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*Hian Teck Hoon
Edmund S. Phleps*

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*Department of Economics
Columbia University
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Hian Teck Hoon[†]

Edmund S. Phelps[‡]

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[†]Professor Hian Teck Hoon, School of Economics and Social Sciences, Singapore Management University, 469, Bukit Timah Road, Singapore 259756. E-mail: hthoon@smu.edu.sg; tel: (65)-6822-0248; fax: (65)-6822-0833.

[‡]Corresponding Author: Professor Edmund S. Phelps, Department of Economics, Columbia University, International Affairs Building, Room 1004, New York , NY 10027. E-mail: esp2@columbia.edu; tel: 1-212-854-2060; fax: 1- 212-854-8059.

Future Fiscal and Budgetary Shocks

Abstract

We study here the effects of future tax and budgetary shocks on present levels of economic activity and real interest rates in a non-monetary and possibly non-Ricardian economy. The paper first takes up an (unanticipated) temporary tax cut to be effective on a given future date—a delayed “debt bomb.” The sudden prospect of this future-dated shock causes at once a drop in the (unit) value placed on the firms’ business asset, the customer, and accordingly on the price of shares—with the result that the hourly wage, hours worked and GDP drop in tandem. This paradox of reduced activity through announcement of future “stimulus” does not hinge on an upward jump of long rates of interest, which may or may not occur: the short rate of return on shares is increased by the initial drop in their price, but the price has so much farther to fall that this is more than offset for a time by the expectation of ongoing capital loss, so short rates of interest actually drop. The paper next studies a future tax cut lacking a “sunset” provision and requiring instead a gradual welfare benefit adjustment to retain solvency. The same negative effects on present activity result. Third, the paper shows that if the tax cut is effective immediately, its effect is ambiguous, as the Marshallian supply-sider effect works the other way. Finally, the paper also examines the new anticipation of a future increase in the number of retirees in a pay-as-you-go social security program. In conclusion, juxtaposing these results against recent US experience, we hypothesize that the legislation of an unsustainable fiscal gap—the cuts in tax rates and the rise of future obligations owing to the cumulative deficit and the approaching bulge in retirement benefits—is an important cause of the decline in hours worked per employee and in the participation rates over the period. (*JEL*: E24, E43, E62, F41)

Key words: Future shocks, business assets, employment

The impetus for this paper was the enactment in summer 2001 of the Economic Growth and Tax Relief Reconciliation Act (EGTRRA), familiarly known as the first Bush tax cut. This bill was another structural tax cut, one interpretable as aimed at boosting rates of investment, thus economic growth, and, it was said, thereby shrinking the medium-term natural unemployment rate.¹ The bill also had the backloading feature, one present to a lesser degree in the multi-stage Reagan tax cut enacted in 1981, of scheduling the largest rate reductions in the future years from 2005 to 2010—yet proponents of the bill said that its backloaded cuts would have announcement effects exerting an expansionary impulse on employment and investment in the present. The bill also had the novelty of providing “sunset” of the tax cuts in 2011. This legislation has left economists unsure and divided about its effects. We suggest that the first step toward clarity, whether or not resolution of our divisions, is to investigate the effects of such an (unanticipated) fiscal shock on the natural, or equilibrium, path of the unemployment rate—or its neoclassical counterpart, hours worked per employee. Accordingly we conduct in this paper an intertemporal-equilibrium analysis of such a fiscal shock in a model that abstracts from monetary channels.

This perspective on the economics of taxation is a marked departure from the postwar tax literature. For Keynesians fiscal policy was all about the deployment of tax rates to moderate swings in business activity—in both supply-side and Keynesian models (Abba Lerner, 1946; Robert Mundell, 1971). Theorists of a more neoclassical persuasion focused on shifts of tax rates appropriate to shifts in circumstances presumed to be permanent. Early investigations, often recalling Ramsey, explored the “neoclassical principles” for the design of fiscal policy (Paul Samuelson, 1951, 1953). One of the last in this genre argued that the (flat) tax rate was best set at the level needed for an unchanging public debt (Robert Barro, 1979).

¹True, those who saw employment as still well above their estimates of its natural level believed that unemployment was on its way up. But even if employment had been seen as below estimates of its natural level, the bill’s proponents would have defended the bill as a fillip to growth and a boost to employment.

But, with the Reagan era, times changed. Tax rates have now been used as a strategic instrument to preempt expansion of welfare entitlements in the expectation that doing so will succeed in lowering future tax rates as well or at any rate lessening their rise. In this new world, the tax rates are more like shocks than responses to shocks and the government's entitlement spending may be endogenous rather than parametric. With this paradigm shift in the way fiscal policy is conceived, the Ramseyan framework needs some changes. However, it is no longer clear that such a framework is well-suited to capture the immediate impacts on rates of investment in business assets that sudden prospects of future tax-rate decreases may have. For example, we would like our model to contain the prices of the one or more business assets in which the existing firms invest. It would be nice to see firms!

We use here a model of the closed non-Ricardian economy in which output is sold on a customer market and, for simplicity, the labor variable is hours worked. A key feature is the shadow price of an incremental customer: a decrease operates to reduce the demand wage and thus decreases hours worked. In this model, we find three effects on present economic activity from enactment of a wage income tax cut effective with or without a delay and with or without a sunset provision. One of these effects is the classical supply-side effect of the reduced tax rate on hours of work supplied. But there are one or two other effects, both operating through the asset price. Except in the Ricardian case, there are Feldstein-Rubin-Summers consequences of the lower tax rates in the future for the future stock of public debt and thus for future short real rates of interest, and these imply an immediate drop of the shadow price in the present, thus decreasing present hours and output—even though the present short real rate drops. Furthermore, if the tax cut in the medium-term future is accommodated by a subsequent tax increase rather than entitlement reductions, there are also consequences for current earnings on the business asset, which also lead to a drop in the present shadow price.

After the exposition of the basic model in section 1 the paper has three parts corresponding to the three shocks, fiscal and budgetary, that we study—a future tax cut with the sunset provision, a delayed as well as an immediate tax cut without

the sunset provision, and a future increase in the number of retirees. In section 2, we begin with a future “debt bomb,” one that is announced at t_0 to occur at t_1 —a “time bomb” of exploding public debt, such as the present enactment of a tax cut to become effective at a future date and with a sunset provision soon thereafter. (Thus there is some small interval over which there is a big government deficit). We show that this shock depresses the (unit) value of the business asset, the price of shares, hours worked and thus the GDP. The drop in share price raises the earnings-price ratio, which (taken alone) increases the rate of return, but generates expectations of capital loss, with the latter dominating, so the Wicksellian natural short rate of interest drops in the present. Contrary to what many in the debate suppose, the paradox of output contraction does not hinge on an immediate rise of the long-term real interest rate, which may or may not occur. In this section, as an addendum, we also show how any inflation-targeting central bank following a Taylor rule would set its short-term interest rate (the federal funds rate in the US) in such a fiscal environment.

In section 3, we investigate the effects of a delayed as well as an immediate permanent tax cut with entitlement spending adjusting gradually to retain solvency. We find that the extent to which entitlement spending as a ratio to GDP will have to be cut as the debt-income ratio rises depends on how far tax revenue falls relative to government interest cost savings when asset prices fall. Even when fiscally sustainable, a delayed permanent tax cut is shown to produce the paradoxical result of employment contraction as asset prices unambiguously decline. The extent of the decline in asset prices is greater the earlier is the future event, rendering ambiguous the net effect on economic activity of an immediate tax cut. We show that hours worked is pulled up by the direct Marshallian impact of the (immediate) decrease of the tax rate on the demand wage but pushed down by the (proportionately greater) drop in stock market capitalization as a ratio to GDP whose depressing effect on the demand wage through its impact on markups dominates the resulting wealth effect on labor supply.

Section 4 extends the basic model to include retirement and studies the problem of a prospective increase in the share of retired people in the population under a

pay-as-you-go social security system. In the concluding section, we argue that a theory of dynamic markup variation, such as the one developed in this paper, is needed on top of changes in marginal tax rates if we are to quantitatively account for labor-leisure distortions in the US at medium-term frequencies.

Roberto Perotti (1999) examines both theoretically and empirically how drastic cuts in government deficits—fiscal consolidation—in countries with exceptionally high levels of the debt-GDP ratio tend to lead to consumption booms.² His paper differs from ours in neglecting employment responses by assuming perfectly inelastic supplies of labor, and relies on a competitive framework. Phelps (1992) develops a closed-economy customer market model and examines a public debt shock but it does not incorporate the distortionary effect of the wage income tax that plays a crucial role in this paper and it neither analyzes the effects of a back-loaded tax cut nor studies the endogenous evolution of the debt-income ratio in a fully specified general-equilibrium system. Based upon their empirical study of the US economy, Paul Beaudry and Franck Portier (2004) make a case for the importance of expectational shocks in explaining business fluctuations. They suggest that these shocks take the form of news regarding shifts in future technological possibilities. Our paper, however, shows that present concerns about future fiscal and budgetary overhangs that do not directly affect technological possibilities might nonetheless also depress current asset prices and contract employment.

1. The Basic Model of the Non-Ricardian Economy With No Retirement

Our basic model describes a closed economy with no retirement. It is not the Ricardian type of economy favored by RBC theorists and some public finance theorists. Instead, we follow the treatment by Blanchard (1985) in which worker-savers toil throughout life, save by buying annuities invested in the shares of the firms, and *die off* exponentially.³ In order to provide a business asset to back the

²Perotti (1999) also provides a review of the related papers on “expansionary fiscal consolidations” but these papers do not rely on the supply-side channels and asset price channels emphasized in our paper.

³This model nests the special case of the Ricardian economy exhibiting Ricardian equiv-

shares of firms, and in order to give a role to the variation of price-marginal cost markups in explaining the big changes in the distortive gap between the value marginal product of labor and the marginal rate of substitution in consumption and leisure (the “marginal value of time” measured in consumption units), we use the customer-market model set up by Phelps and Sidney Winter (1970) and placed in a general-equilibrium setting by Guillermo Calvo and Phelps (1983) and Bruce Greenwald and Joseph Stiglitz (2003).⁴ Owing to frictions in the transmission of price information, the competition of firms for market share will fail to wipe out all pure profit, and so leave the optimal price charged by firms hanging above the average and marginal cost. Firms set mark-ups below the monopolist’s level but above the pure competitor’s level—how high depending upon the value per unit placed on the average and marginal customer. The output supply and thus also employment, we show, is an increasing function of this per-unit asset value and a decreasing function of the tax rate.

Production for the customer market, which is the only commercial market supplied by firms using only labor to produce a single homogeneous good, is carried out by a large (constant) number of atomistic firms in identical (or symmetrical) circumstances. The size of the population and the stock of customers are equal to a positive constant, which we normalize to one. Hence the number of customers per firm is a demographic parameter in our closed economy. There are four marketable assets—shares, which are titles to the stream of profits, private short- and long-

alence obtained by setting a parameter representing the probability of death to zero. We point out later that the influence of a change in the wage income tax rate on current earnings on business assets and thus on share prices does not depend on the non-Ricardian nature of the economy.

⁴Julio Rotemberg and Michael Woodford (1992) argued convincingly that a model featuring imperfect competition in the product market is required in order to explain how aggregate demand changes, such as increases in government purchases, can increase output while at the same time raise the real wage. Our paper goes a step further to argue that fully accounting for variations in the distortive gap between the value marginal product of labor and the marginal rate of substitution in consumption and leisure requires tracking not only changes in the tax rate but also changes in the price-marginal cost markup.

term bonds issued and held by individuals, and government bonds. These non-monetary assets are assumed to be perfect substitutes so arbitrage among them implies that they have the same expected short-term rate of return. It will be innocuous but convenient to suppose that all (non-human) wealth, in equilibrium, is held in the form of shares and public debt. A proportional wage income tax is imposed to finance government expenditure.

Agents derive utility from consumption and leisure, have finite lives and face an instantaneous probability of death θ that is constant throughout life. Let $c(s, t)$ denote consumption at time t of an agent born at time s , $l(s, t)$ the number of hours worked, $w(s, t)$ non-human wealth, and $h(s, t)$ human wealth. Also let $y^g(s, t)$ be welfare entitlement received and $v^h(s, t)$ be the after-tax real hourly wage (both measured in units of output, our numeraire good), where v^h is related to the hourly labor cost to the firm, v^f , by $v^f \equiv (1 + \tau)v^h$, τ being the proportional wage income tax rate. We make the assumption that workers of all age cohorts have the same productivity, face the same tax rate and receive the same entitlement so $v^h(s, t) = v^h(t)$ and $y^g(s, t) = y^g(t)$ for all s . We let $r(t)$ denote the real instantaneous short-term interest rate, $\rho (> 0)$ the pure rate of time preference, and \bar{L} the total time available per worker.

The agent maximizes

$$\int_t^\infty [\log c(s, \kappa) + B \log(\bar{L} - l(s, \kappa))] \exp^{-(\theta + \rho)(\kappa - t)} d\kappa, \quad B \equiv \text{parameter} > 0$$

subject to

$$\frac{dw(s, t)}{dt} = [r(t) + \theta]w(s, t) + v^h(t)l(s, t) + y^g(t) - c(s, t)$$

and a transversality condition that prevents agents from going indefinitely into debt. The solution to the agent's problem is given by

$$\begin{aligned} c(s, t) &= (\theta + \rho)[h(s, t) + w(s, t)], \\ \frac{\bar{L} - l(s, t)}{c(s, t)} &= \frac{B}{v^h(t)}, \end{aligned}$$

where human wealth is given by

$$h(s, t) = \int_t^\infty [l(s, \kappa)v^h(\kappa) + y^g(\kappa)] \exp^{-\int_t^\kappa [r(\nu) + \theta]d\nu} d\kappa.$$

Aggregating across all individuals, dropping the time index t and denoting per capita aggregate variables by capital letters, we obtain

$$C = (\theta + \rho)[H + W], \quad (1)$$

$$\frac{BC}{\bar{L} - L} = v^h, \quad (2)$$

$$\dot{H} = (r + \theta)H - (Lv^h + y^g), \quad (3)$$

$$\dot{W} = rW + Lv^h + y^g - C, \quad (4)$$

where a dot over a variable denotes its time derivative. We note that although every worker faces the same hourly pay, the fact that the members of the labor force are of different ages means that their wealth levels are different, and consequently, the number of hours worked will be different across the different age cohorts. We also note from (3) and (4) that whereas the rate of interest used to discount after-tax wage income and entitlement is $(r + \theta)$, aggregate non-human wealth accumulates at rate r . It is this difference in discount rates that results in the non-neutrality of debt and deficits.

The government's budget constraint can, in general, be expressed as

$$\dot{D} = rD + G + y^g - \tau Lv^h, \quad (5)$$

where D is the per capita level of government debt, G is the per capita amount of government purchases, and tax revenue collected is entirely from wage income taxation. For simplicity, we will throughout set $G = 0$. Assuming that, in equilibrium, agents have zero holdings of private bonds, $W \equiv V + D$, where V is the total value of shares held by individuals. Taking the time derivative of (1), and using (3) and (4), we obtain

$$\dot{C} = (\theta + \rho)[rW + (r + \theta)H - C]. \quad (6)$$

Using (1) in (6), we obtain, after re-arrangement of terms,

$$\frac{\dot{C}}{C} = (r - \rho) - \frac{\theta(\theta + \rho)[V + D]}{C}. \quad (7)$$

We now turn to the firms. We assume that each identically situated symmetric firm faces a technology that converts one unit of labor into one unit of

output. Taking the wage rate, v^f , as given, each firm i has to choose the price at which to sell to its current customers or, equivalently, the output to supply per customer to its consumers. Raising its price causes a decrease, and lowering the price an increase, in the quantity demanded by its current customers according to a per-customer demand relationship, $D(p^i/p, C^s)$. For simplicity, we assume that $D(\cdot)$ is homogeneous of degree one in total sales, C^s , and so we write $C^{si} = \eta(p^i/p)C^s$; $\eta'(p^i/p) < 0$; $\eta(1) = 1$. Each firm chooses the path of its real price or, equivalently, the path of its supply per customer to its consumers, to maximize the present discounted value of its cash flows. The maximum at the i th firm is the value of the firm, V^i , which depends upon x^i :

$$V_0^i \equiv \max \int_0^\infty \left[\left(\frac{p_t^i}{p_t} \right) - v_t^f \right] \eta \left(\frac{p_t^i}{p_t} \right) C_t^s x_t^i \exp^{-\int_0^t r_\nu d\nu} dt.$$

The maximization is subject to the differential equation giving the motion of the stock of customers of the i th firm as a function of its relative, or real, price given by (8) below and an initial x_0^i :

$$\dot{x}^i = g \left(\frac{p^i}{p} \right) x^i; \quad g' < 0, g'' \leq 0; \quad g(1) = 0. \quad (8)$$

The first-order condition for optimal p^i is

$$\eta \left(\frac{p^i}{p} \right) \frac{C^s x^i}{p} + \left[\left(\frac{p^i}{p} \right) - v^f \right] \eta' \left(\frac{p^i}{p} \right) \frac{C^s x^i}{p} + q_m^i g' \left(\frac{p^i}{p} \right) \frac{x^i}{p} = 0, \quad (9)$$

where q_m^i is the shadow price, or worth, of an additional customer. Another two other necessary first-order conditions (which are also sufficient under our assumptions) from solving the optimal control problem are:

$$\dot{q}_m^i = [r - g \left(\frac{p^i}{p} \right)] q_m^i - \left[\left(\frac{p^i}{p} \right) - v^f \right] \eta \left(\frac{p^i}{p} \right) C^s, \quad (10)$$

$$\lim_{t \rightarrow \infty} \exp^{-\int_0^t r_\nu d\nu} q_{mt}^i x_t^i = 0. \quad (11)$$

We note that “marginal q” denoted q_m^i is equal to “average q,” which we denote

as $q_a^i \equiv V^i/x^i$, so $q_m^i = q_a^i \equiv q^i$.⁵

Now we move on to consider the economy's general equilibrium. First, we take note that in the closed economy, the aggregate stock of customers is a fixed constant given by the size of the population, which we have normalized to one. Hence, in the closed economy, $x = 1$. Next, equating p^i to p and setting $q^i = q$ in (9) for a symmetric equilibrium gives:

$$\left[1 + \frac{\eta(1)}{\eta'(1)} - v^f\right] = -\left(\frac{q}{C^s}\right) \left(\frac{g'(1)}{\eta'(1)}\right); \quad \eta(1) = 1, \quad \eta'(1) < 0, \quad g'(1) < 0. \quad (12)$$

The expression in the square brackets in (12) is the algebraic excess of marginal revenue over marginal cost, a negative value in customer-market models as the firm supplies more than called for by the static monopolist's formula for maximum current profit, giving up some of the maximum current profit for the sake of its longer-term interests. Defining the average (gross) markup as $m \equiv 1/v^f$ since $p^i = p$ in $(p^i/p)/v^f$ in a symmetric equilibrium, we can re-arrange (12) to obtain:

$$\left(\frac{m-1}{m}\right) = -\left(\frac{1}{\eta'(1)}\right) \left(\eta(1) + g'(1)\left(\frac{q}{C^s}\right)\right). \quad (13)$$

Equation (13) shows that the optimal markup depends negatively on what may be called, Tobin's Q, q/C^s —the ratio of the present discounted value of acquiring an additional customer relative to the payoff from current consumption. We write $m = \phi(q/C^s)$, with $\phi'(q/C^s) < 0$. An increase in q relative to C^s means that profits from future customers are high relative to payoff from current consumption so that each firm reduces its price (equivalently its markup) in order to increase its customer base.

There is yet another way of expressing (12), which puts a focus on the labor market, that will be useful for developing intuition for the results we obtain in this paper. Noting our simple production technology, $C^s = L$, we can re-express

⁵The proof is as follows: Taking the time derivative of the product $q_{mt}^i x_t^i$, we obtain $d(q_{mt}^i x_t^i)/dt = q_{mt}^i [dx_t^i/dt] + x_t^i [dq_{mt}^i/dt] = r_t q_{mt}^i x_t^i - [(p_t^i/p_t) - v_t^f] \eta(p_t^i/p_t) C_t^s x_t^i$, after using (8) and (10). Integrating, and using (11), we obtain $q_{mt}^i x_t^i = \int_t^\infty [(p_\kappa^i/p_\kappa) - v_\kappa^f] \eta(p_\kappa^i/p_\kappa) C_\kappa^s x_\kappa^i \exp^{-\int_t^\kappa r_\nu d\nu} d\kappa \equiv V_t^i$.

(12) as saying that the representative firm's real demand wage, v_{demand}^f , is negatively related to employment, L , and positively related to the shadow value of an additional customer, q :

$$v_{\text{demand}}^f = 1 + \frac{\eta(1)}{\eta'(1)} + \left(\frac{q}{L}\right) \left(\frac{g'(1)}{\eta'(1)}\right). \quad (14)$$

In a Marshallian employment-real wage diagram (see Figure 1), (14) gives a downward-sloping labor demand curve, with q acting as a labor demand-shifter. An increase in q , the shadow value attached to having an additional unit of the business asset—here, a customer—leads firms to reduce their monopoly power and thus to increase the demand wage.

The other schedule in this plane is the aggregate labor supply curve, which comes from (2). Noting the identity $v^h \equiv (1 + \tau)^{-1}v^f$, (2) can be re-expressed as

$$v_{\text{supply}}^f = \frac{(1 + \tau)BC}{\bar{L} - L}, \quad (15)$$

which says that the real supply wage, v_{supply}^f is positively related to employment, L , given the tax rate and consumption. In the Marshallian employment-real wage diagram, (15) gives an upward-sloping labor supply curve, with the parameter representing the value of leisure (B), the tax rate (τ), and the level of consumption demand (C) acting as labor supply shifters.

Putting together labor demand and supply, we see that a decrease in τ reduces the tax wedge and consequently expands L . To understand the role played by C , we substitute (1) and the definition of non-human wealth into (15) to get

$$v_{\text{supply}}^f = \frac{(1 + \tau)(\theta + \rho)B[q + D + H]}{\bar{L} - L}, \quad (16)$$

where H is human wealth. We see from (16) that an increase in q raises the worker's non-human wealth and as a result raises the supply wage. Since an increase in q increases labor demand but also decreases labor supply, can we determine what the net effect on employment is? To get the answer, we draw upon the condition that, in the closed economy, equilibrium requires that total consumption demand must be equal to the economy's supply of the consumer good, an equilibration that

is brought about through an adjustment in human wealth, H . Since $C^s = L$, we can impose the goods market-clearing condition, $C = C^s = L$, in (15), and then equate the demand wage to the supply wage to obtain

$$\left[1 + \frac{\eta(1)}{\eta'(1)}\right] + \left(\frac{q}{L}\right) \left(\frac{g'(1)}{\eta'(1)}\right) = \frac{B(1 + \tau)L}{\bar{L} - L}, \quad (17)$$

where L , we find, is unambiguously increasing in q : an increase in q induces firms to lower their markups, thus to raise the demand wage, and that effect dominates the wealth effect on labor supply.⁶ We also make the observation from (17) that since the optimal markup, $1/v^f$, chosen by firms depends on q/C^s so the demand wage (given by the LHS of (17)) depends on q/L , and the supply wage (given by the RHS of (17)) is increasing in L , an increase in q brings forth a less than proportionate increase in L . Lastly, we note that the stock of public debt, D , does not appear explicitly in (17) so its effect on L works only indirectly through its influence on q .⁷

We summarize these results in the following lemma:

LEMMA I: We obtain $C^s = L = \Omega(q; \tau)$, with $0 < e_q \equiv d \ln C^s / d \ln q < 1$, where e_q denotes the elasticity of C^s with respect to q , and the partial derivative $\Omega_\tau < 0$. Additionally, $v^f = V^f(q; \tau)$, with the partial derivatives $V_q^f > 0$ and $V_\tau^f > 0$.

⁶Suppose that we are initially at point A in Figure 1 where the level of consumption demand (to which the supply wage curve is indexed) is initially equal to the level of output supply. Let there be an increase in q . Looking at (14), we can infer the extent to which the representative firm is willing to raise the demand wage, v_{demand}^f , at the original level of L . Consider next how the supply wage curve shifts when q is increased. Suppose that at the original level of L , v_{supply}^f is increased by the same amount as v_{demand}^f has increased so that L is not changed. In this situation, however, consumption demand would exceed the unchanged output supply. The term structure of interest must then adjust to reduce H to make the supply wage curve intersect the demand wage curve at a point that is north-east of point A so that $C = C^s = L$ at a higher q .

⁷Given q , an increase in D raises consumption demand so that at the original employment level, consumption demand exceeds supply. The term structure of interest then adjusts to cause H to fall by the increase in D so that $C = C^s = L$ is restored.

Equivalently, the equilibrium markup, m , can be expressed as $m = (V^f(q; \tau))^{-1} = \psi(q; \tau)$ with the partial derivatives $\psi_q < 0$ and $\psi_\tau < 0$.

It will sometimes be useful to use another reduced-form function for output supply or equilibrium employment. For this alternative formulation, we note from setting $L = C^s$ in (17) that since the demand wage is increasing in $\hat{q} \equiv q/C^s$, and the supply wage is increasing in L , therefore, L is increasing in \hat{q} .

LEMMA II: Output supply, equal to employment, can alternatively be expressed as $C^s = L = \Psi(\hat{q}; \tau)$ with $\Psi_{\hat{q}} > 0$ and $\Psi_\tau < 0$.

In a symmetric situation across firms, (10) simplifies to

$$r = \frac{[1 - V^f(q; \tau)]\Omega(q; \tau)}{q} + \frac{\dot{q}}{q} + g(1); \quad g(1) = 0, \quad (18)$$

after using $v^f = V^f(q; \tau)$ and $C^s = \Omega(q; \tau)$. Equation (18) in the firm's instantaneous rate of return to investment in its stock of assets, which are customers, is an inter-temporal condition of capital-market equilibrium: it is entailed by correct expectations of \dot{q} and r at all future dates. We observe that with the q elasticity of C^s being less than unity, and with an increase in q raising the unit cost, the earnings-price ratio is unambiguously decreasing in q .

Finally, equating consumption demand to supply in (7), and noting that non-human wealth is held in the form of shares and public debt and that $\dot{C}^s/C^s = e_q(\dot{q}/q)$, we obtain an expression for the consumer's required rate of interest, r :

$$r = \rho + \frac{\theta(\theta + \rho)(q + D)}{\Omega(q; \tau)} + e_q \left(\frac{\dot{q}}{q} \right); \quad 0 < e_q < 1. \quad (19)$$

If we define the long-term (real) interest rate as the yield on consols paying a constant coupon flow of unity, and let R be their yield and hence R^{-1} be their price, arbitrage between short and long bonds gives the condition $R = r + (\dot{R}/R)$. Equating required rate of interest in (19) to the market rate of return in (18), and noting that $g(1) = 0$, we obtain, for given fiscal parameters, an expression for the

size of capital gains (or loss):

$$\frac{\dot{q}}{q} = (1 - e_q)^{-1} \left[\rho + \frac{\theta(\theta + \rho)(q + D)}{\Omega(q; \tau)} - \frac{[1 - V^f(q; \tau)]\Omega(q; \tau)}{q} \right]. \quad (20)$$

To rule out multiple equilibria and focus on unique rational expectation paths, we make the following assumption:

ASSUMPTION I: (Unique rational expectation paths) Assume that $\theta(\theta + \rho) \left[\frac{d(q/\Omega)}{dq} - \frac{D}{\Omega^2} \Omega_q \right] > 0$.

From (20), using assumption 1, we observe that the capital gain (\dot{q}/q) is increasing in q , D and τ . If we now substitute for the capital gain term in either (18) or (19) using (20), we obtain

$$r = \left(\frac{1}{1 - e_q} \right) \left[\rho + \frac{\theta(\theta + \rho)(q + D)}{\Omega(q; \tau)} - e_q \frac{(1 - V^f(q; \tau))\Omega(q; \tau)}{q} \right], \quad (21)$$

which makes the equilibrium interest rate or market rate of return an increasing function of q , D and τ .

LEMMA III: The natural interest rate function is given by the following: $r = \Upsilon(q; D, \tau)$ with the partial derivatives $\Upsilon_q > 0$, $\Upsilon_D > 0$ and $\Upsilon_\tau > 0$.

An increase in the share price raises the price-earnings ratio, which (taken alone) decreases the market rate of return, but this is associated with increased capital gains, which dominates, so an increase in q raises the equilibrium interest rate. Intuitively, the reason is that an increase of q raises the required rate of interest, necessitating an increase in the rate of capital gain that more than compensates for the decline in the earnings-price ratio.

We now study the economy's equilibrium state given $D > 0$, $y^g > 0$, and $\tau > 0$. We have the following lemma:

LEMMA IV: Given D , y^g , and τ , the rational expectations equilibrium is given by a unique value of q , denoted q_{ss} , that makes the RHS of (20) equal to zero.

Since the elasticity of C^s with respect to q is less than one, the RHS of (20) is increasing in q . Applying the transversality condition, $\lim_{t \rightarrow \infty} [\exp^{-\int_0^t r_s ds} q_t] = 0$, the unique perfect foresight path of q requires that it be stationary at the value that makes $\dot{q} = 0$. Let the unique stationary level of q be denoted by q_{ss} .

To be explicit about the Wicksellian natural rate of interest that corresponds to q_{ss} in the stationary equilibrium, it will be useful to develop a diagram using (18) and (19) after setting $\dot{q} = 0$. In Figure 2, the downward-sloping schedule gives the market rate of return as a negative function of q while the upward-sloping schedule, under assumption 1, gives the consumer's required interest rate as a positive function of q in a stationary equilibrium. We note that in the stationary rational expectations equilibrium of this model, the Wicksellian natural rate of interest is unambiguously positive. (Appendix A.1 shows that in the stationary state, an increase in D increases the natural rate of interest.) Explicitly, the stationary level of natural interest, r_{ss} , given $D > 0$ and $\tau > 0$, is given by

$$r_{ss} = \frac{[1 - V^f(q_{ss}; \tau)]\Omega(q_{ss}; \tau)}{q_{ss}} = \rho + \frac{\theta(\theta + \rho)(q_{ss} + D)}{\Omega(q_{ss}; \tau)} > 0. \quad (22)$$

We observe from (22) that in the stationary state, $r_{ss} - \rho > 0$. Noting that $[dc(s, t)/dt]/c(s, t) = r_{ss} - \rho$, and that given q_{ss} , the worker's real wage is stationary, it must be the case that although aggregate consumption and aggregate labor supply are both constant in the stationary state, individual agents are accumulating over their life and also planning to increase their leisure over their life.⁸

2. Effects of a Future Debt Bomb

⁸A problem can arise in the stationary state of this model, as pointed out recently by Guido Ascari and Neil Rankin (2004) in a different context, that the demand for leisure for some very wealthy individuals might exceed their time endowment, \bar{L} . We avoid this problem in two ways in our paper: first, by considering expectational shocks that get the economy out of the stationary state and that cause r to fall when asset prices decline, possibly below the rate of time preference (see (21)), and thus lead individual economic agents to decumulate wealth outside the stationary state; and second, by introducing mandatory retirement into the model in section 4. Ascari and Rankin (2004) instead propose to use a utility function that makes the demand for leisure independent of wealth.

A. The Analysis

Let us now suppose that at t_0 , it is announced that at t_1 , there will be a temporary tax cut, one that produces a big government deficit over a small time interval, that we dub a debt bomb. As a result of the temporary tax cut, the stock of public debt is suddenly increased by the amount $\Delta \equiv \text{parameter} > 0$. We will explore the effects of the debt bomb under two alternative modes of financing, one where the debt bomb is accommodated by cuts in entitlement spending at t_1 onwards and the other where subsequent wage income tax rates are raised to re-balance the budget. To analyze the effects on asset prices, interest rates, and employment, it is convenient to refer to Figure 3, which depicts the stationary loci of the following pair of equations:

$$\dot{D} = \Upsilon(q; D, \tau)D + y^g - T(q; \tau), \quad (23)$$

$$\frac{\dot{q}}{q} = \Upsilon(q; D, \tau) - \frac{[1 - V^f(q; \tau)]}{q/\Omega(q; \tau)}, \quad (24)$$

where (23) is obtained by using $r = \Upsilon(q : D, \tau)$ and $T \equiv \tau(1 + \tau)^{-1}V^f(q; \tau)\Omega(q; \tau)$ in (5), and (24) is obtained by using $r = \Upsilon(q : D, \tau)$ in (18).

The stationary q locus is downward sloping as an increase in D raises the interest rate, which requires a lower q to raise the earnings-price ratio. Given D , an increase in q above the stationary locus leads to capital gains while a decrease leads to capital loss. As lemma 4 established, use of the transversality condition implies that a rational expectations solution is a unique stationary q at given D . What the negative slope of the stationary locus implies is that the unique stationary q value is decreasing in the stock of public debt. We have the following lemma:

LEMMA V: The unique stationary q , denoted q_{ss} , is decreasing in the stock of public debt, D .

The stationary D locus can be either positively or negatively sloped. In the empirically relevant case where a depressed stock market leads to rising debt-GDP

ratios, as the implied collapse in labor demand leads both to a reduction in total hours worked as well as hourly pay so tax revenue falls at given tax rates and the size of the deficit grows despite government interest cost savings, the stationary D locus is positively sloped. Appendix A.2 establishes that both roots associated with the dynamic system given by (23) and (24) are positive whether or not the debt-GDP ratio rises when equity prices fall.

The economy is initially at point A with (D_{ss}^0, q_{ss}^0) . Working backwards, let us ask, “What is the value of q in the new stationary state after the debt bomb?” Given the dynamic instability of the system, the new stationary state must be attained precisely at t_1 . With the explosion of the public debt at t_1 (the stock of public debt is suddenly augmented by Δ), lemma 5 tells us that the new stationary value of q is lower. The anticipation of the reduced future q , however, causes an immediate drop in present q . The path taken by the economy from t_0 onwards is illustrated in Figure 3. Upon suddenly receiving the news of a future debt bomb, therefore, asset prices fall immediately from q_{ss}^0 to q^B , the value of q that corresponds to point B in Figure 3, and the expected rate of change of q , i.e., the expected capital gains term, goes from zero to a negative value as market participants form a rational expectation of further asset price declines.

As asset prices drop and continue a path of further decline until future time t_1 , the government starts to lose tax revenue even before the temporary tax cut is implemented so the stock of public debt begins to grow from t_0 onwards. At t_1 , the stock of public debt is suddenly augmented by the amount Δ as a result of the debt bomb. In order to retain solvency, we first suppose that the whole constant stream of entitlement spending is reduced from t_1 onwards to accommodate the debt bomb. (The reduction of y^g from t_1 onwards shifts the stationary D locus rightwards to pass through point D in Figure 3.) In essence, public debt is now substituted for social wealth (the present discounted value of the whole stream of entitlements). In a non-Ricardian setup, the stream of government entitlements is discounted at $r + \theta$ but non-human wealth accumulates at r . Consequently, the substitution of public debt for social wealth makes consumers feel wealthier. The stimulus to consumption demand raises the whole path of the short real rate

of interest from t_1 onwards and thus depresses asset price at t_1 . In anticipation, present asset price is also reduced. We obtain the following proposition:

PROPOSITION I: Suppose that the economy is initially in a stationary state with (D_{ss}^0, q_{ss}^0) . At t_0 , there is a sudden announcement that at t_1 , there will be a temporary tax cut causing a debt bomb. The government budget is re-balanced by a subsequent cut in the constant stream of entitlement spending. The asset price immediately drops and continues to fall until it reaches a permanently depressed level. Employment immediately drops and steadily worsens from then on until it reaches a lower plateau at t_1 .

In the alternative financing scheme, the government raises the wage income tax rate to re-balance the budget after the initial splash of public debt. (This also has the effect of shifting the stationary D locus to the right.) This increase in the tax rate has the classic supply-side effect of reducing hours worked through increasing the tax wedge at given q . In addition, as the increase in tax rate reduces current earnings on business assets from future time t_1 onwards, there is a consequent decline in q at t_1 . In anticipation, present q drops. We obtain the following proposition:

PROPOSITION II: Suppose that the economy is initially in a stationary state with (D_{ss}^0, q_{ss}^0) . At t_0 , there is a sudden announcement that at t_1 , there will be a temporary tax cut causing a debt bomb. The government budget is re-balanced by a subsequent permanent increase in the wage income tax rate to service the debt. The asset price immediately drops and continues to fall until it reaches a permanently depressed level. Employment immediately drops and steadily worsens from then on until, at t_1 , there is another abrupt drop to reach a lower plateau as the tax rate is increased.⁹

⁹There is a discontinuous drop in output, and hence employment, at t_1 because q does not jump at t_1 but the wage income tax rate is increased at that point to finance the interest on increased debt. The negative supply-side effect leads to the further decline in employment at t_1 .

For an equal-sized debt bomb, the size of the wealth effect resulting from the use of one method of financing is, *ex ante*, the same as that resulting from the use of the alternative method of financing. To accommodate the debt bomb, human wealth is reduced via a cut in the constant stream of government entitlements in the one case and via a permanent increase in wage income tax rates in the other case. However, there are deadweight losses associated with the imposition of distortionary taxes on wage incomes which are not present with the cut of government entitlements. (In the latter, there is only an income effect whereas in the former, there are both income and substitution effects.) In particular, with higher wage income tax rates, individual agents supply a sub-optimal number of hours at work as leisure is made artificially cheap. As a result unit costs are pushed up and current earnings on each unit of the business asset fall because of the increase in the tax rate. Thus, for an equal-sized debt bomb, asset prices fall further when solvency is retained through a permanent increase in the wage income tax rate rather than through a cut in the constant stream of entitlement spending.

It is also of some interest to ask what are the effects of the future debt bomb when Ricardian equivalence holds. This case is obtained in the formal model simply by setting the parameter that represents the probability of death, θ , to zero. We can readily establish the following proposition:

PROPOSITION III: Suppose that Ricardian equivalence holds, and that the economy is initially in a stationary state. At t_0 , there is a sudden announcement that at t_1 , there will be a temporary tax cut causing a debt bomb. If the government budget is re-balanced by a subsequent cut in entitlement spending, there is no effect on employment, asset price and interest rate. If, however, the government budget is re-balanced by a subsequent increase in the wage income tax rate to service the debt, the asset price immediately drops and continues to fall until it reaches a permanently depressed level. Employment immediately drops and steadily worsens from then on until, at t_1 , there is another abrupt drop to reach a lower plateau as the tax rate is increased.

We find, therefore, that even when Ricardian equivalence holds, if the future tax cut is accommodated by a subsequent tax increase rather than entitlement reductions, there are also consequences for current earnings on the business asset, which lead to a drop in the present asset price.

How can we infer the whole path of the Wicksellian natural rate of interest in response to a future debt bomb in both the Ricardian and non-Ricardian cases accompanied by either a cut in entitlement spending or an increase in the tax rate to retain solvency? We note from (22) the initial value of r before the expectational shock occurs, setting $\theta = 0$ in the Ricardian cases. Then at t_0 when the announcement is made, the value of q drops except in the case of Ricardian equivalence accompanied by a cut in entitlement spending. (In that case, there is no change in q as public debt serves as a perfect substitute for social wealth.) The fall in share price increases the earnings-price ratio, which (taken alone) raises the market rate of return but this is more than offset by the anticipation of capital loss so the short real rate of interest, in fact, falls as we can confirm by inspecting (21). Further inspection of (21) shows us that the further decline in q causes the short real rate of interest to fall further although the gradual build-up of the stock of public debt tends to attenuate the fall when Ricardian equivalence does not hold. Thus it is very possible that the short real rate of interest will remain low for some time between announcement and implementation.¹⁰

It is also worth pointing out that since the short rate, r , initially drops and may

¹⁰We have demonstrated theoretically that the drop in asset price coincides with a drop in the short-term natural rate of interest. Is this relationship observed empirically? Do we tend to see, say, in the past fifteen years, a drop in the stock market being associated with a drop in the short-term real interest rate, and a rise in the stock market associated with an increase in the short-term real interest rate. We suggest that the answer is in the affirmative. A number of authors who have examined the behavior of the Federal Reserve System since the mid-eighties onwards have argued that implicit inflation targeting by the Fed has involved raising the short-term real interest rate as a means of dampening aggregate demand when asset prices increased (see Ben Bernanke and Mark Gertler, 1999; Richard Clarida, Jordi Gali and Gertler, 2000). When stock prices fell in 2001, the Fed lowered the short-term real interest rate in a series of cuts to stimulate aggregate demand.

continue to fall (until t_1) in response to the sudden news of a future debt bomb, the depressed stock market could be accompanied by an initial decline in the long rate, R . Recall that arbitrage ensures $R = r + (\dot{R}/R)$. If the term structure is downward sloping at announcement, R unambiguously falls below r since $\dot{R}/R < 0$. If the term structure is upward sloping at announcement so $\dot{R}/R > 0$, it is still possible that R initially drops if r falls by more than the rate of capital loss on holding a long bond, where R^{-1} is the price of the long-term bond. Hence the paradox of employment contraction does not result from nor imply any immediate increase of the long real interest rate, contrary to the view held by some financial commentators.¹¹

B. Behavior of Inflation-Targeting Central Bank

Let us explore how an inflation-targeting central bank adopting a Taylor rule would behave in a fiscal environment characterized by the future-dated shock just studied. In order to study monetary policy in a world with short-run price-level sluggishness, we now introduce to our basic model the AS equation that would result from the Calvo (1983) staggered pricing model and an expectational AD equation, and suppose that the central bank uses the Taylor rule for setting the short-term nominal rate of interest. Letting z_t represent the output gap, the key Calvo equation describing the rate of change of the inflation rate, π_t , is given by

$$\frac{d\pi_t}{dt} = -\delta^2 z_t, \quad (25)$$

where δ denotes the (constant) probability that a firm receives a signal to reset its price. The expectational AD equation expressed in terms of the output gap, z_t , can be written as

$$\frac{dz_t}{dt} = i_t - \pi_t^e - r_{Nt}, \quad (26)$$

where π_t^e is the expected rate of inflation, i_t is the short term nominal interest rate set by the central bank, and r_{Nt} is the Wicksellian natural rate of interest. Under

¹¹See the editorial entitled, "The Demise of Rubinomics," in the Wall Street Journal (August 28, 2002).

perfect foresight, one can write $\pi^e = \pi$ in (26). Let us write the Taylor rule as

$$i_t = \bar{i}_t + a(\pi_t - \bar{\pi}) + bz_t, \quad (27)$$

where a and b are positive constants and $\bar{\pi}$ is the inflation target. Substituting (27) into (26), and setting $\pi^e = \pi$, we obtain

$$\frac{dz_t}{dt} = \bar{i}_t - a\bar{\pi} - \Upsilon(q; D, \tau) + (a - 1)\pi_t + bz_t, \quad (28)$$

after replacing r_N with the function $r_N = \Upsilon(q; D, \tau)$; $\Upsilon_q > 0$, $\Upsilon_D > 0$ and $\Upsilon_\tau > 0$.

Equations (25), (28) and (20) provide a system of dynamic equations in the three variables: π , z and q . We can readily check that the determinant of the matrix associated with this system of equations evaluated around $\pi = \bar{\pi}$, $z = 0$ and $q = q_{ss}$ is given by $\delta^2\Pi(a - 1)$, where Π defined in appendix A.3 is positive. Hence the determinant would be positive if and only if the Taylor principle holds, namely, that $a > 1$. The trace is given by $(b + \Pi) > 0$. Appendix A.3 provides proof that the three roots are positive so that with all three variables being jumpy, we obtain a unique rational expectations equilibrium with $\pi_t = \bar{\pi}$, $z_t = 0$ and $q = q_{ss}$. Consequently, the equilibrium is characterized by the economy operating at the natural rate of output, and the required path of the short-term nominal interest rate is given by $i_t = r_{Nt} + \bar{\pi}$. By adjusting the short-term nominal interest rate to reflect changes in the natural rate of interest, the economy's actual output path completely reflects movements in the natural output path.

In response to the expectational shock, we have observed that the Wicksellian natural rate of interest drops immediately and may steadily decline between announcement and actual implementation. An inflation targeting central bank, therefore, would implement a whole series of interest rate cuts ahead of t_1 in such a scenario. One factor behind the sequence of interest rate cuts adopted by the Fed in 2001 to mid-2004 may, therefore, have something to do with the depressed stock market brought about by the market's expectations of the future tax shock.¹²

¹²The future pension benefit explosion that we study in section 4 would have similar consequences.

Since there is a lower bound for the nominal interest rate, we see that if the natural rate of interest falls too far into negative territory, it could cause a liquidity trap problem. (See Gauti Eggertsson and Woodford (2004) for a discussion of how optimal monetary and fiscal policy should be conducted when a real disturbance brings the economy into a liquidity trap.)

3. Tax Cuts with Gradual Welfare Payment Adjustment

A. Conditions for Fiscal Sustainability

We now turn to study the effects of future and immediate tax cuts without the sunset provision, that is, future and immediate permanent tax cuts. This immediately raises the question of whether the proposed fiscal changes are sustainable in the sense that the debt-income ratio will not explode when account is taken of all the general-equilibrium effects resulting from the proposed tax cuts (see Blanchard, et al. (1990) for the concept of the sustainability of fiscal policy). We ensure fiscal sustainability by requiring that, for a given tax rate, the primary (non-interest) surplus is made a sufficiently responsive positive function of the debt-income ratio. Conversely, the primary deficit is reduced sufficiently as the debt-income ratio rises in order to retain solvency. We show that it is not possible to launch a fiscally sustainable permanent tax cut while keeping welfare spending constant as a share of GDP. The extent to which policy-makers must reduce the primary (non-interest) deficit, such as through cutting government entitlement programs, depends upon how much a decrease in asset prices decreases the tax revenue to GDP ratio compared to how much it lowers government interest debt burden. In a recent study, Henning Bohn (1998) found significant evidence that for the US the primary surplus taken as a ratio to GDP is an increasing function of the debt-GDP ratio (after controlling for war-time spending and cyclical fluctuations) for 1916-1995 and various subperiods. His estimates of the increase in (non-interest) primary surplus in response to a unit increase in the debt-income ratio range from 0.028 to 0.054 for various sub-periods, with 0.054 for the whole period.

Making the assumption of gradual welfare payment adjustment in response to growing budgetary deficits, it turns out to be convenient to examine a dynamic system where asset price, q , and per capita debt, D , are normalized by GDP per business asset. Defining then $\hat{D} \equiv D/C^s$, $\hat{y}^g \equiv y^g/C^s$, and $\hat{q} \equiv q/C^s$, where \hat{q} has the interpretation of real asset price normalized by GDP per business asset (which is here a customer) or the stock market capitalization as a ratio to GDP, and \hat{D} gives the debt-GDP ratio, we modify (5) and (20), respectively, to yield

$$\dot{\hat{D}} = \mu(\hat{q}, \hat{D})\hat{D} + \hat{y}^g - \hat{T}(\hat{q}; \tau), \quad (29)$$

$$\frac{\dot{\hat{q}}}{\hat{q}} = \mu(\hat{q}, \hat{D}) - \frac{[1 - (\phi(\hat{q}))^{-1}]}{\hat{q}}, \quad (30)$$

where $r - e_q[\dot{q}/q] = \rho + \theta(\theta + \rho)[\hat{q} + \hat{D}] \equiv \mu(\hat{q}, \hat{D})$, which gives the interest rate net of GDP growth. We see that it is increasing in \hat{q} and \hat{D} so $\mu_{\hat{q}} > 0$ and $\mu_{\hat{D}} > 0$. Note also that the tax revenue to GDP ratio is given by $\tau(1 + \tau)^{-1}V^f(q; \tau) = \tau(1 + \tau)^{-1}(\phi(\hat{q}))^{-1} \equiv \hat{T}(\hat{q}; \tau)$; $\hat{T}_{\hat{q}} > 0$, $\hat{T}_{\tau} > 0$. To obtain (29) and (30), we have also used the relationships $\dot{\hat{q}}/\hat{q} \equiv [1 - e_q](\dot{q}/q)$, and $m \equiv (v^f)^{-1} = \phi(\hat{q})$.

To find the condition for fiscal sustainability, we first take note that if the primary surplus (normalized by GDP) given by $\hat{T} - \hat{y}^s$, where $\hat{T} \equiv \tau(1 + \tau)^{-1}(\phi(\hat{q}))^{-1}$ is independent of the debt-GDP ratio, \hat{D} , the steady state of the linearized system given by (29) and (30) is globally unstable so any increase in the debt-GDP ratio above its steady-state value will cause it to increase without bound. Therefore, if the entitlement spending to GDP ratio is held invariant to changes in the debt-GDP ratio, a permanent tax cut would be fiscally unsustainable.¹³ The proof is in appendix A.4.

To achieve fiscal sustainability, we make the (normalized) primary surplus an increasing function of \hat{D} , as the empirical work of Bohn (1998) suggests has been

¹³We would also point out that in the Ricardian equivalence case where $\theta = 0$, the interest rate net of GDP growth is simply given by a parameter, ρ , and so is independent of \hat{q} and \hat{D} . In this case, we would replace the function μ in (29) and (30) by the parameter, ρ . We arrive at the same conclusion that if entitlement spending taken as a ratio to GDP is held invariant to changes in the debt-income ratio, fiscal policy is also unsustainable in the Ricardian equivalence case.

the US practice in the past, so we let entitlement spending as a ratio to GDP decline as the debt-income ratio increases and write $\hat{y}^s = \Phi(\hat{D})$; $\Phi'(\hat{D}) < 0$ since we want to keep τ as a policy parameter. In appendix A.5, we show that the extent to which entitlement spending as a ratio to GDP has to shrink as the debt-GDP ratio rises depends on the extent to which tax revenue to GDP ratio declines relative to interest cost savings when asset prices decline.

In the empirically relevant case where a depressed stock market leads to rising debt-income ratios, as the implied collapse in labor demand leads both to a decline in employment as well as wage earnings so tax revenue falls at given tax rates and enlarges the fiscal deficit despite interest cost savings, we find that in order to achieve saddle-path stability, and hence achieve fiscal sustainability in response to a tax cut, both stationary loci must be negatively sloped in such a way that

$$\left| \frac{d\hat{q}}{d\hat{D}} \Big|_{\dot{\hat{D}}=0} \right| > \left| \frac{d\hat{q}}{d\hat{D}} \Big|_{\dot{\hat{q}}=0} \right|.$$

We show this case in Figure 4.

B. *Effects of Future and Immediate Tax Cuts Without Sunset Provision*

We now establish two propositions in the case of sustainable fiscal policy under the assumption that depressed asset prices lead to a rising debt-GDP ratio (at given tax rates) as the loss in tax revenue exceeds any interest cost savings. The first concerns a future permanent tax cut; the second concerns an immediate permanent tax cut.

PROPOSITION IV: Suppose that the economy is initially in a stationary state with $(\hat{D}_{ss}^0, \hat{q}_{ss}^0)$. At $t_0 = 0$, there is an announcement that at $t_1 = T$, the wage income tax rate will be cut permanently from τ^0 to τ^1 , $\tau^1 < \tau^0$. This leaves the real asset price normalized by GDP per business asset (\hat{q}) permanently depressed notwithstanding the positive supply-side effect so employment could either contract or expand in the long run. Asset prices fall between announcement and implementation with the result that employment and output contract between t_0 and t_1 and the debt-GDP ratio steadily rises throughout.

We observe from (30) that the stationary \hat{q} locus does not shift in response to a tax cut. On the other hand, the stationary \hat{D} locus shifts up since, at a given debt-income ratio and a given size of welfare spending relative to GDP, higher asset prices are required to generate additional tax revenues to offset the direct loss of tax revenue owing to the tax cut. The result of the curve shift is that the new stationary level of \hat{q}_{ss} is lower and \hat{D}_{ss} is higher at $(\hat{D}_{ss}^1, \hat{q}_{ss}^1)$. Intuitively, the pile up of debt resulting from the tax cut leads to higher short real rates of interest in the new stationary state. As a result, the new stationary \hat{q}_{ss} must be reduced to generate a higher market rate of return to match the higher interest rate. To infer what happens to employment, L , we refer to lemma 2, where we have the result that L is increasing in \hat{q} through its influence on markups and decreasing in τ through its supply side influence. Although employment in the new stationary state expands on account of reduced tax rates, it contracts on account of depressed stock market capitalization as a ratio to GDP, a depression that is brought about by a swollen debt-income ratio and resulting higher future short real interest rates.

How does the market respond today in anticipation of the prospective permanent tax cut? We leave to appendix A.6 to prove that for small changes, the Feldstein-Rubin-Summers consequences of higher future short rates overwhelms the positive supply-side effect of a lower wage income tax rate. As a result, stock market capitalization as a ratio to GDP (\hat{q}) drops in anticipation of the future permanent tax cut and current economic activity declines. We illustrate the economy's response to the future permanent tax cut in Figure 5. We show that the prospective tax cut leads to an immediate decline in \hat{q} from point A to point B at t_0 , and continues to fall further from point B to point C , which it reaches at t_1 , the time of implementation.¹⁴ Hence, employment contracts between announcement and implementation. At t_1 , both the asset price as well as the level of debt cannot jump but the tax cut itself leads to an increase in output supply through the positive supply-side effect, which causes $\hat{q} \equiv q/\Omega(q; \tau)$ and $\hat{D} \equiv D/\Omega(q; \tau)$ to

¹⁴Note that at t_0 , the level of debt does not jump while the fall in asset price leads to a fall in output so the debt-GDP ratio rises. Consequently, point B lies south-east of point A .

drop equiproportionately so moving from point C to point D along the ray OX . From that point onwards, the economy travels along the negatively-sloped saddle path to reach a new stationary state with higher debt-GDP ratio and lower market capitalization as a ratio to GDP.

PROPOSITION V: Suppose that the economy is initially in a stationary state with $(\hat{D}_{ss}^0, \hat{q}_{ss}^0)$. At $t_0 = 0$, there is an immediate cut in the wage income tax rate that is understood by all to be permanent, going from τ^0 to τ^1 , $\tau^1 < \tau^0$. The real asset price normalized by GDP per business asset (\hat{q}) immediately drops and continues to fall until it reaches a permanently depressed level. The debt-GDP ratio steadily rises throughout. The immediate impact on employment is ambiguous but it steadily worsens from then on until it reaches a plateau that can either be above or below the original reference path.

In appendix A.6, we show that the extent of the initial drop of the stock market capitalization as a ratio to GDP (\hat{q}) is inversely related to how far away in time the tax cut will be implemented, T . In other words, the depressing effect on \hat{q} is stronger the earlier is the future event. Recall that L is increasing in \hat{q} and decreasing in τ . So hours worked is pulled up by the direct Marshallian impact of the (immediate) decrease of τ on the demand wage; this is the effect relied on by the supply siders. But hours worked is pushed down by the (proportionately greater) drop in \hat{q} whose depressing effect on the demand wage (v^h) through its impact on markups dominates the resulting wealth effect on labor supply. (The increased exercise of monopoly power has unambiguously the net effect of reducing economic activity.) In terms of Figure 5, which we used to illustrate the case of a delayed permanent tax cut, \hat{q} immediately jumps down to a point on the new saddle-path associated with the new stationary state $(\hat{D}_{ss}^1, \hat{q}_{ss}^1)$.¹⁵

4. Prospective Pension Problem in the Model with Retirement

¹⁵Note that with immediate implementation at t_0 , the level of debt does not jump while the fall in asset price leads to a fall in output so the debt-GDP ratio rises. Consequently, the point on the new saddle-path lies south-east of point A .

We introduce mandatory retirement into our basic model so that an agent only works for a length of time t_W after he is born. Hence an agent born at time s receives a wage income at time t of $v^h(t)l(s, t)$ if $t < s + t_W$ and he receives a time-invariant retirement benefit of b^g if $t \geq s + t_W$. The share of retirees in the population at time t is equal to $\int_{-\infty}^{t-t_W} \theta e^{-\theta(t-s)} ds = e^{-\theta t_W}$, and the share of the population in the workforce is $1 - e^{-\theta t_W}$. Supposing that there are only payroll taxes imposed to finance the retirement benefits, the only form of government expenditure here, the government budget constraint is simply given by

$$(1 - e^{-\theta t_W})\tau L v^h = e^{-\theta t_W} b^g, \quad (31)$$

where $(1 - e^{-\theta t_W})L$ is total hours worked and L is hours per worker.

Following Nielsen (1994), we can show that the equation replacing (7) that gives the rate of growth of consumption demand is now (see appendix A.7 for details):

$$\frac{\dot{C}}{C} = (r - \rho) - \frac{\theta(\theta + \rho)q}{C} + e^{-\theta t_W} \left[\frac{\theta(\theta + \rho) \int_t^{t+t_W} (v_\kappa^h L_\kappa - b^g) e^{-\int_t^\kappa r_\nu d\nu} d\kappa}{C} \right]. \quad (32)$$

Noting that $v^h L \equiv v^f L - \tau v^h L$ and using (31), we have $v^h L - b^g = v^f L - (1 - e^{-\theta t_W})^{-1} b^g$, which we can use in (32) to obtain

$$\begin{aligned} \frac{\dot{C}}{C} &= (r - \rho) - \frac{\theta(\theta + \rho)q}{C} \\ &+ e^{-\theta t_W} \left[\frac{\theta(\theta + \rho) \int_t^{t+t_W} [v_\kappa^f L_\kappa - (1 - e^{-\theta t_W})^{-1} b^g] e^{-\int_t^\kappa r_\nu d\nu} d\kappa}{C} \right]. \end{aligned} \quad (33)$$

Equating consumption demand to supply, noting that $\dot{C}^s/C^s = e_q[\dot{q}/q]$, and using (33) together with (18) and (19), we obtain:

$$\begin{aligned} [1 - e_q] \frac{\dot{q}}{q} &= \rho + \frac{\theta(\theta + \rho)q}{\Omega(q; \tau, e^{-\theta t_W})} - \frac{[1 - V^f(q; \tau, e^{-\theta t_W})]\Omega(q; \tau, e^{-\theta t_W})}{q} \\ &- e^{-\theta t_W} \left[\frac{\theta(\theta + \rho) \int_t^{t+t_W} [V^f(q_\kappa; \tau, e^{-\theta t_W})\Omega(q_\kappa; \tau, e^{-\theta t_W}) - (1 - e^{-\theta t_W})^{-1} b^g] e^{-\int_t^\kappa r_\nu d\nu} d\kappa}{\Omega(q; \tau, e^{-\theta t_W})} \right], \end{aligned} \quad (34)$$

where we note that with retirement introduced into the model, $C^s = \Omega(q; \tau, e^{-\theta tw})$ and $v^f = V^f(q; \tau, e^{-\theta tw})$. In appendix A.7, we show that an increase in the share of retirees in the population contracts C^s and increases v^f given q and τ . The reason an increase in the share of retirees in the population, $e^{-\theta tw}$, contracts output and raises unit cost at given τ and q is that it acts to shift the aggregate labor supply curve to the left so moving up the downward-sloping aggregate labor demand curve in the $((1 - e^{-\theta tw})L, v^f)$ plane.

Starting from the assumption of the basic model that the share of retired people in the population is initially zero, let us consider the effect of a sudden anticipation that at future time t_1 , there will occur a sudden increase in the share of retired people in the population. It is straightforward to see from the budget constraint in (31) that the tax rate will then have to be raised to pay for benefits to the retired, hence depressing asset prices at t_1 through lowering current earnings on business assets. In addition, there is another channel acting to depress asset prices. With more people retired, the size of the labor force shrinks and that also acts, at given q , to raise unit cost and thus to reduce quasi-rents, and as a consequence asset prices fall.

To analyze the effects on the whole path of asset prices, interest rates, and employment, it is convenient to refer to Figure 6, which plots the LHS of (34), $(1 - e_q)\dot{q}/q$, against q with the share of retirees in the population initially equal to zero. The initial stationary level of q is equal to q_{ss}^A . The increase in the wage income tax rate and increase in the share of retirees in the population both shift the positively-sloped schedule to the left, leading to a lower level of stationary q via the two channels described above. However, the increase in the share of retirees in the population also works through the presence of the last term on the RHS of (34) to attenuate the leftward shift of the positively-sloped schedule. We observe that this last term, arising from introducing retirement into the original Blanchard (1985) model, acts like a subtractor from non-human wealth, which leads to a fall in the consumer's required rate of interest. This effect tends to shift the positively-sloped schedule to the right. Overall, if a sufficiently big rise in tax rates is required to finance retirement benefits in the future and the shrinkage

of the labor force is significant enough, the net effect is a leftward shift of the positively-sloped schedule. As there can only be one initial jump in q , i.e. at t_0 , and the economy must be at the value of q that makes $\dot{q} = 0$ at t_1 (namely, at q_{ss}^C in Figure 6), the path taken by the economy after the initial jump is given by BC along the old schedule in Figure 6. Upon receiving the news of a future bulge in retirement benefits, therefore, asset prices fall immediately from q_{ss}^A to q^B , and the expected rate of change of q , i.e., the expected capital gains term, goes from zero to a negative value as market participants form a rational expectation of further asset price declines. In fact, asset prices continue to decline at an increasing rate until t_1 when \dot{q} jumps up from a negative value (on the old schedule) to zero (on the new schedule). The stock market value cannot jump at the time of implementation, t_1 , to avoid the possibility of making anticipated infinite rates of capital gain or loss. The implied paths taken by the current short-term real interest rate and employment, which equals output, are given in the following proposition:

PROPOSITION VI: With the sudden news at t_0 that there will be a rise in the share of retirees in the population producing a bulge in retirement benefits that have to be paid for by permanently higher wage income tax rates at future time t_1 , there is an immediate drop in the short real rate of interest, after which it falls steadily between t_0 and t_1 . At t_1 , it jumps to a plateau whose level lies below its pre-shock level. Employment, which equals output, immediately drops and continues its decline until, at t_1 , there is another abrupt drop to reach a lower plateau as the tax rate is increased to finance retirement benefits.¹⁶

¹⁶That the new stationary short real rate of interest is below its pre-shock level can be seen as follows: in the stationary state, the short rate of return is inversely related to the stock market capitalization as a ratio to GDP, $\hat{q} \equiv q/\Omega$. (The term $1 - V^f$ can be re-expressed as $(m - 1)/m$, where one reduced-form expression of the markup is $m = \phi(\hat{q})$.) From (34), an increase in the share of retirees in the population, $e^{-\theta t w}$, from an initial value of zero to a positive number requires an increase in \hat{q} to keep \dot{q}/q equal to zero. Consequently, the short real rate of interest in the new stationary state must be below its pre-shock level. That the new stationary \hat{q} is higher while the corresponding new stationary q is lower means that output supply falls by proportionately more than q falls

5. Concluding Remarks

The supply-siders' thesis that employment activity is predominantly driven by changes in the tax wedge is empirically not the great success that is widely supposed. Casey Mulligan (2002) attempts to establish the part played by public finance distortion in the movements of the supply of labor of American workers over nearly a century, 1889-1996, using the familiar neoclassical model of labor-leisure choice. This leads to the first-order condition $MRS(C, \bar{L} - L) = v^h$, where MRS is the marginal value of time, which is increasing in both consumption C and hours worked L , and v^h is the after-tax hourly wage. The latter is related to the firms' demand wage, v^f , and to the wage income tax rate, τ , by $v^h \equiv (1 + \tau)^{-1}v^f$ and, invoking pure competition, v^f is equated to the marginal product of labor, MPL , which is increasing in capital stock, K , and decreasing in L . Consequently, $MRS(C, \bar{L} - L) = (1 + \tau)^{-1}MPL$.¹⁷ It follows that, given C and K , an increase in τ requires a decrease of L . Mulligan observes from his empirical exercise that the distortion, defined as the gap $(MPL/MRS) - 1$, is indeed correlated with the fiscal wedge, defined as $(1 + \tau) - 1$. But he further observes that the distortions measured in the Great Depression, the Second World War and the 1980s are not well explained by the current fiscal wedge. He concludes that the within-decade aggregate fluctuations in consumption, wages, and labor supply are hard to reconcile with this simple competitive equilibrium model of labor supply and demand.

The present paper has brought in an additional factor: the role of the shadow price attaching to the firm's business asset—in the illustrative model used here, the customer. The firms' inter-temporal perspective makes their current markup via the negative supply side effect of higher tax rates and the contraction of labor force through retirement effect.

¹⁷In principle, the consumption tax rate, say τ_c , also appears on the right-hand side, so $MRS(C, \bar{L} - L) = [(1 - \tau_c)/(1 + \tau)]MPL$. However, with the assumed functional form in Mulligan (2002) as well as in our paper, it is possible to write $MRS(C/(1 - \tau_c), \bar{L} - L) = (1 + \tau)^{-1}MPL$, so that when the measure of consumption used is inclusive of consumption taxes, we do not expect consumption taxes to create a gap between measured MRS and MPL .

m inversely related to q , the shadow price that firms attach to a customer, and also an inverse function of the wage income tax rate τ , $m = \psi(q; \tau)$; $\psi_q < 0$ and $\psi_\tau < 0$. In this imperfectly competitive framework the analogue to Mulligan's labor-equilibrium relationship is $MRS(C, \bar{L} - L) = (1 + \tau)^{-1}[\psi(q; \tau)]^{-1}MPL$, in which an increase of q pulls up the right-hand side (i.e., v^h) and thus induces an increase in hours supplied. The distortive gap, MPL/MRS , is now driven by $(1 + \tau)\psi(q; \tau)$. Given q , an increase of τ increases the distortive gap through the $(1 + \tau)$ term but indirectly decreases the gap through the markup term. Because of these two offsetting effects of a change in τ on the distortive gap, one cannot expect to understand well the medium-term responses of employment (here hours) to wage income tax changes without considering the asset price responses to such shocks.¹⁸ For example, in our framework, an increase in the tax rates introduced in the mid-1990s under the Clinton administration may have helped to *boost* employment, contrary to what would be predicted by Mulligan's competitive equilibrium framework, precisely because the expectation of a decline in the debt-GDP ratio boosted asset prices and thus reduced firms' markups. Our channel, from tax increase to the demand for labor, through which a pay-down of the public debt (relative to income) lowers future short rates and elevates asset prices, including the shadow price of customers, q , whichever way it affects short rates, could have pulled up v^h and L more than the contractionary supply-side effect from the increase of τ pushed them down.

We believe that our framework, by introducing a role for asset prices in the

¹⁸With a utility function such as $\log C + [B/(1 - \eta^{-1})](\bar{L} - L)^{1-\eta^{-1}}$, where η^{-1} gives the constant inter-temporal elasticity of substitution of leisure, an increase in $(1 + \tau)m$ brings about a smaller increase in the demand for leisure at any given level of consumption demand the smaller η^{-1} is. Robert Hall (1997) uses the value $\eta^{-1} = 0.6$ in his numerical simulation while Rotemberg and Woodford (1992) use $\eta^{-1} = 1.3$ in their baseline simulation. The latter cite studies showing that estimated values of the inter-temporal elasticity of substitution of leisure for males are typically near zero while many studies obtain estimates for female workers that fall within the range 0.5-1.5 with two being the upper bound. Our theoretical model in the text assumes $\eta^{-1} = 1$ as also is done by Edward Prescott (2002) in his Ely lecture.

fundamental labor-equilibrium condition, also helps to throw light on some puzzles found by Mulligan in his study of labor-leisure distortions at medium-term frequencies within the competitive equilibrium framework. For example, he found that tax distortions alone could not quantitatively explain the gap between MRS and MPL during the Great Depression. “What drove a 40% [gap] between marginal product and value of time?” he asks. We answer that the part of the increase of MPL/MRS during 1929-33 that cannot be explained by an increase in tax rate is attributable to a decline in asset prices, such as a depressed value placed on a customer, which increases firms’ markups. The recent paper by Varadarajan Chari, et al. (2002) similarly fails to incorporate a role for asset prices in explaining the gap between MRS and MPL during the Great Depression.

Mulligan also found that despite an increase in federal tax rates from practically zero to more than 20% during World War II, leisure during the second world war is lower than implied by the labor-equilibrium condition given by the competitive equilibrium model. Our model suggests that this may be attributable to the fact, highlighted by Mankiw (1985), that the real interest rate was low during the war. Theoretically, the low wartime real interest rate can be explained either by Mankiw’s own introduction of consumer durables into the standard neoclassical growth model or by the introduction of the differences in relative labor intensiveness in the consumer and capital-good producing sectors (see Phelps, 1994). In the former case, an increase in government spending on the aggregative good, which drives capital used in the domestic sector into the commercial sector so reducing the marginal product of capital, and in the latter case, an increase in government spending on the relatively labor-intensive capital good, reduces the real interest rate, and raises asset prices, including the shadow prices firms place on their operating business assets, such as their customers. This counteracts the distortionary

effects of increased federal income tax rates.¹⁹

Finally, Mulligan pointed out that the falling distortive gap during the Reagan years could not be fully explained by the decrease in federal labor income tax rates in the 1980s. Although the Feldstein-Rubin-Summers channel would imply that the stock market should decline if agents formed expectations of a build-up of public debt, authors such as Blanchard and Summers (1984) have argued that in the early eighties, the fiscal expansion in the US was offset by the fiscal contraction in the other major OECD countries so that the aggregate inflation-adjusted deficit as a percent of the group's GNP did not change significantly. They pointed to the strong stock market performance in the 1980s and the strong behavior of investment in the face of increased real interest rates as evidence of a favorable shift in expected profitability. If that inference is correct, the consequent rise in the value placed on customers would cause markups to fall, and thus reduce the distortive gap beyond what was brought about by reduced wage income tax rates.

How well does our model help to explain recent US experience starting from the internet boom from 1995/96? If future prospects of an upward lift to productivity made possible by technological advance in information and communications technology caused the boom, their materialization with a string of outsized productivity gains starting in 2001 should have stabilized the stock market at late-1990s levels and brought employment down for a soft landing at its pre-boom 1996 level. But that was not exactly what occurred. Instead, the stock market suffered a deep decline until regaining in 2003 and 2004 its 1998 level; correspondingly, the unemployment rate, participation rate and work week all overshoot considerably their 1996 marks before achieving a mixed recovery by 2004: unemployment back to its 1996 level but participation and hours still markedly below their 1996 levels. The explanation that the analysis here offers (on top of others such as overinvest-

¹⁹Rotemberg and Woodford (1992) adduce evidence in support of a decline in markups when government purchases increase, including during the two world wars, but they use a different model of dynamic markups from ours. They also acknowledge that the imposition of price controls during World War II places a limitation on one's interpretation of the data.

ment and 9/11) is that the tax cut in 2001 with its backloading feature along with mounting awareness of the bulge in retirement benefits threatening tax rates even above pre-cut levels caused a drop in the stock market starting in 2001, which reduced the demand wage and hours per worker. The low short rates of real interest that we have observed since 2001 are a further consequence. From our structuralist perspective, it comes as no great surprise that the buildup in 2001-2003 of fiscal stimulus and central bank interest rate cuts did not succeed by 2004 in staving off the economy's fall back to pre-boom levels or worse. The reduction of certain marginal tax rates may have had structural impacts every bit as effective as econometric estimates have led us to believe yet the former impacts may have failed to outweigh appreciably, if at all, their perverse impact in worsening the already alarming prospects for future fiscal deficits.

APPENDIX

A.1. To prove that the stationary level of natural interest is increasing in D , we note that setting $\dot{q} = 0$ in (18), we obtain a negatively-sloping relationship relating the RHS of (18) to q (see Figure 2). Next, turning to (19), setting $\dot{q} = 0$ and using assumption 1 gives a positively-sloping relationship relating the RHS of (19) to q . Increasing D shifts the positively-sloping relationship upwards, hence leftwards. The result is that the value of q corresponding to the intersection declines. Substituting the lower value of q back into (18) with $\dot{q}/q = 0$, we see that the new stationary level of natural interest is now higher.

A.2. Technically, the trace of the 2×2 matrix associated with the linearized dynamic system given below is positive, and the determinant is also positive:

$$[\dot{D} \quad \dot{q}]' = A[D - D_{ss} \quad q - q_{ss}]',$$

where $[\cdot \cdot]'$ denotes a column vector, the system (23) and (24) is linearized around the stationary-state values, D_{ss} and q_{ss} , and the 2×2 matrix A contains the following elements:

$$\begin{aligned} a_{11} &\equiv \Upsilon + \Upsilon_D D_{ss}, \\ a_{12} &\equiv \Upsilon_q D_{ss} - T_q, \\ a_{21} &\equiv \Upsilon_D q_{ss}, \\ a_{22} &\equiv q_{ss} \left[\Upsilon_q + \frac{[1 - V^f]}{(q_{ss}/\Omega)^2} \frac{d(q_{ss}/\Omega)}{dq} + \frac{V_q^f}{(q_{ss}/\Omega)} \right]. \end{aligned}$$

We can readily check that $a_{11} > 0$, $a_{21} > 0$ and $a_{22} > 0$ while a_{12} can either be positive or negative. (Consequently, the trace of A ($\text{Tr}(A)$) is clearly positive.) The sign of a_{12} depends upon the relative influence of a change in q on the tax revenue on the one hand, and the interest cost on the other hand. If a rise in the asset price raises the tax revenue by more than it raises the interest cost so a booming stock market leads to declining debt or, conversely, a depressed stock market leads to rising debt, then a_{12} is negative, and the determinant of A ($\text{Det}(A)$), equal to $a_{11}a_{22} - a_{21}a_{12}$, is clearly positive. In the alternative case when a_{12} is positive,

we can check that $(a_{11}/a_{12}) > (a_{21}/a_{22})$ so once again the determinant of A is positive. Therefore, the system represented by (23) and (24) is globally unstable. If the eigenvalues associated with the system represented by (23) and (24) are denoted λ_1 and λ_2 , we have that $\lambda_1\lambda_2 = a_{11}a_{22} - a_{12}a_{21} > 0$.

A.3. The matrix associated with the equation system given by (25), (28) and (20) evaluated around the steady state has the elements: $a_{11} = 0, a_{12} = -\delta^2, a_{13} = 0, a_{21} = a - 1, a_{22} = b, a_{23} = -\Upsilon_q, a_{31} = 0, a_{32} = 0, a_{33} = \Pi$, where

$$\Pi \equiv q_{ss}[\Upsilon_q + \frac{[1 - V^f]}{(q_{ss}/\Omega)^2} \frac{d(q_{ss}/\Omega)}{dq} + \frac{V_q^f}{(q_{ss}/\Omega)}] > 0.$$

The determinant of the matrix is given by $\text{Det} \equiv \delta^2\Pi(a-1)$, which is positive if and only if the Taylor principle holds, that is, $a > 1$. The trace is given by $\text{Tr} \equiv b + \Pi$, which is positive. By an application of Routh theorem, the number of roots of the polynomial, $-\gamma^3 + \gamma^2\text{Tr} - [b\Pi + \delta^2(a-1)]\gamma + \text{Det}$, with positive real parts will be equal to the number of variations of sign in the scheme $-1 \quad \text{Tr} \quad - [b\Pi + \delta^2(a-1)] + [\text{Det}/\text{Tr}] \quad \text{Det}$. We can readily check that $-[b\Pi + \delta^2(a-1)] + [\text{Det}/\text{Tr}] = -[(b + \Pi)b\Pi + b\delta^2(a-1)](b + \Pi)^{-1} < 0$. Consequently, there are exactly three changes in sign, implying that there are exactly three positive roots.

A.4. We prove that if entitlement spending as a ratio to GDP is left invariant to a change in the debt-GDP ratio, fiscal policy is unsustainable. Technically, the trace of the 2×2 matrix associated with the linearized dynamic system given below is positive, and the determinant is also positive:

$$[\dot{\hat{D}} \quad \dot{\hat{q}}]' = A[\hat{D} - \hat{D}_{ss} \quad \hat{q} - \hat{q}_{ss}]',$$

where $[\cdot \cdot]'$ denotes a column vector, the system (29) and (30) is linearized around the stationary-state values, \hat{D}_{ss} and \hat{q}_{ss} , and the 2×2 matrix A contains the following elements:

$$\begin{aligned} a_{11} &\equiv \mu + \mu_{\hat{D}}\hat{D}_{ss}, \\ a_{12} &\equiv \mu_{\hat{q}}\hat{D}_{ss} - \hat{T}_{\hat{q}}, \end{aligned}$$

$$\begin{aligned}
a_{21} &\equiv \mu_{\hat{D}} \hat{q}_{ss}, \\
a_{22} &\equiv \mu + \mu_{\hat{q}} \hat{q}_{ss} - (\phi(\hat{q}_{ss}))^{-2} \phi'(\hat{q}_{ss}).
\end{aligned}$$

We can readily check that $a_{11} > 0$, $a_{21} > 0$ and $a_{22} > 0$ while a_{12} can either be positive or negative. (Consequently, the trace of A ($\text{Tr}(A)$) is clearly positive.) The sign of a_{12} depends upon the relative influence of a change in \hat{q} on the tax revenue-GDP ratio on the one hand, and the interest debt burden on the other hand. If a rise in the stock market capitalization relative to GDP raises the tax revenue-GDP ratio by more than it raises the real interest cost so a booming stock market leads to declining debt-income ratios or, conversely, a depressed stock market leads to rising debt-income ratios, then a_{12} is negative, and the determinant of A ($\text{Det}(A)$), equal to $a_{11}a_{22} - a_{21}a_{12}$, is clearly positive. In the alternative case when a_{12} is positive, we can check that $(a_{11}/a_{12}) > (a_{21}/a_{22})$ so once again the determinant of A is positive. More concretely, we can show that, whether $a_{12} > 0$ or $a_{12} < 0$, we obtain $\text{Det}(A) \equiv \mu\mu_{\hat{q}}\hat{q}_{ss} + \mu_{\hat{D}}\hat{q}_{ss}\hat{T}_{\hat{q}} + [\mu - (\phi'/\phi^2)][\mu_{\hat{D}}\hat{D}_{ss} + \mu] > 0$ since $\mu_{\hat{q}} > 0$, $\mu_{\hat{D}} > 0$, $\hat{T}_{\hat{q}} > 0$ and $\phi' < 0$. Therefore, if the entitlement spending as a ratio to GDP is held invariant to changes in the debt-GDP ratio, the system is globally unstable. Any increase in the debt-income ratio above the steady-state level is bound to lead to an exploding debt-GDP ratio.

A.5. With entitlement spending as a ratio to GDP made a negative function of the debt-income ratio, we have to modify the original value of a_{11} to get $a_{11} \equiv \mu + \mu_{\hat{D}}\hat{D}_{ss} + \Phi_{\hat{D}}$. In the empirically relevant case where a depressed stock market leads to rising debt-income ratios so $a_{12} < 0$, we find that in order to achieve saddle-path stability, it will be necessary though not sufficient for \hat{y}^s to fall in response to an increase in \hat{D} so that a_{11} is negative. In other words, a unit increase in the debt-income ratio necessitates a cut in entitlement spending as a ratio to GDP that more than offsets the rise in interest costs so that the debt-income ratio actually declines. The necessary and sufficient condition for saddle-path stability, and hence fiscal sustainability in response to a tax cut with gradual welfare payment adjustment, in the case when $a_{12} < 0$ is for $-(a_{11}) > -a_{12}(a_{21}/a_{22}) > 0$. Noting

that we can write down the respective slopes of the $\dot{\hat{D}} = 0$ and $\dot{\hat{q}} = 0$ loci as:

$$\begin{aligned}\frac{d\hat{q}}{d\hat{D}}\Big|_{\dot{\hat{D}}=0} &= \frac{-[\mu_{\hat{D}} + (\mu/\hat{D}_{ss}) + \hat{D}_{ss}^{-1}\Phi_{\hat{D}}]}{\mu_{\hat{q}} - (\hat{T}_{\hat{q}}/\hat{D}_{ss})} \equiv \frac{-a_{11}}{a_{12}}, \\ \frac{d\hat{q}}{d\hat{D}}\Big|_{\dot{\hat{q}}=0} &= \frac{-\mu_{\hat{D}}}{\mu_{\hat{q}} + \hat{q}_{ss}^{-1}[\mu - (\phi(\hat{q}_{ss}))^{-2}\phi'(\hat{q}_{ss})]} \equiv \frac{-a_{21}}{a_{22}} < 0,\end{aligned}$$

we obtain saddle-path stability only if both stationary loci are negatively sloped in such a way that

$$\left| \frac{d\hat{q}}{d\hat{D}}\Big|_{\dot{\hat{D}}=0} \right| > \left| \frac{d\hat{q}}{d\hat{D}}\Big|_{\dot{\hat{q}}=0} \right|.$$

If a decline in stock market capitalization as a ratio to GDP leads to bigger cost savings for the government (as a result of a huge drop in interest debt service burden) than its loss of tax revenue (relative to GDP) so $a_{12} > 0$, then the condition for fiscal sustainability is immediately satisfied by a fiscal rule that makes \hat{y}^s fall sufficiently in response to a rise in \hat{D} to make a_{11} negative. (Referring to (29) and (30), this condition says that when $a_{12} > 0$ and $a_{11} < 0$, we are assured of saddle-path stability and the stationary locus for $\dot{\hat{D}} = 0$ is positively sloped in the (\hat{D}, \hat{q}) plane.) If $a_{12} > 0$, the condition that $a_{11} < 0$ is sufficient for fiscal sustainability but it is not necessary. If declining asset prices lead to a smaller loss in tax revenue (relative to GDP) than the government can save from a decline in interest costs so the debt-income ratio actually falls, then, in order to attain fiscal sustainability, big cuts in entitlement spending may not be required when the debt-income ratio rises so a_{11} remains positive so long as the condition $0 < a_{11} < a_{12}(a_{21}/a_{22})$ is satisfied. Referring to (29) and (30), this condition says that when $a_{12} > 0$ and $a_{11} > 0$, we obtain saddle-path stability if both stationary loci are negatively sloped in such a way that

$$\left| \frac{d\hat{q}}{d\hat{D}}\Big|_{\dot{\hat{D}}=0} \right| < \left| \frac{d\hat{q}}{d\hat{D}}\Big|_{\dot{\hat{q}}=0} \right|.$$

In summary, there are three cases where we obtain saddle-path stability. If a drop in \hat{q} leads to a greater loss in tax revenue (relative to GDP) than cost savings from a lower interest debt service burden so $a_{12} < 0$, the only way to achieve saddle-path stability is to cut entitlement spending as a ratio to GDP sharply enough to

make not only $a_{11} < 0$ but also to satisfy the condition: $-a_{11} > -a_{12}(a_{21}/a_{22}) > 0$. However, if a drop in \hat{q} leads to greater interest cost savings for the government than the amount of tax revenue lost (relative to GDP), saddle-path stability is guaranteed for a government that cuts entitlement spending as a ratio to GDP sufficiently to make $a_{11} < 0$. In this case, we have $a_{12} > 0$ and $a_{11} < 0$ so $\text{Det}(A)$ is unambiguously negative. If $a_{12} > 0$, the government can, in fact, attain fiscal sustainability without sharp cuts to entitlement spending as a ratio to GDP so long as $0 < a_{11} < a_{12}(a_{21}/a_{22})$. Letting $\lambda_1 = [\text{Tr}(A) - \sqrt{\text{Tr}(A)^2 - 4\text{Det}(A)}]/2$ be the negative root, the slope of the saddle path is given by $(\lambda_1 - a_{11})/a_{12} = a_{21}/(\lambda_1 - a_{22})$, which is unambiguously negative in all the three cases summarized here. The interested reader can proceed to draw the relevant phase diagrams corresponding to the two cases where a drop in \hat{q} leads to larger interest cost savings for the government than the tax revenue lost (relative to GDP). It is readily checked that the qualitative results regarding the effects on asset prices and employment of the tax shocks we study are similar in all three cases. The differences occur in the short-term movement of the debt-income ratio in response to asset price changes since, at any given \hat{D} , a fall of \hat{q} leads to a gradual buildup of the debt-income ratio when $a_{12} \equiv \mu_{\hat{q}}\hat{D}_{ss} - \hat{T}_{\hat{q}} < 0$ but to a gradual decrease of the debt-income ratio when $a_{12} \equiv \mu_{\hat{q}}\hat{D}_{ss} - \hat{T}_{\hat{q}} > 0$. We, however, conduct our analysis in the main text with the aid of Figure 4 and so focus on the case where $a_{12} \equiv \mu_{\hat{q}}\hat{D}_{ss} - \hat{T}_{\hat{q}} < 0$.

A.6. To establish that an announcement made at $t_0 = 0$ that the tax rate will be permanently reduced from τ^0 to τ^1 , $\tau^1 < \tau^0$, from time $t_1 = T$ onwards causes \hat{q} at t_0 to drop, we proceed as follows. We let the eigenvalues associated with the system represented in Figure 4 and (29) and (30) be denoted λ_1 and λ_2 . The fact that the system is saddle-path stable means that the product $\lambda_1\lambda_2 = a_{11}a_{22} - a_{12}a_{21} < 0$. We shall assume $\lambda_1 < 0$ and $\lambda_2 > 0$. Over the period $0 < t \leq T$, before the tax cut occurs, the solutions for \hat{D}_t and \hat{q}_t are of the form

$$\hat{D}_t = \hat{D}_{ss}^0 + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t},$$

$$\hat{q}_t = \hat{q}_{ss}^0 + \left(\frac{\lambda_1 - a_{11}}{a_{12}} \right) A_1 e^{\lambda_1 t} + \left(\frac{\lambda_2 - a_{11}}{a_{12}} \right) A_2 e^{\lambda_2 t}.$$

We note that because λ_i are eigenvalues,

$$\frac{\lambda_i - a_{11}}{a_{12}} = \frac{a_{21}}{\lambda_i - a_{22}} < 0; \quad i = 1, 2.$$

For the period $t > T$, after the tax cut has occurred, the solutions for \hat{D}_t and \hat{q}_t are

$$\begin{aligned} \hat{D}_t &= \hat{D}_{ss}^1 + A'_1 e^{\lambda_1 t}, \\ \hat{q}_t &= \hat{q}_{ss}^1 + \left(\frac{\lambda_1 - a_{11}}{a_{12}} \right) A'_1 e^{\lambda_1 t}. \end{aligned}$$

Hence after time T , \hat{D}_t and \hat{q}_t must follow the saddle path leading to \hat{D}_{ss}^1 and \hat{q}_{ss}^1 , respectively.

Assuming that at time 0, \hat{D} is at \hat{D}_{ss}^0 implies $A_1 + A_2 = 0$. At time T , since q_T and D_T cannot jump, and noting that $\hat{q} \equiv q/\Omega(q; \tau)$ and $\hat{D} \equiv D/\Omega(q; \tau)$ so \hat{q} and \hat{D} change equiproportionately at time T , we obtain

$$\begin{aligned} [\Omega(q_T; \tau^0)A_1 - \Omega(q_T; \tau^1)A'_1]e^{\lambda_1 T} + \Omega(q_T; \tau^0)A_2 e^{\lambda_2 T} &= \beta_D, \\ \left(\frac{\lambda_1 - a_{11}}{a_{12}} \right) [\Omega(q_T; \tau^0)A_1 - \Omega(q_T; \tau^1)A'_1]e^{\lambda_1 T} + \left(\frac{\lambda_2 - a_{11}}{a_{12}} \right) \Omega(q_T; \tau^0)A_2 e^{\lambda_2 T} &= \beta_q, \end{aligned}$$

where

$$\begin{aligned} \beta_D &\equiv \hat{D}_{ss}^0 \Omega(q_T; \tau^0) \left[\frac{\hat{D}_{ss}^1 \Omega(q_T; \tau^1)}{\hat{D}_{ss}^0 \Omega(q_T; \tau^0)} - 1 \right], \\ \beta_q &\equiv \hat{q}_{ss}^0 \Omega(q_T; \tau^0) \left[\frac{\hat{q}_{ss}^1 \Omega(q_T; \tau^1)}{\hat{q}_{ss}^0 \Omega(q_T; \tau^0)} - 1 \right]. \end{aligned}$$

Solving out A_1 and A'_1 , we obtain

$$A_2 = \left[\Omega(q_T; \tau^0) e^{\lambda_2 T} \right]^{-1} \left[\frac{\beta_q - \left(\frac{\lambda_1 - a_{11}}{a_{12}} \right) \beta_D}{\left(\frac{\lambda_2 - \lambda_1}{a_{12}} \right)} \right].$$

The initial response of \hat{q} at time 0 is given by

$$\hat{q}_0 - \hat{q}_{ss}^0 = \left(\frac{\lambda_2 - \lambda_1}{a_{12}} \right) A_2 = \left[\frac{e^{-\lambda_2 T}}{\Omega(q_T; \tau^0)} \right] \left[\beta_q - \left(\frac{\lambda_1 - a_{11}}{a_{12}} \right) \beta_D \right].$$

We note that $(\lambda_1 - a_{11})/a_{12} = a_{21}/(\lambda_1 - a_{22})$, which gives the slope of the saddle-path, is negative and that $\beta_D > 0$ but the sign of β_q is ambiguous. However, we now show that for small changes, $\beta_q - [(\lambda_1 - a_{11})/a_{12}]\beta_D < 0$, a condition that is satisfied if and only if

$$\left[\frac{\frac{\Omega(q_T; \tau^1) \hat{q}_{ss}^1}{\Omega(q_T; \tau^0)} - \hat{q}_{ss}^0}{\frac{\Omega(q_T; \tau^1) \hat{D}_{ss}^1}{\Omega(q_T; \tau^0)} - \hat{D}_{ss}^0} \right] - \left(\frac{\lambda_1 - a_{11}}{a_{12}} \right) < 0.$$

For small changes evaluated around the original stationary state $(\hat{D}_{ss}^0, \hat{q}_{ss}^0)$, the square bracketed term converges to the value giving the gradient of the $\dot{q} = 0$ locus at $(\hat{D}_{ss}^0, \hat{q}_{ss}^0)$, which is equal to $-a_{21}/a_{22}$. Since, as we can observe from Figure 4, the slope of the saddle-path has the smallest absolute value, that is,

$$\left| \frac{d\hat{q}}{d\hat{D}} \Big|_{\dot{D}=0} \right| > \left| \frac{d\hat{q}}{d\hat{D}} \Big|_{\dot{q}=0} \right| > - \left(\frac{\lambda_1 - a_{11}}{a_{12}} \right),$$

we have that

$$\frac{a_{21}}{a_{22}} > - \left(\frac{\lambda_1 - a_{11}}{a_{12}} \right).$$

Consequently, we establish that \hat{q}_0 drops below \hat{q}_{ss}^0 in response to the tax cut. Moreover, we find that the extent of the initial drop of \hat{q} is inversely related to how far away in time the tax cut will be implemented, T .

A.7. The individual's human capital is given by

$$h(s, t) = \int_t^{\max(t, s+t_W)} v^h(\kappa) l(s, \kappa) R(t, \kappa) d\kappa + \int_{\max(t, s+t_W)}^{\infty} b^g R(t, \kappa) d\kappa,$$

where $R(t, \kappa) \equiv \exp^{-\int_t^\kappa (r(\nu) + \theta) d\nu}$, where the two parts of the integral correspond to the working and retired phases of the individual's life. The aggregate human wealth is then given by

$$H(t) = \int_{-\infty}^t h(s, t) \theta e^{-\theta(t-s)} ds = \int_{-\infty}^{t-t_W} \int_t^{\infty} \theta e^{-\theta(t-s)} b^g R(t, \kappa) d\kappa ds$$

$$\begin{aligned}
& + \int_{t-t_W}^t \int_t^{s+t_W} \theta e^{-\theta(t-s)} v^h(\kappa) l(s, \kappa) R(t, \kappa) d\kappa ds \\
& + \int_{t-t_W}^t \int_{s+t_W}^{\infty} \theta e^{-\theta(t-s)} b^g R(t, \kappa) d\kappa ds.
\end{aligned}$$

The first integral on the RHS gives the present discounted value of benefits received by the fraction of the population that is retired at time t ; the second gives the after-tax wage income received by the working fraction of the population at time t until their retirement at $s + t_W$; and the third integral gives the benefits that the working fraction of the population at time t will receive upon their retirement.

By shifting the order of integration and differentiating, we obtain

$$\dot{H}(t) = (r(t) + \theta)H(t) - (1 - e^{-\theta t_W})v^f L + \theta e^{-\theta t_W} \int_t^{t+t_W} (v^h(\kappa)L(\kappa) - b^g) e^{-\int_t^\kappa r(\nu) d\nu} d\kappa.$$

The law of motion describing the aggregate non-human wealth accumulation is given by

$$\dot{W}(t) = r(t)W(t) + (1 - e^{-\theta t_W})v^f(t)L(t) - C(t).$$

Using $C = (\theta + \rho)(H + W)$ and $\dot{C} = (\theta + \rho)(\dot{H} + \dot{W})$, and assuming that non-human wealth is entirely in share holdings, we then obtain (32).

To prove that an increase in the share of retirees in the population, $e^{-\theta t_W}$, contracts output supply and increases unit cost, v^f , we proceed as follows: The supply wage relation we obtain after substituting the condition that consumption demand is equal to supply, and using the production function $C^s = (1 - e^{-\theta t_W})L$ is

$$v_{\text{supply}}^f = \frac{B(1 + \tau)}{\frac{\bar{L}}{(1 - e^{-\theta t_W})L} - \frac{1}{(1 - e^{-\theta t_W})}},$$

which at given total hours worked, $(1 - e^{-\theta t_W})L$, is increasing in B , τ and $e^{-\theta t_W}$. The demand wage relation is given by

$$v_{\text{demand}}^f = 1 + \frac{\eta(1)}{\eta'(1)} + \left(\frac{q}{(1 - e^{-\theta t_W})L} \right) \left(\frac{g'(1)}{\eta'(1)} \right).$$

Equating the demand wage and supply wage, we readily establish that an increase in the share of retirees, $e^{-\theta t_W}$, contracts total employment (here, total hours worked) and the unit cost, v^f . Intuitively, an increase in $e^{-\theta t_W}$ (taken alone) raises

the unit cost and reduces employment because an increase in the share of retirees in the population shrinks the productive workforce and shifts the aggregate labor supply curve leftwards.

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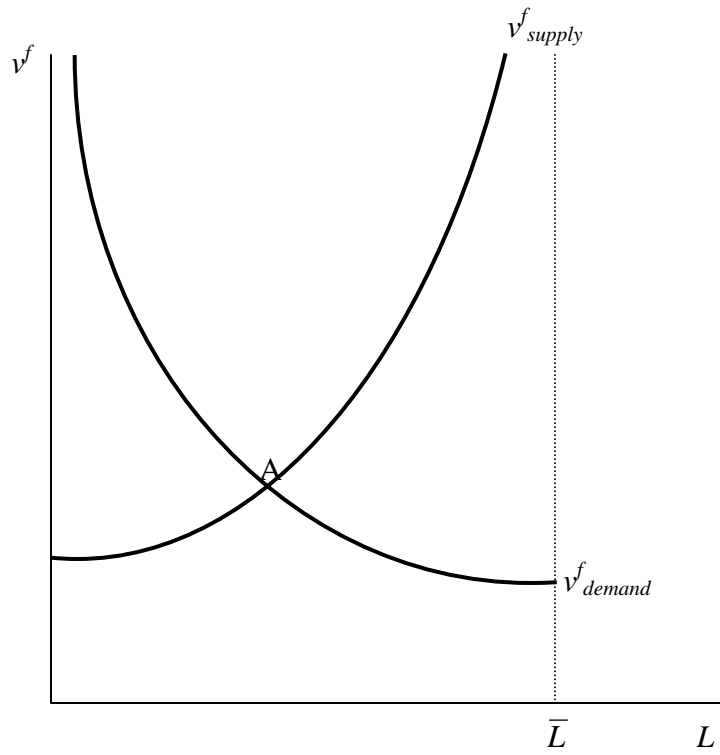


FIGURE 1. LABOR-MARKET DIAGRAM

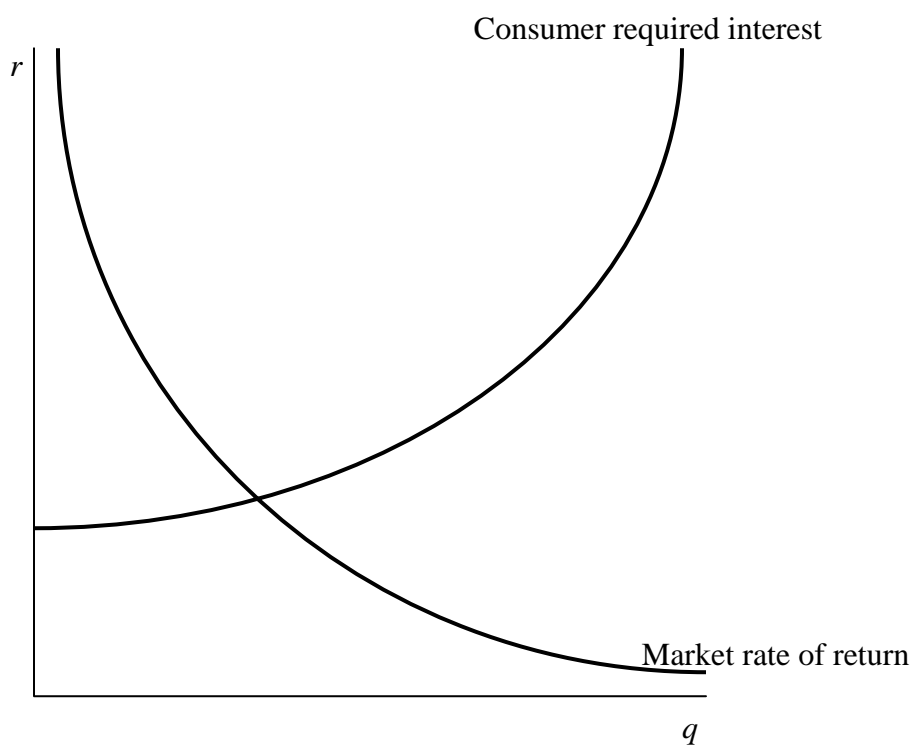


FIGURE 2. DETERMINATION OF INTEREST AND ASSET PRICE IN STATIONARY EQUILIBRIUM

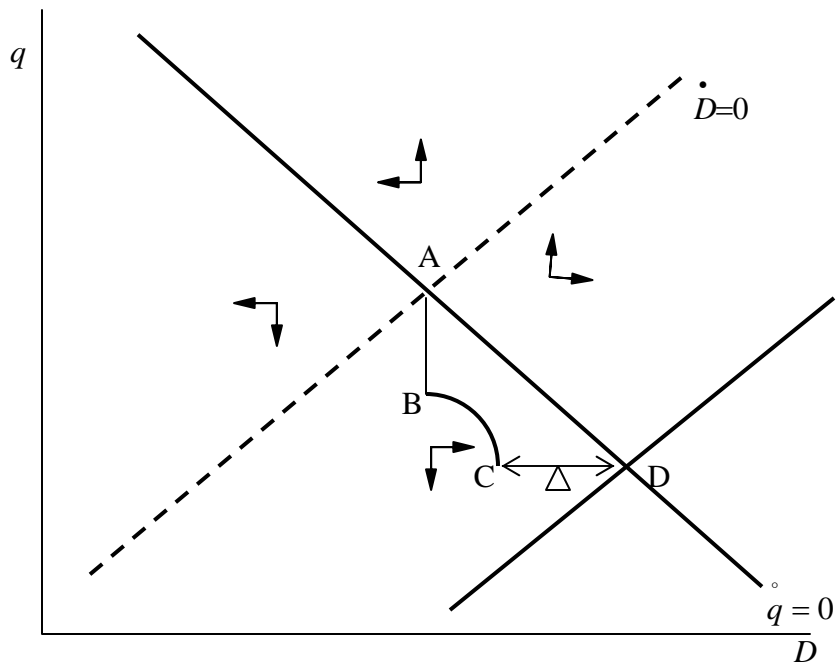


FIGURE 3. A FUTURE DEBT BOMB FINANCED BY SUBSEQUENT CUTS IN ENTITLEMENT SPENDING

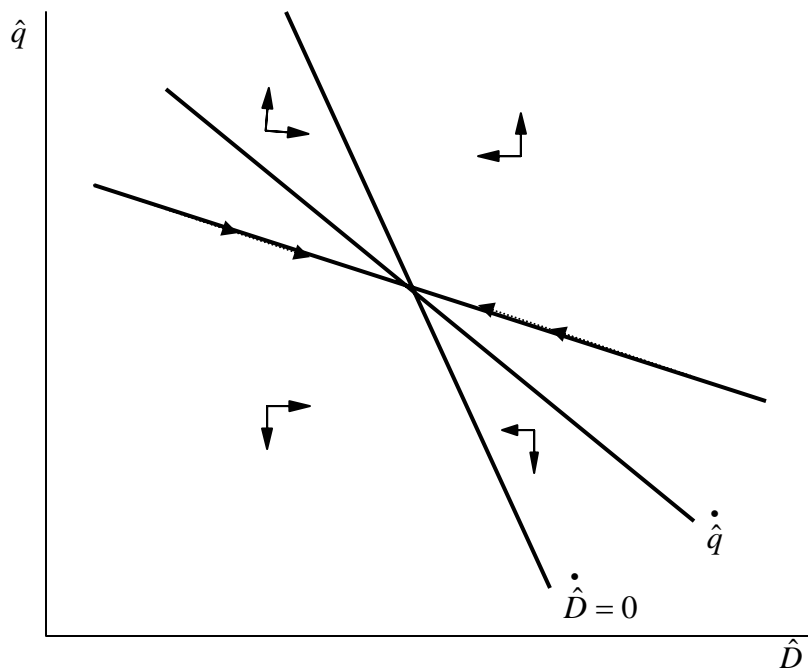


FIGURE 4. SADDLE-PATH STABILITY

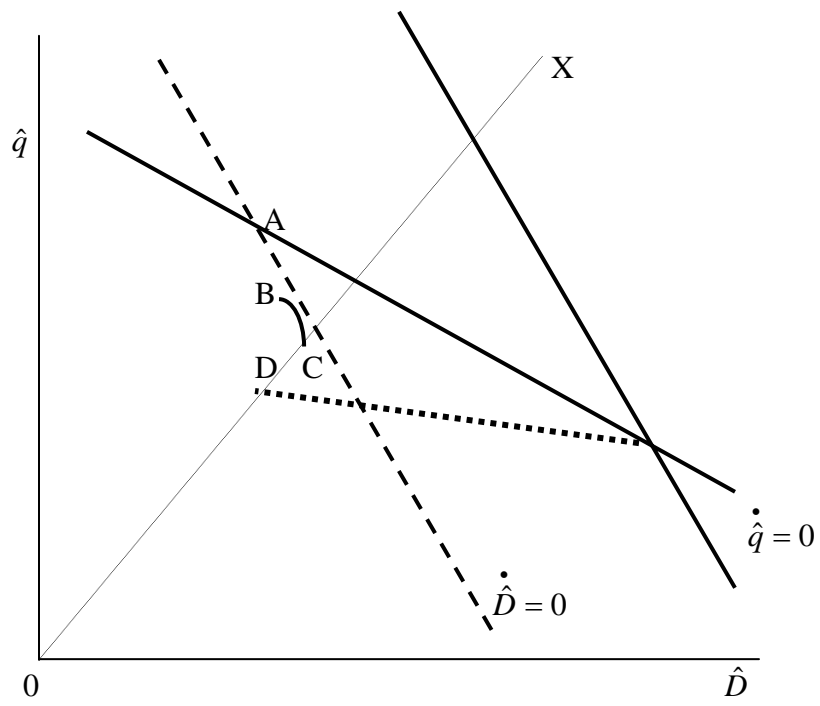


FIGURE 5. FUTURE TAX CUT WITHOUT SUNSET

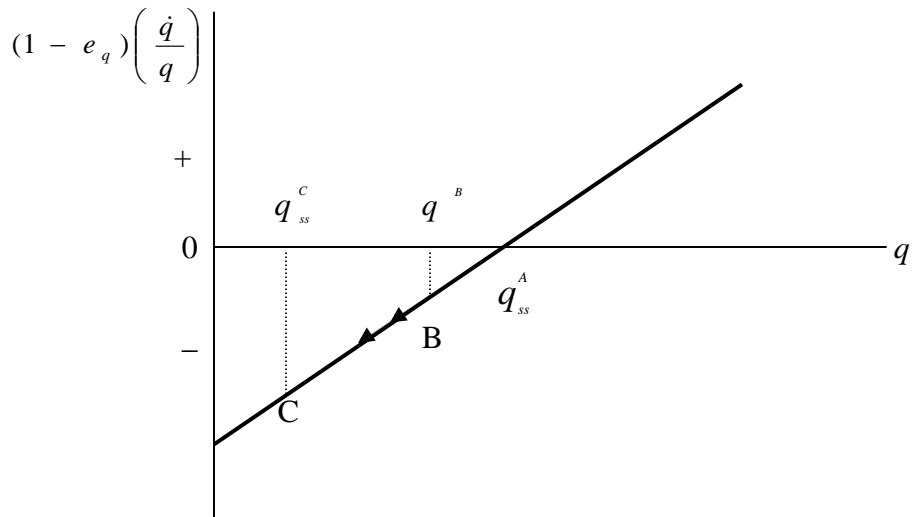


FIGURE 6 FUTURE INCREASE IN SHARE OF RETIREES IN POPULATION