Data Analysis for the E and B EXperiment and Instrumentation Development for Cosmic Microwave Background Polarimetry

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ABSTRACT

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The E and B EXperiment (EBEX) was a balloon-borne instrument designed to measure the polarization of the cosmic microwave background (CMB) while simultaneously characterizing Galactic dust emission. The instrument was based on a two-mirror ambient temperature Gregorian-Dragone telescope coupled with cooled refractive optics to a kilo-pixel array of transition edge sensor (TES) bolometric detectors. To achieve sensitivity to both the CMB signal and Galactic foregrounds, EBEX observed in three signal bands centered on 150, 250, and 410 GHz. Polarimetry was achieved via a stationary wire-grid polarizer and a continuously rotating achromatic half-wave plate (HWP) based on a superconducting magnetic bearing (SMB). EBEX launched from McMurdo station, Antarctica on December 29, 2012 and collected \(~1.3\) TB of data during 11 days of observation.

This thesis is presented in two Parts. Part I reviews the data analysis we performed to transform the raw EBEX data into maps of temperature and polarization sky signals, with a particular focus on post-flight pointing reconstruction; time stream cleaning and map making; the generation of model sky maps of the expected signal for each of the three EBEX signal bands; removal of the HWP-synchronous signal from the detector time streams; and our attempts to identify, characterize, and correct for non-linear detector responsivity. In Part II we present recent developments in
instrumentation for the next generation of CMB polarimeters. The developments we describe, including advances in lumped-element kinetic inductance detector (LEKID) technology and the development of a hollow-shaft SMB-based motor for use in HWP polarimetry, were motivated in part by the design for a prospective ground-based CMB polarimeter based in Greenland.
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Part I

Data Analysis for the E and B EXperiment (EBEX)
Chapter 1

CMB Science

1.1 Cosmological Expansion

Modern physical cosmology traces its roots to Hubble and Humason’s 1929 discovery of a linear relationship between the recessional velocity of galaxies and their distances \[30\]. This evidence is consistent with the uniform expansion predicted for a homogeneous and isotropic Universe assumed by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric[^1]

\[
\begin{align*}
    ds^2 &= -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta \, d\phi^2) \right] \\
\end{align*}
\] (1.1)

where \( s \) is the space-time metric, \( t \) represents proper time, \( k \) is the spatial curvature constant of the Universe, \((r, \theta, \phi)\) are co-moving spatial coordinates, and the scale factor \( a(t) \) encodes the time evolution of physical distances.

Two independent equations follow from substituting the FLRW metric into Einstein’s general relativistic field equations[^2] and modeling the mass-energy content of the Universe.

[^1]: Throughout this chapter we adopt the common practice of setting \( c = 1 \).

[^2]: The Einstein equations relate the curvature of space-time given by the Ricci tensor \( R_{\mu\nu} \) and Ricci scalar \( \mathcal{R} \) to the energy-momentum tensor \( T_{\mu\nu} \) of its mass-energy content, via \( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G T_{\mu\nu} \). The mass-energy content of the Universe is modeled to first order as a perfect fluid (i.e., a fluid with...
the Universe to first order as a perfect fluid with pressure $P$ and density $\rho$. The time-time component of the Einstein field equations yields the Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}$$

(1.2)

in which $G$ is the gravitational constant and $\rho$ represents the total energy density of the Universe, including contributions from dark matter, dark energy, and ordinary matter. By expressing Hubble's law as $\dot{a} = Ha$, we see that the left side of Equation 1.2 is the square of the Hubble parameter $H(t)$, which encodes the time evolution of the scale factor:

$$H(t) = \left(\frac{\dot{a}}{a}\right)$$

(1.3)

With the convention that the present value of the scale factor is $a_0 = 1$, the Hubble constant is defined as $H_0 = \dot{a}(0)$.

The space-space components of the Einstein field equations yield the second equation:

$$2 \ddot{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = -8\pi GP$$

(1.4)

Subtracting Equation 1.2 from Equation 1.4 yields the acceleration equation, which describes the acceleration of the scale factor:

$$\ddot{a} \frac{a}{a} = -\frac{4\pi G}{3} \left(\rho + 3P\right)$$

(1.5)

The equations above can be combined to derive the fluid equation, which gives the time evolution of the energy density:

$$\dot{\rho} + 3 \frac{\dot{a}}{a} \left(\rho + P\right) = 0$$

(1.6)

The first bracketed term corresponds to energy density dilution due to the expansion of space. The second bracketed term corresponds to loss of the fluid’s energy (no viscosity or heat flow) with pressure $P$ and density $\rho$, such that the energy-momentum tensor is $T_{\mu \nu} = \text{diag}(-\rho, P, P, P)$.
to gravitational potential energy as the fluid’s pressure does work as the Universe expands.

The Universe’s precise time evolution depends on the spatial curvature constant $k$; the relative proportion of the Universe’s mass-energy components; and the equation of state for each component, which specifies the relationship between the component’s energy density $\rho$ and its pressure $P$. The observational constraints on these parameters and the generally accepted standard $\Lambda$CDM cosmological model are discussed in §1.3.

Hubble and Humason’s observational evidence for an expanding universe, together with Equations 1.2, 1.5, and 1.6 imply that the Universe has been expanding from an extremely dense state in its distant past. This depiction of evolution of the Universe has come to be known as the Big Bang model.

## 1.2 The Cosmic Microwave Background

Following Lemaître’s first proposal of a Big Bang model in 1931 [41], Gamow, Alpher and Herman [23, 5] predicted that the Universe should be permeated by relic black body radiation. This prediction fell into obscurity for some time. In the early 1960s it was rediscovered by Soviet physicist Yakov Zel’dovich and independently predicted by Princeton physicist Robert Dicke. They reasoned that the hot, dense state of the early Universe would have acted as a black body and would have left a radiation signature observable today in the microwave band, known today as the Cosmic Microwave Background (CMB).

In 1964, while Dicke and his research group were making preparations to search for and measure the CMB, Bell Laboratories astronomers Arno Penzias and Robert Wilson serendipitously discovered the CMB using a horn-reflector radio antenna in Holmdel, New Jersey. Their discovery provided convincing evidence that helped establish the Big Bang theory as the prevailing model of the early Universe.
CHAPTER 1. CMB SCIENCE

1.2.1 The Origin of the CMB

The CMB carries an image of the Universe as it existed approximately 380,000 years after the Big Bang. Prior to that time the Universe was hot and dense, consisting of a photon-baryon fluid, a plasma of photons, free electrons, and free nuclei consisting mainly of free protons. During this epoch the Universe was optically thick, with photons coupled to free electrons via scattering. The photon-baryon fluid continually absorbed and re-radiated photons, acting as a black body.

The Universe cooled as it expanded. When the Universe’s temperature dropped to \( \sim 3,000 \text{ K} \), conditions became favorable for electrons and protons to form neutral hydrogen atoms, an epoch known as recombination. Thereafter the primordial photons were no longer coupled to electrons and the Universe became transparent to radiation. The primordial photons constitute the CMB, observable today as a nearly isotropic microwave glow across the sky. At the time of photon decoupling, the mass-energy components of the Universe were in thermal equilibrium; the temperature of the CMB equaled that of the Universe, and the electromagnetic spectrum of the CMB photons was that of a black body at \( T_{CMB} \approx 3,000 \text{ K} \). The expansion of the Universe has red-shifted CMB photons to their current temperature \( T_{CMB} \approx 2.725 \text{ K} \), with a peak spectral radiance at \( \sim 160 \text{ GHz} \) in the microwave region of the electromagnetic spectrum.

Every point in the Universe is surrounded by a spherical last scattering surface from which CMB photons have been streaming freely since recombination and photon decoupling, i.e., without further scattering by electrons to first order.\(^3\) Measurements of the CMB show a black body spectrum (see Figure 1.1) with nearly uniform temperature across the sky. A temperature map of the Earth-centered last scattering surface

\(^3\)Limited re-scattering occurs during the reionization epoch (when the first luminous objects formed and ionized the predominantly neutral intergalactic medium); and through the Sunyaev-Zel’dovich effect (SZE) (in which the low energy CMB photons receive an energy boost through inverse Compton scattering from high energy electrons, e.g., in galaxy clusters).
as measured by the Planck instrument is shown in Figure 1.2. Temperature variations, which are thought to result from quantum fluctuations in the early universe (see § 1.2.2), are limited to approximately one part in $10^5$.

![Figure 1.1: The spectrum of the CMB as measured by the Far Infrared Absolute Spectrophotometer (FIRAS) instrument on the Cosmic Background Explorer (COBE) satellite [45]. The spectrum very closely matches that of an ideal black body at 2.725 K. The vertical error bars are rendered here at the 100$\sigma$ level for visibility.](image.png)

1.2.2 CMB Temperature Anisotropies

The temperature fluctuations of the CMB as a function of position on the sky are analyzed in terms of the dimensionless temperature anisotropy

$$\frac{\delta T(\theta, \phi)}{\bar{T}} = \frac{T(\theta, \phi) - \bar{T}}{\bar{T}}$$

where $T(\theta, \phi)$ is the temperature of the CMB in a given direction on the sky and $\bar{T}$ is the mean temperature.
Figure 1.2: Temperature map of the CMB as measured by the Planck satellite instrument [54], showing deviation from the mean temperature of 2.725 K. The Galactic emission, as well as the dipole temperature distortion (caused by Doppler shifting due to the net motion of the instrument relative to the reference frame in which the CMB is isotropic), have been subtracted.

When expanded into spherical harmonics, the anisotropy is expressed as

\[
\frac{\delta T(\theta, \phi)}{T} = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{T,\ell m} Y_{\ell m}(\theta, \phi)
\]

where \( Y_{\ell m} \) are the orthonormal Laplace spherical harmonic functions; the multipole value \( \ell \) describes the characteristic angular size of the temperature fluctuation mode, \( \theta \sim 180^\circ/\ell \); the order \( m \) describes the fluctuation mode’s angular orientation; and the multipole coefficients \( a_{T,\ell m} \) satisfy:

\[
a_{T,\ell m} = \int Y_{\ell m}^*(\theta, \phi) \frac{\delta T}{T}(\theta, \phi) d\Omega
\]

As we will discuss in [1.5.1] the CMB temperature anisotropies are caused by primordial perturbations that most models predict to be Gaussian. Under this assumption the \( a_{T,\ell m} \)’s are also Gaussian random variables. Because they represent
deviations from the average temperature, their expectation value is zero:

\[ \langle a_{T,\ell m} \rangle = 0 \]  

(1.10)

Assuming an isotropic Universe, the variance \( C_\ell \) of the multipole moments at a given \( \ell \) is independent of angular orientation \( m \):

\[ \langle a_{T,\ell m} a_{T,\ell' m'}^* \rangle = \delta_{\ell \ell'} \delta_{mm'} C_\ell. \]  

(1.11)

We may therefore obtain an estimator of \( C_\ell \) by averaging over the accompanying \((2\ell + 1)\) values of \( m \). The result is the temperature angular power spectrum \( C_{\ell TT} \), which provides a statistical measure of the temperature fluctuations across angular scales:

\[ C_{\ell TT} \equiv \langle |a_{T,\ell m}|^2 \rangle = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} \langle |a_{T,\ell m}|^2 \rangle. \]  

(1.12)

A plot of the CMB temperature anisotropy angular power spectrum as measured by Planck is shown in Figure 1.3. The structural features of the temperature anisotropy spectrum are traced to several physical processes that took place in the primordial Universe:

- On the largest angular scales (below \( \ell \sim 180 \)), fluctuations are caused principally by the Sachs-Wolfe effect, whereby primordial density fluctuations in the distribution of matter at the time of last scattering causes photons to be gravitationally red-shifted as they climb out of local potential wells or blue-shifted as they fall down local potential hills.

---

4 Note that when measuring the \( a_{\ell m} \) coefficients for a given \( \ell \), each measured \( a_{\ell m} \) is a sampling from the same \( a_{\ell m} \) distribution with variance \( C_\ell \). Therefore even perfect measurements of the \((2\ell + 1)\) coefficients for a given \( \ell \) yields limited information about the variance of the underlying \( a_{\ell m} \) distribution, with larger uncertainty at smaller values of \( \ell \). This fundamental uncertainty, called cosmic variance, scales as the inverse square root of the number of samples. Using the properties of complex Gaussian random variables, it is quantified as \( \Delta C_\ell / C_\ell = \sqrt{2/2\ell + 1} \). Cosmic variance would be reducible if we were able to average over an ensemble of \( a_{\ell m} \) values produced by the same random processes that generated the primordial perturbations visible from Earth, e.g., by averaging over \( a_{\ell m} \)'s measured from last scattering surfaces surrounding other locations in the Universe.
Adiabatic acoustic oscillations arise in the photon-baryon fluid due to cycles of competing inward compression under the gravity of potential wells and outward expansion driven by the ensuing increase in the fluid’s pressure. The acoustic peaks in Figure 1.3 are the result of compression and heating of photon-baryon fluid within potential wells at the time of last scattering.

At the smallest angular scales the spectrum is damped by diffusion damping, whereby photon diffusion from larger- to smaller-density regions tends to equalize temperatures on scales smaller than the mean free path.

1.3 The $\Lambda$CDM Cosmological Model

Upon specifying the equation of state relating the pressure $P$ and density $\rho$ for each of the Universe’s mass-energy components, the Friedmann equation (1.2) and the fluid equation (1.6) can be solved to describe the time evolution of the Universe. Current observational evidence indicates that the total mass-energy density includes contributions from ordinary matter; radiation; dark matter (the gravitational effects of which are observable today in the dynamics of galaxies and star clusters); and dark energy (which drives the current acceleration of the Universe’s expansion).

Dark energy may be included in the Friedmann equation (1.2) as a component of the total mass-energy density $\rho$ with constant energy density $\rho_\Lambda$; the fluid equation (1.6) then implies an associated negative pressure $P_\Lambda = -\rho_\Lambda$. Equivalently, one can include a cosmological constant, $\Lambda$, in an additional term in the Friedmann equation, giving

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

(1.13)

This approach results in an identical additional term of $\Lambda/3$ in the acceleration equation (1.5), demonstrating that a positive cosmological constant gives a positive contribution to the acceleration of the scale parameter $\ddot{a}$. 
Figure 1.3: The CMB temperature (TT) angular power spectrum as measured by the Planck instrument \[61\] as a function of angular scale (top axis) and multipole moment $\ell$ (bottom axis). Red points indicate measurements by Planck. The blue line shows the best fit six-parameter $\Lambda$CDM model to the Planck data. The error bars account for both measurement errors and cosmic variance (see footnote 4). The magnitude of the cosmic variance is indicated by the blue shading surrounding the best fit model. Figure adapted from \[58\].

By introducing the density parameters

$$\Omega = \frac{8\pi G \rho}{3H^2}$$ \hspace{1cm} (1.14)$$

$$\Omega_\Lambda = \frac{\Lambda}{3H^2}$$ \hspace{1cm} (1.15)$$

where $\Omega_\Lambda$ represents the density contribution for the cosmological constant and $\Omega$ represents the contributions from the radiation, matter, and dark matter,
1.13 may be rearranged to give
\[ \Omega + \Omega_\Lambda - 1 = \frac{k}{a^2 H^2}. \] (1.16)

If \( \Omega + \Omega_\Lambda = 1 \), then the Universe’s spatial curvature constant \( k = 0 \).

In recent decades astronomers and observational cosmologists have collected a wide range of evidence that can be used to test cosmological models, including measurements of the Hubble constant [22], the acceleration of the Universe’s expansion [53], CMB anisotropies [9, 58], and baryon acoustic oscillations [52]. The evidence converges on what has come to be called the “\( \Lambda \)CDM model,” in which the Universe has zero spatial curvature (\( k = 0 \)) and the mass-energy content of the Universe is composed of 5% matter; 27% cold dark matter (CDM); a negligible contribution from radiation; and 68% dark energy. Six parameters are sufficient to define the \( \Lambda \)CDM model: the spatial curvature \( k \); the energy density parameters of ordinary matter, dark matter, and dark energy (\( \Omega_m, \Omega_{dm} \) and \( \Omega_\Lambda \)); the scalar spectral index \( n_s \), which describes how primordial density fluctuations vary with scale; and the reionization optical depth \( \tau \).

The CMB temperature anisotropy power spectrum measured by the Planck experiment is shown in Figure 1.3 together with the closely-matching predicted spectrum for the best fit six-parameter \( \Lambda \)CDM model.

1.4 Inflation

Although the \( \Lambda \)CDM cosmological model has withstood rigorous experimental testing, it leaves three important empirical facts in want of explanation:

1. The Flatness Problem. The Universe is observed to be spatially flat (i.e., \( k = 0 \)) to within 1% [59]. Because any initial deviation of \( k \) from zero would magnify rapidly with time, \( k \) is required to be finely tuned to an initial value of almost exactly zero. This fine tuning is assumed, but not explained, in the \( \Lambda \)CDM model.
2. The Horizon Problem. The temperature of the CMB is observed to be isotropic to within one part in $10^5$. This suggests that all parts the last scattering surface must have been in causal contact prior to decoupling, such that they could establish thermal equilibrium. Because information cannot travel faster than the speed of light, points on the surface that are sufficiently separated spatially must be causally disconnected. Given the age of the Universe and its rate of expansion over time as predicted by the ΛCDM model, any points on the last scattering surface that are separated by an angular distance of more than $\sim 2^\circ$ would not have been able to interact and establish thermal equilibrium. The ΛCDM model must include the Universe’s extraordinary uniformity in temperature at the time of decoupling as an ad hoc assumption.

3. The Relic Particle Problem. In particle physics, Grand Unified Theories that model the unification of fundamental forces predict that the extreme temperature and density of the very early Universe should have generated a copious abundance of heavy relic particles, such as magnetic monopoles. Yet the Universe has an apparent dearth of such particles.

In 1981 Alan Guth, then working as a theorist in the field of particle physics, proposed a revision to the standard Big Bang cosmological model called inflation that solves all three problems via a single mechanism. Guth proposed that the Universe underwent a brief period of exponentially increasing expansion when it was much less than one second old; recent estimates place the inflationary epoch during the first $10^{-34} - 10^{-30}$ s after the Big Bang [18]. During inflation the scale factor was accelerating rapidly, with $\ddot{a}(t) \gg 0$. The acceleration equation (1.5) then implies that

$$\rho + 3P < 0 \quad (1.17)$$

with the result that the pressure $P$ during inflation was negative. The $\rho$ and $k$ terms on the right-hand side of the modified Friedmann equation (1.13) rapidly diminish as
a increases. The remaining Λ term, however, is constant and quickly dominates. The Friedmann equation during inflation may then be approximated as

\[
H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3}.
\]

(1.18)

Solving for \(a\) demonstrates that the scale factor expands exponentially with time,

\[
a(t) = \exp\left(\frac{\Lambda}{3}t\right)
\]

(1.19)

with the result that the Universe expands by many orders of magnitude during the small fraction of a second that inflation is active.

The momentary, but exponentially rapid expansion driven by inflation solves each of the three problems outlined above:

- Prior to inflation, regions of the Universe were sufficiently close to one another to be in causal contact and come to thermal equilibrium. Rapid expansion during inflation then brought them out of causal contact. This explains how portions of the last scattering surface that are apparently beyond causal contact can share the same temperature.

- Any initial spatial curvature in the Universe was greatly reduced during inflation, such that the curvature constant \(k\) that is observable today is close to zero.

- Inflation dilutes the number density of the extremely heavy relic particles that were generated during the most energetic stages of the very early Universe, such that the probability of detecting them today is vanishingly small.

The mechanism that drives inflation is thought to have its origins in quantum field theory. Inflation models posit the existence of an “inflaton field” \(\phi\) and an associated potential \(V(\phi)\) that drives inflation. A variety of inflationary models have been proposed with varying fields and associated potentials. In the simplest “slow roll” models, \(\phi\) is a homogeneous scalar field in an initial state of high potential energy
and $V(\phi)$ is modeled to return $\phi$ slowly ($\dot{\phi} \gg \ddot{\phi}$) to its minimum potential. Inflation then terminates and the time evolution of the Universe proceeds according to the standard, non-inflationary $\Lambda$CDM model.

Inflation predicts that microscopic quantum mechanical fluctuations occurred in the inflaton field and the corresponding metric during inflation. In Fourier space these fluctuations can be decomposed into scalar, tensor and vector perturbations. Scalar perturbations generate density fluctuations in the primordial fluid, thereby producing the CMB temperature anisotropies and seeding the growth of large-scale structure through gravitational instability. Tensor perturbations are the source of primordial gravitational waves that are predicted to have imprinted a faint signature in the polarization of the CMB, as discussed in §1.5. The tensor-to-scalar ratio, $r$, is the ratio of the power spectral amplitudes of the tensor and scalar perturbations. In the simplest inflation models it can be shown that the energy scale of the inflation potential $V$ at the time of inflation is related to $r$ by:

$$V^{\frac{1}{4}} \sim 10^{16} \left( \frac{r}{0.01} \right)^{\frac{1}{2}} \text{GeV}. \tag{1.20}$$

In recent years CMB research has focused on searching for the primordial gravitational wave signature in the CMB polarization. A measurement of this signal would allow cosmologists to determine $r$, ascertain the energy scale of inflation, and probe the physics of the Universe when it was a small fraction of a second old.

### 1.5 CMB Polarization

Maps of CMB polarization on a given sky area can be decomposed into two modes that take their names from the properties they share with electric and magnetic fields: the curl-free $E$-mode polarization and the divergence-free $B$-mode polarization. A

---

5 Vector perturbations, which are produced by vortex motion of the primordial fluid, are negligible in most inflationary models.
Figure 1.4: Diagram showing examples of pure E-mode and pure B-mode polarization fields.

Diagram of pure E- and B-mode polarization patterns is shown in Figure 1.4. The E-mode pattern is invariant upon reflection about a line passing through its center and has positive parity, while the B-mode pattern changes sign upon reflection and has negative parity [29].

Separating the polarization field into E- and B-mode patterns is useful because each is produced by different underlying physical mechanisms, as described below in § 1.5.1. Section 1.5.2 describes the Stokes parameter representation of polarization that is commonly used in CMB research for ease of measurement. Section 1.5.3 outlines the method of calculating E- and B-modes from the Stokes parameters and calculation of the E- and B-mode angular power spectra.

### 1.5.1 Sources of CMB Polarization

Polarization in the CMB is produced by Thomson scattering of CMB photons by free electrons. When electromagnetic radiation encounters a free electron, the Lorentz force induced on the electron causes the electron to oscillate in the direction of the incoming radiation’s electric field, i.e., its polarization. The electron then re-emits the incident radiation through dipole radiation. The intensity of the scattered radiation
peaks in the direction perpendicular to the incident radiation and parallel to the incident polarization.

If the incoming radiation were isotropic, the polarization produced by photons incoming from perpendicular directions would cancel and there would be no preferred direction for the net polarization. In the presence of quadrupolar temperature anisotropies, however, the incoming flux of photons scatters from an electron with a net linear polarization in the manner illustrated in Figure 1.5. One can show by symmetry that linear polarization of CMB photons can be produced only by local quadrupolar anisotropies [29].

Figure 1.5: Linear polarization of CMB photons produced by Thomson scattering of radiation with a quadrupolar anisotropy. Hot photons (blue) incoming from the $\pm \hat{x}$ direction undergo Thomson scattering from the electron $e^-$ and produce radiation propagating in the $+\hat{y}$ direction and polarized along the $\hat{z}$ direction. Cold photons (red) incoming from the $\pm \hat{z}$ direction produce radiation propagating in the $+\hat{y}$ direction and polarized along the $\hat{x}$ direction. The result is a net linear polarization observable in radiation propagating in the $+\hat{y}$ direction. Figure adapted from [29].
1.5.1.1 Primordial Temperature and Polarization Anisotropies

Primordial temperature anisotropies in the CMB result from three main causes:

- **Adiabatic perturbations**: quantum fluctuations result in density fluctuations $\delta \rho$ in the distribution of the photon-baryon fluid. The resulting temperature fluctuation is given by
  \[
  \frac{\Delta T}{T} = \frac{1}{3} \frac{\delta \rho}{\rho}.
  \]  
  \[\text{(1.21)}\]

- **Gravitational perturbations** caused by the Sachs-Wolfe effect (see §1.2.2), in which photons are redshifted (blueshifted) as they climb out of (into) potential wells surrounding primordial density inhomogeneities.

- **Kinetic perturbations**: variation in the velocity of the primordial fluid causes photons to undergo a Doppler shift, resulting in a temperature fluctuation that is proportional to the velocity $v$ of the fluid relative to the observer:
  \[
  \frac{\Delta T}{T} \propto v.
  \]  
  \[\text{(1.22)}\]

As described above, primordial polarization anisotropies are generated by quadrupolar temperature anisotropies. Quadrupolar anisotropies can be generated by three physical mechanisms operating in the primordial universe:

- **Scalar perturbations** produce density fluctuations in the photon-baryon fluid. The density fluctuations give rise to velocity gradients within the fluid, producing quadrupolar CMB temperature anisotropies in the rest frame of the free electrons [27]. Scalar perturbations are capable of generating only curl-free E-mode polarization.

- **Tensor perturbations** are caused by gravitational waves passing through the fluid. Tensor perturbations give rise to both E-mode and B-mode polarization, but the polarization they produce is much weaker than the E-modes generated by scalar perturbations [29]. As outlined in §1.4, gravitational waves are a
predicted effect of inflation. Detection of an “inflationary B-mode” signal would provide confirming evidence of the inflationary model.

- **Vector perturbations** are caused by vortex motion in the primordial fluid. Like tensor perturbations, they are capable of generating both E-mode and B-mode polarization. Vector perturbations are damped rapidly by inflation, however, and are thought to be negligible [29].

In spherical harmonic representation, the scalar, vector, and tensor quadrupole moments correspond respectively to the $m = 0$, $m = \pm 1$, and $m = \pm 2$ quadrupole ($\ell = 2$) spherical harmonics.

### 1.5.1.2 Secondary Anisotropies

Secondary anisotropies result when CMB photons encounter perturbations between the last scattering surface and the observer. They may be classified broadly into two categories: scattering effects, such as the SZE and rescattering during the reionization epoch (see footnote [3]); and gravitational effects. A detailed review of secondary anisotropies and their effects on the E- and B-mode power spectra is provided in [27].

The effects of *gravitational lensing* on CMB photons is of particular importance to B-mode search experiments such as EBEX. In addition to the primordial “inflationary B-mode” signal generated by tensor perturbations, B-modes can arise through lensing of CMB photons by massive objects such as galaxies and clusters. Distortion in the CMB caused by gravitational lensing transforms E-modes into B-modes. “Lensing B-modes” dominate the expected B-mode angular power spectrum at high $\ell$ (see Figure 1.6) and have been detected recently by the ACTpol [49], BICEP2-Keck Array [69], POLARBEAR [4], and SPTpol [25] experiments.
1.5.1.3 Foregrounds

Polarized CMB signals are obscured by several sources of foreground contamination that must be removed from CMB maps. The most important contributor for the majority of current B-mode search experiments is Galactic dust emission, which generates a B-mode signal that can be misidentified as an inflationary B-mode signature. In March 2014 the BICEP2 collaboration reported the detection of inflationary B-modes [10]. The detected signal was subsequently found to be consistent with B-modes produced by Galactic dust emission [3]. Additional sources of foreground contamination include synchrotron radiation generated by the acceleration of high-energy electrons in the Galaxy’s magnetic field and free-free emission produced by the scattering of free electrons from free ions in the Galaxy.

Each foreground source has its own characteristic spectral index, which allows the source to be distinguished from the CMB signal. By making observations of the sky at multiple frequencies, foreground contamination can be measured and subtracted from CMB maps.

1.5.1.4 The EE and BB Power Spectra

As outlined below in §1.5.3, the E-mode and B-mode angular power spectra can each be computed in a manner analogous to that outlined in §1.2.2 for the temperature spectrum. Figure 1.6 shows the resulting E-mode (EE) and B-mode (BB) angular power spectra as measured by several experiments. The E-mode spectrum is approximately two orders of magnitude weaker than the temperature spectrum (cf. Figure 1.3). The B-mode spectrum is roughly two orders of magnitude weaker still at high $\ell$ modes, and has yet to be well characterized at low $\ell$. 
Figure 1.6: Top: CMB polarization EE power spectra measurements by various experiments. The solid line indicates the best-fit ΛCDM EE spectrum. Figure adapted from [1]. Bottom: Upper limits on the BB spectrum from several CMB experiments (triangles); measurements of the gravitational lensing components by the ACTpol [49], BICEP2-Keck Array [69], POLARBEAR [4], and SPTpol [25] experiments (squares). Dashed red lines show the predicted inflationary B-mode signal for $r = 0.01$ and $r = 0.05$. The solid black line shows the best fit lensing B-mode spectrum. The solid red line shows the total B-mode signal for $r = 0.05$. Figure adapted from [65].
1.5.2 Stokes Parameter Representation

In CMB research electromagnetic polarization is typically formulated in terms of the Stokes parameters for ease of measurement. For an electromagnetic plane wave with wavenumber \( k \) propagating in the \( \hat{z} \) direction, the real part of the electric field can be expressed as

\[
\vec{E}_k(z, t) = E_x \cos(kz - \omega t + \phi_x) \hat{x} + E_y \sin(kz - \omega t + \phi_y) \hat{y}
\]  

(1.23)

where \( \omega = kc \) is the wave’s angular frequency, \( E_x \) and \( E_y \) are the amplitudes of the electric field’s \( x \) and \( y \) components, and \( \phi_x \) and \( \phi_y \) are the phases of the electric field’s \( x \) and \( y \) components. The Stokes parameters are defined as

\[
I \equiv E_x^2 + E_y^2 
\]  

(1.24)

\[
Q \equiv E_x^2 - E_y^2 
\]  

(1.25)

\[
U \equiv 2E_xE_y \cos(\phi_y - \phi_x) 
\]  

(1.26)

\[
V \equiv 2E_xE_y \sin(\phi_y - \phi_x). 
\]  

(1.27)

In the case of linear polarization \( \phi_x = \phi_y \), with the result that \( V = 0 \) and \( U = 2E_xE_y \).

The \( I \) parameter represents total intensity. \( V \) represents circular polarization while \( Q \) and \( U \) represent linear polarization as measured along two sets of coordinate axes separated by a 45° rotation, as shown in Figure [1.7].

The polarization fraction \( P \) and polarization angle \( \alpha \) are defined as:

\[
P = \sqrt{Q^2 + U^2}/I 
\]  

(1.28)

\[
\alpha = \frac{1}{2} \tan^{-1}\left(\frac{U}{Q}\right). 
\]  

(1.29)
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Figure 1.7: Polarization as represented by Stokes parameters in the pure $\pm Q$, $\pm U$ and $\pm V$ states. The $\pm V$ states are circularly polarized in opposing directions. $\pm Q$ are linear polarization along perpendicular axes. The $\pm U$ linear polarization axes are rotated by $45^\circ$ from $Q$.

1.5.3 Calculating the EE and BB Angular Power Spectra

$Q$ and $U$ polarization maps are transformed into E- and B-mode maps by forming the combinations $Q \pm iU$. These combinations transform as spin-$\pm 2$ quantities and can be decomposed into spin-weighted spherical harmonics $\pm 2Y_{\ell,m}$:

$$Q \pm iU = \sum_{\ell,m} a_{\pm 2\ell,m} \pm 2Y_{\ell m}(\theta, \phi). \quad (1.30)$$

Through applying spin raising and lowering operators to the $U \pm iQ$ combinations, it can be shown that the E- and B-mode multipole coefficients for a given multipole $\ell$ are [43, 36]:

$$a_{E\ell,m} = -\frac{1}{2}(a_{2\ell,m} + a_{-2\ell,m}) \quad (1.31)$$

$$a_{B\ell,m} = -\frac{i}{2}(a_{2\ell,m} + a_{-2\ell,m}). \quad (1.32)$$

The angular power spectra are then calculated by summing over $m$ for each multipole $\ell$, yielding

$$C_{\ell}^{EE} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} \langle |a_{E\ell,m}|^2 \rangle \quad (1.33)$$
\[ C_B^\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} \langle |a_{B,\ell,m}|^2 \rangle. \] (1.34)

The polarization anisotropy multipole coefficients \( a_{E,\ell,m} \) and \( a_{B,\ell,m} \) and the temperature anisotropy multipole coefficients \( a_{T,\ell,m} \) from equation 1.9 are used to calculate the auto-correlation spectra \( C_T^\ell, C_E^\ell, \) and \( C_B^\ell, \) as shown in equations 1.12, 1.33, and 1.34. In addition they can be used to calculate the cross-correlation spectra \( C_{TE}^\ell, C_{TB}^\ell, \) and \( C_{EB}^\ell. \) Because B-modes have negative parity, the \( C_{TB}^\ell \) and \( C_{EB}^\ell \) are, in the absence of exotic physics, expected to be zero. Their calculation is still useful, however, to identify and characterize instrumental errors and separate primordial signals from lensing-induced E- and B-modes. The amplitude of the \( C_B^\ell \) spectrum provides a measure of the tensor-to-scalar ratio \( r. \)
Chapter 2

The E and B EXperiment

The E and B EXperiment (EBEX) was a balloon-borne telescope designed to measure the E- and B-mode polarization of the CMB while simultaneously measuring Galactic dust emission over the range $30 < \ell < 1500$ of the angular power spectrum. To achieve sensitivity to both the CMB polarization signal and galactic foregrounds, EBEX observed in three signal bands centered on 150, 250, and 410 GHz.

EBEX launched from McMurdo station, Antarctica on December 29, 2012, circumnavigating the continent at an altitude of $\sim 35$ km and landing 25 days later on January 23, 2013. EBEX collected data during the first 11 days of flight before its cryogens were depleted. The instrument and its flight disks, containing $\sim 1.3$ TB of data, were collected shortly thereafter.

This chapter provides a brief overview of the EBEX instrument and its primary components, its design scan strategy, and the sky coverage and scan strategy achieved in flight.

2.1 Gondola and Instrument Overview

The $\sim 6,000$ lb EBEX instrument comprised a telescope and polarimetric receiver mounted on an aluminum structure called the “gondola.” The EBEX gondola design
was based on previous CMB balloon-borne experiments, including MAXIMA \[62\] and BLAST \[51\]. It consisted of two primary components: a rope-suspended outer frame that moved the entire gondola in azimuth, and an inner frame containing the telescope and receiver that moved in elevation.

A drawing and photograph of the instrument are shown in Figure 2.1. The outer frame was suspended by four synthetic fiber ropes from a triangular aluminum spreader. The spreader was suspended from a pivot motor and universal joint that connected to the flight train and balloon. The inner frame consisted of a box beam structure that was connected to the receiver. The inner frame supported the telescope’s primary and secondary mirrors.

In addition to the telescope and receiver, the gondola supported the following components:

- Attitude control sensors and motors to track and control the telescope’s orientation, described in § 2.2.

- Four detector readout electronics crates, cooled by a liquid cooling system that transferred heat from inside the electronics crates to external radiator panels (see Figure 2.1 right).

- Two redundant flight computers running custom software to tune and control the TES bolometers; collect, store, and telemeter data; and manage automated and pre-scheduled tasks, such as cryogenic refrigerator cycling and execution of predetermined sky observation events.

- A telemetry system to downlink data and uplink commands to the flight computers. Communication occurred via three pathways: line of sight communication at a bandwidth of 1 Mbps (available only during the first 24 hours of flight); tracking and data relay satellites (TDRSS) at 6-75 kbps; and uplinking commands via the Iridium phone network at 2 kbps.
A power system, consisting of lithium-ion batteries charged during flight by 30 solar panels. The power system was designed to be capable of generating at least 2.3 kW. The total peak power consumption of the instrument’s components as measured on the ground was 1.7 kW.

- Baffles, consisting of double-layers of aluminized mylar, to shield the telescope and receiver from illumination from the Sun and Earth (see Figure 2.1 right).

Additional details regarding the instrument’s design are provided in [68; 66; 67; 63; 11; 17; 26].

Figure 2.1: Left: drawing of the EBEX gondola and the primary components of the instrument. Part of the liquid cooling system radiators has been removed for clarity. Right: photograph of the instrument showing placement of the baffles and solar panels. Figures from [67].

## 2.2 The Attitude Control System

The Attitude Control System (ACS) was responsible for tracking and controlling the telescope’s orientation on the sky. The EBEX ACS was adapted from that used in
BLAST [51]. It consisted of pointing sensors and actuators controlled by software control algorithms running in feedback loops. The pointing sensors measured the instantaneous orientation of the telescope, which the control algorithms used to estimate the telescope’s instantaneous attitude. The control algorithms compared the estimated instantaneous attitude to the target attitude defined by the predetermined telescope scan strategy, then issued commands to the actuators to align the two.

The primary pointing sensors included two redundant star cameras, running custom software [13; 12] that determined the attitude by comparing images of star fields to a catalog of star field patterns; and two redundant 3-axis fiber optic rate gyroscopes that were integrated to estimate pointing between star camera image captures. Secondary sensors included two Sun sensors, two redundant magnetometers, a differential global positioning system (dGPS), an elevation encoder, and an inclinometer. The secondary sensors were used to provide coarse attitude information to the star camera software, which could be used to generate a pointing guess for the star camera images. In addition to providing real-time control of the telescope attitude, the pointing sensor data was used to produce a post-flight pointing reconstruction (see Chapter 3) to project the detector data onto maps of the sky.

Actuators included an elevation actuator with ends attached to the inner and outer frames; an active pivot motor to provide coarse azimuth control by torquing the entire gondola relative to the flight train and balloon; and a motorized reaction wheel to provide fine-grain azimuth control.

Further detailed information about the ACS can be found in [67; 17; 11; 13; 12; 16].

2.3 Optics and Receiver

The EBEX telescope optics comprised warm primary and secondary mirrors and a series of cold lenses and filters located inside the cryogenically cooled receiver (see
The warm mirrors were situated in an off-axis Gregorian Mizuguchi-Dragone configuration to minimize polarized systematics\footnote{24}. Incoming light was reflected from the 1.5 m parabolic primary mirror to the 1.10 m by 0.98 m elliptical secondary mirror, which then reflected the light into the cryogenically cooled receiver.

A drawing of the receiver is shown in Figure\ref{fig:EBEX-receiver}. Light entering the receiver cryostat first encountered an 0.02 in thick, 30 cm diameter vacuum window made of ultra-high molecular weight polyethylene (UHMWPE). The cryostat included five progressively cooler stages: ambient temperature ($\sim$ 300 K) at the window; 77 K and 4 K stages cooled by liquid cryogens; and a 1 K inner stage and the 0.25 K detector focal plane cooled by closed-cycle helium adsorption fridges. Low-pass filters at the first three stages reduced thermal loading on the inner stages.

Inside the cryostat, the field lens and a series of pupil and camera lenses focused the incoming rays onto two focal planes. An intermediate continuously rotating half-wave plate, together with a polarizing grid mounted at 45$^\circ$ to the optical path, split the
beam into independent polarizations and directed them to independent (“horizontal” and “vertical”) focal planes. The detectors were organized on each focal plane into wafers, each of which was layered with a low-pass metal mesh filter. Conical feed horns and cylindrical wave guides then coupled the light to the detectors.

Figure 2.3: Drawing of the EBEX receiver. Light (green lines) entered through a vacuum window and passed through a series of filters and lenses and the rotating half-wave plate. A polarizing grid transmitted one polarization state to the horizontal focal plane (“H”) and reflected the other to the vertical focal plane (“V”). The focal planes operated near 0.25 K (blue dashed box); the internal optics were maintained near 1 K (volume shown in green); all components inside the red dashed line were cooled to liquid helium temperature (∼ 4 K). Figure from [68].

The low-pass filters on the wafers and the wave guides in each feed horn served to define the EBEX frequency bands at the high and low edges, respectively. The three bands were centered on 150, 250 and 410 GHz and each had a beam size of 8′ and a
band-pass of \( \sim 13\% \) as measured on the ground before flight. A preliminary analysis indicates that the in-flight beam sizes were 16', 15', and 24' at 150, 250, and 410 GHz respectively. Work is ongoing to identify the source of the discrepancy between the ground and in-flight beam measurements.

Additional information regarding the optical system and the receiver appears in [68 48 74].

### 2.4 Detectors and Readout

Each EBEX focal plane employed an array of transition-edge sensor (TES) bolometric detectors arranged into seven hexagonal wafers, with four 150 GHz wafers, two 250 GHz wafers, and one 410 GHz wafers per focal plane (see Figure 2.4). Each wafer contained 139 TES detectors fabricated by thin-film deposition and optical lithography. Due to limited fabrication yield and a variety of secondary problems, only 1,107 out of the total possible 1,946 detectors were operational during flight [44].

![Figure 2.4](image)

Figure 2.4: *Left:* drawing of one of the two identical EBEX focal planes. TES detectors were arranged into seven wafers, each with a frequency band centered on 150, 250 GHz, or 410 GHz. After traversing a band-defining filter, light entered the detectors via conical feed horns and cylindrical wave guides. *Right:* drawing of a single TES detector, with superconducting material at the center of a “spiderweb” absorber. Figures from [66].
TES bolometers detect changes in the electrical resistance of superconducting material due to small temperature changes that follow radiation absorption. A photo of an EBEX TES bolometer is shown in Figure 2.4 right. Superconducting Al-Ti at the center of a “spiderweb” pattern of absorber is is maintained at its superconducting transition point by a voltage bias. When incoming radiation is absorbed by the web, a small temperature increase in the Al-Ti causes a change in electrical resistance, as shown in Figure 2.5. This causes a measurable change in the current through the Al-Ti.

Figure 2.5: Left: Conceptual drawing of a bolometer. Incident power $P$ is absorbed by a thermal mass with heat capacity $C$ weakly connected to a temperature bath at constant temperature $T_{bath}$ through a link with thermal conductance $G$. The temperature increase in the thermal mass $\Delta T = P/G$ is recorded by the observer. The temperature gradually returns to the bath temperature with an intrinsic thermal time constant $\tau = C/G$. Right: A superconductor biased at its transition point experiences large changes in electrical resistance with small changes in temperature.

The current read from the detectors was amplified by arrays of superconducting quantum interference devices (SQUIDs), with 16 detectors frequency multiplexed on
pairs of wires to reduce power consumption and thermal loading. Detector readout and voltage biasing were controlled by 28 “digital frequency domain multiplexing” (DfMUX) electronics boards containing field-programmable gate array (FPGA) processors, digital-to-analog converters (DACs), and analog-to-digital converters (ADCs). Additional information regarding the detectors and readout are provided in [66; 7; 47].

2.5 Polarimetry

EBEX achieved polarimetry via a stationary wire-grid polarizer and a 24 cm diameter continuously rotating achromatic half-wave plate (HWP) composed of a stack of five birefringent sapphire crystal disks following a Pancharatnam design [50]. The HWP was mounted to the 4 K cryostat stage to reduce thermal emission. In order to stabilize the rotational speed and minimize friction-induced thermal noise, the HWP rotated on a superconducting magnetic bearing (SMB). The HWP was driven by a tensioned Kevlar belt at a frequency of 1.23 Hz. Readout of the HWP’s angular orientation was provided by an LED shining through a slotted encoder onto a cryogenic photodiode detector. The EBEX polarimetry system is described in additional detail in [68; 38].

Figures 2.6 and 2.7 provide an overview of HWP polarimetry. Figure 2.6 illustrates the effect of an ideal HWP on linearly polarized incident radiation. The incident radiation can be decomposed into orthogonal components with polarization vectors aligned parallel and perpendicular to the fast optical axis of the birefringent sapphire crystal. The parallel component propagates through the crystal at a faster speed than the perpendicularly polarized component. A phase delay accumulates between the two polarization states as they propagate, causing the net polarization to rotate as the radiation travels through the crystal. The emerging polarization states are phase shifted by a half-wavelength. The net result is that the incident electric field
is reflected about the fast axis; incident radiation with an electric field at an angle $\theta$ to the fast axis emerges with an electric field that is rotated by an angle of $2\theta$.

Figure 2.6: Diagram showing the effect of an ideal HWP on linearly polarized incident radiation (red) with a polarization vector (red) at an angle $\theta = 45^\circ$ relative to the fast axis of the sapphire crystal. The component that is polarized parallel to the fast axis of the crystal (green) propagates through the plate slightly faster than the perpendicular component (blue). The two components accumulate a $180^\circ$ phase delay at the far side of the plate. The net electric field of the emerging radiation is rotated by $2\theta = 90^\circ$ relative to the optical axis. Figure from [2].

Figure 2.7 shows an illustration of polarimetry using a rotating HWP and a stationary wire-grid polarizer. Incident radiation linearly polarized at an angle $\theta$ to the fast axis of the HWP emerges rotated by an angle of $2\theta$. Rotating the HWP at a constant frequency $f$ therefore causes the electric field of the incoming signal to exit the HWP rotating at $2f$. The stationary wire-grid polarizer on the far side of the HWP separates the emerging polarized signal into horizontally and vertically polarized states. (In EBEX the polarizer was oriented at $45^\circ$ to the beam path, such that each state was directed to one of the two focal planes.) The linearly polarized signal transmitted by the polarizer therefore encounters the detector with an electric
field oscillating at $2f$. The detector itself, however, is phase insensitive; it detects the power dissipated within its absorber, which is proportional to the intensity of the incoming radiation. The dissipated power is therefore proportional to the square of the amplitude of the incoming radiation, and oscillates at $4f$. The net result is that a constant polarized signal incident on the HWP rotating at $f$ yields a detected signal oscillating at $4f$.

Figure 2.7: Conceptual illustration of the EBEX half-wave plate (HWP) polarimetry. The HWP rotating at frequency $f$ causes incoming linearly polarized light to rotate at $2f$. The stationary wire-grid polarizer transmits only one (horizontal or vertical) polarization state. The power dissipated in the detectors oscillates at $4f$. In EBEX the wire-grid polarizer was oriented at an angle of $45^\circ$ relative to the beam path, transmitting one polarization state to one focal plane and reflecting the other polarized state to the second focal plane. Figure adapted from [31].

The combination of HWP modulation and the scanning motion of the telescope across the sky affects the detector time streams in ways that are useful for mitigating systematic errors. Figure 2.8 shows a conceptual diagram of the components in a detector time stream PSD, including white noise and $1/f$ noise (blue), Stokes $I$ (red) and $Q/U$ (green) signal from the sky, and signal residing in selected harmonics of the
HWP rotational frequency $f_{hwp}$ (purple)\(^1\) (Example PSDs of EBEX detector time streams are shown in Figures 4.2 and 6.5). If the telescope boresight were stationary, Stokes $I$ sky signal would appear in the detector time streams at 0 Hz, while $Q$ and $U$ sky signals would be modulated by the HWP to $4f_{hwp} \approx 5$ Hz. The scanning motion of the telescope across $I$ ($Q/U$) sky sources causes the sky signal to appear in the bands surrounding 0 Hz ($4f_{hwp}$) in the PSD. A Gaussian sky feature with an angular scale $\theta$ has a Gaussian PSD component with a frequency domain width

$$\sigma_f = \frac{1}{2\pi} \frac{\text{scan speed}}{\theta}.$$  \hspace{1cm} (2.1)

The FWHM of the sky component signals is therefore larger for faster scan speeds or for sky features with smaller angular scales\(^2\).

Half-wave plate modulation mitigates errors in two ways. First, the combination of $4f$ modulation and the scanning motion of the telescope moves incoming polarized sky signals away from low frequency $1/f$ noise and into the sidebands surrounding $4f$ (see Figure 2.8). Second, it enables polarized emission from the instrument to be distinguished from polarized sky signals. Polarized emission by instrument components on the detector side of the HWP are unmodulated, and are therefore found at 0 Hz in the detector time streams; polarized emission by instrument components on the other side of the HWP are found at exactly $4f$; and polarized sky signals are found in the sidebands surrounding $4f$.

### 2.6 Observation Strategy and Flight Performance

During its 25-day flight EBEX was carried westward around the Antarctic continent by the polar vortex winds, following the trajectory shown in Figure 2.9. EBEX ob-

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\(^1\)The origin of the HWP-synchronous signal and the procedure for removing it from the detector time streams is discussed in Chapter 6.

\(^2\)In order to avoid overlap of the $I$ and $Q/U$ bands, the detector time streams are frequency filtered and flagged for selected ranges of scan speeds as described in §§ 4.1.2 and 4.2.
served at an altitude of \( \sim 35 \) km where the atmosphere is thin. This was intended to enable EBEX to measure the CMB with a high signal-to-noise ratio compared to ground-based experiments. The thin atmosphere at float results in lower optical loading from atmospheric emission as well as smaller noise from atmospheric fluctuations, allowing the use of detectors with higher sensitivity. Atmospheric absorption at high frequencies is also reduced, enabling greater sensitivity to galactic foregrounds in the instrument’s high frequency channel.

The cryogenic system that cooled the receiver was active for 11 days before its cryogens were depleted. During that time it was intended that EBEX would perform science observations on a 400 deg\(^2\) patch, centered on right ascension 4.8 h and declination \(-45.5^\circ\), that was selected for its low foreground contribution. The primary scan pattern intended for this patch was a raster scan, consisting of alternating azimuthal scans at constant elevation, pausing at the start and finish of each scan to take star camera images, followed by a step to the next elevation. The EBEX de-
Figure 2.9: The EBEX flight trajectory. Over a 25 day period the EBEX instrument followed the trajectory over Antarctica shown in red. The solid red portion of the trajectory corresponds to the 11 days of science observations prior to depletion of the cryogens used to cool the receiver. Green triangles demarcate 24 hour periods beginning at launch.

The sign scan strategy also included occasional elevation dips to characterize atmospheric loading and scans of the star cluster RCW 38 for signal calibration.

Due to a thermal modeling error, the pivot motor controller that was responsible for controlling azimuth motion was provided with inadequate radiative shielding. During flight the pivot motor controller overheated and automatically shut down. As a result EBEX floated freely in azimuth for a majority of the flight. This had several consequences that impacted EBEX’s ability to perform science observations.
First, loss of azimuth control rendered EBEX unable to execute its planned scan strategy. The telescope’s azimuth motion was determined by the rotation of the balloon and the spring constant of the flight train. The resulting azimuth motion was a superposition of 360° rotations every 15 to 60 minutes and oscillations of variable amplitude with a period of \( \sim 80 \) s. After loss of azimuthal control the telescope elevation was fixed at 54° to maintain an angular separation of \( \sim 15° \) between the telescope boresight and the Sun’s maximum elevation. The resulting sky coverage is shown in Figure 2.10. It consists of a 5,700 deg\(^2\) strip of sky delimited by declination \(-67.9°\) and \(-38.9°\).

Figure 2.10: The sky coverage of the EBEX flight shown in equatorial coordinates. The color scale indicates the number of detector samples per pixel for all detectors from all three frequency bands.

Second, azimuthal rotations resulted in periodic exposure of the flight computers to direct sunlight. The flight computers required daily shutdowns for one- to two-hour
periods to allow them to cool. Finally, solar illumination of the instrument also may have caused the bolometers to saturate, resulting in non-linear detector responsivity (see Chapter 7).

The azimuthal speed was less than $1^\circ/s$ over more than $97\%$ of the entire flight. Fortuitously, the $80$ s natural azimuthal oscillation period matched that of the EBEX design scan strategy. As a result the gondola came to rest every $\sim 40$ s, allowing the star cameras to take images in the stationary position that is optimal for imaging.
Chapter 3

Post-Flight Pointing Reconstruction

3.1 Introduction

Pointing reconstruction consists of assigning each time sample in detector data time streams a “pointing solution” such the sample can be binned into the correct pixel on a map of the sky. A pointing solution comprises the values and associated uncertainties for the right ascension (RA), declination (Dec), and Roll angles (see Figure 3.1) of one of the two EBEX star cameras, each of which is approximately aligned to the telescope boresight with measured angular offset values. Because each detector has a different alignment with the star camera’s axis by virtue of occupying a different location on the focal plane, correct mapping of the detector time streams onto the sky requires the camera’s Roll angle in addition to the RA and Dec equatorial coordinates. The RA, Dec, and Roll angles together comprise a set of Euler angles that completely describe the rotation between the sky reference frame and the star camera reference frame, which in turn is mechanically referenced to the telescope boresight and then

1The Roll angle is also used in polarization analysis to determine the angle of the half-wave plate axis with respect to the celestial coordinate system (see Chapter 6).
As discussed in §2.2, the ACS that provided real-time determination and control of the telescope’s pointing also stored information from the pointing sensors to enable post-flight pointing reconstruction. Pointing reconstruction was accomplished using data from the two primary ACS sensors: the two redundant star cameras that were mounted on either side of the inner frame and approximately aligned with the telescope beam; and the two redundant gyroscope boxes mounted on the inner frame, each containing three nearly orthogonal fiber optic rate gyroscopes that measured angular velocity along their respective axes.

EBEX pointing reconstruction was achieved by the following recursive steps, the details of which are elaborated further below:

1. **Star field identification**: Star camera image processing software attempts to determine the location on the sky for each time-stamped star field image stored
CHAPTER 3. POST-FLIGHT POINTING RECONSTRUCTION

by the star cameras. The location on the sky, referred to as the “pointing solution,” is obtained by matching image star field patterns to a catalog of known star patterns. For brevity we refer to images with successfully matched star field patterns as “solved” images.

2. **Reconstruction:** The location on the sky for time samples between solved images are estimated by integrating the gyroscope angular velocities. As explained in §3.4, a pointing reconstruction at each time sample is generated using an Unscented Kalman Filter (UKF) \[71\] to combine gyroscope data and the pointing solutions for solved images, as well as a least-squares optimizer to determine the transformation matrix that rotates from the star camera reference frame to the reference frame defined by the three gyroscopes.

3. **Recursion:** To improve the yield of solved star camera images, the star field identification software is run iteratively by using the pointing reconstruction from step 2 as an initial pointing guess in step 1 of the next iteration. As explained below, because the initial pointing guess provides the approximate location of the image star fields, the requirements for matching the star fields to the catalog of known star patterns are relaxed, resulting in a higher yield of solved images. An updated pointing reconstruction is then produced as in step 2.

As explained in §3.4, pointing uncertainties grow with gyroscope rate integration between solved images. Recursively increasing the yield of solved images results in shorter time periods between solved images, and therefore produces an updated pointing reconstruction with smaller pointing uncertainties. The iterative star field identification and pointing reconstruction is repeated until no additional star camera images can be matched to the catalog of known star patterns.

Section 3.2 reviews the constraints on pointing uncertainties that are required to meet EBEX’s science goals. Section 3.3 describes the procedures used to identify the
location on the sky of star field images captured by the two EBEX star cameras. Section 3.4 describes the procedure for generating the post-flight pointing reconstruction from the combined star camera image pointing solutions and the gyroscope data. We end this chapter in §3.5 by applying the formalism described in §3.2 to show that the pointing uncertainties for the post-flight pointing reconstruction meet the required limits.

### 3.2 Pointing Requirements

Random error in the telescope pointing, called *pointing jitter*, induces a spurious signal in the CMB B-mode power spectrum. We refer to this spurious B-mode signal as the *induced B-modes* created by pointing jitter. To meet the EBEX science goals we require the induced B-modes be restricted to below 10% of the expected CMB lensing and inflationary B-mode signal over the EBEX $\ell$-range of interest $30 < \ell < 1500$ for $r = 0.05$. We evaluated the sufficiency of the EBEX pointing reconstruction by determining whether the induced B-modes expected from the pointing reconstruction uncertainties met this requirement. We show below that this translates to a map domain requirement that the error on the mean of all pointing samples within a $\sim 1.7'$ pixel ($\text{nside} = 2048$) must be smaller than $\sim 10''$. In Appendix A we show that this requirement corresponds to a time domain requirement that the root-mean-square (rms) of the pointing error for all samples in a typical 40 s azimuth scan is less than $54''$.

This section reviews the formalism for analyzing pointing jitter induced B-modes as developed in [28, 72]. Its application to estimate the induced B-modes for EBEX is discussed below in §3.3. I contributed to the software and analysis for estimating the induced B-modes of the EBEX flight pointing, which differed greatly from the design scan strategy (see §2.6). A previous analysis of the induced B-modes expected for the EBEX design scan strategy is described in [17].
In the formalism presented in \cite{28, 72}, the random deviation $\delta \vec{\theta}$ of the true telescope pointing from the measured pointing $\hat{n} = (\text{RA}, \text{Dec})$ results in erroneous measurements $\tilde{Q}$ and $\tilde{U}$ of the true polarization Stokes parameters $Q$ and $U$. For a single time sample $i$ the measured $\tilde{Q}_i$ and $\tilde{U}_i$ are:

\begin{align*}
\tilde{Q}_i(\hat{n}) &= Q(\hat{n} + \delta \vec{\theta}_i) \approx Q(\hat{n}) + \delta \vec{\theta}_i \cdot \vec{\nabla}Q(\hat{n}) \\
\tilde{U}_i(\hat{n}) &= U(\hat{n} + \delta \vec{\theta}_i) \approx U(\hat{n}) + \delta \vec{\theta}_i \cdot \vec{\nabla}U(\hat{n})
\end{align*}

(3.1)

With successive scans in the same vicinity, multiple measurements in the direction $\hat{n}$ are binned into its corresponding map pixel and then averaged. As a result the error terms in the measured $\tilde{Q}$ and $\tilde{U}$ average down; over $N$ hits in the pixel corresponding to $\hat{n}$, the measured $\tilde{Q}$ is:

\begin{align*}
\tilde{Q}(\hat{n}) &= \frac{1}{N} \sum_{i=1}^{N} \tilde{Q}_i(\hat{n}) = Q(\hat{n}) + \frac{1}{N} \left( \sum_{i=1}^{N} \delta \vec{\theta}_i \right) \cdot \vec{\nabla}Q(\hat{n}) \\
&= Q(\hat{n}) + \delta \vec{\theta}_{map} \cdot \vec{\nabla}Q(\hat{n})
\end{align*}

(3.2)

and similarly for $\tilde{U}$, where $\delta \vec{\theta}_{map}$ is the averaged pointing error in the map domain:

\begin{equation}
\delta \vec{\theta}_{map} = \frac{1}{N} \left( \sum_{i=1}^{N} \delta \vec{\theta}_i \right). 
\end{equation}

(3.3)

To calculate the induced B-modes we begin by decomposing the E- and B-modes in the flat sky approximation:

\begin{equation}
(\tilde{E} \pm i\tilde{B})(\vec{\ell}) = \int (\tilde{Q} \pm i\tilde{U})(\hat{n}) e^{\mp 2i\Phi_{\ell}} e^{-i\vec{\ell} \cdot \hat{n}} d\hat{n}
\end{equation}

(3.4)

where $\Phi_{\ell} = \cos^{-1}(\ell_x / \ell)$ and $\ell_x$ and $\ell$ are the x-component and magnitude of $\vec{\ell}$. By substituting the expressions for $\tilde{Q}$ and $\tilde{U}$ from equation \ref{3.2} into equation \ref{3.4} and retaining only first-order terms, the induced B-modes can be expressed as the sum of
two contributions $\delta \tilde{B}(\vec{\ell})_A$ and $\delta \tilde{B}(\vec{\ell})_B$:

$$
\delta \tilde{B}(\vec{\ell})_A = \int \frac{d^2 \vec{\ell}_1}{(2\pi)^2} \tilde{E}(\vec{\ell} - \vec{\ell}_1) \delta_{\text{map}}(\vec{\ell}_1) \left( (\vec{\ell} - \vec{\ell}_1) \cdot \hat{\ell}_1 \right) \sin(2(\Phi_{\ell \cdot \ell_1} - \Phi_{\ell})) \\
\delta \tilde{B}(\vec{\ell})_B = \int \frac{d^2 \vec{\ell}_1}{(2\pi)^2} \tilde{E}(\vec{\ell} - \vec{\ell}_1) \delta_{\text{map}}(\vec{\ell}_1) \left( (\vec{\ell} - \vec{\ell}_1) \times \hat{\ell}_1 \right) \cdot \hat{z} \sin(2(\Phi_{\ell \cdot \ell_1} - \Phi_{\ell}))
$$

where the Fourier transform $\delta_{\text{map}}(\vec{\ell})$ is calculated assuming isotropic pointing errors. Because the pointing errors generally are not isotropic between the RA and Dec directions, this assumption requires the induced B-mode contributions from RA and cross-Dec errors to be calculated separately.

To compute the power spectrum of the induced B-modes $\delta C_{\ell}^{BB}$, we use the general definition of the power spectrum of a field $X$:

$$
\langle X^*(\vec{\ell})X(\vec{\ell}_1) \rangle = (2\pi)^2 \delta(\vec{\ell} - \vec{\ell}_1) C_{\ell}^{XX}
$$

The power spectrum contributions, written as $p_A$ and $p_B$, solve as:

$$
p_A : \quad \delta C_{\ell}^{BB} = \int \frac{d^2 \vec{\ell}_1}{(2\pi)^2} C_{\ell_2}^{EE}(\sigma_{\text{beam}}) C_{\ell_1}^{\theta\theta} \left[ \vec{\ell}_2 \cdot \hat{\ell}_1 \sin(2(\Phi_{\ell_2} - \Phi_{\ell})) \right]^2
$$

$$
p_B : \quad \delta C_{\ell}^{BB} = \int \frac{d^2 \vec{\ell}_1}{(2\pi)^2} C_{\ell_2}^{EE}(\sigma_{\text{beam}}) C_{\ell_1}^{\theta\theta} \left[ (\vec{\ell}_2 \times \hat{\ell}_1) \cdot \hat{z} \sin(2(\Phi_{\ell_2} - \Phi_{\ell})) \right]^2
$$

where $\vec{\ell}_2 = \vec{\ell} - \vec{\ell}_1$, $C_{\ell}^{\theta\theta}$ is the power spectrum of the pointing error map $\tilde{\theta}_{\text{map}}$, and $C_{\ell}^{EE}(\sigma_{\text{beam}})$ is the E-mode power spectrum $C_{\ell}^{EE}$ smoothed by the telescope’s Gaussian beam width $\sigma_{\text{beam}} = \frac{\text{FWHM}}{\sqrt{8 \ln 2}}$:

$$
C_{\ell}^{EE}(\sigma_{\text{beam}}) = C_{\ell}^{EE} \exp\left( -\ell(\ell + 1)\sigma_{\text{beam}}^2 \right)
$$

The induced B-modes for the EBEX flight can then be estimated by using the UKF-estimated pointing error time streams to generate a map of pixel-averaged pointing errors $\delta \tilde{\theta}_{\text{map}}$; computing the map’s power spectrum $C_{\ell}^{\theta\theta}$; smoothing a known E-mode power spectrum $C_{\ell}^{EE}$ (e.g., as measured by WMAP) by the EBEX beam width $\sigma_{\text{beam}}$ using equation (3.8) and performing the integrals in equation (3.7).

The requirement that the error on the mean of all pointing samples within a pixel must be smaller than $\sim 10''$ is calculated from a white noise model of the pointing
uncertainties, as defined in [28] for a pointing error map with magnitude \( \sigma_{\theta_{\text{map}}} \) and coherence scale \( \ell_s \):

\[
C_{\ell,WN}^{\theta\theta} \simeq \frac{2\pi\sigma_{\theta_{\text{map}}}^2}{\ell_s^2} \exp\left(\frac{-\ell(\ell + 1)}{2\ell_s^2}\right)
\]  

(3.9)

Substituting \( C_{\ell,WN}^{\theta\theta} \) into equation 3.7 yields:

\[
\delta C_{\ell}^{BB} \simeq \frac{1}{2} \frac{\sigma_{\theta_{\text{map}}}^2}{\ell_s^2} \int_0^{\ell_s} d\ell_1 \ell_1^3 C_{\ell_1}^{EE}(\sigma_{\text{beam}})
\]

(3.10)

When evaluated for a beam with the EBEX FWHM = 8', the induced \( \delta C_{\ell}^{BB} \) is maximal for the coherency scale \( \ell_s \sim 500 \). Using this worst-case-scenario value of \( \ell_s \), the requirement that \( \delta C_{\ell}^{BB} < 0.1 \times C_{\ell}^{BB} \) yields a maximum allowable value of \( \sigma_{\theta_{\text{map}}} \sim 10'' \). This translates to the time domain restriction that the rms of the difference between the true pointing and the reconstructed pointing over a typical 40 s azimuth throw must be less than 54'' (see [16]; Appendix A).

3.3 Star Field Identification

3.3.1 Procedural Overview

The custom software used to identify star fields in EBEX star camera images, called the Star Tracking Attitude Reconstruction Software (STARS), was developed by D. Chapman and is treated in detail in [11; 12; 13]. It produced real-time, in-flight pointing solutions for star camera images by identifying triplets or pairs of image sources that are consistent with point sources convolved with our point spread function (i.e., objects identified in the image that potentially correspond to stars) and matching them to a catalog of known stars, as follows:

- STARS begins by identifying the brightest triplet of image sources. It then calculates the angular distance along the legs of the triangle formed by the triplet and compares the leg lengths to those formed by star triplets in the catalog.
• If a match to known stars in the catalog is identified, STARS performs a least-squares fit to align the image source triplet with the triplet of stars. It then attempts to match additional image sources to the catalog.

• A tentative pointing solution is produced by performing an updated least-squares fit using all matched image sources.

• If the tentative pointing solution meets certain user-defined criteria discussed below, STARS accepts the solution. Otherwise, STARS rejects the tentative pointing solution and selects a new triplet or pair of image sources. STARS continues to match image sources to the catalog until every unique triplet of the brightest seven sources is exhausted, and then until every pair combination from the brightest seven sources is exhausted.

In addition to providing real-time pointing solutions for in-flight attitude control, STARS was used to generate post-flight image pointing solutions for use in pointing reconstruction. The user-defined criteria for accepting or rejecting an image pointing solution differed between these two cases.

During flight, data from the ACS’s secondary pointing sensors—the magnetometers, Sun sensors, dGPS, elevation encoder, and inclinometer—were used when available to provide a coarse pointing estimate to limit the region of sky STARS searched for source-star matches. STARS rejected the tentative pointing solution if it failed to fall within specified limits of the coarse pointing estimate, or if fewer than four or five matches were identified. When secondary pointing sensor data was unavailable, STARS ran in its “lost-in-space” mode; no coarse pointing estimate limited the sky region STARS searched for matches, and seven or eight matches were required [11].

During recursive post-flight pointing reconstruction, interim pointing reconstructions were used in place of secondary pointing sensor data to provide STARS with pointing estimates. This was accomplished by using an ancillary program to load each
star camera image, identify its corresponding time sample, load the reconstructed pointing at the time sample, and feed the reconstructed pointing and user-defined parameters to the STARS program. I contributed to rewriting the ancillary program and experimenting to identify the optimal parameters for star field identification. We found that the yield of solved images was maximized by using the following algorithm and user-defined parameters:

- In the first iteration of star field identification, STARS is run in its “lost-in-space” mode for each image: no pointing guess is provided to the pattern matching algorithm, and eight sources within the image must be matched to the catalog of known stars.

- After an interim pointing reconstruction has been produced from the first iteration of solved image pointing solutions and gyroscope data, STARS is run again using the pointing reconstruction as a pointing guess. Because the approximate pointing at the time of the image is known, the parameters for pattern matching may be relaxed.

- The parameters for pattern matching are adjusted according to the uncertainty of the pointing reconstruction at the time of the star camera image, with pattern matching proceeding in the following ordered steps: If the pointing uncertainty is less than 0.5°, STARS searches within a 2° radius of the pointing guess and is required to match five stars to the catalog; otherwise, if the uncertainty is less than 3°, STARS searches within a 12° radius and is required to match six stars; otherwise, STARS solves in lost-in-space mode and is required to match eight stars.

\(^2\)The flight computers stored the state of each star camera shutter (open or closed) in a “trigger line” data time stream at a sampling rate of 100.16 Hz. The time sample assigned to an image was the average of the start and end times of the image’s trigger event, defined as the time samples corresponding to the rising and falling edges of the trigger line. Star camera exposure times were set to 300 ms to allow detection of stars with apparent magnitude 7.3 or brighter \[^11\].
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The ancillary program records the pointing solution for each image in a data table that is stored to disk. The data table is used to provide image pointing solutions to the pointing reconstruction software discussed in §3.4, which is used in turn to produce an updated iteration of image pointing solutions. As pointing solutions for additional image star fields are identified, the pointing reconstruction uncertainty of the surrounding pointing reconstruction decreases (see 3.4.1), allowing the parameters for pattern matching to be relaxed in subsequent iterations as described above. This results in an increased yield of solved images with each iteration.

3.3.2 Results

The two star cameras took a combined total of 41,262 images. We ran three iterations of recursive star field identification, using the image pointing solutions from each iteration to produce an updated pointing reconstruction for use as a pointing guess during the next iteration. The results are shown in Table 3.1. When we exclude images that are pointing within 30° of the Sun or that our image processing software identifies as saturated, the percentage of solved images for each successive iteration was 92.6%, 92.4%, and 92.9%. We performed no additional iterations because we found that most of the remaining unsolved images were unsolvable for various reasons (e.g., the camera was pointed at the balloon, or the image processing software failed to identify an image as saturated), and because the yield of solved images rendered a pointing uncertainty that was sufficiently low for our purposes (see §§3.4.3 3.5).

3.4 Pointing Reconstruction

The EBEX pointing reconstruction software comprised two main elements. The first was a UKF that estimated the pointing and gyroscope bias offset values at each time sample by combining star camera image pointing solutions, gyroscope velocity data, and a given transformation matrix between the star camera reference frame and the
### Number and Percentage of Star Camera Images Solved

<table>
<thead>
<tr>
<th>Solving Iteration</th>
<th>All images (none excluded)</th>
<th>Saturated images excluded</th>
<th>Saturated and close-to-Sun images excluded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iteration 1</td>
<td>35,266 of 41,262</td>
<td>35,242 of 38,647</td>
<td>33,124 of 35,779</td>
</tr>
<tr>
<td></td>
<td>85.6%</td>
<td>91.2%</td>
<td>92.6%</td>
</tr>
<tr>
<td>Iteration 2</td>
<td>35,353 of 41,262</td>
<td>35,329 of 38,647</td>
<td>33,202 of 35,779</td>
</tr>
<tr>
<td></td>
<td>85.7%</td>
<td>91.4%</td>
<td>92.8%</td>
</tr>
<tr>
<td>Iteration 3</td>
<td>35,410 of 41,262</td>
<td>35,385 of 38,647</td>
<td>33,252 of 35,779</td>
</tr>
<tr>
<td></td>
<td>85.8%</td>
<td>91.6%</td>
<td>92.9%</td>
</tr>
</tbody>
</table>

Table 3.1: Number and percentage of solved star camera images after successive iterations of image solving as described in § 3.3.1. Saturated images are defined as having a median pixel value of less than 3500 out of the range 0 (black) to 4095 (white). Close-to-Sun images are defined as having a great circle distance from the Sun of less than 30°. Note that the baffles were designed to operate at great circle distances of greater than 60° from the Sun.

The Unscented Kalman Filter

Pointing solutions from solved star camera images occur at times when the gondola was stationary at the end of azimuth throws, which are rotations in azimuth at a constant elevation. The duration of most azimuth throws was \( \sim 40 \text{ s} \). The UKF used image pointing solutions and gyroscope rate time streams to produce a pointing solution at each time sample, at the 100.16 Hz sampling rate of the flight computer’s...
A Kalman filter is a time series analysis algorithm that uses Bayesian inference to produce estimates of unknown values from noisy or inaccurate measurements. The algorithm includes two steps. In the prediction step, the filter produces an estimate of a system’s state variable vector $\hat{x}$ and its covariance matrix $P$ at the current time step $k$ by evolving the state from the previous time step $k-1$ according to a specified model. During the update step, a subsequent measurement provides a new estimate of the state vector, with its own associated uncertainty. A weighted average of the predicted estimate and the measurement estimate is used to update $\hat{x}$ and $P$, with greater weight assigned to estimates with lower uncertainty. This produces an updated estimate that lies between the predicted and measured estimates, and has a smaller uncertainty than either estimate alone. The process repeats recursively, using the updated $\hat{x}$ and $P$ to generate the predicted estimate in the next time step $k+1$.

In EBEX pointing reconstruction, the state vector $\hat{x}$ comprises six state variables: the three pointing angles, RA ($\theta$), Dec ($\psi$), and Roll ($\phi$); and the bias values for the three gyroscopes, $b_1$, $b_2$, and $b_3$. The bias values are constant angular velocities that are fitted over each throw and added to the measured gyroscope rates to correct for systematic error from 1/$f$ noise. In our case the update step corresponds to a measurement provided by star camera image pointing solutions, which does not occur at every time step. The prediction steps correspond to calculating the pointing angle values at each time sample between image solutions by integrating the gyroscope velocities.

The measured estimated state vector $\hat{x}_{k|k}$ at the update step $k$ is:

\footnote{The frequency spectrum of the gyroscope velocity contains a white noise component of $\sim 40''/s$ and a 1/$f$ noise component. Integration of the white noise component introduces a random walk about the true gyroscope velocity; this causes the pointing uncertainty to grow as $\sqrt{N}$, where $N$ is the number of integrated samples. The 1/$f$ component can be thought of as a slowly-drifting bias. Measurements show it to be constant ($\sim 20''/s$ or less) on timescales shorter than 200 s \cite{11}. It is therefore modeled as a constant over each $\sim 40$ s azimuth throw.}
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\[ \hat{x}_{k|k} = \begin{pmatrix} \theta \\ \psi \\ \phi \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} \]  

(3.11)

where \( k|k \) denotes the inclusion of the step \( k \) measurement in the step \( k \) estimate.

The prediction estimate at step \( k \) generated from the previous state at step \( k - 1 \), \( \hat{x}_{k|k-1} \), is:

\[ \hat{x}_{k|k-1} = \hat{x}_{k-1|k-1} + \begin{pmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{\phi} \\ 0 \\ 0 \\ 0 \end{pmatrix} \Delta t \]  

(3.12)

where \( \Delta t \) is the time step between the 100.16 Hz samples and the equatorial angular velocities \( \dot{\theta}, \dot{\psi}, \) and \( \dot{\phi} \) are calculated from the gyroscopes’ measured angular velocities \( \omega_1, \omega_2 \) and \( \omega_3 \) using Euler angle kinematics for rotating frames:

\[ \begin{pmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} \cos \psi & 0 & \sin \psi \\ \sin \psi \tan \theta & 1 & -\cos \psi \tan \theta \\ -\sin \psi \sec \theta & 0 & \cos \psi \sec \theta \end{pmatrix} \begin{pmatrix} \omega_1 + b_1 \\ \omega_2 + b_2 \\ \omega_3 + b_3 \end{pmatrix} \]  

(3.13)

Here \( O \) is an orthogonalization matrix that orthogonalizes the (nearly orthogonal) rotational axes of the three gyroscopes, which depends on three small misalignment angles; \( R \) denotes the rotation matrix that rotates the orthogonalized gyrooscope frame into the star camera frame, which depends on three rotation angles; and the first matrix on the right-hand side of equation 3.13 is the rotation matrix between the star camera and equatorial coordinate frames.
A standard Kalman filter is designed to track the state vector of a system that varies according to linear equations. Because equation [3.13] is nonlinear in $\theta$, $\psi$, and $\phi$, we use an Unscented Kalman Filter in place of a standard Kalman filter. The UKF uses the current estimate of the covariance matrix $P$ to select a sampling of values surrounding the current state vector $\hat{x}$ using a technique called the unscented transform [35]. The sampled values are propagated through the nonlinear equations and the mean and covariance of the propagated values are used to calculate $\hat{x}$ and $P$ at the next step.

As the UKF propagates the state vector, the covariance grows with each integration of the gyroscope velocity. When the UKF reaches the next star camera measurement an update is performed: the gyroscope biases $b_1$, $b_2$, and $b_3$ are fitted by comparing the measured pointing angles to the values predicted by gyroscope integration from the previous star camera measurement, and the pointing angles $\theta$, $\psi$, and $\phi$ are updated to the weighted average of the predicted state and the measured state. Because the covariance of the updated pointing angles is dominated by smaller (more strongly weighted) errors of the star camera measurement, the covariance drops.

When the UKF is run forward in time, the filter’s estimate of the state vector at step $k$ is informed only by time steps preceding $k$. To ensure that the pointing reconstruction at each time sample benefits from the maximum information available, the filter is also run backward in time, from later to earlier star camera measurements, with the backward-running covariance growing in the reverse temporal direction as the gyroscope velocities are integrated from later to earlier time steps. The final pointing reconstruction is an average of the forward and backward reconstructions at each time sample, weighted by the inverse of the filter-generated covariances of each

\footnote{The EBEX pointing reconstruction software uses pykalman, a UKF written in Python (available at \url{https://pykalman.github.io/}), that we adapted to C++.
}
reconstruction. The pointing error $\sigma_i$ at each time sample $i$ is given by:

$$\frac{1}{\sigma_i^2} = \frac{1}{\sigma_{i,f}^2} + \frac{1}{\sigma_{i,b}^2}$$  \hspace{1cm} (3.14)$$

where $\sigma_{i,f}$ and $\sigma_{i,b}$ are the errors for the reconstructions in the forward and backward directions.

Figure 3.2 shows an example Dec angle reconstruction in the vicinity of a star camera image pointing solution. The final reconstruction (solid blue) is the weighted average of the forward (solid red) and backward (solid green) reconstructions. The filter-generated uncertainty for the forward (backward) reconstruction, indicated by the red (green) dashed lines, grows as gyroscope noise is integrated, but collapses when the image pointing solution is used to update the estimated pointing. The final reconstruction benefits from using the information available from both the forward and backward reconstructions to decrease the resulting uncertainty (dashed blue).

### 3.4.2 The Least-Squares Optimizer

As shown in equation 3.13 pointing reconstruction requires knowledge of the three misalignment angles to rotate the gyroscopes into an orthogonal frame, as well as the three rotation angles required to rotate the orthogonalized gyroscope frame into the star camera frame. The values of these six constant parameters are determined by running the UKF through a least-squares optimizer.

To optimize the six parameters, a metric must first be defined that the least-squares optimizer can minimize. This is accomplished by estimating the error of the filter after integrating the gyroscope over an azimuth throw. This error is estimated by calculating the difference between the star camera image pointing solution at the end of the azimuth throw and the predicted estimate of the pointing before the UKF updates the estimated pointing. The metrics for all throws over the entire flight are then combined to produce a single metric for use in the least-squares optimizer.

The UKF was modified to calculate the metric and feed it to the least-squares
Figure 3.2: Example UKF reconstruction of the Dec pointing surrounding a star camera image solution (yellow star) shown on a longer time scale (top panel, near $t \sim 42$ s) and a close-up view of the same on a shorter time scale (bottom panel). The reconstruction (solid blue) is the weighted average of the forward (solid red) and backward (solid green) reconstructions. The colored dashed lines indicate the filter-generated uncertainty for each reconstruction.
optimizer. The optimizer uses a standard Levenberg-Marquardt algorithm to vary the six parameters and determine the values that minimize the metric. The optimizer runs the UKF over the entire flight multiple times, producing a metric at each iteration, until converging on the optimal parameter values. Simulations have shown that the optimizer determines each of the angles to within $\sim 3.4'$ [17].

3.4.3 Results

We generated and compared several pointing reconstructions for possible use in data analysis. An initial reconstruction was generated for the pointing of first of the two stars cameras ("xsc0") without using the least-squares optimizer to determine the optimal misalignment and rotation angles. We also produced two pointing reconstructions with misalignment and rotation angles determined by the least-squares optimizer: a reconstruction of the xsc0 pointing, and a reconstruction of the pointing for the second camera ("xsc1"). We used two related methods to compare the pointing errors of the three reconstructions.

First, we compared histograms of the filter error metric described in §3.4.1 (i.e., the difference between the image pointing solution and the predicted pointing estimate generated by the UKF prior to incorporating the image pointing in an updated estimate) for image pointing solutions at the end of representative azimuth throws, for both the forward- and backward UKF-generated reconstructions. Figure 3.3 shows histograms of the cross-Dec, Dec, and Roll filter error metric (blue) for the forward reconstruction for image pointing solutions separated by 28 to 52 seconds—a time interval centered on the $\sim 40$ s duration of a typical throw—for each of the three reconstructions. Results for cross-Dec, Dec, and Roll are shown for three pointing reconstructions in the forward direction: an initial, non-optimized xsc0 reconstruction (top) and least-squares optimized reconstructions of the xsc0 (middle) and xsc1

---

5Cross-declination, defined as $\Delta RA \times \cos(Dec)$, provides an approximate measure of the angular displacement in RA at a given Dec angle in the flat sky approximation.
(bottom) star camera pointing. The same plots for the three pointing reconstructions run in the backward direction are shown in Figure 3.4. Gaussian fits are shown in red; the mean and 1σ values of the fits are shown in red solid and dashed vertical lines. Histograms of the UKF-estimated errors are overlaid in green for comparison, with the mean value indicated by a vertical dashed green line; this provides an estimate of the expected location of the 1σ spread of the blue histogram. The larger variance in the Roll histograms occurs because the Roll angle is inherently more difficult to constrain; the size of its error is dependent on the distance between the observed stars in the star camera image, which acts as a lever arm that can produce a large error in Roll from a small displacement error (see Figure 3.5).

Note that the histograms in Figures 3.3 and 3.4 were used to evaluate the non-optimized and optimized reconstructions against one another, not to determine whether each reconstruction met the pointing requirements set by the EBEX science goals. That determination is made ultimately by comparing the spurious induced B-modes to the expected B-mode signal (see §§ 3.2 and 3.5). These histograms examine the error metric for each of the unidirectional reconstructions, not for the averaged reconstruction that benefits from information from both unidirectional reconstructions. Also note that the histograms overstate the error of the unidirectional pointing reconstruction over the azimuth throw; the error is smaller mid-throw, as shown in Figure 3.2. Evaluating the sufficiency of the three pointing reconstructions in the time domain requires estimating the rms error over a 40 s azimuth throw for the averaged forward and backward unidirectional reconstructions and comparing it to the 54″ limit calculated from the induced B-mode requirement. Analysis performed by others \cite{16, 11, 17} presents simulations and theoretical calculations showing that the rms error over a typical azimuth throw is ~ 25″ for the non-optimized reconstruction and ~ 11″ to 15″ for the optimized reconstructions.

The second comparison we performed among the three reconstructions involved examining plots of the estimated uncertainty generated by the UKF as a function of
Figure 3.3: Histograms of the difference between the image pointing and the forward-running UKF-estimated pointing for images at the end of 28 – 52 s azimuth throws.
Figure 3.4: Histograms of the difference between the image pointing and the backward-running UKF-estimated pointing for images at the end of $28 - 52$ s azimuth throws.
Figure 3.5: Diagram illustrating the variability of Roll errors produced by small displacement errors in the estimated location of stars within a star camera image. In the simplified examples shown, the displacement $d$ between the true and estimated vertical location on the image of the red (blue) star (exaggerated here for clarity) causes a Roll error $\Delta \phi_1$ ($\Delta \phi_2$) that depends on the distance $L_1$ ($L_2$) from the black star. In the small angle approximation the Roll error $\Delta \phi = d/L$.

time. An example plot is shown in Figure 3.6. It illustrates the growth of pointing errors as the length of time from the nearest star camera image pointing solution grows. This occurs when star camera image solutions are unavailable: for example, at the start of the segment before the cameras are turned on; or when pointing solutions cannot be found for star camera images, e.g., when images are unsolvable due to saturation (gray dots), proximity to the Sun (yellow dots), or when the cameras are aimed at part of the gondola ($\sim 15000 - 18000$ s).

These comparisons showed that the least-squares optimized reconstructions had smaller pointing errors than the non-optimized initial reconstruction. The least-squares optimized xsc0 and xsc1 reconstructions were generally comparable, but dif-
Figure 3.6: Time stream plot of the UKF-estimated reconstructed Dec error for a \( \sim 1 \) hr segment of data, shown for three pointing reconstructions: an initial reconstruction using the xsc0 star camera pointing, made without least-squares fitting of misalignment and rotation angles (green); and reconstructions for the xsc0 (cyan) and xsc1 (magenta) pointing, made with least-squares optimization. Dots indicate star camera image times, with solved (unsolved) images located on the upper (lower) horizontal line of dots. Blue (red) dots indicate solved xsc0 (xsc1) images.

Figure 3.6: Time stream plot of the UKF-estimated reconstructed Dec error for a \( \sim 1 \) hr segment of data, shown for three pointing reconstructions: an initial reconstruction using the xsc0 star camera pointing, made without least-squares fitting of misalignment and rotation angles (green); and reconstructions for the xsc0 (cyan) and xsc1 (magenta) pointing, made with least-squares optimization. Dots indicate star camera image times, with solved (unsolved) images located on the upper (lower) horizontal line of dots. Blue (red) dots indicate solved xsc0 (xsc1) images.

The time ordered pointing uncertainties for the xsc1 reconstruction generally fell below those of the xsc0 reconstruction, as shown in Figure 3.6. This is due in part to the more frequent availability of solved xsc1 images. The filter error metric histograms in Figure 3.3, however, showed that the xsc1 reconstruction had wider distributions and an unexplained thick negative tail in the Dec distribution. The induced B-mode analysis described in § 3.5, as well as the equivalent time
domain analysis presented in [16; 11; 17], showed that each of the least-squares optimized reconstructions had pointing uncertainties that were sufficiently small to meet our science requirements. We therefore selected the xsc0 reconstruction with the more well-defined uncertainties for use in data analysis.

3.5 Induced B-modes from Pointing Jitter

In §3.2 we reviewed the formalism for calculating the magnitude of pointing jitter induced B-modes. This section applies the formalism to the EBEX pointing reconstruction to demonstrate that the induced B-modes are restricted to below 10% of the expected CMB lensing and inflationary B-mode signal over the EBEX \( \ell \)-range of interest \( 30 < \ell < 1500 \) for \( r = 0.05 \).

As described in §3.2, estimating the induced B-modes for the EBEX flight involves using the time ordered UKF-estimated pointing errors to generate a map of pixel-averaged pointing errors \( \delta \vec{\theta}_{\text{map}} \). The power spectrum of the map, \( C_{\ell}^{\theta \theta} \), is then computed and substituted into the integrals in equation 3.7, yielding the power spectrum of the induced B-modes, \( \delta C_{\ell}^{BB} \).

To create a map of estimated pointing errors \( \delta \vec{\theta}_{\text{map}} \), we first generate time streams of the estimated pointing errors from the post-flight pointing reconstruction; the pointing error \( \delta \vec{\theta}_i \) at each time sample \( i \) is sampled from a Gaussian distribution with a standard deviation equal to the UKF-estimated pointing error for that time sample. The time streams are then binned into a sky map and averaged within each pixel. We then compute the power spectrum of the map, \( C_{\ell}^{\theta \theta} \), using the anafast function from NASA’s Healpix\(^6\) software.

A map of the sky coverage for a single detector is shown in Figure 3.7 top. The presence of empty pixels within the overall scanning area results in an artificially low estimate of the \( C_{\ell}^{\theta \theta} \) power spectrum, even after accounting for the sky fraction covered

\(^6\)http://healpix.jpl.nasa.gov/
CHAPTER 3. POST-FLIGHT POINTING RECONSTRUCTION

by the scan. To remove this bias we estimated its magnitude using simulations. We first generate a test power spectrum $C_{\ell,WN}^{\theta\theta}$ for white noise pointing errors as modeled in equation 3.9, substituting $\sigma_{\theta,\text{map}} = 10''$ for the standard deviation of the pointing error map and $\ell_s \approx 500$ for the map’s coherence scale. We then use the Healpix `synfast` function to create a $\delta\vec{\theta}_{\text{map}}$ realization map from the $C_{\ell,WN}^{\theta\theta}$ test power spectrum using the flight hit map. We apply the `anafast` function to calculate the $C_{\ell}^{\theta\theta}$ of the sparsely covered realization map. Comparing the spectrum of the sparse realization map to the known input test power spectrum $C_{\ell,WN}^{\theta\theta}$ allows us to compute a corrective scaling factor $\sim 1.7$ that can then be applied to the $C_{\ell}^{\theta\theta}$ calculated from flight data. The corrected spectra for Dec and cross-Dec are shown in Figure 3.7 bottom.

Having obtained the $C_{\ell}^{\theta\theta}$ power spectrum, we compute the integrals in equation 3.7 and add the $p_A$ and $p_B$ contributions to find the total induced $\delta C_{\ell}^{BB}$. This process is repeated separately to determine the induced B-mode power spectra for the Dec and cross-Dec errors, which are averaged to determine the final $\delta C_{\ell}^{BB}$ power spectrum.

The resulting $\delta C_{\ell}^{BB}$ power spectrum is shown in Figure 3.8 (red points), together with the expected B-mode spectrum for the predicted CMB lensing and inflationary B-mode spectrum for $r = 0.05$ (solid black). The induced B-modes fall below 10% of the expected sky signal (dashed black) over the EBEX $\ell$-range of interest $30 < \ell < 1500$.

The preliminary analysis outlined in this section can be improved upon in several ways. First, this analysis was performed using the pointing of a single detector. A complete analysis would include multiple detectors, which would increase the number of hits per pixel and decrease the level of induced B-modes by average down $\delta\vec{\theta}_{\text{map}}$. The sky coverage shown in Figure 3.7 would also be increased due to different loc-

---

7We found that the ratio of the input test power spectrum to the spectrum of the sparse realization map was roughly constant over the $\ell$-range of interest, and was never greater than 1.7. To produce the most conservative estimate of the induced B-modes, we scaled the $C_{\ell}^{\theta\theta}$ calculated from flight data by a factor of 1.7 over the entire $\ell$-range.
Figure 3.7: *Top:* sky coverage for a single detector over the duration of the EBEX flight, shown in equatorial coordinates for Healpix nside = 2048. The presence of empty (gray) pixels within the overall coverage introduces a bias in the $C_\ell^{\theta\theta}$ power spectrum that must be corrected. *Bottom:* corrected $C_\ell^{\theta\theta}$ spectra for the Dec (green) and cross-Dec (red) pointing errors.
Figure 3.8: The power spectrum of spurious B-modes induced by the estimated pointing jitter for a single detector over the duration of the EBEX flight.

locations on the focal plane have slightly different pointing. In addition, the analysis above underestimates $\delta\tilde{\theta}_{\text{map}}$ by assuming that the pointing errors for repeated hits in the same pixel are uncorrelated and therefore average down according to equation 3.3. Pointing errors within a given pixel are correlated, however, for hits generated from the same scan. A complete analysis would either characterize and account for this correlation, or down-sample the pointing error time streams to avoid this limitation. We chose not to refine the analysis presented here because the pointing reconstruction uncertainties are far better than we require given the sensitivity of our detector measurements.
Chapter 4

Map Making

This chapter reviews the data analysis pipeline used to transform raw bolometer data in the time domain into sky maps of temperature and polarization, with a particular focus on detector time stream cleaning. Section 4.1 provides an overview of the principal stages of the analysis pipeline, as well as detailed descriptions of the signal filtering and map making procedures that are employed to evaluate the quality of the detector data in the time- and map-domains. Section 4.2 describes the various data flagging and time stream cleaning algorithms that are applied to exclude contaminated time samples from map making and other stages of data analysis. The flagging statistics for the EBEX flight data and the glitch removal algorithm are reviewed in detail. Section 4.3 describes the time- and map-domain visualization tools we developed to inform the design of the time stream cleaning algorithms.

4.1 The EBEX Data Analysis Pipeline

The EBEX data analysis pipeline converts raw time ordered bolometer data into sky maps of temperature and polarization. A detailed diagram of the analysis pipeline is shown in Figure 4.1. Its stages include the following:

1. *Flight base construction:* the conversion of raw data time streams into time-
CHAPTER 4. MAP MAKING

synchronized, usable data structures, described in detail in [11].

2. **Pointing reconstruction:** the use of star camera images and attitude sensor data to determine the location on the sky at which the telescope is pointing a function of time, described in Chapter 3.

3. **Half-wave plate template removal:** fitting and subtracting HWP-synchronous signal from the detector time streams, described below in Chapter 6.

4. **Time stream cleaning:** flagging of contaminated detector time stream samples to enable their exclusion during data analysis, described in detail in §§ 4.3–4.2.

5. **Signal calibration:** the use of temperature sky maps to calibrate detector time streams from readout system “count” units to temperature units using a constant conversion factor per detector per “segment” of data (defined below). Signal calibration is described in detail in [8].

6. **Detector non-linearity correction:** correcting for reduced detector responsivity due to operation of the bolometers in a non-linear detector regime, described in Chapter 7.

7. **I, Q and U signal extraction:** processing of the detector time stream to determine incident the Stokes $I$, $Q$ and $U$ sky signal time streams, described in § 4.1.2.

8. **Map making:** the conversion of the $I$, $Q$ and $U$ time streams into temperature and polarization sky maps, described below in § 4.1.3.

9. **Instrumental polarization removal:** removal of temperature-to-polarization signal leakage due to the instrument’s polarization of incoming unpolarized light, described in detail in [17].

Section 4.1.1 describes the data structures comprising the EBEX flight base that are used to generate detector time streams and sky maps. The extraction of Stokes
CHAPTER 4. MAP MAKING

\( I, Q \) and \( U \) time streams from detector data time streams is reviewed in §4.1.2. A procedural overview of the generation of sky maps from the \( I, Q \) and \( U \) detector time streams is presented in §4.1.3.

4.1.1 EBEX Flight Base Data Sets

During the flight, the EBEX flight computers received and recorded to disk \( \sim 1.3 \) TB of raw data from four categories of multiple, generally asynchronous time streams:

- Attitude Control System (ACS) streams, sampled at 100 Hz, containing timing and data time streams for the ACS, as well as housekeeping information for other subsystems.

- Bolometer streams, sampled at 191 Hz, containing timing and detector time streams. The bolometer streams are divided among 28 asynchronous readout boards, each containing data for 64 detectors.

- HWP streams, sampled at 3050 Hz, containing timing and HWP encoder data, which are used to reconstruct the HWP angle time stream.

- Slow streamer (SS) board streams, sampled at 0.99 Hz, containing timing and housekeeping information for each of the asynchronous 28 detector boards.

As described in [11], after flight these raw time streams were extracted, time-aligned, and organized into a Dirfile system, with data from each of the two flight computers merged such that there were no redundant data. The data are organized into segments and subsegments. A segment consists of data between intentional power cycling of the flight computers. Segments range between 3 to 20 hours in duration, and are named according to their starting times in the YYY-MM-DD--hh-mm-ss format.

\[ \text{A Dirfile is a file-system based database for time-ordered data; see } \text{http://getdata.sourceforge.net}\]
Figure 4.1: Diagram of the EBEX data analysis pipeline. Data products are shown in blue; analysis software programs are shown in green; and data flow is indicated by arrows. Contributions to analysis software made by the work described in this thesis are indicated by a red star. Figure adapted from [11].
Within each segment the data are divided into numbered subsegments, consisting of blocks of continuous, uninterrupted data. Data within each raw time stream is referred to as a data channel. Each channel is written to disk in its native binary format, organized into Dirfile directories by subsegment. The ACS flight base, for example, is structured as follows:

acs/ [ACS base folder]  
  2012-12-31--13-17-57/ [segment folder]  
    subsegment0/ [Dirfile]  
      channel1 [binary file]  
      channel2 [binary file]  
      ...  
    subsegment1 [Dirfile]  
      channel1 [binary file]  
      channel2 [binary file]  
  2012-01-01--13-52-48/ [segment folder]  
    ...

The EBEX collaboration works largely in a Python software framework called the Long-duration EBEX Analysis Pipeline (LEAP), a repository of analysis software and supporting data resources stored on a server at Columbia University. LEAP includes software tools for loading data channels from Dirfiles for specified segments and time ranges for use by analysis software. LEAP was developed by D. Chapman and J. Didier and is described in detail in [11; 17].

4.1.2 Extraction of I, Q and U Signal Time Streams

To make sky maps of the Stokes \( I \), \( Q \) and \( U \) parameters the data time stream for each detector, \( s(t) \), is processed into three corresponding time streams, \( I(t) \), \( Q(t) \), and \( U(t) \). This is accomplished by modeling the detector time stream as the sum of
CHAPTER 4. MAP MAKING

three contributions:

\[ s(t) = s_{\text{sky}}(t) + s_{\text{hwp}}(t) + n(t) \] (4.1)

where \( n \) is the noise contribution, \( s_{\text{sky}} \) is the incoming signal from the sky, and \( s_{\text{hwp}} \) denotes the HWP-synchronous signal called the HWP “template.”

The HWP template is described further in Chapter 6 and is modeled as the sum of harmonics of the HWP rotational frequency \( \omega_{\text{hwp}} \):

\[ s_{\text{hwp}}(t) = \sum_{n=1}^{20} (C_{1,n} + C_{2,n} t) \cos(n\theta_{\text{hwp}}(t)) + (S_{1,n} + S_{2,n} t) \sin(n\theta_{\text{hwp}}(t)) \] (4.2)

where the HWP angle at time \( t \) is \( \theta_{\text{hwp}} = \omega_{\text{hwp}} t \) and the \( S \) and \( C \) symbols are constants.

The noise contribution \( n(t) \) includes white noise and 1/\( f \) components. The power spectral density (PSD) of the detector noise is modeled as

\[ PSD = \left( 1 + \left( \frac{f_{\text{knee}}}{f} \right)^\alpha \right) \cdot NET \] (4.3)

where \( f_{\text{knee}} \) is the knee frequency of the 1/\( f \) component, \( \alpha \) is a dimensionless spectral index, and \( NET \) is the noise equivalent temperature of the white noise component.

Table 4.1.2 lists the median parameters for the detectors in each of the three EBEX frequency bands.

<table>
<thead>
<tr>
<th>Frequency Band (GHz)</th>
<th>( f_{\text{knee}} ) (Hz)</th>
<th>( \alpha ) (dimensionless)</th>
<th>( NET ) (( \mu K ) cmb ( \sqrt{s} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>0.2</td>
<td>2.3</td>
<td>432</td>
</tr>
<tr>
<td>250</td>
<td>0.2</td>
<td>2.3</td>
<td>1046</td>
</tr>
<tr>
<td>410</td>
<td>0.2</td>
<td>2.2</td>
<td>17003</td>
</tr>
</tbody>
</table>

Table 4.1: Median measured noise parameters for the detectors in each EBEX frequency band.

The sky signal component \( s_{\text{sky}} \) for a linearly polarized incoming signal can be expressed [17] in terms of the incoming signal’s Stokes vector in the sky frame \( \vec{S} = (I, Q, U, 0) \) as:

\[ s_{\text{sky}}(t) = \frac{1}{2} \left( I + Q \cos(4\theta_{\text{hwp}} + 2\psi) + U \sin(4\theta_{\text{hwp}} + 2\psi) \right) \]

\[ = \frac{1}{2} I \left( 1 + P \cos(4\theta_{\text{hwp}} - 2\alpha + 2\psi) \right) \] (4.4)
where $\psi$ is the telescope’s Galactic Roll angle, $P$ is the polarization fraction $P = \sqrt{Q^2 + U^2}/I$, and $\alpha$ is the polarization angle $\alpha = \frac{1}{2} \tan^{-1}(U/Q)$. Here the Roll angle is defined as the angle between the Galactic meridian and the symmetry axis of the focal plane in the positive elevation direction; and the Stokes $Q$ and $U$ parameters are defined according to WMAP conventions [39], with $Q > 0$ and $U = 0$ for polarization parallel to the Galactic meridian.

The procedure for extracting the $I$, $Q$, and $U$ signals from a cleaned detector time stream is as follows:

1. **Calibration** from readout count units to temperature, expressed in black body equivalent temperature units $K_{CMB}$. This procedure is described in detail in [8].

2. **Deconvolution** of bolometer time constants: as described in § 2.4, after absorbing incoming radiation a bolometer returns to its initial temperature with an intrinsic thermal time constant. The time constant acts effectively as a low-pass filter, which can be modeled in Fourier space as a single-pole RC filter for a time constant with a half power point $f_{3dB}$:

   $$ s_{out}(f) = s_{in}(f) \frac{1}{1 + i \frac{f}{f_{3dB}}} $$ \quad (4.5)

   This perturbs time-varying $I$ signals and rotates the polarization angle $\alpha$ of incoming radiation. Work is underway to to measure the detectors’ time constants, which pre-flight tests indicated are in the range $f_{3dB} \sim 10–15$ Hz [37], corresponding to a negligible distortion in $I$ and a polarization angle rotation of $\sim 10^\circ$ [17].

3. **HWP template removal**: fitting and subtracting $s_{hwp}(t)$ from the detector time stream (see Chapter 6).

4. **Pre-demodulation frequency filtering**: the detector time stream is then frequency
filtered, with different filtering applied for temperature \((I)\) versus polarization \((Q, U)\) signal extraction (see Figure 4.2):

(a) When extracting the \(I\) time stream, an order 10 Butterworth band-pass filter with low- and high-frequency cutoffs of 33 mHz and 3.0 Hz is applied. The low-frequency cutoff rejects \(1/f\) noise, while the high-frequency cutoff removes high-frequency noise and unremoved higher harmonics of the HWP template (see Chapter 6). The resulting time stream \(s_I(t)\) is the \(I(t)\) temperature time stream; no further signal processing is performed.

(b) When extracting \(Q\) and \(U\) time streams, the detector time stream is first band-pass filtered with cutoffs at 2.0 Hz and 8.0 Hz to prevent \(1/f\) and high-frequency noise from leaking into the polarization signal bandwidth surrounding \(4f_{hwp} \sim 5\) Hz. The resulting signal then undergoes further processing as described in steps 5 and 6.

5. **Demodulation:** The “demodulated” \(Q\) and \(U\) time streams, \(s_Q(t)\) and \(s_U(t)\), are obtained by multiplying the detector time stream \(s(t)\) by the cosine and sine of the modulation angle \((4\theta_{hwp} + 2\psi)\). As shown below, this moves the \(Q\) and \(U\) signals from the sidebands surrounding \(4f_{hwp}\) to lower frequencies, while also moving the \(I\) signal to a higher frequency bandwidth centered on \(4f_{hwp} \sim 5\) Hz and generating additional signal components oscillating near \(\sim 8f_{hwp}\). This allows the \(Q\) and \(U\) time streams to be extracted by filtering the \(s_Q(t)\) and \(s_U(t)\) time streams in step 6.

6. **Post-demodulation frequency filtering:** After demodulation into \(s_Q(t)\) and \(s_U(t)\) signal time streams in step 5, each signal is band-pass filtered with low- and high-frequency cutoffs of 50 mHz and 1.5 Hz to yield the \(Q(t)\) and \(U(t)\) time streams (see Figure 4.2). As explained below, the low-pass filtering removes leakage from \(I\) into \(Q\) and \(U\), while the high-pass filtering removes residual template that resides near 0 Hz in the demodulated time streams.
The demodulation in step 5 allows for direct extraction of the $Q$ and $U$ signals from the detector time stream, without the need for differencing detectors with sensitivity to orthogonal polarization states. When multiplied by $4 \cos(4\theta_{hwp} + 2\psi)$, the detector time stream in equation 4.4 becomes:

$$s_Q(t) \equiv 4s(t) \cos(4\theta_{hwp} + 2\psi)$$

$$= 2I(t) \cos(4\theta_{hwp} + 2\psi) + 2Q(t) \cos^2(4\theta_{hwp} + 2\psi)$$

$$+ 2U(t) \sin(4\theta_{hwp} + 2\psi) \cos(4\theta_{hwp} + 2\psi)$$

$$= Q(t) + 2I(t) \cos(4\theta_{hwp} + 2\psi) + Q(t) \cos(8\theta_{hwp} + 4\psi)$$

$$+ U(t) \sin(8\theta_{hwp} + 4\psi)$$

where the final line follows from applying the identities $\sin 2\theta = 2\sin\theta \cos\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$. The low-pass filtering in step 6 rejects the sine and cosine terms in the bottom line of equation 4.6 — including both the $I$-leakage term oscillating at $4f_{hwp}$ and the modulated $Q$ and $U$ terms oscillating at $8f_{hwp}$ — leaving only the $Q(t)$ time stream. The $U(t)$ time stream is extracted similarly by multiplying the detector time stream $s(t)$ by $4 \sin(4\theta_{hwp} + 2\psi)$.

The high pass filtering in step 6 is applied to remove any residual template signal that resides at $4f_{hwp}$ in the pre-demodulated time stream $s(t)$. The demodulation in step 5 moves the residual template to $\sim 0$ Hz, while the $Q$ and $U$ signals are moved from the sidebands surrounding $4f_{hwp}$ to low frequencies above 0 Hz. The high-pass filtering at 50 mHz therefore rejects the demodulated template fourth harmonic, while passing the demodulated $Q$ and $U$ signals.

Note that although low-pass filtering the demodulated $s_Q(t)$ and $s_U(t)$ time streams rejects the leakage from $I$ into $Q$ and $U$ coming from the $I(t) \cos(4\theta_{hwp} + 2\psi)$ term in equation 4.6, the same effect is accomplished via repeated scans of a given map.

Note that including the Galactic roll, $\psi$, in the modulation angle $(4\theta_{hwp} + 2\psi)$ produces $Q$ and $U$ time streams in the sky frame; excluding $\psi$ produces the time streams as observed in the reference frame of the instrument.
Figure 4.2: Top panel: PSD of an example detector time stream after HWP template removal (blue), showing the bandwidths of the bandpass filter applied to extract the I time stream (blue shading) and the pre-demodulation (gray shading) and post-demodulation (yellow shading) filters applied to extract the Q and U time streams. Bottom panel: close-up view of the region surrounding the 4th harmonic of the HWP template 4f_{hwp} (red). This example detector has significant residual template at 4f_{hwp}, and was chosen specifically to illustrate the utility of the post-demodulation high-pass filter that rejects the residual 4th template harmonic.

Pixel at different angles. The sine and cosine terms from different time samples in the same pixel average down, leaving only the Q(t) and U(t) terms in the respective expressions for s_Q(t) and s_U(t).
4.1.3 Map Making Overview

The “LEAP map-making” procedure\(^3\) assigns each pixel in a sky map the weighted mean of the integrated signal within the pixel resulting from multiple re-visits to that location on the sky, both by the same detector and by other detectors in the focal plane. A sky map of the Stokes $Q$ signal for multiple detectors, for example, is produced by assigning each time sample in the detectors’ extracted $Q(t)$ signal time streams (as obtained via the procedure outlined in §4.1.2) with a noise weighting, described below; the weighted samples are then binned into the their appropriate map pixels as determined by the reconstructed pointing (see Chapter 3); the weighted samples within each pixel are then summed and averaged by dividing by the total weight of the samples in the pixel:

$$Q(p) = \frac{\sum_{i=1}^{N} w_i \cdot Q_i}{\sum_{i=1}^{N} w_i}$$

(4.7)

where $Q(p)$ is the weighted mean $Q$ signal in pixel $p$; $w_i$ is the noise weighting assigned to an individual detector time sample $i$; $Q_i$ is the value of the $Q(t)$ time stream for the sample $i$; and $N$ is the number of hits in pixel $p$.

Because the $I$, $Q$ and $U$ time streams are noise dominated, their variance provides a direct estimate of the noise for each sample. The noise weight $w_i$ assigned to a time stream sample $i$ from a particular detector is obtained by computing a moving

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\(^3\)The frequency filtering described in step 4 above is used only in the LEAP map-making procedure outlined in this chapter. The EBEX collaboration also explored using a more sophisticated Destriper map maker that estimates and removes noise without filtering. Only the procedure and results for LEAP map-making are presented here, as using the Destriper map maker did not result in much improvement on the results presented here.
variance of the time stream with a window size of 20 minutes, i.e., $w_i = 1/\sigma_i^2$ and

$$Q(p) = \frac{\sum_{i=1}^{N} Q_i/\sigma_i^2}{\sum_{i=1}^{N} 1/\sigma_i^2}. \quad (4.8)$$

The $I(p)$ and $U(p)$ pixel values are calculated similarly.

### 4.2 Data Flagging

The LEAP data analysis software library employs a flag generator to exclude contaminated data samples from map making and other stages of data analysis. Given an input time stream, the flag generator creates a Boolean array of identical length for each user-specified category of contaminated data outlined below. A flag array for a given category indicates whether each sample in the input time stream is “valid” (1) or “invalid” (0). The output flag array is the bitwise AND combination of the individual flags. Invalid samples are excluded from sky maps and, where appropriate, from other stages of data analysis.

The following is a descriptive list of the categories of detector time stream samples that are excluded by the individual flags:

- **Bolo not tuned:** samples where the detector is not properly “tuned,” i.e., biased at its transition point such that its electrical resistance is sensitive to changes in temperature caused by radiation absorption (see §2.4).

- **Valid pointing:** samples where the instrument pointing cannot be reconstructed, e.g., due to lack of ACS sensor data.

- **Remove saturation event:** samples within a single segment of data when the detectors were saturated due to heating by Solar illumination of a portion of the primary mirror near the receiver.
• **HWP transition cuts**: data samples that occur when the half-wave plate is accelerating to/decelerating from its target rotational frequency \( f_{hwp} = 1.23 \text{ Hz} \).

• **Valid template**: samples where the HWP template has not been removed (see Chapter 6), e.g., when the HWP is stationary. Because HWP modulation is needed only to extract the Stokes \( Q \) and \( U \) signals (see §4.1.2), this flag is not used for Stokes \( I \) analysis.

• **Template removal glitch**: samples that are identified as glitches during the template removal procedure outlined in Chapter 6.

• **Bad SQUID DC**: samples where the direct current in the detector’s SQUID amplifier is not nominal.

• **SQUID jumped**: samples where the SQUID amplifier is operating at the edge of its linear response.

• **Max covariance dec**: samples where the instrument pointing has been properly reconstructed, but the uncertainty of the reconstructed pointing is greater than 5', e.g., where star camera image solutions are unavailable (see Chapter 3).

• **Glitch cuts**: samples surrounding spikes or “glitches” that occur in the time stream after template removal has been applied. The deglitching algorithm is discussed in detail in §4.2.1 below.

• **El step**: samples near changes in the elevation of the telescope boresight. To completely mask elevation steps, the el step flag excludes samples where either the magnitude of the elevation velocity is greater than 0.25°/s or where the magnitude of the elevation acceleration is greater than 0.75e−6°/s^2.

• **Latched bolo**: samples where a detector has “latched,” i.e., has become unresponsive to incoming radiation due to having transitioned entirely into its superconducting state. Latching requires the bolometer to be re-tuned. Several
bolometers exhibited latching behavior late in the flight for reasons that we are still investigating.

- **Max velocity:** samples where the total angular velocity of the telescope boresight is above a threshold value. The threshold velocities for the temperature $I$ and polarization $(Q$ and $U)$ time streams, $1.5^\circ/s$ and $0.8^\circ/s$ respectively, are selected to prevent overlap of the $I$ and $(Q, U)$ signal bandwidths.

- **Valid T calib:** samples with outlier temperature calibration values. This can occur, e.g., due to insufficient detector responsivity or insufficient sky coverage of the calibration source.

- **Min velocity:** samples where the total angular velocity of the telescope boresight is slower than a threshold value. The threshold velocity ($0.01^\circ/s$) is selected to prevent overlap of the $I$ signal bandwidth with low-frequency noise, as well as to limit overlap between the $(Q, U)$ signal bandwidth and the bandwidth of residual template.

- **Stimulator cuts:** samples when the “stimulator,” a power source inside the receiver used to measure detector responsivity, is active. The stimulator flashes occurred throughout the flight at approximately 20 minute intervals.

### 4.2.1 The Deglitching Algorithm

Among the data flagging algorithms I was responsible for developing, the glitch flag and its underlying deglitching algorithm required continuing and extensive revision until it functioned properly (i.e., without under-flagging contaminated samples or over-flagging uncontaminated portions of the detector time streams). For this reason, and because the deglitching procedure plays an important role in the template removal procedure discussed in Chapter 6, this section describes the deglitching algorithm in detail.
The deglitching algorithm aims to identify and remove transients, i.e., brief and sudden increases in the power detected by the bolometers, which appear in the template removed time streams as sudden outlier samples. Such “glitches” are thought to arise from two physical sources; isolated glitches of the kind shown in Figure 4.3 are thought to originate from the absorption of cosmic rays, while clusters of glitches of the kind shown in Figure 4.4 are believed to originate from electrical burst noise.

The deglitching algorithm comprises three steps:

1. **Glitch identification**: locating a transient in the template removed detector time stream.

2. **Glitch replacement**: the insertion of a noise realization in place of the transient and a buffer of neighboring samples. This is required to prevent discontinuities in the time stream, which must be avoided when performing Fast Fourier Transforms during subsequent frequency filtering (e.g., Butterworth band-pass filtering during $I$, $Q$, and $U$ signal extraction).

3. **Glitch flagging**: generating a flag array in which the noise realization and a surrounding buffer of samples for every glitch are flagged as invalid. This ensures that these data points are excluded from map making, while also preventing the white noise realization from biasing both the data and the estimate of the detector noise.

Glitch identification is accomplished using the Median Absolute Deviation (MAD) of the calibrated, template removed detector time stream. The MAD is calculated as follows:

- The moving median (shown in solid red in Figure 4.3) of the template removed, calibrated time stream (blue) is computed using a 5 s window.

- The absolute deviation of the time stream from the moving median is calculated.
Figure 4.3: Example glitch removal for an isolated glitch in a 150 GHz detector, believed to originate from absorption of a cosmic ray. The calibrated, template removed time stream is shown in blue; its moving median is shown in red. Glitch rejection ranges are indicated by the dashed red, green and yellow lines. The glitch (large spike at $\sim 9$ s) and a surrounding buffer of samples are replaced by a noise realization (solid cyan line). The glitch, the noise realization, and samples in a surrounding 1 s buffer are then flagged as invalid (gray line/cyan shading).

- The MAD is calculated by computing the moving median of the absolute deviation using a 5 s window.

For data with a Gaussian distribution, the standard deviation $\sigma \approx 1.48$ MAD.

Glitches are identified as samples that lie beyond a threshold number of MADs from the moving median. We refer to the threshold number of MADs as the rejection factor. Glitches are identified using a rejection factor of 6.5, i.e., samples that are beyond 6.5 MAD ($\sim 10\sigma$) from the moving median are identified as glitches. In Figure 4.3 the rejection range for a rejection factor of 6.5 is demarcated by the red dashed lines. For samples that lie within $\pm 4^\circ$ of the Galactic plane, a relaxed rejection factor of 8.0 is used (dashed green lines) to avoid confusing strong signals from
Figure 4.4: Example glitch removal for a cluster of glitches, believed to originate in electrical burst noise. The large glitch near $\sim 45$ s may be an independent glitch (similar to that shown in Figure 4.3) caused by cosmic ray absorption.

Galactic sources with transients.

Burst noise can bias the MAD in its vicinity (see Figure 4.4). To avoid under-flagging of glitches within or near burst noise, we implemented an additional glitch identification method that exploits the fact that glitches always occur in the direction of increased detected power (see Figure 4.5). This method first computes the moving minimum of the time stream using a 5 s window (lower dashed yellow line). Glitch identification uses the deviation of the moving minimum below the moving median as an indicator of the acceptable range for valid samples above the moving median. We refer to the deviation of the moving minimum from the moving median as the “floor deviation.” Glitches are identified as samples that lie more than 1.6 times the floor deviation above the moving median (upper yellow dashed line).

Once glitches have been identified, glitch replacement occurs as follows:

- Each glitch is flagged for replacement by a noise realization.
- If a glitch lies within 30 s of a neighboring glitch, all data samples between
Figure 4.5: Zoom-in of an example glitch removal plot showing the “floor deviation” of the calibrated detector time stream, i.e., the distance between the moving median (solid red) and the moving minimum (lower dashed yellow line). Because glitches occur in the direction of increased detected power, the floor deviation provides an indicator of the acceptable range for valid samples above the moving median. Samples that lie more than 1.6 times the floor deviation above the moving median (i.e., above the upper yellow dashed line) are flagged as glitches.
the two are also flagged for replacement by a noise realization. This is done to ensure that the entirety of burst noise glitches is flagged as invalid.

- To ensure complete flagging of the glitch, a 30 sample buffer before and after it are also flagged for replacement by noise realization.

- The noise realization (solid cyan line in Figures 4.3 and 4.4) is generated by adding a white noise component to the moving median of the time stream. The white noise component is calculated using the standard deviation of the data to the left and right of the buffered glitch on timescales equal to the duration of the buffered glitch. The noise realization is then inserted in place of the original samples in the time stream.

After the replacement of glitches with noise realizations, a final glitch flag is generated. In the final glitch flag each glitch and its surrounding noise realization are flagged as invalid. In addition, a 1 s buffer is added to the left and right of each set of contiguous flags. This is done to avoid ringing when filtering the deglitched time stream due to any mismatch at the interface between the noise realization and the original time stream. In Figures 4.3 and 4.4 the samples flagged by the final glitch flagging are rendered in gray. The extent of the flagging is also indicated in cyan shading for ease of visual identification when viewing the time stream on longer time scales.

4.2.2 Flagging Summary Statistics

To assess the amount of data that is excluded by each flag we generated the flagging statistics shown in Figure 4.6 for all detectors for the entire duration of the flight. The flagging statistics are calculated using the set of all potentially “usable samples” of detector data, defined as all samples for which the bolometers are tuned, there is valid pointing data, and there is no saturation event (indicated by the remove saturation event flag defined above).
CHAPTER 4. MAP MAKING

The horizontal axis labels indicate the individual flags that are applied to the detector time streams at each flagging step from left to right, while the vertical axis indicates the percentage of data that is unflagged (i.e., valid and usable in analysis and map making). Star markers (⋆) indicate the percentage of the total usable data excluded by the individual flags, while dots connected by solid lines indicate the percentage of usable data remaining after the cumulative application of all flags up to and including the individual flag (i.e., the individual flag and all flags to its left). Because the maximum velocity cutoff value differs for the \( I \) and \((Q, U)\) signal bands, two data sets are shown; flagging statistics for the Stokes \( I \) time stream are shown in blue, while statistics for the Stokes \((Q, U)\) time streams are shown in red.

Figure 4.6: Flagging statistics for the Stokes \( I \) (blue) and \( Q \) and \( U \) (red) time streams. Star markers (⋆) indicate the percentage of data that is unflagged after the application of each individual flag. Dots connected by solid lines show the percentage of unflagged data after cumulative application of all flags up to and including the individual flag.
4.3 Refinement of the Data Cleaning Algorithms

Contaminated detector time stream samples include transient spikes ("glitches") caused by cosmic rays, bursts of noise from the readout electronics, and other sources; residual HWP template signal due to incomplete template removal (see Chapter 6); and a variety of other spurious data samples outlined in § 4.2.

Because the detector time streams are noise dominated, we were able to examine them by eye for significant outliers without confusing them with sky signal. This allowed us to inform and refine the data cleaning algorithms described above in § 4.2 without biasing our results. This section describes the time- and map-domain visualization tools we developed for this purpose.

4.3.1 Time Domain

Contaminated data is characterized by large jumps, spikes, and other unexpected, but visually identifiable behavior in the noise dominated $I$, $Q$, and $U$ time streams. To assist with identifying instances of failed or incomplete removal of contaminated data samples, we generated four-panel time stream plots of the kind shown in Figure 4.7.

Each panel in Figure 4.7 displays identical time samples in the detector time stream at a different signal processing stage in the data analysis pipeline. The upper-left (right) panel shows the time stream after template removal but before (after) calibration from readout “counts” to temperature units ($K_{CMB}$). The bottom left (right) panel shows the extracted Stokes $I$ ($Q$ and $U$) signals, with shading to indicate samples within $\pm 3^\circ$ Galactic latitude where the expected signal can be strong. The strong spikes in the $I$, $Q$, and $U$ time streams ($t \sim 12$ s) correspond to a glitch in the template-removed time stream from which they were extracted (top left panel). Although the earliest version of our glitch removal code failed to identify the glitch, its presence is readily apparent in the plot. Reviewing the time streams for each detector allowed us to refine the data flagging procedures outlined above in § 4.2.
Figure 4.7: Example four-panel time-domain plot used in time stream cleaning, shown here for a single 250 GHz detector. Visual identification of large spikes in the processed $I$, $Q$ and $U$ time streams (bottom panels) revealed a less apparent glitch in the template-removed time stream from which they were generated (top left: uncalibrated; top right: close-up view after calibration from counts to $K_{CMB}$). This led to refinement of the deglitching procedure outlined in § 4.2.1. Samples within ±3° Galactic latitude are shaded to avoid confusing strong signals from sources on the Galactic plane with glitches.

4.3.2 Map Domain

To streamline the identification of contaminated time stream samples within the ~1000 detectors across the 11 days of science observations, we generated sky maps of the root-mean-square (rms) signal within each pixel divided by the pixel sensitivity. Identifying pixels with unusually large signal-to-sensitivity ratios allowed us to focus our initial time domain examination on the data samples that mapped to the outlier pixels. This allowed us to identify unflagged contaminated data samples in the time domain, such as cosmic ray glitches, that caused the anomalously large pixel signal-to-sensitivity ratio. Refining the flagging and deglitching algorithms to properly remove
the contaminated time stream samples resulted in eliminating the anomaly in the map
domain. Note that this procedure does not result in removing real sky signal from the
cleaned time streams; rather, it is a method for efficiently identifying contaminated
samples within large sets of time ordered data which, without proper removal, would
significantly bias sky signal maps.

We produced signal-to-sensitivity ratio maps for the $I$, $Q$, and $U$ time streams
within each segment of data, for the combined detectors for each readout board.
Detectors within each board were combined because we found that the maps for
individual detectors had an insufficient number of hits per pixel to provide a useful
measure of signal-to-sensitivity.

The code we wrote to generate signal-to-sensitivity maps first creates a map of the
rms signal in each pixel, $s_{\text{rms}}(p)$, within each segment for the combined detectors on
each readout board. A separate map of the pixel sensitivity $S(p)$ is then generated.
Dividing the two maps within each pixel produces a map of the unitless signal-to-
sensitivity ratio, i.e., $s_{\text{rms}}(p)/S(p)$. This procedure is carried out separately for each
of the three $I$, $Q$, and $U$ time streams.

The rms signal map for $I$, $Q$, or $U$ is simply the root-mean-square of the $I(t)$,
$Q(t)$, or $U(t)$ data samples assigned to each pixel $p$, i.e.,

$$s_{\text{rms}}(p) = \sqrt{\frac{1}{N} \sum_{i} (s_i - \bar{s})^2}$$  \hspace{1cm} \text{(4.9)}

where $\bar{s}$ is the mean signal in pixel $p$ and $N$ is the number of hits. The $I$, $Q$, or
$U$ sensitivity map for the combined detectors is calculated by dividing the combined
noise equivalent temperature (NET) for the detectors in units of $\mu K \sqrt{s}$ by the square
root of the time duration in the pixel $t(p)$. The pixel time duration equals the number
of hits $N$ divided by the sampling frequency $f_s$=191 Hz:

$$S(p) = \frac{\text{NET}_{\text{tot}}}{t(p)} = \frac{\text{NET}_{\text{tot}}}{\sqrt{N/f_s}}$$  \hspace{1cm} \text{(4.10)}

The total NET is obtained by combining the individual detector NETs, which add in
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quadrature:

\[
\frac{1}{\text{NET}_{\text{tot}}^2} = \sum_{\text{bolos}} \frac{1}{\text{NET}_{\text{bolo}}^2}
\]  \hspace{1cm} (4.11)

The \( s_{\text{rms}}(p) \) signal map and the \( S(p) \) sensitivity map are then divided within each pixel.

Figure 4.8: Example rms signal-to-sensitivity map used in initial time stream cleaning, shown here in galactic coordinates for the Stokes I signal for all detectors within a single 150 GHz board during a 5.4 h segment of data.

Figure 4.8 shows an example rms signal-to-sensitivity map for all 150 GHz detectors within a single 5.4 hour segment of data prior to complete time stream cleaning. Identification of contaminated data is facilitated by reviewing time domain plots of data samples that map to pixels with the largest rms signal-to-sensitivity ratios. Identifying the contaminated time stream samples that caused large signal-to-sensitivity ratios at the map level allowed us to refine the automated data flagging procedures outlined above. After the flagging procedures were sufficiently refined to remove anomalies at the signal-to-sensitivity map level, we continued to review each detector time stream in detail using plots of the kind shown in Figure 4.7.
Figure 4.9 shows preliminary Stokes $I$, $Q$, and $U$ maps of the Galactic plane. The Stokes $I$, $Q$, and $U$ signals were extracted from the detector time streams using the techniques described in §4.1.2 and were cleaned using the procedures outlined in §§4.2-4.3. The maps were generated using all flight data for the 250 GHz detectors using Healpix nside=512, and were smoothed to the 15' beam size of the EBEX 250 GHz channel.

Figure 4.9: Preliminary maps of Stokes $I$, $Q$, and $U$ signal on a patch of the Galactic plane (Galactic latitude = $-2^\circ$ to $+2^\circ$ and Galactic longitude = $-100^\circ$ to $-10^\circ$), made using all flight data for the 250 GHz detectors. The maps are generated using Healpix nside=512 and are smoothed to 15'.
Chapter 5

EBEX Model Sky Maps

5.1 Overview

In order to test the performance of the EBEX data analysis pipeline it is useful to generate model maps of the expected sky signal for the three EBEX frequency bands centered on 150, 250, and 410 GHz. Such maps can be used to generate simulated detector time streams to characterize the effects of the various pipeline stages for a known input sky signal. For example, the template removal procedure outlined in Chapter 6 is tested by using reference sky maps for the EBEX frequency bands, together with the EBEX pointing reconstruction and a model template signal, to generate simulated detector time streams for a scan of the reference sky maps. Because the simulated time streams have known sky signal and template contributions, the template removal procedure can be tested by comparing the template removed output to the expected sky signal.

We employed two methods to generate model sky maps for the EBEX frequency bands. The first method generated maps using the Planck Sky Model (PSM)\footnote{http://www.apc.univ-paris7.fr/~delabrou/PSM/psm.html} a pre-launch model of the sky produced by the Planck collaboration. The PSM was designed to simulate the predicted sky emission in the frequency range of typical CMB...
CHAPTER 5. EBEX MODEL SKY MAPS

experiments. It generates full sky maps for a user-specified frequency, band transmission, and beam size using a parametric model of various component sky sources (e.g., CMB, free-free emission, and thermal dust emission). Each component is modeled using interpolations and extrapolations of data sets from multiple experiments that were available at the time the Planck mission launched [15]. No Planck observation data are used to model the sky emission. The CMB component is a random realization from measured $C_\ell$ values, and thus does not match observed CMB anisotropies at every point on the sky. Other members of our collaboration produced PSM-generated temperature and polarization maps for the EBEX frequency bands. We refer to these maps as EBEX Sky Model (ESM) maps.

The second method produced model sky maps using component-separated maps generated from Planck mission data. The Planck collaboration’s component separation analysis models the observed sky signal as the sum of contributions from distinct physical components, each of which scales with frequency by a component-specific power law [57]. I was responsible for producing model EBEX temperature sky maps by scaling each Planck component map from its reference frequency to the EBEX frequency band, integrating the scaled component map over the EBEX band transmission, and then smoothing the map to the effective EBEX beam size. The frequency-scaling procedure is detailed below in §5.2. It is possible to generate frequency-scaled $Q$ and $U$ polarization maps by scaling and coadding the three dominant sources of polarization signal — thermal dust emission, synchrotron emission, and the CMB — using analogous scaling models described in [55]. Frequency-scaled polarization maps are not reviewed here, as we generally used ESM polarization reference maps when generating simulated EBEX detector $Q$ and $U$ time streams.

To characterize the PSM’s performance, we compared the temperature map observed by Planck at 143 GHz to a PSM-generated temperature map at the same frequency. We found that on the Galactic plane, the PSM-generated map was slightly brighter than the Planck observation map, with the two differing in some locations
by tens of mK (see Figure 5.1). This is not unexpected, as the Planck component separation model is tailored to modeling the sky signal in CMB-dominated regions off the Galactic plane, and is fitted with masks that exclude Galactic regions [57, 59].

5.2 Component Frequency Scaling Models

The Planck Collaboration employs a Bayesian component separation algorithm described in [57] to model the sky emission \( s(p) \) in each map pixel \( p \) as the sum of several contributing components:

\[
s(p) = s_{\text{cmb}}(p) + s_{\text{lf}}(p) + s_d(p) \tag{5.1}
\]

where the subscripts \( \text{cmb} \), \( \text{lf} \), and \( d \) denote respective contributions from the CMB, combined “low-frequency” sources (free-free, synchrotron, and spinning dust emission), and thermal dust emission. The Planck component separation model includes an additional term for line emission from carbon monoxide which was not implemented in the frequency-scaled model EBEX sky maps.

The contributions \( s_i(p) \) have pixel-dependent signal amplitudes \( A_i(p) \). The low-frequency and dust components scale with frequency \( \nu \) according component-specific power laws with a pixel-dependent spectral indices \( \beta_{\text{lf}}(p) \) and \( \beta_d(p) \). The Planck component separation model provides reference maps of the amplitudes \( A_i(p) \), spectral indices \( \beta_i(p) \), and thermal dust temperature \( T_d(p) \) as fitted by the component separation algorithm. The reference maps are scaled from their respective reference frequencies \( \nu_i \) as described below.

5.2.1 The Thermal Dust Contribution

The Planck dust component amplitude reference map \( A_d(p) \) is provided in K\textsubscript{cmb} units. Following the LEAP standard of performing computations in SI units, the map-scaling
Figure 5.1: **Left:** the Planck Sky Model (PSM) temperature map at 143 GHz for a 5° × 5° patch on the Galactic plane. The PSM map was generated by fitting observational data from multiple experiments that was available prior to launch of the Planck mission. **Middle:** the Planck observed 143 GHz temperature map for the same sky region. **Right:** the difference between the two, with a rescaled color map for visibility.
code first converts the map from $K_{\text{cmb}}$ to intensity units of $\frac{W}{\text{sr m}^2 \text{Hz}}$ by multiplying by the temperature derivative of the Planck black body spectral radiance $B_{\nu}(\nu, T)$ evaluated at the CMB temperature $T_{\text{cmb}}$:

$$
\frac{d}{dT} B_{\nu}(\nu, T) \bigg|_{T_{\text{cmb}}} = \frac{d}{dT} \left( \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \right) \bigg|_{T_{\text{cmb}}} = \frac{2h^2\nu^4}{c^2 kT_{\text{cmb}}} \left( \frac{e^{h\nu/kT_{\text{cmb}}}}{e^{h\nu/kT_{\text{cmb}}} - 1} \right)^2
$$

(5.2)

In SI units the amplitude $A_d$ scales in frequency as a modified black body with spectral index $\beta_d$:

$$
A_d(\nu) \propto \nu^{\beta_d} B_{\nu}(\nu, T_d) = \nu^{\beta_d} \left( \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT_d} - 1} \right) \propto \nu^{\beta_d + 3} \left( \frac{e^{h\nu/kT_d}}{e^{h\nu/kT_{\text{cmb}}} - 1} \right)^2
$$

(5.3)

To scale the dust reference map $A_d(p)$ from the dust reference frequency $\nu_{0,d} = 353$ GHz to a desired frequency $\nu$, we obtain a scaling factor by taking the ratio of equation (5.3) evaluated at the frequencies $\nu$ and $\nu_{0,d}$:

$$
A_d(\nu) = A_d \left( \frac{e^{h\nu_{0,d}/kT_d}}{e^{h\nu/kT_d} - 1} \right) \left( \frac{\nu}{\nu_{0,d}} \right)^{\beta_d + 3}
$$

(5.4)

where $T_d$ is the pixel-dependent dust temperature fitted by the component separation model.

To determine the sky contribution as observed by the EBEX instrument, the intensity is converted to $K_{\text{cmb}}$ units by dividing by equation (5.2) and integrated numerically over the measured EBEX transmission $\tau(\nu)$:

$$
\int d\nu \, \tau(\nu) \left\{ A_d \frac{dB_{\nu}(\nu, T)}{dT} \bigg|_{\nu, T_{\text{cmb}}} \right. \left( \frac{e^{h\nu_{0,d}/kT_d}}{e^{h\nu/kT_d} - 1} \right) \left( \frac{\nu}{\nu_{0,d}} \right)^{\beta_d + 3} \left. \cdot \frac{dT}{dB_{\nu}(\nu, T)} \bigg|_{\nu, T_{\text{cmb}}} \right\}
$$

(5.5)
5.2.2 The Low-Frequency Contribution

The Planck low-frequency amplitude reference map $A_{lf}(p)$ is provided in Rayleigh-Jeans temperature units $K_{RJ}$ and scales by the spectral index $\beta_{lf}$. Upon converting to SI units via the temperature derivative of the Rayleigh-Jeans spectrum,

$$\frac{d}{dT} B_{\nu, RJ} = \frac{d}{dT} \left(2kT \left(\frac{\nu}{c}\right)^2\right) = 2k \left(\frac{\nu}{c}\right)^2 \quad (5.6)$$

the $A_{lf}$ reference map scales as the ratio of equation 5.6 evaluated at $\nu$ and $\nu_{0,lf} = 30$ GHz:

$$A_{lf}(\nu) = A_{lf} \left(\frac{\nu}{\nu_{0,lf}}\right)^{\beta_{lf}+2} \quad (5.7)$$

When converted to $K_{cmb}$ units and integrated over the EBEX transmission $\tau(\nu)$, the low-frequency contribution becomes:

$$s_{lf} = \int d\nu \tau(\nu) A_{lf} \left|\frac{d B_{\nu, RJ}(\nu, T)}{dT}\right|^{\beta_{lf}+2}_{\nu_{0,lf} T_{cmb}} \left|\frac{d T}{dB_{\nu}(\nu, T)}\right|^{\beta_{lf}+2}_{\nu T_{cmb}} \quad (5.8)$$

5.2.3 The CMB Contribution

The CMB reference map is provided in $\mu K_{cmb}$ units. Because the CMB is a black body, its temperature in thermodynamic units is independent of frequency. The CMB reference map is therefore added directly to the final output map without any scaling.

5.3 Results and Comparison to ESM Maps

Figure 5.2 shows the thermal dust, low-frequency, and CMB component temperature maps (upper left, upper right, and lower left, respectively) scaled to the 150 GHz

\footnote{In the Planck 2013 release \[57\], the contributions of the free-free, synchrotron, and spinning dust components are modeled as a single “low-frequency” contribution. In the 2015 release \[55\], the three components are modeled separately; each has a unique reference amplitude map and spectral index map that scales in frequency according to the same model outlined here.}
EBEX band for the same region of the Galactic plane shown in Figure 5.1 (center) for the Planck 143 GHz band. The maps are converted to a common Healpix nside resolution and co-added to produce the final output map, which is then smoothed to the 8′ EBEX beam size (lower right).

A comparison of the 150 GHz frequency-scaled temperature map in Figure 5.2 to the 150 GHz ESM map is shown in Figure 5.3. As in the comparison between the PSM and Planck 143 GHz maps (Figure 5.1), the ESM 150 GHz map is brighter along the Galactic plane than the corresponding map made from frequency-scaled component maps. We were careful to note this difference when generating simulated time streams from ESM and frequency-scaled component maps, and avoided combining simulations generated using the two methods.

Figure 5.4 shows a full-sky comparison of the 150 GHz ESM and frequency-scaled temperature maps. The strong temperature difference along the Galactic plane observed in the previous plots is clearly visible, with higher ESM signal toward the center of the Galaxy and lower ESM signal toward the edges. As noted above, this discrepancy occurs because the component separation model is tailored to modeling sky regions dominated by CMB signal, and is fitted with masks that exclude Galactic regions that are dominated by thermal dust emission. The temperature difference in CMB-dominated regions is on the order of the observed signal because the PSM-generated CMB component is a random realization of the CMB from measured $C_\ell$ values.

Corresponding maps for the 250 and 410 GHz frequency bands are shown in Figures 5.5 and 5.6. The difference between the ESM and component-constructed maps is most apparent in the 410 GHz maps, in which the thermal dust emission is strongest.
Figure 5.2: Component-separated temperature maps for thermal dust emission (top left), low-frequency emission (top right), and the CMB (bottom left) scaled from Planck component map reference frequencies to the EBEX 150 GHz band, for the same sky region in Figure 5.1. Each component map is displayed with a different color scale for visibility. The sum of the component maps is shown at bottom right.
Figure 5.3: \textit{Left:} the ESM temperature map at 150 GHz for the region of the Galactic plane shown in Figures 5.1 and 5.2. \textit{Middle:} the sum of the Planck component separated maps scaled to 150 GHz. \textit{Right:} the difference between the two, with a re-scaled color map for visibility.
Figure 5.4: Top: a full-sky map of the ESM temperature signal at 150 GHz, shown in Galactic coordinates. Middle: the sum of the Planck component separated maps scaled to 150 GHz. Bottom: the difference between the two. Because the ESM generates a random CMB realization from $C_\ell$ values, the temperature difference is on the order of the observed CMB signal in CMB-dominated regions.
Figure 5.5: Full-sky EMS (top) and component-constructed (middle) temperature maps at 250 GHz, and the difference between the two (bottom).
Figure 5.6: Full-sky EMS (*top*) and component-constructed (*middle*) temperature maps at 410 GHz, and the difference between the two (*bottom*).
EBEX’s use of a continuously rotating achromatic half-wave plate (HWP) and a stationary wire grid to achieve polarimetry is described in §2.5. Recall that the HWP rotating at the frequency $f_{hwp} \sim 1.23$ Hz causes the polarization of incident linearly polarized light to exit the HWP rotating at $2f_{hwp}$; the stationary wire grid polarizer transmits one (vertical or horizontal) polarization state to each of the two EBEX focal planes; and the power dissipated in the detectors oscillates at $4f_{hwp}$ (see Figure 2.7).

This chapter describes removal of the HWP template from the detector time streams. The HWP template consists of HWP-synchronous signal originating from three principal sources: polarized emission from the telescope mirrors; instrumental polarization, i.e., polarization of unpolarized incident light by the instrument, which was larger than anticipated before flight\(^1\); and thermal emission from the HWP itself.

\(^1\)The measured leakage from Stokes $I$ to $Q$ and $U$ due to instrumental polarization was 8.5$\%$ for 150 GHz and 18$\%$ for 250 GHz. Chapter 7 describes the use of the HWP template to attempt to characterize and remove instrumental polarization due to suspected non-linear detector responsivity. For a general discussion of the analysis and removal of instrumental polarization, see [17].
Template removal consists of fitting the HWP template observed by each detector in
the time domain and subtracting it from the detector time stream. Note that mod-
ulation by the rotating HWP allows polarized sky signals to be distinguished from
polarized emission from the instrument. As described in § 2.5, the scanning motion
of the telescope moves the polarized sky signal into the sidebands surrounding \( 4 f_{hwp} \),
while polarized emission from the instrument on the detector (sky) side of the HWP
appears at 0 Hz (\( 4 f_{hwp} \)).

Figure 6.1 shows plots of the calibrated time stream for 4 s of data from a single
detector as a function of time (top panel) and of HWP angle (bottom panel). The
HWP template has an amplitude of \( \sim 3 \) K and is two to three orders of magni-
tude larger than the detector noise and the incident sky signal. The HWP template
is dominated by the 4th harmonic of \( f_{hwp} \); the period for one major oscillation is
\( 1/4 f_{hwp} \approx 0.2 \) s, as can be seen in the top panel plot. The plot in the bottom
panel shows that the HWP template is synchronous with the HWP rotation; each
sinusoidal cycle corresponds to a 90° quarter-rotation of the HWP.

### 6.1 The HWP Template Model

Following Johnson, et al. [32], we model the HWP template time stream \( h(t) \) as the
sum of \( N \) harmonics of the HWP-synchronous signal:

\[
h(t) = \sum_{n=1}^{N} (C_{1,n} + C_{2,n}t) \cos(n\theta(t)) + (S_{1,n} + S_{2,n}t) \sin(n\theta(t)) \tag{6.1}
\]

where the \( S \) and \( C \) symbols are constant parameters to be fitted and \( \theta(t) \) is the angle
of the HWP’s optical axis with respect to the wire grid polarizer (see Figure 2.7). For an ideal HWP spinning at a perfectly constant frequency, \( \theta(t) = 2\pi f_{hwp}t \). In
practice the HWP rotational frequency undergoes small perturbations about \( f_{hwp} \)
and the angle \( \theta(t) \) is measured by the HWP encoder readout. As described below in
§ 6.2.3, we found that template removal for the EBEX detector time streams requires
fitting and removing the first \( N = 20 \) harmonics.
Figure 6.1: Plots of the calibrated time stream for 4 s of data from a single detector, versus time (top panel) and versus HWP angle (bottom panel). The HWP template is synchronous with the rotation of the HWP, and is dominated by the 4th harmonic of the HWP rotational frequency ($4f_{\text{hwp}} = 4.92 \text{ Hz}$). The large HWP template amplitude ($\sim 3 \text{ K}$) dominates the detector time stream.
Because the detector noise and the incident sky signal are several orders of magnitude smaller than the HWP template, we neglect these contributions when fitting the HWP template. Our approach uses a maximum likelihood method to fit for $\vec{h}(t)$ as a function of the HWP angle $\theta(t)$. In matrix form equation [6.1] may be expressed as

$$\vec{h}(t) = \Omega_{tn} \mathcal{C}_n$$

(6.2)

where $\mathcal{C}_n$ encodes the $4N$ parameters for the stationary sine and cosine amplitudes $\{C_{1,n}, S_{1,n}\}$ and for the non-stationary amplitudes $\{C_{2,n}, S_{2,n}\}$,

$$\vec{h}(t) = \begin{pmatrix} h(t_0) \\ h(t_1) \\ h(t_2) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \quad \mathcal{C}_n = \begin{pmatrix} C_{1,1} \\ C_{2,1} \\ S_{1,1} \\ S_{2,1} \\ C_{1,2} \\ C_{2,2} \\ S_{1,2} \\ S_{2,2} \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

(6.3)

and the HWP angle sine and cosine terms are encoded in the matrix $\Omega_{tn}$:

$$\Omega_{tn} = \begin{pmatrix} \cos(\theta(t_0)) & \sin(\theta(t_0)) & t_0 \cos(\theta(t_0)) & t_0 \sin(\theta(t_0)) & \cos(2\theta(t_0)) & \sin(2\theta(t_0)) & t_0 \cos(2\theta(t_0)) & t_0 \sin(2\theta(t_0)) & \cdots \\ \cos(\theta(t_1)) & \sin(\theta(t_1)) & t_1 \cos(\theta(t_1)) & t_1 \sin(\theta(t_1)) & \cos(2\theta(t_1)) & \sin(2\theta(t_1)) & t_1 \cos(2\theta(t_1)) & t_1 \sin(2\theta(t_1)) & \cdots \\ \cos(\theta(t_2)) & \sin(\theta(t_2)) & t_2 \cos(\theta(t_2)) & t_2 \sin(\theta(t_2)) & \cos(2\theta(t_2)) & \sin(2\theta(t_2)) & t_2 \cos(2\theta(t_2)) & t_2 \sin(2\theta(t_2)) & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \end{pmatrix}$$

(6.4)

The maximum likelihood estimator for $\mathcal{C}_n$ is:

$$\hat{\mathcal{C}}_n = (\Omega_{tn}^T \Omega_{tn})^{-1} \Omega_{tn}^T \vec{h}(t)$$

(6.5)
To remove the HWP template for a given detector, we first obtain a processed detector time stream $h(t)$ from its raw time stream $s(t)$ using the procedure described below in §6.2. We use the processed time stream and the measured HWP angles $\theta(t)$ to calculate the estimated parameters $\hat{C}_n$ via equation 6.5. We construct the template using equation 6.1 and the estimated parameters $\hat{C}_n$. Finally, we subtract the template from the raw, unprocessed detector time stream $s(t)$ to yield a “template removed” time stream for the detector.

6.2 The HWP Template Removal Algorithm

The HWP template removal procedure begins by pre-processing the raw time stream $s(t)$ for a given detector to obtain the time sample vector $\vec{h}(t)$, which is then used to obtain the HWP template coefficients via equation 6.5. The pre-processing comprises the following steps:

1. **Chunking:** To minimize processing time the raw detector time stream is first divided into 60 minute sections. Within each section the data is divided into “chunks” of continuous valid data. For this purpose data samples are defined as invalid where there is an undefined HWP angle or detector time stream value, or where data has been flagged for bolometer latching, bolometer tuning, elevation steps, or stimulator flashes (see §4.2). The resulting chunks range from several minutes to 1 h in duration. Each chunk is processed individually according to the remaining steps.

2. **Baseline subtraction:** The slowly-varying drift of the detector time stream, which we refer to as the signal baseline, is calculated using the procedure outlined below in §6.2.1. The baseline is then subtracted from the raw detector time stream.

3. **Glitch masking:** A Boolean array is generated to mask time stream samples that
are identified as glitches during the baseline subtraction procedure. (Glitches, which are thought to be caused by cosmic rays and electrical burst noise, are discussed in detail in § 4.2.1.) The glitch mask is used as described below in bandpass filtering and template fitting.

4. **Bandpass filtering:** To remove any residual baseline and high frequency noise while retaining signal in the sky signal bandwidths (see § 4.1.2), the baseline subtracted time stream is bandpass filtered using order-8 low- and high-pass Butterworth filters with respective frequency cutoffs of 0.01 and 40 Hz. Prior to bandpass filtering each glitch masked sample is replaced by a linear interpolation of its neighboring unmasked samples. This is done to avoid filter ringing that would result from Fourier transforming a time stream containing a large discontinuity.

5. **Template fitting:** The filtered time stream is divided into 60 s slices. The template coefficients in equation 6.5 are fitted for each slice using the maximum likelihood method described above in § 6.1. This process is described in further detail in § 6.2.2 below. The 60 s duration was determined empirically; we found that the fitted template coefficients vary over longer time scales.

6. **Template subtraction:** The fitted template coefficients are used to calculate the HWP template via equation 6.1. The template is subtracted from the raw detector time stream, yielding the “template removed” time stream.

I was closely involved in revising the template removal algorithm and writing the software that implemented it. Detailed descriptions of the baseline subtraction and template fitting procedures are presented below. The performance of the revised template removal algorithm is evaluated in § 6.3.

## 6.2.1 Calculating the Signal Baseline

The signal baseline for a chunk of raw detector time stream is obtained follows:
• An initial estimate of the baseline is computed using an order-10 Butterworth\(^2\) low-pass filter with a cutoff frequency of 0.2 Hz. This cutoff frequency is chosen to match the knee frequency of the measured detector noise (see Table 4.1.2). The initial baseline is then subtracted from the raw time stream.

• To enable the identification of glitches—which are often smaller in amplitude than the template itself—the time stream undergoes a “temporary template removal”: a temporary template is computed using the maximum likelihood method described above in §6.1 and is subtracted from the time stream.

• A glitch mask is generated by passing the temporary template-subtracted time stream to the deglitching algorithm described in §4.2.1. The temporary template is then discarded and is not used in any further analysis or time stream processing.

• The final baseline is calculated by computing the moving mean of the template-subtracted time stream. The window size of the moving mean filter is set to 4.87 s, corresponding to six complete rotations of the HWP.

Figure 6.2 shows the calculated signal baseline (top panel, red) for an example detector. Prior to baseline subtraction the raw time stream (top panel, blue) follows a slowly-varying downward drift. After baseline subtraction (bottom panel) the time stream is centered on zero.

### 6.2.2 Template Fitting

After a detector time stream chunk has undergone baseline subtraction and bandpass filtering, the chunk is divided into 60 s slices for template fitting and removal. As

\(^2\)The order of the Butterworth filter was selected empirically by observing the residual template in simulated detector time streams.
Baseline subtraction removes the long-term drift in the time stream such that it is centered on zero. Right: the HWP template, which has not yet been removed, is visible when the same data is plotted on smaller time scales.

mentioned above, glitches in the detector time stream are replaced by linear interpolations to enable bandpass filtering during pre-processing. When fitting the template, the glitch mask generated during the signal baseline calculation is used to mask all samples corresponding to glitches. A buffer of the neighboring 25 samples (~0.13 s) before and after each glitch sample is also masked. This buffer is included to avoid any filter ringing effects caused by replacing the glitch with a linear interpolation between neighboring valid data samples.

The template coefficients in equation 6.5 are then fitted for each time stream slice using the maximum likelihood method described above in § 6.1. Note that the HWP template model in equation 6.1 and the maximum likelihood estimator for its parameters in equation 6.5 assume that the DC offset of the template is exactly zero. Previous versions of the template removal software attempted to ensure that the preprocessed signal was centered on zero by subtracting the mean of the signal. We
found that this yielded poor results because the signal mean generally does not equal the DC level; for example, a time stream slice that contains a template with a DC offset of zero, but spans a non-integer number of HWP rotations will have a non-zero mean (see Figure 6.3).

To address this problem we extended the template model in equation 6.1 by adding a constant DC offset parameter that is fitted using the maximum likelihood method in equation 6.5. This requires extending the parameter vector $C_n$ to include the additional DC parameter, and adding a final column of 1s to the matrix $\Omega_{tn}$:

$$C_n = (C_{1,1}, \ C_{2,1}, \ S_{1,1}, \ S_{2,1}, \ldots \ \text{DC})^T$$  \hspace{1cm} (6.6)

$$\Omega_{tn} = \begin{pmatrix}
\cos\theta(t_0) & \sin\theta(t_0) & t_0\cos\theta(t_0) & t_0\sin\theta(t_0) & \cos2\theta(t_0) & \sin2\theta(t_0) & t_0\cos2\theta(t_0) & t_0\sin2\theta(t_0) & \ldots & 1 \\
\cos\theta(t_1) & \sin\theta(t_1) & t_1\cos\theta(t_1) & t_1\sin\theta(t_1) & \cos2\theta(t_1) & \sin2\theta(t_1) & t_1\cos2\theta(t_1) & t_1\sin2\theta(t_1) & \ldots & 1 \\
\cos\theta(t_2) & \sin\theta(t_2) & t_2\cos\theta(t_2) & t_2\sin\theta(t_2) & \cos2\theta(t_2) & \sin2\theta(t_2) & t_2\cos2\theta(t_2) & t_2\sin2\theta(t_2) & \ldots & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ldots & 1
\end{pmatrix}$$  \hspace{1cm} (6.7)

The parameters, including the DC offset, are then fitted in the usual way using equation 6.5. We tested the performance of the template fitting code by fitting and removing a known input signal, consisting of 60 s of pure template constructed from template coefficients fitted for an example detector, with the DC offset manually set to zero. In this instance successful template fitting should yield a template removed signal of zero. Figure 6.4 shows the template removed signal (red) for the pure template input signal (blue). Attempting to remove the DC offset by subtracting the signal mean results in a residual with amplitudes on the order of $10 \mu$K (left plots) and an overall offset of $\sim -0.3 \text{ mK}$. When the DC offset is fitted instead as an additional factor, the results are significantly improved.

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3We also examined the alternative solution of fitting the template on data slices containing an integer number of HWP rotations and subtracting the mean of the signal. Template removal tests with simulated time streams showed that this yielded poorer results than fitting a constant DC offset parameter, as the discrete sampling of HWP angle values results in data slices that virtually never contain precisely an integer number of rotations.
Figure 6.3: Example illustrating that the mean of the HWP template generally does not equal the DC offset. The solid blue curve shows the reconstructed template for an example detector in calibrated units ($K_{CMB}$). Although the DC offset of the template is zero (red), the mean (dashed blue) is positive due to the presence of samples to the right of the full HWP rotation (vertical black line).

Parameter in equation 6.5 (right plots), the residual amplitude is effectively zero to within computational precision ($\sim 1e-13$ K) and is centered correctly on zero.

### 6.2.3 Fitting Higher Harmonics

Previous versions of the template removal code fitted and removed $N = 10$ template harmonics. This number was chosen in part on the basis of MAXIPOL’s experience with HWP polarimetry; their collaboration found that fitting $N = 8$ harmonics yielded sufficiently robust template removal, although their template amplitudes ($\sim 1$ to 600 mK) were much smaller than in EBEX [32]. It was also thought initially that any signal residing in higher harmonics would be unimportant to analysis given that they are excluded by frequency filtering at various stages in the analysis pipeline (see § 4.1).
Figure 6.4: Plots of the residual signal (red) after fitting and removing a test input signal (blue) consisting of pure $\sim 4$ K amplitude template constructed from the template coefficients for an example detector time stream; the DC level is set manually to zero, such that the ideal residual signal is zero. *Left:* when the DC offset is estimated as the mean of the input signal, the residual has an amplitude of $\sim 10 \mu$K and is offset by $\sim -0.3$ mK. *Right:* when the DC offset is fitted instead as an additional parameter in the maximum likelihood template fitting method, the residual amplitude is effectively zero to within computational precision ($\sim 1e-13$ K) and is centered on zero.

Subsequent investigation showed that the phases of the higher harmonics align such that they interfere to produce periodic structure at lower frequencies; for example, the comb of spikes in the template removed time stream in Figure 6.5 (*top panels*) occurs with a period of $1/f_{hwp} = 0.8$ s. These features are removed by fitting $N = 20$ template harmonics or higher (*bottom panels*).

The presence of higher harmonics in the detector time streams is problematic for at least three reasons. First, if the residual template time stream features in Figure 6.5 are not removed, they can cause the deglitching algorithm (see § 4.2.1) to misidentify and flag extended portions of the time stream as transient glitches.
Second, the presence of large amplitude residual template features can mask the presence of glitches, preventing the deglitching algorithm from properly flagging them. Finally, it is possible that the signal residing in the higher harmonics is caused by operation of the detectors in a non-linear regime; if so, the non-linearity must be characterized and corrected (see Chapter 7).

The revised template removal code fits and removes $N = 20$ harmonics. Currently, harmonics above $N = 20$ are not removed for several reasons: the higher harmonics are filtered out during later steps in the data analysis pipeline; removing up to 20 harmonics eliminates the periodic time stream spikes that otherwise cause the deglitching code to malfunction; and the template removal code’s processing time increases as additional harmonics are fitted.

### 6.3 Comparison of the Previous and Revised HWP Template Removal Methods

At the time that I joined the EBEX data analysis team, the EBEX collaboration employed a template removal procedure and accompanying code that other collaboration members had adapted from Johnson, et al. [32]. I was closely involved in improving upon the existing template removal procedure and revising the template removal code. The revised template removal procedure differed in the following ways from the previous version:

- The baseline subtraction method outlined in §[6.2.1] replaced an earlier version.

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4In addition to the changes described here, the revised template removal procedure used an updated HWP angle reconstruction generated by other members of the EBEX collaboration. In the updated reconstruction a small number of non-idealities in the time ordered angle values were identified and removed (e.g., misreading of the angle over a single HWP rotation due to a skipped sample in the angle encoder readout). These non-idealities were infrequent, but caused large residuals in the template removed time streams where they occurred.
Figure 6.5: The template removed time stream (left) and time stream PSDs for an example detector before (right, blue) and after (right, red) fitting and removing $N = 10$ template harmonics (top plots) versus $N = 20$ harmonics (bottom plots). Peaks in the PSDs occur at integer multiples of $f_{\text{hwp}} = 1.23$ Hz. The bottom panels in the PSD plots depict the PSD in the vicinity of $4f_{\text{hwp}}$. The periodic spikes in the top left time stream occur at intervals of $1/f_{\text{hwp}} \sim 0.8$ s, and are removed by fitting $N = 20$ harmonics.
that calculated a baseline by performing piece-wise polynomial fits on 1 s slices of data, which were then smoothed by a moving mean filter with a 5 s window. The previous method had not been tested thoroughly and was computationally intensive.

- Glitch masking was achieved using the revised deglitching procedure outlined in §4.2.1, which proved to be more robust at identifying glitches than the previous procedure.

- The DC offset was properly fitted for each 60 s time stream slice, as outlined in §6.2.2. Previously the DC offset was estimated as the mean value of the 60 s slice.

- The number of template harmonics fitted, $N$, was increased from 10 to 20.

I was responsible for generating the revised template removed bolometer time streams for all detectors for the duration of the flight, and for reviewing and comparing them to the previous template removed time streams. We found that the revised template removed time streams displayed similar overall structures to the previous time streams, but were improved in two ways. First, the removal of periodic spikes residing in the higher template harmonics eliminated the problem of the deglitching algorithm improperly flagging the detector time streams. Second, glitches that were previously hidden within the residual template were revealed, allowing the deglitching algorithm to properly identify and flag them. Figure 6.6, for example, shows a comparison of the previous and revised template removal for an example detector on long (top) and short (bottom) time scales. A series of electrical burst noise glitches are apparent in the revised version. In the previous version they were hidden within residual template, preventing the deglitching algorithm from flagging them (bottom, far right).

Despite the improvements we implemented in the template removal procedure, we found that in a significant portion of the data the PSDs of the template removed time
Figure 6.6: Comparisons of the revised (new) and previous (old) template removal procedures applied to an example detector, shown for long (*top plot*) and short (*bottom plot*) time scales. From top to bottom, the four panels in each plot depict the revised template removed time stream; the previous template removed time stream; the difference (“residual”) between the two; and the change in HWP angle between time samples, which is useful for identifying errors in the HWP angle reconstruction.
Figure 6.7: PSDs for an example detector with significant residual template. Shown are PSDs of the time stream before (blue) and after (red) template removal, with a noticeable residual in the vicinity of $4f_{\text{hwp}}$ (bottom panel, red).

streams show residual template at $4f_{\text{hwp}}$ (see Figure 6.7). In §7.2.3 we show that the template amplitude is correlated with the angular distance between the telescope boresight and the Sun. We also show that the loss of azimuth control early in flight (see §2.6) caused the instrument to operate periodically well beyond the specified 60° minimum angular distance from the Sun for which the baffling was designed. This suggests solar radiation as a primary candidate explanation for the observed residual template.
Chapter 7

Non-Linear Detector Responsivity

In Chapter 6 we saw that the PSDs of the detector time streams contain power in a multitude of higher harmonics of the HWP frequency $f_{\text{hwp}} = 1.23$ Hz. We also saw that residual template features with large amplitudes remain in the template removed time streams unless all harmonics of $f_{\text{hwp}}$ up to at least $N = 20$ are fitted and removed (see Figure 6.5).

This chapter examines the possibility that the presence of higher harmonics of $f_{\text{hwp}}$ in the detector time streams results from non-linear detector responsivity caused by the operation of the bolometers in a non-linear detector regime. For brevity we refer to non-linear detector responsivity simply as “non-linearity.” Section 7.1 presents the detector response model that was used to investigate, characterize, and correct for non-linearity. The evidence for the presence of non-linearity is reviewed in §7.2. Section 7.3 describes an approach we developed to characterize and correct for non-linearity. The results of the non-linearity correction method as applied to EBEX detector time streams are presented in §7.4.
7.1 The Non-Linearity Model

In a bolometer operating in the optimal regime, the relationship between the incident power and the response of the bolometer is linear; when properly calibrated from readout system “counts” to $K_{CMB}$ temperature units, equal increments of incident power result in equal increments in the power registered by the detector, regardless of the total incident power. In a bolometer that has become exposed to an excessively large incident signal, however, the relationship is no longer linear. A bolometer operating in a non-linear detector regime becomes less responsive to increases in incident power and registers only a fraction of the incident signal. For brevity we refer to the signal registered by the non-linear response of the detector as the “detected signal,” in contrast to the “incident signal” falling on the detector.

This relationship is illustrated for the simulated data plotted in Figure 7.1. Here the detected signal $s_{nl}$ is modeled as a 3rd-order polynomial function $f_{nl}$ of the incident signal $s$:

$$s_{nl} = f_{nl}(s) = C_0 + C_1 s + C_2 s^2 + C_3 s^3 \quad (7.1)$$

where the parameters $C_i$ are constant coefficients with values that ensure that $s_{nl} \approx s$ for the lowest values of $s$. In this plot the constants $C_i$ have been chosen to match those estimated for an example detector using the procedure described below in § 7.3.

At low incident power ($s \sim -4 \, K_{CMB}$) the detector is in its “linear regime,” where the incident and detected signals are nearly equal. The deviation of the detected signal from the incident signal, which we refer to as “compression” of the incident signal, increases with increasing incident power.

In the non-linearity model it is assumed that the incident signal $s$ is composed solely of the dominant physically motivated harmonics of the HWP frequency $f_{hwp}$, with other harmonics resulting from signal compression at large incident power due to non-linear responsivity. Physically motivated harmonics include 40:

- Rotation-synchronous signals at $1 f_{hwp}$. 


Figure 7.1: An example detector response curve in which the detected power is modeled as a third-order polynomial function of the incident power. In this example the polynomial coefficients have been selected to match those estimated for an example EBEX bolometer (see §7.3).

- Contributions at $2f_{hwp}$ arising from two possible sources: differential transmission of incident radiation through the HWP due to the different loss tangents along the HWP’s ordinary and extraordinary axes; and polarized thermal emission from the HWP resulting from differential emissivity along the two axes.

- Contributions at $4f_{hwp}$. These can arise from modulation by the HWP of scan-synchronous incident polarized sky signal. In addition, spurious contributions at $4f_{hwp}$ can arise from modulation of the $2f_{hwp}$ harmonic due to non-uniformity in the HWP’s anti-reflective coating, or due to a small misalignment of the HWP axis from the angle encoder axis.

Figure 7.2 illustrates the generation of higher harmonics in the frequency domain (right) due to compression (i.e., under-detection) of the incident signal in the time domain (left). Shown in blue is a simulated incident signal composed of harmonics 1, 2, and 4 of the HWP rotation frequency, with the amplitude of each harmonic taken...
from those measured for an example EBEX bolometer. The PSD of the incident signal contains power only at these three harmonics. The detected signal (red) is the result of applying the non-linear responsivity model from Figure 7.1 to the incident signal. The detected signal matches the incident signal at low incident power (\(\sim -4\, K_{CMB}\)), but is compressed at high incident power. The PSD for the detected signal shows that it contains power at each of the harmonics of \(f_{hwp}\).

Figure 7.2: Simulated time stream (left) and PSD (right) for an input signal containing only HWP harmonics 1, 2, and 4 (blue), with template amplitudes matching those fitted for an example EBEX detector; and the detected signal (red) that results from applying the non-linear response function shown in Figure 7.1 to the input signal. The PSD of the detected signal contains power at all HWP harmonics. Note that the time streams are plotted in \(K_{CMB}\) units, with a signal minimum corresponding to 4 K below the combined signals from the CMB and atmospheric loading.

The unexpected presence of higher harmonics of \(f_{hwp}\) in the PSDs of the EBEX bolometer time streams (see Figure 6.5) is consistent with non-linear detector responsivity\(^1\). This motivated several lines of investigation into the possible presence of non-linearity, the results of which are summarized in \(\S\) 7.2.

\(^1\)This effect was also observed recently in the POLARBEAR experiment \([64]\). Like EBEX, POLARBEAR used a continuously rotating HWP to achieve polarimetry.
7.2 The Evidence for Non-Linearity

7.2.1 Comparing Galactic Temperature Maps at High vs. Low Template

The fact that the HWP template amplitude is large and dominates the detector time streams may be exploited to test for non-linearity. As shown in Figure 7.2 left, a detector with non-linear responsivity under-detects incident signal, with the strongest effects where the template signal is at its peak value. In the example shown, the detected signal is $\sim 2 \, \text{K}_\text{CMB}$ weaker than the incident signal where the template signal is maximal (e.g., at $t \sim 0.6 \, \text{s}$). Like the HWP template itself, incident sky signal that coincides with peaks in the HWP template will also be compressed maximally by the non-linear detector response. The sky signal recovered after template removal therefore will be weaker than the incident sky signal where the template signal is at its peak value. Where the template signal is lowest, however, the incident and detected signals coincide (e.g., at $t \sim 0.2 \, \text{s}$); the effect of non-linearity is minimal, and the detected sky signal will roughly equal the incident sky signal.

To test for the presence of non-linearity, we generated and compared two temperature maps of the Galaxy at 250 GHz, on a portion of the Galactic plane where the expected $I$ signal ($\sim 30 \, \text{mK}_\text{CMB}$) is an order of magnitude stronger than the CMB-dominated off-Galaxy signal: a map constructed from only detector time stream samples where the template signal is maximal (“high”), and a map constructed only from samples where the template signal is minimal (“low”). For this purpose we defined samples with high (low) template signal as samples where the template signal is within the highest (lowest) 20% of the template signal range. We then produced a “difference map” by subtracting the high-template map from the low-template map.

If present, non-linear detector responsivity would compress the signal in the high-template map, resulting in a temperature map that is cooler than the low-template map. We would therefore expect to see a warm signal across the Galactic plane in
the difference map. Without non-linear detector responsivity we would expect no systematic difference between the high- and low-template maps and no discernible signal on the Galactic plane in the difference map.

Because the template amplitudes and phases vary among the detectors, we generated the difference maps for each detector separately. We then co-added the individual detector maps into a final difference map, with each pixel weighted by its pixel variance. The pixel error for a single detector’s difference map is:

\[ \sigma_{\text{pixel}} = \sqrt{\frac{\sigma_{\text{high}}^2}{N_{\text{high}}} + \frac{\sigma_{\text{low}}^2}{N_{\text{low}}}} \]

(7.2)

where \( \sigma_{\text{high}} \) and \( \sigma_{\text{low}} \) are defined as:

\[ \sigma_{\text{high}} = \sqrt{\frac{1}{N_{\text{high}}} \sum_{i=1}^{N_{\text{high}}} (\bar{s}_{\text{high}} - s_{\text{high},i})^2} \]

(7.3)

\[ \sigma_{\text{low}} = \sqrt{\frac{1}{N_{\text{low}}} \sum_{j=1}^{N_{\text{low}}} (\bar{s}_{\text{low}} - s_{\text{low},j})^2} \]

where \( s_{\text{high},i} \) (\( s_{\text{low},j} \)) is the template removed signal at the \( i \)th (\( j \)th) high- (low-) template sample, \( \bar{s}_{\text{high}} \) (\( \bar{s}_{\text{low}} \)) is the sample mean within the pixel, and \( N_{\text{high}} \) (\( N_{\text{low}} \)) is the number of high- (low-) template samples in the pixel. Note that the high-template and low-template hit maps for a given detector do not coincide perfectly; because the difference map for the detector is undefined where either of the hit maps contains an empty pixel, the coverage of the difference map tends to be sparse.

Figure 7.3 presents the coadded difference maps for all bolometers in a 250 GHz wafer during a single detector tuning, shown for three cases: noiseless simulated data, consisting of sky signal and an artificial template containing only HWP template harmonics 1, 2, and 4 (with constant harmonic amplitudes that match those measured for an example detector) (top); the same simulated data with non-linearity added via the function shown in Figure 7.1 (middle); and real data from flight (bottom). In contrast to the difference map for the simulated data without non-linearity, both the
difference map for real data and the difference map for simulated data with non-linearity show a clear warm signal on the Galactic plane. This suggests the presence of non-linearity in the EBEX detectors. (The vertical stripes of dark pixels in the difference map for real flight data correspond to pixels with large pixel error values, as shown in Appendix B.)

The same conclusion can be drawn from the accompanying histograms shown in Figures 7.3 and 7.5. The histograms display the distribution of pixel values inside and outside the red lines demarcating the Galactic plane ($\pm 3^\circ$ Galactic latitude). In the difference maps for real flight data (bottom) and for simulated data with non-linearity (middle), the histograms show systematic positive biases on the Galactic plane where the effect of non-linearity is expected to be strongest (Figure 7.4). They show little difference off the Galactic plane where non-linearity is expected to be weak (Figure 7.5).

The low-template and high-template sky maps, hit maps, and pixel error maps corresponding to the maps in Figure 7.3 are included in Appendix B.

### 7.2.2 Temperature to Polarization Leakage in Sky Maps

Figure 7.6 shows plots of the Stokes $I$, $Q$, and $U$ signal (top, middle, and lower panel, respectively) on the Galactic plane for the EBEX 250 GHz channel (left) and for a simulated scan of the Planck 217 GHz sky map (right). The EBEX $Q$ and $U$ maps show excess polarization signal where the $I$ temperature signal is strong, suggesting signal leakage from temperature into polarization. Similar effects are observed in

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2The color gradient in $Q$ and $U$ along the Galactic plane occurs because the $I$ signal leaks into $Q$ and $U$ as observed in the reference frame of the instrument. The instrument reference frame rotates on average with respect to the sky frame as the instrument executes circular scans across the Galactic plane. Because the elevation of the boresight was approximately constant during flight (see § 2.6), the range of Roll angles at which the instrument scanned each pixel is limited. The average Roll angle at which the instrument scanned each pixel differs on the left- versus the right-side of the Galactic plane.
Figure 7.3: Low-template minus high-template maps of a $10^\circ \times 30^\circ$ portion of the Galactic plane for all active detectors in a 250 GHz wafer for a single data segment, shown for simulated noiseless data without non-linearity (top); with non-linearity added (middle); and for real data from flight (bottom).
Figure 7.4: Histograms showing the distribution of pixel values inside the boxes marked in red in the low-template minus high-template maps in Figure 7.3.
Figure 7.5: Histograms showing the distribution of pixel values outside the boxes marked in red in the maps in Figure 7.3.
EBEX maps of RCW38 and in co-added maps of degree-scale CMB hot- and cold-spots [17].

Figure 7.6: Maps of the Stokes $I$, $Q$, and $U$ signal within $\pm 3^\circ$ of the Galactic plane for the 250 GHz EBEX channel (left), and for a simulated scan of the Planck 217 GHz sky map that has been processed and filtered using the EBEX data analysis pipeline (right). Each map is smoothed to 16' and binned to Healpix $n_{\text{side}}=512$. The EBEX $Q$ and $U$ maps show excess polarization signal where the $I$ signal is strong. This suggests leakage from $I$ to $Q$ and $U$, which can be caused by detector non-linearity.

The material in the remainder of this subsection summarizes the analysis by J. Didier [17] to evaluate two potential models of the observed temperature to polarization leakage: instrumental polarization (IP) (i.e., polarization of unpolarized incident radiation by the instrument) caused by elements in the instrument’s optical system, such as the field lens and the telescope mirrors; and conversion of $I$ into $Q$ and $U$ caused by detector non-linearity.

Software modeling of the instrument’s optics\(^3\) shows that the field lens is the dominant contributor of IP in the optical system. The modeling predicts that field lens IP should contribute $\sim 3-4 \, K_{CMB}$ to the HWP template amplitude, which is in rough

\(^3\)Optical modeling was performed using the CODE V optical design software by Synopsys: https://optics.synopsys.com/codev/
agreement the observed template amplitude. It also predicts the variation we observe across the focal plane of the amplitude and the polarization angle \( \alpha = \frac{1}{2} \tan^{-1}(Q/U) \) of the HWP template’s 4th harmonic. As shown in Figure 7.7, the amplitude of the 4th harmonic observed by the detectors increases with the radial distance from the center of the focal plane (left), while the polarization angle \( \alpha \) of the 4th harmonic (right) shows a strong 1:1 correlation with the polar angle on the focal plane. Both of these relationships are consistent with the field lens IP model predictions [17].

Figure 7.7: Plots of detector positions on focal plane V, with color scales indicating the amplitude (left) and polarization angle \( \alpha \) (right) of the 4th HWP template harmonic. With the exception of the detectors on the central 410 GHz wafer, the amplitude increases with radial distance from the center. This relationship, as well as the observed dependence of \( \alpha \) as a function of polar angle on the focal plane (right), are in agreement with field lens IP model predictions.

Optical modeling indicates that polarization by the field lens produces a maximum polarization fraction of 2-3%, however, which is far less than the excess polarization observed in EBEX (\(~ 8%\) at 150 GHz and \(~ 18%\) at 250 GHz) [17]. This suggests that IP produced by the optical system is not the dominant source of excess polarization.

As explained below in Figure 7.8 and the accompanying text, detector non-
linearity is an additional mechanism that converts unpolarized incident radiation into $Q$ and $U$ polarization signals. Polarization from detector non-linearity is distinguishable, however, from field lens induced IP by the phase offset between the polarization angle $\alpha$ of the excess polarization and the $\alpha$ of the 4th harmonic of the HWP template.

Figure 7.8 shows the extraction of $I$, $Q$, and $U$ time streams (the “reconstructed” signal) for a simplified simulated scan of an unpolarized Gaussian sky source (the “input signal”) by a detector operating in the non-linear regime. The simulation uses a simplified HWP template consisting solely of the dominant 4th harmonic. The simulated detector time stream is modeled as:

$$s_{nl}(t) = f_{nl}\left(\frac{1}{2}I + A_4\cos(4\theta_{hwp} + \phi_4)\right)$$  \hspace{1cm} (7.4)

where $I$ is the unpolarized incident sky signal, $f_{nl}$ is the non-linear response function from equation 7.1, and the cosine term corresponds to the simplified HWP template consisting solely of the 4th harmonic.

The time streams in each panel of Figure 7.8 are plotted versus time. Panel A shows the input $I$ signal from the unpolarized Gaussian sky source (red); the non-linear response of the detector after template removal (magenta); and the $I$ signal extracted from the template removed signal (blue). Panel B shows the HWP template. Panels D, E, and F (right) show the $Q$, $U$, and polarization power $P$ of the input sky signal (red) and of the time streams extracted from the template removed signal (blue). Panel C shows the polarization angle $\alpha$ of the 4th harmonic HWP signal (green) and of the extracted $Q$ and $U$ signals (blue). The polarization angle of the unpolarized input sky signal is not plotted, as it is undefined.

Because the total signal incident on the detector is the sum of the HWP template and the incident $I$ signal from the sky, it is maximally (minimally) compressed where the HWP template is maximal (minimal). After template removal has been applied to the $s_{nl}(t)$ signal, the template removed signal (panel A, magenta) contains a component oscillating at $4f_{hwp}$, even though the sky source is unpolarized. This $4f_{hwp}$
Figure 7.8: The extraction of $I$, $Q$ and $U$ time streams for a simulated scan of an unpolarized Gaussian source by a bolometer exhibiting non-linear detector response, for a HWP template containing only the dominant 4th harmonic. Shown are (A) the input $I$ signal from the sky (red), the non-linear response after template removal (magenta), and the reconstructed $I$ signal (blue); (B) the HWP template, consisting solely of the $4f_{hwp}$ harmonic; (D, E, F) the $Q$, $U$, and polarization power $P$ input by the sky source (red) and extracted from the non-linear detector time stream (blue); and (C) the polarization angle $\alpha$ of the HWP 4th harmonic (green) and of the excess polarization signal (blue). Figure adapted from [17].

Component is perfectly out of phase with the HWP template because the maximal signal compression (and thus the minimal detected signal) occurs where the HWP template is maximal. The plots show that:

1. Panel A: the reconstructed $I$ signal (blue) is weaker than the incident $I$ signal from the sky (red);
2. Panels D and E: leakage from $I$ generates spurious $Q$ and $U$ signals in the reconstructed signal (blue); and

3. Panel C: the polarization angle $\alpha$ for the polarization leakage (blue) is offset by $90^\circ$ from the $\alpha$ of the HWP 4th harmonic (green).

In contrast to the $90^\circ$ offset between the $\alpha$ of the HWP 4th harmonic and the $\alpha$ of the excess polarization produced by non-linearity, the $\alpha$ of the excess polarization induced by the field lens is directly in phase with the $\alpha$ of the 4th harmonic. The field lens is located in the optical system at an image of the focal plane (see Figure 2.2). Unpolarized radiation incident on the field lens at a radius $r$ from its center and an angle $\alpha$ with respect to its axis produces an instrumental polarization signal on the focal plane, correlated with the unpolarized incident signal, at the same radius $r$ and polar angle $\alpha$ on the focal plane. The field lens IP model and the non-linearity model are therefore testable by measuring the phase offset between the $\alpha$ of the HWP 4th harmonic and the $\alpha$ of the excess polarization. A phase offset of 0° is consistent with the field lens IP model, while a phase offset of $90^\circ$ is consistent with the non-linearity model.

Figure 7.9 shows a plot of the HWP 4th harmonic polarization angle (red) and the excess polarization angle for each detector in the vertical (green) and horizontal (blue) focal planes as a function of polar angle on the focal plane. The $\alpha$ values for the 4th harmonic and for the excess polarization are clearly offset by $90^\circ$ as predicted by the detector non-linearity model, and in contradiction with the field lens IP model prediction. This strongly suggests that detector non-linearity or an as-yet unidentified source of $T$-to-$P$ leakage is the dominant contributor of excess polarization observed by the EBEX detectors, and suggests that IP from the optical system is not the dominant source.
Figure 7.9: Plot of the polarization angle $\alpha$ for the HWP 4th harmonic (red) and for the observed excess polarization for detectors in the vertical (green) and horizontal (blue) focal plane as a function of polar angle on the focal plane. The $90^\circ$ offset between the HWP and excess polarization $\alpha$ values is consistent with detector non-linearity, but inconsistent with excess polarization produced by the field lens. Figure from [17], courtesy of J. Didier.

7.2.3 Solar Radiation as a Possible Cause of Non-Linearity

As described above in §7.1, non-linearity results when an excessively large incident signal causes a bolometer to operate in its non-linear regime. Solar radiation suggests a plausible physical origin of excess power in the detectors that gives rise to non-linearity. Figure 7.10 shows plots in the time domain of the great circle distance between the telescope pointing and the Sun (blue) and of the HWP template amplitude (red), defined here as the moving maximum of the template magnitude with a 5 s window. The two are clearly correlated, modulo a time lag.

Contamination via solar radiation—either through heating of and re-emission by the instrument, or by reflection of solar radiation into the optical system—is all the more plausible given that the telescope baffling (see §2.1) was design specified to operate at a minimal distance of $60^\circ$ from the Sun. Figure 7.10 shows that the telescope boresight frequently operated beyond this limit.

A review of temperature sensor data from sensors on the focal planes and on the
Figure 7.10: Plots for an example detector showing correlation between the HWP template amplitude (red) and the great circle distance between the Sun and the telescope boresight (blue) over time, modulo a time lag.

mirrors showed no obvious correlation with proximity to the Sun. Temperature data for the field lens would be informative, but is unavailable. Solar radiation nonetheless remains a prime candidate explanation for the excess power in the detectors that gives rise to non-linearity.

### 7.3 The Non-Linearity Correction Algorithm

We developed two approaches to characterize and correct for the effects of detector non-linearity. The first method utilizes the HWP template harmonics to estimate detector response curves, an example of which is shown in Figure 7.1. The second method, which we refer to as the “stacked map IP removal method,” applied a technique described in [39] to measure and remove IP—including IP from sources other than non-linearity—using coadded maps of CMB hot- and cold-spots as observed in
the reference frame of the instrument. This second method is not presented here, but its application to EBEX detector data is discussed in detail in [17]. This method is not expected to correct completely for detector non-linearity, as it measures and removes $T$-to-$P$ leakage without accounting for compression of the incident signal. The stacked map method proved successful at removing a significant fraction of the observed excess polarization at the map level, as measured by the amplitude of a Gaussian fit of the stacked map hot- and cold-spots: 60% of the observed 8.5% leakage in the 150 GHz band is removed, and 87% of the observed 18% leakage in the 250 GHz band is removed [17].

Because I was closely involved in its development and application, the remainder of this section is devoted to describing the first non-linearity correction method. This method uses the HWP template harmonics to estimate the non-linear response function for each detector. The incident signal is recovered from the measured, compressed signal by inverting the estimated response function. This process is illustrated in Figure 7.1 for a simplified simulated incident time stream consisting solely of the 4th HWP harmonic, i.e., the harmonic that dominates the EBEX detector time streams (top panel, red). Applying the non-linear response function in Figure 7.1 to the incident signal results in a detected signal that is compressed at high incident power (top panel, blue). By applying the non-linearity correction method described below, we obtain a recovered signal (top panel, cyan) that closely matches the incident signal.

To recover the corrected signal, we obtain and apply a “correction function” to the measured, compressed signal according to the following steps, as illustrated in Figure 7.11 bottom:

1. The signal baseline (see § 6.2) is subtracted from the detector time stream. Bandpass filters with an 0.6 Hz bandwidth are then used to obtain each of the HWP template harmonics in the measured, compressed signal that are assumed to appear in the incident, non-compressed signal. As explained above, these include the physically motivated harmonics 1, 2, and 4; in this simplified
Figure 7.11: Top: The non-linearity correction method applied to a simulated incident time stream (upper panel) containing only the dominant 4th HWP harmonic (red) which has been compressed (blue) by the non-linear response function in Figure 7.1. The method recovers the incident signal (cyan) from the signal measured by the detector. In this simplified simulation the correction eliminates the difference between the detected and incident signals to within 1 mK (lower panel). Bottom: Correction algorithm plots for the signals shown at top, described in the accompanying text.
example we consider harmonic 4 only.

2. The compressed, baseline-removed signal is plotted against its summed harmonic components (blue). Had no non-linearity been present, the 4th harmonic of the incident signal would be the only harmonic present; data points would follow a line with a slope of 1 (dashed black line).

3. The slope at low power is measured by performing a least-squares 3rd order polynomial fit for all data points in the lowest 20% of the compressed signal (yellow curve). The tangent line at the bottom-most point on the polynomial fit is shown in red. Note that by hypothesis, non-linearity is weak at low incident power; had the compressed signal been plotted against the *uncompressed (incident)* 4th harmonic signal, the slope of the blue curve at low power would equal 1. In this plot the compressed signal has been plotted against the 4th harmonic of the *compressed (measured)* signal and the slope of the blue curve at low power is therefore greater than 1.

4. We then reconstruct a plot of the compressed signal against the *uncompressed* incident signal by forcing the slope at low power to equal 1. This is achieved by performing an origin translation (not shown) to the bottom-most point on the blue curve and then applying a scaling factor, equal to the slope of the tangent line (red), to the compressed 4th harmonic signal plotted on the $x$-axis. The scaling moves the blue data points to the magenta data points. (Note that the data plotted on the $x$-axis is different for the blue versus the magenta curve; the blue curve plots the compressed signal against its own 4th harmonic component, while the magenta curve plots the compressed signal against the scaled or “corrected” 4th harmonic.)

5. A *correction function* is obtained that takes the compressed signal as input and gives the corrected 4th harmonic (i.e., the incident signal) as output. This is achieved by fitting an order-3 polynomial to the magenta curve (shown in green).
Note that the polynomial is fitted by taking the compressed signal plotted on the $y$-axis as input, and the corrected 4th harmonic plotted on the $x$-axis as output.

6. Applying the correction function to the compressed signal recovers the corrected signal shown in cyan. In this plot the corrected signal is plotted against the corrected 4th harmonic, such that the slope of the cyan plot is 1 (dashed black line), matching the expected plot of the incident signal had there been no non-linearity. (When plotted against the compressed 4th harmonic, the corrected signal traces the red tangent line.)

The time stream plots in Figure 7.11 top show that the recovered, corrected signal (cyan) traces the incident signal (red) almost exactly. In this simplified simulation the non-linearity correction reduces the maximal difference between the incident and detected signals by $>99\%$, from $\sim 1$ K to $< 1$ mK.

Similar results obtain for simulated time streams containing all three incident HWP harmonics 1, 2, and 4 with harmonic amplitudes taken from measurements of an example EBEX detector (see Figure 7.12). The additional harmonics cause the template amplitude (top, upper panel) to vary over the course of a full HWP rotation ($\sim 0.8$ s), with larger compression occurring in different quarter-rotations (cf. the red and blue curves at $t \sim 12.05$ s and $t \sim 12.25$ s). This introduces a slight branching in the correction plots that is visible on small scales (bottom, right panel). As a result the 3rd order polynomial fit is a close, but imperfect estimate of the correction function. The correction nonetheless reduces the maximal difference between the incident and detected signals by $\sim 95\%$ (top, lower panel), from $\sim 2$ K to $\sim 0.1$ K.

Applying the non-linearity correction method above allows us to estimate the response curve for detectors. Figure 7.13 left shows the non-linearity correction plot for real data from an example detector. In the right panel the detected non-linear, compressed signal is plotted against the estimated signal incident on the detector, i.e., the corrected signal obtained using the method described above. The result is a plot
Figure 7.12: Top: The non-linearity correction method applied to a simulated incident time stream containing HWP harmonics 1, 2, and 4 (red) which has been compressed by non-linearity (blue). The corrected signal (cyan) traces the $\sim 4$ K incident signal to within $\sim 0.1$ K. Bottom: Correction algorithm plots for the same signal, shown for large (left) and small scales (right).

of the estimated response curve for the detector, demonstrating a linear response at low incident power and increasing compression due to non-linearity with increasing incident power. By fitting a third-order polynomial to the response curve, we obtain the coefficients $C_i$ in equation 7.1 for the non-linear response function for the detector, in which the detected signal $s_{nl}$ is expressed as a function $f_{nl}$ of the incident signal $s$. 
The non-linear response function for a representative detector may then be used in simulations to introduce non-linear compression in simulated detector time streams. Comparing simulated time streams without non-linear compression to simulated time streams that were first compressed and then corrected allowed us to measure the effect of the non-linearity correction methods we developed.

Figure 7.13: Left: the non-linearity correction plot for data from an example detector. Right: the estimated response curve for the detector, showing the detected signal as a function of the estimated incident signal obtained by applying the non-linearity correction method.

7.4 Non-Linearity Correction Results

7.4.1 The Correction Applied to Simulated Data

We tested the effectiveness of the non-linearity correction method in the map domain by generating simulated scans of EBEX Sky Model maps (see Chapter 5). The simulated time streams consisted of the sky signal added to a simplified template containing only HWP harmonics 1, 2, and 4. We applied the non-linearity function in equation 7.1 to generate a compressed signal. We then applied the non-linearity correction to the compressed signal. Figure 7.14 shows maps of the Stokes $I$, $Q$, and $U$
signals (first, second, and third rows, respectively) on a portion of the Galactic plane for the uncompressed incident signal (left), the compressed, non-linearized signal (middle), and the corrected signal (right) for the detectors on a 250 GHz wafer for a \(~13\) h segment of data, and the accompanying hit map for the same (bottom). We found that the non-linearity correction removed \(\sim 70 - 80\)% of the non-linearity-induced signal as measured by pixel values along the Galactic plane.

The correction is substantial, but imperfect for two reasons. First, as noted in the example shown in Figure 7.12 above, the 3rd order polynomial fit (green) is an imperfect fit to the correction plot (magenta), and therefore provides an imperfect estimate of the correction function. Second and more importantly, the magnitude of the correction is highly sensitive to errors in the estimated slope of the correction plot at low incident power (i.e., the slope of the red tangent line). We found that artificially fine-tuning the estimated slope for each detector yielded a larger correction. We did not seek to perfect the slope estimation procedure any further due to limitations we encountered when applying the correction method to real data (see § 7.4.2 below).

### 7.4.2 The Correction Applied to EBEX Data

In contrast to the results observed in simulations, we found that the non-linearity correction resulted in minimal noticeable effect when applied to EBEX detector data. Upon further investigation we found that although some of the detector time streams were susceptible to correction via the algorithm outlined in § 7.3, others exhibited behavior that is unaccounted for in our modeling.

Figure 7.15 shows maps of the Stokes $I$, $Q$, and $U$ signals from EBEX flight data for the same detectors and data segment shown in the maps of simulated time streams in Figure 7.14. Maps of the time streams are plotted before (left) and after (right) applying the non-linearity correction. (Maps of the uncompressed signal incident on the detectors are not plotted, as it is unknown.) Both maps show excess polarization signal along the Galactic plane. As compared to simulations, only modest improvement
Figure 7.14: Stokes $I$, $Q$, and $U$ maps (1st, 2nd, and 3rd row) and hit map (bottom) showing the effect of non-linearity correction on simulated time streams generated with a simple template containing only HWP harmonics 1, 2, and 4. Maps are shown for the time stream before (left) and after (middle) applying the non-linearity function in equation 7.1, and after non-linearity correction (right).
(∼ 30%) is observable after applying the non-linearity correction.

We expected that including data from additional detectors might yield improvement, as increasing the number of repeated scans of each map pixel results in better averaging down of the sinusoidally modulated $I$ to $Q$ and $U$ leakage signal in the map making pipeline (see §4.1.2). (Note that in simulations, the pixels with low hit counts tend to exhibit poorer non-linearity correction; see Figure 7.14.) Full-flight maps, however, showed little discernible difference when generated from time streams with versus without the non-linearity correction applied (see Figure 7.16). As we describe below, further investigation showed that although a significant fraction of detectors behaved as expected under the non-linearity model outlined in §7.3, many bolometers exhibited behavior unaccounted for in the model. This is thought to result in cancellation of over- and under-correction effects among the bolometer time streams.

A review of the non-linearity algorithm correction plots of the kind shown in Figure 7.13 left reveals a variety of behavior among the EBEX detectors that can be divided roughly into the following three overlapping categories:

1) **Expected behavior:** Approximately one out of every three detectors has a well-defined correction plot curve that behaves as expected under the non-linearity model (see Figure 7.17). These detectors exhibit either no apparent compression, or linear behavior at low incident power and compression at high incident power. The correction plots have a clear, well-defined slope at low power that can be measured and used to estimate a non-linearity correction function. The slight hysteresis-like effect visible in the right plot appears when plotting data on long time scales. It is thought to be due to small changes in the detector responsivity over time, which may occur due to small changes in the focal plane temperature on long time scales, or due to small drifts in the tuning point of the detector.

2) **Looping behavior:** More than 50% of detectors contain four well-defined “loops” or branches of the kind shown in Figure 7.18, which occur even when plotting data
Figure 7.15: Maps of EBEX detector Stokes $I$, $Q$, and $U$ time streams (1st, 2nd, and 3rd row) and hit map (bottom) for the same data segment and 250 GHz detector wafer shown in the simulated maps in Figure 7.14. Maps are shown with (left) and without (right) the non-linearity correction applied, showing only modest improvement compared to simulations.

on short time scales. As shown in the bottom panels, each loop corresponds to one quarter-rotation of the HWP. In many cases there is no well-defined low power slope to be measured. In some cases the four loops have distinct linear slopes that can be measured and averaged to perform an estimated non-linearity correction for the en-
Figure 7.16: Maps of EBEX detector Stokes $I$, $Q$, and $U$ time streams (1st, 2nd, and 3rd row) and hit map (bottom) for all 250 GHz detectors over the entire flight, for the same sky region shown in Figures 7.14 and 7.15. Maps are shown with (left) and without (right) the non-linearity correction applied, showing little discernible difference. Many detectors exhibit behavior unaccounted for in the non-linearity model, resulting in cancellation between over- and under-corrected time streams.

tire bolometer time stream. In such cases, however, the unexpected looping behavior indicates a missing element in the non-linearity model.
Figure 7.17: Non-linearity correction plots showing the measured (compressed) signal plotted against the combined HWP harmonic 1, 2, and 4 components for two detectors exhibiting “expected” behavior: a linear detector, showing no discernible compression (left) and a detector showing compression at large incident power (right).

We made several attempts to adjust our simulated data and the non-linearity model to capture the looping behavior seen in real data, with limited success. In some bolometers the looping collapses when the 3rd HWP harmonic is included in the assumed harmonics in the incident signal (see Figure 7.19 for a comparison of simulated and real data). The physical motivation for including the 3rd harmonic in the incident signal is unclear, however, and other detectors exhibit persistent looping behavior when the 3rd harmonic and/or additional harmonics are included in the incident signal (see Figure 7.19). This suggests the incident signal contains two or more components of the same harmonic separated by a phase offset. Thus far, however, we have been unable to analyze or replicate the observed looping behavior by including phased harmonics in the incident signal.

(3) Smearing behavior: ∼25% of detectors contain a large number of branches smeared into the distinctive shape shown in Figure 7.20. The presence of this shape renders it impossible to measure a well-defined low power slope and generate a well-defined non-linearity correction function. The physical origin of this behavior is un-
CHAPTER 7. NON-LINEAR DETECTOR RESPONSIVITY

Figure 7.18: Top panels: non-linearity correction plots exhibiting “looping” behavior. Middle: plots of a detector time stream with low-power samples in each HWP quarter-rotation rendered in red, yellow, cyan, and magenta. Bottom: correction plot (left) and zoom-in (right) showing the colored loops corresponding to HWP quarter rotations. Colored lines show the ∼ linear slopes of each loop; in this instance the average slope (black) can be used as an estimated low power slope in the correction algorithm.
known. We examined the possibility that such behavior arises from drifts in the
tuning point of the bolometers over time, but found no evidence to support this
hypothesis.

* * *

When applied to simulated time streams containing HWP harmonics 1, 2, and 4
in the non-compressed incident signal, the non-linearity correction method removes
a substantial fraction of the non-linearity-induced $T$-to-$P$ leakage from sky maps.
The method functions as expected when applied to a significant subset of EBEX
detector time streams. Most detector time streams, however, show features that
are unexpected under the non-linearity model described above. For reasons that
remain unclear, a majority of the EBEX detector time streams appear to contain
additional incident harmonics, possibly including two or more contributions from the
same harmonic separated by phase offsets. Although the correction method described
here fails to achieve the desired results when applied to the full flight EBEX data set,
it may prove useful to other HWP polarimeter experiments in which the behavior of
the incident HWP harmonics is better understood.

If the excess polarization can be removed successfully from the EBEX detector
time streams—either by revising the non-linearity model to account for the observed
behavior of the detector time stream harmonics, or by applying the second “stacked
map” IP removal method [17] alluded to above—data analysis would proceed by
generating maps of the Stokes $I$, $Q$, and $U$ sky signal using the methods described in
Chapter 4. The frequency-dependent foreground signals would then be estimated and
removed using sky maps at the three EBEX frequency bands. Foreground-removed
maps of the CMB $Q$ and $U$ signal would be used to form the combinations $Q \pm iU$,
which can be decomposed into spherical harmonics using equation 1.30. The E-
and B-mode power spectra would be calculated from the multiple coefficients via
equations 1.31-1.34. The B-mode spectrum would then be used to place an upper
limit on the tensor-to-scalar ratio $r$. 
Figure 7.19: Top: correction plots for a simulated time stream containing HWP harmonic 3 in addition to the physically-motivated harmonics 1, 2, and 4. Looping appears when the signal is plotted against harmonics 1, 2, and 4 (left), but collapses when plotted against all four harmonics (right). Middle: an example EBEX bolometer showing similar behavior. Bottom: an EBEX bolometer showing looping that persists when harmonic 3 is included in the assumed incident signal harmonics.
Figure 7.20: Example detector time streams with correction plots exhibiting “smearing” behavior of unknown physical origin. The non-linearity correction algorithm fails when applied to these detector time streams, as the plots have no well-defined slope at low power.
Part II

Instrumentation and Hardware Development
Chapter 8

Motivation: A Ground-Based LEKID Polarimeter

The remaining Chapters in this thesis explore the Columbia University Experimental Cosmology Group’s advances in hardware development for CMB polarization experiments. Our research was motivated in part by the design of a proposed ground-based polarimeter I presented at the Society of Photographic Instrumentation Engineers (SPIE) Astronomical Telescopes and Instrumentation conference in July 2014. This Chapter provides a brief overview of the instrument and its contemplated observation strategy. Further details, including the results of a feasibility study we performed with collaborators at other institutions, are provided in [6].

Section 8.1 describes the design of the instrument, which is based on a multiple kilo-pixel array of Lumped Element Kinetic Inductance Detectors (LEKIDs) and a half-wave plate (HWP) rotated by a hollow-shaft motor based on a superconducting magnetic bearing (SMB). Section 8.2 describes the rapid scan strategy contemplated by the instrument’s design, which is enabled by the short time constants and fast readout system of the LEKIDs. The readout system for the LEKID detectors is described in Chapter 9. Chapter 10 describes the development of the hollow-shaft SMB-based motor.
CHAPTER 8. MOTIVATION: A GROUND-BASED LEKID POLARIMETER

8.1 Instrument Overview

Figure 8.1 shows a conceptual overview of the instrument and the observatory design. The instrument design is based on an F/2.4, 500 mm aperture crossed Dragone telescope that is mounted inside a cryogenic receiver. Full-circle azimuthal motion is enabled by mounting the receiver on a rotating platform that turns on a commercial precision rotary air bearing. The receiver is cooled below 4 K by a closed-cycle $^4$He sorption refrigerator backed by a pulse tube cooler (PTC). The telescope focal plane contains a multiple kilo-pixel array of LEKIDs (see Chapter 9). To enable extraction of foreground contamination, the instrument is designed to observe in three spectral bands centered on 150, 210, and 267 GHz.

Polarimetry is achieved via a stationary polarizing grid and a 500 mm diameter, continuously rotating metal-mesh HWP mounted at the aperture stop. The HWP is rotated at a frequency of $\sim 10$ Hz to modulate incoming linearly polarized sky signals. In order to reduce vibrations and eliminate heating from friction, the HWP is rotated by hollow-shaft SMB-based motor (see Chapter 10).

As in EBEX, the continually rotating HWP plays an important role in mitigating systematic errors. As described in §2.5, the combination of the HWP rotation and the scan speed on the sky modulates incoming linearly polarized sky signals into sidebands surrounding $4f_{\text{hwp}}$ (see Figures 2.7 and 2.8), allowing for rejection of signals outside the sky signal band. Note that unlike in EBEX, all imaging optical elements are

1The mirror arrangement of the crossed Dragone system is described in [70]. It is designed to enable a large field of view and a flat focal plane without the need for refractive reimaging optics.

2As described in [6], the feasibility study we conducted considered observations using two detector focal plane configurations. The first configuration includes a focal plane array of 2,317 detectors for observing in a spectral band centered on 150 GHz. The second configuration is tailored for high frequency observations of Galactic dust emission. It includes a focal plane array of 3,283 detectors in bands centered on 210 and 267 GHz.

3The metal-mesh HWP was developed by our collaborators at Cardiff University, and is described in [73].
on the detector side of the HWP. This arrangement ensures that systematics from optical elements appear outside of the sky signal band surrounding $4_{hwp}$. For example, differential reflection of orthogonal polarization states by the mirrors produces an unmodulated signal, while polarized emission from the HWP produces a signal at $2f_{hwp}$. These systematics can therefore be distinguished from polarized sky signals and rejected during analysis.

The LEKID detectors on the focal plane are arranged among seven modules. An adiabatic demagnetization refrigerator (ADR) backed by a second, dedicated PTC maintains the detectors at 100 mK. As described in Chapter 9, the design of LEKIDs allows for frequency multiplexed readout of large numbers of detectors that are coupled to a single transmission line. Our design contemplates frequency multiplexing the detectors in each module into a dedicated readout transmission line, such that the entire focal plane array can be read using seven pairs of coaxial cables and seven cryogenic low noise amplifiers.
8.2 Observations and Scan Strategy

Our feasibility study contemplated observing the CMB from Isi Station, a 3,210 m altitude research site in Greenland. We selected the Isi site to enable observation the Northern Galactic hemisphere, which Planck observes to be significantly cleaner with respect to thermal dust emission than the Southern hemisphere [60, 56].

The instrument scan strategy is based on rapid, full-circle azimuthal scans at a rate of 2°/s on the sky, performed over a range of fixed elevation angles. Scans occur in 24 hour segments. The elevation varies between 30° – 75° among segments, but is held constant within each segment to reduce atmospheric fluctuations.

When combined with the sky rotation at the latitude of the Isi site, the motion of the platform on which the receiver is mounted produces the scan pattern shown in Figure 8.2 right. The pattern is shown for a scanning period of 10 days and covers 34% of the sky. Note that the scan pattern in this figure was rendered using an artificially reduced rotation speed of 0.01 RPM in order to reveal its structure; the pattern would otherwise appear uniform and smooth to the eye. Figure 8.2 left shows a comparison of the instrument’s observation field with the SPIDER and BICEP/KECK fields, as well as the design specified EBEX observation field.

As shown in Figure 8.2 right, the scan pattern is highly cross-linked. With the exception of pixels at the outer edge of the observation field, most pixels are observed for a wide range of scan directions. In addition, the scan pattern connects each pixel with pixels on the opposite side of the field on all time scales between ∼ 45 s and several months. Note that the rapid scan strategy contemplated here would not be feasible with detectors with long time constants. To avoid under-sampling of sky pixels, the instrument must use detectors—such as LEKIDs—that have both a short time constant and a fast readout rate.
Figure 8.2: *Left:* comparison \[20\] of the observation field contemplated for the Greenland-based instrument (upper left quadrant) with the EBEX, BICEP/KECK and SPIDER observation fields. *Right:* the scan path shown for a period of 10 days. The rotation speed has been reduced artificially to 0.01 RPM for clarity; absent this reduction, the scan pattern appears dense and smooth. Figures from \[6\].
Chapter 9

Kinetic Inductance Detector Development

Transition edge sensor (TES) bolometers of the kind used in EBEX (see §2.4) are the current standard detector technology for CMB polarization studies. The Columbia University Experimental Cosmology Group, together with collaborators at other institutions, is actively pursuing the development of Lumped Element Kinetic Inductance Detectors (LEKIDs) as an alternative detector technology for potential use in future CMB experiments. I was involved in developing elements of the baseband readout system we developed to monitor commercially fabricated prototype LEKIDs. Section 9.1 provides a brief overview of LEKID technology and its underlying physics. The baseband readout system is described in §9.2.

9.1 Lumped-Element Kinetic Inductance Detector Physics

Microwave Kinetic Inductance Detectors (MKIDs) are thin-film superconducting resonators designed to detect absorbed photons via the changes they induce in the resonant frequency and internal quality factor of the resonator. MKIDs were first de-
scribed in the literature in 2003 [14]. Early MKID prototypes were designed to absorb photons through antenna coupling, which can introduce a significant efficiency loss. Lumped-Element Kinetic Inductance Detectors (LEKIDs) are a variety of MKIDs designed to address this limitation and first published in 2008 [19]. LEKIDs use the inductor in a single-layer \( LC \) resonator to absorb incident radiation directly without the use of a coupled antenna.

LEKIDs detect the absorption of incident radiation via changes in the surface impedance of a thin-layer superconducting film. Absorbed photons with energies greater than the superconducting gap break Cooper pairs into pairs of unbound electrons (\textit{quasiparticles}). The resulting change in the quasiparticle density within the device affects the kinetic inductance\(^1\) of the superconducting film, shifting the resonant frequency \( f_0 \) and quality factor \( Q \) of the resonator. The shifts in \( f_0 \) and \( Q \) are measured by monitoring changes in the amplitude and phase of a microwave probe signal (“probe tone”) tuned to drive the resonator near \( f_0 \).

LEKIDs offer significant advantages over TES detector technology that make them attractive for use in future CMB experiments. LEKIDs are easily and inexpensively fabricated photolithographically using thin-films on silicon wafers (see, e.g., [46], [34]). LEKIDs are also characterized by large quality factors and near-perfect transmission off resonance. These features enable frequency multiplexing of large arrays of LEKIDs by lithographically tuning the resonant frequency of each detector and coupling all of them to a single transmission line that carries the probe tone. The inherent scalability of LEKIDs makes them a natural candidate for use in future multiple kilo-pixel experiments.

Figure 9.1 left shows a schematic of the lithographic etching for a single horn-coupled LEKID described in [46]. The meandered inductor in the resonator (blue)

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\(^1\)The \textit{kinetic inductance} is a contribution to the inductance of the resonator resulting from the inertial mass of mobile charge carriers. In an alternating electric field the inertia of the charge carriers opposes changes in the emf, manifesting in the resonator as an equivalent series inductance.
serves simultaneously as a photon absorber; the area exposed to the feed horn aperture is bounded in gray. The resonant circuit is completed by the interdigitated capacitor (IDC) shown in red. The probe tone generated by the detector readout system is capacitatively coupled to the resonator through a transmission line shown in green.

The accompanying resonator circuit diagram is shown in the right panel. The resonator is capacitively coupled with capacitance $C_c$ to the transmission line with impedance $Z_0$. The resonant frequency $f_0$ is determined by the capacitance $C$ of the IDC and the total inductance $L = L_g + L_k$, where $L_g$ is an inductance contribution set by the geometry of the inductor and $L_k$ is the kinetic inductance. Photon absorption changes the quasiparticle density in the device and alters $L_k$. The resulting shift in the resonant frequency and quality factor are measured by monitoring changes in the amplitude and phase of the probe tone, which is amplified using a cryogenic microwave low-noise amplifier (LNA).

Note that because the transmission far from the resonance frequency is essentially unaffected by the resonators, an array of multiple LEKIDs lithographically designed to have slightly different resonant frequencies can be multiplexed in the frequency domain using a single transmission line.

9.2 The 150 MHz Baseband Readout System

Our first attempts at LEKID testing used commercially fabricated prototype LEKIDs with lower resonance frequencies in the range of $\sim 150$ Mz. This simplified testing arrangement avoided the use of mixers to convert the frequency of the baseband readout signals to the higher resonator frequencies ($\sim 1-6$ GHz) typical of MKIDs. I was involved in constructing the intermediate frequency (IF) board for the 150 MHz “baseband” readout system we used to monitor the prototype LEKIDs. We are now performing additional tests of devices with higher resonance frequencies. For these

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2STAR Cryoelectronics, Santa Fe, NM
Figure 9.1: Illustration showing the lithographic pattern (left) and accompanying resonator circuit diagram (right) for a single LEKID. Photons absorbed by the inductor (blue) alter the kinetic inductance $L_k$ of the resonator. Shifts in the resonant frequency and quality factor of the resonator are measured by monitoring the amplitude and phase of a probe tone capacitatively coupled to the resonator through a transmission line (green) and amplified by a cryogenic low-noise amplifier (LNA). Figure adapted from [46].

devices we use an $0.5 - 4$ GHz heterodyne readout system, described in [34], that is not presented here.

The baseband readout system monitors the LEKID resonators by injecting sinusoidal probe tones near the resonant frequency $f_0$ into the capacitatively coupled transmission line and measuring the amplitude and phase of the emerging waveform. The $Q$ and $f_0$ of a resonator are first measured by sweeping the frequency of the probe tone through the resonance. After $f_0$ has been determined, the probe tone frequency is tuned to this resonant frequency. The readout then records a complex voltage time series, which can then be decomposed into fluctuations of $f_0$ and $Q$.

Figure 9.2 shows a schematic of the signal path for the probe tones. The readout system comprises the following primary elements:
• The Reconfigurable Open Architecture Computing Hardware (ROACH) signal processing board (green), developed by the CASPER collaboration\footnote{The Collaboration for Astronomy Signal Processing and Electronics Research: \url{https://casper.berkeley.edu/}}, which hosts a Xilinx field-programmable gate array (FPGA). The ROACH board is controlled by an independent Linux-based PC running open-source software\footnote{\url{https://github.com/ColumbiaCMB/kid_readout}} developed by our collaboration.

• The 12-bit analog-to-digital converter (ADC) and 16-bit digital-to-analog converter (DAC) card (red) originally developed for Caltech’s MUltiwavelength Sub/millimeter Inductance Camera (MUSIC) instrument.

• Room temperature analog signal conditioning elements hosted on an intermediate frequency (IF) board.

The IF board (see Figure \ref{fig:if_board} and Table \ref{table:if_board}) consists of a series of connectorized amplifiers, low-pass filters, fixed attenuators, and digital step attenuators mounted on an aluminum plate. All circuit components are made by Mini-Circuits, Inc. The amplifiers and digital step attenuators are powered by linear power supplies and voltage regulators.

The elements of the IF board are selected and arranged to allow for a wide range of powers useful for resonance probing to reach the LEKID devices. The DAC outputs $\sim 0$ dBm (1 mW) of power. The maximum total attenuation between the DAC and the devices, including $\sim -2$ dB insertion loss for each digital step attenuator and $-6$ to $-10$ dB loss in the cabling, is $\sim -110$ dB. If the single transmission line is used to read out a series of 1000 devices, the power incident on each device is reduced by a further 30 dB. On the return path the noise temperature of the readout chain is dominated by that of the LNA ($\sim 4$ K), which itself is subdominant to the detector noise.
Figure 9.2: Block diagram of the 150 MHz baseband readout system. All items external to the cryostat (blue) operate at room temperature. All room temperature elements other than the ROACH board (green) and ADC/DAC cards (red) are hosted on the intermediate frequency (IF) board.

The baseband readout system presented here remains in active use at Columbia University, and was used to take data presented in [46] and [21].
Figure 9.3: Photograph of the 150 MHz LEKID readout system intermediate frequency (IF) board. The ribbon cable visible in the upper left connects to an external PC (not shown) for control of the digital step attenuators.
### IF Board Components

<table>
<thead>
<tr>
<th>Description</th>
<th>Part No.</th>
<th>Voltage [V]</th>
<th>Specified (Measured) Current [mA]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplifier (+13 dB)</td>
<td>ZX60-33LN-S+</td>
<td>5</td>
<td>80 (55)</td>
</tr>
<tr>
<td>Amplifier (+17 dB)</td>
<td>ZX60-V83-S+</td>
<td>5</td>
<td>82 (66)</td>
</tr>
<tr>
<td>Amplifier (+40 dB)</td>
<td>ZKL-1R5+</td>
<td>12</td>
<td>115 (102)</td>
</tr>
<tr>
<td>*Attenuator (-3 dB)</td>
<td>VAT-3+</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>*Attenuator (-6 dB)</td>
<td>VAT-6+</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>*Attenuator (-10 dB)</td>
<td>PN-2082-6</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Digital attenuator (0 to -31.5 dB)</td>
<td>ZX76-31R5-SN+</td>
<td>3</td>
<td>1.5 (-- )</td>
</tr>
<tr>
<td>Low pass filter</td>
<td>SLP-200+</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 9.1: Table of IF board components listing Mini-Circuits Inc. part numbers. Voltages and specified (measured) current draws are shown for the amplifiers and digital step attenuators. Components marked with an asterisk (*) were used in previous versions of the IF board, but are not used in the final design.
Chapter 10

Half-Wave Plate Polarimetry Development

EBEX achieved polarimetry using a stationary wire grid polarizer and an achromatic HWP that was mounted to the 4 K cryostat stage and rotated on a superconducting magnetic bearing (SMB) (see §2.5). The EBEX SMB is described in detail in [68]. It consisted of a stator made of tiles of the high-temperature superconductor yttrium barium copper oxide (YBCO); and a rotor, made of segments of NdFeB magnet, on which the HWP was mounted. The use of an SMB in place of mechanical bearings reduced power dissipation, allowing the HWP to be maintained at a low temperature and minimizing its thermal emission. The HWP was rotated, however, by a tensioned Kevlar belt connected to a drive pulley, which was coupled in turn to a motor external to the cryostat.

The Columbia University Experimental Cosmology Group has been developing a cryogenic hollow-shaft motor based on an SMB for use in HWP polarimetry. In our design the HWP is mounted on a rotor that makes no physical contact with the driving mechanism or any other equipment. In place of a tensioned Kevlar belt, the rotor is driven by a pulsed electromagnet. The thermal path to the cryostat’s exterior that existed in the EBEX design is therefore eliminated. The angular position of the
HWP is measured using an optical encoder system, described in §10.3.3, that requires no electrical signals to enter the cryostat.

I contributed to the design and construction of the motor, the encoder readout, and the cryogenic chamber we used for laboratory testing. Section 10.1 describes the prototype motor we constructed to test the motor driving mechanism. A description of the design and wiring of the cryogenic testing chamber is provided in §10.2. Section 10.3 reviews the design and performance of the cryogenic motor. Further details on the hardware design and initial testing results for the cryogenic motor appear in [33].

10.1 The Mechanical Bearing Prototype Motor

I was closely involved in designing and constructing the room-temperature prototype mechanical bearing motor we built as a proof-of-concept for the cryogenic drive system. I also wrote the data acquisition and control software used for the prototype motor drive system. The cryogenic motor drive system (see §10.3.4) was adapted from the prototype motor drive system by F. Columbro and others.

Figure 10.1 shows photographs of the prototype motor. The rotor consists of a 12 in OD × 11 in ID × 0.5 in steel ring mounted on a 12 in OD × 0.75 in ID × 0.5 in aluminum wheel. The stator consists of an 18 in × 18 in × 0.5 in aluminum base plate, and a top plate of identical dimensions supported by four 0.5 in diameter aluminum corner posts. Material on the top plate is removed to for visibility (see Figure 10.1 left). The rotor is attached to an 0.75 in diameter aluminum shaft that turns on steel tapered-roller bearings (McMaster-Carr part nos. A4138, A4050). The rotor and stator were fabricated by Rochester Tool & Die Corp.

Aluminum mounts attached to the base plate host an electromagnet (“driving coil”) and an identical non-driven wire coil (“emf sensor”), each situated at an adjustable distance (∼ 0.5 cm) above the edge of the rotor. Each coil (MotiCont part
no. HVMC-019-016-003-02) has a diameter of 0.625 in and a length of 0.75 in, and contains 132 turns (∼ 20 ft) of AWG 29 copper wire. As described further below, the magnetic field generated by current in the driving coil exerts a rotational force on permanent magnets mounted on the rim of the rotor. The voltage across the non-driven coil is monitored to measure the back emf produced by the motion of the permanent magnets.

The angular position of the rotor is measured via a US Digital E2 optical encoder mounted to the upper end of the shaft. The encoder readout produces a quadrature output that enables determination of the direction of rotation. The readout interfaces with an external Linux-based PC via a US Digital QSB-M Quadrature-to-USB Adapter. Custom software I wrote in Python allows the PC to issue commands to the encoder readout to monitor the angular position and speed in real time, and record time stamped angular position values to file for later analysis.

![Figure 10.1: Photos of the prototype motor based on a mechanical bearing, shown from above (left) and from the side (right).](image)

Figure 10.2 shows a conceptual drawing of the motor drive mechanism as viewed from the edge of the rotor wheel. Twenty-four permanent magnets are mounted along the rim of the rotor (gray) in an alternating North-South orientation with a uniform
spacing of 15°. The magnetic field lines emanating to and from the magnets’ lower poles are partially dispersed within the rotor, such that the magnetic field above the rotor is dominated by the field lines emanating to and from the upper, exposed poles. Voltage is applied across the stationary driving coil situated ∼ 0.5 cm above the rim, inducing a magnetic field in the coil. As the rotor turns from left to right, the voltage is varied such that the polarity of the coil’s magnetic field flips as each permanent magnet passes beneath the coil. The applied voltage $V$ (red) is a sinusoidal function of the angular orientation $\theta$ of the rotor wheel with amplitude $V_0$. The voltage is phased such that the voltage is zero when the coil is directly above a magnet and maximum (minimum) when the coil is at the midpoint between adjacent North-South (South-North) pairs of poles, i.e.,

$$V = V_0 \sin\left(\frac{N_{\text{mag}} \theta}{2} + \phi_0\right)$$

(10.1)

with $N_{\text{mag}} = 24$ magnets and the phase $\phi_0$ chosen such that $V = 0$ when the coil is directly above the permanent magnets. The magnetic field generated by the coil exerts a driving force $F$ (green) on the adjacent pair of exposed magnet poles that propels them in the same rotational direction, regardless of the angular orientation of the wheel. The driving force is zero when the coil is above a magnet and is maximum at the midpoint between magnet pairs.

The rotational speed and direction of the rotor are controlled by a PI (proportional-integrated) feedback loop running on the external PC as shown in Figure 10.3. The PI loop was implemented in custom software I wrote in Python. The rotor is accelerated to a desired set point frequency $f_{sp}$ by varying the amplitude $V_0$ of the driving voltage waveform; a positive amplitude accelerates the rotor as outlined above, while

\footnote{We experimented with a number of driving voltage wave forms, including square waves and intermittent voltage pulses. As discussed below, substantial friction limited the rotor’s rotational speed; we found that the sinusoidal waveform allowed us to achieve the largest rotational speeds ($\sim 2$ to 3 Hz).}
a negative amplitude reverses the direction of the driving force by inverting the polarity of the driving coil’s magnetic field. At 19 ms sampling intervals\(^2\), the PI loop software calculates the instantaneous rotational speed of the rotor \(f(t)\) using buffered angular position samples read by the encoder; calculates the error term for the PI loop, \(e(t) = f(t) - f_{sp}\); and updates the voltage amplitude \(V_0(t)\) to

\[
V_0(t) = K_p e(t) + K_i \int e(t) \, dt \tag{10.2}
\]

where the proportional term constant \(K_p\) and integral term constant \(K_i\) are tuned empirically to dampen oscillations about the set point frequency. Although the feedback loop software is capable of implementing a complete PID loop by including an additional derivative term with constant \(K_d\), we found that this yielded no discernible improvement in performance.

A LabJack U3-HV USB DAQ is used to generate the instantaneous driving voltage signal \(V_0(t)\) within a range of ±2.5 V. The signal is then amplified to ±5 V by a Linear Technology LT1210 linear voltage amplifier. The analog inputs on the LabJack are

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\(^2\)The sampling rate is determined by that of the optical encoder readout.
used simultaneously to monitor the current in the driving coil and in the emf sensor coil by measuring the voltage drop across 1 Ω power resistors wired in series with each coil.

Figure 10.3: Schematic of the PI loop used to control the prototype motor’s rotational speed and direction.

We found that the prototype motor generally performed to expectation. Figure 10.4 shows an example of the encoder readout data for a set point of $f_{sp} = 0.5$ Hz. The rotor is initially set in motion by timed voltage bursts in the driving coil and a second, redundant driving coil placed on the opposite side of the rotor (see Figure 10.1 left). The frequency settles to the set point frequency within the first $\sim 100$ s, with the error term dampening to within 10% of the set point frequency within the first two minutes of rotation.

The performance of the prototype motor was limited, however, by several factors. First, a machining imperfection in the shaft caused a slight misalignment with the rotational axis of the rotor, resulting in excess friction between the rotor and stator. We reduced, but could not eliminate the excess friction by experimentally applying different lubricants to the roller bearings. Second, the surface of the rotor was slightly warped, resulting in an imbalance that exacerbated the excess friction. The warped surface also caused the vertical distance between the driving coil and the rim of the rotor to vary by $\sim 3$ mm as the rotor turned, altering the driving force on the permanent magnets and prolonging oscillations about the set point frequency. With
Figure 10.4: Example encoder readout data for the prototype motor for a set point frequency of 0.5 Hz, showing time streams for the angle (top panel) and frequency (middle panel) of the rotor and the PI loop error term (bottom panel).

these limitations we found that the rotational frequency of the rotor was limited in practice to 2 to 3 Hz.

After completing testing of the motor driving mechanism using the room-temperature mechanical bearing prototype motor described above, we expected that implementing the driving mechanism on the virtually frictionless, cryogenic hollow-shaft SMB-based motor would allow us to achieve larger speeds using a smaller driving voltage.

10.2 The Cryogenic Testing Chamber

I was closely involved in the design and construction of the cryogenic testing chamber we used to test the hollow-shaft SMB-based motor. The testing chamber consists of a 21 in OD \times 20 in ID \times 14 in aluminum cryostat with an aluminum cold plate and inner shield, cooled by a two-stage Gifford-McMahon (GM) cryocooler. The cryostat was made by Precision Cryogenics Systems (model no. PCS 21.0/14.0). The
CHAPTER 10. HALF-WAVE PLATE POLARIMETRY DEVELOPMENT

GM cooler’s 50 K stage is used to cool the motor system; the second, 20 K stage is unused. To provide thermal insulation we fabricated multi-layered insulation (MLI) blankets to cover the underside of the aluminum cold plate and the outer surface of the shield. The MLI blankets consist of eight layers of aluminized mylar, between which are sandwiched seven layers of polyester insulating veil.

Thermometry is provided by four Lakeshore 4-wire temperature sensors, including a single absolutely calibrated platinum resistor thermometer (model PT-103-AM-2S) and three relatively calibrated thermometers (model PT-103-AM), placed on the cold head and at three locations on the cold plate. The thermometers are read out by a Lakeshore 218 Temperature Monitor.

The cryostat contains four KF50 ports. One port is used for vacuum pumping. The remaining three are used for power and signal I/O wiring as follows:

- One port hosts a USB 2.0 vacuum feedthrough made by MDC Vacuum Products. This port is used to provide power and signal I/O wiring for USB-powered cameras that are used to monitor the motor (see §10.3.3).

- The second port hosts the fiber optic encoder cable feedthrough discussed in §10.3.3.

- The third port hosts a 41-pin connector feedthrough that is used for all other wiring, including wiring for the four 4-wire thermometers, as well as the linear actuator, two redundant driving coils, and emf sensor described in §10.3. Wiring is also provided for an additional fifth thermometer should one need to be installed in the cryostat. An Amphenol Industrial Operations model PT06SE-20-41S(SR) connector is used on the air side of the port; a Ceramtek 41-pin NW50KF connector is used on the cryostat side of the port. To minimize thermal loading, we wired the Ceramtek connector with .008 gauge manganin 290 wire made by the California Fine Wire Company. Copper wiring is used for the Amphenol connector and all external wiring.
A schematic of the power and signal I/O wiring for the cryogenic testing chamber is shown in Figure 10.5.

Figure 10.5: Schematic of the signal I/O and electrical power wiring for the cryogenic testing chamber.

10.3 The Hollow-Shaft SMB-Based Motor

Photographs of the cryogenic hollow-shaft motor are shown in Figure 10.6. The motor comprises a rotor, consisting of a ring magnet on which an encoder wheel is mounted,
levitating above a stator, consisting of an aluminum ring in which YBCO disks are embedded. The entire system is cooled to 50 K. While the system is cooling, a linear actuator is used to extend mechanical grippers that hold the rotor fixed above the stator. After the YBCO transitions the actuator retracts, releasing the rotor and allowing it to float freely above the stator. A slotted encoder wheel is mounted to the rotor and read by fiber optic pickups. Permanent magnets used by the driving system are also mounted on the rotor in an alternating North-South pole orientation. The driving force is provided by a pulsed electromagnetic coil. The direction of the current in the coil is timed to alternate such that the emf drives the permanent magnets in the desired rotational direction at a pre-defined target speed of 5 to 10 Hz.

10.3.1 The Superconducting Magnetic Bearing

The stator consists of fourteen YBCO disks that are embedded with equal spacing in an aluminum ring. The 25 mm diameter YBCO disks were manufactured by CAN.
Superconductors s.r.o. and are specified to generate a 60 N levitation force at a height of 9 mm and a temperature of 77 K.

The ring magnet on which the rotor is based is an N42 grade NdFeB magnet made by Applied Magnetics. It has dimensions 6 in OD × 4 in ID × 0.5 in and a nominal pull force of 210 lb. A triple-layer Ni-Cu-Ni coating provides durability and protection against corrosion. After the YBCO has transitioned, the rotor levitates above the stator with a gap size of ∼ 2 mm.

10.3.2 The Actuator

We selected an UltraMotion Digit NEMA-17 linear actuator to operate the gripping mechanism. The linear actuator has a resolution of 2400 steps per inch and 200 steps per revolution, and is designed to function at cryogenic temperatures.

The actuator is powered by Applied Motion HT17-75 2-phase, high-torque hybrid step motor. The step motor is controlled by an Applied Motion step motor driver (part no. ST5-Si-NN). The motor driver is pre-programmed to extend or retract the actuator by 1 in upon activation. Vendor-supplied limit switches serve as failsafes to limit the actuator’s range of motion.

10.3.3 The Encoder and Readout

The encoder system consists of a laser-cut stainless steel encoder wheel made by Thin Metal Parts, read by two titanium optical pickups\(^3\) (see Fig. 10.7 left). The 0.155 in × 12 in × 24 in encoder wheel contains 360 equally spaced, 200 µm-wide contiguous slits arranged on an outer track and a single index position slit on an inner track. The index position slit is used to reset the angular position of the encoder readout to zero upon a complete rotation of the encoder wheel.

\(^3\)The optical pickups were custom manufactured by Micronor Inc. with assistance and design input from Robert Rickenbach.
Figure 10.7: Photos of the optical encoder pickups (left) and cryogenic test chamber cameras (right).

Figure 10.8 illustrates the operation of the encoder readout system. The slits on the outer track of the encoder wheel are read by a quadrature pick-up in order to determine the rotational direction. The pickup works as a photo interrupter. Light is emitted from a fiber optic cable and reflected from a mirror on the opposite side of the encoder wheel slits. Color filters in front of the mirror produce reflected light of two different wavelengths. The slotted encoder wheel interrupts the two colors with a quarter-wave phase offset, generating a quadrature optical readout signal that travels back through the fiber optic cable and outside the cryostat for readout by a Micronor MR320 Encoder Controller. The photosensitive diodes that detect the optical signal are located in the readout controller outside the cryostat, rather than in the titanium optical pickups near the encoder wheel. The encoder system therefore does not require any electrical signals to enter the cryostat. The position index slit on the inner track is used to reset internal counter of MR320 controller. A second optical sensor, which is read out by a MR382-1 Fiber Optic U-Beam Controller Module, is used for this purpose. The optical encoder readout system uses 62.5/125 um fiber optic cables fed through a hermetic fiber optic feedthrough made by Douglas Electrical Components.

We installed two Leopard Imaging USB-powered cameras (part no. LI-OV5640-
Figure 10.8: Diagram illustrating the optical encoder readout. *Left:* light emitted from a fiber optic cable reflects from a mirror opposite the encoder wheel slits (“Shutter”). Color filters pass two different wavelengths of light back into the fiber. *Right:* the slots (*top*) interrupt the two colors at different times, generating two optical signals with a quarter-wave phase offset (*middle*). The encoder readout transforms the optical signals into a quadrature electrical output (*bottom*). Figures courtesy of Micronor Inc.

USB-72) mounted on custom aluminum interface plates to monitor the motion of the encoder and rotor (see Fig. 10.7 *right*). Ceramic standoffs are used to reduce thermal conduction between the cameras and the interface plates. Illumination is provided by a lighting strip of white LEDs

10.3.4 The Driving Mechanism

The driving mechanism for the cryogenic motor was developed from the prototype motor driving mechanism discussed in §10.1. As in the prototype design, the periodic permanent magnet structure on the rotor is driven by an electromagnetic coil. In similar fashion, the rotational speed is controlled by a PI control loop comprising

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4The LEDs that accompany the cameras ceased to function nearly immediately after cooling, and provided insufficient illumination in any case.
the optical encoders, the coil, and a driving voltage source. The LabJack DAC and LT1210 linear voltage amplifier in the prototype motor design are replaced by an H-bridge pulse width modulation (PWM) motor driver controlled by an Arduino 2 programmable microcontroller. The driving mechanism is targeted to operate at a rotational speed of 5 to 10 Hz.

The cryogenic motor has achieved stable operating speeds in excess of 11 Hz during laboratory testing. Figure 10.9 shows the results of a spin-up test in which the set point frequency was increased from 1 to 5 Hz in 1 Hz increments every 5 minutes. The performance of the motor and the driving mechanism are discussed in further detail in [33].

![Figure 10.9: Results of a spin-up test of the cryogenic motor. Figure from [33].](image-url)
Conclusions

In these final pages we present a brief summary of our findings.

Chapters 2 through 7 describe several challenges we faced in performing data analysis for EBEX and the methods we developed to meet them. As described in § 2.6, the unexpected overheating and shutdown of the pivot motor controller that was responsible for controlling the instrument’s azimuth motion significantly impacted EBEX’s ability to perform science observations. Loss of azimuth control prevented EBEX from executing its planned scan strategy, and caused the instrument to operate frequently beyond the telescope baffling’s design-specified minimum angular distance from the Sun. The data collected by the attitude control system nonetheless allowed for robust post-flight reconstruction of the instrument’s instantaneous attitude (see Chapter 3). Analysis shows that the spurious B-mode signal induced by the attitude variance is restricted to below 10% of the expected lensing and inflationary B-mode signal over the $\ell$-range of interest $30 < \ell < 1500$ for a value of $r = 0.05$.

Application of the map making techniques described in Chapter 4 and comparison to observations by the Planck satellite instrument revealed significant temperature-to-polarization ($T$-to-$P$) leakage in EBEX sky maps. This leakage is consistent with instrumental polarization caused by elements in the instrument’s optical system, and/or compression of the EBEX detector time streams due to non-linear detector responsivity. In addition, analysis of the half-wave plate (HWP) template revealed the unexpected presence of significant power in higher harmonics of the HWP rotation frequency (see Chapter 6). This was unexpected based on experience from MAX-
IPOL, but is consistent with non-linear detector responsivity. These findings motivated several lines of investigation into the possible presence of non-linear detector responsivity, which may have resulted from excess loading from solar radiation. The results of our investigations suggest that non-linear detector responsivity is the dominant contributor to the observed $T$-to-$P$ leakage, and that instrumental polarization by the optical system is not the dominant source (see §7.2).

Section 7.3 describes a method we developed to characterize and correct for the effects of non-linear detector responsivity via analysis of the HWP harmonics present in the signal incident on the detectors. We found that the method removed a substantial fraction of $T$-to-$P$ leakage in simulations and in a significant subset of the EBEX detector time streams. The majority of the detector time streams were not susceptible to correction by this method, however, as they exhibited behavior that we were unable to capture in our time stream modeling. It is our hope that the method nonetheless will prove useful to HWP polarimetry experiments in which the harmonics of the signal incident on the detectors are better understood.

In Chapters 9 and 10 we explored recent developments in instrumentation for CMB experiments, which were motivated in part by the design of a proposed ground-based application first presented in [6] and described in Chapter 8. In Chapter 9 we reviewed the physics of Lumped Element Kinetic Inductance Detectors (LEKIDs), thin-film superconducting resonators that detect absorbed photons via induced shifts in their resonant frequencies and quality factors. We also presented the readout system we designed to test and monitor prototype LEKIDs. The advantages LEKIDs offer over TES detector technology, including their short time constants and the ease of frequency multiplexing large arrays of detectors, make them an attractive candidate for use in future multiple kilo-pixel experiments.

In Chapter 10 we presented the development of a prototype cryogenic hollow-shaft motor based on a superconducting magnetic bearing (SMB) for HWP polarimetry. Our design builds from the EBEX design, in which the HWP was mounted on a rotor
that turned on a virtually frictionless SMB. In the EBEX design the rotor was torqued by a tensioned Kevlar belt coupled to a driving motor that was mounted outside the cryostat. In our design the rotor is driven instead by a pulsed electromagnet. The rotor therefore makes no physical contact with the driving mechanism or any other equipment, thereby eliminating paths for mechanical vibrations and the thermal path to the cryostat’s exterior that existed in the EBEX design. In addition, the angular position of the HWP is measured using a cryogenic optical encoder system that requires no electrical signals to enter the cryostat. Preliminary tests show that the device achieves stable rotation in the targeted frequency range of 5 to 10 Hz.

The proposed ground-based instrument described in Chapter 8 incorporates both the LEKID technology described in Chapter 9 and the cryogenic hollow-shaft motor described in Chapter 10. The rapid scan strategy contemplated by the instrument’s design requires the use detectors — such as LEKIDs — that have both a short time constant and a rapid readout rate.
Bibliography


Appendix A

Time Domain Pointing Error Constraints

This Appendix expands upon the analysis presented in [16] to show that the map domain pointing error requirement described in § 3.2 (i.e., that the error on the mean of all pointing samples in a pixel must be smaller than \( \sim 10'' \)) implies a time domain requirement that the \( \text{rms} \) of the difference between the true pointing and the reconstructed pointing over a typical 40 s azimuth throw must be less than 54''.

If \( N \) is the number of time samples in an azimuth throw, the \( \text{rms} \) error for the throw is defined as

\[
\text{rms}^2 = \frac{1}{N} \sum_{i=1}^{N} e_i^2 \quad (A.1)
\]

where \( e_i \) is the difference between the reconstructed pointing and the true pointing for time sample \( i \).

Next we consider a pixel that receives a total of \( P \) hits from independent scans. In this analysis we make the reasonable assumption that although the errors for samples within the same scan of a given pixel are highly correlated, the errors are uncorrelated among samples from different scans of the same pixel. The distribution of pointing errors \( e_i \) from independent scans will be centered on zero and will have a standard deviation we will call \( \sigma_P \). The distribution of the accompanying measured pointing
angles $\theta_i$ in the pixel will be centered on the true angle, and will have the same standard deviation $\sigma_P$. The error on the mean of all pointing samples $\theta_i$ within a pixel, $\sigma_P/\sqrt{P}$, is therefore the same as the error on the mean of all pointing errors $e_i$.

For convenience we will call the sum of the squared pointing errors from independent scans $S^2$:

$$S^2 = \sum_{j=1}^{P} e_j^2$$  \hfill (A.2)

where $e_j$ is the difference between the true and reconstructed pointing for independent pixel hit $j$. Equations [A.1] and [A.2] show that $S$ is related to the $rms$ by:

$$S = rms \cdot \sqrt{P}.$$  \hfill (A.3)

The standard deviation of the pointing errors is:

$$\sigma_P = \sqrt{\frac{1}{P} \sum_{j=1}^{P} e_j^2} = \frac{S}{\sqrt{P}}.$$  \hfill (A.4)

The error on the mean of all pointing samples in the pixel is the same as the error on the mean of the pointing errors $\sigma_P$, which is given by:

$$\sigma_P = \frac{\sigma_P}{\sqrt{P}} = \frac{S}{P} = \frac{rms}{\sqrt{P}}.$$  \hfill (A.5)

By estimating the number of hits per pixel from independent scans $P$ for the EBEX hit map (see Figure 2.10), equation [A.5] can be used to translate the map domain requirement that $\sigma_P \sim 10''$ to a limit on the $rms$ pointing error on an azimuth throw. In an $nside = 2048$ hit map for the EBEX flight, the $\sim 1.7'$ pixels contain 1600 hits on average. Not all of the 1600 hits come from independent scans; as noted above, consecutive hits from within the same throw contain highly correlated pointing errors that do not average down. The number of consecutive hits within a pixel from
the same throw can be estimated from the median velocity of the telescope, 0.1°/s, and the 191 Hz detector sampling rate. It takes \( \sim 0.28 \) s to traverse a 1.7′ pixel at a velocity of 0.1°/s, yielding \( \sim 53 \) consecutive pixel hits at the 191 Hz sampling rate. The number of pixel hits from independent scans is therefore \( P = \frac{1600}{53} \sim 30 \).

With this value for \( P \), the map domain requirement that \( \sigma_P \sim 10'' \) corresponds to the time domain requirement that the \( \text{rms} \) error over a typical 40 s azimuth throw must be less than \( \sigma_P \cdot \sqrt{P} \sim 54'' \).
Appendix B

High- and Low-Template Galaxy Maps

The following maps accompany the discussion in § 7.2.1 of the low-template minus high-template temperature maps of the Galactic plane shown in Figure 7.3. Included are signal maps, hit maps, and pixel error maps for the detectors in a 250 GHz wafer (readout boards 64 and 65) during a single detector tuning (segment 2013-01-03-13-07-11) that have been coadded as described in § 7.2.1. Maps are shown for both simulated flight data (Figures B.1, B.2, and B.3) and for EBEX flight data (Figures B.4, B.5, and B.6), for four cases running from the top to the bottom panel of each Figure:

1. Maps made with all data, regardless of template amplitude;

2. Maps made solely from time samples where the template signal is within the lowest 20% of the total template magnitude, where compression from non-linearity is expected to be weak;

3. Maps made solely from time samples where the template signal is within the highest 20% of the total template magnitude, where compression from non-linearity is expected to be strong; and
4. Coadded maps of the differenced low- and high-template maps for each detector.

The coverage for the difference map is sparse because the coverage of each coadded detector is sparse; high- and low-template hit maps for a given detector do not coincide perfectly, resulting in undefined difference map pixels wherever either one of the hit maps is empty. Red lines mark the area defined for the pixel histograms in Figures 7.4 and 7.5.

The simulated time streams are noiseless and contain only HWP harmonics 1, 2, and 4, with constant amplitudes matching those measured for a representative detector. The difference maps for both the real data and simulated data with non-linearity show a clear warm signal on the galactic plane, suggesting that non-linearity is present in the EBEX detectors.
Figure B.1: Simulated data: coadded signal maps made from all detector data (top panel), low-template data (2nd panel), and high-template data 3rd panel; and differenced low- minus high-template data (bottom).
Figure B.2: Simulated data: pixel error maps corresponding to the signal maps in Figure B.1 for all detector data (top panel), low-template data (2nd panel), and high-template data 3rd panel; and differenced low- minus high-template data (bottom).
Figure B.3: Simulated data: hit maps corresponding to the signal maps in Figure B.1 for all detector data (top panel), low-template data (2nd panel), and high-template data 3rd panel; and differenced low- minus high-template data (bottom).
Figure B.4: EBEX data: coadded signal maps made from all detector data (top panel), low-template data (2nd panel), and high-template data 3rd panel; and differenced low- minus high-template data (bottom).
Figure B.5: EBEX data: pixel error maps corresponding to the signal maps in Figure B.4 for all detector data (top panel), low-template data (2nd panel), and high-template data 3rd panel; and differenced low- minus high-template data (bottom).
Figure B.6: EBEX data: hit maps corresponding to the signal maps in Figure B.4 for all detector data (top panel), low-template data (2nd panel), and high-template data 3rd panel; and differenced low- minus high-template data (bottom).