Mathematical modeling in algebra textbooks at the onset of the Common Core State Standards

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Submitted in partial fulfillment of the
requirements for the Degree of Doctor of Philosophy
under the Executive Committee of the
Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2014

ABSTRACT

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Student achievement in mathematics continues to be compared internationally, with the results indicating that students in other developed countries are outperforming students from the United States. Mathematical modeling is an expectation in both the new Common Core State Standards and the Programme for International Student Assessment (PISA). This study seeks to find the differences in expectations for students in mathematical modeling between the United States and Singapore, which is one country that regularly outperforms the U.S. on international assessments. Since teachers and students regularly use textbooks for curriculum, homework, and other resources, this study compares two textbooks from the U.S. with the high school series adopted in Singapore. More specifically, the aim of this study is to compare frameworks of mathematical modeling and code to-be-solved problems in algebra textbooks using characteristics common to all frameworks. While the U.S. textbooks explicitly state which word problems address the expectation of mathematical modeling, the Singapore program does not have this attribute. So, an equivalent chapter (in objective and number of to-be-solved problems) in all three textbooks will be coded for evidence of the expectations of mathematical modeling.

The results of this study indicate that there is no standard framework for mathematical modeling, but there are multiple areas of overlap. This study found that the ratio of word problems to numerical problems was comparable in the three textbooks, although the U.S. algebra textbooks used in a one-year course had the same number of to-be-solved problems as the four-year Singapore series. Results also indicate that to-be-solved problems in the Singapore textbook series do not provide students with more explicit mathematical modeling instructions than do the U.S. textbooks.

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Acknowledgements

To God be the glory. Jeremiah 13:16

I would like to thank Dr. Pollak for his thoughtful support on this project. I learned so much from you about mathematical modeling, but also about being a considerate colleague and a thoughtful parent. I am honored to know more of your story. I thank you for being such a wonderful example, and for seeing me through to the end of this journey. I would like to also thank the department members Dr. Smith, Dr. Walker, and Dr. Karp for being wonderful mathematicians, educators, and role models. I look forward to implementing the ideas I have learned from you as I enter academia.

Thank you to my Mercy family. Thank you, Dr. Posamentier, for never giving up on me. To Bill and Meghan and Amanda and Howard: you have been so very compassionate and more than accommodating to my questions and need. I would be remiss if I did not acknowledge my Adelphi family, who have guided me and molded and cheered me on towards the place where I am today. I am forever indebted to Dr. Feige, Dr. Thornburg, Dr. Snaweart, Dr. Cohen, Dr. Linne, Professor Welsh, Professor Lambert, Provost Insler, and President Scott. I hope to now collaborate with all of you as colleagues, and to continue your legacy.

To the TC students who I've met along the way: Lydia, Mark, Ronnie, Eddy, Krystle, AJ, Heather and Susan: your collegiality and your help have been integral to my success. Thank you.

My friends have listened to me when I needed an ear, reached out to me when I needed a hand, and helped me along when I needed a push. Thank you so much, Carla, Dan and Rose.

To my family: you already know just how much I love you and how much you mean to me. For my parents: you are and always will be the reason for who I am and what I have been able to do. I wish to express much appreciation to my grandmas and my aunts and uncles and cousins for being so very supportive, for checking in regularly and helping me focus to stay the course. Thank you to my husband's family, who I now consider mine, for all of your prayers and love and acceptance. To my sons: thank you for your smiles and your hugs; your spirits have kept me smiling through an arduous process. To Matt: Thank you for caring for and loving our sons so that I could do what needed to be done. Last, and surely most importantly, I thank my God-given husband Ed. There are no words to thank you for the countless moments of gentle reinforcement and endless support. I share this accomplishment with each of you, who have always believed in me and continue to push me to greater heights.

CHAPTER I: INTRODUCTION

Need for the Study

There is no lack of data or commentary on the state of American students' mathematics achievement, particularly when compared to other developed countries (Lemke et al., 2004; Mullis, Martin, Gonzalez, & Chrostowski, 2004; NCES, 2003; Schmidt, 2002). The proficiency levels of students here in the United States have been said to evoke "both a sense of despair and of hope" (National Research Council [NRC], 2001). With this in mind, the National Governors Association and the Council of Chief State School Officers embarked on an initiative to create national standards, with the goal of increasing student achievement. As of now, according to the Web site dedicated to this body of work, www.corestandards.org, "Forty-five states, the District of Columbia, four territories, and the Department of Defense Education Activity have adopted the Common Core State Standards."

While the CCSS may have been intended to positively affect student achievement, those implementing classroom instruction at the school level rely much more heavily on their textbooks than on the standards when planning and teaching lessons (Garner, 1992). Textbooks continue to be the most influential planning tool and the key means of imparting new information to students (Ball & Cohen, 1996; Ben-Peretz, 1990; Britton, Woodward, & Binkley, 1993; Budiansky, 2001; Chandler & Brosnan, 1994; Garner, 1992; Mayer et al., 1995; Osborne, Jones, & Stein, 1985; Porter, 1989; Thomas & Fleming, 2004; Valvarde et al., 2002; Valverde & Schmidt, 1998). There has been a shift within the progressive movement towards the acquisition of conceptual knowledge from what is called "traditional teaching" that focused on computation and procedures (Schoenfeld, 2002). However, most textbooks have not changed to meet the new standards, since textbooks underscore the content with heavily teacher-directed lessons in "well over 80 percent of the textbooks used in schools" (Van de Walle, 2007, page). The assumption that standards will change what is happening in classrooms, in other words, may be false. To that end, there must be an examination of what is taught in light of the new standards (Chatterji, 2002; Nolet & McLaughlin, 2005), since when compared to textbooks from other countries, those used in the U.S. are deficient in their "focus and coherence" and do not embed "meaningful connections between the big ideas of mathematics" (Valverde & Schmidt, 1998, page). Regardless of the beliefs of teachers, research indicates that textbooks are essential to teaching and learning of mathematics. While some progressive educators are proponents of eliminating mathematics textbooks, one example from the Netherlands concludes that the textbook is central to increasing student achievement (Van den Heuvel-Panhuizen, 2000).

One of the new emphases in the Common Core State Standards for Mathematics (CCSSM) is on Mathematical Practices, or "varieties of expertise" that teachers "should seek to develop in their students." There are eight such practices, including make sense and persevere, reason abstractly and quantitatively, construct viable arguments, and model with mathematics (NGA, 2011). Practice #4 is "Model with Mathematics" (NGA, 2010). Modeling has been included purposefully throughout the K-12 standards, and as a specific designation in the Standards for High School. In recent years, the use of modeling in mathematics education has begun to surface as a way to increase student achievement and understanding (Blum, Galbraith, Henn, & Niss, 2007; Matos, Blum, Houston, & Carreira, 2001). A number of mathematics education researchers have also addressed the topic (Confrey & Doerr, 1994; Doerr & English, 2003; Doerr & Tripp, 1999; Lesh & Doerr, 2003). There is evidence that learning mathematical models has additional implications, such as philosophical and historical relevance (Dear, 1995; Sepkoski, 2005).

Sfard and Linchevski (1994) contend that the "objectification of symbols is the necessary process for learning algebra." More specifically, Dias (2006) stresses the need for "increased attention to the validation phase of modeling," citing a lack of student attention to such detail (page number). While a number of researchers (Blum & Niss, 1991; Doerr, 1996; Galbraith & Clatworthy, 1990; Lesh & Harel, 2003; Pollak, 1997; Preston, 1997; Tanner & Jones, 1994) have defined the process of mathematical modeling, this study uses the definitions outlined by CCSSM and Dr. Henry Pollak, a pioneer in the field. Pollak has long been a proponent of modeling, advocating for an emphasis on mathematical modeling in curricula as early as 1965.

While other countries do not have explicit initiatives to promote mathematical modeling, the international assessments measure students' abilities to model. In determining the effects that this new modeling initiative may have on student achievement in mathematics, there is an opportunity to make a comparison between the practices in the United States and those of higher-performing countries. In this study, the presence of mathematical modeling in U.S. textbooks, as well as the nationally adopted textbooks used in Singapore, will be examined.

Purpose of the Study

The purpose of this study is to determine the extent to which mathematical modeling is found in commonly used algebra textbooks with respect to the algebra standards designated as modeling by the CCSSM. Given that Singapore is an example of a country that is among the top 10 based on Programme for International Student Assessment (PISA) achievement, this study will also investigate the presence of algebraic modeling-related questions in the state-approved Singapore textbooks.

Research Questions

1. What are the differences between four frameworks of mathematical modeling:

- Common Core State Standards (CCSS) (NGA, 2010);
- Modeling expert Henry O. Pollak (Pollak, 1997);
- Programme for International (PISA) (OECD, 2003); and,
- Australian team Gloria Stillman, Peter Galbraith, Jill Brown, and Ian Edwards (Stillman et al., 2007).

2. What portion of to-be-solved problems from two CCSS-modified algebra textbooks from major publishers are word problems, and what portion of these are designated as mathematical modeling questions?

3. How are the word problems for chapters teaching CCSS-designated modeling standards in CCSS-modified algebra textbooks aligned with the major components common to the four frameworks of mathematical modeling?

4. In the area of algebraic modeling, how do the to-be-solved problems in textbooks used in the United States compare to the to-be-solved problems in textbooks utilized in Singapore, a country whose students consistently score higher than students from the U.S. on the PISA?

Procedures for the Study

The four definitions of mathematical modeling will be investigated for common components and key differences. Components of each definition will be compared and contrasted. Two textbooks will be chosen on the basis of the claim that they have been "updated" to relate to the Common Core State Standards. Based on a review of the leading publishing companies' Web sites, Pearson/Prentice Hall and Glencoe have "updated" their textbooks to meet the newly adopted Common Core Standards. On the basis that these texts claim to be aligned to the Common Core State Standards, and given that they are published by two of the four largest textbook companies in the United States, these texts have been chosen for study.

The third textbook chosen for this study is the Singapore secondary mathematics series. Singapore has consistently outperformed the U.S. on the PISA, and they do have a national curriculum. The mathematics curriculum has been revised to support student achievement in word problems, and was fully implemented in all grades by 2003. The Singapore texts were chosen also due to the fact that they are written in English; this allows for a direct comparison with no language translation issues.

The Glencoe/McGraw Hill text studied is the 2012 version, entitled "Glencoe Algebra 1," with the designation of "Common Core State Standards edition" on the copyright page. The publisher's Web site requests that the reader enter his or her U.S. state prior to entering and browsing available texts. There exist multiple editions of the texts, with different purposes and guiding principles. The Pearson text examined is entitled "Algebra 1 Common Core," and has a copyright date of 2010. Finally, the Singapore series is entitled "New Syllabus Mathematics 1-4 6th edition," and its copyright pages note that the series has been "approved by the Ministry of Education for use 2008-2013."

All chapters in all three textbooks had their to-be-answered problems analyzed and compared, and all to-be-answered problems were tallied in all three textbooks. Further, all word problems were tallied and compared as a percentage of total to-be-answered problems. Finally, all to-be-answered problems designated as addressing mathematical modeling were inventoried and analyzed.

Singapore does not have an initiative or specific standard that addressed mathematical modeling, thus there are no modeling-designated word problems in their textbooks. In order to compare the expectations of the to-be-solved problems in Singapore to those in the U.S. textbooks, an entire chapter was chosen and coded for the components of modeling. This chapter was chosen based on the following criteria: it would have the same learning objective and curricular topic in all three textbooks, as well as a similar number of to-be solved word problems. The sample chapter will serve to compare the expectations of mathematical modeling in the three textbooks.

The standards designated with an asterisk from the Common Core State Standards (CCSS) Mathematics Standards for High School Algebra section were reviewed as a focus for this study. This list includes all standards and was coded using the CCSS abbreviations, e.g. A-SSE.1.a; it is included as Appendix A. Each chapter in the textbook was analyzed for evidence that the chapters and/or to-be-solved problems within the chapter are connected to mathematical modeling in two ways: 1) by requiring students to engage in mathematical modeling as described in the Mathematical Practices, and 2) by addressing the standards designated with an asterisk.

Modeling-designated problems in both textbooks used in the United States as well as the sample chapter will be investigated further for attributes of the definition of mathematical modeling. After analysis of the four definitions, components will be coded in three categories.

Each to-be-solved problem will be reviewed for the expectation that students will engage in all, two, one, or none of the categories. Four mathematics educators reviewed each problem to determine inter-rater reliability. Two of the raters are experienced mathematics educators, and two of the raters are first-year teachers and recent graduates of a master’s degree program in mathematics education.

CHAPTER II: BACKGROUND FOR THE STUDY

Introduction

This study investigates mathematical modeling as addressed by secondary school textbooks. It has been preceded by research in multiple areas, including mathematical modeling as a need within secondary classrooms, the teaching of mathematical modeling, and the impact of textbooks on teaching and learning. This chapter begins by addressing the need and process of mathematical modeling, as well as access (or a lack of access) to strategies for mathematical modeling. The chapter addresses the significance of teaching mathematical modeling, along with the difficulties and successes of applying modeling in the classroom. Finally, studies telling of the impact of textbooks on teaching are included.

In mathematics, one way to ensure that students have experiences that are relevant is to have them engage in mathematical modeling. There are many advantages to having students engage in modeling. In his seminal work *Education and Experience*, John Dewey emphasized "practical consequences or real effects to be vital components of meaning and truth in education" (Dewey, 1938, page). From an early age, children are exposed to articles and advertisements using data to make a point or sell a product, and "it is important for informed citizens to be able to make sense of the graphs, diagrams, tables and other types of mathematical artifacts that increasingly fill publications" (Lesh, 2008, page). Modeling is a complicated and multi-step activity. It "provides concrete embodiments of mathematical concepts, develops reliable computation and checking, develops multiple connections inside and outside mathematics, and so on" (Antonius, Haines, Jensen, & Niss, 2007, page). These connections have their benefits; for example, it has been found that "relating context to task is essential to increase human attention and extend long-term memory" (Kali, 2008).

In an innovative approach to secondary mathematics, authors Garfunkel, Godbold, and Pollak authored a series dedicated to mathematical modeling entitled *Mathematics: Modeling Our World*. The authors state that the “ability to transfer ideas from one context to another—to make connections—is ultimately the skill that makes mathematics valuable” (Garfunkel, Godbold, & Pollak, 1998). More recently, with technology so rapidly changing, “the capacity to perceive structural data, model that structure, and make decisions regarding its implications is rapidly becoming the most important of quantitative literacy skills (Lesh, 2008). As one researcher puts it, “Curriculum designers need to consider the development of tasks that engage students in meaningful problem situations where students’ thinking processes are revealed via their representations and justifications as they engage with the task and self-evaluate the quality of their answers” (Doerr, 2006, page).

Students often protest that the mathematics they use in schools is not useful in the real world. They incessantly question secondary educators: “When am I ever going to use this?” Many textbooks break down mathematics into dozens of chapters, forcing students to learn skills in isolation; however, “understanding that mathematics is founded on reasoning and is not just a collection of rules to apply is an important message to convey to students” (Stacey & Vincent, 2009). Teachers do attempt to create problems and exercises with a context to support student learning; however, such examples can range from well-intentioned inaccurate problems to unrealistic situations. However, this range of mathematical problems cannot be the norm, as “it has been agreed that mathematics teachings should not be reduced to just reality-based examples but that these should play a central role in education” (Kaiser, 2010). Finally, mathematics educators generally wish to spark curiosity and appreciation for their subject, and mathematical modeling “potentially involves both deeper understanding of known curricular mathematics and the motivation to learn new mathematics” (Zbiek, 2006).

One of the benefits of mathematical modeling is its accessibility to children. As a population, we model and interact with models in more ways than we recognize. For example:

> Children estimate the amount of food in their dish, comparing it with their siblings' portions. They measure their growth by marking height on a wall. They count to make sure they have a “fair” number of sweets. So school mathematical education has much to build on, should it choose to do so. (Burkhardt, 2006)

This accessibility can be used to combat the culture of ability grouping, as well as the prevailing “anxiety” that mathematics has seemed to produce:

> A curriculum that emphasizes modeling can perhaps keep students together through most, if not all, of their elementary and secondary school mathematics education. The universality of mathematical modeling can become a major unifying force in mathematics education and perhaps in society as a whole. (Pollak, 2003)

Another benefit of mathematical modeling is the flexibility allowed for solutions. “A great variety of models can be constructed for any given thing,” allowing students to use the tools and knowledge they possess to approach a problem situation (Hestenes, 1993). Within this process, students need to review their own work, review others' work, and be reviewed by others, in that “there should be multiple opportunities for modelers to verify, or at least monitor and share their progress, including communication with self as well as communication with others” (Zbiek, 2006). Verification is integral to the modeling process, and can allow for the development of communication skills: “Modeling is inherently a social enterprise, and significant forms of generalizability and transferability are involved” (Lesh, 2003). This entire process forges connections and forces links between and within concepts and skills in curricular standards, as “a conception of modeling should maintain appreciation of the complexity of mathematical modeling rather than stress details of seemingly disconnected sub processes or oversimplify the complex undertaking" (Zbiek, 2006).

In its redesign of mathematics standards in 2000, the National Council of Teachers of Teachers of Mathematics (NCTM) set modeling as a priority: "Use of mathematical representations to model and interpret physical phenomena and solve problems is one of the major teaching objectives in [the] high school math curriculum" (NCTM, 2000). Overall, the shift in mathematics and science instruction "is best exemplified by a transition from pedagogical approaches based on learning facts and procedures to those oriented around constructing, evaluating, and revising models" (Petrosino, 2003). Clearly influenced by both NCTM and the educational climate, the authors of the CCSS-M emphasize modeling, by making "Model with Mathematics" one of its Standards for Mathematical Practice, which replace the NCTM "Process" strands. They take the following position:

> Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol. (Citation)

Notably, 192 instances (or derivatives) of the word "model" appear in the 93-page document.

Even though standards emphasis on modeling has been explicitly encouraged by the standards for over a decade, there has been a lack of synergy between written expectations and implementation. As one researcher puts it:

> For many students in the middle grades mathematics is just computational work, often only a slight extension of what is discovered in earlier grades. They do not see it as an exploratory, dynamic, evolving discipline but rather as a rigid, closed body of rules to be memorized. (Sendova, 2005)

Students of mathematics and science, given the nature of the textbooks they are provided with, are often "confounded…[and] think the game is to collect and memorize facts" (Hestenes, 1992).

Unfortunately, the result is a lack of harmony or cohesiveness; “without a thorough representation of the physical phenomena and the mathematical modeling processes undertaken, problem solving unintentionally appears as simple algorithmic operations” (Sokolowski, 2011). Connections between subjects areas such as “data analysis, chance, and modeling are often severed in school mathematics, yet, in a wide variety of professions, data modeling is integral to practice” (Lehrer, Kim, & Schauble, 2007). The lack of cohesion leaves students with limited understanding and insight into mathematics, even though it is clearly “insufficient to simply impart competencies for applying mathematics only within the framework of the school curriculum” (Kaiser, 2010). Taking a more fatalistic view, Zbiek finds that “the curricular context of schooling in the USA does not readily admit the opportunity to make mathematical modeling an explicit topic in the K-12 mathematics curriculum” (Zbiek, 2006).

Given its benefits, there is clearly interest in promoting mathematical modeling in K-12 schools. However, a number of studies have shown a lack of understanding of modeling by the teaching force in the United States as well as abroad. Findings show that veteran teachers, “though they share the general notion that a model is a simplified representation of reality, may have quite different cognitions about models and modeling” (Van Driel, 2010). One issue arises because “the main load of the modeling process lies on its intricate mathematization part, in which a large number of idealizations, assumptions, and simplifications had to be made, requiring the modeler to be able to navigate in muddled waters” (Niss, 2010). This proves difficult, since “knowledge of the majority of teachers of models and modeling [is] not very pronounced” (Van Driel, 2010). Another obstacle facing even the most willing teachers: “What is usually missing is the understanding of the original situation, the process of deciding what to keep and what to throw away, and the verification that the results make sense in the real world" (Pollak, 2003).

While this study focuses on the impact of modeling in mathematics, and more specifically in secondary algebra, it is necessary to emphasize the importance of the connection between mathematics and science as pertains to modeling. Science as taught in schools in the United States emphasizes laboratory activities, where students repeatedly utilize and are tested on the "scientific method." Even unbeknownst to them, the data collection and representation students participate in demonstrate components of mathematical modeling. As early as 1976, the commissioned Unified Science and Mathematics for Elementary Schools Project encouraged (without naming specifically) modeling activities (Pollak, 2003). Similarly, a number of studies have sought to explore the knowledge of and emphasis on modeling in science curricula. For example, Van Driel found that science "teachers emphasized different functions and characteristics of models" (Van Driel, 2010). Similarly, Petrosino notes, "Modeling is central to both mathematics and science" (2003, page).

Mathematical modeling as a subject for study was introduced at the college level prior to being emphasized at the elementary and secondary levels. More specifically,

> In mathematics education at the school level, the phrases mathematical or model building appeared in 1966, did not appear where one might have expected them in 1970 and 1975, and reappeared in 1976. At the college level, they appeared in 1965. (Pollak, 2003, page)

"A Sourcebook of Applications of School Mathematics," published by committees of the Mathematics Association of America and the National Council of Teachers of Mathematics in 1980, presented modeling examples as well as applications. Soon after, Hugh Burkhardt authored *The Real World and Mathematics*, a text that was

> Not envisaged as a whole new approach to the teaching of mathematics but rather as an important additional activity, which might take perhaps 20 per cent of school

> mathematics time, justified by both its inherent value and by the reciprocal benefits it will bring the rest of the mathematics curriculum through improved motivation, extra practice and better conceptual understanding through "concrete" illustration. (Burkhardt, 1981, page number)

Subsequently, when PISA published their framework in 2003, they defined "Mathematical Processes" as a key component within Mathematical Literacy, delineating the processes as "the use of mathematical language, modeling and problem-solving skills" (OECD, 2003, page).

While modeling has moved from collegiate classrooms to secondary standards, there has been comparatively little mention of the skill in elementary textbooks and teacher education programs. This is contrary to developmental thinking, since "one might expect that [modeling] would be emphasized from the earliest years of instruction and developed over time, not postponed until high school or beyond" (Petrosino, 2003, page). Students at every age are exposed to modeling situations; "everyone has been modeling with mathematics from an early age" (Burkhardt, 2006). Keeping modeling out of the curriculum impedes the development of these skills, as well as opportunities for growth. As one researcher puts it, "Integrating the learning and creative process by means of visual modeling could contribute to a new learning style in elementary education" (Sendova, 2005). Similarly, "One of the foundational pillars of a modeling perspective is the belief that early reasoning about models is anchored in children's invention and use of a broad variety of representational devices, such as maps, data displays, drawings, or photographs" (Petrosino, 2003). As another team of researchers concludes, "Reasoning, explanation and proof at an appropriate level of sophistication should therefore be a prominent part of learning mathematics for students of all ages" (Stacey & Vincent, 2009).

By learning how to model using mathematics, students will benefit in terms of their abilities, knowledge, understanding, and efficacy. Zbiek explains, "The primary goal of including mathematical modeling activities in students' mathematics experiences within our schools typically is to provide an alternative—and supposedly engaging—setting in which students learn mathematics without the primary goal of becoming proficient modelers" (2006). When students are engaged in "context, where students are offered tools for collecting, visualizing, processing and analyzing data, [they] have less difficulty comprehending the role of mathematical representations and modeling in interpreting physical phenomenon and solving problems" (Sokolowski, 2011). One of the issues teachers believe inhibits learning is a lack of engagement in the process. Mathematical modeling allows for increased engagement, "motivat[ing] students to study mathematics by showing them the real-world applicability of mathematical ideas" (Zbiek, 2006). In addition, "The need for students to take responsibility for their own learning often connects this aim with empowering the students to read, write, and discuss mathematics intelligently" (Lingefjard & Holmquist, 2005).

Research finds that "most students learn effectively in active-engagement environments" (Redish, 1999). For example, "Models and modeling activities build on Piaget's structuralist views about the holistic and constructed nature of the conceptual systems that children develop to make sense of their mathematical experiences" (Lesh, 2003). While opponents might argue that the curriculum does not allow for modeling activities, or that modeling activities would reduce the quality of mathematics taught, "most of traditional high school mathematics was in fact necessary for modeling a variety of situations in which all students could be interested " (Pollak, 2003).

Students also often complain that they are un-involved in the process; expected to listen to a lecture, and then to answer numerous questions in succession without feedback or interaction. However, "model building can provide opportunities for the expression of student voice through the process of representing concepts, defining relationships among components, and exploring consequences of these relationships" (Confrey & Doerr, 1994). Further, model-eliciting activities "can assist in the democratization of access to powerful and important mathematics" (Keut, 2002). Just as students, "practitioners have long been aware of the process and its importance" (Pollak, 2003).

> The most effective types of activities we have found have come to be known as model-eliciting activities…because the most important and neglected aspects of thinking…tend to be the continually evolving interpretation systems that people develop to make sense of their experiences. (Lesh, 2008, page)

Other researchers agree. "As students engage with purpose, mathematical entities and situations in various mathematical modeling subprocesses, there are opportunities for them to grow as knowers and doers of curricular mathematics" (Zbiek, 2006). To summarize: "Engagement in mathematical modeling activities supports three different types of motivation: confirmation that real-world situations appeal to (some) learners, to study mathematics in general, and to learn new mathematics" (Zbiek, 2006).

Modeling cannot be taught in a classroom where the teacher simply writes on the board and students obediently take notes, asking few questions. The modeling process is reciprocal and includes feedback, revision, and discussion: "Successful modeling requires the ability to generate multiple possibilities from a single setting and to raise alternative assumptions for consideration" (Garfunkel, Godbold, & Pollak, 1998). The roles of student and teacher are altered as well, since "modeling can only be taught through a teaching approach that is investigative and student-centered… the traditional teacher's role as the prime source of explanation, demonstration and correct answers is no longer appropriate" (Antonius, Haines, Jensen, & Niss, 2007).

The assignments must also be carefully chosen when teaching modeling. While textbooks include one-step problems with one solution, "the model building process is one that alternates and cycles through these activities, providing a framework for the systematic inquiry that results in the development of a larger conceptual structure that can be used in problem solving" (Confrey & Doerr, 1994). Tasks "need to offer sufficient support for typical teachers, who are mostly inexperienced in teaching modeling, without undermining authority" (Antonius, Haines, Jensen, & Niss, 2007). Unfortunately, there "is a conflict among the wish to create opportunities for unconstrained expression for students, the obvious need for standardization that allows communication, and the danger of meaningless descriptions and narratives" (Yerushalmy, 1997). Assignments and models must be carefully selected, so as to generate investment on the part of the student:

> Students cannot be expected to see, let alone believe in, the relationships between data they have generated and the physical equation unless the data really do fit the equation. This is where the opportunity for data analysis, reflection, and the generation of new data become crucial. (Woolnough, 2000, page)

While the benefits of modeling are numerous, though, so are the potential difficulties. A number of studies have investigated veteran teachers, novice teachers, and teacher candidates at universities across the globe, along with students of mathematics at the secondary and university levels. The findings point to poor understanding, execution, and understanding of mathematical modeling. A study of 223 graduate students in mathematics departments in Poland, for example, noted that "students found the task both difficult and unusual as only nine of them were able to succeed fully" (Trelinski, 1983).

This lack of understanding can have negative impacts. Many students enter courses with inadequate preparation in their lower grades (Blomhoj & Jensen, YEAR; Kaiser & Maass, 2007; Rodruiguez & Gallego, 2007). One study found that "half the students entering Year 11 physics cannot determine the slope of a simple straight line graph [and] many of them find it difficult to develop a good understanding of proportionality," both of which are key concepts necessary to modeling (Woolnough, 2000). Another study found that students "have difficulties connecting real-world events with particular characteristics to graphs" (Zbiek, 1998). Students bring their prior knowledge to the classroom, and the experience of "the interaction between students' modeling abilities and the process components of modeling is continuous and reciprocal. Students' modeling abilities influence the way students solve a modeling problem" (Mousoulides, Christou, & Sriraman, 2008). Similarly, participants in another "study show[ed] disconnected skills when faced with a quite straightforward realistic situation" (Maull & Berry, 2001). Additionally, "research on problem solving suggests that many students tend to give up rather quickly when presented with novel or unfamiliar problem solving tasks" (Doerr, 2006). As another researcher notes, "Students struggle to make the leap from the language of the word problem to the symbolic language of mathematics (Yerushalmy, 1997).

Mathematics is often taught in the United States in minute components through disjointed problems, not allowing students to conceive of generalized applications. Comments one researcher, "The fact that no students entering year 11 physics assigned any units to their calculation of slope would indicate to us that they have not yet begun to develop a conceptual link between the mathematics and the physical world" (Woolnough, 2000). Given their early experiences with mathematics, the concept of modeling can be confusing to students:

> The source of students' difficulty with developing their theoretical modeling skills is probably linked to a comparison with their experiences in mathematics classes. Here the subject is presented as being neat, precise, and logical where one step follows nicely from the previous one. Modeling is much more messy! (Maull & Berry, 2001)

A similar study found "our senior secondary students are operating in three distinct contexts; the real world, the physics world, and the mathematics world, each with different characteristics and belief systems" (Woolnough, 2000). Students are unable to self-teach in such an environment, as "without guidance or access to external clues individuals have great difficulties expressing relationship between mathematical ideas and real-world situations as equations or functions" (Zbiek, 1998).

Despite such difficulties, though, students have found success and inspiration from mathematical modeling. In one study, given the choice, "62 percent of students make a plea to include these kinds of modeling examples in their usual mathematics teaching [with] the most important reason for the rejection [being] time constraint" (Kaiser, 2009). Another study found that "modeling concepts and abilities are likely to be unusually empowering…and unusually accessible [to a range of] students" (Lesh, 2008). Through adversity, students prevailed, as in one study from Kaiser, who explains: "Although at some time during the modeling week the students had made the experience of helpless, lacking orientation, and insecurity, these impressions did not change their positive judgment of the whole modeling experience" (Kaiser, 2009). Within a variety of modeling options, one study found that "one result from the mathematical-modeling test was that the students seemed to handle problems with some sort of visualization much better than problems with just text or symbols" (Lingefjard & Holmquist, 2005).

Today, businesses plead for students to be prepared for the workforce by mastering "21st Century Skills." Mathematical modeling could certainly be considered a key skill. Students would engage in "extensive and focused group work with emphasis on a group's responsibility for delivering a solution to a mathematical modeling problem created a positive learning atmosphere, especially in developing group work skills and personal skills of communication and presentation" (Houston & Lazenbatt, 1999). Just as concerns can arise regarding student ownership, research indicates that modeling provides a solution, in that "overall, students take much more responsibility for their work and their solutions" (Antonius, Haines, Jensen, & Niss, 2007).

There is much that needs to be improved in the teaching of mathematical modeling. For instance, examples of modeling are often not explicitly taught, and components of modeling are consistently omitted. Some researchers have found it to be "unusual to invite the students to actively construct and revise models" (Van Driel, 2010). In addition, in many cases "the focus is on the content of the models being taught and learned, yet the nature of the models is not always explicitly discussed" (Van Driel, 2010). There also needs to be more teacher education and understanding: "A necessary condition for such learning is that the teacher is him/herself competent in all these aspects of modeling" (Justi, 2002).

Assessment of modeling is difficult; "a much more complicated task than anticipated before one starts to teach mathematical modeling" (Lingefjard & Holmquist, 2005). This can partially be attributed to the lack of uniformity or "one correct answer" since in modeling, "students' approaches to learning vary with context" (Lingefjard & Holmquist, 2005). Teachers without knowledge of the underlying mathematics can be hesitant. As a researcher observing one teacher noted, her

> Uncertainty reflect[ed] the limits of her existing pedagogical knowledge of how student models might develop from their early non-mathematical attempts to more sophisticated strategies…[This teacher] did not appear to have in mind a range of strategies for how students might approach the problem. (Doerr, 2006, page)

Comments another researcher, "The most significant challenge to the teacher is to strike a proper balance between student autonomy and teacher intervention…too little feedback can cause frustration and insecurity with the students" (Antonius, Haines, Jensen, & Niss, 2007). Teachers must become proficient modelers before embarking on the act of teaching, and have the ability "to develop appropriate mathematical models to make sense of the world" (Petrosino, 2003).

Clearly, “developing modeling skills should be an important part of an undergraduate degree program but is often overlooked as courses concentrate on teaching mathematical knowledge and skills and introducing standard models” (Maull & Berry, 2001). Further, “Teaching modeling needs a wider range of teaching strategies than most teachers use in delivering the essentially-imitative curriculum that dominates classrooms in most countries” (Burkhardt, 2006).

Student input and interaction is often ignored; to that end, “much more attention needs to be paid to students’ points of view in modeling” (Confrey & Doerr, 1994). Further, “There is a need for studying the working styles of students of all ages in modeling contexts and we need to develop more problems and strategies to encourage the development of good cyclic mathematical modeling skills” (Maull & Berry, 2001). One study found “an alarmingly high number of teachers were either ignorant of or did not pay attention to their students’ ideas about models and modeling” (Justi & Gilbert, 2001). Concluded another study, “It is obviously not enough to ask the teacher to avoid giving a solution to their problem. The teacher needs more information, training, and supervision about how to act in specific situations” (Lingefjard & Meier, 2010).

In order to change what happens in American classrooms, teachers will need to change the activities that they plan and enact, and in which students participate. Note one team of authors, “The choice of mathematical tasks for students to tackle, in the classroom and in assessment, epitomizes any curriculum” (Antonius, Haines, Jensen, & Niss, 2007). Similarly, “Various researchers have revealed that the teaching approaches adopted by teachers and those embodied in the textbooks used in their classrooms were often highly alike” (Fan & Zhu, 2007). Unfortunately, “most school mathematics curricula are fundamentally imitative—students are only asked to tackle tasks that are closely similar to those they have been shown exactly how to do" (Burkhardt, 2006). In order for the tasks and activities to change, the textbooks must change, as "textbooks are considered a de facto national curriculum" (Sood & Jitendra, 2007). To that end, "Improving textbook programs used in American schools is an essential step toward improving American schooling" (Osborn, Jones, & Stein, 1985). Today, "Teacher-directed instruction accounts for well over 80 percent of the textbooks used in schools" (Van de Walle, 2007). However, "The development of performance in modeling processes requires a much richer range of learning activities that the explanation-example-exercises 'ritual' that dominates" (Antonius, Haines, Jensen, & Niss, 2007).

As Valverde and Schmidt put it, "The unfocused curriculum is not a curriculum of high achievement [and] is also a curriculum of very little coherence" (1998, page). The origins and motives behind the curriculum and textbooks are in need of review; as Burkhardt notes, "Curriculum design in mathematics is mainly driven by people whose core interest is in mathematics itself, not in its use" (2006, page). Other researchers agree: "A large body of research that seems to be ignored all too often by those responsible for planning textbook programs comes from the area of instructional design" (Osborn, Jones, & Stein, 1985). However, this view may be diluted by the fact that creating a curriculum is extremely intricate and difficult, forcing decisions and questions at every page and word. As Henry Pollak once commented while participating in the School Mathematics Study Group (SMSG, 1958-1972), "I found writing school mathematics so that it made sense surprisingly challenging!" (Burkhardt, 2006).

Improvements in American textbooks are also necessary: "International comparisons can provide a partial picture of not only what is taught but also how it is taught across nations" (Mayer, Sims, and Tajika, 1994). To that end, "In 2004, the 10th International Congress on

Mathematical Education (ICME-10) organized a Discussion Group specifically focused on textbooks" (Fan & Zhu, 2007). Some of the criticisms are clearly articulated in the research; for example, textbooks "should focus on enhancing students' conceptual understanding, problem solving skills, analytic skills, and transference skills, while simultaneously reducing lengthy calculations [and] this revision should begin in lower level mathematics textbooks" (Sokolowski, 2011). Textbooks tend to focus on facts and concrete information in the early years, which does not prepare students for critical thinking later in their educational careers. As one researcher notes, "It is not necessary to emphasize the processes of observing, ordering, and categorizing the directly perceivable and the concrete, while relegating scientific investigation to later years" (Metz, 1995).

There is also a need to reduce the volume and length of textbooks, as "U.S. textbooks cover far more topics in grades four and eight than do 75 percent of the nations participating in TIMSS and contain an average of 530 pages, versus 170 pages" (Valverde & Schmidt, 1998). In addition to length, the usability of textbooks comes into question: "Many textbooks are difficult to read" (Garner, 1992). It may seem to be common sense that "the more organized and readable a text, the more students will learn from it" (Osborn, Jones, & Stein, 1985). There are clear international differences in this area:

> In Japan, the major use of page space is to explain mathematical procedures and concepts in words, symbols, and graphics, with an emphasis on worked-out examples and concrete analogies. In U.S. textbooks… the major use of page space is to present unexplained exercises in symbolic form for students to solve on their own. (Mayer, Sims, & Tajika, 1994)

Extraneous information and pictorials also plague U.S. textbooks: "Lessons are supplemented with attention-grabbing graphics that are, unlike those in the Japanese textbooks, interesting but irrelevant" (Mayer, Sims, & Tajika, 1994). Finally, "One way of improving education is to ensure that curriculum materials are of high quality and are error-free" (King, 2010).

Textbooks also articulate an unfortunate lack of consistency across disciplines and general inaccuracies. In one survey of secondary science textbooks, "a total of 453 instances of 'error/oversimplification' were noted" (King, 2010). In modeling scientific phenomena, for example, the textbooks should use accurate and precise language. However, one study found that "many of the definitions and science concepts presented to students [in pre-calculus textbooks] lack consistency with their physical counterparts" (Sokolowski, 2011). Further, "Current science textbooks contain many examples of scientific models, usually presenting these models as static facts" (Van Dreil & Verloop, 2002).

Studies have also found a lack of correlation between the intended outcomes and assessments. One Ohio study found "several areas of mismatch between mathematics textbooks and current content emphasis in the Ohio Ninth Grade Proficiency Test, with significant differences" (Chandler & Brosnan, 1995). Even with the NCTM Focal Points and calls for an increased focus on problem solving, the U.S. continues to fall behind other developed countries in the area of mathematics. This may be attributed to an issue within our materials; for example, "the amount of space in mathematics textbooks that is devoted to meaningful explanation of problem-solving strategies may be an important determinant of students' mathematical problem-solving competence" (Mayer, Sims, & Tajika, 1994). In U.S. materials examined, "The content in the mathematics textbooks was disproportionate to the content of the proficiency test studied" (Chandler & Brosnan, 1995). A Turkish study found, "It has been fixed that mathematics textbooks; which were examined within the scope of this research, are generally far from students' needs, are isolated from real life, only done the determined procedure, treatment, and algorithm calculation, and written in abstract and academic style" (Dede, 2006). Further, "Cognitive modeling of problem solving processes is emphasized more in Japan than in the United States, whereas drill and practice on the product of problem solving is more emphasized in the U.S. than Japan" (Mayer, Sims, & Tajika, 1994).

Mathematics textbooks attempt and fail to make the connection between the real world and mathematics. Textbooks are littered with inadequate problems; one study "highlights the importance of curricula analyses, not simply in terms of the scope and sequence of topics, but rather in terms of the particulars of problem contexts and formats" (McNeil et al., 2006). More specifically, researchers found in examining textbooks that

> Perhaps the most abundant of all are those [problems] that use words from everyday life outside mathematics to make the problem sound good…the statement of such problems rarely questions the honesty and genuineness of the connection to the real world, but the connection is often false in one or more ways. (Pollak, 1969, page)

Other researchers have come to similar conclusions: "In mathematics textbooks, problem solving is mostly conceptualized as the activity of solving traditional word problems. These problems usually present simplified forms of a decontextualized world based situation, with the purpose of exercising a specific type of mathematical learning" (Mousoulides, Christou, & Sriraman, 2008). Along similar lines, "The applications of mathematics…have all been simple specific problems whose solution required only the direct translation of the story into mathematical terms and the application of standard mathematical technique. Actual applications of mathematics, of course, are often not as simple as that" (Pollak, 1969).

One study dedicated itself to a comparison of textbooks used in China with those used in the United States, and found:

> U.S. textbooks have a much more narrow view of the distributive property…[it] limit[s] students' understanding of the distributive property to whole number concepts with a regular direction. They also focus more on surface strategies rather than the

> principle underlying the strategies. The word problems (often one-step) are not used to provide contextual support to learn the distributive property In contrast, Chinese text aligns well with Curriculum Focal Points, developing students' intuitive sense from the second grade…using diverse word problem contexts and constructive approaches such as asking deep questions to prompt students to understand the distributive property. "The dominant context for the distributive property in both U.S. texts is computation, in contrast, the Chinese texts guide students to discover and abstract the principle of the distributive property through many comparison problems. (Din & Li, 2010, page)

It should be noted, though, that "developmental factors alone cannot always account for children's misunderstandings" (McNeil et al., 2006). For example, one study found that four popular middle-school mathematics textbooks do not often utilize the equal sign with operations on both sides. This has lasting results for school-aged children: "Even in eighth grade, many students continue to interpret the equal sign as an operational symbol" (McNeil et al., 2006). Concluded one team of researchers, "Our cross-cultural findings suggest alternatives for improving the U.S. elementary text approaches and potentially inspiring U.S. teachers to develop better classroom activities" (Din & Li, 2010).

CHAPTER III: PROCEDURES

The purpose of this study is to examine the presence of mathematical modeling in algebra textbooks in the United States and Singapore. At their onset in the United States, 48 out of 50 states have adopted the Common Core State Standards. The reliance on textbooks is clear from prior research, and this study seeks to find how the textbooks have included and addressed the new Modeling expectation of the CCSSM.

Content Analysis

This investigation involves content analysis of algebra textbooks as they address the CCSSM-Algebra standards designated as addressing mathematical modeling. The framework for review includes four definitions of mathematical modeling. Content analysis is defined as a "research technique for making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use" (Krippendorff, 2004, page).

While studies of textbooks may be considered more recent, the process of using content analysis as a research tool has can be found as early as the start of the 20th century. The method was used, for example, in 1923 to examine high-school texts. Another example of textbook content analysis can be seen in Berelson's 1952 study evaluating the representation of foreign countries. These studies were able to examine balance and accuracy, and uncovered data that pointed to an inferred bias in textbooks. In my study, content analysis facilitated the examination of the textbooks, allowing me to evaluate the presence (or absence) and types of reading strategies in teacher editions of mathematics textbooks., It provided me with the methodological structure to examine the textbooks, arrive at thematic patterns, and formulate conclusions for the improvement of textbooks (Holsti, 1969; Krippendorff, 2004).

Content analysis research can be advantageous when compared to alternate research methods. To begin, data are in print and readily available for review at any time. This minimizes human error and the possibility of distortion of data; therefore, the process of data collection in content analysis has a higher degree of reliability (Babbie, 2007). By nature, content analysis is unobtrusive; the material is in print and cannot be influenced or conditionally altered. As Krippendorff comments, "Content analysis is an empirically grounded method, exploratory in process, and predictive or inferential in intent" (2004, page).

Choice of Textbooks

Over the past years, the number of major publishers in the United States has shrunk to four, as Pearson, McGraw-Hill, Reed Elsevier, and Houghton Mifflin have

> Absorbed dozens of independent textbook companies…including Macmillan, Merrill, and Glencoe (imprints of McGraw-Hill); Prentice-Hall, Silver Burdett, Ginn, Addison Wesley, Longman, and Scott Foresman (imprints of Pearson); Holt, Rinehart and Winston (imprint of Harcourt); and D.C. Heath and McDougal Littell (imprints of Houghton Mifflin). (Sewall, 2005, page)

Two of the major companies have published editions that claim to be aligned to the Common Core State Standards for 2012: McGraw-Hill/Glencoe and Prentice Hall. Both versions will be reviewed in this study. Additionally, Singapore, a country that consistently outperforms the United States in international assessments has English translations of their textbooks available. This study will also analyze the nationally approved secondary textbook series used in Singapore.

Choice of To-Be-Solved Problems

All problems designated by the authors of the U.S. textbooks as "mathematical modeling" were analyzed in this study. The Singapore textbook series does not designate problems as mathematical modeling. So, an entire chapter in each textbook will be chosen as the basis for a comparison that can also include an international comparison, in this case Singapore. These chapters will have the same learning objective and a similar quantity of problems to be solved by students.

Table 1: Common Chapter Across Three Textbooks

Textbook	Lesson	Pearson Chapter Title	Modeling (N)	Word Problems (N)	Total Problems (N)	Word Problems (%)
Pearson	6-4	Applications of Linear Systems	1	15	28	53.6
Glencoe	6-5	Applying Systems	1	10	26	38.5
Singapore	2-5	Problem Solving Involving Simultaneous Equations	N/A	26	26	100.0

Evidence of Mathematical Modeling

CCSS Algebra standards were reviewed, and standards designated with (★) were extracted:

> Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). (National Governors Association, 2011)

These standards are listed in Appendix A. Next, the researcher identified chapters in each textbook that cover the identified Common Core Standards. These chapters or subchapters are listed in Table 4.2 in the Findings section. The researcher then reviewed the chapter for word problems, or exercises in the chapter that include more than numbers and symbols.

In order to create a common definition of mathematical modeling, the four definitions were distilled into three main characteristics of modeling. All selected problems will be coded for evidence of these three characteristics of mathematical modeling: 1) formulating a model; 2) employing mathematics; and, 3) interpretation of results.

Reliability

Two coders were recruited based on their experience with teaching algebra and knowledge of mathematics. One (R1) is a high-school teacher in a high-performing district with 10 years of teaching experience, as well as post-graduate degrees. The other (R2) is a college professor who has a PhD in Mathematics Education. Two additional first-year teachers were recruited as third (R3) and fourth (R4) raters.

Ten sample questions were developed, each with a different result (no evidence, one attribute, etc.), and used to train each rater. Each rater completed the 10 questions, and discussed the results. After training, each of the four coders individually reviewed all word problems from the sample chapters of each textbook and all modeling-designated problems from the U.S. textbooks. In total, each coder and the researcher (R) analyzed 222 problems.

To measure the level of agreement of the raters measuring the three modeling characteristics, the researcher calculated Cohen’s Kappa Agreement Index. This statistic is used to assess inter-rater reliability when observing or otherwise coding qualitative or categorical variables. Kappa is considered to be an improvement over using percent agreement to evaluate this type of reliability, since it addresses one-to-one agreement of each characteristic. For example, two raters might each have coded a characteristic for 50 percent of the sample, but they might have only coded the characteristic for the same examples at a rate of 25 percent.

The Kappa Index is calculated with the following components: expected frequencies, number of agreement between raters, and number of observations (N). The amount of the index indicate the approximation of the level of agreement between raters (0 to 1), so the higher the Kappa statistic, the stronger the raters' agreement. The Statistical Package for Social Sciences (SPSS) Version 19 was used to calculate Kappa. Through use of this software, the researcher identified the level of agreement between the five raters coding the three characteristics of modeling in the same sample of 222 word problems was estimated. Steps for using SPSS can be found in Appendix G.

CHAPTER IV: FINDINGS

Research Question 1

What are the differences between four frameworks of mathematical modeling? There are a number of frameworks for describing the process of mathematical modeling. The four frameworks described in this section are:

1. Common Core State Standards;
2. Modeling expert Henry Pollak;
3. Programme for International Student Assessment (PISA); and,
4. Australian team (Gloria Stillman, Peter Galbraith, Jill Brown, and Ian Edwards).

The Common Core State Standards includes the following steps in the modeling cycle:

Table 2: Common Core State Standards for Mathematical Modeling

The basic modeling cycle…involves (C1) identifying variables in the situation and selecting those that represent essential features, (C2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (C3) analyzing and performing operations on these relationships to draw conclusions, (C4) interpreting the results of the mathematics in terms of the original situation, (C5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (C6) reporting on the conclusions and the reasoning behind them. (C*) Choices, assumptions, and approximations are present throughout this cycle.

Dr. Henry Pollak includes the following steps in the modeling cycle:

Table 3: Henry Pollak Framework for Mathematical Modeling

H1. We identify something in the real world we want to know, or understand. The result is a question in the real world.
H2. We select "objects" that seem important in the real-world question and identify relations among them. The result is the identification of important concepts in the real-world situation.
H3. We decide what we will keep and what we will ignore about the objects and their interrelations. We cannot take everything into account. The result is an idealized version of the original question.
H4. We translate this idealized version into mathematical terms and obtain a mathematical formulation of the idealized question. This is called a mathematical model.
H5. We identify the field or fields of mathematics that are relevant to the model and bring to bear our instincts and knowledge about these fields.
H6. We use mathematical methods and insights to get results. Out of this step may come new techniques, interesting examples, solutions, approximations, theorems, algorithms.
H7. We take all these results and translate back to the real world. We now have a theory about the idealized question.
H8. Now comes the reality check. Do we believe what's being said? Are the results practical, the answers reasonable, the consequences acceptable?
(a) If yes, the real-world problem solving has been successful. Our next job—namely, to communicate with potential users—is both difficult and extraordinarily important.
(b) If no, we go back to the beginning. Why are the results impractical or the answers unreasonable or the consequences unacceptable? Because the model was not right. We examine what went wrong, try to see what caused it, and start again.

Organisation for Economic Co-operation and Development (OECD) has published the following:

Table 4: PISA 2012 Mathematics Framework

	P1 Formulating situations mathematically	P2 Employing mathematical concepts, facts, procedures, and reasoning	P3 Interpreting, applying, and evaluating mathematical outcomes
Devising strategies for solving problems	P1.1 Select or devise a plan or strategy to mathematically reframe contextualized problems	P2.1 Activate effective and sustained control mechanisms across a multi-step procedure leading to a mathematical solution, conclusion, or generalization	P3.1 Devise and implement a strategy in order to interpret, evaluate, and validate a mathematical solution to a contextualized problem
Using symbolic, formal, and technical language and operations	P1.2 Use appropriate variables, symbols, diagrams, and standard models in order to represet a real-world problem using symbolic/formal language	P2.2 Understand and utilize formal constructs based on definitions, rules, and fomal ssystems as well as employing algorithms	P3.2 Understand the relationship between the context of the problem and representation of the mathematical solution. Use this understanding to help interpret the solution in the context and gauge the feasibility and possible limitations of the solution
Using mathematical tools	P1.3 Use mathematical tools in order to recognize mathematical structures or to portray mathematical relationships	P2.3 Know about and be able to make appropriate use of various tools that may assist in implementing processes and procedures for determining mathematical solutions	P3.3 Use mathematical tools to ascertain the reasonableness of a mathematical solution and any limits and constraints on that solution, given the context of the problem

Galbraith, Stillman, Brown, & Edwards developed the following (Stillman et al., 2007)

Table 5: Framework of Mathematical Modeling

G1. MESSY REAL WORLD SITUATION >REAL WORLD PROBLEM STATEMENT:
1.1 Clarifying context of problem
1.2 Making simplifying assumptions
1.3 Identifying strategic entit(ies)
1.4 Specifying the correct elements of strategic entit(ies)
G2. REAL WORLD PROBLEM STATEMENT > MATHEMATICAL MODEL:
2.1 Identifying dependent and independent variables for inclusion in algebraic model
2.2 Realizing independent variable must be uniquely defined
2.3 Representing elements mathematically so formulae can be applied
2.4 Making relevant assumptions
2.5 Choosing technology/mathematical tables to enable calculation
2.6 Choosing technology to automate application of formulae to multiple cases
2.7 Choosing technology to produce graphical representation of model
2.8 Choosing to use technology to verify algebraic equation
2.9 Perceiving a graph can be used on function graphers but not data plotters to verify an algebraic equation
G3. MATHEMATICAL MODEL >MATHEMATICAL SOLUTION:
3.1 Applying appropriate symbolic formulae
3.2 Applying algebraic simplification processes to formulae to produce more sophisticated functions
3.3 Using technology/mathematical tables to perform calculation
3.4 Using technology to automate extension of formulae application to multiple cases
3.5 Using technology to produce graphical representations
3.6 Using correctly the rules of notational syntax (whether mathematical or technological)
3.7 Verifying of algebraic model using technology
3.8 Obtaining additional results to enable interpretation of solutions
G4. MATHEMATICAL SOLUTION > REAL WORLD MEANING OF SOLUTION:
4.1 Identifying mathematical results with their real world counterparts
4.2 Contextualizing interim and final mathematical results in terms of RW situation (routine [R] complex versions)
4.3 Integrating arguments to justify interpretations
4.4 Relaxing of prior constraints to produce results needed to support a new interpretation
4.5 Realizing the need to involve mathematics before addressing an interpretive question
G5. REAL WORLD MEANING OF SOLUTION>REVISE MODEL OR ACCEPT SOLUTION:
5.1 Reconciling unexpected interim results with real situation
5.2 Considering Real World implications of mathematical results
5.3 Reconciling mathematical and Real World aspects of the problem
5.4 Realizing there is a limit to the relaxation of constraints that is acceptable for a valid solution
5.5 Considering real-world adequacy of model output globally

Comparing all four side by side, we can see that there are a number of areas that are directly related:

Table 6: Comparison of Modeling Characteristics in Four Frameworks.

CCSS	Pollak	PISA	Galbraith, Stillman, Brown, & Edwards
	H1. We identify something in the real world we want to know, or understand. The result is a question in the real world.		
(C1) Identifying variables in the situation and selecting those that represent essential features	H2. We select "objects" that seem important in the real-world question and identify relations among them. The result is the identification of important concepts in the real-world situation.		G1. MESSY REAL WORLD SITUATION > REAL WORLD PROBLEM STATEMENT
(C*) Choices, assumptions, and approximations are present throughout this cycle	H3. We decide what we will keep and what we will ignore about the objects and their interrelations. We cannot take everything into account. The result is an idealized version of the original question.	P1.1 Select or devise a plan or strategy to mahtematically reframe contextualized problems	G1.2 Making simplifying assumptions
(C2) Formulating a model by creating	H4. We translate this idealized version into mathematical terms and obtain a mathematical formulation of the idealized question. This is called a mathematical model.	P1.2 Use appropriate variables, symbols, diagrams, and standard models in order to represet a real-world problem using symbolic/formal language	G2. REAL WORLD PROBLEM STATEMENT > MATHEMATICAL MODEL
(C2) And selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables	H5. We identify the field or fields of mathematics that are relevant to the model and bring to bear our instincts and knowledge about these fields.	P2.2 Understand and utilize formal constructs based on definitions, rules, and formal systems as well as employing algorithms	G2. REAL WORLD PROBLEM STATEMENT > MATHEMATICAL MODEL
(C3) Analyzing and performing operations on these relationships to draw conclusions	H6. We use mathematical methods and insights to get results. Out of this step may come new techniques, interesting examples, solutions, approximations, theorems, algorithms.	P2.1 Activate effective and sustained control mechanisms across a multi-step procedure leading to a mathematical solution, conclusion, or generalization	G3. MATHEMATICAL MODEL >MATHEMATICAL SOLUTION

CCSS	Pollak	PISA	Galbraith, Stillman, Brown, & Edwards
(C4) Interpreting the results of the mathematics in terms of the original situation	H7. We take all these results and translate back to the real world. We now have a theory about the idealized question.	P3.1 Devise and implement a strategy in order to interpret, evaluate, and validate a mathematical solution to a contextualized problem	G4. MATHEMATICAL SOLUTION > REAL WORLD MEANING OF SOLUTION:
(C5) Validating the conclusions by comparing them with the situation	H8. Now comes the reality check. Do we believe what's being said? Are the results practical, the answers reasonable, the consequences acceptable?	P3.2 Understand the relationship between the context of the problem and representation of the mathematical solution. Use this understanding to help interpret the solution in the context and gauge the feasibility and possible limitations of the solution	
(C6) Reporting on the conclusions and the reasoning behind them	(a) If yes, the real-world problem solving has been successful. Our next job—namely, to communicate with potential users—is both difficult and extraordinarily important.	P3.3 Use mathematical tools to ascertain the reasonableness of a mathematical solution and any limits and constraints on that solution, given the context of the problem	G5. REAL WORLD MEANING OF SOLUTION>ACCEPT SOLUTION:
(C5) And then either improving the model	(b) If no, we go back to the beginning. Why are the results impractical or the answers unreasonable or the consequences unacceptable? Because the model was not right. We examine what went wrong, try to see what caused it, and start again.		G5. REAL WORLD MEANING OF SOLUTION>REVISE MODEL

Research Question 2

What portion of to-be-solved problems from two CCSS-modified algebra textbooks from major publishers are word problems, and what portion of these are designated as mathematical modeling questions? The following table includes the number of to-be-answered problems in each textbook.

Table 7: Word Problems in To-Be-Solved Problems

	Total Problems	Modeling-Designated Word Problems (N)	(%) Modeling-Designated Word Problems	Word Problems (N)	(%)Word Problems
Glencoe One-Year Algebra	4771	40	0.84%	757	15.87%
Pearson Prentice Hall One-Year Algebra	4768	116	2.43%	658	13.80%
Singapore Four-Year Textbook Sequence	4183	N/A	N/A	802	19.17%

Research Question 3

How are the word problems for chapters teaching CCSS-designated modeling standards in CCSS-modified algebra textbooks aligned with the components common in the four frameworks of mathematical modeling? This table represents the number of problems and their corresponding attributes for each subgroup of problems analyzed. The following problems were designated as corresponding to the CCSM-Modeling expectations by the publisher.

Table 8: Modeling Characteristics in U.S. Modeling To-Be Solved Problems (N)

	Modeling Characteristic				Number of Characteristics			
Number	N	Formulate	Employ	Interpret	0 of 3	1 of 3	2 of 3	3 of 3
PPH Modeling	106	32	83	9	11	70	21	4
Glencoe Modeling	36	13	31	9	0	21	13	2

Table 9: Modeling Characteristics in U.S. Modeling To-Be Solved Problems (%)

	Modeling Characteristic				Number of Characteristics			
Percentage	N	Formulate	Employ	Interpret	0 of 3	1 of 3	2 of 3	3 of 3
PPH Modeling	106	30%	78%	8%	10%	66%	20%	4%
Glencoe Modeling	36	36%	86%	25%	0%	58%	36%	6%

Research Question 4

In the area of algebraic modeling, how do the to-be-solved problems in textbooks used in the United States compare to the to-be-solved problems in textbooks utilized in Singapore, a country whose students consistently score higher than students from the U.S. on the PISA? This table represents the number of problems and their corresponding attributes for each subgroup of problems from the Applications of Systems of Linear Equations that the researcher analyzed from each textbook:

Table 10: Modeling Characteristics in Common Chapter (N)

	Modeling Characteristic				Number of Characteristics			
Number	N	Formulate	Employ	Interpret	0 of 3	1 of 3	2 of 3	3 of 3
Singapore Systems Chapter	27	0	27	0	0	27	0	0
PPH Systems Chapter	28	1	21	12	0	23	4	1
Glencoe Systems Chapter	25	9	20	4	4	12	6	3

This table represents the data as percentages of the total problems (N):

Table 11: Modeling Characteristics in Common Chapter (%)

	Modeling Characteristic				Number of Characteristics			
Percentage	N	Formulate	Employ	Interpret	0 of 3	1 of 3	2 of 3	3 of 3
Singapore Systems Chapter	27	0%	100%	0%	0%	100%	0%	0%
PPH Systems Chapter	28	4%	75%	43%	0%	82%	14%	4%
Glencoe Systems Chapter	25	36%	80%	16%	16%	48%	24%	12%

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Inter-Rater Reliability

Table 12: Inter-Rater Reliability for Formulate

Formulate				
Kappa	**R2(F)**	**R3(F)**	**R4(F)**	**R(F)**
R1(F)	**0.41***	0.05	-0.67	**0.27***
R2(F)		0.045	-0.053	**0.655***
R3(F)			**0.264***	0.042
R4(F)				-0.011
*significant with alpha .01				

Table 13: Inter-Rater Reliability for Employ Mathematics

Employ Mathematics				
Kappa	**R2(E)**	**R3(E)**	**R4(E)**	**R(E)**
R1(E)	**0.644***	0.101	0.011	**0.764***
R2(E)		0.01	0.028	**0.722***
R3(E)			**0.656***	0.03
R4(E)				0.022
*significant with alpha .01				

Table 14: Inter-Rater Reliability for Interpret Results

Interpret Results				
Kappa	**R2(I)**	**R3(I)**	**R4(I)**	**R(I)**
R1(I)	**0.798**	0.091	0.048	**0.465***
R2(I)		0.071	0.057	**0.508***
R3(I)			**0.534***	0.084
R4(I)				0.104
*significant with alpha .01				

Table 15: Analysis of Kappa Index

Analysis of Kappa Index	
K	**Strength of Agreement**
< **0.20**	Poor
0.21 – 0.40	Weak
0.41 – 0.60	Moderate
0.61 – 0.80	Good
0.81 – 1.00	Very Good

The highest levels of reliability were found in the experienced educators (R1, R2) and the researcher (R).

To-Be-Solved Problems

This study also examined to-be-solved problems, and found a number of areas for improvement. These areas centered around five major themes: misprints, not realistic, dimensional inaccuracy, lack of precision, and problems with missed opportunities.

Misprint

Some of the modeling problems analyzed would not make sense, but the problem could easily be fixed with a keystroke, so we make the assumption that the problem was misprinted. For example, one problem in the solving quadratic trinomials chapter noted that a child threw a discus that followed the path $h = 16t^2+95t+6$.

The following problem indicates the approximate height of a diver in meters after t seconds, but seems to have forgotten to divide the force of gravity in half: *Alexis jumps from a diving platform upward and outward before diving into the pool. The function* $h = -9.8t^2+4.9t+10$ *approximates Alexis's dive…*

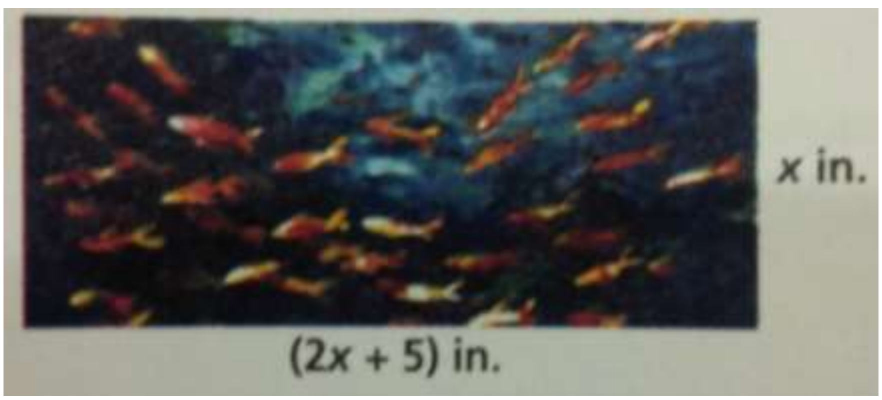
The wording of another problem, below, renders it nonsensical. The quantities of data are noted as "up to", making the division of the two numbers meaningless: *A recent cell phone study showed that company A's phone processes up to* 7.95×10^5 *bits of data every second. Company B's phone processes up to* 1.41×10^6 *bits of data every second. Evaluate and interpret* $(1.41 \times 10^6)/(7.95 \times 10^5)$.

This problem can be found in a chapter entitled "Solving Systems by Graphing": *One satellite radio service charges $10 per month plus an activation fee of $20. A second service charges $11 per month plus an activation fee of $15. In what month was the cost of service the* *same?* The textbook indicates a correct answer of "5 months." However, since the monthly charges are finite and unequal, the "cost of service" will never be the same.

Not Realistic

More than a few problems in the textbook lacked the quality of being true to life. The following, for example, is labeled as being an application of "art" and mathematical modeling—clearly a situation that does not arise regularly (if at all) in the life of an artist.

The painting shown has an area of 420 sq in. What is the value of x?



The next problem asks students to calculate "commission" for cutting wood planks. Generally, woodcutters would not be paid by plank, but would be paid at an hourly or salaried rate. On the other hand, a salesperson might be more likely to earn commission on planks sold. Also, the 2 x 4's might be prepared at a sawmill. *The table shows how Ryan is paid at his lumber yard job. A. Write a function to represent Ryan's commission. B. Graph the function and determine the domain.*

Linear Feet of 2x4 Planks Cut	10	20	30	40	50	60	70
Amount Paid in Commission ($)	8	16	24	32	40	48	56

The following problem is designated as mathematical modeling but has no basis in reality nor does is show mathematical modeling. *Che is building a dog house for his new puppy. The upper face of the dog house is a trapezoid. If the height of the trapezoid is 12 inches, find the area of the face of this piece of the dog house.*

This problem asks students to draw a "step graph." There is no indication of what the purpose of drawing the graph or a question that might be answered; therefore there is no modeling involved: *The United States Postal Service increases the rate of postage periodically. The table shows the cost to mail a letter weighing 1 ounce or less from 1995 through 2009. Draw a step graph to represent the data.*

Year	1995	1999	2001	2002	2006	2007	2008	2009
Cost ($)	0.32	0.33	0.34	0.37	0.39	0.41	0.42	0.44

The following problem does not occur in the "real world": *The radius of a cylindrical gift box is (2x+3) in. The height of the gift box is twice the radius. What is the surface area of the cylinder? Write your answer in standard form.*

Dimensions

In some problems, the units or dimensions do not seem to make sense, or do not translate to common usage for the context.

The following problem does not indicate whether "size" indicates the perimeter, length, width, or area. The problem could be misinterpreted, since the dimensions are not identified:

Jameka is enlarging a photograph to make a poster for school. She will enlarge the picture repeatedly at 150%. The function $P=1.5^x$ models the new size of the picture being enlarged, where x is the number of enlargements. How many times as big is the picture after 4 enlargements?

The following problem indicates that a golf ball begins at three feet off the ground. It does not seem likely to have a tee that is three feet high. *The height of a golf ball in the air can be modeled by the equation* $h = -16t^2+60t+3$ *where h is the height in feet of the ball after t seconds. A. How long was the ball in the air? B. What is the ball's maximum height? C. When will the ball reach its maximum height?*

This problem includes a graph with no indication of what the dimensions are or what the numbers are meant to represent. *A path for a new city park will connect the park entrance to Main Street. The path should be perpendicular to Main Street and will pass through the park entrance. What is an equation of the line that represents the bike path?*

Precision

Some problems were clearly made too simplistic for the sake of attending to the mathematics of the chapter. As CCSSM researcher and author Phil Daro noted in his talk on the Common Core, "Japanese teachers look at problems and ask: 'How can I use this problem to teach the mathematics I want students to learn as opposed to American teachers who ask, 'How can I teach students to learn how to solve this problem?'" (SERP, 2011). The paradigm that Daro describes clearly influences the design of questions in textbooks used in the United States.

The following problem assumes that temperature rises and falls at a consistent rate: *An artist completed an ice sculpture when the temperature was -10°C. The equation t=1.25h-10 shows the temperature h hours after the sculpture's completion. If the artist completed the sculpture at 8am, at what time will it begin to melt?*

The following problem assumes that temperature will increase indefinitely: *The temperature at sunrise is 65°F. Each hour during the day, the temperature rises 5°F. Write an equation that models the temperature y, in degrees Fahrenheit, after x hours during the day. What is the graph of the equation?*

The following problem makes an assumption that the costs create a continuous function. However, since the fictional person cannot buy a portion of a can of paint or a fraction of a bed linen set, the problem does not accurately represent the situation described: *Sybrina is decorating her bedroom. She has $300 to spend on paint and bed linens. A gallon of paint costs $14, while a set of bed linens costs $60. A. Write an inequality for this situation. B. How many gallons of paint and bed linen sets can Sybrina buy and stay within her budget?*

Missed Opportunity

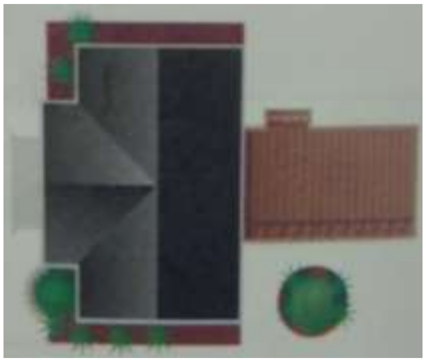
Many problems had the makings of realistic, age-appropriate, applicable modeling problems but lacked one or more steps from the modeling process.

The following problem is indicated as a "Financial Literacy" modeling problem. It missed the opportunity for students to decide whether or not to include many of the expenses of owning a vehicle, such as gas, repairs, oil changes, insurance, registration, taxes, and fees for the car. The situation could also consider whether Keisha's entire paycheck is going towards the purchase price of the car. Some babysitters are employees if they earn a certain amount of money. Finally, some people tithe or save a percentage of their earnings. *Define a variable, write an inequality, and solve each problem. Then interpret your solution. Keisha is babysitting at $8 per hour to earn money for a car. So far she has saved $1300. The car that Keisha wants to buy costs at least $5440. How much money does Keisha need to earn to buy the car?*

This problem shares an interesting application of machine replacement of human work. There are so many factors to be accounted for that a problem such as this could be rich and engaging. Some aspects for students to consider in addition to the questions asked might be: How do the nurses' salaries compare or vary? What is the effect on support staff, such as pharmacists and nurse aides; or administrators, such as human resources and nursing directors? *The TOBOR robot saves 120 minutes of a nurse's time n and 180 minutes of support staff time s each day. Another robot that aids stroke patients' limbs is estimated to save 90 minutes of nursing time and 120 minutes of support staff time each day. A. To be cost effective, TOBOR must save a total of 1500 minutes per day. Write an equation that represents this relationship. B. To make stroke assistant cost effective, it must save a total of 1050 minutes per day. Write an equation that represents this relationship. C. Solve the system of equations, and interpret the solution in the context of the situation.*

Designing a deck for a home might be an ideal modeling problem situation. This problem, as indicated, could use a number of improvements. For example, the lumber for railing, stairs, and supports is not indicated. Also, the photo does not represent the problem, which only describes a rectangular deck.

Collin is building a deck on the back of his family's home. He has enough lumber for the deck to be 144 square feet. The length should be 10 more feet than its width. What should the dimensions of the deck be?



One problem asks students to calculate the dimension of a frame given the dimensions of a photograph, a mat surrounding the photograph, and a length of a wood frame. The problem does not account for the width of the wood or the unseen overlap of the wood frame over the mat.

This is an additional carpentry problem with the potential for students to model: *A carpenter is designing a drop-leaf table with two drop leaves of equal size. The lengths of the table when one leaf is folded up and when both leaves are folded up are shown. How long is the table when no leaves are folded up?*

CHAPTER V: Conclusions and Recommendations

Summary

Across the globe, countries outperform the U.S. in mathematics testing. Mathematical modeling is one area that has become a focus within the new Common Core State Standards. When students in other developed countries do better on international assessments than their American counterparts, the results make news headlines, and the education system in the U.S. is questioned. One resource important to both teachers and students is the textbook. Research has found that until textbooks change, teaching will not change. Taking that into account, this study analyzed textbooks for evidence of components of the mathematical modeling across three textbooks: two from major U.S. publishers, and one nationally adopted in Singapore.

The study found that there is no common framework applied to the mathematical modeling cycle. Multiple frameworks exist, and four were compared in this study. Many characteristics of the modeling cycle were found to overlap, but a few components were not ubiquitous. For example, the framework written by a mathematician from the field included a pre-step to modeling; namely, determine something that you would like to know or understand. The other three frameworks did not identify this as a step. Three of the four frameworks acknowledged that the figures found in the real world would not be sufficient for creating a model. One of the frameworks does not include a step that requires students to improve upon the model when necessary. More specifically, modelers must make choices, estimations, and assumptions in order to make a problem into a usable model.

All three textbooks included a similar ratio of to-be-solved problems that were numerical and that included contexts and scenarios, although the Singapore textbook series had about one quarter of the overall number of problems when compared to the U.S. textbooks. This study also found that the modeling problems in the U.S. textbooks asked students to explicitly engage in components of the modeling cycle: formulate a model, employ mathematics, and interpret the results. Most problems had the attribute of "employ mathematics," although a few did not employ any operations or functions. The U.S. textbooks showed no pattern in the problems that were labeled as "modeling" when compared to the problems without the "modeling" label.

Conclusions

Research Question 1: *What are the differences between four frameworks of mathematical modeling?*

The description and process of mathematical modeling differs based on the context, purpose and audience. Strands of commonalities can be found in the four frameworks, but the discrepancies were glaring when the expectation was that they should be more similar. After coding and organizing the commonalities between the definitions, some differences were revealed. One discrepancy can be found in the number of steps or categories in each definition. CCSS delineates six steps, with an additional note indicating approximations and assumptions are made within the process. Pollak describes an eight-step modeling process. The PISA framework pares down the process to three categories: Formulating, Employing, and Interpreting. Galbraith, Stillman, Brown, and Edwards' framework outlines five steps.

Pollak's definition is the only one to include the identification of something that we want to know as a step in the modeling framework (H1). It can be interpreted that the other three frameworks were created for the classroom setting, and that therefore this step could be eliminated. Pollak, on the other hand, approached the framework through the lens of himself as a modeler in the workplace setting (Henry Pollak, personal communication, February 11, 2014).

The formulation of the problem or re-stating of the real-world situation is noted as a step in three out of the four frameworks. For example, H3: "Decide what we will use and what we will ignore about the objects and interrelations," and G1.2: "Making simplifying assumptions." On the other hand, the CCSS generalize the process of making suppositions into a statement applying to all steps: C* "Choices, assumptions, and approximations are present throughout this cycle."

The PISA framework also does not make explicit that the model should be evaluated, then either accepted or reviewed and modified when necessary. The process of validation and rejection or acceptance is required in all three of the other frameworks. One further distinction found between Pollak and the other three frameworks was the verbiage describing important values. Dr. Pollak's second step (H2) reads: "We select 'objects' that seem important in the real-world questions and identify relations among them." The other three frameworks refer to the identification of variables (C1, G2, P1.2). Here we find an assumption that three out of the four frameworks make: that algebraic interpretation is necessary in mathematical modeling.

Pollak's description also includes the step where "we identify something in the real world we want to know." This was not included in any of the remaining descriptions of mathematical modeling. No evidence was found that students are asked to review a situation or the world around them, and to come up with a question or idea that they would like to investigate.

Research Question 2: ***What portion of to-be-solved problems from two CCSS-modified algebra textbooks from major publishers are word problems, and what portion of these are designated as mathematical modeling questions?***

There are more to-be-answered problems included in each one-year textbook used in the U.S. than there are in the entire four-year sequence of texts used in Singapore. This is not a new finding; however, the significance of this result lies in the claims of the CCSSM that the newly revised standards are "internationally benchmarked" and "fewer, higher, clearer." Analysis from this study shows that the textbooks have not reduced the amount of topics, nor have they reduced the number of problems presented in the chapters. The textbooks do not, in this case, reflect the nature of the expectations of the to-be-completed student problems found in Singapore.

Research Question 3: ***How are the word problems for chapters teaching CCSS-designated modeling standards in CCSS-modified algebra textbooks aligned with the components common in the four frameworks of mathematical modeling?***

There was no significant difference in the percentage of to-be-solved problems that were word problems between the three textbooks. The Glencoe algebra textbook has the highest percentage of problems that include the three modeling characteristics. No pattern was found in the U.S.-based textbooks when comparing to-be solved word problems designated as "modeling" and those not designated. For example, the following was designated as "modeling" in one textbook*: Describe three real-world situations: one with a positive correlation, one with a negative correlation, and one with no correlation*. Additionally, multiple similar problems were found in the to-be-solved exercises, yet only one was marked as "modeling." In other instances, a to-be-solved word problem addressed more of the characteristics of mathematical modeling than one marked as “modeling” by the publisher.

Research Question 4: ***In the area of algebraic modeling, how do the to-be-solved problems in textbooks used in the United States compare to the to-be-solved problems in textbooks utilized in Singapore, a country whose students consistently score higher than students from the US on the PISA?***

The Singapore textbook did not show evidence of asking students explicitly to formulate models or interpret their solutions. Further, the results of this study did not determine that the to-be-solved problems in textbooks resulted in higher achievement on the PISA mathematics section.

Additional Conclusions

There was a low level of reliability found in the raters for this study. This may be related to the knowledge of, and assumptions made by, experienced teachers. For example, teachers with experience understand the implicit need to require students to formulate equations as well as interpret responses. Novice educators, by contrast, may assume that the questions must explicitly note these requests in order to ensure that students will provide such responses. Textbook authors may make the assumption that teachers deserve “professional respect,” precluding them from incorporating patronizing directions (Sol Garfunkel, personal communication, January 28, 2014). This result is in keeping with a study that found a disparity in the interpretation of textbooks and supporting materials (Remillard, 2000).

There is a distinct difference between the modeling description found in the Standards for Mathematics Practice and the Modeling Standard found in the U.S. High School standards. Descriptors in the High School Standard for Modeling relate directly to algebraic models (NGA, 2010). In addition, there seemed to be a lack of emphasis on memorization in the Singapore textbooks. Specifically, there was no glossary, and there were no text boxes, words defined repeatedly, or words defined out of context of the chapter descriptive paragraphs. This was also true for formulas.

Finally, no evidence was found that students made approximations and assumptions in translating a real-world problem from a description of a real-life, messy situation. In most cases, the data or information leading to an equation was provided. Students did not have the opportunity to become familiar with the process of eliminating unnecessary information, nor were they confronted with determining the consequences of disregarding real phenomena in order to determine a usable result. This conclusion aligns with Pollak's assertion that students need to engage in the practice of what to keep from the real-world, messy situation, and what to disregard or put aside (Pollak, 2003).

Recommendations

A study like this would benefit from a number of improvements. To begin, obtaining and analyzing additional textbooks would provide more depth to the results. Including additional countries, learning objectives, subjects, and publishers would also be useful. The textbooks examined here were so varied in their approach to mathematical modeling that more research would provide a broader context for understanding how mathematical modeling is being presented. Additionally, this study could have been conducted well from the perspective of mathematics educators instead of as a textbook analysis. Since the raters and the researcher had such low agreement on what the to-be-solved problems were asking of students, this study might have investigated how teachers approach teaching modeling in the context of the new standards and with the materials that are provided to them. Since textbooks in the U.S. are adapted to multiple versions, this study could address regional or level-based discrepancies in textbooks with the same subject matter and objectives.

Implications

This study has a number of implications for educators, researchers, and policymakers. More attention to the mathematical modeling cycle could produce rich problems in which students make and defend suppositions, analyze their work, and decide feasibility. This result is in line with other researchers' conclusion that students need to validate their answers more frequently in order to increase understanding of mathematics (Dias, 2006). Textbook problems, based on those examined, show clear potential for improvement. To-be solved problems, for example, might be reviewed for missed opportunities. Textbook problems might also ask students more often to improve upon a model or determine reasonableness, make assumptions, report conclusions, turn the mathematics into statements for sharing, revise or change assumptions, and justify conclusions and feasibility.

Since the textbook problems studied did not indicate that Singapore textbooks prepared students better for mathematical modeling than the U.S. textbooks, an opportunity for further study would be to look at the differences in teacher preparation and teaching methodologies in the two countries. Finally, the rater discrepancies indicate that interpretation of components of mathematical modeling is not universal. There exists the opportunity to investigate mathematics educators' interpretations of components of the mathematical modeling process in word problems. Early in their careers, mathematics teachers may need different directions or guidance than those who are more experienced.

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Appendix A: Common Core State Standards for Mathematics Algebra Modeling Standards

Common Core State Standards for Mathematics Algebra Modeling Standards

A-SSE Seeing Structure in Expressions

Interpret the structure of expressions.

A-SSE 1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients.

A-SSE 1. Interpret expressions that represent a quantity in terms of its context b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

Write expressions in equivalent forms to solve problems.

SSE 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 3a. Factor a quadratic expression to reveal the zeros of the function it defines.

SSE 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. SSE 3b. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

SSE 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. SSE 3c. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Use the properties of exponents to transform expressions for exponential functions. *For example the* *expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

SSE 4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

A-CED Creating Equations

Create equations that describe numbers or relationships.

A-CED 1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-CED 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R*.*

A-REI Reasoning with Equations & Inequalities

Represent and solve equations and inequalities graphically.

A-REI 11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Appendix B: Algebra Modeling Standards Textbook Correlation

The following table lists the table of contents from each textbook used in this study.

Table of Contents

Glencoe McGraw Hill	**Pearson Prentice Hall**	**Singapore #2**	**Singapore #3**
0. Preparing for Algebra	1. Foundations for Algebra	1. Congruence and Similarity	1. Solutions to Quadratic Equations
1. Expressions, Equations, and Functions	2. Solving Equations	2. Direct and Inverse Proportions	2. Indices and Standard Form
2. Linear Equations	3. Solving Inequalities	3. Expansion and Factorisation of Algebraic Expressions	3. Linear Inequalities
3. Linear Functions	4. An Introduction to Functions	4. Algebraic Manipulation & Formulae	4. Coordinate Geometry
4. Equations of Linear Functions	5. Linear Functions	5. Simultaneous Linear Equations	5. Matrices
5. Linear Inequalities	6. Systems of Equations and Inequalities	6. Pythagoras' Theorem	6. Application of Mathematics in Practical Situations
6. Systems of Linear Equations and Inequalities	7. Exponents and Exponential Functions	7. Volume and Surface Area	7. Linear Graphs and Their Applications
7. Exponents and Exponential Functions	8. Polynomials and Factoring	8. Graphs of Linear Equations in Two Unknowns	8. Congruent and Similar Triangles
8. Quadratic Expressions and Equations	9. Quadratic Functions and Equations	9. Graphs of Quadratic Equations	9. Area and Volume of Similar Figures and Solids
9. Quadratic Functions and Equations	10. Radical Expressions and Equations	10. Set Language and Notation	10. Trigonometrical Ratios
10. Radical Functions and Geometry	11. Rational Expressions and Functions	11. Statistics	11. Further Trigonometry
11. Rational Functions and Equations	12. Data Analysis and Probability	12. Probability	12. Measuration – Arc Length, Sector Area, Radian Measure
12. Statistics and Probability			13. Geometric Properties of Circles

Appendix C: Glencoe Textbook Analysis

The following chart includes information recorded about the Glencoe textbook. Column A lists the lesson found in each chapter. Column B includes the title of each lesson. Column C tells us if the textbook author has designated the chapter as addressing the CCSSM SMP 4: Modeling with Mathematics. Column D lists the mathematical modeling standard addressed, where applicable. Column E lists the problems that the textbook authors cite as addressing mathematical modeling. Column F lists the quantity of to-be-completed exercises in the chapter (not including cumulative review). Column G lists the quantity of mathematical modeling problems found in the lesson. Column H lists the quantity of word problems that are not designated as modeling problems in the lesson. Column I tallies the total number of non-modeling-designated word problems in the chapter. Finally, Column J calculates the percentage of word problems that are designated as modeling problems in the chapter.

A	B	C	D	E	F	G	H	I	J
Lesson	Title	SMP 4	CCSSM*	Modeling Prob #	To Be Completed	N(Modeling)	N(Non-Modeling Designation Word Problems)	N(Non)	% Modeling
1-1	Variables and Expressions	1	A-SSE-1a		42		6		
1-2	Order of Operations	0	A-SSE-1b		64		9		
1-3	Properties of Numbers	0	A-SSE-1b		60		11		
1-4	The Distributive Property	0			59		6		
1-5	Equations	0	A-CED-1	34	70	1	7		
1-6	Relations	0		7,8,17,18	42	4	16		

1-7	Functions	0	A-CED-2^		55		5		
1-8	Interpreting Graphs of Functions	0			22		19	79	6.3%
2-1	Writing Equations	0	A-CED-1	10	50	1	5		
2-2	Solving 1-Step Equations	0			78		14		
2-3	Solving Mult-Step Equations	0			57		6		
2-4	Solving Eq q/Variables on Both Sides	0			47		6		
2-5	Solving Eq w/Abs Value	0			64		10		
2-6	Ratios and Proportions	0			50		11		
2-7	Percent of Change	0		13,41	49	2	10		
2-8	Literal Equations and Dim Analysis	0	A-CED-4		40		9		
2-9	Weighted Averages	1		12	26	1	20	91	3.3%
3-1	Graphing Lin Eq	0			64		10		
3-2	Solving Lin Ed by Graphing	1		37	50	1	6		
3-3	Rate of Change and Slope	0			52		7		
3-4	Direct Variation	0			50		10		
3-5	Arithmetic Seq as Linear Fn	0		23	36	1	6		
3-6	Proportional and Nonprop Relations	0			18		4	43	4.7%
4-1	Graphing EQ Pt Slope Form	0			66		9		
4-2	Writing EQ SlpINt Form	0		22	52	1	13		
4-3	Writing Eq PtSlope Form	0			54		3		
4-4	Parallel and Perpendicular Lines	0			48		7		
4-5	Scatter Plots and Lines of Fit	1			17		12		
4-6	Regression and Median Fit	0			18		14		

4-7	Inverse Linear Fns	0	A-CED-2		43		4	62	1.6%
5-1	Solv Ineq by Addition and Subt	1	A-CED-1	34-40	56	7	1		
5-2	Solv Ineq by Mult and Division	0	A-CED-1		47		10		
5-3	Solving Multi-Step Ineq	0	A-CED-1		59		7		
5-4	Solving Compound Ineq	0	A-CED-1		43		8		
5-5	Ineq Involving Abs Value	0	A-CED-1		47		6		
5-6	Graphing Ineq in 2 Variables	0	A-CED-3	30	50	1	3	35	22.9 %
6-1	Graphing Systems of Eq	0	A-CED-3, A-REI-11^	9, 26	52	2	3		
6-2	Substitution	0	A-CED-3		31		5		
6-3	Elimination +, -	0	A-CED-2		39		6		
6-4	Elimination x	0		25	33	1	7		
6-5	Applying Systems	1		15	26	1	9		
6-6	Systems of Inequalities	0		26	43	1	4	34	8.8%
7-1	Mult Properties of Exponents	0			67		5		
7-2	Division Properties of Exponents	0			65		6		
7-3	Rational Exponents	0		85	93	1	5		
7-4	Scientific Notation	0		66	75	1	13		
7-5	Exponential Functions	0	A-REI-11^	20	45	1	5		
7-6	Growth and Decay	1	A-SSE-3c^		20		15		
7-7	Geometric Sequences as Exp Fns	0			42		6		
7-8	Recursive Formulas	0		22	35	1	4	59	6.8%
8-1	+, - Polynomials	0	A-SSE-1a		65		6		
8-2	x Poly by Monomial	0		42	50	1	4		
8-3	Multiplying Polynomials	0			49		5		

8-4	Special Products	0			61		3		
8-5	Using Distributive Property	0	A-SSE-3a		56		7		
8-6	Solving x^2+bx+c=0	0	A-SSE-3a		49		5		
8-7	Solving ax^2+bx+c=0	1	A-SSE-3a	9	44	1	4		
8-8	Differences of Squares	0	A-SSE-3a		63		4		
8-9	Perfect Squares	0	A-SSE-3a		60		5	43	4.7%
9-1	Graphing Quadratic Fns	0			74		5		
9-2	Solving Quadratics by Graphing	0		36	46	1	4		
9-3	Transformations of Quadratics	0	A-SSE-3b		40		5		
9-4	Solving by Completing Square	1	A-SSE-3b^	9	52	1	3		
9-5	Quadratic Formula	0		41	58	1	4		
9-6	Successive Differences	0			35		6		
9-7	Special Function	1		16	60	1	6	33	12.1%
10-1	Square Root Functions	0			52		3		
10-2	Simplifying Radical Expressions	0			57		5		
10-3	Operations w Radical Expressions	0			39		3		
10-4	Radical Equations	1	A-CED-2		37		5		
10-5	Pythagorean Theorem	0			50		7		
10-6	Trigonometric Ratios	0			55		6	29	0.0%
11-1	Inverse Variation	0			56		8		
11-2	Rational Functions	0	A-CED-2		47		6		
11-3	Simplifying Rational Expressions	0			48		5		
11-4	x, / Rational Expressions	0			59		14		

11-5	Dividing Polynomials	0			50		4		
11-6	+, - Rational Expressions	0			75		9		
11-7	Mixed Expressions and Complex Fractions	1		33	47	1	6		
11-8	Rational Equations	1	A-CED-2, A-REI-11^		40		10	62	1.6%
12-1	Samples and Studies	0	0		41	0	41		
12-2	Statistics and Parameters	0		4	21	1	38		
12-3	Distributions of Data	0			17	0	10		
12-4	Comparing Sets of Data	0			25	0	9		
12-5	Simulations	1		8	16	1	13		
12-6	Permutations and Combinations	0		10	43	1	31		
12-7	Probability of Compound Events	0		14	38	1	28		
12-8	Probability Distributions	1			20	0	17	187	0.5%
			^=LAB/EXTENSION		4855	40	757		5%

Appendix D: Pearson Prentice Hall Textbook Analysis

The following chart includes information recorded about the Pearson Prentice Hall textbook. Column A lists the lesson found in each chapter. Column B includes the title of each lesson. Column C tells us if the textbook author has designated the chapter as addressing the CCSSM SMP 4: Modeling with Mathematics. Column D includes the quantity of to-be-completed exercises in the chapter (not including cumulative review). Column E notes the pages that include teacher directions regarding the modeling problems. Column F cites the problems that the textbook authors cite as addressing mathematical modeling. Column G lists the quantity of mathematical modeling problems found in the lesson. Column H lists the quantity of word problems that are not designated as modeling problems in the lesson. Column I tallies the total number of non-modeling-designated word problems in the chapter. Finally, Column J calculates the percentage of word problems that are designated as modeling problems in the chapter.

A	B	C	D	E	F	G	H	I	J
Lesson	Title	SMP 4	Total problems	Modeling Teacher Notes Pg	Modeling Prob #	N(Modeling)	N(Non-Modeling Designation Word Problems)	N(Non)	% Modeling
1-1	Variables and Expressions		41	8	26	4	6		
1-2	Order of Operations and Evaluating Expressions		60	14	44	1	6		
1-3	Real Numbers and the Number Line		71	21	63	1	14		
1-4	Properties of Real Numbers		57	27	37	1	4		
1-5	Adding and Subtracting Real Numbers	1	69	34	61	1	5		

1-6	Multiplying and Dividing Real Numbers	1	69	43	63	1	7		
1-7	The Distributive Property		90	50	68	1	6		
1-8	An Introduction to Equations		66	57	66	1	7	13	
1-9	Patterns, Equations, and Graphs		35	65	30, 34	2	3	58	22.4%
2-1	Solving One-Step Equations		78	86	50,74	2	9		
2-2	Solving Two-Step Equations		65	92	56, 57, 61	3	5		
2-3	Solving Multi-Step Equations		68	98	54, 65	2	8		
2-4	Solving Equations With Variables on Both Sides		56	106	19, 42	2	7		
2-5	Literal Equations and Formulas	1	50	109, 113	47	1	10		
2-6	Ratios, Rates, and Conversions		44	120	10, 41	2	15		
2-7	Solving Proportions		59	128	37	1	12		
2-8	Proportions and Similar Figures		30	134	17	1	16		
2-9	Percents		53	143	8, 30	2	12	18	
2-10	Change Expressed as Percent		41	149	35, 17	2	7	108	16.7%
3-1	Inequalities and their Graphs	1	64	168	7, 41, 43	3	2		
3-2	Solving Inequalities using Addition or Subtraction		80	174	66, 72a	2	6		
3-3	Solving Inequalities using Multiplication		64	182	63	1	6		

	or Division								
3-4	Solving Multi-Step Inequalities		57	190	47	1	6		
3-5	Working with Sets		52			0	0		
3-6	Compound Inequalities		52	205	40	1	6		
3-7	Absolute Value Equations and Inequalities		82	211	70	1	10	10	
3-8	Unions and Intersections of Sets		45	219	26	1	3	49	20.4 %
4-1	Using Graphs to Relate Two Quantities		20	238	4, 16, 17, 18	4	8		
4-2	Patterns and Linear Functions		20	244	17	1	4		
4-3	Patters and Nonlinear Functions		22	250	20	1	2		
4-4	Graphing a Function Rule	1	41	258	57	1	5		
4-5	Writing a Function Rule	1	32	265	22, 27	2	10		
4-6	Formalizing Relations and Functions		40	272	23	1	3	11	
4-7	Arithmetic Sequences	1	75	280	70	1	6	49	22.4 %
5-1	Rate of Change and Slope		58	298	5, 41	2	7		
5-2	Direct Variation		47	305	25	1	8		
5-3	Slope-Intercept Form		62	312	35	1	5		
5-4	Point-Slope Form		32	319	29	1	3		
5-5	Standard Form		63	326	33	1	5		
5-6	Parallel and Perpendicular Lines		37	334	25	1	5		
5-7	Scatter Plots	1	22	341	11, 16	2	10	10	

	and Trend Lines								
5-8	Graphing Absolute Value Functions		37	349	21	1	0	53	18.9 %
6-1	Solving Systems by Graphing	1	42	368	21	1	3		
6-2	Solving Systems Using Substitution		43	376	24	1	6		
6-3	Solving Systems using Elimination		44	382	14	1	6		
6-4	Applications of Linear Systems	1	28	391	10	1	14		
6-5	Linear Inequalities		41	398	31	1	5	6	
6-6	Systems of Linear Inequalities		38	403	38	1	6	46	13.0 %
7-1	Zero and Negative Exponents		72	422	64	1	4		
7-2	Multiplying Powers with the Same Base		53	430	21	1	6		
7-3	More Multiplication Properties of Exponents		73	437	64	1	4		
7-4	Division Properties of Exponents		93	443	31	1	10		
7-5	Rational Exponents and Radicals		54	451	53	1	6		
7-6	Exponential Functions		55	457	29	1	7		
7-7	Exponential Growth & Decay	1	42	464	27	1	10		
7-8	Geometric Sequences		53	471	34	1	4	51	15.7 %
8-1	Adding and		51	490	35	1	4		

	Subtracting Polynomials								
8-2	Multiplying and Factoring		43	495	42	1	6		
8-3	Multiplying Binomials		50	502	30	1	7		
8-4	Multiplying Special Cases		58	508	19	1	2		
8-5	Factoring x^2 + bx+c		60	516	44	1	4		
8-6	Factoring ax^2 + bx+c		51	521	21	1	3		
8-7	Factoring special cases		57	527	43	1	2		
8-8	Factoring by grouping		46	532	40	1	4	32	25.0 %
9-1	Quadratic Graphs and Their Properties	1	49	550	26	1	5		
9-2	Quadratic Functions		36	557	26	1	6		
9-3	Solving Quadratic Equations		57	564	42	1	8		
9-4	Factoring to Solve Quadratic Equations		45	571	27	1	6		
9-5	Completing the Square		49	580	31	1	5		
9-6	The Quadratic Formula & the Discriminant		49	587	42	1	4		
9-7	Linear, Quadratic, and Exponential Models		27	593	19	1	6		
9-8	Systems of Linear and Quadratic Equations		36	600	17	1	5	45	8.9%
10-1	Pythagorean Theorem		42	617	29	1	7		
10-2	Simplifying Radicals		78	624	35	1	7		
10-3	Operations with		63	630	37	1	8		

	Radical Expressions								
10-4	Solving Radical Equations		52	637	50	1	7		
10-5	Graphing Square Root Functions		68	642	41	1	5		
10-6	Trigonometric Ratios		63	650	43	1	6	40	15.0 %
11-1	Simplifying Rational Expressions		53	668	44	1	6		
11-2	Multiplying and Dividing Rational Expressions		69	674	60	1	7		
11-3	Dividing Polynomials		59	682	51	1	4		
11-4	Adding and Subtracting Rational Expressions		54	688	44	1	4		
11-5	Solving Rational Equations		51	696	45	1	9		
11-6	Inverse Variation		49	703	42	1	10		
11-7	Graphing Rational Functions		49	711	23	1	4		
12-1	Organizing Data Using Matrices		32	730	30	1	5		
12-2	Frequency and Histograms		33	736	21	1	22		
12-3	Measures of Central Tendency and Dispersion		30	743	29	1	20		
12-4	Box-and-Whisker Plots		26	750	24	1	11		
12-5	Samples and Surveys		40	757	37	1	36		
12-6	Permutations and		65	766	37	1	14		

	Combinations								
12-7	Theoretical and Experimental Probability		46	776	9,36	2	13		
12-8	Probability of Compound Events		45	781	6, 42, 43	3	6	131	35.8%
			4768			116	658		18%

Appendix E: Singapore Textbook Results

This table quantifies the results of the Singapore textbook. Column A tells us the abbreviation for each lesson. The "S" stands for "Singapore." The first number tells us which book or year the lesson comes from. The second number tells us which chapter within the book the lesson is from. The third number indicates the lesson or sub-heading within the chapter. For example, "S3.4.7" refers to Singapore Year 3, fourth chapter, seventh lesson. Column B lists the title for the chapter. Column C lists the number of word problems, or problems that include a context or words as opposed to just variables and quantities. Column D counts the number of to-be-completed problems in the chapter. Finally, Column E is the percentage of the chapter that can be considered word problems.

A	B	C	D	E
Abbr	Title	Word Problems (N)	Total Problems (N)	%
S1.1.1	Factors and Multiples	0	0	
S1.1.2	Prime Numbers and Composite Numbers	3	31	10%
S1.1.3	Prime Factorisation	0	0	
S1.1.4	Index Notation	0	26	0%
S1.1.5	Highest Common Factor (HCF)	4	23	17%
S1.1.6	Least Common Multiple (LCM)	6	31	19%
S1.1.7	Square and Square Roots	0	0	
S1.1.8	Cube and Cube Roots	4	31	13%
S1.1.9	Mental Estimation	0	0	
S1.1.10	The Use of Calculators	0	45	0%
S1.2.1	Negative Numbers	0	0	
S1.2.2	Integers	0	0	
S1.2.3	The Number Line	0	0	
S1.2.4	Absolute Value of an Integer	19	31	61%
S1.2.5	Addition of Integers	3	21	14%
S1.2.6	Subtraction of Integers	0	18	0%
S1.2.7	Multiplication of Integers	0	0	

S1.2.8	Division of Integers	0	10	0%
S1.2.9	Rules for Operating on Integers	0	21	0%
S1.3.1	Rational Numbers	0	0	
S1.3.2	Ordering of Rational Numbers	2	6	33%
S1.3.3	Addition and Subtraction of Rational Numbers	8	17	47%
S1.3.4	Multiplication and Division of Rational Numbers	7	10	70%
S1.3.5	Arithmetical Operations on Rational Numbers	0	13	0%
S1.3.6	Problem Solving Involving Rational Numbers	6	6	100%
S1.3.7	Terminating and Recurring Decimals	0	12	0%
S1.3.8	Use of Calculators on Real Numbers	0	32	0%
S1.4.1	Estimation and Rounding	3	13	23%
S1.4.2	Approximations in Measurements and Accuracy	0	0	
S1.4.3	Rounding Off a Number to a Given Number of Decimal Places	0	13	0%
S1.4.4	Accuracy and Significant Figures	0	0	
S1.4.5	Rounding Off a Number to a Given Number of Significant Figures	0	37	0%
S1.4.6	Rounding and Truncation Errors	3	10	30%
S1.5.1	Notation in Algebra	12	23	52%
S1.5.2	Evaluation of Algebraic Expressions	0	15	0%
S1.5.3	Some Rules in Algebra	0	19	0%
S1.5.4	Use of Brackets in Simplification	0	30	0%
S1.5.5	Addition and Subtraction of Algebraic Expressions	0	16	0%
S1.5.6	Linear Algebraic Expressions with Fractional Coefficients	0	16	0%
S1.5.7	Factorisation	0	15	0%
S1.5.8	Factorisation by Grouping	0	10	0%
S1.6.1	Number Sequences	0	20	0%
S1.6.2	General Term in a Number Sequence	0	6	0%
S1.6.3	Problem Solving	7	7	100%
S1.7.1	Open Sentences	0	0	
S1.7.2	Simple Equations	0	16	0%
S1.7.3	Solving Simple Equations	0	29	0%
S1.7.4	Further Examples on Equations	0	46	0%
S1.7.5	Formulae	0	16	0%
S1.7.6	Construction of Formulae	7	10	70%

S1.7.7	Writing Algebraic Expressions	14	14	100%
S1.7.8	Problem Solving with Algebra	0	27	0%
S1.7.9	Inequalities	0	16	0%
S1.7.10	Properties of Inequalities	0	0	
S1.7.11	Equations and Inequalities	5	6	83%
S1.8.1	Units of Area	5	40	13%
S1.8.2	Area of a Parallelogram	0	0	
S1.8.3	Area of a Trapezium	3	26	12%
S1.9.1	Concept of Volume	0	0	
S1.9.2	Volume of Fluids	5	22	23%
S1.9.3	Right Prisms	0	0	
S1.9.4	Volume of a Prism	0	0	
S1.9.5	Surface Area of a Prism	7	19	37%
S1.9.6	Cylinders	0	0	
S1.9.7	Volume of a Cylinder	0	0	
S1.9.8	Surface Area of a Cylinder	7	14	50%
S1.10.1	Ratio	0	0	
S1.10.2	Equivalent Ratios	7	26	27%
S1.10.3	Increase and Decrease in Ratio	10	17	59%
S1.10.4	Rate	17	17	100%
S1.10.5	Average Rate	6	6	100%
S1.10.6	Time	13	15	87%
S1.10.7	Speed and Average Speed	0	0	
S1.10.8	Problems Involving Speed and Average Speed	19	19	100%
S1.11.1	Percentages, Fractions, and Decimals	9	9	100%
S1.11.2	Expressing One Quantity as a Percentage of Another	0	0	
S1.11.3	Finding the Percentage of a Number	6	10	60%
S1.11.4	Comparing Two Quantities by Percentages	0	0	
S1.11.5	Percentages Greater than 100%	11	11	100%
S1.11.6	Increasing/Decreasing a Quantity by a Given Percentage	4	19	21%
S1.11.7	Discount	0	0	
S1.11.8	Commission	0	0	
S1.11.9	Value-added Tax and GST	22	22	100%
S1.12.1	Rectangular Coordinates in Two Dimensions	0	0	
S1.12.2	The Rectangular or Cartesian Plane	0	0	
S1.12.3	Coordinates of a Point	0	24	0%
S1.12.4	The Idea of Functions	0	0	

S1.12.5	Ordered Pairs Satisfying a Function	0	0	
S1.12.6	Gradient of a Straight Line	0	13	0%
S1.13.1	Introduction to Numerical Data	0	0	
S1.13.2	Collection, Organization and Interpretation of Data	0	0	
S1.13.3	Collection of Data Using a Questionnaire	0	0	
S1.13.4	Pictograms	0	0	
S1.13.5	Bar Graphs	8	8	100%
S1.13.6	Collection of Data Through Observation	0	0	
S1.13.7	Pie Charts	0	0	
S1.13.8	Collection of Data Through Interviews	10	10	100%
S1.13.9	Line Graphs	3	3	100%
S1.13.10	Frequency Tables and Histograms	0	0	
S1.13.11	Collection of Data by Measuring	6	6	100%
S1.13.12	Collection of Data by Using Electronic Means and the Internet	0	0	
S1.13.13	Grouped Frequency Table	5	5	100%
S1.14.1	Points	0	0	
S1.14.2	Lines, Rays, Line Segments	0	0	
S1.14.3	Planes	0	0	
S1.14.4	Intersecting Lines	0	0	
S1.14.5	Angles	0	0	
S1.14.6	The Protractor and Angle Measure	0	0	
S1.14.7	Different Kinds of Angles	0	0	
S1.14.8	Complementary Angles	0	0	
S1.14.9	Supplementary Angles	0	0	
S1.14.10	Adjacent Angles on a Line	0	0	
S1.14.11	Vertically Opposite Angles	0	26	0%
S1.14.12	Parallel Lines, Alternate Angles, Corresponding Angles, Interior Angles	0	16	0%
S1.15.1	Polygons	0	0	
S1.15.2	Triangles	0	0	
S1.15.3	Angle properties of Triangles	0	0	
S1.15.4	Exterior and Interior Opposite Angles	0	37	0%
S1.15.5	Quadrilaterals	0	20	0%
S1.15.6	Convex Polygons	0	0	
S1.15.7	Sum of Interior Angles of a Convex Polygon	0	0	
S1.15.8	Sum of Exterior Angles of a Convex Polygon	0	29	0%
S1.16.1	Geometrical Constructions	0	0	
S1.16.2	Use of Compasses	0	0	

S1.16.3	Bisecting an Angle	0	0	
S1.16.4	Bisecting a Line Segment	0	31	0%
S2.1.1	Congruent Figures and Objects	0	23	0%
S2.1.2	Similar Figures and Objects	2	9	22%
S2.1.3	Similarity and Enlargement	0	0	
S2.1.4	Similarity and Scale Drawings	16	22	73%
S2.2.1	Direct Proportion	9	19	47%
S2.2.2	More on Direct Proportion	0	0	
S2.2.3	Graphical Representation of Direct Proportion	10	25	40%
S2.2.4	Other Forms of Direct Proportion	3	23	13%
S2.2.5	Inverse Proportion	17	17	100%
S2.2.6	More on Inverse Proportion	0	0	
S2.2.7	Graphical Representation of Inverse Proportion	6	15	40%
S2.2.8	Other Forms of Inverse Proportion	1	20	5%
S2.3.1	Expansion of Algebraic Expressions	0	40	0%
S2.3.2	Further Algebraic Expansions	0	30	0%
S2.3.3	Perfect Squares and Difference of Two Squares	0	38	0%
S2.3.4	Factorisation of Algebraic Expressions	0	30	0%
S2.3.5	Factorisation of Algebraic Identities	0	72	0%
S2.3.6	Factorisation of Quadratic Expressions	0	40	0%
S2.3.7	Solving Quadratic Equations by Factorisation	0	40	0%
S2.3.8	Problem Solving Involving Quadratic Equations	5	14	36%
S2.4.1	Simple Algebraic Fractions	0	34	0%
S2.4.2	Multiplication and Division of Algebraic Fractions	0	12	0%
S2.4.3	Further Examples on Simplifications of Algebraic Fractions	0	14	0%
S2.4.4	Addition and Subtraction of Algebraic Fractions	0	15	0%
S2.4.5	Further Addition and Subtraction of Algebraic Fractions	0	30	0%
S2.4.6	Equations Involving Algebraic Fractions	0	20	0%
S2.4.7	Problem Solving Involving Algebraic Fractions	20	20	100%
S2.4.8	Changing the Subject of a Formula	0	32	0%
S2.4.9	Further Examples on Changing the Subject of a Formula	0	28	0%
S2.4.10	Finding an Unknown in a Formula	0	20	0%

S2.5.1	Simultaneous Linear Equations	0	0	
S2.5.2	Solving Simultaneous Linear Equations Using Elimination Method	0	38	0%
S2.5.3	More Examples on Elimination	0	30	0%
S2.5.4	Solving Simultaneous Linear Equations Using Substitution Method	0	18	0%
S2.5.5	Problem Solving Involving Simultaneous Equations	26	26	100%
S2.6.1	Pythagoras' Theorem	0	44	0%
S2.6.2	Applications of Pythagoras' Theorem	17	17	100%
S2.7.1	Pyramids	0	0	
S2.7.2	Volume of Pyramids	0	0	
S2.7.3	Total Surface Area of Pyramid	1	19	5%
S2.7.4	Cones	0	0	
S2.7.5	Comparison Between Pyramid and Cone	0	0	
S2.7.6	Volume of Cones	0	0	
S2.7.7	Surface Area of Cone	0	24	0%
S2.7.8	Volume of Sphere	0	0	
S2.7.9	Surface Area of Sphere	6	35	17%
S2.8.1	Choice of Appropriate Scales for Graphs	0	10	0%
S2.8.2	Graphs of Equations of the Form y = c	0	0	
S2.8.3	Graphs of Equations of the Form x = a	0	37	0%
S2.8.4	Graphs of Equations of the Form y = mx	0	0	
S2.8.5	Graphs of Equations of the Form y = mx + c	0	24	0%
S2.8.6	Solving Simultaneous Linear Equations Using Graphical Method	0	12	0%
S2.9.1	Quadratic Equations in Two Variables of the Form y = ax2	0	0	
S2.9.2	Graphs of General Quadratic Equations in Two Variables	0	16	0%
S2.9.3	Problem Solving Involving Quadratic Graphs	0	7	0%
S2.10.1	Introduction to Sets	0	0	
S2.10.2	Number of Elements in a Set	0	24	0%
S2.10.3	Venn Diagrams	0	10	0%
S2.10.4	Intersection of Sets	0	0	
S2.10.5	Union of Sets (U)	0	6	0%
S2.11.1	Collection, Organisation and Interpretation of Data	0	0	
S2.11.2	Dot Diagram	0	0	
S2.11.3	Grouped Data	0	0	

S2.11.4	Stem and Leaf Diagram	5	5	100%
S2.11.5	Mode	0	0	
S2.11.6	The Mean	0	0	
S2.11.7	Median	0	0	
S2.11.8	Comparison of the Mean, Median and Mode	14	36	39%
S2.11.9	Averages for Grouped Data	0	0	
S2.11.10	Using Calculator to Find Mean	6	6	100%
S2.12.1	Introduction (to Probability)	0	0	
S2.12.2	Experiments and Sample Space	0	0	
S2.12.3	Definition of Probability	35	35	100%
S3.1.1	Solving Quadratic Equations by Factorisation	0	22	0%
S3.1.2	Solution by Completing the Square	0	34	0%
S3.1.3	General Solution to a Quadratic Equation	0	15	0%
S3.1.4	Equations Reducible to a Quadratic Equation	0	16	0%
S3.1.5	Problems Involving Quadratic Equations	20	20	100%
S3.2.1	Multiplication Law of Indices	0	19	0%
S3.2.2	Division Law of Indices	0	24	0%
S3.2.3	Power Law of Indices	0	0	
S3.2.4	More Laws of Indices	0	34	0%
S3.2.5	Zero and Negative Indices	0	40	0%
S3.2.6	Fractional Indices	0	54	0%
S3.2.7	The Standard Form	0	0	
S3.2.8	Common Prefixes Used in Everyday Life Situation	0	32	0%
S3.2.9	Use of Calculator	4	33	12%
S3.3.1	Inequalities	0	0	
S3.3.2	Solving Inequalities	0	29	0%
S3.3.3	Difference between < and ¡Ü	0	20	0%
S3.3.4	Problem Solving Involving Inequalities	8	8	100%
S3.3.5	Linear Inequalities in One Variable	2	46	4%
S3.4.1	Revision	0	0	
S3.4.2	Length of Line Segment	0	12	0%
S3.4.3	Gradient of a Straight Line	0	14	0%
S3.4.4	Equation of a Straight Line	0	28	0%
S3.5.1	Introduction to Matrices	0	0	
S3.5.2	Some Special Matrices	0	42	0%
S3.5.3	Addition and Subtraction of Matrices	0	0	
S3.5.4	Rules for Matrix Addition and Matrix Subtraction	0	44	0%

S3.5.5	Multiplication of a Matrix by a Real Number	1	30	3%
S3.5.6	Multiplication of Matrices	0	17	0%
S3.5.7	Use of Matrices in Solving Everyday Life Problems	8	8	100%
S3.6.1	Profit and Loss as a Percentage of Cost/Sale Price	11	11	100%
S3.6.2	Further Examples on Percentages	9	9	100%
S3.6.3	Simple Interest	17	17	100%
S3.6.4	Compound Interest	9	9	100%
S3.6.5	Hire Purchase	14	14	100%
S3.6.6	Money Exchange	24	24	100%
S3.6.7	Taxation	5	5	100%
S3.6.8	Personal and Household Finances	13	13	100%
S3.6.9	Interpretation and Use of Tables and Charts	0	0	
S3.6.10	Strategies in Problem Solving	12	12	100%
S3.7.1	Applications of Graphs in Practical Situations	0	0	
S3.7.2	Conversion Graphs	5	5	100%
S3.7.3	Travel Graphs	5	5	100%
S3.7.4	Drawing of Graphs	8	13	62%
S3.7.5	Problem Solving Involving Linear Graphs	9	9	100%
S3.8.1	Congruent Triangles	0	0	
S3.8.2	Congruency Tests	0	59	0%
S3.8.3	Simple Applications of Congruent Triangles	5	5	100%
S3.8.4	Similar Triangles	0	0	
S3.8.5	Tests for Similarity between Two Triangles	3	41	7%
S3.9.1	Area of Similar Figures	2	20	10%
S3.9.2	Volumes of Similar Figures	5	18	28%
S3.10.1	Trigonometrical Ratios	0	12	0%
S3.10.2	Value of Trigonometrical Ratios	0	0	
S3.10.3	Use of Calculator	0	17	0%
S3.10.4	Solving Right-Angled Triangles Using Trigonometrical Ratios	0	18	0%
S3.10.5	Finding the Value of an Angle with Trigonometrical Ratios	0	26	0%
S3.10.6	Practical Applications of Trigonometry	16	16	100%
S3.10.7	More Examples on Applications of Trigonometry	9	14	64%
S3.11.1	Trigonometrical Ratios	0	24	0%

S3.11.2	Area of Triangle	0	17	0%
S3.11.3	The Sine Rule	2	31	6%
S3.11.4	The Cosine Rule	0	19	0%
S3.11.5	Bearings	6	25	24%
S3.11.6	Three-Dimensional Problems	0	7	0%
S3.11.7	Further Examples of Three-Dimensional Problems	1	7	14%
S3.12.1	Area and Circumference of a Circle (Revision)	2	16	13%
S3.12.2	Length of Arc and Area of Sector	4	35	11%
S3.12.3	The Radian (rad)	0	27	0%
S3.12.4	Radian, Arc Length and Area of Sector	0	21	0%
S3.13.1	Symmetrical Properties of Circles	0	14	0%
S3.13.2	Angle of Properties of Circles	0	29	0%
S3.13.3	Angles in Opposite Segments of a Circle	0	14	0%
S3.13.4	Problems on Angle Properties of Circles	0	5	0%
S3.13.5	Tangents	0	0	
S3.13.6	Tangents from an External Point	0	23	0%
S4.1.1	Graphs of Cubic Functions	0	0	
S4.1.2	Graphs of Reciprocal Functions	0	0	
S4.1.3	Graphs of the Function y = a/x2	0	0	
S4.1.4	Graphs of Exponentials Functions	0	13	0%
S4.1.5	Sketches of Some Important Graphs	0	0	
S4.1.6	Sketching Graphs of Quadratic Functions	0	0	
S4.1.7	Graphs of the form y = ± (x - a) (x - b)	0	0	
S4.1.8	Graphs of the form y = ± (x – p)2 + q	0	0	
S4.1.9	Graphical Solution of Quadratic Equations	0	32	0%
S4.2.1	Linear Distance-Time Graphs	0	0	
S4.2.2	Gradient of a Curve	0	0	
S4.2.3	Gradient of a Distance-Time Curve	15	25	60%
S4.2.4	Speed-Time Graphs	0	0	
S4.2.5	Graphs in Practical Solutions	16	16	100%
S4.3.1	Scalar and Vector Quantities	0	0	
S4.3.2	Terminologies and Notation of Vectors	0	0	
S4.3.3	Equal Vectors	0	0	
S4.3.4	Vectors which are Opposite	0	0	
S4.3.5	Column Vectors	0	18	0%
S4.3.6	Addition of Vectors	0	0	
S4.3.7	Zero Vectors	0	0	
S4.3.8	Subtraction of Vectors	0	91	0%
S4.3.9	Scalar Multiple of a Vector	0	51	0%

S4.3.10	Expression of a Given Vector in Terms of Two Vectors	0	11	0%
S4.3.11	Position Vectors	0	19	0%
S4.4.1	Mean (Revision)	6	6	100%
S4.4.2	Standard Deviation	10	14	71%
S4.5.1	Cumulative Frequency Table	0	0	
S4.5.2	Cumulative Frequency Curve	6	6	100%
S4.5.3	Median, Quartiles and Percentiles	0	0	
S4.5.4	Interquartile Range	6	10	60%
S4.5.5	Box-and-Whisker Plots	9	9	100%
		802	4183	19%

Appendix F: Comparable Chapters Across Three Textbooks

Lesson	Pearson Chapter Title	#Modeling	#WordProbs	Total Problems	% Word Probs	Lesson	Glencoe Chapter Title	#Modeling	#WordProbs	Total Problems	% Word Probs	Singapore Chapter Title	Year	Chapter	Page	# Word Probs	Total Probs	% Word Prob
1-1	Variables and Expressions	4	10	41	24.4	1-1	Variables and Expressions	2	6	42	14.3	Notations in Algebra	1	5	91	17	23	73.9
1-4	Properties of Real Numbers	1	5	57	8.8	1-3	Properties of Numbers	0	11	60	18.3	Rules for Operating on Integers & Formulae	1	2	45	0	21	0.0
1-7	The Distributive Property	1	7	90	7.8	1-4	The Distributive Property	0	6	59	10.2	Use of Brackets in Simplification	1	5	98	0	30	0.0
2-5	Literal Equations and Formulas	1	11	50	22.0	2-8	Literal Equations and Dim Analysis	0	9	40	22.5	Changing the Subject of a Formula	2	4	138	0	32	0.0
2-10	Change Expressed as Percent	2	9	41	22.0	2-7	Percent of Change	2	12	49	24.5	Increasing/ Decreasing a Quantity by a Given Percentage	1	11	255	16	20	80.0
5-1	Rate of Change and Slope	2	9	58	15.5	3-3	Rate of Change and Slope	0	7	52	13.5	Gradient of a Straight Line	1	12	280	0	24	0.0
5-3	Slope-Intercept Form	1	6	62	9.7	4-2	Writing EQ SlpINt Form	1	14	52	26.9	Equation of a Straight Line	3	4	82	0	28	0.0
6-1	Solving Systems by Graphing	1	4	42	9.5	6-1	Graphing Systems of Eq	2	5	52	9.6	Solving Simultaneous Linear Equations using Graphical Method	2	8	250	0	12	0.0
6-2	Solving Systems Using Substitution	1	7	44	15.9	6-2	Substitution	0	5	31	16.1	Solving Simultaneous Linear Equations using Substitution Method	2	5	161	0	18	0.0
6-4	Applications of Linear Systems	1	15	28	53.6	6-5	Applying Systems	1	10	26	38.5	Problem Solving Involving Simultaneous Equations	2	5	164	26	26	100.0
7-2	Multiplying Powers with the Same Base	1	7	53	13.2	7-1	Mult Properties of Exponents	0	5	67	7.5	Multiplication Law of Indices	3	2	23	0	20	0.0
7-4	Division Properties of Exponents	1	11	93	11.8	7-2	Division Properties of Exponents	0	6	65	9.2	Division Law of Indices	3	2	24	0	24	0.0
8-3	Multiplying Binomials	1	8	50	16.0	8-3	Multiplying Polynomials	0	5	49	10.2	Further Algebraic Expansions	2	3	77	0	30	0.0
8-4	Multiplying Special Cases	1	3	58	5.2	8-4	Special Products	0	3	61	4.9	Perfect Squares and Difference of Two Squares	2	3	81	1	38	2.6
8-7	Factoring special cases	1	3	57	5.3	8-8	Differences of Squares	0	4	63	6.3	Factorisation Using Algebraic	2	3	89	0	62	0.0

												Identities						
9-1	Quadratic Graphs and Their Properties	1	6	49	12.2	9-1	Graphing Quadratic Fns	0	5	74	6.8	Graphs of General Quadratic Equations in Two Variables	2	9	264	0	16	0.0
9-5	Completing the Square	1	6	49	12.2	9-4	Solving by Completing Square	1	4	52	7.7	Solutions by Completing the Square	3	1	5	0	20	0.0
9-6	The Quadratic Formula & the Discriminant	1	5	49	10.2	9-5	Quadratic Formula	1	5	58	8.6	General Solution to a Quadratic Equation	3	1	9	0	15	0.0
10-1	Pythagorean Theorem	1	8	42	19.0	10-5	Pythagorean Theorem	0	7	50	14.0	Pythagorean Theorem & Applications	2	5	177	17	61	27.9
11-1	Simplifying Rational Expressions	1	7	53	13.2	11-3	Simplifying Rational Expressions	0	5	48	10.4	Simple Algebraic Fractions	2	4	119	0	34	0.0
11-2	Multiplying and dividing Rational Expressions	1	8	69	11.6	11-4	x, / Rational Expressions	0	14	59	23.7	Multiplication and Division of Algebraic Fractions	2	4	122	0	12	0.0
11-3	Dividing Polynomials	1	5	59	8.5	11-5	Dividing Polynomials	0	4	50	8.0	Further Examples on Simplification of Algebraic Fractions	2	4	123	0	14	0.0
11-4	Adding and Subtracting Rational Expressions	1	5	54	9.3	11-6	+, - Rational Expressions	0	9	75	12.0	Addition and Subtraction of Algebraic Fractions	2	4	126	0	30	0.0
		28	**165**	**1248**	**13.2**			**10**	**161**	**1234**	**13.0**					**77**	**610**	**12.6**

Appendix G: Kappa Statistic

To calculate the Index between two raters in SPSS, follow the steps below:

- Enter your rater's data.
- Select in the menu the option named "analyze."
- Select "descriptive statistics."
- Select the "options" crosstab.
- Designate the raters for the columns and rows.
- Select the Kappa Statistic.

The program will output the Kappa Statistic.