articles

Spatial and Psychoacoustic Factors in Atonal Prolongation

By Fred Lerdahl

Consider the sequences of letters in example 1 and think of them as strings of objects perceived and parsed in space or time. In case (a), each object is distinct. There may be degrees of similarity among them, measured by whatever means, but no member of the string is a point of reference for the others. In case (b), X_1 repeats literally as X_2. One might say that X extends in space or is prolonged in time. In cases (c) and (d), the repetition of X creates a frame or context for Y. The two Xs connect perceptually and Y is perceived inside that connection. In other words, Y is subordinate within the context X_1–X_2. If moving from one object to the next is experienced as a path, the motion $X_1 \rightarrow Y$ represents a departure and $Y \rightarrow X_2$ represents a return. If for some reason, say relative temporal proximity, Y groups with X_1, as in case (c), Y belongs to X_1 in the context X_1–X_2; similarly, in case (d) Y belongs to X_2 in the context X_1–X_2. In these instances one can speak of a constituent hierarchy (that is, the subordinate element is not merely subordinate within its context but is subordinate to a single superordinate element). Sometimes it is not proximity but patterns of repetition that cause the internal grouping, as in case (e). But let us leave to one side the issue of subsegments within a string and look at a few other whole patterns. In case (f), the pattern causes a nesting of departure and return: Z belongs within the context Y_1–Y_2, and Y_1–Y_2 within the context X_1–X_2. In case (g), X returns in a modified form, symbolized as X′; Y is now subordinate within the context X–X′. Here X and X′ must be experienced as different versions of the same object. Cases (h) and (i) introduce the factor of salience, symbolized by drawing X larger than Y. For X to be more salient than Y, X must stand out perceptually in comparison to Y; for example, X might be bigger or longer or louder than Y. Then, if X and Y group together and all else is equal, X is judged as dominating Y, with Y either to the right as in (h) or to the left as in (i). Cases (j) and (k) introduce the contrasting factor of stability, symbolized by tilting Y to suggest its relative instability. Again, if X and Y group together and all else is equal, X is judged as dominating Y, with Y either to the right as in (j) or to the left as in (k). X provides the context for Y; X is the prototype against which Y is experienced.

Example 1: Prolongations abstractly considered.

(a) UVWXYZ (b) $X_1\,X_2$ (c) $X_1\,Y\ \ X_2$ (d) $X_1\ \ Y\,X_2$ (e) $W_1\,X\,W_2\ \ Y_1\,Z\,Y_2$

(f) $X_1\,Y_1\,Z\,Y_2\,X_2$ (g) $X\,Y\,X'$ (h) $\mathbf{X}\,Y$ (i) $Y\,\mathbf{X}$ (j) $X\,\mathit{Y}$ (k) $\mathit{Y}X$

In all these cases, simple hierarchical relationships arise that are extensible to longer and more complex strings. They pertain to visual as well as to aural objects. In music, these letters stand for sequences of pitches or chords, whether tonal or atonal. As example 1 suggests, minimal amounts of repetition and grouping are required for the inference of a pitch hierarchy. Because music flows in time, a repeat of an event "prolongs" the event; the connection $X_1 \rightarrow X_2$ constitutes a prolongation of X. This is ordinary English usage, and one that resonates with its historically shifting music-theoretic meaning, even if it differs in some ways with Schenkerian usage. The "away" events in example 1—the events that are subordinate in the various cases—might or might not be closely related to their superordinate contexts, in ways that could be specified. Although it is significant how subordinate and superordinate events are related, there is a level at which this factor can be abstracted away.

As these remarks imply, I resist attempts to restrict the idea of prolongation to its Schenkerian usage and to limit the "away" material to standard tonal treatments (as in Straus 1987). To be sure, it matters to define and use terms precisely. If someone wants to make different use of the terms "prolongation" and "away," I have no objection. The distinctions at issue, unless they prove to be pointless, must be made anyway, regardless of terminology. In my view, however, generalizing these terms, as long as one is clear, is a conceptual gain.

From a complementary angle, I agree with Straus that the Schenkerian notion of prolongation is locked into the tonal idiom. In my view, it is essential to set Schenkerian orthodoxy aside in any consideration of atonal prolongation. For instance, the basic concept of a *Zug* depends on a hierarchical pitch space, specifically a scale level and a chord level. A *Zug* moves stepwise at the diatonic level between two framing pitches that are also pitches of the prolonged chord. But, in genuinely atonal music, there is no referential hierarchical pitch space, hence no scale and chord levels. In these cases it is inappropriate to search for atonal *Züge*.

More broadly, instead of tying ourselves to conventional meanings of "prolongation" and "away," it is more productive to examine how humans hierarchize in general, and from that perspective to tailor our theories of particular musical idioms. This is the approach taken in Lerdahl and Jackendoff (1983, hereafter *GTTM*) and which I have subsequently taken in my ideas about atonal prolongational structure. I see little interest in making a theory of atonal music that cannot find its place in a general theory of music. One normally listens to Bach, Brahms, Bartók, and Boulez with the same ears, adjusting for the manifest differences. A music theory should reflect this continuity.

Along the lines of example 1, I define prolongational connections in general terms: repetition, partial repetition, departure, and return. Example 1 also assumes a parsing of events into units. As it is prolongational structure that is under consideration, I call these units "prolongational regions." The importance of establishing regions of analysis has been insufficiently appreciated. In pitch-class set theory this is the familiar segmentation problem: which pc sets are picked out in analysis, and why? But it is also a problem for Schenkerian analysis: what are the frames within which tonal lines and chords are elaborated? If this second question has been less recognized, it is because there is greater tacit agreement about what the regions of analysis are in tonal music. Schenkerian methodology does not specify these hierarchically organized spans. However, *GTTM* provides a strict procedure for doing so, first by establishing a hierarchical time-span segmentation based on the interaction of meter and grouping, then by deriving prolongational regions from global to local levels of the segmentation. My theory of atonal prolongational structure adopts this procedure (Lerdahl 1989). Within the resulting regions for an atonal piece, superordinate events are selected not by principles of stability, as is done for tonal music, but by psychoacoustic salience. Thus, the prolongational representations for tonal and atonal music are of the same type, reflecting the need for theoretical integration; but the analyses are derived by contrasting criteria, reflecting differences between the two idioms.

This approach to atonal music forces the realization that even in tonal music the criteria used in determining hierarchical importance are a combination of stability and salience. Most crucial is stability: a dissonant neighbor elaborates a chord tone, a dominant elaborates a tonic, and so forth. But we also hear pitches in the soprano and bass voices as more structural than those in inner voices. Likewise, all else being equal, we select a metrically, durationally, or dynamically emphasized event over one that is not so emphasized. These are criteria of salience. How important is salience in tonal prolongation? It would be curious to perform a reductional analysis of a Beethoven piece entirely according to salience criteria.

Probably the result would be as inapposite as doing a Roman-numeral analysis of Schoenberg's Fourth Quartet. For classical tonal music, salience criteria are supplementary to stability criteria.

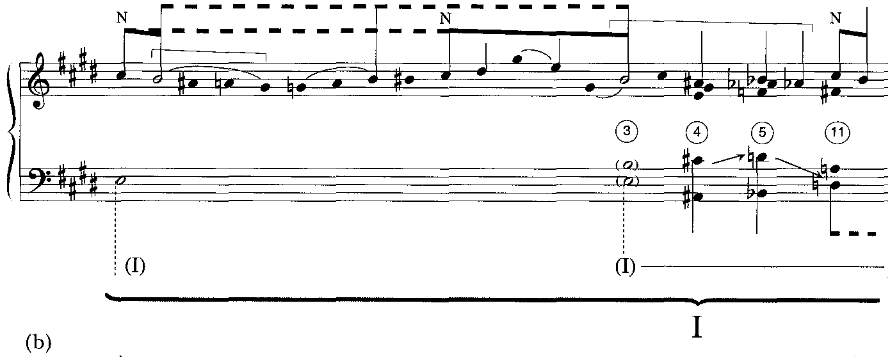
The balance starts to shift for chromatic tonal music. Consider the contrasting significance ascribed to the opening C♯ of Debussy's *Prélude à l'après-midi d'un faune.* As shown in example 2a, Salzer (1962), relying exclusively on criteria of stability and on the view that the piece prolongs an E-major tonic triad from beginning to end, treats the C♯ as a local upper neighbor to the immediately following B in m. 1. At the other extreme, Brown (1993) relies implicitly on the greater salience of the C♯ compared to the B, by virtue of the C♯'s greater duration, pitch height, metrical position, and position at a grouping boundary. As illustrated in example 2b, he treats the C♯ as prolonged over the Bs in the first phrase and through the next phrase, where the melody is harmonized beginning with a D-major triad, until its resolution to B in m. 13. I prefer an intermediate interpretation: the arrival on a clear arpeggiation of an E-major triad in bar 3, ending on a B of some duration, resolves the opening C♯ both melodically and harmonically. The second phrase repeats this C♯–B motion with overt harmonization. In this view, the B in m. 1 is too fleeting and the implied harmony too unstable for the C♯ to resolve as suggested by Salzer; yet the stability of the tonic in m. 3 is enough to override Brown's hearing of the opening C♯ as governing mm. 1–13 in their entirety. This interpretation balances criteria of stability and salience. But the point is less to argue which of these interpretations is correct than to observe that they all lie on a single conceptual continuum, with stability criteria at one end and salience criteria at the other. It is doubtful that such a continuum would be useful for Beethoven. Its relevance to Debussy demonstrates that, with the weakening of tonality brought about by chromaticism, salience has infiltrated the system as an organizing principle. Later on, when chromatic tonality gives way to full atonality, the balance tips still further and stability no longer plays a major role.

This picture of stability and salience in forming prolongational analyses can be developed further, either by extending the scope of stability or by supplementing salience with other principles of perceptual organization. I will now explore these two directions, concentrating more on concepts than on derivational details.

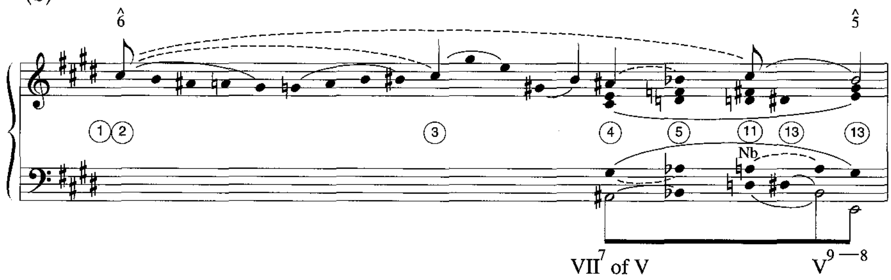
To explain how the scope of stability can be extended, it is first necessary to review some features of my theory of diatonic pitch space (Lerdahl 1988), which models the cognitive distance of any pitch, chord, or region from any other pitch, chord, or region. The basic diatonic space appears in example 3. More stable pcs at one level appear at the next higher level. The configuration in example 3 is oriented to the tonic chord in C major

Example 2: Constrasting interpretations of the opening C♯ at the beginning of Debussy's *Prélude à l'après-midi d'un faune:* (a) Felix Salzer's; (b) Matthew Brown's.

(a)



(b)



(C = 0, C♯ = 1, . . . , B = 11). Different configurations represent different chords and regions. The chord distance algorithm in example 4 transforms the structure in example 3 into other structures inside or outside the tonic region. Its variables are two cycle-of-fifths operators—one to transform diatonic scales along the chromatic scale, the other to move triads around the diatonic collection—and a third factor to track pc nonduplications that result from these transformations. If values are calculated for all the chords within a region, the closest distances fall on the cycle of fifths and the next closest on the cycle of diatonic thirds. Projected geometrically, the result is the chordal space given in example 5a, which, if extended along each axis, can be expressed toroidally. Likewise, if distance values are calculated for all the regions, the geometry takes a similar form, shown in the partial representation of regional space in example 5b. (Regions are designated in boldface.) This space duplicates Weber's (1817–21) and Schoenberg's (1954) regional charts. Both spaces correlate with Krumhansl's (1990) data representations based on experiments on the perceived distances of pitches, chords, and keys from

Example 3: The basic diatonic space.

Octave level:	0											
Fifth level:	0							7				
Triadic level:	0				4			7				
Diatonic level:	0		2		4	5		7		9		11
Chromatic level:	0	1	2	3	4	5	6	7	8	9	10	11

Example 4: The chord distance algorithm.

$\delta(C_1 \rightarrow C_2) = i + j + k$,
where $\delta(C_1 \rightarrow C_2)$ = distance from chord C_1 to chord C_2; i = the number of t7 (mod 12) steps on the chromatic level of the basic space, applied to the diatonic level; j = the number of t4 (mod 7) steps on the diatonic level of the basic space, applied to the triadic, fifth, and octave levels; k = the number of new or "noncommon" pcs, counted at all levels, in C_2 compared to those in C_1.

Example 5: Geometric projections of distances derived from the chord distance algorithm: (a) a portion of chordal space; (b) a portion of regional space; (c) a portion of chordal/regional space.

(a)

iii	V	vii°
vi	I	iii
ii	IV	vi

(b)

e	**G**	**g**
a	**C**	**c**
d	**F**	**f**

(c)

III	V	vii°	iii	V	vii°
VI	**e**	III	vi	**G**	iii
ii°	iv	VI	ii	IV	vi
III	V	vii°	iii	V	vii°
VI	**a**	III	vi	**C**	iii
ii°	iv	VI	ii	IV	vi
III	V	vii°	iii	V	vii°
VI	**d**	III	vi	**F**	iii
ii°	iv	VI	ii	IV	vi

an induced tonic. Example 5c combines the two models into one, with the tonic chord of each region represented by its regional letter name and the regions arrayed as in example 5b.

This spatial organization provides a basis for reductional choices. Take the abstract sequence of events W, X, Y, and Z at any prolongational level, and imagine that X and Y occur within a prolongational region bounded by superordinate W and Z. What prolongational connections do X and Y make? As *GTTM* explains, events cannot connect outside their superordinate endpoints. X could conceivably attach to W, Y, or Z; Y could attach to W, X, or Z. Suppose now that X's least distance in the space is to W and Y's least distance is to Z. Then that is how they attach. If the least distances were otherwise, the attachments would be otherwise. Events are inter-

Example 6: The melodic attraction algorithm.

$\alpha(p_1 \rightarrow p_2) = {}^{s2}/_{s1} \times {}^{1}/_{n^2}$,

where p_1 and p_2 are nonidentical pitches; $\alpha(p_1 \rightarrow p_2)$ = the attraction of p_1 to p_2; s_1 = the anchoring strength of p_1 and s_2 = the anchoring strength of p_2 in the basic space; and n = the number of semitone intervals between p_1 and p_2. (The anchoring strength of the chromatic level is 1, that of the diatonic level is 2, etc.)

preted by the least distance in their prolongational context. I call this powerful factor the "principle of the shortest path." The length of the path between events also quantifies the extent to which the progression tenses or relaxes.

The basic diatonic space also provides a framework for intuitions of attraction between individual pitches (Lerdahl 1996). For instance, in the context I/**C**, leading-tone B is more attracted to tonic C than C is to B, and B is more attracted to C than it is to Bb or A or G. The attraction algorithm in example 6 quantifies these intuitions for any virtual or realized melodic or voice-leading context. The operative factors in the algorithm are semitone proximity and the ratio of the depth of embedding of the two pitches in the current configuration of the basic space.

All of these concepts and structures transfer—in principle and, it is hoped, eventually with empirical support—to post-diatonic styles in which there are both a scalar level and a chordal level built from stable elements in the scalar level. For example, octatonic tonal music comes in three varieties, depending on whether the chordal level is filled by a half-diminished seventh chord, as in Wagner, a French sixth, as in late Scriabin, or a triad, as in neoclassic Stravinsky. Substitution within a single scalar or chordal level in one of these spaces effects a kind of modulation across spaces, as, for instance, van den Toorn (1983) has explored for Stravinsky. Any of these spaces can be modeled by the distance algorithm by changing the cyclic operator for its variable j so that chordal motion is capable of being saturated within the collection. For the diatonic collection the cyclic operator for variable j is t4 (mod 7) over the scale level, or the cycle of fifths; however, for the octatonic collection it is t2 (mod 8) over the scale level, or the cycle of minor thirds. Repeated transposition by fifth moves the triad to all possible positions in a diatonic collection. Analogously, repeated transposition by minor third moves a chord—whether a half-diminished seventh, a French sixth, or a triad—to all possible positions of the given structure in an octatonic collection.

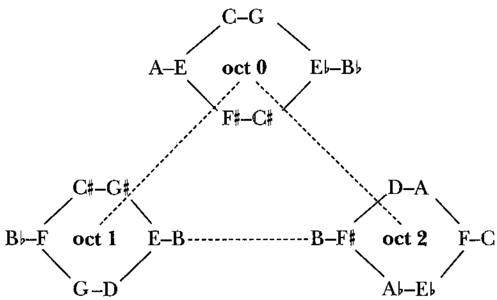
Consider as illustration the opening section of the fifth movement of Bartók's Fourth Quartet. In analyzing this passage, Morrison (1991) reviews the stability-salience distinction and proposes stability criteria involving "disposition pairs" (as proposed earlier by Benjamin 1978) and progression within an octatonic collection. These criteria enable him to mount a prolongational sketch. The pitch-space model can provide a foundation for his approach. Simplifying somewhat, in this passage the diatonic collection is replaced by the octatonic collection and the triadic level is suppressed, resulting in the basic space in example 7. Variable j in the distance algorithm is set at t2 (counting at the scale level) and variable k is unchanged. The treatment of variable i is elementary, for there are only three mutually equidistant octatonic regions. The resultant spatial mapping appears in example 8: the fifth level forms squares and the octatonic regions form a triangle. In the example, the three octatonic regions are indicated by "**oct 0**," "**oct 1**," and "**oct 2**," respectively. This structure resembles Lendvai's (1971) well-known axis system. The purpose of presenting it here is not to repeat what has been said elsewhere but to demonstrate that it derives, with small changes, from the same principles that generate the diatonic structures in example 5.

Added to this system are the disposition pairs derived by Morrison from a statement by Bartók (1976), who completes the aggregate by a combination of Lydian and Phrygian modes built on a single tonic. This Lydian/Phrygian polymode yields double leading tones around the tonic and dominant pitches, indicated by the arrows in example 9. Although one could employ this chromatic voice-leading complex directly (as, for instance, Schoenberg did in his Second String Quartet), Bartók thought in terms of diatonic modes. These four leading tones do not fit within a

Example 7: Octatonic basic space with the usual chordal level suppressed.

Octave level:	0											
Fifth level:	0							7				
Octatonic level:	0	1		3	4		6	7		9	10	
Chromatic level:	0	1	2	3	4	5	6	7	8	9	10	11

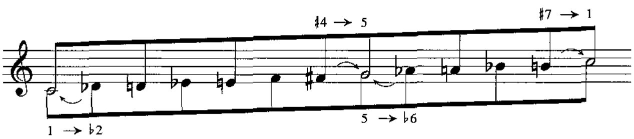
Example 8: Chordal/regional space for Bartók's Fourth Quartet, V.



single octatonic scale. Thus, the music at hand employs two overlapping pitch organizations: the octatonic collection and the Lydian/Phrygian polymode. Calculations of the attraction algorithm would demonstrate what is intuitively transparent: among all the pitch pairs in the Lydian/Phrygian polymode, the greatest attractions are from the double chromatic leading tones to the first and fifth scale degrees.

A full prolongational analysis of the Bartók passage would involve a complete time-span segmentation as well as a step-by-step reduction over the associated prolongational regions. I will provide just a few snapshots of the opening section. The material spins out repetitively, making C salient. The role of salience here is not just to establish tonicity by emphasis but to provide orientation within a stability system that is, as implied by examples 8 and 9, intrinsically symmetrical. The asymmetrical diatonic system, by contrast, does not require salience to establish orientation. Level 1 in example 10 distills the crucial elements in the unfolding discourse: at (a), the C–G fifth, in bass and soprano, with leading tones D♭ and F♯ frozen into the sonority; at (b), the viola-cello ostinato, adding A♭ to the leading-tone mix; at (c), the initial line in the violins, which continues in the same octatonic mode as the music proceeds; at (d), transposition within **oct 0** to a chord on A; and at (e), further transposition within **oct 0** to a chord on F♯. In (d), the grace note F, and, in (e), the frozen chord-tone D, arise from transpositions of the Lydian/Phrygian polymode, which unfolds simultaneously with transpositions in **oct 0**. The spelling of the chords in (a), (b), and (d) is Bartók's, indicating that he conceives the foreign tones as having leading-tone function. Level 2 reduces out these frozen leading tones in (a), (b), (d), and (e), on the basis of the hierarchy provided by the octatonic basic space in conjunction with the attraction algorithm's operation on the Lydian/Phrygian polymode. The assumption is that the leading tones are so attracted to the roots and fifths of the chords that they perceptually merge with them. Unlike diatonic tonal practice, the nonharmonic tones thus reside within the prevailing harmony; there is reduction within as well as across events. The melody in (c) reduces to the C♯–F♯ fourth that comprises the frozen leading tones over the prolonged

Example 9: Bartók's Lydian/Phrygian polymode, with double leading tones indicated by the arrows (from Morrison 1991).



Example 10: Fragments of a prolongational analysis of the first section of the fifth movement of Bartók's Fourth Quartet.

C–G pedal. Here the basis is salience: C♯ and F♯ are boundary tones, and F♯ has the greatest stress and duration. Level 3 adjusts the registers of the prolonged fifths that govern the section, displaying a harmonic progression around the minor-third cycle. In terms of example 8, the path is counterclockwise with C-centricity and within **oct 0**.

Morrison argues for the relative hierarchical significance of the F♯ area, so that the area on A nests within a larger prolongational connection from C to F♯. This interpretation is supported by the pitch commonality between the opening chord built over C at (a) and its tritone transposition at (e). Leaving aside complications arising from the overlapping use of the Lydian/Phrygian polymode, the tritone transposition of (0 1 6 7) duplicates itself, while the minor-third transposition at (d) yields the other pcs of **oct 0,** (3 4 9 10). Hence (a) is closer to (e) than (d) is to either. By the shortest path, (a) connects to (e) in the derivation, with (d) as passing. This is shown at level 3 in example 10.

This discussion has sketched how aspects of Bartók's music are amenable to strict prolongational treatment along the lines of the corresponding tonal theory. The attraction algorithm is unchanged, and likewise (although I have not shown it) with the method for constructing prolongational regions. The crucial differences concern adjustments in the basic space and, consequently, in the distance algorithm. Stability remains a crucial factor. Despite its nondiatonic and nontriadic structures, this music remains tonal in fundamental ways.

When an atonal surface does not elicit a hierarchical pitch space of the type in examples 3 and 7, the distance algorithm fails to apply, and the listener reverts to all-purpose perceptual strategies to make sense of the input. In Lerdahl (1989), I provide a list of interactive factors that mark a pitch event as salient in its context, such as relative loudness, density, duration, registral extremity, and so forth. The psychological premise is that, other things being equal, contextually prominent elements are the most attended-to and the most connected in memory. From relative salience within hierarchically organized prolongational regions, the listener constructs a prolongational reduction of limited depth.

As Boss (1994) points out, however, there is more to atonal prolongation than the contribution of psychoacoustic salience. He proposes two additional ways of distinguishing structural from ornamental events: limiting which events are accepted as structurally connected, and limiting how subordinate events can be ornamental. Unless a musical surface can fit into a hierarchical pitch space like that of the Bartók, the strategy of limiting structural connections comes down to privileging motivically related intervallic structures; that is, closely related motives or pc sets are picked out as prolongationally connected, while unrelated motives or sets are treated as subordinate within that prolongational context. I am uncomfortable with this strategy, at least in the strong form advocated by Forte (1988). A space such as that in example 7 applies, with minor variants, to many pieces; but motives are associational, not hierarchical, structures that are realized in particular ways in particular pieces. Motivic associations play a local, secondary role in tonal reduction; they should do likewise in atonal music. My position in this regard is more Schenkerian than Schoenbergian.

The limitation of ornament types is more promising, for it relates to the standard typology of dissonances for tonal music and can be general in approach. In addition to salience, four principles of perceptual organization help distinguish ornamental from structural tones.

First is streaming. The auditory system automatically parses the incoming auditory signal into simultaneous, continuous streams of activity (e.g., a speaking voice, a humming air vent, and car traffic outside the window).

Bregman (1990) has explored the variables that govern this ubiquitous process, and in ways that impinge on music perception. Generally, the further apart two pitches are registrally and the less similar their acoustic characteristics, the more the ear hears them as belonging to separate streams.

Second is the anchoring principle, articulated in psychological terms by Bharucha (1984). Briefly stated, listeners expect a dissonant pitch to anchor on a subsequent, proximate, and more consonant pitch. The relation between consonance and dissonance is asymmetrical: unstable pitches are judged to be closer to stable pitches than the reverse. Subsequent resolution assimilates the dissonance to the consonance. The attraction algorithm in example 6 models this phenomenon, explaining the asymmetry not by differences in distance but by differences in attraction. The rules of dissonance treatment in tonal syntax implicitly rely on the anchoring principle.

Third is virtual pitch theory (Terhardt 1974). When hearing a chord, the ear tries to match the pitches to a harmonic template corresponding to the natural overtone series. Virtual pitches account for the perception of missing fundamentals and chordal roots. If a chord does not fit the template well, the ear weakly infers multiple roots. If the chord fits the template perfectly, the ear infers a single root. (Incidentally, virtual pitches—which were not postulated at the time—underlie intervallic roots in Hindemith's (1942) theory of harmony, rather than the difference (or combination) tones upon which he relied.)

Fourth is the "critical band" and the associated phenomenon of roughness. When a periodic signal reaches the inner ear, an area of the basilar membrane is stimulated, the peak of which fires rapidly to the auditory cortex, causing the perception of a single pitch. If two periodic signals simultaneously stimulate overlapping areas, the perturbation causes a sensation of "roughness." In most pitch ranges this sensation arises from intervals between a unison and a minor third. This area of overlapping is called the critical band. Plomp and Levelt (1965) modernized Helmholtz's (1885) beating theory of dissonance by demonstrating the role of the critical band in judgments of sensory consonance.

The first two perceptual principles, streaming and anchoring, together identify an important kind of atonal ornament: the resolution of a melodic pitch to a subsequent proximate pitch. If two pitches in a sequence are at least a minor third apart, they potentially coexist as members of different melodic streams or as members of an arpeggiated chord. (The pitches may vary in salience, but that is not the present concern.) However, if the two pitches are a minor or major second apart, they unequivocally fall within the same stream. In line with the anchoring principle, and all else being equal, the first pitch is more ornamental and can be reduced out.

Since the anchoring principle in its tonal application relies not just on subsequent proximity, but on the relative consonance of the second pitch, it may be asked whether an extension of the principle to atonal melodic sequences, where it is assumed (for present purposes) that both pitches are equal with respect to consonance and dissonance, is justified. A brief answer is that because anchoring is pervasive in tonal music, listeners transfer it by habit to superficially similar contexts in atonal music; yet, because there is no resolution of dissonance in an atonal context, the effect is comparatively weak. A compatible but more probing answer is that anchoring depends on attractions, and the attraction algorithm (ex. 6) has two parts: $^{s2}/_{s1}$, which represents levels of stability (or relative consonance), and $^{1}/_{n^2}$, which represents proximity. In an atonal context, it is assumed that pitch space is flat, so both pitches are at the same level and the effect of $^{s2}/_{s1}$ is neutralized: $^{s2}/_{s1} = {}^{1}/_{1} = 1$. But $^{1}/_{n^2}$ nevertheless remains operative. If two pitches are a semitone apart, $^{1}/_{n^2} = {}^{1}/_{1^2} = 1$, which expresses a moderate attraction; if they are two semitones apart, $^{1}/_{n^2} = {}^{1}/_{2^2} = {}^{1}/_{4}$, which is a rather weak attraction; if they are three semitones apart, $^{1}/_{n^2} = {}^{1}/_{3^2} = {}^{1}/_{9}$, which is a miniscule attraction. Thus, even without the role of $^{s2}/_{s1}$, melodic attractions are felt, but the effect of the inverse-square factor renders inconsequential any attractions between pitches a minor third or more apart. The distance between a unison and the major-second/minor-third boundary can be called the attraction band. Attractions within the band are strong enough to justify the practice of atonal anchoring.

The attraction band agrees with what is established elsewhere concerning proximal pitch perception. Its extent corresponds to that of the perceptual attention band, within which humans focus on proximate more than on non-proximate pitches, much as they notice objects near to each other in the visual field more quickly than objects that are far apart (Scharf et al. 1987). Bharucha (1996) builds his recent connectionist approach to anchoring on the notion of attentional selectivity: a dissonant pitch is relatively salient (particularly if it falls on a strong beat), and draws attention to itself and to nearby stable tones; the focus of attention increases activation at that point in the neural net, and this activation heightens the appetite for resolution. It is noteworthy that the width of the attention band corresponds to that of the critical band. Scharf et al. hint at a common basis for the attention and critical bands, but the physical mechanisms underlying it are not yet fully understood. Evidently, the neural firing that gives rise to the critical band is an aspect of the attention process. Thus, there is a convergence between a critical threshold in values predicted by the attraction algorithm, frequency-selective auditory attention, and the frequency range that gives rise to sensory roughness.

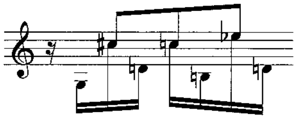
Example 11 illustrates the operation of streaming and atonal anchoring for melodic lines taken from Schoenberg's orchestral song "Seraphita," Op. 22, no. 1; these fragments are discussed by Boss (1994). In a radio talk (published as Schoenberg 1965), the composer beams the violin line as in example 11a, following his intuitive awareness of streaming. Boss evaluates intervals between the two streams, but from a psychoacoustic standpoint such intervals are less important than those within a stream. In example 11b, Schoenberg emphasizes the pitches marked by an "x," connecting them as lines, again apparently on the basis of streaming. In the second fragment of example 11b he treats the second pitch F as a neighboring ornament between E and E♭. From the present perspective, this is so because the F and E♭ occur in the same time-span segment and within a minor third. By the anchoring principle, the F is a quasi-appoggiatura and reduces out. The first note E is structural in comparison to the F on the ground of salience, for it abuts a grouping boundary. Schoenberg and Boss are both interested in bringing out motivic relationships, but the distinctions in question hold anyway. Boss similarly treats the two circled As in mm. 1–2 of example 11c as embellishing neighbors. Again, by the anchoring principle, the D♯ in m. 3 is a quasi-appoggiatura to C♯. The A in m. 3 resolves to A♯ by the same factor, and likewise with C in m. 4 to B in m. 5, and with E♭ to D in m. 5. Although A♯ to G♯ in m. 4 could be treated in parallel fashion, in this case the comparative salience of the A♯ seems to override the anchoring principle. That is, the anchoring principle and salience vie for dominance, in the manner of *GTTM*'s preference rules or of competing activations in a neural net. Under this view, the A♯ wins because of its relative length and the G♯ functions as a quasi-*échappée*. Similarly, the B in m. 1 is sufficiently salient by virtue of its beginning the phrase, that it dominates the B♭ later in the measure.

Example 11d slots these choices for example 11c into a prolongational analysis of the entire line, divided into two streams. Observe that the structural notes of the two lines form intervallic successions that are inversionally equivalent: B–C♯–A♯ in the upper stream, C♯–B–D in the lower. In this analysis, motivic relatedness is a consequence rather than a cause of reductional levels. (At the next reductional level, the two streams would coalesce into one, showing a motion from the opening B to the closing D.)

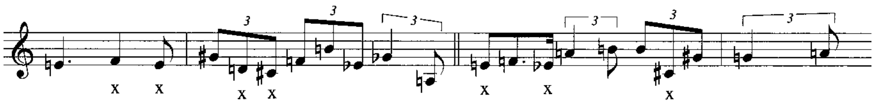
The third perceptual principle, the extraction of virtual pitches, comes into play not for melodic sequences but for harmonic contexts that include relatively consonant chords. Example 12a, Webern's Op. 7, no. 1, provides a good illustration. Any convincing analysis must address the

Example 11: Application of perceptual principles for distinguishing ornamental from structural tones for melodic lines from Schoenberg's Op. 22, no. 1: (a–b) are taken from Schoenberg (1965:9, 19); (c) from Boss (1994:205); (d) is my prolongational analysis of (c).

(a)

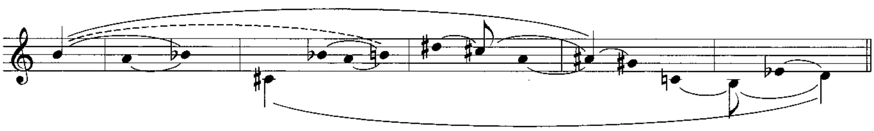


(b)



(c)

(d)



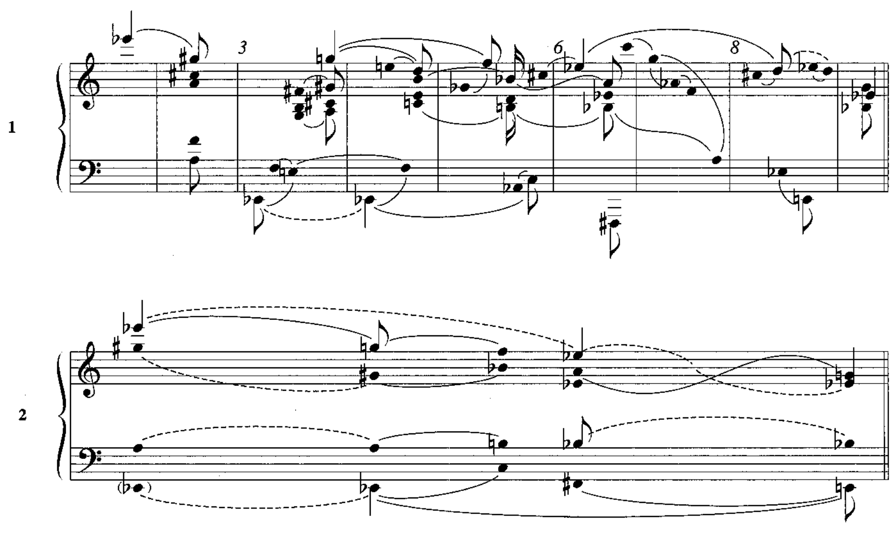
presence of the striking ending on an E♭ major triad. As Berry's (1987) analytic sketch projects, the piece prolongs E♭, first by a high E♭ pedal, then by undulation between E♭ and C♯, with C♯ embellishing E♭ within the attraction band, and finally by the E♭ triad. But the pitch E♭ is in an inner voice; why is it, more than the G that overlays it, recovered at a global level? The reason is that, even in this atonal context, E♭ is the unambiguous virtual pitch, or root, of the sonority. The major triad lends the E♭ psychoacoustic, and therefore hierarchical, prominence.

Example 12b gives a prolongational analysis of the piece. Level 1 presents local connections. An interesting detail concerns the local anchoring of the undulating E♭ and C♯ on D in the violin in m. 8. In a more global perspective, however, this D embellishes the E♭ back in mm. 6–7. The D in m. 8 is then left hanging, in symmetry around E♭ with the low E in the piano; both express an unrealized attraction to E♭ in their own registers.

Example 12: Webern, Op. 7, no. 1, and its prolongational analysis.

(a)

(b)



Level 2 shows global connections between pitches that are superordinate at level 1. Pitches at level 2 are "normalized" (roughly as in Rothstein 1990) into five sonorities, in agreement with the grouping structure. The motion is essentially stepwise in all voices: in the soprano, E♭ by octave transfer throughout, with an inner-voice G–F–E♭ in mm. 3–6 in the violin; in the alto, G♯–B♭–A–G; in the tenor, A–B–B♭; in the bass, E♭ to E, elaborated by a neighboring F♯. From the present viewpoint, such motion takes place within the attraction band, creating the attractional pull that channels these pitches into individual streams.

This is not the occasion to discuss how this analysis is derived. It should be noted, however, that such a derivation cannot be as decisive as one for a tonal piece, because of the inapplicability of the distance algorithm and the weakened applicability of the attraction algorithm. This partial indeterminacy in the analysis reflects the perceptual/cognitive reality of the difficulty in processing atonal surfaces. Nonetheless, the analysis is not merely subjective but is generated by formal procedures. These procedures could be instantiated in a computer program and could be submitted to empirical testing.

The fourth perceptual principle, the critical band and roughness, is important to atonal prolongational theory—beyond its apparent connection to attractions and attention—in the following way. In Lerdahl (1989) I assume that, because dissonance is not syntactically controlled in atonal music, pitch space is flat. However, pitch space is never completely flat, not only because some pitches are more salient than others, but because sequences of simultaneous combinations of pitches yield varying degrees of roughness. Pressnitzer et al. (in press) conduct experiments using complex sonorities in which salience is neutralized and roughness is varied. They find that listeners reliably correlate roughness and tension. The implication for the present theory is that low roughness might partly take the place of high salience in atonal prolongation. That is, within each prolongational region, relatively rough events might be reduced out, leaving more consonant events for the next reductional stage. A potential difficulty with this hypothesis is that relatively rough events also tend to be relatively salient. As a result, salience may conflict with and even overwhelm sensory consonance as a reductional factor. The hypothesis is attractive, however, in its appeal to intuitions of tension and relaxation, which are the starting point in *GTTM*'s conception of tonal prolongation. In this sense, prolongational theory comes full circle. The difference is that for tonal music the chief measure of tension, once local dissonance is reduced out, is a cognitive one, based on distances from triad to triad in the elaborate mental schema that is tonal pitch space, whereas for atonal music the chief measure of tension is psychoacoustic at all levels.

Example 13: The ending of Bartók's Fourth Quartet, together with its prolongational analysis.

The Bartók movement discussed above projects a combination of cognitive and psychoacoustic tension. Cognitively, it creates pitch-space paths in octatonic and other quasi-tonal spaces; it prolongs and resolves to C as tonic. Psychoacoustically, it modulates through degrees of roughness throughout. With the closing gesture shown in example 13, the dissonant (0 1 4) trichords on the upbeat and downbeat move to an implied C-major triad, partly resolving the roughness. This is accomplished by the melodic descent to C (E♭–D♭–C), accompanied by the soprano F♯ moving to E. The final melodic C arrives while, in other streams, G and E linger in memory.

Throughout most of the Webern piece, roughness remains at a more or less even state between pure consonance and crunching dissonance. At the end, however, as the violin ceases and the low E in the piano fades (see ex. 12a), the E♭ triad brings an unprecedented level of sensory consonance. This moment provides a simulacrum of tonal closure, not by cognitive resolution to a tonic in a chromaticized basic space, as in the Bartók, but by sensory resolution to a euphonious sonority within an otherwise unhierarchized space. Although in other pieces Webern often seeks an open form, in this instance his goal is prolongational resolution. To this end, the E♭ triad implicates two psychoacoustic principles: virtual pitch theory for the centrality of E♭ (as mentioned above), and nonroughness for the effect of resolution. Both are needed: a triad with any other root would create an open form by not prolonging the E♭, and a dissonant chord containing a salient E♭ might manifest prolongation but not closure.

In these brief analyses I have ignored pc-set approaches to atonal analysis. I see analysis of that kind as supplementary to a prolongational approach. To take two simple instances: in the Bartók the musical unfolding could not take place without the possibility of partitioning the octatonic collection into two (0 1 6 7) tetrachords; and, as Morris (1994) discusses, the (0 1 4 7) tetrachord and closely related sets are prevalent throughout the Webern. It is the E in the bass, placing the E♭ chord in a (0 1 4 7) context, that allows a triad to conclude the piece without violating harmonic consistency. By itself, however, a pc-set analysis leaves a piece in a tangle of fragments that are not particularly accessible to the ear. A prolongational theory, freed of encumbering baggage from tonal theory, is required to account for the ways in which atonal pieces act as unified wholes that progress from beginning to end.[1]

Notes

1. This paper is a revised version of a talk given at a session on atonal pitch organization at the 1996 annual meeting of the Society for Music Theory in Baton Rouge, LA.

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