ESSAYS ON MACROECONOMICS AND LABOR MARKETS

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Chapter 1 of my dissertation focuses on the effectiveness of fiscal policy in stabilizing the business cycle. Both government purchases and transfers figure prominently in the use of fiscal policy for counteracting recessions. However, existing representative agent models including the neoclassical and New Keynesian benchmark rule out transfers by assumption. This paper provides a role for transfers by building a borrower-lender model with equilibrium credit spreads and monopolistic competition. The model demonstrates that a broad class of deficit-financed government expenditures can be expressed in terms of purchases and transfers. With flexible prices and in the absence of wealth effects on labor supply, transfers and purchases have no effect on aggregate output and employment. Under sticky prices and no wealth effects, fiscal policy is redundant to monetary policy. Alternatively, in the presence of wealth effects, multipliers for both purchases and transfers will depend on the behavior of credit spreads, but purchases deliver a higher output multiplier to transfers under reasonable calibrations due to its larger wealth effect on labor supply. When the zero lower bound is binding, both purchases and transfers are effective in counteracting a recession, but the size of the transfer multiplier relative to the purchases multiplier is increasing in the debt-elasticity of the credit spread.

The second chapter of my dissertation examines the relationship between shifts in the Beveridge curve, sector-specific shocks and monetary policy. In this joint work with Dmitriy Sergeyev, we document a significant correlation between shifts in the US Beveridge curve in postwar data and periods of elevated sectoral shocks. We provide conditions under which sector-specific shocks in a multisector model augmented with labor market search frictions generate outward shifts in the Beveridge curve and raise the natural rate of unemployment. Consistent with empirical evidence, our model also generates cyclical movements in aggregate matching function efficiency and mismatch across sectors. We calibrate a two-sector version of our model and demonstrate that a negative shock to construction employment calibrated
to match employment shares can fully account for the outward shift in the Beveridge curve experienced in the Great Recession (2007-2009).

The final chapter of my dissertation considers the decline in labor market turnover experienced in the US in the Great Recession, and its link to the housing crisis. In this joint work with Dmitriy Sergeyev, we analyze the behavior of job flows to test the hypothesis that the housing crisis has impaired firm formation and firm expansion by diminishing the value of real estate collateral used by firms to secure loans. We exploit state-level variation in job flows and housing prices to show that a decline in housing prices diminishes job creation and lagged job destruction. Moreover, we document differences across firm size and age categories, with middle-sized firms (20-99 employees) and new and young firms (firms less than 5 years of age) most sensitive to a decline in house prices. We propose a quantitative model of firm dynamics with collateral constraints, calibrating the model to match the distribution of employment and job flows by firm size and age. Financial shocks in our firm dynamics model depletes job creation and job destruction and replicates the empirical pattern of the sensitivity of job flows across firm age and size categories.
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To my parents
Chapter 1

Fiscal Policy Stabilization: Purchases or Transfers?
1.1 Introduction

The Great Recession has brought renewed attention to the possibility of using fiscal policy to counteract recessions. Since 2007, policymakers have adopted a series of historically large fiscal interventions in an attempt to raise output, reduce unemployment, and stabilize consumption and investment. In addition to some increases in government purchases, policymakers have also relied heavily on transfers of various forms - to individuals, institutions, and state and local governments - as instruments of fiscal policy. Table 1.1 provides the Congressional Budget Office breakdown of the various components of the Recovery Act and estimates for the associated policy multiplier. Transfers account for more than half of the expenditures in the Recovery Act.

Table 1.1: Outlays and estimated multipliers for American Recovery and Reinvestment Act

<table>
<thead>
<tr>
<th>Category</th>
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<th>Estimated Multiplier (Low)</th>
<th>Outlays</th>
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<tr>
<td>Purchases of goods and services by the federal government</td>
<td>2.5</td>
<td>1.0</td>
<td>$88 bn</td>
</tr>
<tr>
<td>Transfers to state and local governments for infrastructure</td>
<td>2.5</td>
<td>1.0</td>
<td>$44 bn</td>
</tr>
<tr>
<td>Transfers to state and local governments not for infrastructure</td>
<td>1.9</td>
<td>0.7</td>
<td>$215 bn</td>
</tr>
<tr>
<td>Transfers to persons</td>
<td>2.2</td>
<td>0.8</td>
<td>$100 bn</td>
</tr>
<tr>
<td>One-time Social Security payments</td>
<td>1.2</td>
<td>0.2</td>
<td>$18 bn</td>
</tr>
<tr>
<td>Two-year tax cuts for lower and middle income persons</td>
<td>1.7</td>
<td>0.5</td>
<td>$168 bn</td>
</tr>
<tr>
<td>One-year tax cuts for higher income persons (AMT fix)</td>
<td>0.5</td>
<td>0.1</td>
<td>$70 bn</td>
</tr>
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</table>

In contrast to government purchases, the effectiveness of transfers as an instrument of stabilization has only recently garnered attention in the literature. Empirical work by Johnson, Parker, and Souleles (2006) demonstrate that an economically significant portion of tax rebates (intended as stimulus) are spent. The authors track changes in consumption in the Consumer Expenditures Survey and use the timing of rebates as a source of exogenous variation. Agarwal, Liu, and Souleles (2007) provide additional evidence of sizable consumption effects by examining spending and saving behavior of households using credit card data. This literature finds an economically significant and persistent response of household consumption to rebates. Recent work by Oh and Reis (2012) and Giambattista and Pennings (2012) have emphasized the important role of transfers in recent stimulus programs and have posited models to determine the effect of these programs. Similarly, work by Kaplan and Violante (2011) and Bilbiie, Monacelli, and Perotti (2012) have further examined the channels by which
transfers effect aggregate output, employment and consumption.

In this paper, I examine the role of transfers as an instrument of fiscal policy with an emphasis on purchases and transfers as alternative policies. To provide a role for transfers, the model features patient and impatient households along with a credit spread which generates borrowing and lending in steady state. The model allows for flexible or sticky prices to determine how the conclusions of the representative agent RBC and New Keynesian models carry over to a multiagent setting. Additionally, the model demonstrates how a broad class of deficit-financed government expenditures can be represented as some combination of government purchases and transfers.

My analysis reveals that several insights from the representative agent setting carry over to a multiagent setting with credit spreads. Under flexible prices, fiscal policy only affects output and employment through a wealth effect on labor supply. If preferences or the structure of labor markets eliminate wealth effects on labor supply, neither purchases nor transfers will have any effect on output or employment. However, even in the presence of wealth effects, the deviations from the representative agent benchmark are small for plausible calibrations. The government purchases multiplier on output is positive and driven by the negative wealth effect on labor supply, while the transfers multiplier is close to zero as wealth effects lead to offsetting movements in hours worked by the households that provide and receive the transfer. A sensitivity analysis reveals that the variability of the credit spread does not affect these results.

Under sticky prices, fiscal policy now has both an aggregate supply effect (via wealth effect on labor supply) and an aggregate demand effect (via countercyclical markups). In the absence of wealth effects, a Phillips curve can be derived in terms of output and inflation. So long as the instrument of monetary policy is not constrained, the central bank may implement any combination of output and inflation irrespective of the stance of fiscal policy. In this sense, fiscal policy is irrelevant for determining aggregate output or inflation as monetary policy is free to undo any effect of fiscal policy. More generally, the tradeoff between purchases and transfers will depend on the monetary policy rule. In the presence of wealth effects,
purchases or transfers may lower wages and shift the Phillips curve. Under a Taylor rule and a standard calibration, transfers continue to have small effects on output and employment relative to purchases. The primacy of monetary policy in determining the effect of fiscal policy is analogous to the conclusions of Woodford (2011) and Curdia and Woodford (2010). The presence of a credit spread and intermediation alters the implementation of monetary policy (rule) but not the feasible set (Phillips curve).

When the instrument of monetary policy is constrained by, for example, the zero lower bound on the nominal interest rate, the choice between purchases and transfers once again becomes relevant and monetary policy cannot substitute for fiscal policy. Moreover, the behavior of the credit spread and its dependence on endogenous variables such as aggregate borrowing and income will determine the relative merits of purchases versus transfers. In the model, an exogenous shock to the credit spread causes the zero lower bound to bind. Under the calibration considered, purchases act more directly to increase output and inflation while transfers allow for a faster reduction in private sector debt. Both types of policies allow a faster escape from the zero lower bound relative to no intervention due to the endogenous effect of debt reduction on credit spreads, and consumption multipliers for each policy are typically positive. A credit spread that is more elastic to changes in private sector debt favors transfers, while a spread that is more elastic to borrower income favors purchases.

The paper is organized as follows: Section 1.2 briefly summarizes related literature on fiscal policy in a non-representative agent setting and its role in stabilizing business cycles. Section 1.3 presents the model and introduces credit spreads and fiscal policy. Section 1.4 compares purchases and transfers in the case of no wealth effects on labor supply. Alternatively, Section 1.5 considers purchases and transfers in the presence of wealth effects. Section 1.6 examines the effect of purchases and transfers at the zero lower bound and Section 1.7 concludes.

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1 The Appendix relates the credit spread model considered here to models with rule-of-thumb households, models with borrowing constraints, and overlapping generations models.
1.2 Related Literature

The model of patient and impatient agents draws on the borrower-saver model used in Campbell and Hercowitz (2005), Iacoviello (2005), and Monacelli (2009) where different rates of time preference among households allow for borrowing and lending in steady state. Differing rates of time preference are a staple in financial accelerator models such as Bernanke, Gertler, and Gilchrist (1999), but these models typically go further and link the discount rate to the structure of production. The structure of model considered here closely relates to the model used by Bilbiie, Monacelli, and Perotti (2012) which focuses on the aggregate effects of income redistribution. My work differs in considering the role of credit spreads on the choice of purchases and transfers and the analysis of alternative fiscal instruments at the zero lower bound. Also, like Eggertsson and Krugman (2012), I also analyze fiscal policy in a two-agent setting where an exogenous debt shock causes the zero lower bound to bind and borrowers reduce consumption as debt is repaid. However, my model differs in considering deficit-financed fiscal policy, credit spreads that are partly determined endogenously, and analyzing the importance of wealth effects on labor supply both at the zero lower bound and away from the zero lower bound.

The effect of fiscal policy has also been examined in models with rule-of-thumb agents - agents who do not participate in financial markets and simply consume their income each period. Mankiw (2000) analyzes the effects of changes in taxation in a savers-spenders framework, noting that such a model provides a justification for temporary reductions in taxes as stimulus. Gali, Lopez-Salido, and Valles (2007) examine the effect of rule-of-thumb consumers on the government purchases multiplier, and find that the presence of these agents can boost the multiplier above one. However, the effect of nominal rigidities and labor market frictions in their model have substantial effects on the government purchases multiplier even in the absence of rule-of-thumb consumers. In a model with rule-of-thumb agents, Giambattista and Pennings (2012) also compare the transfers multiplier to the government purchases multiplier finding cases in which the former can exceed the later. In contrast to a rule-of-thumb model, my model allows for intertemporal optimization on the part of both households and better
fits the empirical evidence on tax rebates by allowing for a persistent response to temporary

My work also relates to a literature on the effects of the public debt and transfers on pro-
duction in settings with credit frictions such as borrowing constraints and incomplete markets.
Aiyagari and McGrattan (1998) examine the optimal level of public debt in a heterogenous
agent model with idiosyncratic earnings risk and capital as the variable factor of production.
A higher level of public debt can increase welfare by easing liquidity constraints but tends
to lower output by reducing precautionary saving and decreasing capital. Woodford (1990)
presents a stylized overlapping generations model with capital to illustrated that increases in
the public debt can both increase welfare and increase output, counteracting the view that high
levels of debt must necessarily crowd out investment. This paper differs from this literature
by considering an aggregate demand channel for changes in the public debt and focusing on
short-term rather than long-run effects of fiscal policy.

A small literature has studied the conduct of fiscal policy for stabilization purposes in
heterogenous agent models with incomplete markets. Heathcote (2005) considers the short-
run effect of tax cuts in a model with idiosyncratic income risk, but where both hours and
capital are variable factors of production. He finds that a tax rebate has a multiplier of 0.15,
and somewhat higher multipliers when considering reductions in distortionary taxes. His
work does not consider the aggregate demand effect of alternative fiscal policies. Moreover,
the output effect comes from investment rather than hours since he assumes GHHI preferences
and no wealth effects on labor supply. Similarly, Oh and Reis (2012) consider the effect
of targeted transfers as fiscal stimulus and find very low transfer multipliers. The increase
in hours worked by households that experience a negative wealth shock does not offset the
decrease in hours worked by households that receive transfers. The model considered here
differs by treating only hours as a variable factor, considering sticky prices as the nominal
rigidity, and using credit spreads as opposed to borrowing constraints to allow for financial
intermediation.
1.3 Model

The model consists of two types of household, monopolistically competitive firms, a monetary authority that sets the deposit rate as its policy instrument, and a fiscal authority. The two-agent model facilitates the introduction of sticky prices and monetary policy to examine aggregate demand effects, and allows for the use of log-linearization to understand the key mechanisms at work. To generate borrowing and lending in steady state, the lender and borrower household are assumed to differ in their rates of time preference. An equilibrium credit spread is introduced to ensure that both agent’s Euler equations are satisfied in steady state.

1.3.1 Households

A measure $1 - \eta$ of patient household chooses consumption and real savings to maximize discounted expected utility:

$$\max_{\{C^s_t, N^s_t, D_t\}} E \sum_{t=0}^{\infty} \beta^t U (C^s_t, N^s_t)$$

subject to

$$C^s_t = W_t N^s_t + \frac{1 + i^d_t d_{t-1}}{\Pi_t} D_{t-1} - D_t + \Pi^f_t - T_t$$

where $D_t$ is real savings of the patient household and $\Pi^f_t$ are any profits from the real or financial sectors\(^2\). The government may collect non-distortionary lump sum taxes $T_t$ that are levied uniformly across households. The period utility function $U (C, N)$ is twice continuously differentiable, increasing, and concave in consumption: $U_c (C, N) > 0$, $U_{cc} (C, N) < 0$ and decreasing and convex in hours: $U_h (C, N) < 0$, $U_{hh} (C, N) < 0$. While patient households could choose to borrow, for sufficiently small shocks, the interest rate on borrowings would be too high and the patient household only saves.

A measure $\eta$ of impatient household chooses consumption and real borrowings to maximize

---

\(^2\)If equity in the firms and intermediaries were traded and short-selling ruled out, the patient household would accumulate all shares in steady state. For sufficiently small shocks, the assumption that patient households own all shares would continue to hold in the stochastic economy.
discounted expected utility:

$$\max\{C_t^b, N_t^b, B_t\} \quad E_0 \sum_{t=0}^{\infty} \gamma^t U \left( C_t^b, N_t^b \right)$$

subject to $C_t^b = W_t N_t^b + B_t - \frac{1 + i_{t-1}^b}{\Pi_t} B_{t-1} - T_t$

where $B_t$ is the real borrowings of the impatient household. The impatient household’s discount rate $\gamma < \beta$ ensures that the household chooses not to save and to only borrow in the neighborhood of the steady state. The impatient household’s optimality conditions are analogous to those of the patient household and standard:

$$\lambda_t^s = U_s \left( C_t^s, N_t^s \right) \quad (1.1)$$
$$\lambda_t^i W_t = -U_h \left( C_t^i, N_t^i \right) \quad (1.2)$$
$$\lambda_t^s = \beta E_t \lambda_{t+1}^s \frac{1 + i_t^d}{\Pi_{t+1}} \quad (1.3)$$
$$\lambda_t^b = \gamma E_t \lambda_{t+1}^b \frac{1 + i_t^b}{\Pi_{t+1}} \quad (1.4)$$

for $i \in \{s, b\}$ in equations (1.1) and (1.2). The difference between the borrowing rate and the deposit rate allows both agents Euler equations to be satisfied in the non-stochastic steady state, with the interest rates determined by the patient and impatient household’s discount rates.

Aggregate consumption $C_t$ and labor supply $N_t^{sup}$ are simply the weighted sum of each household’s consumption and labor supply:

$$C_t = \eta C_t^b + (1 - \eta) C_t^s \quad (1.5)$$
$$N_t^{sup} = \eta N_t^b + (1 - \eta) N_t^s \quad (1.6)$$

As my analysis demonstrates, wealth effects play a critical role in determining the effect of fiscal policy on output, employment and consumption.

**Definition 1.1.** Wealth effects are absent from household labor supply if the household’s labor
supply has the following representation:

\[ W_t = v_i \left( N_i^t \right) \]

for some function \( v_i \) that is increasing.

Wealth effects on labor supply are eliminated under the preference specification considered by Greenwood, Hercowitz, and Huffman (1988):

\[ U(C, N) = \left( \frac{C - \gamma N^{1 + \frac{1}{\phi}}}{1 - \sigma} \right)^{1 - \sigma} \]

where \( \varphi \) is the Frisch elasticity of labor supply. Under GHH preferences, labor supply takes the form shown in the definition:

\[ W_t = \gamma \left( 1 + \frac{1}{\varphi} \right) \left( N_i^t \right)^{1/\varphi} \]

Aside from GHH preferences, wealth effects on labor supply would also be absent in a model with labor market rigidities. Under a rigid real wage, the labor supply relation no longer holds for each household:

\[ W_t > -\frac{U_h \left( C_i^t, N_i^t \right)}{U_c \left( C_i^t, N_i^t \right)} \]

for \( i \in \{s, b\} \). In a model where wages remained constant - the case of perfect wage rigidity considered by (Blanchard and Gali, 2010) and Shimer (2012) - fiscal multipliers are determined exclusively by firm’s labor demand condition. Under wage rigidity, household’s labor supply can be represented (locally) by a constant function \( v_i \left( N_i^t \right) = c = \bar{W} \) satisfying the definition of no wealth effects.

To obtain an aggregate labor supply curve and an aggregate IS curve, I must log-linearize the household’s labor supply and Euler equations. In the general case with wealth effects, labor supply is a function of the aggregate wage and the household’s consumption:

\[ w_t = \frac{1}{\varphi_i} n_i^t + \frac{1}{\sigma_i} c_i^t \]
for $i \in \{s, b\}$ where the lower case variables represent log deviations from steady state, $\varphi_i$ is the household’s Frisch elasticity and $\sigma_i$ is the household’s intertemporal elasticity of substitution.

Solving for each agent’s labor supply $n^i_t$, aggregate labor supply is the weighted sum of each agent’s log-linearized labor supply (where the weight is the steady state share of employment for each household). Similarly, an aggregate IS equation can be obtained by a weighted sum of each agent’s log-linearized Euler equation:

$$
\frac{1}{\bar{\varphi}^i} n^i_t + l^i_b \frac{\varphi^b}{\bar{\varphi}^b} c^b_t + (1 - l^i_b) \frac{\varphi^s}{\bar{\varphi}^s} c^s_t = \frac{1}{\bar{\varphi}} n_t + \frac{1}{\bar{\varphi}^b} c^b_t + \frac{1}{\bar{\varphi}^s} c^s_t \tag{1.7}
$$

$$
c_t = E_t c_{t+1}^b + s_b \sigma^b c^b_t + (1 - s_b) \sigma^s c^s_t - \tilde{\sigma} E_t \pi_{t+1} \tag{1.8}
$$

with $l_b = \eta \bar{N}_b / \bar{N}$ and $s_b = \eta \bar{C}_b / \bar{C}$. The parameters $\bar{\varphi}^i = l^i_b \varphi^b + (1 - l^i_b) \varphi^s$ and $\tilde{\varphi} = s_b \sigma^b + (1 - s_b) \sigma^s$ are the appropriate weighted aggregate Frisch elasticity and aggregate intertemporal elasticity of substitution respectively. Relative to a standard representative household model, the labor supply curve depends on the distribution of consumption (as opposed to just the level of consumption) and the IS curve depends on the real borrowing rate (in addition to the real deposit rate).

### 1.3.2 Credit Spreads

The credit spread - the difference between the borrowing rate and deposit rate - is treated as a reduced form equation:

$$
\frac{1 + i^b_t}{1 + i^d_t} = 1 + \omega_t = E_t \Gamma \left( B_t, W_{t+1} N^b_{t+1}, Z_t \right) \tag{1.9}
$$

The function $\Gamma$ is assumed to be weakly increasing in its first and last arguments and weakly decreasing in its middle argument. The assumption that the spread is increasing with the level of household debt $B_t$ is needed to ensure determinacy of the rational expectations equilibrium and is analogous to the stationarity conditions needed in small open economy models\(^3\). The effect of expected borrower income, $W_{t+1} N^b_{t+1}$ on credit spreads is consistent with the observed

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\(^3\)See discussion in Schmitt-Grohé and Uribe (2003).
countercyclicality of credit spreads and the fact that spreads lead the business cycle. The dependence of the spread on borrower income would emerge in a model where lending is subject to adverse selection or limited commitment.

The shock $Z_t$ is an exogenous financial shock that can increase spreads. The financial shock may be interpreted as either a shock to the supply or demand side of the credit market. On the supply side, if financial intermediaries’ capacity to raise funds is constrained by their own net worth, a depletion of equity due to an unexpected loss on the asset side of the balance sheet will cause an increase in borrowing rates. Alternatively, on the demand side, a shock to borrower collateral can likewise make borrowers less creditworthy thereby raising spreads. In particular, in a model with housing as collateral, a shock to house prices would reduce the value of collateral and raise credit spreads for the borrower household.

The log-linearized credit spread can be summarized by two parameters: the elasticity of the spread to private borrowings and the elasticity of the spread to borrower income with $\chi_b > 0$ and $\chi_n \geq 0$:

$$\omega_t = \chi_b b_t - \chi_n E_t \left( w_{t+1} + n_{t+1}^b \right) + z_t$$

The elasticity on debt strictly exceeds zero to ensure stationarity. The credit spread may rise due to an exogenous increase in $z_t$ or may rise due to some fundamental shock that drives up the level of debt or decreases borrower’s household income. The log-linearized credit spread is flexible enough to incorporate the type of interest rate spreads seen in a broad class of models.

When $\chi_n = 0$, the model exhibits a debt elastic spread as in standard small open economy models. When $\chi_b = \chi_n > 0$, the credit spread varies with the leverage of the borrower household. The canonical financial accelerator model of Bernanke, Gertler, and Gilchrist (1999) features a leverage elastic spread. Finally, when $\chi_n > \chi_b > 0$, the credit spread may be described as income elastic strengthening comovement with the business cycle. Variations in these parameters will be used to determine the effect of credit spreads on the choice among fiscal instruments.
1.3.3 Fiscal and Monetary Policy

The instruments of fiscal policy consist of a set of uniform nondistortionary taxes, government consumption, and transfers. The fiscal authority may also run a budget deficit subject to a fiscal rule that ensures that the debt returns to its steady state level and subject to an intertemporal solvency condition:

\[ G_t = B_t^g - \frac{1 + i_{t-1}^d}{\Pi_t} B_{t-1}^g + T_t \]  
(1.10)

\[ T_t = \phi_b \left( B_{t-1}^g - \overline{B}_g \right) - reb_t \]  
(1.11)

\[ 0 = \lim_{T \to \infty} E_T \frac{P_t}{P_T} \frac{B_T^g}{\Pi_t (1 + i_{t-1}^d)} \]  
(1.12)

where \( reb_t \) is a lump sum tax rebate delivered to all households. The instruments of fiscal policy are government purchases \( G_t \) and a reduction in lump sum taxes \( reb_t \). The government’s cost of funds is the policy rate \( i_t^d \), not the borrowing rate \( i_t^b \). This assumption best fits larger economies like the United States where the government controls the currency. For small open economies and countries in a currency union (such as the Eurozone), the rate at which the government borrows may carry a premium to the policy rate.

The monetary authority is assumed to set a rule for monetary policy so long as its instrument of policy, the deposit rate \( i_t^d \), is not constrained by the zero lower bound. I will consider when monetary policy follows a standard Taylor rule (without interest rate smoothing) or pursues perfect inflation stabilization:

\[ \left( \frac{i_t^d}{\overline{r}_d} \right) = (\Pi_t)^{\phi_y} \left( \frac{Y_t}{\overline{Y}_t} \right)^{\phi_y} \]  
(1.13)

\[ \Pi_t = 1 \]  
(1.14)

When monetary policy is constrained by the zero lower bound, I assume that the deposit rate is set at zero or inflation is perfectly stabilized.
1.3.4 Firms

Monopolistically competitive firms set prices periodically and hire labor in each period to produce a differentiated good. Cost-minimization for firms and production function play a key role in examining the effects of various fiscal policy shocks and are given below:

\[
MC_t = \frac{W_t N_t}{\alpha Y_t} \quad (1.15) \\
Y_t = N_t^\alpha \quad (1.16)
\]

where \(\alpha\) is the labor share, \(N_t\) is labor demand and \(MC_t\) is the firm’s marginal cost which varies over time depending on the rate of inflation and the stance of monetary policy.

Prices are reset via Calvo price setting where \(\theta\) is the likelihood of firm to reset it’s prices in the current period. When \(\theta = 1\), prices are set each period, monopolistic competitive firms set prices as a fixed markup over marginal costs:

\[
\frac{P_t}{P_t} = \frac{\nu}{\nu - 1} MC_t
\]

where \(\nu\) is the elasticity of substitution among final goods in the Dixit-Stiglitz aggregator. If the initial price level is unity, then prices will be normalized to unity, and marginal costs will be fixed at all periods \(MC_t = \overline{MC} = 1/\mu_p\). When \(\theta < 1\), firms will set prices on the basis of future expected marginal costs. The firms prices problem and the behavior of the price level are summarized by the following equations:

\[
F_t = \mu_p \lambda_t^s MC_t Y_t + \theta \beta E_t \Pi_{t+1}^{\nu-1} F_{t+1} \\
K_t = \lambda_t^s Y_t + \theta \beta E_t \Pi_{t+1}^{\nu-1} K_{t+1} \\
1 = \theta \Pi_t^{\nu-1} + (1 - \theta) \left( \frac{K_t}{F_t} \right)^{\nu-1}
\]

Firms are owned by the saver households and therefore future marginal costs are discounted by the saver household’s stochastic discount factor.

When prices are flexible, marginal costs are fixed and, to a log-linear approximation,
When prices are sticky, a log-linear approximation to the firm’s pricing problem around a zero inflation steady state implies the standard expectations-augmented Phillips curve:

\[ \pi_t = \kappa mc_t + \beta E_t \pi_{t+1} \]

where \( \kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta} \).

### 1.3.5 Equilibrium

Asset market clearing requires that real saving equals real borrowing:

\[ \eta B_t + B^g_t = (1 - \eta) D_t \]

Combining the household’s budget constraints and the government’s budget constraint and firm profits implies an aggregate resource constraint of the form:

\[ Y_t = C_t + G_t \tag{1.17} \]

Labor market clearing requires:

\[ N_t = N^s = \eta N^b_t + (1 - \eta) N^s_t \tag{1.18} \]

**Definition 1.2.** An equilibrium is a set of allocations \( \{Y_t, N_t, C_t^s, C_t^b, N_t^s, N_t^b, \lambda^s_t, \lambda^b_t, B_t, F_t, K_t\} \), a price process for \( \{W_t, \Pi_t, \hat{c}_t, \hat{i}^b_t, M C_t\} \), a fiscal policy \( \{B^g_t, T_t, G_t, reb_t\} \), and initial values for private debt \( B_0 \) and public debt \( B^g_0 \) that jointly satisfy the equilibrium conditions listed in the Appendix.

The fiscal policy considered consists of government purchases and tax rebates, as opposed to transfers. However, deficit-financing of these fiscal policies is equivalent to a transfer from saver to borrower households and back again.

**Proposition 1.1.** Consider an equilibrium under a deficit financed fiscal policy \( \{B^g_t, T_t, G_t, reb_t\} \). There exists a set of household-specific taxes \( T^b_t \) and \( T^s_t \) that implement the same equilibrium.
and satisfies a balanced budget: \( G_t = \eta T^b_t + (1 - \eta) T^s_t \)

**Proof.** Since the saver household purchases the issuance of government debt, the saver’s budget constraint may be expressed using the asset market clearing condition and substituting out for taxes using the government’s budget constraint (1.10):

\[
C^s_t + \frac{1}{1 - \eta} (\eta B_t + B^g_t) = W_t N^b_t + \Pi^f_t + \frac{1 + i^d_{t-1}}{\Pi_t} \frac{1}{1 - \eta} (\eta B_{t-1} + B^g_{t-1}) + B^g_t - \frac{1 + i^d_{t-1}}{\Pi_t} B^g_{t-1} - G_t
\]

Rearranging, we may define saver specific tax \( T^s_t \):

\[
C^s_t + \frac{\eta}{1 - \eta} B_t = W_t N^b_t + \Pi^f_t + \frac{1 + i^d_{t-1}}{\Pi_t} \frac{\eta}{1 - \eta} B_{t-1} - T^s_t
\]

\[
T^s_t = \frac{\eta}{1 - \eta} \left( B^g_t - \frac{1 + i^d_{t-1}}{\Pi_t} B^g_{t-1} \right) + G_t
\]

For the borrower household, we may define the borrower specific tax \( T^b_t = G_t - \left( B^g_t - \frac{1 + i^d_{t-1}}{\Pi_t} B^g_{t-1} \right) \).

It is readily verified that the household specific taxes satisfy the balanced budget constraint.

\[ \blacksquare \]

The proposition illustrates an equivalence relation between deficit-financing and transfers between agents. As the budget deficit increases, taxes fall for the borrower household and rise for the saver. A tax rebate represents a pure transfer from savers to borrowers despite the fact that both households receive the tax rebate. A deficit financed increase in purchases represents a combination of both transfers and purchases.

However, the transfer cannot be one way. As the debt is stabilized or decreased, the transfer reverses - borrowers make a transfer back to savers. Thus, in general, the converse of the proposition will not hold. A fiscal authority that can levy household specific taxes can implement a richer set of policies than a fiscal authority constrained to uniform taxation and deficit financing. For example, a one-way transfer cannot be implemented as a deficit-financed rebate. Moreover, the capacity of the fiscal authority to engineer transfers depends
on the initial level of debt - with high levels of public debt, an increase in transfers requires an increase to higher debt levels where the overall transfer will be blunted by the size of interest payments.

1.4 Case of No Wealth Effects on Labor Supply

In this section, I examine the effect of purchases and transfers in a setting where household preferences or the structure of labor markets eliminate wealth effects on labor supply. The absence of wealth effects eliminates any effect of fiscal policy on aggregate supply. With prices set freely each period, firms’ incentives to hire labor are not changed because neither its marginal costs nor its production technology are affected by the change in fiscal policy. When prices are changed only periodically, changes in fiscal policy will have an effect on aggregate demand. When prices are fixed, producers must meet demand at posted prices raising marginal costs. However, the monetary authority is always free to tighten interest rates and dampen demand so long as it is not constrained by the zero lower bound.

1.4.1 Flexible Prices

When producers are free to set prices each period, prices are a constant markup over marginal costs. Since price is normalized to unity, marginal costs are constant: \( MC = \frac{1}{\mu_p} \) in all periods.

**Proposition 1.2.** If labor supply depends only on the wage for both households, then output and employment are determined independently of fiscal policy

*Proof.* For each household, labor supply is determined by (2):

\[ W_t = v_i (N_t^i) \]

for \( i \in \{s, b\} \). Under the assumptions in Section 1.3, the function \( v \) is strictly increasing. Therefore, its inverse exists and combining the labor supply equation with labor market clearing:

\[ N_t = \eta v_b^{-1} (W_t) + (1 - \eta) v_s^{-1} (W_t) \]
Using the firm’s production function (11) and labor demand condition (10), wages can be expressed in terms of employment:

\[ W_t = \alpha MC N_t^{\alpha - 1} \]

Replacing wages, aggregate employment is determined independent of fiscal policy. The production function implies that output is also determined independent of fiscal policy. Importantly, the irrelevance of fiscal policy holds irrespective of any of the properties of the credit spread, and would continue to obtain in a model with other types of financial frictions (such as borrowing constraints) or a larger number of agents so long as the labor supply relation holds for each agent. Using the economy’s resource constraint (1.17), it follows that a tax rebate or transfer has no effect on aggregate consumption while an increase in government purchases is offset by an equivalent decrease in consumption. Significantly, the insights of the representative agent model are unchanged in the multiple agent setting.

Wealth effects on labor supply are eliminated under the preference specification considered by (Greenwood, Hercowitz, and Huffman, 1988):

\[ U(C, N) = \frac{\left( C - \gamma N^{1+\frac{1}{\nu}} \right)^{1-\sigma}}{1 - \sigma} \]

where \( \nu \) is the Frisch elasticity. The labor supply conditions for each household take the following form:

\[ W_t = \gamma \left( 1 + \frac{1}{\nu} \right) \left( N_t^i \right)^{1/\nu} \tag{1.19} \]

for \( i \in \{s, b\} \).

Aside from GHII preferences, wealth effects on labor supply would also be absent in a model with labor market rigidities. Under a rigid real wage, the labor supply relation no longer holds for each household:

\[ W_t > -\frac{U_h (C_t^i, N_t^i)}{U_c (C_t^i, N_t^i)} \]
for \( i \in \{s, b\} \). In a model where wages remained constant - the case of perfect wage rigidity considered by (Blanchard and Gali, 2010) and Shimer (2012) - fiscal multipliers are determined exclusively by firm’s labor demand condition. As long as wages exceed the marginal rate of substitution, output and employment will be unaffected by fiscal policy. More generally, if the wage only adjusts gradually to changes in the households’ marginal rates of substitution, the effect of fiscal shocks can be made arbitrarily small.

1.4.2 Sticky Prices

Under sticky prices, marginal costs are no longer constant and fiscal shocks will affect output and employment through the aggregate demand channel. However, monetary policy can also affect output and employment via the aggregate demand channel, and, since the feasible set of combinations of output and inflation is unchanged by the presence of credit spreads, monetary policy and fiscal policy are redundant.

To show that the Phillips curve is unchanged, I use a log-linear approximation to the equilibrium conditions to obtain the output inflation tradeoff. Under GHH preferences, the household’s log-linearized labor supply conditions imply:

\[
w_t = \frac{1}{\nu} n_t^{i}
\]

for \( i \in \{s, b\} \). Aggregating using a log-linearized version of (1.18) and eliminating \( w_t \) using (1.15):

\[
m_c_t = n_t - y_t + \frac{1}{\nu} n_t
\]

Eliminating \( n_t \) using the log-linearized production function (1.16) and using the equation for \( m_c_t \), an expectations-augmented Phillips curve is obtained:

\[
\pi_t = \frac{\kappa}{\alpha} \left(1 - \alpha + \frac{1}{\nu}\right) y_t + \beta E_t \pi_{t+1}
\]
The case of wage rigidity is simply the case of \( \nu \to \infty \):

\[
\pi_t = \frac{\kappa}{\alpha} (1 - \alpha) y_t + \beta E_t \pi_{t+1}
\]

If monetary policy seeks to stabilize some combination of output and inflation, the targeting rule for optimal monetary policy will be unaffected by the presence of credit spreads or their variability. Formally, if the central bank chooses a path of \( \pi_t, y_t \) to minimize a loss function of the form:

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t \left( \pi_t^2 + \lambda y_t^2 \right)
\]

subject to the Phillips curve given above, then the target criterion is the standard one

\[
\pi_t + \frac{\lambda}{\vartheta} (y_t - y_{t-1}) = 0
\]

where \( \vartheta \) is the slope of the Phillips curve\(^4\). Though the loss function here does not follow from a second-order approximation of average utility in a multiple household economy, it seems reasonable to assume that the central bank will be primarily concerned with maintaining aggregate output rather than distributional considerations. The primacy of monetary policy in determining the effect of fiscal shocks is similar to the conclusions in Woodford (2011). He showed that the government purchases multiplier could be larger or smaller than the neoclassical multiplier depending on how aggressively monetary policy responds to inflation.

Though, the inflation/output tradeoff is unchanged by credit spreads, the implementation of monetary policy will be affected. This result is analogous to the results presented in Curdia and Woodford (2010) who show that the presence of financial intermediation does not affect the targeting rule for optimal monetary policy but may affect the implementation of optimal monetary policy. In general, setting the correct policy rate \( i_t^d \) to implement optimal policy will require the monetary authority to take into account changes in the credit spread. A log-linear approximation to the household’s Euler equations (1.1) and (1.3) - (1.4) and a log-linear approximation to the household’s Euler equations (1.1) and (1.3) - (1.4) and a log-linear

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\(^4\)The optimal targeting criterion features output instead of the output gap because the natural rate of output is simply steady state output.
approximation to the resource constraint (1.17) can be combined to derive an aggregate Euler equation:

\[ i_t^d = E_t \pi_{t+1} - \frac{1}{s_c \bar{\sigma}} (y_t - g_t - E_t (y_{t+1} - g_{t+1})) - \frac{s_b \sigma_b}{\bar{\sigma}} \omega_t \]

where \( \omega_t \) is the credit spread, \( \sigma_b \) is the borrower household’s intertemporal elasticity of substitution, \( \bar{\sigma} \) is a weighted average of households intertemporal elasticity of substitution, \( s_b \) is the share of borrower’s consumption in total consumption in steady state, and \( s_c \) is the share of private consumption in total output in steady state. Fiscal policy will directly affect the determination of interest rates through government purchases and also affect interest rates via the spread. So long as the zero lower bound on nominal interest rates is not binding, there exists a path of interest rates consistent with the target path of output and inflation set by the monetary authority. Any changes in fiscal policy can be accommodated by suitable adjustment of the interest rate. Since a path of output implies a path of employment, monetary policy and fiscal policy are redundant in determining those quantities when the zero lower bound is not binding. Importantly, monetary policy and fiscal policy cannot achieve the same equilibrium allocations and are not equivalent in terms of the distribution of consumption. Fiscal policy may still have a role in achieving some distribution of consumption or level of private debt.

1.5 Case of Wealth Effects on Labor Supply

In this section, I consider the more conventional case of government purchases and transfers in the presence of wealth effects on labor supply. The canonical RBC and New Keynesian models typically feature wealth effects ensuring both an aggregate supply and an aggregate demand channel for fiscal policy. While the conclusions in this section are not as strong as the case with no wealth effects, the insights from the previous section carry over in the calibrated examples analyzed in this section.
1.5.1 Representative Agent Benchmark

To allow for wealth effects on labor supply, I consider standard preferences where the level of consumption affects agent’s labor supply. To a log linear approximation, each agent’s labor supply condition relates the wage to hours worked and consumption:

\[ w_t = \frac{1}{\varphi_i} n^i_t + \frac{1}{\sigma_i} c^i_t \]

for \( i \in \{s, b\} \), where \( \varphi_i \) is the Frisch elasticity of hours with respect to the wage and \( \sigma_i \) is the elasticity of intertemporal substitution. The labor supply approximation given above holds irrespective of whether utility is separable in consumption and hours. To examine how credit spreads affect fiscal multipliers, it is useful to derive a representative agent benchmark for comparison. In a representative agent model, marginal utilities must be equalized across agents implying that \( c^s_t = c^b_t = c_t \). Solving each agent’s labor supply equation in terms of \( n^i_t \) and aggregating labor using (1.18), an aggregate labor supply relation can be derived:

\[ w_t = \frac{1}{\tilde{\varphi}} n_t + \frac{1}{\tilde{\sigma}} c_t \]

\[ \tilde{\varphi} = l_b \varphi_b + (1 - l_b) \varphi_s \]

\[ \tilde{\sigma} = \frac{\tilde{\varphi}}{l_b \varphi_b + (1 - l_b) \frac{\varphi_b}{\sigma_b}} \]

where \( l_b \) is the share of borrower’s hours in total hours worked. Given this aggregate labor supply condition, the output multiplier can be obtained by solving for consumption and the wage in terms of output (since \( mc_t = 0 \)) and substituting into the resource constraint (1.17):

\[ y_t = \frac{\alpha}{\alpha + s_c \tilde{\sigma} \left( 1 - \alpha + \frac{1}{\tilde{\varphi}} \right) g_t} \]

where \( s_c \) is the share of consumption in GDP and \( \tilde{\varphi} \) is the average Frisch elasticity and \( \tilde{\sigma} \) is the representative agent’s intertemporal elasticity of substitution. Government spending increases output via a negative wealth effect, but the government spending multiplier is necessarily less
than one. Transfers and deficit-financing have no affect on output.

The representative agent model also admits a representation for the Phillips curve. Eliminating $m c_t$ using the labor demand equation (1.15) and eliminating $n_t$ using the production function (1.16) provides a Phillips curve representation:

$$\pi_t = \kappa \left( w_t + \frac{1 - \alpha}{\alpha} y_t \right) + \beta E_t \pi_{t+1}$$

Using the resource constraint (1.17) to eliminate $c_t$ and the production function, wages can be expressed in terms of output and government purchases. Replacing the wage in the Phillips curve provides the relationship between output and inflation:

$$\pi_t = \frac{\kappa}{\alpha} \left( \frac{1}{\varphi} + \frac{1}{s_c \bar{\sigma}} + 1 - \alpha \right) y_t - \frac{\kappa}{s_c \bar{\sigma}} g_t + \beta E_t \pi_{t+1}$$

An increase in government purchases shifts back the Phillips curve by increasing labor supply and lowering wages - purchases raise the natural rate of output.

### 1.5.2 Flexible Prices

In the case of a model with credit spreads, the labor supply relations can be solved for consumption $c^i_t$ in terms of the wage $w_t$ and hours worked $n^i_t$ for each agent. Substituting into the resource constraint and eliminating the wage using (1.15), output can be expressed in terms of purchases and hours worked by each agent:

$$y_t = \frac{\alpha}{\alpha + s_c \bar{\sigma} (1 - \alpha)} g_t - \frac{s_c}{\alpha + s_c \bar{\sigma} (1 - \alpha)} \left( \frac{s_b \sigma_b}{\varphi_b} n^b_t + \frac{(1 - s_b) \sigma_s}{\varphi_s} n^s_t \right)$$

where $\bar{\sigma} = s_b \sigma_b + (1 - s_b) \sigma_s$ is a weighted average elasticity of intertemporal substitution and other parameters as defined earlier.

The expression for output can be further simplified by solving for $n^b_t$ from labor market clearing (1.18), giving output as a function of government purchases and the saver household’s
labor supply:

\[ y_t = \frac{\alpha}{\alpha + s_c \sigma (1 - \alpha) + s_c \sigma_b \varphi_b} g_t + \frac{\alpha s_c}{\alpha + s_c \sigma (1 - \alpha) + s_c \sigma_b \varphi_b} \left( \frac{(1 - l_b) s_b \sigma_b}{l_b \varphi_b} - \frac{(1 - s_b) \sigma_s}{\varphi_s} \right) n_t^s \]

**Proposition 1.3.** Transfers and the means of financing any government expenditure have no effect on output and employment if:

1. Preferences are linear in hours worked as in Hansen (1985) and Rogerson (1988)

2. Labor supply by households is coordinated: \( n_t^s = n_t^b \)

3. Preferences satisfy the following condition:

\[ \frac{1 - l_b \varphi_s}{l_b \varphi_b} = \frac{1 - s_b \sigma_s}{s_b \sigma_b} \]

**Proof.** In the first case, as the Frisch elasticities \( \varphi_s = \varphi_b \to \infty \), the coefficient on the second term in the expression for output goes to zero, and output is only affected by purchases. In the second case, hours worked by the saver hours equal aggregate hours: \( n_t^s = n_t = \frac{1}{\alpha} y_t \) and output is solely a function of purchases. In the last case, the coefficient on hours of the saver household is zero.

The proposition illustrates that, even with wealth effects on labor supply, transfers and deficit-financing may have little effect on output or employment. The deviations from the representative agent benchmark stem solely from the second term in the output expression. If households are sufficiently homogenous - that is, if household do not differ appreciably in underlying parameters and shares of consumption and hours, the coefficient on the second term is likely to be small. If this coefficient is positive, fiscal policies that strengthen the negative wealth effect on the saver household will boost the output multiplier relative to the representative agent benchmark. In particular transfers away from the saver household should boost multipliers. However, if the coefficient is negative, fiscal policies that boost the negative
wealth effect on borrowers will increase multipliers.

1.5.3 Sticky Prices

In the case of sticky prices, fiscal policy has both an aggregate supply element that reduces marginal costs and an aggregate demand element that raises marginal costs. Monetary policy does not face a stable Phillips curve relation between inflation and output, and the choice of fiscal policy may shift the Phillips curve in favorable or unfavorable ways. As before, the Phillips curve can be expressed in terms of both output and wages:

$$\pi_t = \kappa \left( w_t + \frac{1 - \alpha}{\alpha} y_t \right) + \beta E_t \pi_{t+1}$$

However, unlike the representative agent model, in the presence of wealth effects, wages cannot generally be expressed in terms of aggregate output.

In the cases considered in the previous proposition, transfers have no effect on aggregate output and the Phillips curve can be represented in terms of inflation, output, and government purchases as in the representative agent model. Since transfers do not shift the Phillips curve, credit spreads do not affect the Phillips curve and the output-inflation tradeoff is unchanged. As before, households labor supply equations can be aggregated into an aggregate labor supply equation:

$$w_t = \frac{1}{s_c \sigma} (y_t - g_t) + \frac{1}{\sigma} \left( \frac{s_b \sigma_b}{\varphi_b} n_t^b + \frac{(1 - s_b) \sigma_s}{\varphi_s} n_t^s \right)$$

where the first term gives the wealth effect on labor supply and the second term gives the substitution effect. Because government purchases act to directly lower the wage while transfers cause offsetting movements in hours between households, purchases are likely to have a greater downward effect on wages. A reduction in wages will provide the monetary authority with a more favorable output and inflation tradeoff and allow for a less restrictive monetary policy. In this sense, one can conjecture that purchases may be better than transfers for boosting output and employment.
Table 1.2: Calibration summary

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>Intertemporal elasticity</td>
<td>$\sigma_i$</td>
<td>1</td>
<td>Deposit rate</td>
<td>$i_d$</td>
<td>1.02^{0.25}</td>
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<td>Frisch elasticity</td>
<td>$\varphi_i$</td>
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<td>Borrowing rate</td>
<td>$i_b$</td>
<td>1.06^{0.25}</td>
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<td>Calvo parameter</td>
<td>$\theta$</td>
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<td>Borrower share</td>
<td>$\eta$</td>
<td>50%</td>
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<tr>
<td>Markup</td>
<td>$\mu_p$</td>
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<td>Debt elasticity</td>
<td>$\chi_b$</td>
<td>0.1</td>
</tr>
<tr>
<td>Wage bill</td>
<td>$\bar{W}/\bar{N}/\bar{Y}$</td>
<td>0.70</td>
<td>Income elasticity</td>
<td>$\chi_n$</td>
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<td>Gov’t purchases</td>
<td>$\bar{G}/\bar{Y}$</td>
<td>0.20</td>
<td>Taylor rule (inflation)</td>
<td>$\phi_\pi$</td>
<td>1.5</td>
</tr>
<tr>
<td>Debt/GDP</td>
<td>$\bar{B}/\bar{Y}$</td>
<td>2</td>
<td>Taylor rule (output)</td>
<td>$\phi_y$</td>
<td>0.25</td>
</tr>
<tr>
<td>Household debt</td>
<td>$\bar{B}/\bar{W}\bar{N}_b$</td>
<td>4</td>
<td>Fiscal rule</td>
<td>$\phi_b$</td>
<td>0.2</td>
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</table>

1.5.4 Calibration

As I have shown, in the presence of wealth effects on labor supply, fiscal policy will have both aggregate supply and aggregate demand channels. To assess the degree to which the multiple agent model differs from the representative agent model, I calibrate the model with wealth effects and examine the effect of deficit-financed purchases and tax rebates. While each deficit-financed policy can be expressed as a balanced budget combination of purchases and transfers, the deficit-financed policies considered here are closest to fiscal policy in practice and avoid issues of incentive compatibility\(^5\).

The baseline calibration assumes standard separable utility function of the form

$$U(C, N) = \frac{C^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \nu N^{1+\varphi^{-1}}$$

with standard values for the Frisch elasticities and intertemporal elasticities of substitution. In the baseline calibration these values are equal across agents with $\varphi_b = \varphi_s = 2$ and $\sigma_b = \sigma_s = 1$. In steady state, output $\bar{Y}$ is normalized to 1 and the disutility of labor supply for each household $\nu_s$ and $\nu_b$ is set to ensure that each household supplies labor such that $\bar{N}_b = \bar{N}_s = 1$. The markup due to monopolistic competition is set at 25\% and the labor share $\alpha$ is set to ensure that the wage bill is equal to 70\% of GDP, consistent with U.S. data. The Calvo

\(^5\)In the case of household specific taxes and transfers, household have an incentive to mask their type and represent themselves as borrowers or lenders based on the proposed fiscal policy.
parameter $\theta$ is set to 0.75 so that firms change prices every 4 quarters on average. The rates of time preference $\beta$ and $\gamma$ are set to target an annual deposit rate of 2% and an annual borrowing rate of 6%. The disutilities of labor supply, the rates of time preference, and the markup do not enter the log-linearized equilibrium conditions and, therefore, do not affect the dynamics of the model. In steady state, the consumption of the borrower household is less than that of the saver household since the saver household earns both wage income and profits from the firm. Government spending is 20% of GDP in steady state. The steady state public debt is 50% of GDP consistent with recent U.S. levels. In steady state, the household debt for the borrower household is equal to annual household income consistent with data on household wealth from the Survey on Consumer Finances.

The nonstandard parameters for the model include the credit spread parameters $\chi_b$ and $\chi_n$ that control, respectively, the endogenous response of spreads to private sector debt and expected borrower income and the share of borrower households $\eta$ in the economy. In the baseline case, I will consider a debt-elastic spread such that $\chi_b = 0.1$ and $\chi_n = 0$ - a calibration that implies a 1% increase in debt raises spreads by roughly 50 basis points. In general, a regression of spreads on measures of indebtedness and income in aggregate data is unlikely.
to accurately estimate these elasticities given that common shocks may induce a comovement of income and spreads even though $\chi_n = 0$. As shown in Section 1.6, the financial shock $z_t$ causes income and spreads to comove even with $\chi_n = 0$. As it turns out, these credit spread elasticities have little effect on the experiments here suggesting that spreads may have a fairly small effect on fiscal policy transmission away from the zero lower bound. The share of borrower household $\eta$ is set to 50% as in Curdia and Woodford (2010); this parameter has no obvious analogue in the data and is selected conservatively to minimize heterogeneity. The calibration values are summarized in Table 1.2.

1.5.5 Fiscal Policy Experiments and Sensitivity

The first experiment in Figure 1.1 considers the effect of a 1% of GDP increase in government purchases (top panel) and a 1% of GDP increase in tax rebates (bottom panel), each with a persistence of $\rho = 0.9$. The figure also shows the response of the representative agent economy with parameters as defined in Section 1.5.1. The fiscal authority runs a budget deficit and taxes follow a fiscal rule - taxes adjust upwards to return the public debt to its steady state level. The response parameter in the fiscal rule $\phi_b$ is close to the rule used in Gali, Lopez-Salido, and Valles (2007), which is based on VAR estimates for U.S. data. Prices are reset each period and, therefore, firm markups are constant.

In this environment, the effect of purchases and rebates is driven by wealth effects on labor supply. Under the baseline calibration where the Frisch elasticity and intertemporal elasticity of substitution are equal, the only source of heterogeneity is the share of borrower consumption $s_b < \frac{1}{2}$ since the borrower household pays interest to the intermediary and does not receive any profits from firms. Under this calibration, the coefficient on saver’s hours (in the output expression Section 1.5.2) is negative. As a result, the tax rebate multiplier is slightly negative - the fall in hours worked by the borrower household is not fully offset by the rise in hours by the saver household. The transfer acts to dampen aggregate incentives to work. Likewise,

---

6Steady state government purchases are financed by a tax on patient households to reduce differences in steady state levels of consumption (through a tax on capital holdings). However, it is assumed that both household pay taxes proportional to their size in the economy to finance government purchases in excess of steady state levels. In steady state, $\bar{C}_s/\bar{C}_b \sim 1.3$. 

the government purchases multiplier on output is slightly lower than the representative agent multiplier since the labor supply effects for the borrower are dampened by the increase in the deficit. As the second column shows, the response in hours worked by each household is quite different reflecting the transfer component of fiscal policy. However, these movements wash out in the aggregate - the difference in aggregate hours between the representative agent model and the multiagent model is miniscule. The dynamics of public debt illustrate the degree of transfers from the saver household - periods of increasing debt represent net transfers to borrowers, while periods of stabilizing and falling debt represent transfers from borrowers back to savers. Importantly, these policies do not imply the same debt dynamics since changes in the interest rate have an effect on debt accumulation in a calibration with a positive steady state level of debt. With zero debt and a linear approximation, both policies would imply the same path of the public debt. Government purchases have larger output multipliers than tax rebates simply because purchases have a larger wealth effect on labor supply. Output and employment rise as the wage falls due to the increased willingness of both households to work.

Figures 1.2 and 1.3 examine how sensitive these results are to the credit spread elasticities.
χ_b and χ_n and to heterogeneity in wealth effects across households by adjusting the relative intertemporal elasticities of substitution. Figure 1.2 show that different models of the spread have little effect on the deviations of output multipliers from the representative agent benchmark - in particular that tax rebate multiplier is still negative and close to zero. Figure 1.3 considers three cases: debt elastic spreads (χ_b = 0.5, χ_n = 0), leverage elastic spread (χ_b = χ_n = 0.5), and income elastic spread (χ_b = 0.1, χ_n = 0.5). In all cases, the purchases and rebate multipliers deviates by less than 5% from the representative agent benchmark. In each case, the behavior of hours and spreads differs, but the aggregate effect on output, hours, wages, and consumption are all close to the representative agent benchmark. The second column shows that saver’s hours respond strongly to the tax rebate shock, but the borrower’s response almost fully offsets this rise in hours resulting in little net effect.

Figure 1.3 examines the effect of variations in the relative intertemporal elasticity of substitution holding the average intertemporal elasticity fixed at ˜σ = 1 where ˜σ is as defined in Section 1.5.2. In the case of “high borrower elasticity,” wealth effects for the borrower household are diminished by choosing an intertemporal elasticity of substitution three times higher than that of the saver household. Alternatively, in the case of “high saver elasticity,”
the borrower household has an intertemporal elasticity of substitution one-third the size of the saver household and, therefore, the borrower’s labor supply is more sensitive to changes in wealth. When the borrower household exhibits smaller wealth effects, the tax rebate multiplier becomes positive. Savers respond to the negative wealth shock by working harder while borrowers reduce their hours but by less than in the baseline case. As a result, aggregate hours and output rises. The opposite occurs in the case of high saver elasticity for the same reason. As before, the government purchases multiplier is an order of magnitude higher than the tax rebate multiplier simply because of the stronger wealth effects on aggregate labor supply under purchases.

Figure 1.4: Deficit-financed purchases and tax rebates under a Taylor rule

Figure 1.4 relaxes the assumption of flexible prices and examines the effect of an increase in purchases and tax rebates when monetary policy follows a Taylor rule. To ensure that a tax rebate is expansionary, the calibration used in Figure 1.4 assumes the case of a high borrower elasticity of intertemporal substitution - that is, \( \sigma_b/\sigma_s = 3 \). As the experiment demonstrates, fiscal multipliers rise sizably under an operative aggregate demand channel. Moreover, a more elastic credit spread raises multipliers further - when the elasticity of the spread to debt rises from \( \chi_b = 0.1 \) to \( \chi_b = 0.5 \), the output multiplier on purchases rises from 0.69 to 0.82.
Likewise, the output multiplier for tax rebates rises from 0.05 to 0.15. The falling credit spread dampens the transmission of monetary policy as the rise in the deposit rate is not fully incorporated into the borrowing rate (since spreads are falling). However, as noted earlier, bigger multipliers come only at the cost of higher inflation as seen in the last column. This rise in inflation is due to the fact that the Phillips curve has not shifted, and larger multipliers are the product of an accommodative stance of monetary policy. As before, the purchases multiplier is an order of magnitude larger than the transfers multiplier. However, if monetary policy responds asymmetrically to different fiscal shocks, it is possible to obtain cases where the tax rebate multiplier is as high or higher than the purchases multiplier. Finally, purchases are preferred to tax rebates in that sense that purchases generate a larger rise in output and employment for a given amount of inflation. The negative wealth effect of purchases raises labor supply, reduces marginal costs, and improves the Phillips curve tradeoff.

1.6 Zero Lower Bound

In this section, I examine how credit spread shocks may cause the zero lower bound to bind and consider the effect of government purchases and transfers on output and consumption. Consistent with evidence from representative agent models, the government purchases multiplier is above unity at the zero lower bound. Additionally, transfers (implemented by tax rebates) may be similarly effective as purchases in stabilizing output and consumption. The choice among policies depends on the endogenous feedback of debt and income on the credit spread.

Representative agent models typically rely on preference shocks or other reduced form shocks to the natural rate of interest to cause the zero lower bound to bind. However, in a model with multiple agents, disruptions to the financial system that raise the credit spread may also cause the zero lower bound to bind. As shown in Section 4.2, an aggregate IS-equation can be obtained by summing the agent’s Euler equations:

\[ i_t^d = E_t \pi_{t+1} - \frac{1}{s_{t+1}} (y_t - g_t - E_t (y_{t+1} - g_{t+1})) - \frac{s_b \sigma_b}{\sigma} \omega_t \]
Any shock to the credit spread $\omega_t$ will drive down the interest rate when monetary policy seeks to maintain $y_t = \pi_t = 0$, and for sufficiently large shocks, the interest rate will fall to the zero bound. The reduced form credit spread depends endogenously on debt and borrower income and, exogenously, on a financial shock. Any underlying shock that drives up debt and/or decreases borrower income may reduce the deposit rate, but I will consider an exogenous financial shock as the shock that binds the zero lower bound.

The special case of real wage rigidity and a credit spread with zero debt elasticity ($\chi_b = 0$) illustrates the role of purchases versus transfers in determining output and inflation. Under these conditions, output and inflation are solely determined by the Phillips curve and the intertemporal IS curve:

$$
\pi_t = \kappa \left( \frac{1 - \alpha}{\alpha} \right) y_t + \beta E_t \pi_{t+1} \\
y_t = E_t (y_{t+1}) + E_t (g_{t+1} - g_t) - s_c \sigma \left( i_t^d - E_t \pi_{t+1} \right) - s_c s_b \sigma_b \left( z_t - \frac{X_n}{\alpha} E_t y_{t+1} \right)
$$

where the credit spread is replaced by the log-linearized version of (1.9). By setting the debt-elasticity of the spread to zero, the law of motion for debt and the distribution of consumption between saver and borrower households is decoupled from the determination inflation and output. A zero lower bound episode is caused by a temporary increase in $z_t$ to $\bar{z}$ that reverts to zero with probability $1 - \rho$ in each period caused the zero lower bound to bind: $i_t^d = -\bar{r}$. Given the absence of a state variables, this two-equation system is forward-looking and multipliers may be computed explicitly as in Woodford (2010):

$$
y_{zlb} = \nu g \bar{y} - \zeta \\
\nu_g = \frac{(1 - \rho)(1 - \beta \rho)}{(1 - \beta \rho)(1 - \rho - s_c s_b \sigma_b \rho \frac{X_n}{\alpha}) - s_c \sigma \frac{X_n}{\alpha} (1 - \alpha) \rho} \\
\zeta = \frac{s_c (1 - \rho)(s_b \sigma_b \sigma - \sigma \sigma)}{(1 - \beta \rho)(1 - \rho - s_c s_b \sigma_b \rho \frac{X_n}{\alpha}) - s_c \sigma \frac{X_n}{\alpha} (1 - \alpha) \rho}
$$

The constant term gives the decrease in output in the absence of any policy intervention and under the assumption that monetary policy ensures that $y_t = \pi_t = 0$ after the financial
shock dissipates. If the credit spread does not respond (or responds weakly) to changes in private debt, transfers and tax rebates have no effect on output and inflation and are ineffective as fiscal stimulus. Government purchases are effective in counteracting the effects of a financial shock, but the mechanism is essentially the same as any representative agent model of government purchases at the zero lower bound. The means of financing the increase in purchases are irrelevant. In fact, the multiplier is identical to the multiplier in Woodford (2011) except for the endogenous effect of output on the spread through $\chi_n$. When $\chi_n > 0$, the multiplier on government purchases is higher as is the negative effect of the financial shock.

This simple example highlights how transfers operate through the credit spread. To the extent that transfers decrease the credit spread by lowering household indebtedness, transfers will have a positive multiplier. This analysis suggests that deficit-financed government purchases will be preferred to purchases financed by taxes because the transfer component of the policy further reduces credit spreads; indeed, this result holds in the numerical examples considered in the next section. Moreover, since transfers only operate through the spread, the transfers multiplier is unlikely to exceed the government purchases multiplier unless purchases worsen the rise in spreads. In the calibrated examples considered next, purchases reduce private sector indebtedness and the credit spread.

1.6.1 No Policy Intervention

The experiment here roughly attempts to capture the type of disruption experienced in the U.S. after the collapse of Lehman Brothers in 2008. The model and calibration are the same as considered in the previous section, however, for simplicity, that monetary authority is assumed to follow perfect inflation stabilization $\pi_t = 0$ for all periods after the zero lower bound ceases to bind. While monetary policy may, in principle, mitigate the effects of the financial shock by committing to higher future inflation (as discussed extensively in Eggertsson and Woodford (2003)), I assume that time inconsistency diminishes the effectiveness of these commitments. Additionally, steady state public debt is assumed to be zero to ensure that each policy implies
the same path for the public debt (to a linear approximation) and the shock generates no endogenous movements in debt or taxes.

Figure 1.5 shows the effect of a financial shock that raises (annualized) credit spreads 16 percentage points\(^7\). The model is solved using the solution algorithm described in the appendix of Eggertsson and Woodford (2003) and the financial shock is assumed to have a persistence of \(\rho = 0.8\). The point at which the economy exits the zero lower bound depends on the endogenous behavior of private sector debt and the value of the elasticity \(\chi_b\). For high elasticities, a faster rate of deleveraging will cause the credit spread to fall faster hastening the exit from the zero lower bound. However, as shown in Figure 1.5, agents actually increase their debt loads since the elasticity of the spread to debt is fairly low (\(\chi_b = 0.1\)).

A 16 percentage point financial shock raises credit spreads by 20 percentage points and leads to a very large fall in output and consumption in excess of 20%. The negative wealth effect drives down wages 40% and the fall in demand and wages combines to cause a very steep deflation. The zero lower bound episode lasts for 10 quarters or two and half years, and

\(^7\)The actual rise in the credit spread is larger because of the endogenous component due to the increase in private sector debt.
rates gradually normalize as debt and spreads fall. After the ZLB ceases to bind, inflation remains at zero and output is marginally positive due to wealth effects that keep wages lower than their steady state level.

Given the role of wealth effects in determining the sharp fall in wages and inflation, I also consider a similar shock in a model without wealth effects on labor supply. In the presence of a perfectly rigid wage, changes in household wealth have no affect on output or employment. Figure 1.6 provides the impulse responses to a larger 20 percentage point financial shock. Under rigid wages, the fall in output and inflation are significantly dampened with output falling 6% and (annualized) inflation falling 1% on impact - values that are comparable to the U.S. output and inflation response in the fourth quarter of 2008. The zero lower bound ceases to bind in 12 quarters and, since the Phillips curve is independent of spreads, output, consumption and inflation jump to their steady state levels. Households deleverage throughout the crisis period and only begin to releverage after three years; interest rates remain below their steady state level for the entire period of 24 quarters or six years. The output and inflation response in the rigid wage model suggest that some degree of wage rigidity might be desirable for fitting the model to the current recession.
1.6.2 Policy Intervention

I consider deficit financed fiscal policies where the intervention ends as soon as the zero lower bound ceases to bind. Fiscal policy may either raise government purchases or reduce taxes by some level so long as the zero lower bound binds. The choice of a flexible or rigid wage has significant implications for the efficacy of policy. Figure 1.7 shows the effect of a 1% of GDP increase in government purchases and a 1% decrease in taxes for all periods that the zero lower bound binds. Small policy interventions have very large effects relative to no intervention. For both government purchases and tax rebates, the economy exits the zero lower bound within a year instead of 2.5 years. Under the tax rebate, the fall in output is 2.5% versus a 22% fall absent intervention. For government purchases, the fall in output only last a quarter with output falling by only 0.5%. Instead of continuing to increase leverage, households deleverage between 3% and 5% and the intervention reduces the rise in spreads by 1/3. Deficit-financed purchases are preferred to tax rebates both in terms of output and consumption. Purchases act more directly to raise output and inflation, reducing the real interest rate faced by saver households and “crowding-in” consumption.

Figure 1.8 shows the much more limited effect of a 1% increase in purchases and tax rebates
in the rigid wage model. Both policies are successful in boosting both output and consumption relative to a policy of inaction, but these policies carry much smaller multipliers than the case of flexible wages. Government purchases limit the fall in output to 3.8% and rebates limit the fall in output to 5.3% relative to the 5.7% fall absent any intervention. Purchases, once again, act more directly to boost inflation and raise the consumption profile of the saver household while the borrowers consumption path is a function primarily of the credit spread. Output and inflation actually rise above their steady state values before jumping to those values once the zero lower bound stops binding. Relative to no intervention, the economy exits the zero lower bound only one period earlier.

1.6.3 Role of Credit Spreads

The choice between purchases and transfers/tax rebates depends, in part, on which policy is more effective in reducing credit spreads. Since the solution for the model at the zero lower bound is nonlinear, the model response to various fiscal shocks will not be invariant to the size of the shock. In other words, a 2% of GDP deficit-financed increase in purchases is not simply a scaled version of the 1% of GDP experiment; therefore, a fiscal multiplier is not readily
defined. Moreover, given that the model features endogenous state variables, the response of various variables of interest will depend on the persistence and shape of the fiscal response, even if the total size of the fiscal intervention is held constant. However, to provide insight into the role of credit spreads, I consider a simple metric for computing output multipliers:

$$M_t = \frac{\sum_{t=0}^{24} \left( y_{t}^{pol} - y_{t}^{nopol} \right)}{\sum_{t=0}^{24} x_t}$$

where $y_{t}^{pol}$ is the response of output, in log-deviations, under a particular fiscal intervention, $y_{t}^{nopol}$ is the response of output under no intervention, and $x_t$ is the total expenditure on the policy. The multiplier is a equally weighted sum over 24 quarters (6 years) of the deviations of output from the path it would have taken absent any intervention, conditional on the same underlying shock.

Two natural variables of interest are output (which is also employment in this model) and aggregate consumption. To isolate the effect of variations in the debt and income elasticity of the credit spread, I consider fiscal interventions of the following form: a 1% of GDP increase in government purchases financed by current taxes or a 1% transfer from saver to borrower households in all periods. The underlying shock is a 5 percentage point increase in the financial shock that decays deterministically at rate $\rho = 0.9$, and fiscal interventions end once the zero lower bound stops binding. For simplicity, only the model with perfect wage rigidity is considered.

Table 1.3 provides output and consumption multipliers and the time to exit for different values of the debt and income elasticity parameters in the credit spread. As Table 1.3 shows, the output multiplier on government purchases exceeds the output multiplier on transfers, though transfers may be more effective in boosting aggregate consumption than government purchases. The effectiveness of transfers relative to purchases rises with the debt-elasticity of the credit spread $\chi_b$, and the time to exit the zero lower bound falls with $\chi_b^8$. Absent

---

8Evidence from Edelberg (2006) suggest very low elasticities of risk premia on consumer loans with respect to debt and borrowing. Using data from the Survey on Consumer Finances, even for large differences in income and personal debt, spreads vary by less than two percentage points. A naive extrapolation suggests elasticity of spreads with respect to borrowing and income of less than 0.01 - an order of magnitude lower than shown
any fiscal intervention, a higher debt elasticity leads to faster deleveraging by borrowers and a quicker exit from the zero lower bound as the endogenous component offsets the financial shock component in the credit spread. Transfers facilitate this process of deleveraging allowing for quicker exits from the zero lower bound. As the debt elasticity rises, transfers are more effective in reducing credit spread and mitigating the effects of the financial shock. Unlike transfers, an increase in government purchases leaves the path of the credit spread largely unchanged along with the timing of exit. Consistent with the analytical results shown earlier for a debt inelastic spread, transfers are ineffective when $\chi_b = 0$; while transfers redistribute consumption from savers to borrowers, aggregate output and inflation will be unaffected.

An increase in the income-elasticity of the credit spread $\chi_n$ (shown on the right-hand side of Table 1.3) tends to magnify the effect of either type of fiscal intervention with output multipliers approaching the values estimated by the Congressional Budget Office. The time to exit the zero lower bound increases because the fall in output from the financial shock feeds back into the credit spread, amplifying its effect. The amplification of both the underlying shock and the effect of fiscal policy is consistent with the analytical results shown earlier. Despite the importance of the debt and income elasticity parameters for fiscal policy, a simple regression of credit spread measures on output and private sector borrowing is unlikely to accurately estimate these parameters since the error term is likely to be highly correlated with output and borrowing.

---

Table 1.3: Output and consumption multipliers

<table>
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<tr>
<th>Purchases</th>
<th>$\chi_n = 0.0$</th>
<th>$\chi_n = 0.0$</th>
<th>$\chi_n = 0.1$</th>
<th>$\chi_n = 0.3$</th>
<th>$\chi_n = 0.2$</th>
<th>$\chi_n = 0.0$</th>
<th>$\chi_n = 0.1$</th>
<th>$\chi_n = 0.2$</th>
<th>$\chi_n = 0.3$</th>
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<tr>
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<td>$\chi_b = 0.0$</td>
<td>$\chi_b = 0.1$</td>
<td>$\chi_b = 0.3$</td>
<td>$\chi_b = 0.2$</td>
<td>$\chi_b = 0.0$</td>
<td>$\chi_b = 0.1$</td>
<td>$\chi_b = 0.2$</td>
<td>$\chi_b = 0.3$</td>
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<tr>
<td>Output</td>
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<td>$\chi_b = 0.0$</td>
<td>$\chi_b = 0.1$</td>
<td>$\chi_b = 0.3$</td>
<td>$\chi_b = 0.2$</td>
<td>$\chi_b = 0.0$</td>
<td>$\chi_b = 0.1$</td>
<td>$\chi_b = 0.2$</td>
<td>$\chi_b = 0.3$</td>
</tr>
<tr>
<td>Consumption</td>
<td>$\chi_b = 0.0$</td>
<td>$\chi_b = 0.0$</td>
<td>$\chi_b = 0.1$</td>
<td>$\chi_b = 0.3$</td>
<td>$\chi_b = 0.2$</td>
<td>$\chi_b = 0.0$</td>
<td>$\chi_b = 0.1$</td>
<td>$\chi_b = 0.2$</td>
<td>$\chi_b = 0.3$</td>
</tr>
<tr>
<td>Transfers</td>
<td>$\chi_b = 0.0$</td>
<td>$\chi_b = 0.0$</td>
<td>$\chi_b = 0.1$</td>
<td>$\chi_b = 0.3$</td>
<td>$\chi_b = 0.2$</td>
<td>$\chi_b = 0.0$</td>
<td>$\chi_b = 0.1$</td>
<td>$\chi_b = 0.2$</td>
<td>$\chi_b = 0.3$</td>
</tr>
</tbody>
</table>

in the numerical experiments in this section. However, given the extensive use of non-price tools such as credit limits, downpayments, and credit history in rationing credit, these elasticities should be viewed as a lower bound rather than an upper bound on credit spread elasticities.
1.6.4 Savers and Borrowers Policy Preferences

Given that the model features distinct agents, optimal fiscal and monetary policy will, in general, depend on the weighting of each agent in the social welfare function. Moreover, even in the absence of financial shocks, the fiscal authority may wish to redistribute income among agents by changes in the level of the public debt\(^9\). However, even at the zero lower bound - where fiscal policy is most relevant - savers and borrowers may disagree on which fiscal interventions they prefer and may, in some cases, prefer no intervention at all. Furthermore, when fiscal policies are financed by deficits, preferences will differ on the rate with which the deficit is returned to its steady state. As shown earlier, when the credit spread is fairly debt inelastic, tax rebates and transfers simply have the effect of redistributing income and consumption with little effect on aggregates and savers may prefer inaction to any fiscal intervention. Importantly, the financial shock itself is redistributive - saver households benefit from the increase in credit spreads because, under the assumption that intermediary profits flow to the savers and absent no default (no real costs of intermediation), intermediary profits increase with a rise in spreads.

A simple metric, analogous to the output multiplier, for gauging household’s preferences over various policy options is the difference in household consumption under a fiscal intervention relative to no intervention:

\[
\frac{1}{24} \sum_{t=0}^{24} \left( c_{t,\text{pol}} - c_{t,\text{nopol}} \right) \epsilon s,b
\]

Table 1.4 displays each households’ consumption relative to the no intervention baseline under two alternative fiscal policies:

1. Debt-financed increase in government purchases by 1% of GDP for all periods the zero lower bound binds

2. Debt-financed tax rebates to all households of 1% of GDP for all periods the zero lower bound binds

---

\(^9\)For instance, a higher government debt implies greater inequality in steady state given higher interest payments to borrowers. Aiyagari and McGrattan (1998) consider the optimal level of public debt in a model with idiosyncratic risk and incomplete markets.
Table 1.4: Relative consumption (% deviation)

<table>
<thead>
<tr>
<th>Fiscal Parameter: $\varphi_b$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases $= 1%$ of GDP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver Household</td>
<td>-0.03</td>
<td>0.06</td>
<td>0.09</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Borrower Household</td>
<td>1.01</td>
<td>0.81</td>
<td>0.78</td>
<td>0.71</td>
<td>0.65</td>
</tr>
<tr>
<td>Tax Rebates $= 1%$ of GDP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver Household</td>
<td>-0.12</td>
<td>-0.04</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>Borrower Deviation</td>
<td>0.72</td>
<td>0.52</td>
<td>0.41</td>
<td>0.33</td>
<td>0.28</td>
</tr>
</tbody>
</table>

bound binds

Under the assumption of zero initial public debt, these policies imply equivalent fiscal cost for the government, with the debt increasing during the ZLB episode and subsequently converging back to zero. The rate at which taxes are raised to pay back the public debt is controlled by the parameter $\varphi_b$ with higher values leading to a faster increase in taxes. As Table 1.4 indicates, saver households prefer policies that minimize the increase in the public debt, and therefore minimize the degree of transfers. Borrower households instead prefer a greater degree of deficit-financing which increases disposable income in the near term when the cost of credit is high. In the particular calibration considered ($\chi_b = 0.1, \chi_n = 0$), any fiscal intervention offers a higher consumption path for borrowers relative to no intervention. In contrast, savers often enjoy a higher consumption path absent any fiscal intervention. Under this calibration, savers never prefer tax rebates (even though savers receive this rebate and are paid market rate on public debt), and only prefer purchases if the transfer component is limited. Deficit-financed purchases are the policy that most frequently increases the consumption path of both households relative to the baseline of no intervention. The preference for government purchases in this example is somewhat driven by the low debt elasticity of the credit spread, which renders transfers less effective for stabilization. Though this analysis abstracts from welfare costs of labor supply and inflation, it illustrates the potential for disagreement over fiscal policy and deficits.
1.7 Conclusion

Existing representative agent models, by Ricardian equivalence, rule out any role for transfers, tax rebates, or deficit-financing as tools for stabilizing business cycles. This paper analyzes a borrower-lender model with credit spreads to examine transfers as an instrument of fiscal policy and compare transfers to government purchases. I showed that deficit-financed policies such as an increase in purchases or temporary lump sum tax rebates financed budget deficits can be expressed as a combination of two fiscal instruments: purchases and transfers, and I distinguished between two important channels for these instruments: aggregate supply and aggregate demand. I find, in general, that government purchases are a more effective means of boosting output and employment than transfers/tax rebates, primarily because of its large wealth effects on labor supply.

The aggregate supply channel for fiscal policy is the wealth effect on labor supply. Purchases or transfers will boost output only to the extent that the policy increases labor supply. In the absence of wealth effects, the aggregate supply channel is inoperative and neither purchases nor transfers have any effect on output. In the presence of wealth effects, purchases increase output by making both agents poorer, but transfers (as implemented by tax rebates) have little effect on aggregate labor supply. On the whole, the aggregate supply channel favors purchases over transfers for boosting output and employment.

The aggregate demand channel for fiscal policy stems from countercyclical markups due to sticky prices. While both fiscal policy instruments can boost demand, monetary policy can undo or amplify any fiscal shock. When monetary policy is unconstrained, fiscal policy is redundant for aggregate demand management. While fiscal policy may play a role in the distribution of production or consumption, monetary policy is sufficient to manage the inflation-output tradeoff. However, when monetary policy is constrained, fiscal policy becomes the sole tool for managing aggregate demand. As a result, the choice between purchases and transfers or some combination thereof is not inconsequential. As the numerical experiments in Section 6 illustrate, both tools of fiscal policy can have substantial effects on aggregate demand at the zero lower bound, and the choice between these policies will depend on the details of
the credit spread. However, the low estimated elasticities of credit spreads to borrowing and income as reported in Edelberg (2006) suggest that tax rebates and transfers are unlikely to match government purchases in boosting output, employment or consumption.

Given the role of the credit spread elasticities in determining the multiplier on transfers relative to government spending, the details of financial intermediation are likely to be important for determining relative multipliers. While there is no single consensus model of financial intermediation, particularly among households, business cycle models of intermediation have generally focused on the importance of either collateral constraints for the borrower or the net worth of the intermediary\(^\text{10}\). In a model where quantity is constrained by the value of collateral, fiscal multiplier depend on the effect of policy on the shadow price of the collateral constraint. Both transfers and government purchases by boosting disposable income could ease these constraints temporarily or may boost the value of collateral, with the multiplier depending on how much each policy boosts borrower’s disposable income. Moreover, given the importance of housing as household collateral, relative multipliers are also likely to depend heavily on the effect of these policies on housing values. Even for relatively large fiscal outlays, these effects are likely to be small absent a policy directed towards housing.

Alternatively, in a model where the cost of financial intermediation depends on the net worth of intermediaries, fiscal multipliers are likely to operate via the default channel. If the net worth of financial intermediaries is low, the cost of credit will only fall if intermediaries are able to restore their net worth. Absent direct recapitalizations or changes in dividend policy, fiscal policy is only likely to impact intermediary net worth by reducing default rates. By raising disposable income either by raising output or through direct transfers, fiscal policy may reduce default rates and increase the rate at which intermediaries recapitalize. Alternatively, if fiscal policy leads to more defaults via households’ decisions to strategically default, could increase the cost of credit resulting in negative multipliers. In the end, more direct fiscal instruments that address the cause of the rise in credit costs - whether declining house prices or insufficient intermediary net worth - are likely to be more effective than indirect policies.

\(^{10}\)For the former, see Kiyotaki and Moore (1997) or Iacoviello (2005). For the latter, see Bernanke, Gertler, and Gilchrist (1999).
such as transfers and government purchases.

Given multiple instruments of policy, a natural extension will be analysis of optimal fiscal policy at the zero lower bound and further analysis of disagreement among agents over policy. Endogenizing the credit spread will be another important extension given the dependence of policy on the behavior of the credit spread. These extensions are ongoing research.
Chapter 2

Sectoral Shocks, the Beveridge Curve and Monetary Policy

with Dmitriy Sergeyev

\(^1\)We would like to thank Andreas Mueller, Ricardo Reis, Jón Steinsson and Michael Woodford for helpful discussions and Nicolas Crouzet, Hyunseung Oh, Andrew Figura, Emi Nakamura, Serena Ng, Bruce Preston, Stephanie Schmitt-Grohe, Luminita Stevens, Martin Uribe, Gianluca Violante, and Reed Walker for useful comments.
2.1 Introduction

You can’t change the carpenter into a nurse easily, and you can’t change the mortgage broker into a computer expert in a manufacturing plant very easily. Eventually that stuff will work itself out. . . . Monetary policy can’t retrain people. Monetary policy can’t fix those problems.

Charles Plosser, President of the Federal Reserve Bank of Philadelphia

Though the Great Recession ended in the middle of 2009, the US labor market remains weak three years later with an unemployment rate near 8%. Some have speculated that a slow recovery is inevitable as the labor force must reallocate from housing-related sectors to the rest of the economy. Proponents of this view have cited the shift in the US Beveridge curve as evidence for sectoral shocks leading to labor reallocation. The view that Beveridge curve shifts reflect sectoral disruptions and periods of increased labor reallocation was first elucidated by Abraham and Katz (1986) and Blanchard and Diamond (1989). Figure 2.1 displays unemployment and vacancies since 2000 using vacancy data from the Job Openings and Labor Turnover Survey (JOLTs). The Beveridge curve has shifted during the recovery period with the unemployment rate rising 1.5-2 percentage points at each level of vacancies. Vacancy rates in 2012 are consistent with an unemployment rate of less than 6% on the pre-recession Beveridge curve. The observed shift in the Beveridge curve has prompted disagreement on what implications, if any, this shift may have for monetary policy. Kocherlakota (2010) and Plosser (2011) suggest that, if sectoral shocks require labor reallocation and that process is costly and prolonged, then the natural rate of unemployment has risen, implying that further monetary easing would be inflationary.

We investigate the relationship between sector-specific shocks, shifts in the Beveridge curve, and changes in the natural rate of unemployment. In particular, we address three

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See Kocherlakota (2010), and Plosser (2011).

See Barnichon, Elsby, Hobijn, and Şahin (2010) for measurement of the shift in the empirical Beveridge curve using JOLTs data. Exact size of the shift depends on the definition of the vacancy rate: job openings rate used in JOLTs is $V/(N + V)$ or alternative is vacancy to labor force ratio $V/L$ (analogous to the unemployment rate).
questions: Has the US labor market experienced sector-specific disruptions? Can sectoral shocks account for the shift in the Beveridge curve? Do sectoral shocks raise the natural rate of unemployment? We build a measure of sector-specific shocks using a factor analysis of sectoral employment and augment a standard multisector model with labor market search to analyze the relationship between sector-specific shocks, the Beveridge curve, and the natural rate of unemployment.

Our first contribution is a new index of sector-specific shocks that measures the dispersion of the component of sectoral employment not explained by an aggregate employment factor. Our measure is distinct from the Lilien (1982) measure of employment dispersion and addresses the Abraham and Katz (1986) critique that asymmetric responses of sectoral employment may be attributable to differing sensitivities of sectors to aggregate shocks. We confirm that the recovery from the Great Recession is characterized by a substantial increase in sectoral shocks that matches the timing of the shift in the Beveridge curve. Moreover, we show that shifts in the US Beveridge curve in postwar data are correlated with periods in which sector-specific shocks are elevated as measured by our index.
Our second contribution is to define the Beveridge curve in a multisector model and examine its behavior in the presence of sectoral shocks. The Beveridge curve is defined as the set of unemployment and vacancy combinations traced out by changes in real marginal cost, which captures the effect of a variety of aggregate disturbances. We show that sectoral productivity or demand shocks will, in general, shift the Beveridge curve. Sectoral shocks shift the Beveridge curve through two channels: a composition effect and a mismatch effect. The former channel is operative if a sectoral shock shifts the distribution of vacancies towards a sector with greater hiring costs, thereby increasing unemployment for any given aggregate level of vacancies. The latter channel stems from decreasing returns to the matching function and costly reallocation: a sectoral shock that leaves overall vacancies unchanged raises unemployment because the reduction in vacancies in one sector increases unemployment by more than the corresponding fall in unemployment in the other sector. Our model validates our empirical strategy and verifies the hypothesized relationship between our sector-specific shock index and shifts in the Beveridge curve.

Our third contribution is to clarify the relationship between the Beveridge curve and the natural rate of unemployment. In the baseline model with exogenous sectoral productivity or demand shocks, shifts in the Beveridge curve necessarily imply a movement in the natural rate of unemployment in the same direction as the shift in the Beveridge curve. However, the converse need not hold: for example, a negative aggregate productivity shock raises the natural rate of unemployment without shifting the Beveridge curve. Changes in the natural rate affect monetary policy by changing the inflation-employment tradeoff for the central bank.

We calibrate a two-sector version of our model to data on the construction and non-construction sectors of the US labor market to quantify the effect of sectoral shocks on the Beveridge curve and the natural rate of unemployment. A sector-specific shock to construction of sufficient magnitude to match movements in construction’s employment share generates a shift in the Beveridge curve that quantitatively matches the shift observed in the US. Moreover, the shock to construction raises the natural rate of unemployment by 1.4 percentage
points - insufficient to fully explain the rise in unemployment observed in the current recession and of similar magnitude to the estimates in Sahin, Song, Topa, and Violante (2010) who examine the contribution of mismatch to overall unemployment.

Our final contribution is an extension of the model to incorporate financial frictions. In this environment, it is no longer the case that a Beveridge curve shift implies a change in the natural rate. We show that financial shocks or systematic changes in monetary policy increase mismatch in the same way as a sector-specific productivity or demand shocks. Events like a binding zero lower bound could act like a sector-specific shock, generating a shift in the Beveridge curve while not implying any change in the natural rate of unemployment. Given our analysis, we conclude that a Beveridge curve shift is not sufficient to draw any conclusions about the behavior of the natural rate of unemployment.

Our paper is organized as follows. Section 2.2 describes our method for constructing a long-run sector-specific shock index and its correlation with historic shifts in the Beveridge curve. Section 2.3 lays out our baseline model: a sticky price multisector model augmented with labor market search within sectors and costly reallocation across sectors. Analytical results establishing the relationship between sectoral shocks, labor reallocation, and the Beveridge curve along with implications for the natural rate are described in Section 2.4. Section 2.5 describes our calibration strategy and shows the effect of sectoral productivity shocks in a two-sector model. Section 2.6 extends the multisector model to incorporate financial frictions and illustrates how financial frictions and changes in the monetary policy rule can act as sectoral shocks and shift the Beveridge curve. Section 2.7 concludes.

### 2.2 Empirics on Sectoral Shocks and the Beveridge Curve

To examine the relationship between sectoral shocks and the Beveridge curve, we construct the long-run US Beveridge curve and build a summary measure of sector-specific shocks. Since vacancies data from the JOLTs survey is only available after 2000, the Conference Board’s Help-Wanted Index is frequently used as a proxy for the vacancy rate prior to 2000. Figure 2.2 displays the Beveridge curve using the Help-Wanted Index (HWI) normalized by the labor
force as a proxy for the vacancy rate⁴. Figure 2.2 shows that the historic Beveridge curve exhibits periods when the vacancy-unemployment relationship is stable and periods when it appears to shift.

Historic shifts in the US Beveridge curve are documented in Bleakley and Fuhrer (1997) and Valletta and Kuang (2010). Importantly, shifts in the Beveridge curve are not a business cycle phenomenon with some recessions accompanied by shifts but other shifts occurring during expansions - the behavior of vacancies and unemployment in the mid 1980s provides a good example. Like the Beveridge curve obtained using JOLTs data, the composite HWI Beveridge curve exhibits an upward shift since 2009.

2.2.1 Existing Measures of Sector-Specific Shocks

Lilien (1982) proposed the dispersion in sectoral employment growth as a measure for sector-specific shocks, arguing that these shocks are an important driver of the business cycle given the strong countercyclical behavior of his measure. Figure 2.3 plots the Lilien measure using

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⁴After 1996, the HWI is the composite index derived in Barnichon (2010) and updated to 2011, which adjusts for the shift away from newspaper advertising of vacancies to online advertising.
monthly sectoral employment data\textsuperscript{5}. The figure demonstrates the strongly countercyclical behavior of the series including most recent recessions that have featured a slower recovery in the labor market in comparison to past recessions. In the current recession, the Lilien measure peaks in the summer of 2009 at the recession trough.

Abraham and Katz (1986) questioned the Lilien measure by arguing that increases in the dispersion of employment growth could be attributed to differences in the elasticity of sectoral employment to aggregate shocks. As an alternative, Abraham and Katz argued that sector-specific shocks should result in periods in which vacancies and unemployment are both rising and showed that the Lilien measure does not comove positively with vacancies.

2.2.2 Constructing Sector-Specific Shock Index

To derive a measure of sector-specific shocks, we conduct a factor analysis of sectoral employment. The factor analysis addresses the Abraham and Katz critique by allowing sectoral employment to respond differently to aggregate shocks.

\textsuperscript{5}The Lilien measure is: $\sigma_t = \left( \sum_{i=1}^{K} (g_{it} - g_t)^2 \right)^{1/2}$ where $g_{it}$ is the growth rate of employment in sector $i$ and $g_t$ is the growth rate of aggregate employment.
We estimate the following approximate factor model:

\[ n_t = \epsilon_t + \lambda F_t, \]

where \( n_t \) is a \( N \times 1 \) vector of employment by sector, \( \epsilon_t \) is a \( N \times 1 \) vector of mean-zero sector-specific shocks, \( F_t \) is a \( K \times 1 \) vector of factors, and \( \lambda \) is a \( N \times K \) matrix of factor loadings.

As is standard in the approximate factor model discussed in Stock and Watson (2002), we assume that \( n_t \) and \( F_t \) are covariance stationary processes, with \( \text{Cov}(F_t, \epsilon_t) = 0 \). As shown by Stock and Watson (2002), the approximate factor model allows for serial correlation in \( F_t, \epsilon_t \), and weak cross-sectional correlation in \( \epsilon_t \) - the variance-covariance matrix of \( \epsilon_t \) need not be diagonal. The factor analysis implicitly identifies the sector-specific shock by assuming that loadings on the aggregate factor are invariant over time; that is, sectoral employment responds in a similar manner over the business cycle to aggregate fluctuations.

The sectoral residual \( \epsilon_{it} \) represents the sector-specific shock, and we construct an index to examine the time variation in sector-specific shocks by measuring cross-sectional dispersion, squaring the sectoral employment residuals from our factor analysis:

\[ S_{t}^{\text{dis}} = \frac{1}{K} \left( \sum_{i=1}^{K} \epsilon_{it}^2 \right)^{1/2}. \]

Given that variances are normalized to unity before estimating, the sector specific shocks need not be weighted by their employment shares. We also construct an alternative measure of employment dispersion as the sum of the absolute values of the residuals from our factor analysis:

\[ S_{t}^{\text{abs}} = \frac{1}{K} \sum_{i=1}^{K} |\epsilon_{it}|. \]

This measure of sector-specific shocks is always positive and weights all sectors equally.
2.2.3 Data

To estimate the sectoral shock index, we use long-run US data on sectoral employment. These data are available for the US from January 1950 to July 2012 on a monthly basis for 14 sectors that represent the first level of disaggregation for US employment data. Due to its relatively small share of employment, we drop the mining and natural resources sector. The sectoral data is taken from the Bureau of Labor Statistics establishment survey. While, in principle, we could use sectoral data on variables like real output, relative prices, or relative wages, employment data offers the longest available history at the highest frequency and is presumably measured with the least error. The principal concern with this data set is the small number of cross-sectional observations relative to the number of observations in the time dimension. While traditional factor analyses draw on highly disaggregated price, output, or employment data, these series are not available before the 1970s. Given our aim of investigating shifts in the Beveridge curve and the relative infrequency of these events, we try to construct the longest possible series for sector-specific shocks.

The log of monthly sectoral employment is detrended to obtain a mean-zero stationary series and the variance of each series is normalized to unity. This normalization ensures that no series has a disproportionate effect on the estimation of the national factor.

We detrend employment in each sector by means of a cubic deterministic trend. The underlying trend in sectoral employment differs substantially among sectors, and employment shares are nonstationary over the postwar period. For example, manufacturing employment falls as a share of total employment over the whole period, but even decreases in absolute terms starting in the 1980s. Sectors, such as construction and information services show a general upward trend in levels characterized by very large and long swings in employment that are longer than simple business cycle variation. Higher-order deterministic trends fit certain sectors much better than a simple linear or quadratic trend. Moreover, most of the sectoral employment series obtained by removing a linear or quadratic trend fail a Dickey-Fuller test at standard confidence levels. For robustness, as will be shown in the next section, we also consider detrending by first-differences, computing quarter-over-quarter or year-over-
year growth rates for each sector, normalizing variances, and then estimating the factor model.

Given that our full sample from 1950-2012 has a small number of cross-section observations relative to the time dimension, we also estimate the same model using a larger cross-section of 85 sectoral employment series at the 2-digit NAICS level available monthly since 1990. We find the same pattern for the shock index as in our larger sample.

2.2.4 Sectoral Shock Index and Shifts in the Beveridge Curve

The sector-specific shock index shown in Figure 2.4 displays several notable features. First, the shock index rises rapidly in late 2009. The rise in the shock index occurs at the beginning of the recovery, not at the beginning of the recession, matching the timing of the shift in the Beveridge curve. Second, the sector-specific shock index is not a business cycle measure. Its correlation with various monthly measures of the business cycle is highlighted in Table 2.1, with all correlations below 0.15. Third, the sectoral shock index displays a low and negative

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6The rise in the index in the recovery period after the Great Recession is also consistent with the elevated dispersion in labor market conditions highlighted by Barnichon and Figura (2011) and sectoral dispersion measures computed by Rissman (2009).
correlation with the Lilien measure. Finally, the average level of the shock index is higher in the Great Moderation period as shown by the gray line in Figure 2.4. This behavior is consistent with the behavior of sectoral employment documented in Garin, Pries, and Sims (2010).

Just as the current shift in the Beveridge curve coincides with a rise in the sector-specific shock index, historic shifts in the Beveridge curve are also correlated with elevated levels of sector-specific shocks. We illustrate this correlation between shifts in the Beveridge curve and the sector-specific shock index by plotting the shock index against the intercept of a 5-year rolling regression of vacancies on unemployment (five-year trailing window). Absent any shifts in the Beveridge curve, the intercept should be constant. Therefore, variation in the intercept series captures movements in the Beveridge curve. Figure 2.5 shows a clear correlation between movements in the intercept of the Beveridge curve and the sector-specific shock index. This correlation in monthly data calculated from 1956-2012 is 0.363 and is shown in the last column in Table 2.1. This result is robust to the use of a 4th order trend, though somewhat weaker. Our evidence provides support for the mechanism described by Abraham and Katz where sector-specific shocks generate a shift the Beveridge curve.

To examine the robustness of this correlation, we also estimate the Beveridge curve aug-

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Table 2.1: Correlation of shock index with business cycle measures

<table>
<thead>
<tr>
<th>Detrend with time trend</th>
<th>Business Cycle Measures</th>
<th>Lilien measure</th>
<th>Beveridge Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Industrial production growth</td>
<td>Employment growth</td>
<td>Unemployment rate</td>
</tr>
<tr>
<td>Cubic</td>
<td>-0.02</td>
<td>0.12</td>
<td>0.00</td>
</tr>
<tr>
<td>Quartic</td>
<td>-0.05</td>
<td>0.08</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Detrend with growth rates</th>
<th>Business Cycle Measures</th>
<th>Lilien measure</th>
<th>Beveridge Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter over quarter</td>
<td>-0.17</td>
<td>-0.12</td>
<td>-0.07</td>
</tr>
<tr>
<td>Year over year</td>
<td>-0.32</td>
<td>-0.36</td>
<td>0.09</td>
</tr>
</tbody>
</table>

---

Footnote: For the index obtained using growth rates, the correlation with business cycle measures and the Lilien measure is markedly higher than the time trend specifications. This correlation is driven by the behavior of the index in the first half of the sample. The correlation of the sectoral shock index with the Lilien measure drops to 0.18 from 0.56 in the Great Moderation period.
mented with our sector-specific shock index:

\[ v_t = c + \beta(L) u_t + \gamma(L) S_t + \eta_t \]

where \( v_t \) is log vacancies, \( u_t \) is log unemployment, \( \beta(L) \) and \( \gamma(L) \) are lag polynomials, \( c \) is a constant, and \( \eta_t \) is a mean zero error term\(^8\). The Beveridge curve is estimated with four lags of unemployment to control for the persistence of both vacancies and unemployment and with Newey-West standard errors (4 lags) to account for serial correlation in \( \eta_t \). We consider several variants of our sector-specific shock index using both the dispersion measure (Panel A) and the absolute-value measure (Panel B). Employment is detrended with either time trends and growth rate trends. Given the persistence exhibited by the sector-specific shock indices obtained from time detrending, we estimate specifications both with and without an additional lag of the shock index.

Figure 2.5: Correlation of Beveridge curve shifts and sector-specific shocks

![Figure 2.5: Correlation of Beveridge curve shifts and sector-specific shocks](image)

Table 2.2 displays the estimates for the coefficient \( \gamma \) on the sector-specific shock index. This coefficient enters significantly for most of the time trend specifications we consider. Our

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\(^8\)An earlier version of this paper estimates the Beveridge curve using vacancies and unemployment rates in levels. Given the nonlinear nature of the Beveridge curve, the log specification is preferred. However, the use of log or levels does not greatly affect the estimation.
baseline cubic detrending is highlighted in bold in the table with positive and statistically significant coefficients in all cases. The shock index based on growth rate detrending delivers a significant negative coefficient in the case of the year-over-year specification. While our reduced form model makes no prediction about the sign of the coefficient $\gamma$, we show in section 2.4.3 that our model-implied measure of Beveridge curve shifts delivers coefficients that are consistent in sign across all specifications. We defer further discussion until then.

<table>
<thead>
<tr>
<th>Panel A: Dispersion Index</th>
<th>Panel B: Absolute Value Index</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Detrend with time trend</strong></td>
<td><strong>Detrend with time trend</strong></td>
</tr>
<tr>
<td>Cubic**</td>
<td>Cubic**</td>
</tr>
<tr>
<td>Cubic w/1 lag**</td>
<td>Cubic w/1 lag**</td>
</tr>
<tr>
<td>Quartic**</td>
<td>Quartic</td>
</tr>
<tr>
<td>Quartic w/1 lag**</td>
<td>Quartic w/1 lag</td>
</tr>
<tr>
<td>Coeff</td>
<td>t-stat</td>
</tr>
<tr>
<td>0.82</td>
<td>2.64</td>
</tr>
<tr>
<td>0.80</td>
<td>2.58</td>
</tr>
<tr>
<td>0.64</td>
<td>2.11</td>
</tr>
<tr>
<td>0.62</td>
<td>2.07</td>
</tr>
<tr>
<td><strong>Detrend with growth rates</strong></td>
<td><strong>Detrend with growth rates</strong></td>
</tr>
<tr>
<td>Quarter over quarter</td>
<td>Quarter over quarter</td>
</tr>
<tr>
<td>Year over year**</td>
<td>Year over year**</td>
</tr>
<tr>
<td>-0.08</td>
<td>-0.45</td>
</tr>
<tr>
<td>-1.36</td>
<td>-4.18</td>
</tr>
</tbody>
</table>

Tables indicate coefficients of sectoral shock index in Beveridge curve estimation under alternative detrending procedures. W/1 lag specification includes lagged value of index with coefficient expressed as sum of contemporaneous and lagged effect of sectoral shock index on vacancies. ** indicates significance at 5% level. The number of time series observations is $T = 726$.

### 2.3 Multisector Model with Labor Reallocation

We augment a multisector sticky-price model as in Aoki (2001) or Carvalho and Lee (2011) with search and matching frictions in the labor market as in Shimer (2010).

There model features four types of agents: continuum of identical households, intermediate good producers, wholesale firms and retailers. The households hold preferences over consumption and leisure, they trade state-contingent assets, own all firms, and provide different types of workers to intermediate goods producers through a frictional labor market.

The intermediate goods producers are competitive and hire labor to produce an intermediate good. Each intermediate firm operates in one of several sectors. A firm working in
a particular sector hires sector-specific workers and produces a sector-specific intermediate good. Their production is subject to sector-specific productivity shocks. The intermediate goods producers sell their output to the wholesale firms.

Wholesale firms are competitive and combine sector-specific intermediate goods into a homogenous final good. Their production is subject to sector-specific demand shocks, which affects the relative demand of the wholesale firms for different intermediate goods. The wholesale firms sell their final goods to retailers.

Retailers are monopolistically competitive. A retailer buys final goods from the wholesale firms, costlessly differentiates these goods, and sells their good to a household exploiting their market power to set prices in excess of marginal cost. In a sticky price version of the model, we assume that prices for these differentiated goods are updated à la Calvo.

The labor market is sector-specific and subject to search and matching frictions. Each sector has a pool of unemployed workers who search for jobs in this sector. Intermediate goods producers from this sector search for workers in the unemployment pool of this sector. The households can reallocate its unemployed workers among different sectors. Sectors may, in principle, conform to geographies, industries, occupations or other dimensions of worker heterogeneity. Households reallocate their workers across sectors subject to a utility cost of changing the distribution of the labor force.

Next we provide a detailed description of the agents problems followed by analysis of wage setting and the definition of equilibrium.

2.3.1 Households

Households supply labor across $K$ distinct sectors and invest in a full-set of state-contingent securities. While hiring in each sector is subject to search frictions, the household is free to reallocate workers across sectors subject to a utility cost of changing the distribution of labor. This utility cost captures costs associated with worker retraining, relocation, or the loss of industry-specific skills. With costly reallocation, the household’s problem differs from the standard labor market search model since the household has an active margin of adjustment
by reallocating the pool of available workers across sectors. As a result, the initial distribution of the labor force is a state variable for the household in addition to the last period distribution of employment.

Household behavior can be expressed by the following optimization problem:

\[
\max_{\{C_t, B_{t+1}, L_t, N_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t, N_t) - \sum_{i=1}^{K} R(L_{i,t-1}, L_{i,t}) \right\},
\]

\[
\text{s.t.} \quad P_tC_t = \sum_{i=1}^{K} W_{i,t} N_{i,t} + B_t - E_t Q_{t,t+1} B_{t+1} - T_t
\]

\[
\quad + \Pi^{f}_t + \sum_{i=1}^{K} \Pi^{Int}_{i,t} + \int_{l=0}^{1} \Pi^{ret}_{i}(l) dl,
\]

\[
N_{i,t} = (1 - \delta_i) N_{i,t-1} + p_{i,t} U_{i,t},
\]

\[
L_{i,t} = N_{i,t-1} + U_{i,t},
\]

\[
\sum_{i=1}^{K} L_{i,t} = 1,
\]

\[
(2.1)
\]

\[
(2.2)
\]

\[
(2.3)
\]

\[
(2.4)
\]

\[
(2.5)
\]

where \(N_t = \{N_{i,t}\}_{i=1}^{K}\) and \(L_t = \{L_{i,t}\}_{i=1}^{K}\) for \(t \geq 0\) are \(K \times 1\) vectors of sectoral employment and the sectoral distribution of the labor force respectively,

\[
N_t = \sum_{i=1}^{K} N_{i,t},
\]

\[
(2.6)
\]

and \(C_t\) is an index of the household’s consumption of the differentiated goods. The initial conditions for this problem are \(\{B_0, N_{-1}, L_{-1}\}\). The household maximizes utility net of reallocation costs subject to a standard budget constraint \((2.2)\), where \(P_t\) is an index of the prices of the differentiated goods, \(W_{i,t}\) is the nominal wage that workers receive when working in sector \(i\), \(\Pi^{f}_t, \Pi^{Int}_{i,t}, \Pi^{ret}_{i}(l)\) represent wholesale, intermediate and retailer firms nominal profits distributed to households, \(B_t\) are nominal payments from state contingent securities and \(Q_{t,t+1}\) is an asset-pricing kernel\(^9\). For each sector, sectoral employment \(N_{i,t}\) evolves by

\[^9\text{The existence and uniqueness of the asset-pricing kernel is guaranteed by the absence of arbitrage opportunities in equilibrium.}\]
a law of motion (2.3) where $p_{i,t}$ is the job-finding rate in sector $i$ and $\delta_i$ is a sector-specific separation rate. Sectoral unemployment is the difference between the labor force allocated in that sector $L_{i,t}$ and last period sectoral employment (2.4). The total labor force of the household is normalized to unity (2.5). The household takes the sectoral job-finding rate and profits from firms as exogenous.

Following Dixit and Stiglitz (1977), we assume that the index $C_t$ is a constant-elasticity-of-consumption aggregator

$$C_t = \left[ \int_0^1 C_t(l)^{(\zeta-1)/\zeta} dl \right]^{\zeta/(\zeta-1)}$$

with $\zeta > 1$, and $P_t$ is the corresponding price index

$$P_t = \left[ \int_0^1 P_t(l)^{1-\zeta} dl \right]^{1/(1-\zeta)}.$$

We assume that the cost of reallocation of a worker from sector $i$ to sector $j \neq i$ depends on the current and past labor force in sector $i$ and on the current and the past employment in sector $j$. The function $R(\cdot, \cdot)$ is assumed to be continuous and differentiable in its arguments and minimized when $L_{i,t-1} = L_{i,t}$ for any sector $i$.

The optimal choice of assets purchases and consumption implies the following relation determining the nominal one-period interest rate:

$$1 + i_t = \beta^{-1} \left\{ \mathbb{E}_t \left[ u_c(C_{t+1}, N_{t+1}) \frac{P_t}{P_{t+1}} \right] \right\}^{-1}. \tag{2.7}$$

See Appendix B.1.1 for a detailed discussion of the household optimality conditions.

Optimal choice of the allocation of the labor force across sectors implies

$$p_{i,t} \lambda_{2,t,i} = \lambda_{3,t} + R_2(L_{i,t-1}, L_{i,t}) + \beta \mathbb{E}_t R_1(L_{i,t}, L_{i,t+1}), \tag{2.8}$$

where $\lambda_{2,t,i}$ is a Lagrange multiplier on constraint (2.3), $\lambda_{3,t}$ is a Lagrange multiplier on constraint (2.5). $\lambda_{2,t,i}$ represents the utility value of an additional employed worker in sector...
for the household given the equilibrium path for wages $\{W_{i,t}\}_{t=0}^{\infty}$, while $\lambda_{3,t}$ represents the utility value of an increase in the labor force by one worker for the household. This first order condition states that the household equalizes the costs and benefits when choosing to allocate an additional worker to sector $i$. The left-hand side represents the utility benefit of an additional worker employed $\lambda_{2,t,i}$ weighted by the probability of finding a job $p_{i,t}$. The right-hand side is the cost of an additional worker in sector $i$, which is the sum of the shadow value of a person for the household $\lambda_{3,t}$ plus the adjustment costs of the labor force in sector $i$: $R_2(L_{i,t-1}, L_{i,t})$ gives the immediate costs of adjustment while the term $\beta \mathbb{E}_t R_1(L_{i,t}, L_{i,t+1})$ takes into account the affect on future adjustment costs.

Optimality with respect to $N_{i,t}$ gives a recursive formula for $\lambda_{2,t,i}$

$$
\lambda_{2,t,i} = u_N(C_t, N_t) + \frac{W_{i,t}}{P_t} u_c(C_t, N_t) + \beta \mathbb{E}_t [(1 - \delta_i - p_{i,t+1}) \lambda_{2,t+1,i}] + u_i(C_t, N_t) p_{i,t} 
$$

(2.9)

This expression states that the value of an additional employed worker equals the sum of the disutility from working $u_n(C_t, N_t)$, the utility value of the nominal wage $W_{i,t}$, and the expected discounted value from having this worker employed in the next period weighted by the probability of retaining a job $\beta \mathbb{E}_t [(1 - \delta_i) \lambda_{2,t+1,i}]$ less the expected discounted value that the worker could be worth next period if he was not employed in the current period $\beta \mathbb{E}_t [p_{i,t} \lambda_{2,t+1,i}]$.

It will prove useful to introduce a variable closely related to $\lambda_{2,t,i}$ that will be used to determine workers wages. Let $J_{i,t}(\tilde{W})$ denote the marginal utility for a household at the equilibrium level of employment of having one additional worker employed at a wage $\tilde{W}$ in period $t$ rather than unemployed and with the wage returning to an equilibrium sequence from the next period for this worker. We can express this new variable as follows:

$$
J_{i,t}(\tilde{W}) = \lambda_{2,t,i} + \frac{u_c(C_t, N_t)}{P_t} (\tilde{W} - W_{i,t})
$$

This expression states that the value of an additional worker employed at wage $\tilde{W}$ equals the value of a worker employed at the equilibrium wage, the first term, plus a gain from receiving
wage $\tilde{W}$ rather than $W_{i,t}$ expressed in units of marginal utility, the second term.

Two extreme cases for labor reallocation will prove useful in our analysis and are defined here.

**Definition 2.1.**

- **Costless reallocation:** $R(L_{i,t-1}, L_{i,t}) = 0$ for all $L_{i,t-1}, L_{i,t} \geq 0$, $i = 1, 2, \ldots, K$ and $t \geq 0$.
- **No labor reallocation:** $R(L_{i,t-1}, L_{i,t}) = \infty$ for any $L_{i,t-1} \neq L_{i,t} \geq 0$, $i = 1, 2, \ldots, K$ and $t \geq 0$.

If reallocation is costless, then the right-hand side of equation (2.8) is always equalized across sectors to $\lambda_{3,t}$. Alternatively, if there is no reallocation the labor force is fixed across sectors and equation (2.8) becomes redundant.

Also, for reference in later sections, we define the case of no wealth effects on labor supply.

**Definition 2.2.** Let

$$-u_n(C_t, N_t)/u_c(C_t, N_t) = f(N_t)$$

for some function $f$. That is, the marginal rate of substitution does not depend on consumption $C_t$. Then, labor supply does not exhibit wealth effects$^{10}$.

### 2.3.2 Retailers

The consumption goods are sold to households by a set of monopolistically competitive retailers who can costlessly differentiate the single final good purchased from wholesale firms. These retailers periodically set prices à la Calvo at a markup to marginal cost, which is the real cost of the final good $P_{ft}/P_t$, where $P_{ft}$ is the nominal price of the final good. The

---

$^{10}$The standard search and matching model, see, for example, Mortensen and Pissarides (1994), assumes neither wealth effects nor any variable disutility of labor supply. This conforms to the case of $f(N) = z$ for some constant reservation wage $z$. 
retailers problem is standard to any New Keynesian model:

\[
\max_{P_t(l)} \Pi^{ret}_t(l) = \mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} [P_t(l) - P_{fT}] Y_T(l),
\]

s.t. \( Y_T(l) = Y_T \left( \frac{P_t(l)}{P_T} \right)^{-\zeta} \),

where \( P_t(l) \) is the nominal price chosen by a retailer that sells differentiated good \( l \) and who faces a downward sloping demand schedule and discounts future profits by the nominal stochastic discount factor \( Q_{t,T} \). Parameter \( \chi \) is the Calvo parameter governing the degree of price stickiness. The optimality condition for price-setting is given by:

\[
\mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} P_T^{*} Y_T \left( P_T^{*} - \frac{\zeta}{\zeta - 1} P_{fT} \right) = 0,
\]

Which implies

\[
\frac{P_T^{*}(l)}{P_t} = \frac{K_t}{F_t}, \tag{2.10}
\]

where

\[
K_t = \frac{\zeta}{\zeta - 1} \mathbb{E}_t \sum_{T=t}^{\infty} u_c(C_t, N_t)(\beta \chi)^{T-t} P_{fT} \left( \frac{P_T}{P_t} \right)^{\zeta-1} Y_T,
\]

\[
F_t = \mathbb{E}_t \sum_{T=t}^{\infty} u_c(C_t, N_t)(\beta \chi)^{T-t} \left( \frac{P_T}{P_t} \right)^{\zeta-1} Y_T.
\]

The last two relations can be expressed in recursive form:

\[
K_t = \frac{\zeta}{\zeta - 1} u_c(C_t, N_t) \frac{P_{fT}}{P_t} Y_t + \beta \chi \mathbb{E}_t \Pi_{t+1}^{\zeta} K_{t+1}, \tag{2.11}
\]

\[
F_t = u_c(C_t, N_t) Y_t + \beta \chi \mathbb{E}_t \Pi_{t+1}^{\zeta-1} F_{t+1}. \tag{2.12}
\]

where \( \Pi_t \equiv \frac{P_t}{P_{t-1}} \). The inflation rate is derived from the Calvo assumption with a fraction \( 1 - \chi \) of firms resetting their prices to \( P_t(l) / P_t \):

\[
P_t = \left\{ \chi P_{t-1}^{1-\zeta} + (1 - \chi) (P_T^{*})^{1-\zeta} \right\}^{\frac{1}{1 - \chi}}.
\]
The last equation implies:
\[
\frac{1 - \chi \Pi_{t}^{\zeta-1}}{1 - \chi} = \left( \frac{K_{t}}{F_{t}} \right)^{1-\chi}
\]  
(2.13)

At the zero inflation steady state, a log-linearization of these equilibrium conditions delivers the standard New Keynesian Phillips curve.

### 2.3.3 Wholesale Firms

The final good purchased by retailers is sold by wholesale firms who purchase an intermediate output good produced by firms in each sector. We assume a finite set of sectors that produce an intermediate good that is transformed into the final good using a constant elasticity of substitution aggregator:

\[
\Pi_{t}^{f} = \max_{Y_{i,t}} P_{ft} Y_{i,t} - \sum_{i=1}^{K} P_{i,t} Y_{i,t},
\]

s.t.: \( Y_{t} = \left( \sum_{i=1}^{K} \phi_{i,t} Y_{i,t}^{\eta-1} \right)^{1/(\eta-1)}, \)

(2.14)

where \( \phi_{i,t} \) represents a relative preference shock (or relative demand shock) and \( \eta \) is the elasticity of substitution among intermediate goods. Optimization by final good firms provides demand functions for each intermediate good:

\[
Y_{i,t} = \phi_{i,t} Y_{t} \left( \frac{P_{i,t}}{P_{ft}} \right)^{-\eta}. \forall i = 1, 2, \ldots, K
\]  
(2.15)

For \( \eta = 1 \), the CES aggregator is Cobb-Douglas and intermediate goods are neither complements nor substitutes. If \( \eta < 1 \), intermediate goods are complements, while if \( \eta > 1 \), intermediate goods are substitutes. The aggregate price index for the final good can be expressed as follows

\[
\frac{P_{ft}}{P_{t}} = \left\{ \sum_{i=1}^{K} \phi_{i,t} \left( \frac{P_{i,t}}{P_{ft}} \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}}
\]  
(2.16)
2.3.4 Intermediate Good Firms

Intermediate goods are produced by competitive firms in each sector who hire labor and post vacancies subject to a linear production function and a law of motion for firm employment. The production function has linear form:

\[ Y_{i,t} = A_{i,t} N_{i,t}, \quad (2.17) \]

where \( A_{i,t} \) is sector-specific productivity. Firms in each sector take sectoral productivity shocks, wages, separation rates, and a job-filling rate as given. The firm solves the following problem:

\[ \Pi_{i,t}^{\text{Int}} = \max_{\{V_{i,T} N_{i,T}\}_{T=t}^{\infty}} \mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \left[ P_{i,T} A_{i,t} N_{i,t} - W_{i,T} N_{i,T} - \kappa V_{i,T} P_T \right], \quad (2.18) \]

s.t.:

\[ N_{i,t} = (1 - \delta_i) N_{i,t-1} + q_{i,t} V_{i,t}. \quad (2.19) \]

where \( q_{i,t} \) is the vacancy yield or job-filling rate. Optimal choice of vacancies is determined as follows:

\[ q_{i,t} \lambda_{4,t,i} = \kappa, \quad (2.20) \]

where \( \lambda_{4,t,i} \) is the Lagrange multiplier on (2.19) expressed in real terms; this multiplier can be interpreted as the value of an additional hired worker in period \( t \) at the equilibrium wage. This condition states that the cost of posting a vacancy, the right-hand side, equals the value that it brings if the firm meets a worker with probability \( q_{i,t} \). Optimality with to employment leads to:

\[ \lambda_{4,t,i} = \frac{P_{i,t}}{P_t} A_{i,t} - \frac{W_{i,t}}{P_t} + E_t Q_{t,t+1} (1 - \delta_i) \lambda_{4,t+1,i} \quad (2.21) \]

where \( Q_{t,t+1} \) is the stochastic discount factor of the representative household between period \( t \) and \( t + 1 \). The condition states that the value of an additional employed worker equals the revenue this worker brings net of wage costs plus the future value of the worker tomorrow conditional on not separating.
It will prove useful to introduce a variable closely related to \( \lambda_{4,t,i} \) that determines workers’ wages. Let \( J_{i,t}^{Int}(\bar{W}) \) be the value for an intermediate goods firm at equilibrium employment levels of having one additional worker employed at a wage \( \bar{W} \) in period \( t \) and with the wage returning to an equilibrium sequence from the next period for this worker. We can express the new variable as follows:

\[
J_{i,t}^{Int}(\bar{W}) = \lambda_{4,t,i} - \frac{(\bar{W} - W_{i,t})}{P_t}.
\]

This expression states that the value of an additional worker employed at wage \( \bar{W} \) equals the value of a worker employed at the equilibrium wage net of a gain from paying wage \( \bar{W} \) rather than \( W_{i,t} \).

### 2.3.5 Labor Market and Wages Determination

Hiring is mediated by a sectoral matching function that depends on the level of vacancies and unemployment in each sector. We allow sectoral matching functions to differ in matching function productivity, but require the matching function to display constant returns to scale and share a common matching function elasticity \( \alpha \). The job-filling rate can be defined as follows:

\[
q_{it} \equiv \frac{H_{it}}{V_{it}} = \varphi_i \left( \frac{V_{it}}{U_{it}} \right)^{-\alpha} \tag{2.22}
\]

The job-finding probability is taken as exogenous by the household and is determined in equilibrium by the sectoral matching function and the level of vacancies and unemployed persons in each sector:

\[
p_{it} \equiv \frac{H_{it}}{U_{it}} = \varphi_i \left( \frac{V_{it}}{U_{it}} \right)^{1-\alpha} \tag{2.23}
\]

Wages are determined via Nash bargaining in each sector. Assuming that there are gains from trade, i.e., \( J_{i,t}^{Int}, J_{i,t} \geq 0 \), the bargained wage solves the following problem

\[
\max_{\bar{W}} \left[ J_{i,t}^{Int}(\bar{W}) \right]^{1-\nu} \left[ J_{i,t}(\bar{W}) \right]^\nu
\]
Nash-bargaining implies that the sectoral wage satisfies the following condition:

$$
\nu J_{i,t}^\text{Int} \left( \bar{W} \right) = (1 - \nu) \frac{J_{i,t} \left( \bar{W} \right)}{u_c(C_t, N_t)}.
$$

In equilibrium it will be true that $$\bar{W} = W_{i,t}$$ which implies that $$J_{i,t}^\text{Int} \left( \bar{W} \right) = \lambda_{4,t,i}$$ and $$J_{i,t} \left( \bar{W} \right) = \lambda_{2,t,i}$$. Hence,

$$
\nu \lambda_{4,t,i} = (1 - \nu) \frac{\lambda_{2,t,i}}{u_c(C_t, N_t)}.
$$

(2.24)

2.3.6 Shocks

Our model features both aggregate and sector-specific shocks. We consider two types of sector-specific shocks: sectoral productivity shocks $$A_{i,t}$$ and sectoral preferences (or demand) shocks $$\phi_{i,t}$$. Fluctuations in government purchases $$G_t$$ provide an aggregate demand shock, though, as we will show, other types of demand shocks like preference shocks or monetary shocks could be considered without affecting our conclusions. A uniform change in $$\{A_{i,t}\}_{i=1}^K$$ can be an example of aggregate productivity shock.

Since our model features a finite number of sectors, it is necessary to account for the aggregate component of variation in $$A_{i,t}$$ and $$\phi_{i,t}$$. In the absence of productivity shocks and assuming a uniform level of productivity, i.e., $$A_{i,t} = A_{h,t} = A_t$$ for $$i, h = 1, 2, \ldots, K$$, the only sector-specific shock is the product share $$\phi_{i,t}$$ in the CES aggregator. Naturally, a sector-specific shock is any change in the distribution of $$\phi_{i,t}$$ subject to the restriction that $$\sum_{i=1}^K \phi_{i,t} = 1$$. However, given that sectors have nonzero mass, an increase in sectoral productivity will have aggregate effects if not offset by declines in sectoral productivity elsewhere. Moreover, the size of the offsetting shock depends on the degree of substitutability for goods across sectors. For example, if goods are perfect complements and productivity is initially equalized across sectors, a negative shock to one sector shifts in the production possibilities frontier of the economy even if offset by a corresponding positive shock to another sector. We address this issue by redefining aggregate productivity and sectoral shocks as follows:

Definition 2.3.
1. **aggregate productivity** is given by $A_t \equiv \left\{ \sum_{i=1}^{K} \phi_i,t A_{i,t}^{\eta-1} \right\}^{\frac{1}{\eta-1}}$.

2. **sector-specific productivity** is given by $\bar{A}_{i,t} \equiv A_{i,t}/A_t$.

3. **sector-specific demand** is given by $\bar{\phi}_{i,t} \equiv \phi_i,t \bar{A}_{i,t}^{\eta-1}$.

Note that $\bar{A}_{i,t}$ and $\bar{\phi}_{i,t}$ are functions of the underlying sectoral shocks $A_{i,t}$ and $\phi_{i,t}$. Also note that $A_t, \{\bar{A}_{i,t}\}_{i=1}^{K}, \{\bar{\phi}_{i,t}\}_{i=1}^{K}$ represent only $2K - 1$ independent variables because of the restrictions $\sum_{i=1}^{K} \bar{\phi}_{i,t} = 1$ and $\sum_{i=1}^{K} \bar{\phi}_{i,t}/\bar{A}_{i,t}^{\eta-1} = 1$. Let these independent variables be $\{A_t, \{\bar{A}_{i,t}\}_{i=1}^{K-1}, \{\bar{\phi}_{i,t}\}_{i=1}^{K-1}\}$, where we removed $\bar{A}_{K,t}$ and $\bar{\phi}_{K,t}$.

This definition of aggregate and sector-specific shocks is motivated by a simple decomposition of the CES aggregator where output can be expressed in terms of aggregate productivity, aggregate employment, and a misallocation term that reflects the deviation from the equilibrium allocation in the absence of the labor market frictions\textsuperscript{11}. Formally (omitting time subscripts),

$$Y = \left\{ \sum_{i=1}^{K} \phi_i A_i^{\eta-1} \right\}^{\frac{\eta}{\eta-1}} = \left\{ \sum_{i=1}^{K} \phi_i \left( A_i N_i \right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} = AN \left\{ \sum_{i=1}^{K} \phi_i \left( \frac{A_i}{A} \right)^{\eta-1} \frac{N_i}{N} \right\}^{\frac{\eta}{\eta-1}} \leq AN,$$

where the last inequality follows from the fact that both $\phi_i (A_i/A)^{\eta-1}$ and $N_i/N$ must sum to one\textsuperscript{12}. When the distribution of productivity is uniform, a sector-specific preference shock satisfies the typical CES condition that product shares sum to one.

---

\textsuperscript{11}In the absence of sectoral reallocation costs and a search-and-matching friction the following condition $\phi_{i,t} A_{i,t}^{\eta-1} = \text{const} \cdot N_{i,t}$ for $i = 1, 2, \ldots, K$ holds in equilibrium. This implies that the output of the wholesale firms can be expressed as $Y_t = A_t N_t$.

\textsuperscript{12}The fact that $\phi_i (A_i/A)^{\eta-1}$ and $N_i/N$ sum to one follows directly from the definition of $A_t$ and $N_t$. The inequality formally follows from the application of Holder’s inequality: $\sum_{i=1}^{K} x_i^{1/p} y_i^{1/q} \leq \left( \sum_{i=1}^{K} x_i \right)^{1/p} \left( \sum_{i=1}^{K} y_i \right)^{1/q}$ with $x_i, y_i \geq 0$ and $1/p + 1/q = 1$, see, for example, Kolmogorov and Fomin (1970). In our case $x_i = \phi_i (A_i/A)^{\eta-1}, y_i = N_i/N, p = \eta, q = \eta/\eta - 1$. 


2.3.7 Government Sector

We assume that the central bank can control riskless short-term nominal interest rate \( i_t \) and the zero lower bound on \( i_t \) never binds. The central bank follows the variant of Taylor rule

\[
\log \left( 1 + \frac{i_t^d}{i_t} \right) = \log \left( 1 + i_d \right) + \phi_x \log(\Pi_t) + \phi_y \log \left( \frac{Y_t}{Y} \right)
\]

(2.25)

The fiscal authority chooses a sequence of government purchases \( G_t \). We assume that the fiscal authority insures intertemporal government solvency regardless of the monetary policy chosen by the central bank.

2.3.8 Equilibrium

A competitive equilibrium is a set of aggregate allocations \( \{Y_t, N_t, C_t, K_t, F_t, \lambda_{3,t}\}_{t=0}^{\infty} \), sectoral allocations \( \{\{Y_{i,t}, N_{i,t}, U_{i,t}, V_{i,t}, L_{i,t}, \lambda_{2,t,i}, \lambda_{4,t,i}\}_{i=1}^{K}\}_{t=0}^{\infty} \), sectoral prices \( \{\{W_{i,t}/P_t, P_{t,t}/P_t\}_{i=1}^{K}\}_{t=0}^{\infty} \) and aggregate prices \( \{P_{f,t}/P_t, i_t^d, \Pi_t\}_{t=0}^{\infty} \), job-finding and job-filling rates \( \{p_{it}, q_{it}\}_{i=1}^{K}\}_{t=0}^{\infty} \), initial values of sectoral employment, unemployment, and the labor force \( \{N_{i,-1}, U_{i,-1}, L_{i,-1}\}_{i=1}^{K}\), exogenous processes \( \{G_t, A_t, \{\tilde{A}_{i,t}, \tilde{\phi}_{i,t}\}_{i=1}^{K-1}\}_{t=0}^{\infty} \) that jointly satisfy:

1. (2.3) - (2.9) (household optimization)
2. (2.11) - (2.13) (retailers optimization and inflation dynamics equation),
3. (2.14), (2.15) (wholesale firms optimization),
4. (2.17), (2.20), (2.21) (intermediate goods firms optimization),
5. (2.22), (2.23) (job-filling and job-finding rates),
6. (2.24) (wages are determined by Nash bargaining),

---

13See Woodford (2003) for the analysis of monetary policy in the absence of the demand for central bank liabilities.

14See Eggertsson and Woodford (2003) for the analysis of the consequences of the binding zero lower bound constraint on short-term nominal interest rate.
7. (2.25) (monetary policy rule),

8. \( Y_t = C_t + \sum_{i=1}^{K} \kappa V_{i,t} + G_t \) (goods-market clearing),

### 2.4 Sectoral Shocks, the Beveridge Curve and Unemployment Rate

In this section, we characterize the Beveridge curve in a multisector model and provide analytical results relating sectoral shocks, the Beveridge curve, and the natural rate of unemployment.

#### 2.4.1 Preliminaries

The definition of equilibrium implies that the economy is characterized by \( 11K + 9 \) endogenous variables with \( 11K + 9 \) equilibrium conditions, \( 2K + 1 \) exogenous shocks and \( K \) initial values for \( \{N_{i-1}\}_{i=1}^{K} \). The aggregate productivity shock is derived from the sectoral shocks using Definition 2.3.

Substituting the relation determining Nash wages (2.24) into the dynamic equation for the household value of an additional worker (2.9), we can express the wage in terms of the job-filling rate and job-finding rates in each sector:

\[
W_{i,t} = \frac{u_n(C_t, N_t)}{u_c(C_t, N_t)} + \frac{\nu}{1 - \nu} \kappa \left[ \frac{1}{q_{i,t}} - E_t Q_{t,t+1} (1 - \delta_t - p_{t+1}) \frac{1}{q_{t+1}} \right] 
\]

(2.26)

While the optimality condition for worker reallocation (2.8) may appear cumbersome, the costless reallocation limit is instructive. When reallocation is costless or in the nonstochastic steady state, the right hand side of the reallocation condition is equalized across sectors and household surpluses are equalized for all sectors. In particular, this condition implies the Jackman-Roper condition that labor market tightness must be equalized across sectors\(^1\).

\(^1\)The condition that labor market tightness is equalized across sectors was postulated in Jackman and Roper (1987) as a benchmark for measuring the degree of structural unemployment.
**Proposition 2.1.** Let \( R(L_{i,t-1}, L_{i,t}) = 0 \) for all \( L_{i,t-1}, L_{i,t} \geq 0 \). Then, for any sectors \( i \) and \( j \), \( \theta_{i,t} = \theta_{j,t} \) where \( \theta_{i,t} = V_{i,t}/U_{i,t} \).

**Proof.** Observe that for any two sectors, household optimality and Nash-bargaining imply:

\[
p_{i,t}J_{i,t} = p_{j,t}J_{j,t} \Rightarrow \kappa \frac{\nu}{1 - \nu} \frac{p_{i,t}}{q_{i,t}} = \kappa \frac{\nu}{1 - \nu} \frac{p_{j,t}}{q_{j,t}} \Rightarrow \frac{V_{i,t}}{U_{i,t}} = \frac{V_{j,t}}{U_{j,t}},
\]

where the first equality follows from the relation of firm surplus and household surplus from Nash-bargaining and the second equality follows from the definition of \( p_{i,t} \) and \( q_{i,t} \).

This result requires bargaining power and flow vacancy costs to be equalized across sectors but places no restriction on the parameters of the matching function or separation rates. In contrast to the environment considered by Jackman and Roper (1987), our results show that this condition continues to hold in a fully dynamic setting and allowing for greater heterogeneity in hiring costs across sectors. More generally, if bargaining power or vacancy posting costs differ across sectors, a generalized Jackman-Roper condition will obtain where sectoral tightness will be equalized up to a wedge term reflecting differences in bargaining power and vacancy costs. This condition is analogous to the generalized Jackman-Roper condition derived in Sahin, Song, Topa, and Violante (2010).

When reallocation is costly, the probability-weighted household surplus will generally fail to be equalized across sectors and the household will have an incentive to transfer workers to sectors with a higher surplus or a greater job-finding rate. In the no reallocation limit with a fixed labor force distribution, tightness across sectors will generically depart from the Jackman-Roper condition.

### 2.4.2 Defining the Beveridge Curve

For the US, labor market flows are large and vacancies and unemployment quickly converge to their flow steady state. To derive the Beveridge curve, we treat the sectoral equations determining vacancies, unemployment and employment as steady state conditions. In particular, in the analysis that follows, equations (2.3) - (2.5), (2.21) and (2.26) are assumed to be at
their flow steady state\textsuperscript{16}.

In the standard one-sector model (i.e., \( K = 1 \)), the Beveridge curve is a single equation defining the relationship between unemployment and vacancies and given by the steady state of the employment flow equation (2.3):

\[
\delta(1 - U) = \varphi U^{\alpha} V^{1-\alpha}.
\]

Only changes in the separation rate \( \delta \) and matching function productivity \( \varphi \) shift the Beveridge curve, while other shocks like aggregate productivity shocks simply move unemployment and vacancies along the pair of points defined by this equation. This relation also explains why the one-sector Beveridge curve is the same irrespective of real or demand-driven business cycles.

In a multi-sector model, an analytical relationship between \( U \) and \( V \) does not exist, and the aggregate steady state Beveridge curve is an equilibrium object. It is useful to construct the multisector analog of the one-sector steady state employment flow equation. Summing over sectoral employment in equation (16), we obtain a single equation relating sectoral vacancies and sectoral unemployment:

\[
L - U = \sum_{i=1}^{K} \frac{\varphi_i}{\delta_i} U_i^{\alpha} V_i^{1-\alpha} \Rightarrow \frac{L - U}{V} \theta^\alpha = \sum_{i=1}^{K} \frac{\varphi_i}{\delta_i} \left( \frac{\theta_i}{\theta} \right)^{-\alpha} \frac{V_i}{V}
\]

where \( \theta = V/U \) is aggregate labor market tightness and \( \theta_i = V_i/U_i \) is sectoral labor market tightness. The left-hand side is an expression solely in terms of aggregate unemployment and vacancies but the right-hand side will generally depend on both the type of aggregate shocks and the distribution of sectoral shocks. This term is the source of shifts in the Beveridge curve.

In a solution to our model, aggregate vacancies and unemployment are a function of the exogenous shocks: government purchases, aggregate productivity and the full set of sectoral shocks.

---

\textsuperscript{16}Impulse responses for the multisector model calibrated to monthly data show that unemployment and vacancies converge to the log-linearized Beveridge curve within 3 months. The rapid convergence of the labor market to the steady state Beveridge curve explain the high correlation of vacancies and unemployment in the calibration exercise in Shimer (2005).
productivities \( \tilde{A}_{i,t} \) and preferences \( \tilde{\phi}_{i,t} \):

\[
U = U \left( G_t, A_t, \{\tilde{A}_{i,t}, \tilde{\phi}_{i,t}\} \right),
\]

\[
V = V \left( G_t, A_t, \{\tilde{A}_{i,t}, \tilde{\phi}_{i,t}\} \right),
\]

The full set of equations that determine unemployment and vacancies are listed at the beginning of Appendix B.2. We use variations in \( G_t \) as the variable that traces out the Beveridge curve and drop time subscripts:

**Definition 2.4.** The Beveridge curve is a function \( f(\cdot) \) given by \( V \left( G; A, \{\tilde{A}_i, \tilde{\phi}_i\} \right) = f \left( U \left( G; A, \{\tilde{A}_i, \tilde{\phi}_i\} \right) \right) \), where \( G \) is the parameter varying \( U \) and \( V \), holding constant aggregate productivity, sectoral productivity and preferences: \( A, \{\tilde{A}_i\} \) and \( \{\tilde{\phi}_i\} \).

**Aggregate Shocks and the Beveridge Curve**

To separate movements along the Beveridge curve from shifts in the Beveridge curve, it is necessary to choose a single shock as the source of business cycles. Indeed, in the absence of any other aggregate or sectoral shocks, the Beveridge curve in a multisector model never shifts. However, in the presence of several different types of aggregate and sectoral shocks, the Beveridge curve could be equally well-defined as the locus of points in the U-V space traced out by aggregate productivity shocks or shocks to any given sector.

While our definition of the Beveridge curve as the locus of points in the U-V space traced out by government purchases shocks may seem fairly restrictive, a variety of real and nominal shocks trace out the same Beveridge curve. In the absence of wealth effects on labor supply, the equations that determine aggregate vacancies and unemployment and the sectoral distribution of vacancies and unemployment can be decoupled from the remaining equations that determine other endogenous variables.

**Proposition 2.2.** Assume no wealth effects and either costless labor reallocation or no reallocation. For any value of government spending shock \( G \), there exists an \( A \) such that

\[
V \left( G, 1, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K \right) = V \left( 1, A, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K \right) \quad \text{and} \quad U \left( G, 1, \{\tilde{A}\}_{i=1}^K, \{\tilde{\phi}\}_{i=1}^K \right)
\]
= U \left( 1, A, \{\tilde{A}_i\}_{i=1}^K, \{\tilde{\phi}_i\}_{i=1}^K \right).

Proof. See Appendix B.2.1. ■

This proposition shows that an aggregate productivity shock traces out the same Beveridge curve as a government purchases shock. Moreover, the same proposition applies to other types of demand shocks like monetary policy shocks not specified in our model. Indeed, any shock, real or nominal, that does not enter the steady state labor market equations that determine vacancies and unemployment, traces out the same Beveridge curve.

In the absence of wealth effects, holding constant sectoral productivity and preferences, aggregate vacancies and unemployment can be parameterized by real marginal cost times aggregate productivity: \( A_t P_{ft}/P_t \). Real marginal cost, an endogenous variable, is the only link between the block of equations that determine aggregate vacancies and unemployment and the rest of the model equations. Under no wealth effects on labor supply (as in Shimer (2005) or Hagedorn and Manovskii (2008)), our multisector model effectively generalizes the behavior of the one-sector Beveridge curve under aggregate shocks.

Moreover, given the results on aggregate productivity shocks in Proposition 2.2, our conclusions about the relationship between sectoral shocks and shifts in the Beveridge curve continue to hold in a model without sticky prices where business cycle fluctuations are driven by real shocks instead of demand shocks.

**Neutrality of Sector-Specific Shocks**

As our derivation of the Beveridge curve suggests, sectoral shocks can shift the Beveridge curve if these shocks alter the distribution of vacancies or generates mismatch across sectors. However, as showed earlier, when labor reallocation is costless, the Jackman-Roper condition obtains and tightness is equalized across sectors. In this case, we can once again obtain an aggregate Beveridge curve that is identical to the one-sector Beveridge curve:

**Proposition 2.3.** If labor reallocation is costless across sectors and separation rates and matching function efficiencies are the same across sectors (i.e. \( \delta_i = \delta, \varphi_i = \varphi \)), then sector-specific shocks do not shift the Beveridge curve.
Proof. Under costless labor reallocation, the Jackman-Roper condition holds and labor market tightness across sectors is equalized: \( V_{i,t}/U_{i,t} = V_{h,t}/U_{h,t} \) for all \( i, h = 1, 2, \ldots, K \). Summing over the steady state sectoral Beveridge curves (steady state version of equation (2.3)):

\[
\sum_{i=1}^{K} N_i = \sum_{i=1}^{K} \frac{\varphi}{\delta} \theta^{-\alpha} V_i \Rightarrow 1 - U = \frac{\varphi}{\delta} \left( \frac{V}{U} \right)^{-\alpha} V
\]

as required.

As a result, neither aggregate nor sector-specific shocks generate a shift in the Beveridge curve, providing a useful benchmark for our analysis of the effects of sector-specific shocks when reallocation is costly.

The conditions that recover the aggregate Beveridge curve in Proposition 2.3 highlight the two channels through which sector-specific shocks shift the Beveridge curve: the mismatch channel and the composition channel. If sectors share identical hiring technologies and separation rates, a sector-specific shock can only shift the Beveridge curve by changing the distribution of \( \theta_i/\theta \) - in other words, by generating mismatch. When labor market reallocation is costly, a sector-specific shock increases tightness in one sector while decreasing tightness in the other. Because of the decreasing returns to scale of the matching function, the rise in vacancies for the sector experiencing a positive shock exceeds the fall in vacancies for the sector with a negative shock. In contrast, an aggregate shock depresses tightnesses more or less uniformly, lowering vacancies in all sectors. The composition effect is present even when labor reallocation is costless. If some sectors feature greater hiring frictions, a shock favoring those sectors will shift the distribution of vacancies toward that sector, raising overall vacancies relative to a shock that leaves the distribution unchanged. Together, these two channels account for the effect of sector-specific shocks on the Beveridge curve.

2.4.3 Model-Implied Measures of Sectoral Shocks and Beveridge Curve Shifts

Our multisector model provides a useful framework for assessing the validity of empirical measures that rely on the labor market to measure sector-specific disturbances. As discussed
earlier, Lilien (1982) argued that sector-specific shocks could be measured by dispersion in employment growth across sectors, with Abraham and Katz (1986) countering that increases in employment growth dispersion could be generated by aggregate shocks if sectors feature asymmetric responses to aggregate shocks.

Our model verifies that the Lilien measure is a biased measure of sector-specific shocks validating the Abraham and Katz critique. To a log-linear approximation, sectoral employment can be expressed as a function of sectoral shocks and aggregate output. Below, we express sectoral employment under the polar cases of no reallocation $n_{it}^{nr}$ and costless reallocation $n_{it}^{r}$ respectively, assuming no wealth effects on labor supply:

$$n_{it}^{nr} = \lambda_i [\phi_i - (1 - \eta) a_i] + \lambda_i [y_t - (1 - \eta) a_t],$$

$$n_{it}^{r} = [\phi_i - (1 - \eta) a_i] + y_t - (1 - \eta) a_t - \eta [s_i \varphi_i + (1 - s_i) \alpha] \theta_t,$$

where

$$\lambda_i = \left\{ 1 + \eta [s_i \varphi_i + (1 - s_i) \alpha] \frac{L_i/U_i}{1 - \alpha} \right\}^{-1},$$

where $\varphi_i$ is a macro Frisch elasticity that reflects the dependence of the Nash-bargained sectoral wages on labor market tightness and $1 - s_i$ is the steady state size of the surplus. This parameter is a function of steady-state job-finding rates and vacancy-filling rates along with other parameters of the model such as the sectoral separation rate, etc. These expressions for sectoral employment are not materially changed by allowing for wealth effects or convex disutility of labor supply, which would simply add linear functions of $y_t$ and $n_t$ to each expression.

These expressions for sectoral employment show that both sector-specific shocks and aggregate shocks will increase employment dispersion in both the costless reallocation and no reallocation cases. In the case of the latter, the sensitivity of a sector to aggregate and sector-specific shocks increases with the elasticity $\lambda_i$ which is larger for sectors with a lower Frisch elasticity. For example, if household’s bargaining power is zero, wages are set at a constant level and $\varphi_i = 0$ for all sectors. Then sectors with a lower surplus display greater sensitivity.

$^{17}$Specifically, $s_i = \overline{W_i}/\overline{P_iA_i}$. 


to aggregate shocks consistent with the volatility of employment in a one-sector search model as discussed by Hall (2005) and Hagedorn and Manovskii (2008).

Since sector-specific shocks are generally correlated with output, our model shows that the assumptions underlying our factor analysis in Section 2.2 will generally not be satisfied. In short, simply allowing for differential elasticities to aggregate shocks is insufficient to identify sector-specific shocks. However, following the procedure in Foerster, Sarte, and Watson (2011), we can conduct a structural factor analysis by using a calibrated version of the model to correct for the endogeneity problem. For simplicity, assume only sectoral productivity shocks $a_{it}$ and assume that aggregate productivity shocks are simply a linear combination of sectoral productivity shocks. Let $a_t = (a_{1t}, \ldots, a_{Kt})'$ be the vector of sectoral productivity shocks taken as exogenous. Assume a factor decomposition of this exogenous process such that:

$$a_t = \Phi z_t + \epsilon_t,$$

where $\epsilon_t$ is a $K \times 1$ vector of sector-specific productivity shocks and $z_t$ is a scalar defined as the aggregate productivity shock with $cov(z_t, \epsilon_t) = 0$. Combining the expressions for sectoral employment and output, sectoral employment is a function of the vector of sectoral productivity shocks:

$$M n_t = H a_t,$$

where $M$ is a nondiagonal matrix with $1/\lambda_i - \gamma_i$ as its diagonal elements and $-\gamma_j$ as its off-diagonal elements. Similarly $H$ is a nondiagonal matrix with $\eta - 1 + \gamma_i$ as its diagonal elements and $\gamma_j$ as its off diagonal elements. The coefficient $\gamma_i = \frac{1}{\phi_i/\eta} \left( \frac{\bar{Y}_i}{\bar{Y}} \right)^{\eta-1}$ - the steady state share of output for each sector - enters the solution for sectoral employment since $y_t = \sum_{i=1}^{K} \gamma_i (a_{it} + n_{it})$. Unless $M$ is diagonal, a factor analysis of $n_t$ will not accurately identify the sectoral shocks $\epsilon_t$. However, for higher degrees of substitutability, the off-diagonal elements of $M$ and $H$ are dominated by the diagonal elements and the endogeneity correction becomes less important. In the limit, when goods are perfect substitutes, the reduced-form analysis in Section 2.2 is the correct procedure for identifying sector-specific shocks.
Proposition 2.4. Assume the case of no labor reallocation and let $\eta \to \infty$. Then $n_t = Ha_t$ and a factor analysis of employment identifies the sector-specific shock $\epsilon_t$.

Proof. See Appendix B.2.2. ■

To correct for possible endogeneity in our estimates of sector-specific shocks, we calibrate our model to derive the rotation matrix $M$, apply this rotation to sectoral employment data, and then perform a factor analysis on this rotation of the data. The calibration used to derive the matrix $M$ is discussed in the Appendix. Our structural factor analysis follows the same procedure as in Section 2.2 with the exception of applying the rotation $M$ to the data and using quarterly data instead of monthly data before removing the first principal component and computing the sector-specific shock index. As shown in Figure 2.6, the model-implied sectoral shock index displays a strong correlation with our reduced form shock index. As hypothesized, the correlation is stronger when goods are moderate substitutes (the case of $\eta = 2$) because the off-diagonal elements of $M$ are less important. Table 2.3 provides the correlation for alternative specifications of the sector-specific shock index obtained using 4th order detrending or year-over-year growth rates.

---

18We use quarterly data instead of monthly data since, in our model, we assume the labor market is in its...
Table 2.3: Reduced-form and structural sectoral shock index correlation

<table>
<thead>
<tr>
<th>Detrending</th>
<th>Index Type</th>
<th>$\eta = 0.5$</th>
<th>$\eta = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cubic Trend</td>
<td>Absolute value index</td>
<td>0.59</td>
<td>0.91</td>
</tr>
<tr>
<td>Cubic Trend</td>
<td>Dispersion index</td>
<td>0.65</td>
<td>0.91</td>
</tr>
<tr>
<td>Quartic Trend</td>
<td>Absolute value index</td>
<td>0.81</td>
<td>0.95</td>
</tr>
<tr>
<td>Quartic Trend</td>
<td>Dispersion index</td>
<td>0.83</td>
<td>0.96</td>
</tr>
<tr>
<td>Growth rates (year-over-year)</td>
<td>Absolute value index</td>
<td>0.92</td>
<td>0.95</td>
</tr>
<tr>
<td>Growth rates (year-over-year)</td>
<td>Dispersion index</td>
<td>0.89</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Correlation of model-implied shock index with reduced form shock index computed under different detrending procedures and alternative summary measures. See Section 2.

**Sectoral Shock Index and Shifts in the Beveridge Curve**

In Section 2.2, we correlated our sector-specific shock index with movements in the Beveridge curve intercept and showed that the index appears significant in explaining variation in vacancies controlling for the the variation explained by unemployment. Our model can also be used to think about the relationship between sector-specific shocks and movements in the Beveridge curve.

Under the assumption of no reallocation across sectors and log-linearizing around a steady state with $\bar{\theta}_i = \bar{\theta}_h$ for all $i, h = 1, 2, \ldots K$, we can derive an expression for the Beveridge curve augmented with sectoral dispersion:

$$v_t = -\frac{1}{1 - \alpha} \left( \alpha + \frac{\bar{U}}{\bar{N}} \right) u_t + \frac{1}{1 - \alpha} \sum_{i=1}^{K} \left( \frac{\bar{U}_i}{\bar{U}} - \frac{\bar{N}_i}{\bar{N}} \right) n_{it}$$

where $\alpha$ is the matching function elasticity, and the weights on sectoral employment are difference between the unemployment share and employment share in each sector. When matching function parameters are identical, these weights are all zero, and we obtain a standard log-linearized Beveridge curve relating vacancies and unemployment. Positive shocks to sectors with a higher share of employment than unemployment shift in the Beveridge curve since these sectors have lower search frictions while the opposite happens to sectors with a lower employment share then unemployment share.

Using our calibration described in the Appendix B.3, we compute the model-based distri-flow steady state.
Table 2.4: Regression analysis for reduced-form and model-implied index distribution of unemployment and run a regression of vacancies on unemployment and the model-based measure of shifts in the Beveridge curve. Log vacancies (measured by the HWI) and log unemployment are quarterly from 1951 to 2011. We replicate the regression in Section 2.2 using quarterly instead of monthly data.

<table>
<thead>
<tr>
<th>Panel A: Dispersion Index (Full Sample)</th>
<th>Panel B: Model-Implied Index (Full Sample)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detrend with time trend</td>
<td>Detrend with time trend</td>
</tr>
<tr>
<td>Cubic</td>
<td>Cubic**</td>
</tr>
<tr>
<td>0.60</td>
<td>-0.72</td>
</tr>
<tr>
<td>1.27</td>
<td>-2.43</td>
</tr>
<tr>
<td>Cubic w/1 lag</td>
<td>Cubic w/1 lag**</td>
</tr>
<tr>
<td>0.58</td>
<td>-0.73</td>
</tr>
<tr>
<td>1.21</td>
<td>-2.40</td>
</tr>
<tr>
<td>Quartic</td>
<td>Quartic</td>
</tr>
<tr>
<td>0.53</td>
<td>0.17</td>
</tr>
<tr>
<td>1.09</td>
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<tr>
<td>Quartic w/1 lag</td>
<td>Quartic w/1 lag</td>
</tr>
<tr>
<td>0.51</td>
<td>0.19</td>
</tr>
<tr>
<td>1.04</td>
<td>0.40</td>
</tr>
<tr>
<td>Detrend with growth rates</td>
<td>Detrend with growth rates</td>
</tr>
<tr>
<td>Quarter over quarter</td>
<td>Quarter over quarter***</td>
</tr>
<tr>
<td>-0.11</td>
<td>0.58</td>
</tr>
<tr>
<td>-0.35</td>
<td>2.90</td>
</tr>
<tr>
<td>Year over year*</td>
<td>Year over year***</td>
</tr>
<tr>
<td>-0.99</td>
<td>1.10</td>
</tr>
<tr>
<td>-1.93</td>
<td>4.09</td>
</tr>
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</table>

\( T = 242 \)

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Detrend with time trend</td>
<td>Detrend with time trend</td>
</tr>
<tr>
<td>Cubic***</td>
<td>Cubic</td>
</tr>
<tr>
<td>2.24</td>
<td>1.38</td>
</tr>
<tr>
<td>3.98</td>
<td>1.56</td>
</tr>
<tr>
<td>Cubic w/1 lag***</td>
<td>Cubic w/1 lag</td>
</tr>
<tr>
<td>2.19</td>
<td>1.33</td>
</tr>
<tr>
<td>3.77</td>
<td>1.47</td>
</tr>
<tr>
<td>Quartic**</td>
<td>Quartic**</td>
</tr>
<tr>
<td>1.78</td>
<td>1.93</td>
</tr>
<tr>
<td>2.37</td>
<td>7.78</td>
</tr>
<tr>
<td>Quartic w/1 lag**</td>
<td>Quartic w/1 lag**</td>
</tr>
<tr>
<td>1.74</td>
<td>1.95</td>
</tr>
<tr>
<td>2.32</td>
<td>7.70</td>
</tr>
<tr>
<td>Detrend with growth rates</td>
<td>Detrend with growth rates</td>
</tr>
<tr>
<td>Quarter over quarter</td>
<td>Quarter over quarter***</td>
</tr>
<tr>
<td>0.29</td>
<td>1.39</td>
</tr>
<tr>
<td>0.50</td>
<td>4.18</td>
</tr>
<tr>
<td>Year over year</td>
<td>Year over year***</td>
</tr>
<tr>
<td>-1.01</td>
<td>2.36</td>
</tr>
<tr>
<td>-1.21</td>
<td>7.14</td>
</tr>
</tbody>
</table>

Estimation of Beveridge curves using reduced-form and model-implied measures of sectoral shocks. Left panel uses reduced form dispersion index computed in Section 2, right panel uses model-implied measure computed as described in text. Top panel uses full sample from 1950-2011, bottom panel uses subsample 1980-2011. *** indicates significance at 1% level, ** indicates significance at 5% level, * indicates significance at 10% level. The number of time series observations is \( T = 726 \).

Our results are presented in Table 2.4. The top panels A and B compute the Beveridge curve estimate using the reduced form shock index from Section 2.2 and the model-implied index respectively using the full sample. In quarterly data, the reduced-form regressions are similar to the regressions presented in Table 2.2 but feature higher standard errors. Panel B shows that sectoral employment detrended with time trends displays coefficients that are
negative and often insignificant, inconsistent with the predictions of our model. However, in
the case of growth rate detrending, coefficients are positive and significant.

Panels D shows that the negative coefficients on the specifications using detrending via
time trends are driven by the early part of the sample. If we consider a sample only after 1980,
the coefficients are positive, consistent with our model, and frequently greater than one as
predicted by the model. Given that our model is a log-linearization around a steady state and
that our calibration relies on unemployment and employment weights computing averages in
the last decade, our model-implied measure is likely to be less accurate farther back in time.
Given the large movements in employment share across sectors over time, our model-implied
measure should fit better in more recent data. It is also worth noting that our model-implied
measure delivers positive coefficient across all detrending procedures in Panel D, in contrast
to the reduced-form measure considered in Section 2.2.

2.4.4 Beveridge Curve and the Natural Rate of Unemployment

We define the natural rate of unemployment as the unemployment rate at which inflation is
stabilized. This is a policy-relevant variable for a central bank that seeks to lower unemploy-
ment to a point at which inflation remains stable.

Definition 2.5. The natural rate of unemployment is unemployment rate when \( P_{ft}/P_t = 1 \).

Undistorted Initial State

A useful benchmark for assessing the relationship between sector-specific shocks, Beveridge
curve shifts, and the natural rate is the case of an undistorted initial state with no misalloca-
tion of output and no differences in labor market tightness across sectors. The household’s
marginal rate of substitution is assumed to be constant at \( z < 1 \). If sectors share the same
separation rates \( \delta \) and matching function efficiencies \( \varphi \), then hiring costs are equalized, relat-
vies prices \( P_i/P \) are equalized and determined by the inverse markup. In this setting, the
model admits a symmetric solution with \( Y = AN, A_iP_i/P = \mu^{-1}A, N_i = \tilde{\phi}_iN \) where \( \tilde{\phi}_i \) is the
productivity-adjusted product share defined in Section 2.3.6 and \( \mu = \zeta/(\zeta - 1) \) is a markup.
Aggregate employment $N$ and labor market tightness $\theta$ are implicitly defined by a common vacancy posting condition and labor market clearing:

$$
\mu^{-1} A = z + \frac{\kappa}{\varphi} g(\theta),
$$

(2.27)

$$
N = \frac{\varphi \theta^{1-\alpha}}{\delta + \varphi \theta^{1-\alpha}},
$$

(2.28)

where $g$ is an increasing and concave function of labor market tightness $\theta$. Total employment is simply the job-finding rate over the sum of job-finding rate and the separation rate. Moreover, the distribution of labor market variables: employment, unemployment, vacancies and the labor force all equal the productivity-adjusted product share $\bar{\phi}_i$.

**Proposition 2.5.** Assume costless labor reallocation and for $i = 1, 2, \ldots, K$, $\delta = \delta_i$ and $\varphi = \varphi_i$. Then a sector-specific demand or productivity shock does not change the natural rate of unemployment and does not shift the Beveridge curve.

**Proof.** The first result follows from the solution for the undistorted steady state and the joint determination of employment and tightness in the equations (2.27) and (2.28). Observe that sector-specific productivity and preferences shares do not enter these equilibrium conditions implying that total employment is determined independently of any sector-specific shock. The second result is an application of Proposition 2.3. ■

With costless reallocation, a sector-specific shock results in an immediate redistribution of the labor force. Because the cost of hiring is equalized across sectors, a sector-specific shock does not shift the production possibilities frontier leaving aggregate tightness and employment unchanged. Thus, both the Beveridge curve and the natural rate after left unchanged by a sector-specific shock. While the Beveridge curve does not shift under sectoral or aggregate shocks (due to Proposition 2.3), the natural rate of unemployment may change under real aggregate shocks. A negative productivity shock raises the natural rate, but an increase in markups due to a negative aggregate demand disturbance will leave the natural rate of unemployment unchanged. This provides a simple instance in which changes in the natural rate do not imply a shift in the Beveridge curve.
However, the neutrality of sector-specific shocks for both the Beveridge curve and the natural rate of unemployment hinge on the assumption of costless labor reallocation.

**Proposition 2.6.** Assume no reallocation of labor with $\delta_i = \delta$ and $\varphi_i = \varphi$ for $i = 1, 2, \ldots, K$.

Then a sector-specific demand or productivity shock such that $L_i \neq \bar{\varphi}_i$ raises the natural rate of unemployment and shifts the Beveridge curve outward (i.e. for any level of unemployment, aggregate vacancies rise).

**Proof.** See Appendix. ■

In this case, shifts in the Beveridge curve and changes in the natural rate are tightly connected, with an outward shift in the Beveridge curve implying an increase in the natural rate of unemployment. Our proof relies on the properties of convex functions to show how mismatch raises the unemployment rate. Intuitively, a sector-specific shock generates mismatch since labor must be reallocated across sectors to ensure that employment shares equal the product shares. If the labor force cannot be reallocated, tightness rises in the sector where desired employment rises and falls in the other sector. This causes aggregate employment to fall since hiring costs rise faster in the sector that is positively impacted relative to the fall in costs for the sector that is negatively impacted. Similarly, due to the convexity of the matching function, vacancies in the sector with a positive shock rise more than the fall in vacancies in the sector that is negatively hit.

**Distorted Initial State**

When separation rates or matching function efficiency differ across sectors, the relationship between shifts in the Beveridge curve and changes in the natural rate are not as straightforward. Assuming that labor market reallocation is costless, the steady state of the two-sector
version of the model can be summarized in three equation:

\[
\mu^{-1}A = \left[ \tilde{\phi} g_A (\theta)^{1-\eta} + \left( 1 - \tilde{\phi} \right) g (\theta)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{2.29}
\]
\[
1 = N \left[ 1 + \theta^a \left( n_A \delta_A \varphi_A + (1 - n_A) \frac{\delta_B}{\varphi_B} \right) \right] \tag{2.30}
\]
\[
\frac{n_A}{1 - n_A} = \frac{\tilde{\phi}}{1 - \tilde{\phi}} \left[ \frac{g_A (\theta)}{g_B (\theta)} \right]^{-\eta} \tag{2.31}
\]

where labor market tightness \( \theta \), total employment \( N \), and employment share \( n_A \) are the endogenous variables. The function \( g_i \) measures hiring costs (inclusive of wages) and is increasing and concave in labor market tightness. Without loss of generality, if sector \( A \) has a higher relative matching function efficiency or lower relative separation rate, then \( g_A < g_B \) for \( \theta > 0 \).

Differences in hiring frictions across sectors imply that even in the absence of sectoral shocks, employment shares respond asymmetrically to changes in labor market tightness as can be discerned from equation (2.31). If sector \( A \) has lower hiring costs, it follows that \( n_A > \tilde{\phi} \) since relative prices are distorted by the asymmetry in hiring costs. Effectively, sector \( A \) has higher productivity than sector \( B \) and the competitive allocations of labor are distorted toward that sector. A sector-specific shock favoring sector \( A \) lowers hiring costs and shifts out the production possibilities frontier for the economy thereby reducing the natural rate of unemployment. Moreover, this reduction in the natural rate is accompanied by a decrease in the aggregate quantity of vacancies needed to attain a particular level of employment. Since labor market tightness is equalized, shifts in the Beveridge curve due to sectoral shocks in this case stem from a composition channel. Moreover, shifts in the Beveridge curve and changes in the natural rate of unemployment move in the same direction; the Beveridge curve may shift inward or outward depending on the whether or not the sector-specific shock favors the sector with lower hiring costs. The following proposition summarizes this result:

**Proposition 2.7.** Consider the two-sector version of the model with costless labor reallocation and zero bargaining power for households \( \nu = 0 \). Without loss of generality, assume that \( \varphi_A > \varphi_B \) and \( \delta_A = \delta_B \) or vice versa (i.e. sector \( A \) has lower hiring costs than sector \( B \)). Then,
a positive sector sector-specific shock to sector A lowers the natural rate of unemployment (i.e. if $\bar{\phi}_A < \bar{\phi}_A' \Rightarrow N < N'$) and shifts the Beveridge curve inward.

Proof. See Appendix. ■

The assumption of zero bargaining power simply guarantees that the ratio $g_A/g_B$ as a function of $\theta$ is monotonic. For moderate values of $\theta$, the ratio of hiring costs will be locally monotonic with nonzero bargaining power as confirmed in numerical experiments.

With the combination of costly reallocation and asymmetric hiring costs, the connection between the direction of Beveridge curve and the natural rate of unemployment appears to hold in our numerical examples. However, we cannot analytically rule out cases in which a sector-specific shock lowers the natural rate but shifts out the Beveridge curve or vice versa. The analysis here however suggests that this would be the exception rather than the rule.

2.4.5 Alternative Labor Market Measures and Sectoral Shocks

Our model also provides a framework for assessing how well alternative labor market measures capture sector-specific shocks and shifts in the Beveridge curve.

Aggregate Matching Function Efficiency and Mismatch

Recent papers by Sedlacek (2011) and Barnichon and Figura (2011) perform a decomposition analysis of the matching function analogous to measuring the Solow residual in a growth accounting exercise. Constructing measures of unemployment, vacancies, and hires, these authors measure aggregate matching function efficiency as the residual relating these variables

$$\varphi = \frac{H}{U^{\alpha}V^{1-\alpha}}$$

and show that aggregate matching function efficiency is procyclical. In our multisector model, aggregate matching function efficiency can be expressed in terms of mismatch and the distri-
bution of vacancies:

\[
H = \sum_{i=1}^{K} \varphi_i U_i^\alpha V_i^{1-\alpha} \Rightarrow \frac{H}{\varphi U_i^\alpha V_i^{1-\alpha}} = \sum_{i=1}^{K} \frac{\varphi_i}{\varphi} \left(\frac{\theta_i}{\theta}\right)^{-\alpha} V_i, \tag{1}
\]

where \(\bar{\varphi}\) is the average level of matching function efficiency. Changes in mismatch and the distribution of vacancies will lead to variations in measured aggregate matching function efficiency. To a log-linear approximation, mismatch is a function of sectoral employment in our model:

\[
\theta_{it} = \frac{1 + \sum_i U_i}{1 - \alpha} n_{it}^{nr}.
\]

Since sectoral employment is a function of both aggregate and sector-specific shocks, dispersion in mismatch will also be subject to the Abraham and Katz critique. Therefore, fluctuations in matching function efficiency are not, as such, an indicator of either sector-specific shocks or shifts in the Beveridge curve. For a suitably long time series, if the relationship between matching function efficiency and aggregate shocks is stable, then sector-specific shocks could be identified as periods where movements in matching function efficiency are not explained by the business cycle. Unlike a one-sector model with constant matching function efficiency, our multisector model with costly reallocation is consistent with the empirical observation of movements over the cycle in aggregate matching efficiency.

Similarly, work by Sahin, Song, Topa, and Violante (2012) and Lazear and Spletzer (2012) construct mismatch indices by industry, region and occupation to examine whether mismatch has increased in the current recession. Like measurements of matching function efficiency, our model shows that variation in these measures over the cycle is not sufficient to identify sector-specific shocks or Beveridge curve shifts. Instead, these measures are evidence of the feature in our model that generates mismatch: costly labor reallocation. These empirical mismatch indices rely on direct measures of labor market tightness with vacancies data from either the JOLTs or from online vacancy postings collected by the Conference Board. Measures of sectoral or regional unemployment are constructed from the Current Population Survey (CPS). Data availability limits the time series dimension of these measures, with the mismatch
indices beginning in either 2001 or 2006. Since, mismatch can be driven by either aggregate or sectoral shocks, the cyclical increase in mismatch shown in Sahin, Song, Topa, and Violante (2012) is consistent with either aggregate or sectoral shocks.

**Labor Productivity**

Garin, Pries, and Sims (2010) document systematic changes in the behavior of labor productivity in post Great Moderation recessions. Our model supports the view that measured labor productivity behaves differently under sectoral shocks than aggregate shocks. To a log-linear approximation, measured labor productivity is a function of sectoral employment:

\[
y_t - n_t = a_t + \sum_{i=1}^{K} (\gamma_i - \frac{N_i}{N}) n_{it}
\]

where \(\gamma_i\) is the share of sector \(i\)'s output in total output and \(\frac{N_i}{N}\) is sector \(i\)'s employment share. In an undistorted state where these shares are equalized, measured labor productivity equals true productivity, but if these shares are not equalized, measured labor productivity will be a biased indicator of labor productivity and sectoral shocks can both raise or lower labor productivity depending on whether the sector experiencing a positive shock has a larger output share than its employment share. To the extent that sector-specific shocks contribute more to business cycles in the Great Moderation, labor productivity’s correlation with the business cycle will be weakened.

**Okun’s Law**

Our multisector model provides a straightforward relationship between output and the unemployment rate. The typically stable relationship between output growth and the changes in the unemployment rate is labeled as Okun’s Law and, like the Beveridge curve, is a reduced form relationship that occasionally breaks down. Combining the CES aggregator with our definition of sector-specific shocks and total employment, a structural relationship between
output and unemployment can be obtained:

\[
Y_t = A_t N_t \left\{ \sum_{i=1}^{K} \phi_i^{\frac{1}{\eta}} \left( \frac{N_{it}}{N_t} \right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}
\]

\[
= A_t (1 - U_t) \left\{ \sum_{i=1}^{K} \phi_i^{\frac{1}{\eta}} \left( \frac{N_{it}}{N_t} \right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}
\]

where the last term reflects the effect of labor misallocation on output.

The misallocation term is maximized at one - any misallocation must reduce output holding constant the level of unemployment. In this case, sector-specific shocks can disrupt the Okun’s law relationship between output growth and changes in the unemployment rate. If the economy is typically characterized by some steady state level of misallocation, then sectoral shocks can shift Okun’s law relationship in either direction. For example, a sectoral shock that improves the allocation of labor raises output for any level of unemployment - as shown in Proposition 2.7, this case would conform to an inward shift in the Beveridge curve. However, without a direct measure of aggregate productivity, it is not clear how to separate the misallocation channel from changes in aggregate productivity.

### 2.4.6 Reservation Wage Shocks and Implications for Structural Change

We can readily extend our model to consider the effect of exogenous shocks to the reservation wage with no wealth effects. Now, a solution for vacancies and unemployment is a function of the reservation wage \(z\) in addition to the other exogenous shocks described earlier.

**Proposition 2.8.** Assume no reallocation and no wealth effects. Assume that \(A_i = A_j = A, \delta_i = \delta_j, \varphi_i = \varphi_j\) for \(i, j = 1, 2, \ldots K\). For any value of the government spending shock \(G\), there exists a \(z\) such that \(V(G, z_0, A, \phi_i) = V(1, z, A, \phi_i)\) and \(U(G, z_0, A, \phi_i) = U(1, z, A, \phi_i)\).

**Proof.** See Appendix. \(\blacksquare\)

A uniform increase in the reservation wage reduces the surplus in each sector in the same way as a productivity or demand shock leaving aggregate vacancies and unemployment on the same Beveridge curve. This proposition shows that, to the extent that unemployment
benefits act as an increase in the household’s reservation wage, extensions in the duration of unemployment insurance cannot generate a shift in the Beveridge curve.

With some assumptions on functional forms, our multisector model can be augmented to address the effect of structural change in the long-run on labor market variables and employment shares. Structural change refers to the long-run trends in employment and output shares across sectors. Over the postwar period, employment in manufacturing has steadily dropped from nearly 1/3 of total employment to less than 10%. Over the same period, sectors like education, health care and professional services have all steadily grown. Alternatively, sectors like construction have displayed highly persistent fluctuations without any clear time trend. A recent literature highlighted by Acemoglu and Guerrieri (2008) and Ngai and Pissarides (2007) consider the implications of structural change for aggregate growth rates in models without labor market search. Our model extends these models to allow for consideration of structural change on unemployment and vacancies.

Under the assumption of balanced growth preferences (i.e., King-Plosser-Rebelo) and vacancy posting costs that are proportional to the household’s marginal rate of substitution, our model admits a balanced growth path with constant unemployment and vacancy rates and constant growth rates for employment. Wages and output grow at the same rate as aggregate productivity, though, aggregate productivity growth is only asymptotically constant if sectors diverge in their growth rates of productivity. The assumption that vacancy posting costs are proportional to the household’s MRS is a natural one if hiring is an activity that requires labor. Similar assumptions in Blanchard and Gali (2010) and Michaillat (2012) on vacancy costs are justified by assuming that the cost of hiring is proportional to the wage paid to workers.

**Proposition 2.9.** Consider the $K$ sector flexible-price version of the model with costless labor reallocation and identical separation rates and matching function parameters. Additionally, assume that vacancy posting costs are proportional to the household’s marginal rate of substitution: $\kappa_t = -\chi_c U_n(C_t, N_t) / U_c(C_t, N_t)$, preferences are King-Plosser-Rebelo: $U(C, N) = \log(C) - v(N)$, and the number of households grows at a constant rate $g_t$ with
each household supplying a unit measure of labor inelastically. Then, in the labor market steady state:

1. Employment shares equal product shares: \( \frac{N_{it}}{N_t} = \tilde{\phi}_i \)

2. Unemployment rates \( \frac{U_t}{L_t} \) and vacancy rates \( \frac{V_t}{L_t} \) are constant

3. Employment growth \( \Delta N/N \) equals labor force growth \( g_l \)

4. Aggregate output \( \Delta Y/Y \) and consumption growth \( \Delta C/C \) is equal to productivity plus labor force growth: \( g_y = g_c = g_A + g_l \)

5. Wage growth equals productivity growth: \( g_w = g_A \)

If initial productivity is equalized across sectors and grows at the same rate or if \( \eta = 1 \), then \( g_A \) is constant and equal to input-share average of productivity growth across sectors. If sectors grow at different rates, productivity growth is asymptotically constant with \( g_A = \gamma_j \) where \( j = 1, 2, \ldots, K \) is the sector with the highest growth rate if \( \eta > 1 \) or \( j \) is the sector with the lowest growth rate if \( \eta < 1 \).

Proof. See Appendix.

Under KPR preferences and symmetry across sectors in hiring costs, the household reallocates labor to mirror the movements in productivity-adjusted product shares. Since the cost of labor is equalized across sectors, relative prices are equalized and an aggregate vacancy posting condition obtains. The assumption that vacancy posting costs are proportional to the household’s MRS ensures that market tightness and employment have no trend. If real vacancy posting costs did not change over time, productivity growth would result in a downward trend for unemployment. In contrast, US unemployment exhibits, if anything, a slight upward trend. In general, if sectors exhibit persistent differences in matching function efficiency or separation rates, unemployment, vacancies and employment would not exhibit constant growth rates. However, the proposition presented here establishes a useful benchmark for thinking about long-run trends in unemployment and vacancies.
2.5 Quantitative Predictions of the Model

To examine whether a sector-specific shock can account for the observed shift in the Beveridge curve and the rise in the unemployment rate in the Great Recession, we calibrate a two-sector version of our model. In this recession, the construction sector is the largest contributor to the sector-specific shock index and is frequently identified as the sector where the employment dislocation has been most severe and persistent. We calibrate the two-sector model to match various moments on employment, unemployment and vacancies across construction and non-construction sectors. Since construction displays a far higher job-filling rate than the rest of the economy, our calibration requires that construction either feature markedly lower hiring costs or reduced labor market tightness relative to the non-construction sector. We consider each explanation in turn.

2.5.1 Calibration Strategy

The economy is partitioned into construction and non-construction sectors with initial labor market tightness equalized across sectors as would be the case in the model steady state. Several standard parameters in search models are chosen exogenously: the discount rate $\beta = 0.96^{1/12}$ to target an annual interest rate of 4%, and the matching function elasticity $\alpha = 0.5$ is assumed to be the same across sectors consistent with evidence from Petrongolo and Pissarides (2001). We also assume that sectoral productivity is equalized and normalized to unity along with the price markup\(^{19}\).

Parameters unique to our model determine hiring costs in each sector: the sectoral separation rates $\delta_c$ and $\delta_{nc}$, sectoral matching function efficiencies $\varphi_c$ and $\varphi_{nc}$, the cost of posting vacancies $\kappa$, the reservation wage $z$, and the household’s bargaining power $\nu$. Moreover, we must also choose parameters in the CES aggregate - namely the input share of construction $\phi$ in the CES aggregator and the elasticity of substitution $\eta$ that determines the degree of complementarity or substitutability across goods. We fix $\eta = 0.5$ so that construction and

---

\(^{19}\)A positive markup has no effect on our calibration other than changing the average price of each good. Alternatively, if the fiscal authority provides a production subsidy to retailers, the markup will be fully offset in steady state with the price index equal to unity.
non-construction goods are moderate complements. However, we consider other values of $\eta$ in our robustness checks.

Separation rates are set using the 2001-2006 averages of employment-weighted sectoral separation rates in the Job Openings and Labor Turnover survey; construction exhibits a significantly higher separation rate than other sectors. Bargaining power is set at $\nu = 0$ to deliver real wage rigidity as in Hall (2005) to ensure large employment effects from small changes in markups or aggregate productivity. As Hagedorn and Manovskii (2008) emphasize, the key variable determining the variability of employment is the size of the surplus rather than the bargaining power. Moreover, since bargaining power is the same across sectors, the level of bargaining power does not affect the mismatch channel by which sector-specific shocks shift the Beveridge curve.

The remaining five parameters - matching function efficiencies, reservation wage, vacancy posting cost, and product share - are jointly chosen to match the following targets: unemployment rate $U/L = 5\%$, vacancy rate $V/L = 2.5\%$, construction’s share in total employment $N_c/N = 5.7\%$, construction’s share in total vacancies $V_c/V = 3.7\%$, and a product share-weighted average accounting surplus of 10% as in Monacelli, Perotti, and Trigari (2010) and close to the surplus delivered in the calibration of Hagedorn and Manovskii (2008). The construction share of employment is chosen to match the peak of construction employment in 2007 and the vacancy share is the average level of vacancies from 2001-2006. Parameter values and targets are summarized in the Table 2.5.

Under the assumption that labor market tightness is equalized across sectors, the model generates a lower unemployment rate for construction relative to non-construction sectors, 3.3% vs. 5.1%. Because hiring costs are considerably lower in the construction sector under this calibration, the household allocates fewer worker to the construction sector to search in order to equalize labor market tightness, 5.6% vs. 94.4% in non-construction sector. However, using sectoral unemployment shares calculated in the CPS, the level of unemployment in the construction sector appears counterfactually low. Nevertheless, the correspondence

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20The surplus is defined as $A_i P_i / P - z$, the difference between the marginal product of labor and the household’s marginal rate of substitution.
Table 2.5: Summary of calibration parameters

<table>
<thead>
<tr>
<th>Aggregate Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>$\nu$</td>
</tr>
<tr>
<td>Matching function elasticity</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>$\eta$</td>
</tr>
<tr>
<td>Reservation wage</td>
<td>$z$</td>
</tr>
<tr>
<td>Construction share</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Vacancy posting cost</td>
<td>$\kappa$</td>
</tr>
</tbody>
</table>

### Construction
- Monthly separation rates $\delta_c$ | 6.00% 
- Matching function efficiency $\varphi_c$ | 2.48 

### Non-Construction
- Monthly separation rates $\delta_{nc}$ | 3.60% 
- Matching function efficiency $\varphi_{nc}$ | 0.95 

between the CPS measure of sectoral unemployment and the economic concept of sectoral unemployment in the model is unclear. The CPS measures sectoral unemployment by assigning workers to sectors based on the industry of previous employment with those workers outside the labor force or entering the labor force unassigned to any sector. In the model, a worker is unemployed in sector $i$ if that worker is searching for jobs in sector $i$. The CPS measure may not accurately capture the sector in which a worker is searching, particularly among those workers transiting between participation and non-participation. In any case, in the next section, we show that an alternative calibration matching the unemployment and vacancy shares of workers does not substantially alter our results.

#### 2.5.2 Experiment

We depict the shift in the steady state Beveridge curve generated by a permanent shock to the construction share $\phi$ that reduces the share to $\phi' = 0.04$. This reduction in construction share is chosen to match the observed drop in construction employment shares from a pre-recession peak of 5.7% to its 2012 level of 4.1%. The pre-shock Beveridge curve traces out the locus of aggregate vacancies and unemployment rates for different levels of real marginal cost, while the post-shock Beveridge curve traces the same locus with $\phi = \phi'$ leaving the distribution of the labor force either unchanged (in the case of no reallocation) or shifting the distribution
to ensure equalized labor market tightness across sectors (in the case of perfect reallocation).

Figure 2.7 illustrates our main quantitative results. We show that, in the absence of reallocation (left-hand panel of Figure 2.7), a sector-specific shock to the construction sector generates a shift in the Beveridge curve of about 1.3% (horizontal shift - the rise in the unemployment rate at each level of vacancies). This matches the observed shift in the US data on unemployment rates and the vacancy to labor force ratio. A comparison of simple trend lines of $V/L$ on $U/L$ before and after 2009 (using data from December 2001-November 2011) reveals a shift in the horizontal intercept of 1.4%. While analyses using the job-openings rate (a slightly different measure of vacancies then the vacancy to labor force ratio) reveal a somewhat larger shift of 2%, the shift generated in our baseline calibration with no labor reallocation explains a substantial fraction of the observed shift in either case.

In contrast, when reallocation is costless, the Beveridge curve is essentially unchanged after the sector-specific shock. We take each case as bounds on the shift in the Beveridge curve and, as we will argue, the case of no reallocation is both a good approximation for the short-run behavior of the Beveridge curve and will continue to hold over the medium run given evidence on the costs of labor reallocation for displaced workers. So long as the labor force does not overshoot its long-run distribution, vacancies and unemployment along the transition path will lie in the region between these curves\textsuperscript{21}.

In our model, employment shares vary with both changes in the markup and sector-specific shocks, though the movement in employment shares for aggregate shocks is quite small. For a markup shock, employment shares in construction drop because the surplus in construction is lower than that of the non-construction sectors. Lower hiring costs ensure a smaller surplus and, therefore, a greater decrease in the relative surplus for the construction sector. While construction shares displayed somewhat larger cyclical movements in employment shares before 1984, construction shares did not fall in the last recession and recovered quite slowly after the 1990s recession. Our calibration is consistent with small cyclical effects of aggregate shocks on employment shares consistent with evidence in the past three recessions where shares show

\textsuperscript{21}Numerical simulations using a quadratic cost of reallocation in a two-sector model show that the labor force moves monotonically after a permanent shock towards the labor force distribution that equates tightnesses.
little systematic movement in recessions. In this experiment, construction’s employment share falls to 4.2% when overall unemployment is at 9% and predicts that the share would only rise to 4.3% at a 5% unemployment rate (with no labor reallocation). Once reallocation takes place, this sector-specific shock lowers construction’s employment share further to 4.1%.

### 2.5.3 Distorted Initial State and Substitutability

As mentioned, the restriction that initial labor market tightness is equalized across sectors results in a counterfactual sectoral unemployment rate and labor force distribution using measures of these moments from the CPS. If we relax the assumption of equalized labor market tightness, an alternative calibration matches the distribution of employment, unemployment and vacancies. As before, five parameters - matching function efficiencies, the reservation wage, the vacancy posting cost and the product share - are jointly chosen to match the same targets as in Section 2.5.1. For consistency, we modify the targeted employment share of construction at 5.3%, it’s 2000-2006 average. As Table 2.6 shows, aside from the matching function efficiencies, the remaining parameters are largely unchanged.

Figure 2.8 shows the shift in the Beveridge curve for a preference shock that reduces the construction share to $\phi' = 0.04$. This shock generates a shift in the Beveridge curve slightly
smaller than the previous calibration with an average 1% shift in the unemployment rate at each vacancy rate. At higher levels of unemployment, the shift is mitigated since the sector-specific shock favors the non-construction sector which has a lower cost of hiring for a given level of market tightness. Even though job-filling rates are similar under both calibrations, the reasons for the higher job-filling rate for construction in each calibration are quite different. In our baseline calibration, job-filling rates in the construction sector are higher solely due to higher matching function productivity (even after accounting for the higher separation rate). However, in the distorted steady state calibration, job-filling rates are higher because of lower labor market tightness in the construction sector - effectively the labor force is misallocated with too many workers in construction. Absent labor reallocation, the sector-specific shock still shifts the Beveridge curve outward because a negative sector-specific shock worsens the mismatch between construction and non-construction sectors.

In addition to generating a similar shift in the Beveridge curve, employment shares exhibit somewhat greater volatility under aggregate shocks, though the overall volatility remains low. Since the initial level of mismatch is elevated in this case, the surplus is lower in construction than in non-construction sectors. As a result, aggregate shocks have a greater effect on employment and generate larger increases in mismatch and movement in employment shares.

The behavior of employment shares under aggregate shocks is also affected by the degree of complementarity among goods. When goods are complements, aggregate shocks generate relatively small movements in employment shares. This is due to the limited effect of prices
on relative employment shares and can be seen by combining input demand conditions:

\[
\frac{N_A}{N_B} = \frac{\phi}{1 - \phi} \left( \frac{P_A}{P_B} \right)^{-\eta}.
\]

In the limit, when \( \eta \to 0 \), goods are perfect complements and employment shares are constant irrespective of any aggregate shocks. For higher levels of substitutability, employment shares exhibit greater variation with aggregate shocks, but the magnitude of the shift in the Beveridge curve induced by a sector-specific shock decreases. Figure 2.9 displays the shift in the Beveridge curve when \( \eta = 2 \) and \( \eta = 10 \) - moderate and high degrees of substitutability. For the alternative values of \( \eta \), we recalibrate the five parameters discussed earlier to maintain the same aggregate and distributional targets. With a higher degree of substitutability, sectors exhibit greater variation in employment shares over the business cycle but show a smaller shift in the Beveridge curve conditional on a sector-specific shock that delivers the same movement in employment shares from 5.3% to about 4% after labor reallocation. However, in the absence of labor reallocation, sector-specific shocks do not match the observed fall in construction employment shares. With \( \eta = 2 \), construction’s employment share is 4.5% at an unemployment rate of 8% - too high relative to the data. Similarly, for \( \eta = 10 \), construction’s share is 5.3%.
Aside from counterfactually high employment shares in the short-run, high degrees of substitutability imply business cycle variation in employment shares inconsistent with evidence in the Great Moderation period. Aside from trends, employment shares across sectors are typically stable over the cycle with durable goods and service sectors displaying the strongest business cycle movements (durables are countercyclical while services are countercyclical). While construction’s share of employment fell in the early 1990s recession, the construction share remained stable in the 2001 recession before rising and falling with the housing bubble. This suggests that the assumption of mild complementarity or substitutability is not unreasonable in the current recession. Moreover, evidence cited in the growth literature and in studies of durable versus nondurable goods do not support very high levels of substitutability in the CES aggregator. In short, our conclusions that a sector-specific shock to construction account for over 2/3 of the shift in the Beveridge curve hold under alternative assumptions of labor market tightness and for reasonable values of the degree of substitutability.

### 2.5.4 Natural Rate of Unemployment

The experiments considered here also allows for an examination of the quantitative relationship between shifts in the Beveridge curve and changes in the natural rate of unemployment.

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22See Acemoglu and Guerrieri (2008), Carvalho and Lee (2011), and Monacelli (2009).
Table 2.7 shows the natural rate of unemployment before and after a sector specific shock for various specifications of our model. The baseline calibration, which fully accounts for the shift in the Beveridge curve, finds a rise of 1.4 percentage points in the natural rate of unemployment to 6.4%. Once labor reallocation takes place, the sectoral shock to construction has a trivial effect on the unemployment rate, raising the rate to 5.06%. The initial rise in the natural rate of unemployment is similar in magnitude to the estimate in Sahin, Song, Topa, and Violante (2012) of the contribution of mismatch unemployment in the Great Recession. The absence of labor reallocation is responsible for most of the rise in the unemployment rate, while the composition effect accounts for the increase in the unemployment rate once reallocation takes place. This slight long-run rise in the unemployment rate is due to the fact that a sectoral shock shifts employment away from the sector with lower hiring costs. For higher degrees of substitutability, sectoral shocks that deliver the same employment share once reallocation takes place imply similar long-run unemployment rates but also a lower rise in the natural rate even in the absence of labor reallocation. In each case, a higher degree of substitutability implies less movement in employment shares as agents tolerate greater deviations of employment shares from product shares leading to a smaller rise in the natural rate of unemployment. Greater substitutability also generates a smaller shift in the Beveridge curve.

Table 2.7: Natural rate and Beveridge curve shift

<table>
<thead>
<tr>
<th></th>
<th>No Reallocation</th>
<th>Full Reallocation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BC shift</td>
<td>Rate</td>
</tr>
<tr>
<td><strong>Undistorted state</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \eta = 0.5 )</td>
<td>1.3</td>
<td>6.40</td>
</tr>
<tr>
<td>( \eta = 2 )</td>
<td>0.9</td>
<td>6.00</td>
</tr>
<tr>
<td>( \eta = 10 )</td>
<td>0.4</td>
<td>5.12</td>
</tr>
<tr>
<td><strong>Distorted state</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \eta = 0.5 )</td>
<td>0.9</td>
<td>6.04</td>
</tr>
<tr>
<td>( \eta = 2 )</td>
<td>0.5</td>
<td>5.50</td>
</tr>
<tr>
<td>( \eta = 10 )</td>
<td>-0.2</td>
<td>4.92</td>
</tr>
</tbody>
</table>

Beveridge curve shift measured as average increase in unemployment rate at any given level of vacancies. Size of shift and change in unemployment rate measured for various degrees of substitutability among intermediate goods.
As the second panel of Table 2.7 illustrates, in the presence of some initial degree of mismatch, the quantitative relationship between the natural rate and the shift in the Beveridge curve is somewhat weaker. In the baseline case of $\eta = 0.5$, both the shift in the Beveridge curve and the rise in the natural rate are somewhat lower than the undistorted case with a somewhat larger increase in the natural rate than implied by the shift in the Beveridge curve. Moreover, once reallocation takes place, the natural rate actually falls to 4.88% relative to the initial unemployment rate. This reduction in the long-run unemployment rate differs from the undistorted case because hiring costs are now greater in the construction sector relative to the non-construction sector. Therefore, the sectoral shock favors the sector with lower costs. For higher levels of substitutability, movements in the natural rate are attenuated, consistent with the smaller shifts in the Beveridge curve.

Our experiment reveals an approximate one-to-one relationship between shifts in the Beveridge curve and changes in the natural rate of unemployment. Moreover, when labor reallocation is complete, the natural rate of unemployment returns to approximately the same level despite a permanent sector-specific shock and differences across sectors in hiring costs and matching function technology. However, the one-to-one link between Beveridge curve shifts and the natural rate of unemployment does not hold under extensions of the model considered in Section 2.6.

2.5.5 Labor Reallocation

As our quantitative results have emphasized, the ability of sector-specific shocks to explain the shift in the Beveridge curve and generate any economically significant fluctuations in the natural rate of unemployment depends crucially on the speed of labor reallocation across sectors. The available evidence supports slow labor reallocation in the short-run (1-2 years) but evidence on the pace of labor reallocation over the medium-run (2-8 years) is more mixed. We review the available evidence on labor reallocation in both the short-run and medium-run.

Costless labor reallocation is likely to be a poor approximation for the short-run behavior of the labor market. Given the quantitatively small role played by composition effects, costless
reallocation would imply no mismatch across sectors and nearly constant aggregate matching function efficiency over the business cycle. However, the empirical measures constructed in Sahin, Song, Topa, and Violante (2010), Barnichon and Figura (2011), and Sedlacek (2011) show that these variables fluctuate significantly over the business cycle. Moreover, observed vacancy to unemployment ratios using JOLTs and CPS data are not equalized across sectors, which is also inconsistent with the view that labor market reallocation is costless.

Table 2.8: Reallocation rates by education level

<table>
<thead>
<tr>
<th>Education Level</th>
<th>Industry</th>
<th></th>
<th>Occupation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LF</td>
<td>Emp</td>
<td>Unemp</td>
<td>LF</td>
</tr>
<tr>
<td>HS Dropout</td>
<td>2.83</td>
<td>2.85</td>
<td>2.63</td>
<td>3.09</td>
</tr>
<tr>
<td>HS Graduate</td>
<td>2.24</td>
<td>2.21</td>
<td>2.71</td>
<td>2.58</td>
</tr>
<tr>
<td>Some College</td>
<td>2.15</td>
<td>2.14</td>
<td>2.40</td>
<td>2.52</td>
</tr>
<tr>
<td>College Graduate</td>
<td>1.86</td>
<td>1.84</td>
<td>2.25</td>
<td>2.07</td>
</tr>
<tr>
<td>Advanced Degrees</td>
<td>1.28</td>
<td>1.27</td>
<td>1.80</td>
<td>1.30</td>
</tr>
</tbody>
</table>

LF is % of workers in the labor force recording an industry or occupation transition over consecutive months, while Emp and Unemp are transition rates for employed and unemployed workers respectively.

However, to explain a persistent shift in the Beveridge curve, labor reallocation must also be costly over the medium run. Transition rates for workers across sectors suggest large rates of reallocation, while evidence for displaced workers suggest substantial and persistent barriers to reallocation. The most natural measure of reallocation rates across sectors is monthly transition rates for employed and unemployed workers in the CPS. Since the CPS features a rotating panel design, households are tracked for four consecutive months and interviewed again a year later for another four consecutive months. Using matched CPS data from 2003-2006, we measure monthly reallocation rates for both employed and unemployed workers across major industries and major occupations. These monthly transition rates averaged 2.1% and 2.4% for the industry and occupation reallocation rates respectively. As Table 2.8 shows, reallocation rates are decreasing with educational attainment and are generally higher for workers who are currently unemployed. Interestingly, for workers with less than a high school degree, reallocation rates drop for unemployed workers relative to employed workers. This fact may be salient for construction workers since construction exhibits the lowest skill
attainment of any major industry. The left-hand column of Table 2.9 gives the fraction of workers in each industry who are college graduates or higher.

Given our interest in labor reallocation out of construction, we examine transitions for only workers in the construction sector over the same period. Table 2.9 also shows the distribution of transitions from construction to other industries both unconditionally and conditional on the initial skill level. As Table 2.9 reveals, low-skilled construction workers reallocate toward other low skill industries like retail trade and leisure and hospitality. Service-sector industries - like education and health services, financial activities, and government - which account for a significant share of aggregate employment, are relatively underrepresented. While a significant fraction of transitions take place into professional and business services, these transitions may reflect movements into low skilled jobs like janitorial services and office support rather high-skilled occupations like lawyers, scientists, and managers which both belong to this sector.

<table>
<thead>
<tr>
<th>Industry</th>
<th>% College +</th>
<th>Construction Transition</th>
<th>College +</th>
<th>Some College or less</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>14%</td>
<td>3%</td>
<td>2%</td>
<td>3%</td>
</tr>
<tr>
<td>Mining</td>
<td>17%</td>
<td>1%</td>
<td>0%</td>
<td>1%</td>
</tr>
<tr>
<td>Construction</td>
<td>11%</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>23%</td>
<td>18%</td>
<td>14%</td>
<td>19%</td>
</tr>
<tr>
<td>Wholesale and Retail Trade</td>
<td>19%</td>
<td>17%</td>
<td>12%</td>
<td>18%</td>
</tr>
<tr>
<td>Transportation and Utilities</td>
<td>17%</td>
<td>9%</td>
<td>6%</td>
<td>10%</td>
</tr>
<tr>
<td>Information Services</td>
<td>41%</td>
<td>2%</td>
<td>4%</td>
<td>2%</td>
</tr>
<tr>
<td>Financial Activities</td>
<td>40%</td>
<td>5%</td>
<td>11%</td>
<td>5%</td>
</tr>
<tr>
<td>Professional and Business Services</td>
<td>42%</td>
<td>18%</td>
<td>23%</td>
<td>17%</td>
</tr>
<tr>
<td>Education and Health Services</td>
<td>46%</td>
<td>6%</td>
<td>14%</td>
<td>6%</td>
</tr>
<tr>
<td>Leisure and Hospitality</td>
<td>14%</td>
<td>9%</td>
<td>4%</td>
<td>10%</td>
</tr>
<tr>
<td>Other Services</td>
<td>20%</td>
<td>8%</td>
<td>5%</td>
<td>8%</td>
</tr>
<tr>
<td>Public Administration</td>
<td>39%</td>
<td>3%</td>
<td>6%</td>
<td>2%</td>
</tr>
</tbody>
</table>

First column is the percentage of workers within a sector with a college degree or higher. Second column is the sectoral distribution of transitions from construction to non-construction sectors. The third and fourth columns show the sectoral distributions for workers will less than a college degree and for workers with more than a college degree respectively. All values are averages from the Current Population Survey, 2000-2006.

The aggregate industry and occupation transition rates reported here are similar in magnitude to the rates documented in Moscarini and Thomsson (2007) who examine transition rates at a higher level of disaggregation across occupations instead of industries. However,
Kambourov and Manovskii (2004) argue that classification errors are significant in year-to-year transitions in the CPS, leading to spurious transitions. Indeed, measurement error always biases transition rates upwards since a transition is recorded for any consecutive change in recorded industry. Moreover, these transition rates are silent on whether newly transitioned workers are a good substitute for existing workers with industry experience. Therefore, while the raw transition rates suggest large flows across sectors, these transitions are subject to significant measurement error and may not capture whether workers who reallocate are screened by firms. Measurement error and the absence of any measures of match quality may also bias the mismatch measures of Sahin, Song, Topa, and Violante (2012), who record a sharp fall in mismatch in the recovery period despite the shift in the Beveridge curve.

High rates of reallocation in the medium term are also inconsistent with evidence from the literature on displaced workers, which documents persistent effects of job loss on wages and labor force outcomes. Davis and von Wachter (2011) show that, in periods of high unemployment, wage loss is up to three years of pre-displacement earnings. This study and related work relies on higher quality longitudinal data from administrative records that accurately track worker outcomes for extended periods. To the extent that wages accurately reflect a worker’s marginal product, the steep decline in wages suggests that, conditional on finding employment, displaced workers are not as well suited for their new jobs. The most recent Displaced Workers Survey - a occasional supplement to the CPS - shows that 62% of long-tenured displaced workers (i.e. workers employed for over 3 years) from 2007-2009 came from construction, manufacturing, wholesale and retail trade, or professional and business services. These are precisely the same sectors into which construction workers reallocate suggesting that weak labor market conditions in these sectors make them unlikely to absorb transitions from construction. Moreover, in the latest wave of the Displaced Workers Survey, displaced construction workers exhibit among the lowest rate of reemployment in another industry at 23.9% - second lowest next to education and health services at 19.4%. In short,

23Construction workers alone account for 13% of long-tenured displaced workers with a total of 6.8 million workers displaced over the 2007-2009 period. These findings are also supported by Charles, Hurst, and Notowidigdo (2012).
evidence on displaced workers suggests significant costs to reallocation over the medium term.

2.5.6 Skilled and Unskilled Labor

While evidence on the degree of labor reallocation across sectors is mixed, one dimension along which workers cannot readily reallocate is skill level. In this section, we extend our baseline model to include skilled and unskilled workers and show that sector-specific shocks can still shift the Beveridge curve even when industry reallocation is costless. Firms in all sectors now hire both skilled and unskilled workers using a fixed proportions technology to produce sectoral output. Workers at a given skill level can freely reallocate across sectors, but workers cannot reallocate across skill levels.

The intermediate good firm’s problem from Section 2.3.4 is modified as follows:

\[
\Pi_{i,t}^{Int} = \max_{\{V_{i,t}^s, N_{i,t}^s, V_{i,t}^u, N_{i,t}^u\}} \mathbb{E}_t \sum_{T=0}^{\infty} Q_{t,T} \left( P_{i,T} Y_{i,T} - W_{i,T} N_{i,T}^u - W_{i,T} N_{i,T}^s - \kappa P_T V_{i,T}^u - \kappa P_T V_{i,T}^s \right),
\]

subject to:

\[
N_{i,t}^u = (1 - \delta_i) N_{i,t-1}^u + q_t^u V_{i,t}^u,
\]

\[
N_{i,t}^s = (1 - \delta_i) N_{i,t-1}^s + q_t^s V_{i,t}^s,
\]

\[
Y_{i,t} = A_{i,t} \min\left\{ N_{i,t}^s, \nu_i N_{i,t}^u \right\}.
\]

Relative to the baseline model, firms in each sector \( i \) hire both skilled workers \( N_{i,t}^s \) and unskilled workers \( N_{i,t}^u \) subject to a fixed proportions technology where a unit of effective labor requires a constant sector-specific combination of skilled and unskilled labor \( \nu_i \). Firms post vacancies \( V_{i,t}^s \) and \( V_{i,t}^u \) for both types of workers with skill-specific job-filling rates \( q_t^s \) and \( q_t^u \). Given costless reallocation within skill cohorts, the job-filling rates are the same across sector for a given skill level. Wages may differ across skill levels but vacancy posting costs are assumed to be the same.

Optimizing behavior by firms implies a single vacancy posting condition for hiring a fixed
proportion of workers across skill levels:

\[
\frac{P_{dt}}{P_t} A_{dt} = \frac{W^s_{dt}}{P_t} + \frac{1}{\nu_i} \frac{W^u_{dt}}{P_t} + \frac{\kappa}{q^s_{it}} - E_t Q_{t,t+1} \frac{\kappa}{q^u_{it}} (1 - \delta_i) - E_t Q_{t,t+1} \frac{\kappa}{q^u_{it}} (1 - \delta_i),
\]

\[
N^s_{it} = \nu_i N^u_{it}.
\]

This vacancy posting condition generalizes the standard vacancy posting condition. For sectors with a higher ratio of skilled to unskilled labor, wages and search costs for skilled workers account for a larger share of the marginal product of labor. Changes in the marginal product for a sector characterized by a relatively high skill workforce have a greater effect on skilled worker employment than unskilled worker employment.

The household problem is left largely unchanged with households free to assign skilled and unskilled workers to search across sectors but unable to transform unskilled workers into skilled workers or vice versa. At each skill level, workers search in sectors to equate their probability-weighted surplus from finding a job - the same condition as in Section 2.3.1. This optimality condition implies the Jackman-Roper condition with labor market tightness equated across sectors for a given skill level.

We calibrate a two-sector version of this model to demonstrate that sector-specific shocks to the low-skilled sector can generate a quantitatively significant shift in the Beveridge curve. Following the discussion in Section 2.5.4, we partition the economy into two sectors and two skill levels, segmenting workers as either college graduates or workers with less than a four-year college degree. As noted in Table 2.8, sectors differ markedly in the skill composition of their workforce. We define the low-skilled sector as construction, mining, leisure and hospitality, trade and transportation, and other services, assigning all remaining sectors to a composite high-skilled sector. The employment weighted ratio of college graduates to non-college graduates is 0.193 for the low-skilled sectors while this ratio is 0.64 for the other sector and determines the value for the parameter \(\nu_i\).

For the remaining parameters, our calibration strategy largely follows our strategy described in Section 2.5.1. Bargaining power \(\nu\), matching function elasticity \(\alpha\), the elasticity of
Table 2.10: Skilled/unskilled model parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$\beta$ 0.96^{1/12}</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>$\nu$ 0</td>
</tr>
<tr>
<td>Matching function elasticity</td>
<td>$\alpha$ 0.5</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>$\eta$ 0.5</td>
</tr>
<tr>
<td>Reservation wage</td>
<td>$z$ 0.17</td>
</tr>
<tr>
<td>Reservation wage</td>
<td>$z_s$ 0.31</td>
</tr>
<tr>
<td>Low-skilled share</td>
<td>$\phi$ 0.56</td>
</tr>
<tr>
<td>Vacancy posting cost</td>
<td>$\kappa$ 0.8</td>
</tr>
<tr>
<td><strong>Low-Skilled</strong></td>
<td></td>
</tr>
<tr>
<td>Monthly separation rates</td>
<td>$\delta_{ls}$ 5.10%</td>
</tr>
<tr>
<td>Matching function efficiency</td>
<td>$\varphi_{ls}$ 0.63</td>
</tr>
<tr>
<td>Labor share</td>
<td>$L_{ls}$ 70%</td>
</tr>
<tr>
<td>Skill ratio</td>
<td>$\nu_{ls}$ 0.19</td>
</tr>
<tr>
<td><strong>High-Skilled</strong></td>
<td></td>
</tr>
<tr>
<td>Monthly separation rates</td>
<td>$\delta_s$ 3.10%</td>
</tr>
<tr>
<td>Matching function efficiency</td>
<td>$\varphi_s$ 1.85</td>
</tr>
<tr>
<td>Labor share</td>
<td>$L_s$ 30%</td>
</tr>
<tr>
<td>Skill ratio</td>
<td>$\nu_s$ 0.64</td>
</tr>
</tbody>
</table>

Substitution $\eta$ across goods, and the discount rate $\beta$ are the same as in Section 2.5.1. Sectoral separation rates are chosen to match the employment weighted separation rates (2000-2006 averages) reported from JOLTs. The remaining parameters to be chosen are the matching function efficiencies for skilled workers $\varphi_s$ and unskilled workers $\varphi_u$, the reservation wages for skilled workers $z_s$ and unskilled workers $z$, the cost of posting vacancies $\kappa$, and the preference for the low-skilled sector’s good $\phi$. These parameters are chosen to jointly match the following targets: unemployment rate $U/L = 5\%$, vacancy rate $V/L = 2.5\%$, employment share of low-skilled sector $N_{ls}/N = 38.9\%$, vacancy share of low-skilled sector $V_{ls}/V = 37.1\%$, skill premium $z_s/z = 1.82$, and share-weighted average accounting surplus of 10\%. The calibration target for employment shares is 2003-2006 average from the BLS establishment survey, while the calibration target from vacancy shares is the average share of vacancies for low-skilled sectors from the JOLTs data over the same period. The skill premium is chosen from estimates in Goldin and Katz (2007), while the share-weighted average accounting surplus is the same as the baseline calibration\textsuperscript{24}. The labor share for the skilled sector $L_s = 30\%$ matches

\textsuperscript{24}See Table A8.1, data for 2005.
the 2003-2006 average share of college graduates in the CPS. The model generates an unemployment share of 51% for the low-skilled sector (versus 50% in the CPS) and unemployment rates by skill level of 4.5% and 5.2% for high skilled and low-skilled workers respectively. Our calibration is summarized in Table 2.10.

The experiment we conduct is a preference shock that reduces the share of low-skilled employment from 38.9% to 38% corresponding to the reduction observed in the current recession. This fall in employment share is driven largely by construction and partially offset by increases in the other constituent sectors classified as low-skilled. A shock that reduces the input share to $\phi' = 0.547$ reduces the employment share to 38%, raises the unemployment rate to 5.12% and raises vacancies from 2.5% to 2.72% accounting for a sizable outward shift in the Beveridge curve. As seen in Figure 2.10, this shock increases the unemployment rate by 0.5 percentage points holding vacancies constant, explaining a bit over 1/3 of the observed shift in the Beveridge curve. For higher levels of unemployment, the shift is smaller analogous to the shape of the Beveridge curve observed in the calibration with a distorted initial state. Moreover, in contrast to the construction/non-construction calibration, the sector-specific shock in this calibration delivers an increase in the natural rate of unemployment that is just a quarter of the shift in the Beveridge curve confirming that the size of Beveridge curve shifts and changes in the natural rate are not necessarily one for one.
2.6 Financial Disruptions as Sectoral Shocks

In this section, we extend our baseline model to illustrate how sector-specific shocks could be represented as financial shocks. If financial shocks are responsible for the shift in the Beveridge curve, then Beveridge curve shifts no longer necessarily imply any changes in the natural rate of unemployment. In particular, it is now possible for monetary easing to counteract any shift in the Beveridge curve since changes in the conduct of monetary policy in and of itself could generate a shift in the Beveridge curve. We show that a binding zero lower bound on the policy rate—effectively a departure from the unconstrained monetary policy rule—operates as a financial shock that disproportionately impacts the financially constrained sector.

2.6.1 Financial Frictions on the Firm Side

To model the effect of financial shocks on the production side, we now assume that some sectors face a working capital constraint of the form considered in Christiano, Eichenbaum, and Evans (2005). Financially constrained firms have to borrow to pay wages and the cost of posting vacancies. For these firms, their optimization problem is slightly modified from the baseline model by introducing a borrowing rate $i^b_t$:

$$\Pi_{i,t} = \max_{\{V_{i,t}, N_{i,t}\}} \mathbb{E}_t \sum_{T=0}^\infty Q^b_{t,T} \left[ P_{t,T} Y_{i,t} - \left( 1 + \frac{i^b_T}{P_T} \right) (W_{i,t} N_{i,t} - \kappa P_T V_{i,T}) \right],$$  \hspace{1cm} (2.32)

s.t. \hspace{0.5cm} N_{i,t} = (1 - \delta_i) N_{i,t-1} + q_{i,t} V_{i,t},  \hspace{1cm} (2.33)

$$Y_{i,t} = A_t N_{i,t}. \hspace{1cm} (2.34)$$

Financially constrained firms’ vacancy posting condition now includes the borrowing rate and changes in expected future borrowing rates:

$$\frac{P_{i,t}}{P_t} \frac{A_t}{1 + i^b_t} = \frac{W_{i,t}}{P_t} + \frac{\kappa}{q_{i,t}} - E_t Q^b_{i,t+1} (1 - \delta_i) \frac{\kappa}{q_{i,t+1}} \frac{1 + i^b_{t+1}}{1 + i^b_t}. \hspace{1cm} (2.35)$$

---

25To introduce financial frictions, we now assume that firms are operated by a distinct set of agents with stochastic discount factor $Q^b_t$. Given our focus on labor market steady states, the entrepreneur’s stochastic discount factor does not enter into the steady state vacancy posting condition.
In steady state, the second term with expected future borrowing rates drops out and changes in the borrowing rate are isomorphic to a negative sector-specific productivity shock as in equation (2.21). We show in the appendix that a collateral constraint as opposed to a working capital constraint would imply the exact same vacancy posting condition. In Curdia and Woodford (2010), and Mehrotra (2012), the borrowing rate is endogenous to monetary policy as the sum of the nominal deposit rate - the instrument of monetary policy - and an exogenous credit spread less changes in expected inflation:

$$1 + i^b_t = (1 + \omega_t) \left( 1 + i^d_t \right) / E_t \Pi_{t+1}.$$

While the credit spread is exogenous, the borrowing rate is not and credit spread shocks may be offset by a reduction in the policy rate or increases in inflation expectations. The presence of a working capital constraint (or other type of financial friction) creates a channel for increasing labor market mismatch between financially constrained and unconstrained sectors, while the effect of the deposit rate on the borrowing rate renders movements in mismatch partially endogenous.

### 2.6.2 Financial Frictions on the Household Side

Analogous to the production side, financial frictions on the household side can generate the same change in relative prices as a sector-specific preference shock does in our baseline model. The most realistic financial friction on the household side involves costs of borrowing for purchasing durable goods as modeled in Campbell and Hercowitz (2005) or Monacelli (2009). Since durable goods are lumpy purchases, households typically borrow to make these purchases.

However, the correspondence between sector-specific preference shocks and financial frictions on the household side can be established in a simpler cash-in-advance type setting. We modify our existing model with two types of households and incomplete markets. Assume that a subset of patient households enjoys a fixed share of national income and carries positive wealth from period to period (in the form of government debt). These households provide
loanable funds in our setup. The impatient households in our economy supply labor (subject to the search frictions and reallocation frictions detailed earlier) and carry zero wealth from period to period since they are subject to a nonnegative wealth constraint that will bind in the steady state. The impatient household consume two types of goods: $C_t$ and $D_t$, but the impatient household must borrow at the beginning of the period to purchase $D_t$, and repay this loan at the end of the period out of income earned from working.

In this setting, the impatient household faces a static optimization problem (in addition to the labor allocation decision detailed in Section 2.3.1):

$$\max_{C_t, D_t, B_t} u(C_t, D_t),$$

subject to:

$$\frac{P_{ct}}{P_t} C_t + \left(1 + i^b_t\right) B_t = \sum_{i=1}^{K} (W_{it}N_{it} + \Pi_{it}),$$

$$\frac{P_{dt}}{P_t} D_t = B_t,$$

where $i^b_t$ is the net borrowing rate and the last constraint requires that borrowing inclusive of interest be repaid in full by the end of the period. Instead of a single set of retailers selling a continuum of differentiated goods, we now assume retailers for both types of goods as in Monacelli (2009). These retailers are identical implying the same markup in each sector.

The optimality conditions for the impatient household determine the relative demand for each good. Under the assumption that $u(C_t, D_t)$ is separable:

$$\lambda_t u_c(C_t) = \frac{P_{ct}}{P_t},$$

$$\lambda_t u_d(D_t) = \left(1 + i^b_t\right) \frac{P_{dt}}{P_t}.$$

Relative consumption demand is now a function of both prices and the borrowing rate:

$$\frac{u_c(C_t)}{u_d(D_t)} = \frac{P_{ct}}{\left(1 + i^b_t\right) P_{dt}}.$$
condition that is analogous to the relative employment condition in our baseline model. When patient households relative demand for consumption goods is small or is very similar to that of the impatient household, it follows that a shock to the borrowing rate changes relative employment shares in the same manner as a sector specific shock to \( \phi \):

\[
\frac{N_{ct}}{N_{dt}} \approx \frac{C_{t}}{D_{t}} = \frac{\phi}{1 - \phi} \left(1 + \frac{\phi}{i_{t}}\right) \left(\frac{P_{ct}}{P_{dt}}\right)^{-1}.
\]

While a shock to the borrowing rate is not isomorphic to a preference shock \( \phi \), changes in borrowing rates shift employment shares and, in the presence of costly labor reallocation, will increase mismatch across sectors.

### 2.6.3 Phillips Curve and Mismatch

Since financial frictions on the firm side fits most naturally into our existing model, we illustrate how a change in the monetary policy rule increases mismatch thereby shifting the Beveridge curve. We log-linearize a two-sector version of model where firms in the financially constrained sector are subject to subject to a working capital constraint and there is no reallocation of labor across sectors. When reservation wages are constant, the firms’ log-linearized vacancy-posting conditions are given as follows:

\[
\begin{align*}
    p_{ct} &= i_{t} + s_{c} \alpha \theta_{ct}, \\
    p_{ut} &= s_{u} \alpha \theta_{ut},
\end{align*}
\]

where \( c \) indexes the financially constrained sector, \( u \) indicates the unconstrained sector and \( 1 - s_{i} \) is the surplus in sector \( i \). When \( s_{c} = s_{u} \), the borrowing rate constitutes a wedge between relative prices; an increase in the borrowing rate drives up the prices disproportionately in the financially constrained sector.

An aggregate Phillips curve is obtained by combining the price index and the log-linearized equilibrium conditions of the retailers, with the latter delivering the standard New Keynesian
Phillips curve along with equations defining aggregate output and relative employment:

\[
\pi_t = \kappa \left\{ \nu \left[ \nu_t^d + \omega + \frac{\alpha}{1 - \alpha} s_c (1 + \epsilon_c) n_{ct} \right] + (1 - \nu) \frac{\alpha}{1 - \alpha} s_u (1 + \epsilon_u) n_{ut} \right\} + \beta E_t \pi_{t+1},
\]

(2.36)

\[
y_t = \gamma n_{ct} + (1 - \gamma) n_{ut},
\]

(2.37)

\[
n_{ct} - n_{ut} = -\eta \left[ \nu_t^d + \omega + s_c (1 + \epsilon_c) \frac{\alpha}{1 - \alpha} n_{ct} - s_u (1 + \epsilon_u) \frac{\alpha}{1 - \alpha} n_{ut} \right],
\]

(2.38)

where \(\gamma\) is the steady state share of output for the constrained sector, \(\nu\) is the steady state share of the price index for the constrained sector, and \(\epsilon_i\) is the ratio or employment to unemployment in each sector. The three equations summarize the supply block of the two-sector model with financial frictions where labor markets are in their flow steady state. If the initial steady state is distorted (i.e. \(P_{ct} \neq P_{ut}\)), output and price level shares need not be equalized. Moreover, these shares will generally differ from employment shares and vacancies shares. In the special case where \(\gamma = \nu = \phi\), these three equations simplify to two equations:

\[
\pi_t = \kappa \left[ \nu \left( \nu_t^d + \omega_t \right) + \frac{\alpha}{1 - \alpha} s (1 + \epsilon) y_t \right] + \beta E_t \pi_{t+1},
\]

(2.39)

\[
n_{ct} - n_{ut} = -\eta \left[ \nu_t^d + \omega + s_c (1 + \epsilon_c) \frac{\alpha}{1 - \alpha} \left( \nu_t^d + \omega_t \right) \right]
\]

(2.40)

and the inflation/output tradeoff is decoupled from the determination of employment shares.

The model is closed by adding the household’s aggregate IS condition and specifying a monetary policy rule. We assume that the exogenous credit shock also affects some subset of borrower households as described in the model of Mehrotra (2012). In that setting, an increase the credit spread delivers a business cycle: a decrease in output, inflation, consumption, and employment. Monetary policy is assumed to follow a standard Taylor rule:

\[
y_t = E_t y_{t+1} - \sigma \left( \nu_t^d + E_t \pi_{t+1} \right) - \sigma_b \omega_t,
\]

(2.41)

\[
\nu_t^d = \phi \pi_t + \phi_y y_t,
\]

(2.42)
where $\sigma$ is the average intertemporal elasticity of substitution and $\sigma_b$ is the elasticity of substitution for the borrower household. A solution to this five equation system (2.36) - (2.38) and (2.41) - (2.42) is a process for $\{n_{ct}, n_{ut}, y_t, \pi_t, i_{t}^d\}$ as a function of the exogenous shock $\omega_t$.

To see how a change in the monetary policy rule shifts the Beveridge curve, it is useful to fix the level of employment $n_t$ and observe that equation (2.38) determines the distribution of employment conditional on the response of monetary policy. To a log-linear approximation, steady state employment is $n_t = \tau n_{ct} + (1 - \tau) n_{ut}$ where employment shares $\tau$ need not match output or price level shares in a distorted steady state. Employment in each sector is given by the expressions:

\[
\begin{align*}
n_{ct} &= \frac{1}{\tau} [n_t - (1 - \tau) n_{ut}], \\
n_{ut} &= \frac{(1 + \eta \lambda_c) n_t + \eta (i_{t}^d + \omega_t)}{(1 + \eta \lambda_c) \frac{1 + \tau}{\tau} + (1 + \eta \lambda_u)},
\end{align*}
\]

where $\lambda_i$ is composite of the other parameters like the sectoral surplus $s_c$. A weaker policy response (decrease in $i_{t}^d$) to the increase in spreads $\omega_t$ will increase the share of employment at unconstrained firms so long as similar size shocks $\omega_t$ are needed to deliver the same level of employment under each policy$^{26}$. This change in the distribution of employment shifts the Beveridge curve since total vacancies are also a function of the distribution of employment.

As shown below, vacancies are equal to:

\[
\begin{align*}
v_t &= \frac{1}{V_c} \frac{1 + \alpha \epsilon_c}{1 - \alpha} n_{ct} + \frac{1}{V_u} \frac{1 + \alpha \epsilon_u}{1 - \alpha} n_{ut}, \\
&= \frac{1}{V_c} \frac{1 + \alpha \epsilon_c}{1 - \alpha} n_t + \left( \frac{1}{V_c} \frac{1 + \alpha \epsilon_u}{1 - \alpha} - \frac{1 - \tau}{\tau} \frac{1}{V_c} \frac{1 + \alpha \epsilon_c}{1 - \alpha} \right) n_{ut},
\end{align*}
\]

where the second equality is obtained by expressing employment in the constrained sector in terms of total employment and employment in the unconstrained sector. So long as uncon-
strained firms face a tighter labor market or account for a disproportionate share of vacancies (relative to their employment share), the coefficient on $n_{it}$ will be positive and the increase at vacancies at these firms will more than offset the fall in vacancies at the constrained firms shifting the Beveridge curve outward.

In addition to offering an explanation for the shift in the Beveridge curve, the interaction of the zero lower bound and financial frictions at the firm level also offers a potential explanation for the relative stability of inflation in the US despite persistently high unemployment. A credit shock, by affecting firms’ costs of production, raises marginal costs for constrained firms. This rise in costs for constrained firms partially offsets the fall in marginal costs from decreasing employment. The financial frictions channels dampens downward pressure on prices, limiting the degree of deflation and, depending on the relative strength of these channels, possibly generating higher inflation. Standard ZLB models in the spirit of Eggertsson and Woodford (2003) have difficulty generating long-lasting zero lower bound episodes without predicting counterfactually high levels of deflation (see Mehrotra (2012)). While extreme downward rigidity in wages could also explain the absence of outright deflation, the presence of a supply-side channel for financial frictions offers another realistic channel to account for stable inflation at the zero lower bound.

2.7 Conclusion

Discussions about the slow recovery in the US following the Great Recession have raised the possibility of sectoral shocks. Proponents of this view have cited the disproportionate impact of the recession on housing-related industries and the shift in the Beveridge curve as evidence of sector-specific shocks. We investigate the role of sector-specific shocks and their impact on the Beveridge curve empirically and theoretically.

On the empirical side, a factor analysis of sectoral employment in the postwar data is used to isolate sector-specific shocks while addressing the Abraham and Katz critique. We derive a sector-specific shock index and show that this index is elevated in the current period and distinct from the business cycle or the Lilien measure of sectoral shocks. Moreover, we show
that this measure of sector-specific shocks is elevated in those periods when the Beveridge curve shifts.

On the theoretical side, we build a multisector model with labor market search to investigate how sector-specific shocks affect equilibrium variables like the aggregate Beveridge curve and the level of employment. Our model shows that sector-specific shocks generally shift the Beveridge curve through a composition channel due to differences in hiring costs and hiring technology across sectors and a mismatch channel due to segmentation in labor markets. We show analytically that, through the composition effect, sectoral shocks can raise or lower the natural rate of unemployment, while the mismatch effect always raises the natural rate of unemployment. Moreover, in our baseline model, sectoral shocks that shift the Beveridge curve must also change the natural rate of unemployment.

We calibrate a two-sector version of our model and show that a negative preference shock to the construction sector that matches the distribution of employment shares at the recession trough generates a shift in the Beveridge curve that matches the magnitude of the shift observed in the data. This shock raises the natural rate of unemployment by a quantitatively similar level as the shift in the Beveridge curve - the natural rate rises 1.4 percentage points and results are robust if goods are moderate substitutes instead of complements.

Finally, we show that financial shocks act like sector-specific shocks and can also generate a shift in the Beveridge curve if a subset of firms is financially constrained. In this richer setting, a change in the conduct of monetary policy can generate a shift in the Beveridge curve by magnifying the effect of financial constraints. For example, if monetary policy switches from a Taylor rule to a fixed nominal rate due to a binding zero lower bound, financial constraints will lead to a higher level of mismatch across sectors. These changes in mismatch due to a binding zero lower bound can still be addressed through unconventional monetary policy such as price level targets or credit easing.

As noted in our quantitative results, the assumption of costly or no labor reallocation is crucial in generating the observed persistence of the shift in the Beveridge curve. Existing evidence suggests somewhat contradictory findings on the pace of labor reallocation. Ob-
erved transition rates in the CPS and the size of cross-sector flows suggest relatively frequent transitions across sectors. However, evidence from the Displaced Worker Survey and an extensive literature studying labor market outcomes after job loss point to fairly high costs to reallocation. Future research will seek to reconcile these findings to determine the business cycle cost of labor reallocation and dimensions of heterogeneity along which workers do not readily transition.
Chapter 3

Job Flows and Financial Shocks

with Dmitriy Sergeyev

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1We would like to thank Frederic Mishkin, Andreas Mueller, Emi Nakamura, Ricardo Reis, Jón Steinsson and Michael Woodford for many helpful discussions and seminar participants in the Columbia University Macro Lunch for their comments. Neil Mehrotra thanks the Kauffman Foundation for financial support through the Kauffman Dissertation Fellowship.
3.1 Introduction

The labor market recovery since the trough of the Great Recession in 2009 has been characterized by a pronounced decline in labor market churn. As shown in Figure 3.1, job creation and job destruction during the recovery have fallen relative to their pre-recession averages, despite positive net job creation during each period. Haltiwanger, Jarmin, and Miranda (2011) also document the sharp fall in job creation in the Great Recession, and Haltiwanger, Jarmin, and Miranda (2012) show an acceleration in the decline of employment and job creation among young firms.

Figure 3.1: US job flows, 2000-2012

A broad literature has documented the importance of overall labor market turnover in the process of labor reallocation and productivity growth\(^2\). Moreover, higher job flow rates in the time series and cross section typically coincide with a healthier labor market, characterized by higher levels of employment growth and lower unemployment rates. The decline in job flows exhibited in the Great Recession may provide clues explaining the slow recovery in the labor market.

Recent work by Haltiwanger, Jarmin, and Miranda (2012) demonstrates that new and

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\(^2\)For an overview, see Haltiwanger (2012) and Davis and Haltiwanger (1999)
young firms account for a disproportionate share of job creation and job destruction due to their up and out pattern of expansion. Using 2000-2006 averages from the Business Dynamics Statistics, new firms account for about 3% of employment but for nearly 17% of job creation. Similarly, young firms (defined as firms 5 years or younger) account for 12% of employment but 16% of job creation and 21% of job destruction respectively. Given the disproportionate contribution of new and young firms to labor market churn, some economists in the business press have speculated that the credit crisis may have had a disproportionate impact on young businesses. They argue that a decline in house prices may impair the formation of new firms and the expansion of existing firms by reducing the value of collateral and thereby restricting their access to finance. Figure 3.2 provides some suggestive support for this hypothesis, showing a strong correlation between the decline in house prices at the state level and the decline in job creation from expanding establishments as measured in the Business Employment Dynamics.

In this paper, we argue in favor of this hypothesis - that the financial crisis and sharp decline in housing prices explain the large and persistent decline in job creation and job
destruction. In particular, we argue that a collateral shock tightens the availability of credit for new firms and young expanding firms leading to a decline in aggregate job creation and job destruction. We investigate this hypothesis both empirically and theoretically.

In our empirical work, we provide direct evidence that a decline in collateral values diminishes job creation and lagged job destruction using state-level data from the Business Dynamics Statistics (BDS). We exploit state-level variation in job flows and housing prices to examine the effects of movements in state housing prices on job flows. To address issues of causality, we control for state and time fixed effects and add direct controls for the state business cycle. We also utilize an IV approach based on the differences across states in their sensitivity to movements in aggregate US house prices. This land supply elasticity approach - used to examine the effect of real estate shocks on investment - is applied here to examine the effect of housing prices on job flows.

To analyze the theoretical effect of a collateral shock on job flows and employment, we build a model of firms dynamics with financial frictions and decreasing returns to scale. Newly born firms and young firms accumulate assets and expand towards their efficient scale. Mature firms are financially unconstrained and are free to expand or contract depending on idiosyncratic shocks to firm productivity. Firms differ in productivity levels so that some businesses remain small without any binding financial constraint. Our model is calibrated to match the average size and age distribution of employment in the Business Dynamics Statistics.

Our empirical results show that a shock to housing prices reduces job creation persistently and reduces job destruction with a lag. These results hold under both the OLS and IV specifications and are robust to alternative controls for the state business cycle. Moreover, we document differences across firm age and size categories in the sensitivity of job flows to housing price shocks. In particular, we find that job creation for middle-sized firms (firms with 20-99 employees) and new firms exhibit the strongest sensitivity to housing price shocks. Similar patterns hold for job destruction with middle-size firms and young firms (less than 5 years old) exhibiting a long-run decline in job destruction in response to a decline in housing prices.
Using our model of firm dynamics, we show that collateral shocks diminish job creation and job destruction in steady state. Moreover, the collateral shock replicates the empirical pattern of job flows sensitivity across firm size and age. The collateral shock reduces job creation for larger firm size categories by reducing employment demand for credit-constrained firms and shifting the size distribution towards smaller firms. This effect is partially offset by the general equilibrium effect of a decline in wages which causes unconstrained firms to become larger. As a result, job flows for middle-sized firms exhibit the greatest sensitivity to housing price shocks. Likewise, a tightening collateral constraint reduces job creation for new and young firms by increasing the required asset level to achieve a given level of employment. Collateral shocks diminish job destruction by shrinking the size of firms leading to lower job destruction from firm deaths. Our model highlights the various mechanisms culminating in lower job flows from a collateral tightening and replicates the patterns in job flow sensitivity documented in our empirical work.

### 3.1.1 Related Literature

Our paper is related to several strands of literature. Firstly, our empirical work is related to a literature documenting the real effect of housing price shocks. Recent papers by Gan (2007) and Chaney, Sraer, and Thesmar (2012) examine the effect of collateral shocks on firm investment. Chaney, Sraer, and Thesmar (2012) use firm-level financial data to show that a decline in the value of real estate for a firm’s headquarters has a statistically significant effect of firm investment. Adelino, Schoar, and Severino (2013) documents that small business starts and employment levels showed a strong sensitivity to increases in housing prices during the boom years from 2002-2007. Both papers use the land supply elasticity instruments proposed in Saiz (2010), and our IV strategy follows a similar approach. Our paper focuses on the effect of housing prices on job flows.

Likewise, our work draws on and contributes to an empirical literature documenting differences in job flows across firm size and age categories. The influence of startups and young firms on job creation and job destruction is documented in Haltiwanger, Jarmin, and Miranda
(2010), but we establish facts about the sensitivity of job flows across firm size and age to housing prices. Our empirical work is closest to contemporaneous work by Fort, Haltiwanger, Jarmin, and Miranda (2012) that examines the cyclical role of housing prices on employment and job flows. Our results are consistent with their results and differ primarily in the use of an instrument variables approach to establish the causal effect of housing price shocks on job flows by firm size and age. Additionally, we reconcile the empirical patterns we document with a firm dynamics model with financial frictions.

Finally, our work is related to an emerging literature on quantitative firm dynamics models. Our model comes closest to Khan and Thomas (2011) who study the effect of a credit shock in a model with collateral constraints and firm-specific capital. They find that credit shock recessions behavior quite differently than productivity-driven recession, but they do not explore the implications for their mechanism on job flows. Gomes (2001) and Cooley and Quadrini (2001) also build firm dynamics models with various financial frictions to fit facts on the firm age and firm size distribution and stylized facts about the financing of small versus large businesses.

The paper is organized as follows: Section 3.2 discusses our data and presents empirical results on the link between collateral values and job creation and job destruction. Section 3.3 presents a simple continuous time firm dynamics model and characterizes firm behavior. Section 3.4 discusses our benchmark discrete time firm dynamics model. Section 3.5 describes our calibration strategy and shows the quantitative effect of a collateral shock. Section 3.6 concludes.

3.2 Empirical Strategy and Results

3.2.1 Empirical Strategy

Any test of the hypothesis that an increase in financial frictions diminishes job flows must overcome several challenges of both measurement and causality. Our empirical strategy addresses these issues by using state-level variation in job flows and financial conditions to determine
the causal effect of increased financial frictions on job flows.

The first issue we confront is finding suitable proxy for financial conditions at the state level. Since financial constraints are not typically observable, we use data on the growth rate of state house prices as a proxy for financial conditions. To the extent that lending to firms is secured by either the firm’s real estate or the owner’s real estate, movements in housing prices should directly affect the ability of a firm to obtain financing. For firms with access to corporate debt and equity markets, housing price movements may be a poor proxy for financial conditions. However, the vast majority of firms do not issue debt or equity securities, instead relying upon bank financing or other forms of collateralized finance. Fairlie and Krashinsky (2012) provide direct evidence for changes in housing equity on entrepreneurship using data from the Current Population Survey. Data from the Survey of Small Business Finances and aggregate balance sheet data from the Flow of Funds also suggests the importance of real estate collateral for firm credit. Moreover, as noted in our discussion of related literature, an extensive literature documents the importance of real estate prices for both investment and employment.

In addition to finding a suitable proxy for financial frictions, the relative dearth of time-series data on aggregate job flows limits the analysis of the relationship of financial frictions and job flows in the aggregate data. Instead, we exploit state-level variation in job flows and housing prices to improve the power of our estimates and increase useful variation from state and regional housing price booms.

The most significant challenge in establishing a causal effect of housing price movements on job flows is ruling out an aggregate demand channel that drives the correlation between job flows and housing prices. We address this concern in several ways. Firstly, we include state and time fixed effects to account for the business cycle and differences across states in job flows. Secondly, to control for state-specific demand shocks, we include measures for the state business cycle. Our baseline regression takes the following form:

$$y_{it} = \alpha_i + \delta_t + \gamma (L) \Delta GSP_{it} + \beta (L) \Delta hp_{it} + \epsilon_{it}$$
where \( y_{it} \) is job creation or job destruction for state \( i \) at time \( t \). \( \Delta GSP_{it} \) represents the growth rate of the state-level business cycle variable, while \( \Delta hp_{it} \) is the growth rate of state housing prices. Our coefficient of interest is the sum of the coefficients \( \beta(1) \) on state housing prices.

Alternatively, we also adopt an IV strategy following the methodology laid out in Saiz (2010) where differences in land supply elasticities generate differential responses in house prices to national shocks. Specifically, we use movements in national housing prices interacted with a state dummy variable as an instrument for state housing prices. The identifying assumption is that whatever causes movements in national house prices is uncorrelated with state-specific aggregate demand shocks. Our IV strategy is similar to the methodology used in Nakamura and Steinsson (2011) in their study of government spending multipliers. The authors use movements in national government defense spending as an instrument for state government spending by exploiting differences in state sensitivity to government defense expenditures. Our IV regression takes the following form:

\[
\begin{align*}
y_{it} &= \alpha_i + \delta_t + \beta(L)\Delta \overline{hp}_{it} + \epsilon_{it} \quad \text{(2nd stage)} \\
\Delta hp_{it} &= \alpha_i + \delta_t + \rho_i(L)\Delta hp_t + u_{it} \quad \text{(1st stage)}
\end{align*}
\]

where \( \Delta \overline{hp}_{it} \) is the fitted value for state house prices obtained from the first-stage regression of state house prices on national house prices. As before, the coefficient of interest is the sum of coefficients \( \beta(1) \) measuring the effect of housing prices on job flows.

Together the OLS specification and IV specification attempt to control for an aggregate demand channel and establish the causal affect of movements in house prices on gross job creation and job destruction.

### 3.2.2 Data

We draw on several distinct data sources for measures of job flows, house prices, and state measures of the business cycle. Data on job flows comes the Business Dynamics Statistics compiled by the US Census Bureau. The Business Dynamics Statistics is drawn from the Census Bureau’s Longitudinal Database (LBD), a confidential database that tracks employ-
ment at the establishment and firm level over time. The Business Dynamics Statistics report job creation and job destruction by firm age and size categories at the state level; prior to the development of BDS, these data were only available to researchers with access to confidential Census microdata. The job flows data in the BDS is drawn from Census Bureau’s Business Register, which consists of the population of firms and establishments with employees covered by unemployment insurance or filing taxes with the Internal Revenue Service³.

Specifically, we use data on gross job creation and job destruction at the state level from 1982-2010, where job creation measures the increase in employment at new firms or expanding firms and job destruction measures the decrease in employment at exiting firms or contracting firms. Firm level employment is recorded in March of each year and job flows are measured with respect to employment in the previous year. Our data set includes job flows from 50 states and the District of Columbia resulting in a balanced panel of 29 x 51 observations.

Our house price data comes from the Federal Housing Finance Agency’s state level house price indices. We use the all-transactions indexes which provide a quarterly time series of housing prices from 1975 to present. These data are not seasonally adjusted, but we use year-over-year changes in the log of the house price index as our measure of state housing price growth. National housing prices are measured in the same way using the national house price index⁴.

State-level business cycle measures come from the Bureau of Economic Analysis (BEA). Our baseline measure for the state business cycle is the growth rate of gross state product. We use measures of annual gross state product and compute the growth rate as the change in the log of gross state product. Since job flows are measured as of March in a given year, we use the growth rate of gross state product in the previous year. For example, an observation of job creation for a given state in 2010 is matched with the growth rate of gross state product in 2009. Since housing prices are reported quarterly, no similar lag is required for house price growth. In addition to gross state product, we also use personal income growth and

³A more complete description of the BDS and access to job flows data is available at http://www.census.gov/cesdataproduc ts/bds/.

⁴Housing price data may be downloaded from: http://www.fhfa.gov.
employment growth as alternative proxies for the state business cycle using BEA regional data on personal income and employment.

3.2.3 Empirical Results

Aggregate Job Flows

Table 3.1 displays the coefficients of state housing price growth on job creation and job destruction at the state-level. State job creation and job destruction are converted to logs and detrended using a linear state-specific time trend. As Table 3.1 shows, both the OLS and IV specifications give statistically significant coefficients for state house prices on job creation on impact and with a lag. For job destruction, the impact effect of house prices is negative, but the lagged coefficient is positive implying that a decline in house prices reduces lagged job destruction. It is worth noting that since the sample ends in March 2010, our conclusions for the effect of house prices on job flows are exploiting variation that does not include any

Table 3.1: Effect of housing prices on job flows

<table>
<thead>
<tr>
<th></th>
<th>Job Creation</th>
<th></th>
<th>Job Destruction</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS (2 lag)</td>
<td>IV (2 lag)</td>
<td>IV (2 lag)</td>
<td>OLS (2 lag)</td>
</tr>
<tr>
<td>Housing price growth</td>
<td>0.211**</td>
<td>0.603**</td>
<td>1.067**</td>
<td>-0.316**</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.140)</td>
<td>(0.122)</td>
<td>(0.084)</td>
</tr>
<tr>
<td>Housing price growth _t-1</td>
<td>0.153**</td>
<td>0.111**</td>
<td>0.943**</td>
<td>-0.013</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.128)</td>
<td>(0.089)</td>
<td>(0.060)</td>
</tr>
<tr>
<td>Housing price growth _t-2</td>
<td>-0.029</td>
<td>-0.115</td>
<td>-0.479**</td>
<td>0.337**</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.127)</td>
<td>(0.094)</td>
<td>(0.070)</td>
</tr>
<tr>
<td>State Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Time Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>1479</td>
<td>1479</td>
<td>1479</td>
<td>1479</td>
</tr>
<tr>
<td>Hprice + Hprice_t-1 + Hprice_t-2</td>
<td>0.335**</td>
<td>0.598**</td>
<td>1.531**</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.121)</td>
<td>(0.081)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>First-stage F-test</td>
<td>2.8</td>
<td>7.4</td>
<td>2.8</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>2.8</td>
<td>5.8</td>
<td>2.8</td>
<td>5.8</td>
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<td></td>
<td>2.7</td>
<td>4.8</td>
<td>2.7</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Notes:
**Coefficient significant at the 5% level
of the slow recovery after the Great Recession.

Table 3.1 also computes the sum of the coefficients on housing prices. For job creation, the sum of the coefficients is positive and statistically significant indicating that house price movements have a persistent effect on job creation. For job destruction, the sum of the coefficients under the baseline OLS and IV specifications is not statistically different from zero. However, excluding time fixed effects, the IV specification delivers a positive long-run coefficient for housing prices on job destruction, implying that housing price shocks diminish job destruction. The specification without fixed effects also displays higher first stage F statistics. While somewhat low first-stage F statistics may raise concerns about weak instruments, since coefficients under the IV specification are higher than the OLS coefficients, our estimated coefficients would be biased downward suggesting that we underestimate the effect of house prices on job creation and lagged job destruction.

Category-Specific Job Flows

To further examine the effect of housing prices on job flows, we decompose the effect of housing prices on job flows by firm size and firm age. As before, we utilize both the OLS and IV specifications. Our OLS specification is a generalization of the state-level job flows regression:

\[ y_{iht} = \alpha_i + \delta_t + \kappa_h + \gamma_h (L) \Delta GSP_{it} + \beta_h (L) \Delta hp_{it} + \epsilon_{iht} \]

where \( y_{iht} \) is job creation or job destruction for state \( i \), in year \( t \) and category \( h \). In addition to state and time fixed effects, we include category fixed effects. In these regressions, we allow state house prices to have differential effects on job flows across categories, and our coefficient of interest is \( \beta_h(1) \) - the sum of coefficients of state house prices by category. The IV specification is analogous to the IV specification for aggregate job flows, where the

\footnote{Figure 3.1 uses a different data set, the Business Employment Dynamics, maintained by the Bureau of Labor Statistics that is available with a shorter delay.}
Table 3.2: Effect of housing prices on job flows by initial firm size

<table>
<thead>
<tr>
<th>Hprice + Hprice_t-1 + Hprice_t-2</th>
<th>Job Creation OLS (2 lag)</th>
<th>IV (2 lag)</th>
<th>Job Destruction OLS (2 lag)</th>
<th>IV (2 lag)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-19 employees</td>
<td>0.158**</td>
<td>0.128</td>
<td>-0.161*</td>
<td>-0.305**</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(0.114)</td>
<td>(0.100)</td>
<td>(0.094)</td>
</tr>
<tr>
<td>20-99 employees</td>
<td>0.600**</td>
<td>1.096**</td>
<td>0.144</td>
<td>0.123</td>
</tr>
<tr>
<td></td>
<td>(0.101)</td>
<td>(0.120)</td>
<td>(0.122)</td>
<td>(0.080)</td>
</tr>
<tr>
<td>100+ employees</td>
<td>0.341**</td>
<td>0.824**</td>
<td>0.005</td>
<td>0.062</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.109)</td>
<td>(0.091)</td>
<td>(0.102)</td>
</tr>
<tr>
<td>State Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Time Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Category Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>4437</td>
<td>4437</td>
<td>4437</td>
<td>4437</td>
</tr>
<tr>
<td>H = 20-99 employees - 1-19 employees</td>
<td>0.442**</td>
<td>0.968**</td>
<td>0.305**</td>
<td>0.428**</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.118)</td>
<td>(0.051)</td>
<td>(0.064)</td>
</tr>
</tbody>
</table>

Notes:
**Coefficient significant at the 5% level
*Coefficient significant at the 10% level

instrument is now national house price movements interacted with a state-category dummy:

\[ y_{iht} = \alpha_i + \delta_t + \kappa_h + \beta_h(L) \Delta \hat{h}_{pit} + \epsilon_{iht} \]  
(2nd stage)

We first consider job flows by initial firm size, and consider three categories: small firms (1-19 employees), medium-sized firms (20-99 employees), and large firms (100+ employees). Initial firm size assigns firm size categories based on employment in the previous year to avoid reclassification bias (see Moscarini and Postel-Vinay (2012) for a discussion) which would conflate job flows for firms with differing initial conditions.

Table 3.2 displays the results from the category-specific regression of job creation and job destruction on housing prices. The table shows the sum of coefficients on state housing prices, \( \beta_h(1) \) under the OLS and IV specifications respectively. For job creation, middle-sized firms exhibit the highest sensitivity to housing prices, followed by large firms and small firms respectively. In the case of the IV specification, the coefficient of housing prices on job creation
for small firms is not statistically different from zero. For job destruction, the order of the coefficients on housing prices across categories follows the same ordering as for job creation. Job destruction for middle-sized firms display a positive coefficient on housing prices, but the coefficient is not statistically significant. In contrast, the coefficient on job destruction for small firms is negative and statistically significant.

Table 3.2 also shows that the difference in coefficients between middle-sized firm and small firms is statistically significant across all specifications for both job creation and job destruction. In contrast, the difference for middle and large sized firms is generally not significant. In summary, we find evidence that job creation at middle-sized firms is most sensitive to housing price movements and job destruction at middle-sized firm responds negatively to a decrease in housing prices.

We also consider job flows by firm age categories: new firms, young firms (1-5 years of age), and mature firms (6+ years of age). These firm age categories are same categories used in Haltiwanger, Jarmin, and Miranda (2010). Table 3.3 shows that job creation at new firms

Table 3.3: Effect of housing prices on job flow by firm age

<table>
<thead>
<tr>
<th></th>
<th>Job Creation</th>
<th>Job Destruction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS (2 lag)</td>
<td>IV (2 lag)</td>
</tr>
<tr>
<td>Births</td>
<td>0.608**</td>
<td>0.932**</td>
</tr>
<tr>
<td>(0.128)</td>
<td>(0.135)</td>
<td></td>
</tr>
<tr>
<td>Young Firms (1-5 yrs)</td>
<td>0.337**</td>
<td>0.661**</td>
</tr>
<tr>
<td>(0.129)</td>
<td>(0.148)</td>
<td></td>
</tr>
<tr>
<td>Mature Firms (6+ yrs)</td>
<td>0.311**</td>
<td>0.624**</td>
</tr>
<tr>
<td>(0.095)</td>
<td>(0.118)</td>
<td></td>
</tr>
<tr>
<td>State Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Time Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Category Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>4437</td>
<td>4437</td>
</tr>
<tr>
<td>H = Births - Mature/Young - Mature</td>
<td>0.297**</td>
<td>0.309**</td>
</tr>
<tr>
<td>(0.090)</td>
<td>(0.099)</td>
<td>(0.084)</td>
</tr>
</tbody>
</table>

Notes:
**Coefficient significant at the 5% level
exhibit the strongest response to housing prices followed by job creation at young and mature firms respectively, with all coefficients statistically significant under both specifications. By definition, new firms have zero job destruction. Job destruction at young firms shows a positive and statistically significant coefficient on housing prices, while the sensitivity of job destruction at mature firms to housing prices is not different from zero. As the last row of Table 3.3 shows, the difference in coefficients on job creation for new firms versus mature firms is statistically significant, as is the difference in coefficients on job destruction for young firms versus mature firms.

**Firm Entry and Exit**

In addition to documenting the effect of housing prices on job flows, we can also examine the behavior of firm entry and exit due to housing price shocks. Firm entry or exit on the state level is expressed in logs. The first column of Table 3.4 displays the effect of housing prices on firm entry rates, showing a strong effect of housing price movements on firm entry rates under both OLS and IV specifications. The second column of Table 3.4 looks at firm death and exit for firms of differing age.
rates for firms 1 year of age. We find some evidence that housing price shocks initially lead to a higher death rate but subsequently reduce death rates in the next period. This is consistent with the effect of house price shocks on firm entry - if a negative house price shock reduces entry on impact, then it should lower firm exits for firms 1 year of age with a lag. Similarly, the last column of Table 3.4 provides evidence that house price shocks reduce firm exit with a 2 period lag consistent with a strong effect of housing prices on firm entry. The sum of the coefficients for house prices on firm exits are either poorly estimated or inconclusive in sign providing no definitive evidence of an effect of housing prices on firm deaths.

3.3 Simple Model

In this section, we build a simple continuous time firm dynamics model to characterize the behavior of a collateral shock on asset accumulation, employment, and job flows. This simple model shows the mechanisms at work that causes a collateral shock to diminish employment and job creation at young firms and middle-sized firms.

The core framework is a real business cycle model. To this we add (i) a financial friction that limits the amount of borrowing, (ii) firm heterogeneity, (iii) a non-CRS production technology.

The economy consists of three types of agents: households, heterogeneous firms, and intermediaries. Each household consumes, supplies labor and trades in a market for capital. The household consists of measure $n$ of workers. Workers supply labor to firms and return their wages to the household. Each firm hires workers from households and buys capital from intermediaries to produce. Intermediaries issue one-period real risk-free bonds and rent capital to firms. Every period $\sigma$ firms die and transfer their assets to the household and $\sigma$ new firms are born; these entrepreneurs receive an initial transfer of assets from the household. There is a single consumption good in the economy that serves as the numeraire good. There are two types of assets in the economy: capital and the risk-free one period real bonds. Capital can be freely converted to the consumption good and vice versa using one-to-one technology.

There is no aggregate uncertainty in the economy. The only idiosyncratic uncertainty in
the economy is the risk of death for individual firms.

3.3.1 Households

Let \( c(t) \) be consumption and \( n(t) \) be labor supply. Then household preferences are given by the following expression

\[
\int_0^\infty e^{-\rho t} U [c - v(n)] dt. \tag{3.1}
\]

The household faces a budget constraint

\[
\dot{a} = wn + ra + \Pi - c, \tag{3.2}
\]

where \( r \) is the return on household assets \( a \), \( \rho \) is the rate of time preference, \( \Pi \) is net payouts to the household from the ownership of firms, \( wn \) is household labor income. We assume preferences with no wealth effect on labor supply to simplify the analysis of equilibrium in the labor market.

The household takes its initial assets and the equilibrium behavior of prices as given. In addition, for the problem to be well-defined we add the natural debt limit constraint

\[
a(t) \geq - \int_t^\infty \left[ w(t)n(t) + \Pi(t) \right] \left[ - \int_t^s r(s) ds \right] ds.
\]

3.3.2 Firms

The economy is composed of a measure 1 of firms which produce homogeneous output. Firms behave competitively on the output, capital and labor markets. Each firm faces an exogenous rate of exit \( \sigma \) in which case the firm transfers its assets to the household and disappears. Every period \( \sigma \) firms exit and \( \sigma \) new firms are born with an initial endowment of assets \( a_0^6 \).

Each firm has productivity of \( z\epsilon \), where \( z \) is common across firms while \( \epsilon \) is a firm-specific productivity. Both values are constant over time for a given firm. We assume that \( \epsilon \in \{\epsilon_L, \epsilon_H\} \)

\[6^a_6 \text{a}_0 \text{ will be chosen to match the average share of employment at new firms in the US.} \]
with $\epsilon_L < \epsilon_H$. We also assume that the probability of being born with a high firm-specific productivity is $\mu$, i.e., $Pr(\epsilon = \epsilon_H) = \mu$.

Let the firms use $\Lambda_{t,t+\tau} = e^{-\rho \tau} U'(c_{t+\tau}) / U'(c_t)$ as their discount factor between periods $t$ and $t + \tau$. Each firm maximizes its terminal wealth. Formally, each firm maximizes

$$\max_{\{n_{t+\tau}, k_{t+\tau}\}} \int_0^\infty e^{-\sigma \tau} \Lambda_{t,t+\tau} a_{t+\tau} d\tau,$$

where $a_{t+\tau}$ is wealth of the firm in period $t + \tau$, $k$ is the amount of capital the firm decides to rent in period $t$. The firm faces two constraints. First, the wealth accumulation equation:

$$\dot{a} = \pi + ra,$$

where

$$\pi = z\epsilon (k^\alpha n^{1-\alpha})^\phi - r_k k - wn,$$

where first term $z\epsilon (k^\alpha n^{1-\alpha})^\phi$ represents diminishing-returns-to-scale production function. The second and the third terms represents cost of capital and labor inputs respectively.

Second, the firm faces financial constraint of the following form:

$$k \leq \chi a,$$

where $\chi$ denotes the borrowing capacity which is common across firms; $\chi = \infty$ corresponds to frictionless capital rental markets, and $\chi = 1$ to self-financing. This specification reflects the prediction of financial contracts in models with limited contract enforcement. See Evans and Jovanovic (1989) for an early use of this specification of the financial constraint.\(^7\)

\(^7\)From Buera and Shin (2011): “Our collateral constraint can be derived from the following limited enforcement problem. Consider an individual with financial wealth $a \geq 0$ deposited in the financial intermediary at the beginning of a period. Assume that the firm rents $k$ units of capital. Then it can escape with fraction $1/\lambda$ of the rented capital. The only punishment is that the firm will lose its financial wealth $a$ deposited in the intermediary. In particular, the firm will not be excluded from any economic activity in the future. The firm is even allowed to instantaneously deposit the stolen capital $k^\alpha$ and continue operating. (This assumption is essential for obtaining the simple static collateral constraint. If there are any dynamic considerations, the constraint will also depend on the shock realization and its persistence.) In the equilibrium, the financial intermediary will rent capital only to the extent that no firm will renege on the rental contract: $k/\lambda \leq a.$”
3.3.3 Intermediaries

Households and firms have access to competitive intermediaries that receive deposits (issue risk-free one-period bonds) and rent out capital at rate $r_k$ to firms\(^8\). The zero profit condition of the competitive intermediaries implies:

$$r_k = r + \delta,$$

(3.7)

where $\delta$ is the depreciation rate of capital.

3.3.4 Competitive Equilibrium

A competitive equilibrium is paths for \(\{c(t), a^H(t), n^H(t), a^F(t), n^F(t), k(t), w(t), r(t), r_k(t)\}_{t=0}^{\infty}\) such that

1. households optimize: solve (3.1) and (3.2) given initial level of assets $a^H(0)$ taking prices \(\{w(t), r(t)\}_{t=0}^{\infty}\) as given

2. firms optimize: solve (3.3) - (3.5) given initial level of assets $a^F(0)$ taking prices \(\{w(t), r(t)\}_{t=0}^{\infty}\) as given

3. markets clear:

   (a) the capital market clears:

   $$\mu \sigma \int_0^\infty e^{-\sigma t} k(t, \epsilon_H) dt + (1 - \mu) \sigma \int_0^\infty e^{-\sigma t} k(t, \epsilon_L) dt$$
   
   $$= \mu \sigma \int_0^\infty e^{-\sigma t} a^F(t, \epsilon_H) dt + (1 - \mu) \sigma \int_0^\infty e^{-\sigma t} a^F(t, \epsilon_L) dt + a^H(t) \text{ for all } t;$$

   (b) the labor market clears:

   $$n^H(t) = \mu \sigma \int_0^\infty e^{-\sigma t} n(t, \epsilon_H) dt + (1 - \mu) \sigma \int_0^\infty e^{-\sigma t} n(t, \epsilon_L) dt \text{ for all } t.$$

\(^8\)Following Buera and Shin (2011) we assume that firm cannot borrow intertemporally.
### 3.3.5 Characterization

In this section we characterize the steady state equilibrium of the economy in which prices are constant over time but firms enter and exit at rate $\sigma$.

Household optimality requires:

$$\dot{c} = -\frac{u'(c)}{u''(c)}(r - \rho), \quad (3.8)$$

$$w = v'(n). \quad (3.9)$$

The first equation is the standard continuous time Euler equation. The second line is the labor supply equation which equates real wage with the marginal disutility of working. See Appendix C.1 for the derivation details. Equation (3.8) immediately implies that the real interest rate is equal to the discount factor in the steady state, i.e., $r = \rho$.

To describe firm problem solution we specify the Hamiltonian for the firm’s problem:

$$\mathcal{H} = e^{-\sigma t}A_{0,t}a + \lambda_F \left[ z(1-\alpha)n(1-\alpha) - r_k k - wn + ra \right] - \eta[k - \chi a]$$

The maximum principle implies:

$$\mathcal{H}_k = \lambda_F [z(1-\alpha)n(1-\alpha) - r_k] - \eta = 0, \quad (3.10)$$

$$\mathcal{H}_n = \lambda_F [z(1-\alpha)n(1-\alpha) - w] = 0, \quad (3.11)$$

$$\dot{\lambda}_F = - \left\{ e^{-\sigma t}A_{0,t} + \lambda_F r + \eta \chi \right\}, \quad (3.12)$$

$$k \leq \chi a, \quad \eta \geq 0, \quad \eta[k - \chi a] = 0. \quad (3.13)$$

**Case 1** Assume that the collateral constraint binds. This implies that $\eta > 0$ and $k = \chi a$.

Optimality with respect to labor, equation (3.11), implies:

$$n = \left[ \frac{z(1-\alpha)}{w} \chi^{\alpha\phi a}\right]^{1/(1-\alpha)}. \quad (3.14)$$
Substituting optimal employment and capital in output function we can rewrite the law of motion for assets as follows

\[ \dot{a} = Aa^\psi - Ba, \]

where

\[ A = (z\epsilon\chi^{\alpha\phi})^{\frac{1}{1-\alpha\phi}} \frac{\phi(1-\alpha)}{w} \]
\[ B = r_k \chi - r; \psi = \phi\alpha/(1-\phi)(1-\alpha) < 1. \]

**Lemma 3.1.** The solution to the law of motion of a capital-constrained firm is

\[ a = \left\{ \frac{A}{B} - \left( \frac{A}{B} - a_0^{-\psi} \right) e^{-B(1-\psi)t} \right\}^{1/(1-\psi)}, \]

(3.15)

where \(a_0\) is the initial level of wealth.

Solution \(a(t, \chi, \epsilon)\) is monotonic in \(t\), can be convex or concave in \(t\), non-monotonic in \(\chi\) and increasing in \(\epsilon\) (see Appendix C.1.3 for the details). Because assets and labor demand are related by equation (3.14) the same conclusion can be reached about properties of employment. \(n(t, \chi, \epsilon)\) is monotonic in \(t\), may be convex or concave in \(t\), is non-monotonic in \(\chi\), and increasing in \(\epsilon\).

**Case 2** The collateral constraint does not bind. This implies \(\eta = 0\) and \(k < \chi a\). Optimality with respect to labor and capital implies

\[ \frac{\alpha}{1-\alpha} \frac{n}{k} = \frac{r_k}{w} \]

(3.16)

Thus, the optimal level of capital and labor can be expressed in terms of prices:

\[ n^* = (z\epsilon\phi)^{1/(1-\phi)} \left( \frac{\alpha}{r_k} \right)^{(1-\alpha)/(1-\phi)} \left( \frac{1-\alpha}{w} \right)^{(1-\alpha)/(1-\phi)} \]

\[ k^* = (z\epsilon\phi)^{1/(1-\phi)} \left( \frac{\alpha}{r_k} \right)^{(1-\alpha)/(1-\phi)} \left( \frac{1-\alpha}{w} \right)^{(1-\alpha)/(1-\phi)} \]
Combining the two cases The optimal capital and labor choices are \( k(t) = \min \{ \bar{k}(t), k^* \} \) and \( n(t) = \min \{ \bar{n}(t), n^* \} \), where \( \bar{k}(t), \bar{n}(t) \) are constrained optimal choice of capital and labor. Figure 3.3 shows the firm-level employment dynamics for two firms with different level of productivities. The more productive firm cannot achieve its optimal level immediately and has to grow before it reaches its optimal employment level \( n^*(\epsilon_H) \). In contrast, the low-productivity firm can immediately jump to its optimal level of employment \( n^*(\epsilon_L) \).

The assets of an unconstrained firm continue to grow according to \( \dot{a} = \pi^* + (1 + r)a \), where \( \pi^* \) is the profit level of an unconstrained firm. Figure 3.3 demonstrates the role of age versus size in identifying the effect of a collateral shock. The collateral constraint is irrelevant for low productivity firms; a tightening of the collateral constraint (at least locally) has no impact on employment for these firms. In contrast, for high-productivity firms, which grow from small to large firms, the collateral constraint impacts their rate of growth while leaving the optimal level employment unchanged.

Let’s \( \bar{t} \) denote the moment in time when a firm grows out of its financial constraint (assuming that the firm was financially constrained at the beginning of its life). This time \( \bar{t} \) solves the equation \( \bar{k}(\bar{t}) = k^* \).

Lemma 3.2. Consider two financial constraint parameters \( \chi_L, \chi_H \) where \( \chi_L < \chi_H \). Denote
$T(\chi_L, \chi_H) : \bar{k}(T(\chi_L, \chi_H), \chi_L) = \bar{k}(T(\chi_L, \chi_H), \chi_H)$ - that is, $T(\chi_L, \chi_H)$ is the time at which the employment path of the credit constrained high-productivity firm crosses under $\chi_L$ and $\chi_H$ respectively. Assume that $\bar{t}(\chi_L) < T(\chi_L, \chi_H)$. Then it follows:

1. $n(t, \chi_L) \leq n(t, \chi_H)$,
2. $\bar{t}(\chi_H) < \bar{t}(\chi_L)$.

The results of the lemma are presented in Figure 3.4. The lemma shows that a collateral shock depresses employment for the high-productivity, credit constrained firm and extends the time it takes for the firm to reach its optimal size. As a result, employment at the high-productivity firm is depressed at every age level after an adverse collateral shock. However, since the optimal size of the firm is unchanged, job creation for any given firm is unchanged over its lifecycle conditional on surviving long enough to reach its optimal size.

Figure 3.4: Firm employment dynamics: comparative statics with respect to $\chi$

If the firm’s optimal size is left unchanged, how does a collateral shock depress job creation and job destruction? Given the constant hazard rate of exit $\sigma$, since it takes longer to reach optimal size, fewer firms survive, thereby lowering aggregate job creation. Since job destruction in this model is attributable solely to firm exits, the decrease in job destruction
is due to fewer firms surviving to any given level of employment. It can be shown that job destruction \( JD = \sigma N \) where \( N \) is aggregate employment.

It is also worth noting, that an aggregate productivity shock \( z \) has a qualitatively different effect on employment paths for firms than a collateral shock \( \chi \). A productivity shock depresses employment at all ages; while the optimal size of the firm is independent of the collateral constraint, productivity directly affects the optimal size. As such, a productivity shock will affect both high and low-productivity firms, and constrained and unconstrained firms. Like Khan and Thomas (2011), a productivity shock will interact with the financial constraints to have asymmetric effects on firm employment across age and size categories.

**Aggregate labor demand**

Summing across firms leads to

\[
N^d = \mu \left[ \sigma \int_0^\tau n(t, \epsilon_H, w, r, r_k, \chi)e^{-\sigma t}dt + \sigma \int_\tau^\infty n^*(\epsilon_H, w, r, r_k)e^{-\sigma t}dt \right] \\
+ (1 - \mu) \left[ \sigma \int_0^\tau n(t, \epsilon_L, w, r, r_k, \chi)e^{-\sigma t}dt + \sigma \int_\tau^\infty n^*(\epsilon_L, w, r, r_k)e^{-\sigma t}dt \right].
\]

(3.17)

In a dynamic steady state equilibrium \( r \) and \( r_k \) are pinned down by the household preferences. Hence, any change in \( \chi \) will have a direct effect on the aggregate labor demand and an indirect effect through an equilibrium change in wages \( w \).

**Assumption 3.1.** Parameters of the model are such that in equilibria to be considered all firms with low productivity are not credit constrained.

This assumption implies that the aggregate demand for labor can be expressed as follows

\[
N^d = \mu \left[ \sigma \int_0^\tau n(t, \epsilon_H, w, r, r_k, \chi)e^{-\sigma t}dt + \sigma \int_\tau^\infty n^*(\epsilon_H, w, r, r_k)e^{-\sigma t}dt \right] \\
+ (1 - \mu)n^*(\epsilon_L, w, r, r_k)
\]

(3.18)

**Lemma 3.3.**
\(• (PE) \) Assume that \( w \) is constant and \( T(\chi_L, \chi_H) < T(\chi_L, \chi_H) \), then \( N^d(w, \chi_L) < N^d(w, \chi_H) \).
\(• (GE + No \ wealth \ effect) \) Assume that \( T(\chi_L, \chi_H) < T(\chi_L, \chi_H) \), then \( N(w(\chi_L), \chi_L) < N(w(\chi_H), \chi_H) \).

Proof: See Appendix C.1.4 for a proof. ■

As Lemma 3 shows, under certain mild conditions, a tightening in the collateral constraint reduces aggregate labor demand when wages are held constant. Employment at low-productivity firms is unchanged, while employment at the high-productivity firms falls for firms that are credit constrained and remains unconstrained for firms with sufficient assets. Since employment is weakly lower after the collateral shock, aggregate employment falls. Wage adjustment partially offsets the direct effects of the collateral shock. A lower wage raises optimal size of all firms increasing employment for unconstrained firms. So long as the labor supply is upward-sloping, wage adjustment will not fully undo the direct effect of the collateral shock.

### 3.4 Benchmark Model

In this section, we describe a dynamic stochastic general equilibrium model with endogenous firm entry and exit. The agents, markets, assets and production technology are the same as in the simple model. We add uncertainty to aggregate productivity and the collateral constraint as well as uncertainty to firm-specific productivity.

Time is discrete and is indexed by \( t = 1, 2, \ldots \). The common component of productivity \( z_t \) follows a Markov process that takes values in the set \( Z \) with conditional distribution \( H(z_{t+1}|z_t) \). The idiosyncratic component of productivity \( \epsilon_t \) follows a Markov process that takes values in \( E \) and has conditional distribution \( G(\epsilon_{t+1}|\epsilon_t) \). Financial shock \( \chi_t \) also follows a Markov process with values in \( K \) and conditional distribution \( Q(\chi_{t+1}|\chi_t) \).

In addition to the aggregate states, a firm’s problem will be characterized by the firm’s initial level of assets and its current level of idiosyncratic productivity. Thus, the aggregate state of the economy \( x_t \) consists of \( \{z_t, \chi_t, a^H_t, \mu_t\} \), where \( \mu_t \) is the distribution of firms over assets and idiosyncratic shocks and \( a^H_t \) is the wealth of the representative household. Observe
that if the households were heterogenous we would need to keep track of the households
distribution over their state space. However, because households are identical they can be
summarized by a single state variable $a_t^H$.

Denote $\mu' = \Gamma(z, \chi, a^H, \mu)$ as the law of motion for the firm distribution. Also denote
$\Phi(x_{t+1} | x_t)$ as the conditional distribution of the aggregate state. This conditional distribution
can be expressed as follows

$$d\Phi [z', \chi', (a^H)' , \mu'| z, \chi, a^H, \mu] = \frac{1}{(a^H)'} \cdot dH(z'|z)dQ(\chi'|\chi).$$

(3.19)

3.4.1 Households

The household problem can be summarized as follows:

$$V^H(a, x) = \max_{c,n,a'} \left\{ u[c - v(n)] + \beta \int V^H(a', x')d\Phi(x'|x) \right\},$$

s.t. $c + a' = wn + (1 + r)a + \Pi$.

Households choose consumption $c$, labor supply $n$, and next period assets $a'$ subject to a
standard budget constraint taking firm profits as given.

3.4.2 Firms

All operating firms must pay fixed cost $c_F > 0$ every period. The incumbent firm solves the
following problem:

$$V^F(\epsilon, a, x) = \max_{k,n,a'} \left\{ 0, -c_F + \max_{k,n,a'} \int \left[ \sigma \Lambda a' + (1 - \sigma)V^F(\epsilon', a', x') \right] d\Phi(x'|x)dG(\epsilon'|\epsilon) \right\},$$

s.t. $a' = z(\epsilon k^{\alpha} n^{1-\alpha}) - r_k k - wn + (1 + r)a$,

$k \leq \chi a$. 

Incumbent firms operate a decreasing returns to scale production technology subject to idiosyncratic and aggregate productivity shocks. Firms choose whether to exit or maximize a weighted average of next period assets and their continuation value, where the weight $\sigma$ reflects the exogenous probability of exit. Firms choose capital $k$, next period assets $a'$, and employment $n$ subject to a standard accumulation equation for assets and the same constraint on renting capital described in the previous section.

There are two possibilities for entry. Firstly, firms enter exogenously: each period a measure $\sigma$ of firms enter without paying the cost of entry and immediately start producing. Secondly, measure $\sigma_0$ of prospective firms can pay a fixed cost $c_E$ to enter. The firm that enters in the current period starts producing immediately. The decision to pay $c_E$ occurs after firms learn their current level of productivities. The value of a firm that has learned its initial productivity $\epsilon$ is $V^F(\epsilon, a_0, x)$. Firms that pay to enter will enter if and only if:

$$V^F(\epsilon, a_0, x) \geq c_E.$$  

We assume that the initial productivity is drawn from stationary distribution $G_0(\epsilon)$.

### 3.4.3 Intermediaries

Perfectly competitive intermediaries operate identically as described in the previous section. The zero-profit condition for intermediaries implies:

$$r_k = r + \delta$$

### 3.4.4 Recursive Equilibrium

A recursive equilibrium is a collection of functions $V^H(a, x), V^F(\epsilon, a, x), c(a, x), a'_H(a, x), n(\epsilon, a, x), k(\epsilon, a, x), a'_F(\epsilon, a, x), w(x), r(x), r_k(x), \Gamma(x), \Lambda(x)$ such that:

1. households, firms, intermediaries optimize;

2. capital, labor, goods markets clear;
3. Γ: for all Borel $\mathcal{E} \times \mathcal{A} \in \mathbb{R}^+ \times \mathbb{R}^+$

$$
\mu'(\mathcal{E} \times \mathcal{A}) = (1 - \sigma) \int_{\mathcal{E}} \int_{(\epsilon,a) \in \mathcal{B}(x,\mathcal{A})} d\mu(\epsilon,a)dG'(\epsilon) \\
+ 1(a_0 \in \mathcal{A}) \sigma \int_{\epsilon \in \mathcal{B}_0(x,a_0) \cap \mathcal{E}} dG_0(\epsilon) \\
+ 1(a_0 \in \mathcal{A}) \sigma_0 \int_{\epsilon \in \mathcal{B}_E(x,\mathcal{A}) \cap \mathcal{E}} dG_0(\epsilon),
$$

(survived incumbents)

(exogenous entry + no exit)

(3.20)

(endogenous entry)

where

$$
\mathcal{B}_E(x,a_0,\mathcal{A}) = \{ \epsilon : V^F(\epsilon, a_0, x) \geq c_E \},
$$

$$
\mathcal{B}_0(x,a_0) = \{ \epsilon : V^F(\epsilon, a_0, x) > 0 \},
$$

$$
\mathcal{B}(x,\mathcal{A}) = \{ (\epsilon,a) : V^F(\epsilon,a,x) > 0, \pi(x,\epsilon,a) + (1+r(x))a \in \mathcal{A} \},
$$
given $\mu_0$.

### 3.4.5 Stochastic Steady State without Endogenous Firms Entry and Exit

In this section we assume that $c_E = \infty$ and $c_F = 0$, i.e., firms enter and exit exogenously. A stochastic steady state is an equilibrium in which there are no aggregate shocks, i.e., $\chi$ is constant and $z$ is constant, and prices, distribution of firms and assets of households do not change over time.

Household optimality requires that in the steady state equilibrium the real interest rate equals the discount factor:

$$
\frac{1}{\beta} = 1 + r,
$$

(3.20)

and the wage equals marginal disutility of working:

$$
w = v'(n)
$$

(3.21)
Firm optimality with respect to capital and labor implies standard factor demand conditions:

\[ z_t \phi^\alpha k_t^\phi (1 - \alpha) n_t (1 - \alpha)^\phi = r_k + \frac{\eta_t}{\lambda_t}, \]  
\[ z_t \phi (1 - \alpha) k_t^\alpha n_t (1 - \alpha)^\phi - 1 = w. \]  

These conditions are the discrete time equivalent of the firm’s policy functions described in the previous section.

### 3.5 Quantitative Predictions of the Model

To explore the quantitative implications of our model, we calibrate the version of our benchmark model without endogenous entry and exit and examine the effect of a collateral shock in our model on job flows and the distribution of job creation and job destruction across firm size and firm age categories.

#### 3.5.1 Calibration Strategy and Targets

Our calibration strategy chooses several common parameters from the literature. Given that our empirical evidence on job flows is observed in annual data, we use annual values for several common parameters. As shown in Table 3.5, the household’s discount rate \( \beta \), the depreciation rate of capital \( \delta \), and the capital share \( \alpha \) are all standard. The parameter \( \phi \) governing the degree of decreasing returns to scale is set at 0.95, comparable to values chosen in Cooley and Quadrini (2001) and Khan and Thomas (2011). The Frisch elasticity \( \nu \) is chosen at two extreme values to gauge the importance of labor supply response in our quantitative experiment. A Frisch elasticity of zero conforms to the case of a vertical labor supply curve, while an infinite Frisch elasticity conforms to the case of a horizontal labor supply curve. In the former case, wages adjust so that total employment is unaffected by the collateral shock. In the latter case, wages are unchanged so employment is demand determined. In effect, this case conforms to the partial equilibrium effect of the collateral shock or, equivalently, the effect of a collateral shock with perfect real wage rigidity.
Table 3.5: Calibration values

<table>
<thead>
<tr>
<th>Aggregate Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate $\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>Depreciation rate $\delta$</td>
<td>0.1</td>
</tr>
<tr>
<td>Capital share $\alpha$</td>
<td>0.3</td>
</tr>
<tr>
<td>Decreasing returns $\phi$</td>
<td>0.95</td>
</tr>
<tr>
<td>Frisch elasticity $\nu$</td>
<td>$0, \infty$</td>
</tr>
<tr>
<td>Initial assets $a_0$</td>
<td>8</td>
</tr>
<tr>
<td>Collateral constraint $\chi$</td>
<td>20</td>
</tr>
</tbody>
</table>

It remains to choose an initial level of assets $a_0$, the collateral constraint parameter $\chi$, firm exit rate $\sigma$, and a support and distribution of idiosyncratic productivity levels $\epsilon$, and a transition process for the idiosyncratic productivity shocks. For simplicity, we assume that firms’ idiosyncratic productivity level does not change so there are no transitory idiosyncratic productivity shocks. We select the distribution of the idiosyncratic productivity levels to target the distribution of employment by mature firms in the data. In our model, firm that survive sufficiently long converge towards an optimal level of employment. We take averages of employment share by firm size categories for firms over 21 years of age in the Business Dynamics Statistics from 2000-2006. We back out the implied level idiosyncratic productivity so that the optimal employment size of the firm is at the midpoint of the employment bin range. We target the share of employment by firm size in the data by computing the distribution over idiosyncratic productivity levels. Table 3.6 shows the size bins used and the employment shares that our calibration targets. The last column shows the implied distribution of firms that matches the employment shares we are targeting.

Instead of choosing a single constant exit rate for firms, we choose time-dependent exit rates for the first five years before a constant exit rate for firms older than five years. In the absence of endogenous entry and exit, the firm’s policy functions and the steady state are unaffected by this assumption. We choose entry and exit rates to match the empirical age distribution of firms using 2000-2006 averages from the BDS. Table 3.7 provides the age distribution of firms and the distribution implied by our calibration. By construction, the
empirical distribution and model distribution match for firms age 0-5, but differs for older ages when a constant exit rate is assumed. The exit rate for firms older than age 5 is $\sigma = 0.069$ and implies a model age distribution that does well in matching the empirical distribution.

Table 3.6: Idiosyncratic shock calibration

<table>
<thead>
<tr>
<th>Size Bins</th>
<th>Employment Share</th>
<th>Firm Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>2.53%</td>
<td>43.99%</td>
</tr>
<tr>
<td>7</td>
<td>3.62</td>
<td>22.47</td>
</tr>
<tr>
<td>14.5</td>
<td>5.10</td>
<td>15.27</td>
</tr>
<tr>
<td>37.5</td>
<td>8.46</td>
<td>9.80</td>
</tr>
<tr>
<td>74.5</td>
<td>7.10</td>
<td>4.14</td>
</tr>
<tr>
<td>174.5</td>
<td>9.83</td>
<td>2.45</td>
</tr>
<tr>
<td>374.5</td>
<td>6.99</td>
<td>0.81</td>
</tr>
<tr>
<td>749.5</td>
<td>6.42</td>
<td>0.37</td>
</tr>
<tr>
<td>1749.5</td>
<td>8.56</td>
<td>0.21</td>
</tr>
<tr>
<td>3750</td>
<td>41.40</td>
<td>0.48</td>
</tr>
</tbody>
</table>

The final parameters that we choose are the initial level of assets $a_0$ and the collateral constraint parameter $\chi$. We jointly choose these parameters shown in Table 3.5 to best match the distribution of employment by firm age and size. The empirical and model distributions are shown in Table 3.8. Our calibration closely matches the age distribution of employment and does a reasonable job matching the size distribution of employment. Our calibration has a somewhat lower distribution of employment among new and younger middle-sized and larger firms and consequently too large employment share for small firms.

3.5.2 Collateral Shock Experiment

We consider the effect of a 20% tightening of the collateral constraint parameter from $\chi = 20$ to $\chi = 16$. This tightening roughly conforms to the magnitude of the drop experienced in US housing prices during the Great Recession. Table 3.9 displays the effect of this shock on the distribution of job creation and job destruction by firm age and size categories. The top panel shows the effect of the collateral shock on job creation and the bottom panel shows the effect on job destruction. The first three columns show job flows as a percentage of total labor supply: the first column is the baseline, the second column reflects the case of a horizontal
Table 3.7: Exit rate calibration

<table>
<thead>
<tr>
<th>Firm Age</th>
<th>Firm Shares (Data)</th>
<th>Firm Shares (Model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10.22%</td>
<td>10.22%</td>
</tr>
<tr>
<td>1</td>
<td>7.69</td>
<td>7.69</td>
</tr>
<tr>
<td>2</td>
<td>6.55</td>
<td>6.55</td>
</tr>
<tr>
<td>3</td>
<td>5.76</td>
<td>5.76</td>
</tr>
<tr>
<td>4</td>
<td>5.16</td>
<td>5.16</td>
</tr>
<tr>
<td>5</td>
<td>4.67</td>
<td>4.67</td>
</tr>
<tr>
<td>6-10</td>
<td>18.03</td>
<td>17.89</td>
</tr>
<tr>
<td>11-15</td>
<td>12.61</td>
<td>12.60</td>
</tr>
<tr>
<td>16-20</td>
<td>8.82</td>
<td>9.68</td>
</tr>
<tr>
<td>21-25</td>
<td>6.17</td>
<td>6.43</td>
</tr>
<tr>
<td>26+</td>
<td>14.40</td>
<td>13.37</td>
</tr>
</tbody>
</table>

demand curve and shows the effect when wages do not adjust, the third column shows the effect of wage adjustment on job flows when labor supply is vertical and wages adjust to ensure the same level of employment as in the baseline case. The last two columns express columns two and three as a percentage of the baseline (first column).

As the first row shows, the collateral shock depresses overall job creation and job destruction (in steady state job creation equals job destruction). Even with a fall in wages that ensures the same level of total employment, overall job creation still falls. This is due to the fact that firm exit rates are declining with age. A fall in wages raises the optimal size of mature firms, which raises their employment demand, but the tighter credit constraint lowers job creation among new and young firms. As a result, job creation falls even though wages adjust to ensure the same level of employment.

**Effect on Job Creation**

The effect of the collateral shock for job creation by age and size largely mirrors the empirical patterns we document. For firm age categories, younger firms exhibit the greatest response to a collateral shock followed by new firms and mature firms. This ordering differs from our empirical work where new firms exhibited the greatest decline in job creation followed by young and mature firms respectively. However, the absence of an active entry margin
Table 3.8: Distribution of employment by firm size and age

<table>
<thead>
<tr>
<th></th>
<th>1-19 emps</th>
<th>20-99 emps</th>
<th>100+</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Births</td>
<td>1.5</td>
<td>0.8</td>
<td>0.6</td>
<td>2.8</td>
</tr>
<tr>
<td>1-5 years</td>
<td>5.7</td>
<td>3.4</td>
<td>3.0</td>
<td>12.1</td>
</tr>
<tr>
<td>6+ years</td>
<td>12.1</td>
<td>13.6</td>
<td>59.3</td>
<td>85.0</td>
</tr>
<tr>
<td>Total</td>
<td><strong>19.3</strong></td>
<td><strong>17.8</strong></td>
<td><strong>62.9</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1-19 emps</th>
<th>20-99 emps</th>
<th>100+</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Births</td>
<td>2.9</td>
<td>0.2</td>
<td>0.0</td>
<td>3.0</td>
</tr>
<tr>
<td>1-5 years</td>
<td>6.1</td>
<td>6.1</td>
<td>1.6</td>
<td>13.7</td>
</tr>
<tr>
<td>6+ years</td>
<td>11.1</td>
<td>15.6</td>
<td>56.6</td>
<td>83.2</td>
</tr>
<tr>
<td>Total</td>
<td><strong>20.0</strong></td>
<td><strong>21.8</strong></td>
<td><strong>58.2</strong></td>
<td></td>
</tr>
</tbody>
</table>

in our calibration is likely crucial for this discrepancy. Given our results on firm entry and house prices, an active entry margin should increase the sensitivity of job creation among new firms. In the absence of wage adjustment, all age categories experience a decrease in job creation. Job creation falls the most for young firms due to binding collateral constraint, while job creation falls least for mature firms since job creation is simply delayed due to a collateral shock. In the partial equilibrium case, optimal size is unchanged so even with a tighter constraint surviving firms still ultimately create the same number of jobs.

For firm size categories, our model with wage adjustment matches the ordering of sensitivity across size categories with middle-sized firms experiencing the biggest decline in job creation relative followed by large firms and small firms respectively. Moreover, in the case of small firms, a collateral shock generates either a small decline or an increase in job creation in the partial equilibrium and general equilibrium cases respectively. Our empirical evidence on job creation by small firms showed a response to housing price shocks that was not statistically different from zero. A collateral shock has a stronger effect on medium-sized and
Table 3.9: Effect of collateral shock on job flows by firm size and age

<table>
<thead>
<tr>
<th>Job Creation</th>
<th>Share of Total Employment (%)</th>
<th>Shock Effect (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline (BL)</td>
<td>Partial Eq (PE)</td>
</tr>
<tr>
<td>Aggregate</td>
<td>8.09</td>
<td>7.27</td>
</tr>
<tr>
<td>Births</td>
<td>38</td>
<td>33</td>
</tr>
<tr>
<td>1-5 years</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>6+ years</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>1-19 emps</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>20-99 emps</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>100+ emps</td>
<td>42</td>
<td>36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Job Destruction</th>
<th>Baseline (BL)</th>
<th>Partial Eq (PE)</th>
<th>General Eq (GE)</th>
<th>PE/BL</th>
<th>GE/BL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-5 years</td>
<td>25</td>
<td>22</td>
<td>23</td>
<td>87</td>
<td>93</td>
</tr>
<tr>
<td>6+ years</td>
<td>75</td>
<td>68</td>
<td>76</td>
<td>91</td>
<td>101</td>
</tr>
<tr>
<td>1-19 emps</td>
<td>25</td>
<td>25</td>
<td>28</td>
<td>102</td>
<td>112</td>
</tr>
<tr>
<td>20-99 emps</td>
<td>24</td>
<td>21</td>
<td>22</td>
<td>86</td>
<td>92</td>
</tr>
<tr>
<td>100+ emps</td>
<td>51</td>
<td>44</td>
<td>49</td>
<td>86</td>
<td>96</td>
</tr>
</tbody>
</table>

large firms because the collateral constraint is more relevant to these firms in comparison to small firms. Low productivity firms need not accumulate a great deal of assets to achieve their optimal size, while high productivity firms must wait to accumulate capital to achieve optimal size. These higher productivity firms transit through the middle-sized employment category and therefore exhibit the effects of a tighter collateral constraint on their growth rates. When wages adjust, the effect on large firms is partially offset by increased job creation among unconstrained firms - lower wages imply larger optimal size and increases job creation among these high productivity, unconstrained firms.

**Effect on job destruction**

Our model also does a good job of matching the empirical patterns of housing prices on job destruction. Job destruction falls for young firms relative to mature firms, consistent with the
empirical ordering we documented. Moreover, in the general equilibrium case, job destruction among mature firms increases relative to baseline, consistent with the insignificant estimates for job destruction in our firm age regressions. Given that young firms become smaller after the collateral shock, the jobs destroyed by these firms when they exit also fall. In other words, since these firms create fewer jobs, they also destroy fewer jobs when they exit. In contrast, since mature firms continue to grow to their optimal size, job destruction falls by less for this category. There are two competing effects in the general equilibrium case: given exogenous exit rates, fewer firms survive to their optimal size reducing job destruction, however, as wages fall, optimal size increases leading to greater job destruction for firms that exit after reaching their optimal size.

For firm size categories, we find declines in job destruction for middle-sized and large firms while an increase in job destruction for small firms. This ordering is consistent with our empirical evidence and consistent with the finding that collateral shocks raise job destruction for small firms. The ordering and signs are also consistent across both partial equilibrium and general equilibrium cases. Job destruction falls for middle-sized and larger firms because constrained firms become smaller after a collateral shock, while unconstrained firms do not become bigger. As a result, higher productivity firms spend a longer period of time as smaller firms. When these firms exit, this decreases job destruction among middle-sized firms but increases job destruction for small firms. In general equilibrium, lower wages offset this effect for the highest productivity firms mitigating the effect of the collateral shock on job destruction for large firms relative to middle-sized firms.

3.6 Conclusion

The US housing crisis has raised concerns the depressed real estate values may inhibit firm formation and expansion, disrupting the process of innovation and labor market turnover that characterizes a healthy economy. An extensive literature has documented the importance of real estate collateral for new firms to obtain lending and for small businesses to obtain financing for expansion. Moreover, recent work also documents the disproportionate contribution of
new and young firms to overall labor market turnover. Given these facts, it stands to reason that job flows may be particularly sensitive to a decline in collateral values.

In this paper, we provide support for this hypothesis illustrating the empirical and theoretical link between job flows and housing prices. Using state-level variation in job flows and housing prices, we show that both job creation and lagged job destruction decline in response to fall in housing prices. We control for aggregate demand effects by introducing direct controls for the business cycle and using a land supply elasticity approach common in the empirical literature on the real effects of collateral shocks. We also document size and age patterns in the sensitivity of job flows to housing prices, showing that job flows for new and young firms (0-5 years of age) are most sensitive to housing prices shocks as are job flows for medium-sized firms (20-99 employees).

We build a simple firm dynamics model with collateral constraints and examine the effect of a collateral shock on job flows and the distribution of job flows by firm size and age. We show analytically in a simple version of our model that a collateral shock must reduce employment, job creation and job destruction, and demonstrate why a collateral shock should have stronger effects for young firms and medium-sized firms. We calibrate our benchmark model to match the distribution of employment by firm size and age seen in the data. Our calibrated model replicates the empirical pattern of job flow sensitivity to a collateral shock by firm size and age categories.

Future work will extend our numerical work to include transitory productivity shocks to match the overall level of job flows and better match the distribution of job flows across age and size categories. Given the importance of house prices on firm entry, we will also extend our quantitative work to include an endogenous entry margin. Moreover, examination of the transition path under a collateral shock may provide further clues about whether a collateral shock can account for the prolonged labor market recoveries associated with a housing price collapse.
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Appendix A

Fiscal Policy Stabilization: Purchases or Transfers?

A.1 Extensions

In this section, I briefly consider fiscal policy in a model where a subset of the population operates as rule of thumb agents who simply consume current income each period and a model where agents face a borrowing constraint. This section relates to a literature on fiscal policy and rule of thumb agents developed by Mankiw (2000) and Gali, Lopez-Salido, and Valles (2007), and a literature examining the effects of monetary policy when agents face borrowing constraints such as Iacoviello (2005) and Monacelli (2009). As this section illustrates, the credit spread model considered in this paper can easily be related to rule of thumb or borrowing constraint models and, therefore, the policy implications are likely to carry over to a broader class of DSGE models.
A.1.1 Rule of Thumb Agents

Rule of thumb agents face a static optimization problem and choose hours period-by-period facing a simple budget constraint with consumption equal to current disposable income:

\[ U_c(C_y^t, N_y^t) W_t = -U_h(C_y^t, N_y^t) \]  
(A.1)

\[ C_y^t = W_t N_y^t - T_t \]  
(A.2)

Log-linearizing these equilibrium conditions and combining with the equilibrium conditions for the firms and saver households discussed earlier delivers a closed form solution for output in terms of government purchases and taxes:

\[
y_t = \frac{\alpha}{\left(\alpha + s_c \tilde{\sigma}(1 - \alpha) + s_c \frac{1 - s_y}{1 - \tilde{\nu}} \frac{\sigma_s}{\tilde{\varphi}_s} - s_c(1 - \alpha) \phi \nu\right) g_t}
- \frac{\alpha \phi \frac{\gamma}{\sigma_y \tilde{c}_y} / \frac{\nu}{\varphi_y}}{\left(\alpha + s_c \tilde{\sigma}(1 - \alpha) + s_c \frac{1 - s_y}{1 - \tilde{\nu}} \frac{\sigma_s}{\tilde{\varphi}_s} - s_c(1 - \alpha) \phi \nu\right) \text{tax}_t}
\]

\[
\phi = s_y \frac{\sigma_y}{\varphi_y} - (1 - s_y) \frac{\sigma_s}{\varphi_s} \frac{l_y}{1 - l_y}
\]

\[
v = \left(1 - \frac{w y}{\sigma_y \tilde{c}_y}\right) / \left(\frac{1}{\varphi_b} + \frac{w y}{\sigma_y \tilde{c}_y}\right)
\]

The multiplier on government spending has several terms similar to the multiplier derived in Section 5, with the parameters \(\phi\) and \(v\) as the new terms. For Frisch elasticities less than unity, \(v < 1\), and for households with sufficient symmetry, \(\phi \approx 0\). Therefore, a tax reduction for the borrower household has negligible effect on output for plausible calibrations, and the government spending multiplier remains below unity, consistent with the numerical experiments shown in Figure 1. Intuitively, a transfer from one household to the other has offsetting effects on the labor supply of each household, leaving total labor supply relatively unchanged and, therefore, output unchanged. To the extent that \(\phi > 0\), tax rebates will be expansionary and the government spending multiplier will be larger than in the representative agent benchmark.

Under sticky prices, analytical solutions with rule of thumb agents can be obtained under the assumption of GHH preferences or wage rigidity that eliminates a labor supply effect. For
simplicity and comparability to the rest of the paper, I consider the case of rigid wages. The Phillips curve from Section 4.2 obtains along with an intertemporal IS curve of the form:

\[ y_t - g_t = E_t (y_{t+1} - g_{t+1}) - s_c \sigma_s (1 - s_y) \left( i_t^y - E_t \pi_{t+1} \right) - s_c s_y \left( E_t C_{t+1}^t - c_t^y \right) \]

\[ c_t^y = \frac{\omega_{t,y}}{\epsilon_y} y_t - \frac{\gamma}{\epsilon_y} t a x_t \]

\[ \pi_t = \frac{\kappa}{\alpha} (1 - \alpha) y_t + \beta E_t \pi_{t+1} \]

The last term in the IS equation can be treated as equivalent to the credit spread \( \omega_t \), and responds to both changes in income and transfers. A temporary increase in transfers that is gradually withdrawn, as in the case of a debt-financed tax rebate, is equivalent to a fall in the credit spread that eventually becomes positive as the transfer turns negative when taxes are raised to return the public debt to its steady state. Relative to the credit spread model, transfers appear directly in the intertemporal IS equation instead of operating indirectly through private sector debt. As before, when monetary policy is unconstrained, the Phillips curve is unchanged and monetary policy is free to target any combination of inflation and output subject to the Phillips curve tradeoff.

When monetary policy is constrained by the zero lower bound, both purchases and transfers may be used for stabilization and an explicit tax rebate multiplier can be derived when there is a constant probability that the shock causing the zero lower bound to bind disappears. While a financial shock no longer appears because of the absence of intermediation, any of the shocks that cause the zero lower bound to bind in representative agent models - like a discount rate shock - would suffice here\(^1\). The solution for output at the zero lower bound is

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\(^1\)We can easily reintroduce the financial shock and credit spread by simply adding a measure of rule-of-thumb consumers to the existing saver/borrower model. Goods market clearing then implies that \( Y_t = \eta_s C_t^s + \eta_b C_t^b + (1 - \eta_s - \eta_b) C_t^y \). As before, under the assumption of zero debt elasticity of the credit spread, the log-linearized economy at the zero lower bound is summarized by an aggregate intertemporal IS curve and the standard Phillips curve. Moreover, in a lifecycle model with distinct borrowing and credit spreads, the stochastic steady state would be characterized by saver households, borrower households, and households living in autarky.
similar to the solution derived in Section 6 with the addition of a multiplier on the tax rebate:

\[ y_{zlb} = \nu^*_g \left( g_{zlb} - tax_{zlb} \right) - \zeta \]

\[ \nu^*_g = \frac{\left(1 - \rho\right) \left(1 - \beta \rho\right)}{\left(1 - \rho\right) \left(1 - \beta \rho\right) \left(1 - inc_y \frac{1}{\alpha} \right) - sc \left(1 - s b\right) \sigma_s \frac{\kappa}{\alpha} \left(1 - \alpha\right) \rho} \]

where \( inc_y \) is the rule-of-thumb agents share of wage income in national income. Comparison to the multiplier derived in section 6 reveals that the multiplier \( \nu_g \) may be higher or lower; the effect of higher inflation reducing real interest rates (the last term in the denominator) is attenuated relative to the saver/borrower model while the presence of rule-of-thumb agents raises the direct effect of government spending on the consumption of rule-of-thumb agents (the \( inc_y \) term) and the multiplier. Unlike an old-style Keynesian model, the government spending multiplier and tax rebate multiplier are the same, and the balanced budget multiplier is zero.

The reason the multiplier is the same for both government spending and tax rebates is that both affect the savers consumption in the same way. A rise in government spending or equivalent fall in tax rebates raises aggregate demand by the same amount, and equilibrium in the goods market requires either a rise in output or a fall in the savers consumption induced by a rise in the real interest rate. With the nominal rate held constant and no direct effect of either policy on the Phillips curve, the savers consumption response is the same and, therefore, the output multiplier is the same for each policy. When the government’s budget is balanced, the aggregate demand effects cancel out and the savers consumption decision is unchanged.

Finally, it’s worth relating this equilibrium analysis of the zero lower bound with rule-of-thumb agents to the extensive literature on the determinants of consumption and the aggregate consumption function where the real interest rate is taken as fixed and exogenous.\(^2\) The multipliers attached to any particular fiscal policy are heavily dependent on the behavior of the real interest rate, and therefore conclusions regarding fiscal multipliers are inherently general equilibrium questions. In the same way that the credit spread - absent wealth effects

---

\(^2\)See for example Carroll (2001) and Kaplan and Violante (2011).
- does not alter the Phillips curve, a more complex (and realistic) theory of consumption is unlikely to alter the effects of fiscal policy away from the zero lower bound. Unless fiscal stabilization has large effects on the production side of the economy - that is, incentives to supply labor and capital - monetary policy can achieve the same aggregate demand objectives of fiscal policy away from the zero lower bound. The nature of the aggregate consumption function will only become relevant at the zero lower bound where fiscal policies that have larger affect on desired consumption will be preferred to policies with a smaller effect.

A.1.2 Borrowing Constrained Agents

A broad range of models consider a class of agents that are constrained either by an exogenous or endogenous borrowing constraint but assume a single rate for lending and borrowing funds. These models often assume that the borrowing constraint binds at all times and solve for the dynamics of the model by log-linearizing around a binding constraint. Relative to the rule of thumb model in the previous section and assuming an exogenous borrowing constraint, the equilibrium conditions become:

\[
U_c \left( C^b_t, N^b_t \right) W_t = -U_h \left( C^b_t, N^b_t \right)
\]

\[
C^b_t + \frac{1 + \bar{i}_t}{\Pi_t} B_{t-1} = W_t N^b_t + B_t
\]

\[
B_t \geq \bar{B}
\]

To a log-linear approximation, the borrower’s budget constraint differs from the rule-of-thumb budget constraint only by including the lagged interest rate. If steady state interest payments are small, this term can be safely disregarded and the fiscal multipliers obtained in Section 7.1 remain a good approximation in the case of exogenous constraints. Without further assumptions on the model, a general characterization of fiscal multipliers with an endogenous borrowing constraint is difficult.

Under sticky prices and a demand driven labor market, a similar Phillips curve and intertemporal IS curve determine output and inflation. When borrowers are constrained by
an endogenous or exogenous constraint, their optimal choice of borrowing is governed by an Euler equation with nonzero Lagrange multiplier on the binding constraint:

$$\lambda_t^b = \gamma E_t \lambda_{t+1}^b \frac{1 + i_t^d}{\Pi_{t+1}} + \Theta_t$$

where the Lagrange multiplier represents the shadow price of the constraint. Since the constraint is assumed to be always binding for sufficiently small shocks, the borrower’s Euler equation can be log-linearized and summed with the saver’s Euler equation to obtain an intertemporal IS curve of the form:

$$y_t - g_t = E_t (y_{t+1} - g_{t+1}) - s_c \sigma_i (i_t^d - E_t \pi_{t+1}) - s_c s_b \sigma_b \theta_t$$

As before, the last term can be regarded as the credit spread, and changes in fiscal policy will shift the credit spread depending on the nature of the borrowing constraint. Importantly, the multiplier is likely to be changed by policy given that any change in income, wages, or taxes will affect the shadow price of the borrowing constraint. Though the mapping of a borrowing constraint model into the credit spread model will depend on further assumptions, the insights on fiscal policy from the credit spreads model should carry over to alternative models of borrowing and lending.

### A.1.3 Housing and Credit Spreads

I maintain the assumption of patient and impatient households, but I now assume a single market interest rate for savers and households. Instead of a credit spread, impatient household are constrained to borrow only a possibly time-varying fraction of the value of their residence.

The impatient household’s chooses :

$$\max \left\{ C_t^b, N_t^b, B_t, H_t^b \right\}$$

subject to

$$C_t^b = W_t N_t^b - \frac{1 + i_{t-1}^d}{\Pi_{t-1}} B_{t-1} + B_t - T_t + Q_t \left( H_{t-1}^b - H_t^b \right)$$

$$B_t \leq \chi Q_t H_t^b$$
Relative to the equilibrium conditions in Section 1.3, the Euler equation changes and a housing
Euler equation is introduced:

$$
\lambda^b_t = \gamma E_t \lambda^b_{t+1} \left( \frac{1 + \delta^d_t}{\Pi_t} + \Theta_t \right)
$$

$$
\lambda^b_t Q_t = \gamma E_t \lambda^b_{t+1} \left( \gamma_{h,t+1}^b + Q_{t+1} \right) + \Theta_t \chi_t Q_t
$$

Furthermore, if impatient households are the only agents that demand housing services and
the supply of housing is fixed, the housing Euler equation will determine the market-clearing
price of housing. We can log-linearize the model around a steady state assuming that the
collateral constraint is always binding. Under these assumptions, an aggregate IS equation of
the same form as the rule-of-thumb case emerges. To a log-linear approximation:

$$
y_t - g_t = E_t (y_{t+1} - g_{t+1}) - s_c \sigma_s (1 - s_b) \left( \delta^d_t - E_t \pi_{t+1} \right) - s_c s_b \left( E_t c_{t+1}^b - c_t^b \right)
$$

$$
c_t^b = \frac{w_m}{\bar{c}_b} \frac{1}{\alpha} y_t - \frac{\bar{V}}{\bar{c}_b} (\chi_t + q_t) - \frac{(1 + \tau) \bar{B}}{\bar{c}_b} (\chi_{t-1} + q_{t-1})
$$

$$
\pi_t = \frac{\kappa}{\alpha} (1 - \alpha) y_t + \beta E_t \pi_{t+1}
$$

The preceding equations along with a monetary policy rule do not fully specify the equilibrium
of the economy; the borrower household’s Euler equation and housing Euler equation are
needed to determine the dynamics of housing prices and the Lagrange multiplier on the
borrowing constraint.

The growth rate of borrower’s consumption takes the place of the credit spread in the
aggregate IS equation just as in the case of rule-of-thumb households:

$$
E_t \left( c_{t+1}^b - c_t^b \right) = \gamma_y E_t (y_{t+1} - y_t) - \gamma_{tax} E_t (tax_{t+1} - tax_t)
$$

$$
+ \gamma_b \left( E_t \chi_{t+1} - (2 + \tau) \chi_t + (1 + \tau) \chi_{t-1} \right)
$$

$$
+ \gamma_b \left( E_t q_{t+1} - (2 + \tau) q_t + (1 + \tau) q_{t-1} \right)
$$

where $\gamma_y$, $\gamma_{tax}$, and $\gamma_b$ are the appropriate constants. An exogenous tightening of the collateral
constraint can be represented as a fall in $\chi_t$ and, ignoring the equilibrium dynamics of housing
prices, will act like an increase in the credit spread so long as the stochastic process for \( \chi_t \) is dominated by the middle term for some period of time. In particular, an AR(3) process of \( \chi_t \) could generate an AR(1) process for the “interest-rate” shock represented by borrower consumption growth in the aggregate IS equation.

The inclusion of housing dynamics further complicates matters since simply a fall in housing prices does not guarantee a rise in borrower consumption growth beyond the initial period. Nevertheless, it appears plausible that a collateral shock could cause act in the same manner as a credit spread shock in the aggregate IS equation even with endogenous house prices. Stronger conclusions require greater structure placed on the saver household’s demand for housing and residential investment which will both determine the market clearing housing price.

A.2 Equivalence with Overlapping Generations Model

In this section, I show that the steady state of the model with infinitely-lived agents with differing degrees of time preference is isomorphic to the steady state of a model with finitely lived agents who share the same rate of time preference but differ in effective labor over the life cycle.

Household live \( T \) periods with variation in the disutility of labor supply over the life cycle, and each generation that dies in a period is replaced by a generation of equal measure in the next period so that the total population is constant. Household choose consumption, hours worked, and whether to borrow or save in each period. Formally, for each generation
\[ \{0, 1, \ldots, T\}, \text{ household’s optimization problem is:} \]

\[
\max \quad E_0 \sum_{t=0}^{T-i} \beta^t \{ u(C_t(i)) - \theta_t \}
\]

\[
C_t(i) = W_t N_t(i) + B_t(i) - \frac{1 + i_{t-1}^b}{\Pi_t} B_{t-1}(i) - D_t(i) + \frac{1 + i_{t-1}^d}{\Pi_t} D_{t-1}(i) + \Pi_t^f - T_t
\]

\[
D_t(i) \geq 0
\]

\[
B_t(i) \geq 0
\]

\[
B_{T-i}(i) = 0
\]

where \( \theta_t \) is an exogenous process for effective labor supply that captures the hump-shaped profile of earnings over the lifecycle. The household is prohibited from borrowing in the final period of life. The first-order conditions characterizing the household’s optimal consumption and savings decisions are given below:

\[
u_c(C_t(i), N_t(i)) = \lambda_t(i)
\]

\[
-u_n(C_t(i), N_t(i)) = \lambda_t(i) W_t \theta_t
\]

\[
\lambda_t(i) = \beta E_t \lambda_{t+1}(i) \frac{(1 + i_t^d)(1 + \omega_t)}{\Pi_{t+1}} - \phi_t^b(i)
\]

\[
\lambda_t(i) = \beta E_t \lambda_{t+1}(i) \frac{1 + i_t^d}{\Pi_{t+1}} + \phi_t^d(i)
\]

\[
\lambda_{T-i}(i) = -\phi_{T-i}^b(i)
\]

\[
\lambda_{T-i}(i) = \phi_{T-i}^d(i)
\]

\[
\phi_t^b(i) B_t(i) = 0
\]

\[
\phi_t^d(i) D_t(i) = 0
\]

Household optimality requires that households do not borrow or save in the final period. Subtracting the Euler equation for borrowing from the Euler equation for deposits shows that households never simultaneously borrow and save, but may find it optimal to live in autarky:

\[
0 = \beta E_t \frac{\lambda_{t+1}(i) \omega_t}{\Pi_{t+1}} - (\phi_t^d + \phi_t^b)
\]
I consider a steady allocation of consumption, borrowing and labor supply across generations where wages, interest rates, and the price level are constant, and assume that the utility functions and distribution of $\theta_i$ over the generations are sufficient to guarantee that a steady state exists.

The firm’s problem, the intermediaries problem, fiscal policy, and monetary policy are unchanged from the discussion in Section 1.3. Market clearing requires:

$$Y_t = \sum_{i=0}^{T} C_t(i) + G_t \quad (A.11)$$

$$N_t = \sum_{i=0}^{T} N_t(i) \quad (A.12)$$

A steady state of the overlapping generations model with credit frictions is a set of aggregate quantities $\{\bar{Y}, \bar{N}, \bar{C}, \bar{F}, \bar{K}, \bar{\Pi}_f\}$, a distribution of consumption, labor supply, deposits and borrowings over generations $\{C_i, N_i, D_i, B_i, \lambda_i, \phi_i^d, \phi_i^b\}_{i=0}^{T}$, a set of prices $\{\bar{W}, \bar{\Pi}, \bar{i}_d, \bar{\omega}, \bar{MC}\}$, a fiscal policy $\{\bar{B}_g, \bar{T}, \bar{G}, \bar{reb}\}$ that jointly satisfy the steady state versions of:

1. Household optimality conditions (A.3) - (A.10)
2. Household budget constraints
3. Firm optimality conditions in (1.15)
4. Government budget constraint, fiscal rule, and solvency condition (1.10)
5. Monetary policy rule (1.13)
6. Market-clearing conditions

Given a definition for the steady state of the overlapping generations model, for suitable choices of the distribution of $\theta_i$ and other model parameters, the steady state of the infinite horizon model is equivalent to the steady state of the overlapping generations model.
**Proposition A.1.** Consider a steady state of the overlapping generations model. There exists a set of discount rates and functions for household utility that give the provide steady state in the infinite horizon model.

*Proof.* Since the firm’s problem, intermediaries’ problem, fiscal and monetary policy are unchanged in the overlapping generation model, a steady state in the OLG model satisfies parts 3-5 of the steady state version of the definition of an equilibrium in the infinite horizon model. It remains to show that household optimality conditions and market clearing conditions may be satisfied.

Let $\Omega$ is the set of borrowers in $i \in \{0, 1, \ldots, T\}$. Savers and borrowers consumption and labor supply can be defined in the OLG model and will satisfy the corresponding market clearing conditions (12) - (13) in the infinite horizon model:

\[
\begin{align*}
\bar{C}_s &= \frac{1}{1 - \pi_b} \sum_{i \in \Omega^c} C_i \\
\bar{C}_b &= \frac{1}{\pi_b} \sum_{i \in \Omega} C_i \\
\bar{N}_s &= \frac{1}{1 - \pi_b} \sum_{i \in \Omega^c} N_i \\
\bar{N}_b &= \frac{1}{\pi_b} \sum_{i \in \Omega} N_i
\end{align*}
\]

For suitable definitions of the utility functions for each household, household’s labor supply conditions hold in steady state:

\[
\begin{align*}
U^s_c (\bar{C}_s, \bar{N}_s) \bar{W} &= -U^b_h (\bar{C}_s, \bar{N}_s) \\
U^b_c (\bar{C}_b, \bar{N}_b) \bar{W} &= -U^b_h (\bar{C}_b, \bar{N}_b)
\end{align*}
\]

Under the assumption that firm profits are only paid to savers and the assumption that $\theta_i$ implies only one switch from borrowing to saving midway through the lifecycle, summing the
budget constraints of borrower household:

$$\sum_{i \in \Omega^c} C_i = W \sum_{i \in \Omega^c} N_i + \sum_{i \in \Omega^c} B_i \left(1 - \frac{1 + i_b}{\Pi}\right) - T \sum_{i \in \Omega^c} 1[i \in \Omega^c]$$

$$\Rightarrow B = \frac{1}{\pi_b} \sum_{i \in \Omega^c} B_i$$

Finally, the interest rate and borrowing rate from the OLG model determine the discount rates in the infinite horizon model:

$$\beta = \frac{1}{1 + i_d}$$

$$\gamma = \frac{1}{(1 + i_d)(1 + \omega)}$$

A.3 Equilibrium Conditions

Household equilibrium conditions and relevant transversality conditions for $i \in \{s, b\}$:

$$\lambda_t^i = u_c(C_t^i, N_t^i)$$

$$\lambda_t^i W_t = -u_n(C_t^i, N_t^i)$$

$$\lambda_t^s = \beta E_t \lambda_{t+1}^s \frac{1 + i_d^{t+1}}{\Pi_{t+1}}$$

$$\lambda_t^b = \gamma E_t \lambda_{t+1}^b \frac{(1 + i_d^t)(1 + \omega_t)}{\Pi_{t+1}}$$

Law of motion for public sector and private sector debt:

$$B_t = C_t^b - W_t N_t^b + \frac{1 + i_b^{t-1}}{\Pi_t} B_{t-1} + T_t$$

$$B_t^g = G_t + \frac{1 + i_d^{t-1}}{\Pi_t} B_{t-1}^g - T_t$$
Firm production, cost minimization, price setting and price level determination:

\[ Y_t = N_t^\alpha \]
\[ W_t = \alpha \frac{Y_t}{N_t} MC_t \]
\[ 1 = \theta \Pi_t^{\nu-1} + (1 - \theta) \left( \frac{K_t}{F_t} \right)^{\nu-1} \]
\[ F_t = \frac{\nu}{\nu-1} \lambda^s_t MC_t Y_t + \theta \beta E_t \Pi_{t+1}^{\nu} F_{t+1} \]
\[ K_t = \lambda^s_t Y_t + \theta \beta E_t \Pi_{t+1}^{\nu-1} K_{t+1} \]

Monetary and fiscal policy rules and solvency condition:

\[ \left( \frac{i_d^t}{\bar{T}_d} \right) = \left( \Pi_t \right)^{\phi_y} \left( \frac{Y_t}{Y_t^n} \right)^{\phi_y} \]
\[ T_t = \phi_y \left( B_{t-1}^q - B_g \right) - reb_t \]
\[ 0 = \lim_{T \to \infty} E^t_0 \frac{P_t}{P_T} \frac{B_T^q}{\Pi_T^f} (1 + i_{t-1}^d) \]

Credit spread determination:

\[ 1 + \omega_t = E_t \Gamma \left( B_t, W_{t+1} N_{t+1}^b, Z_t \right) \]

Market-clearing conditions:

\[ Y_t = \eta C_t^b + (1 - \eta) C_t^s + G_t \]
\[ N_t = \eta N_t^b + (1 - \eta) N_t^s \]

Exogenous processes:

\[ \log \left( \frac{G_t}{\overline{G}} \right) = \rho_g \left( \frac{G_{t-1}}{\overline{G}} \right) + \epsilon_t^g \]
\[ reb_t - \overline{reb} = \rho_{reb} \left( reb_{t-1} - \overline{reb} \right) + \epsilon_t^{reb} \]
Appendix B

Sectoral Shocks, the Beveridge Curve and Monetary Policy

B.1 Model Details

B.1.1 Household’s Problem

\[
V_0(B_0, N_{-1}) = \max_{\{C_t, B_{t+1}, L_i, N_i\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t, N_t) - \sum_{i=1}^{K-1} R(L_{i,t-1}, L_{i,t}) \right\},
\]

s.t. \[
P_tC_t = \sum_{i=1}^{K} (W_{i,t}N_{i,t} + \Pi_{i,t}) + B_t - E_tQ_{t,t+1}B_{t+1}, \quad (\lambda_{1,t})
\]

\[
N_{i,t} = (1 - \delta_i) N_{i,t-1} + p_{i,t}(L_{i,t} - N_{i,t-1}), \quad (\lambda_{2,t,i})
\]

\[
\sum_{i=1}^{K} L_{i,t} = 1. \quad (\lambda_{3,t})
\]
The Lagrangian for this problem is

\[
\mathcal{L}_0 = \mathbb{E}_0 \sum_{i=0}^{\infty} \beta^i \left\{ u(C_t, N_t) - \sum_{i=1}^{K-1} R(L_{i,t-1}, L_{i,t}) - \lambda_{1,t} \left[ P_t C_t - \sum_{i=1}^{K} (W_{i,t} N_{i,t} + \Pi_{i,t}) - B_t \right] \\
+ \mathbb{E}_t Q_{t,t+1} B_{t+1} \right\} - \sum_{i=1}^{K} \lambda_{2,t,i} [N_{i,t} - (1 - \delta_i) N_{i,t-1} - p_{i,t}(L_{i,t} - N_{i,t-1})] \\
- \lambda_{3,t} \left[ \sum_{i=1}^{K} L_{i,t} \right] \right\}.
\]

The optimality condition with respect to consumption index is

\[
u_c(C_t, N_t) = \lambda_{1,t} P_t.
\] (B.1)

The first order condition with respect to \( B_{t+1} \) in any possible state at \( t+1 \), taking into account the previous expression, leads to

\[
\beta \frac{u_c(C_{t+1}, N_{t+1})}{P_{t+1}} = \frac{u_c(C_t, N_t)}{P_t} Q_{t,t+1}.
\] (B.2)

The riskless one-period nominal interest rate can be expressed as follows

\[
\frac{1}{1 + \delta_t} = \mathbb{E}_t Q_{t,t+1}.
\]

The optimality condition with respect to labor force in sector \( i \) is

\[
\lambda_{2,t,i} p_{i,t} = \lambda_{3,t} + R_2(L_{i,t-1}, L_{i,t}) + \beta \mathbb{E}_t (L_{i,t}, L_{i,t+1})
\] (B.3)

The optimality condition with respect to the employed labor in sector \( i \) can be written as follows

\[
\lambda_{2,t,i} = u_N(C_t, N_t) + \lambda_{1,t} W_{i,t} + \beta \mathbb{E}_t [(1 - \delta_i - p_{i,t+1}) \lambda_{2,t+1,i}].
\] (B.4)

Note that \( \lambda_{2,t,i} \) represents the utility value of additional employed worker in sector \( i \) for the household conditional on the equilibrium path of wages \( \{W_{i,t}\}_{t=0}^{\infty} \).
### B.1.2 Retailer’s Problem

The retailers problem is similar to the standard specification in Woodford (2003). Monopolistically competitive retailers set prices to maximize profits:

$$\max_{P_t(l)} \Pi_{ret}^r(l) = \mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} \left[ P_t(l) - P_{FT} \right] Y_T(l),$$

s.t. $Y_T(l) = Y_T \left( \frac{P_t(l)}{P_T} \right)^{-\zeta}$,

where $P_t(l)$ is the nominal price chosen by retailer that sells differentiated good $l$ and who faces a downward sloping demand schedule and discount future profits by the nominal stochastic discount factor $Q_{t,T}$. Parameter $\chi$ is the Calvo parameter governing the degree of price stickiness. The optimality condition for price-setting is given by:

$$\mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} P_T^{\zeta} Y_T \left( \frac{P_t^*(l)}{P_T} - \frac{\zeta}{\zeta - 1} P_{FT} \right) = 0.$$

Which implies

$$\frac{P_t^*(l)}{P_t} = \frac{\zeta}{\zeta - 1} \frac{\mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} P_T^{\zeta} Y_T}{\mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} P_T^{\zeta} Y_T}.$$

The inflation rate is derived from the Calvo assumption with a fraction $1 - \chi$ of firms resetting their prices to $P_t(l) / P_t$:

$$P_t = \left\{ \chi P_{t-1}^{1-\zeta} + (1 - \chi) (P_t^*)^{1-\zeta} \right\}^{\frac{1}{1-\zeta}}.$$

In a zero inflation steady state, a log-linearization of these equilibrium conditions delivers the standard New Keynesian Phillips curve.
B.2 Additional Proofs

For several proofs, we will refer repeatedly to the equilibrium conditions that determine the steady state Beveridge curve and the natural rate of unemployment in the $K$-sector model.

A solution of the multisector model with “fast-moving” labor markets is a value for aggregate output $Y_t$, real marginal cost $P_{ft}/P_t$, consumption $C_t$, state-variables in the retailers pricing problem $K_t$, $F_t$ and sectoral prices and quantities $\{Y_{i,t}, N_{i,t}, U_{i,t}, V_{i,t}, P_{i,t}/P_t, W_{i,t}/P_t, p_{i,t}, q_{i,t}\}_{i=1}^K$ that satisfy the following static equilibrium conditions

\[
Y_t = \left\{ \sum_{i=1}^{K} \phi_{i,t} Y_{i,t}^{\frac{n}{\eta-1}} \right\}^{\frac{\eta}{\eta-1}} \Rightarrow Y_t = A_t \left\{ \sum_{i=1}^{K} \phi_{i,t} N_{i,t}^{\frac{n}{\eta-1}} \right\}^{\frac{\eta}{\eta-1}}, \tag{B.5}
\]

\[
\frac{P_{ft}}{P_t} = \left\{ \sum_{i=1}^{K} \phi_i \left( \frac{P_{i,t}}{P_t} \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}} \tag{B.6},
\]

\[
Y_{i,t} = \phi_{i,t} Y_t \left( \frac{P_{i,t}}{P_{ft}} \right)^{-\eta} \Rightarrow Y_{i,t} = \bar{\phi}_{i,t} A^{1-\eta} Y_t \left( \frac{P_{i,t}}{P_{ft}} \right)^{-\eta}, \tag{B.7}
\]

\[
Y_{i,t} = A_{i,t} N_{i,t} \Rightarrow Y_{i,t} = \bar{A}_{i,t} A N_{i,t}, \tag{B.8}
\]

\[
\frac{P_{i,t}}{P_t} A_{i,t} = \frac{W_{i,t}}{P_t} + \frac{\kappa}{q_{i,t}} [1 - \beta (1 - \delta)] \tag{B.9},
\]

\[
W_{i,t} = f(N_t) + \frac{\nu}{1 - \nu q_{i,t}} [1 - \beta (1 - \delta - p_{i,t})] \tag{B.10},
\]

\[
\delta_i N_{i,t} = \varphi_i U_{i,t}^{\alpha} V_{i,t}^{1-\alpha} \tag{B.11},
\]

\[
q_{i,t} = \varphi_i \left( \frac{V_{i,t}}{U_{i,t}} \right)^{-\alpha} \tag{B.12},
\]

\[
p_{i,t} = \varphi_i \left( \frac{V_{i,t}}{U_{i,t}} \right)^{1-\alpha} \tag{B.13}.
\]
and the following dynamic conditions:

\begin{align}
1 &= \nu \Pi_t^{\zeta-1} + (1 - \nu) \left( \frac{K_t}{F_t} \right)^{\zeta-1}, \\
K_t &= \frac{\zeta}{\zeta - 1} u_c(C_t, N_t) \frac{P_t}{p_t} Y_t + \beta \chi \Pi_t^{\zeta} \Pi_{t+1}^{\zeta-1} K_{t+1}, \\
F_t &= u_c(C_t, N_t) Y_t + \beta \chi \Pi_t^{\zeta-1} F_{t+1}, \\
1 &= \beta E_t \frac{u_c(C_{t+1}, N_{t+1})}{u_c(C_t, N_t)} (1 + i_t^d) / \Pi_{t+1}, \\
Y_t &= C_t + \sum_{i=1}^{K} \kappa V_{i,t} + G_t,
\end{align}

in terms of the exogenous variables: aggregate productivity $A_t$, government spending $G_t$, and sector-specific productivity and demand $\{ \tilde{A}_{i,t}, \tilde{\phi}_{i,t} \}_{i=1}^{K-1}$. We consider either the case of no reallocation or the case of costless reallocation. With no reallocation

\begin{equation}
L_{i,t} = N_{i,t-1} + U_{i,t}
\end{equation}

and with costless reallocation

\begin{equation}
1 = N_t + U_t,
\end{equation}

\begin{equation}
V_{i,t}/U_{i,t} = V_{j,t}/U_{j,t}, \quad \text{for} \ i, j = 1, 2, \ldots, K.
\end{equation}

**B.2.1 Proof of Proposition 2.2**

To that aggregate productivity shocks $A_t$ trace out the same Beveridge curve as government spending shocks $G_t$, we must show that for any value of the government spending shock $G_t$, there exists an aggregate productivity shock $A_t$ that implies the same level of aggregate vacancies and unemployment holding constant $\{ \tilde{A}_{i,t}, \tilde{\phi}_{i,t} \}_{i=1}^{K-1}$. 

Observe that equations (B.5) and (B.7) - (B.9) can be combined to derive the following
modified sectoral demand conditions and vacancy posting conditions:

\[ A_t \bar{A}_{i,t} N_{i,t} = \bar{A}_{i,t}^{1-\eta} \frac{1}{\varphi_{i,t}} A_t \left\{ \sum_{i=1}^{K} \frac{1}{\phi_{i,t}^{1-n}} N_{i,t}^{\frac{n-1}{n}} \right\} \eta^{\frac{n}{\eta-1}} \left( \frac{P_{i,t}}{P_{ft}} \right)^{-\eta}, \quad (B.22) \]

\[ \frac{P_{i,t}}{P_{ft}} A_t \bar{A}_{i,t} = \frac{W_{i,t}}{P_t} + \frac{\kappa}{q_{i,t}} [1 - \beta (1 - \delta_t)]. \quad (B.23) \]

Collectively, equations (B.10) - (B.13) for \( K \) sectors, the \( K \) equations (B.19) (or (B.20) and (B.21)), and the \( K \) equations in (B.22) and (B.23) define the quantities \( \{N_{i,t}, U_{i,t}, V_{i,t}, P_{i,t}/P_{ft}, W_{i,t}/P_t, p_{i,t}, q_{i,t}\}_{i=1}^{K} \) as a function of \( \{P_{ft}/P_t, A_t\} \). Thus, aggregate vacancies and unemployment are the same conditional on the same combinations of \( P_{ft}/P_t \), an endogenous variable, and \( A_t \), an exogenous variable. The absence of wealth effects on labor supply is important, otherwise household consumption \( C_t \) would tie these equations back to the rest of the equilibrium conditions.

Since vacancies and unemployment are functions solely of \( A_t P_{ft}/P_t \), any combinations of \( G_t \) and \( A_t \) that implies the same value for marginal cost times aggregate productivity implies the same values for vacancies and unemployment. Define the function \( \frac{P_t}{P_f} (G, A) \) as the endogenous value of real marginal cost for different combinations of the aggregate shocks holding sectoral shocks constant. Choose, \( A_t = \bar{A} \) such that \( \frac{P_t}{P_f} (G, 1) = \frac{P_t}{P_f} (G, 0, \bar{A}) \).

Then, it follows that:

\[ V \left( G, 1, \{\bar{A}_{i,t}\}_{i=1}^{K}, \{\tilde{\phi}_{i,t}\}_{i=1}^{K} \right) = V \left( G_0, \bar{A}, \{\bar{A}_{i,t}\}_{i=1}^{K}, \{\tilde{\phi}_{i,t}\}_{i=1}^{K} \right), \]

\[ U \left( G, 1, \{\bar{A}_{i,t}\}_{i=1}^{K}, \{\tilde{\phi}_{i,t}\}_{i=1}^{K} \right) = U \left( G_0, \bar{A}, \{\bar{A}_{i,t}\}_{i=1}^{K}, \{\tilde{\phi}_{i,t}\}_{i=1}^{K} \right). \]

\[ \blacksquare \]

**B.2.2 Proof of Proposition 2.4**

We show that under perfect substitutability, sectoral employment has a factor representation in terms of the exogenous sectoral productivity process. Under perfect reallocation, the relative price of goods across sectors must be equalized. From equation (B.6), \( P_i/P = \mu^{-1} \)
for all sectors $i = 1, 2, \ldots, K$. For simplicity, assume no aggregate demand shocks and set $\mu^{-1} = 1$. The firm’s vacancy posting condition is given by equation (50):

$$A_i = W_i + \frac{\kappa}{q_i} [1 - \beta (1 - \delta_i)].$$

Log-linearizing equations (50) - (55) and combining, we have:

$$a_{i,t} = (1 - s_i) \tilde{\alpha}_i \frac{L_i/U_i}{1 - \alpha} n_{it},$$

where $s_i$ is the steady state surplus and $\tilde{\alpha}_i$ is a composite parameter that depends on the matching function elasticity $\alpha$ and other matching function parameters when bargaining power is nonzero. The diagonal matrix $H$ is obtained by simply inverted the expression to solve for sectoral employment.

B.2.3 Proof of Proposition 2.6

In this proof, we show that sector-specific shocks raise the natural rate of unemployment and shift outward the Beveridge curve in the absence of labor market reallocation. We begin by listing the equilibrium conditions that determine aggregate employment. Under the assumption of no heterogeneity in matching function efficiencies or separation rates, the system of equations determining employment are given by the following conditions where time subscripts are dropped for simplicity:

$$Y = A \left\{ \sum_{i=1}^{K} \phi_i^\frac{n+1}{\eta} N_i^\frac{\eta}{\eta-1} \right\}$$

$$AN_i = \tilde{\phi}_i Y A^n g (\theta_i)^{-\eta}$$

$$\delta N_i = \varphi \theta_i^{1-\alpha} (L_i - N_i)$$

where $g$ is an increasing and concave function of sectoral labor market tightness. These $2K + 1$ equations determine equilibrium output $Y$, sectoral employment $N_i$, and sectoral labor market
tightness $\theta_i$ in terms of the labor force distribution $L_i$ taken as given and constant, sectoral shocks $\tilde{\phi}_i$, and an aggregate productivity shock $A$ that traces out the Beveridge curve.

To prove that the natural rate of unemployment must increase, we normalize $A = 1$ and eliminate $\theta_i$:

$$Y = \left\{ \sum_{i=1}^{K} \frac{1}{\phi_i^\eta N_i^{\eta-1}} \right\}^{\eta-1}$$

$$N_i = \tilde{\phi}_i Y g \left( \left( \frac{\delta N_i}{\varphi L_i - N_i} \right)^{1/(1-\alpha)} \right)^{-\eta}$$

Eliminating $Y$, rearranging and summing across sectors, we have:

$$\left\{ \sum_{i=1}^{K} \frac{1}{\phi_i^\eta n_i^{\eta-1}} \right\}^{\eta-1} = \sum_{i=1}^{K} n_i g \left( x_i^{1-\alpha} \right)^\eta = \sum_{i=1}^{K} n_i h (x_i) \tag{B.24}$$

where $x_i = \frac{L_i/N-n_i}{n_i}$, an expression of aggregate employment $N$, sectoral employment shares $n_i$, and the distribution of the labor force $L_i$. The function $h$ is defined in terms of the function $g : h = g \left( x^{-\alpha} \right)^\eta$ where $g$ is given by:

$$g (\theta) = z + \frac{1}{1-\nu} \frac{\kappa \theta^\alpha (1 - \beta (1 - \delta))}{\varphi} + \frac{\nu}{1-\nu} \kappa \beta \theta$$

It is readily shown that $h$ is a decreasing and strictly convex function for standard assumptions on the matching function parameters which ensure the coefficients on the polynomial terms of $\theta$ in the function $g$ are positive.

Let $N_0$ be the level of employment when $L_i = \tilde{\phi}_i$ and let $N_1$ be the level of employment when $L_i \neq \tilde{\phi}_i$. When $L_i = \tilde{\phi}_i$, labor market tightness $\theta_i$ is equalized across sectors and the left-hand side of equation (66) is equal to unity. Therefore, $N_0$ is implicitly defined by the function $h$:

$$1 = h \left( \frac{1}{N_0} - 1 \right)$$
Using our definitions of $x_i$ and the fact that $h$ is a convex function, we have:

$$\left\{ \sum_{i=1}^{K} \frac{1}{\phi_i} n_i \right\}^{\frac{1}{\eta}} = \sum_{i=1}^{K} n_i h(x_i) > h \left( \sum_{i=1}^{K} n_i x_i \right) = h \left( \frac{1}{N_1} - 1 \right)$$

where the first strict inequality follows from the strict convexity of $h$ and the fact for some sectors $i$ and $j$, it must be the case that $x_i \neq x_j$. The second equality follows from the definition of $x_i$.

The left-hand side of the previous equation is bounded above by 1. This can be shown by considering the cases of $\eta < 1$ and $\eta > 1$ separately, and applying the properties of convex or concave functions. If $\eta < 1$, then:

$$\sum_{i=1}^{K} n_i \left( \frac{\tilde{\phi}_i}{n_i} \right)^{1/\eta} \geq 1$$

$$\Rightarrow \left( \sum_{i=1}^{K} n_i \left( \frac{\tilde{\phi}_i}{n_i} \right)^{1/\eta} \right)^{\eta/\eta - 1} \leq 1$$

and vice versa in the case of $\eta > 1$.

Thus, we conclude that:

$$h \left( \frac{1}{N_0} - 1 \right) > h \left( \frac{1}{N_1} - 1 \right) \Rightarrow \frac{1}{N_0} - 1 < \frac{1}{N_1} - 1 \Rightarrow N_0 > N_1$$

and the natural rate of unemployment must rise in the case that $L_i \neq \tilde{\phi}_i$ as required.

It can be readily verified that when $L_i = \tilde{\phi}_i$, then $N_i = \tilde{\phi}_i N$ with aggregate tightness and
employment implicitly defined by the following equations:

\[
\begin{align*}
A &= g(\theta) \\
N &= \frac{\varphi \theta^{1-\alpha}}{\varphi \theta^{1-\alpha} + \delta}
\end{align*}
\]

Since sectoral shocks do not appear in these equations, aggregate shocks keep tightness equalized across sectors even if reallocation is costly. To show that vacancies rise under a sector-specific shock, we derive an expression for aggregate vacancies in terms of aggregate employment and sectoral tightness:

\[
V = \frac{\delta}{\varphi} N \sum_{i=1}^{K} (\theta_i)^{\alpha} \frac{N_i}{N}
\]

In the case of aggregate shocks, tightness is equalized across sectors and given by the expression:

\[
\theta = \left( \frac{\delta}{\varphi} \frac{N}{1-N} \right)^{\frac{1}{1-\alpha}}
\]

Let \( N' = N \left( 1, \tilde{\phi}_i \right) \) and \( \mathcal{N} = N \left( A', \tilde{\phi}_i \right) \). Define the share of employment in a given sector under the sectoral shock as \( n_i = N'_i/N' \) and ratio of labor to employment as \( l_i = L_i/N' = \tilde{\phi}_i/N' \). Then, sectoral tightness is given by:

\[
\theta_i = \left( \frac{\delta}{\varphi \tilde{l}_i - n_i} \right)^{\frac{1}{1-\alpha}} = \left( \frac{\varphi \tilde{l}_i - n_i}{\delta n_i} \right)^{-\frac{1}{1-\alpha}}
\]

Define \( V' = V \left( 1, \tilde{\phi}_i \right) \) and \( \mathcal{V} = V \left( A', \tilde{\phi}_i \right) \). Then:

\[
\begin{align*}
V' &= \frac{\delta}{\varphi} N' \sum_{i=1}^{K} \left( \frac{\varphi \tilde{l}_i - n_i}{\delta n_i} \right)^{-\frac{\alpha}{1-\alpha}} n_i > \frac{\delta}{\varphi} N' \left( \sum_{i=1}^{K} \frac{\varphi \tilde{l}_i - n_i}{\delta n_i} n_i \right)^{-\frac{\alpha}{1-\alpha}} \\
&= \frac{\delta}{\varphi} N' \left( \frac{\delta}{\varphi 1 - N'} \right)^{\frac{\alpha}{1-\alpha}} = \mathcal{V}
\end{align*}
\]
where the first inequality follows from the strict convexity of the inverse labor market tightness and the last equality follows from the fact that \( N' = \overline{N} \), which follows from the assumption that unemployment is equalized.

\[ \]
where the bar superscript signifies the sector-specific shock. It cannot be the case that $\theta = \bar{\theta}$, since equation (B.26) would not be satisfied. If $\theta < \bar{\theta}$, then $n_A > \pi_A$. Taking ratios of equation (B.26), it must be the case that:

$$
\left( \frac{g_A}{g_B} \right)^{-\eta} < 1
$$

$$
\Rightarrow \left( \frac{\bar{g}_A}{g_A} \right)^{\eta} > 1
$$

which is a contradiction since the ratio $g_A/g_B$ is decreasing in tightness. Therefore, it must be the case that $\theta > \bar{\theta}$ and $n_A < \pi_A$. Under costless reallocation, vacancies simply $V = \theta(1 - N)$. Since $N = \bar{N}$, but $\theta > \bar{\theta}$, $V (\bar{\phi}', 1) < V (\bar{\phi}, \mu')$ and the Beveridge curve shifts inward.

\[ \blacksquare \]

### B.2.5 Proof of Proposition 2.8

Holding constant $\{A_i, \phi_i\}_{i=1}^K$, we define $V (G, z)$ and $U (G, z)$ as aggregate vacancies and unemployment for given values of the government spending shock $G$ and the common reservation wage $z$. We wish to show that $\forall G > 0$, there exists a $\bar{z}$ such that $V (\bar{G}, z_0) = V (1, \bar{z})$ and $U (\bar{G}, z_0) = U (1, \bar{z})$.

The government spending shock only affect vacancies and unemployment via the real marginal cost. Let $\bar{\mu}^{-1} = \frac{P_i}{\bar{P}} (\bar{G})$. Relative prices are equalized in steady state since sectoral productivities and hiring costs are equalized. Therefore, the surplus in each sector is the same:

$$
P_i \frac{1}{P} A_i = z + g (\theta_i)
$$

$$
\mu^{-1} A - z = g (\theta)
$$

For each sector $\theta = g (\mu^{-1} A - z)^{-1}$ where $g$ is an increasing and concave function. If $\mu = \bar{\mu}$, then $\bar{z} = A - (\bar{\mu}^{-1} A - z)$ ensures the same labor market tightness in each sector when $\mu^{-1} = 1$, which is the case of no government spending shocks, and tightness is invariant to combinations of $\mu$ and $z$. As a result, aggregate vacancies and unemployment are equalized as required.

If labor reallocation is costless, then Proposition 2.3 applies. However, in the absence of
labor reallocation, sectoral shocks will shift the same Beveridge curve as shown in Proposition 2.6. Therefore, the fact that two aggregate shocks, government spending shocks and reservation wage shocks, trace out the same Beveridge curve does not follow because no shocks shift the Beveridge curve.

\[ Y_t = A_t N_t \left\{ \sum_{i=1}^{K} \frac{1}{\tilde{\phi}_{it}} \left( \frac{N_{it}}{N_t} \right)^{\frac{\eta-1}{\eta}} \right\} \]

\[ 1 = \left\{ \sum_{i=1}^{K} \tilde{\phi}_{it} \left( \frac{\tilde{P}_{it}}{P_t} \right)^{1-\eta} \right\}^{1/(1-\eta)} \]

\[ A_t N_{it} = \tilde{\phi}_{it} Y_t \left( \frac{\tilde{P}_{it}}{P_t} \right)^{1-\eta} \]

\[ \frac{\tilde{P}_{it}}{P_t} A_t = W_t + \frac{\kappa_t}{\varphi} \theta_t^\alpha (1 - \beta (1 - \delta)) \]

\[ W_t = \nu' (N_t) C_t + \frac{\nu}{1 - \nu} \frac{\kappa_t}{\varphi} \theta_t^\alpha (1 - \beta (1 - \delta - \varphi \theta_t^{1-\alpha})) \]

\[ Y_t = C_t + \kappa_t V_t \]

\[ \theta_t = V_t / U_t \]

\[ 1 = N_t + U_t \]

where \( \tilde{P}_{it}/P_t = P_{it} A_{it}/A_t \) is the productivity-adjusted relative price of sector \( i \)'s output and

\[ A_t = \left\{ \sum_{i=1}^{K} \phi_{it} A_{it}^{\eta-1} \right\}^{1/(\eta-1)} \]

Since hiring costs are equalized, it must be the case that \( \tilde{P}_{it}/P_t = 1 \) and \( N_{it}/N_t = \tilde{\phi}_{it} \). Combining the vacancy-posting condition, Nash-bargained wages and the assumption for
vacancy costs, we obtain the following:

\[ A_t = v'(N_t) C_t \left( 1 + \frac{X}{\varphi} h(\theta_t) \right) \]

\[ \Rightarrow 1 = v'(N_t) \frac{N_t}{1 + \chi V_t v'(N_t)} \left( 1 + \frac{X}{\varphi} h(\theta_t) \right) \]

This vacancy posting condition combined with labor market clearing and the definition of market tightness jointly determine labor market variables \( N, U, V, \theta \) where the time subscript is dropped since none of these variables is a function of exogenous variables that change over time: namely \( A_t \) and \( L_t \).

Since growth in the labor force is modeled as a net addition of new households, the labor market variables have a per capita interpretation and each variable grows at the rate \( g_l = \Delta L/L \). Thus, the unemployment rate, vacancy rate, and employment rate are constant. It is straightforward to compute the growth rates of per household output, consumption and wages given the resulting expressions:

\[ Y_t = A_t N \]

\[ Y_t = C_t + \kappa_t V \]

\[ W_t = v'(N) C_t + \frac{\nu}{1 - \nu} \varphi \kappa_t \theta_t^\alpha (1 - \beta (1 - \delta - \varphi \theta^{1 - \alpha})) \]

with \( g_y = g_c = g_w = g_A \).

However, these growth rates are constant only in the special case when sectoral productivities are equalized and grow at the same rates. Since the expression for aggregate productivity is a sum, different growth rates across sectors will generally change the growth rate of aggregate productivity. Moreover, changes in preference shares over time will also alter productivity growth rates. If all structural change is driven by changes in product shares, all per capita growth rates are zero and all aggregates grow only with the labor force. Employment shares will mirror their productivity-adjusted product shares along the growth path.

More generally, if sectoral TFP growth rates differ, then output, consumption and wage
growth will be asymptotically constant. If $\eta > 1$, then $\lim_{t \to \infty} \Delta A/A = \gamma_{\text{max}}$ where $\gamma_{\text{max}}$ is the TFP growth rate of the fastest growing sector. Alternatively, if $\eta < 1$, then the opposite holds and TFP growth converges to the growth rate of the slowest growing sectors. These results are analogous to the asymptotic growth rates computed in Acemoglu and Guerrieri (2008). If $\eta = 1$, the TFP aggregator is Cobb-Douglas and the aggregate TFP growth rate is a weighted average of each sector’s TFP growth rate.

B.3 Calibration and Model-Based Measures

B.3.1 Structural Factor Analysis

To a log-linear approximation, sectoral employment can be expressed by solving the equations that determine the steady state Beveridge curve in our model (shown at the beginning of the appendix):

$$Mn_t = Ha_t = H(\Phi z_t + \epsilon_t)$$

where $n_t = (n_{1,t}, \ldots, n_{K,t})'$ is the vector of log-linearized sectoral employment expressed in terms of the exogenous variables, the vector $a_t = (a_{1,t}, \ldots, a_{K,t})'$ of sectoral productivity shocks. As argued, the exogenous sectoral productivity process can be decomposed into its first principal component and a vector of sectoral shocks $\epsilon_t = (\epsilon_{1,t}, \ldots, \epsilon_{K,t})'$ with $\text{cov}(z_t, \epsilon_{i,t}) = 0$ for $\forall i = 1, 2, \ldots, K$.

The matrix $M$ is determined by the model parameters and the steady state values of labor market variables. To compute this matrix, it is necessary to choose parameters and solve for the model steady state. We calibrate an 11-sector version of our model where the sectors conform to the NAICS supersectors for which there is readily available data on employment, unemployment and vacancies. Our reduced form sector-specific shock index was computed using 13 NAICS sectors, but we use only 11 sectors since retail trade, wholesale trade, transportation and utilities are combined into a single sector in the data on unemployment and
vacancies from the CPS and JOLTs respectively.

To calibrate the 11 sector version of the model, some parameters are chosen directly while some parameters are chosen to match targets. As in the calibrations shown earlier, the household’s discount rate \( \beta \), matching function elasticity \( \alpha \), and bargaining power \( \nu \) are all set to the values described in Section 2.5.1. Separation rates for the 11 sectors are set to match the 2000-2006 average in the JOLTs data. We chose matching function efficiencies \( \phi_i \), CES product shares \( \phi_i \), reservation wage \( z \), and the vacancy posting cost \( \kappa \) to match the following targets: the distribution of vacancies \( V_i/V \), the distribution of employment \( N_i/N \), an unemployment rate \( U/L = 5\% \), a vacancy rate \( V/L = 2.5\% \), and a share-weighted accounting surplus of 10\%. Vacancy shares and employment shares are set using 2000-2006 averages from the JOLTs and payroll survey respectively. Initial labor market tightness is equalized across sectors so that unemployment shares match vacancy shares. The table below summarizes the calibration targets, parameters, and components of the matrix \( M \) that is used to rotate the sectoral employment data. We consider two possible values for the elasticity of substitution \( \eta \), with \( \eta = 0.5 \) and \( \eta = 2 \). Table B.1 summarizes the calibration for the case of complementary goods:

When goods are substitutes the product shares, output shares, and diagonal elements of \( M \) are changed. For brevity, the employment shares, vacancy shares, separation rates, and matching function efficiencies are omitted from this table as they are the same as in Table B.1. These new steady state values are summarized in Table B.2.

### B.3.2 Relation of Sector-Specific Shock Index and the Beveridge Curve

Consider a steady state where \( \bar{\theta}_i = \bar{\theta}_h \) for all sectors \( i, h = 1, 2, \ldots, K \). In the absence of labor market mismatch, it follows that unemployment shares and vacancy shares are equalized. To a log linear approximation, aggregate vacancies, unemployment and employment are given by the following equations:

\[
v_t = \sum_{i=1}^{K} V_i V_{it}, \quad u_t = \sum_{i=1}^{K} U_i U_{it}, \quad n_t = \sum_{i=1}^{K} N_i N_{it}.
\]
Table B.1: Calibration

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<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Discount rate</td>
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<td>Bargaining power</td>
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<td>Matching function elasticity</td>
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<td>Elasticity of substitution</td>
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<td>Vacancy posting cost</td>
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<td>Reservation wage</td>
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Panel B

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<tr>
<th>Sectors</th>
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<th>Vacancy Share</th>
<th>MFE</th>
<th>Separations Share</th>
<th>Product Share $\phi$</th>
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<td>19.5%</td>
<td>19.3%</td>
<td>1.93</td>
</tr>
</tbody>
</table>

Employment shares and vacancy shares are 2000-2006 averages from the CES and JOLTs respectively. Separation rates are 2000-2006 averages from JOLTs. Matching function efficiency is chosen to equate labor market tightness across sectors and target overall vacancy rate of 2.5%. Product shares are chosen to match the sectoral distribution of employment. Output shares and diagonal entries of the $M$ matrix are model-implied values.

A log-linear approximation to the sectoral Beveridge curve provides the following expression:

$$n_{it} = \alpha u_{it} + (1 - \alpha) v_{it}$$

Using the expressions for aggregate vacancies and unemployment and the fact that $U_i / U = V_i / V$, we have:

$$\sum_{i=1}^{K} \frac{U_i}{U} n_{it} = \alpha u_{t} + (1 - \alpha) v_{t}$$

Adding and subtracting aggregate employment and rearranging, we obtain the following relation:

$$v_{t} = \frac{1}{1 - \alpha} \left\{ - \left( \alpha + \frac{U}{N} \right) u_{t} + \sum_{i=1}^{K} \left( \frac{U_i}{U} - \frac{N_i}{N} \right) n_{it} \right\}$$
It is worth noting that in our numerical calibration, the aggregate component of $n_{it}$ approximately cancels out, and we are left with an expression in terms of the sectoral shocks:

$$v_t = \frac{1}{1 - \alpha} \left\{ - \left( \alpha + \frac{\bar{U}}{\bar{N}} \right) u_t + \sum_{i=1}^{K} \left( \frac{\bar{U}_i}{\bar{U}} - \frac{\bar{N}_i}{\bar{N}} \right) \epsilon_{it} \right\}$$

### B.4 Collateral Constraint

Our result demonstrating an equivalence between sector-specific shocks and shocks to the borrowing rate in a model with a working capital constraint can be generalized to other types of financial shocks. A common shock considered in the literature is a Kiyotaki and Moore type shock to the value of collateral. We modify the problem of the intermediate goods producer to include a time-varying collateral constraint that limits the ability of the firm to borrow to
finance the wage bill and the cost of posting vacancies:

\[
\Pi_{i,t}^{int} = \max_{\{V_i,t, N_i,t\}_{t=1}^\infty} \mathbb{E}_t \sum_{T=t}^{\infty} Q_{i,T} \left( P_{i,T} Y_{i,T} - \left( 1 + i_t^b \right) \left( W_{i,T} N_{i,T} - \kappa V_{i,T} P_T \right) \right),
\]

s.t. \( N_{i,t} = (1 - \delta_i) N_{i,t-1} + q_{i,t} V_{i,t} \),

\[ Y_{i,t} = A_{i,t} N_{i,t}, \]

\[ \lambda_t K \geq W_{i,t} N_{i,t} + \kappa V_{i,t} P_t. \]

Fluctuation in \( \lambda_t \) can represent a tightening of lending standards by financial institutions or a fall in the value of collateral like real estate or other forms of capital. For simplicity, we continue to assume that labor is the only variable factor of production and that constrained firms have some fixed endowment of capital. The vacancy posting condition in this setting is similar to the vacancy posting condition (2.21) and can be written as follows:

\[
\frac{P_{i,t}}{P_t} \frac{A_t}{1 + \varphi_t} = \frac{W_{i,t}}{P_t} + \frac{\kappa}{q_{i,t}} - \mathbb{E}_t \left[ Q_{i,t+1} (1 - \delta_i) \frac{\kappa}{q_{i,t+1}} \frac{1 + \varphi_{t+1}}{1 + \varphi_t} \right],
\]

where \( \varphi_t \) is the Lagrange multiplier on the collateral constraint and replaces the interest rate on borrowed funds. In steady state, the Lagrange multiplier on the constraint enters as a sector-specific productivity shock for any sector that faces a working capital constraint. A decrease in the value of \( \lambda_t \) tightens the constraint and raises the Lagrange multiplier. Therefore, our choice of modeling the financial shock as an interest rate shock instead of a shock to collateral values has no qualitative effects on the behavior of firms.
Appendix C

Job Flows and Financial Shocks

C.1 Simple Model: Characterization

C.1.1 Household

The Hamiltonian for the household problem is a sum of instantaneous utility function and the right-hand side of the wealth evolution equation multiplied by the Lagrange multiplier $\lambda_H$

$$\mathcal{H} = e^{-\rho t}[u(c) - v(n)] + \lambda_H[wn + ra + \Pi - c]$$

Maximum principle necessarily implies

$$\mathcal{H}_n = -e^{-\rho t}u'(n) + \lambda_H w = 0$$

$$\mathcal{H}_c = -e^{-\rho t}u'(c) - \lambda_H = 0$$

$$\dot{\lambda}_H = -\lambda_H r$$

If we substitute out the Lagrange multiplier we get

$$\dot{c} = -\frac{u'(c)}{u''(c)}(r - \rho) \quad \text{(C.1)}$$

$$w = \frac{v'(n)}{u'(c)} \quad \text{(C.2)}$$
C.1.2 Proof of Lemma 3.1

To solve equation
\[ \dot{a} = Aa^\psi - Ba, \]
introduce the following change of variables \( y = \log a \). Hence,
\[ \dot{y} = Ae^{(\psi-1)y} - B. \]

We can rewrite this equation as follows
\[ \frac{dy}{Ae^{(\psi-1)y} - B} = dt. \]

Rearranging we get
\[ \frac{1}{B(\psi - 1)} \left( \frac{d(Ae^{(\psi-1)y} - B)}{Ae^{(\psi-1)y} - B} - d[(\psi - 1)y] \right) = dt. \]

Integrating this equation leads to
\[ \log \left[ Ae^{(\psi-1)y} - B \right] - (\psi - 1)y = B(\psi - 1)t + \text{const}. \]

Transforming back to original variable
\[ \log[A - Ba^{1-\psi}] = B(\psi - 1)t + \text{const}. \]

Since \( a(t = 0) = a_0 \) we have
\[ \log \left[ \frac{A - Ba^{1-\psi}}{A - Ba_0^{1-\psi}} \right] = B(\psi - 1)t. \]

This can be expressed as
\[ a = \left\{ \frac{A - (A - Ba_0^{-\psi})e^{-B(1-\psi)t}}{B} \right\}^{1/(1-\psi)}. \]
C.1.3 Properties of $a(t, \chi, \epsilon)$

**Monotonicity in $t$.** By taking first order derivative of (3.15) with respect to time we can show that depending on the initial condition the following three cases are possible

\[
a'(t) \begin{cases} 
> 0 & \text{for all } t \text{ if } a_0 < \left( \frac{A}{B} \right)^{1/(1-\psi)} \\
= 0 & \text{for all } t \text{ if } a_0 = \left( \frac{A}{B} \right)^{1/(1-\psi)} \\
< 0 & \text{for all } t \text{ if } a_0 > \left( \frac{A}{B} \right)^{1/(1-\psi)}. 
\end{cases}
\]

Note that once we combine the two cases it we will not be optimal to borrow up to firms borrowing limit $k = ka$ if $a_0 > (A/B)^{1/(1-\psi)}$. This implies that in equilibrium all the firms will never decrease its level of capital rentals.

**Convexity in $t$.** $\dot{a} = Aa^\psi - Ba$ implies $\ddot{a} = (\psi Aa^{\psi-1} - B) \dot{a} = (\psi Aa^{\psi-1} - B) (Aa^\psi - Ba)$. Hence,

\[
a''(t) \begin{cases} 
> 0 & \text{if } a < \left( \frac{A}{B} \right)^{1/(1-\psi)} \\
< 0 & \text{if } a \in \left( \left( \frac{A}{B} \right)^{1/(1-\psi)}, \left( \frac{A}{B} \right)^{1/(1-\psi)} \right) \\
> 0 & \text{if } a > \left( \frac{A}{B} \right)^{1/(1-\psi)}. 
\end{cases}
\]

**Monotonicity in $\chi$.**

\[
\frac{da^{1-\psi}}{d\chi} = \frac{\partial a^{1-\psi}}{\partial A} \frac{dA}{d\chi} + \frac{\partial a^{1-\psi}}{\partial B} \frac{dB}{d\chi} \\
= \frac{A}{B} \left( 1 - e^{-B(1-\psi)t} \right) \left[ \frac{A\chi}{A} - \frac{B\chi}{B} \right] + \left( \frac{A}{B} - a_0^{1-\psi} \right) e^{-B(1-\psi)t} (1 - \psi)tB\chi \\
< 0 \quad \leq 0
\]

because

\[
\frac{A\chi}{A} - \frac{B\chi}{B} = \frac{\alpha \phi}{1 - \phi(1 - \alpha)} \chi - \frac{r_k}{r_k \chi - r} < 0.
\]

Hence,
• if \( a_0 < \left(\frac{A}{B}\right)^{1/(1-\psi)} \) then there exists \( T_a < \infty \) such that
  \[
  a_2(t, \chi) = \begin{cases} 
  > 0 & \text{if } t < T_a, \\
  = 0 & \text{if } t = T_a, \\
  < 0 & \text{if } t > T_a,
  \end{cases}
  \]

• if \( a_0 > \left(\frac{A}{B}\right)^{1/(1-\psi)} \) then \( a_2(t, \chi) < 0 \).

\[
\Box
\]

C.1.4 Proof of Lemma 3.3

**PE effect.**

\[
\int_0^{\tau(\chi_L)} n(t, \epsilon_H, w, r, r_k, \chi_L)e^{-\sigma t} dt + \int_0^{\tau(\chi_L)} n^s(\epsilon_H, w, r, r_k)e^{-\sigma t} dt < \int_0^{\tau(\chi_H)} n(t, \epsilon_H, w, r, r_k, \chi_L)e^{-\sigma t} dt + \int_0^{\tau(\chi_H)} n^s(\epsilon_H, w, r, r_k)e^{-\sigma t} dt
\]

**GE effect.**

In the absence of wealth effect the equilibrium on labor market can be expressed as follows

\[
N = N^d(w, \chi) = N^s(w).
\]

Taking full derivative we obtain

\[
\frac{dN}{d\chi} = N^d_1(w) N^d_2(w, \chi) > 0,
\]

where \( N^s_1(w) > 0, N^d_2(w, \chi) > 0, N^d_1(w, \chi) < 0 \) are the derivative of the corresponding functions. \( \Box \)
Job creation and job destruction. Firm-level job creation is

\[ jc(t) = \max\{\dot{n}(t), 0\} \begin{cases} = \dot{n}(t) & \text{if } t < \bar{t} \\ = 0 & \text{if } t \geq \bar{t} \end{cases} \]

Note that for any firm that reaches its optimal size it is true that

\[ n^*(t) = \bar{n}(0) + \int_0^t jc(t) dt \quad \text{(C.3)} \]

The first term takes into account that at the moment of birth the firm increases its employment from 0 to \( \bar{n}(0) \).