EVENTS, TOPOLOGY AND TEMPORAL RELATIONS*

We are used to regarding actions and other events, such as Brutus' stabbing of Caesar or the sinking of the Titanic, as occupying intervals of some underlying linearly ordered temporal dimension. This attitude is so natural and compelling that one is tempted to disregard the obvious difference between time periods and actual happenings in favor of the former: events become mere "intervals cum description".[1] On the other hand, in ordinary circumstances the point of talking about time is to talk about what actually happens or might happen at some time or another. We talk about 'now' and 'then' in an effort to put some order in our description of what goes on. And since different events seem to overlap in so many different ways, a full account of their temporal relations seems to run afoul of a reductionist strategy.

This raises two philosophical questions. The first is whether we can actually *go beyond* time, as it were, i.e., whether we can take events as *bona fide* entities and deal with them directly, just as we can deal with spatial entities such as physical bodies or masses without confining ourselves to their spatial representations. This is a controversial issue (though probably not as controversial as it used to be), and ties in with a number of unsettled problems concerning, e.g., the structure of causality or the definition of adequate identity and individuation criteria for events.[2] The second question is whether we can perhaps *do without* time, i.e., whether we can dispense with time points or intervals as an independent ontological category and focus only on actual or potential happenings, in opposition to the form of reductionism mentioned above—in short, whether we can account for the temporal dimension in terms of suitable relations among events. This is also a highly controversial issue, and relates to the classical dispute concerning relational vs. absolutist conceptions of (space and) time.[3]

It is this second question that we intend to focus on here. Even if the acceptance of events as part of our ontological inventory is in itself a matter of dispute, we shall assume there to be enough good arguments to justify a positive attitude in this regard: events are part of the furniture of the world, whatever their exact ontological make up. We shall actually assume that events are individuals, as opposed to, for instance, states of affairs.[4] The focus of our concern is whether this assumption allows one to answer also the second question in the af-

"Events, Topology, and Temporal Relations" by Fabio Pianesi and Achille C. Varzi,
The Monist, vol. 79, no. 1, pp. 89–116.

firmative. More specifically, our purpose is to study the possibility of giving a positive answer in the following form: if events are assumed as *bona fide* individuals in our basic ontological inventory, then the basic temporal relations can be explained away without resorting to an additional set of temporal primitives (instants and/or periods).

One classical suggestion in this direction is of course the account of Russell [1914, 1936], where time instants are construed as maximal sets of pairwise simultaneous (or partially simultaneous) events. This treatment is echoed in various later accounts, from Whitehead [1929] and Walker [1947] to Kamp [1979] and van Benthem [1983], where events are taken as primary entities inducing periods as secondary (and instants as merely tertiary) entities. More recently, Thomason [1989] argued that the mathematical connection between the way events are perceived to be ordered and the underlying temporal dimension is essentially that of a free construction (in the category-theoretic sense) of linear orderings from event orderings, induced by the binary relation “wholly precedes”. Furthermore, Forbes [1993] explored a modal account whereby the time-series of a given world w is construed as a sequence of equivalence classes of entities existing at w, the classes being determined by facts about the possible worlds that branch from w.

In this paper we present an alternative account, based primarily on the basic network of formal ontological relations—specifically, mereological and topological relations—that a domain of events must arguably satisfy. The motivations for this approach are quite general and lie beyond the specific issue of temporal constructions. Among other things, we also believe it may shed light on the first question above. In the following, however, we shall not go much beyond the main issue that we just outlined; our only concern will be to show how mereological and topological reasoning—which we take to be among the basic tools for ontological analysis—provides adequate grounds for the construction of temporal relations.

Our argument is therefore conditional: *if* individual events are countenanced, then temporal relations can be construed out of them. Moreover, this should be taken quite generally, and should not carry any commitment to the view that events are basic entities rather than, say, secondary entities supervening on their participants. Our construction of temporal relations is grounded on the mereotopological properties of events; it does not, however, extend to an account of those very properties, which may therefore be taken as either primitive or derived.

1. Combining Mereology and Topology

We see mereology and topology as closely interconnected. Thus, on the one hand, we sympathize with the view that the theory of parts—as rooted in the work of Leśniewski [1916] and Leonard and Goodman [1940]—provides a resourceful alternative to set theory for the analysis of the objects and events of ordinary experience.[5] At the same time, we are also persuaded that a purely mereological outlook is too restrictive unless it is integrated with some concepts and principles of a topological nature.[6] There are several reasons for this. For instance, mereologically there is no way to distinguish between a one-piece, self-connected whole, such as a stone or a whistle, and a scattered entity made up of several disconnected parts, such as a broken glass, a soccer tournament, or Brutus' repeated stabbing of Caesar. Moreover, mereology alone cannot account for some very basic spatio-temporal relations among the entities of ordinary discourse, such as the relationship between a material object and its surface, or the relation of continuity between two successive actions or events, or the relation of something being inside, abutting, or surrounding something else. All of these are phenomena that run afoul of plain parthood relations, and their systematic account requires a topological machinery of some sort.

There are to be sure various ways in which the two domains of mereology and topology can be combined together.[7] One can see them as two independent provinces (following in the footsteps of *inter alia* Tiles [1981] and Lejewski [1982]); or one may grant priority to topology and characterize mereology derivatively, for instance defining parthood in terms of topological connection (as in Clarke [1981]). The latter approach is nowadays rather popular, particularly in the artificial intelligence community.[8] Indeed it proves fit to account for a fair deal of mereotopological reasoning if we confine ourselves to an ontology of temporal intervals and/or spatial regions. If, however, we are to take an open-faced attitude towards full-fledged entities and actual happenings (without identifying them with their respective spatio-temporal co-ordinates), then the reduction of mereology to a distinguished chapter of topology seems untenable, as different entities can be located at the very same spatio-temporal regions. An object can be wholly located inside a hole, hence totally connected with it, without bearing any mereological relation to the hole itself. Or two events may have exactly the same topological connections and yet be mereologically distinct, as with the rotating and the becoming warm of a metal ball that is simultaneously rotating and becoming warm.[9]

In general, therefore, we are inclined to favor the first option mentioned above, treating mereology and topology as two conceptually independent domains. Formally this will be reflected in our use of two distinct primitives—a pure mereological notion of *part*, and a purely topological notion of *boundary*. Of course, several other sets of primitives could serve the job, but our choice here is not entirely arbitrary, at least for the topological part. For instance, various examples of our favored strategy for combining mereology and topology can also be found in some linguistics-oriented work on tense and aspect, such as Kamp's [1979], Bach's [1986], or Link's [1987] algebraic semantics for event structures, where a mereological relation defined on a field of events is typically matched with an independent ordering of temporal precedence. For our present purposes, however, such a course would have a flavor of circularity. If the point is to provide an account of the temporal dimension based on the fundamental mereotopological features of event structures, it is essential that we take these features to be of a most general, time-independent nature. So precedence won't do. By contrast, reference to parts and boundaries appears to be a natural and independently motivated option, both ontologically and from a cognitive standpoint.[10]

1.1 Background Assumptions. The entire mereotopological machinery can be developed within a first-order language with identity and descriptions. We shall use '$\neg$', ' ', ' ', '$\rightarrow$', and '$\leftrightarrow$' as connectives for negation, conjunction, disjunction, implication, and material equivalence; ' ' and ' ' for the universal and existential quantifiers; and 'ι' for the definite descriptor. This last operator will not have any use *per se*. However, it will play a crucial role in the definition of the term-forming operator of general sum by means of which several basic mereological and topological notions will be characterized. (Alternatively one could use set variables along with a set fusion functor, as in Tarski [1937] or Leonard and Goodman [1940], but this method would introduce additional complications and would mix up mereology with set theory.) Since this sum operator may be undefined for some arguments, the underlying logical apparatus requires therefore some means of accounting for the possibility of non-denoting expressions.

This can be done in a number of ways. Among the alternatives available (including Leśniewski's [1916] original approach) we find it convenient to rely on the minimal free description theory stemming from Lambert [1962], consisting in assuming

(1) $\quad y(y=\iota x\varphi[x] \leftrightarrow (\varphi[x/y] \quad x(\varphi[x] \rightarrow x=y))$

as the only characteristic principle for descriptions. This theory is "minimal" insofar as it only captures the logic of 'ι' with respect to descriptions that are denotationally successful, leaving open the issue of what principles should continue to hold in the presence of referential gaps. It does, however, allow us to treat descriptive terms as genuine singular terms, and this is all we need for our present purposes. For definiteness, we shall also fix the underlying free quantification theory: it is the system resulting from the classical predicate calculus by replacing the principle of Universal Instantiation with its universal closure:

(2) $\quad y(\quad x\varphi[x] \rightarrow \varphi[x/y])$.

This minimalistic strategy has already been exploited in the context of mereological theorizing by Simons [1991], and will prove sufficient also for our purposes in spite of its failing to reveal the whole truth. In any case, to facilitate comparisons we shall try to highlight those points where the choice of a different strategy would affect the argument.

1.2 Mereology. We symbolise the primitive mereological relation of parthood by ‘P’, so that ‘$\mathsf{P}(x, y)$’ reads “x is (a) part of y”. Derived notions, such as identity, overlapping, and the like, or the operations of sum, product, difference, etc. are defined as usual:

DP1	$x=y$	$=_{df}$	$\mathsf{P}(x, y)\quad \mathsf{P}(y, x)$	x is identical with y
DP2	$\mathsf{O}(x, y)$	$=_{df}$	$\quad z\ (\mathsf{P}(z, x)\quad \mathsf{P}(z, y))$	x overlaps y
DP3	$\mathsf{X}(x, y)$	$=_{df}$	$\mathsf{O}(x, y)\quad \neg\mathsf{P}(x, y)$	x crosses y
DP4	$\mathsf{PO}(x, y)$	$=_{df}$	$\mathsf{X}(x, y)\quad \mathsf{X}(y, x)$	x properly overlaps y
DP5	$\mathsf{PP}(x, y)$	$=_{df}$	$\mathsf{P}(x, y)\quad \neg\mathsf{P}(y, x)$	x is a proper part of y
DP6	$\sigma x\varphi[x]$	$=_{df}$	$\iota z\quad y\ (\mathsf{O}(y, z) \leftrightarrow\quad x\ (\varphi[x]\quad \mathsf{O}(x, y)))$	sum of φers
DP7	$\pi x\varphi[x]$	$=_{df}$	$\sigma z\quad x\ (\varphi[x] \rightarrow \mathsf{P}(z, x))$	product of φers
DP8	$x+y$	$=_{df}$	$\sigma z\ (\mathsf{P}(z, x)\quad \mathsf{P}(z, y))$	sum of x and y
DP9	$x\times y$	$=_{df}$	$\sigma z\ (\mathsf{P}(z, x)\quad \mathsf{P}(z, y))$	product of x and y
DP10	$x-y$	$=_{df}$	$\sigma z\ (\mathsf{P}(z, x)\quad \neg\mathsf{O}(z, y))$	difference of x and y
DP11	$\sim x$	$=_{df}$	$\sigma z\ (\neg\mathsf{O}(z, x))$	complement of x
DP12	U	$=_{df}$	$\sigma z\ (z=z)$	universe

Note that the functors/terms based on DP6 may be partially defined (i.e., correspond to improper descriptions) unless we go with the fiction of a null individual that is part of everything. For instance, non-overlapping entities will have no product and the universe will have no complement. This introduces a significant deviation in the obvious correspondence between mereological operations and the standard set-theoretic operations of union, intersection, difference, etc.

As mereological axioms we assume the following two, along with the standard axioms for identity:

AP1 $\quad \mathsf{P}(x, y) \leftrightarrow\quad z\ (\mathsf{O}(z, x) \rightarrow \mathsf{O}(z, y))$

AP2 $\quad x\varphi[x] \rightarrow\quad y\quad z\ (\mathsf{O}(z, y) \leftrightarrow\quad x\ (\varphi[x]\quad \mathsf{O}(x, z)))$.

AP1 secures that parthood is an extensional partial ordering while AP2 (the “fusion axiom”) guarantees that every satisfied condition φ picks out an entity consisting of all φers. This yields a classical mereology as usually understood, corresponding to a Boolean algebra with zero deleted. A sample selection of theorems relevant to the following is listed below:

TP1 $\quad \mathsf{P}(x, x)$

TP2 $\quad \mathsf{P}(x, y)\quad \mathsf{P}(y, z) \rightarrow \mathsf{P}(x, z)$

TP3 $\quad x=y \leftrightarrow\quad z\ (\mathsf{P}(z, x) \leftrightarrow \mathsf{P}(z, y))$

TP4 $x{=}y \leftrightarrow \quad z\ (\mathsf{P}(x, z) \leftrightarrow \mathsf{P}(y, z))$
TP5 $x{=}y \leftrightarrow \quad z\ (\mathsf{O}(x, z) \leftrightarrow \mathsf{O}(y, z))$
TP6 $\mathsf{PP}(x, y) \rightarrow \quad z(\mathsf{P}(z, y) \quad \neg\mathsf{O}(z, x))$
TP7 $\neg\mathsf{P}(x, y) \rightarrow \quad z(\mathsf{P}(z, x) \quad \neg\mathsf{O}(z, y))$
TP8 $\quad x\varphi[x] \rightarrow \quad y(y = \sigma x\varphi[x])$.

It may be worth recalling that none of these principles is uncontroversial. For instance, since Rescher [1955] several authors have had misgivings about such straightforward consequences of AP1 as TP2 (transitivity of 'P') or TP3–TP5 (extensionality).[11] In both cases, however, the objections involve reasoning about the variety of part-whole relations that may be distinguished (e.g., between components and complex, or quantity and mass) and may therefore be disregarded as long as we remain at a sufficiently general level of analysis. Even such an apparently innocent consequence of AP1 as TP1 (reflexivity of 'P') might on some conditions be objected to, particularly insofar as the logical background that we are assuming allows for non-denoting terms. For instance, in this regard Simons [1991] suggests applying the falsehood principle of Fine [1981] to deny that '$\mathsf{P}(x, x)$' may be true when x does not exist (i.e., when 'x' does not denote). However, such a stance would introduce a disturbing asymmetry between parthood and other basic predicates such as identity—of which parthood is a generalization—unless we are also ready to make a self-identity statement '$x{=}x$' depend on the existence of x. This is in contrast with our general attitude towards the logic of singular terms, which in fact follows the customary policy of assuming a standard identity theory. Therefore, this type of objection also will be disregarded in the following.

The second axiom is not uncontroversial either. For one thing, in the presence of AP1 it implies the so-called "supplementation" principles expressed by TP6-TP7, which some authors found reason to deny.[12] There are indeed cases where restrictions on the domain might involve violations of such principles (a disc with a disc removed is not a disc), but this is already a matter of regional ontology and need not concern us for the moment. As a matter of generality, we take it that the existence of a remainder between a whole and a proper part cannot be denied: otherwise it would be possible for an entity to have a single proper part, and that is beyond our understanding of this notion. Likewise, AP2 has been disputed for having counter-intuitive instances when φ is true of scattered or otherwise ill-assorted entities, such as the totality of red things, or Brutus' birth and his stabbing of Caesar.[13] From a purely mereological perspective, however, this criticism also seems off target. As Lewis [1991: 81] put it, if we already have certain things, allowing for their sum is no further commitment: the sum *is* those things. (That the sum is always *uniquely* defined is guaranteed by TP8.) In any case, one may feel uncomfortable with treating unheard-of mixtures as individual wholes; but which wholes are more "natural" than others is not a mereological issue. As noted above, the question of what constitutes an integral whole cannot even be formulated in mereological terms: it is precisely here that topology—the theory of boundaries—comes in.

1.3 Topology. The primitive topological notion of boundary is symbolized by 'B', so that '$\mathsf{B}(x, y)$' reads "x is a boundary for y". (We say "boundary *for*" (rather than *of*) to avoid a reductive interpretation of boundaries as *maximal* boundaries. In general, any boundary *for* something is a boundary *of* some part thereof.) Some useful derived notions can be introduced as follows:

DB1	$\mathsf{b}(x)$	$=_{df}$	$\sigma z\ (\mathsf{B}(z, x))$	maximal boundary of x
DB2	$\mathsf{c}(x)$	$=_{df}$	$x+\mathsf{b}(x)$	closure of x
DB3	$\mathsf{i}(x)$	$=_{df}$	$x-\mathsf{b}(x)$	interior of x
DB4	$\mathsf{e}(x)$	$=_{df}$	$\sim x-\mathsf{b}(x)$	exterior of x
DB5	$\mathsf{Cl}(x)$	$=_{df}$	$x=\mathsf{c}(x)$	x is closed
DB6	$\mathsf{Op}(x)$	$=_{df}$	$x=\mathsf{i}(x)$	x is open
DB7	$\mathsf{BP}(x, y)$	$=_{df}$	$\mathsf{P}(x, y) \quad \mathsf{B}(x, y)$	x is a boundary part of y
DB8	$\mathsf{IP}(x, y)$	$=_{df}$	$\mathsf{P}(x, \mathsf{i}(y))$	x is an interior part of y
DB9	$\mathsf{C}(x, y)$	$=_{df}$	$\mathsf{O}(\mathsf{c}(x), y) \quad \mathsf{O}(\mathsf{c}(y), x)$	x is connected to y
DB10	$\mathsf{EC}(x, y)$	$=_{df}$	$\mathsf{C}(x, y) \quad \neg\mathsf{O}(x, y)$	x is externally connected to y
DB11	$\mathsf{ST}(x, y)$	$=_{df}$	$\quad z(\mathsf{IP}(x, z) \rightarrow \mathsf{X}(z, y))$	x straddles y
DB12	$\mathsf{Cn}(x)$	$=_{df}$	$\quad y \quad z\ (x=y+z \rightarrow \mathsf{C}(y, z))$	x is self-connected

Again, these functors, predicates, and relations may be undefined for some arguments, as they may involve improper (e.g., denotationless) definite descriptions.

Note that nothing in these definitions implies that boundaries are always part of the entities they bound. In fact, we accept the standard topological distinction between open and closed entities, allowing for entities with external boundaries. For example, in Pianesi and Varzi [1994a] we examine a characterization of the standard classification of event types (Vendler [1957]) by treating processes (such as *John's climbing*) as non-closed events, accomplishments (*John's climbing of the mountain*) as closed processes, and achievements (*John's reaching of the top*) as parts or the corresponding boundaries. And in Casati and Varzi [1994], holes are regarded as immaterial bodies spatially bounded from the outside: the boundary of a hole is the surface of its material host.

In this regard, our basic axiomatization is therefore in line with standard topology. More precisely, we assume the following axioms:

AB1 $\quad \mathsf{B}(x, y) \rightarrow \mathsf{B}(x, \sim y)$

AB2 $\quad \mathsf{B}(x, y) \quad \mathsf{B}(y, z) \rightarrow \mathsf{B}(x, z)$

AB3 $\quad \mathsf{P}(z, x) \quad \mathsf{P}(z, y) \rightarrow (\mathsf{P}(z, \mathsf{b}(x\times y)) \leftrightarrow \mathsf{P}(z, \mathsf{b}(x)+\mathsf{b}(y)))$, which are easily seen to be tantamount to (the mereologized version of) the familiar Kuratowski axioms for topological closure:

TB1 $\mathsf{P}(x, \mathsf{c}(x))$
TB2 $\mathsf{P}(\mathsf{c}(\mathsf{c}(x)), \mathsf{c}(x))$
TB3 $\mathsf{c}(x+y) = \mathsf{c}(x)+\mathsf{c}(y)$.

This gives us a straightforward reformulation of much of standard topology based on mereology instead of set theory, provided only that we take care in handling undefined operators. In particular, with AB1 we assume boundaries to be always symmetrical, in the sense that every boundary of an entity is also a boundary of the entity's complement—if that complement exists.[14] This ensures that boundaries are always connected to the things they bound. Here is a list of further theorems that can be proved from AB1–AB3 and that will be required in the following:

TB4 $\mathsf{C}(x, x)$
TB5 $\mathsf{C}(x, y) \rightarrow \mathsf{C}(y, x)$
TB6 $\mathsf{P}(x, y) \quad \mathsf{IP}(y, z) \rightarrow \mathsf{IP}(x, z)$
TB7 $\mathsf{IP}(x, y) \quad \mathsf{P}(y, z) \rightarrow \mathsf{IP}(x, z)$
TB8 $\mathsf{B}(x, y) \leftrightarrow \quad z(\mathsf{P}(z, x) \rightarrow \mathsf{B}(z, y))$
TB9 $\mathsf{B}(x, y) \leftrightarrow \quad z(\mathsf{P}(z, x) \rightarrow \mathsf{ST}(z, y))$
TB10 $\mathsf{IP}(x, y) \leftrightarrow \mathsf{P}(x, y) \quad \neg \quad z(\mathsf{P}(z, x) \quad \mathsf{BP}(z, y))$
TB11 $\mathsf{BP}(x, y) \leftrightarrow \mathsf{P}(x, y) \quad \neg \quad z(\mathsf{P}(z, x) \quad \mathsf{IP}(z, y))$
TB12 $\mathsf{P}(x, y) \rightarrow \quad z\,(\mathsf{C}(z, x) \rightarrow \mathsf{C}(z, y))$
TB13 $\quad x(\varphi[x] \rightarrow \mathsf{B}(x, y)) \rightarrow \mathsf{B}(\sigma x \varphi[x], y)$.

The last two of these theorems are especially noteworthy. TB12 highlights the main connection between mereological and topological notions. As already hinted at above, systems in the tradition of Clarke [1981] also assume the converse of this principle, with the effect of reducing mereology to a part of topology. By contrast, the possibility that topologically connected entities bear no mereological relationship to one another leaves room for a much richer taxonomy of basic mereotopological relations than usually recognized.[15] For instance, the relations of connection, overlapping, parthood, and interior parthood introduced above, and common to most known systems, can be integrated by the following:

DB13	$\mathsf{E}(x, y) =_{df} \quad z\,(\mathsf{C}(z, x) \rightarrow \mathsf{C}(z, y))$	*x* is enclosed in *y*
DB14	$\mathsf{S}(x, y) =_{df} \quad z\,(\mathsf{E}(z, x) \quad \mathsf{E}(z, y))$	*x* is superimposed on *y*
DB15	$\mathsf{A}(x, y) =_{df} \mathsf{C}(x, y) \quad \neg\mathsf{S}(x, y)$	*x* abuts *y*
DB16	$\mathsf{W}(x, y) =_{df} \mathsf{E}(x, \mathsf{i}(y))$	*x* is enclosed within *y*

Evidently, S is implied by O, E by P, and W by IP, but the converses do not generally hold. (The rotation of the metal ball and its simultaneous getting warm

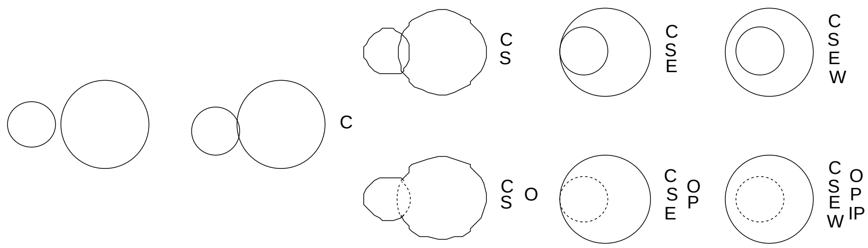

Figure 1. Some basic mereotopological relations exploiting the distinction between mereological parthood (overlapping, etc.) and mere topological enclosure (superposition, etc.).

are perfectly superimposed, but do not overlap; the stone is enclosed in the hole, not part of it.) The resulting taxonomy of relations is depicted in Figure 1.

As for TB13, it shows that boundaries are closed under general sum and therefore under all mereological properties. Following Smith [1993], however, we also wish to capture some further common-sense intuitions that go beyond the repertoire of standard topology. In particular, we need at least a rendering of the intuitive Aristotelian-Brentanian idea that boundaries are "parasitic" entities, i.e., cannot exist independently of the larger entities they bound.[16] This is in contrast with the standard set-theoretic conception of boundaries as sets of ordinary, ontologically independent points. More specifically, we assume that every self-connected boundary is a boundary part of some larger self-connected entity with non-empty interior:

AB4 $\quad \mathsf{Cn}(x) \quad y\ \mathsf{B}(x, y) \rightarrow \quad z\ (\mathsf{BP}(x, z) \quad \mathsf{Cn}(z) \quad w\ \mathsf{IP}(w, z)).$

Thus, to continue with our earlier examples, an achievement such as John's reaching of the top cannot occur except as a culmination of a process (John's climbing), i.e., except as boundary part of an accomplishment (John's climbing of the mountain).

It is understood that further principles would be needed in order to obtain at least a rough approximation of the folk theory of spatio-temporal continua. Here, however, we shall content ourselves with AP1–AP2 and AB1–AB4, regarding this as a minimal theory.

2. Event Structures

Let us now see how this general mereotopological framework can be specialized to a domain of events. As already pointed out, on a rather popular conception events are regarded as intrinsically dependent on the temporal dimension.

This conception ranges from the strong reductionist view that events are nothing but temporal intervals [17] to weaker forms that take events as primitive entities endowed with some primitive temporal relation [18] defined on them. Work in the tradition of Kim's [1969] theory of events as property-exemplifications (that is, exemplifications of specific properties by specific individuals at specific times) also treats time periods as primitive entities upon which events supervene.[19] By contrast, in the following we shall argue that events can be given an independent characterization that accords with our intuitions about their mutual relations and derives the temporal ordering by imposing suitable restrictions on the underlying mereotopological structure. More precisely, we will show that the formal connection between the way events are supposed to be ordered and the underlying temporal dimension is essentially that of a construction of a linear ordering from the mereotopological properties of an oriented domain structure in which events are included as *bona fide* individuals. In other words, temporal relations are a by-product of what we call an *event structure*.

2.1 Divisors. Our characterization of event structures relies on the auxiliary concept of a *divisor.* Intuitively, a divisor is an event that separates the entire domain of events into two disconnected parts, thus making it possible to choose one part as corresponding to the sum of all events that temporally precede the divisor, and the other as the sum of all events that follow it. This is illustrated in Figure 2. (Of course, the choice of precedence vs. following—past vs. future—will be arbitrary, as long as successive choices for different divisors be done in a consistent way. If we think of an event domain as comprising the totality of all happenings, there is no *a priori* way to fix the temporal orientation. But our concern here is the temporal ordering, not the direction of time.)

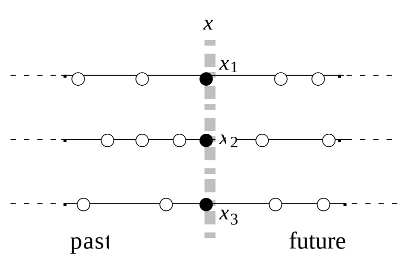

Figure 2. A divisor, x, separating the domain of events into two disconnected parts. (Here and below we use dots to represent events; divisors are represented by (groups of) solid dots and are highlighted by shaded dashed lines. For instance, here x is a divisor consisting of three parts, x_1, x_2, and x_3, none of which is itself a divisor. Thin lines are, intuitively, courses of events as determined by the relation of topological connection.)

As a preliminary characterization, this intuition can be expressed with the help of both mereological and topological notions by requiring every divisor to split all of its neighborhoods into disconnected parts:

DE1 $\mathsf{D}(x) =_{df} \quad y\ (\mathsf{IP}(x, y) \rightarrow \neg \mathsf{Cn}(y{-}x))$.

Within certain limits that will soon become clear, this is sufficient to capture the intuitive distinction between "global" and "local" events. We may think of a divisor as an all-encompassing event made up of all that happens during a certain "period". By contrast, an action such as Brutus' stabbing of Caesar, or an incident such as the sinking of the Titanic, are "local" events: many other actions and events occur at the same time, but in different places. This is of course a crucial distinction arising from our acceptance of events as distinct from the intervals they occupy. Events are full-fledged entities.[20]

Note that if we refer to divisors for the purpose of characterizing the notions of past and future, these latter will be deprived of any absolute meaning and become relative notions: given any divisor x, the suggestion is to interpret the events on one side of x as past events, and the ones on the other side as future events, *relative to x*. There is no past or future except with respect to some division of the whole of history. It will follow that we will need to guarantee that such a relativistic account be nevertheless suited to the task; that is, we must make sure that all the relevant divisors partition past from future events in a *coherent* may. (For instance, an event that lies in the past relative to a given divisor x must also lie in the past relative to any divisor y that lies in the future of x.) This means that 'D' must be refined, since nothing so far prevents two divisors from generating orthogonal divisions.

We also want our construction to allow for a certain degree of control on its grain. Intuitively, any two events that are part of the same divisor should count as simultaneous. However, for that purpose the general notion defined by DF1 is far too unconstrained. For one thing, it allows for divisors with all sorts of topologies, including undesired ones (for instance, multiply-connected divisors, i.e., divisors with "holes", such as the sum of a line and a tangential circle, or a plane and a tangential torus). Second, we need some means for associating each event in the domain with the "right" divisor, viz. the *minimal* one containing it. (Otherwise, for any two events x and y we could pick out a sufficiently large divisor z containing both, thereby making them simultaneous.) Moreover, the predicate 'D' picks out the distinguishing property of a divisor relative to the full mereotopological structure of the given domain of events, but of course there are domains admitting of an indefinite, potentially infinite number of divisors, only *some* of which can (or need) be collected in a temporally coherent structure.

In short, not all divisors can or need be considered together. Rather, every domain can be associated with various dividing devices δ (including divisors that ignore some mereological distinctions, treating as punctual events, for instance, entities that do in fact have a part structure), giving rise to a variety of event structures. Furthermore, the possibility of *varying* the grain itself may be a welcome one. This can be helpful, for instance, in accounting for the various degrees of precision that natural language permits when talking about events and time.[21]

2.2 Structures. Given all of this, we define an *event structure* quite generally as an ordered pair $\langle E, \delta \rangle$ made up of a non-empty domain of events, *E*, along with a distinguished predicate, δ, to be thought of as singling out a coherent set of divisors. The specific conditions are as follows.

First, we assume *E* to be mereotopologically connected:

AE1 $\exists z(\mathsf{O}(z, x) \wedge \mathsf{O}(z, y)) \rightarrow \mathsf{C}(x, y)$.

We are here taking *E* as a domain closed under all mereological and topological principles set forth in the previous section, and we understand individual variables to range over this domain. (Thus, for instance, not only do we assume that the sum of any number of events exists, by AP2; we also assume that it is an event.[22] If one feels more comfortable with a less liberal use of the word "event", one can think of *E* as the mereotopological closure of the domain of *bona fide* events, whatever sorts of entity that might involve.) Accordingly, condition AE1 could also be written as

AE1' $\mathsf{Cn}(\mathsf{U})$,

which effectively corresponds to the statement that the whole universe (the universal event) is self-connected. This is stipulative and reflects the idea that there are no gaps in history: something is always happening, whether remarkable or not.[23]

Second, the divisor-specific predicate δ is to pick out an exhaustive set of mutually coherent divisors incorporating the above-mentioned requirements of minimality and granularity. Generally speaking, it can be characterized as a maximal predicate satisfying at least the following conditions:

AE2 $\delta(x) \rightarrow \mathsf{D}(x)$
AE3 $\delta(x) \wedge \delta(y) \rightarrow . \, \delta(x+y) \leftrightarrow \mathsf{C}(x, y)$
AE4 $\delta(x) \wedge \delta(y) \rightarrow . \, \delta(x \times y) \leftrightarrow \mathsf{O}(x, y)$
AE5 $\delta(x) \wedge \delta(y) \rightarrow . \, \delta(x - y) \leftrightarrow \mathsf{X}(x, y)$.

In other words, the events fulfilling δ form a maximal class of divisors closed under the mereological operations of sum, product, and difference (within certain obvious limits). These latter conditions do not generally hold for 'D', but are easily motivated by the intended meaning of "divisor" that we are considering (Figure 3). Generally speaking, if every event must be included in some divisor (with some limit exceptions to be discussed shortly), then any two connected divisors must make up a ("thicker") divisor; and if divisors are to divide the domain into *two* main parts (intuitively, past and future), then they must be "minimal", in the sense of not consisting of two or more disconnected divisors. (They may, of course, consist of two or more disconnected *parts*, as in Figure 3(*b*); likewise, since the assumption of connectedness of the universe is not preserved locally, there is no guarantee that a divisor always splits its complement into two self-connected parts: see Figure 3(*a*)). This explains AE3. Moreover, if all

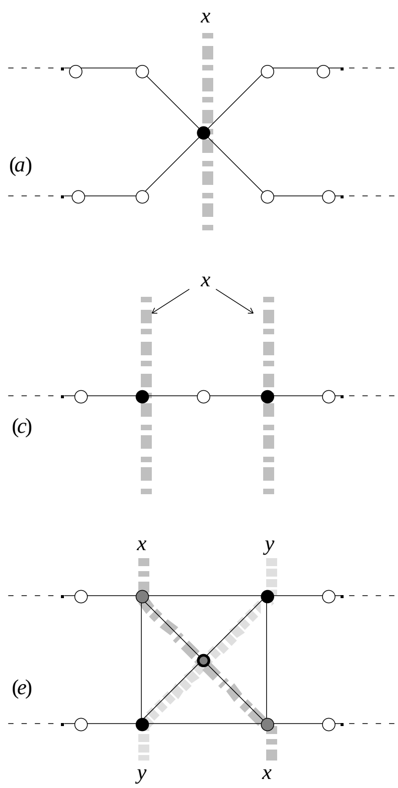

Figure 3. Some effects of AE3–AE6: (*a*) a divisor, *x*, separating its complement into two self-disconnected parts; (*b*) a self-disconnected and yet legitimate (minimal) divisor; (*c*) an illegitimate self-disconnected divisor by AE3 (sum of two separated divisors); (*d*) a non-minimal self-connected divisor by AE3 (sum of two minimal adjacent divisors); (*e*) two illegitimate divisors by AE4 (product is not a divisor); (*f*) an illegitimate divisor by AE5 (subtracting it from any divisor overlapping the event in the middle does not yield a divisor).

divisors must operate in a "parallel" fashion, i.e., have a uniform temporal orientation, then the common part of any two overlapping divisors (AE4) and the difference between any two crossing divisors (AE5) must themselves be divisors.

Conditions AE3 and AE4 can be strengthened in an obvious way by requiring δ to be closed under infinitary sums and infinitary products of connected/overlapping divisors:

AE3* $\quad x(\varphi[x] \rightarrow \delta(x)) \rightarrow .\ \delta(\sigma x\, \varphi[x]) \leftrightarrow \mathsf{Cn}(\sigma x\, \varphi[x])$

AE4* $\quad x(\varphi[x] \rightarrow \delta(x)) \rightarrow .\ \delta(\pi x \varphi[x]) \leftrightarrow \quad z \quad x(\varphi[x] \rightarrow \mathsf{P}(z,x))$.

We are not sure that AE3* is immune from unpalatable consequences, so we refrain from including it in our basic characterization of event structures. (It would imply that the entire universe is a divisor by AE1, so some restriction would be required.) On the other hand, given the intuitive notion of a divisor that δ is meant to capture, AE4* provides a natural extension of AE4, and we shall assume it unrestrictedly. In set-theoretic terms, it has the effect of making the set of divisors into a closure system, and this is really what we mean when we say that the common part of any number of overlapping divisors picked up by δ must itself be a divisor. The corresponding closure operator associates each event x with the smallest divisor containing x. We shall call it the *divisor of x*:

DE2 $\quad \mathsf{d}(x) =_{df} \pi z(\delta(z) \quad \mathsf{P}(x, z))$.

That d is indeed a closure operator is essentially guaranteed by the following theorems, which assert the analogues of the usual increasing, idempotency, and monotonicity properties:

TE1 $\quad \mathsf{P}(x, \mathsf{d}(x))$

TE2 $\quad \mathsf{d}(\mathsf{d}(x)) = \mathsf{d}(x)$

TE3 $\quad \mathsf{P}(x, y) \rightarrow \mathsf{P}(\mathsf{d}(x), \mathsf{d}(y))$.

However, d is only a partial operator, as it is not defined for arguments that are not contained in any divisor in δ. This is as it should be in case we are talking of, say, the universal event U, for the whole of history has no past and no future. On the other hand, if we consider events that are indefinitely extended in the past but not in the future, or vice versa, then we do want to say that such events precede every future event, or vice versa that they follow every past event. Likewise, if U is bounded, then the problem arises of explaining the temporal location of its boundaries. There is no such thing as the divisor of the Big Bang; yet we want to say that nothing happened before, and everything followed. To cover these and similar cases, we extend d to a total operator d^*:

DE3 $\quad \mathsf{d}^*(x) =_{df} x + \sigma z \quad y\ (\mathsf{P}(y, x) \quad z{=}\mathsf{d}(y))$.

This new operator satisfies TE1–TE3 and includes d as a special case.

2.3 Oriented structures. The notion of an event structure provides a characterization of the notion of an event (or a family of events) separating in some intuitive sense what has already happened from what is still to come. It does not, however, say anything concerning the way in which the separation is performed. That is, it guarantees—as we shall see—that the relative notions of past and future behave coherently throughout, but it remains neutral with respect to the question of whether a given event actually lies in the past or in the future of another event. In short, event structures are not temporally oriented.

We can make this more precise with the help of some additional terminology. Let us say that an event x is *flanked* by two events z_1 and z_2 if these lie on two opposite sides, as it were, of the divisor of x; and let us say that x is complemented by z_1 and z_2 (or that x *separates* z_1 from z_2) if these are *maximal* among the events by which x is flanked:

DE4 $\quad \mathsf{F}(z_1, x, z_2) =_{df} \neg\mathsf{O}(z_1+z_2, \mathsf{d}(x)) \quad \neg\mathsf{C}(\mathsf{d}^*(z_1), \mathsf{d}^*(z_2))$

DE5 $\quad \mathsf{S}(z_1, x, z_2) =_{df} \mathsf{F}(z_1, x, z_2) \quad z_1+z_2 = {\sim}\mathsf{d}(x)$.

(It is easily verified that DE5 implies that neither z_1 nor z_2 be a divisor, or a part of a divisor. In fact, both events must extend all the way in one of the two directions induced by the double-sidedness of the divisor predicate δ.) The point is then that whenever x is a "dividing" event, i.e., an event whose minimal divisor $\mathsf{d}(x)$ is defined, we can find a unique pair of events z_1 and z_2 such that $\mathsf{S}(z_1, x, z_2)$. But since such a pair also satisfies $\mathsf{S}(z_2, x, z_1)$ (i.e., S is a relation that is symmetric with respect to its middle argument), there is no way of telling which is the past and which the future of x.

We can, however, do the following (Figure 4). Given an initial triple z_1, x, z_2 such that $\mathsf{S}(z_1, x, z_2)$, we can define a function g associating every dividing event y with the totality of events that lie—as it were—on the same relative side as z_1 and z_2, respectively:

DE6 $\quad g(y, z_i) =_{df} \iota z \quad w(\mathsf{S}(z, y, w) \quad (\mathsf{O}(y, z_i) \rightarrow \mathsf{P}(z, z_i)) \quad (\neg\mathsf{O}(y, z_i) \rightarrow \mathsf{P}(z_i, z)))$.

Clearly, $g(y, z_1)$ and $g(y, z_2)$ are perfectly symmetric and interdefinable:

TE4$_1$ $\quad \mathsf{S}(g(y, z_1), y, g(y, z_2))$

TE4$_2$ $\quad \mathsf{S}(g(y, z_2), y, g(y, z_1))$

TE5$_1$ $\quad g(y, z_1) = \sigma z\ (\mathsf{P}(y,\ g(z, z_2))$

TE5$_2$ $\quad g(y, z_2) = \sigma z\ (\mathsf{P}(y,\ g(z, z_1))$.

And, clearly, there is still no way to say which is the past and which the future. However, we can now say that *if* z_1 occurs, say, before x (and z_2 after), *then* $g(y, z_1)$ must always occur before y (and $g(y, z_2)$ after) no matter how y is chosen. In other words, the choice of some initial point of view relative to some arbitrary event is all we need to *fix* a coherent orientation for the whole event structure.

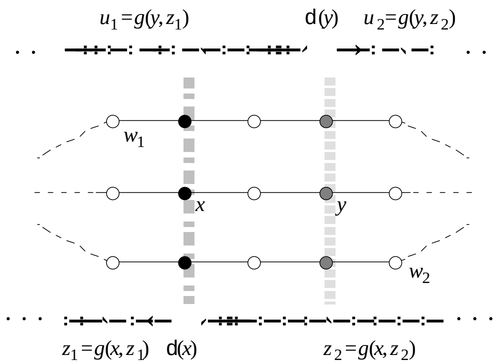

Figure 4. Pictorial illustration of DE4–DE6. Both x and y are flanked by w_1 and w_2 ; x separates z_1 from z_2, while y separates u_1 from u_2 ; these latter in turn coincide with $g(y, z_1)$ and $g(y, z_2)$, respectively, while $g(x, z_1)$ and $g(x, z_2)$ reduce to z_1 and z_2.

This leads to the following general definition. We say that a triple E, δ, e is an *oriented event structure* iff E, δ is an event structure as defined in the previous section, and e is a distinguished element of E such that

AE6 $x \quad y$ (S(e,x,y)).

If E, δ, e is such a structure, we can define a pair of (possibly partial) maps f and f' in the obvious way using $TE5_1$ (or $TE5_2$):

DE7 $f(x) =_{df} g(x,e)$
DE8 $f'(x) =_{df}$ σz (P(x, $f(z)$)).

We then just *stipulate* that e represents the past. That is, we simply treat f as a function of temporal orientation associating each event in the domain with the totality of events that temporally precede it; and, correspondingly, we treat f' as a function associating each event with the totality of what follows it. (We take it that if $f(x)$ is not defined, then x has no past, and likewise for $f'(x)$.) This is admittedly a conventional choice—the alternative stipulation would do just as well. But the following facts (for instance) show that it is a coherent choice:

TE6 $f(x) = f$(d(x))
TE7 $f(x)+f'(x) =$ ~d(x)
TE8 P(x, $f(y)$) → P($f(x)$, $f(y)$)
TE9 δ(x) δ(y) O(y, $f(x)$) O(y, $f'(x)$) → P(x, y).

TE6–TE7 relate to the intuitive interpretation of the divisor operator δ; TE8 relates to the underlying mereology; and TE9 relates to both, reflecting the provi-

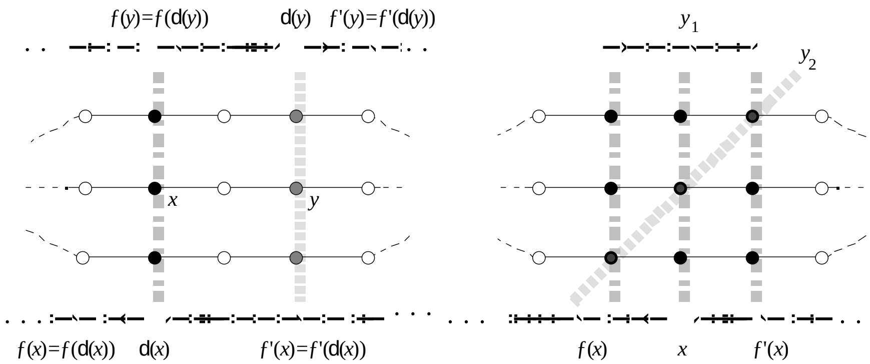

Figure 5. Left: pictorial illustration of the mereotopological relations expressed by TE6–TE8 (TE6'–TE8'). Right: x and y_1 satisfy, whereas x and y_2 violate, the general requirement on divisors expressed by TE9 (compare also Figure 3(e)).

sion that all divisors be uniformly oriented (Figure 5). Moreover, DE8 guarantees perfect duality between f and f'. Thus, just as TE7 and TE9 are symmetric with regard to these functions, the analogues of TE6 and TE8 for f' hold too:

TE6' $\quad f'(x) = f'(\mathsf{d}(x))$

TE8' $\quad \mathsf{P}(x, f'(y)) \rightarrow \mathsf{P}(f'(x), f'(y))$.

Note how the stipulation of the "anchor" element e is grounded on the emerging directionality of event structures. For one can easily verify that if E, δ, e_1 and E, δ, e_2 are two oriented event structures, with orientation functions f_1, f_1' and f_2, f_2' (respectively), then the following holds:

TE10 $\quad \mathsf{P}(e_1, e_2) \quad \mathsf{P}(e_2, e_1) \rightarrow f_1 = f_2 \quad f_1' = f_2'$.

That is, oriented structures whose anchor elements are related by parthood induce the *same* ordering. Since complementation reverses parthood, there are therefore only two ways of orienting an event structure E, δ, and these are given by any pair of oriented structures E, δ, e_1 and E, δ, e_2 whose anchor elements e_1 and e_2 do not overlap. Furthermore it is easy to prove that such structures satisfy

TE11 $\quad f_1 = f_2' \quad f_1' = f_2$

i.e., they induce two opposite orientations.

2.4 Special Structures. Further constraints on E, δ, or e can of course be added to select (oriented) structures with special characteristic properties. For example, one may want to rule out the possibility that there be atomic events (i.e., events with no proper parts) that are open. Or one may want to impose stronger conditions on the granularity of δ: for instance, one may want to require that an event's divisor be never "thicker" (i.e., intuitively, of longer duration) than the event itself; or one may ask for maximum granularity, requiring δ to be "closed downwards" (i.e., to include every D-divisor that is included in some element). Likewise, it is possible to impose stronger conditions on the topology of *E* than just self-connectedness, such as density or, alternatively, discreteness: the former asserts that between any two successive closed events (or, equivalently, between any two successive open events) there is always a third one; the latter amounts to the opposite requirement that every closed (open) event has some closed (open) immediate successor and some closed (open) immediate predecessor. In our formalism these conditions would correspond to the following postulates respectively, the last two corresponding to the two sides of discreteness:

AE7 $\mathrm{Op}(x) \rightarrow \quad y\ \mathrm{PP}(y, x)$
AE8 $\mathrm{O}(x, \mathrm{d}^*(y)) \leftrightarrow \mathrm{O}(\mathrm{d}^*(x), \mathrm{d}^*(y))$
AE9 $\delta(x) \quad \mathrm{P}(y, x) \quad \mathrm{D}(y) \rightarrow \delta(y)$
AE10 $\mathrm{P}(\mathrm{c}(x), f(\mathrm{c}(y))) \rightarrow \quad z\ (\mathrm{P}(\mathrm{c}(x), f(z)) \quad \mathrm{P}(z, f(\mathrm{c}(y))))$
AE11 $\quad y\ \mathrm{P}(\mathrm{c}(x), f(\mathrm{c}(y))) \rightarrow$
$\quad z\ (\mathrm{P}(\mathrm{c}(x), f(\mathrm{c}(z))) \quad \neg \quad w\ (\mathrm{P}(\mathrm{c}(x), f(w)) \quad \mathrm{P}(w, f(\mathrm{c}(z)))))$
AE11' $\quad y\ \mathrm{P}(\mathrm{c}(x), f'(\mathrm{c}(y))) \rightarrow$
$\quad z\ (\mathrm{P}(\mathrm{c}(x), f'(\mathrm{c}(z))) \quad \neg \quad w\ (\mathrm{P}(\mathrm{c}(x), f'(w)) \quad \mathrm{P}(w, f'(\mathrm{c}(z))))).$

We shall not pursue these lines of development here. For our present purposes, let us only stress once again that the operators and mappings introduced above may possibly be undefined for some arguments. Different models of the proposed conditions may therefore be obtained according to the underlying logical apparatus.

3. The Construction of Time

We are now ready to see how the temporal dimension can be retrieved from the mereotopological properties of an oriented event structure. More precisely, let *E*, δ, *e* be such a structure, and let *f* and *f*' be the corresponding functions of orientation (we shall keep these parameters fixed throughout this section). Then we shall show that the members of *E* bear relations to one another comparable to the temporal relations commonly assumed in connection with time ontology and temporal reasoning.

To this end, note first of all that along with T6–T9, the following facts hold for all events *x* and *y* in the given domain *E* :

TE12 $\mathrm{P}(x, f(y)) \rightarrow \neg\mathrm{O}(x, f'(y))$
TE12' $\mathrm{P}(x, f'(y)) \rightarrow \neg\mathrm{O}(x, f(y))$
TE13 $\mathrm{P}(x, f(y)) \rightarrow \mathrm{P}(y, f'(x))$

TE13' $\mathsf{P}(x, f'(y)) \rightarrow \mathsf{P}(y, f(x))$
TE14 $\mathsf{P}(x, y) \rightarrow \mathsf{P}(f(y), f(x))$
TE14' $\mathsf{P}(x, y) \rightarrow \mathsf{P}(f'(y), f'(x))$.

The first pair assert that past and future (relative to a given event x) do not overlap; the second pair assert that whatever happens before a certain event y must be such as to include y among its future events and, vice versa, whatever happens after an event x must include x among its past events; the last pair assert that whenever an event is included in another, the past and the future of the latter must be included in the past and in the future of the former, respectively.

As we have it, of course, only events that have a (minimal) divisor can be fully matched for precedence, since f or f' may otherwise be undefined. This is an important consequence of TE7:

TE15 $y\ (y=f(x)) \quad y\ (y=f'(x)) \rightarrow \quad y\ (y=\mathsf{d}(x))$.

In particular, it follows that $f(f(x))$ and $f'(f'(x))$ are always undefined. (For what would it *mean* to say that “the past” and “the future” of an event have a past and future of their own?) Furthermore, note that if x is a non-dividing event, for instance an open-ended event reaching infinitely backwards or forwards, then $f(x)$ is defined only if $f'(x)$ is not defined, and vice versa. Accordingly, when considering principles such as TE12–TE14', we should always keep in mind that the underlying semantics may be only partially defined. Alternatively, we can think of such non-dividing events as having a degenerate future or a degenerate past, respectively. This is captured in the following equivalences:

TE16 $f(x) = \sigma z\ (\mathsf{P}(x, f'(z))) = \pi z\ (\ y\ \mathsf{PP}(y, x) \quad z=f(y))$
TE16' $f'(x) = \sigma z\ (\mathsf{P}(x, f(z))) = \pi z\ (\ y\ \mathsf{PP}(y, x) \quad z=f'(y))$.

Thus, f yields the “limit” past of an event, which becomes undefined in the case of events infinitely extending backwards—likewise for f'.

This provides grounds for the intended interpretation of the relative properties expressed by TE12–TE14'. Two further interesting facts are the following:

TE17 $f'(f(x)) = {\sim}(f(x))$
TE17' $f(f'(x)) = {\sim}(f'(x))$.

These theorems, in a sense, generalize TE7 yielding the corollaries:

TE18 $\mathsf{P}(x, f'(f(x)))$
TE18' $\mathsf{P}(x, f(f'(x)))$.

Together with TE14 and TE14', these ensure that f and f' behave as a pair of Galois connections in *E*. Moreover, it is worth observing that composing f and f' induces corresponding topological structures:

TE19 $\mathsf{P}(x, ff'(x))$
TE20 $ff'(ff'(x)) = ff'(x)$
TE21 $ff'(x+y) = ff'(x)+ff'(y)$
TE22 $\mathsf{P}(x, y) \rightarrow \mathsf{P}(ff'(x), ff'(y))$

(likewise for $f'f$). Thus, the interaction of f and f' yields a pair of well-behaved topological closure operators. In particular, TE20 shows that their composition reaches a fixed point immediately after two applications.

All these facts, then, support our intended interpretation of E as a domain of events and they allow us to think intuitively of f and f' as functions of temporal orientation, as desired. In particular, we can say that an event x *temporally precedes* (wholly) an event y just in case x is part of $f(y)$, while x and y *temporally overlap* just in case they overlap or have some parts whose minimal divisors overlap:

DE9 $\mathsf{TP}(x, y) =_{df} \mathsf{P}(x, f(y)))$
DE10 $\mathsf{TO}(x, y) =_{df} \mathsf{O}(\mathsf{d}^*(x), \mathsf{d}^*(y))$.

(In DE9, the same definiens can of course be used to introduce the converse relation of y *temporally following* x; by TE13, this will be tantamount to:

DE11 $\mathsf{TF}(y, x) =_{df} \mathsf{P}(y, f'(x))$.

As for DE10, reference to d^* covers the case when x and y have no proper divisor.) Note that parthood excludes temporal precedence (following) whereas overlap implies temporal overlap:

TE23 $\mathsf{P}(x, y) \rightarrow \neg\mathsf{TP}(x, y)$
TE24 $\mathsf{O}(x, y) \rightarrow \mathsf{TO}(x, y)$.

Note also that these relations make it possible to introduce an entire family of additional notions, in analogy with the basic mereological setting. For instance, one can define relations of *temporal inclusion*, *temporal coincidence*, *temporal spanning*, etc. in an obvious way:

DE12 $\mathsf{TI}(x, y) =_{df} \quad z\ (\mathsf{TO}(x, z) \rightarrow \mathsf{TO}(y, z))$
DE13 $\mathsf{TC}(x, y) =_{df} \mathsf{TI}(x, y) \quad \mathsf{TI}(y, x)$
DE14 $\mathsf{TS}(x, y) =_{df} \mathsf{TO}(x, y) \quad \neg\mathsf{TI}(x, y)$.

These can easily be seen to behave in analogy to the corresponding mereological relations of parthood, identity, crossing, etc. For instance, TI is a partial ordering and TC an equivalence relation. In fact, it is apparent that in the case of divisors, these temporal relations reduce to their basic mereological correlates:

TE25 $\delta(x) \quad \delta(y) \rightarrow .\ \mathsf{TO}(x, y) \leftrightarrow \mathsf{O}(x, y)$
TE26 $\delta(x) \quad \delta(y) \rightarrow .\ \mathsf{TC}(x, y) \leftrightarrow x=y$

TE27 $\delta(x) \quad \delta(y) \rightarrow . \; \mathsf{TI}(x, y) \leftrightarrow \mathsf{P}(x, y)$

TE28 $\delta(x) \quad \delta(y) \rightarrow . \; \mathsf{TS}(x, y) \leftrightarrow \mathsf{X}(x, y).$

All of this sheds light on the temporal relations implicit in our oriented event structures, where non-dividing events can be simultaneous or partially simultaneous without necessarily bearing mereological relations to one another.

At this point we need not go any further. We do not, to be sure, have a full characterization yet. (This will ultimately depend on the specific event structure under consideration.) Nevertheless the above suffices to establish a substantial point. For we can now prove that regardless of the specific choice of E, δ, and e, the following general conditions always hold:

TE29 $\mathsf{TO}(x, x)$

TE30 $\mathsf{TO}(x, y) \rightarrow \mathsf{TO}(y, x)$

TE31 $\mathsf{TP}(x, y) \rightarrow \neg\mathsf{TO}(x, y)$

TE32 $\mathsf{TP}(x, y) \rightarrow \neg\mathsf{TP}(y, x)$

TE33 $\mathsf{TP}(x, y) \quad \mathsf{TP}(y, z) \rightarrow \mathsf{TP}(x, z)$

TE34 $\mathsf{TP}(x, y) \quad \mathsf{TO}(y, z) \quad \mathsf{TP}(z, t) \rightarrow \mathsf{TP}(x, t)$

TE35 $\mathsf{TP}(x, y) \quad \mathsf{TP}(y, x) \quad \mathsf{TO}(x, y).$

These are the mereological counterparts of the seven axioms for strict linear orders employed by Kamp [1979] in his construction of time instants out of ordered events, axioms which are common to most recent theories of temporal reasoning. (Counterparts of the axioms for interval structures of van Benthem [1983], for instance, can be established by taking divisors as the counterparts of intervals and then reasoning in terms of the relations of temporal precedence and overlapping as introduced above.) In other words, TE29–TE35 correspond to the seven principles usually assumed to axiomatize the fundamental temporal relations. Since they can now be seen to follow from our basic axioms AE1–AE6 (the proof is routine), it follows that the temporal characterization that we were after is actualy complete.

4. Concluding Remarks

We must stress that deriving relational properties such as those expressed by TE29–TE35 does not amount to construing *time* out of events. What we have is a construction of *temporal relations*—strictly speaking, relations that can be *interpreted as* temporal relations—out of mereotopological relations among events. Even so, we believe the above supports the claim that the notion of an oriented event structure, albeit silent about time, permits at last a full retrieval of the temporal dimension.

This eventually answers our initial question. At least for the purpose of talking about what actually happens or might happen, time need not be posited as an independent notion—be it as a primary ontological category (intervals or instants) or in the form of some primitive, irreducible relation of temporal precedence. Rather, time can simply be viewed as a by-product of the possibility of orienting the domain of all happenings. If one will, this can then be read as evidence in favor of a (somewhat moderate) form of relationalism. More precisely, our construction allows one to see the dispensability of time as a function of three main factors: (i) the assumption of events as *bona fine* individual entities (this is a matter of ontology), (ii) the notion of a divisor (which is intrinsically mereotopological), and (iii) the choice of an anchor event relative to which the "arrow of time" can be oriented (and this is formally a stipulative matter). A full assessment of these factors in relation to the absolutist/relativist dispute goes beyond the aims of this paper. But it will be interesting to see how much of (i), (ii), and (iii) constitutes a necessary provision for any relational undertaking. Particularly, it will be interesting to look further into the bearing of the relevant mereological and topological underpinnings. And it will be interesting to see whether, or to what extent, a similar machinery can be applied to other domains—for instance, to provide an analogous construction of spatial relations out of the fundamental mereotopological properties of common-sense spatial entities such as physical objects and chunks of matter.

Fabio Pianesi

Istituto per la Ricerca Scientifica e Tecnologica
Povo (Trento), Italy

Achille C. Varzi

Columbia University

NOTES

[*] This is a much revised and extended version of a short conference paper which originally appeared as Pianesi and Varzi [1994b]. Section 1 also draws from a conference paper which appeared as Pianesi and Varzi [1994a]. We are thankful to participants in both conferences for helpful discussion. We are also indebted to Nicholas Asher, Roberto Casati, Gennaro Chierchia, Carola Eschenbach, Enrico Franconi, Alessandra Giorgi, Nicola Guarino, Jean Petitot, Chris Pinon, Graham Priest, Barry Smith, Laure Vieu, and Graham White for illuminating exchanges and comments on various drafts. (The final version is of course the result of joint work. For institutional requirements, however, F. Pianesi takes responsability for sections 1 and 3, and A. C. Varzi for the remainder.)

[1]. The expression is from van Benthem [1983: 113]. Much AI work in planning and temporal reasoning can be viewed as embodying this reductionist

approach, particularly under the impact of Allen [1984]; see the papers collected in Allen *et al.*, eds. [1990] and Ford and Anger, eds. [1991] for some indicative examples.

[2]. A selection of classic papers on these issues is collected in Casati and Varzi, eds. [1996].

[3]. We refer here to the classic works of Sklar [1976], Newton-Smith [1980], Earman [1989].

[4]. This aligns us with Davidson [1967] as opposed to, e.g., Chisholm [1970].

[5]. Specifically with respect to events, this view goes back to Whitehead [1919] and is by now rather popular. Among other things, it played a role in the debate on event identity (see, e.g., Davis [1970], Thalberg [1971], Thomson [1977]) and is rooted in much recent work on natural language semantics (compare Landman [1991] and Moltmann [1996], and references therein).

[6]. This was clearly pointed out by Cartwright [1975] and Tiles [1981] and, more recently, by Simons [1987], Bochman [1990], and Smith [1993] *inter alia.*

[7]. See Varzi [1994, 1996b] for a first assessment.

[8]. Applications include spatiotemporal reasoning (Randell and Cohn [1989], Randell *et al.* [1992a]), naive physics (Randell *et al.* [1992b]), and the semantic analysis of spatial prepositions in natural language (Aurnague and Vieu [1993]).

[9]. The hole example is from Casati and Varzi [1994], ch. 7; the ball example from Davidson [1969: 232]. Both cases are admittedly controversial, and will obviously fail if one treats holes or events as material entities (as in Lewis and Lewis [1970] and Quine [1960], respectively). The issue is taken up in Casati and Varzi [1995, 1996].

[10]. We refer here to the material reviewed in Smith [1994].

[11]. Against transitivity see, e.g., Cruse [1979] and Winston *et al.* [1987]; on extensionality see Wiggins [1979] and Simons [1987].

[12]. See, e.g., Chisholm [1978: 201] on Brentano's views.

[13]. The view that scattered entities may not have individual sums may be traced back to Whitehead's restricted mereology of events [1919] and underlies much later literature on the topic; see, e.g., Lowe's [1953] and Goodman's response in [1956].

[14]. On non-symmetric boundaries see Chisholm [1984, 1992/93] and Smith [1996]; for discussion we refer to Varzi [1996a].

[15]. Compare Varzi [1993, 1996b].

[16]. Brentano [1976]. See Chisholm [1984], Bochman [1990], and especially Smith [1993, 1996] for discussion and elaborations.

[17]. See, e.g., Allen [1984] and the other works cited in note 1.

[18]. For instance, the strict ordering of Kamp [1979] or Thomason [1989].

[19]. A noteworthy exception is Chisholm [1990], who puts forward a time-free variant of the property exemplification view.

[20]. Some authors deal with this distinction by treating events as endowed with a "multidimensional" part structure, distinguishing for instance between spatial and temporal mereological relations. See Moltmann [1991] for a proposal in this spirit.

[21]. See Pianesi and Varzi [1996] for some work in this direction.

[22]. In this strong form, AP2 is endorsed, e.g., by Thomson [1977]; misgivings in Taylor [1985: 25] and Lombard [1986: 25]. (Lewis is not so liberal either: see his [1986].) Compare also the discussion in Bennett [1988], ch. 10.

[23]. It may be objected that dismissing the possibility of temporal vacua, i.e., time intervals in which nothing happens (a possibility that some authors have argued to be logically coherent: see Shoemaker [1969] for an influential argument) is question begging. This is true if we assume that every event must involve some change (as advocated, e.g., by Quinton [1979], Taylor [1985], or Lombard [1986]). If we do not make such an assumption, however, changeless time periods may in principle be viewed as involving "static" events. Alternatively, the possibility of changeless time could be accommodated by reasoning in terms of *possible* events, rather than actual happenings. This would mean adding a modal dimension to the construction presented below, and would correspond to a rather common account of the relationalist view as opposed to a pure reductionist account (see, e.g., Butterfield [1984], Teller [1991], and Forbes [1993]). On the other hand, if the universe consisted of two or more separate parts, one could apply the reasoning below to each of them, ending up with a family of disconnected worlds each having its own temporal ordering. Here we shall not pursue this possibility for reasons of simplicity.

REFERENCES

Allen J. F., 1984, "Towards a General Theory of Action and Time," *Artificial Intelligence* 23, 123–54.

Allen J. F., Hendler J., Tate A. (eds.), 1990, *Readings in Planning*, San Mateo, CA: Morgan Kaufmann.

Aurnague M., Vieu L., 1993, "A Three-Level Approach to the Semantics of Space," in C. Z. Wibbelt (ed.), *The Semantics of Prepositions: From Mental Processing to Natural Language Processing*, Berlin: Walter de Gruyter, pp. 393–439.

Bach E., 1986, "The Algebra of Events," *Linguistics and Philosophy* 9, 5–16.

Bennett J., 1988, *Events and Their Names*, Indianapolis: Hackett.

Bochman A., 1990, "Mereology as a Theory of Part-Whole," *Logique et Analyse* 129/30, 75–101.

Brentano F., 1976, *Philosophische Untersuchungen zu Raum, Zeit und Kontinuum*, ed. by S. Körner and R. M. Chisholm, Hamburg: Meiner (Eng. trans. by B. Smith, *Philosophical Investigations on Space, Time and the Continuum*, London: Croom Helm, 1988).

Butterfield J., 1984, "Relationism and Possible Worlds," *British Journal for the Philosophy of Science* 35, 101–13.

Cartwright R., 1975, "Scattered Objects," in K. Lehrer (ed.), *Analysis and Metaphysics*, Dordrecht: Reidel, pp. 153-71.

Casati R., Varzi A. C., 1994, *Holes and Other Superficialities*, Cambridge, MA: MIT Press.

— 1995, "Basic Issues in Spatial Representation," in M. De Glas and Z. Pawlak (eds.), *Proceedings of the 2nd World Conference on the Fundamentals of Artificial Intelligence*, Paris: Angkor, pp. 63–72.

— 1996, "The Structure of Spatial Location," *Philosophical Studies*, forthcoming.

Casati R., Varzi A. C. (eds.), 1996, *Events*, Aldershot: Dartmouth Publishing.

Chisholm R. M., 1970, "Events and Propositions," *Noûs* 4, 15-24

— 1978, "Brentano's Conception of Substance and Accident," in R. M. Chisholm and R. Haller (eds.), *Die Philosophie Brentanos*, Amsterdam: Rodopi, pp. 197–210.

— 1984, "Boundaries as Dependent Particulars," *Grazer Philosophische Studien* 10, 87–95.

— 1990, "Events Without Times. An Essay on Ontology," *Noûs* 24, 413–28.

— 1992/93, "Spatial Continuity and the Theory of Part and Whole. A Brentano Study," *Brentano Studien* 4, 11–23.

Clarke B. L., 1981, "A Calculus of Individuals Based on 'Connection'," *Notre Dame Journal of Formal Logic* 22, 204–18.

Cruse D. A., 1979, "On the Transitivity of the Part-Whole Relation," *Journal of Linguistics* 15, 29–38.

Davidson D., 1967, "The Logical Form of Action Sentences," in N. Rescher (ed.), *The Logic of Decision and Action*, Pittsburgh: University of Pittsburgh Press, pp. 81-95.

— 1969, "The Individuation of Events," in N. Rescher (ed.), *Essays in Honor of Carl G. Hempel*, Dordrecht: Reidel, pp. 216–34.

Davis L., 1970, "Individuation of Actions," *Journal of Philosophy* 67, 520–30.

Earman J., 1989, *World Enough and Space-Time*, Cambridge, MA: MIT Press.

Fine K., 1981, "Model Theory for Modal Logic, Part III: Existence and Predication," *Journal of Philosophical Logic* 10, 293–37.

Forbes G., 1993, “Time, Events and Modality,” in R. Le Poidevin and M. MacBeath (eds.), *The Philosophy of Time*, Oxford: Oxford University Press, pp. 80–95.

Ford K. M., Anger F. D., 1991, *Temporal Reasoning*, Part A and Part B, Special Issues of the *International Journal of Intelligent Systems* 6 (4–5).

Goodman N., 1956, “A World of Individuals,” in J. M. Bochenski, A. Church, and N. Goodman, *The Problem of Universals. A Symposium*, Notre Dame: University of Notre Dame Press, pp. 13–31.

Kamp H., 1979, “Events, Instants, and Temporal Reference,” in R. Bäuerle, U. Egli, and A. von Stechow (eds.), *Semantics from Different Points of View*, Berlin/Heidelberg: Springer, pp. 376–417.

Kim J., 1969, “Events and Their Descriptions: Some Considerations,” in N. Rescher (ed.), *Essays in Honor of Carl G. Hempel*, Dordrecht: Reidel, pp. 198–215.

Lambert K., 1962, “Notes on E! III: A Theory of Descriptions,” *Philosophical Studies* 13, 51–59.

Landman F., 1991, *Structures for Semantics*, Dordrecht/Boston/London: Kluwer.

Lejewski C., 1982, “Ontology: What’s Next?,” in W. Leinfellner, E. Krämer, and J. Schank (eds.), *Language and Ontology*, Vienna: Hölder-Pichler-Tempsky, pp. 173–85.

Leonard H. S., Goodman N., 1940, “The Calculus of Individuals and Its Uses,” *Journal of Symbolic Logic* 5, 45–55.

Leśniewski S., 1916, “Podstawy ogólnej teoryi mnogości. I,” *Prace Polskiego Koła Naukowego w Moskwie*, Sekcya matematyczno-przyrodnicza, 2 (Eng. trans. by D. I. Barnett, ‘Foundations of the General Theory of Sets. I’, in S. Leśniewski, *Collected Works*, ed. by S. J. Surma, J. Srzednicki, D. I. Barnett, and F. V. Rickey, Dordrecht: Nijhoff, 1992, Vol. 1, pp. 129–73).

Lewis D. K., 1986, “Events,” in his *Philosophical Papers, Volume 2*, New York: Oxford University Press, pp. 241–69.

— 1991, *Parts of Classes*, Oxford: Basil Blackwell.

Lewis D. K., Lewis S. R., 1970, “Holes,” *Australasian Journal of Philosophy* 48, 206–12.

Link G., 1987, “Algebraic Semantics for Event Structures,” in J. Groenendijk, M. Stokhof, and F. Veltman (eds.), *Proceedings of the 6th Amsterdam Colloquium*, Amsterdam: ILLC, pp. 243–62.

Lombard L., 1986, *Events: A Metaphysical Study*, London: Routledge and Kegan Paul.

Lowe V., 1953, “Professor Goodman’s Concept of an Individual,” *Philosophical Review* 62, 117–26.

Moltmann, F., 1991, “The Multidimensional Part Structure of Events,” in A. L.

Halpern (ed.), *Proceedings of the 9th West Coast Conference on Fornal Linguistics*, Stanford: CSLI, pp. 361–78.
—— 1996, *Parts and Wholes in Semantics*, New York: Oxford University Press, forthcoming.
Newton-Smith W. H., 1980, *The Structure of Time*, London: Routledge and Kegan Paul.
Pianesi F., Varzi A. C., 1994a, "The Mereo-Topology of Event Structures," in P. Dekker and M. Stokhof (eds.), *Proceedings of the 9th Amsterdam Colloquium*, Amsterdam: ILLC, pp. 527–46.
—— 1994b, "Mereotopological Construction of Time from Events," in A. G. Cohn (ed.), *Proceedings of the 11th European Conference on Artificial Intelligence*, Chichester: John Wiley and Sons, pp. 396–400.
—— 1996, "Refining Temporal Reference in Event Structures," *Notre Dame Journal of Formal Logic*, forthcoming.
Quine W. V. O., 1960, *Word and Object*, Cambridge, MA: MIT Press.
Quinton A., 1979, "Objects and Events," *Mind* 88, 197–214.
Randell D. A., Cohn A. G., 1989, "Modelling Topological and Metrical Properties in Physical Processes," in R. J. Brachman, H. Levesque, and R. Reiter (eds.), *Principles of Knowledge Representation and Reasoning. Proceedings of the First International Conference*, Los Altos, CA: Morgan Kaufmann, pp. 357–68.
Randell D. A., Cohn A. G., Cui Z., 1992a, "A Spatial Logic Based on Regions and Connections," in B. Nebel, C. Rich, and W. Swartout (eds.), *Principles of Knowledge Representation and Reasoning. Proceedings of the Third International Conference*, Los Altos, CA: Morgan Kaufmann, pp. 165–76.
—— 1992b, "Naive Topology: Modelling the Force Pump," in B. Faltings and P. Struss (eds.), *Recent Advances in Qualitative Physics*, Cambridge, MA: MIT Press, pp. 177–92.
Rescher N., 1955, "Axioms for the Part Relation," *Philosophical Studies* 6, 8–11.
Russell B. A. W., 1914, *Our Knowledge of the External World*, London: Allen and Unwin.
—— 1936, "On Order in Time," *Proceedings of the Cambridge Philosophical Society* 32, 216–28.
Shoemaker S., 1969, "Time Without Change," *Journal of Philosophy* 66, 363–81.
Simons P. M., 1987, *Parts. A Study in Ontology*, Oxford: Clarendon Press.
—— 1991, "Free Part-Whole Theory," in K. Lambert (ed.), *Philosophical Applications of Free Logic*, New York: Oxford University Press, pp. 285–306.
Sklar L., 1976, *Space, Time, and Space-Time*, Berkeley: University of California Press.
Smith B., 1993, "Ontology and the Logistic Analysis of Reality," in N. Guarino and R. Poli (eds.), *International Workshop on Formal Ontology in Con-*

ceptual Analysis and Knowledge Representation, Padova: CNR, pp. 51–68. Revised and expanded version as "Mereotopology: A Theory of Parts and Boundaries," *Data & Knowledge Engineering*, forthcoming.

—— 1994, "Topological Foundations of Cognitive Science," in C. Eschenbach, C. Habel, and B. Smith (eds.), *Topological Foundations of Cognitive Science* (Papers from the Workshop at the Buffalo FISI-CS), University of Hamburg, Reports of the Program in Cognitive Science, No. 37, pp. 3–22.

—— 1996, "Boundaries," in L. Hahn (ed.), *Roderick Chisholm*, La Salle: Open Court.

Tarski A., 1937, "Appendix E," in J. E. Woodger, *The Axiomatic Method in Biology*, Cambridge: Cambridge University Press.

Taylor B., 1985, *Modes of Occurrence: Verbs, Adverbs and Events*, Oxford: Blackwell.

Teller P., 1991, "Substance, Relations and Space-Time," *Philosophical Review* 100, 363–97.

Thalberg I., 1971, "Singling Out Actions, Their Properties and Components," *Journal of Philosophy* 68, 781–86.

Thomason S. K., 1989, "Free Construction of Time from Events," *Journal of Philosophical Logic* 18, 43–67.

Thomson J. J., 1977, *Acts and Other Events*, Ithaca: Cornell University Press.

Tiles J. E., 1981, *Things That Happen*, Aberdeen: Aberdeen University Press.

Van Benthem J., 1983, *The Logic of Time*, Dordrecht/Boston/London: Kluwer.

Varzi A. C., 1993, "Spatial Reasoning in a Holey World," in P. Torasso (ed.), *Advances in Artificial Intelligence*, Berlin/Heidelberg: Springer, pp. 326–36.

—— 1994, "On the Boundary Between Mereology and Topology," in R. Casati, B. Smith, and G. White (eds.), *Philosophy and the Cognitive Sciences*, Vienna: Hölder-Pichler-Tempsky, pp. 423–42.

—— 1996a, "Boundaries, Continuity, and Contact," *Noûs*, forthcoming.

—— 1996b, "Parts, Wholes, and Part-Whole Relations: The Prospects of Mereotopology," *Data & Knowledge Engineering*, forthcoming.

Vendler Z., 1957, "Verbs and Times," *Philosophical Review* 66, 143–60.

Walker A. G., 1947, "Durées et instants," *Revue Scientifique* 85, 131–34.

Whitehead A. N., 1919, *An Enquiry Concerning the Principles of Human Knowledge*, Cambridge: Cambridge University Press.

—— 1929, *Process and Reality. An Essay in Cosmology*, New York: Macmillan.

Wiggins D., 1979, "Mereological Essentialism: Asymmetrical Essential Dependence and the Nature of Continuants," *Grazer Philosophische Studien* 7, 297–315.

Winston M., Chaffin R., Herrmann D., 1987, "A Taxonomy of Part-Whole Relations," *Cognitive Science* 11, 417–44.