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Volatility in the Knowledge Economy

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Abstract

We seek to explain the economic volatility of the last 6 years, in particular the rapid expansion and contraction of the knowledge sectors. Our hypothesis is that these sectors amplify the business cycle due to their increasing returns to scale, growing faster than others in an upswing and contracting faster in a downswing. To test this hypothesis we postulate a general equilibrium model with two sectors: one with increasing returns that are external to the firm and endogenously determined – the knowledge sector – and the other with constant returns to scale. We introduce a new measure of volatility of output, a 'real beta', and derive a 'resolving' equation, from which we prove that the increasing return sectors exhibit more volatility then other sectors. We validate the main results on US macro economic data of real GDP by industry (2-3 digits SIC codes) of the 1977-2001 period, and provide policy conclusions.

Keywords: Volatility, Macroeconomics, Applied Microeconomics, General Equilibrium, Knowledge Economy, Knowledge revolution, Increasing Returns to Scale, External Economies of scale, Business Cycles

JEL: D5, D58, E10, L50, L52, O38, O51

1 Introduction

In the last six years the technology sectors of the US economy went through an unprecedented 'boom' and a 'bust', followed by a period of high unemployment and a jobless recovery[1]. This piece reflects on this extraordinary period, and the policy lessons we learned from it. We develop an explanation based on an economy with two sectors, one of which has increasing returns.[2] Our hypothesis

[1] Most notable was the performance of the *dot.com* sector.

[2] This article relates to David Cass' work with Herakles Polemarchakis[7], "Convexity and Sunspots: A Remark", which showed that sunspots disappear in economies with non convexities. One needs therefore a 'real' explanation for the phenomenon of boom and bust, other than sunspots. We owe this comment to Paolo Siconolfi.

is that increasing returns sectors amplify the business cycle in the sense that they grow faster than other sectors during expansions, and contract faster during downturns. In this sense they exhibit more 'volatility' than other sectors[3].

To test our hypotheses we define a measurement of volatility 'real beta' that is a statistical relative of the 'beta' used in financial markets, but it measures volatility in real macro data on output rather than in stock prices. Examining US figures, we capture the connection of the model with macro economic data observed during this period.

The economy is represented by a general equilibrium model with two sectors, one with constant returns to scale and the other with *external* economies of scale, similar to that in Chichilnisky [8], [9] and [10]. In both sectors the firms are competitive, but the 'knowledge sector' has increasing returns to scale (IRS) that are *external* to the firm and endogenously determined. The externalities arise from "learning by doing," and from the free transfer of skills from one firm to the other through job turnover or freely shared R&D. Increasing returns are endogenous because they depend on the output in the knowledge sector in equilibrium[4]. We show that IRS sectors exhibit more volatility than other sectors, an outcome we test empirically using US data[5]. The results are related to the 'virtuous and vicious' cycles in economies with increasing returns to scale that were developed in Chichilnisky and Heal [11] and in Heal [15].

The article proceeds as follows: first we define the 'real beta' measure of macroeconomic volatility for the different sectors of the economy, and then provide the general equilibrium model. We solve this model analytically, finding all prices, production and employment levels in equilibrium, by means of a single 'resolving' equation and two 'nested' fixed point arguments to resolve the issue of endogeneity of returns to scale. Using the 'resolving' equation, we show that the 'real beta' is higher for the increasing returns sectors than for others. Finally we provide details of the data used, and validate the results on the observed real betas during 1977-2001 period. The last section suggests policy conclusions.

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[3] The effect of the business cycle on IRS sectors has not been analyzed in the literature. Real Business Cycle models include IRS in order to generate cyclical productivity (see for example S. Basu and J. Fernald [3] and [4] for a review of the literature). In international trade, IRS is seen as determining the pattern and the factor content of trade (Krugman [18], [19], [20], [21], Panagariya [22], Antweiler et al. [1]). In growth literature, learning by doing leads to endogenous growth in the economy (see Arrow[2], Romer[25], and Rivera-Batiz and Romer[24]). In theoretical macrodynamic general equilibrium models, IRS may lead to unstable systems and may generate "vicious" and "virtuous" cycles in the economy (see Chichilnisky and Heal [11], and Heal [15], [14]).

[4] Chipman (1970)[13] provides an excellent general equilibrium model with external economies of scale. The main difference with our model is in that it has only one factor of production and the extent to economies of scale are exogenously given, whereas here (and in Chichilnisky 1993, 1994, and 1998) there are two factors and the extent of IRS is endogenously determined in one sector of the economy– the other has constant returns.

[5] Our hypothesis is difficult to prove in complete generality, because the data of the last ten years is complex: industry classifications have changed and, as is well know, when using nominal data the increasing returns sectors (whose prices drop when production expands) are underrepresented in GDP. This observation is related to comments by William Baumol on productive and unproductive sectors, and to the November 2002 publication of Survey of Current Business, U.S. [29].

2 Volatility in the Knowledge Sectors

Our hypothesis is that increasing returns to scale sectors (IRS) amplify the business cycles: during an upswing, they grow faster than the rest of the economy and during downturns, they contract faster than the rest. To test this intuitive hypothesis we introduce a measure of volatility of output, and explore its behavior in a general equilibrium model of the economy.

2.0.1 Volatility Index– the 'real beta'

We define an index to measure volatility that is independent of the scale of the variables, denoted 'real beta'. This is a statistical relative of the financial markets concept of 'beta' that is frequently used to measure volatility of stock prices:

$$\beta = \frac{Cov(X, Y)}{Var(Y)} \tag{1}$$

Here X and Y are outputs rather than stock prices, X representing the value of the output of the sector and Y, the value of GDP. β_{IRS} denotes the real beta associated with the increasing returns sector and β_{CRS} that of constant returns to scale sector. Our hypothesis can now be stated as:

Hypothesis: $\beta_{IRS} > \beta_{CRS}$.

2.0.2 Internal and External Increasing Returns to Scale

A firm or an industry has *increasing returns to scale* (IRS) when unit costs fall with increases in production[6]. Economies of scale are *internal* to the firm when a firm becomes more productive, i.e. more efficient, in utilizing its resources, as its own size increases, as is typical to firms with large fixed costs such as aerospace, airlines, and oil refineries[7]. In contrast, increasing returns to scale are *external* to the firm when the increased productivity comes about as a result of decreasing unit costs at the level of the industry as a whole. In the latter case, each firm could have constant unit costs as its production increases, and behave competitively. Yet as the industry as a whole expands, positive *externalities* among the firms are created leading to increased productivity for all firms in the industry[8]. The free movement of skilled workers from one firm to another can have this effect, as a firm may benefit at no cost from training a worker received in another firm[9]. Equally, a firm can benefit from unspecific

[6] Sometimes they are defined by 'average' unit costs that decrease with production.

[7] This type of increasing returns can lead to monopolistic competition due to high entry costs.

[8] For example, in the period between 1990 and 2000, the expansion in output in the computer hardware industry led to yearly doubling of the computing power available per dollar, leading to an exponential increase in CPUs per dollar (a standardized measure of processing power) and to the corresponding rapid increase in demand and consumption of CPUs across the entire economy.

[9] Workers in the knowledge sectors move between firms more than others, on average two years or less.

research and development innovations developed in other firms, which are accessible to it at little or no cost. These positive 'knowledge spillovers' often originate from innovations generated during the course of production. As the new knowledge spreads to all the firms in the industry, total productivity in the industry increases and unit costs fall[10].

2.0.3 General Equilibrium with External Increasing Returns to Scale.

The economy produces and trades two goods B and I. B is a traditional constant returns to scale (CRS) industry, whereas I is produced under *external* increasing returns to scale (IRS). Both goods are produced using two inputs, labor L and capital K, and the firms in each industry are perfectly competitive. They minimize their costs given the market prices. Consumers maximize their utility given their budged constraints. Walras' law is satisfied, so the value of excess demand is equal to zero. At equilibrium all markets for goods and factors clear. Firms production functions are given as:

$$B^s = L_1^\alpha K_1^{1-\alpha} \text{ and } I^s = \gamma L_2^\beta K_2^{1-\beta} \tag{2}$$

where $\alpha, \beta \in (0, 1)$. L_1, K_1 are inputs in the B sector, and L_2, K_2 are inputs in the I sector. The total amount of labor and capital in the economy are L^s and K^s respectively.

The parameter γ in the production function for I is taken as a constant by each firm within this industry. However, at the industry level, γ is endogenously determined and increases with the output of I, i.e. $\gamma = \gamma(I^s)$. For example, $\gamma = I^\sigma$. In this case, when externalities are taken into account the production function for this sector is $I^s = L_2^{\frac{\beta}{1-\sigma}} K_2^{\frac{1-\beta}{1-\sigma}}$, although at the firm's level the technology that determines the firm's behavior is $I^s = \gamma L_2^\beta K_2^{1-\beta}$. Observe that $\sigma > 1$ leads to negative marginal products, while $\sigma < 0$ leads to decreasing returns. Therefore, when the sector I has increasing returns, σ satisfies $0 < \sigma < 1$, which we now assume[11]. In Proposition 1 below, the parameter σ in the definition of γ is key to prove that the 'real betas' of IRS sectors are larger than those in CRS sectors, when $0 < \sigma < 1$ and $\sigma \to 1$. Notice that returns to scale in the sector I are endogenous because the parameter $\gamma = I^\sigma$ is unknown until the equilibrium value of I is determined.

[10] Any industry that depends on knowledge or skilled labor could benefit from such knowledge spillovers and external economies of scale. In the growth literature related phenomena are known as 'learning by doing', a concept introduced by Arrow [2] and developed further by Romer [25] in a one sector growth model. 'Learning by doing' often refers to increasing returns that are internal to the firm. We focus instead on increasing returns that are external to the firm, endogenously determined and internal to one industry within a general equilbirum model with two goods and two factors, and in which a second sector has constant returns to scale.

[11] From the Appendix' equation (28) the marginal product of labor is $MPL = \frac{1}{1-\sigma}(\Phi(p_B)^{\frac{\sigma}{1-\sigma}})(\frac{\partial \Phi}{\partial p_B})(\frac{\partial p_B}{\partial L})$. When $\sigma > 1$ marginal product is negative. When σ is less than or equal to 0, the I sector exhibits decreasing or constant returns to scale, and $0 < \sigma < 1$ is IRS.

Prices for I and B are p_I and p_B respectively. We assume that I is the numeraire so that $p_I = 1$. Then

$$Y^s = I^s + p_B B^s \tag{3}$$

indicates the value of total production in the economy.

Factor prices are denoted as usual, w for wages and r for rental on capital. We assume for simplicity that demand for B is a function of initial endowments and p_B[12],

$$B^d = B^d(L^S, K^S, p_B) \tag{4}$$

By Walras Law, demand for industrial goods in equilibrium is given by:

$$I^{d*} = (w^* L^* + r^* K^* - p_B^* B^{d*}) \tag{5}$$

In equilibrium all markets clear:

$$\begin{gathered} p_B^* B^{s*} + I^{s*} = w^* L^* + r^* K^* \text{ (zero profits)} \qquad (6) \\ K^* = K^s = K_1 + K_2 \text{ (capital market clears)} \\ L^* = L^s = L_1 + L_2 \text{ (labor market clears)} \\ B^{s*} = B^{d*} + X_B^* \text{ (}B\text{ market clears)} \\ I^{s*} = I^{d*} + X_I^* \text{ (}I\text{ market clears)} \end{gathered}$$

where X_B^* and X_I^* are equilibrium levels of net exports in B and I sectors respectively. We assume $X_I^* = 0$ and $X_B^* = 0$ but the results are true for any given X_I^* and X_B^*.

2.0.4 Solving the Model

We show in the Appendix[13] that there is one nonlinear equation $F(p_B) = 0$, depending solely on p_B, from which an equilibrium value p_B^* is obtained, and from this the equilibrium values of all other variables can be computed. This equation is therefore called a 'resolving' equation. The computation of this equation is lengthy and requires two 'nested' fixed point arguments: we find a solution for the general equilibrium of the economy (depending on the scale parameter γ) and at the same time resolve for the endogenous parameter γ which determines the returns to scale in the I sector. To find a 'resolving' equation we start by computing explicitly the market clearing condition in I combining equations (5) and (2); at an equilibrium, the I market must clear

$$F(p_B) = I^d(p_B) - I^s(p_B) = 0 \tag{7}$$

We need to reduce (7) to a function of a single variable p_B. The computation requires a logarithmic transformation by which all the endogenous variables are

[12] The results of the model do not depend on the demand specification.

[13] Chichilnisky[8], [9], and [10] provide step by step solution of a similar model where all sectors have IRS.

written as a function of p_B and of the parameter γ. The function $F(p_B)$ depends on all the 6 exogenous parameters of the model, $\alpha, \beta, \sigma, B^d, L^s$ and K^s. There is an additional level of complexity since as explained the output of I in equilibrium depends on the scale parameter γ – but this scale parameter is itself unknown until the output of I has been computed at an equilibrium. We resolve this by performing a 'fixed point' argument, in which the scale parameter and the level of equilibrium output I are simultaneously determined. Below we indicate how this is achieved; for the complete computation the reader is referred to the Appendix. In more detail, we derive $F(p_B)$ as follows: taking (23) and (24) from the Appendix into account, I^{s*} can be written as functions of the price p_B and the parameter γ, where

$$I^{s*} = \gamma[L^s - \frac{e^A L^s}{(e^A - e^B)} - \frac{e^A e^B K^s p_B^{\frac{1}{\beta-\alpha}}}{(e^A - e^B)}]^{\beta}[K^s - \frac{L^s p_B^{\frac{1}{\alpha-\beta}}}{(e^A - e^B)} - \frac{e^B K^s}{(e^A - e^B)}]^{1-\beta} = \gamma\Phi(p_B) \tag{8}$$

In the Appendix, equation (19), it is shown that the parameters A and B in (8) are constants depending on the technology parameters α and β that are exogenously given. The parameter $\widetilde{\gamma}$ that appears in (19) is a constant since it is given by the marginal conditions of firms, who regard the technology parameter γ as constant. Therefore, in (8), Φ is a function that depends solely on p_B. Similarly one obtains

$$B^{s*} = \Psi(p_B) \tag{9}$$

In view of (8), equation (7) can be rewritten as a function of p_B and γ:

$$F(p_B) = I^{d*}(p_B) - \gamma[L^s - \frac{e^A L^s}{(e^A - e^B)} - \frac{e^A e^B K^s p_B^{\frac{1}{\beta-\alpha}}}{(e^A - e^B)}]^{\beta}[K^s - \frac{L^s p_B^{\frac{1}{\alpha-\beta}}}{(e^A - e^B)} - \frac{e^B K^s}{(e^A - e^B)}]^{1-\beta} = 0 \tag{10}$$

where $I^{d*}(p_B)$ defined in equation (5) above, follows from Walras' Law and it is a function of p_B alone as shown in Appendix in equation (30). In (10) the parameter γ is not yet explicitly computed as a function of p_B, since it is unknown until the equilibrium level of output of I sector is found. This requires a small fixed point argument to resolve the equation (10) as a function of p_B alone. From $\gamma = I^{\sigma}$ and equation (8) we obtain

$\gamma = (\gamma\Phi(p_B))^{\sigma}$ or $(\gamma)^{1-\sigma} = \Phi(p_B)^{\sigma}$ so that $\gamma = \Phi(p_B)^{\frac{\sigma}{1-\sigma}}$. Therefore,

$$I^{s*} = \Phi(p_B)^{\frac{1}{1-\sigma}} \tag{11}$$

Finally, using (11), (10) is written as a function of p_B alone and becomes the 'resolving' equation for this model:

$$F(p_B) = I^{d*}(p_B) - \left\{[L^s - \frac{e^A L^s}{(e^A - e^B)} - \frac{e^A e^B K^s p_B^{\frac{1}{\beta-\alpha}}}{(e^A - e^B)}]^{\beta}[K^s - \frac{L^s p_B^{\frac{1}{\alpha-\beta}}}{(e^A - e^B)} - \frac{e^B K^s}{(e^A - e^B)}]^{1-\beta}\right\}^{\frac{1}{1-\sigma}} = 0 \tag{12}$$

Using (12) one can now solve fully the equilibrium of the model, first computing the equilibrium price, p_B^*, and from this, all other equilibrium values $(K_1^*, K_2^*, L_1^*, L_2^*, B^{s*}, I^{s*}, I^{d*}, \gamma^*)$ can be found[14].

2.1 Volatility: the effects of shocks on different returns to scale.

To study volatility we assume that there are random shocks to the 'fundamentals' of the model (technologies, preferences, demand, initial endowments of capital and labor). The fundamentals thus vary from period to period, although there are no intertemporal links between one period or another. Since the equilibrium value of p_B^* varies with the fundamentals, this produces fluctuations in all equilibrium values of the model, and in particular in the outputs I^{s*} and B^{s*}. Using the general equilibrium model, we study the attendant variations in I^{s*} and B^{s*}, exploring the extent to which sector I is systematically more volatile than sector B. From the 'resolving' equation, we know how outputs of I^{s*} and B^{s*} fluctuate with p_B^* and we are particularly interested in finding out whether the increasing returns sector I exhibits more volatility than the constant returns sector B according to the measurement provided by the 'real beta':

Proposition 1 *The volatility of the increasing returns to scale sector I is larger than that of the constant returns to scale sector B, i.e.* $\beta_{IRS} > \beta_{CRS}$, *when increasing returns are large enough, i.e.* $0 < \sigma < 1$ *and* $\sigma \sim 1$.

Proof. We want to prove the following inequality:

$$\beta_{IRS} = \frac{Cov(I_t, Y_t)}{Var(Y_t)} > \beta_{CRS} = \frac{Cov(p_{Bt}B_t, Y_t)}{Var(Y_t)} \tag{13}$$

or equivalently, because the denominators are equal and positive:

$$Cov(I_t, Y_t) > Cov(p_{Bt}B_t, Y_t) \tag{14}$$

For simplicity of notation, let p_{Bt} be expressed as $p_{B.}$ Rewriting (14):

$$\sum_{t=1}^{T}(I_t - \bar{I})(Y_t - \bar{Y}) > \sum_{t=1}^{T}(p_B B_t - \overline{p_B B})(Y_t - \bar{Y}) \tag{15}$$

where $\bar{I}, \overline{p_B B}$,and $\bar{Y}$ denote time averages: $\overline{p_B B} = \frac{1}{T}\sum_{t=1}^{T}(p_B B_t)$.

Rearranging the terms (14) becomes:

$$\sum_{t=1}^{T}\left\{\left[(I_t - \bar{I}) - (p_B B_t - \overline{p_B B})\right](Y_t - \bar{Y})\right\} > 0$$

[14] There may be more than one equilibrium, but this does not alter our results.

Substituting for Y_t and $\bar{Y}$ from (3):

$$\sum_{t=1}^{T} \left\{ \left[(I_t - \bar{I}) - (p_B B_t - \overline{p_B B})\right] \left[(I_t + p_B B_t) - \overline{(I + p_B B)}\right] \right\} > 0, \text{ or}$$

$$\sum_{t=1}^{T} \{(I_t^2 - (p_B B_t)^2\} + T\{\overline{(p_B B})^2 - (\bar{I})^2\} > 0$$

From (12), (11), and (9) we obtain:

$$\sum_{t=1}^{T} \{(\Phi(p_B)^{\frac{1}{1-\sigma}})^2 - (p_B \Psi(p_B))^2\} + T\{\overline{(p_B B})^2 - (\bar{I})^2\} > 0 \qquad (16)$$

For every t, as $\sigma \rightarrow 1$ and $\Phi_t(p_B) > 1$, the inequality (16) is satisfied since the first term $(\Phi_t(p_B)^{\frac{1}{1-\sigma}})^2$ dominates the equation. It follows therefore, that $\beta_{IRS} > \beta_{CRS}$. ■

3 Empirical Issues

3.0.1 Data Sources and Structure

We use data provided by the *Survey of Current Business (SCB)*[15], prepared by the *U.S. Department of Commerce, Bureau of Economic Analysis* (BEA), in Washington, DC. This survey offers data on GDP by industry (2-3 digit Standard Industry Codes (SIC))[16]. There is a "break" in the time series of this data due to the SIC reclassification. Thus, 1977-1987 data uses 1972 SIC classification, whereas 1987-2001 uses 1987 SIC classification (the estimates of 1977-1987 have not been adjusted to 1987 SIC code due to "lack of adequate data" according to BEA May 2003 publication [28]).

[15] Monthly government publications to be found in www.bea.gov.
[16] Detailed data series are available at the BEA webpage: http://www.bea.gov/bea/dn2/gpo.htm

3.0.2 IRS and Traditional Industries

We reviewed the literature and adopted its findings about IRS sectors.[17]. We also use a simple correlation between quantities produced and prices charged on the level of industry to identify IRS sectors (the sector is considered to exhibit IRS if correlation is negative)[18]. Figure 1 lists the IRS sectors identified in the above mentioned studies, showing in each case the respective sources.

Figure 1: List of Increasing Returns to Scale Industries Identified by the Literature Review and Correlation Coefficient Specification.

	1977-2001	1977-1987	1987-2001	Identification
Agriculture, forestry, and fishing	1.13	1.64	1.08	AT
Credit agencies other than banks	3.97		3.97	corr=<0
Coal mining	1.28	0.86	1.56	AT
Communications	1.46	0.92	1.89	BF
Construction	0.69	0.45	0.84	BF
Electronic equipment and instruments	2.05	0.98	3.07	BF, AT
Fabricated metal products	0.61	0.52	0.73	BF
Furniture and fixtures	0.67	1.27	0.7	BF
Machinery, except electrical	2.18	0.93	3.09	AT
Metal mining	1.88	0.33	2.25	BF
Motor vehicles and equipment	0.58	0.56	1.18	BF
Oil and gas extraction	0.06	-0.01	-0.24	AT
Paper and allied products	0.4	0.75	0.02	BF
Petroleum and coal products	0.8	2.59	0.33	AT
Primary metal industries	-0.02	-2.54	0.71	BF
Retail trade	1.24	1.43	1.55	BF
Security and commodity brokers	2.87	0.85	4.65	corr<0
Services	1.06	1.14	1	BF
Telephone and telegraph	1.55	1.08	2.16	corr<0
Transportation	1.1	0.8	1.25	BF
Wholesale trade	1.55	1.45	1.94	BF

Source: U.S. Department of Commerce, Bureau of Economic Analysis. Detailed annual series can be found at: http://www.bea.gov/bea/dn2/gpo.htm.
Note: BF stands for S. Basu and J. Fernald study of 1997.
AT stands for W. Antweiler and D. Trefler study of 2000.

[17] A study by S. Basu and J. Fernald [3] of US private economy (2-3 SIC) find IRS in: 1. Metal Mining, 2. Construction, 3. Furniture, 4. Paper, 5. Primary Metals, 6. Fabricated Metals, 7. Electrical Machinery, 8. Motor Vehicles, 9. Transportation, 10. Communication, 11. Electric Utilities, 12. Wholesale and Retail, 13. Services (various).

Work by W. Antweiler and D. Trefler [1] examine 27 manufacturing and 7 non-manufacturing industries (no services) for 71 countries over 1972-1992 period and identified 11 industries with increasing returns: 1. Petroleum and Coal Products, 2. Pharmaceuticals, 3. Electric and Electronic Machinery, 4. Petroleum Refineries, 5. Iron and Steel Basic Industries, 6. Instruments, 7. Non-Electric Machinery, 8. Forestry, 9. Livestock, 10. Crude Petroleum and Natural Gas, 11. Coal Mining. Their general equilibrium model estimates scale for these industries in the rage of 1.10 to 1.20.

Paul and Siegel [23] find that scale economies are prevalent in US manufacturing. In particular, this study finds evidence of external economies of scale due to supply-side agglomeration.

[18] Specifically, we used chain-type quantity index for GDP by industry and chain-type price index for GDP by industry from BEA for our correlation computations. Detailed data files can be downloaded at: http://www.bea.gov/bea/dn2/gpo.htm.

From the list in Figure 1 we separated out the industries with economies of scale that are internal to the firm, or industries that have well-known high fixed costs, in order to conform to the model specification. We were left with 7 industries with *external* economies of scale:

1. Credit agencies other than banks (SIC 61)
2. Electronic equipment and instruments (36, 38)
3. Machinery, except electrical (35)
4. Retail Trade (52-59)
5. Security and commodity brokers (62)
6. Telephone and telegraph (481, 482, 489)
7. Wholesale Trade (50, 51).

3.0.3 Empirical Results

GDP's 'real beta', by definition, is equal to one. Figure 2 provides real betas for industries with external economies of scale and for some traditional industries. What is immediately apparent is that betas for IRS industries are larger than one and are larger than that of traditional industries. In addition, breaking the data into two sub-periods, 1977-1986 and 1987-2001 (chosen arbitrarily at the break of the series) provides another interesting view of the data. The betas for IRS are larger in the second period than in the first. This phenomenon could be explained by our model: as $\sigma \to 1$ beta of the IRS sector becomes larger. Figures 3 and 4 illustrate graphically the differences in volatilities in the IRS and traditional sectors of the economy.

4 Conclusions

We showed that increasing returns to scale industries are on the whole more volatile than others, in the sense of having higher 'real betas'. These sectors expand faster in an upswing and contract faster in a downswing than other sectors. The general equilibrium model on which these results are proven has one sector I with increasing economies of scale that are external to the firm and internal to the industry, and another B with constant returns to scale[19].

The results seem to confirm an intuitive view of a modern economy with rapidly growing productivity in knowledge sectors that could however destabilize the economy. The most productive sectors could be the most volatile. Obviously one does not want to miss the productivity gains of the IRS sectors – while at the same time it seems desirable to curb their volatility. What could be a policy solution for this dilemma?

One possibility is to develop financial mechanisms that can 'smooth' the volatility of the increasing returns sectors as the economy goes through the

[19] The empirical results may hold for other forms of increasing returns as well, beyond those which appear in our general equilibrium model. This could include increasing returns that are *internal* to the firm and are based on fixed costs– such as those in oil refineries and the airspace industry.

business cycle. In the financial sectors, 'futures' markets served historically this role for commodity markets, which are notoriously volatile. It has been shown that they can do this job well if properly managed, see Jerome Stein [27] and others. The policy of choice could involve creating the equivalent of futures markets for moderating the volatility of output in the technology industry, rather than for moderating the volatility of the stock and the prices of commodities. Another solution would be to create an institution that uses the Law of Large Numbers (such as Ginnie Mae was created to do). For example, an organization that is 50% government owned and 50% privately owned— with a public aim (in this case, to control volatility) while at the same time being a for-profit organization.

The results need to be expanded to explain the changes in international trade patterns as developing countries 'leapfrog' their industrial counterparts and take advantage of the external returns to scale industries, while the US continues to outsource its high tech work overseas. A good example is software development that is based on knowledge and yet requires little capital or equipment, therefore having small fixed costs but as a knowledge industry exhibits 'learning by doing'. India's Bangalore region is a leader in software exports and fits this pattern.

5 Appendix

5.0.4 Finding a resolving equation for the General Equilibrium Model

This Appendix draws on the results of G. Chichilnisky [8] pages 189 -195, which is a general equilibrium model with IRS in both sectors. In contrast, our model has one industry, I, with external economies of scale, and the other, B, with constant returns to scale. To solve our model, there are three prices to be determined: the price p_B^*, and the two factor prices w^* and r^*. The quantities to be determined in an equilibrium are: the use of factors in each sector $K_{1,}^*K_{2,}^*L_{1,}^*L_2^*$, the outputs of the two goods B^{s*} and I^{s*}, the parameter γ^* determining the external economies of scale in I, and the demand for good I, I^{d*}. The model has 11 endogenous variables to be computed.

We solve the model by finding an explicit function of one variable, p_B, (called a 'resolving' equation), which depends on 6 exogenously given parameter of the economy: $\alpha, \beta, \sigma, B^d, L^s$ and K^s. To obtain the resolving equation we write the market clearing conditions in the I market, demand equals supply, and find a way to express them as a function of only one variable: p_B. Solving this equation gives the equilibrium value of p_B^*,from which all other endogenous variables listed above can be found[20]. Since the model has constant returns to scale at the level of the firms, we derive the equilibrium relations between supplies and prices from the marginal conditions and full employment of factors.

By assumption each firm takes the scale parameter γ as given; denoting $l_1 = \frac{L_1}{K_1}$ and $l_2 = \frac{L_2}{K_2}$, from the production functions (2), marginal conditions

[20]Note that we have given no supply behavior outside of an equilibrium; in particular, there is no information for carrying out stability analysis.

and zero profits imply:

$w = \alpha (l_1)^{\alpha-1} p_B$ and $r = (1-\alpha)(l_1)^{\alpha} p_B$ (derived by firms of B sector), and $w = \gamma\beta(l_2)^{\beta-1}$ and $r = \gamma(1-\beta)(l_2)^{\beta}$ (derived by firms of I sector).

Indicating logarithms with the symbol "$\sim$" the four equations above can be rewritten as:

$$\widetilde{w} = (\alpha-1)\widetilde{l_1} + \widetilde{\alpha} + \widetilde{p_B} \text{ and } \widetilde{r} = \alpha\widetilde{l_1} + \widetilde{(1-\alpha)} + \widetilde{p_B} \tag{17}$$

$$\widetilde{w} = (\beta-1)\widetilde{l_2} + \widetilde{\beta} + \widetilde{\gamma} \text{ and } \widetilde{r} = \beta\widetilde{l_2} + \widetilde{(1-\beta)} + \widetilde{\gamma}$$

so that $(\alpha-1)\widetilde{l_1} - (\beta-1)\widetilde{l_2} = \widetilde{\beta} - \widetilde{\alpha} - \widetilde{p_B} + \widetilde{\gamma}$

and $\alpha\widetilde{l_1} - \beta\widetilde{l_2} = \widetilde{(1-\beta)} - \widetilde{(1-\alpha)} - \widetilde{p_B} + \widetilde{\gamma}$.

Solving for $\widetilde{l_1}$ and $\widetilde{l_2}$ we obtain:

$$\widetilde{l_1} = \frac{(\widetilde{\beta} - \widetilde{p_B} - \widetilde{\alpha} + \widetilde{\gamma})(-\beta) - (1-\beta)[\widetilde{(1-\beta)} - \widetilde{p_B} - \widetilde{(1-\alpha)} + \widetilde{\gamma}]}{[\beta-\alpha]}$$

$$\widetilde{l_2} = \frac{(\alpha-1)[\widetilde{(1-\beta)} - \widetilde{p_B} - \widetilde{(1-\alpha)} + \widetilde{\gamma}] - \alpha(\widetilde{\beta} - \widetilde{p_B} - \widetilde{\alpha} + \widetilde{\gamma})}{[\beta-\alpha]}$$

or

$$\widetilde{l_1} = \frac{\widetilde{p_B}}{(\beta-\alpha)} + A \text{ and } \widetilde{l_2} = \frac{\widetilde{p_B}}{(\beta-\alpha)} + B \tag{18}$$

where A and B are constants such that:

$$A = \frac{(\widetilde{\beta} - \widetilde{\alpha})(-\beta) - (1-\beta)[\widetilde{(1-\beta)} - \widetilde{(1-\alpha)}] - \widetilde{\gamma}}{[\beta-\alpha]} \tag{19}$$

$$B = \frac{(\alpha-1)[\widetilde{(1-\beta)} - \widetilde{(1-\alpha)}] - \alpha[\widetilde{\beta} - \widetilde{\alpha}] - \widetilde{\gamma}}{[\beta-\alpha]}$$

and observe that $A > 0$ and $B < 0$ if $\beta < \alpha$. Therefore,

$$l_1 = e^A p_B^{\frac{1}{\beta-\alpha}} \text{ and } l_2 = e^B p_B^{\frac{1}{\beta-\alpha}} \tag{20}$$

One can also show that by combining (20) and marginal condition equations:

$$w = \alpha e^{A(\alpha-1)} p_B^{\frac{\beta-1}{\beta-\alpha}} \text{ and } r = (1-\alpha)e^{A\alpha} p_B^{\frac{\beta}{\beta-\alpha}} \tag{21}$$

Since $l_2 = \frac{L^S - L_1}{K^S - K_1}$ or $L_1 = L^S - l_2(K^S - K_1)$, and at same time, $l_1 = \frac{L_1}{K_1}$.

So that $L^S - l_2(K^S - K_1) = l_1 K_1$.

Thus, the quantity of K and L demanded in the B sector are:

$$K_1 = \frac{L^S - l_2 K^S}{(l_1 - l_2)} \text{ and } L_1 = \frac{l_1}{(l_1 - l_2)}(L^S - l_2 K^S) \tag{22}$$

From (20) and (22) we obtain:

$$L_1 = \frac{e^A L^s}{(e^A - e^B)} - \frac{e^A e^B K^s p_B^{\frac{1}{\beta-\alpha}}}{(e^A - e^B)} \tag{23}$$

$$K_1 = \frac{L^s p_B^{\frac{1}{\alpha-\beta}}}{(e^A - e^B)} - \frac{e^B K^s}{(e^A - e^B)} \tag{24}$$

which are functions of a single variable p_B. Equations (23) and (24) hold for any level of γ. In particular, taking $\gamma = 1$, we denote production of B and I as $\Psi(p_B)$ and $\Phi(p_B)$ respectively. Therefore, from (2), (23) and (24) we obtain the equilibrium level of output as a function of equilibrium price p_B^*:

$$B^{s*}=[\frac{e^A L^s}{(e^A - e^B)} - \frac{e^A e^B K^s p_B^{\frac{1}{\beta-\alpha}}}{(e^A - e^B)}]^{\alpha}[\frac{L^s p_B^{\frac{1}{\alpha-\beta}}}{(e^A - e^B)} - \frac{e^B K^s}{(e^A - e^B)}]^{1-\alpha} = \Psi(p_B^*) \tag{25}$$

$$I^{s*}= \gamma[L^s - \frac{e^A L^s}{(e^A - e^B)} - \frac{e^A e^B K^s p_B^{\frac{1}{\beta-\alpha}}}{(e^A - e^B)}]^{\beta}[K^s - \frac{L^s p_B^{\frac{1}{\alpha-\beta}}}{(e^A - e^B)} - \frac{e^B K^s}{(e^A - e^B)}]^{1-\beta} = \gamma\Phi(p_B^*)$$

For industry I, (25), does not express output as an explicit function of equilibrium prices alone as we wished, because $\gamma = \gamma(I)$, and $I = I(\gamma, p_B)$. In order to obtain output as explicit functions of equilibrium prices we must therefore find out the equilibrium value of the scale parameter γ^*. As already mentioned in the text, this is an additional "fixed point" problem, since γ depends on I, while I depends on γ. We solve this as follows.

The industry I has increasing returns which are external to the firms in this industry, and the parameter γ increases with the level of output of I. We postulated that

$$\gamma = I^{\sigma}, \text{ where } 0 < \sigma < 1. \tag{26}$$

At an equilibrium, equations (25) and (26) must be simultaneously satisfied, i.e. $\gamma = [\gamma\Phi(p_B)]^{\sigma} = \gamma^{\sigma}\Phi(p_B)^{\sigma}$ or, $\gamma^{1-\sigma} = \Phi(p_B)^{\sigma}$.

Thus,

$$\gamma = \Phi(p_B)^{\frac{\sigma}{(1-\sigma)}} \tag{27}$$

Therefore at an equilibrium from (25) and (27) we obtain a relation between the outputs of I and p_B:

$$I^{s*} = \Phi(p_B^*)^{\frac{1}{(1-\sigma)}} \tag{28}$$

so that

$$I^{s*} = \left\{ [L^s - \frac{e^A L^s}{(e^A - e^B)} - \frac{e^A e^B K^s p_B^{\frac{1}{\beta-\alpha}}}{(e^A - e^B)}]^{\beta}[K^s - \frac{L^s p_B^{\frac{1}{\alpha-\beta}}}{(e^A - e^B)} - \frac{e^B K^s}{(e^A - e^B)}]^{1-\beta} \right\}^{\frac{1}{1-\sigma}} \tag{29}$$

We can now give explicitly the 'resolving' equation for the model:

$$F(p_B) = I^{d*}(p_B) - \left\{ [L^s - \frac{e^A L^s}{(e^A - e^B)} - \frac{e^A e^B K^s p_B^{\frac{1}{\beta-\alpha}}}{(e^A - e^B)}]^{\beta} [K^s - \frac{L^s p_B^{\frac{1}{\alpha-\beta}}}{(e^A - e^B)} - \frac{e^B K^s}{(e^A - e^B)}]^{1-\beta} \right\}^{\frac{1}{1-\sigma}} = 0$$

where from (4), (5) and (21), $I^{d*}(p_B)$ is a function of p_B alone:

$$I^{d*}(p_B) = (\alpha e^{A(\alpha-1)} p_B^{\frac{\beta-1}{\beta-\alpha}}) L^s + ((1-\alpha) e^{A\alpha} p_B^{\frac{\beta}{\beta-\alpha}}) K^s - p_B^* B^{d*}(p_B^*) \qquad (30)$$

Solving the equation $F(p_B) = 0$, gives an equilibrium value of p_B^* from which all equilibrium values of other variables $(K_{1,}^* K_{2,}^* L_{1,}^* L_2^*, w^*, r^*, B^{s*}, I^{s*}, I^{d*}, \gamma^*)$ can be computed. The model is thus solved.

5.0.5 Data Issues

In 1996 the BEA revised national income and product accounts introducing chain-type annual-weighted indexes, also known as Fisher indexes, to measure real output and prices. This new measure takes into account the changes in the relative prices and in the composition of the output over time. Thus, the chain-type index eliminates the major source of the bias in the previously used fixed-weighted, or Laspeyres' measure. "The chain-type estimates provide users with dollar-denominated measures of real GDP by industry, but they do not provide accurate estimates of industry shares of real GDP or of industry contributions to real GDP growth," see SCB November 2002 publication [29]. Due to this nonadditivity property of the data we could not compute an index of IRS industries. In this November 2002 publication the BEA advises to use nominal shares for such computations, but it warns that such measures will understate the share, especially for fast growing industries with plummeting prices, such as information technology, (the problem lies exactly in the industries of our primary interest). But even though real GDP series should not be used for share computations, real GDP level or growth rates data is preferred to nominal GDP. "The chain-type indexes eliminate an understatement of growth in investment spending in the past and an overstatement in current periods. It also avoids misstatement of growth by industry." [31]. In view of this, in computing our 'real betas' we used real GDP by industry index.

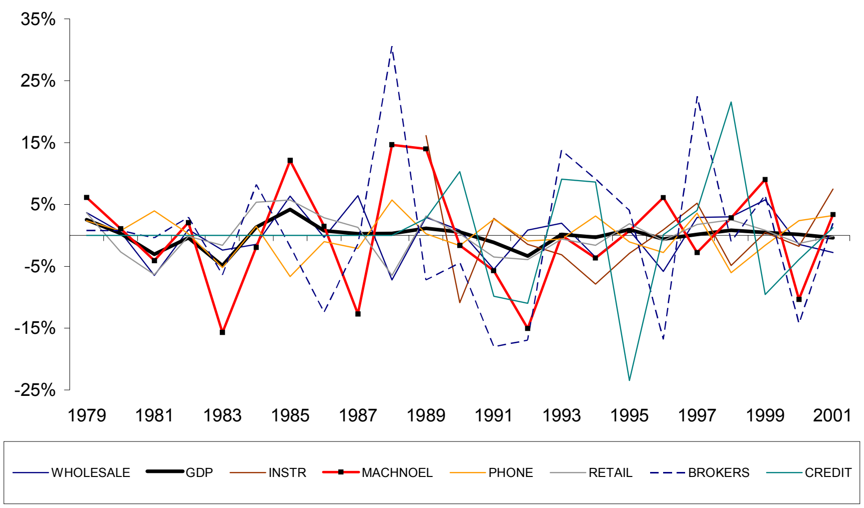
5.1 Charts and Figures

Figure 2: List of All External Increasing Returns to Scale and Traditional Industries Used in This Study, Their 'Real Betas' and Nominal Shares

	1977-2001		1977-1987		1987-2001	
Industry (SIC)	**beta**	**shares**[1]	**Beta**	**shares**	**beta**	**shares**
IRS						
Credit agencies other than banks (61)	2.9	0.9	0.8	0.6	4.7	1.1
Electronic equipment and instruments (36, 38)	2	2.6	1	2.7	3.1	2.6
Machinery, except electrical (35)	2.2	2.1	0.9	2.5	3.1	1.8
Retail trade (52-59)	1.2	9	1.4	9.1	1.5	8.9
Security and commodity brokers (62)	4	0.5		0.4	4	0.5
Telephone and telegraph (481, 482, 489)	1.5	2.2	1.1	2.3	2.2	2.1
Wholesale trade (50, 51)	1.6	6.8	1.5	6.9	1.9	6.7
Total Share of IRS Industry		*24.8*		*25*		*24.6*
Traditional						
Apparel and other textile products	0.1	0.5	0.7	0.6	-0.5	0.4
Chemicals and allied products	0.9	1.8	0.5	1.7	0.7	1.9
Food and kindred products	0.5	1.7	1.4	1.8	0.1	1.5
Furniture and fixtures	0.7	0.3	1.3	0.3	0.7	0.3
Leather and leather products	-1.1	0.1	-2.6	0.1	-1.3	0.1
Lumber and wood products	0.2	0.6	1.6	0.7	-0.4	0.5
Miscellaneous manufacturing industries	1	0.3	0.8	0.4	0.9	0.3
Paper and allied products	0.4	0.8	0.8	0.8	0	0.7
Printing and publishing	-0.1	1.2	1.1	1.2	-0.6	1.2
Real estate	0.9	11	1	10.4	0.9	11.3
Textile mill products	0.4	0.4	0.6	0.5	0	0.3
Total Share of Traditional Industry		18.7		18.5		18.5

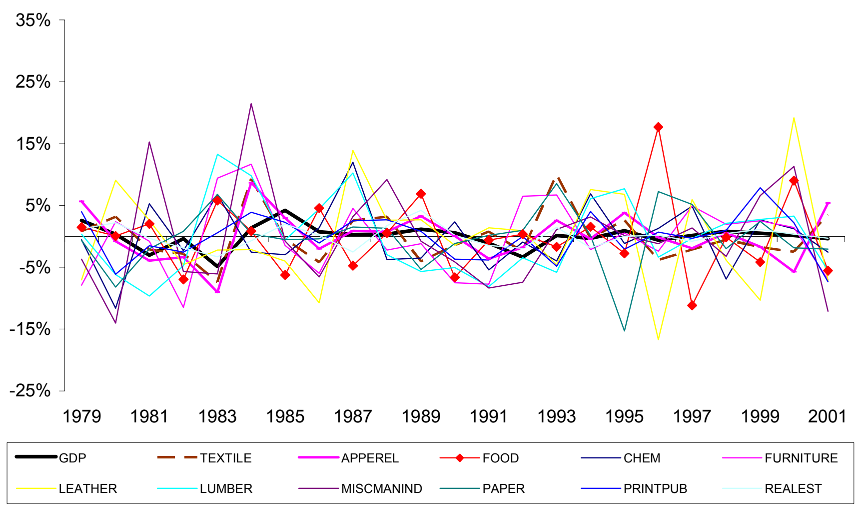
Source: U.S. Department of Commerce, Bureau of Economic Analysis. Detailed annual series can be found at: http://www.bea.gov/bea/dn2/gpo.htm.
Note: Gross Product Originating by Industry Share of Gross Domestic Product in percent.

Figure 3: Graphical Illustration of Volatility in the Increasing Returns to Scale Sectors



Source: U.S. Department of Commerce, Bureau of Economic Analysis. Detailed annual series can be found at: http://www.bea.gov/bea/dn2/gpo.htm.
Note: 1. The graphed series are the growth rates in real GDP by industry detrended using the Hodrick-Prescott filter.
2. For detail on industries chosen for this figure please see the text and Figure 3.

Figure 4: Graphical Illustration of Volatility in the Traditional Sectors



Source: U.S. Department of Commerce, Bureau of Economic Analysis. Detailed annual series can be found at: http://www.bea.gov/bea/dn2/gpo.htm.
Note: 1. The graphed series are the growth rates in real GDP by industry detrended using the Hodrick-Prescott filter.
2. For detail on industries chosen for this figure please see the text and Figure 3.

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