Essays on Open Economy Macroeconomics

Seunghoon Na

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Abstract

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Seunghoon Na

This Ph.D. dissertation contains three essays on Open Economy Macroeconomics. The first chapter investigates monetary policy problem of emerging economies known as the Tošovský Dilemma, which says that when an emerging economy experiences a boom associated with capital inflows and exchange rate appreciation, it is not appealing to tighten monetary policy to counteract inflationary pressures as this might further exacerbate inflows and appreciation. In the chapter, I develop an intertemporal general equilibrium framework of the monetary transmission mechanism to investigate how this dilemma shapes optimal monetary policy. In the model, financing is decentralized and collateralized by physical capital, which is nontradable and costly to adjust over time. The Dilemma materializes when there is a positive external shock that increases capital inflows and generates real exchange rate appreciation and inflation in the nontradable sector, all of which are inefficient. Contrary to conventional wisdom, the Ramsey optimal monetary policy calls for lowering the policy rate in such circumstances in order to suppress capital inflows and appreciation, while accepting inflation in the nontradable sector. If the capital flows can be controlled by an additional policy instrument, then optimal policy becomes countercyclical, as in the conventional framework without the Dilemma.

The second and third chapters focus on dynamics of labor shares over the business cycles in small open economies. The second chapter uses annual labor shares data of 40 years for 35 small open economy countries and finds three empirical regularities. First, labor shares are not constant, but they are as volatile as output. Second, labor shares in emerging economies are about twice as volatile as labor shares in advanced economies. Third, labor shares in emerging economies are procyclical on average, whereas they are countercyclical in most advanced economies. The empirical findings offer a skeptical view of the conventional beliefs

about the unitary elasticity of substitution between capital and labor, and countercyclical labor shares in the short-run.

The third chapter paper builds a theoretical model which can comprehensively explain the empirical findings in the second chapter. The model is a dynamic stochastic general equilibrium, small open economy, composed of tradable and nontradable sectors with CES production functions. In the model, there are two margins of labor share fluctuations over the business cycles, which are fluctuations of the capital-labor ratio in each sector and fluctuations in the relative value of sectoral production. The estimated models show a countercyclical labor share and volatility near that of output in Canada, and procyclical and excessively volatile labor share in Mexico, all of which are in line with the data.

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Chapter 1

A Theory of the Tošovský Dilemma

1.1 Introduction

In emerging economies business cycles, economic booms are typically associated with capital inflows and real exchange rate appreciation. The inflows and appreciation are thought to be excessive, endangering economic and financial stability, thus motivating a need for appropriate policy intervention. This feature constraints possible monetary policy reactions for stabilizing domestic inflation. Policymakers often argue that during an economic boom, a monetary policy intended to reduce inflationary pressure, such as tightening the domestic interest rate, can encourage further capital inflows and real appreciation which are not desirable. This policy dilemma is sometimes referred to as the Tošovský Dilemma, named after the former governor of Czech National Bank, Josef Tošovský. Tošovský was concerned about expansionary capital surges in transition countries in Central and Eastern Europe during the mid-1990s that constrained the ability of the countries to tighten interest rates in order to reduce domestic inflation (see Lipschitz, Lane, and Mourmouras (2002)).

The Dilemma is not unique to those transition countries during that particular period. The commodity price booms in emerging market commodity exporters such as Brazil, Chile, and Colombia during the mid-2000s caused domestic economic booms accompanied by huge capital inflows and real appreciation, which causes the same monetary policy dilemma. Blanchard, Ostry, Ghosh, and Chamon (2016) describe the Dilemma as the general phenomenon of the emerging economies as follows:

> *"Emerging market policymakers ... see capital inflows as leading to credit booms and an increase in output, [the boom] can only be offset by an increase in the policy rate. They point to a policy Dilemma: while the direct effect of an increase in the policy rate is limit the increase in output, the indirect effect is to encourage even more capital inflows, potentially dominating the direct effect."*

The Dilemma cannot be captured in a closed-economy, New Keynesian (NK hereafter) framework of monetary policy (for example, in Woodford (2003)) in which the capital account is closed, while workhorse open economy NK models (see, for example, Clarida, Gali, and Gertler (2001) and Gali and Monacelli (2005)) also provide few insights into the Dilemma. These open economy versions of the NK model usually restrict attention to the special case in which the equilibrium exchange rate and the current account reflect efficient allocation outcomes across countries. As long as the international resource allocation is efficient, there is no conceptual difference between closed and open economies in the optimal monetary policy regime that achieves domestic price stability.

In this paper, I develop a dynamic general equilibrium model of the monetary transmission mechanism of an open economy to study how monetary policy is shaped by the Tošovský Dilemma. The model is a decentralized, small open economy in which nominal and financial frictions coexist. Income in the economy come from endowments of tradable goods and the production of nontradable goods and collateral. Staggered price adjustment of nontradable good producers generates the nominal friction, while the financial friction is due to the collateralized borrowing of households, which uses physical capital needed for nontradable goods production. Nontradable capital decouples in valuation the capital from the price of goods traded, which creates a wealth effect through the interplay of the real exchange rate and the collateral constraint. Since capital is costly to adjust, the price of capital co-moves with investment demand.

In a stationary equilibrium around the steady state in which the collateral constraint is binding, an exogenous increase in the tradable endowment, which can be thought as a positive terms of trade and/or commodity price shock, generates a real appreciation, namely an increase in the relative price of nontradable goods to tradable goods, which raises the value of collateral and in this way encourages additional capital inflows by relaxing the collateral constraint. Capital inflows expand demand for the tradable good which transmits to an expansion of demand for the nontradable good. The expansion of aggregate demand increases the price of capital, which encourages additional capital inflows by relaxing the collateral constraint further. This self-reinforcing process leads to excessive aggregate external indebtedness, which happens because households' borrowing decisions are decentralized and they fail to internalize the collective effect of their borrowing decisions on the aggregate external debt of the economy. Since the economy eventually comes back to the steady state without the relaxation of the constraint in the long-run, the excessive external debt leads to consumption volatility that deteriorates welfare. In addition, the expansion of demand for nontradable goods leads to inflationary pressure in the nontradable sector. This dis-equalizes households' marginal rate of substitution between consumption and worked hours and the marginal rate of transformation by the price rigidity, generating another deterioration in welfare.

The two inefficiencies in the model economy give rise to the Tošovský Dilemma. With respect to the positive tradable endowment shock, monetary tightening against inflationary pressure in the nontradable sector is not supported as the optimal policy since it causes further real appreciation which encourages additional capital inflows. Instead, the Ramsey optimal monetary policy is to ease the domestic interest rate to smooth external debt by creating a real depreciation. The monetary easing takes on inflation in the nontradable sector, which implies that the Dilemma shapes optimal monetary policy towards financial rather than price stability.

In an extension of the model, I investigate how optimal monetary policy changes if the external debt is denominated in a local currency or capital flows can be controlled by another policy instrument, namely a capital control tax. In both cases, the model predicts that the optimal interest rate policy returns to a counter-cyclical regime focused on price stability. My findings from the extension with the capital control tax are supportive of recent studies on monetary and financial policy coordination (see, for example, Klein and Shambaugh (2015), Aoki, Beningno, and Kiyotaki (2016), and Davis and Presno (2017)).

To the best of my knowledge, this paper is the first attempt to investigate the Tošovský Dilemma by developing an intertemporal general equilibrium framework of the monetary transmission mechanism. Although the Dilemma has been known for almost two decades (see, for example, Laurens and Cardoso (1998), Lipschitz et al. (2002), Brunnermeier et al. (2008), De Gregorio (2010), and Blanchard et al. (2016)), a rigorous model for economic analysis has not been developed. First, the conventional open economy model broadly relies on an intertemporal approach of current account in which international resource allocation is efficient or near-efficient. Second, state-of-the-art literature on the Dilemma has focused on speculative capital flows stemming from the risk-taking behavior of foreign investors, such as the profitable carry trade, real exchange rate bubbles, and market sentiments. Although this channel appears to have empirical relevance, developing an intertemporal general equilibrium framework that incorporates this channel is difficult as it embeds behavioral elements that violate assumptions of rational expectations and no-arbitrage. Third, state-of-the-art NK models for monetary policy analysis mainly constrain the monetary policy reaction function to satisfy the Taylor principle, which inherently embeds an aggressive, anti-inflation regime into monetary policy.

The model in this paper seeks to maintain the simplest framework by taking a stylized approach to a rational expectations equilibrium. The modeling strategy for describing the Dilemma appeals to real, financial, and nominal frictions that are stylized and commonly used in the macro and monetary literature. For analysis of the optimal monetary policy, this paper does not assume a specific form for the interest rate reaction function and retrieves the policy function as the solution of the Ramsey problem.

The remainder of the paper proceeds as follows. Section 1.2 relates the contribution of the paper to the previous literature. Section 1.3 develops the model of the monetary transmission mechanism in a small open economy. Section 1.4 analyzes the Tošovský Dilemma in the model. Section 1.5 discusses the Ramsey optimal interest rate policy. Section 1.6 examines an extension of the model with local currency denominated external debt. Section 1.7 extends the model by allowing for a tax on the capital flows. Section 1.8 concludes.

1.2 Related Literature

This paper contributes to five branches of literature.

Optimal monetary policy. This paper builds a dynamic general equilibrium framework of the monetary transmission mechanism for optimal monetary policy analysis, which is the fundamental agenda of the monetary model developed in Woodford (2003) and the subsequent New Keynesian literature. Unlike the standard NK models which typically assume a closed economy, this paper explores the optimal monetary policy in the small open economy context with an open capital account in the spirit of Mundell (1963) and Fleming (1962). The model investigates the imperfections in international capital markets and concludes that optimal monetary policy yields a non-trivial deviation from the traditional regime of price stability. This result yields different policy implications from workhorse open economy models such as Clarida et al. (2001) and Gali and Monacelli (2005).

Breakdown of divine coincidence in open economies. The model described in this paper considers the case in which the divine coincidence in open economies, i.e, that capital account openness does not yield different monetary policy trade-offs from the closed economy context, breaks down due to the Tošovský Dilemma. A strand of literature that describes the breakdown of the divine coincidence suggests a channel of currency misalignment caused by local currency pricing or pricing-to-market (for example, Corsetti et al. (2011), Burstein and Gopinath (2014), and Casas et al. (2017)). Unlike this literature, this paper investigates a breakdown of the divine coincidence due to a financial wedge in international financial markets.

This financial wedge is related to the idea of balance sheet effects of exchange rates motivated by empirical findings in Hausmann et al. (2001) and Calvo and Reinhart (2002). Unlike the early literature of modeling the balance sheet effect (see, for example, Cespedes et al. (2004) and Cook (2004)) which assumes heterogeneous households and focuses on the financial accelerator channel in the spirit of Bernanke et al. (1999), the baseline model of this paper keeps identical households and utilizes the Fisherian amplification channel of asset prices. The balance sheet effect of the exchange rate in this paper comes from the non-tradability of collateral which yields a wealth effect via the real exchange rate that is absent in the literature. The model shows strong balance sheet effects via the exchange rate and predicts optimal monetary policy towards stabilizing balance sheets, without concerns about the distributional effects of monetary policy.

The monetary policy dilemma in emerging economies. This paper formalizes the notion of the monetary policy dilemma due to cross-border capital flows in emerging economies, a widely discussed topic by policymakers. The existing literature documents the dilemma (see, for example, Laurens and Cardoso (1998), Lipschitz et al. (2002), and Blanchard et al. (2016)), but give loose arguments without a rigorous model - this paper contributes by providing a theoretical framework of the dilemma. One candidate channel of the monetary policy dilemma is the risk-taking behavior of international investors (see, for example, Brunnermeier et al. (2008), De Gregorio (2010)). The model in this paper instead formalizes the channel of the dilemma through the lens of inefficient behavior of domestic borrowers. The strategy allows the model to be easily extended to an intertemporal setting for dynamic analysis while generating similar equilibrium phenomena in line with the investors' risk-taking behavior channel.

Fisherian amplification of asset prices. This paper uses the Fisherian channel of asset price amplification (Fisher (1933)) to formalize the financial friction. The recent literature that develops the idea for business cycle analysis for emerging economies (see, for example, Auernheimer and Carcía-Saltos (2000), Mendoza (2010), and Korinek (2011)) assumes a real economy without a monetary transmission mechanism. In contrast, the Fisherian channel of asset prices in this paper interplays with monetary policy. The paper thus provides richer insight into the financial leverage of an economy interconnected with domestic monetary policy, which is absent in the literature.

Monetary policy and financial stability. This paper enters the lively debates on the role of monetary policy in achieving financial stability. One side (for example, see Stein (2012) and Adrian and Liang (2016)) supports the role of monetary policy *leaning against the wind* in achieving financial stability by controlling risk-taking behavior in the financial sector. The other side (see, for example, Svensson (2016)) argues that monetary policy and financial stability policies are different. It claims that although there can be room for coordination between the two policies, financial stability should be achieved by macroprudential policy, while monetary policy should focus on its original goal of price stability. The literature, however, is mainly based on the closed economy. This paper claims that the discussion from the emerging economy perspective is quite different because of the existence of cross-border capital flows. This paper supports the role of monetary policy for financial stability, but it is not necessarily leaning against the wind policy. The optimal monetary policy by the Dilemma becomes pro-cyclical to stabilize the asset price in international prices. This open economy specific result gives richer insight into this lively topic.

Capital controls. An extension of the model considers a capital control tax as another Ramsey policy instrument. The extension relates to the recently growing literature on capital control taxes for business cycle stabilization (see, for example, Bianchi (2011), Farhi and Werning (2012), Schmitt-Grohé and Uribe (2016)). Unlike the literature, which typically assumes a real model, this paper includes a capital control tax in a nominal model to investigate the interplay with monetary policy, in line with the recent NK literature on monetary and financial policy coordination (for example, see Aoki et al. (2016) and Davis and Presno (2017)). This paper formally solves the Ramsey optimal coordination of the two instruments and supports the finding that coordination enhances the role of monetary policy for price stability.

1.3 The Model

In this section, I develop a theoretical framework for the analysis of optimal interest rate policy, under circumstances in which an economy faces the Tošovský Dilemma. The framework is dynamic stochastic general equilibrium model of the monetary transmission mechanism in a small open economy. The economy has multiple sectors, one for tradable and another for nontradable goods. The tradable sector is assumed to follow a stochastic endowment process, while the nontradable sector has a production structure subject to staggered price adjustment by monopolistically competitive firms. Households' financing decisions are decentralized, and the financing must be collateralized. I focus on the equilibrium in which the borrowing constraint is always binding, which yields a welfare cost due to a pecuniary externality. I begin by describing the households' problem.

1.3.1 Households

The economy is populated by a continuum of identical households on the real interval $[0, 1]$. A household's preferences are a function of its consumption and labor supply, which are identical across households. Each household seeks to maximize its lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, h_t\right),$$

where $U(\cdot, \cdot)$ is a periodic utility function that is strictly positive in its first argument, strictly negative in its second argument, and strictly concave. Variable c_t is the final consumption basket in real terms, and h_t is the number of worked hours supplied by the household. The final consumption basket is a composite of tradable consumption c_t^{τ} and nontradable consumption c_t^n. The aggregation technology is assumed to be

$$c_t = A\left(c_t^{\tau}, c_t^n\right),$$

where $A(\cdot, \cdot)$ is a CES-Armington (see Armington (1969)) aggregator that satisfies $A_1(\cdot, \cdot) > 0$, $A_2(\cdot, \cdot) > 0$, $A_{11}(\cdot, \cdot) < 0$, $A_{22}(\cdot, \cdot) < 0$, and $A_{12}(\cdot, \cdot) = A_{21}(\cdot, \cdot) > 0$, where a subscript with a number $j \in \{1, 2\}$ denotes the partial derivative with respect to the j^{th} argument.

Each household trades domestic and international assets. D_t^d is the position of domestic debt denominated in domestic currency, assumed at time t and due at $t + 1$ and d_t is the position of international debt denominated in foreign currency and follows the same maturity structure as domestic debt. Variables i_t and r_t^* refer to the nominal rates of interest for the domestic and international assets in each currency unit, respectively. I assume that the rest of the world has zero inflation so that the nominal international financial variables, d_t and r_t^*, are real variables in foreign currency units. $\mathcal{E}_t$ denotes the nominal exchange rate that determines the units of domestic currency needed to purchase one unit of foreign currency.

It is assumed that the household owns physical capital k_t. The evolution of physical capital has the following law of motion with costly investment,

$$k_{t+1} = (1-\delta)k_t + \mathcal{S}(iv_t, iv_{t-1}), \tag{1.1}$$

where $\delta \in (0,1)$ is the actualized depreciation rate of physical capital, iv_t is investment in physical capital, and $\mathcal{S}(iv_t, iv_{t-1})$ is investment net of adjustment costs. The household also owns firms such that firms' profits are rebated to the household in every period. The household's period-by-period budget constraint in domestic currency is given by

$$P_t^\tau c_t^\tau + P_t^n c_t^n + P_t iv_t + (1+i_{t-1})D_{t-1}^d + \mathcal{E}_t(1+r_{t-1}^*(d_{t-1}))d_{t-1}$$

$$= P_t^\tau y_t^\tau + (1-\tau_t^p)\left(W_t h_t + R_t^k k_t\right) + D_t^d + \mathcal{E}_t d_t + \Psi_t + T_t,$$

where P_t^τ and P_t^n are the nominal prices of tradable good and nontradable good, respectively. P_t is the consumer price index (CPI), R_t^k is the nominal rate of return of physical capital, and W_t is the nominal wage rate for hours worked by the household. It is assumed that the factor income $W_t h_t + R_t^k k_t$ can be taxed by the government at rate τ_t^p. Variable Ψ_t refers to the nominal profit of firms rebated to the household, and T_t refers to a nominal lump sum transfer by the government. The interest rate of foreign debt $r_t^* \equiv r_t^*(d_t)$ is an explicit function of the foreign debt position of each household. This form of internal debt elastic interest rate (IDEIR) induces stationarity in the external debt position (see Schmitt-Grohé and Uribe (2003)).

The nominal variables have unit roots. To stationarize them, I divide the budget constraint by the aggregate CPI P_t. The transformed constraint is given by

$$p_t^\tau c_t^\tau + p_t^n c_t^n + iv_t + \frac{1+i_{t-1}}{1+\pi_t} d_{t-1}^d + e_t(1+r_t^*(d_{t-1}))d_{t-1}$$

$$= p_t^\tau y_t^\tau + (1-\tau_t^p)\left(w_t h_t + r_t^k k_t\right) + d_t^d + e_t d_t + \psi_t + t_t, \tag{1.2}$$

where $p_t^\tau \equiv \frac{P_t^\tau}{P_t}$ is the price of the tradable good relative to CPI, $p_t^n \equiv \frac{P_t^n}{P_t}$ is the price of the nontradable good relative to CPI, $1+\pi_t = \frac{P_t}{P_{t-1}}$ is the gross CPI inflation rate, and $e_t = \frac{\mathcal{E}_t}{P_t}$ is a real exchange rate, the nominal exchange rate divided by CPI. Variables $w_t \equiv \frac{W_t}{P_t}$ and $r_t^k \equiv \frac{R_t^k}{P_t}$ are the real wage and the rate of return of capital, respectively. Variable $d_t^d \equiv \frac{D_t^d}{P_t}$ is the real position of home debt, $\psi_t \equiv \frac{\Psi}{P_t}$ is real profit, and $t_t \equiv \frac{T_t}{P_t}$ is the real government transfer.

When the household makes its financing decision, the amount borrowed is constrained to be at most a fraction $\kappa > 0$ of the price of physical capital. The collateral constraint is given by

$$\mathcal{E}_t d_t + D_t^d \leq \kappa Q_t k_{t+1},$$

where Q_t is the Tobin's q of physical capital times CPI. The transformed constraint divided by CPI is

$$e_t d_t + d_t^d \leq \kappa q_t k_{t+1}, \tag{1.3}$$

where $q_t \equiv \frac{Q_t}{P_t}$ is the Tobin's q. The collateral constraint comes from an incentive problem between lenders and households. The parameter κ is interpreted as the fraction of the collateral that the lenders can liquidate when the households are delinquent on their debt obligation. Therefore the collateral constraint reflects an incentive compatibility constraint for the households which prevents delinquency on their debt obligation in equilibrium (see Kiyotaki and Moore (1997)). The households' borrowing decisions are decentralized and thus each household fails to internalize the effect of its financial decisions on the aggregate indebtedness of the economy.

The household is also subject to No-Ponzi conditions for debt accumulation, which are

$$\begin{aligned}
\lim_{j\to\infty} \mathbb{E}_t \left[\frac{d_{t+j}^d}{\prod_{s=0}^{j} \frac{1+i_{t+s}}{1+\pi_{t+s+1}}} \right] &\leq 0, \\
\lim_{j\to\infty} \mathbb{E}_t \left[\frac{d_{t+j}}{\prod_{s=0}^{j} (1+r_{t+s}^*)} \right] &\leq 0.
\end{aligned}$$

The Lagrangian of the household's problem is given by

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ U\left(A\left(c_t^{\tau}, c_t^{n}\right), h_t\right) + \lambda_t \Big(p_t^{\tau} y_t^{\tau} + (1-\tau_t^{p})(w_t h_t + r_t^{k} k_t) + d_t^{d} + e_t d_t + \psi_t + t_t$$

$$- p_t^{\tau} c_t^{\tau} - p_t^{n} c_t^{n} - iv_t - \frac{1+i_{t-1}}{1+\pi_t} d_{t-1}^{d} - e_t \left(1 + r_{t-1}^{*}(d_{t-1})\right) d_{t-1} \Big)$$

$$+ \lambda_t q_t \left((1-\delta)k_t + \mathcal{S}\left(iv_t, iv_{t-1}\right) - k_{t+1}\right) + \lambda_t \Theta_t \left(\kappa q_t k_{t+1} - e_t d_t - d_t^{d}\right) \Big\},$$

where $\beta^t \lambda_t$ and $\beta^t \lambda_t q_t$, $\beta^t \lambda_t \Theta_t$ are Lagrange multipliers associated with the sequential budget constraint (1.2), law of motion of capital accumulation (1.1), and the household's collateral constraint (1.3), respectively. At each time t, each household takes $\{d_{t-1}^{d}, d_{t-1}, k_t, y_t^{\tau}\}$ and prices as given, and chooses $\{c_t^{\tau}, c_t^{n}, h_t, iv_t, k_{t+1}, d_t^{d}, d_t\}$. The corresponding first order necessary conditions are

$$
\begin{aligned}
U_1(c_t, h_t) A_1(c_t^{\tau}, c_t^{n}) &= \lambda_t p_t^{\tau}, && (1.4)\\
\frac{A_2(c_t^{\tau}, c_t^{n})}{A_1(c_t^{\tau}, c_t^{n})} &= \frac{p_t^{n}}{p_t^{\tau}}, && (1.5)\\
-\frac{U_2(c_t, h_t)}{U_1(c_t, h_t) A_1(c_t^{\tau}, c_t^{n})} &= \frac{(1-\tau_t^{p}) w_t}{p_t^{\tau}}, && (1.6)
\end{aligned}
$$

$$1 = q_t \mathcal{S}_1\left(iv_t, iv_{t-1}\right) + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \mathcal{S}_2\left(iv_{t+1}, iv_t\right) \right], \tag{1.7}$$

$$(1-\kappa\Theta_t) q_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left((1-\delta) q_{t+1} + (1-\tau_{t+1}^{p}) r_{t+1}^{k}\right) \right], \tag{1.8}$$

$$\lambda_t \left(1 - \Theta_t\right) = \beta \mathbb{E}_t \left[\frac{1+i_t}{1+\pi_{t+1}} \lambda_{t+1} \right], \tag{1.9}$$

$$\mathbb{E}_t \left[\frac{1+i_t}{1+\pi_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \right] = \mathbb{E}_t \left[\left(1 + r_t^{*}(d_t) + r_t^{*\prime}(d_t) d_t\right) \frac{\lambda_{t+1}}{\lambda_t} \frac{e_{t+1}}{e_t} \right] \tag{1.10}$$

where $\mathcal{S}_j(\cdot,\cdot), j = 1, 2$ is the partial derivative of the function with respect to its first and second argument, respectively. In addition, the equilibrium should satisfy no-satiation condition which is given by

$$\lim_{j\to\infty} \mathbb{E}_t \left(\frac{e_{t+j} d_{t+j}}{\prod_{s=0}^{j} \frac{e_{t+s}(1+r^*_{t+s})}{e_{t+s-1}}} + \frac{d^d_{t+j}}{\prod_{s=0}^{j} \frac{1+i_{t+s}}{1+\pi_{t+s+1}}} \right) = \kappa \lim_{j\to\infty} \mathbb{E}_t \frac{q_{t+j} k_{t+1+j}}{\prod_{s=0}^{j} \frac{e_{t+s}(1+r^*_{t+s})}{e_{t+s-1}}}.$$

With a restriction on the stationarity of capital, $\lim_{j\to\infty} \mathbb{E}_t k_{t+1+j} < \infty$, and a restriction on the price of collateral to ensure a no-bubble equilibrium, $\lim_{j\to\infty} \mathbb{E}_t \frac{q_{t+j}}{\prod_{s=0}^{j} \frac{e_{t+s}(1+r^*_{t+s})}{e_{t+s-1}}} = 0$, the no-satiation condition becomes

$$\lim_{j\to\infty} \mathbb{E}_t \left(\frac{e_{t+j} d_{t+j}}{\prod_{s=0}^{j} \frac{e_{t+s}(1+r^*_{t+s})}{e_{t+s-1}}} + \frac{d^d_{t+j}}{\prod_{s=0}^{j} \frac{1+i_{t+s}}{1+\pi_{t+s+1}}} \right) = 0.$$

I assume the following form of CES-Armington aggregator,

$$A(c^\tau_t, c^n_t) = \left(\chi_\tau^{\frac{1}{\eta}} \left(c^\tau_t\right)^{1-\frac{1}{\eta}} + (1-\chi_\tau)^{\frac{1}{\eta}} \left(c^n_t\right)^{1-\frac{1}{\eta}} \right)^{\frac{1}{1-\frac{1}{\eta}}},$$

where $\chi_\tau \in (0,1)$ is the share parameter of the consumption of tradable good and $\eta > 0$ governs the intratemporal elasticity of substitution between tradable consumption and non-tradable consumption. The optimal composition of tradable and nontradable consumption maximizes the following profit function of aggregating

$$P_t A(c^\tau_t, c^n_t) - P^\tau_t c^\tau_t - P^n_t c^n_t,$$

and the CPI of the final consumption good c_t is derived as follows,

$$P_t = \left(\chi_\tau \left(P^\tau_t\right)^{1-\eta} + (1-\chi_\tau) \left(P^n_t\right)^{1-\eta} \right)^{\frac{1}{1-\eta}},$$

and dividing both sides by P_t yields the stationary relation of relative price of tradable good and non-tradable good,

$$1 = \left(\chi_\tau \left(p_t^\tau\right)^{1-\eta} + \left(1-\chi_\tau\right)\left(p_t^n\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{1.11}$$

Similarly to the final consumption basket, it is assumed that the final investment basket iv_t is composed of an aggregation of tradable and nontradable components. The aggregator of the investment basket has the same CES-Armington form, $iv_t = A\left(iv_t^\tau, iv_t^n\right)$. Given the prices and final investment basket iv_t, the household calculates the optimal composition of iv_t^τ and iv_t^n which maximize the profit from the aggregation

$$A(iv_t^\tau, iv_t^n) - p_t^\tau iv_t^\tau - p_t^n iv_t^n,$$

and the optimal decomposition for iv_t^τ and iv_t^n is then determined by

$$\frac{A_2\left(iv_t^\tau, iv_t^n\right)}{A_1\left(iv_t^\tau, iv_t^n\right)} = \frac{p_t^n}{p_t^\tau}, \tag{1.12}$$

$$iv_t = p_t^\tau iv_t^\tau + p_t^n iv_t^n. \tag{1.13}$$

1.3.2 Sectors

There are tradable and nontradable sectors in the economy. The tradable sector is assumed to follow a stochastic endowment process, while the nontradable sector has a monopolistically competitive production with producers' pricing decisions subject to some degree of staggered adjustment.

Tradable Endowment

There is a stochastic endowment in the tradable sector following a first order Markov process

$$\ln\left(y_{t+1}^\tau\right) = \rho_\tau \ln\left(y_t^\tau\right) + \epsilon_t^\tau, \tag{1.14}$$

where ρ_τ is the persistence of the process and ϵ_t^τ is the orthogonal innovation of the process, which has mean zero and standard deviation σ_ϵ^τ. The stochastic nature of the tradable endowment aims to describe foreign demand and supply shocks such as terms of trade and commodity price shocks which are considered to be external sources of business cycles in emerging economies. It can also be interpreted as productivity shocks in the tradable sector which create the Balassa-Samuelson effect.

Nontradable Production

In the nontradable sector, there is a continuum of monopolistically competitive firms on the real unit interval, $[0,1]$. Each firm $j \in [0,1]$ produces an intermediate good y_{jt}^n which is an ingredient in the final nontradable good y_t^n. The nontradable good is the standard Dixit-Stiglitz aggregator of the intermediary goods which takes the form

$$y_t^n = \left(\int_0^1 \left(y_{jt}^n \right)^{1-\frac{1}{\mu}} dj \right)^{\frac{1}{1-\frac{1}{\mu}}},$$

where μ is the constant elasticity of intratemporal substitution between the intermediaries reflective of a firm's market power. Here $\mu > 1$ is assumed to ensure positive marginal revenue for each firm. The optimal price index of nontradable good P_t^n is the result of intermediaries' $\{y_{jt}^n\}$ maximization of

$$P_t^n y_t^n - \int_0^1 P_{jt}^n y_{jt}^n dj,$$

where P_{jt}^n is the nominal price of each intermediary j. The necessary first order conditions yield the Marshallian demand function for each intermediary,

$$y_{jt}^n = y_t^n \left(\frac{P_{jt}^n}{P_t^n} \right)^{-\mu}, \tag{1.15}$$

as well as the price index of the nontradable good P_t^n in equilibrium,

$$P_t^n = \left(\int_0^1 \left(P_{jt}^n \right)^{1-\mu} dj \right)^{\frac{1}{1-\mu}},$$

which satisfies $P_t^n y_t^n = \int_0^1 P_{jt}^n y_{jt}^n dj$. Each firm j has the production function with fixed cost

$$\mathcal{F}(k_{jt}, h_{jt}) - \varphi,$$

where $\mathcal{F}(\cdot,\cdot)$ is a function that is homogeneous with degree one, and is strictly concave in its first and second arguments. Parameter $\varphi > 0$ is a constant fixed cost of production that prevents positive profits in the nonstochastic steady state. The production function $\mathcal{F}(\cdot,\cdot)$ and the fixed cost parameter φ are assumed to be same across firms.

Since households own the firms, the firms share the stochastic discount factor of the households. I transform the λ_t to Λ_t by dividing CPI,

$$\Lambda_t \equiv \frac{\lambda_t}{P_t},$$

then the nominal stochastic discount factor, $\Lambda_{t,t+s}$, is represented by

$$\Lambda_{t,t+s} \equiv \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} = \beta^s \frac{\lambda_{t+s}}{\lambda_t} \frac{P_t}{P_{t+s}}.$$

Each firm j's nominal factor cost is

$$W_t h_{jt} + R_t^k k_{jt},$$

where h_{jt} and k_{jt} are the inputs of labor and physical capital for the production of firm j, respectively. It is assumed that there is no under-production in each intermediary, which means

$$\mathcal{F}\left(k_{jt}, h_{jt}\right) - \varphi \geq y_{jt}^{n}. \tag{1.16}$$

The factor market is perfectly competitive and factor prices are fully flexible. Each firm j chooses factor inputs $\{h_{jt+s}, k_{jt+s}\}_{s=0}^{\infty}$ to maximize the following present value of expected profit

$$\mathbb{E}\sum_{s=0}^{\infty}\Lambda_{t,t+s}\left(P_{jt+s}y_{jt+s}^{n} - W_{t+s}h_{jt+s} - R_{t+s}^{k}k_{jt+s} + MC_{jt+s}\left(\mathcal{F}\left(k_{jt+s}, h_{jt+s}\right) - \varphi - y_{jt+s}^{n}\right)\right),$$

and the first order conditions yield the following time invariant conditional factor demand functions,

$$\begin{aligned} MC_{jt}\mathcal{F}_1(k_{jt}, h_{jt}) &= R_t^k, \\ MC_{jt}\mathcal{F}_2(k_{jt}, h_{jt}) &= W_t, \end{aligned}$$

where the shadow price MC_{jt} is the nominal marginal cost of the firm j. The marginal cost is identical across firms for all the time, $MC_{jt} = MC_t, \forall j, t$, because firms have the same functional form $\mathcal{F}(\cdot,\cdot)$ which is homogeneous with degree one, and factor costs R_t^k and W_t are same across the firms, resulting in the same capital-labor ratio across firms. Dividing the above two equations by P_t gives their real forms,

$$mc_t\mathcal{F}_1(k_t, h_t) = r_t^k, \tag{1.17}$$

$$mc_t\mathcal{F}_2(k_t, h_t) = w_t, \tag{1.18}$$

where $mc_t \equiv \frac{MC_t}{P_t}$ is the real marginal cost in terms of the CPI.

Supply

Firms in the nontradable sector are price setters because they have some degree of market power in the goods market. To generate an intertemporal Keynesian aggregate supply in the nontradable sector, I borrow the framework of staggered adjustment of nominal prices in the spirit of Calvo (1983) and Yun (1996).

In each period, a $1-\theta \in (0,1)$ fraction of firms can reset their prices $\tilde{P}^n_{jt}$, whereas the remaining fraction θ of firms are forced to keep their previous prices. I also assume that in each period, the firms who can reset their prices are independently and identically drawn among the continuum of the real interval, so that the draw is time and state independent. At each time t, a firm j who is able to set its product price sets $\tilde{P}^n_{jt}$ to maximize its expected profit, assuming it would not be able to reset the price again in the future. Then the firm j's corresponding Lagrangian is given by

$$\mathcal{L}^j \equiv \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\theta^s \left(\tilde{P}^n_{jt} y^n_{jt+s} - W_{t+s} h_{jt+s} - R^k_{t+s} k_{jt+s} + MC_{t+s} \left(\mathcal{F}\left(k_{jt+s}, h_{jt+s}\right) - \varphi - y^n_{jt+s} \right) \right).$$

The necessary first order condition with respect to $\tilde{P}^n_{jt}$, with replacing the $\{y_{jt}\}$ of the Lagrangian by the Marshallian demand function of the intermediary good (1.15), becomes

$$\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\theta^s y^n_{t+s} \left(\frac{\tilde{P}^n_{jt}}{P^n_{t+s}} \right)^{-\mu} \left((1-\mu)\tilde{P}^n_{jt} + \mu MC_{t+s} \right) = 0,$$

which implies that the firm's pricing seeks to equalize the present value of marginal revenue with the present value of marginal cost in the case where the firm can never reset its price again in the future. The optimality condition's equivalence between the two present values collapses to the equivalence between the static marginal revenue and cost when nominal prices are fully flexible, i.e., $\theta = 0$. Thus the draw of the fraction θ is time and state independent, and all firms who can change the price P^n_{jt} set an identical price, $\tilde{P}^n_{jt} = \tilde{P}^n_t$. Rearranging the first order condition with respect to $\tilde{P}^n_{jt}$ and applying $\tilde{P}^n_{jt} = \tilde{P}^n_t$ gives the equation

$$\tilde{P}_t^n = \frac{\mu}{\mu-1}\frac{\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\theta^s y_{t+s}^n \left(\frac{\tilde{P}_t^n}{P_{t+s}^n}\right)^{-\mu} MC_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\theta^s y_{t+s}^n \left(\frac{\tilde{P}_t^n}{P_{t+s}^n}\right)^{-\mu}}. \tag{1.19}$$

The evolution of the price index of the nontradable good P_t^n evolves according to

$$\begin{aligned}
(P_t^n)^{1-\mu} &= \int_0^1 P_t^n(j)^{1-\mu} dj \\
&= \int_{j\in\boldsymbol{\theta}} P_{t-1}^n(j)^{1-\mu} dj + \int_{j\in\boldsymbol{\theta^c}} P_t^n(j)^{1-\mu} dj \\
&= \theta\left(P_{t-1}^n\right)^{1-\mu} + (1-\theta)\left(\tilde{P}_t^n\right)^{1-\mu},
\end{aligned}$$

where $\boldsymbol{\theta}$ is the set of firms who cannot reset the current price and $\boldsymbol{\theta}^c$ is the complementary set. Dividing both sides of the above equation by P_t^n gives

$$1 = \theta(1+\pi_t^n)^{\mu-1} + (1-\theta)\left(\bar{p}_t^n\right)^{1-\mu}, \tag{1.20}$$

in which I define $\pi_t^n \equiv \frac{P_t^n}{P_{t-1}^n} - 1$ and $\bar{p}_t^n \equiv \frac{\tilde{P}_t^n}{P_t^n}$, which are the rate of inflation in the nontradable sector and desired nominal price of a $1-\theta$ fraction of producers relative to the price level in the nontradable sector, respectively.

Dividing equation (1.19) by P_t gives

$$\tilde{p}_t^n = \frac{\mu}{\mu-1}\frac{z_t}{f_t}, \tag{1.21}$$

where

$$\begin{aligned}
z_t &= \mathbb{E}_t \sum_{s=0}^{\infty} \frac{\lambda_{t+s}}{\lambda_t}(\beta\theta)^s y_{t+s}^n \left(\bar{p}_t^n \prod_{k=1}^{s} \frac{1}{1+\pi_{t+k}^n}\right)^{-\mu} mc_{t+s}, \\
f_t &= \mathbb{E}_t \sum_{s=0}^{\infty} \frac{\lambda_{t+s}}{\lambda_t}(\beta\theta)^s y_{t+s}^n \left(\prod_{k=1}^{s} \frac{1}{1+\pi_{t+k}}\right)\left(\bar{p}_t^n \prod_{k=1}^{s} \frac{1}{1+\pi_{t+k}^n}\right)^{-\mu},
\end{aligned}$$

where $\tilde{p}_t^n \equiv \frac{\tilde{P}_t^n}{P_t}$ refers to desired price of the firms in real terms, z_t and f_t describe the present value marginal cost and marginal revenue, respectively. Since $p_t^n \equiv \frac{P_t^n}{P_t}$, the two relative terms of the desired prices $\bar{p}_t^n$ and $\tilde{p}_t^n$ should be connected by

$$p_t^n = \frac{\tilde{p}_t^n}{\bar{p}_t^n}. \tag{1.22}$$

The variables z_t and f_t can be rewritten in the following recursive forms,

$$z_t = y_t^n (\bar{p}_t^n)^{-\mu} mc_t + \beta\theta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\bar{p}_t{}^n}{\bar{p}_{t+1}^n} \frac{1}{1+\pi_{t+1}^n} \right)^{-\mu} z_{t+1} \right], \tag{1.23}$$

$$f_t = y_t^n (\bar{p}_t^n)^{-\mu} + \beta\theta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1}{1+\pi_{t+1}} \right) \left(\frac{\bar{p}_t^n}{\bar{p}_{t+1}^n} \frac{1}{1+\pi_{t+1}^n} \right)^{-\mu} f_{t+1} \right]. \tag{1.24}$$

1.3.3 Fiscal Policy

In every period, the fiscal authority collects tax revenue $\tau_t^p \left(W_t h_t + R_t^k k_t \right)$ and gives lump-sum transfers T_t to households. I assume that the fiscal authority commits to a Ricardian regime of fiscal policy such that at the end of each period, the lump-sum transfer T_t to each household is equal to fiscal revenue. Thus, the following equation should hold in each period:

$$T_t = \tau_t^p \left(W_t h_t + R_t^k k_t \right),$$

and in real terms, it becomes

$$t_t = \tau_t^p \left(w_t h_t + r_t^k k_t \right).$$

1.3.4 Foreign Interest Rate

The interest rate on the external debt of the economy r_t^* is exogenously set by foreign lenders from the rest of the world. As described in section 1.3.1, the interest rate function has an internal debt elastic interest rate (IDEIR) form, specifically assumed to be

$$r_t^* = \bar{r} + \psi\left(e^{d_t - \bar{d}} - 1\right) + e^{\epsilon_t^r - 1} - 1, \tag{1.25}$$

where $\bar{r}$ and ψ are parameters and variable ϵ_t^r is an exogenous shock process following a first order Markov process

$$\log\left(\epsilon_{t+1}^r\right) = \rho_r \log\left(\epsilon_t^r\right) + \nu_{t+1}^r, \tag{1.26}$$

where $\rho_r \in (0,1)$ is the persistence of the process and ν_t^r is the orthogonal innovation with mean zero and standard deviation σ_r. The shock can be interpreted as a country premium shock or monetary policy shock from the rest of the world.

1.3.5 Equilibrium

I assume that the price of the tradable good and nominal exchange rate perfectly coincide such that law of one price (LOP) holds for tradable goods, which implies $P_t^\tau = \mathcal{E}_t$. The real form of the LOP divided by P_t is

$$p_t^\tau = e_t. \tag{1.27}$$

The inflation rate in the nontradable sector π_t^n and CPI inflation π_t are connected by the equation

$$\pi_t^n = \log\left(\frac{p_t^n}{p_{t-1}^n}\right) + \pi_t. \tag{1.28}$$

Aggregate profit from nontradable firms rebated to households is

$$\Psi_t \equiv \int_0^1 \Psi_{jt} di = P_t^n y_t^n - \left(W_t h_t + R_t^k k_t\right),$$

and stationarized, it is

$$\psi_t = p_t^n y_t^n - \left(w_t h_t + r_t^k k_t\right).$$

Integrating the Marshallian demand function of intermediaries across firms gives

$$\int_0^1 y_{jt}^n dj = y_t^n \int_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\mu} dj.$$

And the measure of price dispersion by the Calvo-Yun price rigidity x_t becomes

$$\begin{aligned} x_t \equiv \int_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\mu} dj &= \int_{\boldsymbol{\theta}} \left(\frac{P_{jt-1}^n}{P_t^n}\right)^{-\mu} dj + \int_{\boldsymbol{\theta}^c} \left(\frac{\tilde{P}_{jt}^n}{P_t^n}\right)^{-\mu} dj \\ &= \theta x_{t-1}(1+\pi_t^n)^{\mu} + (1-\theta)(\bar{p}^n)^{-\mu}. \end{aligned} \tag{1.29}$$

Integrating production function $\mathcal{F}(k_{jt}, h_{jt})$ across firms gives

$$\begin{aligned} \int_0^1 \mathcal{F}(k_{jt}, h_{jt}) dj &= \int_0^1 h_{jt} \mathcal{F}\left(\frac{k_{jt}}{h_j t}, 1\right) dj = \mathcal{F}\left(\frac{k_t}{h_t}, 1\right) \int_0^1 h_{jt} dj \\ &= \mathcal{F}\left(\frac{k_t}{h_t}, 1\right) h_t = \mathcal{F}(k_t, h_t). \end{aligned}$$

And since $y_{jt}^n = \mathcal{F}(k_{jt}, h_{jt}) - \varphi$, integrating both sides across firms yields

$$y_t^n = x_t^{-1} \left(\mathcal{F}(k_t, h_t) - \varphi\right). \tag{1.30}$$

Market clearing in the nontradable sector equates production of the nontradable good and nontradable absorption, which is

$$y_t^n = c_t^n + iv_t^n. \tag{1.31}$$

Domestic debt is cleared domestically, $d_t^d \equiv \int_0^1 d_t^d(i) di = 0$. The economy-wide resource constraint in equilibrium then becomes

$$c_t^{\tau} + iv_t^{\tau} + (1 + r_{t-1}^*(d_{t-1}))d_{t-1} = y_t^{\tau} + d_t. \tag{1.32}$$

Finally, the collateral constraint of the economy collapses to $e_t d_t \leq \kappa q_t k_{t+1}$. I restrict my attention to the equilibrium around the deterministic steady state in where the collateral constraint is always binding. This gives

$$e_t d_t = \kappa q_t k_{t+1}. \tag{1.33}$$

I define the competitive equilibrium as follows.

Definition 1.3.1 (Competitive Equilibrium) The competitive equilibrium of the economy is a set of processes $\{c_t^\tau, c_t^n, h_t, iv_t, k_t, d_t, e_t, p_t^\tau, p_t^n, iv_t^\tau, iv_t^n, mc_t, r_t^k, w_t, \pi_t^n, \bar{p}_t^n, \tilde{p}_t^n, z_t, f_t, x_t, \pi_t, y_t^n, r_t^*\}_{t=0}^\infty$ that satisfy equations (1.1), (1.2), (1.4)-(1.13), (1.17)-(1.18), (1.20)-(1.25),(1.27)-(1.33), given the policy processes i_t and τ_t^p, as well as the exogenous processes y_t^τ and ϵ_t^r that satisfy equations (1.14) and (1.26).

1.4 The Tošovský Dilemma

As shown in equation (1.33), I restrict my attention the case where $\Theta_t > 0$, which implies that the collateral constraint is binding in equilibrium. For consistency of the assumption in the steady state, it is required that the Lagrange multiplier associated with the collateral constraint, Θ, should be positive in the steady state.

Assumption 1.4.1 $\Theta > 0$.

This yields impatience for the households, which gives a discount factor smaller than the one under a risk-free market rate.

Lemma 1.4.1 Given Θ and $\bar{r}^*$, $\beta = \frac{1-\Theta}{1+r^*}$, where $r^* = \bar{r}^* + \psi\left(e^{d-\bar{d}} - 1\right)$.

Proof. See Appendix A.3.1.

The assumption and lemma describe imperfection and underdevelopment in the financial market which typically prevail in emerging economies.

1.4.1 Inefficiencies

When there is an external shock that relaxes the households' budget constraint, such as an increase in y_t^τ or decrease in r_t^*, the economy faces distortions. The inefficiency from the distortions encourage the social planner to intervene in the competitive equilibrium.

(1) External Indebtedness. Because of the collateral constraint, the households do not engage in consumption smoothing as in the standard intertemporal approach of the current account. The households' desire to borrow always exceeds the borrowing limit, $\kappa q_t k_{t+1}$. When the shock relaxes the households' budget constraint, each household increases demand in consumption c_t and investment iv_t. Recall the household's individual sequential budget constraint

$$c_t^\tau + iv_t^\tau + (1 + r_{t-1}^*(d_{t-1}))d_{t-1} = y_t^\tau + d_t.$$

The investment in physical capital causes an additional effect. The increase in physical capital stock k_{t+1} from the investment relaxes the collateral constraint,

$$e_t d_t = \kappa q_t k_{t+1}.$$

Thus, the households' investment in physical capital is determined by a combination of the standard investment channel and a collateral channel. Recall that the intertemporal condition for investment and demand in physical capital (equations (1.7) and (1.8)) are rewritten as

$$q_t = \frac{1 - \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \mathcal{S}_2 \left(iv_{t+1}, iv_t \right) \right]}{\mathcal{S}_1 \left(iv_t, iv_{t-1} \right)},$$

$$q_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left((1-\delta) q_{t+1} + r^k_{t+1} \right) \right] + \kappa \Theta_t q_t.$$

Equation (1.7) describes the standard theory of costly investment. Equation (1.8), on the other hand, describes the optimality condition that equates an increase in the marginal utility of wealth from an additional unit of capital and an increase in the future marginal utility from the resale price of capital and capital gains, *plus* an increase in the marginal utility of wealth from relaxing the collateral constraint. It creates a Fisherian feedback loop to the price of capital, q_t. Since all households are identical, they all want to invest in physical capital, and since capital is costly to adjust, there is an increase in the price of capital q_t. An increase in q_t directly increases the collateral value $q_t k_{t+1}$ and thereby yields further relaxation of the collateral constraint Θ_t. It affects the capital cost and triggers a self-reinforcing spiral on the price of capital until the capital market clears. The economic environment in which the households' decision is decentralized and the binding collateral constraint yields excessive and inefficient dynamics in aggregate external indebtedness.

(2) Inflation in the Non-Tradable Sector. The increases in consumption and investment and complementarity between tradables and nontradables causes an increase in nontradable demand. Under the nominal price rigidity in the nontradable sector, the increase in demand generates inflationary pressure in the nontradable sector π^n_t, which yields an efficiency loss in nontradable production via equation (1.29),

$$x_t = \theta x_{t-1} (1 + \pi^n_t)^\mu + (1-\theta)(\bar{p}^n_t)^{-\mu},$$

which is also the mirror image of the dis-equalization of households' MRS between consumption and labor supply and the MRT caused by the price rigidity, which violates optimality condition between consumption and production of the economy.

1.4.2 The Dilemma

If the nominal interest rate i_t is the only instrument available to the social planner, this environment creates a policy dilemma. An increase of i_t that aims to suppress inflation in the nontradable sector changes the real exchange rate e_t via the interest rate parity condition,

$$\mathbb{E}_t\left[\frac{1+i_t}{1+\pi_{t+1}}\frac{\lambda_{t+1}}{\lambda_t}\right]=\mathbb{E}_t\left[\left(1+r_t^*(d_t)+r_t^{*'}(d_t)d_t\right)\frac{\lambda_{t+1}}{\lambda_t}\frac{e_{t+1}}{e_t}\right],$$

which becomes the uncovered interest rate parity condition in a first order approximation,

$$\hat{e}_t \simeq -\sum_{j=0}^{\infty}\left(\hat{rr}_{t+j}-\hat{rr}^*_{t+j}\right), \tag{1.34}$$

where $rr_t^* \equiv (1+r_t^*)$ is the gross foreign real interest rate. The equation (1.34) shows that the real appreciation $\hat{e}_t < 0$ is associated with a positive current and anticipated real interest rate differential $\sum_{j=0}^{\infty}\left(\hat{rr}_{t+j}-\hat{rr}^*_{t+j}\right) > 0$. Conditional on fixing other variables, the real appreciation relaxes the collateral constraint since it decreases the unit cost of borrowing in the collateral constraint. The collateral constraint (1.33) can be rewritten as

$$d_t = \kappa \underbrace{\frac{q_t}{e_t}}_{\equiv q_t^{\tau}} k_{t+1},$$

where q_t^{τ} is the price of physical capital in terms of tradable units. Tightening monetary policy yields

$$q_t^{\tau 1}k_{t+1}^1 > q_t^{\tau 0}k_{t+1}^0, \tag{1.35}$$

where superscript 1 denotes the case with monetary policy intervention and superscript 0 denotes the case without intervention. When (1.35) materializes, there are further capital inflows from the monetary tightening which contradicts the policy objective of preventing excessive external indebtedness. On the other hand, if the monetary authority conducts loosene monetary policy to prevent (1.35), it contradicts the policy objective of suppressing inflation in the nontradable sector. I define the Tošovský Dilemma as follows.

Definition 4.1 (The Tošovský Dilemma) Given a positive external shock, the set of monetary reactions $\{R_t\} \subset \mathcal{R}$ that causes $rr_t^1 > rr_t^0$ and $q_t^{\tau 1} k_{t+1}^1 > q_t^{\tau 0} k_{t+1}^0$ is the policy set of the Tošovský Dilemma.

The Dilemma comes from two properties of the collateral constraint: i) the collateral (physical capital) is nontradable, and ii) investment in physical capital is costly. To make the point clear, let us consider the case in which physical capital is tradable and investment is not costly. Then the collateral constraint collapses to

$$d_t = \kappa k_{t+1}.$$

In such a case, any policy $\{R_t\} \subset \mathcal{R}$ which yields $rr_t^1 > rr_t^0$ does not bring $k_{t+1}^1 > k_{t+1}^0$, since Tobin's q is 1 and the increase in the real interest rate increases the cost of capital, suppressing investment. In other words, the Dilemma never materializes. If collateral is nontradable but investment is costly, the intertemporal relation of investment becomes

$$1 = \bar{q}_t \mathcal{S}_1 \left(iv_t, iv_{t-1} \right) + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \underbrace{\frac{e_{t+1}}{e_t}}_{*} \bar{q}_{t+1} \mathcal{S}_2 \left(iv_{t+1}, iv_t \right) \right],$$

which implies that the pricing of capital in tradable goods $\bar{q}_t$ already reflects the expected movement of real exchange rate which is highlighted in term (*). In this case, there is no effect to the collateral constraint from the real appreciation, and the Dilemma does not materialize. The case in which the collateral is tradable and costless investment is ambiguous. In such a case, the collateral constraint collapses to

$$e_t d_t = \kappa k_{t+1},$$

and there is no Fisherian spiral in price of capital since the Tobin's q is always one. However, the real appreciation continues to create a collateral effect. Another case for the for currency denomination of external debt to the wealth effect on the collateral constraint is analyzed in section 1.6.

Note that the Dilemma is a particular case of the trade-offs between price stability and financial stability, which is largely discussed in cuent literature (for example, see Curdia and Woodford (2016), Aoki et al. (2016), Davis and Presno (2017)). The Dilemma in this model describes a policy paradox in which a policy that corrects one distortion exacerbates another. How does the Dilemma shape optimal monetary policy? I discuss it in the next section by solving the Ramsey problem.

1.5 Optimal Monetary Policy

In this section, I derive the Ramsey optimal monetary policy. The Ramsey planner seeks to maximize his lifetime utility, taking the competitive equilibrium conditions of the decentralized economy as constraints. It is also assumed that the Ramsey planner has a device to commit to policy promises made previously. In particular, I assume that commitment is optimal from a timeless perspective (see Giannoni and Woodford (2003) and Woodford (2003)), to achieve time-invariant optimality conditions in the Ramsey equilibrium.

1.5.1 The Ramsey Equilibrium

The periodic utility function of the Ramsey planner is given by

$$U^R\left(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{\gamma}_t, \boldsymbol{s}_t, \boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}, \boldsymbol{\gamma}_{t+1}, \boldsymbol{s}_{t+1}\right),$$

where $\boldsymbol{x}_t$ is a vector of predetermined endogenous variables at time t,

$$\boldsymbol{x}_t = (iv_{t-1}, k_t, d_{t-1}, x_{t-1}, p^n_{t-1}, r^*_{t-1})',$$

and $\boldsymbol{y}_t$ is a vector of nonpredetermined endogenous variables,

$$\boldsymbol{y}_t = \left(c^\tau_t, c^n_t, h_t, e_t, p^\tau_t, iv^\tau_t, iv^n_t, mc_t, r^k_t, w_t, \pi^n_t, \bar{p}^n_t, \tilde{p}^n_t, z_t, f_t, \pi_t, y^n_t\right)'.$$

The vector $\boldsymbol{\gamma}_t$ is a policy vector determined by the Ramsey planner, and $\boldsymbol{s}_t$ is an exogenous stochastic process vector which follows the stationary Markov process

$$\boldsymbol{s}_{t+1} - \bar{\boldsymbol{s}} = \boldsymbol{\Phi}_s\left(\boldsymbol{s}_t - \bar{\boldsymbol{s}}\right) + \boldsymbol{\nu}^s_{t+1},$$

where $\bar{\boldsymbol{s}}$ is a vector for steady state values of the process, $\boldsymbol{\Phi}_s$ is a vector of coefficients, and $\boldsymbol{\nu}^s_{t+1}$ is a vector of random innovations with mean zero and variance matrix $\boldsymbol{\Sigma_\nu}$. In our model, those are

$$\begin{aligned}
\boldsymbol{\gamma}_t &= (i_t), \\
\boldsymbol{s}_t &= (y^\tau_t, \epsilon^r_t)', \\
\boldsymbol{\Phi_s} &= \begin{pmatrix} \rho_\tau & \rho_{\tau r} \\ \rho_{r\tau} & \rho_r \end{pmatrix}, \quad \boldsymbol{\Sigma_\nu} = \begin{pmatrix} \sigma^\tau_\epsilon & \sigma^{\tau r}_\epsilon \\ \sigma^{r\tau}_\epsilon & \sigma^r_\epsilon \end{pmatrix}.
\end{aligned}$$

Given a policy instrument $\boldsymbol{\gamma_t}$, the competitive equilibrium conditions at time t can be denoted as

$$\mathbb{P}\left(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{\gamma}_t, \boldsymbol{s}_t, \boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}, \boldsymbol{\gamma}_{t+1}, \boldsymbol{s}_{t+1}\right) = \mathbf{0}, \tag{1.36}$$

where $\mathbf{0}$ is a zero vector.

The Lagrangian of the Ramsey planner with a commitment device optimal from a timeless perspective is given by

$$\boldsymbol{\mathcal{L}}^R \equiv \mathbb{E}_t \sum_{s=-\infty}^{\infty} \beta_R^{t+s} \left(U^R \left(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{\gamma}_t, \boldsymbol{s}_t, \boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}, \boldsymbol{\gamma}_{t+1}, \boldsymbol{s}_{t+1}\right)\right.$$

$$\left. + \boldsymbol{\Lambda}_{t+s}^{R\ \prime} \mathbb{P}\left(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{\gamma}_t, \boldsymbol{s}_t, \boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}, \boldsymbol{\gamma}_{t+1}, \boldsymbol{s}_{t+1}\right)\right), \tag{1.37}$$

where β_R is the subjective discount factor of the Ramsey planner and $\beta_R^t \boldsymbol{\Lambda}_t$ is a Lagrange multiplier of the Ramsey problem associated with the competitive equilibrium conditions $\mathbb{P}$. I define the Ramsey equilibrium as follows.

Definition 5.1 (The Ramsey Equilibrium) Let $\boldsymbol{u}_t \equiv \left(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t, \boldsymbol{\gamma}_t\right)'$. The Ramsey equilibrium is the sequence of processes $\{\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{\gamma}_t, \boldsymbol{s}_t\}$ that satisfies the solution of the system

$$\frac{\partial \boldsymbol{\mathcal{L}}^R}{\partial \boldsymbol{u}_t} = \mathbf{0}, \tag{1.38}$$

subject to the competitive equilibrium conditions (1.36). I assume that the utility function and the discount factor of the Ramsey planner are the same as the households $U^R = U, \beta_R = \beta$, following the convention that assumes that the Ramsey planner is the representative household of the economy. In computing the Ramsey optimal monetary policy, I set the time invariant tax rate on the production $\tau^p = -\frac{1}{\mu - 1}$, so that it eliminates the distortion from the monopolistic competition and isolates the distortion from the price stickiness. Since

$\mu > 1$, the tax rate becomes the production subsidy.

1.5.2 Calibration

In section 1.3.1, the CES-Armington aggregator is assumed to be given by

$$A\left(a_t^\tau, a_t^n\right) \equiv \left(\chi_\tau^{\frac{1}{\eta}} a_t^{\tau 1-\frac{1}{\eta}} + \left(1-\chi_\tau\right)^{\frac{1}{\eta}} a_t^{n 1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}}$$

I assume that the periodic utility function of the households is given by Greenwood-Hercowitz-Huffman (GHH) preferences

$$U\left(c_t, h_t\right) = \frac{\left(c_t - \frac{h_t^\omega}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}.$$

The functional form of costly investment is taken from Christiano et al. (2005), as given by

$$\mathcal{S}\left(iv_t, iv_{t-1}\right) = \left(1 - \frac{\phi}{2}\left(\frac{iv_t}{iv_{t-1}} - 1\right)^2\right) iv_t,$$

where ϕ governs the elasticity of the adjustment cost with respect to investment. The production function of the nontradable good is assumed to be the Cobb-Douglas function

$$\mathcal{F}\left(k_t, h_t\right) = k_t^\alpha h_t^{1-\alpha},$$

where $\alpha \in (0,1)$ is the parameter that governs the capital share of nontradable good production.

Table 1.1 shows the benchmark calibration of the structural parameters. I set the inverse of the intertemporal elasticity of substitution σ to be 2 and labor supply elasticity ω to be 1.445, taking standard values in the business cycle literature. The frequency of the model is quarterly. The parameter for the world interest rate $\bar{r}^*$ is set to $1.04^{1/4} - 1$, which implies that the quarterly risk free world interest rate is 1 percent. The parameter for IDEIR, $\bar{d}$, is set to 4.0495, which gives a 1 percent steady state trade balance to output ratio in the steady state where the economy has an external debt elastic interest rate function (EDEIR). The elasticity of the foreign interest rate with respect to the debt position, ψ, is set to 0.0000335, following the calibrated value in Uribe and Schmitt-Grohé (2017). With these parameters, the steady state level of foreign interest rate becomes

$$r^* = \bar{r}^* + \psi \left(e^{d-\bar{d}} - 1 \right).$$

The steady state impatience parameter Θ is set to 0.014, which sets the households' discount factor $\beta \equiv \frac{1-\Theta}{1+\bar{r}^*+\psi e^{d-\bar{d}}d}$ at 0.9764. The β targets $\beta^{-4}-1 = 0.10$, which is the average of annual country interest rates across emerging economies estimated in Uribe and Schmitt-Grohé (2017). The elasticity of substitution between tradable goods and nontradable goods is set to be 0.5, in line with estimates in Akinci (2011). I set the bias toward the tradable goods parameter, χ_τ, to 0.3, capital share in nontradable goods production α to 0.25, and elasticity of substitution across nontradable intermediaries μ to 6, all of which are consistent with the related literature. The fixed cost parameter in nontradable production τ is calibrated to 0.4859 to ensure zero profit in the steady state. The steady-state tradable endowment is normalized to 1. The collateral constraint parameter κ is set to be 0.1936 to match the external debt position implied in the IDEIR interest rate function.

The parameter that governs the elasticity of costly capital adjustment ϕ does not affect steady state solutions but does affect the endogenous propagation mechanism. I set $\phi = 0.1$ in this section, which is an arbitrary and small number compared to the related literature. This point will be discussed in section 1.5.5. The parameters for the exogenous processes of tradable endowment ρ_y and σ_ϵ^y are taken from estimates in Uribe and Schmitt-Grohé (2017), who use Argentine data over the 1983 Q1 - 2001 Q3 period. The parameters for the exogenous processes of the world real interest rate ρ_r and σ_r are estimated by fitting an AR(1) model to quarterly U.S. real interest rates from 1948 Q2 - 2013 Q3. The data is taken from the updated dataset in Uribe and Yue (2006).

In the following subsections, I investigate the properties of optimal monetary policy in the long-run (non-stochastic steady state) and in the short-run (cyclical fluctuation around the steady state). I examine two models, a baseline model with the collateral constraint (labeled as 'Tošovský') and a model without the collateral constraint (labeled as 'Standard'). The calibrated parameters in the Standard model are described in Table A.1 in Appendix A4.

Table 1.1: Benchmark Calibration

Parameter	Value	Description
σ	2	Inverse of the intertemporal elasticity of substitution
ω	1.455	Labor supply elasticity parameter
$\bar{r}^*$	$1.04^{1/4} - 1$	Risk free world interest rate
$\bar{d}$	4.0495	IDEIR parameter
ψ	0.0000335	Elasticity of r_t^* with respect to debt adjustment
Θ	0.014	Impatience parameter of borrowers
β	0.9764	Subjective discount factor
η	0.5	Elasticity of substitution between T and NT goods
χ_τ	0.3	Preference bias toward the tradable good
α	0.25	Capital share of nontradable good production
θ	0.7	Calvo-Yun parameter
μ	6	Elasticity of substitution across NT intermediaries
τ^p	-0.2	Production Tax Rate (Subsidy)
τ	0.4859	Fixed cost of production in NT sector
$\bar{y}^\tau$	1	Tradable endowment in the steady state
κ	0.1936	Collateral constraint parameter
ϕ	0.1	Elasticity of costly capital adjustment
ρ_y	0.79	Persistence of the log endowment shock
ρ_r	0.87	Persistence of the log foreign interest rate shock
σ_ϵ^y	0.0351	Standard deviation of the log endowment shock
σ_ϵ^r	0.0026	Standard deviation of the log foreign interest rate shock

1.5.3 Optimality of Price Stability in the Long-Run

To investigate the long-run properties of the Ramsey equilibrium, I characterize the Ramsey equilibrium in the non-stochastic steady state. One question of interest is the optimal rate of inflation in the steady state. This is not a trivial question, particularly in the Tošovský model because there are two frictions: staggered adjustment of nontradable prices and the collateral constraint. The following proposition shows that the economy yields a zero optimal inflation rate in the steady state, which means that monetary policy engaging in price stability is the optimal monetary policy in the long-run regardless of the existence of the collateral constraint.

Proposition 1.5.1 *There is zero inflation in the Ramsey optimal steady state.*

Proof. See Appendix A.3.2.

Table 1.2 below shows several key economic indicators in the Ramsey optimal steady state under benchmark calibration.

Table 1.2: The Ramsey Optimal Steady State

Model	π^n	i	e	$\frac{c}{y}$	$\frac{iv}{y}$	$\frac{k}{y}$	$\frac{c^n}{y^n}$	$\frac{iv^n}{y^n}$
Tošovský	0	4.0	1.08	0.88	0.11	4.49	0.89	0.11
Standard	0	4.0	1.24	0.85	0.14	5.84	0.86	0.14

Notes. 'Tošovský' model refers to the model with the collateral constraint. 'Standard' model refers to the model without the collateral constraint. Variable π^n refers to the annualized net inflation rate in the nontradable sector (in percent) and variable i refers to the annualized net nominal interest rate (in percent). Variable e refers to real exchange rate as the nominal exchange rate divided by the CPI. Variables $\frac{c}{y}$, $\frac{iv}{y}$, $\frac{k}{y}$, $\frac{c^n}{y^n}$, and $\frac{iv^n}{y^n}$ refer to the consumption-output ratio, investment-output ratio, capital-output ratio, consumption-output ratio in the nontradable sector, and investment-output ratio in the nontradable sector, respectively.

In the Tošovský model, the annual inflation rate in the nontradable sector is zero and the nominal interest rate is the same as the world interest rate, which is 4 percent in annual terms. The steady state real exchange rate, defined as the nominal exchange rate divided by the CPI, $e \equiv \frac{\mathcal{E}}{P}$, is 1.08. The percentage of aggregate consumption in GDP, $\frac{c}{y} \times 100$, is 88, the percentage of aggregate investment in GDP, $\frac{iv}{y} \times 100$, is 11, and the capital-output ratio, $\frac{k}{y}$, is 4.49. In the nontradable sector, the percentage of nontradable consumption in gross nontradable output, $\frac{c^n}{y^n} \times 100$, is 89, and the percentage of nontradable investment in gross nontradable ouput, $\frac{iv^n}{y^n} \times 100$, is 11.

The standard model without the collateral constraint also yields a zero inflation rate and a nominal domestic interest rate that is the same as in the Tošovský model.

1.5.4 Breakdown of Price Stability in the Short-Run in the Tošovský Model

To compute policy functions of economic indicators, I approximate the equilibrium system up to first and second orders around the Ramsey steady state using perturbation methods (see, for example, Schmitt-Grohé and Uribe (2004)). Table 1.3 shows the unconditional second moments of interest rates and price indicators under the benchmark calibration. The table shows that the standard deviation of the annual inflation rate in the nontradable sector is 4.42 percentage points, a nontrivial deviation from the standard monetary policy regime of price stability.

Result 1.1 (Breakdown of the Regime of Price Stability) *The standard deviation of inflation in the nontradable sector in the baseline model is nonzero and quantitatively high.*

On the other hand, in the standard model without the constraint, the standard deviation of nontradable inflation is 0 percentage points. The subsequent proposition follows.

Table 1.3: Unconditional Second Moments, Tošovský and Standard Models

Model	$\sigma\left(\pi_t^n\right)$	$\sigma\left(\hat{d}_t\right)$	$\sigma\left(i_t\right)$	$\rho\left(i_t, y_t^\tau\right)$
Tošovský	4.42	4.10	9.52	-0.62
Standard	0	94.62	1.13	0.02

Notes. 'Tošovský' model refers to the model with the collateral constraint. 'Standard' model refers to the model without the collateral constraint. Symbol $\sigma(x)$ refers to the standard deviation of variable x in percentage points. Symbol $\rho(x, y)$ refers to the correlation coefficient between variables x and y.

Proposition 1.5.2 (From Nominal To Real in the Standard Model) *The standard model with optimal monetary policy can be represented by a current account model only with real variables.*

Proof. See Appendix A.3.3.

Table 1.3 shows more than the breakdown of price stability. The standard deviation of the policy instrument, the annualized nominal interest rate, is 9.52 percentage points in the Tošovský model, which is far bigger than the 1.13 percentage points in the standard model without the collateral constraint. Furthermore, the correlation coefficient between the nominal interest rate and tradable endowment is -0.62 in the Tošovský model, which is negative, whereas the correlation in the standard model is 0.02 which is almost zero. This implies that the Ramsey planner in the Tošovský model gives up the conventional counter-cyclical regime of monetary policy towards price stability regime when he faces the policy Dilemma.

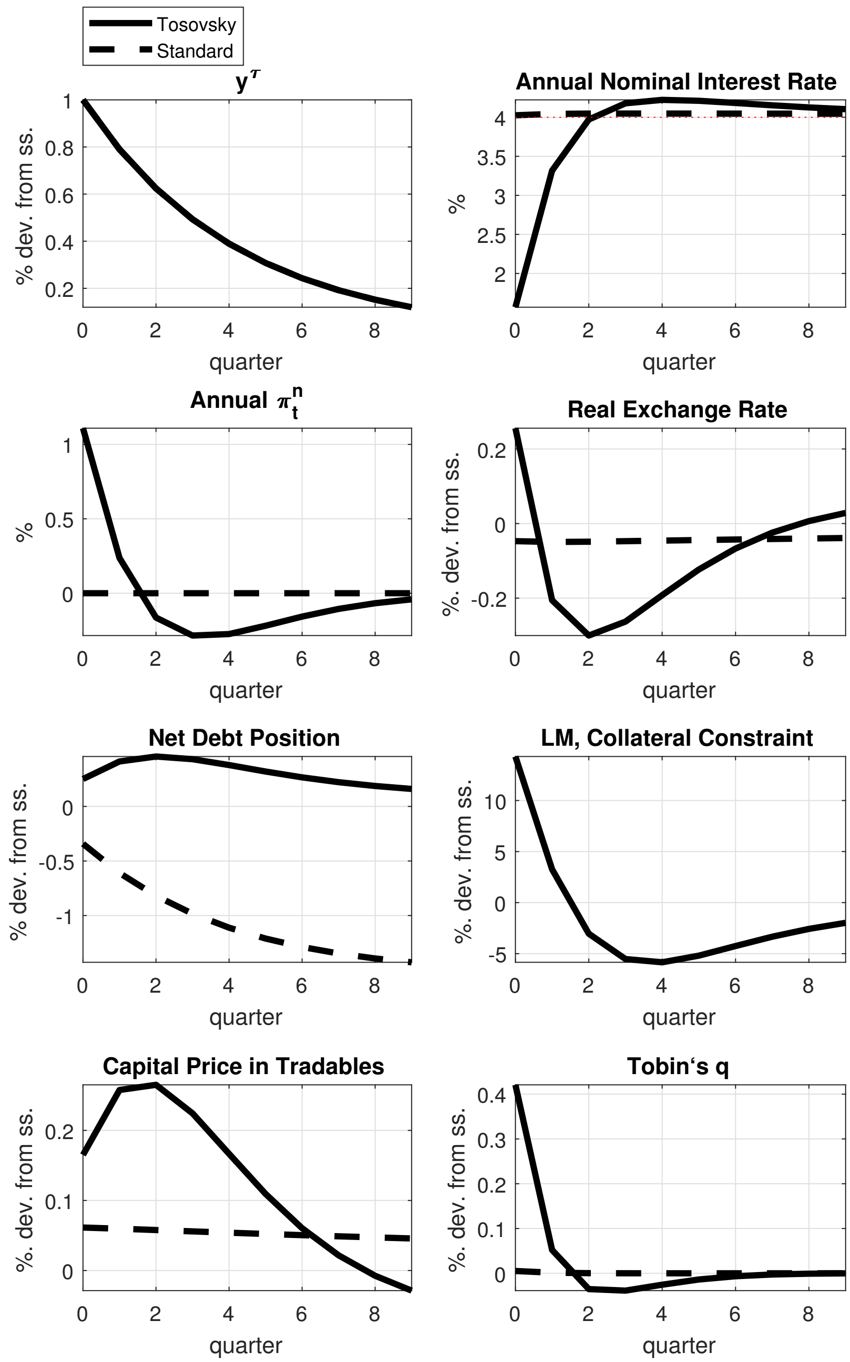
Result 1.2 (Ramsey Planner's Choice to the Tošovský Dilemma) *The Ramsey optimal interest rate policy in the baseline model is negatively correlated with the tradable endowment.*

It is important to note that Results 1.1 and 1.2 are contrast with the standard NK regime of monetary policy. Impulse response functions of the models give clearer insight. Figures 1.2 and 1.3 show the theoretical impulse response functions of key economic indicators with respect to a 1 percent positive tradable endowment shock from the steady state level. The responses of the Tošovský model (solid line) and the standard model (dotted line) are substantially different. With respect to the endowment shock, the Ramsey response of the nominal interest rate in the Tošovský model is significant easing, whereas the standard model without the collateral constraint shows modest tightening. The annualized nominal interest rate in the Tošovský model decreases to 1.56 percentage points. This monetary easing results in an increase in the inflation rate in the nontradable sector and real depreciation. The annual inflation rate in the Tošovský model increases to 1.1 percent, whereas it is perfectly stabilized (zero) in the standard model. The real exchange rate, as the nominal exchange rate divided by the aggregate price index, shows a hump shaped trajectory in the Tošovský model. It shows a 0.25 percent initial increase from the steady state level, goes down to a -0.3 percent decrease from the steady state level, and gradually depreciates to its steady state level.

The dynamics of the net debt position of the Tošovský model and the standard model go in opposite directions. In the Tošovský model, there is a modest capital inflow. The inflow of external debt in the Tošovský model is constrained by the monetary easing, which is reflected in the 14 percent initial increase of the Lagrange multiplier of the collateral constraint. Facing the Dilemma, the Ramsey planner chooses to stabilize the collateral constraint, by managing the price of collateral, namely the price of capital in terms of the tradable good, $q_t^\tau \equiv \frac{q_t}{e_t}$. In contrast, in the model without the collateral constraint, the conventional channel of the intertemporal current account dominates. There are capital outflows (saving) in the economy and the Ramsey planner happily allows real appreciation. The dynamics of real appreciation and the net debt position are consequences of efficient allocation, and there are few concerns of financial instability for the Ramsey planner.

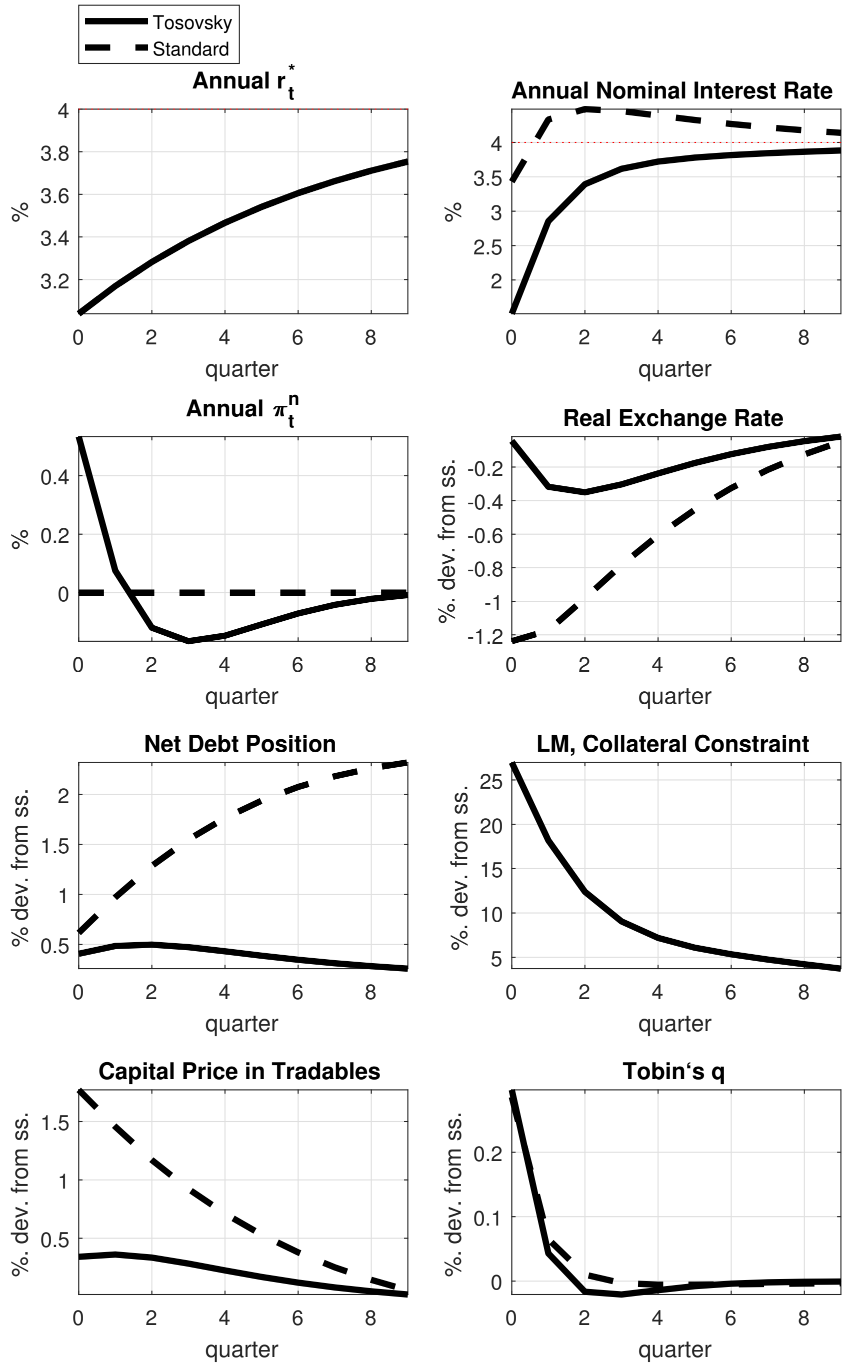
The impulse response functions with respect to a 1 percent decrease in annual foreign interest rate give similar insight. In response to the foreign interest rate shock, the Ramsey planner in the Tošovský model lowers the annual nominal interest rate to 1.5 percent, and the annual inflation rate in the nontradable sector increases to 0.5 percent. In the standard model without the collateral constraint, there is monetary tightening after an initial easing, and the inflation rate is fully stabilized. The heterogeneous responses are related to the Ramsey planner's concern over financial stability. The aggressive monetary easing in the Tošovský model yields much milder real appreciation compared to the standard model without financial frictions. The suppressed real appreciation gives rise to milder dynamics of the price of collateral and capital inflows in the Tošovský model.

Figure 1.1: Impulse Response Functions, 1% Increase in Tradable Endowment



Notes. The solid line refers to the Tošovský model, and the dashed line refers to the standard model. IRFs of the nominal interest rate and the inflation rate in the nontradable sector (π^n) are represented in annual percent terms.

Figure 1.2: Impulse Response Functions, 1% Decrease in Foreign Interest Rate

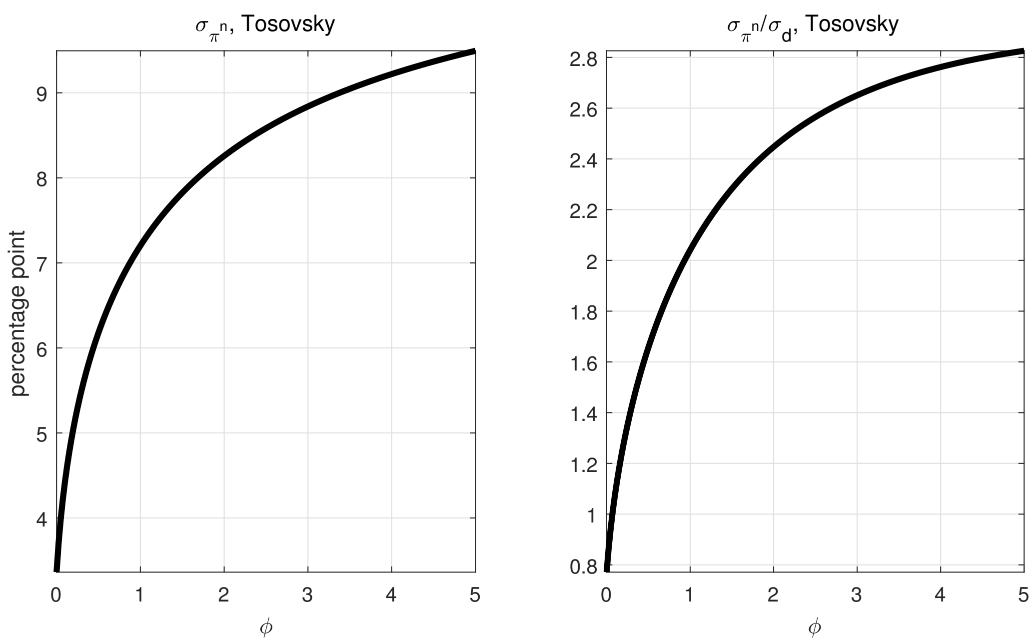


Notes. The solid line refers to the Tošovský model, and the dashed line refers to the standard model. IRFs of the nominal interest rate and the inflation rate in the nontradable sector (π^n) are represented in annual percent terms.

1.5.5 Costly Capital Adjustment, the Dilemma, and Ramsey Responses

As discussed in section 1.5.2, costly adjustment of physical capital crucially contributes to the Dilemma. The baseline calibration assumes a small range for the adjustment cost, $\phi = 0.1$, relative to estimates in previous papers. Altig et al. (2005) estimate it to be 2.79 using quarterly U.S. data over Q2 1959 - Q4 2001, Aguiar and Gopinath (2006) estimtate it to be 1.37 using quarterly Mexican data over Q1 1980 - Q1 2003, and García-Cicoo et al. (2010) estimate it to be 5.6 using annual Argentine data over 1990 to 2005. This section investigates how the Ramsey response depends on the degree of costly capital adjustment.

Figure 1.3: Costly Capital Adjustment and Deviation from Price Stability



Notes. Parameter ϕ is a parameter governing costly adjustment of investment, $\mathcal{S}(iv_t, iv_{t-1}) = \left(1 - \frac{\phi}{2}\left(\frac{iv_t}{iv_{t-1}} - 1\right)^2\right) iv_t$. The left panel shows the unconditional standard deviation of the annual inflation rate in the nontradable sector in the Tošovský model. The right panel shows the ratio between the unconditional standard deviations of annual inflation in the nontradable sector and log linearized external debt, $\hat{d}_t$.

Figure 1.4 shows the unconditional standard deviation of inflation in the nontradable sector, σ_{π^n}, and the relative volatility between nontradable inflation and the net external debt position, $\frac{\sigma_{\pi^n}}{\sigma_d}$, in the Tošovský model. The Ramsey standard deviation of the inflation rate monotonically increases as ϕ increases. Furthermore, the relative volatility between inflation and the net debt position monotonically increases as ϕ increases. This indicates that the Ramsey planner puts more weight on financial stability, $\sigma\left(\hat{d}_t\right)$, than price stability, $\sigma\left(\pi_t^n\right)$, as capital adjustment becomes more costly.

1.6 Local Currency External Debt

In this section, I extend the model to investigate how the currency denomination of external debt alters the wealth effect of monetary policy on the collateral constraint. I assume that the economy borrows from the rest of the world by issuing domestic currency denominated debt.

To examine this channel, I introduce two heterogeneous households: 'arbitrageurs' that can trade local and foreign currency denominated assets, and 'borrowers' who issue international debt denominated in the local currency. Borrowers are subject to a borrowing constraint on international debt, whereas arbitrageurs are not. Furthermore the collateral constraint of borrowers is elastic to the value of domestic collateral in terms of domestic currency. I utilize heterogeneous households because otherwise the foreign interest rate does not play a role in the case of local currency denominated debt, deviating from the small open economy perspective and preventing the model from being closed.

Arbitrageurs

There is a continuum of identical arbitrageurs on the unit interval, $[0, 1]$. The arbitrageurs receive a tradable endowment, consume tradable goods, and trade home and international debt. Each arbitrageur maximizes his expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_a^t U^a(c_t^{\tau a}),$$

where $U^a(\cdot)$ is a utility function which is twice differentiable, strictly positive, and strictly concave. Variable $c_t^{\tau a}$ is the consumption of the tradable good of the arbitrageur in real terms and $\beta_a \in (0,1)$ is the subjective discount factor, common to all arbitrageurs. The sequential resource constraint in domestic currency is

$$P_t^{\tau} c_t^{\tau a} + (1+i_{t-1}) D_{t-1}^d + \mathcal{E}_t (1 + r_{t-1}^*(d_{t-1}^a)) d_{t-1}^a = P_t^{\tau} y_t^{\tau a} + D_t^d + \mathcal{E}_t d_t^a + T_t^a,$$

where variable d_t^a is the arbitrageur's nominal foreign currency debt position which bears gross nominal rate of return, $1 + r_t^*(d_t^a)$, with the same form of IDEIR as in section 4. Variable $y_t^{\tau a} = y^{\tau a}$ denotes the arbitrageur's tradable endowment, which is non-stochastic. Variable T_t^a denotes the nominal lump sum transfer from the government. Dividing the budget constraint by P_t yields the stationary form of the budget constraint

$$p_t^{\tau} c_t^{\tau a} + \frac{1+i_{t-1}}{1+\pi_t} d_{t-1}^d + e_t (1 + r_{t-1}(d_{t-1}^a)) d_{t-1}^a = p_t^{\tau} y_t^{\tau a} + d_t^d + e_t d_t^a + t_t^a. \tag{1.39}$$

Necessary first order conditions of the Lagrangian of the arbitrageur associated with $\{c_t^{\tau a}, d_t, d_t^{*a}\}$ are

$$\begin{aligned}
U^{a\prime}(c_t^{\tau a}) &= \lambda_t^a p_t^{\tau}, && (1.40)\\
\lambda_t^a &= \beta_a \mathbb{E}_t \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \lambda_{t+1}^a, && (1.41)\\
\mathbb{E}_t \left[\frac{1+i_t}{1+\pi_{t+1}} \frac{\lambda_{t+1}^a}{\lambda_t^a} \right] &= \mathbb{E}_t \left[\left(1 + r_t^*(d_t^a) + r_t^{*\prime}(d_t^a) d_t^a\right) \frac{\lambda_{t+1^a}}{\lambda_t^a} \frac{e_{t+1}}{e_t} \right], && (1.42)
\end{aligned}$$

where $\beta_a^t \lambda_t^a$ is the Lagrange multiplier for the resource constraint. The transversality conditions of the two assets hold with equality.

Borrowers

In addition to arbitrageurs, there is continuum of identical borrowers on the unit interval, $[0,1]$. The borrowers are similar to the households in section 1.3 except they only trade one type of asset. The borrowers hold and invest physical capital, k_t, used for production in nontradable sector. Borrowers are more impatient than arbitrageurs, which is represented in the discount factor $\beta_b < \beta_a$. The borrowers' variables are denoted with superscript b, except investment, iv_t, and physical capital, k_t, which belong to borrowers only. Each borrower maximizes his lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_a^t U^b \left(c_t^b, h_t\right),$$

where $U^b(\cdot,\cdot)$ is a function which is strictly positive in its first argument, strictly negative in its second argument, and stictly concave. Variable c_t^b is the final consumption basket in real terms and h_t is hours worked by the borrower. As in the model in section 1.3, the final consumption basket follows the CES-Armington aggregator

$$c_t^b = A\left(c_t^{\tau b}, c_t^n\right),$$

which shares the same functional properties as before. Variable $c_t^{\tau b}$ is a consumption of tradable goods and c_t^n is consumption of nontradable goods by the borrower. The borrower's sequential resource constraint in domestic currency is given by

$$P_t^{\tau} c_t^{\tau b} + P_t^n c_t^n + P_t iv_t + (1+i_{t-1})D_{t-1}^b = P_t^{\tau} y_t^{\tau} + (1-\tau_t^p)\left(W_t h_t + R_t^k k_t\right) + D_t^b + \Psi_t + T_t^b,$$

where variable D_t^b is the nominal domestic currency denominated international debt position of the borrower assumed at time t and due at $t+1$, $y_t^{\tau a}$ denotes the borrower's tradable endowment which follows the same stochastic process as in section 1.3, and T_t^b is the nominal lump sum transfer to the borrower. The factor income of the borrower $W_t h_t + R_t^k k_t$ can be taxed by the government at rate τ_t^p. Variable Ψ_t refers to the nominal profit of firms rebated to the borrower, and T_t^b refers to a nominal lump sum transfer to the borrower from the government. Dividing the budget constraint by P_t yields

$$p_t^\tau c_t^{\tau b} + p_t^n c_t^n + iv_t + \frac{1+i_{i-1}}{1+\pi_t} d_t^b = p_t^\tau y_t^\tau + (1-\tau_t^p)\left(w_t h_t + r_t^k k_t\right) + d_t^b + \phi_t + t_t^b. \tag{1.43}$$

The evolution of physical capital follows the law of motion in equation (1.1). I assume that the borrowers' international debt position should be backed by his stock of physical capital as collateral. The financial friction of the borrower is represented by the following borrowing constraint,

$$D_t^b \leq \kappa Q_t k_{t+1},$$

where $\kappa > 0$ is a parameter, and Q_t is the nominal price of new physical capital, the Tobin's marginal q times CPI. Dividing the collateral constraint by P_t gives

$$d_t^b \leq \kappa q_t k_{t+1}, \tag{1.44}$$

Necessary first order conditions of the Lagrangian problem of the borrower associated with $\{c_t^{\tau b}, c_t^n, h_t, iv_t, k_{t+1}, d_t^b\}$ are

$$U_1^b(c_t^b, h_t) A_1(c_t^{\tau b}, c_t^b) = \lambda_t^b p_t^\tau, \tag{1.45}$$

$$\frac{A_2(c_t^{\tau b}, c_t^b)}{A_1(c_t^{\tau b}, c_t^b)} = \frac{p_t^n}{p_t^\tau}, \tag{1.46}$$

$$-\frac{U_2^b(c_t^b, h_t)}{U_1^b(c_t^b, h_t) A_1(c_t^{\tau b}, c_t^b)} = \frac{(1-\tau_t^p) w_t}{p_t^\tau}, \tag{1.47}$$

$$1 = q_t \mathcal{S}_1\left(iv_t, iv_{t-1}\right) + \beta_b \mathbb{E}_t \left[\frac{\lambda_{t+1^b}}{\lambda_t^b} q_{t+1} \mathcal{S}_2\left(iv_{t+1}, iv_t\right)\right], \tag{1.48}$$

$$(1-\kappa\Theta_t) q_t = \beta_b \mathbb{E}_t \left[\frac{\lambda_{t+1}^b}{\lambda_t^b}\left((1-\delta) q_{t+1} + (1-\tau_{t+1}^p) r_{t+1}^k\right)\right], \tag{1.49}$$

$$\lambda_t^b (1 - \Theta_t) = \beta_b \mathbb{E}_t \left[\frac{1 + i_t}{1 + \pi_{t+1}} \right] \lambda_{t+1}^b, \tag{1.50}$$

$$\Theta_t \left(\kappa q_t k_{t+1} - d_t^b \right) = 0, \quad \Theta_t \geq 0, \tag{1.51}$$

where $\beta_b^t \lambda_t^b$, $\beta_b^t \lambda_t^b q_t$, and $\beta_b^t \lambda_t^b \Theta_t$ are Lagrange multipliers associated with the borrower's resource constraint, law of motion of capital accumulation, and the collateral constraint, respectively. The transversality conditions of foreign debt position and no-bubble condition of price of physical capital hold with equality. The full equilibrium conditions of the model are described in Appendix A1.2.

The Ramsey planner seeks to maximize the lifetime utility of the representative borrowers. This is justified because the the Ramsey planner's instrument, the nominal interest rate, yields no effect on the arbitrageurs' welfare, as described in the following proposition.

Proposition 1.5.3 *The sequence of variables $\{c_t^{\tau a}, r_t^*, d_t^a\}$ is independent of the domestic nominal interest rate.*

Proof. See Appendix A.3.4.

As in model in section 1.3, the Ramsey equilibrium of the arbitrageurs-borrowers model (the AB model hereafter) ensures zero inflation in the nonstochastic steady state.

Proposition 1.5.4 *There is zero inflation in the Ramsey optimal steady state in the AB Model.*

Proof. See Appendix A.3.5.

Table 1.4: The Ramsey Steady States, The AB Model

π^n	i	e	$\frac{c^b}{y^b}$	$\frac{iv}{y^b}$	$\frac{k}{y^b}$	$\frac{c^n}{y^n}$	$\frac{iv^n}{y^n}$
0	4.0	1.11	0.88	0.11	4.60	0.89	0.11

Notes. Variable π^n refers to the annualized net inflation rate in the nontradable sector (in percent) and i refers to the annualized net nominal interest rate (in percent). Variable e refers to real exchange rate as nominal exchange rate divided by CPI price level. Variables $\frac{c^b}{y^b}$, $\frac{iv}{y^b}$ and $\frac{k}{y^b}$ refer to borrowers' consumption-income, investment-income, and capital-income ratios, respectively. Variables $\frac{c^n}{y^n}$, and $\frac{iv^n}{y^n}$ refer to the consumption-output ratio and investment-output ratio in the nontradable sector, respectively.

Table 1.4 shows that the key variables in the AB model. The calibrated parameters are described in Table A.2 in Appendix A.4. In the Ramsey steady state, the annual inflation rate in the nontradable sector is zero and the nominal interest rate is the same as in the models in section 1.3. The steady state real exchange rate, e, is 1.11, and the percentage of borrowers' consumption in income, $\frac{c^b}{y^b} \times 100$, where $y^b = y^{\tau b} + y^n$, is 88, the percentage of aggregate investment in borrowers' income, $\frac{iv}{y^b} \times 100$, is 11, and the capital-income ratio of borrowers, $\frac{k}{y^b}$, is 4.60. The percentage of nontradable consumption in gross nontradable output, $\frac{c^n}{y^n} \times 100$, is 89, and the percentage of nontradable investment in gross nontradable ouput, $\frac{iv^n}{y^n} \times 100$, is 11. All the values are consistent with the values in the Tošovský model in section 1.3, except the capital-income ratio, which is slightly greater than the one in the Tošovský model, 4.49. This is because the external debt position of the borrowers in the steady state the AB model is different due to the local currency denominated debt.

Table 1.5: Unconditional Second Moments, The AB Model

$\sigma\left(\pi_t^n\right)$	$\sigma\left(\hat{d}_t^b\right)$	$\sigma\left(i_t\right)$	$\rho\left(i_t, y_t^\tau\right)$
0.38	8.70	3.81	0.82

Notes. Symbol $\sigma(x)$ refers to the standard deviation of variable x in percentage points. Symbol $\rho(x, y)$ refers to the correlation coefficient between variables x and y.

Table 1.5 shows the unconditional second moments of variables of interest in the AB model. First, the standard deviation of the annual inflation rate in the nontradable sector is 0.38 percentage points, which is much smaller than the one in the Tošovský model, 4.42. The standard deviation of the annualized nominal interest rate, is 3.81 percentage points which is also smaller than the one in the Tošovský model, 9.52. The correlation coefficient between the nominal interest rate and tradable endowment is now 0.82 which is positive, contrary to the negative value found in the Tošovský model, -0.62. Finally, the standard deviation of the external debt position in the AB model is 8.70 percentage points, which is greater than 4.10 percentage points in the Tošovský model. The results can be summarized as follows:

Result 1.1.4 (The Effect of Local Currency Debt) *If the external debt is domestic currency denominated, the optimal monetary policy becomes counter-cyclical as in the conventional framework without the Dilemma.*

The result is intuitive in the sense that one source of the Tošovský Dilemma is the de-coupling of prices of external debt and collateral. In the Tošovský model, external debt is valued at the price in tradable goods, whereas collateral is valued at the price of nontradable goods. Pricing is thus de-coupled and the real exchange rate generates a wealth effect on the collateral constraint. This is not the case in the AB model because there is no de-coupling in pricing between the debt and collateral since external debt is also assumed to be denominated in the local currency. Thus, monetary policy in the AB model does not face the undesirable effects of the exchange rate on the capital flows. As a consequence, the monetary policy regime becomes counter-cyclical as is conventional wisdom in NK models.

1.7 Ramsey Optimal Policies with Two Instruments

This section extends the model by including an additional policy instrument, namely a capital control tax. The sequential budget constraint with with this tax becomes

$$p_t^\tau c_t^\tau + p_t^n c_t^n + iv_t + \frac{1+i_{t-1}}{1+\pi_t} d_{t-1}^d + e_t(1+r_{t-1}^*(d_{t-1}))d_{t-1}$$

$$= p_t^\tau y_t^\tau + (1-\tau_t^p)(w_t h_t + r_t^k k_t) + d_t^d + \left(1-\tau_t^d\right) e_t d_t + \psi_t + t_t, \tag{1.52}$$

where the lump sum transfer in equilibrium

$$t_t = \tau_t^p \left(w_t h_t + r_t^k k_t\right) + \tau_t^d e_t d_t.$$

The Euler equations for international debt and interest parity condition are then modified as follows.

$$\lambda_t e_t \left(1-\tau_t^d - \Theta_t\right) = \beta \mathbb{E}_t \left(1 + r_t^*(d_t) + r_t^{*\prime}(d_t) d_t\right) \lambda_{t+1} e_{t+1}, \tag{1.53}$$

$$\mathbb{E}_t \left[\frac{1+i_t}{(1+\pi_{t+1})(1-\Theta_t)} \frac{\lambda_{t+1}}{\lambda_t}\right] = \mathbb{E}_t \left[\frac{(1+r_t^*(d_t) + r_t^{*\prime}(d_t)d_t)}{1-\tau_t^d - \Theta_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{e_{t+1}}{e_t}\right]. \tag{1.54}$$

Table 1.6 shows the Ramsey steady state of the extended model. The model supports the zero inflation rate as the Ramsey optimal outcome in the steady state.

Table 1.6: The Ramsey Steady States, Tošovský Model with Capital Controls

π^n	i	τ^d	e	$\frac{c}{y}$	$\frac{iv}{y}$	$\frac{k}{y}$	$\frac{c^n}{y^n}$	$\frac{iv^n}{y^n}$
0	6.52	0.63	1.0719	0.88	0.11	4.39	0.89	0.11

Notes. Variable π^n refers to the annualized net inflation rate in the nontradable sector (in percent), i refers to the annualized net nominal interest rate (in percent), and τ^d refers to the tax rate on external debt. Variable e refers to real exchange rate as nominal exchange rate divided by CPI price level. Variables $\frac{c}{y}$, $\frac{iv}{y}$, $\frac{k}{y}$, $\frac{c^n}{y^n}$, and $\frac{iv^n}{y^n}$ refer to consumption-output ratio, investment-output ratio, capital-output ratio, consumption-output ratio in the nontradable sector, and investment-output ratio in the nontradable sector, respectively.

The dynamics in the Ramsey equilibrium in Table 1.7 show that the correlation between nominal interest rate and tradable endowment, $\rho(i_t, y_t^\tau)$ is 0.82, which is positive. Recall that the correlation was -0.62 in the Tošovský model with one instrument in section 1.5. The correlation between the tax and external debt is 0.51, which implies that the tax works in a countercyclical fashion as well. This implies that the coordination between financial and monetary policy helps the interest rate policy regain a conventional counter-cyclical regime towards price stability.

Table 1.7: Unconditional Second Moments, Tošovský Model with Capital Controls

$\sigma\left(\pi_t^n\right)$	$\sigma\left(\tau_t^d\right)$	$\rho\left(i_t, y_t^\tau\right)$	$\rho\left(\tau_t^d, d_t\right)$
3.35	18.8	0.82	0.51

Notes. Symbol $\sigma(x)$ refers to the standard deviation of variable x in percentage points. Symbol $\rho(x, y)$ refers to the correlation coefficient between variables x and y.

Result 1.5 (The Effect of Policy Coordination) *If capital control tax is used as another Ramsey instrument, the optimal monetary policy becomes counter-cyclical as in the conventional framework without the Dilemma.*

1.8 Conclusion

This paper investigates optimal monetary policy when an economy is subject to the Tošovský Dilemma. The optimal monetary policy from the model predicts a pro-cyclical monetary policy response, which is paradoxical and contrasts with the conventional wisdom of monetary policy in standard NK models. The optimal monetary policy aims to stabilize the asset price, namely the price of collateral in terms of the tradable good, while allowing inflation in the nontradable sector. The result shows that the primary objective of monetary policy is achieving financial, rather than price stability. The prediction of this paper formalizes loose arguments made by some policymakers in emerging economies for loosening monetary policy when facing an economic boom associated with capital inflows and real appreciation.

The model in this paper shows the polar case in which the conventional monetary policy regime is disconnected from the concern of financial stability due to cross-border capital flows. The conclusion of the model comes from the assumption of frictions in the financial structure that create a strong wealth effect from the real exchange rate thus resulting in pro-cyclical capital flows. In the real world, the degree of frictions can be moderate and can vary across countries. The investigation of this case, however, clearly provides insight into the existence of open economy-specific trade-offs for monetary policy in emerging economies, which are absent in the standard NK models. The extensions of the model also predict that local currency denominated external debt and adding capital control tax as another instrument, enable monetary policy to once again be countercyclical towards price stability.

The findings of this paper encourage the empirical study of capital flows and monetary policy in future work. There are empirical studies on the connection among domestic monetary and financial policies and cross-border capital flows (see, for example, Ahmed and Zlate (2013), Ghosh et al. (2016), and Ghosh et al. (2017)), but investigating the microstructure of international financial markets is still needed. This extension is left for future research.

Chapter 2

Business Cycles and Labor Shares in Emerging Economies

2.1 Introduction

In modern economics, the labor share is the basic and fundamental measure of income distribution of the economy. It is a well-known fact that modern growth theories and dynamic macroeconomic models have been built based on the assumption of constant factor income shares, which has been perceived as a conventional wisdom.

Recently, there have been several attempts to re-examine the conventional wisdom. One branch of the attempts is studying the fluctuation of the labor share over the business cycle. If the labor share has cyclical properties, then that indicates income distribution of the economy has cyclical properties as well. That is because, the main sources of income of low and middle-income groups are labor incomes, whereas most capital income generally belongs to the high-income group.

Most studies on this topic have focused on advanced economies, including the United States and Western Europe. There has been a wide consensus that the labor share is highly countercyclical (see, for example, Merz (1995), Andolfatto (1996), Ríos-Rŭll and

Santaeulalia-Llopis (2010), and Commission (2007)). The countercyclicality of the labor share indicates that during an economic contraction, the labor share increases temporarily above its long-run trend. Then major cost taker of the economic contraction becomes the high income group since the capital income share shrinks. After the above works, the countercyclicality of the labor share become another conventional wisdom.

Our research starts with a very simple question. Are these conventional wisdoms also valid for emerging economies? There are few studies on this question, in spite of the fact that emerging economies have more than a half of world GDP. Emerging economies take peripheral status in the global economy, and it has been proven that business cycle facts in the economies are qualitatively different from those in advanced economies. To answer our question, we do a comprehensive cyclical analysis of labor shares around the world by using a long period of data of 40 years. In this paper, we consider 15 emerging economies and 20 advanced economies and log quadratically detrend all series as our baseline study.

Strikingly, we learn that evidence around the world does not support the conventional wisdom in general. Instead, we find three empirical regularities of labor shares movements over the business cycle. First, labor shares are as volatile as output so they are rarely stable. Both in emerging and advanced economies, relative standard deviations of labor shares with respect to output are around one. Second, labor shares in emerging economies are about twice as volatile as those in advanced economies. Average of standard deviations of labor shares in emerging economies is 6.1 percentage point, whereas it is 3.3 percentage point in advanced economies in our baseline study. Third, on average, labor shares are procyclical in emerging economies and they are countercyclical in advanced economies. Average of correlations between labor shares and output around emerging economies is 0.15, whereas it is -0.14 in advanced economies.

To affirm the reliability of our deviation from the conventional wisdom, we consider various issues those can potentially be pointed out. Most important one is the relevance of practical measure of the labor share, especially in emerging economies. The bias from an omission of the self-employment sector is well-documented in the seminal paper of Gollin (2002). labor share data which does not include labor income from self-employed sector can give the misguided estimate of the actual labor share of the economy. However, we learn that the point may not be a serious issue when we study cyclical component of the data which is *detrended*. We compare cyclical properties between total labor share data (which has self-employment omission issue) and corporate labor share data (which is free from the issue). We find that cyclical components of two data have systematic correspondence.

Further, we check robustness issues. Low-frequency movement of the labor share is extensively discussed in recent works (see, for example, Raurich et al. (2011), Elsby et al. (2013), Karabarbounis and Neiman (2014), Piketty (2014), Alvarez-Cuadrado et al. (2015)). Also, We discuss the possibility of spurious results when the data is not detrended. We apply other detrending methods and examine the sensitivity of our three empirical regularities. Finally, we discuss the problem caused by the short sample and country choice, those are potentially concerned in the contemporary work of Kabaca (2014).

Beside cyclical study, we also have a discussion on the suggested level of the labor share for the emerging economy, which can be applied to a quantitative macroeconomic model. By comparing several adjustments and their relevance, we propose the level labor share for the emerging economy around 0.70.

The remainder of the paper is organized as follows. Section 2.2 describes data, and section 2.3 shows three empirical regularities of labor shares around the world. Section 2.4 discusses the validity of practical labor share measures for cyclical analysis, section 2.5 discusses some robustness issues, and section 2.6 discusses the level of the labor share in the emerging economy. Section 2.7 concludes and closes the paper.

2.2 Measuring The Labor Share and Data Sources

In empirical work on the labor share, one of the important issues is whether the data is well measured. In economic theory, the labor share, denoted by s_L, is simply defined as

$$s_L = \frac{WL}{Y}, \tag{2.1}$$

where W is the wage, L is the labor force, and Y is the output or gross value added. Variables W and Y can be valued either in real or in nominal. Gross value added splits into labor income and capital income, so the labor share is the fraction of labor income in gross income of the economy. In practice, by using national account data, the *total* labor share (hereafter TLS) is calculated by

$$\text{TLS} = \frac{\text{Compensation of employees}}{\text{GDP (at basic price)}}[1]. \tag{2.2}$$

System of National Account (SNA) offers the standard of measuring denominator and numerator in right hand side, so we can access each component and calculate the TLS. And since many countries around the world have recorded their national account data based on SNA for many times, we can construct a long sample of the TLS.

One issue of the TLS, however, is the underestimation caused by omission of labor income from the self-employment sector, which is pointed in Gollin (2002). SNA does not have compensation for employees from the self-employed sector, such as sole proprietor and unincorporated businesses. TLS treats all of the income as mixed income in which labor income and capital income are not divided. The self-employment sector takes a significant portion of the economy, especially in emerging and developing economies. For example,

[1] In numerator, compensation of employees contains (pre-tax) wage bills and salaries, and employer's welfare contribution for employed workers (sickness, accidents, and retirement, etc.). In the denominator, GDP at basic (factor) price refers to the gross value added (GVA) of the economy. GDP at market (producer) price is calcluated by adding net indirect taxes (sum of product taxes less subsidies) on products and imports to the GDP at basic price. Hereby we denote GDP as GDP at basic price.

from world development indicators (WDI), the percentage of workers among the total employees in Argentina, Mexico, and South Korea in the year of 2011 are 23%, 33%, and 28%, respectively.

In consequence, the TLS tends to underestimate actual labor shares, and the underestimation issue can be potentially severe in emerging and developing economies. There is no simple and clear remedy, because it is implausible to extract labor income from the mixed income exactly. We rely on indirect remedies, and the most famous remedy is to adjust the TLS by assuming that average wage of the self-employed sector is same as the average wage in the corporate sector. This is called the *adjusted* labor share (hereafter ALS) which is calculated by

$$\text{ALS} = \frac{\frac{\text{Compensation of employees in corporate sector}}{\text{Number of employees in corporate sector}} \cdot \text{Total number of workforce}}{\text{GDP}}. \tag{2.3}$$

Organisation for Economic Co-operation and Development (OECD) calculates the ALS for its membership and some non-membership countries. In this paper, we use the ALS primarily, as a conservative measure of the labor share. And for emerging economies which don't provide the ALS, we use the TLS as a secondary option. In section 2.4, we will discuss the cyclical component of the TLS can be a good proxy for cyclical component of actual labor share.

On the other hand, sufficient length of data is crucially needed for valid cyclical analysis. As we will see in section 2.3, the labor share has evident low frequency movement in data. Thus it is necessary to detrend the data to avoid spurious result from the low frequency component. Long sample is needed to decompose secular component and cyclical component properly. Cyclical analysis based on short sample can also induce spurious result, because short sample includes short cycles of the economy.[2] For this reason, in our analysis, we restrict countries those can provide at least 30 years of the labor share data. Table 2.1

[2] García-Cicoo et al. (2010) also point the possibility of spurious results by using short data, when they discuss Aguiar and Gopinath (2006).

provides the list of countries, types of labor share data, number of observations, and sources of data. Countries are divided into two groups, those are emerging economies (hereafter EMs) and advanced economies (hereafter ADVs).[3]. The frequency of all data is annual.

Table 2.1: List of Countries and Description of the labor share Data

EMs	Type	Periods	Obs.	Source	ADVs	Type	Periods	Obs.	Source
Argentina	TLS	1960-2006	47	GK	Australia	ALS	1970-2011	42	OECD
Bolivia	TLS	1970-2011	42	UN	Austria	ALS	1970-2011	42	OECD
Chile	TLS	1974-2011	38	UN	Belgium	ALS	1970-2011	42	OECD
Costa Rica	TLS	1970-2011	42	UN	Canada	ALS	1970-2008	39	OECD
Greece	ALS	1970-2011	42	OECD	Denmark	ALS	1966-2011	42	OECD
Mexico	ALS	1970-2009	42	OECD	Finland	ALS	1970-2011	42	OECD
Peru	TLS	1975-2005	31	KN	France	ALS	1960-2011	52	OECD
Portugal	ALS	1970-2011	42	OECD	Germany	ALS	1970-2011	42	OECD
South Africa	TLS	1977-2011	35	UN	Iceland	ALS	1973-2011	39	OECD
South Korea	ALS	1980-2011	32	OECD	Ireland	ALS	1970-2011	42	OECD
Spain	ALS	1970-2011	42	OECD	Italy	ALS	1970-2011	42	OECD
Taiwan	TLS	1981-2010	31	KN	Japan	ALS	1970-2011	42	OECD
Thailand	TLS	1970-2011	42	UN	Luxembourg	ALS	1970-2011	42	OECD
Turkey	ALS	1970-2006	37	OECD	Netherlands	ALS	1969-2011	43	OECD
Venezuela	TLS	1970-2011	42	UN	New Zealand	ALS	1977-2009	33	OECD
					Norway	ALS	1970-2011	42	OECD
					Sweden	ALS	1970-2011	42	OECD
					Switzerland	ALS	1980-2011	32	OECD
					United Kingdom	ALS	1970-2011	42	OECD
					United States	ALS	1970-2011	42	OECD
Mean			39.1		Mean			41.3	

Notes. GK refers to Grena and Kennedy (2008), UN refers to United Nation database, and KN refers to Karabarbounis and Neiman (2014) dataset. For Bolivia, Chile, Costa Rica, South Africa, Thailand, and Venezuela, we calculate TLS using raw data of compensation of employees and gross value added (GDP at basic price), from SNA database from United Nations.

There are 15 EMs and 20 ADVs in our data. Average number of observations of labor shares is 39.1 years in EMs, and is 41.3 years in ADVs. All of the ALS data are collected from OECD Statistics.[4] We use full sample of the ALS data from OECD, except South Korea, which we discard the periods of 1970s.[5] The TLS come from various sources. For Argentina,

[3]For classifying emerging economies and advanced economies, we follow Uribe and Schmitt-Grohé (2017)

[4]Available online at http://stats.oecd.org/index.aspx?queryid=345. The name of the series is *Labour Income Share Ratios*.

[5]The reason is because ALS in this period is substantially unreliable. The average ALS of South Korea in 1970-1979 is 0.91 which is extremely high, whereas average TLS during the periods is only 0.34. Thus, cyclical results including this period are very sensitive with marginal inclusion and exclusion of year of data

we use data from Grena and Kennedy (2008). For Peru and Taiwan, we use the TLS from Karabarbounis and Neiman (2014) dataset (hereafter KN dataset). For remaining countries, we calculate the TLS by applying equation (2.2), using compensation for employees and gross value added, from the United Nation (hereafter UN dataset).

To decompose secular and cyclical components, we apply log quadratic (hereafter LQ) detrending method and Hodrick-Prescott (hereafter HP) filtering. In HP filtering, we set $\lambda = 100$, following suggestion in Uribe and Schmitt-Grohé (2017) for annual series. Same methods are applied to output. [6]

2.3 Three Empirical Regularities

Using cyclical components of the labor share and output, we calculate standard deviation of each variable, and correlations between the two. After getting second moments of individual countries, we calculate the average of each second moments in EMs and in ADVs. And we test null hypothesis that two second moments are same between the two groups, by applying Student's t test.

Table 2.2 shows the average and median second moments of labor shares (s_L) and output (y) in two groups. First, we can see that there is a striking difference in the volatility of labor shares between the two groups. With LQ detrended series, average of standard deviations of labor shares in EMs ($\sigma(s_L)^{EMs}$) is 6.1 percentage point, whereas it is 3.3 percentage point in ADVs (($\sigma(s_L)^{ADVs}$). The ratio between the two average standard deviations ($\sigma(s_L)^{EMs}/\sigma(s_L)^{ADVs}$) is 1.8, which is close to 2. HP detrended series also show similar results. Average of standard deviations of labor shares is 4.6 percentage point in EMs, and it is 2.2 percentage point in ADVs, so the ratio between the two values of 2.0. For the volatility

of the periods. Thus we decide to use data after 1980.

[6] Real GDP per capita, with local currency unit (indicator code NY.GDP.PCAP.KN). All GDP data are collected from WDI, except Taiwan. GDP data for Taiwan is collected from Penn World Table 7.1 (real gdp per capita with PPP converted international dollar. Variable code *rgdpch*).

of output, the excess volatility of output in EMs is well-documented as a stylized fact. [7]

From these findings, we can see the ratio between average standard deviation of labor shares and that of output ($\sigma(s_L)/\sigma(y)$) is near 1 around the world. With LQ detrended series, $\sigma(s_L)/\sigma(y)$ is 1.0 both in EMs and in ADVs, and with HP filtered series, $\sigma(s_L)/\sigma(y)$ is 1.1 in EMs and is 1.0 in ADVs. These indicate that labor shares are volatile as much as output around the world.

Finally, correlations between labor shares and output are not near-zero (as it is expected in Cobb-Douglas production theory). They do have nonzero correlation and more importantly, the sign of correlations is different between the two groups. With LQ detrended series, $corr(y, s_L)$ is 0.15 in EMs, and it is -0.14 in ADVs. With HP filtered series, $corr(y, s_L)$ gets smaller but is still positive (0.09) in EMs, and it gets more negative in ADVs (-0.19).

Table 2.2: The labor share and GDP Per Capita: Second Moments

	EMs			ADVs			
	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)^{EMs}/\sigma(s_L)^{ADVs}$
LQ							
Average	6.1	1.0	0.15	3.3	1.0	-0.14	1.8
Median	4.3	0.9	0.16	2.9	0.8	-0.12	1.4
HP ($\lambda = 100$)							
Average	4.6	1.1	0.09	2.2	1.0	-0.19	2.0
Median	3.8	1.0	0.09	1.8	0.8	-0.20	2.0

Notes. Variable s_L is the cyclical component of the labor share, and y is the cyclical component of GDP per capita. Symbol $\sigma(\cdot)$ refers to standard deviation of the variable, and $corr(\cdot,\cdot)$ refers to correlation between two variables. Standard deviations are measured in percentage point.

These differences of standard deviations and correlations between EMs and ADVs are statistically significant. Table 2.3 shows t-statistics under the null hypothesis that assumes the second moments between EMs and ADVs are same. The t-statistics calculated under equal variance assumption (t_1) and unequal variance assumption (t_2) both. Table 2.3 shows that t-statistics with respect to $\sigma(s_L)$, and $corr(y, s_L)$ reject null hypothesis with less than

[7] Uribe and Schmitt-Grohé (2017) find excess volatility of output in emerging economies, that is twice as volatile as in advanced economies.

5% significance level. [8]

Table 2.3: t-Statistics under Null Hypothesis

	$\sigma(s_L)$	$corr(y, s_L)$
LQ		
t_1	3.0***	2.9***
t_2	2.7**	3.1***
HP ($\lambda = 100$)		
t_1	3.5***	3.1***
t_2	3.1***	3.9***

Notes. Null hypothesis assumes average second moment ($\sigma(s_L), \sigma(s_L)/\sigma(y), corr(y, s_L)$) in each income group (EMs and ADVs) are same. Variable t_1 refers to t-statistics with equal variance assumption of each moment. Variable t_2 refers to t-statistics with unequal variance assumption. Asterisks **, *** refer to rejection of null hypothesis with 5%, 1% significance level, respectively.

It is necessary to mention that we can see some exceptions in each income group if we look at individual countries. Table 4 shows second moments for individual countries in EMs and in ADVs. In EMs, Greece (-0.07 (LQ), -0.12 (HP)), Portugal (-0.04 (LQ), -0.24 (HP)),

[8] The t-statistic is calculated by applying

$$t_1 = \frac{\bar{X}_{EMs} - \bar{X}_{ADVs}}{S_{X_{EMs}X_{ADVs}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where

$$S_{X_{EMs}X_{ADVs}} = \sqrt{\frac{(n_1 - 1)S^2_{X_{EMs}} + (n_2 - 1)S^2_{X_{ADVs}}}{n_{EMs} + n_{ADVs} - 2}},$$

where $\bar{X}_i$ is the cross sectional average of second moments X_i in country group $i \in \{EMs, ADVs\}$, $S_{X_{EMs}X_{ADVs}}$ is common standard deviation of the two samples X_{EMs} and X_{ADVs}. Variables $S^2_{X_{EMs}}$ and $S^2_{X_{ADVs}}$ are sample variances of second moments of the groups. Variables n_{EMs} and n_{ADVs} indicate the total number of countries in EMs and ADVs, respectively. Under the null hypothesis, t_1 follows Student's t-distribution with $2n-2$ degree of freedom, where $n = n_{EMs} + n_{ADVs}$. With unequal variance assumption, t-statistic is calculated by applying

$$t_2 = \frac{\bar{X}_{EMs} - \bar{X}_{ADVs}}{S_{\bar{X}_{EMs} - \bar{X}_{ADVs}}},$$

where

$$S_{\bar{X}_{EMs} - \bar{X}_{ADVs}} = \sqrt{\frac{S^2_{EMs}}{n_1} + \frac{S^2_{ADVs}}{n_2}}.$$

Then the statistic follows Student's t-distribution approximately with degrees of freedom of

$$df = \frac{(S^2_{EMs}/n_{EMs} + S^2_{ADVs}/n_{ADVs})^2}{(S^2_{EMs}/n_{EMs})^2/(n_{EMs} - 1) + (S^2_{ADVs}/n_{ADVs})^2/(n_{ADVs} - 1)}.$$

South Africa (-0.06 (LQ), -0.25 (HP)), and Venezuela (-0.11 (LQ), -0.21 (HP)) show negative correlations. In ADVs, Australia (0.23 (LQ), 0.06 (HP)) and Iceland (0.41 (LQ), 0.72 (HP)) give positive correlations. In spite of these exceptions, the percentage of countries who have positive $corr(y, s_L)$ is 60% with LQ and is 66% with HP in EMs. On the other hand, the percentage of countries who have negative $corr(y, s_L)$ is 60% with LQ and is 90% with HP in ADVs. The countercyclical labor shares is not major phenomenon in EMs whereas it surely is in ADVs.

We can now build following three empirical regularities of the labor share fluctuation.

Fact 2.1. (Volatility of Labor Shares) *labor shares are volatile as much as output around the world.*

Fact 2.2. (Excess Volatility of Labor Shares in Emerging Economies) *labor shares in emerging economies are about twice as volatile as labor shares in advanced economies.*

Fact 2.3. (The Procyclicality of Labor Shares in Emerging Economies and The Countercyclicality of labor shares in Advanced Economies) *On average, correlation between labor shares and output is positive in emerging economies, and it is negative in advanced economies.*

Figure 2.1 and Figure 2.2 visualize the empirical facts. Figure 2.1 shows historical movements of labor shares in three typical emerging economies; Argentina, Mexico, and South Korea. Before discussing cyclical properties, we should pay attention to the fact that labor shares show low-frequency movements. In three panels of the first column, labor shares in Mexico and South Korea show declining trend, although it is quite modest in Argentina. These imply that cyclical analysis can be spurious if the data is not detrended.

Panels in the second column of Figure 2.1 show cyclical components of the data after

Table 2.4: Second Moments of labor shares and GDP Per Capita: Individual Countries

	LQ			HP				LQ			HP		
EMs	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	ADVs	$\sigma(y)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$
Argentina	13.6	1.5	0.26	10.7	1.8	0.44	Australia	2.5	2.19	0.23	1.9	1.43	0.06
Bolivia	9.7	1.3	-0.04	7.7	2.3	0.17	Austria	5.6	2.13	0.21	2.3	4.56	-0.14
Chile	6.8	0.9	0.37	4.4	0.9	-0.14	Belgium	3.3	2.25	0.19	1.6	1.67	-0.12
Costa Rica	4.2	0.7	0.16	3.8	1.0	0.35	Canada	2.1	3.23	-0.62	1.4	2.21	-0.41
Greece	3.4	0.5	-0.07	3.0	0.8	-0.12	Denmark	2.4	2.89	-0.37	1.6	2.05	-0.29
Mexico	7.8	1.3	0.47	5.0	1.4	0.37	Finland	4.3	5.19	-0.12	2.4	3.65	-0.30
Peru	7.8	1.0	0.50	6.8	1.1	0.41	France	3.0	3.08	0.26	1.2	1.47	-0.50
Portugal	5.7	1.0	-0.04	4.3	1.2	-0.24	Germany	1.8	1.93	-0.09	1.4	1.71	-0.21
South Africa	3.1	0.7	-0.06	2.5	1.0	-0.25	Iceland	6.2	5.92	0.41	4.4	3.69	0.72
South Korea	3.2	1.0	0.25	2.3	0.9	0.35	Ireland	5.7	9.44	-0.70	3.5	3.32	-0.37
Spain	3.0	0.6	0.13	1.8	0.6	0.09	Italy	2.9	2.34	-0.67	1.4	1.76	-0.44
Taiwan	2.2	0.7	0.39	1.5	0.6	0.11	Japan	3.5	3.69	-0.74	1.7	2.11	-0.56
Thailand	4.3	0.5	-0.09	3.4	0.7	0.02	Luxembourg	6.2	7.20	-0.35	4.7	3.53	-0.09
Turkey	13.1	2.8	0.17	9.6	2.4	0.02	Netherlands	2.5	3.01	0.17	1.5	1.78	-0.09
Venezuela	4.1	0.5	-0.11	2.8	0.5	-0.21	New Zealand	2.9	3.48	-0.14	2.5	2.44	-0.13
							Norway	4.8	2.93	-0.21	4.1	1.85	-0.02
							Sweden	3.1	3.41	-0.00	2.6	2.49	-0.13
							Switzerland	1.5	2.34	-0.31	1.4	1.88	-0.37
							United Kingdom	2.2	3.58	-0.12	2.0	2.40	-0.19
							United States	0.8	2.89	0.03	0.7	2.10	-0.21

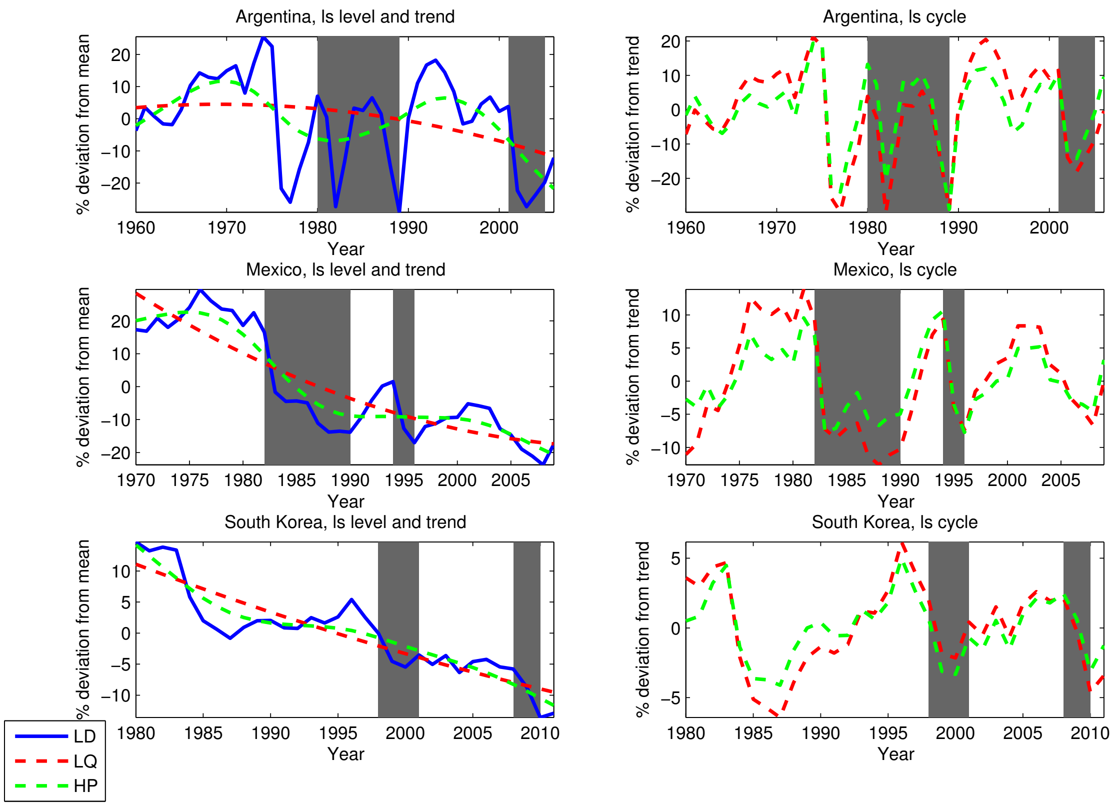
Notes. Variable s_L is the cyclical component of labor share, and y is the cyclical component of GDP per capita. Symbol $\sigma(\cdot)$ refers to standard deviation of the variable, and $corr(\cdot, \cdot)$ refers to correlation between the two variables. Standard deviations are measured in percentage point.

detrending. Cyclical component of the labor share in Argentina exhibits evident procyclical movement. During 1980-1989, Argentine economy was fall in severe economic contraction so called the Lost Decade (La Década Perdida). During the period, Argentina claimed sovereign default in 1982 (which was along with Latin American debt crises), and experienced hyperinflation crisis in 1989. After 21st century, Argentine economy crashed and defaulted again in December 2002. Upper panel in the second column in Figure 1 shows that the labor share in Argentina decreased about 30 percentage point in crises in 1982 and 1989, and was below the mean throughout the Lost Decades, and decreased 15 percentage point during debt crisis in the early 21st century.

Mexico is the same example. During 1982-1990, Mexico also experienced the Lost Decade recession, and claimed sovereign default in 1982. And during 1994-1995, the Mexican Peso Crisis broke out. During the Lost Decade, the labor share in Mexico decreased continuously and the difference between 1982 and 1990 is almost 20 percentage point and there were also big drop of the labor share in during the Peso Crisis. We can see there was almost 15 percentage point drop of the labor share compared to its local pick in 1994.

In the case of South Korea, during 1997-2001 recession from the Asian Financial Crisis in the late 1990s, the labor share dropped almost 5 percentage and recovered with the end of the recession. And, during contraction in 2008-2010 from the Great Recession, the labor share dropped almost 7 percentage point.

Figure 2.1: Labor Shares in Three Typical Emerging Economies

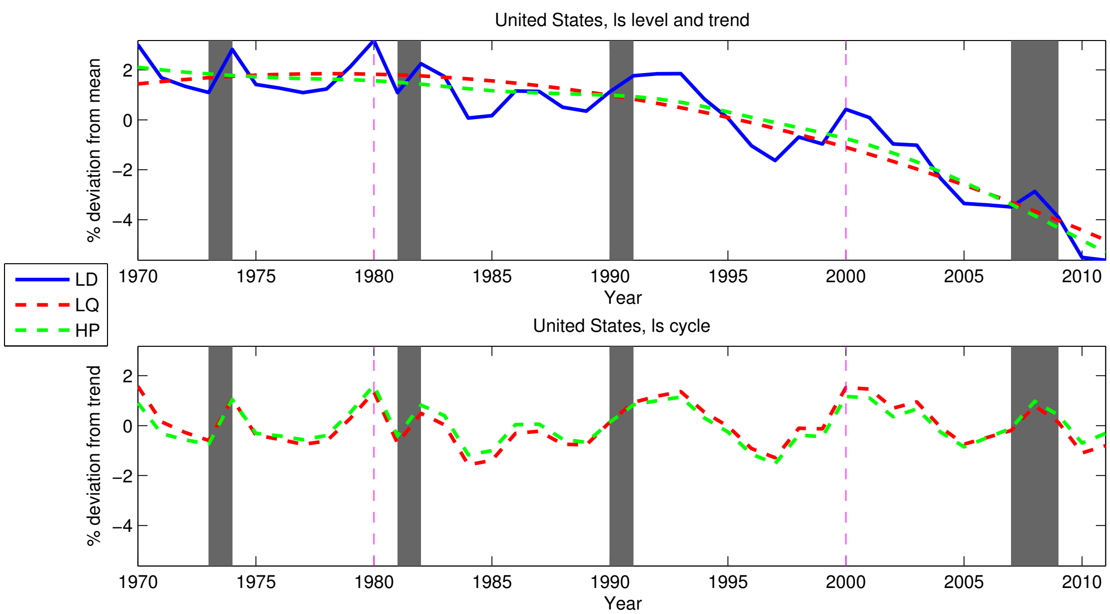


Notes. In all panels, horizontal axes refer to year. In left panels, unit of vertical line is percentage deviation from the mean. In right panels, unit of vertical line is percentage deviation from the trend. In left panels, blue solid line refers to log-demeaned labor share, and red and green dashed lines refer to the secular components of the log-demeaned labor share after applying log quadratic detrending and Hodrick-Prescott (hereafter HP) filtering with weight parameter $\lambda = 100$, respectively. In all panels, grey filled areas indicate periods of recession. Each row displays each individual country. labor shares are total labor shares (TLS) for Argentina, and adjusted labor shares (ALS) for Mexico and South Korea.

In the United States, a typical advanced economy, however, the procylical pattern of the labor share is not captured. Figure 2.2 shows the movement of the labor share in the United States. We can also confirm that the labor share in the United States has also secular movement which is declining (upper panel) so we detrend the data which is shown in the lower panel.

After 1970, the United States have experienced several major recessions. Oil crisis and stock market crash during 1973-1974, the Volcker recession in 1980, early 1980s recession during 1981-1982, early 1990s recession during 1990-1991, the collapse of dot-com bubble in 2000, and the Great Recession. Unlike three emerging economies above, cyclical component of the labor share in the United States increased temporarily during the recessions. Further, the volatility is smaller than three emerging examples. The magnitude of variation of cyclical components is about 2 percentage point, whereas they are more than 10 percentage point in Figure 2.1 countries.

Figure 2.2: Labor Share in the United States

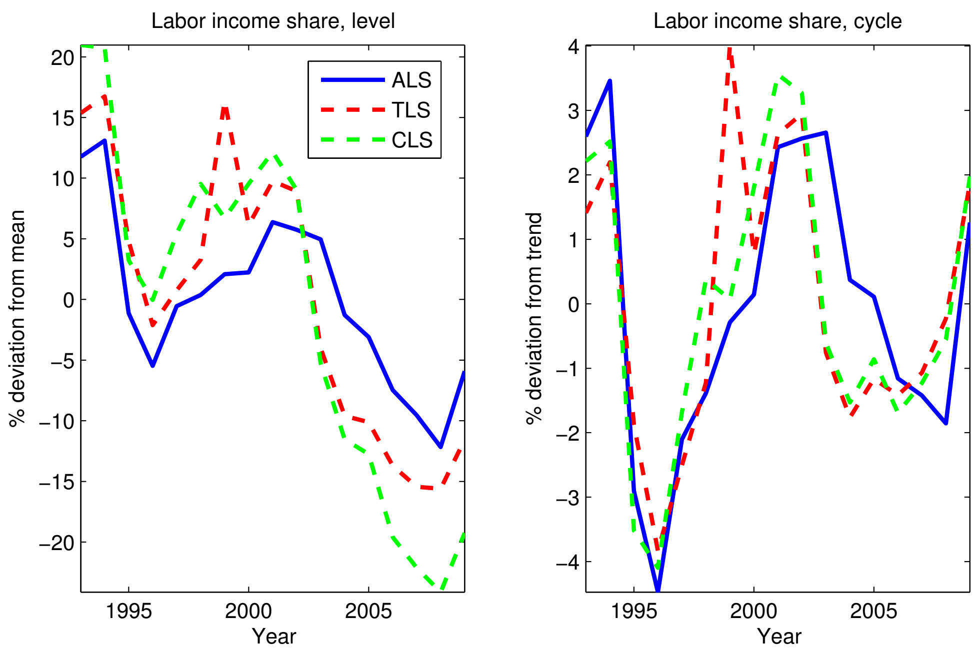


Notes. In all panels, horizontal axes refer to year. In upper panel, unit of vertical line is percentage deviation from the mean. In bottom panel, unit of vertical line is percentage deviation from the trend. In upper panel, blue solid line refers to log-demeaned labor share, and red and green dashed lines refer to the secular components of the Labor Share, after applying log quadratic detrending and Hodrick-Prescott (hereafter HP) filtering with weight parameter $\lambda = 100$, respectively. In lower panel, red and green dashed lines refer to the cyclical component of the log demeaned Labor Share. In all panels, grey filled areas indicates periods of recession. Magenta dashed lines refer to single-year recessions. The labor share is the adjusted labor share (ALS).

2.4 The Validity of the TLS for Cyclical Analysis

As Table 2.1 shows, we use the TLS for 9 emerging countries (Argentina, Bolivia, Chile, Costa Rica, Peru, South Africa, Taiwan, Thailand, Venezuela) since the ALS for those countries is not available. A natural concern is whether the TLS can represent the cyclical behavior of the actual labor share. In this section, we will show that the TLS is a valid proxy for cyclical analysis since it traces the cyclical fluctuation of clean measure of the labor share satisfactory, in spite of its underestimation issue in level pointed in Gollin (2002).

Figure 2.3: Three Labor Shares Data in Mexico, 1993-2009



Notes. Left panel exhibits log-demeaned labor share data, and right panel exhibits LQ detrended data. In both panels, unit of horizontal line is year. In left panel, unit of vertical line is percentage deviation from the mean. In right panel, unit of vertical line is percentage deviation from the trend. Blue solid line is the total labor share (TLS), red dashed line is the adjusted labor share (ALS), and green dashed line is the corporate labor share (CLS).

Figure 2.3 shows dynamics of three different labor share data in Mexico, during 1993-2009. For both panels, blue solid line refers to the ALS and red dashed line refers to the TLS. Green dashed line refers to the corporate labor share (hereafter CLS), which is calculated as

$$\text{CLS} = \frac{\text{Compensation for employees in corporate sector}}{\text{Value added output in corporate sector}}. \tag{2.4}$$

These three labor share measures exhibit similar up-and-down patterns during overlapping periods. All of log-demeaned measures in left panel share significant drops in 1995, the year of peso crisis and reach troughs in 1996. Also, all of them begin to decrease in 2003, and reach troughs in 2008. Cyclical components in right panel share similar patterns as well. All three detrended data drop in 1995 and reach troughs in 1996. And they still closely co-move after then. These motivate the idea that cyclical component of the TLS is a valid proxy for cyclical component of the actual labor share.

In the remaining part of this section, we compare cyclical components between the TLS and the CLS around the world, to examine the idea precisely. Reasons behind choosing the CLS (not ALS) as a counterpart of the TLS are the following. First, the availability of the data. Thanks to comprehensive work in Karabarbounis and Neiman (2014), we can access the CLS dataset in many emerging economies from KN dataset even if the ALS is not available for those countries. Second, the CLS is a clean measure for the labor share in the corporate sector and so it is free from self-employment omission bias problem. Compensation of employees in the TLS contains compensation of employees in the corporate sector and government sector. If we admit that compensation of employees in government sector is relatively negligible, the TLS can be interpreted as a noisy measure of the CLS. Thus, by comparing cyclical components of these two measures we can check whether the noise distorts actual cyclical behavior or not.

We collect countries that have at least 15 years of the CLS and the TLS. We have 26 countries in EMs and 13 countries in ADVs. Table B.1 in Appendix B describes the list of countries, descriptions of the CLS and the TLS. Number of observation is smaller than samples in Table 1, since samples of the CLS in KN dataset is relatively shorter than those of the TLS. We compare the cyclical components of the CLS and the TLS within overlapping periods.

Table 2.5: Average of Correlations between Cyclical Components of the TLS and the CLS

	EMs	ADVs
$corr(s_{TLS}, s_{CLS})_{LQ}$	0.66	0.93
number of outlier	4	0
$corr(s_{TLS}, s_{CLS})_{LQ}$ without outlier	0.79	0.93
$corr(s_{TLS}, s_{CLS})_{HP}$	0.63	0.93
number of outlier	4	0
$corr(s_{TLS}, s_{CLS})_{HP}$ without outlier	0.76	0.93

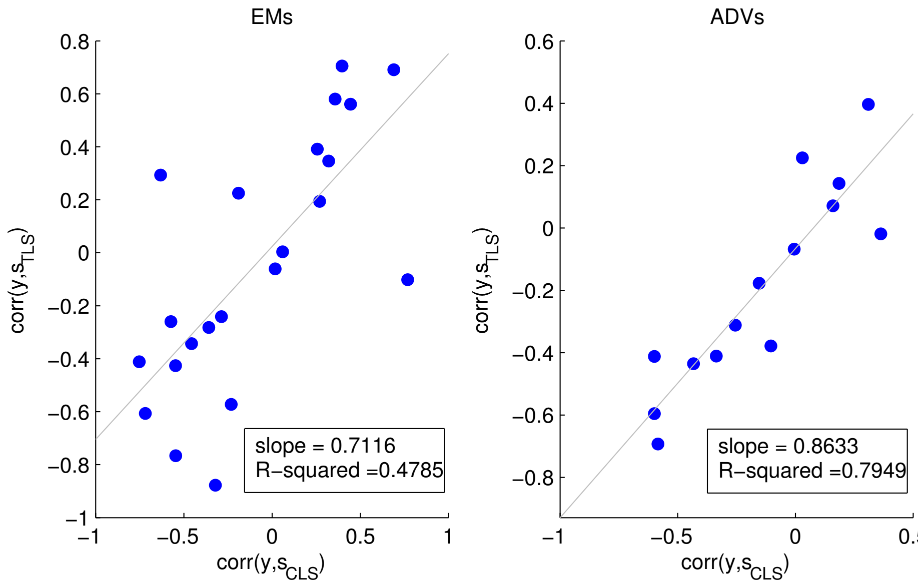
Notes. Variables s_{TLS} and s_{CLS} refer to cyclical component of the TLS and the CLS, respectively. LQ detrending and HP filtering ($\lambda = 100$) are used. For all detrending methods, outliers in EMs are Brazil, China, Colombia, and Tunisia.

Table 2.5 shows the results. With LQ detrended series, correlation between s_{TLS} and s_{CLS} is 0.65 in EMs, and it is 0.93 in ADVs. There are four outliers in EMs whose correlations are less than 0.3 (Brazil (0.17), China (0.28), Colombia (-0.70), and Tunisia (0.13)). If we exclude those outliers, average correlation in EMs jumps to 0.79. HP filtered series gives similar result. Variable $corr(s_{TLS}, s_{CLS})$ in EMs is 0.63, and it is 0.93 in ADVs. If we exclude same 4 outliers in EMs (Brazil (-0.09), China (0.24), Colombia (-0.69), and Tunisia (0.15)), then average correlation in EMs becomes 0.76.

Figure 2.4 shows scatter plots of $corr(y, s_{CLS})$ and $corr(y, s_{TLS})$. All series are LQ detrended, and above four outliers are excluded from EMs. In EMs, when we regress $corr(y, s_{TLS})$ on $corr(y, s_{CLS})$ with constant, we get slope of 0.71, and goodness of fit is $R^2 = 0.47$. These indicate strong and significant positive relation. In ADVs, the relationship gets stronger. The regression coefficient is 0.86 and goodness of fit is $R^2 = 0.79$. Thus cyclical components of the TLS and the CLS are very closely correlated and we can expect that Fact 2.3 is not sensitive to the change of the labor share data.

In addition, Table 2.6 shows average standard deviations of s_{CLS} and s_{TLS} in the overlapping periods. First, we can find that average volatility of s_{CLS} is uniformly less than s_{TLS} (regardless of detrending methods and income groups), which indicates relatively smaller

Figure 2.4: Scatter Plots of $corr(y, s_{CLS})$ and $corr(y, s_{TLS})$



Notes. In both panels, horizontal axes are correlation between output and the CLS, and vertical axes are correlation between output and the TLS. All variables are LQ detrended. Grey linear lines are regression lines. Regressions include intercepts.

Table 2.6: Average Standard Deviations of Cyclical Components of the CLS and the TLS

	EMs		ADVs		$\frac{\sigma(s_L)^{EMs}}{\sigma(s_L)^{ADVs}}$	
	CLS	TLS	CLS	TLS	CLS	TLS
LQ						
$\sigma(s_L)$	6.1	4.2	3.1	2.3	1.9	1.7
$\sigma(s_L)/\sigma(y)$	1.3	0.9	1.1	0.8		
HP						
$\sigma(s_L)$	5.8	3.9	2.5	2.0	2.2	1.9
$\sigma(s_L)/\sigma(y)$	1.3	0.9	1.2	0.9		

Notes. The calculation is based on overlapping periods of two data. Standard deviations are measured in percentage point. Sixth and seventh columns show ratios between average standard deviation of the labor share in EMs and in ADVs.

volatility of the self-employed sector. As we can see in equation (2.4), the denominator in the equation for the CLS is value added in corporate sector, whereas that in the TLS is gross value added of the nation, which includes value added in the self-employed sector.[9] Never-

[9]The potential mechanism is the labor reallocation between the corporate sector and the self-employed

theless, the excess volatility of the CLS compared with the TLS is not huge and $\sigma_{CLS}/\sigma y$ is 1.3 in EMs and 1.1 in ADVs, both of them are near unity. Also, the ratio $\frac{\sigma(s_{CLS})^{EMs}}{\sigma(s_{CLS})^{ADVs}}$ is 1.9 in EMs and 2.2 in ADVs, those are near 2.2. Thus the Facts 2.1 - 2.2 are not sensitive to the change of the data.

In sum, the TLS is a good proxy for cyclical study of the actual labor share.

2.5 Robustness

In this section, we examine the robustness of the Facts 2.1 - 2.3 in chapter 2.3 from detrending and the number of samples. In sections 2.5.1 - 2.5.2, we discuss how the results change if we do not detrend the labor share data or apply other minor detrending methods. In addition, in section 2.5.3, we discuss that cyclical analysis based on short samples can generate imprecise results.

2.5.1 Non-Detrended Labor Shares

In section 2.3, not only output but labor shares are detrended, since we find Labor Shares have evident low frequency movements. However, stationarity (constancy) of the labor share in the long-run has been a conventional wisdom after influential work in Kaldor (1961). It is worthy to keep in mind that even if the benchmark is tremendously controversial, especially in recent decades. This section we assume that the income share itself is a stationary variable. We redo the same exercises of section 2.3, by using log-demeaned labor share.[10] Output is detrended.

Table 2.7 shows average and median of second moments. With log-demeaned labor shares associated with detrended output (either LQ detrended or HP filtered), Fact 2.3 holds, whereas Facts 1-2 are not the cases. The ratios $\sigma(s_L)/\sigma(y)$ are a bit far from unity,

sector over the business cycle.

[10]The reason for using log-demeaned variable is to measure its volatility in percentage point.

Table 2.7: Average and Median Second Moments: Non-Detrended Labor Shares

	EMs			ADVs			
	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)^{EMs}/\sigma(s_L)^{ADVs}$
Log Demeaned s, w/ y_{LQ}							
Average	9.9	1.6	0.09	6.3	2.0	-0.08	1.5
Median	7.3	1.5	0.09	7.1	1.4	-0.06	1.0
Log Demeaned s, w/ y_{HP}							
Average	9.9	4.6	0.07	6.3	4.8	-0.06	1.5
Median	7.3	3.8	0.07	7.1	3.6	-0.07	1.0

Notes. Variable s_L is the cyclical component of the labor share, and y is the cyclical component of GDP per capita. Symbol $\sigma(\cdot)$ refers to standard deviation of the variable, and $corr(\cdot,\cdot)$ refers to correlation between two variables. Standard deviations are measured in percentage point.

and $\sigma(s_L)^{EMs}/\sigma(s_L)^{ADVs}$ is 1.5 which a bit far from 2. If we take median, these discrepancies of Facts 2.1 - 2.2 get worse.

Table 2.8: Second Moments of Individual Countries: Non-Detrended Labor Shares

	y_{LQ}			y_{HP}				y_{LQ}			y_{HP}		
EMs	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	ADVs	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$
Argentina	14.4	1.6	0.24	14.4	2.5	0.37	Australia	7.3	3.3	0.08	7.3	5.1	0.02
Bolivia	10.9	1.5	-0.38	10.9	3.3	0.13	Austria	13.6	6.4	0.09	13.6	8.7	0.01
Chile	7.3	0.9	0.34	7.3	1.5	0.22	Belgium	3.6	1.6	0.17	3.6	2.1	0.06
Costa Rica	7.2	1.2	0.09	7.2	1.9	0.20	Canada	3.7	1.1	-0.35	3.7	1.6	-0.22
Greece	10.6	1.6	-0.02	10.6	2.8	-0.04	Denmark	3.1	1.0	-0.28	3.1	1.5	-0.16
Mexico	16.0	2.7	0.23	16.0	4.5	0.18	Finland	7.1	1.3	-0.07	7.1	1.9	-0.13
Peru	15.6	2.0	0.25	15.6	2.5	0.23	France	7.5	2.4	0.10	7.5	5.1	-0.08
Portugal	5.9	1.0	-0.04	5.9	1.6	-0.10	Germany	3.9	2.0	-0.44	3.9	2.3	-0.05
South Africa	5.8	1.3	-0.03	5.8	2.4	-0.13	Iceland	8.5	1.4	0.30	8.5	2.3	0.46
South Korea	7.0	2.2	0.11	7.0	2.7	0.12	Ireland	10.7	1.1	-0.38	10.7	3.2	-0.26
Spain	6.0	1.2	0.06	6.0	2.1	0.07	Italy	8.4	3.5	-0.23	8.4	4.7	-0.11
Taiwan	4.4	1.4	0.19	4.4	1.8	0.17	Japan	7.3	1.9	-0.35	7.3	3.4	-0.20
Thailand	9.2	1.9	-0.12	9.2	3.4	-0.10	Luxembourg	7.0	0.9	-0.30	7.0	2.0	-0.11
Turkey	14.8	3.1	0.15	14.8	3.7	0.07	Netherlands	4.3	1.4	0.10	4.3	2.4	0.04
Venezuela	6.1	0.8	-0.07	6.1	1.0	-0.13	New Zealand	7.1	2.0	-0.05	7.1	2.9	-0.06
							Norway	10.8	3.7	-0.09	10.8	5.8	-0.04
							Sweden	4.8	1.4	0.00	4.8	1.9	-0.06
							Switzerland	2.7	1.1	-0.17	2.7	1.4	-0.20
							United Kingdom	2.5	0.7	-0.10	2.5	1.0	-0.13
							United States	2.2	0.7	0.01	2.2	1.0	-0.05

Notes. Variable s_L is the cyclical component of labor share, and y is the cyclical component of GDP per capita. Symbol $\sigma(\cdot)$ refers to standard deviation of the variable, and $corr(\cdot,\cdot)$ refers to correlation between the two variables. Standard deviations are measured in percentage point.

These results are clearly spurious, by the secular components of labor shares. The existence of low frequency movements is clear when we compare Table 2.4 and Table 2.8. Second and fifth columns in Table 2.8 show that standard deviations of log-demeaned labor shares in individual countries in EMs, and ninth and twelfth columns show those in ADVs. By comparing the counterparts of LQ detrended labor shares in Table 2.2 (second and ninth columns), we can find that $\sigma(s_L)$s with log demeaned data are far bigger than $\sigma(s_L)$s with LQ detrended data in EMs and in ADVs both. Especially, about 45 percent of each income group (Greece, Mexico, Peru, South Korea, Spain, Taiwan, and Thailand in EMs, Australia, Austria, France, Germany, Italy, Japan, New Zealand, Norway, and United States in ADVs) have more than twice bigger labor share volatility, when we use log-demeaned data. The fact that differences between variances of detrended and non-detrended data are too huge indicates that a lot of proportion of the volatility of the non-detrended data come from the low frequency movement of variables. This implies the necessity of detrending the labor share data.

2.5.2 Alternative Detrending Methods

In this section, we apply other detrending methods. First, we apply $\lambda = 6.25$ for HP filter. Second, we assume that the series have stochastic trends but log of each series are integrated with order one, so their growth rates are stationary. Table 2.9 shows average and median of second moments of the series. In the case of detrended series by HP filter with $\lambda = 6.25$, we can verify that the Facts 2.1 - 2.2 hold well whereas Fact 2.3 doesn't fit exactly. Correlations between output and labor shares are near zero in EMs, and they are highly negative in ADVs (-0.37 on average and -0.49 as a median). These phenomena also occur with log differenced series. The Facts 2.1 - 2.2 also hold but correlations between output and labor shares are near zero in EMs, whereas they are highly negative in ADVs (-0.28 on average, and -0.29 as a median).

How should we interpret the violation of the Fact 2.3? First, these two alternative methods have some issues and may be inappropriate for cyclical study for emerging economies. As Uribe and Schmitt-Grohé (2017) discuss, HP filter with $\lambda = 6.25$ tends to treat many actual cycles as a trend. [11] Log differencing have the same issue. Log differencing assumes all series

[11] For example, HP filter with $\lambda = 6.25$ attributes output drops in the 1989 crisis and 2001 sovereign default in Argentina as secular phenomena, whereas they are considered as cyclical phenomena in many empirical

Table 2.9: Average and Median of Second Moments: Alternative Detrending

	EMs			ADVs			
	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)^{EMs}/\sigma(s_L)^{ADVs}$
HP ($\lambda = 6.25$)							
Average	3.2	1.5	-0.01	1.6	1.1	-0.37	2.0
Median	2.7	1.1	-0.01	1.3	0.9	-0.49	2.0
Growth Rate							
Average	4.9	1.2	0.00	2.4	1.0	-0.28	2.0
Median	4.2	1.1	-0.06	1.9	0.9	-0.39	2.1

Notes. Variable s_L is the cyclical component of the labor share, and y is the cyclical component of GDP per capita. Symbol $\sigma(\cdot)$ refers to standard deviation of the variable, and $corr(\cdot, \cdot)$ refers to correlation between two variables. Standard deviations are measured in percentage point.

have stochastic trends, and the assumption is not in other detrending methods. Deciding which assumption is true is not a simple one. [12] Further, in spite of the violation of the Fact 3, we can still find a qualitative distinction between EMs and ADVs in individual level. In EMs, about a half of countries have positive correlations, whereas 95 percent of countries in ADVs have highly negative correlations. In addition, typical emerging countries in Figure 1 (Argentina, Mexico, and South Korea) and Bolivia, Costa Rica, Peru, who take 40 percent of EMs, always have positive correlations, regardless of detrending methods. These implies pervasive procyclical labor shares in EMs which is not in ADVs at all.

2.5.3 Short Samples and Choice of Countries

It should be discussed that the contemporary work in Kabaca (2014) also claims the Facts 2.2 - 2.3, by using the TLS data for 18 emerging and advanced economies. This may be important since similar statements from contemporary studies can strengthen the power of empirical regularities. However, there are potential problems that the way of collecting data and choosing countries are not rigorous.

grounds.

[12] Nelson and Plosser (1982) initiate the issue, but it is shown that the issue is difficult to solve by the following up studies.

Table 2.10: Second Moments of Individual Countries: Alternative Detrending

	HP ($\lambda = 6.25$)			Growth Rate				HP ($\lambda = 6.25$)			Growth Rate		
EMs	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	ADVs	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$
Argentina	8.7	2.3	0.44	12.7	2.0	0.39	Australia	1.5	1.6	0.31	2.2	1.4	0.34
Bolivia	6.7	6.8	0.24	10.1	3.6	0.11	Austria	1.7	1.5	-0.43	3.0	1.6	-0.19
Chile	2.7	0.8	-0.36	4.2	0.8	-0.17	Belgium	0.8	0.7	-0.54	1.6	0.8	-0.16
Costa Rica	3.0	1.4	0.31	4.3	1.2	0.24	Canada	1.1	0.8	-0.57	1.5	0.7	-0.49
Greece	2.2	1.1	-0.01	3.4	0.9	-0.09	Denmark	1.3	0.9	-0.52	2.0	0.9	-0.48
Mexico	3.2	1.4	0.33	5.0	1.4	0.36	Finland	1.8	0.8	-0.52	2.8	0.9	-0.47
Peru	4.9	1.2	0.28	7.5	1.2	0.25	France	0.7	0.7	-0.63	1.2	0.6	-0.38
Portugal	3.1	1.4	-0.58	4.2	1.2	-0.40	Germany	1.0	0.8	-0.46	1.5	0.8	-0.41
South Africa	1.9	1.3	-0.18	2.9	1.1	-0.15	Iceland	3.3	1.5	0.63	4.8	1.4	0.52
South Korea	1.5	0.7	0.19	2.3	0.6	0.03	Ireland	2.4	1.4	-0.42	3.5	1.0	-0.41
Spain	0.8	0.7	-0.09	1.6	0.7	0.10	Italy	1.0	0.7	-0.58	1.6	0.7	-0.50
Taiwan	1.2	0.6	-0.29	2.1	0.6	-0.16	Japan	1.0	0.7	-0.72	1.9	0.7	-0.40
Thailand	1.8	0.9	-0.27	3.0	0.8	-0.26	Luxembourg	3.4	1.7	-0.47	5.4	1.5	-0.42
Turkey	5.2	2.1	0.00	8.1	2.0	-0.05	Netherlands	1.1	1.0	-0.51	1.8	1.0	-0.34
Venezuela	1.5	0.4	-0.26	2.6	0.4	-0.28	New Zealand	1.7	1.2	-0.03	2.5	1.1	-0.13
							Norway	3.0	2.8	-0.11	4.5	2.3	-0.12
							Sweden	1.5	1.0	-0.43	2.3	1.0	-0.34
							Switzerland	1.1	0.9	-0.61	1.7	0.9	-0.58
							United Kingdom	1.4	1.0	-0.56	1.9	0.8	-0.43
							United States	0.5	0.4	-0.27	0.9	0.4	-0.23

Notes. Variable s_L is the cyclical component of the labor share, and y is the cyclical component of GDP per capita. Symbol $\sigma(\cdot)$ refers to standard deviation of the variable, and $corr(\cdot, \cdot)$ refers to correlation between the two variables. Standard deviations are measured in percentage point.

Kabaca (2014) also uses an annual data, but the minimum requirement of observation is only 10 years. In consequence, average length of labor shares in EMs is only 19 years, and those mainly starts in early 1990s and ends in the year of the beginning of the Great Recession. In the sample, typical Latin American countries, such as Argentina (1993-2007, 15 years), Brazil (1992-2007, 16 years), and Colombia (1992-2007, 16 years) do not contain the samples in the Lost Decade, which it is the most important economic fluctuation in Latin American history. Other emerging countries, Czech (1992-2008, 17 years), Hungary (1995-2008, 14 years), Israel (1995-2007, 14 years), and Philippines (1992-2007, 16 years) also have short observations. Egypt (1996-2006) has only 11 years of observations.

The short samples problem is directly related to the issue of choice of countries. If minimum requirement of data is only 10 years, we can include more countries into EMs. By using same sample requirement in Kabaca (2014), we expand the set of EMs which has 40 countries. We label this expanded group as 40 EMs.[13] All data is the TLS, and we set the earliest and latest years same as in Kabaca (2014), those are 1981 and 2008. Same windows

[13] Table B.2 in Appendix B shows countries those are in 40 EMs, and countries in 18 EMs as in Kabaca (2014). For fair comparison, our 40 EMs excludes Greece and Spain since Kabaca (2014) classifies those countries as advanced countries.

are matched for same countries. We apply HP filter with $\lambda = 6.25$ which is used in the study.

Table 2.11: Average Second Moments by Choice of Countries

	18 EMs	40 EMs	15 Baseline
$corr(y, s_L)$	0.10	- 0.05	-0.01
$\sigma(s_L)$	2.9	2.9	3.2

Notes. All series are detrended using HP filter with $\lambda = 6.25$. Group 18 EMs is the same set of EMs in Kabaca (2014). Group 40 EMs is the expanded EMs. Group 15 Baseline is EMs has same data in Table 1 in section 2. Standard deviations are measured in percentage point.

Table 2.11 shows that the correlation gives different signs between the two different EMs categories. Correlation between output and labor shares is 0.10 in 18 EMs, whereas it is -0.05 in 40 EMs. Thus 40 EMs produce almost acyclical labor shares, conditional on equal terms with Kabaca (2014). Rather, second moments of 40 EMs are coherent to second moments with the 15 EMs with 40 years of data which is -0.01.

Table 2.12: Average Second Moments: LQ and HP ($\lambda = 100$) Detrended Series

	LQ			HP ($\lambda = 100$)		
	18 EMs	40 EMs	15 Baseline	18 EMs	40 EMs	15 Baseline
$corr(y, s_L)$	0.32	0.10	0.15	0.24	0.07	0.09
$\sigma(s_L)$	4.7	4.6	6.1	4.2	4.3	4.6
$\sigma(s_L)/\sigma(y_L)$	1.4	1.3	1.0	1.4	1.2	1.1

Notes. Group 18 EMs is the same set of EMs in Kabaca (2014). Group 40 EMs is the expanded EMs. Group 15 Baseline is EMs with same samples in Table 1 in section 2. Standard deviations are measured in percentage point.

Additional exercise also shows the coherence between 15 Baseline and 40 EMs, and the distance from 18 EMs. Table 2.12 shows average second moments with LQ detrended series and HP filtered series with $\lambda = 100$. We can verify that regardless of two detrending methods, $corr(y, s_L)$s between 15 Baseline and 40 EMs are very similar, whereas $corr(y, s_L)$ in 18 EMs is almost 2-3 times bigger than the former two. This indicates that collecting sufficient number of observations, either long periods or many countries is crucial to reach reliable numbers. Of course, it will be the best if we can approach both.

Kabaca (2014) may give interesting perspective that labor shares in emerging economies may not alike to those in advanced economies. However, the problem described above prevents the perspective to be accepted as a rigorous empirical fact. In spite of this caveat, it is also noteworthy that standard deviations of labor shares between two contemporary works closely correspond each other.

2.6 Discussion on the Level of the Labor Share

So far, we investigate cyclical properties of the labor share in previous sections. In this section, we turn our point and briefly discuss the level of the labor share in emerging economies. The reason of this discussion is to suggest reasonable estimates of the labor share which can be applied to quantitative macroeconomic model. We cannot not plug the TLS estimate into the model because of its well-known underestimation issue.

One of suggestions from Gollin (2002) is to consider all of income from self-employed sector as a labor income. We label this as Adj.1. In the real world, however, capital is also used as a input in the self-employed sector as well, so we consider other variation by assuming 85 percent or 70 percent of income from self-employed sector is labor income. We label these as Adj.2 and Adj.3, respectively. Finally, we assume that labor shares in corporate sector and those in self-employed sector are same, which is also suggested in Gollin (2002). In this case, we can use CLS itself. Table 2.13 shows the average and median of estimates for emerging economies and advanced economies both.[14] Countries and samples are same ones those are used in section 2.4.

Table 2.13 shows that both in EMs and in ADVs, ranges of estimated level are quite wide by adjustments. The range of average level of labor shares in EMs is [0.50, 0.73], and that in ADVs is [0.61, 0.77]. Which values would be reasonable to take? In the business cycle

[14] We do not use the ALS in the exercise, since many of emerging economies don't provide the ALS so we cannot estimate cross sectional mean. For fair comparison, we do not use the ALS of advanced economies neither but calculate four alternative measures in line with the case of emerging economies.

Table 2.13: Average and Median of the Level of Labor Shares

	EMs				ADVs			
	Adj.1	Adj.2	Adj.3	CLS	Adj.1	Adj.2	Adj.3	CLS
Average	0.73	0.66	0.59	0.50	0.77	0.71	0.65	0.61
Median	0.72	0.66	0.60	0.53	0.78	0.72	0.65	0.62

Notes. Indices Adj.1, Adj.2, and Adj.3 are calculated by applying formula $\frac{CLS \cdot GDP_{corporate} + \kappa(GDP - GDP_{corporate})}{GDP}$, where $\kappa = 1, 0.85$, and 0.7, respectively. Average and median are taken to the averages of time-series labor shares of individual countries. All raw data are from KN database.

literature in advanced economies, values between 0.65-0.75 are usually taken, conditional on specific environment of models used. Those values correspond to Adj.3-Adj.1. By picking same adjustments to EMs, we can refine the range to [0.59-0.73] as a reasonable bound. Further, by taking the fact that industries in emerging and developing countries are more labor intensive than those in advanced economies, we can curtail the bound further. We suggest the range of [0.66, 0.73] as a valid approximation of labor shares in emerging economies.

2.7 Conclusion

Over the business cycle, labor shares in emerging economies move differently from those in advanced economies. Using labor shares data of 40 years, we propose three empirical regularities of labor shares. Around the world, labor shares are rarely stable, but they are as volatile as output. In addition, labor shares in emerging economies are twice as volatile as those in advanced economies. Finally, labor shares are procyclical in emerging economies but those are countercyclical in advanced economies. These three empirical facts indicate that the labor share fluctuation is not as simple as the conventional wisdom.

These cyclical behaviors of labor shares offer important welfare implications. Business cycles do have distributional impacts. Economic fluctuations change short-run composition between labor income and capital income of the economy. The degree of change of the composition is almost same as the magnitude of the economic fluctuation. This implies that the business cycle is the cycle not only for aggregate output, but also for distribution of the economy and so inequality.

Further, the direction of the cyclicality indicates which income groups in the economy suffer more from the economic contraction. Interestingly, the main cost taker may differ between emerging and advanced economies. In emerging economies, low and middle income groups whose main sources of income are labor incomes are major cost takers of the economic contraction. In advanced economies, on the other hand, the high income group who holds most of the capital income of the economy, is the major cost taker.

These empirical facts naturally lead us to the following question - why are they so? Yet, we have few theoretical backgrounds that can account for this. This paper raises the necessity of an appropriate theory and this will be partially addressed in next chapter.

Chapter 3

A Model of the Labor Share Fluctuation in Small Open Economies

3.1 Introduction

The labor share, namely the fraction of labor workers' income to the gross income of a country, is a fundamental measure of a country's income distribution. Understanding the evolution of the labor share during economic transitions was a major problem in political economy. However, modern dynamic macro models left this problem largely untouched, as the long-run constancy of the labor share was taken as the stylized empirical fact after Kaldor (1961).

Recent studies challenge this notion. Some studies (see, for example, Karabarbounis and Neiman (2014) and Piketty (2014) focus on the declining long-run patterns of labor shares over periods of economic growth. On the other hand, short-run studies (see, for example, Andolfatto (1996), Ríos-Rŭll and Santaeulalia-Llopis (2010), and Na (2015) show fluctuations of labor shares over the business cycle. These studies indicate that the labor share does not seem to be constant, but has substantial variation during economic transitions. Table 3.1 shows the various patterns of cyclical fluctuations of labor shares between two income groups around the world.

The problem is that standard business cycles models do not give good guidance to analyze short-run labor share fluctuations - the frictionless neoclassical model cannot explain labor share fluctuations at all. Other alternative models mainly rely on the construction of the labor wedge - the gap between marginal productivity of labor and adjusted marginal rate of substitution between consumption and leisure - to generate a cyclical gap between marginal productivity of labor and the real wage.[1] Recently, however, it turns out that empirical support of the labor wedge channels may not be deterministic or significant.[2]

This paper provides a different view of labor share fluctuations ignored by previous studies by proposing two channels of labor share fluctuations over the business cycle: i) cyclical variation of capital-labor ratios (substitution effect) and ii) cyclical variation of the relative value of sectoral production (composition effect). The former implies that the production functions of an economy do not need to have 'unitary' elasticities of substitution between capital and labor, such as in stylized Cobb-Douglas production function. The latter has an open economy context. The economy is composed of tradable and non-tradable sectors, and production cycles in each sector are affected by both domestic and international factors. The production cycles in the two sectors generate cyclical variations of sectoral shares to GDP, which yield an additional margin of the labor share fluctuations in the domestic economy.

For quantitative evaluation, I estimate the model using a likelihood-based Bayesian technique. I use data from Canada and Mexico, as the two countries are typical advanced and emerging small open economies with distinct cyclical features. The estimated models predict the cyclical patterns of the labor share and other business cycle indicators successfully. The

[1]There are many approaches that can generate the labor wedge: i) imperfect competition with inattentive price change and price markup variation (the sticky-price model), ii) search frictions in the labor market and adjustment cost of labor (the search model), iii) firm-specific financial frictions with respect to the labor cost (working-capital constraint).

[2]Nekarda and Ramey (2013) argue that price markups are not countercyclical as in the sticky-price model using manufacturing industry data. Shimer (2005) argues that Mortensen-Pissarides type search models predict only small unemployment fluctuations over the business cycle. Further, Chang and Fernández (2010) and Uribe and Schmitt-Grohé (2017) show that structural estimates of the working capital constraint on labor are not statistically significant and have quantitatively unimportant effects on the business cycle.

Table 3.1: The Labor Share Fluctuation over the Business Cycle

	20 Advanced Countries			15 Emerging Countries			
	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)$	$\sigma(s_L)/\sigma(y)$	$corr(y, s_L)$	$\sigma(s_L)^{EMs}/\sigma(s_L)^{ADVs}$
Average	3.3	1.0	-0.14	6.1	1.0	0.15	1.8
Median	2.9	0.8	-0.12	4.3	0.9	0.16	1.4

Source: Na (2015). Notes. Data is at the annual frequency. All data have at least 30 years of observations. All variables are log quadratically detrended. Variable s_L is the cyclical component of the Labor Share, and y is the cyclical component of GDP per capita. Symbol $\sigma(\cdot)$ refers to the standard deviation of the variable, and $corr(\cdot, \cdot)$ refers to the correlation between two variables. Standard deviations are measured in percentage points.

model shows a countercyclical and near-output volatile labor share in Canada, and shows procyclical and excessively volatile (compared to output) labor share in Mexico, all of which are consistent with the data. The models also match distinct patterns of business cycle indicators of the countries such as the volatilities, correlations, and autocorrelations of the indicators.

In addition, the two-sector model poses a caveat when we estimate the gross elasticity of capital-labor substitution of an economy. The gross elasticity of capital-labor substitution of the whole domestic economy has received a lot of attention in recent literatures in macroeconomics and public economics. However, it is not a structural parameter of an economic model if we consider the multi-sector environment. The model warns that the endogenous behavior of economic indicators such as share of tradable sector, which has substantial cyclical properties, does affect the estimate of the gross elasticity of substitution. Thus, estimating the gross elasticity with a one-sector production function and ignoring this internal transmission mechanism would yield largely biased estimates, especially for small open economies.

The remainder of the paper is organized as follows. Section 3.2 develops the model environment. Section 3.3 explains the two channels of labor share fluctuations. Section 3.4 evaluates the model for Canada and Mexico and examines the performance of the predictions. Section 3.5 proposes the elasticity of substitution between capital and labor in the entire economy by using estimated models. Section 3.6 closes the paper and poses questions for future research.

3.2 A Model with A Time Varying Labor Share

In this section, I develop a model with an endogenous time-varying labor share. The theoretical framework is a stylized two-sector small open economy business cycle model which embeds a deterministic trend, CES production technology, and multiple shocks. I begin by constructing the decentralized decisions of firms and households.

3.2.1 Firms

There are two types of sectors in the economy. Firms in the tradable sector produce tradable goods, which are traded internationally. Firms in the nontradable sector on the other hand produce nontradable goods, which only have domestic demand. Firms in the two sectors use capital and labor as factor inputs and have the following constant elasticity substitution (CES) production functions.

$$Y_t^j = a_t^j \left(\alpha_j K_t^{j\,\frac{\sigma_j - 1}{\sigma_j}} + (1-\alpha_j)(X_t h_t^j)^{\frac{\sigma_j - 1}{\sigma_j}} \right)^{\frac{\sigma_j}{\sigma_j - 1}}, \quad j = T, N. \tag{3.1}$$

where variables a_t^j, $j = T, N$ denote Hicks-neutral technology (often referred to be the stationary TFP) shocks in sectoral production. Assume that the natural logarithm of a_t^j follows a first order Markov processes

$$\ln a_{t+1}^j = \rho_{a,j} \ln a_{t,j} + \epsilon_{t+1}^{a,j} \quad j = T, N, \tag{3.2}$$

where the parameters $\rho_{a,j} \in (-1, 1)$, $j = T, N$ govern the persistence of $\ln a_{t,j}$, $j = T, N$, respectively. Similar to the previous exogenous processes, innovations $\epsilon_{t+1}^{a,T}$ and $\epsilon_{t+1}^{a,N}$ are assumed to follow i.i.d. processes with mean zero and standard deviations σ_a^T and σ_a^N, respectively. All shocks are assumed to be uncorrelated with each other.

In the production function, h_t^j, worked hours, is multiplied by the deterministic trend X_t.[3] The variable X_t is interpreted as nonstationary Harrod-neutral technology (often referred to be the labor augmented technology). Then parameters $\sigma_j \in [0, \infty]$ represent the elasticities of substitution between capital input K_t^j and labor input $X_t h_t^j$ in the production. The Harrod-neutral technology X_t can take a meaning of directed technological progress which is defined in Acemoglu (2009) as follows.

Definition 3.2.1 ***(Directed Technological Change)*** *Let the production function be given by $Y = F(f_i, f_j, X)$, where f_i and f_j are factor inputs and let X be a technology. The change of the technology X is factor i-directed if an increase of X increases the relative marginal product of factor i. That is,*

$$\partial \frac{\frac{\partial F(f_i, f_j, X)}{\partial f_i}}{\frac{\partial F(f_i, f_j, X)}{\partial f_j}} \Big/ \partial X > 0. \tag{3.3}$$

It is important to note that with CES production technology, the elasticity of substitution between capital and labor inputs, σ_j, determines whether labor augmented technological change X_t is capital-directed or labor-directed. If $\sigma_j < 1$, capital and labor are gross complements, and X_t is a labor-directed technology. If $\sigma_j > 1$, on the other hand, capital and labor are gross substitutes, and X_t is a capital-directed technology. With $\sigma_j = 1$, the production function collapses into the Cobb-Douglas form,

$$a_t^j {K_t^j}^{\alpha_j} (X_t h_t^j)^{1-\alpha_j} = a_t^j X_t^{1-\alpha_j} {K_t^j}^{\alpha_j} {h_t^j}^{1-\alpha_j}, \quad j = T, N, \tag{3.4}$$

and X_t is factor neutral so it becomes a Hicks-neutral technology as well. Now I impose the following assumption on the σ_j and X_t.

[3]Some business cycle literatures (see, for example, Aguiar and Gopinath (2006) study the case of stochastic X_t which follows a random walk. However, my 40 years of data for model estimation is not sufficiently long enough to verify this claim because studying the nonstationarity of a stochastic process requires substantially long samples to prevent spurious results (see, for example, García-Cicoo et al. (2010) which use a century of data for the study). As a consequence, the quantitative role of nonstationary Harrod-neutral productivity shock to business cycles diverges across literatures. I thus exclude the case of stochastic trend to prevent our model from falling into the controversial discussion.

Assumption 3.2.1 *Firms share the common labor augmented technology, X_t, across sectors but have different sectoral elasticities of capital-labor substitution σ_j.*

The assumption of common X_t is consistent with the model environment because households are assumed to be identical. More importantly, it guarantees the existence of a balanced growth path,[4], while different σ_j between two sectors is a natural assumption.

Goods and factor markets are perfectly competitive. Firms in each sector take the relative price of nontradables in terms of tradables p_t, real wage rates W_t^T and W_t^N, and rental rates of physical capital r_t^T and r_t^N as given as. In each period, firms in the tradable sector choose $\{K_t^T, h_t^T\}$ to maximize profits

$$\Pi_t^T = Y_t^T - W_t^T h_t^T - r_t^T K_t^T, \tag{3.5}$$

and firms in the nontradable sector choose $\{K_t^N, h_t^N\}$ to maximize

$$\Pi_t^N = p_t Y_t^N - W_t^N h_t^N - r_t^N K_t^N. \tag{3.6}$$

The corresponding necessary first-order conditions are

$$W_t^T = \frac{\partial Y_t^T(a_t^T, X_t, K_t^T, h_t^T)}{\partial h_t^T}, \tag{3.7}$$

$$W_t^N = p_t \cdot \frac{\partial Y_t^N(a_t^N, X_t, K_t^N, h_t^N)}{\partial h_t^N}, \tag{3.8}$$

$$r_t^T = \frac{\partial Y_t^T(a_t^T, X_t, K_t^T, h_t^T)}{\partial K_t^T}, \tag{3.9}$$

$$r_t^N = p_t \cdot \frac{\partial Y_t^N(a_t^N, X_t, K_t^N, h_t^N)}{\partial K_t^N}. \tag{3.10}$$

The assumption that all production functions are homogenous of degree one with respect to

[4] If each sector has a different X_t, then long-run growth of the economy is in general not balanced because each sector grows at different rates. This non-balanced growth is discussed in Baumol (1967) and Acemoglu and Guerrieri (2008) and is used to analyze structural change. However, I do not consider this case because the focus of the present paper is on fluctuations at business cycles frequencies.

factor inputs implies zero profits of all firms.

3.2.2 Households

The economy is populated by a continuum of identical households on a closed interval $[0, 1]$. Each household has Greenwood-Hercowitz-Huffman (GHH) preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \nu_t \beta^t \left(\frac{\left(C_t - s_t X_{t-1} \left(\frac{{h_t^T}^{\omega^T}}{\omega^T} + \frac{{h_t^N}^{\omega^N}}{\omega^N} \right) \right)^{1-\gamma} - 1}{1-\gamma} \right), \tag{3.11}$$

where C_t denotes a composite consumption good, which is given by the Armington aggregator of tradable and nontradable goods

$$C_t = A(C_t^T, C_t^N) = \left(\chi {C_t^T}^{\frac{\xi-1}{\xi}} + (1-\chi) {C_t^N}^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}}, \tag{3.12}$$

where C_t^T denotes consumption of tradable goods, C_t^N denotes consumption of nontradable goods, ξ denotes the elasticity of substitution between the two goods, and χ is a parameter related to the expenditure share. The variable h_t^T denotes hours worked in the tradable sector and h_t^N denotes hours worked in the nontradable sector. The variable ν_t denotes a preference shock which shifts the marginal utility of wealth, and ω^T and ω^N are parameters related to the Frisch elasticities of labor supply in the tradable and nontradable sectors, respectively. Variable s_t denotes an exogenous labor supply (reduced-form labor wedge) shock. The variable X_{t-1} denotes the labor augmented technology in period $t-1$. The labor augmented technology is deterministic and grows at rate g. [5]

[5] We multiply the labor-augmented technology with the labor supply disutility in the model to generate a balanced growth path.

The sequential budget constraint of the household is given by

$$C_t^T + p_t C_t^N + I_t^T + I_t^N + \frac{\phi^T}{2}\left(\frac{K_{t+1}^T}{K_t^T} - g\right)^2 K_t^T + \frac{\phi^N}{2}\left(\frac{K_{t+1}^N}{K_t^N} - g\right)^2 K_t^N + D_t$$

$$= W_t^T h_t^T + W_t^N h_t^N + r_t^T K_t^T + r_t^N K_t^N + \Pi_t^T + \Pi_t^N + \frac{D_{t+1}}{1+r_t^*}. \tag{3.13}$$

All values are expressed in terms of tradable goods. The variable p_t denotes the relative real price of nontradable goods. The variables I_t^T and I_t^N denote investments in physical capital in the tradable and nontradable sectors, respectively. Similarly, K_t^T and K_t^N denote the stocks of physical capital in the two sectors, which are owned by the households. The variable D_{t+1} denotes the households' stock of outstanding external debt in period t which due in period $t+1$. The variables W_t^T and W_t^N denote real wage rates in tradable and nontradable sectors, and r_t^T and r_t^N denote real rates of return of physical capital stocks in the two sectors, respectively. The variables Π_t^T and Π_t^N denote profits from the two sectors which are all owned by households. The variable r_t^* denotes the country specific real interest rate on the stock of the external debt D_{t+1}. The debt is denominated in tradable goods. The parameters ϕ^T and ϕ^N govern cost elasticities of capital adjustment in each sector. The sectoral physical capital stocks evolve via following laws of motion

$$K_{t+1}^T = (1-\delta)K_t^T + \iota_t^T I_t^T, \tag{3.14}$$

$$K_{t+1}^N = (1-\delta)K_t^N + \iota_t^N I_t^N, \tag{3.15}$$

where the parameter $\delta \in (0,1)$ is a common depreciation rate of capital stocks, and ι_t^T and ι_t^N are marginal elasticies of investment (MEI) of the two sectors, where it is assumed that the inverse of each $\xi_t^T \equiv \iota_t^{T^{-1}}, \xi_t^N \equiv \iota_t^{N^{-1}}$ follows

$$\ln \xi_{t+1}^T = \rho_{\xi,T} \ln \xi_t^T + \epsilon_{t+1}^{\xi,T}, \tag{3.16}$$

$$\ln \xi_{t+1}^N = \rho_{\xi,N} \ln \xi_t^N + \epsilon_{t+1}^{\xi,N}. \tag{3.17}$$

Laws of motion of preference and labor supply shocks are

$$\ln \nu_{t+1} = \rho_\nu \ln \nu_t + \epsilon^\nu_{t+1}, \tag{3.18}$$

$$\ln s_{t+1} = \rho_s \ln s_t + \epsilon^s_{t+1}, \tag{3.19}$$

and the labor augmented technology evolves as follows.

$$\frac{X_t}{X_{t-1}} = g, \quad \forall t. \tag{3.20}$$

It is also assumed that shocks ϵ^ν_{t+1} and ϵ^s_{t+1} follow i.i.d processes with zero means and standard deviations σ_ν and σ_s. I assume that the shocks are uncorrelated. Parameters $\rho_\nu, \rho_s \in (-1,1)$ govern the persistence of variables ν_t and s_t, respectively.

In the decentralized economy, each period a household takes $\{D_t, K^T_t, K^N_t, W^T_t, W^N_t, r^T_t, r^N_t, r^*_t, p_t, \Pi^T_t, \Pi^N_t\}$ as given, and chooses $\{C_t, C^T_t, C^N_t, I^T_t, I^N_t, K^T_{t+1}, K^N_{t+1}, h^T_t, h^N_t, D_{t+1}\}$ to maximize his expected lifetime utility (3.11) subject to equations (3.13), (3.14), (3.15), and the following no Ponzi scheme constraint

$$\lim_{k\to\infty} \mathbb{E}_t \frac{D_{t+1+k}}{\prod_{s=0}^{k}(1+r^*_{t+s})} \le 0. \tag{3.21}$$

Let $\beta^t \lambda_t X_{t-1}^{-\gamma}$ be the Lagrange multiplier associated with the sequential budget constraint (3.16). The necessary first-order conditions with respect to $\left\{C^T_t, C^N_t, h^T_t, h^N_t, D_{t+1}, K^T_{t+1}, K^N_{t+1}\right\}$ are

$$\nu_t \left(A(C^T_t, C^N_t)/X_{t-1} - s_t \left(\frac{h_t^{T\omega_T}}{\omega_T} + \frac{h_t^{N\omega_N}}{\omega_N} \right) \right)^{-\gamma} A_1(C^T_t, C^N_t) = \lambda_t, \tag{3.22}$$

$$\frac{A_2(C^T_t, C^N_t)}{A_1(C^T_t, C^N_t)} = p_t, \tag{3.23}$$

$$\frac{h_t^{T\omega_T - 1}}{A_1(C^T_t, C^N_t)} = \frac{W^T_t}{s_t X_{t-1}}, \tag{3.24}$$

$$\frac{h_t^{N^{\omega_N - 1}}}{A_1(C_t^T, C_t^N)} = \frac{W_t^N}{s_t X_{t-1}}, \tag{3.25}$$

$$\lambda_t = \beta \frac{1 + r_t^*}{g^\gamma} \mathbb{E}_t \lambda_{t+1}, \tag{3.26}$$

$$\begin{aligned} &\left(\xi_t^T + \phi^T \left(\frac{K_{t+1}^T}{K_t^T} - g\right)\right) \lambda_t \\ = &\frac{\beta}{g^\gamma} \mathbb{E}_t \lambda_{t+1} \left(r_{t+1}^T + \xi_{t+1}^T (1-\delta) + \phi^T \left(\frac{K_{t+2}^T}{K_{t+1}^T} - g\right) \frac{K_{t+2}^T}{K_{t+1}^T} - \frac{\phi^T}{2} \left(\frac{K_{t+2}^T}{K_{t+1}^T} - g\right)^2\right), \end{aligned} \tag{3.27}$$

$$\begin{aligned} &\left(\xi_t^N + \phi^N \left(\frac{K_{t+1}^N}{K_t^N} - g\right)\right) \lambda_t \\ = &\frac{\beta}{g^\gamma} \mathbb{E}_t \lambda_{t+1} \left(r_{t+1}^N + \xi_{t+1}^N (1-\delta) + \phi^N \left(\frac{K_{t+2}^N}{K_{t+1}^N} - g\right) \frac{K_{t+2}^N}{K_{t+1}^N} - \frac{\phi^N}{2} \left(\frac{K_{t+2}^N}{K_{t+1}^N} - g\right)^2\right), \end{aligned} \tag{3.28}$$

with the transversality condition

$$\lim_{k \to \infty} \mathbb{E}_t \frac{D_{t+1+k}}{\prod_{s=0}^{k} (1 + r_{t+s}^*)} = 0. \tag{3.29}$$

3.2.3 Frictions in International Financial Markets

I assume that this small open economy faces a financial friction when it trades financial assets in the international market. I impose the following external debt-elastic country interest rate (EDEIR) introduced by Schmitt-Grohé and Uribe (2003),

$$r_t^* = \bar{r^*} + \psi \left(e^{\frac{\tilde{D}_{t+1}/X_t - \bar{d}}{\bar{y}}} - 1\right) + e^{\mu_t - 1} - 1, \tag{3.30}$$

where $\tilde{D}_{t+1}$ denotes the cross-sectional average of external debt of the economy in period t, $\bar{r}^*$ denotes the steady-state country-specific real interest rate, $\bar{d}$ denotes the steady-state detrended level of external debt, and $\bar{y}$ denotes the steady-state detrended level of output. The term $r_t^* - \bar{r}^*$ represents the country interest rate premium, which is the gap between the country-specific interest rate and the world interest rate. The country premium characterizes the financial friction of the sovereign economy in the international financial market. The parameter $\psi > 0$ denotes the elasticity of the country specific interest rate to the change of the quantity in the parenthesis. Thus ψ represents the degree of financial frictions in the short-run. The EDEIR is a stationarity inducing device for closing this small open economy model.

The country interest rate is assumed to be subject to an exogenous shock, which is denoted μ_t. As for the other shock processes, I assume it evolves by the following first-order Markov process

$$\ln \mu_{t+1} = \rho_\mu \ln \mu_t + \epsilon^\mu_{t+1}, \tag{3.31}$$

where $\rho_\mu \in (-1, 1)$ governs the persistence of the shock and ϵ^μ_{t+1} is an i.i.d. innovation with zero mean and standard deviation σ_μ, which is uncorrelated to other shocks.

3.2.4 Stationary Competitive Equilibrium

Since there is a continuum of identical households on the unit interval [0,1], the cross-sectional average of the stock of external debt equals the stock of each individual's external debt in equilibrium. This means that

$$\tilde{D}_{t+1} = D_{t+1}. \tag{3.32}$$

And since the sum of sectoral capital and labor should equal aggregate capital and labor to clear factor markets in the economy, we have

$$\begin{aligned} h_t &= h^T_t + h^N_t, && (3.33) \\ K_t &= K^T_t + K^N_t, && (3.34) \end{aligned}$$

and domestic clearance of the nontradable good market yields

$$C^N_t = Y^N_t. \tag{3.35}$$

Since all upper case variables are nonstationary, I stationarize the variables by removing the deterministic trend, by dividing all upper case variables by X_{t-1}. The trend-removed variables are denoted by lower case letters. That is, $v_t = V_t/X_{t-1}$ for any variables V_t which has a deterministic trend. Then previous equilibrium conditions are transformed into the stationary versions of the first-order conditions, sequential budget constraint, market clearing conditions, and transversality condition subject to the initial conditions of endogenous state variables $\{k_{-1}^T, k_{-1}^N, d_{-1}\}$ and sequence of exogenous shocks $\{a_t^T, a_t^N, \xi_t^T, \xi_t^N, r_t^*, \nu_t, s_t\}_{t=0}^{\infty}$. Appendix C.1 describes the stationary competitive equilibrium in detail. Recall that the economy with the competitive equilibrium evolves along the balanced growth path which is defined as follows.

Definition 3.2.2 ***(Balanced Growth Path)*** *The balanced growth path (BGP) is the dynamic competitive equilibrium that features long-run relationships of equal aggregate output-consumption-investment growths, and constant capital-output ratio, real interest rate, and factor shares.*

3.3 The Labor Share and Its Fluctuation over the Business Cycle

The sectoral capital and labor shares $s_t^{k,T}$, $s_t^{h,T}$, $s_t^{k,N}$, and $s_t^{h,N}$ are defined as follows.

$$s_t^{k,T} \equiv \frac{r_t^K K_t^T}{Y_t^T}, \tag{3.36}$$

$$s_t^{h,T} \equiv \frac{W_t^T h_t^T}{Y_t^T}, \tag{3.37}$$

$$s_t^{k,N} \equiv \frac{r_t^N K_t^N}{p_t Y_t^N}, \tag{3.38}$$

$$s_t^{h,N} \equiv \frac{W_t^N h_t^N}{p_t Y_t^N}. \tag{3.39}$$

The following lemma shows that labor shares in the tradable and nontradable sectors (equations (3.37) and (3.39)) are functions of the sectoral capital intensities α_j, effective capital-labor ratios $\frac{k_t^j}{h_t^j}$ (where $k_t = K_t/X_{t-1}$), and σ_j, the elasticities of substitutions between effective capital k_t^j and labor h_t^j.[6]

Lemma 3.3.1 ***(Sectoral Labor Shares)*** *The sectoral labor shares in the tradable and nontradable sectors are given by*

$$s_t^{h,T} = \frac{1-\alpha_T}{(1-\alpha_T)+\alpha_T\left(\frac{k_t^T}{h_t^T}\frac{1}{g}\right)^{\frac{\sigma_T-1}{\sigma_T}}}, \tag{3.40}$$

$$s_t^{h,N} = \frac{1-\alpha_N}{(1-\alpha_N)+\alpha_N\left(\frac{k_t^N}{h_t^N}\frac{1}{g}\right)^{\frac{\sigma_N-1}{\sigma_N}}}. \tag{3.41}$$

Proof. See Appendix C.2.1.

Lemma 3.3.1 indicates that the sectoral labor shares are time varying along the variation of sectoral capital-labor ratios, $\frac{k_t^j}{h_t^j}$, as long as $\sigma_j \neq 1$, that is as long as the elasticity of substitution is not unitary. Further, the lemma shows that the direction of the fluctuations of sectoral labor shares with respect to fluctuations of the capital-labor ratios changes sign at the point $\sigma_j = 1$. If $\sigma_j < 1$, $s_t^{h,j}$ increases with $\frac{k_t^j}{h_t^j}$. If $\sigma_j > 1$, on the other hand, $s_t^{h,j}$ decreases with $\frac{k_t^j}{h_t^j}$. If $\sigma_j = 1$, the Cobb-Douglas case, $s_t^{h,j}$ becomes time invariant and equal to $1-\alpha_j$.

We next turn to the aggregate labor share s_t^h of the entire economy.

Lemma 3.3.2 ***(Aggregate Labor Share)*** *The aggregate labor share is represented by*

$$s_t^h \equiv \frac{W_t^T h_t^T + W_t^N h_t^N}{Y_t}$$

[6] I use a term 'effective' as a labor-augmented technology unit (i.e., effective capital $k_t = K_t/X_{t-1}$). I will often omit the term for simplicity.

$$= \gamma_t^T s_t^{h,T} + (1 - \gamma_t^T) s_t^{h,N}, \quad (3.42)$$

where

$$Y_t \equiv Y_t^T + p_t Y_t^N$$

is the gross value added of the economy in terms of tradable good, and

$$\gamma_t^T \equiv \frac{Y_t^T}{Y_t} = \frac{Y_t^T}{Y_t^T + p_t Y_t^N}$$

is the tradable sector production share of the gross value added of the economy.

Proof. See Appendix C.2.2.

Lemma 3.3.2 shows that there are three time varying components on the right hand side, $s_t^{h,T}$, $s_t^{h,N}$, and γ_t^T, which means that there are two margins of fluctuations of the aggregate labor share over the business cycle. The first margin is the fluctuation of sectoral labor shares $s_t^{h,T}, s_t^{h,N}$ caused by the fluctuations of sectoral capital-labor ratios, which I refer to as the *substitution effect*. The second margin is the fluctuation of the share of tradable sector γ_t^T caused by the fluctuations of the relative market value between tradable and nontradable sectors, which I refer to as the *composition effect*.[7]

Next I describe the fluctuations of the sectoral labor shares and the aggregate labor share over the business cycle in more detail.

3.3.1 Fluctuations of Sectoral Labor Shares

In this subsection, I show how the variation of sectoral capital-labor ratios, the substitution effect, generates fluctuations in sectoral labor shares. Let $\theta_t^j = \frac{k_t^j}{h_t^j}$ be sectoral capital-labor

[7] Remark that regardless of the fluctuation of labor shares in the short-run, they are all constant in the nonstochastic steady state since the stationary competitive equilibrium along BGP ensures the constancy.

ratios. A log-linear approximation of equations (3.40)-(3.41) yields

$$\hat{s}_t^{h,j} = M^j \hat{\theta}_t^j, \quad j = T, N, \tag{3.43}$$

where

$$M^j = \frac{M_1^j}{M_2^j}, \tag{3.44}$$

$$M_1^j = -\frac{\sigma_j - 1}{\sigma_j}\alpha_j \left(\frac{\theta^j}{g}\right)^{\frac{\sigma_j - 1}{\sigma_j}}$$

$$M_2^j = (1 - \alpha_j) + \alpha_j \left(\frac{\theta^j}{g}\right)^{\frac{\sigma_j - 1}{\sigma_j}},$$

and θ^j is the steady state value of θ_t. Then the following lemma describes the fluctuations of sectoral labor shares over the business cycle.

Lemma 3.3.3 ***(Cyclical Fluctuations of Sectoral Labor Shares)*** *The correlation between output growth (with constant relative price of nontradables p) g^Y and sectoral labor shares is*

$$\rho\left(g^Y, \hat{s}_t^{h,j}\right) = \begin{cases} \rho\left(g^Y, \hat{\theta}_t^j\right) & \text{if } M^j > 0 \\ -\rho\left(g^Y, \hat{\theta}_t^j\right) & \text{if } M^j < 0 \\ 0 & \text{if } M^j = 0 \end{cases}, \quad j = T, N, \tag{3.45}$$

and the variances of the sectoral labor shares are

$$\sigma^2\left(\hat{s}_t{}^{h,j}\right) = \left(M^j\right)^2 \sigma^2\left(\hat{\theta}_t^j\right), \quad j = T, N. \tag{3.46}$$

Proof. See Appendix C.2.3.

An important question from lemma 3.3 is what sign are the unconditional correlations between output growth and sectoral capital-labor ratios, $\rho\left(g^Y, \hat{\theta}_t{}^j\right)$. They are very likely to be negative (so the sectoral capital-labor ratios are countercyclical) since capital stock evolves slowly compared to labor over the business cycle as documented in Rotemberg and Woodford (1999).[8]

Another important question is the relationship between the coefficient, M^j, and the elasticity of substitution between capital and labor, σ^j. Recall that in equation (3.44), we can see that M_2^j is always positive, whereas the sign of M_1^j depends on the elasticity of capital-labor substitution σ_j. If $\sigma_j < 1$, M_1^j is positive, whereas if $\sigma_j > 1$, M_1^j is negative. It is zero if $\sigma_j = 1$. The characteristics of M^j, with those of $\rho\left(g^Y, \hat{\theta}_t^j\right)$, govern fluctuations of sectoral labor shares. Figure 1 shows the functional relationships between sectoral elasticity of capital-labor substitution, σ_j, and the coefficient, M^j. The coefficient, M^j monotonically decreases as σ_j increases. And given the threshold of $\sigma_j = 1$, M^j on the left side is positive and the one on the right side is negative. The pattern is the same with the two sectors.

The intuition is simple: the elasticity of capital-labor substitution refers to how the ratio between capital and labor changes when the ratio of their marginal products changes. This is represented by the expression

$$\sigma_j = -\frac{d\ln(k^j/h^j)}{d\ln(MP_K^j/MP_L^j)} = -\frac{\frac{d(k^j/h^j)}{k^j/h^j}}{\frac{d(MP_K^j/MP_L^j)}{MP_K^j/MP_L^j}} = \frac{\frac{d(k^j/h^j)}{k^j/h^j}}{\frac{d(w^j/r^j)}{w^j/r^j}},$$

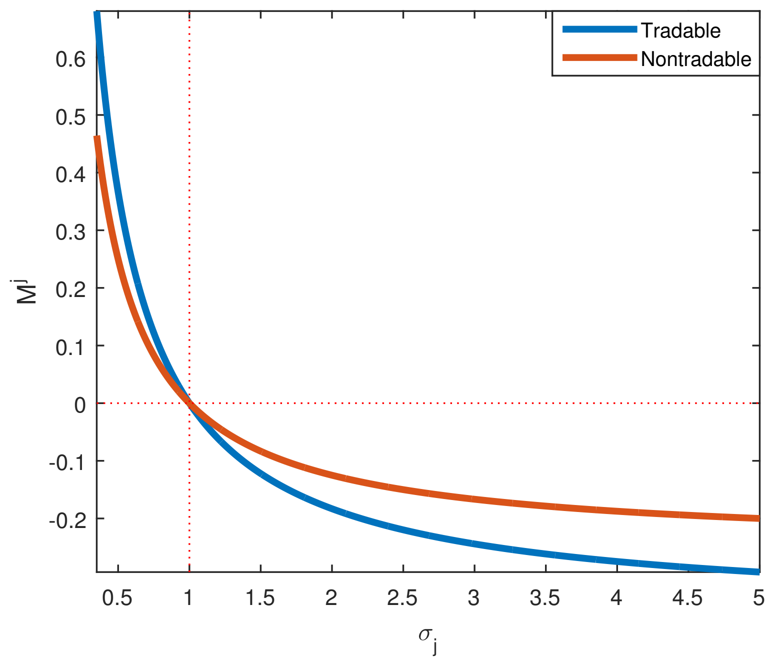
where the last equality comes from competitive and frictionless sectoral factor markets. We can rewrite the equation as

$$\frac{1}{\sigma^j} = \frac{d\ln\left(w^j/r^j\right)}{d\ln\left(k^j/h^j\right)}. \tag{3.47}$$

Equation (3.47) shows that given a one percent increase of capital-labor ratio (k^j/h^j), the relative price of the factors (w^j/r^j) increases by more than one percent if $\sigma_j < 1$, less than one percent if $\sigma_j > 1$, and exactly one percent if $\sigma_j = 1$. Then, given the growth

[8]However, it is noteworthy that negativity is not perfectly guaranteed in the multi-sector economy in principle because of a sectoral reallocation of factors. If elasticity of substitutions between capital and labor are substantially high and the sectoral productivity shocks are substantially persistent, the inflow and outflow of capital in a sector (say, sector T) can be faster than those of labor by the reallocation of factors from the other sector (say, sector N).

Figure 3.1: Coefficient M^j Conditional on the Elasticity of Substitution between Capital and Labor



Note. In this example, $g = 1.01$. Steady state values θ^j, α_j are computed for each σ_j, to match four moment restrictions $\bar{tb/y} = 0.01, \gamma^T = 0.43, s^h = 0.7, s^{h,T} = 0.75$.

of a sector's capital-labor ratio, the sectoral labor share increases if $\sigma_j < 1$, decreases if $\sigma_j > 1$, and remains unchanged if $\sigma_j = 1$. Thus, conditional on the normal scenario of countercyclical sectoral capital-labor ratios (negative $\rho\left(g^Y, \hat{\theta}_t^j\right)$), sectoral labor shares $\hat{s}_t^{h,j}$ become countercyclical (procyclical, constant) if and only if sectoral capital-labor elasticities σ_j are less than (greater than, equal to) one.

The lemma also shows that the deviation of σ_j from unity also affects the volatility of sectoral labor shares. The variance of a sectoral labor share is an increasing function of the square of M^j, given a variance of the capital-labor ratio, and Figure 3.1 implies that this squared term gets bigger as σ_j gets farther from 1.

3.3.2 Fluctuation of the Aggregate Labor Share

The substitution effect determines sectoral labor shares fluctuation. To fully account for the fluctuation of the aggregate labor share of the economy, however, we should take into account another macroeconomic margin, which is the change of the relative size between the two sectors in the domestic economy. The log-linearized version of the aggregate labor share (equation (3.42)) is

$$\hat{s}_t^h = \frac{\gamma^T s^{h,T}}{s^h}\hat{s}_t^{h,T} + \frac{(1-\gamma^T)s^{h,N}}{s^h}\hat{s}_t^{h,N} + \frac{\gamma^T(s^{h,T}-s^{h,N})}{s^h}\hat{\gamma}_t^T, \tag{3.48}$$

where γ^T is the steady state value of the share of the tradable sector, γ_t^T. In equation (3.48), the first two terms on the right hand side represent fluctuations of sectoral labor shares via substitution effect in section 3.3.1, and the last term reflects the fluctuation of the relative size of the two sectors, which I name the composition effect. The following proposition shows the cyclical fluctuation of the aggregate labor share over the business cycle.

Proposition 3.3.4 ***(Cyclical Fluctuation of the Aggregate Labor Share)*** *The correlation between output growth and the aggregate labor share is*

$$\rho\left(g^Y, \hat{s}_t^h\right) = \Phi_T \cdot \rho\left(g^Y, \hat{s}_t^{h,T}\right) + \Phi_N \cdot \rho\left(g^Y, \hat{s}_t^{h,N}\right) + \Phi_\gamma \cdot \rho\left(g^Y, \hat{\gamma}_t^T\right), \tag{3.49}$$

where

$$\Phi_T = \frac{\gamma^T s^{h,T}}{s^h}\frac{\sigma\left(\hat{s}_t^{h,T}\right)}{\sigma\left(\hat{s}_t^h\right)}, \quad \Phi_N = \frac{(1-\gamma^T)s^{h,N}}{s^h}\frac{\sigma\left(\hat{s}_t^{h,N}\right)}{\sigma\left(\hat{s}_t^h\right)}, \quad \Phi_\gamma = \frac{\gamma^T(s^{h,T}-s^{h,N})}{s^h}\frac{\sigma\left(\hat{\gamma}_t^T\right)}{\sigma\left(\hat{s}_t^h\right)},$$

and the variance of the aggregate labor share is

$$\sigma^2\left(\hat{s}_t^h\right) = \Psi_T^2 \cdot \sigma^2\left(\hat{s}_t^{h,T}\right) + \Psi_N^2 \cdot \sigma^2\left(\hat{s}_t^{h,N}\right) + \Psi_\gamma^2 \cdot \sigma^2\left(\hat{\gamma}_t^T\right) + 2 \cdot cov s_t, \tag{3.50}$$

where

$$\Psi_T = \frac{\gamma^T s^{h,T}}{s^h}, \quad \Psi_N = \frac{(1-\gamma^T)s^{h,N}}{s^h}, \quad \Psi_\gamma = \frac{\gamma^T(s^{h,T}-s^{h,N})}{s^h},$$

and

$$covs_t = \Psi_T\Psi_N \cdot cov\left(\hat{s}_t^{h,T}, \hat{s}_t^{h,N}\right) + \Psi_T\Psi_\gamma \cdot cov\left(\hat{s}_t^{h,T}, \hat{\gamma}_t^T\right) + \Psi_N\Psi_\gamma \cdot cov\left(\hat{s}_t^{h,N}, \hat{\gamma}_t^T\right).$$

Proof. See Appendix C.2.4.

The proposition explicitly shows that the composition effect, which is represented by Φ_γ, Ψ_γ, $\rho\left(g^Y, \hat{\gamma}_t^T\right)$, $\sigma\left(\hat{\gamma}_t^T\right)$, $\sigma\left(\hat{\gamma}_t^T\right)/\sigma\left(s_t^h\right)$, and $covs_t$, plays a role in determining the cyclicality and the volatility of the aggregate labor share, $\rho\left(g^Y, \hat{s}_t^h\right)$ and $\sigma\left(\hat{s}_t^h\right)$. It follows that we should know how the composition effect works quantitatively, or more specifically, the signs of the coefficients Φ_γ, Ψ_γ, and the correlation, $\rho\left(g^Y, \hat{\gamma}_t^T\right)$, and how size of the volatility, $\sigma\left(\gamma_t^T\right)$. Remark that all coefficients of the substitution effect Φ_T, Φ_N, Ψ_T, and Ψ_N are nonnegative by themselves, but the signs of the coefficients Φ_γ and Ψ_γ depend on the sign of $s^{h,T}-s^{h,N}$, which is the steady-state difference between the labor shares in the tradable and nontradable sectors. This indicates that, conditional on the given sectoral labor fluctuations, the correlation, $\rho\left(g^Y, \hat{s}_t^h\right)$, increases (decreases, remains unchanged) with $\rho\left(g^Y, \hat{\gamma}_t^T\right)$ if and only if $s^{h,T}-s^{h,N}$ is positive (negative, zero). Empirical studies typically yield estimates of $s^{h,N}$ greater than $s^{h,T}$ so $s^{h,T}-s^{h,N}$ becomes negative. This means that Φ_γ is negative and $\rho\left(g^Y, \hat{s}_t^h\right)$ decreases with $\rho\left(g^Y, \hat{\gamma}_t^T\right)$.

For the moments γ^T, $\rho\left(g^Y, \hat{\gamma}_t^T\right)$, $\sigma\left(\hat{\gamma}_t^T\right)$, and $\sigma\left(\hat{\gamma}_t^T\right)/\sigma\left(s_t^h\right)$, the data also provide useful information. Table 3.2 shows the mean of the moments between the two income groups (emerging and advanced) around the world.[9] The first column shows that the share of

[9] Table 3.2 is constructed by the following steps. The raw data of the share of the tradable sector is collected from UNCTAD database. By summing the shares of value added of agriculture, hunting, forestry, fishing and industry to GDP, I construct the annual share of tradable sector from 1970 to 2011. I consider 15 emerging economies (Argentina, Bolivia (Plurinational State of), Chile, China (Taiwan Province of), Costa Rica, Greece, Korea (Republic of), Mexico, Peru, Portugal, South Africa, Spain, Thailand, Turkey, Venezuela

the tradable sector (γ_t^T) in the emerging group is higher than the advanced group by ten percentage points on average. The second column shows that the share of tradable sectors are procyclical around the world, with that of advanced economies twice as procyclical as in emerging economies. In fact, five countries in the emerging group (Argentina (-0.09), Bolivia (-0.30), Costa Rica (-0.16), Mexico (-0.10), South Africa (-0.28)) and two countries in the advanced group (Australia (-0.14), Norway (-0.18)) in our sample have countercyclical tradable shares. The third and fourth columns indicate that the volatilities of the tradable share are similar between the two groups, relative volatilities to the aggregate labor share is far greater than one, and the relative volatility in advanced economies is about twice as large as in emerging economies.

In sum, proposition 3.3.4 provides us the comprehensive dimensions of aggregate labor share fluctuations. Conditional on the given substitution effect and negative Φ_γ, the composition effect yields a countercyclical (procyclical) aggregate labor share if and only if $\rho\left(g^Y, \hat{\gamma}_t^T\right)$ is positive (negative). And conditional on the given composition effect and countercyclical sectoral capital-labor ratios $\rho\left(g^Y, \hat{\theta}_t^j\right)$, each sectoral labor share yields a countercyclical (procyclical) aggregate labor share if and only if σ_j is less (greater) than one. There is no causal relationship between the two effects - they may be the same or opposite direction.

It is noteworthy that if the substitution effect is eliminated ($\sigma_j = 1$), the aggregate labor share becomes

$$s_t^h = (1-\alpha_T)\gamma_t^T + (1-\alpha_N)(1-\gamma_t^T), \tag{3.51}$$

and a log linear approximation of the above equation yields the fluctuation of the aggregate

(Bolivarian Rep. of)) and 19 advanced economies (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Sweden, Switzerland, United Kingdom, United States). GDP per capita is collected from World Development Indicators (constant local currency unit, code: NY.GDP.PCAP.KN) except Taiwan. GDP of Taiwan is collected from Penn World Table 8.0, by dividing output-side real GDP at chained PPPs (rgdpo) by populations (pop). To calculate historical means, I take an average of time series in each country and then take an average of country averages in each group.

Table 3.2: Mean of First and Second Moments of The Share of Tradable Sector: Data

Economy	γ^T	$corr\left(g^Y, \hat{\gamma}_t^T\right)$	$\sigma\left(\hat{\gamma}_t^T\right)$	$\sigma\left(\hat{\gamma}_t^T\right)/\sigma\left(\hat{s}_t^h\right)$
ADVs	0.35	0.26	0.16	3.29
EMs	0.44	0.12	0.14	1.77

Note. Variable γ^T is the historical mean share of the tradable sector, and variable g^Y is the growth rate of GDP with a constant real exchange rate. Variables $\hat{\gamma}_t^T$ and $\hat{s}_t^h$ are log demeaned share of tradable sector and the aggregate labor share. Symbol $corr(\cdot,\cdot)$ refers to the correlation coefficient, and $\sigma(\cdot)$ refers to the standard deviation in percentage points.

labor share as the following form

$$\hat{s}_t^h = (\alpha_N - \alpha_T)\left(\frac{\gamma^T}{s^h}\right)\hat{\gamma}_t^T, \tag{3.52}$$

where α_N and α_T now become sectoral capital share in nontradable and tradable sectors, respectively. Equation (3.52) means that the fluctuation of the aggregate labor share is *proportional* to the fluctuation of the share of tradable sector in the economy. This implies that conditional on $\alpha_N < \alpha_T$ (higher labor share in nontradable sector than tradable sector), the model predicts the same magnitude but opposite sign of $corr\left(g^Y, \hat{s}_t^h\right)$ to $corr\left(g^Y, \hat{\gamma}_t^T\right)$ and $\sigma\left(\hat{\gamma}_t^T\right)/\sigma\left(s_t^h\right) = \left|\frac{s^h/\gamma_T}{\alpha_N - \alpha_T}\right|$, which are not supported by the data. I revisit this in section 3.4.3.

3.4 Quantitative Analysis

In this section, I perform a quantitative analysis using the theoretical model described in section 3.2. My method was as follows. First, I set some structural parameters based on data and by following standard suggestions from the previous literature. Second, the model is first-order approximated by a perturbation method, and I estimate remaining structural parameters and other nonstructural ones of the model by using a likelihood-based Bayesian method. Finally, using the estimated model with calibrated and estimated parameters, I examine the predictions of the model.[10]

The examination of model prediction contains how well the model matches the major moments of the labor share from the data, whether the model's predictions of the business cycle are successful, and how the estimated model provides information about the source of fluctuations of the labor share and other macroeconomic indicators. We care about not only labor share but also the general performance for predicting business cycles because the theoretical model is a business cycle model by itself. If the model's predictions on the labor share are reasonable, but predictions on the general business cycle statistics are poor, then we should not be in favor of the model..

I estimate the model for two countries, Mexico and Canada. The former is a typical emerging economy, and the latter is a typical advanced economy, both small-open economies.

3.4.1 Calibration

Calibrated parameters are divided into common parameters and country specific parameters. For the common parameters, I set $\omega_T = \omega_N = 1.455$, $\gamma = 2$, and $\delta = 0.12$, by taking common values in standard business cycle literature. I set the elasticity of substitution between tradables and nontradables, ξ to 0.5, by following Uribe and Schmitt-Grohé (2017). For Canada, I set $\bar{r^*} = 0.066$ by taking the mean of annualized net interest rate of government bonds from 1948-2011. Parameter g is set to be 1.02, from the average of data from 1961-2011. Capital intensity parameters in each sector, α_T and α_N are calibrated to match the long-run sectoral labor shares and the aggregate labor share in the steady state.

There are four moment restrictions which pin down the steady state value of relative price of nontradables p^N, and 3 structural parameters α_T, α_N, χ, so the model is partially

[10] In this chapter, the estimated model is the benchmark model in chapter 3.3, but alternative model specifications are also considered. Appendix C.5 shows quantitative results from two other models: i) two sector model with Cobb-Douglas production functions, ii) one sector model with CES production functions. The purpose of the alternative models is to isolate the two effects in chapter 3.3 and to examine the performance of a model with only one channel. The results indicate that the benchmark model in chapter 3.3 performs best in predicting the cyclical patterns of the labor share and business cycles indicators of the two countries.

reparametrized in terms of steady state moment restrictions rather than parameters. For Canada, the restriction $\bar{tb/y} = 0.0135$ is based on the historical mean of trade balance to output ratios from 1961-2011, and restrictions on $s^{h,N}$ and s^h are based on the historical mean of value-added averages of labor shares in construction and service sectors from 1970-2009, and the historical mean of aggregate labor shares during same periods, respectively.

The calibration for Mexico is similar. For country specific parameters, I set $\bar{r^*} = 0.10$ following the value suggested in Uribe and Schmitt-Grohé (2017), and set $g = 1.018$ by taking historical mean of Mexican GDP growth rates and trade balance to GDP ratios from 1961-2011. I set the steady state trade balance to output ratio $\bar{tb/y} = -0.007$ based on the historical mean of Mexican trade balance to GDP ratio from 1960-2011. The steady state labor share in the nontradable sector, $s^{h,N}$, is set to 0.75 by taking estimates from Uribe (1997) and the steady state aggregate labor share in the economy s^h is set to 0.7 by taking estimates from Na (2015). The steady state share of the tradable sector, γ_T, is set to 0.43 by taking average of ratios of the sum of value added in agriculture, hunting, forestry, fishing, and industry to GDP from 1970-2011 from the UNCTAD database. Table 3.3 summarizes all calibrated values.

3.4.2 Estimation

The estimation of the model is conducted via a likelihood based Bayesian method. To construct likelihood function of the structural model, I use six observations: Growth rate of GDP per capita, growth rate of consumption, growth rate of investment, trade balance to output ratio, log demeaned aggregate labor share, and log demeaned share of tradable sector.[11] Data is annual (Canada: 1971-2009, Mexico: 1970-2009). I assume additive mea-

[11]Although some empirical work (e.g. Karabarbounis and Neiman (2014) focus on the trend of the labor share, I do not detrend it in the quantitative study. There are couple of reasons. First, unlike other well known nonstationary macroeconomic level indicators (such as GDP), the ratio indicators (such as factor shares and trade balance to output ratio) are known to be stationary variables, and 40 years of data may not be sufficient to make any conclusions. Here I would like to interpret the 'trend' as the 'persistence' instead. Second and more importantly, if we observe that the labor share has a decreasing trend, it implies that it

Table 3.3: Calibrated Values

	Canada	Mexico
Common Parameters		
γ	2	
ω_T	1.445	
ω_N	1.445	
δ	0.12	
ξ	0.5	
Country Specific Parameters		
$\bar{r^*}$	0.066	0.10
g	1.02	1.018
Moment Restrictions		
tb/y	0.0135	-0.007
$s^{h,N}$	0.74	0.75
s^h	0.64	0.70
γ^T	0.35	0.46

surement errors in all observables to prevent a potential degenerated likelihood function in the Kalman filter iterations in the state space representation of observations of the linearized model.

There are three classes of parameters to be estimated: (i) parameters related to steady state solutions (ii) parameters related to endogenous propagation of shocks that do not change steady state solutions, and (iii) parameters that govern exogenous shock processes. Regardless of the classes of parameters, I assume that all prior distributions are uniform and supports of the prior distribution are all wide. This implies there is no pre-experimental information about the parameters. Thus the joint posterior distribution is mostly driven by the likelihood function, so the Bayesian estimates become similar to the maximum likelihood estimates. Also, since I assume measurement errors for observables, the standard deviation of the measurement errors, which are nonstructural, should be estimated as well. I assume uniform priors for measurement errors and impose a restriction that the maximum supports

converges to zero in the nonstochastic steady state. This is not a case that our economic model wants to think about, because it generates spurious results from the approximated model via local approximation.

Table 3.4: Marginal Prior and Posterior Distributions for Structural Parameters

		Canada		Mexico	
Parameter	Prior	Posterior			
		Median	[5%, 95 %]	Median	[5%, 95 %]
Steady State Related Parameters					
σ_T	Uniform $[0, 10]$	0.89	[0.75, 1.21]	0.95	[0.84, 1.54]
σ_N	Uniform $[0, 10]$	0.49	[4.84, 8.51]	5.12	[2.34, 7.86]
Endogenous Propagation Parameters					
ϕ^T	Uniform $[0, 50]$	1.71	[0.28, 3.72]	28.4	[13.9, 46.1]
ϕ^N	Uniform $[0, 50]$	5.95	[4.30, 7.57]	14.3	[12.7, 16.1]
ψ	Uniform $[0, 20]$	5.92	[3.18, 9.38]	17.5	[12.0, 19.7]
Exogenous Shock Parameters					
$\rho_{a,T}$	Uniform $[-0.99, 0.99]$	0.80	[0.69, 0.87]	0.90	[0.83, 0.95]
$\sigma_{a,T}$	Uniform $[0, 0.2]$	0.035	[0.028, 0.045]	0.02	[0.01, 0.03]
$\rho_{a,N}$	Uniform $[-0.99, 0.99]$	0.44	[-0.65, 0.94]	0.98	[0.98, 0.98]
$\sigma_{a,N}$	Uniform $[0, 0.2]$	0.001	[0.0002, 0.008]	0.037	[0.028, 0.047]
$\rho_{\xi,T}$	Uniform $[-0.99, 0.99]$	0.82	[-0.43, 0.97]	0.30	[-0.70, 0.91]
$\sigma_{\xi,T}$	Uniform $[0, 0.5]$	0.009	[0.0007, 0.03]	0.004	[0.0005, 0.015]
$\rho_{\xi,N}$	Uniform $[-0.99, 0.99]$	0.96	[0.93, 0.97]	0.75	[-0.40, 0.97]
$\sigma_{\xi,N}$	Uniform $[0, 1]$	0.38	[0.24, 0.48]	0.08	[0.0078, 0.41]
ρ_s	Uniform $[-0.99, 0.99]$	0.47	[-0.61, 0.93]	0.94	[0.85, 0.98]
σ_s	Uniform $[0, 0.2]$	0.026	[0.006, 0.05]	0.69	[0.47, 0.96]
ρ_ν	Uniform $[-0.99, 0.99]$	0.54	[0.20, 0.72]	0.84	[-0.52, 0.96]
σ_ν	Uniform $[0, 0.2]$	0.12	[0.08, 0.19]	0.08	[0.014, 0.13]
ρ_μ	Uniform $[-0.99, 0.99]$	0.88	[0.54, 0.97]	0.70	[0.55, 0.77]
σ_μ	Uniform $[0, 0.5]$	0.16	[0.09, 0.28]	0.95	[0.71, 1.32]

Note. All prior distributions are uniform. Posterior distributions are based on draws from the last 1 million draws from 10 million MCMC chain.

of standard deviations of measurement errors should be 6.25 % of the variances of observables.

Let $\boldsymbol{\theta} = [\sigma_T, \sigma_N, \phi^T, \phi^N, \psi, \rho_{a,T}, \sigma_{a,T}, \rho_{a,N}, \sigma_{a,N}, \rho_{\xi,T}, \sigma_{\xi,T}, \rho_{\xi,N}, \sigma_{\xi,N}, \rho_s, \sigma_s, \rho_\nu, \sigma_\nu, \rho_\mu, \sigma_\mu, \sigma^{ME}_{g^Y}, \sigma^{ME}_{g^C}, \sigma^{ME}_{g^I}, \sigma^{ME}_{TB/Y}, \sigma^{ME}_{s^h}]'$ be a vector of parameters to be estimated. For posterior sampling, I construct 10 million MCMC chain for each country, via the random walk Metropolis-Hastings sampler described in Herbst and Schorfheide (2016). The detailed procedure is explained in appendix C.3.2. The construction of MCMC chain requires approximately 10 hours with Intel Core i5-4460 3.20GHz processor with 8 GB RAM.

Table 3.4 shows summary statistics of the prior and simulated posterior distributions. Unsurprisingly, posterior distributions are substantially different between the two countries, in both endogenous propagation parameters and exogenous shock parameters. For sectoral elasticies of capital-labor substitution, σ_T and σ_N, which govern sectoral variation of labor shares, the estimation gives point estimates 0.89 and 0.49 for Canada, and 0.95 and 5.12 for Mexico. With calibrated parameters, these estimates yield heterogeneous quantitative predictions from models which will be shown in the next section.

3.4.3 Predictions from Estimated Models

Table 3.5 summarizes the second moments that characterize business cycles and labor share fluctuations. It shows that the estimated models perform well in matching the moments with the data. First, the models predict a pattern of labor share fluctuations in line with the data. Most importantly, the models predict cyclical behavior of labor shares, consistent with the data. The models predict a near-output volatile labor share in Canada $\sigma(s^h)/\sigma(g^Y)$, and excessive volatility of labor share relative to output in Mexico. In addition, the models predict a countercyclical labor share in Canada and procyclical labor share in Mexico. The correlation between labor share and growth rate of output $\rho\left(s^h, g^Y\right)$ from the model for Canada is -0.20 (-0.11 in data), and 0.09 (0.33 in data) for Mexico.

It is important to note that the model is able to predict cyclical patterns of the labor share in line with the data without relying on the labor wedge channel. Each of the existing labor wedges are intended to generate partial patterns of the labor share fluctuation. [12] Without conclusive empirical support of the existence of these wedges, these prevent theories from generating the general pattern of the labor share fluctuations. Our model, on the other hand, is able to do generate the patterns by opening the new channels of the substitution and composition effects.

[12] For example, search models in the labor market can only explain a countercyclical labor share, whereas a working capital constraint of labor hiring can only account for procyclical labor shares.

Table 3.5: Second Moments: Data and Model

	Canada		Mexico	
	Data	Model	Data	Model
Volatilities				
$\sigma\left(g^Y\right)$	2.00	3.17	3.52	5.80
$\sigma\left(s^h\right)/\sigma\left(g^Y\right)$	1.86	1.76	4.50	2.20
$\sigma\left(g^C\right)/\sigma\left(g^Y\right)$	0.90	1.02	1.22	1.68
$\sigma\left(g^I\right)/\sigma\left(g^Y\right)$	4.07	2.31	3.06	2.35
$\sigma\left(TB/Y\right)$	1.85	2.22	3.21	3.98
$\sigma\left(\gamma^T\right)/\sigma\left(g^Y\right)$	4.15	3.00	2.42	2.00
Correlations with Output Growth				
$\rho\left(s^h, g^Y\right)$	-0.11	-0.20	0.33	0.09
$\rho\left(g^C, g^Y\right)$	0.71	0.87	0.79	0.91
$\rho\left(g^I, g^Y\right)$	0.81	0.72	0.78	0.64
$\rho\left(TB/Y, g^Y\right)$	0.13	0.08	-0.46	-0.18
$\rho\left(\gamma^T, g^Y\right)$	0.27	0.35	-0.07	-0.10
Autocorrelations				
$\rho\left(s_t^h, s_{t-1}^h\right)$	0.91	0.98	0.95	0.98
$\rho\left(g_t^Y, g_{t-1}^Y\right)$	0.32	0.25	0.25	-0.09
$\rho\left(g_t^C, g_{t-1}^C\right)$	0.27	0.34	0.17	-0.28
$\rho\left(g_t^I, g_{t-1}^I\right)$	-0.02	0.01	-0.01	-0.19
$\rho\left(TB_t/Y_t, TB_{t-1}/Y_{t-1}\right)$	0.80	0.83	0.75	0.38
$\rho\left(\gamma_t^T, \gamma_{t-1}^T\right)$	0.93	0.95	0.91	0.83

Note. Standard deviations are measured in percentage points. The prediction is based on the posterior median from the last 1 million draws from 10 million MCMC chain.

The models also match general business cycle statistics successfully. For Canada, the estimated model successfully matches key statistics such as investment more volatile han output, the volatility of trade balance to output ratio, procyclical consumption and investment, and serial correlations of indicators. For Mexico, the model matches excess volatility of consumption and investment, and a countercyclical trade balance to output ratio.[13]

[13] The estimated model naturally predicts the source of fluctuations of macroeconomic indicators. However, unlike the second moments, the decomposition is sensitive to the calibration of σ_T and σ_N. Since we need more work in determining the right values of σ_T and σ_N, I postpone the interpretation of the decomposition result and put it into appendix C.5 (see Tables C.3 - C.4). However, all models consistently predict that the importance of the labor supply (reduce-form labor wedge) shock to the variation of labor share is not of primary importance.

3.5 The Gross Elasticity of Substitution

Although the model has sectoral elasticities of capital-labor substitution σ_T, σ_N, there is a gross elasticity of capital-labor substitution of an economy (which I denote σ henceforth). Estimating the parameter $\hat{\sigma}$ is of interest due to the development of the CES production function, but estimates were basically assumed from a one-sector production economy. Since our model is two-sector open economy, σ is not a structural parameter at all. To get an estimate of σ using the model, I simulate the estimated model. From the definition of the elasticity of capital-labor substitution

$$\frac{1}{\sigma} = \frac{d\log\left(w/r\right)}{d\log\left(k/h\right)},$$

a regression equation is specified such as

$$\log(k_t/h_t) = \beta_0 + \sigma\log(w_t/r_t) + \epsilon_t, \tag{3.53}$$

where

$$\begin{aligned}
k_t &= k_t^T + k_t^N, \\
h_t &= h_t^T + h_t^N, \\
w_t &= \gamma_t^T w_t^T + (1-\gamma_t^T) w_t^N, \\
r_t &= \gamma_t^T r_t^T + (1-\gamma_t^T) r_t^N,
\end{aligned}$$

and ϵ_t is a white noise process. I simulate 1 million series of each variable (after discarding the first 100000 series) and get an OLS estimate $\hat{\sigma}$ of the equation (3.53). Table 3.7 exhibits the estimates $\hat{\sigma}$ of the two countries in comparison to other literature. Our estimated models show that $\hat{\sigma}$ is 0.61 in Canada and 2.53 in Mexico, which means that capital and labor are gross substitutes in Mexico, whereas they are gross complements in Canada. It also implies

Table 3.6: The Elasticity of Capital-Labor Substitution of the Economy

Source	Model		Antrás (2004)	Chirinko (2009)	Raurich et al. (2014)
Country	Canada	Mexico	U.S.	U.S.	Spain
$\hat{\sigma}$	0.55	2.53	(0.4, 0.6)	(0.6, 0.9)	(1.12, 1.56)

Note. The second and three columns show estimates based on regressions with simulated data from estimated models. The estimates from models are all statistically significant at 1 percent significance level.

that the labor-augmented technological progress, X_t, is capital-directed (see definition 2) in Mexico but is labor-directed in Canada in aggregate level. Very interestingly, these estimates are comparable to other estimates using different models and methods.

There have been many attempts to estimate σ using a CES production function, but the estimates have a very wide range, depending on the specified model, data, and countries. Nevertheless, there is a consensus that the estimates are less than one in typical advanced countries. Chirinko (2009) surveys that estimates of σ in the United States have a range of (0.4, 0.6), and the range depends on aggregate/plant level investment data, and specification of the statistical model. Antrás (2004) uses private sector and national account data of the U.S. during 1948-1998, and suggests the typical range of elasticity (0.6, 0.9), based on CES production function with factor-augmented technologies. Our model estimate for Canada, 0.55, is in line with those estimates. There are few empirical studies on estimating σ for emerging countries, but we can document Raurich et al. (2011). By applying the same methodology in Antrás (2004) in Spain, they provide estimates of σ in the range of (1.12, 1.56) which is greater than 1. Our model estimate for Mexico, 2.53, is qualitatively in line with this estimate. However, one important caveat from this exercise is that the estimate of σ depends not only on σ_T and σ_N but also on the relative sectoral share γ_t^T which is time varying and has a substantial cyclicality. This exercise poses a caveat that the estimate from a one-sector economy would miss the important structure when we consider a small open economy that has a non-negligible γ_t^T with cyclical properties.

3.6 Concluding Remarks

This paper proposes an approach to understand labor share fluctuations over the business cycle. Fluctuations of the labor share have important welfare implications as they reflect variation of the income distribution along business cycles. Which income group's (high, middle, or low) income shrinks relatively more in an economic contraction? If we agree that labor income is the main source of income for middle and low income groups, cyclicality of the labor share would give an answer to this question. If the labor share is procyclical, a contraction is more hostile to middle and low income groups, while the opposite holds if it is countercyclical.

This paper finds that sectoral capital-labor substitution and variation of the relative value of sectoral production are important factors of labor share fluctuations. These ingredients however are not new. Capital-labor substitution has always been of interest since the emergence of political economy and the two sector economy composed of tradable and nontradable sectors is one of the standard setups in open-economy macroeconomics. The contribution of this paper is to incorporate the ingredients within the language of dynamic stochastic general equilibrium and provide a comprehensive view to better understand labor share fluctuations without relying on the labor wedge approach. From estimated models using reasonable parametrization, this paper predicts that the aggregate labor share is procyclical and is more volatile than output in Mexico. In Canada, on the other hand, it is countercyclical and is as volatile as output. These findings are consistent with observations from data, reflecting the success of the approach.

An important implication of this paper is the need to obtain micro-based estimates of σ_T and σ_N. This is necessary not only for the accurate prediction of the model, but also for estimating the correct gross elasticity of substitution of the economy. This is not well-explored, especially in countries outside the U.S., and may require comprehensive work using firm or micro-level data. I will leave this as a future research agenda.

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Appendix A

Appendix for Chapter 1

A.1 Competitive Equilibrium

A.1.1 The Tošovský Model

1. Households

$$\begin{aligned} U_1(c_t, h_t)A_1(c_t^\tau, c_t^n) &= \lambda_t p_t^\tau, &\text{(A.1)} \\ \frac{A_2(c_t^\tau, c_t^n)}{A_1(c_t^\tau, c_t^n)} &= \frac{p_t^n}{p_t^\tau}, &\text{(A.2)} \\ -\frac{U_2(c_t, h_t)}{U_1(c_t, h_t)A_1(c_t^\tau, c_t^n)} &= \frac{(1-\tau_t^p)w_t}{p_t^\tau}, &\text{(A.3)} \end{aligned}$$

$$1 = q_t \left[\left(1 - \frac{\phi}{2} \left(\frac{iv_t}{iv_{t-1}} - 1 \right)^2 \right) - \phi \left(\frac{iv_t}{iv_{t-1}} - 1 \right) \frac{iv_t}{iv_{t-1}} \right]$$

$$+ \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \phi \left(\frac{iv_{t+1}}{iv_t} - 1 \right) \left(\frac{iv_{t+1}}{iv_t} \right)^2 \right], \tag{A.4}$$

$$(1 - \kappa\Theta_t)q_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left((1-\delta)q_{t+1} + (1-\tau_{t+1}^p)r_{t+1}^k \right) \right], \tag{A.5}$$

$$\lambda_t (1 - \Theta_t) = \beta \mathbb{E}_t \left[\frac{1+i_t}{1+\pi_{t+1}} \lambda_{t+1} \right], \tag{A.6}$$

$$\mathbb{E}_t\left[\frac{1+i_t}{1+\pi_{t+1}}\frac{\lambda_{t+1}}{\lambda_t}\right] = \mathbb{E}_t\left[\left(1+r_t^*(d_t)+r_t^{*\prime}(d_t)d_t\right)\frac{\lambda_{t+1}}{\lambda_t}\frac{e_{t+1}}{e_t}\right], \tag{A.7}$$

$$\begin{aligned} k_{t+1} &= (1-\delta)k_t + \left(1-\frac{\phi}{2}\left(\frac{iv_t}{iv_{t-1}}-1\right)^2\right)iv_t, && \text{(A.8)} \\ d_t &= \kappa q_t^\tau k_{t+1}, && \text{(A.9)} \end{aligned}$$

$$\frac{A_2\left(iv_t^\tau, iv_t^n\right)}{A_1\left(iv_t^\tau, iv_t^n\right)} = \frac{p_t^n}{p_t^\tau}, \tag{A.10}$$

$$iv_t = p_t^\tau iv_t^\tau + p_t^n iv_t^n. \tag{A.11}$$

2. Firms

$$\begin{aligned} 1 &= \theta(1+\pi_t^n)^{\mu-1} + (1-\theta)\left(\bar{p}_t^n\right)^{1-\mu}, && \text{(A.12)} \\ \tilde{p}_t^n &= \frac{\mu}{\mu-1}\left(\frac{z_t}{f_t}\right), && \text{(A.13)} \\ p_t^n &= \frac{\tilde{p}_t^n}{\bar{p}_t^n} && \text{(A.14)} \\ z_t &= y_t^n(\bar{p}_t^n)^{-\mu}mc_t + \beta\theta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\bar{p}_t{}^n}{\bar{p}_{t+1}^n}\frac{1}{1+\pi_{t+1}^n}\right)^{-\mu}z_{t+1}\right], && \text{(A.15)} \\ f_t &= y_t^n(\bar{p}_t^n)^{-\mu} + \beta\theta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{1}{1+\pi_{t+1}}\right)\left(\frac{\bar{p}_t^n}{\bar{p}_{t+1}^n}\frac{1}{1+\pi_{t+1}^n}\right)^{-\mu}f_{t+1}\right], && \text{(A.16)} \\ y_t^n &= x_t^{-1}\left(k_t^\alpha h_t^{1-\alpha}-\varphi\right) && \text{(A.17)} \\ r_t^k &= mc_t\alpha\left(\frac{k_t}{h_t}\right)^{\alpha-1}, && \text{(A.18)} \\ w_t &= mc_t(1-\alpha)\left(\frac{k_t}{h_t}\right)^{\alpha} && \text{(A.19)} \\ x_t &= \theta x_{t-1}(1+\pi_t^n)^\mu + (1-\theta)(\bar{p}_t^n)^{-\mu}, && \text{(A.20)} \end{aligned}$$

4. Foreign Interest Rate

$$r_t^* = \bar{r} + \psi\left(e^{d_t-\bar{d}}-1\right) + e^{\epsilon_t^r-1} - 1, \tag{A.21}$$

5. Exchange Rate Pass-Through

$$p_t^\tau = e_t, \tag{A.22}$$

6. Aggregations and Prices

$$\begin{aligned}
y_t^n &= c_t^n + iv_t^n, && \text{(A.23)}\\
\pi_t^n &= \log\left(\frac{p_t^n}{p_{t-1}^n}\right) + \pi_t, && \text{(A.24)}\\
1 &= \left(\chi_\tau \left(p_t^\tau\right)^{1-\eta} + (1-\chi_\tau)\left(p_t^n\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}, && \text{(A.25)}\\
q_t^\tau &= \frac{q_t}{e_t}, && \text{(A.26)}
\end{aligned}$$

7. Resource Constraints

$$c_t^\tau + iv_t^\tau + (1 + r_{t-1}^*(d_{t-1}))d_{t-1} = y_t^\tau + d_t, \tag{A.27}$$

8. Exogenous Processes

$$\begin{aligned}
\log\left(y_{t+1}^\tau/\bar{y}^\tau\right) &= \rho_y \log\left(y_t^\tau/\bar{y}^\tau\right) + \nu_{t+1}^n, && \text{(A.28)}\\
\log\left(\epsilon_{t+1}^r\right) &= \rho_r \log\left(\epsilon_t^r\right) + \nu_{t+1}^r. && \text{(A.29)}
\end{aligned}$$

A.1.2 The AB Model

1. Arbitrageurs

$$\begin{aligned}
U^{a\prime}\left(c_t^{\tau a}\right) &= \lambda_t^a p_t^\tau, && \text{(A.30)}\\
\lambda_t^a &= \beta_a \mathbb{E}_t\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\lambda_{t+1}^a, && \text{(A.31)}\\
\lambda_t^a e_t &= \beta_a(1 + r_t^*(d_t^*) + r_t^{*\prime}(d_t^{*a})d_t^{*a})\mathbb{E}_t\lambda_{t+1}^a e_{t+1}, && \text{(A.32)}
\end{aligned}$$

2. Borrowers

$$
\begin{aligned}
U_1\left(c_t^b, h_t\right) A_1\left(c_t^{\tau b}, c_t^n\right) &= \lambda_t^b p_t^\tau, && \text{(A.33)} \\
\frac{A_2\left(c_t^{\tau b}, c_t^n\right)}{A_1\left(c_t^{\tau b}, c_t^n\right)} &= \frac{p_t^n}{p_t^\tau}, && \text{(A.34)} \\
-\frac{U_2\left(c_t^b, h_t\right)}{U_1\left(c_t^b, h_t\right) A_1\left(c_t^{\tau b}, c_t^n\right)} &= \frac{(1-\tau_t^p) w_t}{p_t^\tau}, && \text{(A.35)}
\end{aligned}
$$

$$
\begin{aligned}
1 = q_t &\left[\left(1 - \frac{\phi}{2}\left(\frac{iv_t}{iv_{t-1}} - 1\right)^2\right) - \phi\left(\frac{iv_t}{iv_{t-1}} - 1\right)\frac{iv_t}{iv_{t-1}}\right] \\
&+ \beta_b \mathbb{E}_t\left[\frac{\lambda_{t+1}^b}{\lambda_t^b} q_{t+1} \phi\left(\frac{iv_{t+1}}{iv_t} - 1\right)\left(\frac{iv_{t+1}}{iv_t}\right)^2\right], \qquad \text{(A.36)}
\end{aligned}
$$

$$
(1 - \kappa\Theta_t) q_t = \beta_b \mathbb{E}_t\left[\frac{\lambda_{t+1}^b}{\lambda_t^b}\left((1-\delta) q_{t+1} + (1 - \tau_{t+1}^p) r_{t+1}^k\right)\right], \qquad \text{(A.37)}
$$

$$
\lambda_t^b (1 - \Theta_t) = \beta_b \mathbb{E}_t\left[\frac{1 + i_t}{1 + \pi_{t+1}}\right]\lambda_{t+1}^b, \qquad \text{(A.38)}
$$

$$
\begin{aligned}
k_{t+1} &= (1-\delta) k_t + \left(1 - \frac{\phi}{2}\left(\frac{iv_t}{iv_{t-1}} - 1\right)^2\right) iv_t, && \text{(A.39)} \\
d_t^b &= \kappa q_t k_{t+1}, && \text{(A.40)}
\end{aligned}
$$

$$
\frac{A_2\left(iv_t^\tau, iv_t^n\right)}{A_1\left(iv_t^\tau, iv_t^n\right)} = \frac{p_t^n}{p_t^\tau}, \qquad \text{(A.41)}
$$

$$
iv_t = p_t^\tau iv_t^\tau + p_t^n iv_t^n. \qquad \text{(A.42)}
$$

3. Nontradable Sector

$$
\begin{aligned}
1 &= \theta(1+\pi_t^n)^{\mu-1} + (1-\theta)\left(\bar{p}_t^n\right)^{1-\mu}, && \text{(A.43)}\\
\tilde{p}_t^n &= \frac{\mu}{\mu-1}\left(\frac{z_t}{f_t}\right), && \text{(A.44)}\\
p_t^n &= \frac{\tilde{p}_t^n}{\bar{p}_t^n} && \text{(A.45)}\\
z_t &= y_t^n(\bar{p}_t^n)^{-\mu} mc_t + \beta_b\theta\mathbb{E}_t\left[\frac{\lambda_{t+1}^b}{\lambda_t^b}\left(\frac{\bar{p_t}^n}{\bar{p}_{t+1}^n}\frac{1}{1+\pi_{t+1}^n}\right)^{-\mu} z_{t+1}\right], && \text{(A.46)}\\
f_t &= y_t^n(\bar{p}_t^n)^{-\mu} + \beta_b\theta\mathbb{E}_t\left[\frac{\lambda_{t+1}^b}{\lambda_t^b}\left(\frac{1}{1+\pi_{t+1}}\right)\left(\frac{\bar{p}_t^n}{\bar{p}_{t+1}^n}\frac{1}{1+\pi_{t+1}^n}\right)^{-\mu} f_{t+1}\right], && \text{(A.47)}\\
y_t^n &= x_t^{-1}\left(a_t k_t^\alpha h_t^{1-\alpha} - \tau\right) && \text{(A.48)}\\
r_t^k &= mc_t \cdot a_t\alpha\left(\frac{k_t}{h_t}\right)^{\alpha-1}, && \text{(A.49)}\\
w_t &= mc_t \cdot a_t(1-\alpha)\left(\frac{k_t}{h_t}\right)^{\alpha} && \text{(A.50)}\\
x_t &= \theta x_{t-1}(1+\pi_t^n)^{\mu} + (1-\theta)(\bar{p}_t^n)^{-\mu}, && \text{(A.51)}
\end{aligned}
$$

4. Foreign Interest Rate

$$
r_t^* = \bar{r} + \psi\left(e^{d_t^a - \bar{d}} - 1\right) + e^{\epsilon_t^r - 1} - 1, \tag{A.52}
$$

5. Exchange Rate Pass-Through

$$
p_t^\tau = e_t, \tag{A.53}
$$

6. Aggregations and Prices

$$y_t^n = c_t^n + iv_t^n, \tag{A.54}$$

$$\pi_t^n = \log\left(\frac{p_t^n}{p_{t-1}^n}\right) + \pi_t, \tag{A.55}$$

$$1 = \left[\chi_\tau \left(p_t^\tau\right)^{1-\eta} + (1-\chi_\tau)\left(p_t^n\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{A.56}$$

$$q_t^\tau = \frac{q_t}{e_t}, \tag{A.57}$$

7. Resource Constraints

$$c_t^{\tau a} + (1 + r_{t-1}^*)d_{t-1}^a = y_t^{\tau a} + d_t^a, \tag{A.58}$$

$$c_t^{\tau b} + iv_t^\tau + \frac{1 + i_{t-1}}{p_t^\tau(1+\pi_t)}d_{t-1}^b = y_t^\tau + d_t^b, \tag{A.59}$$

8. Exogenous Processes

$$\log\left(y_{t+1}^\tau/\bar{y}^\tau\right) = \rho_y \log\left(y_t^\tau/\bar{y}^\tau\right) + \nu_{t+1}^n, \tag{A.60}$$

$$\log\left(\epsilon_{t+1}^r\right) = \rho_r \log\left(\epsilon_t^r\right) + \nu_{t+1}^r. \tag{A.61}$$

A.2 Steady State Solutions

A.2.1 The Tošovský Model

Step 1. Under $R = 1 + i$,

$$
\begin{aligned}
y^\tau &= \bar{y}^\tau, \\
d &= \text{the value implied by the IDEIR parameter} \\
r^* &= \bar{r}^* + \psi \left(e^{d-\bar{d}} - 1 \right) \\
\Theta &= \bar{\Theta}, \\
\beta &= \frac{1-\Theta}{1+\bar{r}^* + \psi e^{d-\bar{d}} d}, \\
\pi &= \frac{R}{1+\bar{r}^* + \psi e^{d-\bar{d}} d} - 1, \\
\pi^n &= \pi, \\
q &= 1, \\
r^k &= \frac{1}{1-\tau^p} \left(\frac{1-\kappa\Theta}{\beta} - (1-\delta) \right) q,
\end{aligned}
$$

Then under the given values of p^n and h,

$$
\begin{aligned}
p^\tau &= \left[\frac{1-(1-\chi_\tau)(p^n)^{1-\eta}}{\chi_\tau} \right]^{\frac{1}{1-\eta}}, \\
e &= p^\tau,
\end{aligned}
$$

Then

$$
\begin{aligned}
\bar{p}^n &= \left(\frac{1-\theta(1+\pi^n)^{\mu-1}}{1-\theta}\right)^{\frac{1}{1-\mu}} \\
x &= \frac{(1-\theta)(\bar{p}^n)^{-\mu}}{1-\theta(1+\pi^n)^{\mu}} \\
\tilde{p}^n &= p^n \bar{p}^n, \\
mc &= \frac{\mu-1}{\mu}\tilde{p}^n \left(\frac{1-\beta\theta\left(\frac{1}{1+\pi^n}\right)^{-\mu}}{1-\beta\theta\left(\frac{1}{1+\pi}\right)\left(\frac{1}{1+\pi^n}\right)^{-\mu}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{k}{h} &= \left(\frac{r^k}{\alpha\cdot mc}\right)^{\frac{1}{\alpha-1}}, \\
w &= (1-\alpha)mc\left(\frac{k}{h}\right)^{\alpha}, \\
k &= \frac{k}{h}\cdot h \\
z &= \frac{y^n\left(\bar{p}^n\right)^{-\mu} mc}{1-\beta\theta\left(\frac{1}{1+\pi^n}\right)^{-\mu}}, \\
f &= \frac{y^n\left(\bar{p}^n\right)^{-\mu}}{1-\beta\theta\left(\frac{1}{1+\pi}\right)\left(\frac{1}{1+\pi^n}\right)^{-\mu}}, \\
iv &= \delta k, \\
y^n &= x^{-1}\left(k^{\alpha}h^{1-\alpha}-\tau\right) \\
q^{\tau} &= \frac{q}{e}, \\
d &= \kappa q^{\tau} k.
\end{aligned}
$$

Then from

$$
\left(\frac{1-\chi_\tau}{\chi_\tau}\right)^{\frac{1}{\eta}}\left(\frac{iv^n}{iv^\tau}\right)^{-\frac{1}{\eta}} = \frac{p^n}{p^\tau},
$$

$$
iv = p^\tau iv^\tau + p^n iv^n,
$$

we get

$$\begin{aligned} iv^{\tau} &= \left[p^{\tau} + p^{n}\left(\frac{p^{n}}{p^{\tau}}\right)^{-\eta}\left(\frac{1-\chi_{\tau}}{\chi_{\tau}}\right)\right]^{-1} iv, \\ iv^{n} &= \left(\frac{p^{n}}{p^{\tau}}\right)^{-\eta}\left(\frac{1-\chi_{\tau}}{\chi_{\tau}}\right) iv^{\tau} \end{aligned}$$

then

$$c^{\tau} = y^{\tau} - iv^{\tau} - \frac{\bar{r}^{*}}{1+\bar{r}^{*}} d^{*},$$

and since

$$y^{n} = c^{n} + iv^{n},$$

we get

$$c^{n} = y^{n} - iv^{n}.$$

Then we have

$$\begin{aligned} \frac{p^{n1}}{p^{\tau}} &= \left(\frac{1-\chi_{\tau}}{\chi_{\tau}}\right)^{\frac{1}{\eta}}\left(\frac{c^{n}}{c^{\tau}}\right)^{-\frac{1}{\eta}}, \\ h^{1} &= \left(\chi_{\tau}^{\frac{1}{\eta}} \cdot \left(\chi_{\tau}^{\frac{1}{\eta}} + (1-\chi_{\tau})^{\frac{1}{\eta}}\left(\frac{c^{n}}{c^{\tau}}\right)^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}-1} \frac{(1-\tau^{p})w}{p^{\tau}}\right)^{\frac{1}{\omega-1}}, \end{aligned}$$

until

$$||p^{n} - p^{n1}|| + ||h - h^{1}|| < \textit{very small number}.$$

Then we have

$$
\begin{aligned}
c &= \left(\chi_\tau^{\frac{1}{\eta}} \left(c^\tau\right)^{1-\frac{1}{\eta}} + (1-\chi_\tau)^{\frac{1}{\eta}} \left(c^n\right)^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}}, \\
\lambda &= \frac{\left(c - \frac{h^\omega}{\omega}\right)^{-\sigma} \chi_\tau^{\frac{1}{\eta}} \cdot \left(\chi_\tau^{\frac{1}{\eta}} + (1-\chi_\tau)^{\frac{1}{\eta}} \left(\frac{c^n}{c^\tau}\right)^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}-1}}{p^\tau}.
\end{aligned}
$$

A.2.2 The AB Model

Step 1. Under given $R = 1 + i$,

$$
\begin{aligned}
y^\tau &= \bar{y}^\tau, \\
d^a &= \text{the value implied by the IDEIR parameter} \\
r^* &= \bar{r}^* + \psi\left(e^{d^a - \bar{d}} - 1\right) \\
\beta_a &= \frac{1}{1 + \bar{r}^* + \psi e^{d^a - \bar{d}} d^a}, \\
\pi &= \frac{R}{1 + \bar{r}^* + \psi e^{d^a - \bar{d}} d^a} - 1, \\
\pi^n &= \pi, \\
\Theta &= \bar{\Theta}, \\
\beta_b &= \frac{(1-\Theta)(1+\pi)}{R}, \\
q &= 1, \\
r^k &= \frac{1}{1-\tau^p}\left(\frac{1-\kappa\Theta}{\beta_b} - (1-\delta)\right) q,
\end{aligned}
$$

Then under given value of p^n and h,

$$\begin{aligned}
p^{\tau} &= \left[\frac{1-(1-\chi_{\tau})(p^n)^{1-\eta}}{\chi_{\tau}}\right]^{\frac{1}{1-\eta}}, \\
e &= p^{\tau},
\end{aligned}$$

Then

$$\begin{aligned}
\bar{p}^n &= \left(\frac{1-\theta(1+\pi^n)^{\mu-1}}{1-\theta}\right)^{\frac{1}{1-\mu}} \\
x &= \frac{(1-\theta)(\bar{p}^n)^{-\mu}}{1-\theta(1+\pi^n)^{\mu}} \\
\tilde{p}^n &= p^n \bar{p}^n, \\
mc &= \frac{\mu-1}{\mu}\tilde{p}^n \left(\frac{1-\beta_b\theta\left(\frac{1}{1+\pi^n}\right)^{-\mu}}{1-\beta_b\theta\left(\frac{1}{1+\pi}\right)\left(\frac{1}{1+\pi^n}\right)^{-\mu}}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{k}{h} &= \left(\frac{r^k}{\alpha \cdot mc}\right)^{\frac{1}{\alpha-1}}, \\
w &= (1-\alpha)mc\left(\frac{k}{h}\right)^{\alpha}, \\
k &= \frac{k}{h}\cdot h \\
z &= \frac{y^n\left(\bar{p}^n\right)^{-\mu} mc}{1-\beta_b\theta\left(\frac{1}{1+\pi^n}\right)^{-\mu}}, \\
f &= \frac{y^n\left(\bar{p}^n\right)^{-\mu}}{1-\beta_b\theta\left(\frac{1}{1+\pi}\right)\left(\frac{1}{1+\pi^n}\right)^{-\mu}}, \\
iv &= \delta k, \\
y^n &= x^{-1}\left(k^{\alpha}h^{1-\alpha}-\varphi\right), \\
d^b &= \kappa q k,
\end{aligned}$$

then from

$$\begin{aligned}
\left(\frac{1-\chi_\tau}{\chi_\tau}\right)^{\frac{1}{\eta}}\left(\frac{iv^n}{iv^\tau}\right)^{-\frac{1}{\eta}} &= \frac{p^n}{p^\tau}, \\
iv &= p^\tau iv^\tau + p^n iv^n,
\end{aligned}$$

we get

$$\begin{aligned}
iv^\tau &= \left[p^\tau + p^n\left(\frac{p^n}{p^\tau}\right)^{-\eta}\left(\frac{1-\chi_\tau}{\chi_\tau}\right)\right]^{-1} iv, \\
iv^n &= \left(\frac{p^n}{p^\tau}\right)^{-\eta}\left(\frac{1-\chi_\tau}{\chi_\tau}\right) iv^\tau
\end{aligned}$$

then

$$\begin{aligned}
c^{\tau a} &= y^{\tau a} - \frac{\bar{r}^*}{1+\bar{r}^*} d^a, \\
c^{\tau b} &= y^\tau - iv^\tau - \frac{\pi - i}{p^\tau(1+\pi)} d^b,
\end{aligned}$$

and since

$$y^n = c^n + iv^n,$$

we get

$$c^n = y^n - iv^n.$$

Then get

$$\begin{aligned}
\frac{p^{n1}}{p^\tau} &= \left(\frac{1-\chi_\tau}{\chi_\tau}\right)^{\frac{1}{\eta}}\left(\frac{c^n}{c^{\tau b}}\right)^{-\frac{1}{\eta}}, \\
h^1 &= \left(\chi_\tau^{\frac{1}{\eta}} \cdot \left(\chi_\tau^{\frac{1}{\eta}} + (1-\chi_\tau)^{\frac{1}{\eta}}\left(\frac{c^n}{c^{\tau b}}\right)^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}-1} \frac{(1-\tau^p)w}{p^\tau}\right)^{\frac{1}{\omega-1}},
\end{aligned}$$

Until

$$||p^n - p^{n1}|| + ||h - h^1|| < very\ small\ number.$$

Then get

$$\begin{aligned}
c^b &= \left(\chi_\tau^{\frac{1}{\eta}} \left(c^{\tau a}\right)^{1-\frac{1}{\eta}} + \left(1-\chi_\tau\right)^{\frac{1}{\eta}} \left(c^n\right)^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}}, \\
\lambda^a &= \frac{\left(c^{\tau a}\right)^{-\sigma}}{p^\tau}, \\
\lambda^b &= \frac{\left((1-\varphi)c^b - \frac{h^\omega}{\omega}\right)^{-\sigma} \chi_\tau^{\frac{1}{\eta}} \cdot \left(\chi_\tau^{\frac{1}{\eta}} + \left(1-\chi_\tau\right)^{\frac{1}{\eta}} \left(\frac{c^n}{c^{\tau b}}\right)^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}-1}}{p^\tau}.
\end{aligned}$$

A.3 Proofs

A.3.1 Lemma 1.4.1

The optimality condition of external debt is

$$\lambda_t \left(1 - \Theta_t\right) = \beta \mathbb{E}_t \left[\left(1 + r_t^*(d_t) + r_t^{*\prime}(d_t) d_t\right) \frac{\lambda_{t+1}}{\lambda_t} \frac{e_{t+1}}{e_t} \right],$$

The equation in the nonstochastic steady state is then given by

$$(1 - \Theta) = \beta \left[\left(1 + r^* + r^{*\prime}(d) d\right) \right].$$

which yields

$$\beta = \frac{1 - \Theta}{(1 + r^* + r^{*\prime}(d) d)}.$$

□

A.3.2 Proposition 1.5.1

The relevant competitive equilibrium constraints when the Ramsey planner decides π_t^n and π_t are

$$\begin{aligned}
1 &= \theta(1+\pi_t^n)^{\mu-1} + (1-\theta)(\bar{p}_t^n)^{1-\mu} \\
\tilde{p}_t^n &= \frac{\mu}{\mu-1}\frac{z_t}{f_t} \\
p_t^n &= \frac{\tilde{p}_t^n}{\bar{p}_t^n} \\
z_t &= y_t(\tilde{p}_t^n)^{-\mu} mc_t + \beta\theta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_t^n}{\tilde{p}_{t+1}^n}\frac{1}{1+\pi_{t+1}^n}\right)^{-\mu} z_{t+1}\right] \\
f_t &= y_t(\tilde{p}_t^n)^{-\mu} + \beta\theta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{1}{1+\pi_{t+1}}\right)\left(\frac{\tilde{p}_t^n}{\tilde{p}_{t+1}^n}\frac{1}{1+\pi_{t+1}^n}\right)^{-\mu} f_{t+1}\right] \\
x_t &= \theta x_{t-1}\left(1+\pi_t^n\right)^{\mu} + (1-\theta)(\bar{p}_t^n)^{-\mu} \\
\pi_t^n &= \pi_t + \log\left(\frac{p_t^n}{p_{t-1}^n}\right) \\
(1-\kappa\Theta_t)q_t &= \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left((1-\delta)q_{t+1} + r_{t+1}^k\right)\right] \\
q_t^\tau &= \frac{q_t}{e_t}
\end{aligned}$$

The Lagrangian of the Ramsey planner becomes

$$\mathcal{L}^R = \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big\{U(c_t,h_t) + \lambda_t^{R,1}\left(\theta(1+\pi_t^n)^{\mu-1} + (1-\theta)(\bar{p}_t^n)^{1-\mu} - 1\right)$$

$$+\lambda_t^{R,2}\left(y_t(p_t^n\bar{p}_t^n)^{-\mu} mc_t + \beta\theta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{p_t^n\bar{p}_t^n}{p_{t+1}^n\bar{p}_{t+1}^n}\frac{1}{1+\pi_{t+1}^n}\right)^{-\mu} z_{t+1}\right] - z_t\right)$$

$$+\lambda_t^{R,3}\left(\frac{\mu-1}{\mu}y_t(p_t^n\bar{p}_t^n)^{1-\mu} + \beta\theta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{1}{1+\pi_{t+1}}\right)\left(\frac{p_t^n\bar{p}_t^n}{p_{t+1}^n\bar{p}_{t+1}^n}\right)^{1-\mu}\left(\frac{1}{1+\pi_{t+1}^n}\right)^{-\mu} z_{t+1}\right] - z_t\right)$$

$$+\lambda_t^{R,4}\left(\theta x_{t-1}\left(1+\pi_t^n\right)^{\mu} + (1-\theta)(\bar{p}_t^n)^{-\mu} - x_t\right) + \lambda_t^{R,5}\left(\pi_t + \log\left(\frac{p_t^n}{p_{t-1}^n}\right) - \pi_t^n\right)$$

$$+\lambda_t^{R,6}\left(\beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left((1-\delta)q_{t+1}+r_{t+1}^k\right)\right]-(1-\kappa\Theta_t)q_t\right)$$

$$\left.+\lambda_t^{R,7}\left(\frac{q_t}{e_t}-q_t^{\tau}\right)+\text{others}\right\},$$

where $\lambda_t^{R,j}, j=1,2,...,7$ are Lagrange multipliers associated to the constraints and the term 'others' refers to other remaining constraints with corresponding Lagrange multipliers. The necessary first order conditions with respect to $\pi_t^n, \pi_t, \bar{p}_t^n$, and z_t are

$$\beta^{-1}\lambda_{t-1}^{R,2}\left[\beta\theta\mu\frac{\lambda_t}{\lambda_{t-1}}\left(\frac{p_{t-1}^n\bar{p}_{t-1}^n}{p_t^n\bar{p}_t^n}\right)^{-\mu}\left(\frac{1}{1+\pi_t^n}\right)^{1-\mu}z_{t-1}\right]$$

$$+\beta^{-1}\lambda_{t-1}^{R,3}\left[\beta\theta\mu\frac{\lambda_t}{\lambda_{t-1}}\left(\frac{1}{1+\pi_t}\right)\left(\frac{p_{t-1}^n\bar{p}_{t-1}^n}{p_t^n\bar{p}_t^n}\right)^{1-\mu}\left(\frac{1}{1+\pi_t^n}\right)^{1-\mu}z_{t-1}\right]$$

$$+\lambda_t^{R,1}\left[\theta(\mu-1)(1+\pi_t^n)^{\mu-2}\right]+\lambda_t^{R,4}\left[\theta\mu x_{t-1}(1+\pi_t^n)^{\mu-1}\right]-\lambda_t^{R,5}=0,$$

$$\beta^{-1}\lambda_{t-1}^{R,3}\left[\beta\theta(-1)\left(\frac{1}{1+\pi_t}\right)^2\left(\frac{p_{t-1}^n\bar{p}_{t-1}^n}{p_t^n\bar{p}_t^n}\right)^{1-\mu}\left(\frac{1}{1+\pi_t^n}\right)^{-\mu}z_{t-1}\right]+\lambda_t^{R,5}=0,$$

$$\beta^{-1}\lambda_{t-1}^{R,2}\left[\beta\theta(\mu)(\bar{p}_t^n)^{\mu-1}\left(\frac{\lambda_t}{\lambda_{t-1}}\left(\frac{p_{t-1}^n\bar{p}_{t-1}^n}{p_t^n}\frac{1}{1+\pi_t^n}\right)^{-\mu}\right)z_t\right]$$

$$+\beta^{-1}\lambda_{t-1}^{R,3}\left[\beta\theta(\mu-1)(\bar{p}_t^n)^{\mu-2}\left(\frac{\lambda_t}{\lambda_{t-1}}\left(\frac{1}{1+\pi_t}\right)\left(\frac{p_{t-1}^n\bar{p}_{t-1}^n}{p_t^n}\right)^{1-\mu}\left(\frac{1}{1+\pi_t^n}\right)^{-\mu}\right)z_t\right]$$

$$+\lambda_t^{R,1}\left[(1-\theta)(1-\mu)(\bar{p}_t^n)^{-\mu}\right]$$

$$+\lambda_t^{R,2}\left[(-\mu)(\bar{p}_t^n)^{-\mu-1}\left(y_t(p_t^n)^{-\mu}mc_t+\beta\theta\left(\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{p_t^n}{p_{t+1}^n\bar{p}_{t+1}^n}\frac{1}{1+\pi_{t+1}^n}\right)^{-\mu}z_{t+1}\right)\right)\right]$$

$$+\lambda_t^{R,3}\left[(1-\mu)(\bar{p}_t^n)^{-\mu}\left(\left(\frac{\mu-1}{\mu}\right)y_t(p_t^n)^{1-\mu}\right.\right.$$

$$+\beta\theta\left(\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{1}{1+\pi_{t+1}}\right)\left(\frac{p^n_t}{p^n_{t+1}\bar{p}^n_{t+1}}\right)^{1-\mu}\left(\frac{1}{1+\pi^n_{t+1}}\right)^{-\mu}z_{t+1}\right)\Bigg)\Bigg]$$

$$+\lambda^{R,4}_t\left[(1-\theta)(-\mu)(\bar{p}^n_t)^{-\mu-1}\right]=0,$$

and

$$+\beta^{-1}\lambda^{R,2}_{t-1}\left[\beta\theta\frac{\lambda_t}{\lambda_{t-1}}\left(\frac{p^n_{t-1}\bar{p}^n_{t-1}}{p^n_t\bar{p}^n_t}\frac{1}{1+\pi^n_t}\right)^{-\mu}\right]$$

$$+\beta^{-1}\lambda^{R,3}_{t-1}\left[\beta\theta\frac{\lambda_t}{\lambda_{t-1}}\left(\frac{1}{1+\pi_t}\right)\left(\frac{p^n_{t-1}\bar{p}^n_{t-1}}{p^n_t\bar{p}^n_t}\right)^{1-\mu}\left(\frac{1}{1+\pi^n_t}\right)^{-\mu}\right]-\lambda^{R,2}_t-\lambda^{R,3}_t=0.$$

Let $\pi^n=0$. Then $\pi=0$, $\bar{p}^n=1$, $x=1$ and $z,\bar{p}^n,mc,k,h,iv^n$, and c^n are pinned down. The first order conditions with the steady state solution then become

$$\lambda^{R,2}\theta\mu z+\lambda^{R,3}\theta\mu z+\lambda^{R,1}\theta(\mu-1)+\lambda^{R,4}\theta\mu-\lambda^{R,5}=0, \tag{A.62}$$

$$\lambda^{R,3}\theta z=\lambda^{R,5}, \tag{A.63}$$

$$\lambda^{R,1}(1-\theta)(1-\mu)-\lambda^{R,2}(1-\theta)\mu z+\lambda^{R,3}(1-\theta)(1-\mu)z+\lambda^{R,4}(1-\theta)(-\mu)=0 \tag{A.64}$$

and

$$\lambda^{R,2}=-\lambda^{R,3}. \tag{A.65}$$

These collapse equations (A.62) and (A.64) to the identical equation

$$\lambda^{R,1}(1-\mu)+\lambda^{R,3}\left(\mu z+(1-\mu)z\right)-\lambda^{R,4}\mu=0.$$

□

A.3.3 Proposition 1.5.2

The strategy of proof in this case is to prove that zero inflation rate in the competitive equilibrium is Pareto optimal at all $t\geq 0$. I begin by describing the Pareto planner's problem which maximizes

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$$

subject to the following constraints

$$\begin{aligned} c_t &= A\left(c_t^{\tau}, c_t^{n}\right), \\ c_t^{\tau} &= y_t^{\tau} - iv_t^{\varphi} - (1 + r_{t-1}^*(d_{t-1}))d_{t-1} + d_t, \\ c_t^{n} &= \mathcal{F}(k_t, h_t) - \tau - iv_t^{n}, \end{aligned}$$

$$k_{t+1} = (1-\delta)k_t + \left(1 - \frac{\phi}{2}\left(\frac{iv_t}{iv_{t-1}} - 1\right)^2\right) iv_t,$$

$$iv_t = A\left(iv_t^{\tau}, iv_t^{n}\right),$$

$$r_t^* = \bar{r} + \psi\left(e^{d_t - \bar{d}} - 1\right) + e^{\epsilon_t^r - 1} - 1.$$

Now I turn into the competitive equilibrium. I set $\pi_t^n = 0$ and $\tau_t^p = -\frac{1}{\mu - 1}$. Then $\bar{p}_t^n = 1$ and $x_t = 1$ for all $t \geq 0$. Guess that $mc_t = \frac{\mu-1}{\mu} p_t^n$ for all time. The guess is verified since this satisfies equations for $\tilde{p}_t^n$, z_t, and f_t for all t. And $\frac{p_t^n}{p_t^{\tau}} = \frac{A_2(c_t^{\tau}, c_t^n)}{A_1(c_t^{\tau}, c_t^n)}$ in equilibrium. Then first order conditions for h_t, iv_t, and k_{t+1} from the competitive equilibrium become

$$-\frac{U_2(c_t, h_t)}{U_1(c_t, h_t) A_1(c_t^{\tau}, c_t^{n})} = \frac{A_2(c_t^{\tau}, c_t^{n})}{A_1(c_t^{\tau}, c_t^{n})} \mathcal{F}_2(k_t, h_t),$$

$$\begin{aligned} 1 = q_t &\left[\left(1 - \frac{\phi}{2}\left(\frac{iv_t}{iv_{t-1}} - 1\right)^2\right) - \phi\left(\frac{iv_t}{iv_{t-1}} - 1\right)\frac{iv_t}{iv_{t-1}}\right] \\ &+ \beta \mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \phi\left(\frac{iv_{t+1}}{iv_t} - 1\right)\left(\frac{iv_{t+1}}{iv_t}\right)^2\right], \end{aligned}$$

$$q_t = \beta \mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left((1-\delta) q_{t+1} + a_{t+1} \mathcal{F}_1(k_{t+1}, h_{t+1})\right)\right],$$

and production in the nontradable sector satisfies

$$\mathcal{F}(k_t, h_t) - \varphi = c_t^n + iv_t^n.$$

And since iv_t^τ and iv_t^n in competitive equilibrium always satisfy

$$A(iv_t^\tau, iv_t^n) = iv_t,$$

equilibrium conditions in the competitive equilibrium with zero inflation become identical to the equilibrium of the Pareto planner. Thus, zero inflation rate is Pareto optimal and therefore it is Ramsey optimal for all t. It makes the standard model without the collateral constraints collapses to the real model of current account without nominal variables.

A.3.4 Proposition 1.5.3

Recall the first order conditions of the arbitrageurs

$$U^{a'}(c_t^{\tau a}) = \lambda_t^a p_t^\tau, \tag{A.66}$$

$$\lambda_t^a = \beta_a \mathbb{E}_t \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \lambda_{t+1}^a \tag{A.67}$$

$$\lambda_t^a e_t = \beta_a \left(1 + r_t^*(d_t^a) + r_t^{*\prime}(d_t^a) d_t^a\right) \mathbb{E}_t \lambda_{t+1}^a e_{t+1}. \tag{A.68}$$

In equilibrium, the arbitrageur's sequential budget constraint becomes

$$c_t^{\tau a} + (1 + r_{t-1}^*(d_{t-1}^a)) d_{t-1}^a = y_t^{\tau a} + d_t^{*a}. \tag{A.69}$$

Combining (A.66) and (A.68) yields

$$U^{a'}(c_t^{\tau a}) = \beta_a \left(1 + r_t^*(d_t^*) + r_t^{*\prime}(d_t^a) d_t^a\right) \mathbb{E}_t U^{a'}(c_{t+1}^{\tau a}), \tag{A.70}$$

and because the foreign interest rate r_t^* is internally determined by the debt elastic function,

$$r_t^* = \bar{r} + \psi_a \left(e^{d_t^a - \bar{d}^a} - 1 \right) + e^{\epsilon_t^r - 1} - 1, \tag{A.71}$$

the allocation $\{c_t^{\tau a}, d_t^a, r_t^*\}$ is identical to the solution that maximizes utility function subject to (A.69) and (A.71) as constraints. Note that the simplified problem is invariant to the domestic nominal interest rate, which implies that the policy functions, $c_t^{\tau a}, d_t^a, r_t^*$, are also independent of the domestic nominal interest rate.

A.3.5 Proposition 1.5.4

The proof is isomorphic to the proof in proposition 5.2, except the changing of the discount factor of the Ramsey planner to that of the borrowers, β_b. The remained of the proof is identical.

□

A.4 Additional Tables and Figures

Table A.1: Calibrated Parameters, The Standard Model

Parameter	Value	Description
σ	2	Inverse of the intertemporal elasticity of substitution
ω	1.455	Labor supply elasticity parameter
$\bar{r}^*$	$1.04^{1/4} - 1$	Risk free world interest rate
$\bar{d}$	4.0495	IDEIR parameter
ψ	0.0000335	Elasticity of r_t^* with respect to debt adjustment
β	0.9902	Subjective discount factor
η	0.5	Elasticity of substitution between T and NT goods
χ_τ	0.3	Preference bias toward the tradable good
α	0.25	Capital share of nontradable good production
θ	0.7	Calvo-Yun parameter
μ	6	Elasticity of substitution across NT intermediaries
τ^p	-0.2	Production Tax Rate (Subsidy)
φ	0.5432	Fixed cost of production in NT sector
$\bar{y}^\tau$	1	Tradable endowment in the steady state
ϕ	0.1	Elasticity of costly capital adjustment
ρ_y	0.79	Persistence of the log endowment shock
ρ_r	0.87	Persistence of the log foreign interest rate shock
σ_ϵ^y	0.0351	Standard deviation of the log endowment shock
σ_ϵ^r	0.0026	Standard deviation of the log foreign interest rate shock

Table A.2: Calibrated Parameters, The AB Model

Parameter	Value	Description
σ_a	2	Inverse of the intertemporal elasticity of substitution, Arbitrageurs
σ_b	2	Inverse of the intertemporal elasticity of substitution, Borrowers
ω	1.455	Labor supply elasticity parameter
$\bar{r}^*$	$1.04^{1/4} - 1$	Risk free world interest rate
$\bar{d}$	4.0495	IDEIR parameter
ψ	0.00335	Elasticity of r_t^* with respect to debt adjustment
β_a	0.9902	Subjective discount factor, Arbitrageurs
β_b	0.9764	Subjective discount factor, Borrowers
η	0.5	Elasticity of substitution between T and NT goods
χ_τ	0.3	Preference bias toward the tradable good
α	0.25	Capital share of nontradable good production
θ	0.7	Calvo-Yun parameter
μ	6	Elasticity of substitution across NT intermediaries
τ^p	-0.2	Production Tax Rate (Subsidy)
φ	0.4859	Fixed cost of production in NT sector
$\bar{y}^{\tau b}$	1	Tradable endowment in the steady state, Arbitrageurs
$\bar{y}^{\tau}$	1	Tradable endowment in the steady state, Borrowers
ϕ	0.1	Elasticity of costly capital adjustment
ρ_y	0.79	Persistence of the log endowment shock
ρ_r	0.87	Persistence of the log foreign interest rate shock
σ_ϵ^y	0.0351	Standard deviation of the log endowment shock
σ_ϵ^r	0.0026	Standard deviation of the log foreign interest rate shock

Figure A.1: Impulse Response Functions, 1% Increase in Tradable Endowment (I)

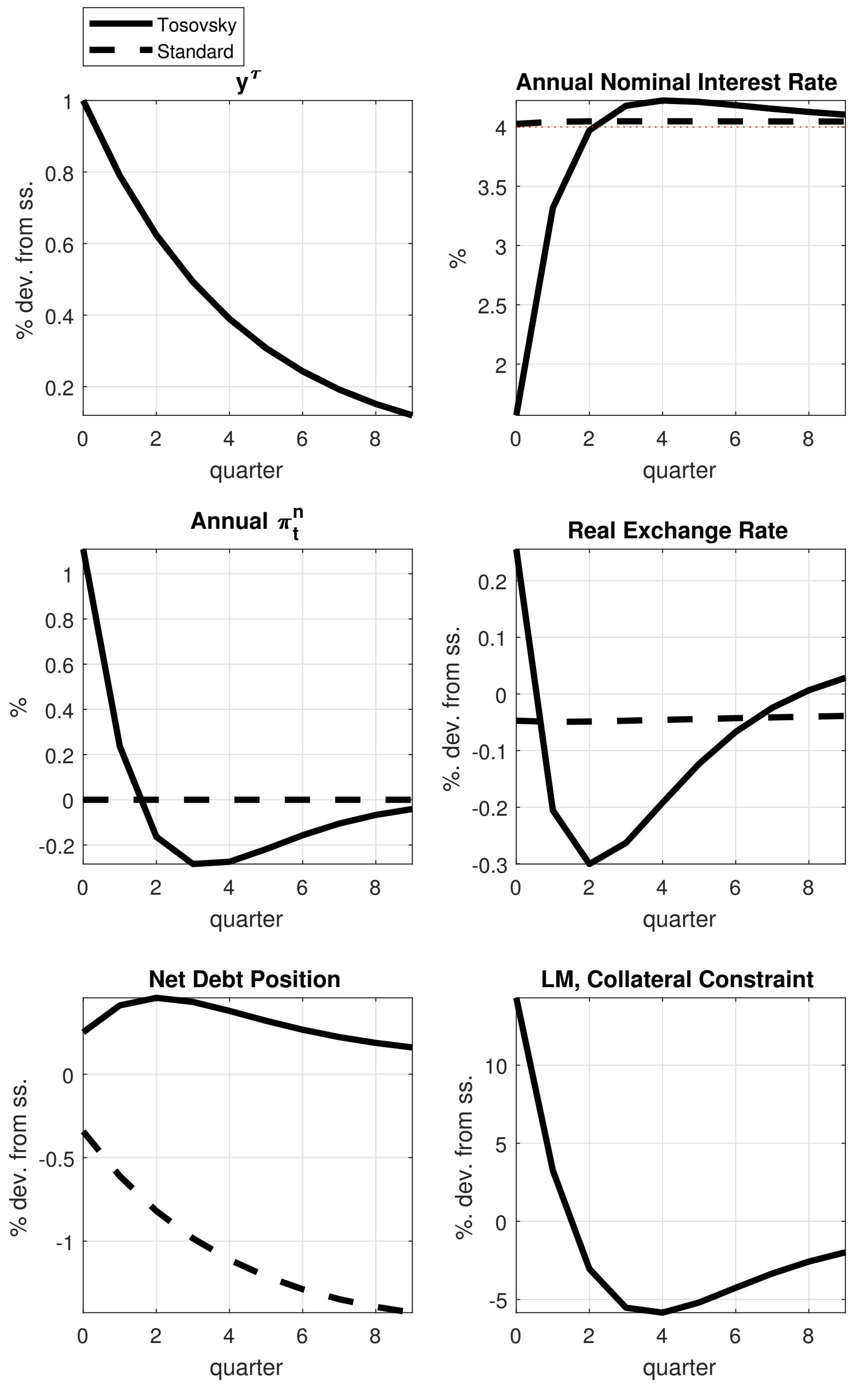


Figure A.2: Impulse Response Functions, 1% Increase in Tradable Endowment (II)

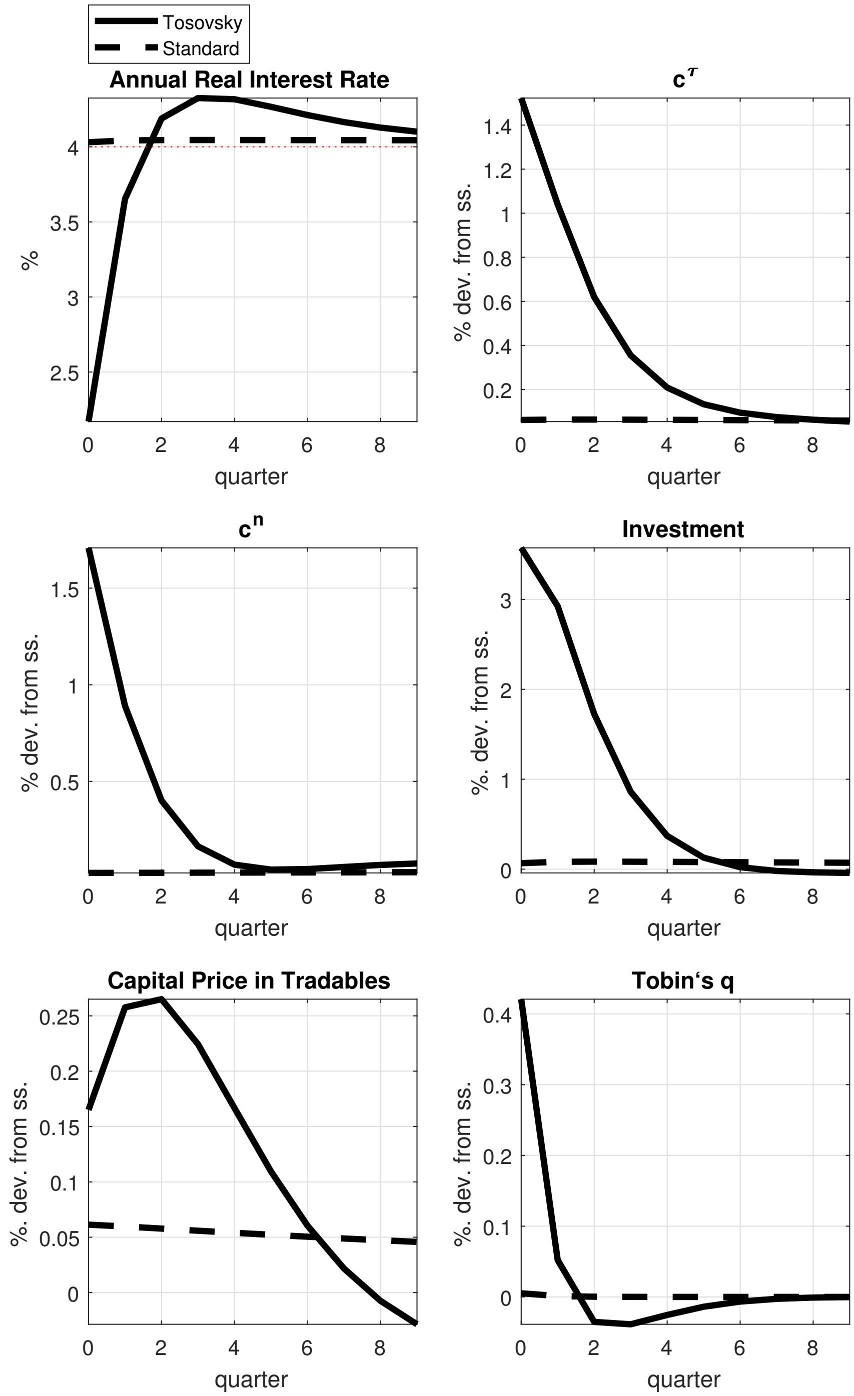


Figure A.3: Impulse Response Functions, 1% Decrease in Foreign Interest Rate (I)

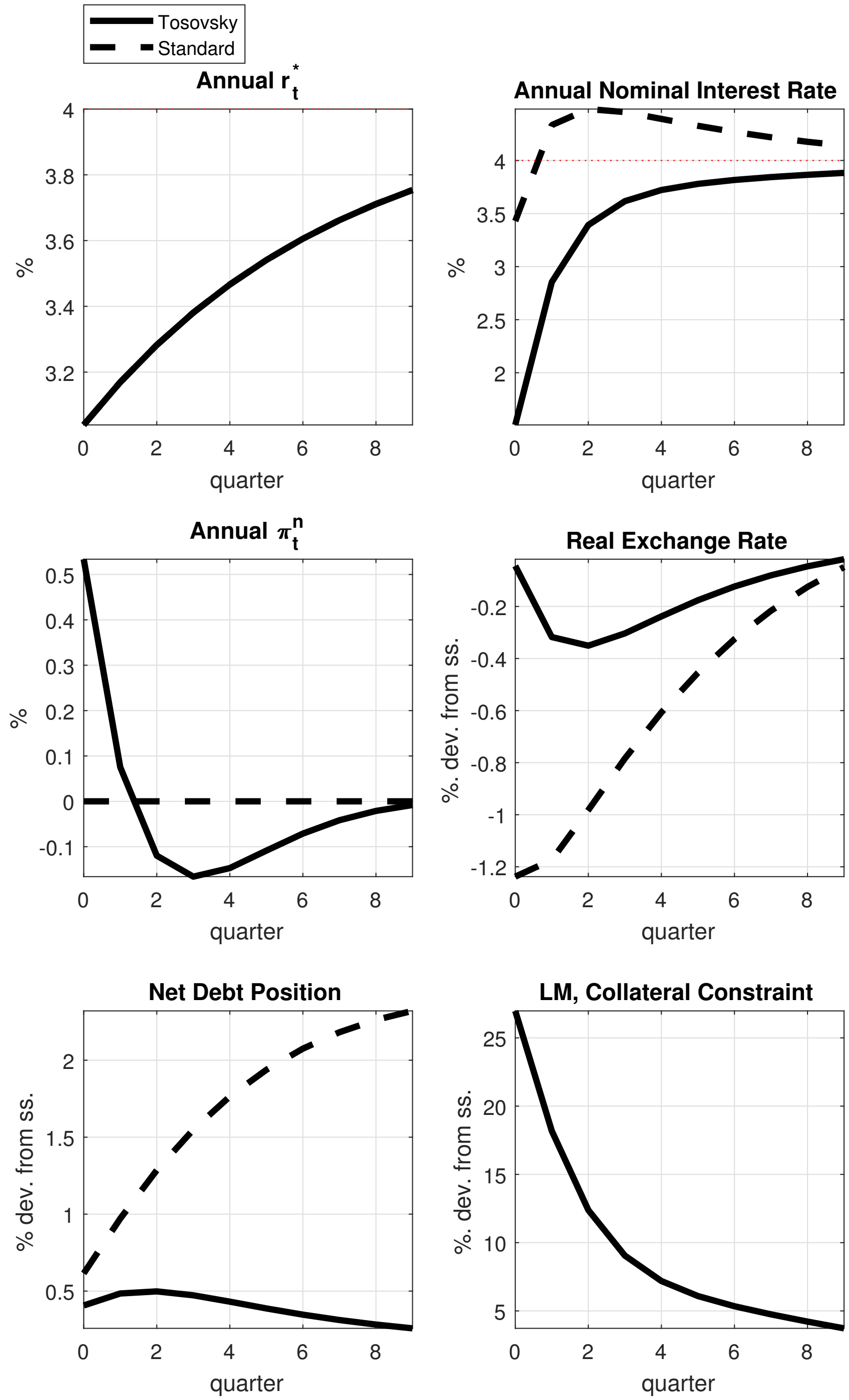


Figure A.4: Impulse Response Functions, 1% Decrease in Foreign Interest Rate (II)

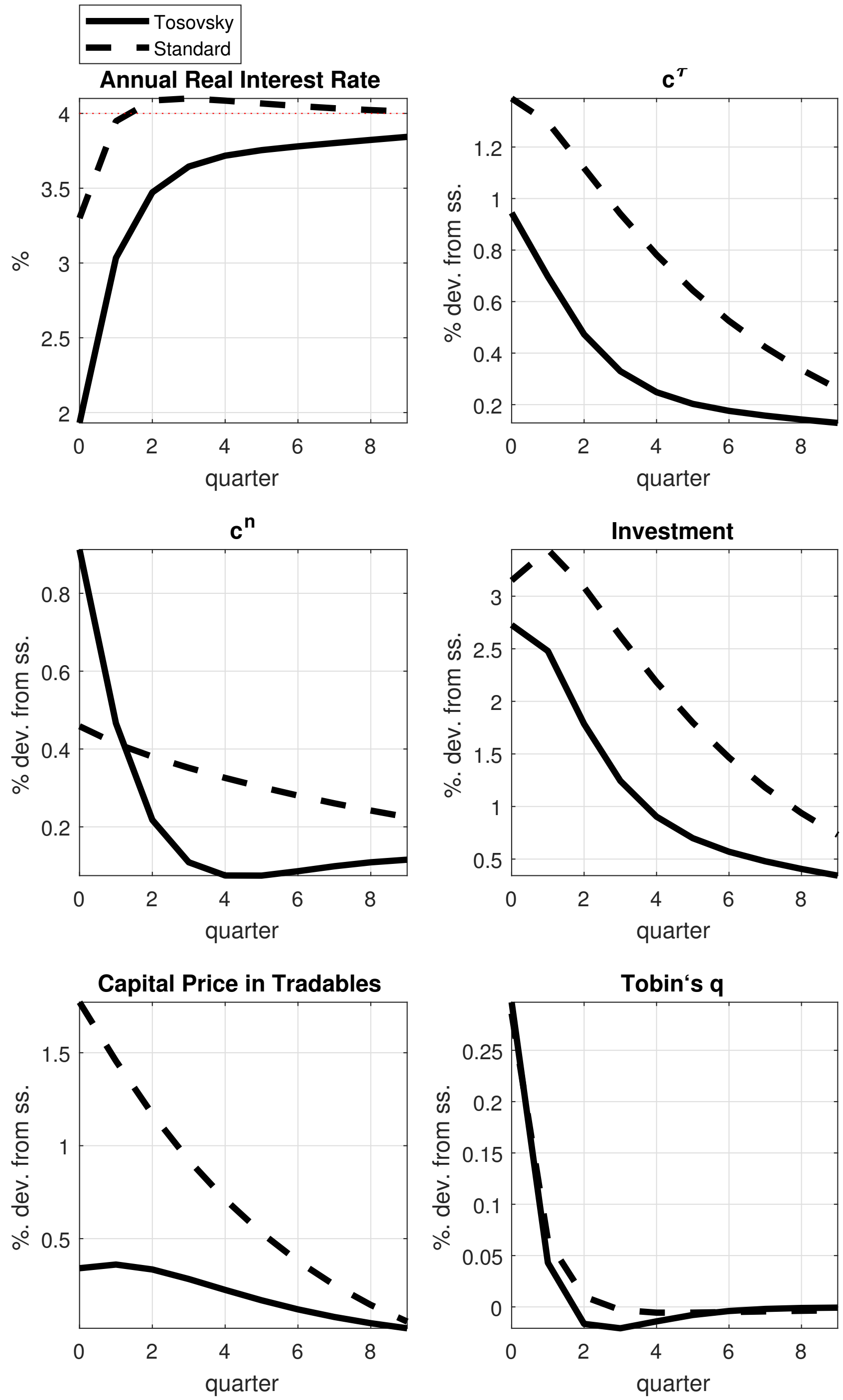


Figure A.5: Impulse Response Functions, 1% Increase in Tradable Endowment (I), The AB Model

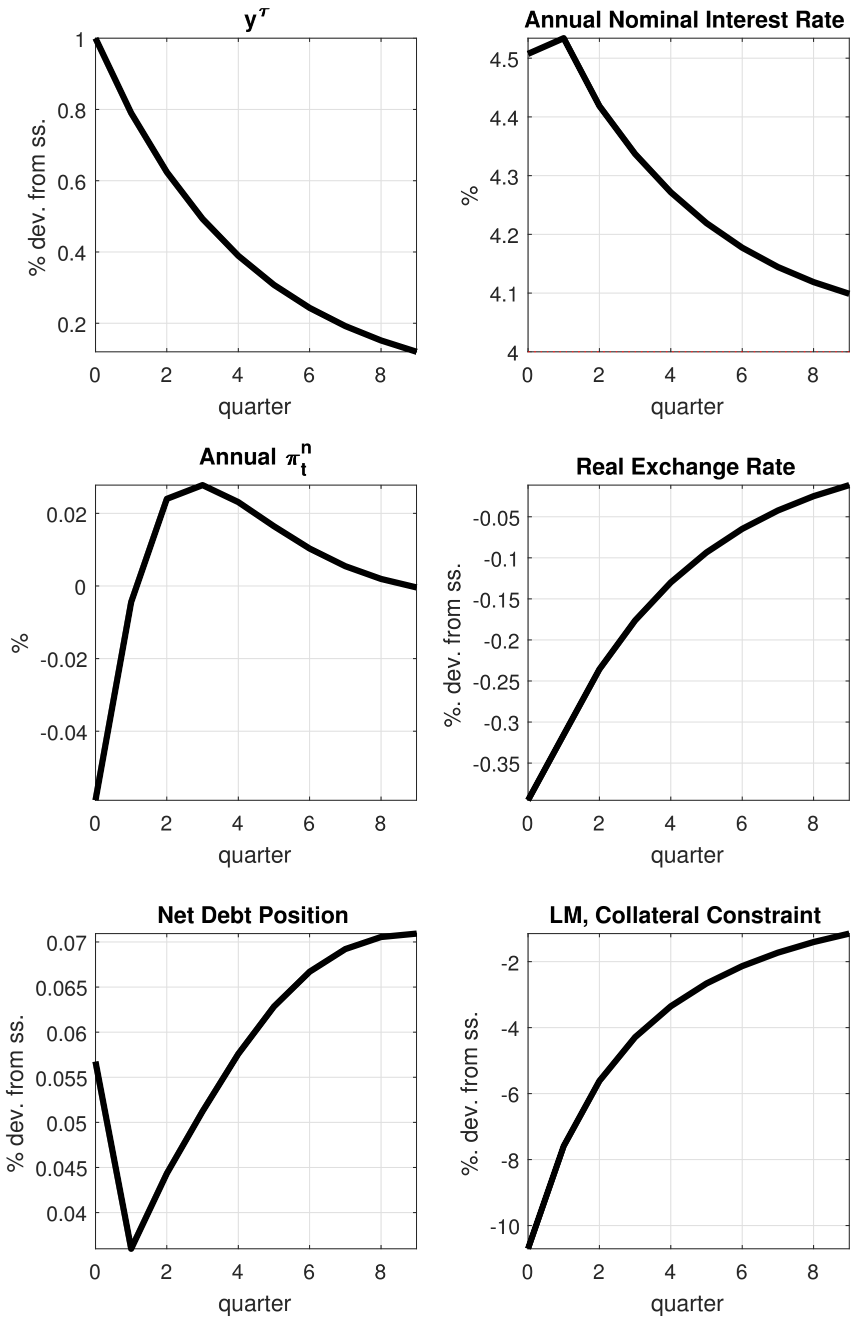


Figure A.6: Impulse Response Functions, 1% Increase in Tradable Endowment (II), The AB Model

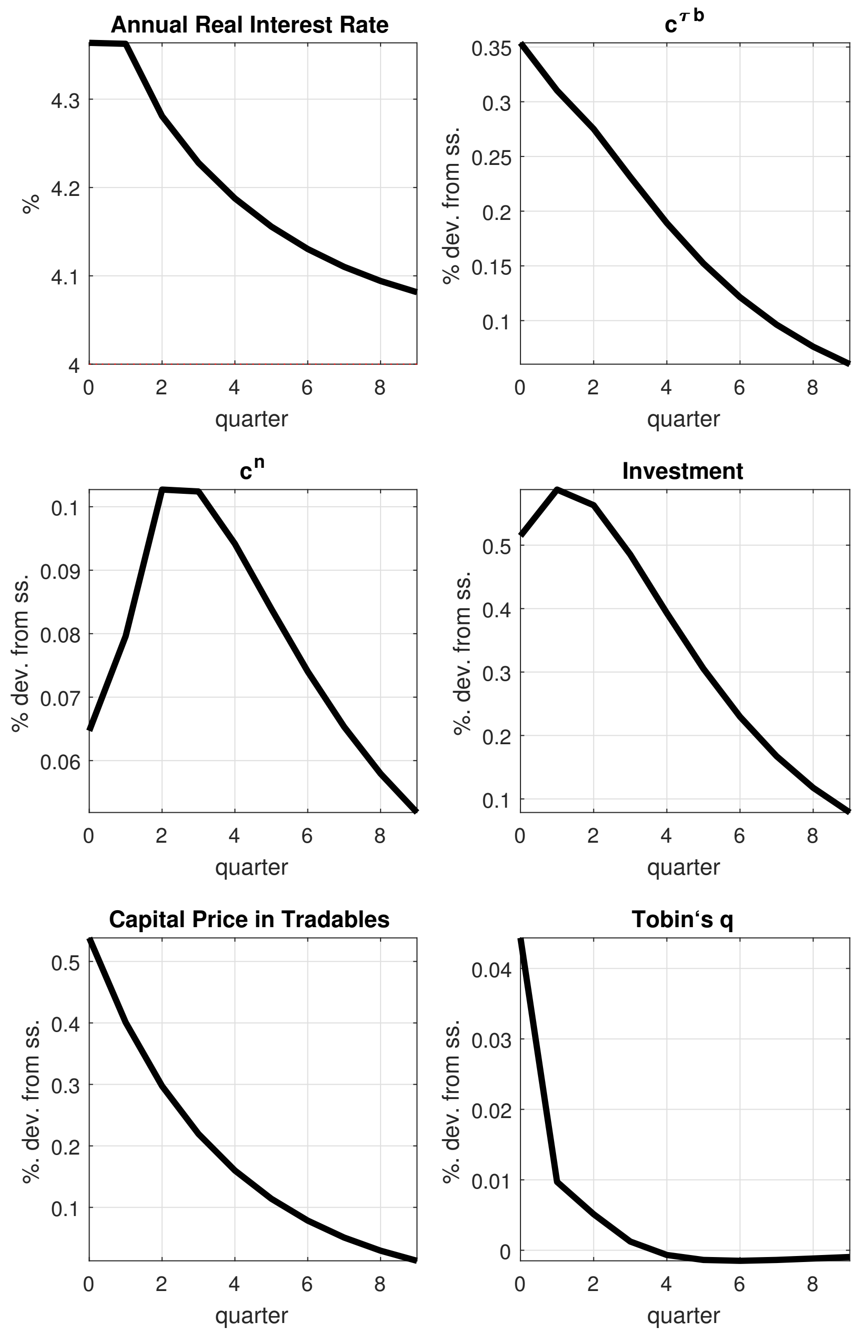


Figure A.7: Impulse Response Functions, 1% Decrease in Foreign Interest Rate (I), The AB Model

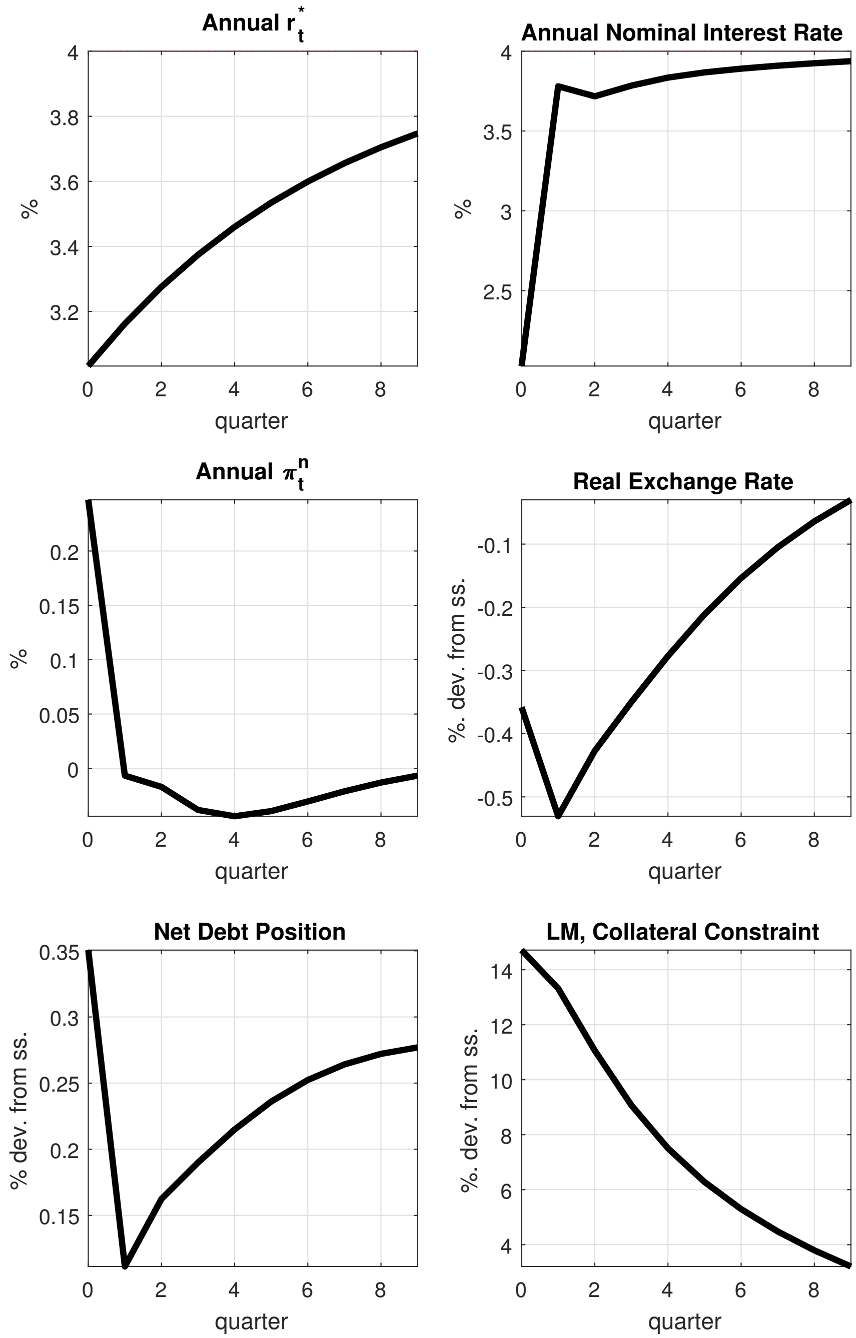
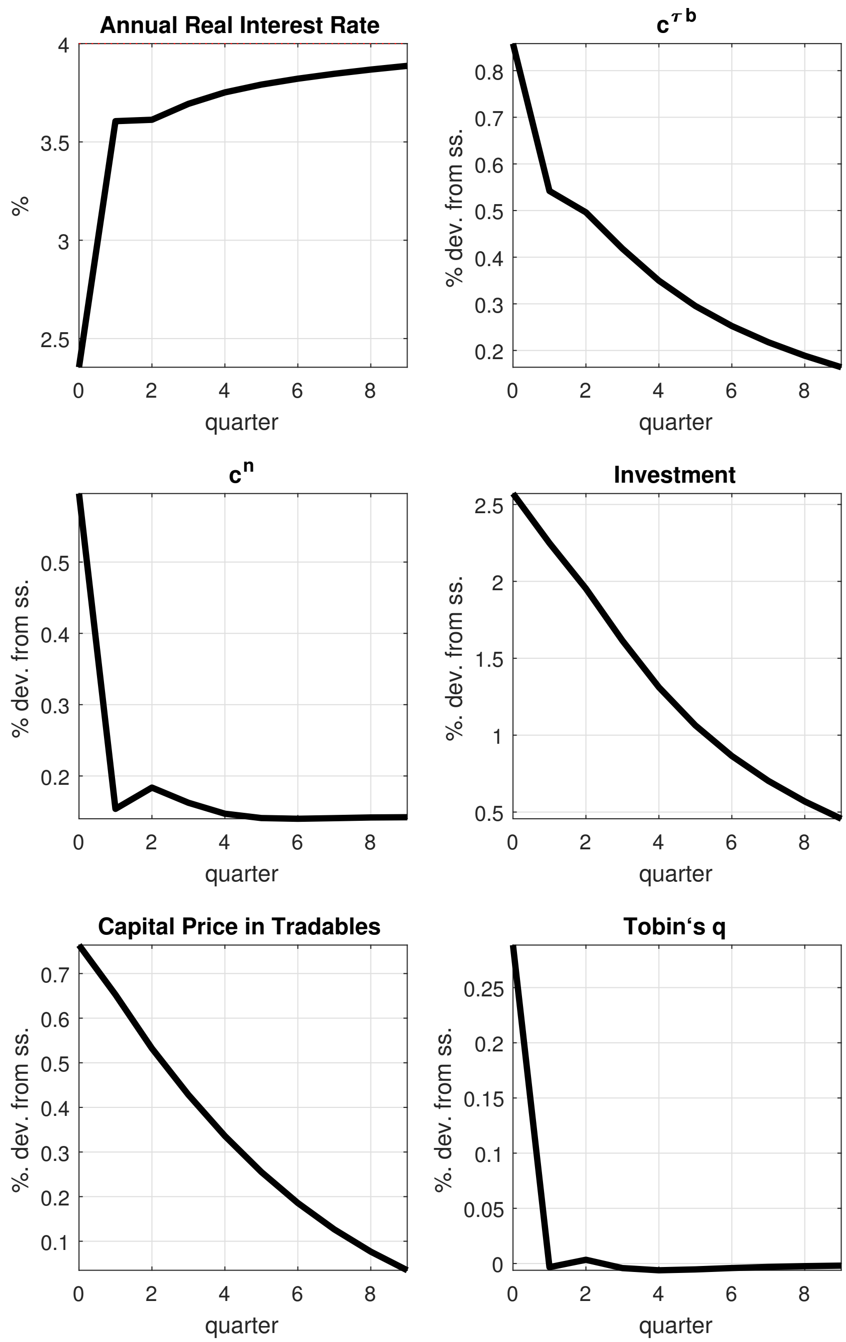


Figure A.8: Impulse Response Functions, 1% Decrease in Foreign Interest Rate (II), The AB Model



Appendix B

Appendix for Chapter 2

B.1 Additional Tables

Table B.1: List of Countries, Periods and Number of Observations of the CLS and the TLS

Country	Periods	Obs.	Country	Periods	Obs.
EMs (26)					
Armenia	1994-2009	16	Lithuania	1995-2011	17
Azerbaijan	1992-2010	19	Mexico	1993-2011	19
Belarus	1995-2010	15	Micronesia	1995-2010	16
Bolivia	1992-2007	16	Moldova	1995-2010	16
Brazil	1995-2009	15	Poland	1995-2011	17
China	1992-2009	18	Portugal	1995-2012	18
Colombia	1994-2010	17	Romania	1995-2009	15
Czech	1993-2011	19	Slovakia	1995-2011	17
Estonia	1996-2011	16	Slovenia	1995-2011	17
Hungary	1995-2011	17	South Africa	1995-2011	17
Kazakhstan	1994-2010	17	Spain	1995-2011	17
Kyrgyz Rep	1994-2010	17	Tunisia	1992-2008	17
Latvia	1994-2011	18	Ukraine	1993-2008	16
Mean		17			
ADVs (13)					
Austria	1995-2011	17	Luxembourg	1995-2009	15
Belgium	1985-2011	27	Netherland	1980-2011	32
Denmark	1981-2011	31	Norway	1978-2012	35
Finland	1975-2011	36	Sweden	1995-2010	16
France	1975-2011	36	United Kingdom	1987-2011	25
Germany	1980-2011	32	United States	1975-2012	38
Italy	1980-2011	32			
Mean		28			

Table B.2: List of Countries, Periods and Number of Observations of the TLS in EMs

Country	Periods	Obs.	Country	Periods	Obs.
40 EMs					
Argentina	1993-2007	15	Lithuania	1995-2008	14
Armenia	1994-2008	15	Mexico	1981-2008	28
Azerbaijan	1994-2008	15	Micronesia	1995-2008	14
Bahrain	1992-2008	17	Moldova	1995-2008	14
Belarus	1995-2008	14	Namibia	1989-2008	20
Bolivia	1992-2007	16	Peru	1986-2006	21
Brazil	1992-2007	16	Philippines	1992-2007	16
Chile	1981-2007	27	Poland	1995-2008	14
China	1992-2008	17	Portugal	1995-2008	14
Colombia	1992-2007	16	Romania	1995-2008	14
Costa Rica	1982-2007	26	Russia	1995-2008	14
Czech	1992-2008	17	Slovakia	1995-2008	14
Egypt	1996-2006	11	Slovenia	1995-2008	14
Estonia	1995-2008	14	South Africa	1981-2008	28
Hungary	1995-2008	14	South Korea	1981-2008	28
India	1981-2002	22	Taiwan	1981-2008	28
Israel	1995-2007	14	Thailand	1981-2008	28
Kazakhstan	1994-2008	15	Tunisia	1992-2008	17
Kyrgyz Rep	1994-2008	15	Turkey	1981-2006	26
Latvia	1994-2008	15	Ukraine	1993-2008	16
Mean		18			
18 EMs					
Argentina	1993-2007	15	Israel	1995-2007	14
Brazil	1992-2007	16	Mexico	1981-2008	28
Chile	1981-2007	27	Peru	1986-2006	21
Colombia	1992-2007	16	Philippines	1992-2007	16
Costa Rica	1982-2007	26	Poland	1991-2008	18
Czech	1992-2008	17	Russia	1995-2008	14
Egypt	1996-2006	11	South Africa	1981-2008	28
Hungary	1995-2008	14	South Korea	1981-2008	28
India	1981-2002	22	Turkey	1987-2006	20
Mean		19			

Appendix C

Appendix for Chapter 3

C.1 Stationary Competitive Equilibrium

The Lagrangian of a household is given by

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left(\nu_t \frac{\left(A(C_t^T, C_t^N) - s_t X_{t-1} \left(\frac{h_t^{T\omega_T}}{\omega_T} + \frac{h_t^{N\omega_N}}{\omega_N} \right) \right)^{1-\gamma} - 1}{1-\gamma} \right.$$

$$+\lambda_t X_{t-1}^{-\gamma} \left(W_t^T h_t^T + W_t^N h_t^N + r_t^T K_t^T + r_t^N K_t^N + \Pi_t^T + \Pi_t^N + \frac{D_{t+1}}{1+r_t^*} - C_t^T - p_t C_t^N \right.$$

$$-\xi_t^T \left(K_{t+1}^T - (1-\delta) K_t^T \right) - \xi_t^N \left(K_{t+1}^N - (1-\delta) K_t^N \right)$$

$$\left. \left. -\frac{\phi^T}{2} \left(\frac{K_{t+1}^T}{K_t^T} - g \right)^2 K_t^T - \frac{\phi^N}{2} \left(\frac{K_{t+1}^N}{K_t^N} - g \right)^2 K_t^N - D_t \right) \right).$$

And stationary competitive equilibrium of the economy is the sequence of $\{c_t, c_t^T, c_t^N, i_t^T, i_t^N,$ $h_t^T, h_t^N, \lambda_t, k_t^T, k_t^N, r_t^T, r_t^N, p_t, y_t^T, y_t^N, h_t, i_t, d_t\}_{t=0}^{\infty}$ which satisfy

$$\nu_t \left(c_t - s_t \left(\frac{h_t^{T\omega_T}}{\omega_T} + \frac{h_t^{N\omega_N}}{\omega_N} \right) \right)^{-\gamma} A_1(c_t^T, c_t^N) = \lambda_t, \tag{C.1}$$

$$\frac{1-\chi}{\chi} \left(\frac{c_t^T}{c_t^N} \right)^{1/\xi} = p_t, \tag{C.2}$$

$$\frac{h_t^{T\omega_T-1}}{A_1(c_t^T,c_t^N)} = s_t^{-1}g(1-\alpha_T)a_t^T\left(\alpha_T\left(\frac{k_t^T}{gh_t^T}\right)^{\frac{\sigma_T-1}{\sigma_T}}+(1-\alpha_T)\right)^{\frac{1}{\sigma_T-1}}, \tag{C.3}$$

$$\frac{h_t^{N\omega_N-1}}{A_1(c_t^T,c_t^N)} = s_t^{-1}p_tg(1-\alpha_N)a_t^N\left(\alpha_N\left(\frac{k_t^N}{gh_t^N}\right)^{\frac{\sigma_N-1}{\sigma_N}}+(1-\alpha_N)\right)^{\frac{1}{\sigma_N-1}}, \tag{C.4}$$

$$r_t^T = \alpha_Ta_t^T\left(\alpha_T+(1-\alpha_T)\left(\frac{gh_t^T}{k_t^T}\right)^{\frac{\sigma_T-1}{\sigma_T}}\right)^{\frac{1}{\sigma_T-1}}, \tag{C.5}$$

$$r_t^N = p_t\alpha_Na_t^N\left(\alpha_N+(1-\alpha_N)\left(\frac{gh_t^N}{k_t^N}\right)^{\frac{\sigma_N-1}{\sigma_N}}\right)^{\frac{1}{\sigma_N-1}}, \tag{C.6}$$

$$\lambda_t=(1+r_t^*)\frac{\beta}{g^\gamma}\mathbb{E}_t\lambda_{t+1}, \tag{C.7}$$

$$\left(\xi_t^T+\phi^T\left(\frac{k_{t+1}^T}{k_t^T}g-g\right)\right)\lambda_t =$$

$$\frac{\beta}{g^\gamma}\mathbb{E}_t\lambda_{t+1}\left(r_{t+1}^T+\xi_{t+1}^T(1-\delta)+\phi^T\left(\frac{k_{t+2}^T}{k_{t+1}^T}g-g\right)\frac{k_{t+2}^T}{k_{t+1}^T}g-\frac{\phi^T}{2}\left(\frac{k_{t+2}^T}{k_{t+1}^T}g-g\right)^2\right), \tag{C.8}$$

$$\left(\xi_t^N+\phi^N\left(\frac{k_{t+1}^N}{k_t^N}g-g\right)\right)\lambda_t =$$

$$\frac{\beta}{g^\gamma}\mathbb{E}_t\lambda_{t+1}\left(r_{t+1}^N+\xi_{t+1}^N(1-\delta)+\phi^N\left(\frac{k_{t+2}^N}{k_{t+1}^N}g-g\right)\frac{k_{t+2}^N}{k_{t+1}^N}g-\frac{\phi^N}{2}\left(\frac{k_{t+2}^N}{k_{t+1}^N}g-g\right)^2\right), \tag{C.9}$$

$$r_t^*=\bar{r^*}+\psi\left(e^{\frac{d_{t+1}-\bar{d}}{\bar{y}}}-1\right)+e^{\mu_t-1}-1, \tag{C.10}$$

$$y_t^T=a_t^T\left(\alpha_Tk_t^{T\frac{\sigma_T-1}{\sigma_T}}+(1-\alpha_T)\left(gh_t^T\right)^{\frac{\sigma_T-1}{\sigma_T}}\right)^{\frac{\sigma_T}{\sigma_T-1}}, \tag{C.11}$$

$$y_t^N=a_t^N\left(\alpha_Nk_t^{N\frac{\sigma_N-1}{\sigma_N}}+(1-\alpha_N)\left(gh_t^N\right)^{\frac{\sigma_N-1}{\sigma_N}}\right)^{\frac{\sigma_N}{\sigma_N-1}}, \tag{C.12}$$

$$y_t=y_t^T+p_ty_t^N, \tag{C.13}$$

$$c_t = c_t^T + p_t c_t^N, \tag{C.14}$$

$$c_t^N = y_t^N, \tag{C.15}$$

$$h_t = h_t^T + h_t^N, \tag{C.16}$$

$$i_t^T = \xi_t^T \left(g k_{t+1}^T - (1-\delta) k_t^T\right), \tag{C.17}$$

$$i_t^N = \xi_t^N \left(g k_{t+1}^N - (1-\delta) k_t^N\right), \tag{C.18}$$

$$i_t = i_t^T + i_t^N, \tag{C.19}$$

$$c_t^T + i_t + \frac{\phi^T}{2}\left(\frac{k_{t+1}^T}{k_t^T} g - g\right)^2 k_t^T + \frac{\phi^N}{2}\left(\frac{k_{t+1}^N}{k_t^N} g - g\right)^2 k_t^N + d_t = y_t^T + g\frac{d_{t+1}}{1+r_t^*}, \tag{C.20}$$

subject to initial conditions of endogenous state variables $\{k_{-1}^T, k_{-1}^N, d_{-1}\}$ and sequence of exogenous shocks $\{a_t^T, a_t^N, \xi_t^T, \xi_t^N, r_t^*, \nu_t, s_t\}_{t=0}^{\infty}$.

C.2 Proof

C.2.1 Lemma 3.3.1

By the optimization condition of the firm, the rental rate of physical capital in the tradable sector is equal to the marginal productivity of capital in the tradable sector, which is

$$r_t^T = MPK_t^T = \alpha_T a_t^T \left(\alpha_T + (1-\alpha_T)\left(\frac{X_t h_t^T}{K_t^T}\right)^{\frac{\sigma_T - 1}{\sigma_T}}\right)^{\frac{\sigma_T}{\sigma_T - 1} - 1},$$

and the tradable production in unit of physical capital in the tradable sector is

$$\frac{Y_t^T}{K_t^T} = a_t^T \left(\alpha_T + (1-\alpha_T)\left(\frac{X_t h_t^T}{K_t^T}\right)^{\frac{\sigma_T - 1}{\sigma_T}}\right)^{\frac{\sigma_T}{\sigma_T - 1}}.$$

And the inverse of physical capital per capita in unit of effective labor is

$$\frac{X_t h_t}{K_t^T} = \frac{h_t}{k_t^T}\frac{X_t}{X_{t-1}} = \frac{h_t}{k_t^T} \cdot g,$$

and then the capital share in the tradable sector becomes

$$s_t^{k,T} \equiv \frac{r_t^K K_t^T}{Y_t^T} = \frac{\alpha_T}{\alpha_T + (1-\alpha_T)\left(\frac{h_t^T}{k_t^T} \cdot g\right)^{\frac{\sigma_T - 1}{\sigma_T}}}$$

and then labor share in the tradable sector becomes

$$s_t^{h,T} \equiv 1 - s_t^{k,T} = \frac{1-\alpha_T}{(1-\alpha_T) + \alpha_T \left(\frac{k_t^T}{h_t^T}\frac{1}{g}\right)^{\frac{\sigma_T - 1}{\sigma_T}}}.$$

Similarly, the rental rate of physical capital in the non-tradable sector is equal to its marginal productivity,

$$r_t^T = MPL_t^T = \alpha_N p_t a_t^N \left(\alpha_N + (1-\alpha_N)\left(\frac{X_t h_t^N}{K_t^N}\right)^{\frac{\sigma_N - 1}{\sigma_N}}\right)^{\frac{\sigma_N}{\sigma_T - 1} - 1}$$

and the inverse of non-tradable production in unit of physical capital is

$$\frac{Y_t^N}{K_t^N} = a_t^N \left(\alpha_N + (1-\alpha_N)\left(\frac{X_t h_t^N}{K_t^N}\right)^{\frac{\sigma_N - 1}{\sigma_N}}\right)^{\frac{\sigma_N}{\sigma_N - 1}}.$$

And then the capital share and the labor share in the non-tradable sector become

$$s_t^{k,N} \equiv \frac{r_t^N K_t^N}{p_t Y_t^N} = \frac{\alpha_N}{\alpha_N + (1-\alpha_N)\left(\frac{h_t^N}{k_t^N} \cdot g\right)^{\frac{\sigma_N - 1}{\sigma_N}}},$$

$$s_t^{h,N} \equiv 1 - s_t^{k,N} = \frac{1-\alpha_N}{(1-\alpha_N) + \alpha_N \left(\frac{k_t^N}{h_t^N}\frac{1}{g}\right)^{\frac{\sigma_N - 1}{\sigma_N}}}.$$

□

C.2.2 Lemma 3.3.2

Becuase of the definitions of sectoral labor shares and $\gamma_t^T \equiv \frac{Y_t^T}{Y_t} = \frac{Y_t^T}{Y_t^T + p_t Y_t^N}$, the proof is trivial. □

C.2.3 Lemma 3.3.3

Since $\hat{s}_t^{h,j} = M^j \hat{\theta}_t^j$ and M^j is a constant, it is straightfoward to derive equations (3.45)-(3.46) by using properties of the correlation coefficient and the variance. □

C.2.4 Proposition 3.3.4

By the functional form of log-linearized aggregate share $\hat{s}_t^h$ in equation (3.48), it is straightforward to derive equations (3.49)-(3.50) by using properties of the correlation coefficient and the variance. □

C.3 Estimation Procedure

C.3.1 Transforming Model Variables with Constant Price Index

In the model, all variables are represented by the tradable good. The original form of output and consumption in terms of the final good are

$$P_t^c y_t = P_t^T y_t^T + P_t^N y_t^N, \tag{C.21}$$

$$P_t^c c_t = P_t^T c_t^T + P_t^N c_t^N, \tag{C.22}$$

where P_t^c is the consumer price index (CPI). If we divide both sides with P_t^T, the two equations are represented in terms of tradables as

$$p_t^c y_t = y_t^T + p_t y_t^N, \tag{C.23}$$

$$p_t^c c_t = c_t^T + p_t c_t^N, \tag{C.24}$$

where p_t is the relative price of nontradables in terms of tradables which is already denoted, and p_t^c is a relative CPI in terms of tradables. Given the market clearing condition of nontradables $c_t^N = y_t^N$, the CPI is derived by solving the maximization problem of a CES packer of the final good

$$\max_{c_t^T} p_t^c A\left(c_t^T, y_t^N\right) - c_t^T - p_t y_t^N, \tag{C.25}$$

which yields

$$p_t^c = \frac{1}{A_1\left(c_t^T, y_t^N\right)},$$

that is,

$$p_t^c = \frac{1}{\chi\left(\chi + (1-\chi)(y_t^N/c_t^T)^{\frac{\xi-1}{\xi}}\right)^{\frac{1}{\xi-1}}}.$$

In the WDI data, all variables are measured in constant prices, namely by dividing variables by the GDP deflator. The GDP deflator is a Paasche index, which is a ratio between current and constant price GDP. The formula for aggregate ouput measured in constant price is

$$P_0^c y_t = P_0^T y_t^T + P_0^N y_t^N, \tag{C.26}$$

and in terms of tradables (by dividing both sides with P_0^T) is

$$p_0^c y_t \quad = \quad y_t^T + p_0 y_t^N, \tag{C.27}$$

The GDP deflator $p_{0,t}$ is defined by

$$p_{0,t} \equiv \frac{p_t^c y_t}{p_0^c y_t} = \frac{y_t^T + p_t y_t^N}{y_t^T + p_0 y_t^N}. \tag{C.28}$$

Then the model variables for output and consumption are transformed with respect to a constant price index by deflating all variables by the GDP deflator to be consistent with the WDI variables,

$$y_t^o \quad = \quad \frac{y_t^T + p_t y_t^N}{p_{0,t}}, \tag{C.29}$$

$$c_t^o \quad = \quad \frac{c_t^T + p_t c_t^N}{p_{0,t}}, \tag{C.30}$$

and investment is same (since it is measured one price)

$$i_t^o = i_t \tag{C.31}$$

The growth rates of aggregate output, consumption, and investment consistent to data become

$$g_t^Y = \frac{y_t^o}{y_{t-1}^o} \cdot g, \tag{C.32}$$

$$g_t^C = \frac{c_t^o}{c_{t-1}^o} \cdot g, \tag{C.33}$$

$$g_t^I = \frac{i_t^o}{i_{t-1}^o} \cdot g. \tag{C.34}$$

C.3.2 Bayesian Estimation using Markov Chain Monte Carlo with Random-Walk Metropolis-Hastings Sampler

Let's denote $\mathbf{Z}_T = \{Z_t\}_{t=1}^T$ as a vector of observables given by sample. Let $p(\boldsymbol{\theta})$ be the prior distribution of parameters, $L(\mathbf{Z}_T|\boldsymbol{\theta})$ the likelihood function of observables to be evaluated via Kalman filter, and $p(\boldsymbol{\theta}|\mathbf{Z}_T)$ the posterior distribution of parameters. Bayes' theorem gives

$$p(\boldsymbol{\theta}|\mathbf{Z}_T) = \frac{p(\boldsymbol{\theta})L(\mathbf{Z}_T|\boldsymbol{\theta})}{K},$$

where $K = \int_{\boldsymbol{\Theta}} p(\boldsymbol{\theta})L(\mathbf{Z}_T|\boldsymbol{\theta})\ d\boldsymbol{\theta}$ is a scale factor. The scale factor may not be expressed in closed form and can be difficult to estimate numerically. We should apply posterior sampling to approach the accurate posterior distribution $p(\boldsymbol{\theta}|\mathbf{Z}_T)$, when available information is only $p(\boldsymbol{\theta})L(\mathbf{Z}_T|\boldsymbol{\theta})$. For posterior simulation, I use the iterative Markov Chain Monte Carlo algorithm.

The idea of Markov Chain Monte Carlo (hereafter MCMC) is to generate a serially correlated sequence of draws of parameters $\left\{\boldsymbol{\theta^i}\right\}$ such that the density of $\left\{\boldsymbol{\theta^i}\right\}$ converges to the posterior density $p(\boldsymbol{\theta}|\boldsymbol{Z}_T)$. To construct the Markov chain, I apply the random walk Metropolis-Hastings (hereafter RW-MH) algorithm as a posterior sampler. Given $i-1^{th}$ draw $\boldsymbol{\theta}^{i-1}$, the RW-MH proposes draw $\boldsymbol{\theta}^*$ for i^{th} by following the random walk process

$$\boldsymbol{\theta}^* = \boldsymbol{\theta}^{i-1} + c \cdot \epsilon \tag{C.35}$$

where $\epsilon \sim N(0, \boldsymbol{\Sigma})$ and $c > 0$ is a constant. After drawing, the RW-MH calculates the ratio

$$r_p(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{i-1}) = \frac{p(\boldsymbol{\theta}^*)L(\mathbf{Z}_T|\boldsymbol{\theta}^*)/q(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{i-1})}{p(\boldsymbol{\theta}^{i-1})L(\mathbf{Z}_T|\boldsymbol{\theta}^{i-1})/q(\boldsymbol{\theta}^{i-1}|\boldsymbol{\theta}^*)}, \tag{C.36}$$

where $q(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{i-1})$ is a proposed density which follows the multivariate normal distribution $N(\boldsymbol{\theta}^{i-1}, c^2\boldsymbol{\Sigma})$. Finally, the RW-MH draws a random number $u \sim U(0,1)$. It accepts the proposed draw $\boldsymbol{\theta}^*$ and sets $\boldsymbol{\theta}^i = \boldsymbol{\theta}^*$ if and only if

$$u < r_p(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{i-1}). \tag{C.37}$$

It rejects the proposed draw and set $\boldsymbol{\theta}^i = \boldsymbol{\theta}^{i-1}$ otherwise.

To match the targeted acceptance rate of the posterior simulation, I implement two-stage RW-MH procedure. In the first stage, I set starting value of covariance matrix $\boldsymbol{\Sigma}_0$ to be identity matrix, set starting value of scaling factor $c_0 = 0.01$, and set initialize starting values of parameters $\boldsymbol{\theta_0}$ to be prior means, except for σ_T and σ_N. I set the starting values of σ_T and σ_N to be 1 both, which represent the Cobb-Douglas production functions. The covariance matrix $\boldsymbol{\Sigma}$ and its scaling factor c of proposal distribution are set to match the targeted acceptance rate in the first stage iteration. Using estimated $\hat{\boldsymbol{\Sigma}}$ and $\hat{c}$ from the first stage, the second stage iteration generates draws of parameters with starting values of $\tilde{\boldsymbol{\theta}}$, which is a posterior mean of parameters from the first stage iteration. I set the targeted acceptance rate as 25 percent.

C.4 Data

- **Canada:**

  (1) Real GDP: World Development Indicators (WDI). Indicator code: *NY.GDP.PCAP.KN*

  (2) Real consumption expenditure: WDI. Indicator code: *NE.CON.PETC.ZS*

  (3) Real investment: WDI. Indicator code: *NE.GDI.TOTL.ZS*

  (4) Trade balance to output ratio: Own calculation, based on WDI statistics for imports and exports of goods and services. Indicator codes: *NE.IMP.GNFS.ZS, NE.EXP.GNFS.ZS*

  (5) Aggregate labor share: OECD database. Indicator name: *Labor Share Ratios*

  (6) Share of tradable sector: Own calculation, based on UNCTAD statistics for value added in agriculture, hunting, forestry, fishing, industry, and GDP.

- **Mexico:**

  (1) Real GDP: World Development Indicators (WDI). Indicator code: *NY.GDP.PCAP.KN*

  (2) Real consumption expenditure: WDI. Indicator code: *NE.CON.PETC.ZS*

  (3) Real investment: WDI. Indicator code: *NE.GDI.TOTL.ZS*

  (4) Trade balance to output ratio: Own calculation, based on WDI statistics for imports and exports of goods and services. Indicator codes: *NE.IMP.GNFS.ZS, NE.EXP.GNFS.ZS*

  (5) Aggregate labor share: OECD database. Indicator name: *Labor Share Ratios*

  (6) Share of tradable sector: Own calculation, based on UNCTAD statistics for value added in agriculture, hunting, forestry, fishing, industry, and GDP.

C.5 Model 2: Two sectors with Cobb-Douglas production functions

Table C.1: Marginal Prior and Posterior Distributions for Structural Parameters, Model 2

Model 2		Canada		Mexico	
Parameter	Prior	Posterior			
		Median	[5%, 95 %]	Median	[5%, 95 %]
Endogenous Propagation Parameters					
ϕ^T	Uniform $[0, 8]$	2.57	[0.57, 4.64]	4.56	[2.15, 7.43]
ϕ^N	Uniform $[0, 8]$	6.64	[4.57, 7.82]	2.77	[1.44, 6.61]
ψ	Uniform $[0, 10]$	5.80	[2.41, 9.54]	0.70	[0.17, 3.06]
Exogenous Shock Parameters					
$\rho_{a,T}$	Uniform $[-0.99, 0.99]$	0.73	[0.59, 0.81]	0.89	[0.79, 0.93]
$\sigma_{a,T}$	Uniform $[0, 0.2]$	0.03	[0.02, 0.04]	0.02	[0.01, 0.03]
$\rho_{a,N}$	Uniform $[-0.99, 0.99]$	0.38	[-0.67, 0.93]	0.98	[0.97, 0.99]
$\sigma_{a,N}$	Uniform $[0, 0.2]$	0.002	[0.0002, 0.007]	0.03	[0.02, 0.04]
$\rho_{\xi,T}$	Uniform $[-0.99, 0.99]$	0.55	[0.27, 0.73]	0.57	[-0.74, 0.97]
$\sigma_{\xi,T}$	Uniform $[0, 0.2]$	0.14	[0.08, 0.18]	0.02	[0.002, 0.06]
$\rho_{\xi,N}$	Uniform $[-0.99, 0.99]$	0.18	[-0.74, 0.83]	0.11	[-0.77, 0.95]
$\sigma_{\xi,N}$	Uniform $[0, 0.2]$	0.06	[0.008, 0.16]	0.01	[0.001, 0.05]
ρ_s	Uniform $[-0.99, 0.99]$	0.29	[-0.70, 0.91]	0.33	[-0.65, 0.92]
σ_s	Uniform $[0, 0.2]$	0.003	[0.0004, 0.01]	0.004	[0.0005, 0.01]
ρ_ν	Uniform $[-0.99, 0.99]$	0.93	[0.87, 0.96]	0.87	[-0.11, 0.93]
σ_ν	Uniform $[0, 0.2]$	0.14	[0.09, 0.19]	0.14	[0.007, 0.19]
ρ_μ	Uniform $[-0.99, 0.99]$	0.69	[-0.47, 0.95]	0.88	[0.76, 0.96]
σ_μ	Uniform $[0, 0.2]$	0.03	[0.009, 0.06]	0.04	[0.01, 0.13]

Note. All prior distributions are uniform. Posterior distributions are based on draws from the last 1 million draws from 10 million MCMC chain.

Table C.2: Second Moments: Data and Model 2

	Canada		Mexico	
	Data	Model 2	Data	Model 2
Volatilities				
$\sigma\left(g^Y\right)$	2.00	2.53	3.52	3.55
$\sigma\left(s^h\right)/\sigma\left(g^Y\right)$	1.86	0.36	4.50	0.13
$\sigma\left(g^C\right)/\sigma\left(g^Y\right)$	0.90	0.87	1.22	1.38
$\sigma\left(g^I\right)/\sigma\left(g^Y\right)$	4.07	2.81	3.06	3.15
$\sigma\left(TB/Y\right)$	1.85	1.81	3.21	2.83
$\sigma\left(\gamma^T\right)/\sigma\left(g^Y\right)$	4.15	2.29	2.42	1.80
Correlations with Output Growth				
$\rho\left(s^h, g^Y\right)$	-0.11	-0.46	0.33	-0.10
$\rho\left(g^C, g^Y\right)$	0.71	0.81	0.79	0.81
$\rho\left(g^I, g^Y\right)$	0.81	0.81	0.78	0.57
$\rho\left(TB/Y, g^Y\right)$	0.13	0.15	-0.46	-0.24
$\rho\left(\gamma^T, g^Y\right)$	0.27	0.46	-0.07	0.10
Autocorrelations				
$\rho\left(s_t^h, s_{t-1}^h\right)$	0.91	0.89	0.95	0.82
$\rho\left(g_t^Y, g_{t-1}^Y\right)$	0.32	0.12	0.25	0.14
$\rho\left(g_t^C, g_{t-1}^C\right)$	0.27	0.19	0.17	-0.01
$\rho\left(g_t^I, g_{t-1}^I\right)$	-0.02	-0.01	-0.01	-0.07
$\rho\left(TB_t/Y_t, TB_{t-1}/Y_{t-1}\right)$	0.80	0.74	0.75	0.55
$\rho\left(\gamma_t^T, \gamma_{t-1}^T\right)$	0.93	0.89	0.91	0.82

Note. Standard deviations are measured in percentage points. The prediction is based on the posterior median from the last 1 million draws from 10 million MCMC chain.

Table C.3: Decomposition of Variances, Baseline Model

	s^h	g^Y	g^C	g^I	TB/Y	γ^T
Canada						
Neutral technologies						
tradable sector	0.50	3.80	0.68	12.8	0.38	0.17
nontradable sector	0.00	0.00	0.00	0.00	0.00	0.00
Marginal Efficiencies of Investment						
tradable sector	0.00	0.00	0.01	1.29	0.08	0.00
nontradable sector	75.5	76.4	85.5	44.4	83.8	92.3
Country premium	0.00	0.01	0.01	0.00	0.00	0.00
Preference	2.25	2.70	1.93	40.8	15.5	1.20
Labor supply	21.7	17.0	11.8	0.20	0.10	6.21
Measurement error	0.00	0.00	0.00	0.33	0.00	0.00
Mexico						
Neutral technologies						
tradable sector	0.01	0.03	0.003	0.02	0.002	0.02
nontradable sector	0.05	0.04	0.02	0.006	0.001	0.02
Marginal Efficiencies of Investment						
tradable sector	0.004	0.02	0.03	0.05	0.00	0.02
nontradable sector	99.3	70.7	56.7	87.6	72.2	88.3
Country premium	0.51	28.8	42.8	12.1	27.7	11.4
Preference	0.01	0.30	0.39	0.09	0.03	0.15
Labor supply	0.00	0.00	0.00	0.00	0.00	0.00
Measurement error	0.01	0.00	0.00	0.00	0.00	0.002

Note. All measures are percentages. The prediction is based on the posterior median from the last 1 million draws from 10 million MCMC chain.

Table C.4: Decomposition of Variances, Model 2

Model 2	s^h	g^Y	g^C	g^I	TB/Y	γ^T
Canada						
Neutral technologies						
tradable sector	1.90	16.3	3.17	15.1	0.77	1.91
nontradable sector	0.00	0.00	0.00	0.00	0.00	0.00
Marginal Efficiencies of Investment						
tradable sector	26.5	44.2	23.2	76.9	92.2	26.6
nontradable sector	0.07	1.10	1.12	0.03	0.01	0.07
Country premium	0.07	0.20	0.68	4.50	1.14	0.07
Preference	70.7	38.0	71.7	2.79	5.78	70.9
Labor supply	0.00	0.00	0.00	0.00	0.00	0.00
Measurement error	0.57	0.00	0.00	0.57	0.00	0.34
Mexico						
Neutral technologies						
tradable sector	0.17	11.3	1.05	14.0	0.57	3.51
nontradable sector	0.02	10.7	5.31	1.36	0.16	0.44
Marginal Efficiencies of Investment						
tradable sector	0.02	0.89	1.73	0.10	0.92	0.52
nontradable sector	0.00	0.03	0.05	0.03	0.00	0.00
Country premium	0.46	10.6	17.9	56.2	87.1	9.29
Preference	4.31	66.3	73.8	28.1	11.1	85.4
Labor supply	0.00	0.01	0.00	0.00	0.00	0.00
Measurement error	94.9	0.01	0.05	0.01	0.00	0.71

Note. All measures are percentages. The prediction is based on the posterior median from the last 1 million draws from 10 million MCMC chain.

Table C.5: The Elasticity of Capital-Labor Substitution of the Economy, Alternative Models

$\hat{\sigma}$	Canada	Mexico
Model 2	1.03	1.01

Note. The first and second columns in row 'Model 2' show estimates based on regressions with simulated data from estimated models. The estimates from models are all statistically significant at 1 percent significance level. On the other hand, the two columns in row 'Model 3' show median of marginal posterior of σ. Posterior distribution is based on draws from the last 1 million draws from 10 million MCMC chain.