STOCKHOLDER UNANIMITY IN MAKING PRODUCTION AND FINANCIAL DECISIONS*

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We show that "spanning" does not imply stockholder unanimity if there is trading in the shares of firms. Each basis vector of the space spanned by all firms' output vectors can be treated like a composite commodity. If, in addition to spanning, firms act as price takers with respect to prices of composite commodities, then there is unanimity. We analyze the spanning assumption for the vector space of contingent claims generated by firms' choices of debt-equity ratios. We show that there is a strong relationship between the Modigliani-Miller theorem, spanning, and the existence of a complete set of markets.

INTRODUCTION

The recent literature on corporate firm behavior has developed conditions under which all stockholders of a firm will unanimously prefer a given production decision over some others. Under the assumption of no trade, stockholder unanimity is proved, provided that any production plan of the firm can be written as a linear combination of the production plans of the other firms, i.e., there is "spanning." This theorem is thought of as an extension of the well-known result that if there is a complete set of state-contingent claims markets, then there is shareholder unanimity.

This paper shows that spanning does not, in general, imply unanimity if there is trading in the shares of firms. We study a model where consumers trade shares after receiving new information about the firm's output between the time of a production decision and the time the output comes out of its machines. We consider the vector space spanned by the output vectors of all firms and choose a basis for the vector space. Each basis vector can be treated in the same way that a composite commodity is treated in consumption theory. Though spanning does not imply unanimity when there is trade due to new information, we give a condition that does imply unanimity. If, *in addition* to spanning, firms are assumed to behave as perfect competitors in the production of the composite commodities that form a basis for the spanned space, then there is unanimity. We call this

* This work was supported by National Science Foundation Grants SOC76-18771 and SOC77-15980. The authors are also indebted to the Dean Witter Foundation for financial support. We are very grateful to Oliver Hart for helpful comments and for correcting an error in an earlier draft of this paper. We are also grateful to an anonymous referee.

The Quarterly Journal of Economics, May 1980 0033-5533/80/0094-0543$02.40

assumption "competitivity." This is the assumption that each firm perceives the market price of the basis (i.e., composite) commodities to be unaffected by its production decision. However, the assumption of both spanning and competitivity leads to the very strong result that all of a firm's shareholders desire to maximize the net market value of their shares. Though this may be a satisfactory theory, we refer to empirical evidence for a particular class of firms that do not value maximize.

Spanning has a further extreme implication when one of the firm's decision variables is its debt-equity ratio. We show that if spanning holds with respect to this decision variable, then there must be a complete set of contingent claims markets, and thus the spanning theorem is no generalization of the well-known result that with complete markets the debt-equity ratio is indeterminate. In addition, we show that there is a strong relationship between the Modigliani-Miller Theorem, spanning, and the existence of a complete set of markets.

The structure of this paper is as follows. Section II describes our three-period economy (where there are two trading periods to allow for the analysis of the effect of trade due to new information). Section III surveys the literature on spanning in models with one trading period in order to point out how the no-trade assumption is used and how competivity can be used to avoid this assumption. Section IV shows that spanning alone does not imply unanimity in a multiperiod context, but that spanning plus competitivity does imply unanimity. Section V discusses unanimity with respect to the choice of the debt-equity ratio, and shows that if the return stream of any bond issued by any firm can be written as a linear combination of the returns of existing firms (i.e., spanning with respect to debt as well as equity), then there must be a complete set of contingent claims markets. Section VI gives conclusions and refers to empirical evidence on value maximization.

II. THE MULTIPERIOD MODEL WITHOUT DEBT

The economy extends for three periods 0, 1, 2. The state of the world ω is composed of a signal t, which is known in period 1 and a final state s, which is known in period 2; $\omega = (t, s)$. There is a single commodity that is available in each period and that is used for consumption and investment in period 0. There is no investment in periods 1 and 2; only consumption occurs in those periods.

In period 0, there are markets for the single current commodity and also for shares in firms. It is assumed that no contingent commodity contracts can be made. In period 1, there are markets for the single current commodity and also for shares in the firms. Consumers also get the signal t which changes their beliefs about s, before trading in period 1. In period 2, consumers are allocated output according to the shares held in firms at the end of period 1.

Let there be I consumers, J firms, T signals, and S final states, indexed by $i = 1, \ldots, I$, $j = 1, \ldots, J$, $t = 1, \ldots, T$, $s = 1, \ldots, S$, respectively. Sometimes I, J, T, S will be referred to as the set of consumers, firms, signals, and final states, respectively.

Consumers

We shall represent consumer i's consumption plan by a vector $x_i = (x_i^0, x_i^1, x_i^2) \in R_+^{1+T+TS}$, where $x_i^0 \in R_+$ is consumption in period 0; $x_i^1(t) \in R_+$ is consumption when there is signal t in period 1; $x_i^2(\omega) \equiv x_i^2(t, s) \in R_+$ is consumption in period 2 when (t, s) was realized. Consumer i is assumed to have a utility function U_i defined on R_+^{1+T+TS}. We shall assume that U_i is strictly quasi-concave and continuously differentiable on the interior of its domain, and $\partial U_i(x_i)/\partial x_i^0 \to \infty$ as $x_i^0 \to 0$, $\partial U_i(x_i)/\partial x_i^1(t) \to \infty$ as $x_i^1(t) \to 0$ for $t = 1, 2, \ldots, T$.

Firms

Firm j's production possibilities are represented by a production set $Y_j \subset R_+^{S+1}$. If $y_j \in Y_j$, we write $y_j = (y_j^0, y_j^2)$, where $y_j^0 \in R_+$ is the input in period 0 and $y_j^2(s) \in R_+$ is the output in period 2 state s. Note that inputs appear as nonnegative numbers. We shall assume that Y_j is convex, closed, and contains the origin. Further, if $y_j \in Y_j$ and $y_j^0 = 0$, then $y_j^2 = 0$.

It is assumed that consumer i has initial endowments $\bar{x}_i^0 \in R_+$, $\bar{x}_i^1 \in R_+^T$ of the commodity in periods 0 and 1, respectively. He also has initial shareholdings, $\bar{\theta}_{ij} \geq 0$ in firm j, where $\sum_i \bar{\theta}_{ij} = 1$ for each j. For notational simplicity we ignore endowments in period 2. Let $\bar{X}^0 = \sum_i \bar{x}_i^0$, and assume that $\bar{X}^0 > 0$.

Equilibrium for Fixed Production Plans

Let y_j be given for each $j = 1, \ldots, J$. The ith consumer maximizes $U_i(x_i^0, x_i^1, x_i^2)$ with respect to $(x_i, \theta_{ij}, \theta_{ij}(t))$, $j = 1, \ldots, J$, $t = 1, \ldots, T$ subject to

$$x_i^2(s, t) = \sum_j \theta_{ij}(t) y_j^2(s) \tag{1}$$

(2) $$x_i^1(t) + \sum_j p_j(t)\theta_{ij}(t) \leq \sum_j p_j(t)\theta_{ij} + \bar{x}_i^1(t)$$

(3) $$x_i^0 + \sum_j p_j\theta_{ij} \leq \sum_j \bar{\theta}_{ij}(p_j - y_j^0) + \bar{x}_i^0,$$

where $x_i^2(s, t)$ is the (s, t)th component of x_i^2 and represents i's planned consumption at date 2 if t is the date 1 signal and s is the date 2 state; $\theta_{ij}(t)$ is the planned holdings of shares of firm j in period 1 when t is the information signal; $x_i^1(t)$ is the tth component of x_i^1; $p_j(t)$ is the price of firm j in period 1, signal t; p_j is the price of firm j in period 0; and θ_{ij} is the desired holdings of shares of firm j in period 0. In (3) we have assumed that inputs y_j^0 are financed by the issuance of equity alone, so that $p_j - y_j^0$ is the net market value of firm j to the initial shareholders. In (1)–(3) we assume that shares of firms provide the only means of purchasing state-contingent income.

With no constraint against short sales (i.e., we do not require θ_{ij} or $\theta_{ij}(t)$ to be nonnegative), necessary and sufficient conditions for a solution to the maximum problem are that there exist multipliers $(\lambda_i^0, \lambda_i^1)$, $\lambda_i^0 \in R_+$, $\lambda_i^1 \in R_+^T$ such that

(4) $$\nabla_0 U_i = \lambda_i^0, \qquad \nabla_1 U_i = \lambda_i^1$$

(5) $$\lambda_i^0 p_j = \sum_t \lambda_{it} p_j(t), \qquad j = 1, \ldots, J$$

(6) $$\nabla_{2t} U_i \cdot y_j^2 = \lambda_{it} p_j(t), \qquad j = 1, \ldots, J, \qquad t = 1, \ldots, T,$$

where

$$\nabla_l U_i \equiv \frac{\partial U_i(x_i)}{\partial x_i^l}, \qquad l = 0, 1,$$

and

$$\nabla_{2t} U_i = \left[\frac{\partial U_i(x_i)}{\partial x_i^2(t, s)}\right]_{s=1}^{S},$$

and λ_{it} is the tth component of λ_i^1.

A competitive exchange equilibrium for the economy, relative to the production plans (y_j), is then a collection (x_i), (θ_{ij}), $(\theta_{ij}(t))$, (p_j), $(p_j(t))$ such that (4)–(6) hold for each consumer and

(7) $$\sum_i x_i^0 + \sum_j y_j^0 = \sum_i \bar{x}_i^0, \qquad \sum_i \theta_{ij} = 1,$$

$$\sum_i \theta_{ij}(t) = 1, \qquad \forall\, t \in T, \qquad \forall\, j \in J$$

and

$$\sum_i x_i^1(t) = \sum_i \bar{x}_i^1(t), \qquad \forall\, t \in T.$$

In the above economy, firms fix their inputs in period 0, and output is realized in period 2. In period 1 consumers get new information about the distribution of firm output in period 2, so they have the opportunity to recontract away from their period zero holdings of firm shares. In period 2, state s, each consumer gets the output that corresponds to the share of the firms he owns.

In the above definition of equilibrium we took as exogenously given the production plans of all firms. We now exposit some methods of determining production plans as well as exchange equilibrium. *We assume that the initial shareholders of a firm have the power to make a legally binding contract for its input and output decisions* (see Grossman and Hart [1979] for an elaboration of the importance of this assumption). For example, the initial shareholders can legally bind the firm to purchase a particular piece of land. We assume that future shareholders cannot alter this purchase (i.e., they cannot renege on the firm's contracts). Thus, we search for a production plan y_j^* such that no initial shareholder prefers some other plan y_j to y_j^*. In this section we shall give a local analysis of each shareholder's problem; in later sections a global analysis is presented. In order to find a locally optimum decision, we derive an initial shareholder's change in utility if the production plan is changed in the direction of an arbitrary $y_j \in Y_j$. That is, for $0 \leqq c \leqq 1, \tilde{y}_j(c) \equiv (1-c)y_j^* + cy_j$ is in Y_j. Note that $\tilde{y}_j^2(c)$ is the vector of date 2 contingent claims. It should not be confused with $y_j^2(s)$, which is output in states at date 2. Define $U_i^*(c)$ as the maximal value of $U_i(x_i^0, x_i^1, x_i^2)$ with respect to $(x_i, \theta_{ij}, \theta_{ij}(t))$ subject to the constraints in (1)–(3) when $y_j \equiv (y_j^0, y_j^2)$ is replaced by $y_j(c)$.

By direct calculation,

$$\text{(8)} \quad \frac{dU_i^*(c)}{dc} = \lambda_i^0(\bar{\theta}_{ij} - \theta_{ij})\frac{dp_j}{dc} + \sum_{t=1}^{T} \lambda_{it}(\theta_{ij} - \theta_{ij}(t))\frac{dp_j(t)}{dc} + \sum_{t=1}^{T} \theta_{ij}(t)\nabla_{2t}U_i \frac{d\tilde{y}_j^2(c)}{dc} - \lambda_i^0\bar{\theta}_{ij}\frac{dy_j^0(c)}{dc}.$$

In deriving equation (8), we have used equations (4)–(6) and the assumption,[1]

$$\text{(9)} \quad \frac{dy_k}{dc} = \frac{dp_k}{dc} = 0 \qquad \text{for } k \neq j.$$

1. Note that there are no terms in (8) for $d\theta_{ij}$ or $d\theta_{ij}(t)$. Direct computation will verify that all such terms drop out. This is a version of the Envelope Theorem: changes in an endogenous variable caused by a change in a parameter cannot increase the magnitude of an objective function, if the endogenous variables are set optimally. Note that we are assuming that the consumers' optimal choice of $(x_i, \theta_{ij}(t), \theta_{ij})$ is a differentiable function of c.

That is, when the jth firm changes its production decision, no change is assumed to occur in all other firms' production decisions and prices. (Here we are already making assumptions about the degree of competition among firms, or implicitly imposing a Nash equilibrium notion on firm behavior.) Note that price changes enter equation (8). The interpretation of equation (8) is that it represents a shareholder's perceptions about his change in utility as a function of perceived price changes. These contemplated changes are caused by a contemplated change in production plan.[2] We shall explain the meaning of each of the terms in (8), below.

Much of the recent literature on firm behavior has attempted to show that (8) has the same sign for all individuals, i.e., that there is unanimity with respect to different individual evaluations of alternative production plans. It is our objective to show that the assumption of "spanning," which is made to get unanimity in a two-period context, is not sufficient in a three-period context. We shall show that in addition to *spanning*, the stronger assumption of *competitivity* (to be defined below) is a sufficient condition for unanimity in a multiperiod trading context. It will be useful first, to review the literature on unanimity in a two-period context (i.e., one trading period).[3]

III. UNANIMITY WITH ONLY ONE TRADING PERIOD

Suppose that we close the market for shares in period 1, or that with the market open, all traders decide not to trade. In this case the firm makes its decision in period 0, and in period 2 consumers get their period 0 shares of the firm's period 2 output. That is, $\theta_{ik} = \theta_{ik}(t)$ for all t, so equation (8) can be written as

$$(10) \qquad \frac{dU_i^*(c)}{dc} = \lambda_i^0 \left[\bar{\theta}_{ij}\left(1 - \frac{dy_i^0}{dc} \Big/ \frac{dp_i}{dc}\right) - \theta_{ij}\right]\frac{dp_i}{dc} + \theta_{ij}\sum_t \nabla_{2t} U_i \frac{d\tilde{y}_j^2(c)}{dc}.$$

The first term on the right-hand side of (10) represents a wealth (i.e., capital gains) effect, while the second term represents a consumption effect (i.e., the change in utility due to a change in the

2. The next two sections will be much more specific about how a firm should perceive the price of its shares to change when it changes its production plan.

3. See Ekern [1973] for some results on multiperiod spanning that are of the same type as the Ekern-Wilson results described in Section III.

composition of date 2 consumption). Stiglitz [1970] considered two alternative technological assumptions under which stockholder unanimity would occur: all traders agree that y_j^2 has a given normal distribution, or alternatively there is multiplicative uncertainty (stochastic homotheticity). Multiplicative uncertainty means that any $y_j \in Y_j$ can be written as ky_j^* for some positive real number k. Thus,

$$\frac{d\tilde{y}_j^2(c)}{dc} = \bar{k}y_j^{2*}, \tag{11}$$

for some real number $\bar{k}$. An increase in c by dc represents an increase in output of $\bar{k}dc$ percent in every state of nature. If this is the case, then the consumption effect in (10) becomes

$$\theta_{ij} \sum_t \nabla_{2t} U_i \frac{d\tilde{y}_j^2(c)}{dc} = \theta_{ij} \sum_t \nabla_{2t} U_i \bar{k} y_j^{2*} = \bar{k}\theta_{ij}\lambda_i^0 p_j, \tag{12}$$

where the second equality follows from (5) and (6). It follows from (12) that all shareholders in firm j agree about the sign of the consumption effect. However, in order for all shareholders to agree to change c, they must agree about the sign of the sum of the consumption and the wealth effect; see (10).

In order to get some intuition as to the sign of this sum, Stiglitz argued that two phases of a corporation's existence can be isolated—an early phase and a mature phase. In the early phase of the firm, a small group of investors own the firm and want to raise capital to expand, and each member wants to diversify his personal wealth into other securities. In this early phase the initial stockholders have very large $\bar{\theta}_{ij}$ (initial holdings) relative to θ_{ij} (desired holdings). Thus, assuming that $\theta_{ij} \cong 0$, we can write (10) as

$$\frac{dU_i^*}{dc} \cong \lambda_i^0 \bar{\theta}_{ij} \frac{d(p_j - y_j^0(c))}{dc}. \tag{13}$$

Clearly in this case all stockholders desire to maximize net market value.

In the mature, steady state stage of a firm's existence, stockholders simply hold their portfolio of shares every period, i.e., in a world with no new information or other shocks hitting the market, a steady state means that $\theta_{ij} = \bar{\theta}_{ij}$. Using $\theta_{ij} = \bar{\theta}_{ij}$ and (12), we can write (10) as

$$\frac{dU_i^*}{dc} = \lambda_i^0 \bar{\theta}_{ij} \left(\bar{k}p_j - \frac{dy_j^0(c)}{dc}\right). \tag{14}$$

Shareholders will unanimously favor a $\bar{k}$ percent increase in y_j^* if $\bar{k}p_j > dy_j^0(c)/dc$. Otherwise, shareholders will unanimously be opposed to such a change. Note that this does not imply that shareholders unanimously favor net market value maximization (i.e., choosing a production plan to maximize $p_j - y_j^0$). Equation (14) would imply net value maximization if the *additional* (competitivity) assumption is made that a $\bar{k}$ percent increase in output in each state leads to a $\bar{k}$ percent increase in the price of the firm's shares. That is, the product this firm is selling to shareholders is a bundle of contingent claims y_j^{2*}. If it produces $\bar{k}$ percent more of this commodity, then its revenue will not go up by $\bar{k}$ percent, unless there is an additional assumption of price-taking behavior. This is further elaborated below.

Note that we have permitted short sales and assumed that shareholders are price takers with respect to their trades in the stock market. It might be thought that competitivity follows immediately from these assumptions because each shareholder can buy or sell a claim to the dividend stream $(1 + \bar{k})y_j^{2*}$ by changing his shareholdings from θ_{ij} to $(1 + \bar{k})\theta_{ij}$. The cost of doing this would be $p_j(1 + \bar{k})\theta_{ij} - p_j\theta_{ij} = \theta_{ij}\bar{k}p_j$, for a price-taking shareholder. Thus, a consumer trading on his own account would face a cost change of $dp_j/dc = \bar{k}p_j$ if he tried to change his dividends to $(1 + \bar{k})y_j^2$. This is misleading in that it confuses competition among shareholders for shares of the firm with competition among firms in producing the output y_j^2. For example, suppose that there is only one state of nature, and y_j^2 is the amount of water a water monopolist sells. Then shareholders are just the consumers of water. Each water consumer can act like a price taker, but the water monopolist owns the only water well, and he realizes that doubling his water sales will lower the price of water.[4]

Ekern-Wilson [1974] and Leland [1974] have generalized the technological conditions under which stockholders are unanimous with respect to the sign of the consumption effect, and called Stiglitz's steady state assumption that $\theta_{ij} = \bar{\theta}_{ij}$, "ex post unanimity." Note that

4. This view is to be contrasted with that of Hart [1979] who argues that if there are an infinite number of consumers and a finite amount of water, then each consumer consumes an infinitesimal amount of water and the price of water is the marginal rate of substitution between water and current consumption, evaluated at zero water consumption. This price will not vary if the firm doubles its water sales because each consumer still consumes an infinitesimal amount of water. In our view each consumer consumes a small enough proportion of the total water output to act as a price taker; however, each consumer would have a much higher marginal rate of substitution if he had to halve his water consumption. This is because even though any consumer's consumption is small relative to total production, it is large relative to his needs. Grossman [1979] analyzes the role of free entry when fixed costs are large in ensuring price-taking behavior, and provides less stringent conditions for perfect competition than does Hart.

in Section II we assumed that initial shareholders made a legally binding commitment on the firm to produce y_j. One interpretation of "ex post unanimity" is that final shareholders make the production decision rather than initial shareholders. A final shareholder, by definition, has no desire to trade so $\theta_{ij} = \bar{\theta}_{ij}$. However, it is clearly in the interest of initial shareholders to choose the production decision that is best for themselves (by definition). Presumably, "ex post unanimity" applies to firms where legally binding commitments are difficult to make. As we shall show in a multiperiod model, final shareholders cannot make the production decision because they do not own the firm until a time in the future subsequent to actual outputs being produced. Ekern and Wilson assume that any feasible change in a firm's output plan can be written as a linear combination of all firm's current output plans; with $\{y_j\}_{j=1}^{J}$ given, they assume there exist real numbers $\hat{\alpha}_{jk}$ such that

$$\text{(15)} \qquad \frac{d\tilde{y}_j^2(c)}{dc} = \sum_{k=1}^{J} \hat{\alpha}_{jk} y_k^2.$$

If (15) holds for $\{y_j\}_{j=1}^{J}$, we say there is spanning at $\{y_j\}_{j=1}^{J}$. Substituting (15) into (10) and using (5) and (6) yields

$$\text{(16)} \quad \frac{dU_i^*}{dc} = \lambda_i^0 \left[\bar{\theta}_{ij}\left(1 - \frac{dy_j^0(c)}{dc} \Big/ \frac{dp_j}{dc}\right) - \theta_{ij}\right] \frac{dp_j}{dc} + \lambda_i^0 \theta_{ij} \sum_{k=1}^{J} \hat{\alpha}_{jk} p_k.$$

We see that the sign of the consumption effect is independent of i, for all shareholders of firm j. Using the no-trade or ex post condition that $\theta_{ij} = \bar{\theta}_{ij}$, we have

$$\text{(17)} \qquad \frac{dU_i^*}{dc} = \lambda_i^0 \bar{\theta}_{ij} \left\{\sum_{k=1}^{J} \hat{\alpha}_{jk} p_k - \frac{dy_j^0(c)}{dc}\right\},$$

the sign of which is independent of i for all shareholders of firm j.

Ekern and Wilson concluded that unanimity can occur, but that it does not imply value maximization, since they saw no relationship between $\sum_k \hat{\alpha}_{jk} p_k$ and dp_j/dc. We shall show below that these two terms are equal if an appropriate form of price-taking behavior occurs, which we call competitivity. We shall show in the next section that the Ekern-Wilson result is a strictly "one trading period" theorem. It does not hold if we allow trade in period 1. Spanning only implies unanimity in an economy where people would not want to trade after production decisions have been made.

Leland [1973, p. 16] and Radner [1974] give general conditions under which there is unanimity on value maximization. In doing so, they assume spanning, but do not impose the no-trade (ex post) condition $\theta_{ij} = \bar{\theta}_{ij}$. Instead, they assume competitivity as Diamond [1967] assumed. Diamond noted that with certain technologies it is possible to analyze a stock market economy as if it is an ordinary Arrow-Debreu economy where composite commodities are traded.

To study their approach to spanning, it is helpful to consider all the sets Z_L of the form $Z_L = R \times L$, where L is a linear subspace of R^S. Let Z be the smallest set Z_L containing all the production sets Y_j, $j = 1, \ldots, J$, i.e., $z \in Z$ iff $Z_L \supset U_j Y_j$ implies that $z \in Z_L$. Z is clearly a linear subspace of R^{S+1} and can be written as $Z = R \times L$ for some L, which is a linear subspace of R^S. Choose a basis for L, say $\{e_k\}_{k=1}^K$, where $e_k \in R^S$.

We shall now define an Arrow-Debreu economy where consumers purchase quantities of the various characteristics $\{e_k\}$, instead of shares of the firm. As in the previous part of this section, we shall assume only one trading period for firm shares, period 0. As there is no trade in firm shares in period 1, $x_i^2 \in R_+^S$, rather than $x_i^2 \in R^{TS}$. If the ith consumer chooses an $x_i^2 \in R_+^S$, which is technologically feasible (i.e., in L), then there exist K real numbers, β_{ik}, $k = 1, \ldots, K$ such that $x_i^2 = \sum_{k=1}^K \beta_{ik} e_k$. Therefore, we can define an implicit utility function for the ith consumer as

$$V_i(x_i^0, x_i^1, \beta_i) \equiv U_i\left(x_i^0, x_i^1, \sum_{k=1}^{K} \beta_{ik} e_k\right), \tag{18}$$

where β_{ik} is the quantity of the kth characteristic that consumer i purchases in period 0 for delivery in period 2.

We can define Arrow-Debreu prices for the characteristics. Let $q \in R^K$ be the price vector for quantities of characteristics. Let q_k be the kth component of q. The object $(\{x_i^*, \beta_i^*\}, \{y_j^*, \alpha_j^*\}, q^*)$ is an Arrow-Debreu equilibrium for characteristics in a one-trading period model if

$$x_i^{2*} = \sum_k \beta_{ik}^* e_k, \qquad x_i^{1*} = \bar{x}_i^1, \tag{19}$$

and

$$\{x_i^{0*}, \beta_i^*\} \quad \text{is a maximizer of} \quad V_i(x_i^0, \bar{x}_i^1, \beta_i)$$

subject to

$$x_i^0 + q^* \cdot \beta_i \leq \bar{x}_i + \sum_j \bar{\theta}_{ij}(q^* \cdot \alpha_j^* - y_j^{*0}); \tag{20}$$

$$y_j^* \in \cdot\, Y_j, \qquad y_j^{2*} = \sum \alpha_{jk}^* e_k$$

and (y_j^{0*}, α_j^*) is a maximizer of $q \cdot \alpha_j - y_j^0$ subject to $(y_j^0, \sum_k \alpha_{jk} e_k) \in Y_j$;

(21) $$\sum_i x_i^{2*} = \sum_j y_j^{2*}, \qquad \sum_i x_i^{0*} + \sum_j y_j^{0*} = \sum_i \bar{x}_i^0.$$

Condition (19) states that consumers maximize utility subject to a budget constraint where their wealth is measured by their initial endowment of goods and their initial shares of firms' profits. Condition (20) states that firms maximize profits. Firms can calculate profits here because they produce characteristics and the market prices characteristics. Thus, if the jth firm produces an additional unit of characteristic k (i.e., increase α_{jk} by one unit), its value increases by q_k. *Competitivity* means that each firm takes the price vector q as given and not affected by its own output decision. If it is the case that the stock market economy is equivalent to the Arrow-Debreu economy in characteristics, then unanimity in favor of value maximization would immediately follow. Obviously, in an Arrow-Debreu economy all shareholders prefer value maximization.

Note that $\hat{\alpha}_j \in R^J$ (see (15)). Further, $\hat{\alpha}_j$ is the representation of firm j's output (or change in output) in terms of the output vectors of all other firms. The assumption of spanning is that the output vector of firm j is in the spanning set generated by the output vectors of all firms. On the other hand, K is the smallest number of vectors needed to represent the output of any firm. The vector α_j represents firm j's output in terms of a basis vector for the K-dimensional space L. In general, K is much larger than J because there are many more states of nature than firms. However, Radner's spanning assumption (below) is that $J = K$.

Radner says that the above economy has the *spanning* property when

(22) $$\{y_j^{2*}\}_{j=1}^J$$

spans L. It is clear that when the Arrow-Debreu economy for characteristics has the spanning property, it is formally equivalent to the stock market economy where shares of firms are traded and firms are valued by

(23) $$p_j = q \cdot \alpha_j,$$

when their output vector is

$$y_j^2 = \sum_k \alpha_{jk} e_k.$$

This is because any vector in L can be achieved by buying appropriate shares of

$$\{y_j^{2*}\}_{j=1}^{J},$$

i.e., by buying appropriate shares of the firms. Suppose that all shareholders believe that the prices of characteristics are parameters beyond any firm's control, then

$$\frac{dp_j}{dc} = q \cdot \frac{d\alpha_j}{dc}, \qquad \left(\frac{dq}{dc} = 0\right). \tag{24}$$

Equations (23) and (24) together make up the "competitivity" assumption in Leland [1973, p. 16] and Radner. In an economy where (22)–(24) hold, it is clear from (19) that each consumer, with $\bar{\theta}_{ij} > 0$, prefers that the jth firm maximize net market value $q^* \cdot \alpha_j - y_j^0$, since the only effect the firm has on consumption opportunities is to change the wealth of the consumer. Leland and Radner call this "ex ante unanimity." By this they mean that even if all consumers do not come to the stockholders' meeting holding the portfolio that is optimal (given that the firm produces y_j^2), they will still be unanimous in valuing any plan. In our interpretation, the initial owners of the firm who have the power to make their production decision legally binding will all be unanimous in their desire to maximize net market value. Further, there is unanimity for both small and large changes in production plans: *All* stockholders want their firm to maximize net market value, computed using (23).

It is helpful to give a calculus proof of unanimity in the above economy. By definition, any firm f has a unique output representation in terms of composite commodities $\{e_k\}$:

$$y_f^2 = \sum_{l=1}^{K} \alpha_{fl} e_l. \tag{25a}$$

Setting $f = j$ in (25a), we see that

$$\frac{dy_j^2}{dc} = \sum_{l=1}^{K} \left(\frac{d\alpha_{jl}}{dc}\right) e_l. \tag{25b}$$

But if we set $f = k$ in (25a) and use (25a) to evaluate (15), we get

$$\frac{dy_j^2}{dc} = \sum_{k=1}^{J} \hat{\alpha}_{jk} \sum_{l=1}^{K} \alpha_{kl} e_l = \sum_{l=1}^{K} \left(\sum_{k=1}^{J} \hat{\alpha}_{jk} \alpha_{kl}\right) e_l. \tag{25c}$$

Equations (25b) and (25c) give two representations of the vector dy_j^2/dc. Since this vector is feasible for a firm, it has a unique representation in terms of the basis vectors $\{e_l\}_l$. Hence

$$\text{(25d)} \qquad \frac{d\alpha_{jl}}{dc} = \sum_{k=1}^{J} \hat{\alpha}_{jk}\alpha_{kl}.$$

But by (23) $dp_j/dc = q \cdot (d\alpha_j/dc) = \sum_l q_l(d\alpha_{jl}/dc)$, where the first equality uses the assumption that the firm takes the prices of characteristics as a parameter beyond its control. So using (25d) and (23)

$$\frac{dp_j}{dc} = \sum_{k=1}^{J} \hat{\alpha}_{jk}p_k.$$

Thus, (16) can be written as

$$\text{(26)} \qquad \frac{dU_i^*}{dc} = \lambda_i^0 \left[\bar{\theta}_{ij}\left(1 - \frac{dy_j^0}{dc} \Big/ \frac{dp_j}{dc}\right) - \theta_{ij}\right]\frac{dp_j}{dc} + \lambda_i^0\theta_{ij}\frac{dp_j}{dc}.$$

From (26) we see that spanning and competitivity imply that $dU_i^*/dc = \lambda_i^0\bar{\theta}_{ij}(d(p_j - y_j^0)/dc)$. Thus, Leland and Radner use competitivity to show that irrespective of whether $\theta_{ij} = \bar{\theta}_{ij}$, all stockholders prefer net-value maximization. By assuming no trade Stiglitz and Ekern-Wilson had no need to examine the sign of the wealth effect, and thus no need to assume competitivity to study dp_j/dc. Under spanning plus competitivity, an increase in the firm's net market value always expands the consumption opportunities of its shareholders as can be seen from (19).

From the above discussion, it is clear that there are two kinds of results in the literature. The first set of results assumes spanning and competitivity and shows that all shareholders want their firm to net-value maximize. The second set of results assumes some kind of "no trade" and only spanning, and proves unanimity. In this case, shareholders need not be unanimous about net-value maximization. In the next section we shall show that none of the "no trade" assumptions make sense in a multiperiod context, and argue that spanning alone is not sufficient to prove unanimity, but spanning and competitivity are sufficient for proving unanimity.

IV. THE MULTIPERIOD ECONOMY

In the last section we assumed that there was only one trading period in order to survey the literature on unanimity. In this section we return to the model of Section II where consumers are permitted to trade in period 1. In equation (8) of Section II, we see that consumer i's preference for a change in the jth firm's production decision dc, depends upon three terms. The first term is the period 0 wealth effect, the second term is the period 1 wealth effect, and the third term is the consumption effect (the fourth term is the wealth effect due to the cost of the new inputs). We can use the spanning assumption (15), along with the first-order conditions (5) and (6) to write (8) as

$$(27)\quad \frac{dU_i^*}{dc} = \lambda_i^0 \left[\bar{\theta}_{ij}\left(1 - \frac{dy_j^0}{dc} \Big/ \frac{dp_j}{dc}\right) - \theta_{ij}\right]\frac{dp_j}{dc} + \sum_{t=1}^{T} \lambda_{it}(\theta_{ij} - \theta_{ij}(t))\frac{dp_j(t)}{dc} + \sum_{t=1}^{T} \lambda_{it}\theta_{ij}(t) \sum_{k=1}^{J} \hat{\alpha}_{jk}p_k(t),$$

where we have used the implicit competitivity assumption that $dp_k/dc = 0 = dp_k(t)/dc$ for $k \neq j$. Note that $\hat{\alpha}_{jk}$ does not depend upon the signal t because none of the firms' output vectors depends upon t. In this model firms must fix their production plans in period 0; no firm produces any output until period 2. Consumers may trade in period 1 based upon new information. From (27) it is clear that there will be unanimity if $\bar{\theta}_{ij} = \theta_{ij} = \theta_{ij}(t)$. However, even if the Ekern-Wilson-Leland assumption is made that $\bar{\theta}_{ij} = \theta_{ij}$, there will not, in general, be unanimity when $\theta_{ij} \neq \theta_{ij}(t)$.

There is one situation in which there will be unanimity and that is when competitivity is assumed. That is, we can construct the Arrow-Debreu economy for characteristics as of period 1. Consider the equilibrium for characteristics, when firms make production decisions that span Z. At date 1, some signal t is observed and shares are traded among consumers. As we showed in the last section, the trading of shares can be thought of as the trading of characteristics. Let q_t be the price vector at date 1, signal t for the vector of characteristics $(e_1, e_2, \ldots, e_K)$, i.e., $q_t \in R_+^K$.

When a consumer buys shares in period 0, he is buying period 1 risk. Let $A = (\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_J)$ be the matrix of composite commodities that firms have chosen to produce in period 2. That is, for each firm j, $y_j^2 = \sum_k \tilde{\alpha}_{jk}e_k$. We now define the space of risky incomes that the consumer will face in holding securities from period 0 to period 1. Let $q \equiv \{q_t\}_{t=1}^T$ be given, and define

$$(28)\quad P_q(A) \equiv \left\{\{p_t\}_{t=1}^T \in R^T \,\middle|\, \sum_{j=1}^{J} \alpha_j(q_t \cdot \tilde{\alpha}_j) = p_t \text{ for some } \{\alpha_j\}_{j=1}^J \in R^J\right\}.$$

As $q_t \cdot \tilde{\alpha}_j$ is the period 1, signal t price of firm j, p_t is the consumer's income from securities in period 1, if his initial portfolio is $\{\alpha_j\}$. $\{p_t\}_t$

is the "signal distribution" of income received in period 1.

We now show that spanning implies that no firm can affect $P_q(A)$ when it changes its output plan:

PROPOSITION. Suppose that

$$p \in P_q(A),\ \tilde{\alpha}'_{jl} \equiv \sum_{k=1}^{J} \hat{\alpha}_{jk}\tilde{\alpha}_{kl},$$

for some real numbers $\hat{\alpha}_{jk}$, and $A' \equiv (\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}'_j, \ldots, \tilde{\alpha}_J)$, then $p \in P_q(A')$.

Proof. We are given $\{\alpha_l\} \in R^J$ such that

$$p_t = \sum_{l \neq j}^{J} \alpha_l q_t \cdot \tilde{\alpha}_l + \alpha_j q_t \cdot \tilde{\alpha}_j,$$

and we must find $\{\alpha'_l\} \in R^J$ such that

$$p_t = \sum_{l \neq j}^{J} \alpha'_l q_t \cdot \tilde{\alpha}_l + \alpha'_j q_t \cdot \tilde{\alpha}'_j.$$

The latter will be true when $\alpha'_l \equiv \alpha_l - \alpha'_j \hat{\alpha}_{jl}$ for $l \neq j$ and $\alpha'_j \equiv \alpha_j \div \hat{\alpha}_{jj}$, as can be verified by direct calculation.

Q.E.D.

Let $\{y^*_j\}$ be the firm output vectors that are assumed to span Z. Let $\{\alpha^*_j\}$ be the characteristics produced $(y^*_j = \sum_k \alpha^*_{jk} e_k)$. Let $A^* \equiv (\alpha^*_1, \alpha^*_j, \ldots, \alpha^*_J)$. Clearly

(29) $$P_q(A) \subset P_q(A^*) \qquad \text{for all } A.$$

Let $\{c_m(q)\}_{m=1}^{M}$ be a basis for $P_q(A^*)$, and consider the following Arrow-Debreu economy. Define $\{x^{0*}_i, x^{1*}_i, \phi^*_i, \beta^*_i\}, \{\gamma^*_j, y^{0*}_j, \alpha^*_j\}, \{r^*, q^*\}$ to be an overall equilibrium in characteristics if $r^* \in R^M_+$, $q^* \equiv \{q^*_t\}_t$, $q^*_t \in R^k_+$, and

Consumer i chooses

(30) $$x^{0*}_i \in R_+, x^{1*}_i \in R^T_+, \{\phi^*_{ij}\} \in R^M, \beta^*_i \in R^{TK}$$

to maximize $V_i(x^0_i, x^1_i, \beta_i)$ subject to

$$x^0_i + \sum_m \phi_{im} r^*_m \leq \bar{x}^0_i + \sum_j \bar{\theta}_{ij}(r^* \cdot \gamma^*_j - y^{0*}_j),$$

and

$$x^1_i(t) + \beta_i(t) \cdot q^*_t \leq \bar{x}^1_i + \sum_m \phi_{im} c^*_{mt},$$

where

$$c^*_{mt} \equiv c_{mt}(q^*).$$

Firm j chooses

$$(\alpha^*_j, y^{0*}_j, \gamma^*_j) \tag{31}$$

to maximize $r^* \cdot \gamma_j - y^0_j$, subject to

$$q^*_t \cdot \alpha_j = \sum_m \gamma_{jm} c^*_{mt} \ \forall \ t \in T$$

and

$$\left(y^0_j, \sum_k \alpha_{jk} e_k\right) \in Y_j.$$

$$\sum_i x^{0*}_i + \sum_j y^{0*}_j = \sum_i \bar{x}^0_i, \quad \sum_i x^{1*}_i = \sum_i \bar{x}^1_i, \tag{32}$$

$$\sum_i \phi^*_{im} = \sum_j \gamma^*_{jm} \qquad \sum_i \beta^*_{ik}(t) = \sum_j \alpha^*_{jk}.$$

In (30) the consumer purchases quantities $\{\phi_{im}\}$ of the period 1 characteristics $\{c^*_m\}$ at prices $\{r^*_m\}$, and he also purchases $\beta_i(t)$ units of the period 2 characteristics at prices q^*_t. Note that we have described the consumer as if he faces two budget constraints. Rather than directly purchasing his date 1 and date 2 consumption at time 0, he purchases enough securities so that he insures himself against date 1 price changes. The equilibrium would be unchanged if we gave the consumer a single budget constraint and allowed him to purchase directly all future consumption at time 0. In (31) the jth firm produces a quantity $\{\alpha_{jk}\}$ of period 2 characteristics $\{e_{jk}\}$, but in so doing also produces $\{\gamma_{jm}\}$ units of the period one characteristics $\{c^*_m\}$.[5] Note that $q^*_t = (q^*_{t1}, q^*_{t2}, \ldots, q^*_{tK})$ is the price vector for characteristics at date 1 given that the information state is t. In (32) we have required that markets for goods and characteristics clear at each date and signal. Using the same argument given in the last section, the economy de-

5. Note that the constraint $q^*_t \cdot \alpha_j = \sum_m \gamma_{jm} c^*_{mt}$ states that the date 1, state t market value of the bundle of the date 2, state s characteristics that firm j produces (which is given by $q^*_t \cdot \alpha_j$) must equal the sum of the date 1, state t incomes it promised to produce as of time zero. That is, there are two equivalent ways of accounting for the firm at date 1, state t. On the one hand, the firm is a claim to date 2 state s income. On the other hand, as of time 0 the firm was a collection of characteristics $\{\gamma_{jm}\}_{m=1}^M$, which represents the random market value that the firm may have at time 1 (because t is random). (Note that γ_{im} is the number of units of characteristic m that a holder of the firm gets at date 1. Characteristic m is the vector $\{c^*_{mt}\}_t$ of date 1 income.) Thus, the total payoff of the firm is $\sum_m \gamma_{jm} c^*_{mt}$ at date 1, state t. It is as if the firm is liquidated at that date for $\sum_m \gamma_{jm} c^*_{mt}$ when t is the state of nature.

fined by (30)–(32) is equivalent to the stock market economy when spanning (equation (22)) holds and the prices in the stock market are given by

$$p_j^* = r^* \cdot \gamma_j^*, \qquad p_j^*(t) = q_t^* \cdot \alpha_j^* = \sum_m \gamma_{jm}^* c_{mt}^*. \tag{33}$$

This follows immediately from the fact that by buying the appropriate portfolio of period 0 shares of firms, a consumer can purchase the same period 1 characteristics at the same prices in the stock market economy as he was purchasing in the characteristics economy. A similar statement holds for the period 2 characteristics.

If we make the competitivity assumption that each firm takes the prices of characteristics as fixed when it changes its production decision,

$$\frac{dr^*}{dc} = \frac{dq_t^*}{dc} = 0, \tag{34}$$

then we can prove an ex ante unanimity theorem. From (30) it is clear that the jth firm should choose γ_j, y_j^0 to maximize the firm's net market value: $r^*\gamma_j - y_j^0$. Increasing the net market value unambiguously increases the consumer's consumption opportunities. Further, by the lemma there is no change in the space of feasible characteristics.

This can also be seen from (27), when we use the Arrow-Debreu prices of characteristics for p_j^*. That is, by (33) and (34) (i.e., competitivity), $dp_j^*(t)/dc = q_t^*(d\alpha_j^*/dc)$. Using (25) and (33), we have $dp_j^*(t)/dc = \sum_k \hat{\alpha}_{jk} p_k^*(t)$. Substitution of this into (27) and using (5) and (6) yields

$$\frac{dU_i^*}{dc} = \lambda_i^0 \left[\bar{\theta}_{ij}\left(1 - \frac{dy_j^0}{dc} \Big/ \frac{dp_j}{dc}\right) - \theta_{ij}\right] \frac{dp_j}{dc} + \lambda_i^0 \theta_{ij} \sum_k \hat{\alpha}_{jk} p_k^*. \tag{35}$$

If the Stiglitz [1970], Ekern-Wilson [1974], and Leland [1974] assumption of initial portfolio equilibrium is made, $\bar{\theta}_{ij} = \theta_{ij}$, then (35) implies unanimity, though not necessarily in the direction of value maximization. It should be clear that in the multiperiod context we are considering it was necessary to assume competitivity as well as spanning to get this unanimity result. In the one-period model spanning alone is enough. We used competitivity to combine the period 1 wealth effect (which represents capital gains and losses due to new information) with the period 2 consumption effect. Without the competitivity assumption the second term on the right-hand side of (27), which gives the period 1 wealth effects, will have different signs for different individuals. This is because different individuals have different tastes for a change in the distribution of their income across signals t. That is, looking at (27), if $dp_j(t)/dc > 0$, then shareholders who are net buyers of shares at state $t(\theta_{ij}(t) > \theta_{ij})$ will be suffering losses, while shareholders who are sellers are realizing more capital gains. The competitivity assumption allows us to connect the capital losses some consumers are realizing in date 1, event t, with the increased consumption benefits they will derive at date 2. In particular, competitivity permits us to derive that $dp_j^*(t)/dc = \sum_k \hat{\alpha}_{jk} p_k^*(t)$; see the discussion just before equation (35). We need competitivity to predict the change in the price of the bundle of characteristics produced by the firm (i.e., $p_j(t)$), which is caused by a change in the composition of the bundle. The competitivity assumption is that the prices of characteristics are not affected when a given firm changes its production plan.

However, because we have assumed competitivity in the production of period 1 characteristics, there is unanimity even if $\bar{\theta}_{ij} \neq \theta_{ij}$. To see this, use (33) and (34) to get $dp_j^*/dc = r^* \cdot (d\gamma_j^*/dc)$. Using an argument similar to the one used in deriving (25d), we can use (15), (25d), the fact that $q_t^* \cdot \alpha_j^* \equiv \sum_m \gamma_{jm}^* c_{mt}^*$ and $\{c_m^*\}$ as a basis to show that[6] $d\gamma_j^*/dc = \sum_k \hat{\alpha}_{jk} \gamma_k^*$. Thus, using (33), $dp_j^*/dc = \sum_k \hat{\alpha}_{jk} p_k^*$. Substituting this into (35) yields

$$\text{(36)} \qquad \frac{dU_i^*}{dy} = \lambda_i^0 \bar{\theta}_{ij} \frac{d(p_j^* - y_j^0)}{dc}.$$

Hence if competitivity and spanning are assumed, then all initial shareholders unanimously desire to maximize net market value, while spanning alone is not, in general, sufficient for unanimity if there is trade in period 0 of period 1.

6. Note that (31) implies that

$$\text{(A)} \qquad q_t^* \cdot \frac{d\alpha_j^*}{dc} = \sum_m \frac{d\gamma_{jm}^*}{dc} c_{mt}^*,$$

but from (25d)

$$\text{(B)} \qquad q_t^* \frac{d\alpha_j^*}{dc} = q_t^* \sum_k \hat{\alpha}_{jk} \alpha_k^* = \sum_k \hat{\alpha}_{jk} q_t^* \alpha_k^* = \sum_k \hat{\alpha}_{jk} \sum_m \gamma_{km}^* c_{mt}^*,$$

where the last equality follows from (31). Rearranging summation signs, (B) becomes

$$\text{(C)} \qquad q_t^* \frac{d\alpha_j^*}{dc} = \sum_m \left(\sum_k \hat{\alpha}_{jk} \gamma_{km}^* \right) c_{mt}^*.$$

But since $\{c_m^*\}$ is a basis, (A) and (C) imply that

$$\frac{d\gamma_{jm}^*}{dc} = \sum_k \hat{\alpha}_{jk} \gamma_{km}^*.$$

V. SPANNING AND THE CHOICE OF A DEBT-EQUITY RATIO

We now return to the one trading period model of Section III. We suppose that the production plan $y_j = (y_j^0, y_j^2)$ has been chosen, and the firm must decide how to finance y_j^0. Let D_j be the period 0 value of the debt issued by firm j. Let $1 - a$ be the fraction of the firm that the initial shareholders sell to raise capital (i.e., new equity). Then

$$y_j^0 \equiv D_j + (1-a)p_j. \tag{37}$$

Let B_j be the principal plus the interest which the firm promises that bondholders will receive in period 2. Let b_{ij} be the fraction of firm j's debt held by consumer i. Then (1) and (3) are replaced by

$$x_i^2(s) = \sum_j \theta_{ij} \max(0, y_j^2(s) - B_j) + \sum_j b_{ij} \min(B_j, y_j^2(s)) \tag{38}$$

and

$$x_i^0 + \sum_j p_j\theta_{ij} + \sum_j D_j b_{ij} \le \sum_j \bar{\theta}_{ij}(p_j - y_j^0 + D_j) + \bar{x}_i^0, \tag{39}$$

respectively. In (38) the consumer's period 2 consumption comes from nondefaulted debt payments and claims to residual output. In (39) an initial shareholder owns $a\bar{\theta}_{ij}$ percent of the firm, which is worth $a\bar{\theta}_{ij}p_j = \bar{\theta}_{ij}(p_j - y_j^0 + D_j)$ by (37).

We assume no trade in period 1, so the consumer maximizes $U_i(x_i^0, \bar{x}_i^1, x_i^2)$ subject to (38) and (39) with respect to x_i^0, θ_{ij}, and b_{ij}. Denote the maximized utility by $U_i^*(B_j)$. Let $B(B_j) = \{s \mid y_j^2(s) < B_j\}$ and $NB(B_j) = \{s \mid y_j^2(s) > B_j\}$ be the sets of states under which bankruptcy and no bankruptcy, respectively, occur. We are interested in a change in the debt-equity ratio keeping the production plan y_j constant. We can calculate $dU_i^*(B_j)/dB_j$ at B_j such that $y_j^2(s) \neq B_j$ for all s. This is given by

$$\frac{dU_i^*(B_j)}{dB_j} = \lambda_i^0(\bar{\theta}_{ij} - \theta_{ij})\frac{dV_j}{dB_j} + (\theta_{ij} - b_{ij})\left[\frac{dD_j}{dB_j} - \sum_{s\in NB} \nabla_s U_i\right], \tag{40}$$

where

$$V_j \equiv D_j + p_j$$

and

$$\nabla_s U_i \equiv \sum_{t=1}^{T} \frac{\partial U_i(x_i)}{\partial x_i^2(t, s)}.$$

The Modigliani-Miller Theorem gives conditions under which a change in the debt-equity ratio leaves the value of the firm unchanged. Explicit in their theory is the idea that since managers desire to maximize net market value, they should be indifferent as to the debt-equity ratio. However, the managers will be indifferent only if the shareholders are indifferent. Hence, the appropriate theorem should be that all shareholders are indifferent about the debt-equity ratio. As can be seen from (40), $dV_j/dB_j = 0$ is not equivalent to $dU_i^*/dB_j = 0$. There is one important and well-known situation where the two are equivalent, that is when $\theta_{ij} = b_{ij}$. This occurs if there is portfolio separation such that all consumers desire to hold the same mutual fund of all risky assets (e.g., when all consumers homogeneously believe that returns on all securities are multivariate normal and there is a risk-free asset) i.e., hold all risky assets in the same proportion.

We feel that the point of the Modigliani-Miller Theorem is that the debt-equity ratio is indeterminate because all shareholders are unanimously indifferent between choices of B_j. For this to be true, it is necessary that $dU_i^*/dB_j = 0$, where it is well defined. As can be seen from (40), this will not be true in general. One situation in which it is true is when there is no trade, i.e., $\bar{\theta}_{ij} = \theta_{ij}$, and strong portfolio separation, i.e., $\theta_{ij} = b_{ij}$. But this will not generalize to a multiperiod model just as ex post unanimity does not generalize.

It might be thought that a spanning argument will lead to unanimity in the above context. It does lead to unanimity, but it also implies that there exist complete markets.[7] To see this, let $\tilde{B}_j(s) \equiv \min(B_j, y_j^2(s))$, the risky payout stream of a bond. Spanning means that no single firm can issue a security that is not in the space spanned by existing securities. Let $L \subset R^S$ be the linear subspace spanned by existing securities. Thus, if $\{W_l\}_1^H$ is a basis for L, then for any B_j there exist real numbers $\{\alpha_l(B_j)\}_1^H$ such that

$$\tilde{B}_j(s) = \sum_l^H \alpha_l(B_j) W_l(s)$$

for all $s \in S$.

We assume that there exists a firm j such that for each $s, s' \in S$, $y_j^2(s) \neq y_j^2(s')$ and $y_j^2(s) > 0$ for all $s \in S$. We now prove that if $\tilde{B}_j$ can be spanned by existing securities for any face value B_j, then $L = R^S$, i.e., markets are complete.

7. Milne [1975, p. 177] proves the result that spanning of all of debt and equity returns by existing securities implies unanimity. He does not show that such spanning also implies that there is a complete set of markets, which is what we show below.

Let $\bar{y}_1 \equiv \min\{y_j^2(s) | s \in S\}$ and define for $k = 2, 3, \ldots, n$, $y_k = \min\{y_j^2(s) | y_j^2(s) > \bar{y}_{k-1}$, and $s \in S\}$: then $0 < \bar{y}_1 < \bar{y}_2 < \cdots < \bar{y}_n$, where n is the number of states in S. Let $\tilde{B}_j^k$ denote the bond that promises to pay $\bar{y}_k$, i.e., $\tilde{B}_j^k(s) = \min(y_j(s), \bar{y}_k)$. Let firm j issue bond $\tilde{B}_j^{n-1}$; this bond has a payout stream that looks like $(\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_{n-1}, \bar{y}_{n-1})$. Note that the payout stream of $\tilde{y}_j^2$ looks like $(\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_{n-1}, \bar{y}_n)$. Since $\tilde{y}_j^2$ has the same payout stream as a bond with face value y_n and since the payout streams of all bonds can be achieved using existing securities, the payout stream of $\tilde{y}_j^2 - \tilde{B}_j^{n-1} = (0, 0, \ldots, 0, \bar{y}_n)$ can be achieved using existing securities. Now consider the payout stream of the bond with face value $\bar{y}_{n-2}$, $\tilde{B}_j^{n-2} = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_{n-2}, \bar{y}_{n-2}, \bar{y}_{n-2})$. By assumption the payout stream of $\tilde{B}_j^{n-1} - \tilde{B}_j^{n-2} = (0, 0, \ldots, 0, \bar{y}_{n-1} - \bar{y}_{n-2}, \bar{y}_{n-1} - \bar{y}_{n-2})$ can be achieved using existing securities. Combining this with the fact that $(0, 0, \ldots, 0, \bar{y}_n)$ is achievable implies that $(0, 0, \ldots, 0, \bar{y}_{n-1} - \bar{y}_{n-2}, 0)$ is achievable.

Continuing the above argument by successively creating bonds with face value $\bar{y}_{n-3}, \bar{y}_{n-4}, \ldots, \bar{y}_1$, we are able to generate n securities each of which pays a nonzero amount in exactly one state, and this state is different for the different securities. Thus, there must be a complete set of markets. (In the above proof the assumption that $\bar{y}_1 > 0$ could be replaced with the assumption that there is some firm with a positive output in the state for which $y_j^2(s) = 0$. If we make the assumptions that (a) bonds can be issued on arbitrary portfolios of securities, (b) for each state s there is some firm i for which $y_i^2(s) > 0$, and (c) for any s, s' there exists a firm j such $y_j^2(s) \neq y_j^2(s')$, then we can use the results of Ross [1976] to show that markets must be complete.)

Thus, spanning implies complete markets, and since it is well-known that with complete markets the Modigliani-Miller Theorem is true, $dU_i^*/dB_j = 0$. Thus, under the assumption of spanning, complete markets is a necessary and sufficient condition for the Modigliani-Miller Theorem to be true.[8] However, this assumption is very strong and leads to the implication that value maximization is una-

8. Fama and Miller [1972, pp. 147–64] wisely emphasized the role of loan collateral as a method of proving the Modigliani-Miller Theorem, when firms can go bankrupt. They correctly argue that if borrowing can be collateralized by shares of firms, then any consumer can reproduce the returns stream of the firm's *risky* debt and equity. An even stronger statement can be made, namely that markets must be complete if these types of secured loans are possible. This is because any consumer can create the risky bond $\tilde{B}_j^k$ described in the text above, by simply promising to pay back $\bar{y}_k$ and securing the loan only with shares in firm j. The relevance of their view depends upon whether it is really true that consumers can create securities as easily as corporations. In particular, if I secure my loan with shares of firm j, how does the lender know that I have not already secured another loan by assigning the rights to the same shares? It

nimously preferred, as we showed previously.

VI. Conclusions

We have attempted to show that most of the unanimity theorems in the literature on spanning make the assumption that there is no trade during the time between the realization of firm's output and the putting of inputs in place. That is, simple spanning theory concerns firms that are not traded on the large stock exchanges of most countries. Stocks traded on these exchanges have a large trading volume everyday because consumers get new information about the probability distribution of firm output (i.e., dividends). We have shown that in such an economy consumers will in general disagree about the production plan that their firm should use. This disagreement arises because different stockholders anticipate different capital gains and losses as new information reaches the market concerning firm output.

We then showed that a unanimity theorem could be proved in a multiperiod context if the assumption of competitivity is made in addition to the assumption of spanning. That is, the output of a firm can be written as if it is a composite commodity composed of various units of basis vectors, called characteristics. We showed that unanimity follows from the assumption that the firm takes the prices of characteristics to be unaffected by its production decision. This is a generalization of the assumption made in Diamond [1967]. Though this may seem like a minor additional assumption, it leads to a very strong implication that spanning above does not imply. That is, if there is spanning and competitivity, then stockholders unanimously desire the firm to maximize net market value.

It is the implication of value maximization that leads us to think that the spanning-competitivity theory may be incomplete. There is one class of firms where it is quite easily seen that net value is not maximized. A closed-end mutual fund is a firm that purchases shares

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can often be difficult and costly to determine whether an asset has some lien against it. Evidence for this is obtained by noting the prevalence of title insurance in the market for land and houses. At the end of Section VI, we refer to further empirical evidence against the conclusion that the debt-equity ratio is irrelevant.

Note that if assumptions about consumer preferences or assumptions about distribution of returns are made that are sufficient to prove portfolio separation, then they are also sufficient to prove the Modigliani-Miller Theorem under an appropriate competitivity assumption. This is because portfolio separation implies that the economy is equivalent to an Arrow-Debreu economy in composite commodities of the type discussed in Sections III and IV. Note that these assumptions of course will also imply that shareholders unanimously desire value maximization.

of other firms on the stock market. Its only productive decision on a given day is to purchase or sell shares of one firm for cash or shares of other firms. The closed-end mutual fund is itself a corporation with shares traded on the stock market. It is a fact that almost all closed-end mutual funds sell at a substantial discount (see Sharpe and Sosin [1975]). That is, the market value of the mutual fund's portfolio is substantially higher than the market value of the mutual fund's own shares. This means that there is a productive decision available to the manager of the firm that would increase its value. The manager can sell off the portfolio of stocks for cash and then distribute the cash to the mutual fund shareholders. As this is not done, we conclude that the shareholders of the mutual fund do not desire the fund to maximize market value.[9]

Most discussions of unanimity involve the choice of production decisions. We argued that the Modigliani-Miller Theorem is the result that with no bankruptcy all shareholders are unanimously indifferent among choices of the debt-equity ratio. We showed that if there is a chance of bankruptcy, then shareholders need not be unanimously indifferent as to the debt-equity ratio. Since the bonds of the same maturity for different firms have different yields, there must be some chance that firms go bankrupt in the real world. It might be thought that the Modigliani-Miller Theorem would still hold if there is spanning for bonds. However, we show that spanning for bonds implies that there is a complete set of markets. This, of course, emphasizes how strong the spanning assumption is. The Modigliani-Miller Theorem is quite hard to test empirically because any debt-equity ratio is consistent with it. However, if the total value of the corporation changes as a consequence of the change in financial structure, then this is strong evidence against the competitivity and completeness of markets. Litzenberger and Sosin [1977] have analyzed the consequences of changes in the financial structure of a type of closed-end mutual fund called a "dual fund." They found strong empirical evidence *against* the hypothesis that markets are complete. They found changes in total value to be a consequence of changes in the financial structure of those corporations.

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9. There is another possibility, namely that the shareholders would all be better off if the closed-end fund liquidated or went open-end, but that the directors would be worse off. Thus, if the directors act in their own interest, rather than in the shareholders' interst, then discounted closed-end funds can persist. Grossman and Hart [1980] analyze this problem further and show that takeover bids cannot successfully be made at the discounted price because of a free rider problem.

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