CdZnTe crystal. These electron-hole pairs are collected in the electrodes (Pt) and generate electric currents. Since the crystal creates an electron-hole pair with a small energy (4.64 eV), the spectral resolution is very good. And its high Z, and thus high stopping power, makes it an efficient detector in the hard X-ray energy band. The CdZnTe sensor was pixelated to 24 × 44 and the size of the sensor is 23.6 mm (width) × 12.9 mm (height) × 2 mm (thickness). A low noise and power ASIC designed at Caltech was attached to each pixel for signal readout [Chen and et al., 2004].

One HEFT detector is composed of two sensors, and thus has 44 × 48 pixels (~25 × 25\text{mm}^2), corresponding to 15' of FoV (figure 2.2), and the main characteristics are shown in table 2.1. The area of each pixel is ~498 × 498 \text{µm}^2, which corresponds to 17". The energy range is 5-100 keV, and the resolution is 900 eV for one-pixel events and 950 eV for two-pixel events at 0°C. The detector is shielded by graded-Z plastic to reduce the ambient background. The detector and the shield together were kept under constant temperature and pressure inside the science pressure vessel during the flight.

<table>
<thead>
<tr>
<th><strong>Sensor</strong></th>
<th>CdZnTe</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dimension</strong></td>
<td>25 × 25 \text{mm}^2</td>
</tr>
<tr>
<td><strong>FoV</strong></td>
<td>15'</td>
</tr>
<tr>
<td><strong>Pixel size</strong></td>
<td>498 \text{µm}</td>
</tr>
<tr>
<td><strong>Thickness</strong></td>
<td>2 mm</td>
</tr>
<tr>
<td><strong>Energy range</strong></td>
<td>5-100 keV</td>
</tr>
<tr>
<td><strong>(\Delta E)</strong></td>
<td>1 keV</td>
</tr>
</tbody>
</table>

Table 2.1: Detector parameters.

Figure 2.2: HEFT detector.
2.2 Optics

The HEFT optics are segmented X-ray mirrors with a conic approximation to the Wolter-I geometry. It requires glass shaped into an upper and a lower cone (an approximation to paraboloid and hyperboloid surface respectively). With an upper (or lower) conical surface alone, optics employing total external reflection cannot focus off-axis photons in an aberration-free fashion [Wolter, 1952] (figure 2.4).

Each optic consisted of 72 layers of glass conic shells stacked radially outwards. Each

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Focal length</td>
<td>6000 mm</td>
</tr>
<tr>
<td>Length of a conic section</td>
<td>200 mm</td>
</tr>
<tr>
<td>Substrates per conic section</td>
<td>2</td>
</tr>
<tr>
<td>Min. aperture radius</td>
<td>40 mm</td>
</tr>
<tr>
<td>Max. aperture radius</td>
<td>120 mm</td>
</tr>
<tr>
<td>Number of layers</td>
<td>72</td>
</tr>
<tr>
<td>Number of substrates</td>
<td>1440</td>
</tr>
<tr>
<td>Field of View</td>
<td>17'</td>
</tr>
</tbody>
</table>

Table 2.2: Optics parameters.
Figure 2.4: Wolter-I geometry (top) and its conic approximation (bottom).

Figure 2.5: The HEFT optics use axially and azimuthally segmented mirrors.
layer was composed of 5 azimuthal sections, and each section had 4 mirror substrates along the axis, two for the upper cone (paraboloid) and two for the lower cone (hyperboloid), as shown in figure 2.5. Each mirror substrate was 10 cm long and $\sim 72^\circ$ wide. Therefore, one optic had in total 1440 mirror substrates [Koglin and et al., 2004].

A mirror substrate is produced by thermally slumping a thin flat piece of glass sheet (300 $\mu$m thick) at the Nevis laboratories of Columbia University [Hailey and et al., 1997]. To enhance the reflectivity at off-axis angles, the mirrors are coated with a few hundred alternating layers of high Z and low Z material at the DTU, utilizing the Bragg’s diffraction, where the alternating layers produce spatially varying index of refraction to satisfy the Bragg’s condition [Jensen and et al., 2003] (figure 2.6). The reflectivity is determined by the coating parameters such as the number of multilayers and the thickness of each layer, and the process control such as the interfacial roughness. The coating parameter was optimized for the best performance. Although each telescope layer could be coated with different parameters, we divided the telescope layers into 10 groups, and used a different set of coating parameters for each group [Mao and et al., 2000].

Those multilayer coated mirror substrates are then stacked on top of a Ti mandrel. Five (or three for some of the inner layers) graphite spacers ($\sim 1.6$ mm ($W$) $\times$ $1.6$ mm ($H$) $\times$ $200$ mm ($L$)) per azimuthal section are bonded with epoxy to the mandrel and precisely
Figure 2.7: Mounting process. In the bottom picture, the optic is fully loaded with the loading fixture, and thus is not visible.

machined to force a cylindrical mirror substrate into the proper conic shape for focusing at that radius, forming a layer of a conic telescope. The next layers are built by repeatedly applying the procedure - bonding graphite spacers, machining the spacers, and mounting glass substrates onto the spacers (figure 2.7).

As we machine the spacers to the correct cone angles and radius, any error occurring in the previous layer does not propagate up to the outer layers. Therefore, this way of assembly (error correcting monolithic assembly and alignment) does not have any stack-up error [Hailey and et al., 2003].

We built three optics for the HEFT experiment. Table 2.2 shows important parameters of the optic. Figure 2.3 shows the HEFT optics, which also shows the mandrel, alignment
<table>
<thead>
<tr>
<th>Sensor</th>
<th>AZ</th>
<th>EL</th>
<th>ROLL</th>
<th>MOUNT LOCATION</th>
<th>TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyro 1</td>
<td></td>
<td></td>
<td></td>
<td>Telescope strut</td>
<td>relative</td>
</tr>
<tr>
<td>Gyro 2</td>
<td></td>
<td></td>
<td></td>
<td>Platform</td>
<td>relative</td>
</tr>
<tr>
<td>Encoder: Gurley A25S</td>
<td></td>
<td></td>
<td></td>
<td>Telescope strut</td>
<td>absolute</td>
</tr>
<tr>
<td>GPS</td>
<td></td>
<td></td>
<td></td>
<td>Platform</td>
<td>absolute</td>
</tr>
<tr>
<td>Magnetometer</td>
<td></td>
<td></td>
<td></td>
<td>Platform</td>
<td>relative</td>
</tr>
<tr>
<td>Star Tracker 1 and 2</td>
<td></td>
<td></td>
<td></td>
<td>Telescope strut</td>
<td>absolute</td>
</tr>
<tr>
<td>Accelerometer</td>
<td></td>
<td></td>
<td></td>
<td>Telescope strut</td>
<td>relative</td>
</tr>
</tbody>
</table>

Table 2.3: Overview of sensors, their position and type of detection. "a" This pitch is the platform pitch and not the elevation pitch (from Madsen, 2007).

cone, spacers and the intermediate mandrel. For the first and the second optic of HEFT (HF1 and HF2 respectively), we used three spacers to support the mirrors for the inner 22 layers, and switched to five spacers after the intermediate mandrel (‘4’ in figure 2.3). For the third optic of HEFT (HF3), we used five spacers from its innermost layer. Therefore, it did not have the intermediate mandrel. The picture of HF3 (figure 2.3) also shows the wagon wheel (see figure 2.7) which guides the pressure loading fixtures during epoxy curing. The optic was thermally shielded in an isolating container to minimize the thermal gradient during the flight and installed as shown in figure 2.1.

2.3 Gondola and Aspect Control System (ACS)

Understanding the aspect control is of crucial importance in the data analysis as is shown in figure 3.1. The goal of the ACS sensors is to provide us with the location at which we are pointing. Each sensor has strengths and weaknesses, and one sensor alone does not give the pointing solution completely. Therefore, we have to understand what the weakness and the strength of each sensor is and how to combine the data from different sensors. This requires a detailed understanding of the sensors. In this section, I will clearly describe each sensor, how it works and what the limit of the sensor is.

General strategies of the aspect control and the reconstruction for balloon-borne missions are well-discussed in references [Craig and et al., 1998], [Gunderson and et al., 2003]. For the
aspect control and the aspect reconstruction, HEFT flew redundant sensors. The sensors and their basic information are listed in table 2.3. For example, Gyro 1 was installed at the telescope strut and gave relative elevation (pitch) (figure 2.8). Many more details about each sensor will be discussed in the following sections.

There are two types of sensors - relative sensors and absolute sensors. A relative sensor gives an aspect relative to the previous aspect, while an absolute sensor gives an aspect relative to a fixed coordinate system. As can be seen in table 2.3, the gyro, the magnetometer and the accelerometer are relative sensors, and the encoder, the GPS and the star tracker are absolute sensors.

Figure 2.8 shows the aspect control flow briefly. Once we set a target pointing, we read the current aspect from the sensors, calculate the directions and the angles to the target and control the servos to rotate the gondola and the telescope; the elevation is controlled by rotating the telescope, and the azimuth by the gondola. For elevation control, we mainly relied on encoders, which are precise and accurate enough. In case the encoders fails, which never happened during the HEFT flight, we had an accelerometer as a backup. For azimuthal control, the primary sensor was the star tracker. But with the star tracker alone, we cannot control the aspect properly because the star tracker gives us the pointing in the equatorial coordinate system - right ascension (RA) and declination (DEC), while the aspect control is done in the horizontal coordinate system (Azimuth-Elevation). Therefore, the star tracker data must be supplemented with the GPS latitude and longitude for the (RA, DEC) to (Az, El) transformation. When the star tracker data was not available (eg. faint star fields) or if precise control was not necessary (slewing), we used the GPS information instead.

In principle, more precise and frequent relative sensor data could be used for the aspect control. The problem with these relative sensors is that they suffer from thermal drift (gyros) or change of local magnetic field (magnetometer). On a short time scale, relative sensors are precise and accurate, but on a long time scale, those are unreliable. Therefore, we corrected those relative sensors using absolute sensors. If neither the star tracker data nor the GPS data were available, we corrected the gyros with the magnetometer.
CHAPTER 2. INSTRUMENT OVERVIEW

15

Aspect Sensors

- Star trackers (RA, DEC)
- Gyros (Yaw, pitch, roll)
- GPS (Az, El, roll, altitude, latitude, longitude)
- Magnetometer (Yaw, pitch, roll)
- Encoders (El)
- Accelerometer (El)

Aspect Control

- Elevation
- Azimuth

Figure 2.8: A schematic view of the HEFT telescope (top) and the aspect control flow chart (bottom).
2.3.1 Star Trackers

Simply speaking, a star tracker is a camera which takes a snapshot of a region of interest in the visible sky. Each star tracker consists of a 2.86 meter stray light baffle (figure 2.1) and a pressure vessel containing a digital camera, lens and PC/104 stack computer (figure 2.9). The design of the star camera is based on the design used by the High Energy Replicated Optics (HERO) developed by the Marshall Space Flight Center [Dietz and et al., 2002].

A photo taken by the camera has some bright stars in the FoV of the camera (figure 2.10). The brightness and configuration of the stars in the photo are then compared to the star catalog which is a sky map of stars. When the camera finds a match between the photo and a portion of the sky in the catalog, it can tell us where we are looking (pointing to) by looking up the catalog (like you can tell the address of a place in a photo by comparing the photo to Google maps). It is impractical and impossible to search the whole sky to find a match because it will take too long, and many portions of the sky may be matched to the photo within the sensitivity of the camera (like you cannot search the whole earth to find the address of the place in your photo). Therefore, it is required that the camera roughly
knows where it is looking (within ±4°).

The pointing solution we find in this way is in the equatorial coordinate system (RA-DEC), which means the solution is not useful unless it is supplemented with longitude and latitude information to transform it to the horizontal coordinate system (Az-El).

HEFT employed two star trackers - a PMI-1401 digital camera/Kodak 1401 CCD for on-axis and Retiga-EXi/Sony ICX285 progressive-scan interline monochrome CCD for off-axis. The on-axis star tracker was co-aligned to the optics while the off-axis one was off by (0.12, 29.39)° in (azimuth, elevation). The pixel sky projection, the spatial resolution of the camera, is 7.79′′ × 7.79′′ (7.39′′ × 7.39′′), and the field of view is 2.86° × 2.24° (2.86° × 2.24°) for the on-axis (off-axis) star tracker. Star trackers take a snapshot of the star field and each star is identified by software (originally written by Ryan McLean at the California Institute of Technology) using the Tycho-2 catalog and supplemented with totals over 2.5 million stars with positions and magnitudes compiled from the ESA Hipparcos satellite combined with the Astrographic Catalog and 143 other ground-based catalogs (figure 2.10). Once the stars are identified, we use a least square minimization to get the pointing solution [Chonko, 2006]. The minimization requires at least three (non co-linear) stars in the field of view,
otherwise one more piece of information (such as Roll of the gondola) must be inputted to get the correct pointing.

It is clear now what the strengths and the weaknesses of the star tracker are. The star tracker provides an absolute, precise and accurate pointing solution. However, the star tracker is slow because it has to find a match, and it needs to be assisted by other sensors.

### 2.3.2 GPS

GPS is commonly used for navigation systems nowadays. It is a constellation of 24 satellites orbiting the Earth and has very accurate clocks and broadcasts the location information in radio bands. An antenna receives signals from those different satellites. Using the signal arrival time from each satellite, the distance to each satellite is calculated \( d = ct \). Since locations of satellites are already precisely known, the location of the antenna can be calculated using trilateration methods. The arrival time (the distance to a satellite) may be inaccurate due to the atmospheric environment, which causes an error in determining the location. A time correction can be made by a stationary antenna at a known location. Since we know the location of the stationary antenna as well as the satellites, we can invert the trilateration methods to calculate the correct time. The correct time is then transferred to the moving antenna for the correct distance calculation. The GPS that uses a stationary antenna is called Differential GPS and has an improved accuracy. This is good for determining the latitude, the longitude and the altitude. In addition to those, HEFT also required the azimuth (rotation around the gravity vector), the pitch (rotation

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Military grade TANS Vector, Trimble Advanced Navigation sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement</td>
<td>Accuracy (baseline of 2m)</td>
</tr>
<tr>
<td>Horizontal</td>
<td>25-100m (5m with Differential mode)</td>
</tr>
<tr>
<td>Vertical</td>
<td>35-150m (8m with Differential mode)</td>
</tr>
<tr>
<td>Pitch/Roll</td>
<td>0.25° (RMS)</td>
</tr>
<tr>
<td>Azimuth (yaw)</td>
<td>0.15° (RMS)</td>
</tr>
</tbody>
</table>

Table 2.4: GPS specification.
around the east-west axis) and the roll (rotation around the north-south axis) information. Therefore, HEFT employed four antennae with a baseline of ∼2 m, whose relative positions provides us with the rotation angles.

Table 2.4 summarizes the GPS of HEFT. HEFT flew a military grade TANS Vector, Trimble Advanced Navigation Sensor [Trimble, 1995]. The GPS antennae were mounted at the top of the gondola (figure 2.1) with a baseline of 2 m. With this baseline, the positional accuracy is 0.15° for azimuth, 0.25° for pitch and roll. The horizontal (vertical) accuracy is 20-100 m (35-150 m) and can be 5 m (8 m) for 95% of the time when the differential mode is enabled.

As discussed in section 2.3.1, longitude and latitude information must be supplied to the star tracker solution to obtain the pointing. GPS took the role of providing longitude, latitude and altitude information to the star tracker. The azimuth information was used for the aspect control when the precision and the accuracy were less important, and the pitch, roll and yaw information was used later for aspect reconstruction as a supplement to the gyro solution.

2.3.3 Gyros

The star tracker and the GPS can completely determine the pointing of the telescope. However, the corresponding aspect solution is necessarily slow and coarse, because the star tracker data need to be processed (slow) and the resolution of the GPS is ∼arcminute (coarse). For the purpose of controlling the aspect (tracking an astrophysical object), these sensors are good enough, considering the FoV of the telescope (15′). As long as the source is in the FoV, we can detect it. However, to reconstruct an image of a source, whose size is ∼1′ and from which the photon arrives at a random time such as the Crab Nebula, this slow and coarse aspect solution is not enough.

The sensor that HEFT employed for fast and accurate aspect solution was a gyro. A gyro is composed of a wheel, a case and an axis (shaft) that penetrates the center of the wheel. The shaft is attached to the wheel, and then to the case frictionlessly by bearings. So when the case is rotating on a vehicle, the wheel does not. By measuring the relative angular velocity of the case and the wheel, we can measure the angular speed of the case,
HEFT flew two Litton G-2000 two axis, strap-down type gyros (figure 2.12). One was mounted on the gondola base, measuring the yaw and the roll of the gondola base, the other was mounted on the telescope strut, measuring the pitch of the telescope. The data rate of gyros is 100 Hz, averaged at 10 Hz, and the resolution is a few arcsecs. Therefore, gyros seem to be excellent sensors for both the aspect control and reconstruction. Although these are excellent sensors, there are several things to notice. First, the gyro is not an absolute sensor. Because it measures the angular speed not the angle, we need to integrate over time, and the integration constant is hard to determine. Second, the dynamic range of the gyro is $2^\circ$/s. When the gondola base moves at a faster rate than this, the gyro cannot measure the speed. Third, the gyro drifts, depending on the temperature. Therefore, it is reliable only over a short time period (when temperature changes little) but not over a long time period. For these reasons, we did not use gyro data for the aspect control, but rather for the post-flight aspect reconstruction.

2.3.4 Magnetometers

The gyro solution needed to be corrected as described in the previous section. While the integration constant could be determined by absolute sensors (the star tracker and the GPS), the temperature dependent drift (bias offset) may not be very well corrected by those sensors because the angular resolution of those sensors is not very good compared
to that of the gyro. Also, there can be an instance when both those sensors do not give a solution (faint star field or no GPS communication). Thus, we needed another sensor that is temperature independent and precise enough.

A magnetometer is temperature independent and precise. It measures the angle between the earth (external) magnetic field and a reference direction of the magnetometer as you can do with a compass (While you are rotating a compass, the reference line rotates with it but the magnetic needle does not. By measuring the angle between the line and the needle, you know the orientation of the line with respect to the needle (earth magnetic field)). Instead of using a permanent magnet as in the case of a compass, the magnetometer uses two electro-magnets, and a secondary coil to measure the current induced by the magnetic field. The electro-magnet is composed of a ferromagnetic core wrapped by a coil. Two electro-magnets are aligned oppositely in such a way that the magnetic field cancels when there is no external magnetic field (no current in the secondary coil). When there is an external magnetic field, the induced magnetic field of the electro-magnet which is aligned to the magnetic field saturates (no change) earlier than the other does. Thus, the induced magnetic field of the aligned electro-magnet does not change while that of anti-aligned one does. This change induces a current in the secondary coil, which can be converted to the angle between the external magnetic field and the magnetometer. By the way it works, it
is temperature independent, and also the precision is almost guaranteed because it involves electronics.

HEFT flew two magnetometers (FGM301/99), and those were mounted close to the pivot point of the telescope. It was not easy to measure the absolute orientation of the gondola because the external magnetic field (earth magnetic field) was not precisely known, and even it was changing due to the metal frame of the gondola. Although it was not possible to measure the absolute aspect, it was possible to measure the change of the aspect over a short time period, because the external magnetic field does not change fast. The magnetometer solution was used to correct the drift bias of the gyros with great success during the HEFT flight.

The magnetometer solution, which was very useful during the flight, is not used for the aspect reconstruction because the corrected gyro solution is more precise and accurate.
2.4 Performance

When we observe a source, the measurement necessarily includes the effect of the telescopes through which we look at an object. If the telescope is blurry, even a clear object will look blurry (angular resolution), and if the telescope is opaque, a bright object will look less bright (effective area). Therefore, knowing the performance of the telescope (effective area and angular resolution) is the key to the data analysis.

Figure 2.13 shows the flow of the performance evaluation. The performance of the telescope is determined mainly by three factors - surface figure, imperfection of the multilayer coatings and the structural obscuration. Surface figure refers to non-flatness and non-roundness of the mounted mirror substrates. Non-flatness reflects the incoming photon
off-focus, making angular resolution worse. Non-roundness may make a shadow, reducing effective area. Imperfection of the multilayer coatings refers to the interfacial roughness and the improper thickness of the coating. Those reduce the reflectivity, and thus effective area. The structural obscuration refers to components that block the telescope. One example is the graphite spacers. Figure 2.3 and 2.7 show how spacers block the opening of the telescope for example. Those factors are measured or calculated in several different ways, and then input to the ray trace calculation, which calculates the trace of each photon that would be seen through the telescope. From the ray trace calculation, we obtain the performance of the telescope.

The 8 keV pencil beam scan was conducted at the DTU. A narrow beam of 8 keV X-rays (pencil beam) produced by a double-axis diffractometer is shone onto a portion of the telescope and detected. The deflection angle and the flux of the detected beam provide information about the mirror. A description of the apparatus and the methodology are discussed elsewhere [Hussain and et al., 1999]. The mechanical probe scan of the optics surface was done at Colorado Precision Products, Inc. (CPPI) during the build. The probe is called a Linear Variable Differential Transformer (LVDT) and commonly used to measure a surface height profile. I will explain this in detail later (chapter 7). In addition to these measurements, ray trace calculations were required for the evaluation. These calculations will be explained in the next section.

In this section, I discuss the effective area (opaqueness), the angular resolution (blurriness) of HEFT, and the sensitivity. These effects of the telescope are verified and unfolded in the data analysis.

2.4.1 The Ray Trace simulation

The measurements of the mechanical probe give us the height profile of the mirror. The height profile (surface figure), when combined with the ray trace simulation, provides us the performance estimation. An incoming X-ray, its reflection off the upper mirror and the lower mirror, finally to the detector are traced in the simulation.

We find the intersection of an X-ray with a conic mirror by solving the simultaneous
equations of a straight line (X-ray) and a cone (upper mirror):

\[
\begin{align*}
\frac{x - x_0}{k_x} &= \frac{y - y_0}{k_y} = \frac{z - z_0}{k_z} \\
 x^2 + y^2 &= (z - z_c)^2 \tan^2 \alpha,
\end{align*}
\]

where \((x_0, y_0, z_0)\) is the initial position of the X-ray, \(\vec{k}\) is the initial propagation vector, \(z_c\) is the vertex of the cone, and \(\alpha\) is the cone angle. At the intersection, we reflect the X-ray using the height profile (or height perturbation according to a distribution) and the reflectivity, and update the position and the propagation vector. We repeat this with the lower mirror, and finally find the position in the detector where the X-ray hits.

“One-bounce” photons, which strike an upper or lower shell only, or which strike the back of a shell, are rejected. The “two-bounce” photons are weighted by the energy dependent reflectivity of the given layer. In this fashion the 2-D Point Spread Function (PSF) can be determined. The PSF is then normalized to unity for the twice-reflected photons, effectively defining the PSF in terms of these photons. The single-bounce photons are separately accounted for in the effective area.
Figure 2.14 is the result of the ray trace. It shows the distribution of X-rays just behind the exit aperture of the HF1 optic. It shows the shadow of the mandrel (central circle), the structure used to hold the whole optic (five wide spokes), spacers (narrow spokes), and the intermediate mandrel (a thin ring) which is used for switching from the three-spacer to the five-spacer configuration.

As we assign the initial propagation vector and the energy to the incoming X-ray, obtaining the optics response as a function of energy and off-axis angle with the ray trace code is straightforward, and those are shown in figure 2.16 and 2.18.

### 2.4.2 Effective area

For HF1, X-ray pencil beam scans with partial areal coverage (~60%) were performed at 8 keV at DTU. Layers 27 and above were also done at 18 keV, the entire telescope was measured at 40 and 50 keV, and 4 layers at 68 keV. Scans at 8 keV were also done on HF3 [Koglin and et al., 2004]. In both cases the LVDT data were available on one quintant per telescope. This data was the basis for constructing the optical throughput model. The throughput model consists of 3 parts; the multilayer X-ray reflectivity; the axial throughput; the obscuration.

The multilayer reflectivity is calculated using the IMD code [Windt, 1998] with the optimized multilayer parameters (section 2.2, [Mao and et al., 2000]) to the mirror substrate. The reflectivity calculations were done with a model [Nevot et P. Croce, 1980]. The axial throughput refers to losses in throughput due neither to reflectivity nor structural obscuration. In this category are included the gaps between the 4 pieces of glass comprising the upper and lower reflectors (see figure 2.4 and 2.5). Also included is the shadowing from the glass. Because the cylindrical glass is forced to a conic shape there is both inward and outward buckling of glass (in-phase roundness errors), and at the smaller graze angles of the inner shells this can lead to substantial shadowing of the X-ray trajectory. The axial throughput can be determined by several independent methods. Firstly the 8 keV pencil beam scans can be used with the calculated X-ray reflectivity to infer the (primarily) axial loss of X-rays. This axial throughput loss can be independently deduced from the LVDT metrology, which provides the surface figure. A ray trace calculation using the LVDT data
thus provides an independent measure of the axial throughput.

The comparison is best done at 8 keV, where reflectivity is high, and thus the error in deducing the throughput small. Measurements were performed on the optics at 8 keV in 2.5 degree intervals. For the LVDT data each quintant sampled was considered representative of the given telescope layer. A comparison of the X-ray and LVDT inferred axial throughput is shown in fig. 2.15. The 2 approaches are in excellent agreement, and as expected there is a marked drop in throughput for the inner layers due to enhanced shadowing at the shallower graze angles. The effect is made worse because the inner 22 layers of the telescopes used only 3 spacers to constrain the glass, leading to a large shadowing effect. The situation is markedly improved after layer 22 where the transition to 5 spacers per glass piece took place, and the shadowing is suppressed due to better control of the glass and the larger graze angle.

The final ingredient of the effective area calculation is the obscuration due to the support structure which supports the optics modules in the gondola, along with obscuration due to the spacers themselves, along with some obscuration due to epoxy which spreads out beyond
the form factor of the spacer. These components of the form factor can be quite precisely measured or estimated, and then the obscuration calculated. The geometric obscuration varies from 10-20% in the HEFT telescopes. Confirmation that this effect is properly accounted for is the excellent agreement between the X-ray data (with obscuration removed) and the LVDT data (which does not probe the obscuration), as shown in figure 2.15. This same data shows that there is no measurable contribution from effects such as scattering due to interfacial roughness or surface contamination due to dust in the machine environment or other sources of contamination.

The high energy effective area was evaluated by using 18 to 70 keV quintant measurements, which were then used, along with 8 keV data, to constrain the parameters of the reflectivity model for each coating recipe [Madsen and et al., 2004]. These multilayer reflectivities, the measured axial throughput and the calculated obscuration were used to
Figure 2.17: HF1 ray traced PSF and PSF measured at 8 keV (projected onto X, Y axis). The measurement is weighted by the effective area of each layer. The measurement is done on 41 layers out of a total of 72 layers of the optic.

calculate the on- and off-axis effective area. The same procedure was used for the other HEFT telescopes.

Those model parameters, the reflectivity, the throughput and the structural obscuration, are then input to the ray trace program to compute the effective area as a function of energy and the off-axis angle of incoming photons. The effective area curves for HF1 and HF3 are given in figure 2.16, computed as described here.

2.4.3 Angular resolution

Several approaches to determining the point spread response function (PSF) and half power diameter (HPD) of the optics were utilized. The most detailed X-ray characterization was done for HF1, where 60% of the telescope was measured with 8 keV pencil beam scans,
layer 27 and above at 18 keV and the entire telescope at 40 and 50 keV. In addition 5 layers were characterized at 68 keV. Similar 8 keV data was obtained on HF2 and HF3. This data was supplemented by the mechanical metrology of the mounted glass.

A linear variable differential transformer (LVDT), a ruby-tipped, inductive transducer, scanned along the back surface of mounted glass pieces. The high parallelism of the front and back surfaces of the glass (< 10") means the LVDT provides an effective tool for directly measuring the slope profile of the mounted glass. On average one quintant from each telescope layer was measured using the LVDT. Provided that the PSF is dominated by lower frequency spatial figure errors, rather than higher spatial frequency effects (eg. scattering), the LDVT metrology should provide an effective means to characterize the PSF. This has been demonstrated in previous prototypes [Koglin and et al., 2004].

Because of the limited X-ray sampling of the telescopes, an analytic model was constructed in order to better estimate the PSF for the telescopes. For the LVDT data a simple ray trace is used, where the distribution of axial surface slopes from a quintant is
considered representative of that particular telescope layer. This data can then be converted to an appropriate King profile of the form \( PSF(r) = \frac{1}{(1 + \frac{(r-r_0)^2}{r_c^2})^\alpha} \), where \( r_0 \), \( r_c \) and \( \alpha \) are fitting parameters. The King profile then provides a statistical distribution for the perturbation of the axial slope, which is then applied to propagation of photons off the upper and lower shells. This approach can be followed for both on-axis and off-axis incident photons.

The same approach is taken in the case of X-ray data. A separate set of King profile parameters was obtained for each multilayer group. The King profiles obtained from the X-ray and the LVDT data agreed within fitting uncertainties for HF1. A direct comparison of the one-dimensional model fit and the X-ray data are shown in figure 2.17. The comparison is shown for 11 layers for which exceptionally high quality X-ray data exist. The LVDT data used in the model was weighted for the effective area of each layer. Because of the excellent agreement between LVDT and X-ray data, the LVDT ray trace was used to construct the PSF.

The ray trace code naturally gives us the PSF as a function of energy and the off-axis angle. The energy dependence of the PSF is weak because the scattering of a photon off a mirror substrate is mainly a geometric effect at these energy bands. However, the dependence on the incoming angle (off-axis angle) is strong due to shadowing effects. These PSFs at different off-axis angle are shown in figure 2.18.

The model of the HF1 and HF3 PSF was compared with in-flight data obtained on Cyg X-1, as discussed in more detail in chapter 3.6. The combined PSF predicted for the combination of HF1 and HF3 above 15 keV is 1.3 arcminutes, which is the value inferred from fitting the in-flight data. This confirms that the degradation in performance of HF3 is a low energy phenomenon, and does not contribute any degradation in in-flight imaging performance [An and et al., 2009].

2.4.4 Sensitivity

Although the effective area and the angular resolution account for the spectral and the spatial performance, they do not directly tell us the general performance of the telescope - how faint a source can it detect?. The sensitivity is a measure of how weak a source an instrument can detect. Because there is background radiation entering the telescope,
collecting a few photons from a source does not imply a detection. This is why we need a larger collecting area and better angular resolution - to suppress background.

Detection of a source is claimed on statistical grounds. That is, if the confidence is 99% (3 $\sigma$), one can claim a source detection. The statistical significance in $\sigma$ is calculated by the following formula.

$$K = \frac{S}{\sqrt{S+B}},$$

(2.1)

where S is source counts and B is background counts. If an assumed source flux is $F(E)$ (cts/s/cm$^2$/keV), and the background flux is $B(E)$ (cts/s/cm$^2$/keV), using the effective area ($A_{eff}$) and the HPD discussed in the previous section, the significance can be expressed as

$$K = \frac{0.5F(E)A_{eff}t\Delta E}{\sqrt{0.5F(E)A_{eff}t\Delta E + HPD^2B(E)t\Delta E}},$$

(2.2)

where, t is the observation time, $\Delta E$ is the energy bandwidth. We require K to be 3 as a source detection criterion and solve equation (2.2) for $F(E)$. Finally, the sensitivity of HEFT is shown as the minimum flux for detection ($K = 3$) in figure 2.19 [Harrison and et al., 2000].
Chapter 3

Analysis of HEFT data

During its flight, HEFT successfully detected two important sources - Cyg X-1 and the Crab Nebula. Cyg X-1 is a point source, and thus very good for calibrating the PSF. The image of the Cyg X-1 observation is the spatial response of the telescope - the PSF. The Crab Nebula is called a “standard candle”, which means its spectrum is very well measured and the same all the time (although it is recently reported that the flux of the Crab Nebula changes [Wilson-Hodge and et al., 2011], it is a small change and thus is not a problem for HEFT), and thus a very good source for calibrating the effective area. At the same time, it is a very interesting astrophysical source, called a pulsar wind nebula (PWN). It is an extended source, whose X-ray image structure can give us a better understanding of the particle acceleration mechanisms in young supernovae.

I have two goals in analyzing the HEFT data. First, I would like to measure the PSF with Cyg X-1 and the effective area with the Crab Nebula observation, and verify our ground measurements and modeling. Second, I would like to investigate the possibility of measuring the shape (asymmetry) of the Crab Nebula, and measure it if possible.

These goals can be attained after a complicated data analysis. Figure 3.1 shows the data analysis flow in a simple and brief way. Once an event (arrival of an X-ray photon at the detector) happens, the detector records the location and the energy of the photon. This recorded location is a relative location of the photon with respect to the telescope pointing. Therefore, to find the true location of the event (event distribution), we have to combine the event data and the telescope pointing solution (red line in figure 3.1). The
energy distribution is obtained directly from the event recording. The event distribution and the energy distribution obtained in this way are not source properties alone. Rather, these are combined properties of the source and everything between the source and the detector, including the detector itself. Therefore, to obtain the source properties, we have to unfold the effects that come from anything other than the source itself. These effects are obtained in the process of data analysis (the blue line and the black line in the figure). Finally, by unfolding the effects from the event distribution and the energy distribution, we obtain the image and the spectrum of the source.

Although the flow chart (figure 3.1) looks relatively simple, executing it is not so simple. Getting quantities in each box of the chart requires detailed understanding of the telescope system. After obtaining those, we have to do integrations, differentiations and coordinate transformations, or solve integro-differential equations to follow the flow
Figure 3.2: The shape of the Crab Nebula at different energy bands.

Some of the contents in the chart were already discussed in the previous chapter. In this chapter, I describe the science object that I am interested in - the Crab Nebula, explain the data analysis process in details, and finally discuss the result of the analysis.

3.1 The Crab Nebula

The Crab Nebula is the remnant of a supernova explosion observed in 1054 and the best studied astronomical object. It is located at 2 kpc (Right Ascension (RA): 83.642°, Declination (DEC): 22.015°) from the earth. It is the brightest persistent source in the sky at energies above 30 keV, which is the reason why it is the best observed and the most important calibration source.

3.1.1 General features of the Crab Nebula

The Crab Nebula is composed of a slightly off-centered pulsar, and the nebula surrounding the pulsar. The pulsar (PSR 0531+21) is a spinning neutron star with a strong dipolar magnetic field. It emits a pulse of radio to ~10 GeV photons with a period of 33 ms [Nolan and et al., 1993]. The nebula which surrounds the pulsar consists of an oval shaped mass of filaments which are mainly composed of He and H. The nebula temperature is 11000 to
18000 K and the density is about 1300 particles/cm$^3$ (top right of figure 3.2). It emits radio to $\sim$100 TeV photons [Aharonian and et al., 2006].

### 3.1.2 X-ray properties of the Crab Nebula

The Crab Nebula was best resolved at soft X-ray energies by Chandra and XMM-Newton (Figure 3.2), which clearly showed the torus and the jet structure of the nebula, whose size is $\sim 2' \times 2'$. However, hard X-ray imaging of the Crab Nebula is still crude. Figure 3.3 shows the hard X-ray image of the Crab Nebula made by a one-dimensional scanning modulation technique [Pelling and et al., 1987].

The theoretical interpretation of the jet-torus structure observed in the X-ray bands is based on the magnetohydrodynamics of the pulsar wind nebula. The pulsar feeds relativistic particles by its strong magnetic field (pulsar wind) [Goldreich and et al., 1969], [Ruderman and et al., 1975]. The pulsar wind creates a termination shock when it contacts the nebula. The shock propagates, compressing the post shock materials and the magnetic
field lines. The compressed magnetic field and the high density charged particles generate the synchrotron emission [Kennel and et al., 1984a] and [Kennel and et al., 1984b]. The pulsar wind energy flux is anisotropic, maximum at the equatorial plane decreasing towards the pole. This anisotropy in the energy flux causes the termination shock to be oblate. Therefore, the post-shock flows are greater in the equator and are diverted to the symmetry axis by magnetic hoop stresses [Bogovalov and et al., 2002], [Lyubarsky, 2002], [Komissarov and et al., 2004], [Zanna and et al., 2004], [Li, 2002] (see figure 3.4).

The soft X-ray (1-10 keV) flux of the pulsar is \(2.5 \times 10^{-9} \text{erg/cm}^2/\text{s}\), which is about 8% of the nebula’s (\(3 \times 10^{-8} \text{erg/cm}^2/\text{s}\)). In the nebula, most of the energy is emitted at the torus and only \(~4%\) percent of the energy is emitted at the jet (volume ratio). The south-west part of the torus is brighter than the north-east part which is due to Doppler boosting and relativistic aberration. The size at energies of 0.4-8 keV measured by Chandra is about 2 arcminute [Mori and et al., 2004]. As the emission process in the X-ray energy band is mainly synchrotron radiation, its size is expected to decrease as the observational energy increases. The energy dependent size from optical to X-ray is empirically given as \(\sim E^{-0.148}\) [Ku and et al., 1976] along the NW-SE direction, and an analytic expression is found in [Kennel and et al., 1984b] along the NE-SW direction (eg. Figure 9. or equation (4.10b) in the reference). These predict the size of the Crab Nebula at the HEFT energy band to be \(~30''\) and \(~60''\) in the NW-SE and the NE-SW direction respectively.

Figure 3.4: The emission from the Crab Nebula.
HEFT made a real 2-D image of the Crab Nebula in the hard X-ray energy band in order to verify whether the model prediction of the size is correct or not.

3.1.3 Theory of asymmetry and predictions

Kennel and Coroniti modeled (from now on KC84) the Crab Nebula as depicted in figure 3.5 based on Ree and Gunn’s model of relativistic wind and shock discontinuity. Kennel and Coroniti developed a self-consistent steady state magneto-hydrodynamic model to explain the various spectral/spatial features of the Crab Nebula. They subdivided the nebula into 6 regions, some of which are shown in figure 3.5.

- Region I: Inside the pulsar magnetosphere, where the pulsar wind is produced (Not denoted in the figure).
- Region II: Outside the light cylinder \( (r > r_L = 1.5 \times 10^8 \text{ cm}) \), where the relativistic wind flows toward the nebula.
- Region III: The nebula proper, which contains a positronic flow that has been decelerated and heated by a MHD reverse shock at \( r_s \sim 3 \times 10^{17} \text{ cm} = 0.1 \text{ pc} \). Nearly all the radiation is generated in this region \((r_s < r < r_N = 2 \text{ pc})\).
- Region IV: The remnant, which has two parts: an inner region that was shock heated because of its interaction with the pulsar, and a surrounding cooler hydrogenic envelope. A weak reverse shock should propagate inward from the outer boundary of Region IV, \( r \sim 5 \text{ pc} \).
- Region V: Contains the small amount of material that has been swept up by the outward-propagating blast wave \((r \sim 5 \text{ pc}, \text{ not denoted in the figure})\).
- Region VI: The interstellar medium (not denoted in the figure).

Further, they assumed negligible plasma interaction around the filament, adiabatic post-shock flow, time independence and spherical symmetry. Using the Rankine-Hugoniot strong shock relation (three conservation laws), they calculated some properties of the upstream and the downstream flow. The analysis showed that the important parameter of the model
Figure 3.5: In the center, there is a pulsar generating a relativistic wind. The pulsar wind flows outward to the nebula (shaded). At the boundary of the wind and the nebula, a standing shock is formed. (from Kennel and et al., 1984)
- $\sigma$, the ratio of magnetic energy flux to particle energy flux - is required to be small to have a significant fraction of the energy flux be converted into thermal energy downstream for observed synchrotron luminosity. Then the nebula flow was calculated with a toroidal approximation. In the nebula, the flow speed decreases as $\sim 1/r^2$ and the magnetic field increases to the distance of $\sim \sqrt{3\sigma} r_s$ then decreases as $\sim 1/r$. They found $\sigma$ to be 0.003 to get agreement with the observations and to match the flow and pressure boundary condition. With the $\sigma$ obtained, the radiation from the nebula was calculated assuming a power-law distribution of electrons, and the synchrotron luminosity was calculated and compared to that observed. The best estimate of parameters was $r_s = 3 \times 10^{17}$ cm, $u_1 = 10^6$ cm/s (upstream speed), $\alpha = 0.6$ (power-law index for the input particles) and $\sigma = 0.003$ although these were not unique.

Although this model does not explain the existence of jets discovered by Chandra and XMM-Newton ([Mori and et al., 2004], [Kirsch and et al., 2006]), it is the basis of most of the recent models which try to explain the jets (eg. [Bogovalov and et al., 2002], [Lyubarsky, 2002], [Zanna and et al., 2004], [Komissarov and et al., 2004], [Shibata and et al., 2003]). Also the spectrum up to $\sim 100$ GeV ([Atoyan and et al., 1996]) and to $\sim 100$ TeV ([Zhang and et al., 2008], [Volpi and et al., 2008]) was successfully modeled by adding inverse Compton scattering of relativistic electrons to KC84 (synchrotron radiation). An interesting consequence of the interpretation of the observation of GeV photons is that more energetic electrons of energy $\sim 10^{15}$ eV are required, which confirms that the particles can be accelerated by the termination shock to the 'knee' in the cosmic particle spectrum.

Among predictions (calculations) of the theories, the most interesting to HEFT are the upper critical frequency (figure 3.33) and the hard X-ray form factor (figure 3.6). The upper critical frequency, $\nu''_c$, is the maximum synchrotron photon frequency that can be emitted at a distance $z (= \frac{r}{r_s}$, where $r$ is the distance from the central pulsar and $r_s$ is the radius of the termination shock) and the hard X-ray form factor is the integral of the volume emissivity along the line of sight. Those analytic solutions from KC84 are plotted in figure 3.6 and 3.33. Once HEFT observes the Crab Nebula, we can directly fit the image to the analytic function to get the parameters.
Figure 3.6: X-ray form factor of the Crab Nebula. Left: The soft X-ray form factor (0.4 keV), Right: The hard X-ray form factor (40 keV). The integral of the volume emissivity along the line of sight normalized to that at the center shown at 40 keV with different $\alpha$. The scale of the x axis is the normalized distance ($z_{\perp}$) of the line of sight from the pulsar (bottom scale) and the angular distance (top). (from Kennel and et al., 1984)
3.2 Aspect Reconstruction

Since the detector records only the relative location of the photon with respect to the telescope pointing, we have to know where the telescope is pointing, at the instance when the data is recorded. This will be the first step of the data analysis.

Aspect reconstruction is the process of finding out where the telescope is pointing. The telescope pointing must change at every instant of time to track a source, as the earth is spinning while the source is stationary. We tracked a source with the aspect control system described in section 2.8, which also controlled the pointing. In addition to this controlled pointing, there was an uncontrolled pointing due to pendulation of the gondola, which was at the arcminute level. The motions of the gondola and the telescope, whether controlled or not, were recorded by various sensors (table 2.3). The goal of aspect reconstruction is to combine the data from different sensors to reconstruct the telescope pointing history.

The aspect reconstruction is done by using the data recorded mainly by three sensors. Although the process looks simple in figure 3.1, it is quite complicated because those three sensors recorded the motion of different things (telescope or gondola) in different reference coordinates, and at different rates. No one sensor provides a perfect solution because each sensor has its own weakness. Therefore, we have to combine the sensor data in such a way that one sensor covers the weaknesses of the others, which requires not only careful investigation of the data of each sensor, but also complicated coordinate transformations between different reference coordinate systems.

3.2.1 Aspect solutions

Table 3.1 summarizes the properties of data taken by three main sensors used for aspect reconstruction - the star tracker, the GPS and the gyro. The star tracker measures the motion of the gondola and telescope together in the equatorial coordinate system (RA-DEC), at every three seconds. It is fairly precise but slow, and loses its accuracy when the star field is faint. The GPS data is coarse and slow but is crucial for the coordinate transformations between the equatorial and the horizontal coordinate system. Gyros give the most precise data at the fastest rate in the gondola frame (about the same as the
Table 3.1: Properties of data taken by three main sensors. G and T in the motion column means that the sensor measures the motion of the gondola (G) or of the telescope (T) or both (G+T).

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Resolution</th>
<th>Coordinate</th>
<th>Data Rate</th>
<th>Motion</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star tracker</td>
<td>8''</td>
<td>equatorial</td>
<td>1/3 Hz</td>
<td>G+T</td>
<td>not good for faint star field</td>
</tr>
<tr>
<td>GPS (r,p,y)</td>
<td>~0.2°</td>
<td>horizontal</td>
<td>~1 Hz</td>
<td>G</td>
<td>sometimes missing</td>
</tr>
<tr>
<td>GPS (lon/lat)</td>
<td>&lt; 10 m</td>
<td>horizontal</td>
<td>~1/284 Hz</td>
<td>G</td>
<td>sometimes missing</td>
</tr>
<tr>
<td>Gyros (r,p,y)</td>
<td>~1&quot;</td>
<td>gondola</td>
<td>10 Hz</td>
<td>G+T, G</td>
<td>not reliable over long period</td>
</tr>
</tbody>
</table>

horizontal coordinate system because the gondola is not tilted much during the flight), but the data is not reliable over a long period.

Considering the strength and the weakness of each sensor, the strategy I developed is to use the star tracker data as a base and filling the 3 secs gaps with the gyro data, where the coordinate transformation is done using latitude and longitude information from the GPS.

3.2.1.1 Star tracker solution

The star tracker measured the combined motion of the gondola and the telescope. While azimuthal motion was possible only with the gondola, elevation motion was possible with both the telescope rotation and the gondola pitch. Each sensor measures the motion of different things as shown in table 3.1, which we can properly combine to get the aspect solution. Nevertheless, what matters in the end is the motion of the telescope and the gondola together, which is given by the star tracker without further manipulations, and which is the reason why its data is so important.

Figure 3.7 shows an example of the elevation and the azimuth measured by a star tracker. The star tracker data, which is in equatorial coordinates (RA-DEC), is converted to horizontal coordinates (Az-El) using the latitude, and longitude of the GPS. During this time, for example, the aspect system is controlled so that the telescope is elevating and the gondola is rotating towards east, tracking a source (the Crab Nebula during this time). The dips in the azimuth plot (right) is related to the azimuthal control of the gondola. Also you can see small wiggles which are caused by uncontrolled pendulation.
CHAPTER 3. ANALYSIS OF HEFT DATA

Figure 3.7: Elevation and Azimuth from the star tracker over $\sim 13$ minutes. Left: Elevation, Right: Azimuth.

The sparse data points of the star tracker need to be filled with gyro data to get a fine pointing solution. Although it is not clear that the star tracker data is sparse in figure 3.7, it becomes clearer when we look at it closer (figure 3.8). Figure 3.8 show the star tracker solution (cross) and the gyro solution (solid) over one minute. The left panels show the El and the Az before correction and the right panels after the correction that I will describe now.

The star tracker solution should agree well with the gyro solution over a short period of time but it does not, as seen in the left panels (before the correction). This disagreement was investigated and attributed to a random shift in the star tracker timing [Madsen, 2007]. Therefore the correction is made to the star tracker timing using the gyro solution which does not suffer from the time shift. Except for the slow drift of the gyro solution, the star tracker solution can be compared to the gyro solution. So we remove the slowly varying part (low frequency) from both the gyro and the star tracker data and fit the star tracker timing to the gyro timing. For the fit, we take every 10 star tracker solutions (30 secs) as a group, and then fit each group to the equivalent length of the gyro solutions (300 data points) to get the time shift of the star tracker and apply it to each group of star tracker solutions (figure 3.8, 3.9). Although figure 3.8 shows elevation and azimuth separately, we fit those simultaneously. After applying the time correction, the star tracker data agrees with the gyro data well as shown in the right column of figure 3.8.

There is another correction needed for the star tracker data. This is especially important
for the Crab observations because there is only one star in the field of view of the star camera due to the faint star field around the Crab. As briefly mentioned in section 2.3.1, we need at least three stars in the Field of View (FoV) of the star camera for accurate determination of the pointing. The Crab star field does not suffice, and thus we need to supplement the star tracker solution with roll information. Since there is no absolute measurement of roll, we assume zero roll when we analyze the star camera to get star tracker solutions. The star tracker solutions obtained in this way can be erroneous (figure 3.10). Figure 3.10 shows the star in the FoV, the true pointing if there is a roll (center of the red rectangle), and the one we obtain assuming zero roll (center of the black rectangle), where the rectangles (drawn in solid line) are the detector of the star tracker with (red) and without (black) roll, and

Figure 3.8: Star tracker time correction (only \( \sim \)1 minute of data shown). Top: Elevation, Bottom: Azimuth, Left: Before the correction, Right: After the correction.
Figure 3.9: Star tracker time correction. We assume zero roll when reconstructing the star tracker data with one star in the FoV (black) but the solution would be different if there is a roll (red), and the error is $\theta_{\text{err}} = \theta_{\text{sp}} \times \theta_{\text{r}}$.

The constant part of the roll which we cannot measure precisely causes a constant offset in star tracker solution, which is a smaller concern. We can offset the pointing solution by such an amount that our events agree with the known location of the Crab Nebula, and this offset does not cause any trouble in imaging. However, the varying part of roll will blur the image if it is not corrected. Fortunately, the varying part of the roll can be measured reliably with the gyro and the GPS, and the correction needed turned out to be very small ($< 6''$).

3.2.1.2 GPS solution

The GPS measures the complete motion (all 6 degrees of freedom) of the gondola but not of the telescope. Since it does not give the motion the telescope, we have to get the motion of the telescope alone by other means, which can be done by the elevation encoder. With the
roll, pitch and yaw of the GPS and the elevation of the encoder, we can fully reconstruct the aspect. However, this is not the method I used because the precision of this method is not good as indicated in table 3.1 and in figure 3.13.

Although its rotational information is not precise, the latitude, the longitude and the altitude of the GPS are precise enough and not measured by the other sensors. As is clear in equations (3.1)-(3.7) (section 3.2.2), the latitude and the longitude are necessary for the transformation between the equatorial and the horizontal coordinate system, and it is also clear that the precision of 10 m ($10 \text{ m} / R_{\text{earth}} \simeq 0.3''$) is good enough. Another useful datum of the GPS is the altitude, which will be used for obtaining the air column density.

Figure 3.11 shows the latitude and the longitude measured in differential mode by the GPS over the flight, which illustrates the first problem of the GPS data - data missing for some periods. This is because the GPS was jammed in the White Sands Missile Range, which did not happen during source observations, and thus is not a concern. When looked at closer (figure 3.12), the data show another problem - the slow data rate, which is also indicated in table 3.1. We have only 3-4 measurements made by the GPS over ~12 minutes. Therefore, we interpolate the GPS solutions assuming a smooth motion of the balloon. The error of the interpolation is proportional to the second derivative of the “true” function ($\text{err} = \frac{f_{\text{true}}''}{8} \Delta t^2$), which is the acceleration in our case. Although we do not know the true
function, the acceleration would be very small, considering the mass of the gondola, and so would the error \( a_{\text{lon}} \sim 0.005 \text{ deg}/s^2, a_{\text{lat}} \sim 0.003 \text{ deg}/s^2 \), when I use the sparsely measured data).

### 3.2.1.3 Gyro solution

Two gyros were employed on HEFT. One was mounted on the telescope strut, measuring the pitch of the gondola and the telescope together, the other was mounted on the gondola base, measuring the yaw and the roll of the gondola. That is, these two gyros measure all the required rotational motions, since elevation involves the gondola and the telescope together, and the roll and the yaw involve the gondola only. Also notice the gyros provide the data with good precision and at a fast rate (table 3.1).

As mentioned earlier, the problem with the gyro data is the stability over long periods of time. Although we make corrections to the gyros during the flight (figure 2.8), the corrections cannot be perfect because some of the sensors to which we correct the gyro data are not precise enough, and this imprecision will accumulate as we integrate the gyro data. Therefore, a post-flight correction to the gain and the offset is performed on the gyro data. This correction was fairly complicated, mainly because the raw gyro voltages were omitted in the down-link stream [Madsen, 2007].
Figure 3.13: GPS and gyro azimuth, roll, and pitch. Left: GPS solution, right: gyro solution. The GPS solution is significantly noisier than that of the gyro. Notice that pitch does not correlate well because GPS measures the pitch of the gondola alone while the gyro measures that of the gondola and the telescope together.
Figure 3.14: Roll scale determination. Left: Roll measured by the GPS (solid) and the gyro (dotted). Right: The variance as a function of the roll scale factor.

Even after the post-flight correction, the thermal drift, which causes a slow drift of the gyro data, cannot be determined. Therefore, the low frequency part of the gyro data is not reliable, and thus I remove it. Two approaches were applied for this - filtering (a 5th order Butterworth filter) and polynomial fitting, and these give the same aspect solution within the accuracy of the sensors. As the two approaches agree to the required accuracy, I chose to use the filtering, and the filtering length scale is determined by comparing the filtered gyro data to the star tracker data.

While doing the analysis, I found out that the gain of the gyro roll is not correct when compared to the star tracker solution and GPS solution. We correct for this by fitting gyro roll to the GPS roll and correcting the gain to 1.62. The fit is done numerically by finding the minimum variance \( (R_{\text{gps}} - A_{\text{scale}} \times R_{\text{gyro}})^2 \). Figure 3.14 shows the data before applying the scaling to the gyro roll (left) and the variance as a function of the scale factor. Also an independent check is done by fitting the gyro roll to the star tracker data using equation (3.12) for the roll scale factor, where we get 1.65 for the scale factor.

3.2.2 Coordinate transformation

I reconstruct the aspect in the horizontal coordinate system for convenience and then finally transform the solution back to equatorial coordinates. This requires two coordinate
transformation formulae.

The first one is the transformation between the equatorial and the horizontal coordinates. This is needed to transform the star tracker data into horizontal coordinates, and to do the final aspect solution back to the equatorial coordinates. The formula is the following:

\[
\begin{align*}
LST & \simeq 100.46 + 0.9857d + \Psi_{long} + 15UT \\
\Psi_{ha} & = LST - \Psi_{RA} \\
x & = -\cos(\Psi_{ha})\cos(\Psi_{dec})\sin(\Psi_{lat}) + \sin(\Psi_{dec})\cos(\Psi_{lat}) \\
y & = -\sin(\Psi_{ha})\cos(\Psi_{dec}) \\
z & = \cos(\Psi_{ha})\cos(\Psi_{dec})\cos(\Psi_{lat}) + \sin(\Psi_{dec})\sin(\Psi_{lat}) \\
tan(\phi) & = y/x \\
tan(\theta) & = \frac{z}{\sqrt{x^2 + y^2}},
\end{align*}
\]

where LST is the local sidereal time in degree, d is the number of days from J2000, \(\Psi_{long}\) is the longitude, UT is Universal time in hours, \(\Psi_{ha}\) is the hour angle, \(\Psi_{RA}\) is the RA, \(\Psi_{dec}\) is the declination, \(\Psi_{lat}\) is the latitude, \(\phi\) is the azimuth, and \(\theta\) is the elevation. Although this
formula gives the basic idea, the code I used includes the effects of the precession and nutation of the earth and so on for better accuracy (http://idlastro.gsfc.nasa.gov/ftp/pro/astro/hor2eq.pro).

The second formula is the transformation between the gondola and the horizontal coordinates. This is used for transforming the gyro data, which is in the gondola coordinate system. When the gondola is tilted by $r_i$ (roll - around x), $p_i$ (pitch - around y), $y_i$ (yaw - around z) (see figure 3.16), the rotation matrix from the gondola to the horizontal coordinate system is given by

$$R_i = \begin{pmatrix} 1 & y_i & p_i \\ -y_i & 1 & r_i \\ -p_i & -r_i & 1 \end{pmatrix}$$

(3.9)

to the first order. The gondola is usually tilted by a small angle as shown in the GPS and the gyro data (figure 3.13). Therefore a first order approximation is sufficient, which simplifies the calculation significantly.

Now that we have the transformation matrices, we can fill in the gap in the star tracker data with the gyro data. If we have star tracker data at $i$, $i+30$, $i+60$, and so on, which means $i$ is a 0.1 sec interval, the pointing solution between two star tracker positions can be found as follows:

$$(\phi, \theta)_{i+1} = R_i R_{i,i+1}(r', p', y') R_i^{-1}(\phi, \theta)_i$$

(3.10)

where $R_i$ is the rotation matrix given in equation (3.9), $\theta$ is the elevation angle, $\phi$ is the azimuthal angle, and $R_{i,i+1}(r', p', y')$ is the rotation of the gondola from the $i$th step to the $i+1$th step measured by the gyro (the matrix in equation (3.9) with $r' = r_{i+1} - r_i$). A rather complicated rotation in equation (3.10) results because yaw ($y$), pitch ($p$) and roll ($r$) occur in the gondola coordinate while the star tracker solution is in the Az-El coordinate. We have to transform the star tracker solution ($(\phi, \theta)_i$) to the gondola coordinate ($R_i^{-1}$), apply yaw, pitch and roll ($R_{i,i+1}(r', p', y')$), and transform back to the Az-El coordinate ($R_i$).

When expanded, this becomes

$$\theta_{i+1} \simeq \theta_i + p'$$

(3.11)

and

$$\phi_{i+1} \simeq \phi_i + y' + r'\tan\theta_i$$

(3.12)
This gives us the fast pointing in the horizontal coordinate system. Figure 3.17 shows the star tracker pointing and the reconstructed pointing. The star tracker takes the data at every three seconds (cross). The gap between star tracker solution is reconstructed using equations (3.11) and (3.12).

The last thing we have to consider is the offset between the telescope and the star tracker. If the star tracker is perfectly aligned to the telescope, equations (3.11)-(3.12) will be the final pointing solution. However, there is a misalignment between them, which is measured to be $(\gamma_\phi, \gamma_\theta) = (0.015^\circ, 0.08^\circ)$ in the gondola coordinate system when the telescope is parallel to the gondola base [Madsen, 2007]. This offset should be applied to the star
tracker pointing to get the telescope pointing.

\[ \theta'_i \simeq \theta_i + \gamma \theta, \] (3.13)

\[ \phi'_i \simeq \phi_i + \gamma \phi \cos(\theta_i + \gamma \theta), \] (3.14)

where \((\phi'_i, \theta'_i)\) is the telescope pointing at the \(i\)th step. Equation (3.11) - (3.14) give the telescope pointing solution which will be used in the next section for the event reconstruction. The cosine factor in equation (3.14) is due to the fact that the azimuthal angle between the telescope and the star tracker changes as they are elevated even if their physical separation does not.

### 3.3 Event Reconstruction

The detector records the event energy and the \(x, y\) coordinate in pixels. After properly offsetting and rotating the detector coordinates (section 2.1), the pixel coordinates can be transformed to \(\Delta \phi_E\) (local azimuthal angle) and \(\Delta \theta_E\) (local elevation angle) with respect to the telescope pointing as following: (figure 3.18).

\[ \Delta \phi_E = \frac{x_d}{F} - \frac{y_d}{F}(r \cos \theta' - y \sin \theta') \] (3.15)

\[ \Delta \theta_E = \frac{x_d}{F}(r \cos \theta' - y \sin \theta') + \frac{y_d}{F}, \] (3.16)

where \(x_d\) and \(y_d\) are the detector \(x\) and \(y\) coordinates of an event in millimeters, \(F\) is the focal length (6000 mm), and the other symbols are defined in the previous section. In equation (3.15) and (3.16), \(x_d\) and \(y_d\) are coupled by effective roll \((r \cos \theta' - y \sin \theta')\), which is different from the gondola roll \((r)\) because the roll and the yaw of the gondola mix together by elevation \((\theta)\). Indeed, equation (3.15) and (3.16) are nothing more than a rotation in the detector plane by an angle of \(r \cos \theta' - y \sin \theta'\), where first order approximation is applied.

We combine this with the pointing (equation (3.13)-(3.14)) according to the following formula, finally getting the event location in the horizontal coordinate system.

\[ \phi_E = \phi' + \frac{\Delta \phi_E}{\cos \theta'}, \] (3.17)

\[ \theta_E = \theta' + \Delta \theta_E, \] (3.18)
Figure 3.18: Telescope pointing and event location on the detector.
Figure 3.19: Event distribution (left) and pointing solution (right) on the flat field background. The crosses are events, dots are pointing solution, and the contours are the flat field background.
CHAPTER 3. ANALYSIS OF HEFT DATA

Figure 3.20: Track of the source (solid) and the reconstructed events (cross). The source is moving from the lower left to upper right in the plot.

Figure 3.19 shows the event distribution (left column) and the pointing (right column) on the detector for the Cyg X-1 observation. This gives us a rough idea of what are source events and what are background events. Finally, to construct the counts map, we subtract the source motion from that of the event. Figure 3.20 shows the trace of the source (Solid line, the Crab Nebula) and the reconstructed event (Cross). The counts map for Cyg X-1 and the Crab Nebula are shown in figure 3.27 and figure 3.30.

The off-axis angle of an observation, which is important in determining the PSF and the effective area for the observation, can be determined by the pointing if the reconstruction is perfect. Unfortunately, this is not the case for the HEFT flight due to the uncertainties in the telescope alignment and the lack of knowledge of absolute roll angles of the gondola. Therefore we use another method for the off-axis angle determination, which will be discussed in section 3.5.

3.4 Construction of Ancillary Response File

Now that we have reconstructed the aspect, we are at the last step of the analysis - unfolding the instrument responses from the data. As shown in figure 3.1, the instrument responses folded with the aspect solution are PSF, Ancillary Response File (ARF) and Redistribution
Matrix File (RMF), which will be explained in this section.

Imagine a photon is coming from an astronomical object. It hits a portion of the telescope, reflects off it, and is detected at the detector. In this process, we do not know which portion of the telescope it hits, and thus where it falls in the detector. Due to the lack of knowledge of the trace of an individual photon, we have to deal with the data in a statistical sense. That is, if many photons are incoming to the telescope at a specific angle $\theta_s$, a photon will reflect to $\theta_{d,1}$, the others to $\theta_{d,2}$, to $\theta_{d,3}$, and so on, depending on the portion of the telescope the photon reflects off, because different portions of the telescope have different surface defects, and thus reflect the photon to a different angle. The probability distribution of $\theta_d$’s is the PSF.

Since photons are not incoming only at one angle but with an angular distribution, what we really have to consider is that $\theta_{s,1}$ goes to $\theta_{d,1}$, $\theta_{d,2}$, ... and $\theta_{s,2}$ goes to $\theta_{d,1}$, $\theta_{d,2}$, ..., and so on. In the simplest mathematical form, this process can be written as following:

$$C(\theta_d) = \int \Psi(\theta_d, \theta_s) f(\theta_s) d\theta_s,$$

where $C(\theta_d)$ is the photon distribution we will see in the detector, $\Psi(\theta_d, \theta_s)$ is the PSF, and $f(\theta_s)$ is the angular distribution of photons at the source. In this sense, the photon distribution of the source is “folded” (convolved) with the PSF, and solving for $f(\theta_s)$ with known $C(\theta_d)$ and $\Psi(\theta_d, \theta_s)$ is “unfolding” (deconvolution). A similar story goes on with the energy distribution too.

In reality, the analysis is more complicated because the incoming angle (energy) is distributed over space (energy) not only because of the distribution of photons at the source but also because of the change in the pointing. The analysis is well decomposed into several measurable quantities and organized in a reference ([Davis, 2000]) which I explain below.

The source counts from the source region $\Omega$ can be calculated by

$$C_\Omega(h, \hat{p}) = \int d\lambda \int_{\Omega} d\hat{p}' \int_0^\infty dt T(\sigma(\hat{p}', t), t) D(\sigma(\hat{p}', t), h, \lambda) F_A(\lambda, \hat{p}', \hat{p}_t) M(\lambda, \hat{p}_t) S(\lambda, \hat{p}),$$

(3.19)

where $\hat{p}_t$ is the direction of a photon exiting the telescope, $\sigma$ is the detector pixel coordinate, $\hat{p}$ is the source location in the sky coordinate, $\hat{p}_t$ is the direction of an incoming photon ($\hat{p}_t = R(t)\hat{p}$) with the sky transformation matrix $R(t)$ which is determined by the aspect
solution, \( h \) is pulse height, \( \lambda \) is the wavelength of the source photon, \( T(\sigma(\hat{p}'_t, t), t) \) is a representation of the so-called “good-time intervals” together with time dependent bad-pixels, \( D(\sigma(\hat{p}'_t, t), t) \) is the detector efficiency, \( F_A \) is the point spread function with the effect of the aspect uncertainties, \( M \) is the multilayer reflectivity, and \( S \) is the source photon distribution (energy and angle).

This basically states that photons are generated at a location in the sky with an energy \( S(\lambda, \hat{p}) \). When those hit the telescope, the number of photons reduces by a factor of \( M(\lambda, \hat{p}_t) \) depending on its energy and incoming angle. Those reduced photons redistribute according the PSF \( (F_A(\lambda, \hat{p}'_t, \hat{p}_t)) \) and are detected at the detector. At the detector, there is another reduction in the number of photons due to the detection efficiency, and there is also a redistribution of energy \( (D(\sigma(\hat{p}'_t, t), h, \lambda)) \). Finally, \( T(\sigma(\hat{p}'_t, t), t) \) is the observation time effect - the longer we observe a source, the more photons we detect.

We further decomposed the detector effect into two factors - reduction in the number of photons and redistribution of energy, assuming that the detector properties do not change over time.

\[
D(h, \sigma, \lambda) = D_R(h, \lambda)Q(\sigma, \lambda),
\]

where \( D_R(h, \sigma, \lambda) \) is the energy redistribution factor called RMF and \( Q(\sigma, \lambda) \) is the loss factor called quantum efficiency (QE). The RMF is constructed from the detector simulation and mostly is diagonal as shown in figure 3.21 with small off-diagonal components due to the escape peaks from Cd, and the QE is produced in a similar way and shown in figure 3.22. More details are discussed in other references such as [Chen, 2008].

Then, the source term \( (S(\lambda, \hat{p})) \) is further decomposed into a image and a spectrum term.

\[
S(\lambda, \hat{p}) = s(\lambda)\rho(\hat{p}).
\]

The pointing is not absolutely stationary, and thus the photon incoming angle changes, which changes the shape of the PSF (see section 2.4.3). Therefore, we construct the ARF for the spectral analysis and the effective PSF for image analysis, taking into account the pointing-dependent change of the PSF. The ARF \( (A_{\Omega}(\lambda)) \) is constructed as the following:

\[
A_{\Omega}(\lambda) = \frac{1}{\tau_{eff}} \int_{\Omega} d\hat{p}' \int_{\Omega} d\hat{p} \int_{0}^{T} dt T(t)Q(\lambda, \sigma(\hat{p}'_t, t))F_A(\lambda, \hat{p}'_t, \hat{p}_t)M(\lambda, \hat{p}_t)\rho(\hat{p}),
\]
Figure 3.21: Redistribution Matrix File. Top left: 2-D surface plot, Top right: 2-D contour plot, Bottom left: 1-D plot at 30 keV, Bottom right: 1-D plot at 50 keV.
where $\tau_{\text{eff}}$ is the effective observation time ($= \int_0^\tau T(t)dt$). Using equation (3.20)-(3.21), and integrating equation (3.19) over $\hat{p}$, we obtain

$$C_{\Omega}(h) = \int d\hat{p}C_{\Omega}(h, \hat{p}) = \tau_{\text{eff}} \int d\lambda D_{R}(h, \lambda) A_{\Omega}(\lambda) s(\lambda),$$

(3.22)

which can be solved with XSPEC [Arnaud, 1996].

For image analysis, we construct the effective PSF($F_{\text{eff}}(\hat{p})$) as following:

$$F_{\text{eff}}(\hat{p}) = \int d\lambda \int d\hat{p}' \int_0^\tau dt T(\sigma(\hat{p}', t), t) D(\sigma(\hat{p}', t), h, \lambda) F_{A}(\lambda, \hat{p}', \hat{p}) M(\lambda, \hat{p}) s(\lambda).$$

(3.23)

With this equation, equation (3.19) becomes

$$C_{\Omega}(\hat{p}) = \int d\hat{p}' F_{\text{eff}}(\hat{p} - \hat{p}') \rho(\hat{p}'),$$

(3.24)

which can be solved by some techniques such as deconvolution and forward folding method.
For the data analysis, the ARF is constructed as defined by equation (3.21) and the effective PSF by equation (3.23). This poses a problem. We have to know the source shape ($\rho(\hat{p})$) to calculate ARF, and the source spectrum ($s(\lambda)$) to calculate the effective PSF. For Cyg X-1, whose shape ($\rho(\hat{p})$) is known, it is straightforward to calculate the ARF. The spectrum is obtained using this ARF and input into equation (3.23) to obtain the effective PSF for the image analysis. For the Crab Nebula, whose spectrum ($s(\lambda)$) is well measured, we calculate the effective PSF first, then the ARF later. One thing to note is that the hard X-ray emission region of the Crab Nebula is small ($r \sim 30''$), and the effective area does not change much over this range, therefore calculating ARF assuming that it is a point source did not make any difference. Therefore I assumed the Crab was a hard X-ray point source to save time in the analysis.

### 3.5 The Optical Axis

The optical axis is found for the Cyg X-1 observation by maximum likelihood analysis [Boese and Doebereiner, 2001]. We assume that the event distribution follows the Poisson distribution, since the probability of photon detection per pixel is small. Thus the likelihood function is derived from the Poisson distribution. The Poisson probability is given by

$$P(n) := \frac{e^{-\Lambda(Q)} \Lambda(Q)^n}{n!},$$

(3.25)

where $\Lambda$ is an expected number of events, $n$ is the measured number of events and $Q$ is a point in phase space (detector X,Y, time, energy). I define average flux ($\lambda$) and local average flux ($\lambda_{i,j,k,l}$, $i,j,k,l = \text{phase space index}$) by

$$\Lambda(\theta_x, \theta_y) := \int_Q \lambda(X, Y, t, E; \theta_x, \theta_y) dXdY dtdE,$$

$$\lambda_{i,j,k,l}(\theta_x, \theta_y) := \frac{1}{\Delta V_{i,j,k,l}} \int_{Q_{i,j,k,l}} \lambda(X, Y, t, E; \theta_x, \theta_y) dXdY dtdE,$$

where $\Delta V_{i,j,k,l}$ is a local phase space volume ($\Delta V_{i,j,k,l} = \Delta A \Delta t \Delta E$, $\Delta A =$ pixel area, $\Delta t = 1$ sec, $\Delta E = 2$ keV in our case). $\lambda$ is an implicit function of off-axis angle ($\theta_x, \theta_y$) due to the optics response in our case.
Then equation (3.25) becomes
\[
P(n_{i,j,k,l}) = \prod_{n_{i,j,k,l}} e^{-\lambda_{i,j,k,l} n_{i,j,k,l} \Delta V_{i,j,k,l}} = \prod_{n_{i,j,k,l}} e^{-\lambda_{i,j,k,l} n_{i,j,k,l} \Delta V_{i,j,k,l}} \prod_{n_{i,j,k,l}=0}^{\lambda_{i,j,k,l}} n_{i,j,k,l}!\]
\[
= P_{\Delta,e} P_{\Delta,0} P_{\Delta,\lambda} P_{\Delta,V} = P_{\Delta},
\]
where
1. \( P_{\Delta,e} = \prod_{n_{i,j,k,l}} e^{-\lambda_{i,j,k,l}(\theta_x, \theta_y) \Delta V_{i,j,k,l}} = e^{-\Lambda(\theta_x, \theta_y)} \) is a constant in \((X,Y,t,E)\).
2. \( P_{\Delta,0} = \prod_{n_{i,j,k,l}=0}^{\lambda_{i,j,k,l}} n_{i,j,k,l}! = 1.\)
3. \( P_{\Delta,\lambda} = \prod_{n_{i,j,k,l}>0}^{\lambda_{i,j,k,l}} \frac{(\lambda_{i,j,k,l} \Delta V_{i,j,k,l})^{n_{i,j,k,l}}}{n_{i,j,k,l}!} \) as \( \Delta \to 0.\)
4. \( P_{\Delta,V} = \prod_{n_{i,j,k,l}>0}^{\Delta V_{i,j,k,l}} \frac{n_{i,j,k,l}!}{n_{i,j,k,l}!} = V_{\Delta} \) is independent of \( \lambda.\)

Renormalizing the probability function with \( V_{\Delta} \) gives us
\[
P(\lambda) : = \lim_{\Delta \to 0} \frac{P_{\Delta}}{V_{\Delta}} = e^{-\Lambda(\theta_x, \theta_y)} \prod_{m=1}^{M} \lambda(X_m, Y_m, t_m, E_m; \theta_x, \theta_y).\]

Finally, the likelihood is defined by taking the log of the renormalized probability.
\[
L(\lambda; \theta_x, \theta_y) := -\int \lambda(X, Y, t, E; \theta_x, \theta_y) dX dY dt dE + \sum_{m=1}^{M} \ln(\lambda(X_m, Y_m, t_m, E_m; \theta_x, \theta_y)).
\]
\[
(3.26)
\]
Simply reparametrizing \( \lambda = \lambda_B + \lambda_S \) (Background term and source term), equation (3.26) becomes,
\[
L(\theta_x, \theta_y) := -\Lambda_B - \Lambda_S(\theta_x, \theta_y) + \sum_{m=1}^{M} \ln(\lambda_B(E_m) + \lambda_S(X_m, Y_m, t_m, E_m; \theta_x, \theta_y)).
\]
\[
(3.27)
\]
At this point we generalize from [Boese and Doebereiner, 2001].
We assume that the background is uniform in the detector pixels and in time. Because we do not know the background, source spectrum and optics response well, we introduce constant correction factors \((\epsilon_B, \epsilon_S)\) in the source and the background terms.
\[
L(\epsilon_S, \epsilon_B, \theta_x, \theta_y) := -\epsilon_B \Lambda_B - \epsilon_S \Lambda_S(\theta_x, \theta_y) + \sum_{m=1}^{M} \ln(\epsilon_B \lambda_B(E_m) + \epsilon_S \lambda_S(X_m, Y_m, t_m, E_m; \theta_x, \theta_y)).
\]
\[
(3.28)
\]
where $\Lambda_B$, $\lambda_B$, $\Lambda_S$ and $\lambda_S$ are calculated as follows.

- $\Lambda_B = \text{counts over a 3 hour long blank sky observation normalized by the source observation time}$

- $\lambda_B = \text{mean background counts per 4-dimensional volume (}\Delta V_{i,j,k,l}\text{)}$

- $\Lambda_S(\theta_x, \theta_y) = \int_E \int_t \int_{\text{pixels}} \int_{\theta_p} \mathcal{P}(\theta_{p,x}, \theta_{p,y}) \Psi(E, x, y; \theta_x - \theta_{p,x}, \theta_y - \theta_{p,y}) A_{\text{eff}}(E, \theta_x - \theta_{p,x}, \theta_y - \theta_{p,y}) \mathcal{E}_{\text{det}}(E, x, y) e^{-\mu E} \mathcal{F}_s(E) d\theta_p dt dN_{\text{pixel}} dE = \text{total expected source counts (}\mathcal{P}: \text{pointing, } \Psi: \text{point spread function, } A_{\text{eff}}: \text{effective area, } \mathcal{E}_{\text{det}}: \text{detector efficiency, } e^{-\mu E}: \text{atmospheric attenuation, } \mathcal{F}_s: \text{assumed Cyg X-1 source flux, } (x,y): \text{detector pixel coordinate, } (\theta_x, \theta_y): \text{off-axis angle from the optical axis, } (\theta_{p,x}, \theta_{p,y}): \text{pointing error, } t: \text{time, } E: \text{detected energy})$

- $\lambda_S(X_m, Y_m, t_m, E_m; \theta_x, \theta_y): \text{mean expected source counts per 4-dimensional volume (}\Delta V_{i,j,k,l}\text{)}$

Now we have four free parameters - $\epsilon_S$, $\epsilon_B$, off-axis angle (azimuth) and off-axis angle (polar). We maximize the log-likelihood (equation (3.28)) with respect to those four free parameters to get the best fit to the optical axis.

We fit the projected Cyg X-1 image with the model PSF (section 2.4.3), a background model determined from flat fields, a source term, and with the source normalization and background normalization as free parameters. Figure 3.27 shows the count distribution. From the one-dimensional distributions of figure 3.27 it is seen the above model provides an acceptable fit to the projected image of Cyg X-1. Note that the observation is done at large off-axis angle (section 3.5) and so the PSF as well as the event distribution are distorted (elongated) due to off-axis effects.

The arbitrary normalization parameters for the signal and background (manifestations of our uncertainty in the ground calibration of the effective area) do not affect our imaging analysis. The normalization, however, is relevant to the flux measurement, which will be discussed below.

Figure 3.23 shows the log likelihood ratio for each optic. The maximum likelihood occurs at (-6.0, 1.4) arcmin for HF1, (2.6, -2.0) arcmin for HF3. This shows both that the optics are
misaligned with the detectors and there is relative misalignment between the optics. The misalignment could be due to inaccurate ground alignment and/or shocks or vibrations introduced in the launch and ascent. The best-fit optical axis obtained from the Cyg X-1 observations is applied to the Cyg X-1 observations and the Crab observations. Figure 3.27 shows the one-dimensional event distribution in RA and DEC coordinates for the instrument model (PSF, background, pointing, detector pixel size) of HF1 and HF3 together with the observed image obtained in the Cyg X-1 observations. The event distribution is statistically consistent with the telescope model PSFs, implying that Cyg X-1 is a point source, and validating the procedure for finding the optical axis.

Figure 3.23: Likelihood ratio contours for HF1 (left) and HF3 (right).
Table 3.2: Optical Axis found for the Cyg X-1 observation with respect to the detector center in arcminutes.

<table>
<thead>
<tr>
<th>Source</th>
<th>Optic</th>
<th>AZ (arcmin)</th>
<th>EL (arcmin)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyg X-1</td>
<td>HF1</td>
<td>-6.0</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>HF3</td>
<td>2.6</td>
<td>-2.0</td>
</tr>
</tbody>
</table>

3.6 Observation

During the 24 hours flight, HEFT successfully detected Cyg X-1 and the Crab Nebula. The main goal of the Cyg X-1 observation was calibrating the optics PSF. Cyg X-1 is a known point source, therefore it can be used for PSF calibration. Using 54 minutes of Cyg X-1 observation data, we find that our PSF modeling is good within the statistical uncertainty.

The Crab Nebula was observed three times during the flight. The duration of each observation was 12 minutes, 15 minutes and 32 minutes. We analyze these observations to measure the size of the Crab Nebula, and to search for the hard X-ray asymmetry in the shape. With this analysis, we measured the size of the Crab Nebula.

3.6.1 Cyg X-1

Cyg X-1 is a high-mass X-ray binary (HMXB) and is a black hole. A blue giant star is orbiting the compact object and the stellar wind of the blue giant star is accreted onto the compact object [Gies and et al., 1986]. Thermal photons generated by the friction of the viscous flow of the accretion disk are Compton scattered up to the hard X-ray band. It is not a good spectral calibration source as the X-ray spectrum is highly variable and unpredictable. However, it is a point source and good for PSF calibration.

3.6.1.1 Image Analysis

HEFT observed Cyg X-1 on May 19th 7:54 to 8:48 UTC at an average elevation of 65°. Figure 3.24 shows the track of the source (solid) and the telescope pointing in elevation and azimuth. The pointing was not very stable but the reconstruction is rather simple as the
star field around the source is bright. We have an average of 10 stars on the star camera. The star tracker time correction for this observation was 0.5 sec on average.

During this observation, there were 1640 events (20-60 keV) recorded over 54 minutes on the HF1 and HF3 detectors together. The pixel coordinates of those events and the telescope pointing are reconstructed to azimuth and elevation angle relative to the source. Figure 3.25 shows the reconstructed azimuth (upper panels) and elevation (lower panels) relative to the source. The source events should correlate with the telescope pointing and are shown as regions of dense events in the figures. When the telescope is pointing off by $\theta$ from the source, the event will be off-centered by $\theta F$, where $F$ is the telescope focal length.

To finally reconstruct the event, we make a pointing correction to the event by subtracting the azimuth and the elevation of the pointing from those of events. Figure 3.26 shows
Figure 3.25: The event and the pointing distribution for the Cyg X-1 observation. Upper panels show the reconstructed azimuth and lower ones show the reconstructed elevation relative to the source for the events (left) and the telescope pointing (right).
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69

the event distribution.

The aspect corrected events are transformed to equatorial coordinates (figure 3.27). We fit the projected Cyg X-1 image with the model PSF (section 2.4.3), a background model determined from flat fields, a source term, and with the source normalization and background normalization as free parameters. Figure 3.27 shows the count distribution. From the one-dimensional distributions of figure 3.27 it is seen that the above model provides an acceptable fit to the projected image of Cyg X-1. Note that the observation is done at large off-axis angle (section 3.5) and so the PSF as well as the event distribution are distorted (elongated) due to off-axis effects. The event distribution is statistically consistent with the telescope model PSFs, implying that Cyg X-1 is a point source, and validating the procedure for finding the optical axis.

The Half Power Diameter (HPD) of the event data and of the PSF are compared. To do this, we reorganize the event data and the PSF as a function of radial distance (Top panel of figure 3.28) and calculate the encircled event fraction for both the event data and the PSF (Bottom panel of figure 3.28). The measured HPD of Cyg X-1 is 96\arcsec and agrees with the model HPD (97\arcsec). This 1-D comparison alone does not guarantee that our PSF modeling is good, but together with the 2-D comparison (figure 3.27) again verifies that our modeling worked very well. Finally, we expect that the HPD would be 81^{+10}_{-8}\arcsec if the observation had been done on-axis, where the quoted error obtained is statistical and is obtained from the encircled energy fraction.

Table 3.3: $\chi^2$/DOF for the Cyg X-1 image fit.

<table>
<thead>
<tr>
<th>Module</th>
<th>X-projection</th>
<th>Y-projection</th>
</tr>
</thead>
<tbody>
<tr>
<td>HF1</td>
<td>1.04</td>
<td>1.25</td>
</tr>
<tr>
<td>HF3</td>
<td>0.85</td>
<td>0.85</td>
</tr>
</tbody>
</table>
Figure 3.26: Event distribution (left column) with its projected histogram (right column) in azimuth (top), elevation (middle), and 2-D (bottom) relative to the source after the pointing correction. For 1-D histograms, a simple fit is overlaid as a guide.
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Figure 3.27: Cyg X-1 image with PSF overlaid. Top: count map (gray scale and white lines) and PSF (red lines, 4 pixels offset upward). Middle and Bottom: 1-D projection of count map (solid black line with error bars), PSF (dashed red) and flat field background (dotted blue). Left column: HF1, Right column: HF3
3.6.1.2 Spectral Analysis

The source spectrum of Cyg X-1 is well explained by Comptonization of thermal photons in a hot electron plasma. The spectrum is highly variable depending on whether Cyg X-1 is in a low-hard state or a high-soft state [Ogawara and et al., 1977]. We use a simple power-law spectrum for the fit. Figure 3.29 shows the data and the fit. From the fit, we obtain \((2.06 \pm 2.00) E^{-1.98 \pm 0.27} \text{cts/keV/s/cm}^2\). It is hard to say in which state Cyg X-1 was during the observation due to the uncertainties.
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3.6.1.3 Conclusion

Cyg X-1 observation and analysis verifies that the spatial responses (optics and detectors) and the analysis procedures are correct. The uncertainty in the width of the PSF is $\sim 10''$, which is small enough to permit a Crab Nebula size determination.

3.6.2 The Crab Nebula

3.6.2.1 Image analysis and the size determination

HEFT observed the Crab Nebula three times during the flight and the total observation time was 59.4 minutes. The aspect reconstruction was done in the same way as is done for Cyg X-1. One thing that was cumbersome is that the star field around the Crab Nebula is faint, and we had only one star in the star camera. As discussed earlier (section 3.2.1.1), we lack the knowledge of the roll offset, which causes a constant offset in the source location. Therefore, after the aspect reconstruction, the location where events originated can be

Figure 3.29: Cyg X-1 spectral fit. Top: Data and the fit, Bottom: the residual.
Figure 3.30: Crab image with PSF overlaid. Top left: count map (gray scale and white lines) and PSF (red lines, 4 pixels offset upward). Top right and bottom left: 1-D projection of count map (solid black line with error bars), PSF (dashed red) and flat field background (dotted blue).

different from the true source location by a few arcminutes, and this offset is corrected manually.

Figure 3.30 shows the count maps and its projection to RA and DEC coordinate for the Crab observations. The event distribution is extended compared to the PSF in the count map as well as the 1-d projections, which qualitatively suggests that Crab is an extended source.

Several approaches have been tried to extract the source image. The first approach is 2-D deconvolution. We tried to deconvolve the PSF from the event distribution by the Maximum Entropy Method (MEM) [Városi and et al., 1993]. We fit the deconvolved image
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Figure 3.31: Deconvolution of the event distribution for Crab observation. Left: Event distribution (black), PSF (red) and the deconvolved image. Right: 2-D Gaussian fit to the deconvolved image.

to a 2-D Gaussian. Figure 3.31 shows the result of the deconvolution. It appears that the deconvolution works fine, but in fact the fit is very unstable. If we change a parameter slightly within the error limits (for example, the alignment of the three observations or the width of the PSF), the deconvolution gives a completely different and non-physical result. This is due to the poor statistics and this method is not used for the following analysis.

The second approach is fitting the 1-D projection. We fit the data with a convolution of the source shape and the PSF, with source parameters as free parameters (the width in x and y direction and the amplitude). Here we assume that the 2-D source profile is a 2-D Gaussian. After the convolution, we project the fit function as well as the event distribution onto the predefined axis. We measure the size at many different projection angles as shown in figure 3.35 to measure the size. Figure 3.32 shows an example of the fit at a projection angle. The left panel shows the fit to the torus projected events and the right shows the fit to the jet projected events. From this analysis we obtain the source size of $55 \pm 9''$ in the torus direction, $23 \pm 12''$ in the jet direction.

Table 3.4 shows the uncertainties estimated for the size determination. Finally we obtain the Crab Nebula size (25-58 keV) of $55^{+14}_{-16}$ arcsec for the torus and $23^{+18}_{-18}$ arcsec for the jet direction. Since the confidence of the size measurement along the jet direction is relatively low, we set the upper limit. With 90% confidence, we set the upper limit of $46''$ along the jet direction [Robertson and et al., 1988]. Uncertainties are discussed in section 3.6.2.2.

The size of the Crab Nebula extrapolated to the HEFT energy band from the Chandra
Figure 3.32: 1-D projection fit results for the torus projection (top) and the jet projection (bottom).

Table 3.4: Uncertainties of the size determination of the Crab Nebula.

<table>
<thead>
<tr>
<th>Uncertainties</th>
<th>Torus direction</th>
<th>Jet direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>$9''$</td>
<td>$12''$</td>
</tr>
<tr>
<td>PSF width</td>
<td>$+9''$</td>
<td>$+10''$</td>
</tr>
<tr>
<td></td>
<td>$-12''$</td>
<td>$-10''$</td>
</tr>
<tr>
<td>Optical axis</td>
<td>$5''$</td>
<td>$5''$</td>
</tr>
<tr>
<td>Fit sensitivity</td>
<td>$1''$</td>
<td>$3''$</td>
</tr>
<tr>
<td>Projection angle</td>
<td>$3''$</td>
<td>$5''$</td>
</tr>
<tr>
<td>Centering</td>
<td>$1''$</td>
<td>$3''$</td>
</tr>
<tr>
<td>Total</td>
<td>$+14''$</td>
<td>$+18''$</td>
</tr>
<tr>
<td></td>
<td>$-16''$</td>
<td>$-18''$</td>
</tr>
</tbody>
</table>
Crab Nebula gets smaller as energy increases. This is easily understood by the synchrotron burn-off, which is a characteristic of synchrotron radiating electrons - they lose energy as they radiate. Quantitatively speaking, the radiating power of a synchrotron electron is 

\[ \frac{dE}{dt} = \frac{4}{3} \sigma_T c \left( \frac{E}{m_e c^2} \right)^2 B^2 \frac{\mu_0}{2} \], and thus the lifetime is \( T = \frac{E}{\frac{dE}{dt}} = \frac{6m_e c^3}{4\sigma_T E B^2} \), where \( E \) is electron energy, \( \sigma_T \) is the Thompson scattering cross section, and \( B \) is the magnetic field strength. Therefore, the lifetime of the electron is inversely proportional to its energy or to the square root of the emitted photon frequency (Recall, \( \nu \sim E^2 \) in synchrotron radiation). In the case of the Crab Nebula, this simple calculation is not strictly applicable, and a more realistic calculation is given in KC84.
3.6.2.2 Uncertainties in size determination

We consider the uncertainties in the size determination listed in table 3.4. The symmetry axis of the torus is tilted by 58° [Mori and et al., 2004] (or 60°, [Willingale and et al., 2001]) from north to west. If the analysis were 2 dimensional, we would get the tilt angle as a result of the analysis (eg. 2-D deconvolution or 2-D Gaussian fit). However, the analysis I did is not fully 2 dimensional, and thus I find the projection angle in the following way.

I vary the projection angle in the analysis and measure the size in the torus and the jet direction, obtaining the size as a function of projection angle. Since we assume that the source is a 2 dimensional Gaussian, the projection angle (position angle) is what makes the torus largest (and/or the jet smallest). Instead of finding one angle which makes the torus largest, I fit the size in both direction to the calculated size of a 1-D projected ellipse:

\[
X = \pm \frac{a^2 \cot^2 \theta}{\sqrt{a^2 + a^2 \cot^2 \theta}}, \quad Y = \pm b\sqrt{1 - \frac{X^2}{a^2}}
\]

\[
L = 2(X \cos \theta + Y \sin \theta),
\]

(3.29) \hspace{1cm} (3.30)

where \(a\) and \(b\) are the semi-major and the semi-minor radius, \(\theta\) is the rotation angle of an ellipse, and \(L\) is the projected size of the ellipse in the x axis (see figure 3.34).

The result is shown in figure 3.35, from which I get the average projection angle of 58 ± 5° (figure 3.35), the size along the torus direction (NE-SW) of 55″ and along the jet
direction (NW-SE) of 23″. This demonstrates why a 2-D imaging telescope is superior to a 1-D scanning telescope with which previous experimenters had to scan along many different projection angles [Pelling and et al., 1987]. With a 2-D imaging telescope, we do not have to “scan” along many different angles - one shot of the sky is enough to analyze the image at different projection angles even when the statistics is not sufficient for full 2-D analysis.

The second uncertainty I consider is the fit range sensitivity. I change the fit range from 4′ to 16′ (90% of the events are included within ~5′), and measure the size as a function of fit range. The fit results were fairly stable over this range (figure 3.36), and the uncertainty is 1″ and 3″ along the torus and the jet direction respectively.

The third uncertainty is that in the optical axis determination (section 3.5). I vary the optical axis by ±2′ in both directions for each module (2 modules × 2 axis = 4, ∆ Optical axis = 0, ±2′, total 81 variations: (-2, -2, -2, -2) → (2, 2, 2, 2)). Change of the optical axes will
Figure 3.36: Fit range sensitivity. The fitting range is varied from 4′ to 16′ (FoV) for the sensitivity study. The vertical line in each plot indicates the diameter where 90% of the events are included.

change the PSF and the flat field background, and thus the result of the fit. Figure 3.37 shows the result of the analysis, where a sudden drop in the plot happens when two of the optical axes change abruptly during the calculation (eg. -2,-2,2,2 → -2,0,-2,-2). From this, I estimate the uncertainty of the optical axis change to be 5′′ and 5′′.

The fourth uncertainty involves centering the 6 observations (2 optics × 3 observations). This is because of the undetermined constant offset. The center of each observation is found by fitting it to a Gaussian (source) plus a polynomial of degree 2 (background). Each observation is aligned so that the centroid of the Gaussian fit is zero. The error in finding center of each observation was ∼3″. Therefore, I generate Gaussian random numbers with a width of 3″ (6 for the torus direction and 6 for the jet direction), randomly misalign the 6 observations with respect to each other by this amount, and measured the size of the combined image. From this analysis, I obtain the uncertainty of the size determination to be 1″ and 3″ for the torus and the jet direction respectively (figure 3.38).

The last uncertainty is from the PSF width, which is the biggest source of error in size determination. I change the size of the PSF by convolving it with a Gaussian (to increase) or by rescaling (to decrease) it. Figure 3.39 shows the result of the analysis, where the x axis is the width of the new PSF (increased or decreased) and the y axis is the size of the Crab.
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Figure 3.37: The uncertainty in finding the optical axis is taken into account. The optical axis is varied by 2' on both the x axis and y axis. The effective PSF and the flat field background are re-calculated for each optical axis.

Figure 3.38: Alignment of the 6 observations is varied by 3'' randomly. 20 simulations are done by randomly misaligning these observations.
Figure 3.39: The width of the PSF is varied as described in the text. The x axis is the width of the PSF and the y axis is the measured size of the Crab Nebula. The star indicates the typical value of the analysis.

Nebula measured with the new PSF. This analysis shows us that the size we determine can change by +9/-12" in the torus direction and +10/-10" in the jet direction if the width of the PSF changes by 10" (obtained by Cyg X-1 analysis).

3.6.2.3 Comparison with the results of the previous missions

Although there have been many missions that measured the size of the Crab Nebula, each mission observed a different energy band. Therefore, it is necessary to do an energy and spatial extrapolation those results to compare with what we obtained. The extrapolation is done mainly using KC84 and the empirical fit of Ku [Ku and et al., 1976]. At the energy band of our interest, the FWHM can be approximated by $78''(\frac{E}{1 \text{keV}})^{-0.148}$ along the torus direction and to $58''(\frac{E}{1 \text{keV}})^{-0.148}$ along the jet direction. We extrapolate the results of other missions by using spectrum weighted averages.

Table 3.5 summarizes the results for different missions. One thing to note is that the ROSAT result is smaller than the others but at the same time it is not clear whether the result is FWHM or $2 \sigma$. Overall, the direct measurement with HEFT agrees well with the energy extrapolated results of other experiments.

1From FITS file, 2[Harnden and et al., 1984], 3[Hester and et al., 1995], 4[Fukada and et al., 1975]
Table 3.5: The size (FWHM) of the Crab Nebula measured by other missions extrapolated to 25-58 keV using spectrum weighted average.

<table>
<thead>
<tr>
<th>Mission</th>
<th>Energy (keV)</th>
<th>Torus (″)</th>
<th>Jet (″)</th>
<th>Torus (″)</th>
<th>Jet (″)</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>25-58</td>
<td>55</td>
<td>&lt;46</td>
<td>55</td>
<td>&lt;46</td>
<td>2-D imaging</td>
</tr>
<tr>
<td>Chandra$^1$</td>
<td>0.3-10</td>
<td>93</td>
<td>56</td>
<td>57</td>
<td>28</td>
<td>2-D imaging</td>
</tr>
<tr>
<td>Einstein$^2$</td>
<td>0.1-4.5</td>
<td>105</td>
<td>72</td>
<td>55</td>
<td>30</td>
<td>2-D imaging</td>
</tr>
<tr>
<td>ROSAT$^3$</td>
<td>0.1-2.4</td>
<td>84</td>
<td>46</td>
<td>44</td>
<td>20</td>
<td>2-D imaging</td>
</tr>
<tr>
<td>Japan-India$^4$</td>
<td>20-70</td>
<td></td>
<td>34</td>
<td>34</td>
<td></td>
<td>lunar occultation</td>
</tr>
<tr>
<td>MIT$^5$</td>
<td>20-150</td>
<td>49</td>
<td>24</td>
<td>49</td>
<td>24</td>
<td>lunar occultation</td>
</tr>
<tr>
<td>Columbia$^6$</td>
<td>0.6-23</td>
<td>many</td>
<td></td>
<td>34</td>
<td></td>
<td>lunar occultation</td>
</tr>
<tr>
<td>Pelling$^7$</td>
<td>22-64</td>
<td>68</td>
<td>28</td>
<td>68</td>
<td>28</td>
<td>1-D scanning modulation</td>
</tr>
</tbody>
</table>

3.6.2.4 Spectral analysis

Figure 3.40 and table 3.7 show the spectral fits for the Crab observation. The spectral fitting is done with XSPEC using a simple power law model. The Crab spectrum obtained with the best ground estimated parameters is $(5.33 \pm 4.45)E^{-2.18\pm0.24} \text{cts/keV/s/cm}^2$. Due to the low counting statistics, the error is relatively large and the confidence contours cover a fairly large region (figure 3.40). The fluxes obtained from the best fit are low compared to previous missions (table 3.6) [Kirsch and et al., 2005] [Weisskopf and et al., 2010], although it was recently reported that the X-ray/gamma-ray flux declined at a level of $\sim$3.5% $yr^{-1}$ from 2008 to 2010 [Wilson-Hodge and et al., 2011], this decline is rather irrelevant to the low flux measured by HEFT which was done in 2005 (MJD 53507) when the flux of the Crab Nebula was close to the typical value quoted in references (figure 3.41).

No one factor seems capable of explaining the low flux measured. The imprecise d-spacing in the multilayer coatings may reduce the effective area. This was discovered during the NuSTAR optics calibration and the effect is being actively investigated. Also higher

$^5$[Ricker and et al., 1975], $^6$[Kestenbaum and et al., 1975], $^7$[Pelling and et al., 1987]
multilayer roughness (table 3.7) would reduce the effective area, and thus flux, but it would produce an unacceptable spectral index. The same is true if we assume our optical axis misalignment is larger, and at any rate, the good fit to the Cyg X-1 image argues against this as well. Possibly there was contamination introduced during the flight which affected the higher energies and not just the 8 keV effective area, which is manifestly low. Although this hypothesis is argued against, since the Mie-scattering is sharply attenuated with increasing energy, such a hypothesis cannot be ruled out. The low flux we measured is likely to be an effect of several factors combined. Unfortunately the observation was too short to disentangle those effects statistically.

More extensive high energy calibrations pre- or post-flight would be the only way to definitively address the effective area discrepancy. This is an important lesson we learned from the HEFT experiment, and thus we built a calibration facility at Nevis laboratories of Columbia University for the NuSTAR mission.

3.6.2.5 Conclusion

The analysis of HEFT data of the Cyg X-1 and the Crab Nebula observation was done after years of efforts. The analysis of Cyg X-1 indicates that the measurement is consistent

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Energy range (keV)</th>
<th>$\Gamma$</th>
<th>$N$</th>
<th>$\int \frac{dN}{dE} dE$ (21-54 keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>21-54</td>
<td>2.18</td>
<td>5.33</td>
<td>2.77</td>
</tr>
<tr>
<td>RXTE HEXTE</td>
<td>15-180</td>
<td>2.09</td>
<td>9.9</td>
<td>7.06</td>
</tr>
<tr>
<td>RXTE PCA</td>
<td>4-60</td>
<td>2.12</td>
<td>11.02</td>
<td>7.08</td>
</tr>
<tr>
<td>INTEGRAL JEM$_{LX}$</td>
<td>5-35</td>
<td>2.136</td>
<td>9.8</td>
<td>5.95</td>
</tr>
<tr>
<td>INTEGRAL ISGRI</td>
<td>25-200</td>
<td>2.253</td>
<td>15.4</td>
<td>6.21</td>
</tr>
<tr>
<td>INTEGRAL SPI</td>
<td>30-1000</td>
<td>2.203</td>
<td>15.9</td>
<td>7.64</td>
</tr>
<tr>
<td>BSAX/HPGSPC</td>
<td>7-30</td>
<td>2.10</td>
<td>9.4</td>
<td>6.47</td>
</tr>
<tr>
<td>BSAX/PDS</td>
<td>13-200</td>
<td>2.126</td>
<td>8.83</td>
<td>5.55</td>
</tr>
<tr>
<td>MIR XEXE</td>
<td>20-200</td>
<td>2.08</td>
<td>8.89</td>
<td>6.57</td>
</tr>
</tbody>
</table>

Table 3.6: Comparison of the spectrum measured by HEFT with that by other missions. The spectrum is given by $\frac{dN}{dE} = NE^{-\Gamma}$. (from Kirsch and et al., 2005)
with the instrument responses folded with the pointing solution, that is, Cyg X-1 is a point source. This verifies that we understand the optics response, the detector response, the aspect, and determine the optical axis properly. Although there is a possibility that errors in different processes in the analysis cancel each other, resulting in the correct answer, it is unlikely.

With the understanding obtained in the Cyg X-1 analysis, we measured the size of the Crab Nebula. From the analysis, we obtained the size of the Crab Nebula, 55" (NE-SW) and an upper limit of 46" (NW-SE, 90% confidence). This result agrees very well with the results of previous mission (table 3.5) as well as the relativistic MHD wind model of Kennel and Coroniti (KC84). The analysis done here confirms that KC84 is properly explaining the phenomena in the PWN, although the parameters used in the model are not unique, and there is controversy about the parameters, especially the magnetization parameter ($\sigma = 0.003$, Chandra estimated it to be 0.01-0.13 [Mori and et al., 2004]). Constraining the parameters with HEFT data was difficult due to the errors in the measurement. Precise measurements of the size, the spectrum, and the morphology in the hard X-ray energy band will be useful for further constraining the parameters and understanding the PWN. This analysis was a step forward in understanding the phenomena in the PWN.

The technique I used for size measurement of the Crab Nebula can be used in general, and will be useful for analyzing data from future 2-D imaging telescopes. When the width
of the PSF of a telescope is a lot smaller than the size of the astrophysical object (as in case of Chandra observation of the Crab Nebula), no further analysis is required for size measurement. In the opposite case, that is, when the width of the PSF of a telescope is a lot larger than the object, it is not possible to measure the size of the object (as in case of gamma-ray observatories nowadays). When the width of the PSF and the size of the object are comparable, this technique will be very useful (or necessary) for the size measurement, since 2-D deconvolution techniques are not transparent and sometimes do not work.

Finally, the analysis proves that the grazing incidence segmented optics and CdZnTe detectors are useful tools to explore the sky at energies above 20 keV.
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Table 3.7: Sensitivity study for relative off-axis and multilayer roughness. Here, Relative off-axis means the off-axis from the best optical axis obtained in section 3.5. Extraction region: 1.85 arcmin in radius, Energy: 21-54 keV.

<table>
<thead>
<tr>
<th>Relative off-axis (x,y) arcmin</th>
<th>ML Roughness</th>
<th>spectral index</th>
<th>Normalization cts/kev/s/cm²</th>
<th>spectral index</th>
<th>Normalization cts/kev/s/cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>4</td>
<td>2.18</td>
<td>5.33</td>
<td>2.10</td>
<td>3.97</td>
</tr>
<tr>
<td>(0,0)</td>
<td>5</td>
<td>1.88</td>
<td>2.03</td>
<td>2.10</td>
<td>4.40</td>
</tr>
<tr>
<td>(0,0)</td>
<td>5.5</td>
<td>1.72</td>
<td>1.29</td>
<td>2.10</td>
<td>4.86</td>
</tr>
<tr>
<td>(2,2)</td>
<td>4</td>
<td>2.10</td>
<td>5.32</td>
<td>2.10</td>
<td>5.40</td>
</tr>
<tr>
<td>(3,3)</td>
<td>4</td>
<td>2.02</td>
<td>4.79</td>
<td>2.10</td>
<td>6.43</td>
</tr>
<tr>
<td>(4,4)</td>
<td>4</td>
<td>1.95</td>
<td>4.51</td>
<td>2.10</td>
<td>7.72</td>
</tr>
<tr>
<td>(2,2)</td>
<td>5</td>
<td>1.79</td>
<td>2.08</td>
<td>2.10</td>
<td>6.11</td>
</tr>
<tr>
<td>(2,2)</td>
<td>5.5</td>
<td>1.63</td>
<td>1.30</td>
<td>2.10</td>
<td>6.62</td>
</tr>
</tbody>
</table>

Figure 3.41: Flickering of the Crab Nebula. Each data set has been normalized to its mean rate in the time interval MJD 54690-54790. (from Wilson-Hodge and et al., 2011)
Part III

The Nuclear Spectroscopic Telescope Array (NuSTAR)
Chapter 4

Introduction

The Nuclear Spectroscopic Telescope Array (NuSTAR) is a NASA Small Explorer experiment (SMEX). It will be the first hard X-ray focusing mission, with an energy bandwidth of 6-79 keV and optics with on-orbit performance of $\sim$50 arcseconds. The two X-ray optics of NuSTAR (figure 4.2) are situated on an optical bench that is deployed in orbit by an extendable mast. The mast provides a 10 meter focal separation between the optics and the focal plane bench (figure 4.1). This is the first use of the extendable mast in a high energy astrophysics mission. NuSTAR has a number of major science goals including a survey of active galactic nuclei (and thus black holes on all mass scales), a survey of the galactic center, the spectral and spatial mapping of $^{44}$Ti in young supernova remnants and correlated observations of ultra-high energy sources (chapter 5). The mission uses a near
Table 4.1: Optics performance parameter.

<table>
<thead>
<tr>
<th>Energy range</th>
<th>5-80 keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular resolution (HPD)</td>
<td>45′′</td>
</tr>
<tr>
<td>Angular resolution (FWHM)</td>
<td>9.5′′</td>
</tr>
<tr>
<td>FoV (50% resp.) at 10 keV</td>
<td>10′</td>
</tr>
<tr>
<td>FoV (50% resp.) at 68 keV</td>
<td>6′</td>
</tr>
<tr>
<td>Sensitivity (6-10 keV) [10^6 s, 3σ ΔE/E=0.5]</td>
<td>2 × 10^{-15} erg/cm^2/s</td>
</tr>
<tr>
<td>Sensitivity (10-30 keV) [10^6 s, 3σ ΔE/E=0.5]</td>
<td>1 × 10^{-14} erg/cm^2/s</td>
</tr>
<tr>
<td>Background in HPD (10-30 keV)</td>
<td>6.8 × 10^{-4} cts/s</td>
</tr>
<tr>
<td>Background in HPD (30-60 keV)</td>
<td>4 × 10^{-4} cts/s</td>
</tr>
<tr>
<td>Strong source (&gt; 10σ positioning)</td>
<td>1.5′′ (1 − σ)</td>
</tr>
<tr>
<td>Temporal resolution</td>
<td>2 μsec</td>
</tr>
<tr>
<td>Target of Opportunity response</td>
<td>&lt; 24 hours</td>
</tr>
</tbody>
</table>

The science goals at which the NuSTAR experiment aims are rather challenging in this energy band. Although HEFT, the forerunner of NuSTAR, demonstrated the possibility of attaining the science goals, NuSTAR requires significant improvements in performance of the optics and the detectors within a limited period of time. Table 4.1 shows the performance parameters of the optic, from which we derive the design parameters of the optic (table 4.2, [Hailey and et al., 2010]). When compared to the HEFT optic (table 2.2), it is clear that the NuSTAR optic will be larger (more mirrors on it) and perform better, which challenges us in many ways. I will mainly discuss two challenges I worked on in this part of the thesis work - the epoxy and the metrology.

Epoxy is one of the main structural and optical components of the optic. As there are many commercially available epoxies with different properties, the challenge was finding the best one for our application within limited time. The epoxy we used for HEFT (Tracon 2113) is very easy to apply, which is the reason we chose it for the HEFT experiment. However, the concern with it is that it outgasses a lot. The outgassed molecules adhere to the optics, degrading the spectral and the angular performance of the optic over time.
at energies below \(\sim 15\) keV. While this was not a problem for HEFT, where the energy band only goes down to \(\sim 17\) keV due to the atmospheric cutoff, it is not acceptable for the NuSTAR mission. Evaluation of many epoxies, including the one we used for HEFT, has been conducted and NuSTAR candidate epoxies selected (chapter 6).

As the process of building optics as well as forming mirror substrates has improved, it is necessary to have better metrology systems for the characterization of the mirrors and optics. The new systems need to be better not only in terms of their ability to characterize the optics and mirrors, but how fast they do it. The development of improved metrology in particular the laser scanner and the Linear Variable Differential Transformer (LVDT), is discussed in chapter 7.

<table>
<thead>
<tr>
<th>Optic Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Length</td>
<td>10.15 m</td>
</tr>
<tr>
<td>Shell Radii</td>
<td>51-191 mm</td>
</tr>
<tr>
<td>Grazing angles</td>
<td>1.3-4.7 mrad</td>
</tr>
<tr>
<td>Mirror thickness</td>
<td>0.21 mm</td>
</tr>
<tr>
<td>Total shell per module</td>
<td>130</td>
</tr>
<tr>
<td>Multilayer (layer 1-89)</td>
<td>Pt/C (\sim 0.5 \mu m)</td>
</tr>
<tr>
<td>Multilayer (layer 90-133)</td>
<td>W/Si</td>
</tr>
<tr>
<td>Composite structure</td>
<td>D263 glass</td>
</tr>
<tr>
<td></td>
<td>DS-4 graphite</td>
</tr>
<tr>
<td></td>
<td>F131 epoxy</td>
</tr>
<tr>
<td>Mandrel/Spider</td>
<td>Titanium</td>
</tr>
<tr>
<td>Mass per optic</td>
<td>37.5 kg</td>
</tr>
</tbody>
</table>

Table 4.2: Optics design parameter.
Chapter 5

Science Objectives

5.1 Extragalactic survey: AGNs

The X-ray background (XRB) was discovered in 1962 [Giacconi and et al., 1962]. It is uniformly distributed over the sky, which suggested an extragalactic origin. With the improvement in spectral and spatial resolution of recent soft X-ray telescopes (ROSAT, Chandra, XMM-Newton), 50% to 90% of the sources are resolved in the soft X-ray energy band (less than 10 keV). This soft X-ray emission is attributed to point sources, especially Active Galactic Nuclei (AGN). Above 10 keV, the resolved fraction of the sources is lower. Resolving sources at higher energy is of crucial importance as seen in figure 5.1. The figure shows that there is a peak at around 30-40 keV and more than ~50% of the energy is coming from >10 keV.

The spectrum of the XRB is explained by Compton-thick AGNs (CTAGN), combined with red-shifted X-rays, as proposed by Setti and Woltjer [Setti and et al., 1989], [Pounds and et al., 1990], [Matsuoka and et al., 1990]. The model is based on unified models of AGN orientation-dependent effects, related to the presence of an absorbing torus around the central source, first introduced by Antonucci and Miller [Antonucci and et al., 1985]. This basic model was modified to be consistent with the statistical properties of AGN X-ray samples and reproduces the XRB spectrum in the 3-100 keV interval [Comastri and et al., 1995], [Gilli and et al., 2007] (figure 5.1). NuSTAR will conduct a survey on AGNs, attempting to measure the XRB from known AGNs. This will answer the question on what
Figure 5.1: The X-ray background spectrum. Data points with error bar are from ROSAT and various experiments and the fits (lines) are from the absorbed AGN model (from Comastri and et al., 1995)
AGN population dominates X-ray background at the peak (30-40 keV).

The existence of the CTAGN with an indication of the reflection hump was first established by Nandra et al. [Nandra and et al., 1989] and Pounds et al. [Pounds and et al., 1989]. Since then, 20 CTAGNs were detected above 10 keV, none of which have well characterized X-ray spectra above 10 keV. Therefore, these objects are not very well understood. A reflection model [Magdziarz and et al., 1995] and a torus model [Murphy and et al., 1995] predict different spectra. NuSTAR is able to measure the spectra of these objects above 10 keV in detail, and we can constrain the models strongly and understand the environments such as column density, element abundances. Also, the timing capability of NuSTAR will allow us to measure the variability of these objects, which will further our understanding on the geometry and the structure of the obscuration.

The absorber surrounding an AGN is related to an interesting phenomenon - AGN feedback - a mechanism to heat the Interstellar Medium (ISM). Although there are some models for the outflow from the Super Massive Black Hole (SMBH) in the center of an AGN [Hopkins and et al., 2010], [Ostriker and et al., 2010], the outflow mechanism is not very well understood. Since the absorber is close to the formation region of the outflows, studies of it will provide a hint to understand the AGN feedback.

5.2 Supernova Remnants

5.2.1 Particle acceleration

Astrophysicists have known of the existence of high energy cosmic rays for ∼70 years (∼10^{15} eV by Pierre Auger [Auger and et al., 1939]). The cosmic ray spectrum is characterized by a power-law. It has a ‘knee’ at ∼3 \times 10^{15} eV, and an ‘ankle’ at ∼10^{18} eV, points at which the power-law index changes [Bhattacharjee and et al., 2000] (figure 5.2). Above 6 \times 10^{19} eV, cosmic rays (protons) interact with the CMB to produce π’s and thus lose energy. Cosmic rays accelerated to this energy farther than ∼150 million light years will lose so much energy via this interaction and not be detected on earth (GZK cutoff, [Greisen, 1966], [Zatsepin and et al., 1966]).

Although a plausible acceleration mechanism for the CR was proposed a long time
Figure 5.2: The cosmic ray, all-particle spectrum. The data represent published results of the LEAP, Proton, Akeno, AGASA, Fly’s Eye, Haverah Park, and Yakutsk experiments. (from Bhattacharjee and et al., 2000)
ago - diffusive shock acceleration [Fermi, 1949] - their origin is still unknown. Recently, supernova remnants (especially shell-type supernova remnants) have been considered as the primary source of the acceleration up to ‘knee’. A strong (the standard) argument that favors supernova remnants as the source of the CR arises from energy considerations. The confinement time ($10^7$ years) and the energy density (1.8 eV cm$^{-3}$) of the CR require energy loss of $2 \times 10^{41}$ ergs$^{-1}$ in the sources. Supernovae, occurring at a rate of 1/30-50 years, emitting $\sim 10^{51}$ erg, and thus $\sim 10^{42}$ ergs$^{-1}$, are the best, if not the only, candidate [Diehl, 2009]. Additional evidence of particle acceleration in supernova remnants is from the observation of radio synchrotron emission ($E_{\text{GeV}} \sim (\nu 16 M \text{Hz})^{-1/2}$, where $\nu$ is photon frequency and $B_\mu$ is the magnetic field strength in micro-Gauss). But this samples electrons far below the ‘knee’, and does not support the acceleration to near or above the ‘knee’.

Recent observations in X-rays and gamma-rays give us a new probe to characterize the acceleration mechanism. The blast wave of a supernova explosion expands, sweeping up the material around the progenitor. As the shock passes through the materials, the density and the temperature increase, producing relativistic particles through diffusive shock acceleration. Since the material behind the shock is hot, it can emit thermal X-ray. At the same time, electrons accelerated through diffusive shock acceleration can emit radio to X-ray photons via synchrotron radiation [Reynolds, 1998], [Ellison and et al., 2001] and gamma-rays via inverse-Compton scattering. Recent observations of supernova remnants such as SN 1006, Cas A, G266.2-1.2, Tycho’s and others show evidence of this idea of diffusive shock acceleration [Eriksen and et al., 2011].

Although these observations support the idea that supernova remnants are the origin of cosmic rays with energies up to the ‘knee’, there are loose ends. With soft X-ray observation alone, the magnetic field strength cannot be determined precisely because it is difficult to decouple the thermal spectrum and the synchrotron spectrum. Hard X-ray observations in the NuSTAR energy band are not contaminated by the thermal component and will provide a precise determination of the magnetic field strength. With the gamma-ray observations only, we cannot tell whether the gamma-rays are from a leptonic process (Inverse Compton scattering) or a hadronic process (pion decay). The acceleration scenario for the supernova remnants works only when the leptonic process is the dominant one. Therefore, broad band
observations are required and NuSTAR will cover the hard X-ray band.

Another way for particles to accelerate in a supernova remnant is through a shock produced by a pulsar wind. The pulsar wind nebulae, where the central pulsar provides wind particles to the nebula, producing a shock at the wind/nebula boundary, has been successfully modeled [Rees and et al., 1974], [Kennel and et al., 1984a]. The shock (termination shock) can accelerate electrons by a factor of 100. The acceleration mechanism may not be the diffusive shock acceleration argued by Arons and Tavani [Arons and et al., 1994] but a resonant cyclotron absorption of the compressed B-field. At the termination shock, the magnetic field is greater than at the blast wave, which makes the termination shock the energetic accelerator. Recently it has been argued that there is evidence that electrons are accelerated to $10^{15}$ eV in the Crab Nebula [Atoyan and et al., 1996]. To further constrain the acceleration mechanism, it is necessary to identify the termination shock and measure the shock parameters such as $\sigma$ - the ratio of Poynting flux to particle flux - to further improve the theory. This can be done by measuring the spectral energy distribution as a function of the distance from the pulsar and directly observing the shock region at various energy bands. Although this is well done by Chandra at the soft X-ray band and by other observations at the radio and optical band for the Crab Nebula, measuring its size at higher energies by imaging will help us to understand the mechanism better.

The unique imaging capability of NuSTAR in the hard X-ray energy band will provide the spatial variation of the spectral energy distribution. This will deepen our understanding of the cosmic ray acceleration.

5.2.2 Supernova Explosion mechanism

As mentioned in the previous section, supernovae can be an important site where particle acceleration occurs. The study on the X-ray line emission in supernova remnants can give us crucial clues as to the details of the explosion mechanism.

The core-collapse supernovae are of particular interests. Massive stars of $> 8 M_\odot$ ($M_\odot$ is the mass of the sun) cannot be supported by degenerate electron pressure. As the mass of the core of such a star reaches the Chandrasekhar mass of 1.4 $M_\odot$, the core becomes unstable. The degenerate electron pressure cannot support the core against gravity, thus
the core implodes to size $\sim 10$ km, increasing the density of the core. When the density gets high enough (nuclear density, $\sim 2 \times 10^{14} g/cm^3$), neutrons drip out of nuclei and become degenerate. The core is barely compressible and bounces the infalling matter back, generating a strong shock front which launches a supernova explosion. (In fact the shock loses its energy by heating materials behind it and generating significant numbers of neutrinos at around $\sim 20$-$30$ km. Therefore, for an explosion to happen, there needs to be a mechanism to re-energize the shock. This is done by the neutrinos in the hot and dense environment.)

Most of the supernova explosions are asymmetric (some of which are jet-like), which is easy to imagine if one considers that expecting spherical symmetry in such an environment is implausible due to Rayleigh-Taylor and Kelvin-Helmholtz fluid instabilities. Many theories explain the current observations. One is jet-induced explosions [Khokhlov and et al., 1999] and the other is rotating convection driven explosion [Fryer and et al., 2000].

Synthesis of $^{44}$Ti is a characteristic of an $\alpha$-rich freeze-out process. This happens when the ejecta cool adiabatically, which commonly happens in supernova explosions. $^{44}$Ti inverse $\beta$ decays to $^{44}$Sc ($\tau = 85.4$ years), which producing 67.9 and 78.4 keV emission lines as it de-excites to the ground state which can be measured by NuSTAR. $^{44}$Ti lines have been measured in two young supernova remnants, Cas A [Iyudin and et al., 1994] (figure 5.3) and GRO J0852-4642 [Iyudin and et al., 1998]. By mapping the $^{44}$Ti lines spatially and
in velocity space, and by measuring the mass in young supernova remnants, NuSTAR will probe the dynamics of the supernova explosion.

5.3 Galactic Survey

The Galactic Center (GC) is an invaluable place for doing soft X-ray astrophysics due to the large number and variety of sources, but in the hard X-ray band it is difficult to resolve the sources spatially. The GC is composed of a super-massive black hole and many X-ray emitting point sources. Located in the nucleus of Sagittarius A (Sgr A*, figure 5.4), the super-massive black hole emits X-rays weakly and variably.

Chandra discovered several thousand X-ray emitting sources which include Cataclysmic Variables (CVs), Low Mass X-ray Binaries (LMXBs), High Mass X-ray Binaries (HMXBs) and so on in a $2^\circ \times 0.8^\circ$ region around the Galactic center [Muno and et al., 2009]. NuSTAR will conduct follow-up observations in the hard X-ray energy band. With the follow-up observations, NuSTAR will provide information about the source population. Especially interesting is the study of the HMXB population and its evolution. From these, we can estimate the numbers of Neutron Star/Neutron Star, Neutron Star/Black Hole and Black hole/Black Hole binaries [Tauris and van den Heuvel, 2006], which can be used by gravity wave detection facilities such as Laser Interferometer Gravitational Wave Observatory.
Another important question that NuSTAR may answer is the relative evolutionary states of the different spiral arms. This can be done by observing other regions of the Galaxy with populations of different ages such as Carina arm and the Norma arm. By finding HMXBs (or HMXB to CV fraction), the evolutionary states can be inferred as suggested by Grimm et al. [Grimm and et al., 2003], which can improve our understanding of Galactic dynamics.

Finally, with its timing and spectral capability, NuSTAR will be able to search for magnetars and rotation powered pulsars in the Galactic Center. If it finds a millisecond pulsar close to Sgr A*, it can provide a precise mass measurement for the Sgr A* black hole.

5.4 GeV/TeV particle accelerators: The Blazars

A blazar is an AGN which has a relativistic jet pointing to the earth, and is believed to be the location where particle acceleration occurs (figure 5.5 and 5.6). Depending on the spectrum, it is further categorized [Urry and et al., 1995], [Ulrich and et al., 1997]. Flat-spectrum radio quasars (FSRQ) are powerful radio Galaxies, while BL Lacertae objects (BL Lacs) are weak ones. Blazar spectra have some characteristic features. The first feature is
the spectral shape, which shows a hump at low energies (radio to $\sim$keV) and another at the energies ($\sim$keV-GeV) (figure 5.7). The intensities of those spectral humps may or may not be correlated depending on the emission processes. Another feature is the variability of the intensities. The variation of the intensities of both the humps is more pronounced above the peak frequency of each hump.

Several models have been proposed to explain the spectral features. The lower energy hump is believed to be due to synchrotron radiation of electrons in the blazar. There are two processes that are capable of reproducing the higher energy hump - leptonic process and hadronic process. The leptonic process is that the synchrotron photons from the lower energy hump interact with the electrons that emit the synchrotron radiation via inverse Compton scattering (synchrotron-self Compton (SSC)) [Band and et al., 1985], [Abdo and et al., 2010], or external photons mostly from the accretion disks Compton scatter with the electrons (external inverse-Compton (EIC)) [Tavecchio and et al., 2000]. The hadronic process is that external high energy cosmic ray protons interact with protons, neutrons, and ions in the blazar to produce pions and the high energy photons [Mücke and et al., 2003],

Figure 5.6: Structure of an active Galactic nucleus. (from Urry and et al., 1995)
To understand blazars, multi-wavelength spectral studies are of crucial importance since the low energy emission and the high energy emission are closely correlated in the leptonic processes but not in the hadronic process. Time variability and the spectral energy density studies will give us a detailed idea of what is powering the blazars in the universe.
Chapter 6

Evaluation of Epoxy

6.1 Introduction

Epoxy is widely used for many different applications. However, the use of epoxy as a major structural component in X-ray optics for a space mission is novel. The application and selection of epoxy for the NuSTAR telescope is the subject of this chapter. In particular I focus on the properties of epoxy in relation to the performance of optics, and the testing and selection of epoxies to obtain nominal X-ray optics performance from the scientific standpoint. This means ensuring that the epoxy employed in the composite optics does not degrade the angular resolution and effective area. In this chapter, I do not discuss the separate (and vitally important) engineering issue of the epoxy’s contribution to the structural integrity of the optic.

The epoxy we used for the HEFT optics was the best selection for a fast, cheap balloon mission. It was easy to apply, which saved time in the process development. However, it has high outgassing, which was not a problem for HEFT optics, which operated only above \( \sim 20 \text{ keV} \) due to the atmospheric attenuation. But the outgassing is a concern for NuSTAR optics, which are required to operate from 6 keV to 79 keV. Therefore, it was necessary to find a new low-outgassing epoxy, and as the new epoxy is likely to be more viscous, it was also necessary to develop a novel application process.

The goal of this research was to develop a general framework for evaluating epoxies and to develop application process not only for the NuSTAR mission but also for future space
missions.

6.2 Performance of optics and Property of epoxy

The grazing incidence optics of NuSTAR telescopes are composed of many segmented mirrors bonded onto graphite spacers. Bonding is done by epoxy and therefore, epoxy is a crucial component that determines the structural and optical performance of the epoxy-graphite-glass composite optic. The bonding strength is a crucial parameter, since the epoxy joints are subject to substantial shear, tensile and peel stresses in a rocket launch. I do not consider this parameter further here.

The epoxy also affects the optical performance. Figure 6.1 shows the relation between the optical performance and the properties of the epoxy. Firstly the epoxy may develop a non-uniform bond line that can distort the glass where it attaches to the spacer. This degrades the figure of the glass and thus the angular resolution of the optic (figure 6.2). Secondly a NuSTAR optic employs a large amount of epoxy (∼0.7 kg). The epoxy in the bond line outgasses, and those outgassed molecules adhere to the mirror. Thus X-ray absorption and scattering can degrade the optics performance unless particular attention
Figure 6.2: Non-uniform bond line (side view). Figure 6.3: Scattering from outgassed molecules. Epoxy may form a non-uniform bond line and deform the mirror. Outgassed epoxy molecules adhere to the mirror, scattering or absorbing the incoming X-ray.

is paid to epoxy outgassing (figure 6.3). And thirdly, viscoelastic creep can degrade the glass figure and angular resolution. For an epoxy cured under conditions of stress (due in this case to the distortion of a cylindrical glass shell to a conic section on the spacers) viscoelastic creep is the irreversible relaxation of the epoxy back to the zero stress state (figure 6.4). The magnitude of the creep increases with time and with temperature.

The main properties of the epoxy which affect optical performance are the viscosity, outgassing rate and glass transition temperature (figure 6.1). The viscosity is related to the flow properties of the epoxy with time and temperature, and will define the bond line thickness (and thus uniformity). Also as a practical matter it is difficult to apply a highly viscous epoxy to the narrow spacers. Outgassing is clearly important, as mentioned above.

The glass transition temperature ($T_g$) is the temperature at which the glass changes from the higher temperature rubber-like phase to the glass phase. The epoxy is basically a super-cooled fluid, and at $T < T_g$ the glass vitrifies.

All the fundamental properties of optical performance that we are interested in are interconnected by $T_g$. A high $T_g$ generally means a lower outgassing rate and high viscosity and a low $T_g$ means a high outgassing rate and low viscosity. The $T_g$ is also connected to viscoelastic distortions (creep) since the (cured) epoxy’s resistance to flow at a given temperature is directly related to $(T_g - T)/T_g$. The farther the temperature is below $T_g$, the more difficult it is for creep to occur. The search for an “ideal” epoxy is thus reduced,
Figure 6.4: Viscoelastic distortion. A cylindrical mirror is forced to a cone by epoxy. The stress in the mirror can be relaxed over time due to the viscoelasticity of the epoxy.

to a first approximation, to a high $T_g$ epoxy whose viscosity is not so high as to render it infeasible to apply to the spacers. Also some care must be taken since a high $T_g$ epoxy is generally more brittle, and fracture toughness is an important engineering consideration.

6.3 Evaluation and selection of epoxy for the NuSTAR optics

6.3.1 Epoxy candidates

There are many commercially developed epoxies, and we had to downselect and/or customize those. Downselection and customization were done by inspecting the manufacturer’s specification and by working with adhesives researchers at Lehigh University. As there is no quantitative relation between the optical performance and the properties of the epoxy, our downselection was rather broad and got narrower as we proceeded with testing.

Epoxies under our consideration were all two-component systems containing a resin and a hardener. Table 6.1 shows the candidate epoxies. The first seven are commercial

| TRABOND-2113 |
| TRABOND-F131 |
| MasterBond EP30-2 |
| STYCAST 2651 |
| TRACAST-3103 |
| TRABOND-2116 |
| TRABOND-216L01 |
| Nanosilica loaded epoxies(5) |

Table 6.1: Epoxy candidates.
formulations with compatible resin and hardener. The rest involved interchanging resins and hardeners among epoxy systems in an attempt to obtain good performance. Nanosilica loaded epoxies are designed to improve almost all properties at minimal viscosity penalty.

Reducing the candidates to a workable number requires examining the relevant parameters. In addition to the fundamental parameters discussed above (outgassing, $T_g$, viscosity, strength) there are additional practical requirements. These include an overnight room temperature cure, sufficient pot life so that epoxy can be applied to glass and graphite before it hardens, ease of mixing and application and good bond line uniformity. In addition, the epoxy must be environmentally robust. It must cure reproducibly and with consistent strength to accommodate the $\sim 50,000$ epoxy bonds in NuSTAR. The epoxy must do so in a sensible range of the relevant conditions (e.g., temperature, humidity, cleanliness). The latter is particularly important on a SMEX mission, since heroic (read costly) measures cannot be employed to ensure environmental stability. This is challenging, since epoxies are notoriously sensitive to all the above conditions. In this section, we describe the procedures we used to reduce this list of epoxies down to several viable candidates.

One of the candidates is the epoxy used for the HEFT balloon optics, TRACON 2113, for example. Although we knew that this epoxy does not work for NuSTAR because of its high outgassing rate, we conducted tests on this epoxy to use the results as a reference, since we had used it for a long time and knew it very well.

### 6.3.2 The application process

The performance of an epoxy is highly dependent on the way it is processed. The application process is developed along with the evaluation iteratively. The process is composed of four steps - mixing, degassing, dispensing and curing. The process we used for HEFT (roller mixing and coarse vacuum degassing) needed to be modified since the candidate epoxies are more viscous, and each process affects the performance. Figure 6.5 shows the application process flow, the requirements, and the related performance. In this section, each of the processes is discussed.
6.3.2.1 Mixing and Degassing

The performance of the bond line is affected by the mixing and degassing. If the epoxy is not well mixed, or is mixed at an incorrect mixing ratio, the unreacted resin and hardener will lead to a weak bond line, and the unreacted components will outgas more. From bond strength tests, we find that the mixing ratio should be controlled to better than ±2%. Also if the epoxy is not well degassed, it will have air bubbles inside (figure 6.6), which will also decrease the bond line strength. Those bubbles are usually squeezed out when we apply pressure on the bond line during curing but on rare occasions some of the large bubbles (visible) are stuck in the bond line.

For mixing and degassing, we tried several different ways such as roller mixing, mixing tubes and vacuum mixing. Roller mixing is the method we used for HEFT, where we mix the two components (the resin and the hardener) in a plastic pouch with a hand roller. The mixing tube is a cylindrical tube which has a structure like a mixing blade along its axis, where the two components run along the structure by pneumatic pressure (or gravity) and mix. Vacuum mixing is the method we used for NuSTAR, where a mixing blade stirs the two components inside a mixing syringe (figure 6.7).

Among those, we find that vacuum mixing/degassing is optimal for us as the epoxy
CHAPTER 6. EVALUATION OF EPOXY

Figure 6.6: Epoxy sample after mixing and degassing. (a) Well degassed sample: No air bubbles, (b) Poorly degassed sample: Air bubbles inside.

Figure 6.7: The vacuum mixing chamber: Inside the small vacuum chamber (∼10^{-4} Torr, we mix and degas the epoxy at the same time.

shows the best performance, and the short process time (two steps are merged into a single step) ensures a long working time during the pot life of the epoxy. Figure 6.7 shows the vacuum chamber and the mixing blade. Inside the mixing chamber, a rod connected to the blade is inserted into a small barrel which contains epoxy. The rod rotates the blade which then mixes the epoxy.

6.3.2.2 Dispensing

Dispensing epoxy onto spacers is done by applying pneumatic pressure to a syringe that contains epoxy (figure 6.8). With our geometry, the dispensing rate has been calculated (eg.
The total dispensing rate is given by the following formula:

\[ Q = \frac{\rho \pi \Delta P R^4}{8 \eta l}, \]  

(6.1)

where \( Q \) is the mass dispensing rate, \( \rho \) is the density, \( \Delta P \) is the difference between applied pressure and the atmospheric pressure, \( R \) is the radius of the nozzle, \( \eta \) is the viscosity, and \( l \) is the length of epoxy along the barrel. With the typical configuration we use (\( \Delta P = 10^5 \) Pascal, \( \eta = 2 \ Pa \cdot s \), \( \rho = 1.4 \ g/cm^3 \), \( R=0.025 \ cm \), \( l=1 \ cm \)), we dispense 12 mg/s.

As one can see in equation (6.1), the dispensing rate is inversely proportional to the viscosity. Therefore, by measuring the dispensing rate as a function of time, we infer how the viscosity of epoxy changes as it cures. Figure 6.9 shows the mass dispensing rate as a function of time. I fit this to an exponential function (\( A e^{-Bt} \)) and obtain \( A = 12.8 \ mg/s \) and \( B = 0.038 \ min^{-1} \). From this we infer that the viscosity increases exponentially in time as \( \sim e^{t/26 \ min} \).

For the bond line, we want to use as little epoxy as possible to minimize outgassing. At the same time, if we use too little, the bond line can have voids and gets weaker. A practical limit on the linear density of epoxy was 2.5 mg/cm obtained by visual inspection and strength measurements. The amount of epoxy dispensed is controlled by the dispensing time with the pneumatic dispenser, although better control could be attained by using a different dispenser such as a positive displacement dispenser, which exploits mechanical
contact pressure not pneumatic pressure.

6.3.2.3 Curing

The optics build procedure requires that the bond line have enough strength on the next day (~16 hours) for machining spacers. There are many factors which affect the strength, among which the curing environment is most important. Potential epoxy resins must cure at room temperature within a day. Although epoxies usually cure faster at elevated temperature, curing at room temperature is required to minimize thermal stress and outgassing.

The curing environment (in fact the whole process) must be very well controlled, otherwise the bond strength reduces. From bond strength (not discussed here) studies, we find that the curing temperature should be greater than 25°C and the relative humidity should be less than 35%. It is very important to cure the epoxy at 25°C or higher to ensure that the chemical reaction proceeds fast enough to get the required bond strength overnight. If it is too cold, the reaction does not proceed fast enough. Also as recommended by the manufacturer, a decent pressure should be applied to the bond while epoxy is curing (figure 6.10).

6.3.2.4 The process

In this section, I describe our application process. Some of the epoxies (TRACON F131, 2113) come as bipax (a plastic pouch that has a separator between the hardener and resin), which have the correct mixing ratio prepared by the manufacturer. We pre-mix the epoxy in the bipax by hand kneading and pour it into a mixing syringe. For other epoxies (eg. EP30-2), the resin and hardener are stored in separate containers, and the proper mixing ratio must be weighed out.

The resin is extracted from the mother pot by a clean, lubricant free syringe and dispensed into a pre-weighed mixing syringe. The mixing syringe and the dispensed resin are weighed by a precision balance accurate to 0.1 mg. Thus the resin mass in the mixing syringe is known. Using a separate clean syringe, the hardener is transferred from its mother pot and is slowly added to the mixing syringe with reweighing, until the proper mass ratio is obtained. The final mixing ratio can be controlled to (an acceptable) 2% in this fashion.
CHAPTER 6. EVALUATION OF EPOXY

Figure 6.10: The curing fixture. (a) For flat telescope prototypes, (b) For the NuSTAR optics. We used spring pressure for flat telescope prototypes but we used pneumatic pressure for the NuSTAR optics.

We take extra care when we pour the epoxy into a syringe so that there is no dust contamination. These steps and those that follow are all done in a clean room on NuSTAR. At this point the process becomes identical for the epoxies that come in bipax and in separate mother pots. The mixing syringe, which now contains the proper resin and hardener amounts, is centrifuged to ensure that any epoxy components along the wall are collected in the bottom of the syringe. We put the mixing syringe into a vacuum chamber, insert a rotating mixing blade (figure 6.7) and mix the epoxy thoroughly for 3 minutes. Since the epoxy is mixed in vacuum (figure 6.7), it degasses as it mixes. We still have adequate time to apply the epoxy since the pot lives of our candidates are all about 30 minutes.

After mixing and degassing the epoxy, we dispense the epoxy on the graphite spacers with a pneumatic dispenser, lay down glass and cure the samples under constant pressure in a curing fixture. The geometry of the curing fixture depends on whether the telescopes are flat or conic (figure 6.10).

6.3.3 Evaluation

6.3.3.1 Bond line characterization

Epoxy bond line thickness uniformity is vital to obtaining proper control of glass figure and thus good X-ray imaging. Testing the epoxy bond line uniformity for a large number of
epoxies using conic-approximation Wolter-I prototypes for each test would be both slow and expensive. Consequently a method was developed for rapidly evaluating epoxy bond lines in the laboratory. It should be noted that bond line uniformity tests do not require X-ray testing of the optic. Using mechanical probes, described in section 7.3, the back surface of the glass mounted in prototypes can be interrogated to assess how well the epoxy is controlling the glass figure along the bond line. This exploits the high degree of parallelism between the front and back surface of the glass. It is a much more rapid means to assess optics performance than doing X-ray tests. The bond line affects the long wave surface figure of the glass, not the X-ray scattering properties.

As an alternative to conic prototype geometries, we utilized a “flat stack” geometry. The flat stacks consist of planar pieces of the D263 glass (0.21 mm thick) which is used for NuSTAR. Flat, rectangular pieces of glass are built up into multi-layer “flat telescopes using flight representative spacers, mounting fixtures and epoxy preparation techniques. An example of such a flat stack is shown in figure 6.11. Sufficient fixtures were available to prepare up to 5 flat stacks per day. This permitted rapid epoxy process development and characterization.

We first estimate the bond line thickness as it is the upper limit to the bond line non-uniformity. The bond line thickness is calculated by solving the Navier-Stokes equation with the assumption of a Newtonian fluid. The geometry is shown in figure 6.12. With this
Figure 6.12: Geometry for epoxy flow. Epoxy (red) is applied on top of the previous layer (x-y plane). A curing pressure ($P_0$) is applied to a spacer (blue, width ‘a’ and length ‘b’) from the top (negative z direction). Epoxy flows to the sides along the y axis.

geometry and the steady flow approximation, the Navier-Stokes equation reads

$$
\vec{v} \cdot \nabla v_i = -\frac{1}{\rho} \partial_i P + \nu \nabla^2 v_i,
$$

(6.2)

where $\nu$ is the kinematic viscosity, $P$ is the pressure, $\rho$ is the density of the fluid, and $v$ is the fluid velocity. With the approximations, $\frac{\partial v_x}{\partial x} \sim \frac{v_x}{b}$, $\frac{\partial v_y}{\partial z} \sim \frac{v_y}{h} \ll \frac{\partial v_x}{\partial y} \sim \frac{v_x}{a}$, the incompressibility condition ($\nabla \cdot \vec{v} = 0$) becomes $\partial_y v_y = 0$. Now, equation (6.2) becomes

$$
\nu \partial_y^2 v_y = \frac{1}{\rho} \partial_y P
$$

(6.3)

$$
\eta \partial_z^2 v_y = \partial_y P,
$$

(6.4)

where $\eta$ is the dynamic viscosity ($\eta = \rho \nu$). This gives

$$
v_y(z) = \frac{P_0 - P_{\text{atm}}}{\eta a} (z^2 - zh).
$$

(6.5)
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<table>
<thead>
<tr>
<th>Epoxy</th>
<th>Thickness $\mu$m</th>
<th>Uniformity $\mu$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRABOND-2113</td>
<td>&lt; 2</td>
<td>&lt; 0.2</td>
</tr>
<tr>
<td>TRABOND-F131</td>
<td>&lt; 2</td>
<td>&lt; 0.2</td>
</tr>
<tr>
<td>MasterBond EP30-2</td>
<td>&lt; 2</td>
<td>&lt; 0.2</td>
</tr>
<tr>
<td>TRACAST-3103</td>
<td>26</td>
<td>2</td>
</tr>
<tr>
<td>STYCAST 2651</td>
<td>42</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 6.13: Microscope image of a bond line.

Figure 6.14: Bond line thickness and its non-uniformity.

And from mass conservation, we obtain

\[
ab \frac{dh}{dt} = \frac{2 \Delta P}{\eta a} \int_0^h (z^2 - zh) dz \tag{6.6}
\]

\[
\frac{dh}{dt} = -\frac{\Delta P h^3}{3 \eta a^2}. \tag{6.7}
\]

Until we know the time dependence of the viscosity, we cannot integrate this. Empirically we know the viscosity exponentially increases as it cures. Therefore, we model the time dependence of the viscosity as an exponential function ($\eta(t) = \eta_0 e^{t/\tau}$). Upon integration, we obtain

\[
\frac{1}{h^2} - \frac{1}{h_0^2} = \frac{2(P_0 - P_{atm}) h^3}{3a^2\eta_0} \int_{5 \text{ min}}^\infty e^{-t/\tau} = \frac{2\tau(P_0 - P_{atm})e^{-5/\tau}}{3a^2\eta_0} \tag{6.8}
\]

\[
h = \sqrt{\frac{1}{\frac{2\tau(P_0 - P_{atm})e^{-5/\tau}}{3a^2\eta_0} + \frac{1}{h_0^2}}}, \tag{6.9}
\]

where $h_0$ is the initial thickness of the epoxy we dispense (5 mins after mixing). The integration is done from 5 mins because other preparations (for example, degassing) need to be done after mixing. Our estimate for $\tau$ is about 20 minutes measured by mass dispensing rate (figure 6.9). After plugging all the appropriate numbers in ($\tau=20 \text{ min}$, $P_0 = 1.4 \text{ atm}$, $a = 1.6 \text{ mm}$, $h_0=0.5 \text{ mm}$), we estimate the thickness to be $\sim 0.4 \mu m$ if the viscosity is 2000 cps ($\text{cps} = 10^{-3} \text{ Pa} \cdot \text{s}$).

The bond line thickness was measured by micrometer and microscope (figure 6.13-6.14), and the bond line uniformity was measured by a mechanical probe (figure 6.16, section 7.3).
Figure 6.15: Measurement of the bond line thickness and its uniformity.

The probe contacts with the surface, scans it and measures the surface height profile (figure 6.17). For the measurements, we clean the glass and spacers and measure the thickness. We epoxy down spacers on a substrate and grind and polish the spacers to have flat and smooth surfaces to $\sim$10-20$''$. Then we measure their thicknesses and the height profile with a micrometer and LVDT (figure 6.15a). After the measurements, we epoxy down thin glass, cure the sample and measure the thickness and the height profile again (figure 6.15b). To extract the bond line thickness only, the thickness of the sample in figure 6.15a and the thin glass is subtracted from the thickness of the sample in figure 6.15b. By doing this, we actually measure the uniformity of the bond line together with the thickness variation of the thin glass. Finally, we cut the sample and look at the cross section with a microscope (figure 6.13). The microscope image and the micrometer independently provide measures of the bond line thickness. For the uniformity, we subtract the height profile in figure 6.15a from that in figure 6.15b. So the uniformity measurement is limited by the small thickness variation of the glass and the uncertainty of the metrology.

The measured bond line thickness is less than $2 \mu m$ and the uniformity is less than $0.2 \mu m$ (and the measured slope error is $< 25''$) for the low viscosity epoxies, which agrees very well with the Navier-Stokes calculation. In the case of the low viscosity epoxies, the bond line is so thin compared to the measurement uncertainties and the thin glass thickness variation that we can only set upper limits. For high viscosity epoxies, the bond line thickness is $\sim$30-40 $\mu m$ and the uniformity is $\sim$2-3 $\mu m$. Since bond line uniformity scales with thickness,
the thicker bond line epoxies proved unacceptable. A summary of measurements is shown in table 6.14 for 5 epoxies. It is an interesting fact that the thickness variation is about 10% of the thickness itself for these epoxies.

The contribution of non-uniformity of the bond line to the angular resolution of the optic will be much less than the 25′′ upper limit. The bond line non-uniformity deforms the glass near the bond line, but this distortion spreads over the whole mirror and attenuates rapidly away from the region of the spacer. Mounting simulations based on the continuity of the surface height and ray trace calculation show that the contribution of ∼20′′ of non-uniformity in the bond lines to the distortion of a 30′′ substrate is estimated to be ∼3′′ (rough estimation can be made by taking the ratio of the spacer area to the mirror area, which yields ∼2′′). Conic approximation Wolter prototypes confirmed that sub-60′′ optics could be built with thin bond line epoxies, but not with the thick bond line epoxies.

6.3.3.2 Epoxy outgassing studies

NuSTAR optics use a large amount of epoxy (∼0.7 kg out of a total optic mass of 37 kg). This unusually large amount of epoxy is directly attributable to the fact that the epoxy is a major component of the composite structure. Consequently an exceptionally low outgassing epoxy is required.

Although we started with more than 10 candidates, many of them are disfavored due
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Figure 6.18: X-ray scattering sample and test results for three epoxies. X-ray testing sample, X-ray result results 1 month after build, F131 scattering result, and EP30-2 scattering result from top left clockwise. In the X-ray scattering sample, (a) The reflecting surface (λ/20 optical flat). (b) Epoxy deposited (~0.4 g), (c) Graphite spacer, (d) Thin flat glass

to bond line non-uniformity or blooming (very high outgassing visible with the naked eye). Therefore, tests were performed on candidate epoxies F131 and EP30-2, along with the HEFT optic epoxy 2113 (table 6.9). The latter was meant to serve as a benchmark since its properties pre- and post-flight were well understood at 8 keV and at higher energies.

Initial tests employed a special flat stack geometry as indicated in figure 6.18a. The flat stack consisted of a thin piece of glass (~8 cm²) coated with 0.4 g of epoxy. A few millimeters away an optical flat supported by graphite spacers was positioned. Epoxy would outgas onto the optical flat (“thick” samples). X-ray scattering studies of the optical flat at 8 keV were done using the Danish Technical University double-crystal monochromator ([Hussain and et al., 1999]). Higher energy X-ray scattering studies (30, 35, 66 keV) were performed using the Spring-8 synchrotron in Japan and the Brookhaven synchrotron. Data
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<table>
<thead>
<tr>
<th>Time (after mixing)</th>
<th>F-131</th>
<th>EP30-2</th>
<th>2113</th>
<th>reference flat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 month</td>
<td>7&quot;</td>
<td>5&quot;</td>
<td>173&quot;</td>
<td>~5&quot;</td>
</tr>
<tr>
<td>8 months</td>
<td>88&quot;</td>
<td>5&quot;</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>9 months</td>
<td>116&quot;</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>14 months</td>
<td>162&quot;</td>
<td>40&quot;</td>
<td>N/A</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2: X-ray scattering measurements (8 keV) on “thick” samples.

<table>
<thead>
<tr>
<th>X-ray energy</th>
<th>F-131</th>
<th>EP30-2</th>
<th>2113</th>
<th>Reference flat</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 keV</td>
<td>8&quot;</td>
<td>7&quot;</td>
<td>43&quot;</td>
<td>6&quot;</td>
</tr>
<tr>
<td>66 keV</td>
<td>8&quot;</td>
<td>7&quot;</td>
<td>20&quot;</td>
<td>9&quot;</td>
</tr>
</tbody>
</table>

Table 6.3: X-ray scattering measurements (30 keV and 66 keV) on “thick” samples two months after mixing.

taken at the higher energies showed nominal X-ray scattering performance; no scattering was detected at the resolution of the measurement (~5") in the F131 and EP30-2 samples (table 6.3). This is not surprising since the X-ray scattering is greatly suppressed at higher energies.

The 8 keV data for 2113, F131 and EP30-2 presented in figure 6.18 and table 6.2. This data is taken very near the lowest energy of the NuSTAR energy band, and thus represents the severest test of the epoxy with regards outgassing and X-ray scattering. Figure 6.18b shows an overlay of the F131, EP30-2 and the 2113 one month after the flat stack build. The 2113 already shows scattering of ~100" in dramatic contrast to the other epoxies. The inferior performance of the 2113 at low energy is not unexpected. The 2113 has anomalously low \( T_g \), and thus high outgassing. It should be noted here and in what follows, that these flat stack samples are not at all representative in terms of the amount of epoxy utilized. In these early tests we utilized areal densities of epoxy some 40-80 times higher (depending on shell radius) than in a NuSTAR bond line.

Measurements on more flight-like samples (“Thin” sample; the same geometry as figure 6.18a with the epoxy only in the bond line), and standard outgassing rate measurements (ASTM E595) were also conducted (table 6.4-6.6). The scattering measurements on the thin
Table 6.4: X-ray scattering measurements (8 keV) on “thin” samples.

<table>
<thead>
<tr>
<th>Epoxy</th>
<th>TML 30°C</th>
<th>CVCM 30°C</th>
<th>TML 60°C</th>
<th>CVCM 60°C</th>
<th>TML 125°C</th>
<th>CVCM 125°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-131</td>
<td>0.22%</td>
<td>0.0095%</td>
<td>0.7%</td>
<td>0.019%</td>
<td>1.48%</td>
<td>0.07%</td>
</tr>
<tr>
<td>EP30-2</td>
<td>N/A</td>
<td>N/A</td>
<td>0.61%</td>
<td>0.013%</td>
<td>0.43%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>

Table 6.5: ASTM E595 test result. Test is conducted in a vacuum chamber (10^{-5} torr) for 24 hours. A Total Mass Loss (TML) and Collected Volatile Condensable Material (CVCM) are measured. For NASA qualification, the TML should be less than 1% and the CVCM less than 0.1%.

samples (table 6.4) show that 2113 is not acceptable. As for F-131 and EP30-2, it is clear that EP30-2 is better in terms of outgassing. However, we cannot tell clearly whether F131 (and/or EP30-2) is acceptable for NuSTAR or not, since we do not have a measurement of the “thin” sample for 2+ years of NuSTAR mission life nor do we know the acceleration factor of the “thick” sample.

This leads us to construct an epoxy outgassing model. The model is based on a theory of Brunauer, Emmett and Teller (BET theory [Brunauer and et al., 1938]), which is a generalization of Langmuir’s equation [Langmuir, 1916]. The thickness of the adsorbed layer is given in the BET theory as the following:

\[ n = \frac{C\beta}{(1 - \beta)(1 + (C - 1)\beta)}, \]  

(6.10)

Table 6.6: TML measurement done in 1 atm (Nitrogen gas).
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Constant | Value | Method
---|---|---
$P_V$ | 3 Torr | Measured at Lehigh
C | 0.2 | “thick” flat sample scattering measurements
F | $\sim 10^{-8} P(torr)$ liters/sec | Geometry
$E_a$ | 7.7 kcal/mol | TML measurement
$P_0/P_V$ | 0.21 | “thin” flat sample scattering measurements

Table 6.7: Determination of unknown constants in the model.

where $n$ is the thickness (in unit of molecular layers), $\beta$ is the epoxy pressure normalized to its vapor pressure ($P/P_V$), which determines the rate at which the molecules strike the surface, and $C$ contains information on the evaporation rate, and is a constant to be determined. The evolution of pressure as epoxy outgasses is given by the following formula:

$$\frac{dP}{dt} = -F \frac{P}{V} + \frac{dP_{out}}{dt},$$ (6.11)

where the first term on the right is a conductance term and the second term is an outgassing term. The first term explains that the outgassed epoxy molecules can exit the optic through the openings, and the constant “$F$” is determined by the geometry of the opening [Roth, 1982]. The outgassing term is obtained from the thermal diffusion model, which is

$$\frac{dP_{out}}{dt} \sim \frac{dM_{out}}{dt} = D_0 e^{-E_a/kT} \sqrt{t},$$ (6.12)

where $M_{out}$ is the mass of the outgassed epoxy, $E_a$ is the activation energy of the epoxy, $k$ is the Boltzmann constant, and $T$ is the temperature. The activation energy determines how easily a molecule can be detached (evaporate) from a surface and the $\sqrt{t}$ is the diffusion term.

Unknown parameters of the model, $P_V$, C, F, $E_a$, and the initial condition, $P_0$ at $t=0$ (including $D_0$), are found by geometrical consideration, measurements and/or fitting the data in the table 6.2-6.6. $E_a$, the activation energy, is obtained by fitting the TML data in table 6.5, and it is 7.7 kcal/mol, and $P_V$ is measured to be 3 Torr by the Adhesive Lab. at Lehigh University. F is calculated using the equation in [Roth, 1982] for the flat stack geometry and the conic geometry. C is found by fitting the thick sample measurements.
The outgassing of the NuSTAR epoxy is investigated in three different cases.

- **CASE 1: CVCM argument**
  Measured CVCM (30°C) of 0.0095% is used
  Epoxy outgasses according to equation (6.12)

- **CASE 2: Continuous outgassing throughout mission life**
  No conductance on the ground
  Both conductance and outgassing on orbit

- **CASE 3**
  Measured CVCM (30°C) of 0.0095% is used
  Epoxy outgasses according to equation (6.12)
• CASE 3: No outgassing after launch
  
  No conductance on the ground
  No outgassing on orbit

  In case 1, the collected mass for the first 1 day is 0.095% (30°C) of the mass of the epoxy in the bond line (2.5 mg/cm × 20 cm × 4 = 200 mg). For two years, collected epoxy on the mirrors will be 200 × 0.0095% × √365 day × 2, where the square root in time is from the integration of equation (6.12). We assume that the outgassed epoxy spreads uniformly over the mirror surface (120 cm² × 2). A simple calculation shows that the thickness of the epoxy layer will be ∼200 Å in this case. Note that the temperature we assumed for the calculation is 30°C. If we use 22°C, the result is ∼20% lower. Case 2 and 3 are studied by solving equation (6.11), and the results are shown in figure 6.19. The results show that the thickness of the outgassed epoxy is 200 Å for case 2 and 10 Å for case 3. The results of case 1-3 correspond to < 2 '' degradation of angular resolution of the optics in the worst case, which will be acceptable for the project. One thing to note is that the epoxy layer on the mirrors increases monotonically in case 1 while it reaches a maximum and decreases in other cases.

### 6.3.3.3 Viscoelastic creep study

Possible long term changes in the position of glass shells due to epoxy creep is a major concern for optics employing large amounts of epoxy. Changes in figure of glass shells at the micron level, due to creep, can affect performance. Worse, the effects of creep can take place over long timescales. For polymorphic polymers, such as epoxy, it has been empirically observed that fundamental quantities such as creep compliance and dynamic shear and extensional modulus follow the principle of time-temperature superposition. Curves of these and other quantities obtained at different temperatures can be shifted in time or frequency to form a superposed time-temperature curve for the quantity of interest. Thus measurements of creep or shear taken at a fixed time but multiple temperatures can be converted to curves of the long term time behavior of the epoxy at its operating temperature (figure 6.20). As a practical matter for NuSTAR, these curves can directly yield the viscoelastic “age” of the epoxy, and thus a measurement of glass figure in an optic can be generated for any future
Figure 6.20: Measurements of relaxation modulus at different temperatures (short segments) are shifted to 90°C for the long term behavior. (from Brinson and et al, 2007)

time (by means of elevated temperature measurements).

The time-temperature relation is given by the Arrhenius relation [Arrhenius, 1889]:

\[
\log(a_T) = \frac{E_a}{R(T - T_0)} \quad (6.13)
\]

\[
a_T = \frac{t(T)}{t_0(T_0)} \quad (6.14)
\]

where \(a_T\) is the shift factor, \(E_a\) is the activation energy of the process, \(R\) is the gas constant, \(t\) (\(t_0\)) is time and \(T\) (\(T_0\)) is temperature. If we know time (\(t_0\)) over which a certain amount of viscoelastic creep happens at temperature (\(T_0\)), we can calculate time (\(t\)) over which the same creep happens at different temperature (\(T\)).

Lehigh University measured the relaxation modulus as a function of time and temperature, obtaining the shift factors with respect to a certain reference temperature. The shift factors are then fitted to equation (6.13), providing the value of \(E_a\). Once \(E_a\) is known,
we can calculate the time-temperature curves referenced at different time and temperature. An example of such a curve is shown in figure 6.21. Each curve is a plot of equation (6.13) with the measured parameter ($E_a$) for each epoxy. This particular curve shows the time-temperature characteristic for a viscoelastic aging of 4 years. By simply reading off the temperature, and the corresponding time, this curve provides the amount of exposure time required at the elevated temperature in order to simulate 4 years of aging (creep) in the epoxy at room temperature.

Once these curves have been obtained by laboratory measurements, it is a simple matter to perform an accelerated aging test on a prototype optic. A simple setup provided the necessary data on a prototype optics. Figure 6.22 shows a homemade oven with temperature control and automatic logging of time and temperature. The optic we used for the test was
NP16 (16th prototype of the NuSTAR project). It has 14 pieces of glass mounted, 6 of which are mounted with F131 and the others with EP30-2. The performance (in HPD) of the mounted mirrors was 23\" to 54\" (average 35\") before the test. The optic is wrapped in a thermal blanket and placed in the oven for the requisite amount of time.

The 6 mirrors were mounted with F131 in October, 2008 and the others in February 2009. Measurements by a mechanical probe were done as an initial condition (pre-bake) just before the first bake (bake 1) in March 2009. The first bake was done at 35\(^\circ\)C for 20 hours (1.5 year at 25\(^\circ\)C equivalent). And the second bake (bake 2) was done at 38\(^\circ\)C for 20 hours (10 year at 25\(^\circ\)C equivalent). Using a mechanical probe the glass surface figure was probed before and after the baking cycle. An example of data acquired with this setup is shown in figure 6.23 and table 6.8.

There is a close correlation between the before and after baking glass figure. Also the
average performance of the mirrors (as well as individual performance of each mirror, not shown in the table 6.8 though) shows no significant change considering the uncertainty of the measurement. It seems as though the performance got worse after the first bake. However, performance returned to the original one after the second bake. This implies that the slight increase in HPD after the first bake is not due to the viscoelastic creep; if it were, it would have been worse after the second bake. The slight increase in HPD after the first bake is more related to the systematic change in the measuring environment such as environmental noise (anyway, they are all within the uncertainty of the measurement).

From data, we estimate a shift in glass surface figure at the level of our measurement error, yielding an upper limit at 12 years (25°C) of 0.15 \( \mu m \) or 5″ (straight degradation, not in

<table>
<thead>
<tr>
<th>Epoxy</th>
<th>pre-bake</th>
<th>bake 1</th>
<th>bake 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>F131</td>
<td>33 (46)</td>
<td>39 (49)</td>
<td>33 (45)</td>
</tr>
<tr>
<td>EP30-2</td>
<td>38 (67)</td>
<td>43 (71)</td>
<td>38 (70)</td>
</tr>
</tbody>
</table>

Table 6.8: Average performance of the mirrors before and after baking. Slope (Phase) removed performance with performance with slope in the parentheses.
quadrature), which is the uncertainty of the measurement. Over the life time of NuSTAR (2+ years), the degradation would be $< 1''$ (linear interpolation, the Maxwell fluid model [Brinson and Brinson, 2007]), and thus both epoxies are acceptable.

In addition to epoxy selection, this study is a guide the parameters of the thermal cycling through which the observatory must go before launch. The thermal cycle is conducted in a vacuum chamber and it is required for any observatory to outgas all the contaminants from all the components in the observatory (figure 6.24). Since contaminants outgas more at higher temperature, it is better to do the thermal vacuum cycling at a high temperature for a long time. However, viscoelastic creep of the epoxy in the optics is a concern if we do it at too high a temperature (or for too long a time). Therefore, the maximum temperature and duration is set by the time-temperature relation which we established for the epoxy creep.

Figure 6.24: The integrated NuSTAR observatory, including the instrument and spacecraft, at Orbital Sciences Corporation (OSC) in Dulles, Virginia on June 29, 2011. The observatory is being prepared for environmental testing, including testing in a thermal vacuum chamber and vibration testing. (from http://www.nustar.caltech.edu/news/35/71/final deployment/d.gallery-detail-template)
Table 6.9: Summary. ○+: excellent, ○: acceptable, ×: unacceptable.

### 6.4 Conclusion

Table 6.9 summarizes a performance matrix used to select the final candidate epoxies. We select F131 as the epoxy for the NuSTAR optics. Another epoxy, EP30-2, which is also very good and superior to F131 in some characteristics, is less favorable because of robustness. Performance (especially the bond line strength) of EP30-2 is very sensitive to the process environment such as mixing ratio, temperature and the relative humidity. The question is how well we can control the environment. Practically speaking, the mounting is an irreversible process, and once the environment is not well controlled by an accident, the consequence is a disaster. Considering this, EP30-2 is disfavored.

Epoxy is utilized as a structural component in the glass-graphite-epoxy composite NuSTAR optics. Elementary considerations have been discussed relevant to epoxy selection and evaluation. Using our optics fabrication approach, the current epoxy candidates are adequate to obtain performance much better than the \(~40''\) level for a NuSTAR optic. The bond line thickness and uniformity are commensurate with sub-20'' performance. The current accuracy of the performance degradation estimation due to the epoxy is not a problem for the NuSTAR optics and even for the IXO soft X-ray optics (requirement: 30'' at 7-40 keV). However, the goal of IXO hard X-ray angular resolution is an order of magnitude better (5'' at 5-40 keV). It will be necessary to estimate the effect of the epoxy very accurately (< 1'').
In order to better understand the ultimate limitations the epoxy bond line uniformity imposes on high performance optics, much better measurement methodologies will have to be developed than have been employed on NuSTAR. Similarly higher quality measurements of viscoelastic creep will be required, since major missions requiring high angular resolution will necessitate demonstrating long term stability over time (temperature). Measuring surface figure changes and bond line uniformity to much better than the current 0.15 microns will be required. One way to attain higher precision with the current mechanical probe system is to calibrate out the motion of the stage in real-time as we measure a surface profile. The resolution of the mechanical probe is in fact 15 nm. What is limiting the accuracy is the mechanical noise from the motion of the stage that carries the probe along the surface to be measured. By adding a secondary probe which scans a reference flat while the primary scans the surface of interest, we can measure the noise of the stage motion and thus subtract it from the measurement.

Outgassing is likely to be less of an issue, especially since our demonstration of the ability to handle and apply high viscosity epoxies opens up a whole new phase space of high $T_g$ epoxies. These epoxies can have ultra-low outgassing.
Chapter 7

Metrologies

7.1 Introduction

Building optics requires many procedures and each procedure introduces errors in the surface figure. Therefore, it is crucial to characterize the error introduced in each step of building an optic in order to control the process and understand the performance of the optic.

Figure 7.1 shows the optics build process and the steps at which we need to characterize mirrors. We had to characterize both the free-standing mirrors and the mounted mirrors for the NuSTAR optics. The small amount of time we had to build the optics and the large numbers of mirrors we had to characterize required fast and reliable metrology systems. As the way we built optics was novel, there were no commercially available metrology systems that satisfy the requirements (table 7.1).

The requirements for the metrology system were quite demanding. Table 7.1 shows the requirements for the metrology system. A mirror shell is 0.21 mm thick, 225 mm long and 60° (or 30° depending on the layer) wide, and the radius varies from 51 mm to 191 mm. This set the requirements on the scan range and the force on the mirror, as the metrology needed to scan the whole surface with a small force so as not to deform (or break) the shell. The angular resolution requirement of the optics was 43″. To have less than 3″ of error when convolved, the resolution of the system needed to be better than 15″ (The characteristic length scale of figure error in the mirror is 10 mm. With this level of resolution, we can measure a figure error of height 0.3 μm. The height scale of figure errors is expected to be
Figure 7.1: Optics build process.

a few microns, therefore a resolution of 15′′ is acceptable).

We had to start scanning a shell in a year from the start of the project. Therefore we had to set up a system in 6 months. We expected \( \sim \) 200 mirror substrates to be delivered a week and needed to scan those in almost real-time, and thus we needed to scan 5 pieces per hour (working 5 days a week, 8 hours a day), which set the requirement for the scan speed.

Several metrologies were investigated as shown in table 7.2 - A laser scanner similar to

<table>
<thead>
<tr>
<th>requirement</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial: ( &gt; 225 \text{ mm} )</td>
<td></td>
</tr>
<tr>
<td>Scan range</td>
<td>Azimuthal: ( &gt; 60° )</td>
</tr>
<tr>
<td>Radial: ( 51 \text{mm - 191mm} )</td>
<td></td>
</tr>
<tr>
<td>Force on the mirror</td>
<td>(&lt; 1 \text{ g (mounted)} ) 0 \text{ g (free-standing)} )</td>
</tr>
<tr>
<td>Resolution</td>
<td>(&lt; 15&quot; ) (Goal: 5&quot;)</td>
</tr>
<tr>
<td>Setup time</td>
<td>(&lt; 6 \text{ months} ) 2.5 \text{ months of delivery not included}</td>
</tr>
<tr>
<td>Scan speed</td>
<td>(&lt; 1 \text{ shell/10 min (free-standing)} ) 1 \text{ layer/4 hour (mounted)}</td>
</tr>
</tbody>
</table>

Table 7.1: Requirements for the metrology systems.
what we used for HEFT (figure 7.2), the Linear Variable Differential Transformer (LVDT), the Keyence confocal laser scanning displacement meter, and the Rainbow probe (electro-optical micro-probe module).

The laser scanner measures the deviation angle of a reflected beam off of the mirror to measure the surface slope. The LVDT is an inductively coupled ruby-tipped mechanical probe, and measures the surface profile that it contacts. The Keyence probe employs a laser diode and uses the confocal principle to measure distance between it and the mirror. The Rainbow probe measures the spectrum shift as a function of separation between it and the mirror.

Each metrology has its own pros and cons. The laser scanner measures the surface slope directly (no differentiation needed to do the performance estimation), but it cannot measure the mounted mirror. The LVDT has very high resolution but applies a force to the mirror, and thus cannot measure the free-standing mirrors. The Keyence probe and the Rainbow probe can measure both the free-standing and the mounted mirror, but with practical installation (separation between them and the mirror greater than 10 mm) the resolution becomes worse. Considering the resolution and our experience on the laser scanner and LVDT, we selected the laser scanner for the free-standing mirrors and LVDT for the mounted mirrors.

In this chapter, I discuss the two metrology systems we built for the NuSTAR project - a laser scanner, and LVDT. I describe the laser scanner in section 7.2 and LVDT in section 7.3.
7.2 The Laser Scanner

7.2.1 Design

For HEFT and R&D on NuSTAR, we used a laser scanner system to develop hard X-ray telescopes. This original system [Jimenez-Garate et al., 2000], which shares the same concept with the one that is described in this section, needed to be modified for the NuSTAR mission. The original system, shown in figure 7.2, operated by running the vertical stage up and down for a scan at an azimuthal position, rotating the vertical stage to the next azimuthal position and then running the vertical stage again. This system put large cantilevered loads on both the rotation and vertical stages, which can cause a stability issue in the straightness of the vertical motion. Because of this, the straightness of the vertical stage had to be calibrated out regularly.

One design goal of the new system was to avoid large loads to the stages and the vertical motion of the stage. The other design goal was to automate the shell alignment. The original system had three manual actuators which were used for aligning the shell to the laser. By replacing those with automatic ones and applying proper procedures (software
CHAPTER 7. METROLOGIES

Table 7.3: Part list for the laser scanner. Only major parts are shown.

<table>
<thead>
<tr>
<th>Component</th>
<th>Manufacturer</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>HeNe Laser</td>
<td>Edmund Optics</td>
<td></td>
</tr>
<tr>
<td>2-D PSD</td>
<td>Hamamatsu</td>
<td>Silicon PIN diode</td>
</tr>
<tr>
<td>Translation Stage</td>
<td>Newport</td>
<td></td>
</tr>
<tr>
<td>Rotation Stage</td>
<td>Newport</td>
<td></td>
</tr>
<tr>
<td>Custom Mirror Type 2</td>
<td>Hellma</td>
<td>rotation</td>
</tr>
<tr>
<td>Custom Mirror Type 1</td>
<td>Hellma</td>
<td>45°</td>
</tr>
<tr>
<td>Mirrors</td>
<td>Newport</td>
<td>steering mirrors</td>
</tr>
<tr>
<td>Controller</td>
<td>Newport</td>
<td></td>
</tr>
<tr>
<td>Non-Polarizing Cubic Beam splitter</td>
<td>Newport</td>
<td></td>
</tr>
<tr>
<td>Plano-convex Cylindrical Lens</td>
<td>Newport</td>
<td></td>
</tr>
<tr>
<td>Pentaprism</td>
<td>Newport</td>
<td></td>
</tr>
<tr>
<td>Laser Line Filter</td>
<td>Andover Corporation</td>
<td></td>
</tr>
<tr>
<td>Signal Processing Circuit</td>
<td>Hamamatsu</td>
<td></td>
</tr>
<tr>
<td>Linear actuator 60 mm</td>
<td>Edmund Optics</td>
<td>manual control</td>
</tr>
<tr>
<td>High torque linear actuator 28 mm</td>
<td>Edmund Optics</td>
<td>manual control</td>
</tr>
<tr>
<td>Additional RS-232C ports</td>
<td>National Instruments</td>
<td></td>
</tr>
</tbody>
</table>

programming), we could automate the alignment and save time. To satisfy these goals, a new design was developed and is shown in figure 7.3.

Figure 7.3 shows the overall structure of the new laser scanner, and table 7.3 shows major parts used. The laser scanner we built is shown in figure 7.4. After being assembled, the new system operates in the following way. The laser beam reflects off two steering mirrors ((a) in figure 7.3) to the linear stage. The first component that the laser beam hits in the linear stage is a pentaprism ((b)). The pentaprism diverts the beam 90° upward to a beam splitter ((c)) which splits the laser beam both up and to the left. The beam that goes to the left side is dumped away, and the upward beam is reflected off of a 45° mirror ((d)), to a rotational mirror ((e)), finally to a free standing mirror substrate ((f)). Due to the local figure of the mirror substrate, the beam deviates from perfect backward reflection. The reflected beam from the substrate is reflected back off the rotational mirror,
Figure 7.3: A schematic view of the NuSTAR laser scanner from above.

Figure 7.4: The NuSTAR laser scanner.
Figure 7.5: Laser scanner - linear stage. A closer view of the optical bed and linear stage. The beam from steering mirror one reflects at the pentaprism, through the beam splitter and onward to the shell. Upon return, the beam strikes the beam splitter on the opposite face, refracts through the cylindrical lens and onto the PSD. The cylindrical lens condenses the return beam into a nearly circular spot after its deformation on the surface of the concave shell.

The 45° mirror and the beam splitter. The beam is finally detected by the position sensitive detector (PSD, (h)), which is a silicon PIN diode utilizing photodiode surface resistance to measure the position where the beam hits. In front of the PSD, there is a cylindrical lens, which is needed because the laser beam is significantly diverging after it is reflected off a cylindrical shell. From the position measurement of the PSD, we calculate the deviation angle of the laser beam.

The way the new system operates makes the improvement clear. The new system runs the linear stage horizontally while the old one runs vertically. The key component allowing this is the 45° mirror, which converts the horizontal motion of the stage to vertical motion of the laser beam. Also, the rotation stage does not rotate the whole linear stage assembly as the old one does. Therefore, the new system guarantees the stability of the linear and the rotational motion better than the old one does. In addition to this, another (and more important) improvement is made by automating the shell alignment, which will be discussed later.
7.2.2 Motion Control

The laser beam scans a shell vertically at an azimuthal position. One a scan is done, the laser beam moves to the next azimuthal position in the shell and does a vertical scan. Therefore, the laser scanner needs to be able to move the beam vertically and azimuthally. In addition to moving the laser beam, the laser scanner needs to control the location and the orientation of the shell for the automatic alignment.

The vertical motion of the laser beam is controlled by a linear stage (figure 7.5). The linear stage translates the optical bed left and right (in the figure 7.5). In the optical bed, there is a pentaprism which diverts the laser beam 90° upward. As is clear in the figure, the horizontal position of the upward going beam changes as the linear stage translates the optical bed. This horizontal motion of the beam is then converted into a vertical motion by the 45° mirror (figure 7.3). Thus, the vertical scan is controlled. The azimuthal motion
Figure 7.7: Shell alignment. Pitch alignment is aligning the axis of a shell to the scan direction by rotating it around the x axis, roll alignment is around the y axis. Radial alignment is locating the shell at its radius from the rotational mirror by translating the shell along the x and the y axis.

of the laser beam is controlled by a rotational stage (see figure 7.6). The rotational stage, mounted on top of a plate ((e)), rotates the rotational mirrors. As the mirror rotates, the azimuthal position of the laser beam on a shell changes.

Control of the tilt and the translation of the platform is more complicated. The platform is connected to a plate ((d)). The translational actuators (actuator 1, 2) are mounted on a plate ((b)), the tilt actuators (actuator 3, 4) are mounted on another plate ((c)). Plate (c) is mounted on the translational actuators and is coupled to plate (d) with a metal ball (at the center) and three springs (Right of figure 7.6). As actuator 1 (or 2) translates plate (c), plate (d) and thus the platform will be translated. Tilt of the platform is accomplished by actuator 3, 4. As actuator 3 (and/or 4) pushes plate (d) upward, the separation between (c) and (d) increases on the right side, does not change in the middle, and decreases on the left side. And thus the platform is tilted.

7.2.3 Automatic Shell Alignment

Building the NuSTAR optics needed to be done at a rate of 24 (or 48 depending on the layer) substrates a day. Considering that we had to measure a shell twice (before and after coating), we needed to be able to scan 50 to 100 substrates a day. Net scanning time is
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About 3 minutes with the new laser scanner but the total time will be a lot longer if a technician has to prepare and align the sample manually. Shell alignment is aligning the axis of a shell to the scan direction (Figure 7.7). For the HEFT laser scanner, a technician needed to align a shell manually by rotating it with the manual actuators on the tip-tilt stage, while looking at the laser beam spot in the display (such as Figure 7.16). As it was difficult to finely adjust the alignment by hand, it took a relatively long time - 5 to 10 minutes.

We installed computer-controlled actuators in the new laser scanner and developed a program to automate the alignment. Therefore, in the new laser scanner system, the alignment is done automatically. The automatic alignment is not only faster (∼3 min) but also more precise.

Possible misalignments can be found in pitch, roll, azimuthal and radial directions. The pitch misalignment is the misalignment between the axis of the shell and the scan direction by an angle around the x axis, the roll misalignment around the y axis (see Figure 7.7). The azimuthal misalignment is the rotation of the shell around the z axis with respect to its own center and the radial misalignment is the shell not being on its radius from the rotational mirror. In this section, I describe each step of the alignment.

7.2.3.1 Pitch alignment

The pitch alignment is obtained by attempting to bring the beam to the center of the PSD using pitch (moving actuators 3 and 4 in the same direction, see Figure 7.6 and 7.7) and x-translation (actuator 1). Note that the beam must be somewhere on the PSD for this to work quickly and accurately.

The first alignment centers the beam spot near the PSD origin. This is done to ensure that the shell is standing upright and parallel to the rotational mirror surface. An example of this alignment is what occurs after the shell is placed on the correct layer on the platform. Should the beam spot appear on the PSD above the x axis and to the right of the y axis, the actuators will pitch up and translate left. This results in a vertical motion of the glass shell, followed by a translation in the x-direction, tangential to the glass surface. If the beam does not yet occupy the region $[-0.5 \text{ mm} < x < 0.5 \text{ mm}, -0.5 \text{ mm} < y < 0.5 \text{ mm}]$,
Figure 7.8: Pitch alignment. The figure shows a beam spot residing at (1.4, -1) on the PSD. The platform pitches down (lowering the platform) and translates left twice to center the beam spot within 0.5mm of the PSD origin.

The scanner pitches and translates repeatedly until the requirement is met. This alignment finishes within seconds.

The travel distance of actuators 3 and 4 ($\Delta x_{act34}$) that corresponds to a pitch angle ($\beta$) is determined by the following calculation. In case of pitch adjustment, we have to drive actuator 3, 4 in the same direction. Let $D_{ba}$ be the distance between the center of actuator 3, 4 and the center of pitch rotation (127 mm, figure 7.6). Then, the relation between the pitch angle and the travel distance is given by

$$\beta = \frac{\Delta x_{act34}}{D_{ba}},$$

where $\beta$ is the pitch angle. The conversion from pitch angle to the y position of the laser beam measured in the detector ($Y_{PSD}$) is given by

$$Y_{PSD} = -2\beta D_{sd} = 2\beta(R + D_{rd}),$$

where $D_{sd}$ is the distance between the shell and the detector, $D_{rd}$ (650 mm) is the distance
between rotational mirror and the detector, and R is the radius of the shell. Combining these two formulas,

\[ Y_{PSD} = -2 \frac{(R + D_{rd})}{D_{ba}} \Delta x_{act34}, \]

or

\[ \Delta x_{act34} = -2 \frac{D_{ba}}{(R + D_{rd})} Y_{PSD}. \] (7.1)

For R=125mm, this becomes

\[ \Delta x_{act34} \approx -0.33 Y_{PSD}. \]

Although it is not necessary at this point, we also attempt to bring the beam to the x center of the detector by translating the shell along the x direction (figure 7.7). This ensures that the beam does not fall off of the detector for the following steps of the alignment.

Figure 7.16 shows the location of the beam spot and its correction during the pitch alignment. The beam spot originally at (1.4 mm, -1 mm) from the center of the detector was moved upward and to the left by pitching down (lowering the platform) and translating the platform, and finally fell near the center of the detector (within 0.5 mm).

### 7.2.3.2 Roll alignment

The roll alignment is to correct the misalignment between the axis of the shell and the scanning direction by roll (rotation around the y axis in figure 7.7). The scanner will begin a z-scan and record data for the alignment. This ensures that the shell is upright with the optical axis of the shell in plane with the scan direction. If there is no roll misalignment, the beam spot on the detector will stay at one x position during the scan (dx/dz=0, ideally). If dx/dz is not less than \( \sim 0.22^\circ \), the platform will roll to minimize the slope created by misalignment, followed by a translation to re-center to beam. This will repeat until the beam produces a minimum dx centered around the PSD origin.

If the optical axis of the shell is not aligned to the axis of the scan (scan direction), the reflected beam deviates from the expected path depending on axial position (see figure 7.15). Change of the x position of the laser beam measured in the detector \( X_{PSD} \) due to this deviation is first order in the tilt angle while that of \( Y_{PSD} \) is second order in the tilt angle. So we will concentrate on \( X_{PSD} \). The deviation in azimuthal angle, which affects
\[ X_{PSD} \text{, is given by} \]
\[ R \Delta \phi = \gamma z, \quad (7.2) \]
where \( \Delta \phi \) is the deviation in the azimuthal angle, \( R \) is the radius of the shell, \( \gamma \) is the roll misalignment angle, and \( z \) is axial position. \( X_{PSD} \) is calculated as follows:
\[ X_c = 2 \Delta \phi (D_{rc} + R) = 2 \frac{\gamma (D_{rc} + R)}{R} z \]
\[ X_{PSD} = \frac{b - f - D_{fd}}{b} X_c = 2 \frac{(b - f - D_{fd}) \gamma (D_{rc} + R)}{b} \frac{R z}{z}, \quad (7.3) \]
where \( a \) is the location of the laser spot (the distance between the shell and the cylindrical lens, \( D_{rc} + R = 650 mm + R \)), \( b \) is the location of the image of the laser spot, \( f \) is the focal length of the lens (100 mm), and the other variables are shown in figure 7.9, where \( b \) is easily calculated using the lens formula and is given by
\[ b = \frac{f(D_{rc} + R)}{(D_{rc} + R) - f} = 115 mm. \]

Therefore, the correction to the roll angle (\( \gamma \)) is obtained by differentiating equation (7.3) and solving it for \( \gamma \).
\[ \gamma = \frac{Rb}{2(D_{rc} + R)(b - f - D_{fd})} \frac{dX_{PSD}}{dz}. \quad (7.4) \]
To roll the platform, we move the actuators 3 and 4 in opposite direction. That is to say, we move one actuator by \( \Delta x_{act34} \), the other by \(-\Delta x_{act34} \), then the roll of the platform (\( \gamma \))
Figure 7.10: Roll alignment. The figure above shows the result of roll alignment. The trace on the left panel is the result of the shell’s optical axis misaligned with the scan direction from an axial scan from -50mm to 50mm. A fitted slope of the trace reveals to which angle the shell must be rolled to align the optical axis. A translation follows to re-center the beam near the PSD center. The process finishes when the slope \( \frac{dX_{PSD}}{dz} \) diminishes to a minimum, as shown in the right panel.

is given by the following:

\[
\gamma = \frac{2\Delta x_{act34}}{D_{3,4}},
\]

where \( D_{3,4} \) is the distance between actuators 3 and 4 (150 mm). Plugging this into equation (7.4), we obtain

\[
\Delta x_{act34} = \frac{D_{3,4}Rb}{4(D_{re} + R)(b - f - D_{fd})} \frac{dX_{PSD}}{dz}.
\]

(7.5)

We measure \( \frac{dX_{PSD}}{dz} \) by doing an axial scan and roll the platform to align the optical axis of the shell to the axis of scanning. For \( R=125 \text{mm} \), equation (7.5) becomes,

\[
\Delta x_{act34} \approx 17.4 \frac{dX_{PSD}}{dz}.
\]

Since the center of rotation for roll is far below the shell (the metal ball in figure 7.6), rolling causes displacement of the shell along the x axis (\( \Delta x_{sh} \)). So whenever we adjust roll, we translate the shell along the x axis at the same time. Translation due to the roll is

\[
\Delta x_{sh} = \gamma D_{bs}
\]

where \( D_{bs} \) is the distance between the center of rotation for roll and the vertical center of the shell (252 mm). We compensate this by translating the platform back by the same
amount:

$$\Delta x_{act1} = \gamma D_{bs}. $$

Figure 7.10 shows the roll alignment process, where the left panel shows the trace of the laser beam before the alignment, and the right after the alignment. Before the alignment, the beam spot on the detector moved along the x axis as we scanned along the z axis ($dX_{psd}/dz$ was not zero). From this scan, we calculated $dX_{PSD}/dz$, used the formula in equation (7.5) and corrected the roll of the platform. After the roll alignment, the laser beam spot stays at the same position as we scan along the z axis (right panel of figure 7.15).

### 7.2.3.3 Radial alignment

The radial alignment is to locate the shell on its radius centered at the rotational mirror. Also during this alignment, the azimuthal misalignment is corrected. Figure 7.11 shows the shell before and after the alignment. The shell is originally placed at a different radial distance and is tilted around its axis (blue arc). The adjustments made during the radial alignment are moving the shell to the right and radially outward, finally locating the shell in its radius (red arc).

These misalignments cause the laser beam spot to deviate along the x axis, and thus create an error in azimuthal slope (not axial). The azimuthal slope error is suppressed by a factor of the graze angle (order of mrad) in a real telescope. Therefore, even if this misalignment is not completely corrected, it will not produce a large error in estimating the performance of the optics.

The laser scanner begins a scan across 60 degrees of the shell surface (generally 30-60 degrees of rotational motion). Actuator 2 will be moved in and out along the y axis in figure 7.7 to reduce $dX_{PSD}/d\phi$ to its smallest value after each successive scan. The shell is then translated along the x direction in such a way that the average of $X_{PSD}$ is zero. If the alignment is successful ($|dX_{PSD}/d\phi| < 0.5 \text{ mm/60}^\circ$, $|X_{PSD}| < 0.25 \text{ mm}$), the scanner will continue on to 31-scans and record data.

The amount of travel distance of actuator 2, 1 is given below. When the shell is not located at its exact radius, the reflected beam spot at the detector ($X_{PSD}$) becomes a function of angular position. We use this fact to relocate the shell at its radius and calculate
the radius of shells. The formula for this is a little bit complicated. The full derivation will be given later in section 7.2.3.4 but to the first order of small angle (scan angle dependent part only), the change of the surface normal of the mirror due to the radial translation is (see the left panel of figure 7.12)

\[
\Delta \phi = \frac{\Delta x_{\text{act}2}}{R} \sin \phi \simeq \frac{\Delta x_{\text{act}2}}{R} \phi,
\]

(7.6)

where \( \phi \) is angular scan position (typically \(-30^\circ \sim 30^\circ\)), \( \Delta \phi \) is the azimuthal deviation angle such as in equation (7.2), and \( \Delta x_{\text{act}2} \) is the travel distance of actuator 2. This \( \Delta \phi \) is directly plugged into equation (7.3), giving us

\[
X_c = 2\alpha(D_{rc} + R) = 2 \frac{\Delta x_{\text{act}2}}{R} \phi(D_{rc} + R)
\]

\[
X_{PSD} = 2\frac{\Delta x_{\text{act}2} \phi(D_{rc} + R)(b - f - D_{fd})}{Rb}.
\]

Upon integrating and solving for \( \Delta x_{\text{act}2} \), this becomes

\[
\Delta x_{\text{act}2} = \frac{Rb}{2(D_{rc} + R)(b - f - D_{fd})} \frac{d(X_{PSD})}{d\phi}.
\]
Figure 7.12: When a shell is misaligned by a translation along the x or the y axis, the misalignment changes the surface normal of the shell by $\Delta \phi$. The red arc represents an aligned shell and the blue arc misaligned one.

At $R=125\text{mm}$, this formula becomes

$$\Delta x_{act2} \simeq 0.23 \times \frac{d(X_{PSD})}{d\phi}.$$ 

Using this formula, we calculate the direction and the travel distance of actuator 2 to minimize $\frac{d(X_{PSD})}{d\phi}$.

The next thing is to translate the shell tangentially (along the x axis in figure 7.7) to make the average $X_{PSD}$ zero. The change in the surface normal due to the tangential translation (actuator 1) is given by the following (see the right panel of figure 7.12):

$$R\Delta \phi \simeq \Delta x_{act1},$$

where $R$ is radius of the shell, $\Delta \phi$ is the angle change due to the translation and $\Delta x_{act1}$ is the translation of actuator 1. This is plugged into equation (7.2) to give $X_c$ and $X_{PSD}$.

$$X_c = 2\alpha D_{sc} = 2\Delta \phi(D_{rc} + R),$$

$$\Delta X_{PSD} = \frac{\Delta X_c(b - f - D_{fd})}{b} = \frac{2(D_{rc} + R)(b - f - D_{fd})\Delta x_{act1}}{Rb}.$$

Solving this for $\Delta x_{act1}$, we find

$$\Delta x_{act1} = \frac{Rb}{2(D_{rc} + R)(b - f - D_{fd})} \Delta X_{PSD}.$$
Figure 7.13: Radial alignment. Two shells and their PSD traces during radial alignment. As the rotational mirror oscillates from -30 to 30 degrees, the scanning software determines the effective radius of the shell and moves actuator 2. This places the shell’s focal point closer to the rotational mirror. The shell used in this example displays a radius gradient across the angular interval, while the shell has a smaller range of radii.

Plugging numbers into the above formula gives us

$$\Delta x_{act1} \simeq 0.23 \times \Delta X_{PSD}$$

at R=125 mm.

If the shell has multiple radii, the scanner may not be able to find the radial position to satisfy the requirement. In that case, it will “break” (virtually separate into two azimuthal sections) the piece into two different scan segments and record data with two different radial positions. Should a shell’s trace be larger than a predetermined interval, the scanning software will separate the scan into two segments. Each segment will be scanned separately with its alignment procedures. So, after the initial alignment determines a shell has too wide a trace at its estimated focal point, the scanner begins aligning itself to scan only the first segment. After alignment and scanning of the first segment, the scanner returns the actuators to the initial alignment positions and begins alignment and scanning of the second segment. In practice, this is scanning a single piece of glass as two separate shells. All actuator positions after each alignment, as well as the angle interval of each segment, are recorded into log files.

Figure 7.13 shows an example of the radial alignment. These were the trace of the
reflected laser beam on the detector during the radial alignment. The shell for the left panel had a radius gradient and could not be radially aligned without “breaking” while the one for the right panel could easily be radially aligned.

When we cannot scan a full 60° sector of a shell, we “break” the shell into two pieces azimuthally and scan in two steps. In this case, we have to do the alignment (roll and pitch) at a different angle than 0°. Since the motion of actuators are aligned to 0° and fixed, we have to do a coordinate transformation to get the correct actuator position.

- tangential/radial translation:

\[
x'_{\text{act}1} = x_{\text{act}1}\cos\phi - x_{\text{act}2}\sin\phi
\]

\[
x'_{\text{act}2} = x_{\text{act}1}\sin\phi + x_{\text{act}2}\cos\phi
\]

- pitch/roll :

\[
\delta'_{\text{pitch}} = \delta_{\text{pitch}}\cos\phi - \delta_{\text{roll}}\sin\phi
\]

\[
\delta'_{\text{roll}} = \delta_{\text{roll}}\cos\phi + \delta_{\text{pitch}}\sin\phi,
\]

where \(\phi\) is the azimuthal position where we align a shell, the primed symbols are the actual amount we have translate or tile, and the un-primed symbols are the amount required at that azimuthal position. During the NuSTAR development and build, there was no shell the laser scanner needed to “break” to scan.

7.2.3.4 Radius measurement

After each alignment is completed, either across the entire shell or each segment, the scanner will calculate the radius of the shell at three \(z\)-positions. It is first done at center and then 50 mm above and below. Each radius measurement is composed of two rotational scans: first the distance traveled by the beam spot on the PSD across 60 degrees (or other set amount) with radius perturbed -1mm from the initial position and an identical scan with the radius perturbed +1mm from the initial position, using actuator two. From these two measurements, the radius at that \(z\)-position can be calculated.

We calculate the radius of a shell by radial scans at two different radial locations. A perfect shell located at its perfect radius is shown by a black arc (center O) in figure (7.14).
To measure the radius of this shell, we have to move the shell along the radial direction by a known amount $l$. The shell might be tilted by $\beta$ around $C'$ from the ideal position. Also the motion of actuator 2 can be tilted by $\alpha$ as shown in the figure (7.14). That is, radial translation is along $OC'$ not $OC$. The red arc (center $O'$) shows the shell that is displaced from the perfect alignment. We calculate the difference between $\theta'$ and $\theta$ and extract radius information. The coordinate of $O'$ in the x-y plane is given:

$$O': x_0 = R\sin\beta + l\sin\alpha \simeq R\beta, y_0 = R(1 - \cos\beta) + l\cos\alpha \simeq l.$$ 

Then we find the intercept between arc $O'$ and OA, which is $A'$. The equation of line OA is

$$y = xcot\theta$$

and the equation of the red arc (circle $O'$) is

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

Solving these for $x$ gives

$$x^2 - 2x_0x + x_0^2 + x^2cot^2\theta - 2y_0xcot\theta + y_0^2 = R^2$$
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\[
csc^2 \theta x^2 - 2(x_0 + y_0 \cot \theta)x + x_0^2 + y_0^2 - R^2 = 0
\]

\[
x = (x_0 + y_0 \cot \theta) \pm \sqrt{(x_0 + y_0 \cot \theta)^2 - (x_0^2 + y_0^2 - R^2) \csc^2 \theta}
\]

\[
x \simeq \frac{y_0 (\gamma + \cot \theta) \pm \sqrt{y_0^2 (\gamma + \cot \theta)^2 + R^2 \csc^2 \theta}}{\csc^2 \theta}
\]

\[
x \simeq y_0 (\gamma + \cot \theta) \sin^2 \theta + R \sin \theta
\]

where \( \gamma \) is \( \frac{x_0}{y_0} \) and \( \frac{1}{R}, \beta, \alpha << 1 \). At the same time, \( A' \) can be expressed in terms of \( \theta' \) in \( x'-y' \) coordinates and can be transformed into \( x-y \) coordinates. \( x' = R \sin \theta', y' = R \cos \theta' \) in \( x'-y' \) coordinates, which become

\[
x = R \sin \theta' + l \alpha - \beta (R \cos \theta' + l), \quad y = \beta (R \cos \theta' + l) + R \sin \theta' + l \alpha
\]

\[
x \simeq R \sin (\theta + \delta \theta) - R \beta \cos (\theta + \delta \theta),
\]

where \( \delta \theta = \theta' - \theta \). Equating \( x \),

\[
R \sin \theta + R \delta \theta \cos \theta - R \beta \cos \theta = y_0 (\gamma + \cot \theta) \sin^2 \theta + R \sin \theta
\]

\[
\delta \theta = \frac{l}{R} (\sin \theta + \gamma \sin \theta \tan \theta) + \beta = \frac{l}{R} \sin \theta + (1 + \sin \theta \tan \theta) \beta
\]

We use this formula to fit the radial scan to get radius. Recall the first term is due to the radial displacement of the shell by \( l \) and is the same as equation (7.6) with \( \Delta x_{act} = l \). We measure \( \delta \theta (\theta) \) at two different \( l \) (\( l_1 \) and \( l_2 \) are unknown but the difference \( l_2 - l_1 \) is known) and fit them to get \( \frac{l_1}{R} = P_1 \) and \( \frac{l_2}{R} = P_2 \). Then,

\[
R = \frac{P_2 - P_1}{l_2 - l_1}
\]

Although measuring the radius was possible in principle, it was difficult to determine it within 1% because of the waviness in the glass and the small misalignment. Also the measurement itself took a few minutes. As a matter of fact, the radius was very well and quickly measured on a paper template (eyeball fit). And the master mandrel onto which we slump a glass piece defined the radius well. In the future, we can improve our approach to make a quick and precise radius measurement, but for NuSTAR we relied on the master mandrel and the paper template fit.
7.2.4 Error budget and optical flat measurement

7.2.4.1 The linear and the rotation stages

Error calculations for the linear and the translation stages are straightforward from the manufacturer’s specification. The azimuthal surface figure will be suppressed by \( \sin \alpha \) when the mirror is mounted at the grazing angle of \( \alpha \), so what contribute to the angular resolution of the optic is the axial slopes. Therefore, we focus on the axial slope measurement error.

As for the linear stage, what contributes to the axial slope are the pitch and flatness of the motion. The specification of Newport ILS250PP says 10.3′′ for pitch. As this happens only between the beam splitter and the detector (125 mm between them) while the slope error is calculated with the distance between the shell and the detector (800 mm), the error due to the pitch of the stage will be suppressed by a ratio of the distances. Therefore, the error due to the pitch of the linear stage becomes 0.8′′ \((10.3'' \times \frac{125}{800} \times \frac{1}{2})\), where the factor of \( \frac{1}{2} \) comes from regarding this as a slope error). In the case of the rotation stage, the wobble of the rotation contributes to the error. The specification of URS75PP/150 says 4.1′′ for the wobble. This wobble deviates the incoming beam as well as the reflected beam. Therefore, the net effect of the wobble will be 8.3′′. However, if the wobble occurs along tangent plane of the shell, it has little effect. Only the radial wobble matters, and therefore there is a factor of \( \sqrt{2} \) suppression, making the effect of wobble 5.8′′.

Practically speaking, the uncertainty due to the motion of the linear stage will be larger than this, especially because it is in motion during scans, which inevitably introduces some mechanical noise. The motion of the linear stage and the noise introduced are measured by scanning an optical flat (section 7.2.5).

7.2.4.2 Roll misalignment

If a shell is misaligned by angle \( \alpha \) in the laser-detector coordinate system(\( x'-y' \)) as seen in figure (7.15), the misalignment will produce an artificial figure. The shell, on its own coordinate system(\( xyz \)), will see the laser beam moving along \( y' \). The off-axis motion of the laser beam will produce an azimuthal deviation in the shell coordinate system.

\[
R \delta \phi = \alpha z,
\]
where \( \alpha \) is the roll angle, \( \delta \phi \) is the change in the surface normal caused by the roll, \( R \) is the radius of the shell, and \( z \) is the axial scan position. To calculate the deviation of the beam in \( x'-y' \), we have to calculate the surface normal at the position of the beam spot in the \( x-y \) coordinate system and transform it into the \( x'-y' \) coordinate system. The surface normal along \( y' \) in the \( x-y \) coordinate system is

\[
\hat{n} = -\hat{x} \sin \delta \phi + \hat{z} \cos \delta \phi.
\]

And in the \( x'-y' \) coordinate system,

\[
\hat{n}' = \begin{pmatrix}
\cos \alpha & -\sin \alpha & 0 \\
\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
-\sin \delta \phi \\
0 \\
\cos \delta \phi
\end{pmatrix} = \begin{pmatrix}
-\cos \alpha \sin \delta \phi \\
-\sin \alpha \sin \delta \phi \\
\cos \delta \phi
\end{pmatrix}
\]

So the deviation angle in \( x' \) and \( y' \) becomes

\[
\theta_x = -2 \cos \alpha \sin \delta \phi \simeq -2 \frac{\alpha}{R} z \\
\theta_y = -2 \sin \alpha \sin \delta \phi \simeq -2 \frac{\alpha^2}{R} z,
\]

(7.7)

where the factor of 2 is due to the reflection from the surface slope. Therefore we ignore this factor when calculating the error. In the shell alignment procedure, we require \( \delta X_{PSD} \) to be less than 1 mm over 170 mm of beam travel length, which corresponds to 0.17\(^\circ\) of misalignment on average at \( R \simeq 100 \text{mm} \). Since \( \theta_x \) will be suppressed by a factor of the grazing angle, I will concentrate on \( \theta_y \). As an example, for \( z = 100 \text{mm}, R = 100 \text{mm} \) and \( \alpha = 0.17\(^\circ\), \( \theta_y \) becomes 1.8\(^\prime\). To calculate the uncertainty due to roll misalignment, we convolve it with the axial figure in quadrature.

\[
HPD_{measured} = \sqrt{HPD_{true}^2 + \delta HPD_{roll}^2} = HPD_{true} \sqrt{1 + \left( \frac{\delta HPD_{roll}}{HPD_{true}} \right)^2} = HPD_{true} + \frac{\delta HPD_{roll}^2}{2 HPD_{true}}
\]

If the true HPD of a shell is 30\(^\prime\), the error due to roll misalignment will become \( \frac{1.8^2}{60} \simeq 0.1^\prime \) for an individual scan.

The next error to be considered regarding the roll misalignment is the relative slope error of the 31 scans. The roll mixes with the pitch as a function of \( \phi \) as follows:

\[
\beta(\phi) = \beta \cos \phi - \alpha \sin \phi,
\]

(7.8)
where $\beta(\phi)$ is the pitch at azimuthal scan position $\phi$, $\beta$ is the pitch angle at $\phi = 0$. The $\beta \cos \phi$ term can be ignored because the pitch alignment is very well done (the next section) and is approximately second order in $\phi$ if we ignore the (irrelevant) constant offset. The $\alpha \sin \phi$ term is the one in equation (7.7) with $\phi$ replacing $\delta \phi$, and is what is producing large ($\sim 100''$) relative slopes between axial scans. It is practically impossible to align the roll of a shell to the desirable level (two orders of magnitude better, that is, $0.0017^\circ$, therefore we remove this later during the data analysis.

### 7.2.4.3 Pitch misalignment

The surface normal in the shell coordinate system (y-z) is transformed into the detector coordinate system as follows:

$$\hat{n} = \hat{z}$$
Figure 7.16: Pitch misalignment. The laser is incoming along the $z'$ axis while the surface normal of the shell is along the $z$ axis.

\[
\hat{n}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \sin\beta \\ \cos\beta \end{pmatrix}
\]

\[
\theta_y = 2\beta,
\]

where $\beta$ is the pitch angle and $\theta_y$ is the deviation angle of the laser beam upon reflection off the shell. Since this is just a constant offset over $z$ scan which will be removed by mean subtraction, the next term that produces $z$ dependence comes from the change in beam travel length. The beam travel length is $D_{sd}$, the distance between the shell to the detector, and becomes $D'_{sd} = D_{sd} + z\beta$. Then the $y$ deviation angle that we measure becomes,

\[
\theta_y = 2\beta + 2\frac{\beta^2}{D_{sd}}z.
\]

(7.9)

Again the first term in $\theta_y$ will be removed. Only the second term will produce error in measuring slope. During the alignment, we require $Y_{PSD}$ to be less than 0.5 mm. This corresponds to $\beta \sim \frac{0.5}{900}$ when $R = 100$ mm. When $z = 100$ mm, this error becomes

\[
\delta\theta_y = 2\frac{0.5^2}{900} \times 100 = 0.025''
\]
which is negligible.

### 7.2.4.4 Misalignment of the axis of the rotational mirror with respect to the laser motion

The alignment of the rotational axis of the rotational mirror to the laser motion can also introduce error to the measurement of a shell as shown below. Let the misalignment angle of the rotational axis be $\alpha$ and $\beta$ with respect to the x axis and the y axis respectively (see figure 7.7). The mirror surface is almost perfectly aligned to the incoming laser (from negative x) when it is facing the laser, and thus the surface normal of the mirror is $\hat{n} = (0, 1, 0)$ (at $\phi = 0$, where $\phi$ is the azimuthal angle). The mirror surface is rotated to $\phi = -\pi/4$ to revert the incoming laser to the shell. This rotation is done with respect to the tilted axis ($\alpha$ and $\beta$). Thus the surface normal after the rotation is given by the following:

$$\hat{n}' = \begin{pmatrix} 1 & 0 & \beta \\ 0 & 1 & -\alpha \\ -\beta & \alpha & 1 \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -\beta \\ 0 & 1 & \alpha \\ \beta & -\alpha & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\cos\phi \\ -\sin\phi \\ \beta\cos\phi - \alpha\sin\phi - \beta \end{pmatrix}.$$  

The laser beam with incoming vector $\vec{L} = (1, 0, 0)$ is reflected by the mirror surface, and the reflected laser vector ($\vec{L}_R$) is

$$\vec{L}_R = (-\cos 2\phi, -\sin 2\phi, 2\beta\cos^2\phi - \alpha\sin 2\phi - 2\beta\cos\phi).$$  

(7.10)

The z component of this vector produces large relative slope between the axial scans as happened in the case of the roll (equation (7.8)). These relative slopes are also removed during the data analysis.

### 7.2.5 Optical flat measurement - repeatability of the linear stage

Section 7.2.4.1 showed the error due to the linear and the rotation stage based on the manufacturer’s specifications. Practically speaking, it will be challenging to attain such a
Table 7.4: Summary of HPD of optical flat at r=150mm. HPDx is ignorable because of $\sin \alpha$ suppression ($\alpha$ is the grazing angle).

<table>
<thead>
<tr>
<th>scan #</th>
<th>HPDx (&quot;)</th>
<th>HPDy (&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
<td>7.6</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>7.9</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>8.6</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>9.3</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>7.9</td>
</tr>
</tbody>
</table>

level of accuracy because as the linear stage moves, the optical components mounted on it will vibrate. We investigated how the accuracy of the system changes as we run the linear stage by scanning an optical flat. The optical flat we used is $\lambda/20$ flat, which was defined as 0" for us (Actual measurement of the flat with different metrologies show 0" within the uncertainty of the apparatus).

We scanned the optical flat several times at different times (for reproducibility) and distances from the rotational mirror (for distance dependence systematics). Every time we scanned it, we dismounted and remounted the optical flat. Table 7.4 shows the Half Power Diameter (HPD) calculated from the surface figures for each scan. For the optical flat which does not have any surface figure, the nonzero HPD is due to the motion of the linear stage. To calculate the non-reproducible part of the motion, we subtracted one surface profile (motion) from another. The difference between scans was 3.6" on average (2 bounce HPD, 1.3" in slope error), we took this as the reproducibility error and calibrated the common figures out for the mirror scan. (This analysis could be done differently. For example, take $\Delta surface = surface_i - surface$ and calculate the HPD of $\Delta surface$.)

This motion seen by the PSD amplifies linearly as the distance between the detector and the reflecting surface (the optical flat) increases. That is, the calibration should be different for different layers. It is impractical to calibrate separately for 133 layers, and thus we got the calibration only at one layer (R=120 mm). As the calibration at different radius
Table 7.5: Summary of HPD of the difference between two scans at r=150 mm of four scans.

<table>
<thead>
<tr>
<th>scan #</th>
<th>HPDx (&quot;)</th>
<th>HPDy (&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 vs 1</td>
<td>0.1</td>
<td>3.7</td>
</tr>
<tr>
<td>0 vs 2</td>
<td>0.5</td>
<td>4.7</td>
</tr>
<tr>
<td>0 vs 3</td>
<td>0.5</td>
<td>3.0</td>
</tr>
<tr>
<td>0 vs 4</td>
<td>0.1</td>
<td>3.3</td>
</tr>
<tr>
<td>1 vs 2</td>
<td>0.1</td>
<td>4.0</td>
</tr>
<tr>
<td>1 vs 3</td>
<td>0.5</td>
<td>3.3</td>
</tr>
<tr>
<td>1 vs 4</td>
<td>0.5</td>
<td>3.0</td>
</tr>
<tr>
<td>2 vs 3</td>
<td>0.1</td>
<td>4.0</td>
</tr>
<tr>
<td>2 vs 4</td>
<td>0.5</td>
<td>4.0</td>
</tr>
<tr>
<td>3 vs 4</td>
<td>0.1</td>
<td>3.0</td>
</tr>
<tr>
<td>average</td>
<td>0.4</td>
<td>3.6</td>
</tr>
</tbody>
</table>

was different, we studied this by measuring the calibration height scale ($S(R)$) for a few layers. The scale was a linear function of the distance (the radius of a mirror substrate + constant).

$$S(R) = A + BR,$$  \hspace{1cm} (7.11)

where $A$ and $B$ are constant to be determined, and $R$ is the distance between the rotation mirror and the reflecting surface.

We measured the surface of the optical flat at 50 mm, 150 mm, 500 mm, 700 mm, 840 mm and 1150 mm away from the rotational mirror five to ten times at each location (table 7.6). Then, we measured the peak to peak value to get the height scale of calibration and fit the data with the formula given in equation (7.11). Figure(7.17) shows the data and the fit. From the fit, we obtained the coefficients and those were $A = 0.041$ and $B = 5.4 \times 10^{-5}$. As we got the calibration only at one radius (120 mm), we had to use the scaling in equation (7.11) to calibrate a measurement done at different radius. Here I calculate the error if we do not use the scaling for HPD measurement of a mirror.
For NuSTAR, the minimum radius is $\sim 50$ mm and the maximum is $\sim 190$ mm, and the calibration was done only at 120 mm. If we calibrate out $7''$ due to the motion of the linear stage at $R=120$ mm, we will have to calibrate $\frac{S(R=50\text{mm})}{S(R=120\text{mm})} \times 7'' = 6.4''$ out at $R=50$ mm and $\frac{S(R=190\text{mm})}{S(R=120\text{mm})} \times 7'' = 7.6''$ at $R=190$ mm. If HPD of a mirror is $30''$ (before calibration) and we calibrate out $7''$, then the HPD we would measure is $\sqrt{30^2 - 7^2} = 29.2''$ when the radius of the mirror is 120 mm. If we apply the calibration scaling properly, the correct HPD is $\sqrt{30^2 - 6.4^2} = 29.3''$ (if $R=50$ mm) or $\sqrt{30^2 - 7.6^2} = 29.0''$ (R=190 mm) depending on its radius. Since we did not apply a different calibration for different radii, we would get 29.2'' regardless of the radius, and thus the error in HPD estimation by not applying the scaling is $0.2''$ for 2 bounce HPD (0.07'' for the slope error).

### 7.2.5.1 Summary

Table 7.7 shows the major uncertainties. The mirror to detector distance and the calibration scaling are systematic uncertainties, and thus directly added to the uncertainties, while the
other errors are statistical and are convolved. The first five terms are added in quadrature to become $6.2''$ ($17.6''$ for 2 bounce HPD), and when convolved with a $30''$ (2 bounce HPD) mirror, it becomes $5.6''$. The other two terms are added directly to be $<0.4''$ ($<1''$ for 2 bounce HPD), and the total uncertainty becomes $<6.6''$ for a $30''$ mirror.

A concern is the large relative slope error ($\sim 100''$) introduced by the roll misalignment and the rotational axis misalignment. Since we do not know the misalignment angles, we subtract the azimuthally (almost linearly) varying relative slopes in the data. Therefore, if there exist azimuthally varying axial slopes in the mirror, we may underestimate the effect of the relative slopes. However, in the approach used for slumping glass (directly onto a precise cylinder) it is unlikely for a mirror to have such slopes. In addition to that, these slopes will be easily suppressed by mounting. Although we cannot properly estimate the effect of the relative axial slope, the uncertainties in table 7.7 should be correct in our mirror fabrication approach.

### 7.2.6 Data Analysis

The position and the laser intensity at the detector plane of the reflected beam is recorded by the Position Sensitive Detector (PSD) at $200$ Hz, and the axial and azimuthal position of the scan is recorded from the linear and rotational stage at $40$ Hz. Since data are taken at different rates, we put a time tag on each data point and interpolate the slower rates to the faster ones, assuming the linear stage is moving linearly in time.
Table 7.7: Summary of the error in measuring slope with the laser scanner. The 2 bounce HPD error is calculated by multiplying $2\sqrt{2}$ with the slope error.

<table>
<thead>
<tr>
<th>component</th>
<th>slope error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational stage wobble</td>
<td>5.8″</td>
</tr>
<tr>
<td>Linear stage motion</td>
<td>1.3″</td>
</tr>
<tr>
<td>Roll misalignment</td>
<td>1.8″</td>
</tr>
<tr>
<td>Pitch misalignment</td>
<td>&lt; 1.0″</td>
</tr>
<tr>
<td>Detector resolution</td>
<td>0.4″</td>
</tr>
<tr>
<td>Mirror to detector distance</td>
<td>&lt; 1% (0.3″ at 30″)</td>
</tr>
<tr>
<td>Calibration scaling</td>
<td>&lt; 0.07″</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td><strong>6.6″</strong></td>
</tr>
</tbody>
</table>

We scanned a mirror substrate over 225 mm along the optical axis at 31 different azimuthal positions and saved each scan at an azimuthal location into a file. Therefore, we had 31 scan files for a mirror substrate. The scan covered the whole NuSTAR mirror axially and sampled 31 azimuthal locations in 60° (or 30°). We ran the linear stage at 47 mm/s, which corresponded to ~150 secs of scanning time and ~1000 data points per scan (one data point per 0.235 mm). From the Nyquist theorem, we would not be able to resolve a length scale smaller than 0.47 mm, which was not a problem as argued before (see section 7.3.4.3 and figure 7.24. Typical length scale of 0.2 mm thick glass is 5 - 10 mm.).

After taking data, the data was converted to the correct unit in the analysis software. The analysis software written in Interactive Data Language (IDL) was developed for the HEFT mission and used for developing and manufacturing the HEFT and the NuSTAR optics. The software produces a 3-D height profile ($dr(\phi,z)$) with the data. Figure 7.18 shows height profiles of 31 scans and Figure 7.19 shows the 3-D height profile of an uncoated mirror substrate in three different ways. The top plot shows the raw profile which gives us the glass shape and it is compared to the laser data taken later after multilayer coating. The other two plots in the middle (slope removed) and the bottom (slope and bow removed) give us a rough idea what the substrate would look like after mounting and are compared
to the LVDT data (section 7.3).

First order performance estimation was done by doing a ray trace simulation with the height profile obtained by the data analysis. Figure 7.20 shows the focal plane counts, its projection, and enclosed energy fraction, from which we obtain the PSF and HPD.
Figure 7.18: Height profiles from individual scans. 31 scans are overlaid. Top: Raw height profile, Middle: Relative slope removed profile, Bottom: Relative slope and bow removed profile. The x axis is the axial position and the y axis is height deviation ($dr$).
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N224A110-072P2-U1: 3D surface profile

Figure 7.19: Surface profile produced by the analysis software. Top: Raw surface profile, Middle: Relative slope removed profile, Bottom: Relative slope and bow removed profile. The x axis is the azimuthal position, the y axis the axial position and the z axis is height deviation ($d_r$). It looks flat because the z axis is $d_r$ not $r$. 
Figure 7.20: Distribution of photons at the focal plane expected from the laser scan analysis. 50% of the events are enclosed in the red circle of the upper left panel.
7.3 The Linear Variable Differential Transformer (LVDT)

The Linear Variable Differential Transformer (LVDT) is an inductively coupled, low force, high resolution mechanical probe that contacts with a surface and measures the height. Inside the tip, it has a ferromagnetic core and three coils in series. The center coil generates an alternating current. As the position of the core changes, the mutual inductance changes, accordingly the induced current in those two outer coils changes depending on the position of the core.

7.3.1 Design

Figure 7.21 shows the LVDT scanning a prototype optic. The LVDT tip was mounted on a linear stage that carries the tip along the axis of the optic. Since substrates are tilted by the graze angle and three times the graze angle, we need to tilt the linear stage by that amount or move the tip radially as we scan axially. We used the former approach for HEFT but switched to the latter for NuSTAR.

We moved the LVDT tip with two linear stages (one radial, one axial) to scan the machined spacers and the back surface of the mounted mirror substrates. When an axial scan was done, the optic rotated by 2.5° where the next axial scan was done. In this way, we scanned at every 2.5° of azimuthal position but it varied depending on whether the scan was close to the spacers or not.
7.3.2 Control System

The scan was controlled by the assembly machine. Encoders in the assembly machine measure the axial, radial, and azimuthal positions and the NI PCI-6601 reads the encoder data. The LVDT data is taken by a National Instrument (NI) USB-6251 Analog Digital Converter (ADC). The ADC is capable of taking data at 1.25 MS/s but we are taking the data at 250 Hz to reduce the file size and processing time. The amplifier for the LVDT is filtering the signal to 100 Hz to reduce the electrical noise, so taking data at 250 Hz is good enough. We filter the data at lower frequency when we analyze the data, which will be discussed in section 7.3.5. The control program written in LabView reads the data from the NI PCI-6601 and NI USB-6251 and saves the data into the control PC. The program is shown in the appendix.

7.3.3 LVDT conversion factor

The amplifier has 4 selectable dynamic ranges - 1.25 $\mu$m, 12.5 $\mu$m, 125 $\mu$m, 1250 $\mu$m and outputs 0-10 V analog signal. Therefore the conversion factor for the amplifier is $\frac{10 \text{ (Volts)}}{\text{Dynamic range (}$\mu$m$)}$. Since we are digitizing the analog signal, we have to consider the conversion of the ADC too. The ADC digitizes $\pm 10$ V into 16 bits ($2^{16}$ integers). So the final conversion from ADC output to height ($\mu$m) is

$$\frac{125 \mu m \times 21 V}{10 V \times 2^{16}} = 0.004 \mu m,$$

if the dynamic range is 125 $\mu$m. Here we use 21 V instead of 20 V due to the way the ADC works.

We quickly checked the conversion factor using a linear stage which is calibrated by gauge blocks. The test setup is shown in figure (7.22) and the results is shown in table (7.8) and in figure (7.23). From the measurement, we obtained the conversion factor of 0.00415 which is 3.5% off from the calculated one. This difference was acceptable and we used the calculated factor in the analysis.
Table 7.8: Measuring the conversion factor - range: 125 \( \mu m \)

<table>
<thead>
<tr>
<th>scan #</th>
<th>offset (( \mu m ))</th>
<th>offset err (( \mu m ))</th>
<th>slope</th>
<th>slope error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6372.21</td>
<td>36.15</td>
<td>-239.720</td>
<td>0.491</td>
</tr>
<tr>
<td>2</td>
<td>6415.96</td>
<td>30.38</td>
<td>-241.443</td>
<td>0.413</td>
</tr>
<tr>
<td>average</td>
<td>6394.085</td>
<td>33.390</td>
<td>-240.581</td>
<td>0.454</td>
</tr>
</tbody>
</table>

7.3.4 Uncertainties

7.3.4.1 Motion of stages

The stages for the LVDT measurements and the mirror mounting are in a custom made assembly machine fabricated by ABTech. The errors in the measurements are produced by the motion of the linear stage that carries the LVDT for a scan. Previous studies on a linear stage whose straightness is 20 \( \mu m \) produced 7' of error (2 bounce HPD). As the stages in the assembly machine are better by some factors than the one we studied, the error would be less than 7'. If the wobble of the rotational stage (spindle) is similar to that of the laser scanner, the uncertainty due to the wobble will be 2.9' (slope error), half of the laser scanner’s (in the case of the laser scanner, the wobble is counted twice because the laser is reflected off the rotational mirror twice, one for the incoming beam and one for the reflected beam).

7.3.4.2 Misalignment

Similar to the laser scanner, the scanning axis of the LVDT system can be misaligned with the optical axis of the shell. The uncertainties due to these misalignments are given in equation (7.7) and equation (7.9). The pitch and roll alignment of the stages was done with a calibration mandrel for the HEFT assembly machine. The alignment for the NuSTAR assembly machine was done to a similar or better level. Recalling that \( \sim 0.1^\circ \) of misalignment produced 1.8'' of slope error for the laser scanner, the quality of the alignment of the assembly machine is of no concern.
The relative axial slopes produced by the roll misalignment (in the case of the laser scanner) are not a concern either. For LVDT measurements, the optic rotates, that is, the axial scans are done all at the same azimuthal position. Therefore, the misalignment between the scan axis and the rotational axis does not change as a function of the azimuthal position of scan.

### 7.3.4.3 Filtering: Surface height noise to HPD

The effect of filtering high frequency (random) noise is analytically estimated below. Let the total length over which we measure the height profile be $L$, the number of data we take be $N$, and the scale length be $a (= L/N)$. Let the height at $x_j$ be $Z(x_j) = Z_j$. We assume the height is not cross-correlated. That is,

$$< Z_j Z_{j+t} > = \frac{1}{N} \sum_j Z_j Z_{j+t} = \sigma_z^2 \delta_{0t},$$
where $\sigma_z$ is the RMS height. To filter the noise, we transform $Z_j$ into the Fourier space, and apply a filter.

\[
\bar{Z}_k = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} Z_j e^{2\pi i k j / N} \tag{7.12}
\]

\[
\bar{Z}'_k = \bar{Z}_k \times F(k), \tag{7.13}
\]

where $F(k)$ is the filter. In our case, we use a 5th order Butterworth filter:

\[
F(k) = \frac{1}{1 + \left(\frac{k}{n}\right)^{10}} \tag{7.14}
\]

which will be approximated to a step function filter in this calculation:

\[
F(k) = \begin{cases} 
1 & \text{if } (k<n) \\
0 & \text{if } (k>n),
\end{cases}
\]

where $n$ is the filtering frequency ($l = L/n$ is the filtering length scale). Let $V'_j$ be the spatial derivative of the filtered height profile (slope) and $\bar{V}'_j$ be its Fourier transformation. Then,

\[
\bar{V}' = \frac{1}{a} \frac{2\pi i k}{N} \bar{Z}',
\]

and

\[
V'_j = \frac{1}{\sqrt{N}} \sum_k \frac{2\pi i}{aN} k \bar{Z}'_k e^{2\pi i k j / N} = \frac{2\pi i}{aN \sqrt{N}} \sum_k \bar{Z}_k e^{2\pi i k j / N} k F(k).
\]
CHAPTER 7. METROLOGIES

From this, we calculate the variance of $V_j$.

$$\sigma_v^2 = \langle V_j'^2 \rangle = \frac{4\pi^2}{a^2N^3} \sum_k k^2 \bar{Z}_k^2 F^2(k) \quad \text{(Parseval’s theorem)}$$

$$= \frac{4\pi^2}{a^2N^3} \sum_k k^2 F^2(k) \frac{1}{N} \sum_j \sum_{j'} Z_j Z_{j'} e^{2\pi ik(j-j')/N}$$

$$= \frac{4\pi^2}{a^2N^3} \sum_k k^2 F^2(k) \frac{1}{N} \sum_j \sum_t Z_j Z_{j+t} e^{2\pi ik(-t)/N}$$

$$= \frac{4\pi^2}{a^2N^3} \sum_k k^2 F^2(k) \sum_t \sigma_z^2 \delta_0 e^{-2\pi ikt/N}$$

$$\approx \frac{4\pi^2}{a^2N^3} \sigma_z^2 \sum_k k^2 F^2(k)$$

$$\approx \frac{4\pi^2}{a^2N^3} \sigma_z^2 \int_0^N k^2 F^2(k) dk$$

$$\approx \frac{4\pi^2}{a^2N^3} \sigma_z^2 \int_0^n k^2 dk$$

$$= \frac{4\pi^2}{a^2N^3} \sigma_z^2 \frac{1}{3} n^2$$

Finally, the slope error of the height profile is given by

$$\sigma_v = \frac{2\pi}{\sqrt{3}} \sqrt{\frac{a}{l^3}} \sigma_z,$$

and when converted into HPD, error in HPD due to this ($\Delta_{HPD}$) is

$$\Delta_{HPD} = 2\sqrt{2} \times 2 \times 0.67\sigma_v = 2.8 \times 10^3 \sqrt{\frac{a}{l^3}} \sigma_z''(n), \quad (7.15)$$

where $\sigma_z$ is in $\mu m$, $a$ and $l$ are in $mm$. Plugging in typical values ($\sigma_z = 0.04 \ \mu m$, $l = 5 \ mm$, $a = 0.16 \ mm$) gives us uncertainty of $4''$ (2 bounce HPD). Equation (7.15) shows that the slope error is proportional to the RMS height and to square root of the data taking interval (inverse of the number of data). The most interesting thing is the dependence on the filtering length scale ($l$). As we decrease the filtering length scale, the error increase, that is to say, we cannot resolve a figure (structure) whose length scale is very small. Therefore, if our mirrors have a significant number of structures with very small length scale, our measurements will be unrealistic since they filter out these length scales.
Figure 7.24: Left: HPD change of a mirror as a function of filtering length scale measured by the laser scanner. Right: Derivative of the HPD with respect to the filtering length scale. Maximum decrease in HPD at 5-10 mm indicates that we are starting to filter out real figure in the mirror.

The proper length scale to filter out was investigated using a less noise-sensitive metrology, the laser scanner. A micron size noise contributes less than 1 arcsec to the slope error of the laser scanner as it is suppressed by the distance between the mirror and the detector (∼800 mm). If we filter laser scan data and calculate the HPD as a function of filtering length scale, the HPD will decrease as we remove the small scale figures. The decrease will be maximum at the length scale at which we start to filter out the real glass figures. Figure 7.24 shows the average HPD of glass measured by the laser scanner (left, average of 93 pieces), and the derivative of the HPD with respect to the filtering length scale (right, average of 792 pieces). The maximum decrease of HPD occurs at 5 mm-10 mm (the right panel of figure 7.24), which suggests this is the length scale of significant real glass figure. However, the slow decrease in HPD at short length scale (<5 mm) could be due to the filtering of real glass figure at these length scale, in which case the difference of unfiltered HPD and 5 mm-filtered HPD should be added to the systematic uncertainty. To estimate the uncertainty conservatively, we add the difference (0.7″, 2 bounce HPD) to the uncertainty.
Table 7.9: Summary of the error in measuring slope with the LVDT. The 2 bounce HPD error is calculated by multiplying $2\sqrt{2}$ with this.

<table>
<thead>
<tr>
<th>component</th>
<th>slope error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational stage wobble</td>
<td>2.9″</td>
</tr>
<tr>
<td>Linear stage motion</td>
<td>&lt;2.5″</td>
</tr>
<tr>
<td>Roll misalignment</td>
<td>&lt;1.8″</td>
</tr>
<tr>
<td>Pitch misalignment</td>
<td>&lt;1.0″</td>
</tr>
<tr>
<td>Filtering (stat.)</td>
<td>1.4″</td>
</tr>
<tr>
<td>Filtering (sys.)</td>
<td>0.25″</td>
</tr>
<tr>
<td>Linearity of the LVDT scale</td>
<td>0.5% (0.15″ at 30″)</td>
</tr>
<tr>
<td>total</td>
<td>&lt;3.8″</td>
</tr>
</tbody>
</table>

### 7.3.4.4 Summary

Table 7.9 shows the major uncertainties. The height scaling is a systematic uncertainty, and thus directly added to the uncertainty, while the others are statistical and are convolved. The first five terms are added in quadrature to become 4.6″ (12.9″ for 2 bounce HPD), and when convolved with a 30″ (2 bounce HPD) mirror, it becomes 2.7″. The filtering (sys.) term and linearity term are added directly to the uncertainty, and the total uncertainty becomes <3.8″ for a 30″ mirror.

### 7.3.5 Data Analysis

The analysis software was written in IDL. It reads in the data files and reconstructs the 3-D surface. Since the data are already heights ($dr$), it is straightforward to interpret. One thing that is tricky is the noise (mechanical, thermal). The motion of the stage generates noise, which contaminate the data and needs to be removed. The typical length scale of height variation for 0.21 mm thick glass is assumed to be 5 mm-10 mm, and we remove the features with the shorter length scales (section 7.3.4.3).

Once the data is read by the analysis software, it directly gives us the 3-D profile of
the surface \(dr(\phi, z)\) unlike the laser scanner data which need to be integrated. There is
the complication of filtering the data, but once the filtering is done, the rest of the analysis
is very straightforward. The filter we apply is a 5th order Butterworth filter. This basic
filtering procedure has been optimized in IDL to have a smooth transition from the end
of the data set back to the start of it by filling in the data to assure \(2^n\) (where \(n\) is an
integer) total data points for optimal Fast Fourier Transform (FFT) speed and to avoid
discontinuities when the data is essentially wrapped around to its start when the FFT is
performed. The filtering length scale can be easily changed when needed.

Figure 7.25 shows LVDT and laser scanner height profile of a mirror substrate at three
different azimuthal locations (rows). The left column shows the surface height profile and
the right column shows the high frequency features. As expected, high frequency features
which are not suppressed by mounting, correlate well between the LVDT and the laser
scanner data while low frequency figure is highly suppressed. The data can also be called
by GUI analysis software which is a more user friendly environment (figure 7.26).

The comparison between the laser scanner data and the LVDT data gave us detailed
information on the mounting process. When done properly, mounting suppressed the low
frequency figure in the mirrors. By comparing the low frequency figure, we could see if
there was torque introduced in the mirrors due to misalignment of the loading fixture. The
left panels in figure 7.25 shows an example of good mounting. However the high frequency
figure in the mirrors is not efficiently suppressed by mounting, and thus stays even after
mounting. Any newly appearing high frequency figure implies dust contamination or badly
machined spacers. The right panels in figure 7.25 shows high frequency figures measured
by LVDT and the laser scanner overlaid and implies that there was no dust contamination
or badly machined spacers.

Also, we estimated the performance of a mirror, a layer, and an optic by ray trace
simulation with the height profile obtained by LVDT scans. Figure 7.27 shows the simulated
focal plane counts (top left), its projection to the x axis and the y axis, and enclosed energy
fraction as a function of diameter. We could obtain the estimated PSF from the focal plane
counts and the HPD from the enclosed energy fraction. This gave us a realistic performance
estimation when combined with X-ray calibration data.
Figure 7.25: LVDT data for individual scans (blue) together with the laser scan data (red) at three different azimuthal locations. Left column: raw height profile, Right column: high frequency features.
Figure 7.26: Graphical User Interface analysis tool.
Figure 7.27: Distribution of photons at the focal plane expected from the LVDT scan analysis. 50% of the events are enclosed in the red circle of the upper left panel.
7.4 Conclusion

The metrology systems we developed played a crucial role in developing and building the NuSTAR optics. During development, these data were used to obtain the optics parameters such as spacer spacing, machining parameters and curing fixture requirement. During optics build, those data were analyzed in real-time to monitor the slumping and mounting process, and to estimate the performance of the optics [Hailey and et al., 2010].
Part IV

Conclusions
The analysis I did for the HEFT experiment showed that a hard X-ray focusing telescope composed of grazing incidence segmented optics and CdZnTe semiconductor detector worked well for astrophysical observations. Although the observation time was short, I was able to show that our understanding of the optics response is good, and was able to measure the size of an extended source - the Crab Nebula. This was the first measurement done with a 2 dimensional focusing telescope on the object.

For the NuSTAR project, I worked on the fabrication and metrology technology. The epoxy we chose guarantees that there will not be any degradation of optics performance. The metrology systems we built/upgraded were used extensively during the optics build for process control and performance estimation. Three optics were finished as of March 2011 and delivered to JPL. The launch of NuSTAR is scheduled for February 2012.
Part V

Bibliography
Bibliography


Part VI

Appendices
Appendix A

The Laser scanner control program

The control program for the laser scanner is written in LabView. The LabView program has two components - the front panel and the block diagram. The front panel is a graphical user interface made of “indicators” and “controls”. An indicator displays a value or a plot and a control is the user input. The block diagram is a real graphical code with functional blocks connected by wires for inputs and outputs.

The program controls 2 stages (a linear stage and a rotation stage), 4 actuators, and takes data from a 2 dimensional PSD. The stages are for scanning a shell (translating and rotating the laser beam), and the actuators are for aligning the shell. The motion of the actuators are described in section 7.2.3.

The data from the PSD is sampled at 200 Hz and those from the linear stage at 40 Hz. These data are recorded separately and later combined with the PC time. Actuator data do not change during a scan, therefore they are saved in the log file.

A.1 The front panel

Figure A.1 shows the front panel for the user interface. A user inputs the geometry of the shell to be scanned, scanning parameters such as scan range and alignment range, and information for file saving. As described in section 7.2.3, the user should roughly align the shell so that the reflected laser beam falls on the detector. This can be done easily with the aid of the real-time display (figure A.2), which shows the location of the laser beam on the
Figure A.1: The front panel of the laser scanner control program
Once the user roughly aligns the shell, finer alignment, scanning and saving data are automatically done by the control program.

A.2 The block diagram

Figure A.3 shows a part of the block diagram. The block diagram is a multi-layered program (5-6 layers) with sub- and subsub- functions cross connected. Therefore it is not practical to shows all the details. The figure shows the main part of the scanning. The left part of the figure is for shell alignment and the right part is for scanning.
APPENDIX A. THE LASER SCANNER CONTROL PROGRAM

Figure A.2: Real-time display for the laser scanner.

Figure A.3: The block diagram of the laser scanner control program
Appendix B

LVDT control program

The LVDT control is done by the assembly machine and the control program is written in G-code. The assembly machine controls the motion of the system and sends the scan position to the DAQ computer. The DAQ program is written in LabView. It receives the position data from the assembly machine, the height profile from the LVDT amplifier, and saves the data into a file.

B.1 The front panel

Figure B.1 shows the front panel of the DAQ program. A user specifies the reading/writing rate and the height range of the height data from the amplifier. The typical reading rate is 100 kHz (maximum 1 MHz) and the writing rate is 250 Hz, and the height range is 125 µm. The user input the directory to save the files and basic scan information.

Once a scan starts, the black panel displays the height profile in real-time for the user to catch any abnormal structure in the mirror. In this way, the DAQ program serves as a real-time diagnostic tool.

B.2 The block diagram

Figure B.2 shows a part of the block diagram of the LVDT DAQ program. The height data from the amplifier is analog voltage data. This analog data is converted to digital and read
Figure B.1: The front panel of the LVDT DAQ program
Figure B.2: The block diagram of the LVDT DAQ program
with an NI-6251 ADC card. The position data from the assembly machine are ticks from
the encoders in the machine. Those are read by an NI-6601 counter. The position data and
the height data are separately saved and later combined by the PC time mark. The upper
part in the figure handles the height data, and the lower part handles the position data.
Appendix C

System assembly and alignment for the laser scanner

As figure 7.3 shows, the new system is very complicated. In this section, I describe how we assemble the new system and align the optical components to attain the best performance. The system assembly and the alignment are done according to the following procedure:

1. Mount the laser and two steering mirrors.

2. Mount the linear stage and the detector.

3. Scan the direct laser beam with the linear stage. Adjust the steering mirrors until the laser beam is stationary at the center of the detector as we scan.

4. Place the pentaprism and have the 45° mirror face downward. Adjust the 45° mirror until we get the perfect backward reflection.

5. Place the beam splitter and adjust it so that the laser hit the center of the detector.

6. Mount the actuators and place the actuator assembly as in figure 7.3.

7. Mount the rotation mirror on the rotation stage and place it on the actuator assembly.

8. Align the rotation mirror to the laser.
At each step, we use a digital level to roughly align the component to 0.1°. The most
difficult and nontrivial alignment is the rotation stage and the rotation mirror (step (8)).
We have to align the rotation axis and the surface normal of the mirror to the coordinate
system defined by the laser beam and the vertical motion of the laser beam. To do this, we
do the following sub-procedure.

(8-1) Place the rotation mirror in such a way that it reflects the laser perfectly backward
(0°). This aligns the surface normal of the mirror, but the rotation axis is still mis-
aligned.

(8-2) Rotate the rotation stage by 90°, run the stage up and down, see whether the vertical
motion of the laser is parallel to the mirror surface, and adjust the rotation stage (do
not adjust the mirror with respect to the rotation stage) to make them parallel.

(8-3) Come back to 0° and adjust the mirror.

(8-4) Place a calibration cylinder on the scanning stage, do the shell alignment (section
7.2.3), and scan over azimuthal angle to adjust α.

(8-5) Iterate (8-1) - (8-4).

These procedures are described by the following calculation. After step (8-1), the surface
normal is \( \hat{n} = (0, -1, 0) \) (figure 7.7). We rotate this vector 90° around the rotation axis
which is tilted by \( \alpha \) and \( \beta \) with respect to the x and y axis respectively. Then we make the
z component of the rotated surface normal zero by adjusting \( \beta \) (\( n'_z = 0 \)).

\[
\hat{n}' = \begin{pmatrix}
1 & 0 & \beta \\
0 & 1 & -\alpha \\
-\beta & \alpha & 1
\end{pmatrix}
\begin{pmatrix}
cos(\pi/2) & -\sin(\pi/2) & 0 \\
\sin(\pi/2) & \cos(\pi/2) & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & -\beta \\
0 & 1 & \alpha \\
\beta & -\alpha & 1
\end{pmatrix}
\begin{pmatrix}
-1 \\
0 \\
0
\end{pmatrix}
= \begin{pmatrix}
0 \\
-\beta - \alpha
\end{pmatrix}
\]

After doing step (8-1) to (8-4), the rotation axis is off by \( \alpha \) for both the x and y axis. The
step (8-4) is rather complicated since it includes the shell alignment. A calculation shows
that the y position of the reflected laser beam measured by the PSD is correlated with \( \phi \)
according to the following formula.

\[
Y_{PSD} = (2\beta \cos^2 \phi - \alpha \sin \phi - 2\beta \cos \phi)D_{sd},
\]
where $\phi$ is the azimuthal angle, and $D_{sd}$ is the distance between the shell and the detector. We make the $Y_{PSD}$ zero at every azimuthal angle by adjusting the $\alpha$ of the stage, which makes $\alpha$ zero. By iterating the sub-procedures, we finally get the system aligned. There are practical difficulties in doing everything precisely, and the errors are discussed (section 7.2.4.4).