Hedge Fund Essays

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ABSTRACT

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This dissertation analyzes hedge fund leverage and its determinants, investigates optimal hedge fund manager behavior induced by hedge fund contracts, and uncovers an evidence of a hedge fund transparency risk premium. The first essay investigates the leverage of hedge funds in the time series and cross-section. Hedge fund leverage is found to be counter-cyclical to the leverage of listed financial intermediaries. Changes in hedge fund leverage tend to be more predictable by economy-wide factors than by fund-specific characteristics. In particular, decreases in funding costs and increases in market values both forecast increases in hedge fund leverage. Decreases in fund return volatilities predict future increases in leverage. In the second essay, I investigate hedge fund compensation from an investor's point of view in a model with a risk neutral fund manager who can continuously rebalance the fund's holdings. I solve for the optimal leverage level in a fund that has a compensation contract with a high-water mark and hurdle rate provisions where management and performance fees are paid at discrete time moments. The compensation contract induces risk-loving behavior with managers often choosing the maximum leverage. Third essay investigates risk premia associated with hedge fund transparency, liquidity, complexity, and concentration over the period from April 2006 to March 2009. Consistent with factor models of risk, we find that during normal times low-transparency, low-liquidity, and high-concentration funds delivered a return premium, with economic magnitudes of 5% to 10% per year, while during bad states of the economy, these funds experienced significantly lower

returns. We also offer a novel explanation for why highly concentrated funds command a risk premium by revealing that the risk premium is mostly prevalent among non-transparent funds where investors are unaware about the exact risks they are facing and hence cannot diversify them away.

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DEDICATION

This dissertation is dedicated to my parents Nataliya and Oleg Gorovyy, who brought me to life and made all of this possible through their love, support, and hard work. Their contribution cannot be overestimated.

Hedge Fund Leverage*

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Abstract

We investigate the leverage of hedge funds in the time series and cross-section. Hedge fund leverage is counter-cyclical to the leverage of listed financial intermediaries and decreases prior to the start of the financial crisis in mid-2007. Hedge fund leverage is lowest in early 2009 when the market leverage of investment banks is highest. Changes in hedge fund leverage tend to be more predictable by economy-wide factors than by fund-specific characteristics. In particular, decreases in funding costs and increases in market values both forecast increases in hedge fund leverage. Decreases in fund return volatilities predict future increases in leverage.

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1. Introduction

The events of the financial crisis over 2007–2009 have made clear the importance of leverage of financial intermediaries to both asset prices and the overall economy. The observed "deleveraging" of many listed financial institutions during this period has been the focus of many regulators and the subject of much research.[1] The role of hedge funds has played a prominent role in these debates for several reasons. First, although in the recent financial turbulence no single hedge fund has caused a crisis, the issue of systemic risks inherent in hedge funds has been lurking since the failure of the hedge fund Long-Term Capital Management L.P. (LTCM) in 1998.[2] Second, within the asset management industry, the hedge fund sector makes the most use of leverage. In fact, the relatively high and sophisticated use of leverage is a defining characteristic of the hedge fund industry. Third, hedge funds are large counterparties to the institutions directly overseen by regulatory authorities, especially commercial banks, investment banks, and other financial institutions which have received large infusions of capital from governments.

However, while we observe the leverage of listed financial intermediaries through periodic accounting statements and reports to regulatory authorities, little is known about hedge fund leverage despite the proposed regulations of hedge funds in the U.S. and Europe. This is because hedge funds are by their nature secretive, opaque, and have little regulatory oversight. Leverage plays a central role in hedge fund management. Many hedge funds rely on leverage to enhance returns on assets which on an unlevered basis would not be sufficiently high to attract funding. Leverage amplifies or dampens market risk and allows funds to obtain notional exposure at levels greater than their capital base. Leverage is often employed by hedge funds to target a level of return volatility desired by investors. Hedge funds use leverage to take advantage of mispricing opportunities by simultaneously buying assets which are

[1] See, for example, Adrian and Shin (2009), Brunnermeier (2009), Brunnermeier and Pedersen (2009), and He, Khang, and Krishnamurthy (2010), among many others.

[2] Systemic risks of hedge funds are discussed by the President's Working Group on Financial Markets (1999), Chan et al. (2007), Kambhu, Schuermann, and Stiroh (2007), Financial Stability Forum (2007), and Banque de France (2007).

perceived to be underpriced and shorting assets which are perceived to be overpriced. Hedge funds also dynamically manipulate leverage to respond to changing investment opportunity sets.

We are the first paper, to our knowledge, to formally investigate hedge fund leverage using actual leverage ratios with a unique data set from a fund-of-hedge-funds. We track hedge fund leverage in time series from December 2004 to October 2009, a period which includes the worst periods of the financial crisis from 2008 to early 2009. We characterize the cross-section of leverage: we examine the dispersion of leverage across funds and investigate the macro and fund-specific determinants of future leverage changes. We compare the leverage and exposure of hedge funds with the leverage and total assets of listed financial companies. As well as characterizing leverage at the aggregate level, we investigate the leverage of hedge fund sectors.

The prior works on hedge fund leverage are only estimates (see, e.g., Banque de France, 2007; Lo, 2008) or rely only on static leverage ratios reported by hedge funds to the main databases. For example, leverage at a point in time is used by Schneeweis et al. (2004) to investigate the relation between hedge fund leverage and returns. Indirect estimates of hedge fund leverage are computed by McGuire and Tsatsaronis (2008) using factor regressions with time-varying betas. Even without considering the sampling error in computing time-varying factor loadings, this approach requires that the complete set of factors be correctly specified, otherwise the implied leverage estimates suffer from omitted variable bias. Regressions can also not adequately capture abrupt changes in leverage. Other work by Brunnermeier and Pedersen (2009), Gorton and Metrick (2009), Adrian and Shin (2010), and others, cites margin requirements, or haircuts, as supporting evidence of time-varying leverage taken by proprietary trading desks at investment banks and hedge funds. These margin requirements give maximum implied leverage, not the actual leverage that traders are using. In contrast, we analyze actual leverage ratios of hedge funds.

Our work is related to several large literatures, some of which have risen to new prominence with the financial crisis. First, our work is related to optimal leverage management by hedge funds. Duffie, Wang, and Wang (2008) and Dai and Sundaresan (2010) derive theoretical models of optimal leverage in the presence of management fees, insolvency losses, and funding costs and restrictions at the fund level. At the finance sector level, Acharya and Viswanathan (2008) study optimal leverage in the presence of moral hazard and liquidity effects showing that due to deleveraging, bad shocks that happen in good times are more severe. A number of authors have built equilibrium models where leverage affects the entire economy. In Fostel and Geanakoplos (2008), economy-wide equilibrium leverage rises in times of low volatility and falls in periods where uncertainty is high and agents have very disperse beliefs. Leverage amplifies liquidity losses and leads to overvalued assets during normal times. Stein (2009) shows that leverage can be chosen optimally by individual hedge funds, but this can create a fire-sale externality causing systemic risk by hedge funds simultaneously unwinding positions and reducing leverage. There are also many models where the funding available to financial intermediaries, and hence leverage, affects asset prices. In many of these models, deleveraging cycles are a key part of the propagating mechanism of shocks.[3] Finally, a large literature in corporate finance examines how companies determine optimal leverage. Recently, Welch (2004) studies the determinants of firm debt ratios and finds that approximately two-thirds of variation in corporate leverage ratios is due to net issuing activity.

The remainder of the paper is organized as follows. We begin in Section 2 by defining and describing several features of hedge fund leverage. Section 3 describes our data. Section 4 outlines the estimation methodology which allows us to take account of missing values. Section 5 presents the empirical results. Finally, Section 6 concludes.

2. The Mechanics of Hedge Fund Leverage

2.1. Gross, Net, and Long-only Leverage

A hedge fund holds risky assets in long and short positions together with cash. Leverage measures the extent of the relative size of the long and short positions in risky assets relative

[3] See, for example, Gromb and Vayanos (2002), He and Krishnamurthy (2009), Brunnermeier and Pedersen (2009), and Adrian and Shin (2010).

to the size of the portfolio. Cash can be held in both a long position or a short position, where the former represents short-term lending and the latter represents short-term borrowing. The assets under management (AUM) of the fund is cash plus the difference between the fund's long and short positions and is the value of the claim all investors have on the fund. The net asset value (NAV) per share is the value of the fund per share and is equal to AUM divided by the number of shares. We use the following three definitions of leverage, which are also widely used in industry:

Gross leverage is the sum of long and short exposure per share divided by NAV. This definition implicitly treats both the long and short positions as separate sources of profits in their own right, as would be the case for many long-short equity funds. This leverage measure overstates risk if the short position is used for hedging and does not constitute a separate active bet. If the risk of the short position by itself is small, or the short position is usually taken together with a long position, a more appropriate definition of leverage can be:

Net leverage is the difference between long and short exposure per share expressed as a proportion of NAV. The net leverage measure captures only the long positions representing active positions which are not perfectly offset by short hedges, assuming the short positions represent little risk by themselves. Finally, we consider,

Long-only leverage or *Long leverage* is defined as the long positions per share divided by NAV. Naturally, by ignoring the short positions, long-only leverage could result in a large underestimate of leverage, but we examine this conservative measure because the reporting requirements of hedge fund positions by the U.S. Securities and Exchange Commission (SEC) involve only long positions.[4] We also investigate if long leverage behaves differently from gross or net leverage, or put another way, if hedge funds actively manage their long and short leverage positions differently.

Only a fund 100% invested in cash has a leverage of zero for all three leverage definitions. Furthermore, for a fund employing only levered long positions, all three leverage

[4] Regulation 13-F filings are required by any institutional investor managing more than $100 million. Using these filings, Brunnermeier and Nagel (2004) examine long-only hedge fund positions in technology stocks during the late 1990s bull market.

measure coincide. Thus, active short positions induce differences between gross, net, and long-only leverage. Appendix A illustrates these definitions of leverage for various hedge fund portfolios.

2.2. How do Hedge Funds Obtain Leverage?

Hedge funds obtain leverage through a variety of means, which depend on the type of securities traded by the hedge fund, the creditworthiness of the fund, and the exchange, if any, on which the securities are traded. Often leverage is provided by a hedge fund's prime broker, but not all hedge funds use prime brokers.[5] By far the vast majority of leverage is obtained through short-term funding as there are very few hedge funds able to directly issue long-term debt or secure long-term borrowing.

In the U.S., regulations govern the maximum leverage permitted in many exchange-traded markets. The Federal Reserve Board's Regulation T (Reg T) allows investors to borrow up to a maximum 50% of a position on margin (which leads to a maximum level of exposure equal to $1/0.5 = 2$). For a short position, Reg T requires that short-sale accounts hold collateral of 50% of the value of the short implying a maximum short exposure of two. By establishing offshore investment vehicles, hedge funds can obtain "enhanced leverage" higher than levels allowable by Reg T. Prime brokers have established facilities overseas in less restrictive jurisdictions to provide this service. Another way to obtain higher leverage than allowed by Reg T is "portfolio margining" which is another service provided by prime brokers. Portfolio margining was approved by the SEC in 2005 and allows margins to be calculated on a portfolio basis, rather than on a security-by-security basis.[6]

Table 1 reports typical margin requirements ("haircuts") required by prime brokers or other counterparties. The last column of the Table 1 lists the typical levels of leverage able to

[5] In addition to providing financing for leverage, prime brokers provide hedge fund clients with risk management services, execution, custody, daily account statements, and short-sale inventory for stock borrowing. In some cases, prime brokers provide office space, computing and trading infrastructure, and can even contribute capital.

[6] Portfolio margining only applies to "hardwired" relations, such as calls and puts on a stock, and the underlying stock itself, rather than to any statistical correlations between different assets.

be obtained in each security market, that are the inverse of the margin requirements. These are obtained at March 2010 by collating information from prime brokers and derivatives exchanges.[7] Note that some financial instruments, such as derivatives and options, have embedded leverage in addition to the leverage available from external financing. The highest leverage is available in Treasury, foreign exchange, and derivatives security markets such as interest rate and foreign exchange swaps. These swap transactions are over the counter and permit much higher levels of leverage than Reg T. These securities enable investors to have large notional exposure with little or no initial investment or collateral. Similarly, implied leverage is high in futures markets because the margin requirements there are much lower than in the equity markets.

Based on the dissimilar margin requirements of different securities reported in Table 1, it is not surprising that hedge fund leverage is heterogeneous and depends on the type of investment strategy employed by the fund. Our results below show that funds engaged in relative value strategies, which trade primarily fixed income, swaps, and other derivatives, have the highest average gross leverage of 4.8 through the sample. Some relative value funds in our sample have gross leverage greater than 30. Credit funds which primarily hold investment grade and high yield corporate bonds and credit derivatives have an average gross leverage of 2.4 in our sample. Hedge funds in the equity and event-driven strategies mainly invest in equity and distressed corporate debt and hence have lower leverage. In particular, equity and event-driven funds have average gross leverage of 1.6 and 1.3, respectively, over our sample.

The cost of leverage to hedge funds depends on the method used to obtain leverage. Prime brokers typically charge a spread over London Interbank Offered Rate (LIBOR) to hedge fund clients who are borrowing to fund their long positions and brokers pay a spread below LIBOR for cash deposited by clients as collateral for short positions. These spreads are higher for less creditworthy funds and are also higher when securities being financed have high credit risk or are more volatile. The cost of leverage through prime brokers reflects the

[7] Brunnermeier and Pedersen (2009) and Gorton and Metrick (2009) show that margin requirements changed substantially over the financial crisis.

costs of margin in traded derivatives markets. We include instruments capturing funding costs like LIBOR and interest rate spreads in our analysis.

In many cases, there are maximum leverage constraints imposed by the providers of leverage on hedge funds. Hedge fund managers make a decision on optimal leverage as a function of the type of the investment strategy, the perceived risk-return trade-off of the underlying trades, and the cost of obtaining leverage, all subject to exogenously imposed leverage limits. Financing risk is another consideration as funding provided by prime brokers can be subject to sudden change. In contrast, leverage obtained through derivatives generally has lower exposure to funding risk. Prime brokers have the ability to pull financing in many circumstances, for example, when performance or NAV triggers are breached. Dai and Sundaresan (2010) show that this structure effectively leaves the hedge funds short an option vis-à-vis their prime broker. Adding further risk to this arrangement is the fact that the hedge fund is also short an option vis-à-vis another significant financing source, their client base, which also has the ability to pull financing following terms stipulated by the offering memorandum.[8] We do not consider the implicit leverage in these funding options in our analysis as we are unable to obtain data on hedge fund prime broker agreements or the full set of investment memoranda of hedge fund clients; our analysis applies only to the leverage reported by hedge funds in their active strategies.[9]

2.3. Reported Hedge Fund Leverage

An important issue with hedge fund leverage is which securities are included in the firm-wide leverage calculation and how the contribution of each security to portfolio leverage is calculated. The most primitive form of leverage calculation is unadjusted balance sheet leverage, which is simply the value of investment assets, not including notional exposure

[8] In many cases, hedge funds have the ability to restrict outflows by invoking gates even after lockup periods have expired (see, for example, Ang and Bollen, 2010).

[9] Dudley and Nimalendran (2009) estimate funding costs and funding risks for hedge funds, which are not directly observable, using historical data on margins from futures exchanges and Chicago Board Options Exchange Market Volatility Index (VIX). They do not consider hedge fund leverage.

in derivatives, divided by equity capital. Since derivative exposure for hedge funds can be large, this understates, in many cases dramatically, economic risk exposure.

To remedy this shortcoming, leverage is often adjusted for derivative exposure by taking delta-adjusted notional values of derivative contracts.[10] For example, to account for the different volatility and beta exposures of underlying investments, hedge funds often beta-adjust the exposures of (cash) equities by upward adjusting leverage for high-beta stock holdings. Likewise, (cash) bond exposures are often adjusted to account for the different exposures to interest rate factors. In particular, the contribution of bond investments to the leverage calculation is often scaled up or down by calculating a 10-year equivalent bond position. Thus, an investment of $100 in a bond with twice the duration of a 10-year bond would have a position of $200 in the leverage calculation. The issues of accounting for leverage for swaps and futures affect fixed income hedge funds the most and long-short equity hedge funds the least. For this reason, we break down leverage statistics by hedge fund sectors.

Funds investing primarily in futures, especially commodities, report a margin-to-equity ratio, which is the amount of cash used to fund margin divided by the nominal trading level of the fund. This measure is proportional to the percentage of available capital dedicated to funding margin requirements. It is frequently used by commodity trading advisors as a gauge of their market exposure. Other funds investing heavily in other zero-cost derivative positions like swaps also employ similar measures based on ratios of nominal, or adjusted nominal, exposure to collateral cash values to compute leverage.

Thus, an important caveat with our analysis is that leverage is not measured in a consistent fashion across hedge funds and the hedge funds in our sample use different definitions of leverage. Our data are also self-reported by hedge funds. These effects are partially captured in our analysis through fund fixed effects. Our analysis focuses on the common behavior of

[10] Many hedge funds account for the embedded leverage in derivatives positions through internal reporting systems or external, third-party risk management systems like RiskMetrics. These risk system providers compute risk statistics like deltas, left-hand tail measures of risk like Value-at-Risk (VAR), and implied leverage at both the security level and the aggregate portfolio level. RiskMetrics allows hedge funds to "pass through" their risk statistics to investors who can aggregate positions across several funds.

leverage across hedge funds rather than explaining the movements in leverage of a specific hedge fund.

3. Data

3.1. Macro Data

We capture the predictable components of hedge fund leverage by various aggregate market price variables, which we summarize in Appendix B. We graph two of these variables in Fig. 1.. We plot the average cost of protection from a default of major "investment banks" (Bear Stearns, Citibank, Credit Suisse, Goldman Sachs, HSBC, JP Morgan, Lehman Brothers, Merrill Lynch, and Morgan Stanley) computed using credit default swap (CDS) contracts in the solid line with the scale on the left-hand axis. This is the market-weighted cost of protection per year against default of each firm. Our selected firms are representative of broker/dealers and investment banking activity and we refer to them as investment banks even though many of them are commercial banks and some became commercial banks during the sample period.

In Fig. 1. we also plot the VIX volatility index in the dotted line with the scale on the right-hand axis. The correlation between VIX and investment bank CDS protection is 0.89. Both of these series are low at the beginning of the sample and then start to increase in mid-2007, which coincides with the initial losses in subprime mortgages and other certain securitized markets. In late 2008, CDS spreads and VIX increase dramatically after the bankruptcy of Lehman Brothers, with VIX reaching a peak of 60% at the end of October 2008 and the CDS spread reaching 3.55% per annum in September 2008. In 2009, both CDS and VIX decline after the global financial sector is stabilized.

Our other macro series are monthly returns on investment banks, monthly returns on the S&P 500, the three-month LIBOR rate, and the three-month Treasury over Eurodollar (TED) spread. The LIBOR and TED spreads are good proxies for the aggregate cost of short-term borrowing for large financial institutions. Prime brokers pass on at least the LIBOR and TED spread costs to their hedge fund clients plus a spread. Finally, we also include the term spread, which is the difference between the 10-year Treasury bond yield and the yield on three-month T-bills. This captures the slope of the yield curve, which under the Expectations Hypothesis is a forward-looking measure of future short-term interest rates and thus provides a simple way of estimating future short-term borrowing costs.

3.2. Hedge Fund Data

Our hedge fund data are obtained from a large fund-of-hedge-funds (which we refer to as the "Fund"). The original data set from the Fund contains over 45,000 observations of 758 funds from February 1977 to December 2009. In addition to hedge fund leverage, our data include information on the strategy employed by the hedge funds, monthly returns, NAVs, and AUMs. The hedge funds are broadly representative of the industry and contain funds managed in a variety of different styles including global macro funds, fundamental stock-picking funds, credit funds, quantitative funds, and funds investing using technical indicators. The hedge funds invest both in specific asset classes, for example, fixed income or equities, and also across global asset classes. Our data include both U.S. and international hedge funds, but all returns, NAVs, and AUMs are in U.S. dollars.

An important issue is whether the hedge funds in the database exhibit a selection bias. In particular, do the hedge funds selected by the Fund have better performance and leverage management than a typical hedge fund? The Fund selects managers using both a "top down" and a "bottom up" approach. The former involves selecting funds in various sector allocation bands for the Fund's different fund-of-funds portfolios. The latter involves searching for funds, or reallocating money across existing funds, using a primarily qualitative, proprietary approach. Leverage is a consideration in choosing funds, but it is only one of many factors among the usual suspects—Sharpe (1992) ratios and other performance criteria, due diligence considerations, network, manager quality, transparency, gates and restrictions, sector composition, investment style, etc. The Fund did not add leverage to its products and only very rarely asked hedge funds to provide a customized volatility target or to provide leverage which differed from the hedge funds' existing product offerings. There is no reason to believe that the Fund's selection procedure results in funds with leverage management practices that are significantly different to the typical hedge fund.

Our Fund database includes funds that are present in TASS, CISDM, Barclay Hedge, or other databases commonly used in research and also includes other funds which do not report to the public hedge fund databases. This mitigates the reporting bias of the TASS database (see Malkiel and Saha, 2005; Ang, Rhodes-Kropf, and Zhao, 2008; Agarwal, Fos, and Jiang, 2010). However, the composition by sector is similar to the overall sector weighting of the industry as reported by TASS and Barclay Hedge. Survival biases are mitigated by the fact that often hedge funds enter the database not when they receive funds from the Fund, but several months prior to the Fund's investment and they often exit the database several months after disinvestment. Our database also includes hedge funds which terminate due to poor performance. The aggregate performance of the Fund is similar to the performance of the main hedge fund indexes.

3.2.1. Hedge Fund Leverage

Leverage is reported by different hedge funds at various frequencies and formats, which are standardized by the Fund. Appendix C discusses some of these formats. Most reporting is at the monthly frequency, but some leverage numbers are reported quarterly or even less frequently. For those funds reporting leverage at the quarterly or at lower frequencies, the Fund is often able to obtain leverage numbers directly from the hedge fund managers at other dates through a combination of analyst site visits and calls to hedge fund managers. The data are of high quality because the funds undergo thorough due diligence by the Fund. In addition, the performance and risk reports are audited, and the Fund conducts regular, intensive monitoring of the investments made in the individual hedge funds.

3.2.2. Hedge Fund Returns, Volatilities, and Flows

We have monthly returns on all the hedge funds. These returns are actual realized returns, rather than returns reported to the publicly available databases. In addition to examining the relation between past returns and leverage, we construct volatilities from the returns. We construct monthly hedge fund volatility using the sample standard deviation of returns over the past 12 months. Fig. 2. plots the volatilities of all hedge funds and different hedge fund strategies over the sample. The volatilities follow the same broad trend and are approximately the same. This is consistent with hedge funds using leverage to scale returns to similar volatility levels.

Fig. 2. shows that at the beginning of the sample, hedge fund volatilities were around 3% per month and reach a low of around 2% per month in 2006. As subprime mortgages start to deteriorate in mid-2007, hedge fund return volatility starts to increase and reaches 4–5% per month by 2009. Volatility stays at this high level until the end of the sample in October 2009. This is because we use rolling 12-month sample volatilities which include the very volatile, worst periods of the financial crisis 12 months prior to October 2009.

Fig. 3. compares the rolling 12-month volatilities of hedge fund returns in the data sample with the rolling 12-month volatilities of hedge fund returns in the Hedge Fund Research, Inc. (HFR) database for the December 2004 – October 2009 time period. We observe that the average volatilities of hedge funds in the data closely track the median hedge fund volatility in the HFR database. Thus, the Fund's hedge funds have very similar return behavior as the typical hedge fund reported on the publicly available databases. Since hedge funds often use leverage to target particular levels of volatility, this partially alleviates concerns that the Fund's hedge funds have atypical leverage policies.

In addition to hedge fund volatility, we also use hedge fund flows as a control variable. We construct hedge fund-level flows over the past three months using the return and AUM information from the following formula:

$$Flow_t = \frac{AUM_t}{AUM_{t-3}} - (1 + R_{t-2})(1 + R_{t-1})(1 + R_t), \tag{1}$$

where $Flow_t$ is the past three-month flow in the hedge fund, AUM_t is assets under management at time t, and R_t is the hedge fund return from $t-1$ to t. The flow formula in Eq. (1) is used by Chevalier and Ellison (1997), Sirri and Tufano (1998), and Agarwal, Daniel, and Naik (2009), among others. We compute three-month flows, as the flows over the past month tend to be very volatile. We also compute past three-month hedge fund flows for the aggregate hedge fund industry as measured by the Barclay Hedge database using Eq. (1).

3.3. Summary Statistics

We clean the raw data from the Fund and impose two filters. First, often investments are made by the Fund in several classes of shares of a given hedge fund. All of these share classes have almost identical returns and leverage ratios. We use the share class with the longest history or the share class representing the largest AUM. Our second filter is that we require funds to have at least two years of leverage observations. The final sample spans December 2004 to October 2009 and thus, our sample includes the poor returns of quantitative funds during Summer 2007 (see Khandani and Lo, 2007) and the financial crisis of 2008 and early 2009. There are at least 63 funds in our sample at any one time. The maximum number of funds at any given month is 163 over the sample period.

Panel A of Table 2 lists the number of observations and number of hedge funds broken down by strategy. The strategies are defined by the Fund and do not exactly correspond to the sector definitions employed by TASS, Barclay Hedge, CISDM, or other hedge fund databases (which themselves employ arbitrary sector definitions). The TASS categories of fixed income arbitrage and convertible arbitrage fall under the Fund's relative value sector. In the relative value sector, hedge funds invest in both developed and emerging markets and can also invest in a variety of different asset classes. Most of the Fund's investments have been in long-short equity funds in the equity category and this is also by far the largest hedge fund sector in TASS, as reported, for example, by Chan et al. (2007). At the last month of our sample, October 2009, the proportion of equity funds reported in Barclay Hedge, not including multi-strategy, other, and sector-specific categories, is also over 40%.

After our data filters, there are a total of 208 unique hedge funds in our sample with 8,136 monthly observations. Over half (114) of the funds in our sample run long-short equity strategies. The number of funds in the areas of credit and relative value are 21 and 36, respectively. The remaining 37 funds are in the event driven strategy, which are mainly merger arbitrage and distressed debt. The number of funds reported in Panel A of Table 2 is large enough for reliable inference when averaged across strategies and across all hedge funds.[11]

In Panel B of Table 2, we report summary statistics of all the hedge fund variables observed in the sample. These statistics should be carefully interpreted because they do not sample all hedge funds at the same frequency and there are missing observations in the raw data. Panel B reports that the average gross leverage across all hedge funds is 2.13 with a volatility of 0.62. This volatility is computed using only observed data and the true volatility of leverage, after estimating the unobserved values, will be lower, as we show below. Nevertheless, it is clear that hedge fund leverage changes over time. Even without taking into account missing observations, this volatility is much lower than the volatility of leverage reported in the estimations of McGuire and Tsataronis (2008) using factor regressions. This discrepancy could possibly result from the large error in their procedure of inferring leverage from estimated factor coefficients in regressions on short samples. Individual gross hedge fund leverage is also persistent, with an average autocorrelation of 0.68 across all the hedge funds. Again because of unobserved leverage ratios, this persistence is biased downwards and we report more accurate measures of autocorrelation taking into account other predictive variables below.

Panel B of Table 2 also reports the summary statistics for the other two leverage measures. The average net leverage of hedge funds is 0.59 and average long-only leverage is 1.36. The raw volatilities of net leverage and long-only leverage are 0.28 and 0.38, respectively, which are significantly lower than the volatility of gross leverage. Thus, in our analysis, we break out gross, net, and long-only leverage separately.

The other variables reported in Panel B of Table 2 are control variables used in our analysis. The average hedge fund return is 29 basis points per month. These returns are autocorrelated, with an average autocorrelation of 0.24 across funds, which indicates that out- or under-performing manager returns are persistent, as noted by Getmansky, Lo, and

[11] The sample also includes commodity trading funds and global macro funds, but we do not break out separate performance of these sectors as there are too few funds for reliable inference.

Makarov (2004) and Jagannathan, Malakhov, and Novikov (2010). The returns are lower than those reported by previous literature because our sample includes the financial crisis during which many hedge funds did poorly.[12] The average 12-month rolling volatility across hedge funds is 2.65% per month. The volatility is computed only when all fund returns in the previous 12 months are observed. This explains why only approximately 70% of fund volatilities are observed. Nevertheless, our volatility estimates are close to those reported in the literature by Ackermann, McEnally, and Ravenscraft (1999) and Chan et al. (2007), among others.

The last two fund-specific variables we include are past three-month hedge fund flows and log AUMs. Flows are on average positive, at 2.2% per month and exhibit a large average autocorrelation of 0.62. The average fund size over our sample is $962 million. The median fund size is $430 million. The difference between mean and median of fund size is explained by the presence of some large funds, with the largest funds having AUMs well over $10 billion in just one share class. Our sample is slightly biased upwards in terms of size compared to recent estimates such as those by Chan et al. (2007) and the Banque de France (2007). This is due to the application of filters which tend to remove smaller funds which are effectively different share classes of larger funds. Our filters also remove funds which are in their infancy. These funds are likely to have lower levels of leverage, with more onerous financing conditions, than more established funds, making the levels of our leverage ratios conservatively biased upwards.

The last column in Panel B, Table 2 lists the proportion of months across all funds where the variables are observed. While we always observe returns, the leverage variables are observed approximately 80% of the time. We do not restrict our analysis to a special subset of data where all variables are observed. Instead, our algorithm permits us to use all the available data and to infer the leverage ratios when they are missing. We now discuss our estimation methodology.

[12] See, among many others, Fung and Hsieh (1997, 2001), Brown, Goetzmann, and Ibbotson (1999), and more recently, Bollen and Whaley (2009).

4. Methodology

4.1. Predictive Model

We specify that leverage over at month $t+1$ for fund i, $L_{i,t+1}$, is predictable at time t by both economy-wide variables, x_t, and fund-specific variables, which we collect in the vector $y_{i,t}$, in the linear regression model:[13]

$$\Delta L_{i,t+1} = c_i + \gamma \cdot x_t + \rho \cdot y_{i,t} + \varepsilon_{i,t+1}, \tag{2}$$

where $\Delta L_{i,t+1} = L_{i,t+1} - L_{i,t}$ is the change in fund i leverage from t to $t+1$, γ is the vector of predictive coefficients on economy-wide variables, ρ is the vector of coefficients on fund-specific variables, and the idiosyncratic error $\varepsilon_{i,t+1} \sim N(0, \sigma^2)$ is independent and identically distributed (i.i.d.) across funds and time. The set of firm-specific characteristics, $y_{i,t}$, includes lagged leverage, $L_{i,t}$, which allows us to estimate the degree of mean reversion of the leverage employed by funds. We capture fund-fixed effects in the constants c_i which differ across each fund.

We estimate the parameters $\theta = (c_i \, \gamma \, \rho \, \sigma^2)$ using a Bayesian algorithm which also permits estimates of non-observed leverage and other fund-specific variables. Appendix D contains details of this estimation. Briefly, the estimation method treats the non-reported variables as additional parameters to be inferred along with θ. As an important byproduct, the estimation supplies posterior means of leverage ratios where these are unobserved in the data. We use these estimates, combined with the observed leverage ratios, to obtain time-series estimates of aggregate hedge fund leverage and leverage for each sector. Since we use uninformative priors, the special case where both the regressors and regressands in Eq. (2) are all observed in the data is equivalent to running standard ordinary least squares (OLS) regression.

An advantage of our procedure is that we are able to use all observations after imposing the data filters. Using OLS would result in very few funds and observations because both

[13] We also investigate the forecastability of proportional leverage changes, $\Delta L_{i,t+1}/(1 + L_{i,t})$, in the same regression specification of Eq. (2). The results are very similar to the results for leverage changes.

the complete set of regressors and the regressand must be observed. Taking only observed leverage produces a severely biased sample as different types of funds report at quarterly or lower frequencies versus the monthly frequency. Sudden stops in leverage reporting correlate with unexpected bad performance. Linearly interpolating unobserved leverage produces estimates that are too smooth because it relies on filling in points based on the mean reversion properties of leverage alone. We show below that other variables significantly predict leverage, both in the time series and cross-section.

4.2. Contemporaneous Model

The model in Eq. (2) is a predictive model where leverage over the next period is forecastable by macro and fund-specific variables at the beginning of the period. We consider an alternative model where leverage is determined contemporaneously with instruments:

$$L_{i,t} = c_i + \gamma \cdot x_t + \rho \cdot y_{i,t} + \epsilon_{i,t}, \tag{3}$$

where we use the same set of macro variables in x_t as in the predictive model (2), but we now assume that the fund-specific variables, $y_{i,t}$, do not include lagged leverage.

In Eq. (3), the potential observable determinants of leverage like VIX, interest rate spreads, hedge fund flows, etc. in x_t and $y_{i,t}$ are persistent. The unobserved determinants, which are in the error term $\epsilon_{i,t}$, are also likely to be persistent so we specify that the errors are serially correlated and follow

$$\epsilon_t = \phi_\epsilon \epsilon_{t-1} + v_t, \tag{4}$$

where $v_t \sim$ i.i.d. $N(0, \sigma^2)$. It can be shown that accounting for the persistence in the regressands in Eq. (3) through VAR or autoregressive specifications produces a reduced-form model of the same form as Eq. (2), except without a lagged leverage term. The relation between Eq. (2) and (3) involves the persistence of the regressands and the strength of the serial correlation, ϕ_ϵ, of the error terms. Appendix D describes the estimation of the contemporaneous system and compares it with the predictive model.

The contemporaneous model (3) can be used to test various theories on the determinants of hedge fund leverage. It is important to note, however, that Eq (3) is not a structural model. Many of the fund-specific variables, and perhaps some of the macro variables, are jointly endogenously determined with hedge fund leverage. Put another way, while Eq. (3) can shed light on contemporaneous correlations between hedge fund leverage and various instruments, it is silent on causation. We can expect that some variables that are contemporaneously associated with hedge fund leverage in Eq. (3) can have the opposite sign when used as a predictor of hedge fund leverage in Eq. (2). Some of this can be due to the effect of the serially correlated errors in the contemporaneous specification or that the contemporaneous vs. predictive relations between certain variables and leverage are indeed different.

5. Empirical Results

5.1. Time Series of Leverage

5.1.1. Gross Leverage

We begin our analysis by presenting the time series of gross leverage of hedge funds. This is obtained using the model in Eq. (2) with all macro and fund-specific variables and fund-fixed effects. We graph gross hedge fund leverage for all hedge funds and the hedge fund sectors in Fig. 4.. We report the posterior mean of gross leverage across all hedge funds in the solid line. Gross leverage is stable at approximately 2.3 until mid-2007 where it starts to decrease from 2.6 in June 2007 to a minimum of 1.4 in March 2009. At the end of our sample, October 2009, we estimate gross leverage across hedge funds to be 1.5. Over the whole sample, average gross leverage is 2.1. As expected from the fairly smooth transitions in Fig. 4., gross leverage is very persistent with an autocorrelation of 0.97.

The patterns of gross leverage for all hedge funds are broadly reflected in the dynamics of the leverage for hedge fund sectors, which are also highly persistent with correlations well above 0.95. Leverage for event-driven and equity funds is lower, on average, at 1.3 and 1.6, respectively, than for all hedge funds, which have an average gross leverage of 2.1 over the sample. Both the event-driven and equity sectors reach their highest peaks of gross leverage in mid-2007 and gradually decrease their leverage over the financial crisis. Event-driven leverage falls below one and reaches a low of 0.8 in December 2008 before rebounding. Credit funds steadily increase their gross leverage from 1.5 at the beginning of 2005 to reach a peak of 3.9 at June 2007. This decreases to 1.1 at the end of the sample.

Fig. 4. shows that the most pronounced fall in leverage is seen in the relative value sector: relative value gross leverage reaches an early peak of 6.8 in April 2006 and starts to cut back in early 2006. This is well before the beginning of the deterioration in subprime mortgages in 2007. In December 2007, gross leverage in relative value funds falls to 4.5 and decreases slightly until a sharp increase over April to June 2008 to reach a local high of 5.8 in June 2008. These periods coincide with increasing turbulence in financial markets after the purchase of Bear Stearns by JP Morgan Chase in March 2008 and the illiquidity of many securitized asset markets.[14] The increasing leverage in early 2008 in relative value is not due to any one fund; several large funds in the database exhibit this behavior and, in general, the leverage of all relative value funds over the financial crisis is volatile. From June 2008 gross leverage of the relative value sector decreases from 5.8 to 2.3 at October 2009. Over the whole sample, relative value gross leverage is 4.8.

5.1.2. Dispersion of Gross Leverage

While Fig. 4. shows the average hedge fund leverage, an open question is how the cross-section of leverage changes over time. We address this in Fig. 5. which plots the median and the cross-sectional interquartile range (25th and 75th percentiles) of gross leverage. The cross-sectional distribution of all leverage measures does change, but is fairly stable across

[14] Relative value strategies (e.g., capital structure arbitrage and convertible bond arbitrage) tend to be more sensitive to the relative relation between securities and asset classes than credit, equity, and event-driven strategies, which tend to be based more on single-security fundamentals. When markets showed signs of normalizing after the Bear Stearns takeover in March 2008, many relative value strategies were quick to reapply leverage to take advantage of the stabilized and converging valuations. This period of improved market conditions was brief as new financial sector shocks occurred during the Summer of 2008, at which time relative value managers quickly brought leverage down.

the sample. Since there are some funds with very large leverage in our sample, the median falls closer to the 25th percentile than to the 75th percentile for all the leverage ratios. During 2005 to early 2007, the interquartile range for gross hedge fund leverage stays in the range 1.0 to 1.3. During mid-2007, the interquartile cross-sectional dispersion increases to 1.6 in May 2007 and then falls together with the overall decrease in leverage during this period. Interestingly, the largest decline in leverage in 2008 during the financial crisis is not associated with any significant change in the cross-section of hedge fund leverage. In summary, although hedge fund leverage is heterogeneous, the cross-sectional pattern of hedge fund leverage is fairly stable and in particular, does not significantly change in 2008 when the overall level of leverage is declining.

5.1.3. Gross vs. Net and Long-only Leverage

In Fig. 6. we plot gross, net, and long-only leverage across all hedge funds (top panel) and for hedge fund sectors (bottom four panels). The lines for gross leverage are the same as Fig. 4. and are drawn so we can compare net and long-only leverage. Fig. 6. shows that the three leverage measures, for all hedge funds and within the hedge fund sectors, are highly correlated and have the same broad trends. Table 3 reports correlations of the gross, net, and long-only leverage and they are all high. In particular, gross, net, and long-only leverage all have pairwise correlations above 0.92 in Panel A.

Panel B of Table 3 reports the correlations of gross, net, and long leverage for the hedge fund sectors. If there are no independent active short bets, then the correlations of all leverage measures should be one. Thus, we can infer the extent of the separate management of long and short positions by examining the correlations between gross and net leverage. The correlation of net and gross leverage is lowest for equity hedge funds, at 0.49, and above 0.80 for the other hedge fund sectors. This is consistent with funds in the equity sector most actively separately managing their long and short bets. In contrast, the highest correlation between net and gross leverage is 0.88 for relative value funds, which indicates these funds are most likely to take positions as long-short pairs.

One difference between the leverage measures in Fig. 6. is that the net and long-only leverage ratios are smoother than gross leverage. For all hedge funds the standard deviation of gross leverage is 0.36, whereas the standard deviations for net and long leverage are 0.14 and 0.25, respectively. Thus, hedge funds manage the leverage associated with active long and short positions in different ways. This pattern is also repeated in each of the hedge fund sectors. The largest difference in the volatility of gross leverage compared to net leverage is for relative value, where gross and net leverage standard deviations are 1.22 and 0.20, respectively. The mean of net leverage for relative value is also much lower, at 0.82, than the average level of gross leverage at 4.84. The low volatility of net leverage for relative value funds is consistent with these funds maintaining balanced long-short positions where a large number of their active bets consist of taking advantage of relative pricing differentials between assets. The stable and low net leverage for relative value funds could also imply that focusing on gross leverage overstates the market risk of this hedge fund sector.

An interesting episode for equity hedge funds is the temporary ban on shorting financial stocks which was imposed in September 2008 and repealed one month later (see Boehmer, Jones, and Zhang, 2009, for details). Equity hedge fund leverage was already trending downwards prior to this period beginning in mid-2007 and there is no noticeable additional effect in September or October 2008 for gross leverage or long-only leverage. However, Fig. 6. shows there is a small downward dip in net leverage during these months with net leverage being 0.48, 0.44, and 0.50 during the months of July, September, and October 2008, respectively. Thus, this event seems to affect the short leverage positions of equity funds, but the overall effect is small. This could be because the ban affected only the financial sector or because these hedge funds were able to take offsetting trades in derivatives markets or other non-financial firms to maintain their short positions.

Finally, we observe a high level of covariation for net and long-only leverage in Fig. 6. across all hedge funds and within sectors. This is similar to the high degree of comovement of gross leverage across sectors in Fig. 4.. We report correlations for all hedge funds and across sectors for each leverage measure in Table 4. These cross correlations are high indicating that each leverage measure generally rises and falls in tandem for each hedge fund sector.

In particular, Panel A shows that although the relative value sector contains the smallest number of funds, the correlation of gross leverage of relative value with all hedge funds is 0.93. The lowest correlation is between relative value and event driven, at 0.65. Put another way, looking at gross leverage across all hedge funds is a good summary measure for what is happening to gross leverage in the various hedge fund sectors. Panels B and C also show that this is true for net and long-only leverage. Thus, sector-level variation in hedge fund leverage is similar to the aggregate-level behavior of leverage across all hedge funds.

5.2. Macro Predictors of Hedge Fund Leverage

In this section, we discuss the ability of various macro and fund-specific variables to predict hedge fund leverage. We first report estimates of the predictive model in Eq. (2) taking only economy-wide variables and report the results in Table 5. We consider gross leverage in Panel A, net leverage in Panel B, and long-only leverage in Panel C. In all regressions we include lagged leverage as an independent variable. Regressions (1)–(8) add each macro variable one at a time together with lagged leverage, while all variables jointly enter regression (9). We use fund-level fixed effects in all regressions. In each panel, the coefficients on lagged leverage are negative with very high posterior *t*-statistics. The lagged leverage coefficients range from -0.20 to -0.31 indicating that hedge fund leverage is strongly mean-reverting.

Panel A, which reports results for gross leverage, shows that all the macro variables, with the exception of aggregate hedge fund flows, significantly predict changes in hedge fund leverage when used in conjunction with past leverage. The largest coefficient in magnitude is on investment bank CDS protection, where for a 1% increase in CDS spreads, next-month hedge fund leverage shrinks by 11.5%, on average. As investment banks perform well (regression (2)) or the S&P 500 posts higher returns (regression (3)), hedge fund leverage tends to increase next month. We observe that when volatility increases, as measured by VIX (regression (4)), or assets become riskier, as measured by the TED spread (regression (6)), hedge fund leverage tends to decrease over the next month. This is consistent with hedge funds targeting a specific risk profile of their returns, where an increase in the riskiness of the assets leads to a reduction in their exposure. In particular, a 1% movement in VIX predicts that gross leverage declines by 0.9% over the next month and a 1% increase in the TED spread predicts gross leverage will fall over the next month by 15.2%.

In regression (5), the sign on LIBOR is unexpectedly positive. We might expect increases in funding rates, of which LIBOR should be a large component, to decrease future leverage. Instead, the coefficient on LIBOR is positive at 4.35. This is surprising given that Fig. 4. shows that hedge fund leverage decreases before and during the financial crisis. However, in the joint regression (9), the coefficient on LIBOR flips sign and is now negative at -6.66. Thus, controlling for other variables, which are significantly correlated especially over the 2007–2009 period, produces the expected negative relation between LIBOR and future leverage changes. In fact, LIBOR, the TED spread, CDS spreads, and VIX are very highly correlated, all around 90%, and capture common effects associated with the financial crisis over the sample period. Thus, it is not surprising that the coefficient on VIX also becomes insignificant in the joint regression (9). In contrast, the term spread coefficients are consistently negative as expected, which implies that higher expected funding costs reduce leverage next period.

In regression (9), where we take all macro variables together, the predictors of hedge fund leverage which have posterior *t*-statistics greater than two in absolute value are investment bank CDS spreads, the lagged S&P 500 return, LIBOR, and the term spread. Increases in current funding costs, as measured by CDS spreads and LIBOR predict decreases in leverage, as do increases in future expected funding costs, as measured by the term spread.

In Panels B and C of Table 5, we report estimates of the same regressions for net and long-only leverage. In Panel B, all the coefficients on the macro variables are significant in the bivariate regressions (1)–(8), with the same signs as Panel A for gross leverage but with smaller magnitudes. However, there are no significant macro predictors of net leverage in the joint regression (9). Thus, overall net leverage is mostly determined only by its lagged value. Said differently, the only significant distinguishing feature of net leverage predictability is that it is highly mean-reverting. In Panel C, long-only leverage is significantly predicted by each individual macro variable in regressions (1)–(8) with the same signs as gross leverage in Panel A. The last column in Panel C for regression (9) reports that increases in the cost of investment bank CDS protection and the term spread significantly lower future long leverage. This indicates that most of the predictability in gross leverage by macro determinants in Panel A is coming from the predictability of long-only leverage by macro variables.

5.3. Fund-specific Predictors of Hedge Fund Leverage

In Table 6 we examine the ability of fund-specific variables to predict hedge fund leverage. All the regressions in Table 6 include the macro predictors used in Table 5 which are not reported as they have the same signs, same significance levels, and approximately the same magnitudes, as the coefficients reported in the macro-only regressions of Table 5.

The main surprising result of Table 6 is that, with one exception, all of the fund-specific variables have insignificant coefficients. This is for both the case of the bivariate regressions (1)–(4), where the fund-specific variables are used together with past leverage, and in the case of the joint regression (5). This occurs for all three measures of leverage in Panels A-C. Moreover, the adjusted R^2s of the macro-only specifications in Table 5 are almost identical to their counterparts in the fund-specific variable specifications in Table 6. This finding suggests that hedge funds exhibit a high degree of similarity in their leverage exposures that depends largely only on the aggregate state of the economy. Said differently, predictable changes in hedge fund leverage are mostly systematic and there are few fund-level idiosyncratic effects.[15]

The only fund-specific variable that has a posterior *t*-statistic larger than two is hedge fund return volatility. In Panel A for gross leverage, this variable has a coefficient of -1.41 in the joint regression (5) with a posterior *t*-statistic of -2.11. The bivariate regression (2) also has a similar coefficient on fund-specific volatility of -1.34 with a posterior *t*-statistic of -

[15] Our filters remove young hedge funds which tend to be smaller and tend to have higher funding costs. Thus, our data filters could account for the lack of a relation between AUM and hedge fund leverage. The lack of a relation between past flows and leverage can be due to notice period, lockups, and gates restrictions (see, for example, Ang and Bollen, 2010), which give managers advance notice of flows before they actually occur.

1.93. In the deleveraging cycles of Brunnermeier and Pedersen (2009) and others, fund return volatility affects margins and since margins correspond to limits in leverage, increases in fund return volatility should lead to lower leverage levels of hedge funds. Thus, our findings confirm the prediction of Brunnemeier and Pedersen of a significantly negative coefficient on return volatility. This is essentially the only significant fund-specific effect and it occurs only for gross leverage.

5.4. Contemporaneous Relations with Hedge Fund Leverage

We now investigate the contemporaneous relations of gross leverage in the model in Eq. (3) with macro and fund-specific variables. Table 7 reports the regression coefficients of the contemporaneous model (3) and compares them with the predictive model (2), which are identical to regression (9) of Table 5 for the macro-only predictors and regression (5) of Table 6 for the fund-specific predictors.

The contemporaneous model has significantly lower adjusted R^2s than the predictive model, at 0.08 vs. 0.13 for the macro-only system and 0.09 vs. 0.13 for the fund-specific variable system. Thus, the fit of the contemporaneous model without lagged leverage is worse than the predictive system with lagged leverage. Hence, the lagged leverage coefficient is an extremely important predictor. The contemporaneous model does have significantly autocorrelated error terms, with estimates of ϕ_ϵ of 0.25 and 0.55 for the macro-only and fund-specific variable cases, respectively. As a specification check, we compute the autocorrelation of error terms in the predictive specification. This turns out to be 0.03. Thus, absorbing the persistence of leverage by past leverage on the right-hand side (RHS) absorbs most of the serial correlation effects—when lagged leverage is included as a regressor, there seems to be little gained by making the error terms autocorrelated.

Table 7 shows two major differences in sign between the predictive model coefficients and the contemporaneous determinants of leverage in the macro-only specification. First, the coefficient on the S&P 500 return is positive at 0.67 in the predictive model and negative at -0.94 in the contemporaneous model. As the stock market increases, leverage contemporaneously decreases—by definition, as asset values increase. But, higher stock returns in the past forecast that hedge fund leverage will increase in the future.

Second, the coefficient on LIBOR is contemporaneously positive, at 3.44, but insignificant, in the contemporaneous model compared to a significantly negative coefficient of -6.66 in the predictive model. We expect the coefficient to be negative, which it is in the predictive regression. The unexpected positive sign in the contemporaneous model could be due to lack of power or the fact that true funding costs could have much shorter duration and be more variable than LIBOR. The LIBOR interest rate is, of course, a valid predictor even though it could be an inferior instrument to proxy for leverage costs in a contemporaneous model.

The coefficient on VIX and on aggregate hedge fund flows have the same sign in the predictive and contemporaneous systems, but while their effects are statistically insignificant in predicting hedge fund leverage, they are significantly contemporaneously correlated. In the contemporaneous model, VIX has a coefficient of -1.43 with a posterior *t*-statistic of -4.79. When VIX increases, it is well-known that asset prices fall (the leverage effect), which accounts for the negative contemporaneous coefficient. This finding is also consistent with the prediction of Fostel and Geanakoplos (2008), among others, where leverage decreases during times of high volatility. It is also consistent with hedge funds increasing (decreasing) leverage during less (more) volatile times to achieve a desired target level of volatility. As a predictor, the forecasting ability of VIX for future leverage is largely subsumed by lagged leverage as a regressor. The finding that aggregate hedge fund flows are contemporaneously correlated with hedge fund leverage goes against Stein (2009), who predicts that the entry of new capital should decrease the leverage of arbitrageurs.

The last two columns of Table 7 report coefficients for fund-specific variables for the predictive and contemporaneous systems, where both estimations control for the macro variables. The results are similar. The only significant variable in both cases is the fund's rolling 12-month volatility of returns. The effect, however, is much stronger contemporaneously (with a coefficient of -4.35 and a posterior *t*-statistic of -2.35) compared to the predictive model (with a coefficient of -1.41 with a posterior *t*-statistic of -2.11). While the negative forecasting ability of fund-specific volatility for future leverage is consistent with deleveraging cycle models, the contemporaneous relation is even stronger. Like the effect of VIX, this can be a reflection of the leverage effect, but it is also consistent with hedge funds using leverage to target a desired level of volatility.

5.5. Hedge Fund Leverage vs. Finance Sector Leverage

In this section we compare hedge fund leverage to the leverage of listed financial companies. We focus on aggregate gross hedge fund leverage, but our previous results show that the net and long-only leverage ratios exhibit similar patterns both for all hedge funds and within hedge fund sectors. We define the leverage of listed firms as the value of total assets divided by market value, that is, we study market leverage. Other authors studying the leverage of financial institutions like Adrian and Shin (2009, 2010), among others, use book leverage rather than market leverage. We use market leverage because the market equity value is closest to the NAV of a hedge fund (see Appendix A). We compare hedge fund leverage to the leverage of banks, investment banks, and the entire finance sector, which we describe in more detail in Appendix B.[16]

Fig. 7. plots the average level of gross hedge fund leverage in the solid line using the left-hand scale and plots the leverage of the financial sectors in various dashed lines on the right-hand scale. The level of gross hedge fund leverage is the same as in Fig. 4. and starts to decline in mid-2007. Gross hedge fund leverage is modest, between 1.5 and 2.5, compared to the leverage of listed financial firms: the average leverage of investment banks and the whole finance sector over our sample are 14.2 and 9.4, respectively. Fig. 7. shows that leverage in each of the banking and investment banking subsectors and the whole finance sector are highly correlated. Finance sector leverage starts to rise when hedge fund leverage starts to fall in 2007, continues to rise in 2008, and then shoots up in early 2009 before reverting back to more normal levels in late 2009. This counter-cyclical behavior of financial leverage,

[16] He, Khang, and Krishnamurthy (2010) contrast the behavior of commercial and investment bank leverage and show they are different. However, many investment banks were either acquired or became commercial banks during the financial crisis. Since our focus is on hedge fund leverage, we choose to contrast hedge fund leverage with the leverage of all of these institutions.

where market leverage increases during bad times, is consistent with the model of He and Krishnamurthy (2009).[17]

The remarkable takeaway of Fig. 7. is that hedge fund leverage is counter-cyclical to the market leverage of financial intermediaries. As hedge fund leverage declines in 2007 and continues to fall over the financial crisis in 2008 and early 2009, the leverage of financial institutions continues to inexorably rise. The highest level of gross hedge fund leverage is 2.6 at June 2007, well before the worst periods of the financial crisis. In contrast, the leverage of investment banks is 10.4 at June 2007 and severely spikes upward to reach a peak of 40.7 in February 2009. During this month, the U.S. Treasury takes equity positions in all of the major U.S. banks. In contrast, hedge fund leverage is very modest at 1.4 at that time. Note that hedge fund leverage started to decline at least six months before the financial crisis began in 2008.

We show the counter-cyclical behavior of hedge fund leverage to finance sector leverage more completely in Table 8. We report correlation matrices of gross, net, and long-only hedge fund leverage in Panels A-C, respectively, with banks, investment banks, and the finance sector. These correlations are very negative. For example, the correlations of gross leverage for all hedge funds with the finance sector are -0.88, -0.82, and -0.88 for banks, investment banks, and the finance sector, respectively. The correlations are very similar for each listed finance sector. The correlations between financial firms and hedge funds are also highly negative for each hedge fund strategy. Clearly, hedge fund leverage moves in the opposite way during the financial crisis to the leverage of regulated and listed financial intermediaries.

There are at least two explanations for the counter-cyclical behavior of hedge fund leverage with respect to listed financial intermediary leverage. First, hedge funds voluntarily reduced leverage much earlier than banks as part of their regular investment process of search-

[17] Other authors like Fostel and Geanakoplos (2008), Adrian and Shin (2009, 2010), and Shleifer and Vishny (2010) emphasize the pro-cyclicality of leverage. Many of these authors focus on accounting or book leverage rather than market leverage. Market leverage increases to very high levels during the financial crisis because stock prices of financial institutions are very low at this time.

ing for trades with excess profitability and funding them. An alternative explanation is that the reduction of hedge fund leverage was involuntary. Hedge funds often obtain their leverage through prime brokers which are attached to investment banks and other financial firms. The change in hedge fund leverage could be caused by the suppliers of leverage to hedge funds curtailing funding. Risk managers in the prime brokerage divisions of investment banks could have been prescient in partially forecasting the turbulent periods in 2008 and forced hedge funds to reduce leverage earlier. Only when times were very bad in late 2008 did investment banks adjust their own balance sheet leverage. While this story cannot be refuted, the substantial lead time of six to eight months, shown clearly in Fig. 7., where hedge funds reduced leverage before 2008 makes this unlikely. Furthermore, anecdotal evidence through the Fund's industry contacts suggests that prime brokers were not substantially increasing funding costs in early to mid-2007.

5.6. Hedge Fund vs. Finance Sector Exposure

We last attempt to measure the dynamic total exposure of the hedge fund industry. We do this by multiplying leverage by AUM to obtain an estimate of the total exposure. This exercise is, of course, subject not only to the estimation error of our procedure, but also the measurement error of total hedge fund AUM. Since hedge funds are not required to report, the estimates of aggregated hedge fund AUM in the public databases are probably conservative. Thus, our estimated levels of hedge fund exposure have to be interpreted carefully.

Fig. 8. plots total hedge fund exposure by taking the estimated gross leverage across hedge funds and aggregated hedge fund AUM reported from the Barclay Hedge database. In the top panel, we plot hedge fund exposure in the solid line (left-hand scale) and hedge fund AUM in the dashed-dot line (right-hand scale) in trillions of dollars. The correlation between the two series is 0.83. Both AUM and exposure increase over 2006 and 2007 and start falling after June 2008. The total hedge fund exposure starts the sample in January 2005 at $2.5 trillion, steadily increases, and then drops from a peak of $4.9 trillion in June 2008 to a low of $1.7 trillion in March 2009. This decrease represents an overall drop of 65%

from peak. The correlations of hedge fund AUM and total exposure with gross leverage are only 0.08 and 0.61, respectively. Note that the decrease in hedge fund leverage from 2007 to 2009 is from around 2.3 to 1.5. Thus, hedge fund exposure is primarily driven by AUM and the dramatic fall in total hedge fund exposure over the financial crisis is caused by investors withdrawing capital from the hedge fund sector. While many studies emphasize the role of leverage cycles, Fig. 8. highlights that inflows and outflows are important components of determining total exposure for hedge funds.

The bottom panel of Fig. 8. plots the total exposure and market value for investment banks for comparison. Exposure is defined as the total amount of assets held on the balance sheet. Investment bank and hedge fund exposure have similar patterns in the top and bottom panels of Fig. 8. and have a high correlation of 0.8. There is a sharp drop in investment bank assets in March 2009 which is due to large writedowns in balance sheets during this quarter. Total assets of investment banks decreased from $6.9 trillion in early 2008 to a low of $3.8 trillion in February 2009. Towards the end of the sample, assets rebounded to $5.2 trillion as financial markets stabilized.

We graph the relative exposure of hedge funds to investment banks and the finance sector in Fig. 9., which is measured as the ratio of hedge fund exposure to total assets for each of the investment banks and finance sector. The ratio of hedge fund exposure to investment banks (the finance sector) is approximately 65% (30%) until early 2008. Then, the events of the financial crisis in 2008 cause hedge fund exposure to decline to 40% and 15% of the total asset base of investment banks and the finance sector, respectively. Thus, total exposure of hedge funds is modest compared with the exposure of listed financial intermediaries, especially recently after the financial crisis, and it is modest even before the start of the financial crisis in mid-2007.

6. Conclusion

This paper presents, to our knowledge, the first formal analysis of hedge fund leverage using actual leverage ratios. Our unique data set from a fund-of-hedge-funds provides us with both a time series of hedge fund leverage from December 2004 to October 2009, which includes the worst periods of the financial crisis, and a cross-section to investigate the determinants of the dynamics of hedge fund leverage. We uncover several interesting and important results.

First, hedge fund leverage is fairly modest, especially compared with the listed leverage of broker/dealers and investment banks. The average gross leverage (including long and short positions) across all hedge funds is 2.1. While there are some funds with large leverage, well above 30, most hedge funds have low leverage partly due to most hedge funds belonging to the equity sector where leverage is low. Gross leverage for other hedge fund sectors like relative value is higher, at 4.8, over the sample.

Second, hedge fund leverage is counter-cyclical to the market leverage of listed financial intermediaries. In particular, hedge fund leverage decreases prior to the start of the financial crisis in mid-2007, where the leverage of investment banks and the finance sector continues to increase. At the worst periods of the financial crisis in late 2008, hedge fund leverage is at its lowest while the leverage of investment banks is at its highest. We find that the dispersion of hedge fund leverage does not markedly change over the financial crisis and that the leverage of each hedge fund sector moves in a similar pattern to aggregate hedge fund leverage. However, we find that the total exposure of hedge funds is similar to the total exposure of investment banks even though the behavior of leverage is different. The main reason for this similar behavior is not the change in hedge fund leverage, but the withdrawal of assets from the hedge fund industry during 2008.

Third, we find that the predictability of hedge fund leverage is mainly from economy-wide, systematic variables. In particular, decreases in funding costs as measured by LIBOR, interest rate spreads, and the cost of default protection on investment banks predict increases in hedge fund leverage over the next month. Increases in asset prices measured by lagged market returns also predict increases in hedge fund leverage. We find the only fund-specific variable significantly predicting hedge fund leverage is return volatility, where increases in fund return volatility tend to reduce leverage. There is little evidence that hedge fund leverage changes are predictable by hedge fund flows or assets under management. Contemporaneously, hedge fund leverage decreases when VIX or fund-specific volatility increase and hedge fund leverage is positively related to aggregate hedge fund flows.

An interesting direction for future work is to study hedge fund leverage and returns, since in theory, when managers perceive better investment opportunities, they should increase leverage. Thus, leverage levels can provide a crude measure of a hedge fund manager's market outlook. Existing empirical work finds little relation at an unconditional level between leverage and returns at the stock level (see, e.g., Bhandari, 1988; Fama and French, 1992), which could be due to not accounting for endogenous leverage and investment choices. Hedge funds are a good laboratory to examine the relation between dynamic leverage management and returns because the underlying asset returns are more easily measured than the asset returns of corporations.

Appendix A. Examples of hedge fund leverage

In order to illustrate how our definitions of leverage differ for various portfolios, we present several simple examples of highly stylized hedge funds. In all our examples, we assume no fees are paid so the gross value of the fund is the same as the net value of the fund. All the transactions are done instantaneously and we report the overall balance sheet of the fund at the same date. For simplicity, assume there is only one share so the NAV per share is the same as the AUM of the fund.

Example 1: Long-only fund

Consider a hedge fund that has just obtained \$10 in cash from investors. The hedge fund manager purchases securities worth \$10. In addition, the hedge fund manager borrows \$50 and invests those proceeds in a \$50 long securities position. The NAV of the hedge fund is the difference between the long and short positions, which is \$10, and is the same as the initial investment by investors. The balance sheet of the hedge fund after these transactions can be represented by:

Long assets	Short assets
\$60 Long securities	\$50 Borrowed cash
	\$10 NAV

In this case, the hedge fund has \$60 of Long securities and \$0 of Short securities on its balance sheet. As a result, gross leverage is $60/10 = 6$, net leverage is $60/10 = 6$, and long-only leverage is also 6. All these leverage measures coincide because there are no risky asset short positions and the long positions are levered by short cash positions.

Note that an unlevered long-only fund, which holds long asset positions between zero and one together with cash, has positive leverage ratios less than one. All three leverage ratios—gross, net, and long-only—also coincide. In comparison, a corporate finance definition of leverage where assets are the sum of debt and equity would result in a zero leverage measure. This is because cash is counted as an asset on corporate balance sheets, but in our leverage definitions, only risky assets are included in the leverage measures.

Example 2: Dedicated long-short fund

Suppose a fund with an initial cash endowment of \$10 uses that cash to purchase a \$10 long security position. In addition, the fund places \$50 in long-short bets in risky assets. The balance sheet of the fund is:

Long assets	Short assets
\$60 Long securities	\$50 Short securities
	\$10 NAV

In this case, gross leverage is $(60 + 50)/10 = 11$, net leverage is $(60 - 50)/10 = 1$, and long-only leverage is $60/10 = 6$. Now all three leverage measures are different because of the presence of the active short position. In particular, the active short bet in this example induces the marked difference between gross and net leverage.

Example 3: General levered fund

Consider a fund with the following balance sheet:

Long assets	Short assets
\$20 Long securities	\$8 Short securities
	\$2 Borrowed cash
	\$10 NAV

In this example, the fund obtains leverage by both a short cash position as well as a short position in risky assets. The gross leverage is $(20 + 8)/10 = 2.8$, net leverage is $(20 - 8)/10 = 1.2$, and long-only leverage is $20/10 = 2$. In this example, the long position is leveraged by both short security positions, which could be active bets or passive hedges, and a short cash position. Note that whereas net leverage in Example 2 is equal to one, the combination of short risky and cash positions causes net leverage to be different from one.

Example 4: Dedicated short fund

Our final example is a dedicated short fund. The fund starts with \$10 cash, which it pledges as a collateral to borrow \$50 worth of assets. This represents a margin (haircut) of 20%. The proceeds from selling the securities result in cash received by the fund. These positions represent \$60 of cash on the asset side of the balance sheet and \$50 of short securities on the liability side of the fund's balance sheet:

Long assets	Short assets
\$60 Long cash	\$50 Short securities
	\$10 NAV

In this case, the hedge fund has \$0 of Long securities and \$50 of Short securities on its balance sheet. Hence, the fund's gross leverage is $(0 + 50)/10 = 50/10 = 5$, the net leverage is $(0 - 50)/10 = -50/10 = -5$, and the long-only leverage is $0/10 = 0$. In the case when net leverage is negative, the fund is said to be net short, otherwise it is said to be net long. Since the fund is taking only active short positions, the leverage on the long-side of the balance sheet is zero.

In the case of a fund buying or selling derivative securities instead of transacting in the physical or cash market, the previous examples hold if the derivatives are decomposed into underlying, but time-varying, positions in physical assets and risk-free securities at the reporting date. At a given time, once the derivatives are decomposed into replicating positions in underlying securities, the same leverage calculations can be performed.

Appendix B. Macro data sources

This appendix describes data sources of the macro variables and the construction of leverage for investment banks, bank holding companies, and the financial sector.

B.1. Macro variables

The list of macro variables is:

Investment bank (IB) CDS protection. We take credit default swap (CDS) spreads on 10-year senior bonds of the following institutions, with tickers in parentheses: Bear Stearns (BSC), Citigroup (C), Credit Suisse (CS), Goldman Sachs (GS), HSBC (HBC), JP Morgan (JPM), Lehman Brothers (LEH), Merrill Lynch (MER), and Morgan Stanley (MS). While several of these firms are mainly commercial banks with relatively small investment banking and proprietary trading activities compared to other firms in the list, we take these firms as representative of broker/dealer and investment banking activity. Merrill Lynch and Bear Stearns ceased to be independent entities in the sample and Lehman Brothers entered bankruptcy. Data on CDS prices are obtained from Bloomberg and market weights are taken from CRSP. The CDS contract is specified so that a buyer of protection pays premiums specified in percentage points per annum of a notional contract amount to a seller of protection. If the credit event (default) occurs, then the seller of protection has to deliver the underlying bond to the buyer of protection. We take CDS on 10-year senior bonds of the listed financial institutions. We market weight the CDS spreads using market capitalization data on common equity for those firms in existence at a given point in time.

Investment bank (IB) returns. We take monthly total returns on the investment banks from CRSP. These are market value weighted.

S&P 500 returns. This is the total return on the S&P 500 index taken from Standard & Poor's Index Services.

VIX. This is the monthly level of the VIX volatility index taken from Yahoo Finance.

LIBOR. We obtain the three-month LIBOR rate from Bloomberg.

TED spread. The TED spread is the difference between the three-month LIBOR yield and the three-month T-bill yield. We obtain the three-month T-bill rate from the St. Louis Fed.

Term spread. The term spread is defined to be the difference between the 10-year Treasury yield and the three-month T-bill. These are obtained from the St. Louis Fed.

Aggregate hedge fund flows. This is the past three-month flow on the aggregate hedge fund industry, at a monthly frequency, constructed from the Barclays Hedge fund database. This is computed following Section 3.2.2.

B.2. Financial sector leverage

We construct leverage for investment banks (BSC, C, CS, GS, HBC, JPM, LEH, MER, and MS), bank holding companies, and the entire financial sector using CRSP and Compustat data. Bank holding companies are defined as U.S.-based institutions with Standard Industrial Classification (SIC) codes which fall between 6000 and 6199. We define the financial sector as all U.S.-based companies with SIC codes between 6000 and 6299.

Leverage for the listed financial sub-sector is defined to be:

$$\frac{\sum_{i \in \text{sub-sector}} A_{i,t}}{\sum_{i \in \text{sub-sector}} MV_{i,t}} \tag{B.1}$$

for firm i at time t, $MV_{i,t}$ is the company i's market value obtained from CRSP as the product of number of shares outstanding and the closing price at the end of the month t, and $A_{i,t}$ is the total assets of the company obtained from COMPUSTAT. The assets are reported quarterly and we use the most recent, observable quarterly balance sheet report. Note that $A_{i,t}/MV_{i,t}$ is the market leverage of company i using the market value of common stock as the value of equity.

Appendix C. Examples of reported hedge fund leverage

Hedge funds report their leverage to investors in several formats, often with several measures of leverage. First, hedge funds periodically send their investors risk reports which list performance and risk statistics over the last reporting period. Table A.1 provides an extract of a risk exposure report from an actual hedge fund. This fund breaks down its exposure into different sectors and reports a gross leverage of 1.11, a net leverage of 0.22, and a long-only leverage of 0.66. This fund reports both long and short positions in each sector. These numbers are received by the Fund every reporting period.

Second, some hedge funds report leverage information in investor letters. An extract of an actual letter is:

> *We made 5.3% on the short book and lost 3.3% on the long book. Having started the month with 7% net long position, we were by mid-month slightly net short for the first time in the fund's history. Around mid-month we suspected that the market falls, triggered by subprime losses in the financial system, were coming to an end and decided to rebuild a modest 18% net long position, which is where we ended the month.*

From the text of the investor letter, we observe that net leverage at the end of the month is 0.18, but gross leverage and long-only leverage are not reported. However, the Fund is able to obtain more details on leverage, and other risk and performance characteristics of each hedge fund than reported in the investor letters by having analysts visit or call the funds to obtain further information. Thus, although the hedge fund officially does not report size of long and short exposure at this month, our data set contains this information.

Appendix D. Estimation

This appendix describes the conditional distributions used in the Gibbs sampler. We treat the unobserved data variables as additional parameters using data augmentation. A textbook exposition of these procedures is Robert and Casella (1999).

D.1. Predictive model

We rewrite the predictive model as:

$$Y_{i,t+1} = c_i + \beta_1 \cdot Y_{i,t} + \beta_2 \cdot X_{i,t} + \varepsilon_{i,t+1}, \tag{D.1}$$

where $Y_{i,t}$ is leverage of fund i at time t, the vector $X_{i,t}$ includes both fund-specific variables and economy-wide variables, and $\varepsilon_{i,t} \sim N(0, \sigma^2)$ and is i.i.d. across funds and time. The constant terms, c_i, captures fund-fixed effects. We are especially interested in the predictive coefficients, $\beta = (\beta_1\, \beta_2)$.

We cast the model in Eq. (D.1) into a measurement equation:

$$Y^*_{i,t+1} = Y_{i,t+1} + w_{i,t+1}, \tag{D.2}$$

where each observation error in $\{w_{i,t+1}\}$ is equal to zero if $Y_{i,t+1}$ is observed and if $Y_{i,t+1}$ is unobserved is distributed as $N(0, \sigma^2_w)$, where the measurement error is i.i.d. across funds and time and is orthogonal to $\varepsilon_{i,t+1}$. This extreme form of measurement error follows Sinopoli et al. (2004) and others and effectively eliminates observations which are observed from the set of measurement equations. This allows us to use a Kalman filter, with extreme heteroskedasticity, in the estimation (see below). We denote

$$\sigma^2_v = \sigma^2 + \sigma^2_w,$$

which is the total variance for observations where leverage is not reported.

We denote the parameters $\theta = (\beta\, \sigma^2\, \sigma^2_v)$ and partition the data $Y = \{Y_{i,t}\}$ and $X = \{X_{i,t}\}$ into observed and unobserved sets, $X = \{X^{obs}\, X^{unobs}\}$ and $Y = \{Y^{obs}\, Y^{unobs}\}$, where we denote the unobserved data with "unobs" superscripts. The set of observed data we denote as $\mathcal{Y} = \{X^{obs}\, Y^{obs}\}$. We use θ_- to denote the set of parameters less the parameter currently being drawn.

The set of conditional distributions in the Gibbs sampler is:

$p(\beta, c_i | \theta_-, \mathcal{Y}, X^{unobs}, Y^{unobs})$.

Conditional on X^{unobs} and Y^{unobs} being observed, Eq. (D.1) is a regular OLS regression and we can use a conjugate Normal draw. The dependent variable has two variances: if the regressor is observed in data the residuals have variance σ^2 and if the regressor is unobserved in data the residual variance is σ^2_v. Thus, we can rewrite Eqs. (D.1) and (D.2) as

$$Y = X\beta + V, \tag{D.3}$$

where $Y = \{Y_{i,t+1} - c_i\}$, $X = \{Y_{i,t}\, X_{i,t}\}$, and $V \sim N(0, \Sigma)$, where Σ is a diagonal covariance matrix with entries σ^2 or σ^2_v depending on whether the regressor is observed in data or not.

We estimate the fixed effects in each iteration by appropriately demeaning both sides of Eq. (D.3). For fund-fixed effects we subtract average values of the left-hand side and right-hand side variables for the observations that correspond to that fund. The fixed effects change in each iteration because the missing Y^{unobs} and X^{unobs} are updated.

$p(\sigma^2, \sigma^2_v | \theta_-, \mathcal{Y}, X^{unobs}, Y^{unobs})$

We draw σ^2 using a conjugate Inverse Gamma distribution given the regression (D.3) taking only the entries where the residual variance is σ^2. We can draw $\sigma^2_v = \sigma^2 + \sigma^2_w$ by taking the entries where the residual variance is σ^2_v. We ensure that $\sigma^2_v > \sigma^2$ in each draw.

$p(Y^{unobs} | \theta, \mathcal{Y}, X^{unobs})$

We can interpret the system for $Y_{i,t}$ as a state equation (D.1) and a measurement equation (D.2). This allows us to use a forward filtering backward sampling (FFBS) draw following Carter and Kohn (1994), except with (extreme) heteroskedasticity and exogenous variables. For notational simplicity, we suppress dependence on fund i below and use a FFBS draw separately on each fund i with missing values.

We run the Kalman filter to determine the conditional distributions of the unobserved variables,

$$Y_{t|t-1} \sim N(\mu_{t,t-1}, V_{t,t-1}),$$

where $Y_{t|t-1}$ is Y_t conditional on the history of observations up to and including $t-1$, which we denote as H_{t-1},

$$\mu_{t,t-1} = c + \beta_1 Y^*_{t-1} + \beta_2 X_{t-1}$$

and

$$V_{t,t-1} = \beta_1^2 V_{t-1,t-1} + \sigma^2,$$

treating the X_t values as exogenous.

When Y^*_t is added to the history, we have the joint distribution

$$\begin{pmatrix} Y_t \\ Y^*_t \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_{t,t-1} \\ \mu_{t,t-1} \end{pmatrix}, \begin{pmatrix} V_{t,t-1} & V_{t,t-1} \\ V_{t,t-1} & V_{t,t-1} + \sigma^2_w \end{pmatrix} \right). \tag{D.4}$$

Note that $\sigma^2_w = 0$ if Y_t is observed. From the moments of a partitioned normal, we have

$$Y_{t|t} = Y_t | Y^*_t, H_{t-1} \sim N(\mu_{t,t}, V_{t,t}), \tag{D.5}$$

where

$$\mu_{t,t} = \mu_{t,t-1} + \frac{V_{t,t-1}}{V_{t,t-1} + \sigma^2_w}(Y^*_t - \mu_{t,t-1}),$$

and

$$V_{t,t} = V_{t,t-1} - \frac{V^2_{t,t-1}}{V_{t,t-1} + \sigma^2_w} = \frac{V_{t,t-1}\sigma^2_w}{V_{t,t-1} + \sigma^2_w}.$$

Note that if $\beta_2 = 0$, this simplifies to a regular Kalman filter. We assume the initial distribution is

$$y_1 \sim N\left(\frac{c + \beta_2 \mathrm{E}X}{1 - \beta_1}, \frac{\sigma^2}{1 - \beta_1^2} \right),$$

which is the stationary distribution for Y_t assuming X_t is exogenous. We update as per a normal Kalman filter to obtain the distribution $y_{T|T}$ and the smoothed conditional values $y_{t|T}$. Once the Kalman filter is run forwards, we backwards sample following Carter and Kohn (1994).

$p(X^{unobs}|\theta, \mathcal{Y}, Y^{unobs})$

We assume that the regressand variables, both observed and unobserved, are all jointly normally distributed $N(\tilde{\mu}, \tilde{\Sigma})$. To draw the unobserved variables for fund i at time t, $X^{unobs}_{i,t}$, we have

$$X^{unobs}_{i,t} | X^{obs}_{i,t}, \theta, \mathcal{Y} \sim N(m, v^2), \tag{D.6}$$

where m and v^2 can be obtained by the mean and variance of a partitioned normal where

$$X_{i,t} = (X^{obs}_{i,t}\; X^{unobs}_{i,t}) \sim N(\tilde{\mu}, \tilde{\Sigma})$$

has been partitioned into the observed and unobserved components. A similar procedure is used by Li, Sarkar, and Wang (2003), except we recognize that $Y_{i,t}$ is endogenously persistent.

We update the values $\tilde{\mu}$ and $\tilde{\Sigma}$ each iteration by a conjugate normal distribution and conjugate Wishart draw, respectively.

We estimate with a burn-in period of 1,000 observations and 2,000 simulations. Convergence is extremely fast. We report in the tables a posterior mean for each parameter and a posterior *t*-statistic which is the ratio of the posterior mean and posterior standard deviation. This is to make inference comparable to a classical OLS estimation, which cannot handle missing observations.

During each iteration we compute adjusted R^2 statistics. We calculate the regular R^2 as

$$R^2 = 1 - \frac{SS_{residual}}{SS_{total}}, \tag{D.7}$$

where $SS_{residual}$ denotes the residual sum of squares, while SS_{total} denotes the total sum of squares. For our model that predicts values $Y_{i,t}$ by producing estimates $\hat{Y}_{i,t}$, $SS_{residual} = \sum_{i,t}(Y_{i,t+1} - \hat{Y}_{i,t+1})^2$ and $SS_{total} = \sum_{i,t}(Y_{i,t+1} - \bar{Y})^2$, where $\bar{Y}$ is the average value of $Y_{i,t}$ and $\hat{Y}_{i,t+1} = c_i + \beta_1 \cdot Y_{i,t} + \beta_2 \cdot X_{i,t}$ from Eq. (D.1). We record the adjusted R^2:

$$\text{adjusted } R^2 = 1 - (1 - R^2)\frac{n-k}{n-p-k}, \tag{D.8}$$

where the number of observations is n, the number of funds is k, and the number of explanatory variables is p. In the tables, we report the posterior mean of the adjusted R^2 statistic computed in each iteration.

D.2. Contemporaneous model

The estimation of the contemporaneous model in Eq. (3) is similar to the predictive model in Eq. (2), except that we must now account for serial correlation in the error terms. The model is

$$Y_{i,t} = \beta' X_{i,t} + \epsilon_{i,t}, \tag{D.9}$$

where for simplicity we ignore the fund-fixed effects. Fund i's idiosyncratic error term, $e_{i,t}$, follows the AR(1) process

$$\epsilon_{i,t} = \phi_\epsilon \epsilon_{i,t-1} + v_{i,t}, \tag{D.10}$$

where $v_{i,t} \sim N(0, \sigma^2)$. Similar to the predictive model, leverage may be unobserved at time t, so we employ the measurement Eq. (D.2).

We follow Chib (1993) in recasting Eqs. (D.9) and (D.10) as a regular OLS equation by defining

$$\begin{aligned} \tilde{Y}_{i,t} &= Y_{i,t} - \phi_\epsilon Y_{i,t-1} \\ \tilde{X}_{i,t} &= X_{i,t} - \phi_\epsilon X_{i,t-1}. \end{aligned} \tag{D.11}$$

This allows us to write

$$\tilde{Y}_{i,t} = c_i + \beta' \tilde{X}_{i,t} + v_{i,t}, \tag{D.12}$$

which now has an i.i.d. error term. The corresponding measurement equation is

$$\tilde{Y}^*_{i,t} = \tilde{Y}_{i,t} + w_{i,t}, \tag{D.13}$$

where the observation error variance is $\sigma_v^2 = \sigma^2 + \sigma_w^2$ where $\tilde{Y}_{i,t}$ is unobserved and σ^2 if $\tilde{Y}_t$ is observed.

The set of conditional draws in the Gibbs sampler we use are:

$p(\beta|\theta_-, \mathcal{Y}, Y^{unobs})$

We draw β using a conjugate normal draw from the regression Eq. (D.12). There are two possible variances, σ^2 in the case $\tilde{Y}_{i,t}$ is observed and σ_v^2 in the case it is unobserved.

$p(\phi_\epsilon|\theta_-, \mathcal{Y}, Y^{unobs})$

Chib (1993) notes that Eq. (D.10) is a standard regression draw with ϵ_t given by Eq. (D.9). We draw ϕ_ϵ with a conjugate normal distribution.

$p(\sigma^2, \sigma_v^2|\theta_-, \mathcal{Y}^*, Y^{unobs})$, We draw σ_v^2 using a conjugate Inverse Gamma distribution from the regression Eq. (D.12). We ensure that $\sigma_v^2 > \sigma^2$ in each draw.

$p(Y^{unobs}|\theta, \mathcal{Y})$

Same as Section D.1.

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Table 1: Margin requirements by security type

	Margin (haircut)	Implied leverage
Treasuries	0.1–3%	33–100
Investment grade corp bonds	5–10%	10–20
High yield bonds	10–15%	6.6–10
Convertible bonds	15–20%	5–6.6
Equities	5–50%	2–20
Commodity futures	10%	10
Financial futures	3%	33
Foreign exchange futures	2%	50
Options (equity)	75%	1.3
Interest rate swaps	1%	100
Foreign exchange swaps	1%	100
Total return swaps	10%	10

The table lists the margin requirements and their implied level of leverage in various security markets. The data are obtained by collating information from prime brokers and derivatives exchanges as of March 2010.

Table 2: Summary statistics of data

Panel A: Number of observations

Strategy	Observations	Funds
Relative value (RV)	1414	36
Credit (CR)	875	21
Event-driven (ED)	1408	37
Equity (EQ)	4439	114
Total hedge funds	8136	208

Panel B: Fund-specific variables

	Mean	Standard deviation	Auto-correlation	% Observed
Observed gross leverage	2.130	0.616	0.680	82.0%
Observed net leverage	0.587	0.278	0.595	82.0%
Observed long-only leverage	1.360	0.382	0.690	82.1%
Past 1-month returns	0.003	0.031	0.241	100.0%
Past 12-month volatility	0.026	0.010	0.828	69.6%
Past 3-month flows	0.022	0.226	0.620	77.4%
Log AUM	8.528	0.143	0.883	85.0%

Panel A lists the number of observations and number of hedge funds broken down by strategy. Panel B reports summary statistics for the hedge fund variables across all funds. We report means, standard deviation, and autocorrelation of the monthly frequency variables. The means and standard deviation are computed using the full observed data while the autocorrelations are computed only using observations with adjacent months for each fund. We compute the variables for each fund and then report the average across funds for each variable. Hedge fund flows are computed using assets under management (AUM) and fund returns over the past three months following Eq. (1). The last column reports the percentage of observations that are observed in the data set. The data sample is from December 2004 to October 2009.

Table 3: Correlations of gross, net, and long-only leverage

	Gross	Net	Long-only	Gross	Net	Long-only
Panel A: All hedge funds						
Gross	1.000					
Net	0.927	1.000				
Long-only	0.994	0.962	1.000			
Panel B: Hedge fund sectors						
Relative value				Equity		
Gross	1.000			1.000		
Net	0.876	1.000		0.490	1.000	
Long-only	0.997	0.910	1.000	0.955	0.725	1.000
Event-driven				Credit		
Gross	1.000			1.000		
Net	0.835	1.000		0.805	1.000	
Long-only	0.974	0.938	1.000	0.981	0.904	1.000

The table reports correlations of the posterior means of gross, net, and long-only leverage for all hedge funds and for hedge fund sectors at a monthly frequency. Gross leverage is a sum of long and short exposures as a portion of assets under management (AUM). Net leverage is a difference of long and short exposures as a portion of AUM. Long-only leverage is the long exposure as a portion of AUM. The hedge fund leverage ratios consist of all observed hedge fund leverage and estimated hedge fund leverage when these are unobserved following Eq. (2) and the estimation method outlined in Appendix D using all macro and fund-specific variables and fund-fixed effects. The data sample contains 8136 monthly observations that cover 208 hedge funds during a period from December 2004 to October 2009.

Table 4: Cross-correlations of hedge fund leverage within sectors

	All hedge funds (HF)	Hedge fund strategies RV	EQ	ED	CR
Panel A: Gross leverage					
All hedge funds (HF)	1.000				
Relative value (RV)	0.930	1.000			
Equity (EQ)	0.761	0.557	1.000		
Event-driven (ED)	0.846	0.650	0.899	1.000	
Credit (CR)	0.836	0.738	0.853	0.786	1.000
Panel B: Net leverage					
All hedge funds (HF)	1.000				
Relative value (RV)	0.780	1.000			
Equity (EQ)	0.932	0.695	1.000		
Event-driven (ED)	0.963	0.657	0.857	1.000	
Credit (CR)	0.921	0.578	0.854	0.879	1.000
Panel C: Long-only leverage					
All hedge funds (HF)	1.000				
Relative value (RV)	0.923	1.000			
Equity (EQ)	0.866	0.683	1.000		
Event-driven (ED)	0.915	0.736	0.920	1.000	
Credit (CR)	0.877	0.751	0.917	0.857	1.000

The table reports correlations of the posterior means of leverage of hedge funds (HF) and average leverage of their specific strategies (RV, EQ, ED, CR) for each of the definitions of hedge fund leverage: Gross leverage (Panel A), Net leverage (Panel B), and Long-only leverage (Panel C) separately at a monthly frequency. Gross leverage is a sum of long and short exposures as a portion of assets under management (AUM). Net leverage is a difference of long and short exposures as a portion of AUM. Long-only leverage is the long exposure as a portion of AUM. The hedge fund leverage ratios consist of all observed hedge fund leverage and estimated hedge fund leverage when these are unobserved following Eq. (2) and the estimation method outlined in Appendix D using all macro and fund-specific variables and fund-fixed effects. The data sample contains 8136 monthly observations that cover 208 hedge funds during a period from December 2004 to October 2009.

Table 5: Macro predictors of hedge fund leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A: Gross leverage									
Past gross lev	-0.2446	-0.2228	-0.2250	-0.2423	-0.2378	-0.2288	-0.2401	-0.2347	-0.2447
	[-32.0]	[-28.8]	[-30.7]	[-31.8]	[-30.0]	[-29.5]	[-31.5]	[-30.9]	[-32.0]
IB CDS	-11.49								-9.3278
	[-12.4]								[-3.54]
IB ret		0.5968							-0.0436
		[6.11]							[-0.26]
S&P 500 ret			1.3684						0.6750
			[7.68]						[2.09]
VIX				-0.9238					-0.1010
				[-11.9]					[-0.51]
LIBOR					4.3489				-6.6629
					[7.66]				[-2.35]
TED spread						-15.19			7.5973
						[-8.64]			[1.90]
Term spread							-6.8214		-10.32
							[-9.54]		[-2.80]
Agg HF flows								7.7129	0.0934
								[1.15]	[0.38]
Adjusted R^2	0.130	0.118	0.121	0.129	0.120	0.122	0.123	0.120	0.131
Panel B: Net leverage									
Past net lev	-0.3114	-0.2931	-0.3003	-0.3013	-0.3053	-0.2965	-0.3036	-0.2959	-0.3052
	[-3.48]	[-3.75]	[-3.31]	[-4.22]	[-3.61]	[-3.49]	[-3.90]	[-3.86]	[-3.82]
IB CDS	-3.3967								-1.1898
	[-3.69]								[-1.04]
IB ret		0.2644							0.1340
		[5.88]							[1.83]
S&P 500 ret			0.5101						0.0784
			[5.92]						[0.57]
VIX				-0.2854					-0.1051
				[-4.83]					[-1.22]
LIBOR					1.4241				0.9969
					[2.75]				[0.85]
TED spread						-4.5400			-0.7010
						[-4.26]			[-0.43]
Term spread							-2.0531		0.5129
							[-3.16]		[0.34]
Agg HF flows								0.3295	-0.0668
								[3.29]	[-0.61]
Adjusted R^2	0.155	0.150	0.151	0.155	0.151	0.149	0.153	0.149	0.156

Table 5 Continued

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel C: Long-only leverage									
Past long lev	-0.2376	-0.2157	-0.2177	-0.2351	-0.2301	-0.2219	-0.2324	-0.2273	-0.2373
	[-31.2]	[-27.0]	[-29.1]	[-31.3]	[-28.4]	[-29.9]	[-29.4]	[-30.1]	[-29.9]
IB CDS	-6.9342								-4.9876
	[-12.6]								[-3.39]
IB ret		0.4228							0.0433
		[6.77]							[0.40]
S&P 500 ret			0.9124						0.3891
			[8.52]						[1.92]
VIX				-0.5741					-0.0918
				[-12.7]					[-0.79]
LIBOR					2.5667				-2.8146
					[7.59]				[-1.58]
TED spread						-9.4262			3.2221
						[-8.51]			[1.31]
Term spread							-4.0850		-4.6731
							[-9.48]		[-2.00]
Agg HF flows								0.6891	0.0152
								[7.81]	[0.10]
Adjusted R^2	0.126	0.116	0.118	0.126	0.116	0.118	0.119	0.117	0.127

The table reports regression coefficients of Eq. (2) to predict changes in gross leverage (Panel A), net leverage (Panel B), and long-only leverage (Panel C) over the next month. Gross leverage is a sum of long and short exposures as a portion of assets under management (AUM). Net leverage is a difference of long and short exposures as a portion of AUM. Long-only leverage is the long exposure as a portion of AUM. The first row in each panel reports the coefficient on the lagged leverage variable and the other right-hand side variables are all macro variables. Each column reports a different regression. “IB CDS” is the equity market-value weighted cost of CDS protection on defaults on 10-year senior bonds of major investment banks (IB), “IB ret” is the return on the market-value weighted portfolio of IB common stocks, “S&P 500 ret” is the monthly total return on the S&P500 index, “Agg HF flows” is the past three-month flow on the aggregate hedge fund industry as reported by Barclay Hedge. All variables are described in detail in Appendix B. The table reports posterior means of coefficients and posterior means of *t*-statistics in square brackets below each coefficient. All estimations have fund fixed effects. Appendix D contains details of the estimation, including the implementation of fixed effects and the calculation of the adjusted R^2. The data sample contains 8136 monthly observations that cover 208 hedge funds during a period from December 2004 to October 2009.

Table 6: Fund-specific predictors of hedge fund leverage

	(1)	(2)	(3)	(4)	(5)
Panel A: Gross leverage					
Past gross lev	-0.2443	-0.2452	-0.2445	-0.2451	-0.2455
	[-30.3]	[-30.5]	[-30.5]	[-30.1]	[-31.1]
Past ret	-0.1288				-0.2151
	[-0.49]				[-0.82]
12-Month vol		-1.337			-1.4139
		[-1.93]			[-2.11]
3-Month flows			-0.0053		-0.0024
			[-0.21]		[-0.10]
Log AUM				-0.0325	-0.0414
				[-1.13]	[-1.43]
Adjusted R^2	0.130	0.131	0.131	0.131	0.131
Panel B: Net leverage					
Past net lev	-0.3107	-0.3066	-0.3106	-0.3089	-0.3098
	[-3.69]	[-3.99]	[-3.59]	[-3.71]	[-3.62]
Past ret	-0.2357				-0.2057
	[-1.93]				[-1.64]
12-Month vol		0.1615			0.0543
		[0.51]			[0.18]
3-Month flows			0.0142		0.0153
			[1.35]		[1.49]
Log AUM				-0.0183	-0.0201
				[-1.41]	[-1.45]
Adjusted R^2	0.157	0.156	0.156	0.157	0.157

Table 6 Continued

	(1)	(2)	(3)	(4)	(5)
Panel C: Long-only leverage					
Past long lev	-0.2371	-0.2372	-0.2373	-0.2381	-0.2375
	[-30.4]	[-32.1]	[-31.1]	[-29.9]	[-30.6]
Past ret	-0.1923				-0.2258
	[-1.20]				[-1.41]
12-Month vol		-0.6278			-0.7289
		[-1.60]			[-1.76]
3-Month flows			0.0048		0.0045
			[0.33]		[0.31]
Log AUM				-0.0236	-0.0284
				[-1.38]	[-1.60]
Adjusted R^2	0.127	0.127	0.127	0.127	0.127

The table reports regression coefficients of Eq. (2) to predict changes in gross leverage (Panel A), net leverage (Panel B), and long-only leverage (Panel C) over the next month. Gross leverage is a sum of long and short exposures as a portion of assets under management (AUM). Net leverage is a difference of long and short exposures as a portion of AUM. Long-only leverage is the long exposure as a portion of AUM. The first row in each panel reports the coefficient on the lagged leverage variable and the other right-hand side variables are fund-specific and macro variables. Each column reports a different regression. “Past ret” is the fund’s return in the past month, “12-Month vol” is the volatility of the hedge fund’s returns computed using monthly data over the past 12 months, “3-Month flows” is the hedge fund flow over the past three months computed using Eq. (1), and “Log AUM” is the logarithm of each hedge fund’s AUM. All the regression specifications also control for the macro predictors used in Table 5: the cost of CDS protection on major investment banks, the return on the market-value weighted portfolio of investment banks, the S&P 500 return, option VIX volatility, LIBOR, the TED spread, the term spread, and aggregate hedge fund flows. All variables are described in detail in Appendix B. The table reports posterior means of coefficients and posterior means of *t*-statistics in square brackets below each coefficient. All estimations have fund-fixed effects. Appendix D contains details of the estimation, including the implementation of fixed effects and the calculation of the adjusted R^2. The data sample contains 8136 monthly observations that cover 208 hedge funds during a period from December 2004 to October 2009.

Table 7: Contemporaneous relations with gross hedge fund leverage

	Predictive	Contemporaneous		Predictive	Contemporaneous
Macro variables			*Fund-specific variables*		
Past leverage	-0.2447		Past leverage	-0.2455	
	[-32.0]			[-31.1]	
IB CDS	-9.3278	-1.3666	Past ret	-0.2151	-0.1123
	[-3.54]	[-0.38]		[-0.82]	[-0.35]
IB ret	-0.0436	-0.2248	12-Month vol	-1.4139	-4.3495
	[-0.26]	[-0.90]		[-2.11]	[-2.35]
S&P 500 ret	0.6750	-0.9419	3-Month flows	-0.0024	-0.0530
	[2.09]	[-2.02]		[-0.10]	[-1.11]
VIX	-0.1010	-1.4324	Log AUM	-0.0414	0.2552
	[-0.51]	[-4.79]		[-1.43]	[1.75]
LIBOR	-6.6629	3.4420			
	[-2.35]	[0.76]			
TED spread	7.5973	8.7629			
	[1.90]	[1.49]			
Term spread	-10.32	-12.237			
	[-2.80]	[-2.09]			
Agg HF flows	0.0934	1.3419			
	[0.38]	[3.13]			
ϕ_ϵ		0.2494	ϕ_ϵ		0.5547
		[32.9]			[45.5]
Adjusted R^2	0.131	0.080	Adjusted R^2	0.131	0.086

The table reports regression coefficients for macro and fund-specific variables of the "Predictive" model in Eq. (2) and the "Contemporaneous" model in Eq. (3) for gross hedge fund leverage. Gross leverage is a sum of long and short exposures as a portion of assets under management (AUM). The predictive model coefficients are identical to regression (9) of Table 5 for the macro-only predictors and regression (5) of Table 6 for the fund-specific predictors. The "Fund-specific variables" regressions control for the macro predictors listed in the "Macro variables" regressions: "IB CDS" is the equity market-value weighted cost of CDS protection on defaults on 10-year senior bonds of major investment banks (IB), "IB ret" is the return on the market-value weighted portfolio of IB common stocks, "S&P 500 ret" is the monthly total return on the S&P 500 index, "Agg HF flows" is the past three-month flow on the aggregate hedge fund industry as reported by Barclay Hedge. For the fund-specific variables: "Past ret" is the fund's return in the past month, "12-Month vol" is the volatility of the hedge fund's returns computed using monthly data over the past 12 months, "3-Month flows" is the hedge fund flow over the past three months computed using Eq. (1), and "Log AUM" is the logarithm of each hedge fund's AUM. All variables are described in detail in Appendix B. The table reports posterior means of coefficients and posterior means of *t*-statistics in square brackets below each coefficient. All estimations have fund-fixed effects. Appendix D contains details of the estimation, including the implementation of fixed effects and the calculation of the adjusted R^2. The data sample contains 8136 monthly observations that cover 208 hedge funds during a period from December 2004 to October 2009.

Table 8: Correlations of hedge fund and finance sector leverage

		Hedge fund strategies			
	All hedge funds	RV	EQ	ED	CR
Panel A: Gross leverage					
Banks	-0.884	-0.820	-0.613	-0.774	-0.658
Investment banks	-0.823	-0.734	-0.536	-0.733	-0.586
Finance sector	-0.884	-0.812	-0.608	-0.776	-0.656
Panel B: Net leverage					
Banks	-0.873	-0.623	-0.740	-0.923	-0.772
Investment banks	-0.845	-0.525	-0.766	-0.891	-0.765
Finance sector	-0.884	-0.610	-0.764	-0.931	-0.789
Panel C: Long-only leverage					
Banks	-0.893	-0.801	-0.735	-0.867	-0.722
Investment banks	-0.840	-0.712	-0.680	-0.828	-0.667
Finance sector	-0.896	-0.791	-0.738	-0.872	-0.726

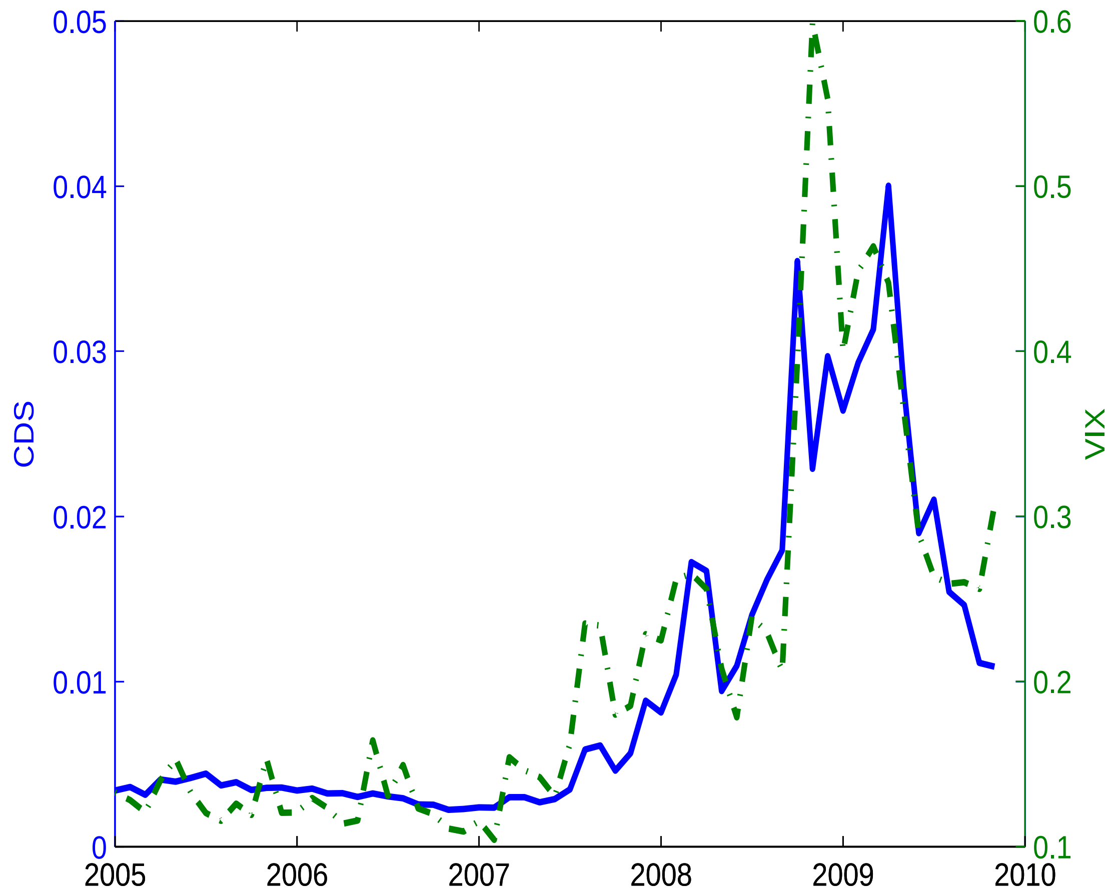
The table reports correlations of average levels of leverage of hedge funds (HF) and average leverage of their specific strategies—relative value (RV), equity (EQ), event-driven (ED), and credit (CR)—with average leverage of bank holding companies (banks), investment banks (Bear Stearns, Citibank, Credit Suisse, Goldman Sachs, HSBC, JP Morgan, Lehman Brothers, Merrill Lynch, and Morgan Stanley), and the finance sector separately for each definition of hedge fund leverage: Gross leverage (Panel A), Net leverage (Panel B), and Long-only leverage (Panel C) at the monthly frequency. We compute the leverage of finance subsectors following Appendix B. The leverage of hedge funds consists of all observed hedge fund leverage and estimated hedge fund leverage when these are unobserved following Eq. (2) and the estimation method outlined in Appendix D using all macro and fund-specific variables and fund-fixed effects. Gross leverage is a sum of long and short exposures as a portion of assets under management (AUM). Net leverage is a difference of long and short exposures as a portion of AUM. Long-only leverage is the long exposure as a portion of AUM. The data sample is from December 2004 to October 2009.

Table A.1: A sample hedge fund risk exposure report

Sector	Gross leverage ratio (%)	Net leverage ratio (%)	Long market value/ Equity (%)	Short market value/ Equity (%)
Consumer discretionary	16.73	1.93	9.33	(7.40)
Consumer staples	9.08	5.16	7.12	(1.96)
Energy	7.84	(1.91)	2.97	(4.87)
Financials	4.20	(2.87)	0.66	(3.53)
Health care	5.01	2.17	3.59	(1.42)
Industrials	22.14	7.28	14.71	(7.43)
Information technology	26.05	5.41	15.73	(10.32)
Materials	1.31	0.46	0.89	(0.43)
Other assets	17.72	3.76	10.74	(6.98)
Telecommunication services	0.69	0.28	0.48	(0.21)
Total	110.78	21.68	66.23	(44.55)

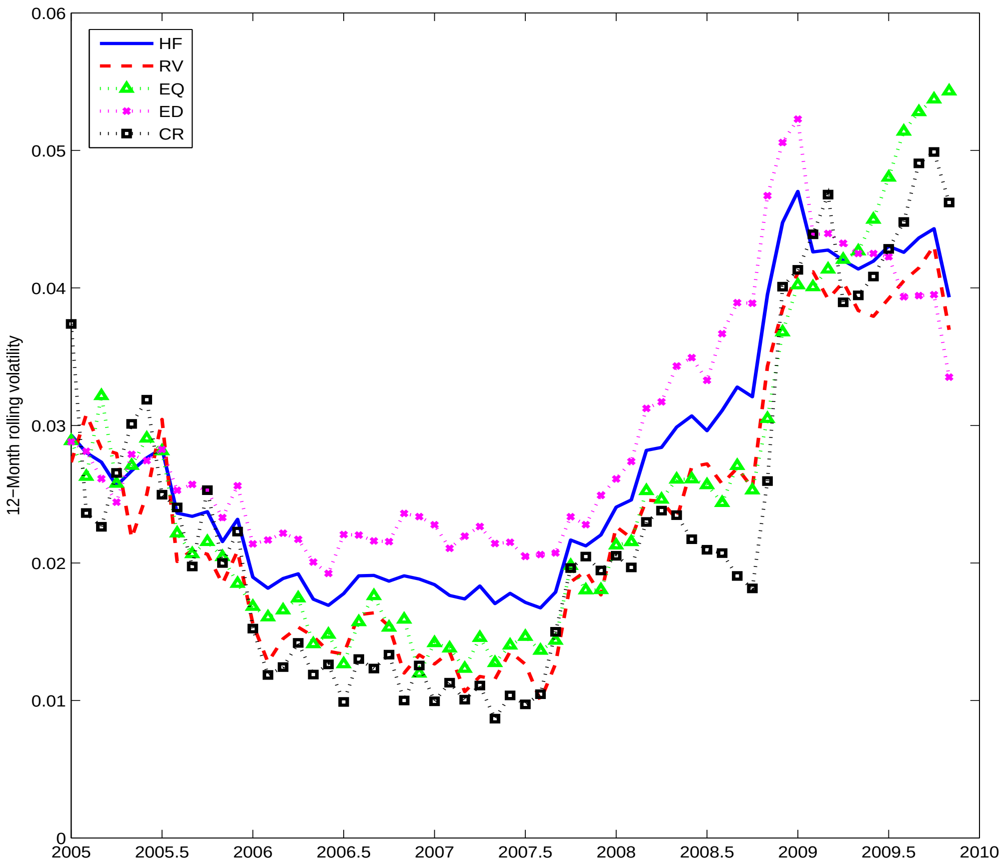
This table shows a sample hedge fund risk exposure report. This fund reports exposures monthly broken down by sector. The reported quantities are percentages of net asset value (NAV). Gross leverage is a sum of long and short exposures as a portion of assets under management (AUM). Net leverage is a difference of long and short exposures as a portion of AUM.

Figure 1.: VIX and CDS protection.



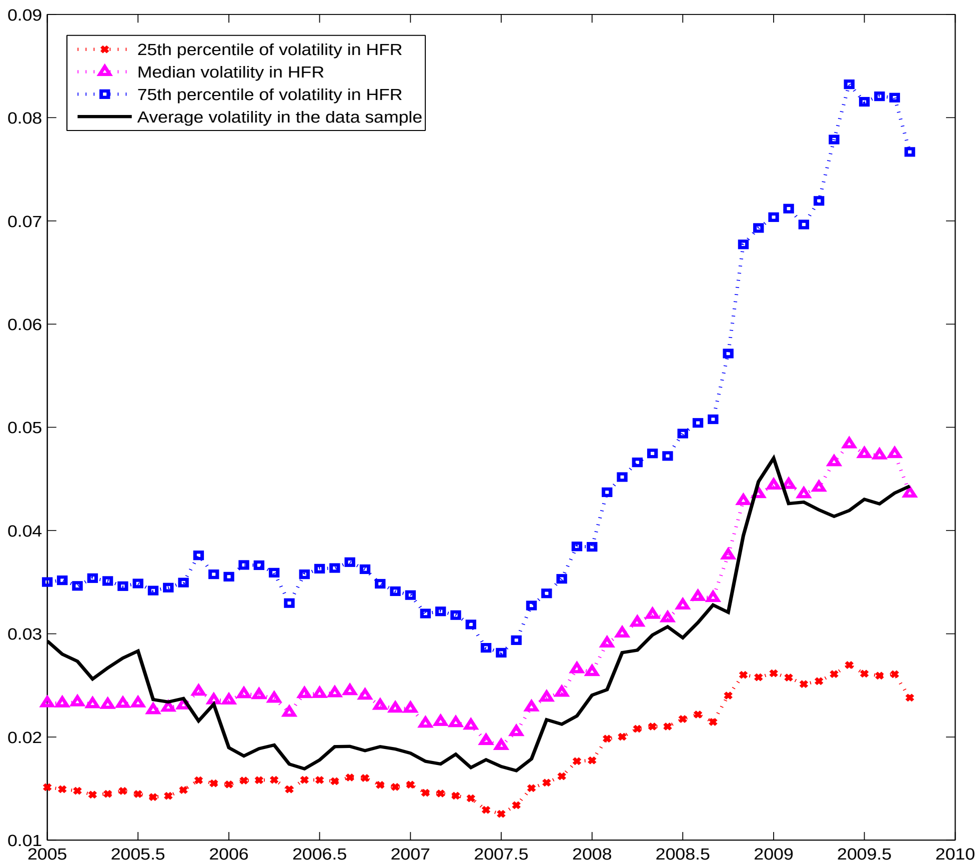
The market-value-weighted credit default swap (CDS) cost of protection for the investment banks (Bear Stearns, Citibank, Credit Suisse, Goldman Sachs, HSBC, JP Morgan, Lehman Brothers, Merrill Lynch, and Morgan Stanley) is shown in the solid line with the axis on the left-hand scale. We plot the VIX volatility index in the dotted line with the axis on the right-hand scale. The data sample is from December 2004 to October 2009 at a monthly frequency.

Figure 2.: Rolling 12-month hedge fund volatilities.



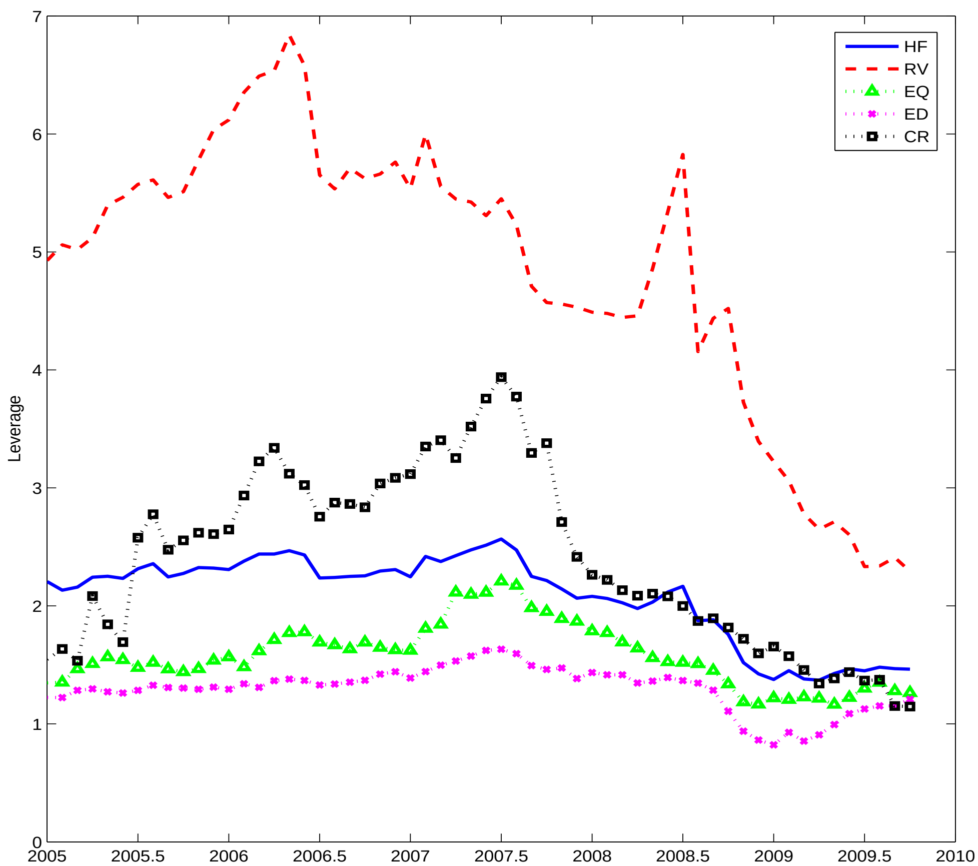
This figure compares volatilities of returns of different hedge fund strategies over the sample period. The monthly volatility for each strategy is constructed as an average value of sample volatilities of returns over the past 12 months for the hedge funds that belong to the strategy. The strategies are relative value (RV), equity (EQ), event driven (ED), credit (CR), and the whole hedge fund sample is denoted HF. The data sample is from December 2004 to October 2009.

Figure 3.: Hedge fund volatilities vs. HFR volatilities.



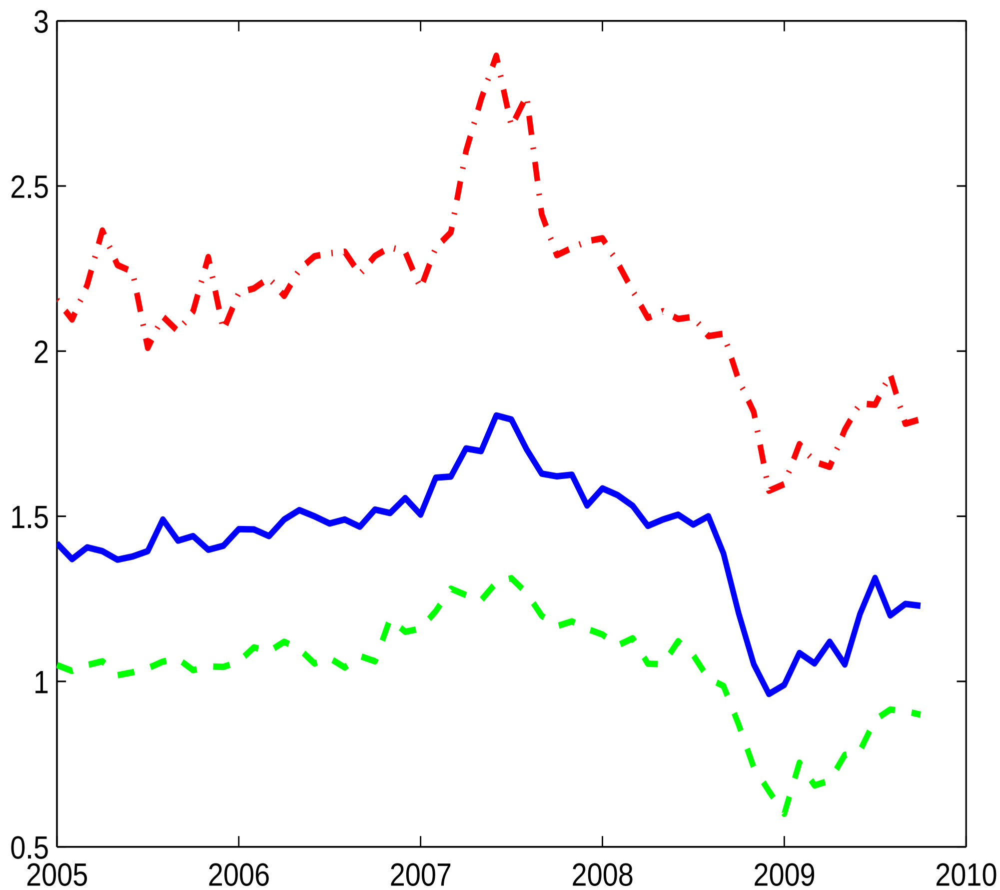
We plot 25th, 50th, and 75th percentile values of 12-month rolling volatilities of returns of funds in the HFR database and the average 12-month rolling volatility of returns of funds in the Fund's database. The data sample is from December 2004 to October 2009.

Figure 4.: Hedge fund gross leverage.



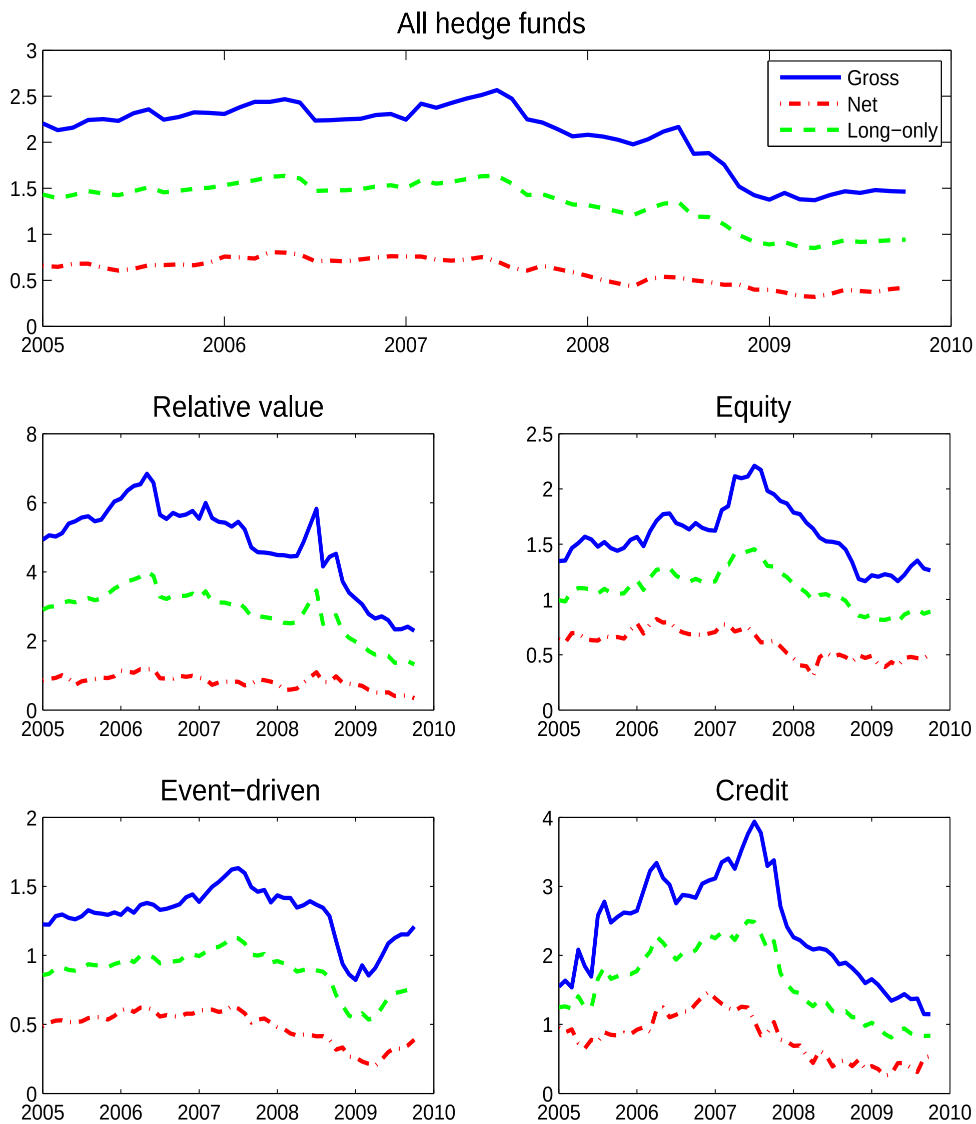
The figure plots hedge fund gross leverage for all hedge funds (HF) and hedge fund sectors. The sectors are relative value (RV), equity (EQ), event driven (ED), and credit (CR). The leverage aggregates all observed hedge fund leverage and estimated hedge fund leverage when these are unobserved following the estimation method outlined in Appendix D. These estimates are obtained using the model in Eq. (2) using all macro and fund-specific variables and fund-fixed effects. The data sample is from December 2004 to October 2009.

Figure 5.: Cross-sectional dispersion of gross hedge fund leverage.



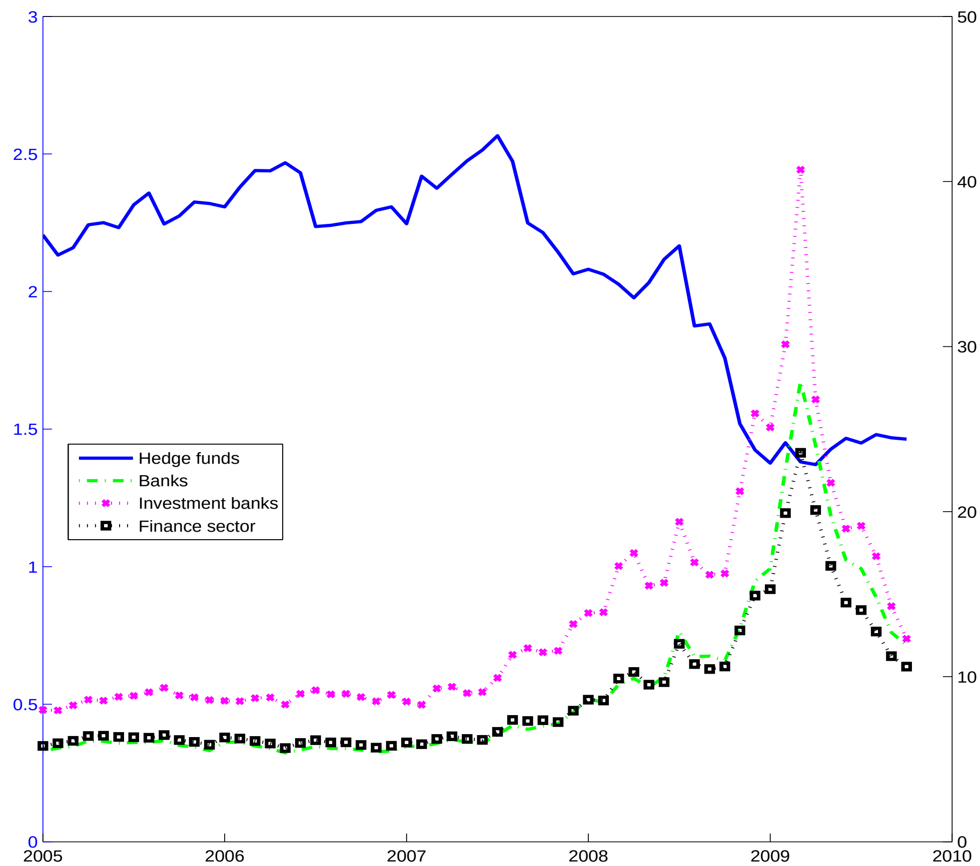
The figure plots the median (solid line) together with the 25th and 75th cross-sectional percentiles (dashed and dashed-dot lines, respectively) of gross hedge fund leverage across all funds. The hedge fund leverage ratios consist of all observed hedge fund leverage and estimated hedge fund leverage when these are unobserved following Eq. (2) and the estimation method outlined in Appendix D using all macro and fund-specific variables and fund-fixed effects. The data sample is from December 2004 to October 2009.

Figure 6.: Gross, net, and long-only hedge fund leverage.



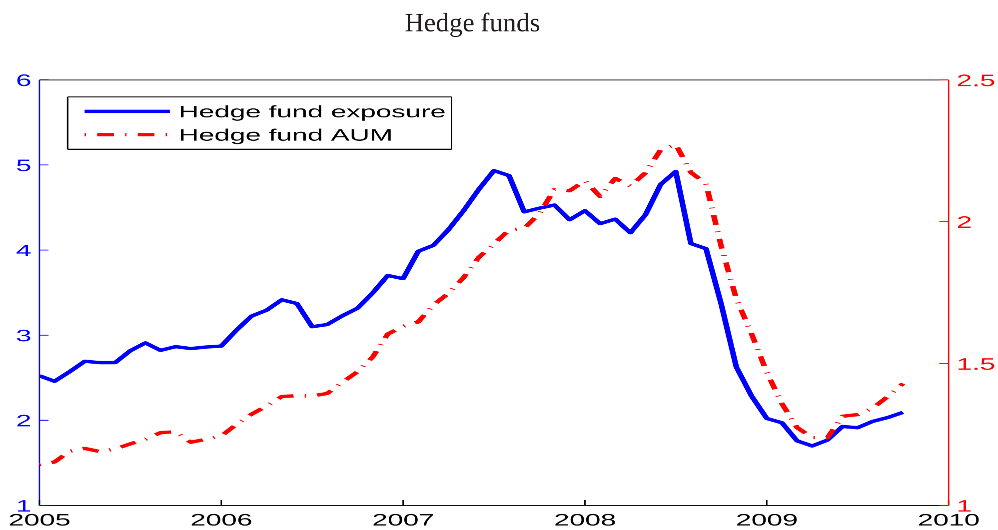
The figure shows the dynamics of the posterior means of gross leverage (solid line), net leverage (dashed-dot line), and long-only leverage (dashed line) for all hedge funds and for hedge fund sectors at the monthly frequency. The hedge fund leverage ratios consist of all observed hedge fund leverage and estimated hedge fund leverage when these are unobserved following Eq. (2) and the estimation method outlined in Appendix D using all macro and fund-specific variables and fund-fixed effects. The data sample is from December 2004 to October 2009.

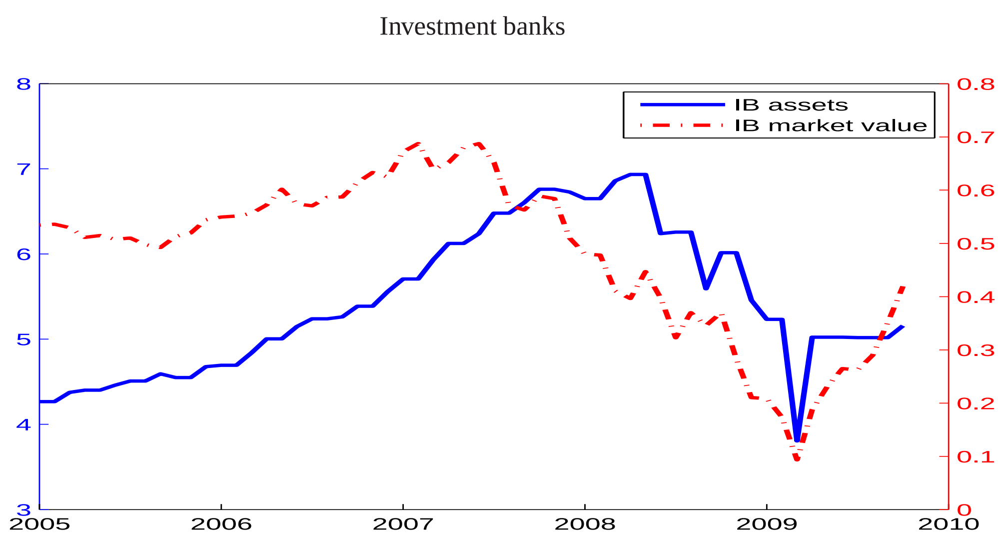
Figure 7.: Hedge fund and finance sector leverage.



We compare average gross hedge fund leverage with the leverage of banks, investment banks, and the finance sector. The left-hand axis corresponds to average gross hedge fund leverage and the right-hand axis corresponds to the leverage of banks, investment banks, and the finance sector. The hedge fund leverage ratios consist of all observed hedge fund leverage and estimated hedge fund leverage when these are unobserved following Eq. (2) and the estimation method outlined in Appendix D using all macro and fund-specific variables and fund-fixed effects. The finance sector leverage is constructing following the method described in Appendix B. The data sample is from December 2004 to October 2009.

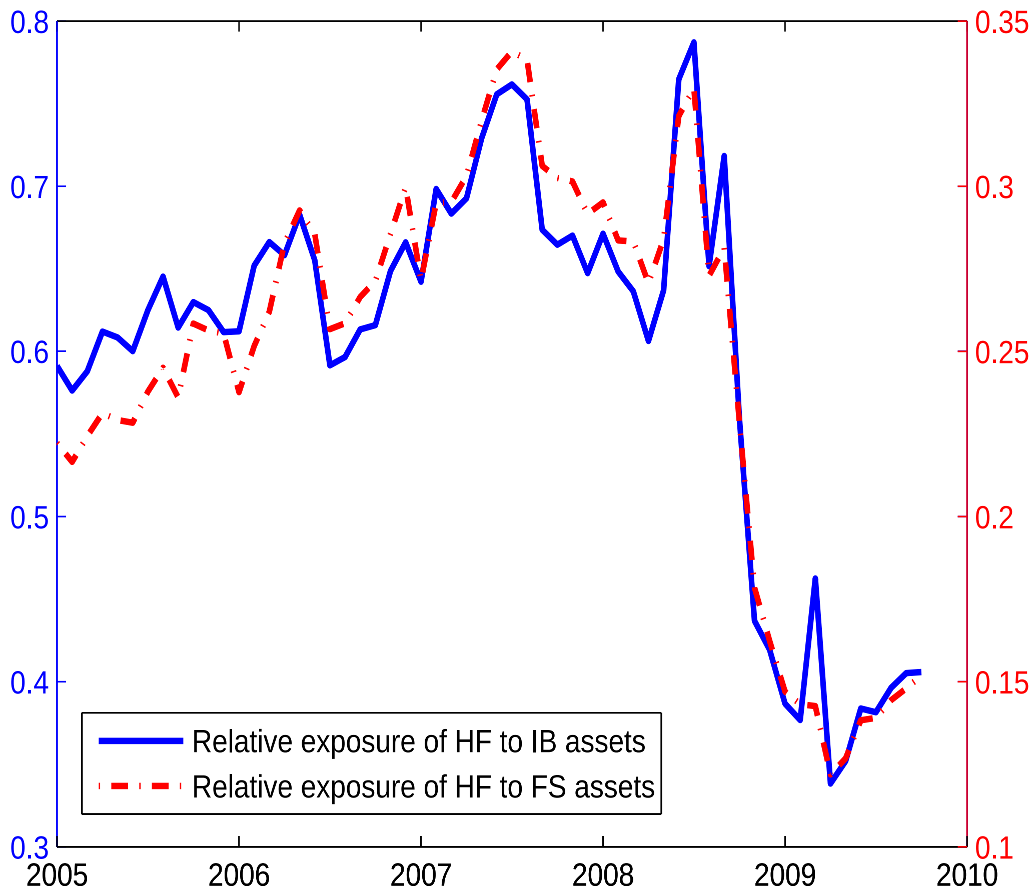
Figure 8.: Hedge fund and investment bank gross exposure and leverage.





We graph the gross exposure and AUM of hedge funds in Panel A and the gross exposure and market value of equity of investment banks (IB) in Panel B. For hedge funds, we take gross leverage across all hedge funds which consists of observed gross leverage and estimated gross leverage when these are unobserved following Eq. (2) and the estimation method outlined in Appendix D using all macro and fund-specific variables and fund-fixed effects. The hedge fund exposure is computed by multiplying the gross leverage by the aggregated AUM of hedge funds from the Barclays Hedge database. Investment bank exposure is the total amount of assets held by investment banks. The left-hand axes in both panels correspond to AUM or equity. The market value of investment banks is the value of common equity. Appendix B contains further details on these variables. The right-hand axes correspond to gross exposure. The scale of both axes is in trillions of dollars. The data sample is from December 2004 to October 2009.

Figure 9.: Relative gross exposures of hedge funds to investment banks and the finance sector.



We plot the ratio of gross exposure of hedge funds (HF) to investment banks (IB) and the finance sector (FS). The gross exposure is computed by multiplying gross leverage and AUM in the case of hedge funds and is total assets in the case of investment banks and the finance sector. For hedge funds, we take gross leverage across all hedge funds which consists of observed gross leverage and estimated gross leverage when these are unobserved following the estimation method outlined in Appendix D using all macro and fund-specific variables and fund-fixed effects. The left-hand axis corresponds to the relative gross exposure of hedge funds to the assets of investment banks, while the right-hand axis corresponds to the relative exposure of hedge funds to the assets of the finance sector. The data sample is from December 2004 to October 2009.

Hedge Fund Compensation*

Sergiy Gorovyy†

Abstract

I investigate hedge fund compensation in a model with a risk neutral fund manager who can continuously rebalance the fund's holdings. I solve for the optimal leverage level in a fund that has a compensation contract with a high-water mark and hurdle rate provisions where management and performance fees are paid at discrete time moments. The compensation contract induces risk-loving behavior with managers often choosing the maximum leverage.

JEL Classification: G11, G23, G32

Keywords: Hedge Fund, Portfolio Choice, Capital Structure, Leverage, Margins

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1 Introduction

Hedge fund compensation contracts are one of the most complicated compensation contracts in the money management industry. Hedge fund fees are among the highest fees relative to the fees of other money managers. For example, mutual funds charge on average 1.7% of Assets Under Management (AUM) per year,[1] while hedge funds usually charge a 2% management fee and an additional 20% performance fee, where the management fee is proportional to the fund's AUM and the performance fee is proportional to the fund's net AUM gain. Hedge fund investors became especially concerned with the higher fees in hedge fund compensation contracts during the financial crisis of 2007 - 2009.[2] Investors paid high fees even during the period when hedge funds experienced large negative returns. Some of the hedge funds did not have provisions that restrict payment of performance fees (high-water mark or hurdle rate provisions) and they charged performance fees for periods when the fund delivered a positive performance even when the fund's long term performance was negative. After the crisis, provisions that restrict payment of the performance fee became widespread.[3]

This paper studies hedge fund compensation contracts and their influence on the investment decisions of hedge fund managers. I formulate a model with a risk-averse investor and a risk-neutral hedge fund manager. The manager has a trading strategy and he is allowed to apply leverage in order to amplify hedge fund returns. The manager can rebalance the fund's portfolio continuously, while the fees for his service are charged at the end of each time period (for example, a month, a quarter, or a year). In the model, I specify a hedge fund compensation contract with management and performance fees that include high-water mark and hurdle rate provisions. The model nests, as special cases, models with compensation contracts that have only management fees, contracts that have performance fees with only the high-water mark provision, and contracts that have performance fees with only the hurdle rate provision.

[1] See Khorana, Servaes, and Tufano (2009).
[2] See, for example, Karmin and Strasburg (2009).
[3] According to HFR database 91.7% of hedge fund compensation contracts contained the high-water mark provision and 13.1% of the contracts contained the hurdle rate provision in 2010.

I find that the hedge fund compensation contracts induce risk-taking behavior in managers in the sense that managers use the maximum leverage possible, which is exogenously specified to be the inverse of a margin requirement. I find that there is a strong correlation between the inverse of margin requirement for S&P 500 futures contract and average hedge fund leverage. Similar to Goetzmann, Ingersoll, and Ross (2003), this paper studies hedge fund fees. I construct an equivalent management fee (EMF) in order to compare various hedge fund compensation contracts. The EMF allows the comparison of complicated hedge fund compensation contracts to simple mutual fund compensation contracts that deliver the same utility for the manager. I find that the hedge fund compensation contract that charges a 2% per annum management fee and 20% performance fee without high-water mark or hurdle rate provisions has an EMF equal to 6.45%, while the hedge fund compensation contract that charges a 2% per annum management fee and 20% performance fee and includes an indexed with respect to hurdle rate high-water mark provision has an EMF equal to 5.31%. Consequently, the cost of the indexed high-water mark provision is equal to 1.14% management fee. I also find that the hedge fund compensation contract that charges monthly a 2% per annum management fee and a 20% performance fee with both high-water mark and hurdle rate provisions has an EMF equal to 2.93%. Consequently, hedge fund managers dislike an increase in the fee payment frequency more than they dislike the indexed high-water mark provision, since an adoption of monthly frequency is equivalent to a sacrifice of a 2.38% management fee.

In contrast to my model which predicts that hedge fund leverage is mostly determined by its maximum possible value, there is no consensus in the academic literature on the correlation between the performance fee and hedge fund leverage. Ackermann et al. (1999) and Brown et al. (2001) report that the correlation between the performance fee and the fund's leverage is low, while Kouwenberg and Ziema (2007) report that it is high. Empirical results on the hedge fund manager compensation have to be carefully interpreted due to the fact that hedge funds do not have to report their compensation schemes. Additional assumptions about the distribution of the high-water marks and the shares invested by managers in their funds are needed in order to draw conclusions about managerial compensation. I find that since, independent of the level of the performance fee, the hedge fund manager chooses the maximum leverage, the correlation between the performance fee and fund's leverage should be low.

Theoretical literature does not also agree on the correlation between hedge fund fees and hedge fund leverage. Carpenter (2000) proposes that the option-like form of compensation contract leads, in some cases, to very high levels of leverage, while in other cases it leads to even more conservative levels than the one the manager would take if investing personal money. Panageas and Westerfield (2009) and Lan, Wang, and Yang (2012) suggest that the risk neutral manager uses leverage conservatively.

The model formulated in this paper is related to the models considered in Hodder and Jackwert (2007), Panageas and Westerfield (2009), and Lan, Wang, and Yang (2012). Similar to Panageas and Westerfield (2009) and Lan, Wang, and Yang (2012), the hedge fund manager has access to risk-free and risky assets and is allowed to continuously rebalance the fund's portfolio. Panageas and Westerfield (2009) conclude that the risk-neutral manager does not place unbounded weights on the risky asset despite the convexity of the compensation contract. Lan, Wang, and Yang (2012) consider an expansion of Panageas and Westerfield (2009) model and find that the manager behaves in a risk-averse way with the risk aversion coefficient determined by a liquidation boundary. I use a suggestion proposed by Hodder and Jackwert (2007) to consider a model where the hedge fund charges fees at discrete time moments, which is also the way hedge funds levy fees in the real world. I show that the presence of margin requirement is required in the model since otherwise the optimal leverage level is unbounded. I consider possibilities of fund liquidation by the investor and by the prime broker as suggested by Dai and Sundaresan (2010) and find no significant effect on the managerial behavior.

Section 2 describes the hedge fund compensation contracts with fee calculations. Section 3 presents the model. Section 4 estimates costs of the high-water mark and hurdle rate provisions and Section 5 concludes.

2 Hedge Fund Fees

Traditional hedge fund compensation contracts contain both management and performance fees.

The management fee is a fee that is proportional to the total value of fund's Assets Under Management (AUM). The management fee is common in money management industry and is the standard mutual fund fee. The management fee can be charged at the beginning or at the end of each time period. In a multi-period model these two possibilities differ only in payment of the first and the last fees. We consider a case where the management fee is charged at the end of each period to make formulas simpler. Later it will become clear that the timing of the management fee payment does not change the conclusions of the paper.

The performance fee is charged based on the performance of the fund during the period. Payment of the performance fee is conditional on the net fund's performance being positive and on satisfaction of some additional constraints (e.g., high-water mark and hurdle rate provisions) if they are present in the compensation contract. Terhune and Lorence (2005) define the performance fee as a fee that is charged on "net value added". "Net value added" is the difference between "gross value added" and expenses that in general include legal, accounting, trustee, administrative, marketing and sales, custodial, and general investment management charges. "Gross value added" is the difference between the value of the current fund's AUM before fees and the value of the previous after-fee fund's AUM. We consider only management and performance fees in this paper, so the expenses subtracted from the "gross value added" constitute only the management fee.

The high-water mark provision requires the fund to outperform the highest Net Asset Value (NAV) in order for the investor to be charged the performance fee. The value of the performance fee is proportional to the outperformance. The value of the high-water mark can be indexed or not indexed depending on the contract specification. We consider a case where the value of the high-water mark is indexed by the hurdle rate.[4]

[4] We selected the indexed high-water mark case because it results in less complicated formulas, while the differences between the outcomes in the indexed and not indexed cases are not economically significant. The formulas in the case where the high-water mark is not indexed are more complex due to the fact that the hedge

The hurdle rate provision states that the fund does not charge the performance fee when the fund's return is below some predetermined value called the hurdle rate (for example, this can be some constant required rate of return, beginning of the period LIBOR, or a treasury yield).

According to January 2010 HFR database 11.1% of hedge fund contracts include high-water mark and hurdle rate provisions, 80.6% of the contracts include only the high-water mark provision, 2.0% of the contracts include only the hurdle rate provision, and 6.3% of the contracts do not include either the high-water mark or the hurdle rate provision.

In order to understand how high-water mark and hurdle rate provisions affect calculations of hedge fund fees consider the following example shown in Table 1. A money manager decides to start a hedge fund at the end of 2006. He meets an investor and they sign a compensation contract with a 2% management fee and a 20% performance fee payable when a high-water mark is outperformed, which is indexed with respect to 4%. The investor allocates $1,000,000 in the hedge fund at the end of 2006.

The hedge fund delivers a 20% gross return which results in a $1,200,000 value of the fund at the end of 2007. The fees are now levied and the numbers in Table 1 state the following series of fee calculations. At first the management fee is charged, which results in a $24,000 management fee. The fund's value becomes equal to $1,176,000 after payment of the management fee. The value of the indexed high-water mark is equal to the initial $1,000,000 indexed by 4% which is equal to $1,040,000. The fund's value outperforms this value by $136,000, so the manager gets a $0.2 \times \$136,000 = \$27,200$ performance fee. The total amount of the fees charged in 2007 is equal to $51,200. The ending value of the fund is $1,148,800 at the end of 2007.

In the column labeled '2008', I detail the next series of calculations for 2008. The fund delivers a -20% gross return. This shrinks the fund from $1,148,800 down to $919,040. In

fund can outperform the value of the high-water mark by delivering return that does not satisfy the hurdle rate provision, so that the value of the high-water mark is updated but the performance fee is not paid. This is not the case in the indexed high-water mark case since both provisions are automatically satisfied by outperforming the value of the indexed high-water mark.

contrast, the indexed high-water mark value increases by 4% and is now $1,194,752. Therefore, since the fund value is below this amount, the manager does not obtain the performance fee. Only the management fee equal to $0.02 \times \$919,040 = \$18,381$ is charged in 2008. The ending value of the fund is $900,659 at the end of 2008.

In the final column of Table 1, the fund delivers a 50% gross return increasing the size of the fund from $900,659 to $1,350,989 in 2009. The high-water mark is again indexed and is now equal to $1,242,542. The performance fee is charged on the outperformance of this value, which is equal to $81,427. The manager obtains $0.02 \times \$1,350,989 = \$27,020$ in management fees and $0.2 \times \$81,427 = \$16,285$ in performance fees which results in a total of $43,305 in fees charged in 2009. The ending value of the fund is $1,307,684 at the end of 2009.

The general case of the total amount of fees charged as a function of the fund's AUM and high-water mark is equal to

$$z(A_t, A_{(t-1)+}, H_{t-1}) = f_m A_t + f_p((1 - f_m)A_t - (1 + h_r)H_{t-1})^+$$

where A_t is the value of the AUM at time t before the fees are paid, $A_{(t-1)+}$ is the value of the AUM after the fees were paid at the end of the previous period, H_{t-1} is the value of the high-water mark after the fees were paid at the end of the previous period, f_m is the management fee, f_p is the performance fee, h_r is the hurdle rate which is also the rate of the high-water mark indexation, and notation $(x)^+$ denotes $\max\{x, 0\}$. The total amount of the fees charged is composed of the management fee that is equal to $f_m A_t$ and the performance fee that is equal to $f_p((1 - f_m)A_t - (1 + h_r)H_{t-1})^+$, where $(1 - f_m)A_t$ is the fund's AUM after payment of the management fee and $(1+h_r)H_{t-1}$ is the value of the indexed high-water mark.

In the previous example $f_m = 2\%$, $f_p = 20\%$, $h_r = 4\%$, $H_0 = A_{0+} = A_0 = \$1,000,000$, $A_1 = \$1,200,00$, $z(A_1, A_{0+}, H_0) = \$51,200$, $A_{1+} = \$1,148,800$, $H_1 = \$1,148,800$, $A_2 = \$919,040$, $z(A_2, A_{1+}, H_1) = \$18,381$, $A_{2+} = \$900,659$, $H_2 = \$1,194,752$, $A_3 = \$1,350,989$, $z(A_3, A_{2+}, H_2) = \$43,305$, and $A_{3+} = \$1,307,684$.

3 Model Setup

We construct a model with a fund manager who is able to dynamically rebalance the fund's portfolio as, for example, in Goetzmann, Ingersoll, and Ross (2003), Panageas and Westerfield (2009), and Lan, Wang, and Yang (2012). The fund manager charges fees at discrete time moments. In Goetzmann, Ingersoll, Ross (2003), Panageas and Westerfield (2009), and Lan, Wang, and Yang (2012) the fees are charged continuously. Payment of the fees at discrete time moments makes an enormous difference because it significantly changes the nature of the compensation options – instead of an infinite number of infinitesimal options in the continuous case the manager has a finite (or countable) number of options of substantial size. The payment of fees at discrete intervals is more realistic as in the real world fees are charged at the end of each month, quarter, or year.

3.1 Model

The investor invests with the hedge fund at time $t = 0$. The hedge fund manager dynamically rebalances the fund's portfolio between a risky and a risk-free asset. The risky asset represents the manager's proprietary strategy. The investor is allowed to withdraw all the capital from the hedge fund at the end of any investment period after the fees for the period are paid. There is no reinvestment afterwards.

There is only one investor in this model who makes only one investment with the hedge fund and therefore the fund's AUM and NAV coincide. This allows to use the AUM instead of the NAV in the high-water mark definition, which significantly simplifies the model.

The risk-free asset delivers a constant rate of return $r > 0$. The price of the risk-free asset evolves according to

$$\frac{dP_{0,t}}{P_{0,t}} = r\,dt.$$

The price of the risky asset evolves according to

$$\frac{dP_{1,t}}{P_{1,t}} = \mu\,dt + \sigma\,dB_t,$$

where $\mu > r$, $\sigma > 0$, and B_t is a standard Brownian motion. The risky asset can be interpreted not only as a particular security, but also as an unlevered trading strategy employed by the fund.

At each time moment t, the manager specifies the leverage level π_t applied to the risky asset, which is effectively the weight of the risky asset of the portfolio. The weight of the risk-free asset is equal to $1 - \pi_t$. We allow the manager the possibility to lever up the risky asset, which corresponds to having $\pi_t > 1$ or to short-sell the risky asset, which corresponds to having $\pi_t < 0$. Since $\mu > r$ the manager prefers to have $\pi_t \geq 0$ for all times t.

We assume an exogenous margin requirement m, which effectively limits the leverage the fund manager can take. A *margin* is a requirement from an institution providing leverage (a prime-broker or an exchange) to an institution (fund) obtaining leverage to post a portion m of the market value of the security purchased to a margin account. Futures contracts exchanges require agents to post specific dollar amounts for each contract to the margin account, but following Dudley and Nimalendran (2011), this can be converted to a portion of the value of the underlying securities. The portion m effectively provides the maximum leverage the fund can take equal to $\frac{1}{m}$.[5] For example, Regulation T(Reg T) in the US requires agents to post at least 50% of the market value of equities bought or sold short to a margin account, which corresponds to $m = 0.5$.

The fund's AUM, A_t, at time t between moments when the fees are paid evolves according to

$$dA_t = A_t \pi_t (\mu \, dt + \sigma \, dB_t) + (1 - \pi_t) A_t r \, dt. \tag{1}$$

At the moment the fees are paid the fund's AUM decreases according to

$$A_{t+} = A_t - z(A_t, A_{(t-1)+}, H_{t-1})$$

where A_t denotes the value of fund's AUM at time t before fees and A_{t+} denotes the value

[5] There are two types of margins available: *initial margin*, which is the amount required to be posted to the margin account when the position is opened and *maintenance margin* — the amount required to be maintained on the margin account while the position is open. We consider only maintenance margin in this paper because it limits the maximum leverage the fund can take over time, while the initial margin provides limitations only at the moment of the trade.

of fund's AUM at time t after fees. $z(A_t, A_{(t-1)+}, H_{t-1})$ denotes the amount of fees paid at the moment t given the current and the previous after-fee AUM as well as the latest level of the high-water mark H_{t-1}. The total value of fees z is given by

$$z(A_t, A_{(t-1)+}, H_{t-1}) = f_m A_t + f_p((1-f_m)A_t - (1+h_r)H_{t-1})^+. \tag{2}$$

The value of the indexed high-water mark is updated according to

$$H_t = \max\{(1+h_r)H_{t-1}, (1+h_r)A_{(t-1)+}, A_{t+}\} = \max\{(1+h_r)H_{t-1}, A_{t+}\}. \tag{3}$$

where the value of the high-water mark is indexed by the hurdle rate h_r. The performance fee is paid if the fund's AUM after the management fee is paid is higher than the value of the indexed high-water mark $(1+h_r)H_{t-1}$.

The investor decides to continue with the investment in the hedge fund after payment of the fees with probability $Q(A_{i+}, A_{(i-1)+}, H_{i-1})$ that depends on the current and previous values of the fund's after-fee AUM and the previous value of the high-water mark. Note that there is no need to include the current high watermark value because it is determined by other included variables. A constant for Q corresponds to a constant disinvestment frequency assumed in Panageas and Westerfield (2009). If the function Q is equal to 1 for all the values above some boundary and it is equal to 0 for all the values below the boundary, then this corresponds to a liquidation boundary considered in Hodder and Jackwert (2007).

The manager maximizes his utility function

$$\begin{aligned} V(A_0, H_0) = \max_{\pi_t \in [0, \frac{1}{m}]} E\Big[&\beta z(A_1, A_{0+}, H_0) + \beta^2 Q(A_{1+}, A_{0+}, H_0) z(A_2, A_{1+}, H_1) + \\ &\beta^3 Q(A_{1+}, A_{0+}, H_0) Q(A_{2+}, A_{1+}, H_1) z(A_3, A_{2+}, H_2) + \dots\Big] \end{aligned} \tag{4}$$

where π_t denotes leverage strategy, m is the margin requirement, z denotes total fund fees, A_t denotes fund's AUM at time t, A_{t+} denotes fund's AUM after fees are paid at time t, H_t denotes the value of the high-water mark at time t, Q denotes the continuation probability, and $\beta = \frac{1}{1+r}$ is the time discount factor.

Table 2 specifies parameter values in the model. The mean $\mu = 10\%$ and the volatility $\sigma = 20\%$ of the risky asset are assumed to match the parameters of S&P 500 index, which is one of the possible investment strategies. The risk-free rate $r = 4\%$ is matched to the average US treasury rate. The time discount $\beta = 0.96$ is inversely related to the risk-free rate and the margin $m = 50\%$ is specified to match the Reg T requirement.

3.2 Solution

The optimization problem (4) can be written in the following recursive form:

$$V(A_{(i-1)+}, H_{i-1}) = \max_{\pi_t \in [0, \frac{1}{m}]} E\left[\beta z(A_i, A_{(i-1)+}, H_{i-1}) + \beta Q(A_{i+}, A_{(i-1)+}, H_{i-1}) V(A_{i+}, H_i)\right], \tag{5}$$

subject to (1), (2), and (3), where $V(A_{(i-1)+}, H_{i-1})$ is the managerial utility, the AUM of the fund at time $i-1$ after fees are paid is equal to $A_{(i-1)+}$, the value of the indexed high-water mark is equal to H_{i-1}, and m is the margin requirement.[6] Later I show that if the margin requirement is dropped in the optimization problem (5) then the optimal leverage is unbounded.

The function z defined by equation (2) represents the total hedge fund fee payment at time t. This function is homogeneous of degree 1, that is

$$z(bA_1, bA_{0+}, bH) = bz(A_1, A_{0+}, H),$$

so

$$z(A_1, A_{0+}, H) = Hz\left(\frac{A_1}{H}, \frac{A_{0+}}{H}, 1\right) = Hz\left(\frac{A_1}{H}, \frac{A_{0+}}{H}\right).$$

Denote $\omega_0 = \frac{A_{0+}}{H_0}$ and $\omega_t = \frac{A_t}{H_0}$ for $t \in (0, 1]$. Assuming the utility function V is also homogeneous of degree 1, that is $V(bA, bH) = bV(A, H)$,

$$V(A, H) = HV\left(\frac{A}{H}, 1\right) = HV\left(\frac{A}{H}\right) = HV(\omega).$$

Equation (5) can be rewritten as

$$H_0 V(\omega_0) = \max_{\pi_t \in [0, \frac{1}{m}]} E\left[\beta H_0 z\left(\frac{A_1}{H_0}, \frac{A_{0+}}{H_0}\right) + \beta Q(A_{1+}, A_{0+}, H_0) H_1 V\left(\frac{A_{1+}}{H_1}\right)\right]. \tag{6}$$

[6] Note, that we can substitute index i in equation (5) and consider interval $[0, 1]$ instead of the interval $[i-1, i]$.

Assuming that the probability that the investor continues to stay invested in the fund depends only on the ratio of the after-fee AUM to the high-water mark, we obtain

$$H_0 V(\omega_0) = \max_{\pi_t \in [0, \frac{1}{m}]} E\left[\beta H_0 z\left(\frac{A_1}{H_0}, \frac{A_{0+}}{H_0}\right) + \beta Q\left(\frac{A_{1+}}{H_1}\right) H_1 V\left(\frac{A_{1+}}{H_1}\right)\right]. \tag{7}$$

From equation (3) that defines the update of the indexed high-water mark value we have

$$H_1 = \max\{(1+h_r)H_0, A_{1+}\},$$

where

$$A_{1+} = A_1 - z(A_1, A_{0+}, H_0) = H_0 \frac{A_1}{H_0} - H_0 z\left(\frac{A_1}{H_0}, \frac{A_{0+}}{H_0}\right) = H_0(\omega_1 - z(\omega_1, \omega_0)).$$

Therefore

$$\frac{A_{1+}}{H_1} = \frac{A_{1+}}{\max\{(1+h_r)H_0, A_{1+}\}} = \min\left\{\frac{A_{1+}}{(1+h_r)H_0}, 1\right\} = \min\left\{\frac{\omega_1 - z(\omega_1, \omega_0)}{1+h_r}, 1\right\}.$$

This allows me to rewrite equation (7) as

$$\begin{aligned} V(\omega_0) = \max_{\pi_t \in [0, \frac{1}{m}]} E\Bigg[&\beta z(\omega_1, \omega_0) + \beta Q\left(\min\left\{\frac{\omega_1 - z(\omega_1, \omega_0)}{1+h_r}, 1\right\}\right) \\ &\cdot \max\{1+h_r, \omega_1 - z(\omega_1, \omega_0)\} V\left(\min\left\{\frac{\omega_1 - z(\omega_1, \omega_0)}{1+h_r}, 1\right\}\right)\Bigg], \end{aligned} \tag{8}$$

where function $z(\omega_1, \omega_0)$ satisfies

$$z(\omega_1, \omega_0) = f_m \omega_1 + f_p((1-f_m)\omega_1 - (1+h_r))^+. \tag{9}$$

Note that the total value of the fees paid depends on ω_0 through ω_1 due to portfolio management strategy that starts from the AUM to high-water mark ratio equal to ω_0.

Appendix A provides an analytical solution of the system of equations (8) and (9) for the case where the function Q is constant. The optimal leverage level is equal to $\frac{1}{m}$, which is the maximum leverage allowed.

In order to solve the system of equations (8) and (9) in the general case, I discretize the time period $[0, 1]$ and the set of feasible leverage levels $[0, m]$. During each of the time moments the manager has a finite set of options with different utility levels and the optimal leverage is the leverage which delivers the highest utility for the manager. After a number of steps going from time $t = 1$ to time $t = 0$ we find the managerial utility at time $0+$ after fee payment. The utility after fees is used to find the optimal managerial decision before the fees are paid. Time moment $t = 0$ is equivalent to time $t = 1$ and we start the process again starting now from the utility obtained for $t = 0$. The procedure is repeated until the solution obtained converges. The numerical procedure is explained in Appendix B.

3.3 Extension: Model Without Margins

In the previous section, I obtained that the margin requirements drive the hedge fund manager behavior. This section investigates managerial behavior in case the margin requirements are relaxed. In this case the only difference between this model and the model considered in Panageas and Westerfield (2009) is the timing of fee payment. In Panageas and Westerfield (2009) the fees are paid continuously in infinitesimal increments, while in this paper the fund manager is paid at discrete time moments. This minor change produces a huge change in optimal leverage.

Consider one possible managerial strategy. Assume the manager chooses a constant leverage level $\pi_t = \pi$ for $t \in [0, 1]$. Then from equation (1) we have

$$dA_t = A_t(r + \pi(\mu - r))dt + A_t \pi \sigma dB_t. \tag{10}$$

Consequently,

$$A_1 = A_{0+} e^{r + (\mu - r)\pi - \frac{\sigma^2}{2}\pi^2 + \sigma \pi \xi}, \tag{11}$$

where ξ is a standard normal variable.
As a result,

$$E[A_1] = A_{0+} e^{r + (\mu - r)\pi}. \tag{12}$$

Therefore, the expected value of the AUM at time $t = 1$ prior to the payment of fees goes to infinity as π goes to infinity.

The dollar value of the management fee is equal to $f_m A_1$, so the expected value of the management fee paid at time $t = 1$ is equal to $f_m A_{0+} e^{r + (\mu - r)\pi}$, which increases to infinity as π increases to infinity.

The dollar value of the performance fee is equal to $f_p((1-f_m)A_1-(1+h_r)H_0)^+$. The expected value of the performance fee satisfies

$$\begin{aligned} E[f_p((1-f_m)A_1-(1+h_r)H_0)^+] &\geq f_pE[(1-f_m)A_1-(1+h_r)H_0] \\ &= f_p((1-f_m)E[A_1]-(1+h_r)H_0). \end{aligned} \tag{13}$$

The right hand side of equation (13) goes to infinity as π goes to infinity. Consequently, the expected value of the performance fee goes to infinity as π goes to infinity.

The reasoning used in inequality (13) with respect to the indexed high-water mark can be used with respect to any level that needs to be outperformed in order for the manager to obtain the performance fee. Therefore, the expected value of the performance based portion of the managerial compensation goes to infinity as leverage π increases to infinity provided $f_p > 0$ and $1-f_m > 0$.

All the fees the hedge fund charges across time are nonnegative, so the total expected utility for the risk-neutral manager goes to infinity since the first fee already goes to infinity. As a result, the manager cannot have an optimal strategy that has a bounded leverage level and delivers a finite utility, since he can always choose a higher leverage level that provides a higher utility than the given strategy.

Consider a case where the manager is restricted to rebalancing portfolio only at $t = 1$ after the fees are paid. In this situation denoting π the weight of the risky asset we obtain

$$A_1 = A_0(\pi e^{\mu-\frac{\sigma^2}{2}+\sigma\xi} + (1-\pi)e^r),$$

so

$$E[A_1] = A_0(\pi e^{\mu} + (1-\pi)e^r) = A_0(e^r + \pi(e^{\mu}-e^r)).$$

$E[A_1]$ goes to infinity as leverage goes to infinity, since $e^{\mu} - e^r > 0$. Considering the management and the performance fees as before, we obtain that the expected value of each of the fees goes to infinity as the leverage π goes to infinity. This shows that the result that the risk-neutral manager does not have an optimal bounded leverage does not depend on the ability to continuously rebalance the fund's portfolio.

Therefore, unlike in the models where the fees are charged continuously we obtain that when the fees are charged at discrete time moments the model needs some additional limitations in order for the manager to use a limited leverage. For example, Panageas and Westerfield (2009) find that a risk neutral manager who has a continuous flow of fees chooses a limited leverage, while I find that a small change to their model where the continuous flow of fees is substituted with payment of fees at discrete time moments, the optimal managerial behavior changes. When the manager faces a continuous flow of fees, he has less incentive to choose high leverage, since the closest payment option is infinitesimal, while a loss would lead to a drop in value of all the future options that add up to a substantial sum. In a discrete case, however, the closest option value is already substantial and infinitely high leverage delivers infinitely high expected utility from this option alone, and therefore the manager chooses to behave differently in this case.

There are a number of arguments for limitations on hedge fund leverage provided in the academic literature: future career concerns, managerial investments in the fund, and liquidation in the case of a poor performance. I consider these and other arguments in terms of the model with the risk neutral hedge fund manager in Appendix C and find that they do not lead to limitations on the optimal leverage level. If the hedge fund manager is risk averse, he chooses a finite leverage, but this result comes from risk aversion rather than these listed reasons. The question of an average risk aversion level of hedge fund managers is an interesting one, but unfortunately, according to my knowledge, there is no empirical study which reports it. There are many books about individual hedge fund manager stories, for example, Richard (2010), who points out that if anything, at least some hedge fund manager behavior is risk-seeking rather than risk-averse. Hedge fund flows are convex (see Chevalier and Ellison, 1997 and Sirri and Tufano, 1998), and this also effectively results in risk-seeking behavior because it increases utility gains from positive results and decreases utility losses from negative results.

Lan, Wang, and Yang (2012) propose that the presence of a liquidation boundary induces limited leverage levels without an exogenous margin requirement. Dai and Sundaresan (2010) provide two possible types of hedge fund liquidation that are relevant to the model:

liquidation by the investor and liquidation by a prime broker. In a continuous fee payment framework with a liquidation boundary, as in Lan, Wang, and Yang (2012), it is impossible to distinguish between these two cases. Below I consider two extensions of the basic model which allow me to study managerial behavior given each of the two possibilities separately.

3.4 Extension: Liquidation by the Investor

Lan, Wang, and Yang (2012) consider a model with a continuous flow of fees and find that a liquidation boundary set to the AUM to high-water mark ratio of 0.685 makes risk neutral manager to behave in a risk-averse way. They also consider a case where the liquidation boundary is set to the ratio equal to 0.5 and find similar results.

I consider an extension of the basic model where the investor sets a liquidation boundary for the fund to AUM to high-water mark ratio of l_b. I assume that liquidation by investor can happen only after the moment the fees are paid, which is a requirement set in order so the investor does not liquidate the fund before the first fees are paid. In reality there are lockup and notification periods that restrict early withdrawal.[7] In the terms of the model the liquidation boundary is set as $Q(\omega) = 1$ for the ratio of AUM to high-water mark ω that is above l_b and $Q(\omega) = 0$ for ω that is below l_b.

Figure 1 shows numerical solution of this problem in the case where the manager can rebalance fund's portfolio only at discrete time moments and $l_b = 0.5$. Each of the separate pictures corresponds to a different intermediate time moment from $t = 0$ to $t = 1$, where I assume that the fund charges fees at the end of each year (at $t = 1$). The manager uses maximum leverage 2 during the first half of the year, because even if the fund underperforms, then there is still the second half of the year the manager can recoup losses. However, at the levels of AUM close to the liquidation boundary he starts to lower leverage, since now the probability to receive performance compensation is small, while there is a high probability of liquidation in case of further losses. Closer to the end of the year manager tries to avoid very costly liquidation by lowering leverage even more. However, if the fund is below the

[7] See, for example, Ang and Bollen (2010).

liquidation boundary then the manager uses all the available leverage in order to gamble for survival, as in Hodder and Jackwert (2007). At the levels of the AUM significantly higher than the liquidation boundary the manager uses all the available leverage since even using the maximum leverage he has a very small probability of being liquidated. When this numerical solution for discrete time is taken to the limit with respect to number of subperiods within a year in order to solve the continuous time case, we find that only at the liquidation boundary at the moment the fees are paid is the optimal leverage equal to zero while everywhere else it is equal to the maximum allowed level. This is because the manager can always lower leverage close to the boundary and avoid liquidation in the continuous time case.

3.5 Extension: Liquidation by the Prime Broker

Hodder and Jackwert (2007) consider a discrete time model with a risk averse manager and a liquidation boundary given by an AUM to high-water mark ratio of 0.5. Liquidation in their model can happen any time moment before the fees are paid, which is the same situation as the liquidation by prime broker case in Dai and Sundaresan (2010). They derive that managerial behavior depends not only on the AUM to high-water mark ration, but also on the time until the fees are paid. Next I examine this case in the context of my model.

Consider now an extension of the main model where the prime broker continuously monitors the hedge fund's operations and liquidates the fund when the ratio of current AUM to high-water mark falls below some predetermined value. This extension assumes that the managerial value function is equal to 0 for ω below some liquidation boundary inside of the $[0, 1]$ time interval instead of just the fee payment moment.

Figure 2 shows a numerical solution of this extension. Each of the separate pictures corresponds to a different intermediate time moment from $t = 0$ to $t = 1$, where I assume that the fund charges fees at the end of each year (at $t = 1$). I find that if the manager is allowed to rebalance the fund's portfolio only at discrete time moments then he maintains leverage close to 0 if the fund's AUM is significantly close to the liquidation boundary in order to preserve the fund. When the fund is liquidated the manager loses all future fees and therefore he tries to avoid liquidation. If the fund's AUM is significantly above the liquidation boundary, then the manager decides to use the maximum leverage, since he will have time to reduce leverage and preserve the fund in case of negative performance, while in a case of a positive performance he faces significant utility gains from the performance fees.

Figure 2 shows that if the fund's AUM is lower than the fund's high-water mark, then the manager chooses a more conservative portfolio before the fee payment than at the beginning of the year in order to preserve the fund's AUM. The reason for this behavior is that even using the maximum level of leverage, the probability to obtain performance fees is small, while a drop in the AUM level may lead to a struggle for survival in the next term. This case differs significantly from the case where liquidation is triggered by the investor, since there is much less dependence on the particular time moment here, while the managerial behavior is more conservative when the fund is close to liquidation boundary inside of the time interval.

For the same AUM to high-water mark ratio considered right after the moment the fees are paid, the leverage is lower in the case of liquidation by prime broker than in the case of liquidation by investor. For example, the AUM to the high-water mark ratio equal to 0.6 in the liquidation by investor case is already high enough for the fund manager who chooses the maximum leverage 2, while in the liquidation by prime broker case the fund's leverage is close to 1. The difference disappears when I take the liquidation by prime broker extension to the limit with respect to the number of intermediate time moments during $[0, 1]$ time interval. This corresponds to the continuous portfolio management case where for the AUM to high-water mark ratio equal to the liquidation boundary the leverage is equal to 0 while everywhere else it is equal to the maximum allowed level. Consequently, in the continuous portfolio management model formulation there is no difference in managerial behavior between the cases of liquidation by the investor and liquidation by the prime broker.

3.6 Extension: Multiple Margin States

Consider an extension of the base model that instead of a constant margin requirement includes several possible margin states $m_1 < m_2 < \ldots < m_k$. The motivation for such extension comes from Dai and Sundaresan (2010) who report that the typical values of the margin requirements changed significantly from April 2007 to August 2008, for example, the margin requirements for AAA ABS CDO changed from 2-4% to 95%, the margin requirements for high-yield bonds changed from 10-15% to 25-40%. In this case the hedge fund may suddenly face much stricter margin requirements with significantly lower maximum level of leverage allowed. Since the hedge fund manager knows there is a possibility of a forced deleveraging in the future with an associated cost, he may chose leverage lower than the maximum allowed leverage.

There is no need to apply lower leverage than $\frac{1}{m_k}$ in the state of the world with margin requirement m_k, since the manager wishes maximum leverage in this state and the forced deleveraging is impossible in this regime. Therefore, in the highest margin state (the lowest maximum leverage state) the hedge fund manager chooses leverage equal to the maximum possible leverage. In other states of the world the manager may decide not to have the highest possible leverage because of the possibility of forced deleveraging. As a result, for the margin levels lower than m_k the manager chooses leverage that depends on the likelihood of forced deleveraging and its timing and therefore the hedge fund leverage is less correlated with the time-varying maximum leverage level than in the state with margin requirement m_k.

3.7 Testable Implications

The main model and the multiple margin states extension produce two testable implications. First, the hedge fund leverage is determined mostly by the maximum leverage level which is equal to the inverse of margin requirement. In order to test this implication we use a proxy for the maximum possible leverage equal to the inverse of Chicago Mercantile Exchange (CME) margin requirements for S&P 500 futures contracts from Dudley and Nimalendran (2011). Figure 3 shows daily relative margin requirements constructed as a ratio of the margin requirement in dollars and the market value of the underlying assets. I also use average gross hedge fund leverage from Ang et al. (2011). Figure 4 shows both series from December 2004 to June 2009. I find that sample correlation between the two series is equal to 0.914, while the sample correlation of changes in the average hedge fund leverage and changes in the inverse of margin requirement is equal to 0.47. These high and significant correlations support the conjecture that the maximum possible level of leverage significantly correlates with hedge fund leverage.

The multiple margin states extension suggests that the correlation between the margin requirement and the hedge fund leverage is higher in the state with the stricter margin requirement. In order to test this hypothesis, I split the data into two periods: 2005-2006 and 2007-2008. During the first period the margin requirement is lower, at 0.05, and therefore a lower correlation between the margin requirement and hedge fund leverage is expected. During 2007-2008, the margin is higher, at 0.1, and a higher level of correlation between the margin requirement and hedge fund leverage is expected. The sample correlation over 2005–2006 period is equal to 0.16, while the sample correlation over 2007-2008 period is equal to 0.97, which confirms this prediction. The correlation between changes in the margin requirement and changes in hedge fund leverage during 2005-2006 is equal to -0.07 and during 2007-2008 it is equal to 0.57, which shows that the correlation between changes in the margin requirement and changes in hedge fund leverage is also higher during the periods with the highest margin requirements.

4 Costs of the High-water Mark and the Hurdle Rate Provisions

Comparing two otherwise identical compensation contracts one with a provision and the other one without, it is clear that on the one hand presence of the provision lowers the fees the manager collects at the moment, but it also increases the fund's remaining AUM, which increases future fees. Since the investor prefers to include provisions that limit payment of the performance fees, it is interesting to determine if the manager dislikes them and to measure their utility cost. The question of costs of individual provisions became more important in light of a recent regulatory debate on the levels of hedge fund fees and provisions included in the compensation contracts.[8] In this section I study the impact of high-water mark and hurdle rate provisions on managerial utility in terms of the model.

For simplicity I consider a baseline case where the investor withdraws his investment from the hedge fund at the end of the time period with a fixed probability Q and stays invested with probability $1 - Q$.

I introduce an equivalent management fee (EMF) in order to compare effects of different fee provisions across contracts. The EMF is the value of the management fee such that the hedge fund manager is indifferent between his current hedge fund contract and a mutual fund contract that charges only a management fee equal to the EMF. Therefore, for each hedge fund compensation contract, there is a corresponding mutual fund compensation contract yields the same utility to the manager. Consequently, management and performance fees from the hedge fund contract together with different provisions can be mapped to one number, which allows simple comparisons across different contracts. This approach allows me to find appropriate fee changes in case the manager or the investor wants to modify the compensation contract by adding or subtracting one or both of the provisions.

A mutual fund that charges only the management fee is a partial case of the model considered in Section 3.3. Equation (8) can be rewritten as

$$V(\omega_0) = E\left[\beta z\left(\omega_1, \omega_0\right) + \beta Q(\omega_1 - z(\omega_1, \omega_0))V\left(1\right)\right], \tag{14}$$

where $V(\omega) = C\omega$ and $z(\omega_1, \omega_0) = f_m \omega_1$. The optimal leverage level π is equal to the inverse of the margin requirement, so

$$\omega_1 = \omega_0 e^{r + (\mu - r)\pi - \frac{\sigma^2 \pi^2}{2} + \sigma \pi \xi}. \tag{15}$$

Therefore,

$$E\left[\omega_1\right] = \omega_0 e^{r + (\mu - r)\pi}. \tag{16}$$

Plugging equation (16) in equation (14) and canceling out ω_0 I obtain

$$C = \beta f_m e^{r + (\mu - r)\pi} + \beta Q(1 - f_m) e^{r + (\mu - r)\pi} C. \tag{17}$$

[8] See, for example, Karmin and Strasburg (2009).

This allows to express the equivalent management fee in terms of the level of the managerial utility for \$1 of AUM and model parameters:

$$f_m = \frac{Ce^{-r-(\mu-r)\frac{1}{m}} - \beta QC}{\beta(1-QC)}. \tag{18}$$

Table 3 reports values of the EMF obtained using equation (18) for different specifications of the hedge fund compensation contracts. The hedge fund compensation contract that pays annually a 2% management fee and a 20% performance fee has an EMF equal to 6.45%. Restriction of the fee payment by the high-water mark provision decreases the EMF to 5.75%. If the payment of the performance fee is restricted by a 4% hurdle rate, then the EMF decreases to 6.11%. If the hedge fund compensation contract contains both the high-water mark and the hurdle rate provisions, then the EMF for this contract is equal to 5.31%. The main result of these calculations is that the addition of the high-water mark provision without indexation is equivalent to a 0.7% drop in EMF from 6.45% to 5.75%, the addition of the 4% hurdle rate to a contract without the high-water mark provision is equivalent to a 0.34% drop in EMF from 6.45% to 6.11% and an addition of the indexed by 4% high-water mark is equivalent to a 1.14% drop in EMF from 6.45% to 5.31%.

I also consider a possibility of change in the frequency of fee payments by increasing the frequency from annual to quarterly or monthly. The values of the parameters used for the quarterly and the monthly models are adjusted so that their annual compounded values are equal to the values for the annual model, in particular, the probability to disinvest in the first year in the monthly model coincides with the probability to disinvest in the first year in the annual model. I find that the increase in the frequency of the fee payment is even more detrimental for the hedge fund managers. A change in the frequency of the fee payments from annual to quarterly leads to a drop in the EMF from 5.31% to 3.73%. A change in the frequency of the fee payments from annual to monthly leads to an even more pronounced drop in the EMF — from 5.31% to 2.93%. The results suggest that hedge fund managers dislike an increase in the fee payment frequency more than they dislike high-water mark and hurdle rate provisions.

5 Conclusions

I consider a hedge fund model with a manager who can continuously rebalance while the fees for his services are paid at discrete time moments. In this model, I consider management and performance fees. Payment of the performance fees can be restricted by high-water mark and/or hurdle rate provisions. The manager faces a margin requirement and a possibility of liquidation in case of poor performance.

I find that if the model does not include the margin requirements then the risk-neutral manager chooses to employ unbounded leverage. Exogenously imposed margin requirements limit the maximum allowed leverage level. The analysis with multiple states where margins can differ suggests that the inverse of the margin requirement is a significant determinant of the hedge fund leverage, since the manager chooses to use the maximum leverage available in at least some states. I test this conjecture using CME S&P 500 futures margins data and average hedge fund leverage and find that the correlation between hedge fund leverage and the inverse of the margin requirement over 2005-2008 is equal to 0.91. The correlation between changes in hedge fund leverage and changes in the inverse of the margin requirement is equal to 0.47.

I construct an equivalent management fee (EMF) — the management fee that the hedge fund manager is indifferent between managing his hedge fund and managing a mutual fund that charges only a management fee of EMF. I use this measure to compare different hedge fund compensation contracts from the point of view of manager utility. I estimate that for a standard 2% per annum management fee and 20% performance fee contract that has no provisions restricting payment of the performance fees, an addition of the high-water mark provision without indexation is equivalent to a 0.7% drop in EMF from 6.45% to 5.75%, an addition of a 4% hurdle rate is equivalent to a 0.34% drop in EMF from 6.45% to 6.11% and an addition of the indexed by 4% high-water mark is equivalent to a 1.14% drop in EMF from 6.45% to 5.31%. I also find that an increase in the frequency of fee payments from annual to quarterly or monthly costs more for the hedge fund manager than these provisions. For example, a change in the frequency of fee payments from annual to monthly in a contract that has a 2% management fee and a 20% performance fee with an indexed by 4% high-water mark provision leads to a drop in EMF from 5.31% to 2.93%.

Hedge fund compensation contracts constitute a very interesting and a fruitful topic for the academic research. An interesting question for further study is the influence of different economic variables on the terms of the hedge fund compensation contracts. For example, the hurdle rate provision is not as popular in new contracts as it was before the financial crisis. Agarwal, Naveen, and Naik (2009) report that during 1994 - 2002 61% of the hedge fund compensation contracts had a hurdle rate provision, while according to HFR 2010 database only 13.1% of hedge fund contracts now have a hurdle rate provision. The high-water mark provision, however, became more popular. Agarwal, Naveen, and Naik (2009) report that during 1994 - 2002, 80% of the hedge fund compensation contracts had the high-water mark provision, while according to HFR 2010 database 91.7% of the current hedge fund compensation contracts have the high-water mark provision.

Appendix

A Analytical Solution of the Constant Q Case

In order to solve the system of equations (8) and (9) at first note that the compensation $f(\omega_1, \omega_0)$ is a convex function of ω_1. Consider the total expected utility function V as a sum of the discounted future compensation options. The future options are multiplied by the constant probability Q, so the future options add convex functions under the expectation sign. Therefore, the total fees over time form a convex function.

Consider a leverage strategy π_t that forms a step function, that is, it consists of intervals where the leverage is constant. Consider one of these intervals $[t_1, t_2]$. If the leverage on this interval π changes to $\frac{1}{m}$ then the expected value of the total fees increases. The value of ω_1 can be written as

$$\omega_1^* e^{r(t_2-t_1)+(\mu-r)\pi(t_2-t_1)-\frac{\sigma^2}{2}\pi^2(t_2-t_1)+\sigma\sqrt{t_2-t_1}\pi\xi}.$$

Denote

$$\overline{\omega_1} = \omega_1^* e^{r(t_2-t_1)+(\mu-r)\pi(t_2-t_1)-\frac{\sigma^2}{2}\pi^2(t_2-t_1)}$$

When we consider two symmetric realizations of the standard normal variable ξ equal to η and $-\eta$, we see that for leverage π this results in $\omega_{11} = \overline{\omega_1}e^{-\sigma\sqrt{t_2-t_1}\pi\eta}$ and $\omega_{12} = \overline{\omega_1}e^{\sigma\sqrt{t_2-t_1}\pi\eta}$, while for leverage $\frac{1}{m}$ it results in $\omega_{21} = \overline{\omega_1}e^{-\sigma\sqrt{t_2-t_1}\frac{1}{m}\eta}$ and $\omega_{22} = \overline{\omega_1}e^{\sigma\sqrt{t_2-t_1}\frac{1}{m}\eta}$. We can rewrite this as $\omega_{21} = \omega_{11}e^{-\sigma\sqrt{t_2-t_1}(\frac{1}{m}-pi)\eta}$ and $\omega_{22} = \omega_{12}e^{\sigma\sqrt{t_2-t_1}(\frac{1}{m}-pi)\eta}$, that is the smaller value is divided and the larger value is multiplied by the same number. Due to the convexity of the sum of total fees paid, the sum of values in ω_{21} and ω_{22} is higher than the sum in ω_{11} and ω_{12}. Therefore, the manager prefers to change leverage π on interval $[t_1, t_2]$ to the maximum level $\frac{1}{m}$. This is true for each of the intervals of the strategy and as the result, manager prefers to have leverage equal to $\frac{1}{m}$ at each point in time.

Consider some strategy π_t^*. It can be approximated by step function strategies that are all less preferable than the strategy $\pi_t = \frac{1}{m}$. Taking this approximation series to the limit we find that the strategy $\pi_t = \frac{1}{m}$ cannot be inferior to the strategy π_t^* and therefore the strategy $\pi_t = \frac{1}{m}$ provides the highest possible utility for the hedge fund manager.

B Numerical Solution

The main restriction of the equation (5) is the hardwired upper bound on the leverage level $\frac{1}{m}$ due to a margin level m.[9] The lower bound on the leverage level is zero, since for the negative weights there is the same normal distribution around the mean, as for the positive weights, but the value of the mean return itself is lower, therefore it is never optimal to have negative weights of the risky asset (assuming $\mu > r$.

The optimization problem is solved on a set of feasible leverage levels $\pi \in [0, \frac{1}{m}]$. Consider a grid of different leverage levels $\frac{i}{mN}$ that spans the interval $[0, \frac{1}{m}]$, where $i = 0, 1, \ldots, N$ and N is some integer number.

The manager solves optimization problems at time moments $\frac{j}{T}$, where $j = 0, 1, \ldots, T$ and T is the number of time intervals. In this numerical solution we use an underlying assumption that on each of the time intervals $[\frac{j}{T}, \frac{j+1}{T}]$ the leverage level is constant. This assumption simplifies modeling since there is no violation of margin requirements during the time intervals between the discrete moments where the manager decides on the leverage. Additionally, it allows to write down simple formulas for the distribution of the new value of the AUM, that is

$$A_{\frac{j+1}{T}} = A_{\frac{j}{T}} e^{\frac{r}{T}+(\mu-r)\pi\frac{1}{T}-\frac{\sigma^2\pi^2}{2T}+\sigma\pi\int_{\frac{j}{T}}^{\frac{j+1}{T}} dB_s} \tag{B-1}$$

that is

$$A_{\frac{j+1}{T}} = A_{\frac{j}{T}} e^{\frac{r}{T}+(\mu-r)\pi\frac{1}{T}-\frac{\sigma^2\pi^2}{2T}+\frac{\sigma\pi}{\sqrt{T}}\xi}, \tag{B-2}$$

[9] The need for this limitation is based on Section 3.3.

where ξ is a standard normal variable.

There are several problems that arise from this approach, since the manager has to know the expected value of the future utility function in order to solve the optimization problem (5). Therefore, at first we need to know the values of the future utility function and then we need to be able to calculate the expected value of the future utility for a normal probability density function.

I use Gauss-Hermite quadrature in order to approximate the value of the integral over the normal density function. The Gauss-Hermite quadrature is a method for approximating integrals

$$\int_{-\infty}^{\infty} f(x)e^{-x^2}dx,$$

using finite sums

$$\int_{-\infty}^{\infty} f(x)e^{-x^2}dx \approx \sum_{i=1}^{n} \omega_i f(x_i)$$

where for each number n we calculate the abscissas x_i and their weights ω_i. This poses a new problem, since we need to know the values of the future utility function at each point and not only on some predetermined grid. The future utility function is obviously increasing in the ratio of the AUM to the high-water mark, since given the same value of the high-water mark, the higher values of the AUM result in higher fees for the manager. I use a spline procedure to estimate values of the function between grid nodes. Since the utility function is monotonous, we know that the error obtained using this estimation procedure is bounded.[10]

At the end of the time interval we add fees and the value of the utility function going forward, which is the function obtained at the beginning of the time interval. This results in the following algorithm:

Step 0. Take random initial values of the utility function going forward for a specified grid of the assets under management to the high-water mark ratio for the time grid node $\frac{0}{T}$.

Step 1. Calculate the value of the utility function at the time $\frac{T}{T}$ as a sum of the discounted fees obtained at the end of the period and of the discounted utility function at the new assets to high-water mark ratio. The utility function used is the utility function for the time grid node $\frac{0}{T}$. The value of the assets under management is the after-fees assets under management and the high-water mark is the updated high-water mark. If there is a probability to continue that may depend on the assets to high-water mark ratio, then it's an additional discounting multiplier used for the utility function going forward. This step defines values of the utility function at the node $\frac{T}{T}$.

Step 2. For the previous node $\frac{j}{T}$ of the time interval find the leverage level that maximizes the expected utility function calculated using Gauss-Hermite quadrature using the expected utility obtained for the time grid node $\frac{j+1}{T}$. Assign the value of the utility function at the current point to this maximum value. Repeat step 2 until the node $\frac{0}{T}$.

Step 3. If the values of the utility function at the node $\frac{0}{T}$ converged, then stop. Otherwise go to Step 1.

C Other Arguments for Limited Leverage

The academic literature has provided a number of arguments for a more conservative use of leverage by hedge fund manager. Some of these models consider risk-neutral hedge fund managers, while other models consider risk averse managers. Here I test each of the presented arguments in light of the main model. There are two

[10] Additionally we consider a method that allows to bound the values of the utility function and to improve the lower and upper bounds in order to check our approximations. The errors come from estimation of values between the nodes, but since the estimated function is increasing, the values in the neighbor nodes provide upper and lower bounds for the values of the utility function between them. Therefore, instead of approximating values of integrals we may calculate the upper and lower bounds on these values. Using the estimation procedure for highest values we obtain the upper bounds on the values of the utility function, while using the lowest values we obtain the lower bounds. Increasing the density of the grid we improve estimations, which decreases the gap between the upper and the lower bounds.

main differences of the main model from the referenced models: my model accounts for both management and performance fees as well as high-water mark and hurdle rate provisions and while the manager can rebalance portfolio continuously, the fees for his service are paid at discrete moments.

C.1 Future career concerns

Brown et al. (2001), Panageas and Westerfield (2009) and Hodder and Jackwert (2007) among others argue that the multi-period career concerns that arise from having a number of compensation options over the long run lead to limitations on the risk taking by the fund manager.

As I showed in Section 3.3, the risk-neutral manager in the case when the fees are charged at discrete time moments can achieve infinitely high utility from the first compensation option already. Since the other options result in nonnegative utilities due to the fees being nonnegative, the manager's total expected utility is infinitely high and therefore there is no need to decrease leverage due to the presence of future options. Thus, the multi-period career concerns by themselves do not make the risk-neutral manager to limit the leverage level.

C.2 Managerial investments in the fund

Kowenberg and Ziema (2007) suggest that a substantial (e.g. $> 30\%$) stake in the fund reduces managerial risk taking. This argument does not apply for the risk-neutral hedge fund manager. When the leverage increases towards infinity the expected values of the management and the performance fees increase towards infinity. The expected value of the personal investment with the fund also increases towards infinity. Therefore the manager is prone to take unbounded leverage in this case.

C.3 Liquidation in case of a poor performance

The fund can be liquidated by the investor, that is the investor can withdraw the capital from the fund. Investors can withdraw money from hedge only after a lengthy notification period (Ang and Bollen, 2010) which significantly limits investors freedom to withdraw money. The model in Section 3 incorporates this in a form of possibility to withdraw money only at the fee payment moments. Hodder and Jackwert (2007) suggest that if the fund's liquidation may be linked to the ratio of the fund's AUM to the high-water mark level, that is if this ratio drops below a predetermined level, then the investor disinvests from the fund. In the model formulated in Section 3 this can be done by defining the probability of continuation with investment equal to 0 for the case when the ratio of the fund's AUM to the high-water mark drops below this threshold.

Considering that the dollar value of the performance fee goes to infinity when leverage goes to infinity and it is payed out only in the case of outperformance of the high-water mark, it is obvious that by using levels of leverage that go to infinity the manager obtains levels of utility that go to infinity. Therefore, the possibility of liquidation by the investor in case of significant underperformance does not lead to limited leverage.

C.4 Different high-water mark levels for different investors

A similar argument to the argument regarding the future career concerns can be made in a case, where we consider a number of investors with different values of the high-water marks. This results in a number of different options for each time moment, but the expected gain from each of these options increases to infinity for leverage level that increases towards infinity. Therefore the managerial utility increases towards infinity and consequently the diversity of the investors in terms of their high-water marks does not make risk neutral manager to apply bounded leverage.

C.5 Conditional probability of continuation with investment

An extension of the case where investor liquidates the hedge fund if the ratio of the fund's AUM to the high-water mark drops below some threshold is the case where the probability that investor will stay invested in the hedge fund depends on the ratio of the fund's AUM to the high-water mark level. However, the hedge fund manager obtains the performance fee for the first period in case of outperformance of the high-water mark and therefore the expected utility from the performance fee goes towards infinity when leverage goes to infinity. Consequently, independent of the ability of the investor to decide if he wants to disinvest from the fund, the hedge fund manager can obtain infinitely high utility. As a result, the ability of investor to disinvest from the fund does lead to limitations on leverage.

C.6 Margin requirements

Duffie et al. (2008) produce a policy implication that requires hedge funds to have a fixed level of maximum leverage they can take. A policy that accomplishes this is already in place.

Margin (haircut) is a requirement from prime-brokers or exchanges for hedge fund managers to post a particular portion m of the value of the assets that manager wants to buy or to short-sell to a margin account. This results in a hardwired bound on leverage equal to $\frac{1}{m}$. Consequently the manager does not have a possibility to get an infinitely high expected utility from the first fees and all the before mentioned reasons may matter when there is a positive margin requirement.

C.7 Continuous monitoring with a possibility of liquidation

We can consider a model with a continuous monitor, presumably a prime-broker, who continuously monitors the hedge fund's performance. This monitor immediately liquidates the fund in case A_t drops below a particular predetermined level. This case is considered in the Section 3.2. The main observation is that the presence of this monitor matters in the discrete case, but in the continuous case the manager will always take unlimited leverage. This is different from the constant optimal leverage solution of manager's problem obtained in Merton (1969), Panageas and Westerfield (2009), and Lan, Wang, and Yang (2012) that depends on the risky asset expected return μ and its volatility σ.

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Table 1: Fee payment example

	Initial Value	2007	2008	2009
Performance		20%	-20%	50%
Fund Size Before Fees		1,200,000	919,040	1,350,989
Management Fee	2%	24,000	18,381	27,020
After Management Fee		1,176,000	900,659	1,323,969
Net Value Added		176,000	-248,141	423,310
High-water Mark		1,000,000	1,148,800	1,194,752
Indexed High-water Mark	4%	1,040,000	1,194,752	1,242,542
Outperformance		136,000	0	81,427
Performance Fee	20%	27,200	0	16,285
Total Fees		51,200	18,381	43,305
Investor	1,000,000	1,148,800	900,659	1,307,684

This table shows a fee payment example for a hedge fund that charges a 2% management fee and a 20% performance fee. The high-water mark value is indexed by a 4% rate. The indexed high-water mark value has to be outperformed in order for the fund to obtain the performance fee. The investor invests $1,000,000 at the end of 2006. This table reports an evolution of his investment and the fees paid over time.

Table 2: Values of parameters used in estimations

Variable	Value
μ	10%
σ	20%
r	4%
β	0.96
m	50%

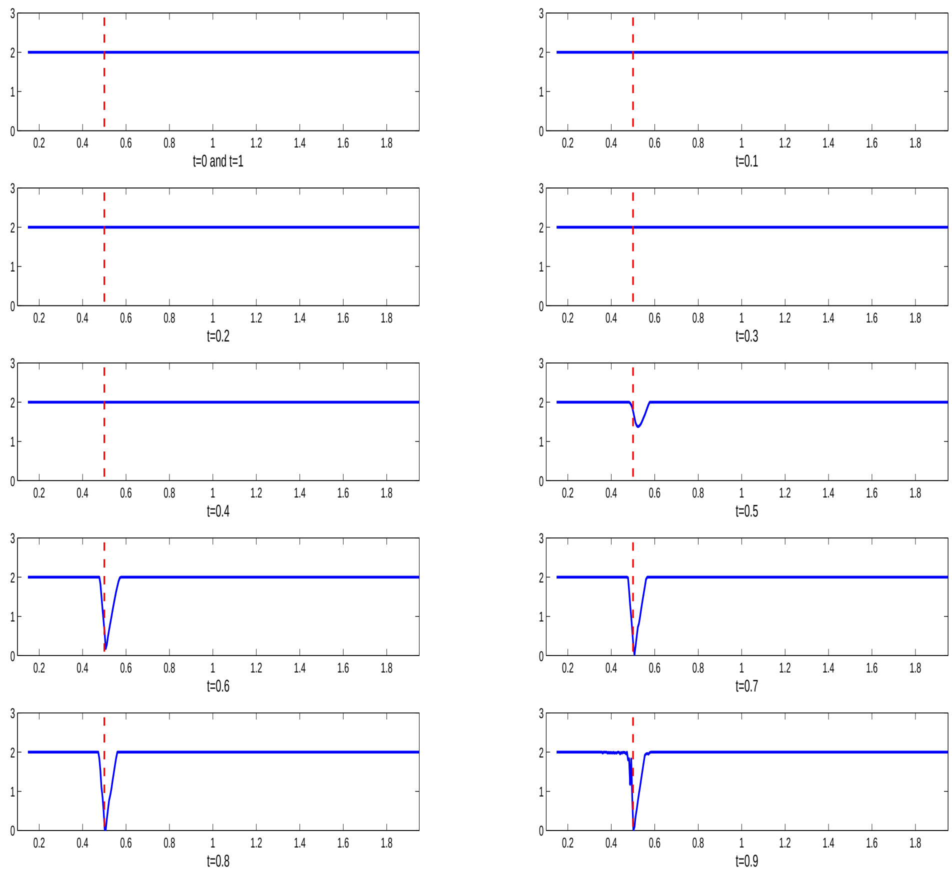
This table reports the values of the parameters used in the numerical examples throughout the paper. Here μ is the mean return of the risky strategy employed by the hedge fund, σ is the volatility of this risky strategy, r is the risk-free rate, β is the time discount factor that is related to the risk-free rate, and m is the value of the margin requirement that bounds the leverage level the hedge fund manager can use by $\frac{1}{m}$.

Table 3: Equivalent no-performance fee contracts

Frequency	f_m	f_p	High-water mark	Hurdle rate	Equivalent Management Fee
Annual	2%	20%	0	0%	6.447%
Annual	2%	20%	1	0%	5.754%
Annual	2%	20%	0	4%	6.107%
Annual	2%	20%	1	4%	5.308%
Quarterly	2%	20%	1	4%	3.723%
Monthly	2%	20%	1	4%	2.926%

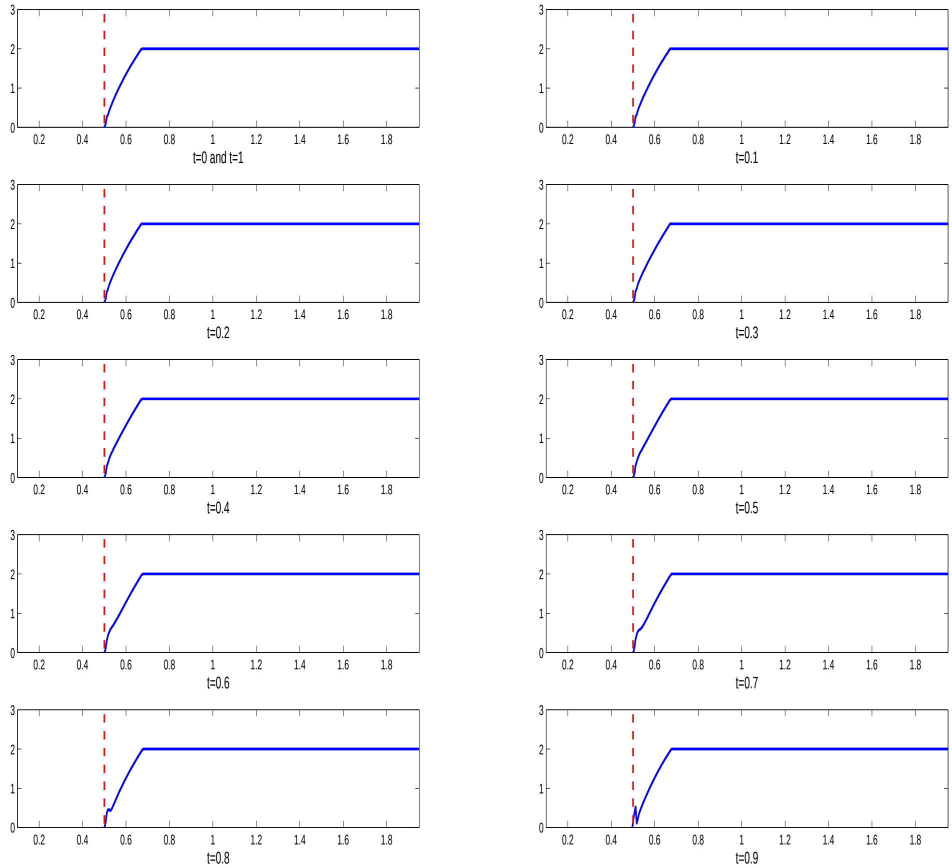
This table reports the values of the equivalent management fee for different specifications of the hedge fund compensation contract. The equivalent management fee is equal to the management fee m_f where the hedge fund manager is indifferent between managing his hedge fund and managing a mutual fund that charges only the management fee m_f. f_m denotes the management fee and f_p denotes the performance fee specified in the compensation contract. The high-water mark column in the table contains an indicator variable that reflects the presence of the high-water mark provision in the compensation contract. The high-water mark is the highest NAV the investor had with the hedge fund. The high-water mark provision requires the manager to outperform the high-water mark in order to obtain the performance fee. The hurdle rate column contains the hurdle rate value that restricts payment of the performance fee to cases where the fund outperformed the hurdle rate. If the high-water mark and the hurdle rate are present in the compensation contract, then we assume that the high-water mark is indexed with respect to the hurdle rate. The hedge fund manager has to satisfy all the existing provisions in order to obtain the performance fees.

Figure 1: Optimal portfolio with liquidation by the investor



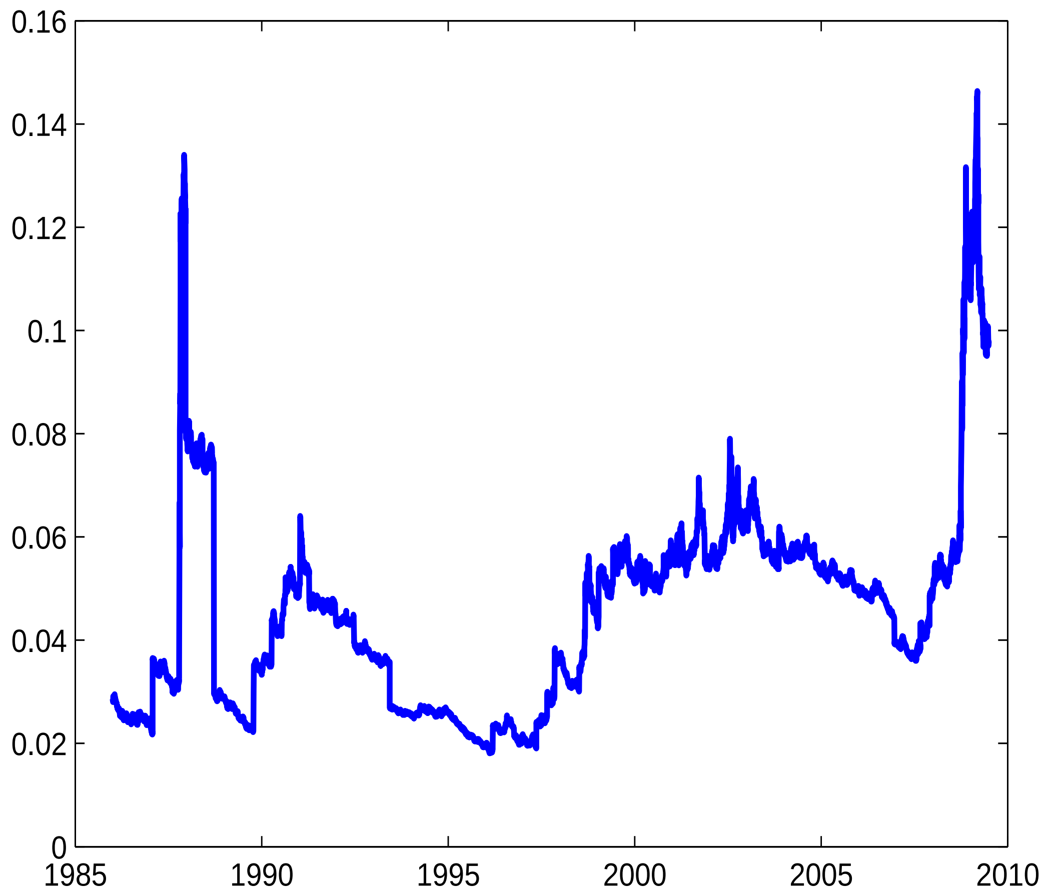
This figure shows the optimal hedge fund leverage level (vertical axis) depending on the AUM to the high-water mark ratio (horizontal axis) for time moments $t = 0$, $t = 0.1$, ... , $t = 0.9$, during the time interval between the fee payment moments that correspond to $t = 0$ and $t = 1$. The pictures correspond to a case where the investor liquidates the fund in case the AUM to the high-water mark ratio drops below 0.5 at the fee payment moment. The dashed line represents the liquidation boundary, that is present only at the moment the fee is paid. The investor can liquidate the fund only after the fees are paid. The margin requirement is assumed to be equal to 0.5 which corresponds to the maximum leverage level equal to 2.

Figure 2: Optimal portfolio with a liquidation by the prime broker



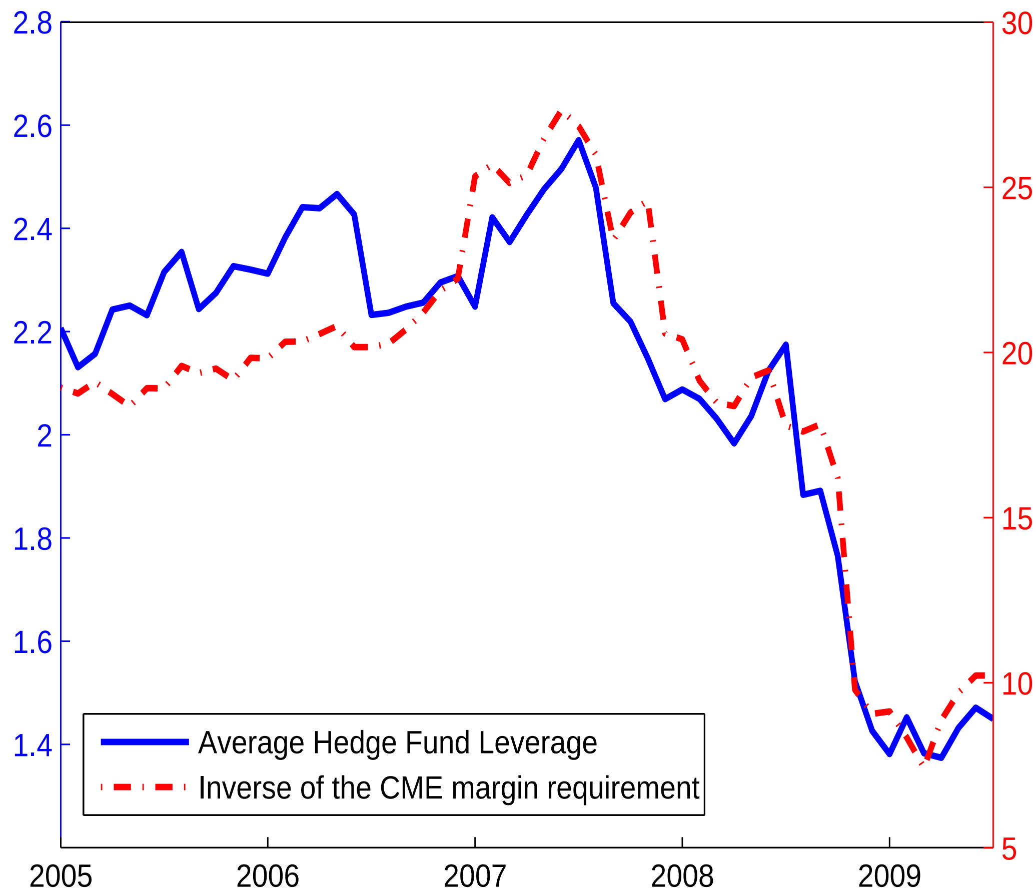
This figure shows the optimal hedge fund leverage level (vertical axis) depending on the AUM to the high-water mark ratio (horizontal axis) for time moments $t = 0$, $t = 0.1$, ... , $t = 0.9$, during the time interval between the fee payment moments that correspond to $t = 0$ and $t = 1$. The pictures correspond to a case where the prime broker liquidates the fund in case the AUM to the high-water mark ratio drops below 0.5 at any point during the $[0, 1]$ time interval. The dashed line represents the liquidation boundary. The margin requirement is assumed to be equal to 0.5 which corresponds to the maximum leverage level equal to 2.

Figure 3: CME margin requirements for S&P 500 futures contracts



This figure shows daily relative margin requirements imposed by Chicago Mercantile Exchange (CME) as a function of time. The relative margin requirements are obtained as a ratio of a maintenance margin requirement in dollars to the value of the underlying S&P 500 futures contracts. The data sample is from January 2, 1986 to June 30, 2009. The horizontal axis corresponds to time and the vertical axis corresponds to the relative margin level.

Figure 4: Comparison of an average hedge fund leverage and an inverse of the CME margin requirement



This figure compares the average hedge fund leverage obtained in Ang et al. (2011) shown in solid line with the inverse of the relative Chicago Mercantile Exchange (CME) margin requirement for S&P 500 futures contracts obtained from the data used in Dudley and Nimalendran (2010) shown in dashed line. The value axis for the average hedge fund leverage is on the left-hand side, while the value axis for the inverse of the CME margin requirement is on the right-hand side. The data sample is monthly from December 2004 to June 2009.

Hedge Fund Risk Premia: Transparency, Liquidity, Complexity, and Concentration*

Sergiy Gorovyy† Olga Kuzmina‡

Abstract

We study risk premia associated with hedge fund transparency, liquidity, complexity, and concentration over April 2006 to March 2009. We directly measure these qualitative characteristics by using the internal grades that a fund of funds attached to all the funds it invested in, and which represents the unique information that cannot be obtained from quantitative data alone. Consistent with factor models of risk premium, we find that during normal times low-transparency, low-liquidity, and high-concentration funds delivered a return premium, with economic magnitudes of 5% to 10% per year, while during bad states of the economy, these funds experienced significantly lower returns. We offer a novel explanation for why highly concentrated funds command a risk premium by revealing that their risk premium is mostly prevalent among non-transparent funds where investors are unaware about the exact risks they are facing and hence cannot diversify them away.

JEL Classification: G01, G11, G23, G32

Keywords: Hedge funds, Risk premia, Transparency, Liquidity, Complexity, Concentration

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1 Introduction

In the modern era of delegated portfolio management hedge funds constitute one of the most interesting and complicated investment vehicles. Usually they operate in a way that does not require them to disclose details about their operations. This does not mean that hedge funds do not disclose this information, but that they are not obliged to do so and as a result the level of disclosure is an internal decision by the hedge fund manager. The fund's structure and disclosure level is rarely modified during fund's life since the fund's investors expect it to maintain the same structure and disclosure level during its operation. After 2008, however, hedge funds began to offer more transparency on demand from government and investors. Sometimes hedge funds use third party aggregation services such as "Riskmetrics" in order to disclose more information on the aggregate risks of the fund without disclosing its particular holdings.

The question of whether hedge funds should be required to disclose information regarding their trades and positions is important, especially in the light of recent regulatory changes, including the Dodd-Frank Wall Street Reform Act passed in July 2010. This act requires managers of hedge funds with more than $150 million in assets under management to register with the Securities and Exchange Commission and become subject to its disclosure rules. Although the consequences of this act are yet to be evaluated, in this paper we attempt to explore the connection between hedge fund reporting level and their returns. The primary goal of this paper is, thus, to determine whether there is a significant return premium associated with more secretive, less transparent hedge funds.

The contribution of our paper is three-fold. First of all, by using a novel proprietary dataset obtained from a fund of funds that spans April 2006 to March 2009, we are able to directly measure the transparency level of a fund, a qualitative characteristic that is missing in public hedge fund databases, use it to uncover and quantify the non-transparency risk premium which amounts to 5.4% per year. The data spans both good and bad states of the economy allowing us to test the risk-premium story against the alternative of better managers being selected into managing low-transparency funds. Second, by investigating how excess returns vary with other fund characteristics, such as fund liquidity, complexity of its strategy, and concentration of its investments, we document the presense of several other risk premia in a cross-section of hedge fund returns. Finally, we explore how transparency, liquidity, complexity, and concentration help explain the fund return volatility and capital inflows.

Few papers in the asset pricing literature have raised the issues of hedge fund transparency, presumably due to the absence of adequate data to explore this question. Anson (2002) outlines different types of transparency and discusses why investors may want higher degree of transparency. Hedges (2007) overviews the key issues of hedge fund investment from a practitioners perspective. Goltz and Schroder (2010) survey hedge fund managers and investors on their reporting practices and find that the quality of hedge fund reporting is considered to be an important investment criterion. Aggarwal and Jorion (2012) study quantitatively effects of hedge funds' decisions on whether to provide or not to provide managed accounts to their investors. Managed accounts contain securities custodial in the client's name, who knows the exact account positions, while commingled accounts contain securities custodial in manager's name and clients do not generally know fund's holdings. Aggarwal and Jorion (2012) interpret the incidence of accepting managed accounts as indicating of the willingness of the fund to offer transparency and do not relate the results to risk premiums. In contrast to these studies, we are able to directly measure the level of transparency of a fund by using proprietary scores that are based on formal and informal interactions with hedge funds, such as internal reports, meetings with managers and phone calls made by a fund of funds. To the best of our knowledge, we are the first paper to explore and quantify the risk premium associated with low transparency.

To illustrate the risk premium channel, let us consider a risk-averse investor who faces two alternative hedge funds. If investing with one is more risky from the point of view of investors, this fund will have to deliver superior returns during normal times in order to attract any investment at all, i.e. investors are compensated for bearing risks. At some point these risks will realize, and this is when the riskier fund underperforms.

To further relate this to transparency, hedge funds that choose to provide less information about their positions and strategy details to investors leave investors uncertain about the underlying risks of investing with these hedge funds. In particular when a transparent fund starts to diverge from its declared strategy, investors can quickly disinvest if they dislike the change, while in case of a non-transparent fund investors will only learn about the change in the fund strategy later and have to face the consequences. This means that risk-averse investors should be compensated for bearing the risks associated with non-transparency. In particular, during normal times, low-transparency hedge funds are expected to perform better than high-transparency hedge funds by delivering an additional non-transparency risk premium. During bad times, on the other hand, the risks associated with non-transparency can realize, meaning that the low-transparency funds may deliver lower returns relative to high-transparency funds.

The time frame of our dataset is April 2006 to March 2009, allowing separately study the return premia over the good and bad states of the economy. In particular, this period covers the collapse of large global investment banks – Bear Stearns and Lehman Brothers, in March and September 2008, respectively. Investors feared being stuck with bad investments leading to a demand for transparency. Therefore, it is realistic to assume that non-transparency risks indeed realized during the later period of our data. Indeed, our empirical results show that during the crisis period from April 2008 to March 2009, more transparent funds outperformed the less transparent funds by about 7.1% per year.

We also document a presence of a hedge fund illiquidity risk premium. This is consistent with a large literature on risk premia associated with illiquidity across a variety of asset classes.[1] In general, an illiquidity premium is a premium for investment in more illiquid assets. For example, when the investor faces two alternative assets with one being more liquid than the other, she is able to disinvest from a more liquid asset with a lower loss when faced with a liquidity shock. Therefore, risk-averse investors invest in less liquid assets only if they expect to obtain superior returns. The most liquid funds in our dataset are the funds that both invest in higher liquidity assets and have fewer restrictions with regard to investment withdrawal (so fewer lockup restrictions). We estimate the illiquidity premium to be about 5.7%

[1] See Amihud and Mendelson (1986) for the seminal contribution, as well as Pastor and Stambaugh (2003), and Acharya and Pedersen (2005).

to 7.8% per year depending on the empirical specification. This is consistent with Liang (1999) and Aragon (2007) who show that funds with longer lockup periods outperform other funds.

Given the richness of our dataset, we are also able to explore the risk premia associated with more complicated strategies used by hedge funds, as well as more concentrated investments. We find that during normal times, high-complexity hedge funds underperformed low-complexity hedge funds by 3.8% per year, while we do not find a significant underperformance afterwards. The complex funds are usually non-transparent and have medium or low liquidity and therefore this complexity result is partially driven by hedge fund transparency and liquidity. This points to a presence of a negative complexity risk premium among hedge funds.

It is interesting to note that concentration of hedge fund investments should not matter in the light of the standard finance theory due to the theoretical ability of investors to diversify away the non-systematic (idiosyncratic) risks. This is in contrast to a recent empirical study by Ang et al. (2009) who find that idiosyncratic volatility bears a significant negative premium. In our paper we are able to offer a novel explanation of why investors may not be able to diversify their risks, by exploring in which funds the concentration premium is most pronounced. Intuitively, hedge fund investors should be compensated for the risks associated with concentrated investments of a fund when they do not know what constitutes these investments, i.e. they do not know which risks to diversify away. Hence, we expect to see a concentration risk premium only among the non-transparent hedge funds. Indeed, we verify this prediction using the interactions between concentration and transparency variables in our empirical setup.

Our paper is close in spirit to Brown, Goetzmann, Liang, and Schwarz (2008) who use SEC filing data to construct an $\omega-$score, which is a combined measure of conflict of interests, concentrated ownership, and leverage. They show that the $\omega-$score is a significant predictor of the projected fund life. In a subsequent paper, Brown, Goetzmann, Liang, and Schwarz (2012) use proprietary due diligence data to construct an operational risk variable as a linear combination of variables that correspond to mistakes in statements, internalized pricing, and presence of an auditor in the Big 4 group. We consider operational risk in a broader sense, where the willingness of hedge fund managers to provide details of their strategies, as well as hedge fund liquidity, investment concentration, and the ability of the investors to understand fund's operations are important.

We also study hedge fund return volatility and capital flows and find that the return volatility can be partially explained by the high degree of hedge fund concentration and liquidity, with up to 37% of the explained variation in the full-sample specification. During each of the periods considered the difference between volatilities of high-concentration versus low-concentration funds constitutes on average 2% per year, while the volatility of high-liquidity funds is on average about 1% lower than the volatility of low-liquidity funds. Both these magnitudes are economically significant given that the average hedge fund volatility over the sample is equal to 11.0% per year.

Finally, we also study how hedge fund capital flows are related to their transparency, liquidity, complexity, and concentration and find that among our qualitative variables only the level of liquidity can robustly explain capital flows across different periods in our sample. In particular, we find that low-liquidity funds experienced heavier outflows, especially during the crisis period from April 2008 to March 2009, where the difference between the flows from low-liquidity and high-liquidity funds amounted to 26.6 percent.

Our paper is organized as follows: Section 2 describes the data and variables used in our study, Section 3 explains the estimation procedure and the empirical setup, Section 4 discusses the main results on the risk premia associated with transparency, liquidity, complexity, and concentration, as well as additional results and robustness checks, and Section 5 concludes.

2 Data

We use a unique dataset obtained from a fund of funds that contains detailed fund-year information over the 2007–2009 period. This fund of funds is one of the largest in the U.S. The data provide information on hedge fund returns net of fees, their assets under management, and long and short exposures. Most importantly, these data include scores for hedge fund transparency, liquidity, complexity, and concentration as rated by the fund of funds on a scale from 1 to 4. Each year, at the end of March, the fund of funds grades all the hedge funds it invests in based on its interactions with them during the previous twelve months. These interactions consist of weekly or monthly reports to the fund of funds, meetings with managers, phone calls, etc. Due to the nature of the scoring process and a significant level of effort put into the construction of the scores we feel confident that they represent unique information about funds' operation that cannot be captured by the quantitative data alone. Such qualitative measures are not present in public hedge fund databases, such as CISDM, HFR, or TASS. Therefore, we think our data are especially well-suited for studying the return premia associated with different qualitative characteristics of hedge funds.

The definitions of transparency, liquidity, complexity, and concentration as used by the fund of funds are natural and intuitive. In particular, hedge fund transparency represents the willingness of the hedge fund manager to share information about the fund's current activities and investments with its investors. Hedge fund liquidity measures the liquidity of investments with the hedge fund from the point of view of investors. It comprises both the liquidity of the fund's assets and restrictions on withdrawal from the fund, such as the presence and the length of lockup periods. Hedge fund complexity corresponds to the complexity of the hedge fund strategy and its operations. For example, an offshore hedge fund that uses derivative instruments and swap agreements is considered to be complicated, since it is very hard for investors to understand exactly the kinds of risks it faces by investing with this fund. Finally, hedge fund concentration represents the level of concentration of hedge fund investments.

After filtering out various versions of the funds we are left with 355 observations of 167 different hedge funds that are evenly spread across the three years, with 121 observations in 2007, 122 – in 2008, and 112 – in 2009. Since our qualitative grades are given at the end of March, we use 2007, 2008, and 2009 to denote April 2006 to March 2007, April 2007 to March 2008, April 2008 to March 2009 periods, correspondingly. For example, the annualized return of a fund from April 2006 to March 2007 is matched to transparency, liquidity, complexity, and concentration grades that the fund of funds issued at the end of March 2007. This approach ensures that all interactions with the hedge fund that constitute the basis for the grades are conducted in the same period when the fund return is realized.

Our time frame is purposefully divided into three very distinct periods, since the risk premium story predicts different funds to perform better during good versus bad states of the economy. According to the Financial Crisis Inquiry Report (2011), the period from April 2006 to March 2007 can be considered a normal year. The beginning of the period from April 2007 to March 2008 also corresponds to a good state of the economy, but the end of this period was already associated with a recession in US. The collapse of Bear Stearns in March 2008 declared the beginning of the financial crisis, so we treat April 2007 to March 2008 as an intermediary period. Finally, April 2008 to March 2009 was clearly a period corresponding to a bad state of the economy, highlighted by the bankruptcy filing by Lehman Brothers, one of the largest investment banks. [2] The exogeneity of the global financial crisis allows us to test the risk premium explanation, since we are able to observe both the return premia during normal times as well as manifestations of the corresponding risks during the crisis period.

Hedge funds in our dataset represent a broad set of strategies. In particular, there are credit (CR), event-driven (ED), equity (EQ), relative-value (RV), and tactical trading (TT) hedge funds. Credit hedge funds trade mostly corporate bonds and CDS on those bonds; event-driven hedge funds seek to predict market moves based on specific news announcements; equity hedge funds trade equities; relative value hedge funds implement pair trades where one asset is believed to outperform another asset independent of macro events; and tactical trading funds speculate on the direction of market prices of currencies, commodities, equities and bonds.

Each fund is identified by a single strategy, which is time invariant for a given hedge fund (at least during the period considered). Panel A of Table 1 tabulates the number of hedge funds by various strategies for each of the periods considered. Approximately half of

[2] It is also worth mentioning, that according to NBER April 2006 to November 2007 was an expansion period while December 2007 to March 2009 was a recession period.

the hedge funds in the database are equity funds, with relative-value and event-driven as the next popular strategies. This distribution of strategies across funds is comparable to other databases, as reported, for example, by Bali, Brown, and Caglayan (2011) for TASS.

Panel B of Table 1 reports the mean, standard deviation, 25-th, 50-th, and 75-th percentiles, and the number of observations for hedge fund annualized returns, volatility, and assets under management (AUM) separately for each of the periods considered. Hedge funds performed well as a group during the normal period from April 2006 to March 2007 delivering on average a 13.59% per annum return with a 6.53% standard deviation. During the intermediate period they delivered on average a 3.72% return with a higher 10.92% volatility, while during the crisis period they delivered on average a negative -16.56% return with a 15.81% volatility.

The funds in our dataset are somewhat larger than funds in CISDM, HFR, or TASS databases, since we filter out copies of the same funds, that although legally constitute different hedge funds are in fact just different versions of the same fund (and hence have same returns, as well as transparency, liquidity, complexity, and concentration scores). An example of such situation would be an onshore and an offshore versions of a fund (different for tax treatment) or versions denominated in different currencies that have identical portfolios. Ang, Gorovyy, and van Inwegen (2011) use the same data to explore hedge fund leverage and note that funds in the dataset are not subject to selection bias. Therefore, we are confident that funds in our dataset are representative of the hedge fund industry.

For each of the qualitative characteristics (transparency, liquidity, complexity, and concentration), we define their High, Medium, and Low levels. The fund of funds gives original grades in such a way that a grade of 1 represents the lowest level of the characteristic from the point of view of risk for an investor. In particular, funds with high levels of transparency and liquidity and funds with low levels of complexity and concentration are rated with a 1.

For consistency purposes and the ease of interpretation we define all the variables in such a way that a High value represents a high level of the *variable itself* rather than a high level of *problem* with that variable. Therefore, whenever we speak of high transparency or high complexity, for example, we always mean a high *level* of transparency and a high *level* of complexity, respectively. We define Medium and Low levels in a similar way. There is a very small percentage of funds that are ever rated with a 4, hence we combine the grades of 3 and 4 into one category in order to ensure that we have a reasonable number of observations in each category.

Panel C of Table 1 reports the pairwise rank correlations between transparency, liquidity, complexity, and concentration, computed using Kendall's (1938) τ_B-method to account for the categorical nature of the variables and ties, for each year. As can be seen from these results, the pairwise correlations are quite robust over time. More transparent funds are also more liquid, with the correlation statistically significant at the 5% level for 2007 and at the 10% level for 2008 and 2009. More transparent and more liquid funds are also less complex on average. Finally, more liquid funds are also less concentrated. These results document the interesting patterns in the cross-sectional distribution of fund characteristics.

3 Empirical Strategy

We study the hedge fund return premia associated with transparency, liquidity, complexity, and concentration using the following empirical specification:

$$\begin{aligned} r_{it} = {} & \alpha^{\mathrm{H}}_{\mathrm{Tran}} D^{\mathrm{H}}_{\mathrm{Tran},it} + \alpha^{\mathrm{H}}_{\mathrm{Liq}} D^{\mathrm{H}}_{\mathrm{Liq},it} + \alpha^{\mathrm{H}}_{\mathrm{Com}} D^{\mathrm{H}}_{\mathrm{Com},it} + \alpha^{\mathrm{H}}_{\mathrm{Con}} D^{\mathrm{H}}_{\mathrm{Con},it} \\ & + \alpha^{\mathrm{M}}_{\mathrm{Tran}} D^{\mathrm{M}}_{\mathrm{Tran},it} + \alpha^{\mathrm{M}}_{\mathrm{Liq}} D^{\mathrm{M}}_{\mathrm{Liq},it} + \alpha^{\mathrm{M}}_{\mathrm{Com}} D^{\mathrm{M}}_{\mathrm{Com},it} + \alpha^{\mathrm{M}}_{\mathrm{Con}} D^{\mathrm{M}}_{\mathrm{Con},it} \\ & + \gamma X'_{it} + d_t + \epsilon_{it} \end{aligned} \tag{1}$$

where r_{it} denotes the annual excess return of fund i in year t. α is a set of regression coefficients with respect to the corresponding indicator variables D_{it}, where the subscript refers to the qualitative characteristic of the fund (transparency, liquidity, complexity, or concentration) and the superscript refers to the level of that characteristic (High or Medium). For example, the indicator variable $D^{\mathrm{H}}_{\mathrm{Tran},it}$ is equal to 1 if fund i in year t has a high level of transparency, and 0 otherwise. Similarly, the indicator variable $D^{\mathrm{M}}_{\mathrm{Com},it}$ is equal to 1 if fund i in year t has a medium level of complexity, and 0 otherwise. In some specifications we also allow for a vector of controls X_{it} that includes the return volatility and the logarithm of the fund's assets under management, to account for a potential difference in performance of funds that have different level of volatility or size.

Since risk premia for transparency, liquidity, complexity, and concentration can be different for different years, we estimate the above relationship separately for each year. Furthermore, in our full-sample results that cover all three years of data we include year fixed effects d_t in order to account for macroeconomic effects that are common to all hedge funds. Finally, ϵ_{it} denotes the error term in the above-specified regression model.

The "Low" levels of our qualitative variables of interest are naturally omitted in the regression specification. Funds with Low levels of transparency, liquidity, complexity, and concentration serve as the base category. α-coefficients can be interpreted as the corresponding risk premia with respect to these groups of funds.

Although there is a panel component to our data, the qualitative characteristics of interest are highly persistent within a fund. For example, among all the funds that have a transparency level present for two years or more, 89% actually have the same level of transparency in all years. Similarly, 91%, 94%, and 83% of funds have the same level of liquidity, complexity, and concentration, respectively, in all years. The observation that the fund disclosure level and its structure in general are rarely modified after the fund's initiation is not surprising, because fund investors expect the fund to maintain the same configuration over time. Given the high persistency of fund qualitative characteristics, we do not attempt to estimate the within-fund return premia for transparency, liquidity, complexity, and concentration, especially since we believe that the cross-sectional relationship in this case is more insightful.

We also include strategy fixed effects to allow for a differential performance of funds pursuing different strategies in some regression specifications. Such specifications allow to explore how fund returns vary with transparency, liquidity, complexity, and concentration across funds of the same strategy or style. Finally, in all our specifications we report standard errors that are robust to heteroskedasticity, as well as within-fund correlation over time in full-sample results.

4 Results

4.1 Univariate Results

We start with univariate regressions of hedge fund performance on the indicator variables corresponding to our qualitative characteristics in order to take a first look at hedge funds with different levels of transparency, liquidity, complexity, and concentration. Table 2 reports the results of such specifications. We see that, consistent with our predictions from Section 1, high-transparency hedge funds and medium-transparency hedge funds considerably underperformed the low-transparency hedge funds during the normal time period from April 2006 to March 2007 (Panel A). This underperformance is statistically significant at the 1% significance level. Moreover, the economic magnitude of this coefficient is large, suggesting for an average difference in returns between low- and high-transparency hedge funds of 5.7% per year. At the same time, medium-transparency hedge funds underperformed low-transparency hedge funds by 4.3% per year.

During the intermediate April 2007 to March 2008 period, the difference in performance becomes less significant both economically and statistically. During the crisis period (April 2008 to March 2009), however, we see a clear reversal in the sign of the difference between high-transparency and low-transparency hedge fund returns. According to the theory, if risks associated with low-transparency funds are realized in this period, we should see the high-transparency funds to be performing better during this period. Indeed, the high-transparency funds outperform the low-transparency funds by 7.1% per year. While economically large, it is insignificant, due to the high volatility of returns during this period (as documented in Panel B of Table 1, with a p-value of 14%.

Turning to our liquidity measure in Panel B, we observe that the difference in performance between high- and low-liquidity hedge funds is even more pronounced than the difference in performance between high- and low-transparency hedge funds. Table 2 reports that during April 2006 to March 2007 period high-liquidity hedge funds underperformed low-liquidity hedge funds by 7.8% per year and medium-liquidity hedge funds underperformed low-liquidity hedge funds by 5.5% year, with both coefficients highly economically and statistically significant. In the intermediate April 2007 to March 2008 period we observe that the signs of the coefficients are reversed with high-liquidity hedge funds outperforming low-liquidity hedge funds by 8.2%. Finally, during the crisis period we observe that high-liquidity hedge funds outperformed low liquidity hedge funds by an extraordinary 28.2%, while medium-liquidity hedge funds outperfomed them by 13.3%. These results are again both highly economically and statistically significant. Consistent with the illiquidity-risk premium story, during the good period low-liquidity funds deliver higher return as a compensation for the illiquidity risk premium, while during the bad period the risk manifests in the underperformance of these funds.

Interestingly, we do not find any evidence for the existence of a risk premium associated with the complexity of the strategies employed by funds, at least in the univariate framework. The results in Table 2, Panel C suggest that there is no statistical or economical difference between returns of high-complexity and low-complexity funds in all periods. This suggests that the risk premium associated with fund complexity is small, if it exists at all.

We also observe a premium for hedge fund concentration reported in Table 2, Panel D. During the normal April 2006 to March 2007 period, highly concentrated hedge funds outperform low-concentration funds by 7.4%, while medium-concentration funds outperfom low-concentration hedge funds by 4.4%. During April 2007 to March 2008, we observe that the realized risk premium is close to zero and during the crisis period of April 2008 to March 2009, we see a reversal with highly concentrated hedge funds underperforming low-concentration hedge funds by 12.3%. These results are consistent with the existence of risk premium associated with more concentrated (less diversified) funds.

In the last column of each panel in Table 2 we consider regressions that include all three time periods and allow for a different average return in each year by including year fixed effects. We observe that the coefficients for transparency and concentration lose their significance. This is not surprising in light of the risk premium story, since our three years both cover the years of expansion and the years of recession. Low and insignificant coefficients for the qualitative variables over time rule out the alternative story where fund managers with persistently better performance are selected into managing low-transparency and/or high-concentration funds. The exogenous variation introduced by the downturn of the economy in 2008-2009 enables us to observe the performance of funds in different states of the world, and to provide a direct support for the risk premium story that is represented by funds earning a positive premium during growth periods and negative premium during crisis periods when the embedded risk manifests.

We observe that the difference in performance between high- and low-liquidity funds is positive and significant, which seems to be driven by the very high difference in performance between high-liquidity and low-liquidity hedge funds during the crisis period. Since the recession years are less frequent than the growth periods, we expect the significance of the liquidity coefficient to drop if the time frame of the study was increased.

In light of the above results it is interesting to explore whether the documented risk premia still exists if we take a more general approach allowing for all of our measures to influence returns at the same time, as well as investigate whether our results are driven by other potential factors such as fund return volatility, size or the strategy employed. This is the approach we take next.

4.2 Multivariate Results

Table 3 reports the results of multivariate regressions that use all of our qualitative variables at the same time, as well as controls for hedge fund size, volatility and strategy. These results are very similar to the results we obtained in univariate regressions. For example, during the normal April 2006 to March 2007 period, high-transparency funds underperformed low-transparency funds by 5.4% per year, controlling for the level of other qualitative characteristics. At the same time, high-liquidity funds underperformed low-liquidity funds by 5.7% per year or 6.1% in the specification which includes additional controls for the size of the hedge fund, its return volatility and its strategy.

It is important to control for all of our qualitative characteristics at the same time, since many of them are correlated with each other, as reported in Panel C of Table 1. The results in Table 3, however, suggest that each of the main variables of interest is important irrespective of the values of other variables, and the risk premium for low-transparency funds, for instance, is not driven by illiquidity or concentration premia. These results are also robust to the inclusion of the logarithm of assets under management (a proxy for the size of the hedge fund), return volatility, and strategy fixed effects, suggesting that the observed risk premia are not driven by funds being larger or more volatile, or by a potentially different performance of funds employing different strategies[3] .

Similar to our univariate results, the regression coefficients are mostly insignificant during the intermediate April 2007 to March 2008 period, while during the crisis period we observe a reversal in the signs of the coefficients for high-transparency and high- and medium-liquidity funds, with the latter two being statistically significant at the 1% level both in the specifications with and without additional controls.

In contrast to the univariate regression results we find some evidence of a low-complexity risk premium. In particular, we observe that high-complexity funds significantly underperformed the low-complexity funds during the normal April 2006 to March 2007 period by about 3.7%-3.9% per year. This suggests that the absence of evidence towards a low-complexity risk premium in the univariate case (Panel C of Table 2) is likely driven by a negative correlation of complexity with transparency and liquidity (as reported by Panel C of Table 1), given that high levels of both command a return premium during normal times. It is therefore important to look at all qualitative variables together in order to implicitly account for interrelations between them. The results in Table 3 can thus be interpreted as the presence of risk premia associated with low transparency, low liquidity, low complexity and high concentration, conditional on the level of all qualitative characteristics as well as additional controls.

During the final period we find a significant negative effect of past return volatility on future fund returns which is connected to higher on average sales of assets belonging to high

[3] Ideally, we would like to estimate a separate specification for each strategy to explore potential differences in magnitudes of the risk premia across various strategies. However, the number of strategy-year observations is too small to fit so many parameters, so we have to leave this intriguing question for future research. Instead, we estimate a set of specifications where we drop one strategy at a time and find that the results are robust.

volatility funds. This is explained in part by withdrawals and in part by reduction in hedge fund leverage. We observe no size premium during any of the three periods.

4.3 Robustness Checks

The data sample consists of observations when the fund of funds actually chose to invest with a given fund in a given year, so a potential concern for our results is that the fund of funds selected a different subsample of funds every year and for this reason some high-transparent funds underperformed some low-transparent funds in the normal period from April 2006 to March 2007 while other high-transparent funds outperformed other low-transparent funds in the crisis period from April 2008 to March 2009. To explore further the issue of the selection and as a robustness check, we also provide the results of estimating the same set of specifications in a balanced panel in Table 4, where we require funds to be present during all three periods. This leaves us with 73 observations per year.

We note that the magnitudes of the risk premia associated with transparency and liquidity are almost identical when we require the funds to be present in all three periods. Furthermore, the picture with regard to complexity and concentration risk premia becomes even more clear. In particular, controlling for other qualitative characteristics, high-complexity funds underperformed low-complexity funds by 5.2% per year during the normal April 2006 to March 2007 period. When we additionally control for volatility, size of the fund, and strategy employed, this coefficient stays highly statistically significant with a simillar economic magnitude of 4.6% per year. Interestingly, high-concentration funds overperformed low-concentration funds by 10.5% per year, or 8.5% per year when additional controls are taken into account. Taken together, the evidence in Table 4 suggests that our results are not driven by a different composition of funds from year to year, but rather by the same funds earning a risk premium during good times and facing a loss when a negative economy shock realizes.

4.4 Concentration and Transparency Interactions

The results in Tables 2, 3 and 4 provide evidence for the presence of various risk premia, in particular the one associated with high levels of concentration of hedge fund investments. Standard finance theory, however, suggests that investors should be able to diversify away all non-systematic (idiosyncratic) risk (see Markowitz, 1952, for the seminal paper). Therefore, such a premium should exist only if investors' diversification capabilities are limited.[4]

To the best of our knowledge the question of why investors do not fully diversify the risks associated with holding a concentrated portfolio has not been explored in the context of hedge funds. Concentration should command a premium when hedge fund investors do not know hedge fund holdings and hence cannot diversify associated risks away. On the other hand, when investors perfectly know what underlying assets the fund is trading, even if the fund is concentrated, they can diversify the corresponding risks and hence concentration should not require a risk premium.

In terms of our empirical framework, this suggests that we should observe a concentration risk premium mainly among low-transparency funds. To test this hypothesis, we regress fund excess returns on their qualitative characteristics (transparency, liquidity, complexity, and concentration) by year, where we additionally introduce all pairwise interactions of the levels of transparency and concentration. Indeed, the results in Table 5 suggest that it is exactly the low-transparency high-concentration funds that command a return premium during normal times.

In particular, during the April 2006 to March 2007 period among the low-transparency funds, high-concentration funds earned 11.7% more than the low-concentration ones, where this difference is significant at a 1% level. At the same time, among the high-transparency funds the return premium of high-concentration funds over the low-concentration funds constituted a mere $11.7\% - 9.6\% = 2.1\%$ per year, which is statistically indistinguishable from zero. The -9.6% difference between these two return premia thus has an interpretation of a difference-in-differences estimate and is significant at a 1% level. Overall, the results of Ta-

[4] See, for example, Merton (1987).

ble 5 corroborate our intuition that investors are in fact able to diversify the risks associated with investing in funds that hold concentrated asset portfolios as long as their portfolios are transparent.

4.5 Hedge Fund Volatility and Flows

In this section, we investigate the effect of transparency, liquidity, complexity, and concentration on hedge fund volatilities and flows. Hedge fund volatility is computed as an annualized sample monthly return volatility using previous 12 monthly observations. Hedge fund 12-month flow is equal to relative change in the fund's AUM adjusted for fund's return following Ang et al. (2011).

Table 6 reports results of multivariate regressions of hedge fund return volatility on transparency, liquidity, complexity, and concentration indicators, for each year as well as for all three years of data controlling for an average level of volatility using year fixed effects in the last column. We observe that some portion of volatility can be attributed to these qualitative variables, with up to 37% of explained variation in the full-sample specification. The signs of the coefficients are in general similar across years. The high-liquidity funds are generally less volatile, with a 0.9% lower annualized volatility as compared to low-liquidity funds. This result is very intuitive since higher levels of liquidity of fund holdings lead to smaller jumps in returns on a month-to-month basis as compared to those of illiquid funds which can experience such jumps due to updates in prices of their assets. This evidence is consistent with the one presented in Huberman and Halka (2001) who document that more liquid stocks have lower idiosyncratic volatilities. As expected, this effect is most pronounced during the crisis period from April 2008 to March 2009, given that the overall propensity to experience sudden changes in asset prices is higher during this period.

We also observe high-concentration hedge funds to be significantly more volatile than the low-concentration funds, with a difference in annualized volatility of about 2% across different specifications. This is intuitive as high-concentration funds diversify less, so similar shocks to prices lead to larger changes in returns of these funds compared to the low-concentration funds. This magnitude is economically significant given that the average hedge fund volatility over the sample is equal to 11.0% per year. Interestingly, we do not find any difference in volatility of hedge fund returns between high-transparency and low-transparency funds.

Finally, we also study how hedge fund capital flows are related to their transparency, liquidity, complexity, and concentration by considering multivariate regressions of hedge fund flows on these variables. Results of the regressions are reported in Table 7. We find that hedge fund flows are in general very volatile and that among our qualitative variables only the liquidity characteristic can robustly explain capital flows across different periods in our sample. In particular, we find that high-liquidity funds experienced bigger inflows than low-liquidity funds, especially during the crisis period from April 2008 to March 2009. Given that the actual values of these flows were negative, we interpret this result as low-liquidity funds experiencing heavier outflows than high-liquidity funds, with the difference of about 26.6 percentage points.

5 Conclusion

We use proprietary data obtained from a fund of funds to study the risk premia associated with hedge fund transparency, liquidity, complexity, and concentration. We directly measure the transparency level of a fund, a qualitative characteristic that is missing in public hedge fund databases, and estimate a non-transparency risk premium of 5.4% per year during normal times. We also have qualitative measures of hedge fund liquidity, complexity, and concentration. We estimate an illiquidity premium of 6.1% per year during normal times. We do not find a premium for hedge fund complexity with high-complexity funds significantly underperforming during normal times. With regard to hedge fund concentration risk premium we find that it is concentrated in high-concentration low-transparency funds. We estimate a high-concentration low-transparency premium of 11.7% during normal times. This is a premium investors require in order to invest in concentrated hedge funds which risks they cannot diversify since investors do not know the area where the risks are concentrated. This result can be interpreted as a novel explanation for why investors cannot diversify away the non-systematic risks.

Importantly, the use of the data that come from both good and bad states of the economy allows us to directly test the risk-premium story against the alternative of better managers being selected into funds which belong to one of the categories. According to the risk premiums story we find that during normal times low transparency, low liquidity, and high concentration and low transparency funds deliver a premium, while during bad times risks manifest and these funds underperform.

Finally, we explore how transparency, liquidity, complexity, and concentration help explain the fund return volatility and capital flows. In particular, the returns of high-liquidity and low-concentration funds are less volatile. This result is not surprising since high concentration of illiquid investments can lead to significant jumps in hedge fund returns. With regard to hedge fund capital flows we find that during the crisis period low-liquidity funds experienced significantly heavier outflows than high-liquidity funds.

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Table 1: Summary Statistics of Data

Panel A: Number of funds by strategy

Strategy	2007	2008	2009
CR	11	13	10
ED	18	19	20
EQ	65	65	51
RV	20	20	25
TT	7	5	6
Total	121	122	112

Panel B: Hedge fund characteristics

Variable	Year	Mean	Std	q_{25}	q_{50}	q_{75}	N
Return	2007	13.59%	8.62%	9.03%	13.32%	18.02%	121
	2008	3.72%	14.52%	-5.00%	2.61%	10.58%	122
	2009	-16.56%	19.64%	-28.30%	-16.21%	-5.32%	112
Volatility	2007	6.53%	4.34%	3.68%	5.89%	7.80%	121
	2008	10.92%	6.70%	6.44%	9.04%	13.09%	122
	2009	15.81%	10.06%	9.30%	12.67%	20.38%	112
AUM	2007	905m	1.67b	128m	364m	1.05b	121
	2008	1.04b	1.86b	145m	399m	1.28b	122
	2009	810m	1.47b	121m	249m	1.03b	112

Panel C: Pairwise rank correlations of qualitative variables by year

Year		Transparency	Liquidity	Complexity	Concentration
2007	Transparency	1.000			
	Liquidity	0.187^{**}	1.000		
	Complexity	-0.144^{*}	-0.155^{*}	1.000	
	Concentration	-0.025	-0.175^{**}	0.090	1.000
2008	Transparency	1.000			
	Liquidity	0.159^{*}	1.000		
	Complexity	-0.335^{***}	-0.159^{*}	1.000	
	Concentration	0.071	-0.140^{*}	-0.135	1.000
2009	Transparency	1.000			
	Liquidity	0.147^{*}	1.000		
	Complexity	-0.269^{***}	-0.241^{***}	1.000	
	Concentration	0.073	-0.193^{**}	-0.269^{***}	1.000

This table reports various descriptive statistics of our data. Panel A reports the number of funds in our sample by strategy by year. CR denotes credit hedge funds, ED – event-driven hedge funds, EQ – equity hedge funds, RV – relative-value hedge funds, and TT – tactical-trading hedge funds. 2007 stands for April 2006 to March 2007, 2008 – for April 2007 to March 2008, and 2009 – for April 2008 to March 2009. Panel B reports the summary statistics of hedge fund returns, volatility, and assets under management (AUM) for each of the time periods. Mean denotes the annualized sample average, Std denotes the annualized sample standard deviation, q_{25}, q_{50}, and q_{75} denote the 25-th, 50th, and the 75th percentiles, respectively. Finally, N denotes the number of observations. Panel C reports the pairwise rank correlations between transparency, liquidity, complexity, and concentration, computed using Kendall's (1938) τ_B-method to account for the categorical type of the variables and ties. *, **, and *** indicate significance at the 10%, 5%, and 1% levels correspondingly.

Table 2: Hedge fund performance: Univariate regression results

Panel A: Transparency

Variable	Level	APR06-MAR07	APR07-MAR08	APR08-MAR09	APR06-MAR09
Transparency	High	-0.057^{***}	-0.039	0.071	-0.015
		(0.020)	(0.046)	(0.048)	(0.024)
	Medium	-0.043^{**}	-0.029	-0.008	-0.024
		(0.018)	(0.035)	(0.043)	(0.022)
Observations		121	122	112	355
Adjusted R^2		0.042	0.007	0.018	0.352

Panel B: Liquidity

Variable	Level	APR06-MAR07	APR07-MAR08	APR08-MAR09	APR06-MAR09
Liquidity	High	-0.078^{***}	0.082^{*}	0.282^{***}	0.105^{***}
		(0.017)	(0.042)	(0.034)	(0.022)
	Medium	-0.055^{***}	0.045	0.133^{***}	0.048^{**}
		(0.018)	(0.032)	(0.035)	(0.019)
Observations		121	122	112	355
Adjusted R^2		0.103	0.036	0.251	0.385

Panel C: Complexity

Variable	Level	APR06-MAR07	APR07-MAR08	APR08-MAR09	APR06-MAR09
Complexity	High	-0.010	0.020	0.031	0.014
		(0.016)	(0.045)	(0.050)	(0.026)
	Medium	0.023	0.023	-0.027	0.004
		(0.017)	(0.027)	(0.043)	(0.020)
Observations		121	122	112	355
Adjusted R^2		0.015	0.006	0.011	0.351

Table 2 Continued

Panel D: Concentration

Variable	Level	APR06-MAR07	APR07-MAR08	APR08-MAR09	APR06-MAR09
Concentration	High	0.074^{*}	0.008	-0.122^{**}	-0.020
		(0.041)	(0.037)	(0.053)	(0.029)
	Medium	0.044^{***}	-0.008	-0.035	0.005
		(0.013)	(0.026)	(0.038)	(0.016)
Observations		121	122	112	355
Adjusted R^2		0.105	0.002	0.059	0.352

This table reports the results of linear univariate regressions of annual hedge fund excess returns on indicator variables representing different fund characteristics, as described in Sections 2 and 3, separately for each time period considered (April 2006 to March 2007, April 2007 to March 2008, and April 2008 to March 2009), as well as for all three years, where the year fixed effects are included. Panel A, B, C, and D report the results for transparency, liquidity, complexity, and concentration, respectively. The base category are the funds with low levels of transparency, liquidity, complexity, and concentration, so that the obtained slope coefficients can be interpreted as the corresponding return premia earned by high- and medium-level funds with respect to the low-level groups of funds. Standard errors, robust to heteroskedasticity, as well as to within-fund correlation in full-sample results, are reported in brackets. *, **, and *** indicate significance at the 10%, 5%, and 1% levels correspondingly.

Table 3: Hedge fund performance: Multivariate regression results

Variable	Level	APR06-MAR07		APR07-MAR08		APR08-MAR09		APR06-MAR09	
Transparency	High	−0.054***	−0.054**	−0.045	−0.022	0.052	0.035	−0.030	−0.009
		(0.019)	(0.022)	(0.048)	(0.046)	(0.040)	(0.039)	(0.024)	(0.024)
	Medium	−0.042**	−0.043**	−0.025	−0.019	−0.027	0.006	−0.030	−0.012
		(0.018)	(0.021)	(0.039)	(0.035)	(0.034)	(0.026)	(0.022)	(0.019)
Liquidity	High	−0.057***	−0.061**	0.102**	0.063	0.279***	0.176***	0.116***	0.047**
		(0.016)	(0.028)	(0.047)	(0.041)	(0.038)	(0.043)	(0.024)	(0.023)
	Medium	−0.051***	−0.052**	0.056	0.032	0.156***	0.132***	0.059***	0.025
		(0.018)	(0.022)	(0.036)	(0.031)	(0.038)	(0.033)	(0.021)	(0.019)
Complexity	High	−0.037***	−0.039**	0.021	−0.075	0.064	−0.012	0.016	−0.053
		(0.013)	(0.020)	(0.049)	(0.063)	(0.043)	(0.052)	(0.024)	(0.033)
	Medium	0.001	−0.004	0.040	0.037	0.024	−0.021	0.016	−0.004
		(0.016)	(0.020)	(0.031)	(0.044)	(0.045)	(0.026)	(0.023)	(0.023)
Concentration	High	0.057	0.088**	0.027	0.018	−0.044	0.050*	0.004	0.062***
		(0.040)	(0.043)	(0.037)	(0.035)	(0.051)	(0.029)	(0.028)	(0.023)
	Medium	0.039***	0.048***	0.004	0.021	−0.002	−0.003	0.015	0.036**
		(0.013)	(0.017)	(0.026)	(0.029)	(0.036)	(0.030)	(0.016)	(0.016)
Ln(AUM)			−0.003		0.016*		0.003		0.008
			(0.004)		(0.010)		(0.008)		(0.005)
Volatility			−1.390		−0.637		−4.705***		−3.581***
			(0.985)		(0.891)		(0.378)		(0.370)
Strategy fixed effects			Y		Y		Y		Y
Observations		121	121	122	122	112	112	355	355
Adjusted R^2		0.222	0.249	0.063	0.290	0.300	0.722	0.394	0.566

This table reports the results of linear multivariate regressions of annual hedge fund excess returns on indicator variables representing different fund characteristics, as described in Sections 2 and 3, separately for each time period considered (April 2006 to March 2007, April 2007 to March 2008, and April 2008 to March 2009), as well as for all three years, where the year fixed effects are included. The base category are the funds with low levels of transparency, liquidity, complexity, and concentration, so that the obtained slope coefficients can be interpreted as the corresponding return premia earned by high- and medium-level funds with respect to the low-level groups of funds. Every other column also includes the controls for the size of the hedge fund (proxied by the logarithm of its assets under management), annualized volatility, and strategy fixed effects. Standard errors, robust to heteroskedasticity, as well as to within-fund correlation in full-sample results, are reported in brackets. *, **, and *** indicate significance at the 10%, 5%, and 1% levels correspondingly.

Table 4: Hedge fund performance: Balanced panel multivariate regression results

Variable	Level	APR06-MAR07		APR07-MAR08		APR08-MAR09		APR06-MAR09	
Transparency	High	−0.058**	−0.052**	−0.096*	0.008	0.065	0.038	−0.035	−0.015
		(0.024)	(0.025)	(0.050)	(0.044)	(0.042)	(0.035)	(0.029)	(0.026)
	Medium	−0.045**	−0.043**	−0.080*	−0.038	−0.043	−0.007	−0.047*	−0.020
		(0.018)	(0.018)	(0.042)	(0.036)	(0.038)	(0.025)	(0.027)	(0.022)
Liquidity	High	−0.059***	−0.061**	0.137**	0.114**	0.278***	0.147***	0.115***	0.062**
		(0.018)	(0.027)	(0.067)	(0.049)	(0.048)	(0.045)	(0.036)	(0.030)
	Medium	−0.043**	−0.041**	0.043	0.023	0.164***	0.112***	0.055*	0.023
		(0.017)	(0.020)	(0.049)	(0.040)	(0.037)	(0.026)	(0.030)	(0.025)
Complexity	High	−0.052***	−0.046***	0.044	−0.004	0.042	−0.063	0.008	−0.057
		(0.016)	(0.016)	(0.071)	(0.072)	(0.044)	(0.047)	(0.036)	(0.044)
	Medium	−0.025	−0.024*	0.029	0.059	0.075	0.007	0.018	0.008
		(0.015)	(0.014)	(0.050)	(0.049)	(0.050)	(0.027)	(0.029)	(0.028)
Concentration	High	0.105***	0.085***	−0.006	−0.024	−0.089	0.010	−0.006	0.056**
		(0.025)	(0.030)	(0.049)	(0.057)	(0.059)	(0.029)	(0.038)	(0.025)
	Medium	0.036***	0.031**	0.031	0.049	−0.019	−0.005	0.019	0.047**
		(0.013)	(0.014)	(0.034)	(0.038)	(0.041)	(0.027)	(0.021)	(0.021)
Ln(AUM)			−0.005		0.032**		0.010		0.018***
			(0.004)		(0.014)		(0.006)		(0.005)
Volatility			0.467*		0.116		−1.371***		−1.104***
			(0.238)		(0.511)		(0.100)		(0.115)
Strategy fixed effects			Y		Y		Y		Y
Observations		73	73	73	73	73	73	219	219
Adjusted R^2		0.480	0.520	0.168	0.410	0.340	0.831	0.511	0.683

This table reports the results of linear multivariate regressions of annual hedge fund excess returns on indicator variables representing different fund characteristics, as described in Sections 2 and 3, separately for each time period considered (April 2006 to March 2007, April 2007 to March 2008, and April 2008 to March 2009), as well as for all three years, where the year fixed effects are included, on a balanced panel of funds. The base category are the funds with low levels of transparency, liquidity, complexity, and concentration, so that the obtained slope coefficients can be interpreted as the corresponding return premia earned by high- and medium-level funds with respect to the low-level groups of funds. Every other column also includes the controls for the size of the hedge fund (proxied by the logarithm of its assets under management), annualized volatility, and strategy fixed effects. Standard errors, robust to heteroskedasticity, as well as to within-fund correlation in full-sample results, are reported in brackets. *, **, and *** indicate significance at the 10%, 5%, and 1% levels correspondingly.

Table 5: Hedge fund performance: Transparency and concentration interaction results

Variable	Level	APR06-MAR07	APR07-MAR08	APR08-MAR09	APR06-MAR09
Transparency	High	-0.043^{*}	0.093	0.078	0.021
		(0.023)	(0.074)	(0.061)	(0.035)
	Medium	-0.040^{*}	0.094	-0.007	0.008
		(0.023)	(0.074)	(0.059)	(0.035)
Liquidity	High	-0.059^{***}	0.131^{***}	0.284^{***}	0.124^{***}
		(0.018)	(0.050)	(0.041)	(0.025)
	Medium	-0.051^{***}	0.083^{**}	0.161^{***}	0.067^{***}
		(0.019)	(0.042)	(0.039)	(0.023)
Complexity	High	-0.034^{**}	0.065	0.069	0.030
		(0.016)	(0.055)	(0.048)	(0.026)
	Medium	0.002	0.060^{*}	0.030	0.024
		(0.022)	(0.034)	(0.048)	(0.024)
Concentration	High	0.117^{***}	0.277^{***}	-0.035	0.085^{*}
		(0.024)	(0.081)	(0.031)	(0.050)
	Medium	0.025	0.145^{*}	0.013	0.065^{*}
		(0.033)	(0.082)	(0.061)	(0.038)
Interactions	High&High	-0.096^{***}	-0.268^{***}	0.024	-0.078
		(0.033)	(0.125)	(0.072)	(0.061)
	High&Med	0.005	-0.217^{**}	-0.083	-0.096^{*}
		(0.048)	(0.108)	(0.090)	(0.050)
	Med&High	-0.062	-0.273^{***}	-0.016	-0.091
		(0.074)	(0.096)	(0.094)	(0.065)
	Med&Med	0.018	-0.156^{*}	-0.036	-0.051
		(0.036)	(0.093)	(0.072)	(0.043)
Observations		121	122	112	355
Adjusted R^2		0.237	0.119	0.305	0.400

This table reports the results of linear multivariate regressions of annual hedge fund excess returns on indicator variables representing different fund characteristics, as described in Sections 2 and 3, separately for each time period considered (April 2006 to March 2007, April 2007 to March 2008, and April 2008 to March 2009), as well as for all three years, where the year fixed effects are included. Additionally the regressions include the interactions between transparency and concentration variables. The first level in the interaction terms notation represents the level of transparency, while the last one corresponds to the level of concentration. For example, High&Med is a dummy variable that is equal to 1 if a fund has a high level of transparency and a medium level of concentration. The base category are the funds with low levels of transparency, liquidity, complexity, and concentration, so that the obtained slope coefficients can be interpreted as the corresponding return premia earned by high- and medium-level funds with respect to the low-level groups of funds. Standard errors, robust to heteroskedasticity, as well as to within-fund correlation in full-sample results, are reported in brackets. *, **, and *** indicate significance at the 10%, 5%, and 1% levels correspondingly.

Table 6: Hedge fund return volatility: Multivariate regression results

Variable	Level	APR06-MAR07	APR07-MAR08	APR08-MAR09	APR06-MAR09
Transparency	High	0.0041	0.0035	−0.0046	0.0026
		(0.0036)	(0.0062)	(0.0063)	(0.0036)
	Medium	0.0026	−0.0002	0.0064	0.0035
		(0.0026)	(0.0042)	(0.0044)	(0.0024)
Liquidity	High	0.0029	−0.0131**	−0.0165***	−0.0090***
		(0.0041)	(0.0061)	(0.0049)	(0.0036)
	Medium	0.0010	−0.0112**	−0.0027	−0.0043
		(0.0030)	(0.0051)	(0.0060)	(0.0033)
Complexity	High	−0.0037	−0.0046	0.0008	−0.0024
		(0.0035)	(0.0046)	(0.0050)	(0.0028)
	Medium	−0.0093***	−0.0116***	−0.0042	−0.0076**
		(0.0027)	(0.0035)	(0.0072)	(0.0032)
Concentration	High	0.0225***	0.0177***	0.0223**	0.0205***
		(0.0046)	(0.0045)	(0.0095)	(0.0042)
	Medium	0.0049***	−0.0002	−0.0020	0.0012
		(0.0020)	(0.0033)	(0.0047)	(0.0021)
Observations		121	122	112	355
Adjusted R^2		0.342	0.298	0.213	0.373

This table reports the results of linear multivariate regressions of annual hedge fund return volatilities on indicator variables representing different fund characteristics, as described in Sections 2 and 3, separately for each time period considered (April 2006 to March 2007, April 2007 to March 2008, and April 2008 to March 2009), as well as for all three years, where the year fixed effects are included. The base category are the funds with low levels of transparency, liquidity, complexity, and concentration, so that the obtained slope coefficients can be interpreted as the corresponding volatility difference between high- and medium-level funds as compared to the low-level groups of funds. Volatility is equal to an annualized sample monthly return volatility calculated using previous 12 months. Standard errors, robust to heteroskedasticity, as well as to within-fund correlation in full-sample results, are reported in brackets. *, **, and *** indicate significance at the 10%, 5%, and 1% levels correspondingly.

Table 7: Hedge fund flows: Multivariate regression results

Variable	Level	APR06-MAR07	APR07-MAR08	APR08-MAR09	APR06-MAR09
Transparency	High	0.222	0.453	−0.031	0.205
		(0.237)	(0.420)	(0.077)	(0.173)
	Medium	0.096	0.100	−0.070	0.046
		(0.157)	(0.230)	(0.058)	(0.095)
Liquidity	High	0.157	0.240	0.266***	0.224
		(0.370)	(0.392)	(0.092)	(0.197)
	Medium	−0.265*	−0.067	0.155**	−0.056
		(0.152)	(0.211)	(0.062)	(0.093)
Complexity	High	0.283*	0.297	0.120	0.235*
		(0.168)	(0.361)	(0.077)	(0.140)
	Medium	−0.061	0.265	0.156**	0.139
		(0.174)	(0.175)	(0.069)	(0.085)
Concentration	High	0.233	−0.026	0.091	0.087
		(0.201)	(0.194)	(0.068)	(0.098)
	Medium	0.280*	0.062	0.045	0.141
		(0.163)	(0.197)	(0.063)	(0.094)
Observations		109	107	95	311
Adjusted R^2		0.133	0.055	0.123	0.189

This table reports the results of linear multivariate regressions of annual hedge fund inflows (measured as a percentage of past assets under management) on indicator variables representing different fund characteristics, as described in Sections 2 and 3, separately for each time period considered (April 2006 to March 2007, April 2007 to March 2008, and April 2008 to March 2009), as well as for all three years, where the year fixed effects are included. The base category are the funds with low levels of transparency, liquidity, complexity, and concentration, so that the obtained slope coefficients can be interpreted as the corresponding volatility difference between high- and medium-level funds as compared to the low-level groups of funds. Hedge fund flow is equal to a relative change in the fund's AUM adjusted for fund's returns during previous 12 months. Standard errors, robust to heteroskedasticity, as well as to within-fund correlation in full-sample results, are reported in brackets. *, **, and *** indicate significance at the 10%, 5%, and 1% levels correspondingly.