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Strategic Militarization, Deterrence and Wars

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Abstract

We study countries choosing armament levels and then whether or not to go to war. We show that if the costs of war are not overly high or low, then all equilibria must involve "dove," "hawk," and "deterrent" strategies and the probability of war is positive (but less than one) in any given period. Wars are between countries with differing armament levels and the frequency of wars is tempered by the presence of armament levels that are expressly chosen for their deterrent properties. As the probability of winning a war becomes more reactive to increased armament, the frequency of wars *decreases*. Finally, as it becomes increasingly possible to negotiate a credible settlement, the probability of peace increases, but the variance of armament levels increases and war becomes increasingly likely when negotiation is not available. This matches observed patterns in the data over time.

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1 Introduction

Wars are costly and destructive and yet they recur, even where it is clear that the involved parties are rational. Given the political and economic impact of war, there are a variety of explanations for why wars occur. However, most explanations (with the exception of a couple discussed below) take the military power of the countries involved in any dispute as given.[1] Militarization is as much a strategic variable as going to war, and the frequency of war is sensitive to the relative power chosen by the disputing countries. By the same token, the strategic militarization choices are affected by the ability to bargain in any given crisis. Thus, studying strategic militarization and war and peace choices together improves our understanding of each of them. This is the main focus of this paper.

Our model is relatively simple and tractable, but still includes the critical strategic variables of interest. At the beginning of a period, countries simultaneously choose a level of militarization. This is a costly "guns/butter" choice such that an increase in military spending leads to a decrease in consumption. Countries then observe each other's militaries and subsequently choose whether or not to go to war. War is costly and the outcome can be random, but is such that an increase in a country's militarization improves its chances of winning a war. If a war occurs, then there is an outcome, and the victor takes over the loser's country and gains a portion of the loser's future consumption and the game ends. If peace prevails, then the game starts afresh in the next period. Countries try to maximize the overall expected discounted stream of consumption, and we solve this dynamic game using game theoretic reasoning.

Our analysis leads to a series of new insights. First, beyond proving existence of (Markov perfect) equilibrium, we provide a characterization for the existence of three types of equilibria. There is an obvious type of pure peace equilibrium in cases where the costs of war are overwhelmingly high and war is never worthwhile. There

[1] This is true for most of the "rational" or "realist" explanations, including those based on: differences in beliefs about the outcome by the involved participants (e.g., von Clausewitz (1832), Blainey (1973), Gartzke (1999), and Wagner (2000)); incomplete information about the possible outcome due to inability to signal strength, or desire to hide it (e.g. Fearon (1995, 1997)); the spiraling of events and arms races (e.g., Waltz (1959), Schelling (1960), Jervis (1976, 1978), Baliga and Sjostrom (2003)); differences between the incentives of leaders and their countries (e.g., Lake (1992), Bueno de Mesquita (2003), Jackson and Morelli (2007)); internal political competition and signaling (e.g., Schultz (1998)), incentives due to preventive or preemptive strategic advantage (e.g, Jervis 1976); and any bargaining friction or commitment problems (e.g., Fearon (1995), Powell (2006), Kirshner (2000), Cai (2003), and Slantchev (2005)).

is another type of equilibrium that arises in cases where the costs of war are very low. There, costs are low enough so that countries attack each other, even when evenly matched, to avoid future costs of armament. The third type of equilibrium exists in a middle range of costs. Here equilibrium must lead to war with a positive probability but also to peace with a positive probability. Thus, war is inevitable, but it occurs at a random time and there can be peace for an arbitrarily long time before war occurs. More importantly, we show that such equilibria involve mixing over different armament levels, and that three different types of armament choices all must play a role in equilibrium: namely "dove," "hawk" and "deterrent" levels. That is, in every equilibrium there must be some militarization level such that the countries using it are sometimes attacked but never attack (doves), some militarization level such that the countries using it are sometimes attackers and never attacked (hawks), and some middle-range militarization levels that are not optimal against lower levels, but deter attacks from higher levels.[2] While the existence of hawk and dove levels is relatively easy to deduce, the necessity of deterrent levels is more subtle and an insight that seems new to a general class of competitive games. Third, we analyze how the probability of war versus peace in any period depends on the war technology and the costs of armament. For example, improving the advantage of militarization (by increasing the probability of winning as a function of increased militarization, or decreasing the cost of militarization) actually decreases the probability of a war occuring. The change in the advantage of militarization, leads to a greater chance that military choices are made that offset each other and lead to peace. While this increases the probability of peace, overall welfare effects can be ambiguous, as there can be increased spending on militarization.

Beyond these results about equilibrium choices of militarization, the probability of war, and how these respond to technology and costs, we also consider how strategic militarization levels and war choices are affected by settlement opportunities, where there can be bargaining and transfers between countries and credible commitments to peace. The base model is one where there is no bargaining, so that countries cannot credibly commit not to attack one another. If countries can sign binding agreements not to attack each other possibly in exchange for some compensation, then wars would

[2] Technically, this is a characterization result concerning the mixed strategy equilibria of two countries. However, this can also be interpreted as a setting in which each country, at the time of armament, does not know exactly who its opponent will be, and in equilibrium consists of the variation in militarization levels across the set of countries. Thus, an informal interpretation is that we should observe coexistence of hawks, deterrents and doves, and war between hawks and doves.

not occur; an point made convincingly by Fearon (1995). We examine how settlement opportunties affect the game by introducing a probability that the countries can bargain and sign binding agreements to avoid a war after the militarization levels have been chosen. When this probability of settlement increases, (1) the overall probability of peace increases, but (2) the number of potential disputes between hawks and doves increases and (3) the variance of armament levels across countries increases. We then examine how these three implications compare to available data from the "Correlates of War". We find that these changes match what has happened between the years of 1816 to 2000 (the available data). It is also interesting to note that as the probability of bargaining increases, the ex ante incentives to arm can adjust in such a way to decrease overall welfare, as arming increases to improve bargaining positions or to counter a potential adversary's bargaining position.

Relation to the Literature

Since our model is an infinite horizon contest game, it has foundations in the "contests" literature. That literature can be thought of as examining games where competitors make costly investments in order to improve their relative chances of winning a prize. Within this literature, most papers either take the existence of a future contest as given or directly black box the outcome as a payoff so that the only inefficiency of equilibria is excessive arming, and decisions of whether or not to engage in war are never made.[3] In other words, either war is assumed, or else the outcome is simply modeled by some settlement function, with no predictions about the incidence of war. Our model explicitly handles the choice of war, and this ends up providing the interesting feedback to militarization choices, predictions about incidence of war over time, and also provides insights regarding deterrence as a necessary part of a strategy, among other things.

Powell (1993) (see also Kydd (2000)) is an important exception to the previous literature in that he allows for both guns-butter choices and war-peace choices, and hence Powell's work is the closest antecedent to our's.[4] In his model countries take turns adjusting their militarization and choosing whether to attack each other. The timing of his model leads to very different equilibrium behavior and conclusions.

[3] See e.g. Skaperdas (1992), Hirshleifer (1995), and the survey by Garfinkel and Skaperdas (1996). There are also papers where the stronger players can simply take from weaker players, such as in Piccione and Rubinstein (2007) and Jordan (2006).

[4] There is also recent work by Slantchev (2005) which provides interesting new ideas for how endogenous militarization can lead to inefficient outcomes and war. His focus is on an incomplete information bargaining process and potential bluffing, and so his model is less closely related to ours.

Effectively, it results in an absence of any war: a country must arm to a given level knowing that the other country will be able to respond to it in the next period without the first country being able to make any adjustments. Countries follow pure strategies which result in a stalemate and perpetual peace. This provides some foundation for how peace can result in an anarchic world, but does not rationalize different militarization strategies or the prevalence of war. As we will see, the timing of militarization and war choices are extremely consequencial, since with our timing choices there cannot exist any equilibrium with perpetual peace. Thus, our results stand in strong contrast to Powell's. One way to view the key difference, is that if there is some aspect of one country's militarization that is not fixed when another country responds, then war is inevitable.

Our result about the impossibility of perpetual peace is similar to that in a recent paper by Meirowitz and Sartori (2007). In their model armament levels are not observable at the time of the war-peace decisions, and in fact the uncertainty of militarization is one of the central points that they make. That leads to a different reasoning behind wars. In their setting if there were some levels of arms that ensured peace, then since the militarization is not observed, a country could save its costs and not invest in any military and still have peace. In our model, the absence of perpetual peace comes from a different insight. If countries were matching arms levels and not having any war, then one country could slightly lower its militarization and not risk being attacked (even if the other country saw this deviation, given that war is costly enough). Moreover, beyond the fact that both our model and the Meirowitz and Sartori model predict that war must occur with some probability, our other results (e.g., about deterrence, comparative statics, and bargaining) are unlike anything in their model or in the previous literature.

The paper is organized as follows: we first introduce the general model, proving existence of Markov equilibrium, the impossibility of perpetual peace, and the necessary coexistence of three types of militarization strategies. Then we analyze a simpler version of the model with only three armament options, and we provide some comparative statics predictions, about which some empirical evidence is provided. All proofs are in Appendix 1.

2 The Model

2.1 Consumption and Arming

Two countries interact over a discrete set of times $t \in \{1, 2, \ldots\}$. In each period country i begins with a level of resources $X_i > 0$. The resources can either be consumed or converted into arms.

Spending on arms in period t is $A_{it} \in [0, X_i]$ and consumption is $X_i - A_{it}$.

Countries are risk neutral and derive payoffs that are the discounted expected sum of future consumption, where the discount factor for country i is $\delta_i \in [0, 1)$.

2.2 The Costs and Benefits of War

If either country chooses to go to war, then the outcome of the war is determined by a function $P(A_{it}, A_{jt})$ which indicates the probability that a country with armament A_{it} wins against a country with armament A_{jt}. The war results in one side "winning", so that $P(A_{jt}, A_{it}) = 1 - P(A_{it}, A_{jt})$.

This is a symmetric technology, so that asymmetries between countries come from armament choices, and arise endogenously.

P is continuous and increasing in its first argument and decreasing in its second argument.[5]

Country i incurs a present-value cost of war of $c_i \geq 0$ regardless of the outcome.

The victor obtains a fraction $v \in [0, 1]$ of the loser's future productive value, while the loser maintains a fraction $\ell \in [0, 1]$ of its productive value, where $v + \ell \leq 1$. A special case of winner-take-all is such that $\ell = 0$. We allow for inefficient sharing of the loser's productive value, so that it is possible that $v + \ell < 1$. Thus, there are two potential inefficiencies of war: having a direct cost of war $c_i > 0$ and/or having resources destroyed in war $v + \ell < 1$.

If i goes to war at time t' and wins then its continuation utility is

$$\sum_{t=t'+1}^{\infty} \delta_i^{(t-t')} \left(X_i + vX_j\right) - c_i$$

or

$$\frac{\delta_i \left(X_i + vX_j\right)}{1 - \delta_i} - c_i. \tag{1}$$

[5] The results have analogs with weak monotonicity conditions, but the added indifference complicates the analysis without adding insight.

If country i loses then its continuation utility is

$$\sum_{t=t'+1}^{\infty} \delta_i^{(t-t')} \ell X_i - c_i.$$

or

$$\frac{\delta_i \ell X_i}{1-\delta_i} - c_i. \tag{2}$$

Given (1) and (2) we can write the expected utility of country i from an arms level A_i and going to war as

$$U_i^W(A_i, A_j) = X_i - A_i - c_i + \left(\frac{\delta_i}{1-\delta_i}\right) \left[P(A_i, A_j)(X_i + vX_j) + (1 - P(A_i, A_j))\, \ell X_i\right]. \tag{3}$$

Another useful benchmark is the utility from perpetual peace at an armament level of A_i:

$$U_i^P(A_i) = \frac{X_i - A_i}{1-\delta_i}. \tag{4}$$

2.3 Timing and Equilibrium

Each period is divided into phases. First, the countries simultaneously choose arms levels. Second, the countries each observe both arms levels. Third, the two countries choose whether to go to war. Here, one country moves before the other and it is irrelevant which moves first or second.[6] If the countries have not gone to war in any previous periods, the process repeats itself. Otherwise, there is an outcome of the war and the countries obtain their respective shares of consumption forever after.

Given the Markovian structure of this setting, we focus on Markov perfect equilibria. In this game a Markov strategy is one where, whenever countries choose arms, their strategies are history independent and all decisions of whether or not to go to war depend only on the current armament levels. A Markov perfect equilibrium, or Markov equilibrium for short, is a subgame perfect equilibrium such that countries select Markov strategies (on and off the equilibrium path).

3 Guns and Butter in Markov Equilibria

We first prove the existence of Markov equilibria.

[6] Having the countries move sequentially here simply circumvents a technical issue of having to use a refinement to rule out countries going to war even when neither would like to, due to a simultaneous move.

3.1 Existence of Markov Equilibria

We do not know of any result that shows the existence of equilibria (Markov Perfect or even subgame perfect) for the class of games that we analyze here, with a continuum of armament levels and payoffs that are discontinuous given decisions of whether or not to go to war (in a way so that neither a lower nor upper semicontinuity condition is satisfied).

To see just one part of the challenge in establishing the existence of equilibrium, consider the following instance. Suppose that country 2 will choose some armament level A_2. Let us ignore the issue of understanding how continuation strategies depend on current choices, which also presents a challenge, and simply fix those for now. In that case, country 2 will have some militarization level A_1^* such that if country 1 arms above that level then country 2 prefers peace and if country 1 arms below that level country 2 prefers war. This presents a discontinuity for country 1, especially depending on how country 2 acts when just indifferent. If country 2 ever goes to war when indifferent, then country 1 might like to arm above the level A_1^*, but as close to it as possible. So a best response does not always exist. This makes it impossible to directly apply fixed point arguments to best response correspondences, a standard technique for establishing existence of equilibrium. One has to use techniques that deals with such discontinuities and establishes how players must act at such points. In this particular example, it would be that country 2 has to always choose peace when indifferent at A_1^*. However, more generally it might involve some other behavior.

THEOREM 1 *There exists a Markov perfect equilibrium.*

The proof technique is to first prove existence when strategies are restricted to a finite set of armament levels. Then to find the limit (along a subsequence) of equilibria as the finite set of possible armament levels comes to approximate the whole set $[0, X_i]$, and then to argue that the limit point has to be an equilibrium of the full game. This is based on a technique developed by Simon and Zame (1990) to prove existence of equilibria in static discontinuous games.[7] However, their theorem does not apply here, even when the game is simplified in a way that translates it into a static game. Thus, we have to offer a direct proof dealing with the Markovian structure. Certain steps of our proof involve constructions similar to those in Simon

[7] Simon and Zame's theorem results in an endogenous sharing rule in a class of games where discontinuous points involve some exogenous payoff sharing procedure. Here, discontinuities involve strategies of the players and so no sharing rules are necessary.

and Zame's work, and so we focus on the issues dealing with the Markovian structure, and parts that can be filled in as more direct extensions of parts of Simon and Zame's proof are referred to, but those details are omitted.

We note that the technique of establishing Markov perfect equilibrium for this game only makes limited use of the specific structure of the game and can be extended to prove existence of equilibria in a variety of repeated and stochastic games where the stage games are discontinuous.

3.2 War and Peace

We now present examples and results that start to outline the structure of equilibrium.

A useful benchmark is what we call the "forced-war game." In this game, the two contries choose arms just once and then a war is forced to occur. So, the payoff to country i when the armament levels are A_i, A_j is $U_i^W(A_i, A_j)$. This is continuous, and given the compact armament space, there exists an (possibly mixed) equilibrium of this game by standard arguments.

We begin with an example that shows that it is possible to have perpetual peace in some settings. However, the example is fragile and "non-generic" in a well-defined sense, so that in most settings such perpetual peace is not possible.

Example 1 *A Markov Equilibrium with Perpetual Peace*

Let $X_i = X$, $c_i = c$, and $1 > \delta_i = \delta \geq 1/2$ for each country. Let $0 < v < 1$ and $\ell = 0$, so that the winner gets some portion of the loser's future resources and the rest is lost. Let c be small for each i, as specified below.

Consider the following war technology described by the following P function.[8] Let $A^* = \frac{X(1-v)}{2} + \frac{c(1-\delta)}{\delta}$ and let

- $P(A_i, A_j) = 1$ if $A_i \geq A^*$ and $A_j < A^*$,
- $P(A_i, A_j) = 0$ if $A_i < A^*$ and $A_j \geq A^*$, and
- $P(A_i, A_j) = 1/2$ otherwise.

[8]This P is discontinuous, which makes the example more transparent. We can approximate P by continuous and monotone functions in a way that does not change A^* being a best response to itself in the forced-war game.

So, if one country reaches the arms threshold A^* and the opponent does not, then the first country wins. If both are below the threshold, or both reach it, then it is an even match, and further armament is not beneficial.

Therefore,

$$U^W(A^*, A^*) = X - A^* - c + \frac{\delta}{1-\delta}\left(\frac{X(1+v)}{2}\right) \tag{5}$$

and

$$U^P(A^*) = X - A^* + \frac{\delta}{1-\delta}(X - A^*) = X - A^* + \frac{\delta}{1-\delta}\left(\frac{X(1+v)}{2} - \frac{c(1-\delta)}{\delta}\right).$$

Thus, $U^W(A^*, A^*) = U^P(A^*)$. Note also that the continuation values having sunk A^*, from peace and war are exactly the same.

The following is a Markov equilibrium for small enough c. In every period each country arms to a level A^*. The decision is to attack if the country has a probability of 1 of winning, and not to attack otherwise. Without offering a full proof that this is an equilibrium, the essential things to note are the following. Raising arms above the given level provides no advantage and an increased cost. Lowering arms below the given level leads to being attacked and a sure loss. The best payoff to lower arms is simply X (setting arms to 0 and consuming only for one period, and then losing the war). The payoff to A^* is given by (5), which is at least as high as not arming whenever

$$\frac{\delta}{1-\delta}\left(\frac{X(1+v)}{2}\right) \geq A^* + c$$

or

$$\frac{X}{2(1-\delta)}(2\delta - 1 + v) \geq \frac{c}{\delta}.$$

The left hand side is positive since $\delta \geq 1/2$ and $v > 0$, and so this is satisfied for small enough c. Changing the attack decision at the given arms levels does not have any effect on payoffs given the exact indifference between the continuation values from peace and war.[9]

What is special about this equilibrium? Two things. First, A^*, A^* forms a pure strategy equilibrium to the forced-war game (for small enough c). Next, the resulting utilities from war at those arms levels are exactly the same utilities as from perpetual

[9]Note that the one-stage deviation principle holds for our class of games (e.g., see Theorem 4.2 in Fudenberg and Tirole (1993)), so we only need to check for single deviations at any given node.

peace. When these are both true, then there is an equilibrium that supports perpetual peace at those arms levels. If a country increases its arms, it would then prefer to go to war, but that cannot be improving since A^*, A^* is an equilibrium conditional on going to war. Lowering arms will lead to the other country attacking, and then again that cannot be an improvement since A^*, A^* is an equilibrium conditional on going to war. Peace at A^* is sustained since the countries are exactly indifferent between perpetual peace and a current war when both use that armament level.

While such equilibria exist, they are non-generic, in the sense that a slight change in the discount rate or cost of war will break the indifference between peace and war, so that a country will prefer war to peace at the given arms levels. As we will show, the only way to sustain peace in a Markov equilibrium is to have the utility from a pure strategy equilibrium of the forced-war game yield exactly the same utilities as perpetual peace at the same armament levels.

There are also pure strategy war equilibria. Consider the same example, but change the war/peace decisions so that each country chooses war with probability one whenever they have a probability of at least 1/2 of winning. That is also an equilibrium. There are more robust pure war equilibria, in that they exist whenever we have a situation where an equilibrium of the forced-war game offers higher utility than peaceful continuation, whereas the peace requires exact indifference.

To get a better feel for the spectrum of equilibria and how they depend on the setting, consider the following example.

Example 2 *Three Types of Equilibria*

Consider a symmetric setting where $X_i = X$, $\delta_i = \delta$, $c_i = c$, $v \geq 0$, and $\ell = 0$, so the winner takes some portion of the losers wealth but the loser loses everything.

Let[10]

$$P(A_i, A_j) = \frac{A_i^\beta}{A_i^\beta + A_j^\beta},$$

where $\beta > 0$. Here,

$$U^W(A_i, A_j) = X - A_i - c + \frac{\delta X(1+v)A_i^\beta}{(1-\delta)\left(A_i^\beta + A_j^\beta\right)}$$

[10] Set $P(0,0) = 1/2$. To ensure continuity at 0, we could set $P(A_i, A_j) = \frac{A_i^\beta + \varepsilon}{A_i^\beta + A_j\beta + 2\varepsilon}$, which leads to similar results with more complicated expressions.

and

$$U_P(A_i) = \frac{X - A_i}{1 - \delta}.$$

First, note that if c is high enough then not arming and choosing not to attack is an equilibrium. If a country deviates, arms slightly and attacks, it earns just less than $X - c + \frac{\delta X(1+v)}{1-\delta}$, while not arming and peace leads to $\frac{X}{1-\delta}$. So, there is a pure peace equilibrium when

$$c \geq \frac{\delta v X}{1 - \delta}.$$

That is, there is a pure peace equilibrium when the cost of war outweighs the gains when a country knows it will win the war with certainty and with arbitrarily small arms costs.

At the other extreme, when c is very low, we find equilibria where war is certain. Note that the best response of i anticipating $A_j > 0$ and war is characterized by the first order condition

$$-1 + \frac{\delta X(1+v)\beta A_i^{\beta-1} A_j^{\beta}}{(1-\delta)\left(A_i^{\beta} + A_j^{\beta}\right)^2} = 0$$

Solving for a symmetric forced-war equilibrium where $A_i = A_j = A$ leads to $A = \frac{\beta \delta X(1+v)}{4(1-\delta)}$. If $\beta \geq \frac{4(1-\delta)}{\delta(1+v)}$, then there is a corner solution of $A = X$ and it is easy to see that war is then supported as an equilibrium in the continuation. So, for the case where $\beta < \frac{4(1-\delta)}{\delta(1+v)}$, let us examine when there is an equilibrium with war. By attacking, the utility is $X - A - c + \frac{\delta X(1+v)}{2(1-\delta)}$, while never attacking at these arms levels leads to a utility of $\frac{X-A}{1-\delta}$. Thus, a necessary condition for this armament and a war to be a Markov equilibrium is that

$$\frac{\beta \delta^2 X(1+v)}{4(1-\delta)^2} - \frac{\delta X(1-v)}{2(1-\delta)} \geq c.$$

This turns out also to be sufficient (see Theorem 2 below).

When the above inequality holds with equality we also find another pure peace equilibrium like the one outlined in Example 1, where the countries arm to $A = 2\beta X$ but then never attack.

In the remaining region where

$$\frac{\delta v X}{1-\delta} > c > \frac{\beta \delta^2 X(1+v)}{4(1-\delta)^2} - \frac{\delta X(1-v)}{2(1-\delta)},$$

the only equilibria are a type of mixed strategy equilibrium that we study in detail below, where there is war with a probability strictly between 0 and 1. For instance, if $\delta = 1/2$, $\beta = 1$, then this holds when $\frac{X}{2} > c > \frac{X}{8}$.

This pattern of equilibrium as a function of the costs of war is typical, as we now show.

We say that the costs of war are *overwhelmingly high* if either $U_i^W(A_i, 0) \leq U_i^P(0,0)$ for all A_i or $U_j^W(A_j, 0) \leq U_j^P(0,0)$ for all A_j. Thus, costs of war are overwhelmingly high if there is a country which would never want to arm and go to war even when the other country is not arming at all. If costs of war are overwhelmingly high, then not arming and never going to war is an equilibrium. To consider interesting cases where there is some possibility of conflict, we need to examine cases where the costs of war are not overwhelmingly high.[11]

THEOREM 2 *If the costs of war are not overwhelmingly high, then*

(I) there are Markov equilibria of the overall game that lead to perpetual peace if and only if *there exists a pure strategy equilibrium A_1, A_2 to the forced-war game such that $U_i^W(A_i, A_j) = U_i^P(A_i)$ for each i.*

(II) there are Markov equilibria of the overall game that lead to certain war in the first period only if *there exists a (possibly mixed) strategy equilibrium σ_1, σ_2 to the forced-war game such that for almost every realization of A_1, A_2 there is at least one i for which $U_i^W(A_i, A_j) \geq X_i - A_i + \delta_i U_i^W(\sigma_i, \sigma_j)$,*

(III) there are Markov equilibria of the overall game that lead to certain war in the first period if *there exists a (possibly mixed) strategy equilibrium σ_1, σ_2 to the forced-war game such that $U_i^W(A_i, A_j) \geq X_i - A_i + \delta_i U_i^W(\sigma_i, \sigma_j)$ for each i and every realization of A_1, A_2, and*

(IV) there are symmetric pure strategy Markov equilibria of the overall game that lead to certain war in the first period if and only if *there exists a symmetric pure strategy equilibrium A, A to the forced-war game such that $U_i^W(A, A) \geq U_i^P(A)$ for each i,*

(V) if all equilibria of the forced-war game fail to satisfy the condition in (II), then in any Markov equilibrium of the overall game and any period there is a probability of war that lies strictly between 0 and 1.

[11] The idea that peaceful equilibria exist if the cost of war is overwhelmingly high is clear and common to any model of war, regardless of whether arms are observed or not (e.g., see Meirowitz and Sartori (2007) for a model where arms are private information).

While the equilibrium characterization involves a series of different scenarios, the overall picture can be summarized succinctly as follows.

- If costs[12] are high enough, then there exist equilibria where neither country arms and there is perpetual peace,
- if costs are low enough, then there exist equilibria where at least one country arms and war is certain,
- in a middle range of costs, all equilibria involve mixed strategies and a probability of war strictly between 0 and 1.

The idea that high costs must lead to peace is obvious. The fact that there are pure war equilibria, and also that there exist mixed equilibria is a bit more subtle. The pure war equilibria rely effectively on a coordination problem. Countries arm to a high level because they expect the other country to arm to a high level. At that point, if the cost of war is low enough, the countries prefer to battle and save future expected costs of arms rather than be peaceful and have to continue to invest in high levels of arms. The rough idea behind the mixed strategy equilibria is that costs of war are high enough so that when both countries arm to a high level, then they prefer to avoid war. As such, the high level of arms are not stable. A country would like to lower its arms to save on costs as it realizes that the other country would also like to avoid war. But then low arm levels are not stable either, as then a country can gain by arming to a higher level and having a high chance of winning a war.

The mixed equilibria turn out to be fairly rich in both structure and intuition. Also, they provide empirical insight, and we investigate mixed strategy equilibria in much more detail in the following sections.

3.3 Some Remarks About Mixed Strategies

The fact that the middle range equilibrium involves mixed strategies might seem strange, as it would seem to imply that countries must actively randomize in deciding on how large a military to have. There are several remarks to be made here. First, there are two ways to interpret mixtures that provide some foundation for the structure here. One is to purify the equilibrium so that countries are choosing pure

[12] Note that we are using the term "costs" to refer to the costs c and any lost resources $1 - v - \ell$, but *not* including the costs of armament, which we are comparing things to.

strategies, but specific aspects of their decision process are not viewed externally and so are "as if" the countries are mixing as viewed from each other's point of view. This rationalization has a rich history in game theory, so we will not debate it here. Another perspective is that although our model only examines two countries, the real world has countries interacting with a variety of others over time. In such a setting, the world of countries would match the equilibrium distribution, but any given country could be playing a pure strategy. Indeed, in the data (see Section 5), there is a wide range of armament choices across countries, which would be consistent with this interpretation, especially as countries seem to specialize (at least over short time horizons).

3.4 Properties of Markov Equilibria

We now provide more insight into the structure of the equilibria. We define three key types of armament levels relative to an equilibrium.

- An armament level is a *dove* level if,
  - (i) when used, the country does not wish to go to war against any level of armaments in the opponent's support;
  - (ii) when used, the country is sometimes attacked; and
  - (iii) each arms level used by the opponent such that the opponent chooses war in some equilibrium situations leads the opponent to choose war against this arms level.
- An armament level is a *hawk* level if,
  - (i) when used, the country chooses to go to war for some levels of armaments in the opponent's support,
  - (ii) the opponent never chooses to go to war against this armament level, and
  - (iii) any arms level of the opponent that is not attacked by this arms level is never attacked in the equilibrium.
- An armament level is a *deterrent* level if
  - (i) it is *not* a best response against the opponent's mixture over arms levels below this level,

(ii) it is not attacked by some armament levels in the opponent's support that sometimes attack in that equilibrium.

Part (i) of the definition of deterrent requires that it is an armament level not being chosen for its offensive advantage, while part (ii) requires that the armament level serve the purpose of deterring attacks by some armament levels used by the opponent that would attack some lower armament levels.

It is possible, under these definitions, to have a level serve as a "deterrent" and a "hawk" strategy, but only in a very special case. This is a case where the high armament level is only chosen because of its *defensive* advantage against high armament levels of the opponent, and is only chosen because the opponent is also choosing a high level of arms that the country needs to deter. This is the implication of (i) in the definition of deterrent. That is, the deterrent level is selected because of its deterrence properties and not its offensive advantages.

It follows from Theorem 2 (V) that any symmetric Markov equilibrium when costs are not overwhelming and in the absence of condition (II) (so there is no pure war equilibrium and thus when costs are not too high or too low) must involve both dove and hawk armament levels. This is shown by taking the min and max of the support, respectively, of necessarily mixed strategies over arms levels, and with a bit of proof that these satisfy the definitions. What is less obvious is that it must also involve deterrent armament levels.

THEOREM 3 *If the costs of war are not overwhelmingly high, and the condition in (II) does not hold (so war costs are high enough so that there is no certain war equilibrium), then every symmetric Markov equilibrium involves levels that are doves, hawks, and deterrents in its support.*[13]

Theorem 3 provides new insights into the structure of armament levels. To our knowledge, this equilibrium characterization, especially the presence of deterrent arms levels, is new to such conflict games more generally. From Theorem 2 we know that the equilibrium must involve some mixing. However, an equilibrium with just hawks and doves, without any deterrence, cannot hold together. To get the intuition, suppose

[13] Having a type of armament in the support does not mean that the strategy gets positive probability. However, the strategy must have positive probability in some neighborhood of the strategy. Strategies near a dove need not be a dove, as they might attack some lower levels, but given the continuity of payoffs and the strict monotonicity, the nearby strategies are "nearly" doves.

to the contrary that there are no deterrence levels, and for simplicity suppose that just two levels of armament are used, a low and a high one. In this case, Theorem 2 implies that war must sometimes occur between low and high, but the symmetry and costs of war ensure that war does not occur between low and low or high and high. Thus, the high level qualifies a hawk and the low level as a dove. Then a hawk must be at least as good a response against a dove as a dove since otherwise it would qualify as a deterrent. We then argue that a hawk strategy must be a strictly better response against a hawk than a dove is, by arguing that the benefit from changing to hawk from dove against hawk is even greater than it is against dove, since it has the same change in probability of war but saves a cost of war. This implies that the hawk level is a better response than dove against the mixture, which contradicts equilibrium. The presence of a deterrence armament lowers the payoff to being a hawk and then justifies a dove strategy. Also, having hawks around justifies deterrence, and having doves around means that countries are willing to be hawks. The proof is more complicated as it must show that equilibria that involve mixing over many and even continua of hawk and dove armament levels (and possibly other armament levels that are none of the above) without any deterrence armaments cannot hold together.

Although Theorem 3 discusses three types of arms levels, we point out that an arms level could be both a hawk and deterrent level at the same time. This only happens when the "hawk" level is not being chosen for its offensive strength, in cases where it is not the best response against lower armament levels (by part (i) of deterrent), but is instead chosen to deter the other country. Thus, there are two different types of mixed strategy equilibria that can arise. One type of equilibrium involves at least three different armament levels, some of which are doves, some of which are deterrents, and some others that are hawks that are "offensive" hawks in that they are the best response to lower armament levels. A second type of equilibrium still has "hawk" armament levels, but these are not offensive, but are defensive ones and are actually chosen as part of a bad coordination: countries choose these levels only to deter the other country from attacking with their high levels of armaments.

4 Three Armament Levels

We now illustrate the interplay of these three types of actions by examining a setting where only three armament levels are possible.[14] We do this since solving for equilibria in general settings can involve weight on an infinite number of arms levels which makes comparative statics complicated. Focusing on three arms levels allows us to analyze the adjustment in the roles and interplay of the three main armament *types* (hawks, doves, and deterrents) as the setting changes. Here, we concentrate on equilibria where hawk armament levels are offensive and separate from deterrent levels.

Consider a symmetric game in which the following three militarization levels are feasible in each period: $L < M < H$. Let $X_1 = X_2 = X$, $c_1 = c_2 = c$, and $\delta_1 = \delta_2 = \delta \in (0,1)$.

Since we have fixed the three militarization levels rather than allowing all militarization levels, some conditions are necessary to ensure that all three are used in equilibrium. For instance if M and H are very close together, then it might be that only M is used and never H (or vice versa).

The following proposition provides sufficient conditions for all three armament levels to be used in equilibrium, and also for the equilibrium to be unique.

PROPOSITION 1 *Suppose that the costs of war are not overwhelmingly high,*[15] $U^P(M) > U^W(M,L)$, *and* $U^P(H) > U^W(H,M)$. *Then there is a unique Markov Equilibrium. It is symmetric and such that* L, M *and* H *all have positive weight.*

The fact that the costs of war are not overwhelmingly high implies that a hawk is willing to attack a dove. The conditions that $U^P(M) > U^W(M,L)$ and $U^P(H) > U^W(H,M)$ imply that the deterrent armament level would not like to attack a dove, but is enough to deter a hawk.

With these conditions in hand, we now can analyze how the probability of peace depends on the underlying structure.

[14] It is easy to find P functions such that only three armament levels would ever be used. To do this, start with a P which is discontinuous, so that it is constant when a country increases its armament above one of the prescribed levels, but below the next level. Under the conditions in the propositions below, we can approximate such a P function by a continuous function that maintains the equilibrium found with just the three levels.

[15] Here, consider L to fit the role of "0" in the definition of costs not being overwhelmingly high.

Proposition 2 *Under the conditions of Proposition 1, in the unique Markov equilibrium, the probability of peace increases (and the weights on both H and L decrease while the weight on M increases) if*

- *the probability that a hawk beats a dove ($P(H, L)$) increases,*
- *the relative gain to a hawk compared to a dove increases (v increases and ℓ decreases so that $v + \ell$ remains constant),*
- *if the discount rate increases (given low enough c),*
- *the cost of arming at a dove level L is increased and/or the cost of arming at a hawk level H is decreased (holding $P(H, L)$ constant).*

We explain these effects as follows. Things that improve the payoffs of H at the expense of L increase the relative payoffs from playing H and decrease the relative payoff from playing L, holding all else constant. Thus, it must be that L's meet H's with lower frequency in order to keep them willing to play L; and to keep countries from shifting weight entirely to H it must be that H's expect to meet L's with lower frequency. So, weight shifts from both strategies to M. Changes in the discount rate affect the payoffs to all three strategies, and so it can be difficult to disentangle, but for low enough c we still see the overall impact, as noted in the last part of the proposition.

We remark that no claim is made about how equilibrium changes in response to changes in the cost of war c. An increase in the direct cost of war c hurts both high and low armament levels. That has an ambiguous impact on peace. It increases the probability of L and decreases the probability of H, but depending on the starting point this could lead to higher or lower probabilities of war.

We can also comment on how things adjust as we change the costs of L, M and H. If we increase M, the cost of deterrence, then the changes are again ambiguous. The weight on hawk goes up, and the weight on dove goes down, but the weight on deterrence and the probability of peace are ambiguous. Things are clearer if we increase L or H (while holding $P(H, L)$ constant), where the effects are as follows. When H is increased, we see a higher weight on Dove and a lower weight on deterrent (this keeps countries willing to play H). This increases the probability of war. When L is increased, we see an increase in the weight on deterrence and a decrease in the weight on hawks, and a decrease in the probability of war.

4.1 The Role of Settlement Possibilities

So far, we have considered situations in which no settlement of a crisis between hawks and doves is ever possible, for example because the dove cannot credibly commit to transfers necessary to appease the hawk.

In this section, we analyze the role of potential settlement opportunities in the following simple way. We suppose that there is a probability π that a binding settlement can be agreed to under which one country makes a transfer to the other and the second country agrees not to attack. For simplicity, we assume that the minimum transfer to avoid war is made, although qualitatively similar results hold if larger transfers are made.[16] The countries do not know whether a binding settlement will be possible at the time of armament choices, but do take this possibility into account when choosing their arms. The agreement only lasts the given period, so that the next period if the countries re-arm (possibly to new levels), they must write a new treaty or risk going to war.

Proposition 3 *Under the conditions of Proposition 1 there exists $\overline{\pi} \in (0,1]$ such that if $\pi < \overline{\pi}$ then there is a unique Markov equilibrium. Such a unique equilibrium is symmetric and such that L, M and H each get positive weight, with the same comparative statics as in Proposition 2. Moreover, as π increases (but remains below $\overline{\pi}$)*

- *the equilibrium weight on H increases, the weight on M decreases, and weight on L is unchanged,*
- *the overall probability of war decreases, but the probability of war conditional on the absence of a settlement opportunity increases, and*
- *total ex ante welfare is unchanged.*

Once π is high enough (above $\overline{\pi}$) there is no more room for deterrence because now a dove can expect to be able to pay off a hawk and avoid a war and this leads to a higher payoff than arming at the deterrent level. Above that level, there are different regions in terms of equilibria: there is always an symmetric equilibrium where each

[16] In particular, the transfer only occurs from an L to an H and then is the amount that makes the H indifferent between declaring war or forgoing war for the transfer plus the equilibrium continuation value. The explicit expression appears in the appendix.

country mixes between L and H, and above some level there are are also two pure strategy equilibria where one country plays H and the other plays L.

So, the comparative statics pointed out in Proposition 2 continue to hold when settlement opportunities become possible, until the probability of settlement reaches a threshold. To understand the comparative statics on the equilibrium weights and the probability of war, note the following: The expected payoffs to the M strategy is unaffected by the presence of the settlement possibility. Also, the expected payoff to H is unchanged since H still meet L with the same frequency (as we shall show) and the transfers they receive if settlement is possible makes them indifferent between war and peace. It is only the L strategy which has its payoffs directly affected. Here, the possibility of transfers makes them better off with settlement than without when they meet an H type. Thus to maintain the equilibrium indifference, there must be an increase on the probability that L types meet H types, since H types still provide a lower payoff to an L type than meeting an M or L type (where there is peace, and without need for any transfers). Thus, the increase in weight on H stems not from a change in the payoff of that type, but from a need to balance the incentives of the L types who benefit from the bargaining. This then translates into changes in war probabilities that decrease due to settlement possibilities, but increase in the absence of settlement. This happens since when π increases, the cost of hawk versus dove armament levels is lower, and hence the overall frequency of these armament levels must increase, which can be interpreted as an increase in the number of "disputes." Historical evidence for such an increase in the number of disputes and a decreased total number of conventional wars between nations is discussed in Section 5.

The result on the total welfare being unchanged comes from the fact that the welfare in equilibrium is just the expected utility from perpetually using the deterrent arms level. This is still used in all equilibria such that $\pi < \overline{\pi}$. The nature of equilibrium changes above $\overline{\pi}$, and as π approaches 1 then the equilibrium approaches one where just L is used and so welfare increases.

5 Historical Evidence

Although an in-depth examination of the empirical implications of the model are beyond the scope of this paper, we briefly discuss some general trends in armaments, wars, and disputes that have been noted in the literature that are consistent with the model's predictions. In particular, some of the interesting and novel predictions can be examined using the "Correlates of War" data set (henceforth, "CoW", see Jones, Bremer, and Singer (1996) for a description).

Even without any observation about how the probability of settlement π has evolved over time, we can deduce things about co-movements of other observable variables. Proposition 3 indicates that as the probability of settlement increase, the frequency of disputes and the variance of armament levels should increase, whereas the frequency of war should decrease. We recall the intuitive explanation for these co-movements: As countries increase their assessment that any given dispute will be settled, the deterrence strategy becomes less attractive and a hawk strategy becomes more attractive. This leads to an increased variance in militarization, and also leads to more hawk-dove confrontations so the number of disputes rises. However, given the increased availability of settlements, the number of disputes that precipitate into wars goes down.

The CoW contains data on disputes, wars, and armament levels from 1816 to 2004, but we considered only the data until 2000 because of missing data for the last four years of the data.[17] The data include between 92 and 755 countries depending on the year, with the number of countries generally increasing over time. We divide the data in periods of 4 years, 1816 to 1820, and so on.[18] For each group of four years, we compute the frequency of wars per country, the frequency of militarized international disputes (MID) per country, and the ratio of military personnel over population for each country and then the standard deviation of this across countries.[19] The following figures provide the data and fitted trend lines.

The trends in the disputes/country and standard deviation in military personel per country are statistically significant at the 5 percent level (with p-values of .00005 and .04, respectively), and the trend in wars/country is significant only at the ten percent level (with a p-value of .09), but has been confirmed by other recent studies (see e.g. Mueller (1990)).[20]

[17]There is evidently a lag between events and the date of entry of the correspondent data in the data set.

[18]The subdivisions are inconsequential to the trend lines and significance, but allow for easier graphing.

[19]The military personel per capita is clearly only one measure of armament, and the technology of war is changing over time. The average of military personel per capita has the advantage of being roughly constant over time across the whole data set at about three quarters of one percent (with notable exceptions in the first and second world wars where it rises to about 2 percent on average). Thus, the change in variance indicates more dramatic differences across countries later in the data.

[20]The R^2's on the regressions are .31 for disputes, .07 for wars, and .09 for standard deviation

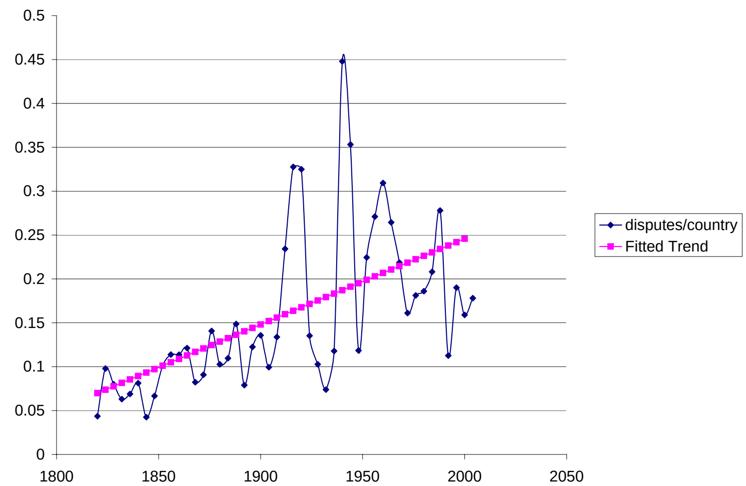

Figure 5 Militarized Disputes per Country: Correlates of War Data, 1816-2000

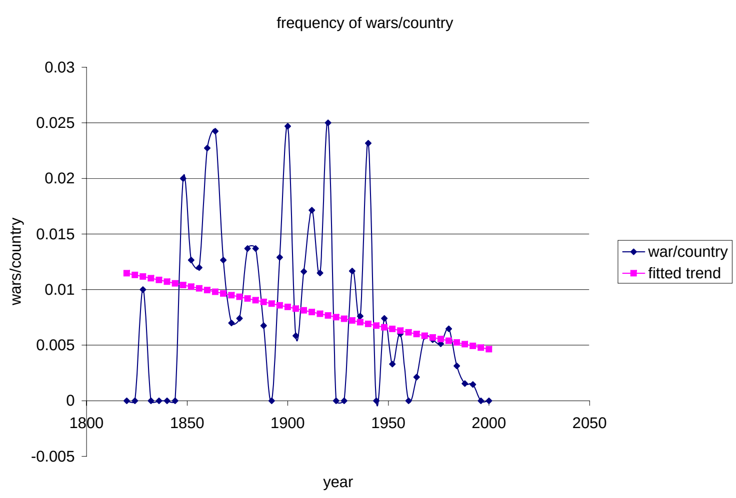

Figure 5 Wars per Country: Correlates of War Data, 1816-2000

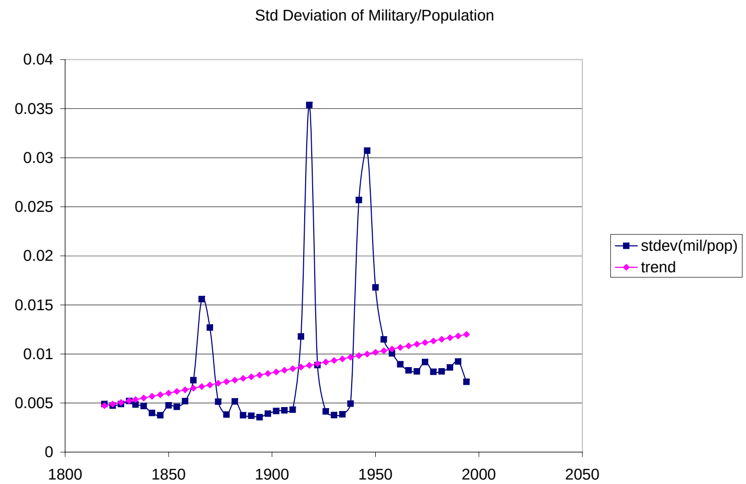

Figure 5 Standard Deviation in Military Personel per Population across Countries: Correlates of War Data, 1816-2000

There are many caveats to intepreting these data, and so we provide these figures only to suggest that there are interesting empirical angles to pursue in future work on this subject, and that such a model can help suggest combinations and patterns that we should expect to see in the data that are not as obvious without examining equilibrium comparative statics. The caveats mainly have to do with the substanial nonstationarities that exist in the world over this time period, which include changes in the number of countries, their resources, their interaction patterns, and the technology of war, among other things. In addition, while the definition of war can be operationalized in terms of a given number of casualties (in the CoW data, it involves a cumulative 1000 deaths in battle), the definitions of militarized disputes are open to more discretion. While these are carefully detailed by Jones, Bremer and Singer (1996), there are many disputes that are included in the data which might fall short of the kind of dispute that we envision in this paper. Moreover, there are data on how the disputes end in the CoW data set, and as Jones, Bremer, and Singer (1996)

in military. So, there is a great deal of noise around the trends, and one has to be cautious in interpreting the significance numbers.

point out, negotiated settlements have decreased over time, while more disputes simply end without any explicit agreements. This might be inconsistent with the trends being due to increased settlement opportunities, but that requires a more detailed examination of what really happens when a conflict ends with an explicit agreement and it could be that countries still reach a point where they credibly believe they will no longer be attacked by an opponent even without having an overtly negotiated settlement.

6 Concluding Remarks

> When crises arise, hawks counsel resolve, and the implementation of steps to make the deterrent threat credible. Their motto is 'peace through strength'. ... Doves worry about arms as such, and the irresistible momentum of military preparations, ..., because threats that are intended to deter may instead provoke. ... There is a point at which greater military strength is transformed from being a deterrent threat into a provocative threat.
>
> (Allison, Carnesale, and Nye (1985), 584-585).

As evident from this quotation, the terms hawk and dove in international relations often refer to a propensity to fight rather than arms levels or military capabilities (see also Smith [34]). Hawkish versus dovish foreign policies indicate different "attitudes" about how to face international relations in general. In contrast, we define hawks, doves and deterrents as militarization decisions and these are defined relative to an equilibrium. We have shown that in equilibrium the range of militarizations observed must include each of hawks, doves and deterrent, and have examined some comparative statics predictions of the model.

Among the interesting extensions that we are investigating in further research, is the possibility of *build-up*. That is missing from the current model, where armament resets to 0 in each period. One might conjecture that the probability of war goes down if countries can build up armaments slowly in response to each other. That turns out not to be so clear. Our basic intuition appears to be fairly robust: if a country is sure to have peace, then it should be decreasing (or increasing) its arms level slightly so that it sees some tradeoff between arms reductions and potential war. Even if that turns out to involve small probabilies of war in any given period, it can lead to nontrivial probabilities when considered over time. Even working through simple examples in such settings is quite complex, as now the past armament level becomes important and much of the Markovian structure that simplified our analysis is lost.

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Appendix

Proof of Theorem 1:

First, let us represent an action for i as a triplet A_i, t_i, q_i, where $A_i \in [0, X_i]$ is an armament level, $t_i \in [0, X_j]$ is a threshold, so that i chooses war if j's armament is below t_i and peace if j's armament level is above t_i, and where $q_i \in [0, 1]$ is the probability with which war is chosen if $A_j = t_i$.

By allowing for general mixtures over these actions, we capture all possible Markov strategies, but subject to making decisions to go to war in a monotone way based on the armaments of the other player. This will turn out to be a best response in equilibrium, and so the equilibria we find under this restriction are also equilibria of the more general game where players are not restricted to make war decisions in a monotone manner.

Next, let us examine a related game where the actions are restricted to some finite subset as follows. For each i, A_i is restricted to come from an arbitrary finite subset $Z_i \subset [0, X_i]$, t_i is restricted to come from the set Z_j, and q_i is restricted to be either 0 or 1. So this is a finite set of strategies. Let σ_i denote a mixed strategy in this game. We can view such a strategy as a finite vector indicating a probability placed on every pure action. Note that by mixing, in this game, players can still induce any probability of going to war contingent on an armament level hitting a threshold, so the restriction of q_i to be 0 or 1 is without loss of generality when mixed strategies are allowed. Thus the only real restriction is of armament and thresholds to lie in finite sets.

Given a pair of mixed strategies (σ_1, σ_2), Let $B_i(\sigma_1, \sigma_2)$ be the set of best responses of i presuming that (σ_1, σ_2) will be played in every period in the continuation (if a war has not ensued), and that σ_2 will be played in the current period. Following standard arguments for finite games this correspondence is nonempty, closed and convex-valued for each choice of (σ_1, σ_2). The correspondence is also upper hemi-continuous as a function of (σ_1, σ_2). This can be established via an argument that best responses vary upper-hemicontinuously as the payoffs to actions are varied continuously. For example, one can apply Theorem 2 in Jackson, Simon, Swinkels and Zame (2002),[21] noting that the continuation payoffs depend continuously on (σ_1, σ_2) as these are

[21] The theorem is applied to a degenerate case where the type distribution has weight one on a given type. That theorem established upper-hemi continuity of the equilibrium correspondence, but can be applied to best responses by taking some players strategies to be part of the specification of the game and then just looking at the remaining player.

mixtures.

Thus, applying Kakutani's fixed point theorem, there is a fixed point to the game restricted to finite armament and threshold levels. A fixed point is a Markov perfect equilibrium, and so for any finite Z_i there is an equilibrium σ_1, σ_2. It is easily checked that these remain an equilibrium even if we allow players to use strategies where the war decisions are functions that indicate a probability of going to war in response to each pair of armaments A_1, A_2, instead of restricting them to threshold functions as we have done.[22]

Now consider a sequence of finite pairs of subsets of admissible armaments (Z_1^n, Z_2^n) such that each point in $[0, X_i]$ is within $1/n$ of a point in Z_i^n for each i and n. Thus, we are letting the Z_i's "converge" to $[0, X_i]$. For each n, as argued above there exists a Markov equilibrium σ_1^n, σ_2^n. To embed these all in the same space, now view σ_i^n as a Borel measure on $[0, X_i] \times [0, X_j] \times [0, 1]$. Taking a subsequence if necessary, σ_1^n, σ_2^n converges (in the sense of weak convergence of measures) to a limit σ_1, σ_2.

Since we only have weak convergence, we may need to modify σ_1, σ_2 on sets of measure 0 to ensure that it is an equilibrium. We do this as follows. Associated with σ_1, σ_2 are continuation values. Fix those. Fixing those, view the game as a one shot game, where the q_i's are viewed as endogenous to the game (in the sense of endogenous sharing rules as in Simon and Zame [30][23]) Then following steps 3 to 6 in the proof of Simon and Zame [30], or else invoking their theorem where the q_i's are viewed as sharing rules, we obtain an equilibrium σ_1^*, σ_2^* of the stage game, fixing the continuation values. Moreover, σ_1^*, σ_2^* are only adjusted on sets of measure 0 and so lead to the same continuation values as σ_1, σ_2, and hence form a Markov perfect equilibrium. ∎

Let

$$V^W(A_1, A_2) = \frac{X\delta}{1-\delta}\left[P(A_1, A_2)(1+v) + (1 - P(A_1, A_2))\ell\right] - c$$

be the expected continuation value from going to war given specific armament levels after armament costs are sunk (so $U^W(A_1, A_2) = X - A_1 + V^W(A_1, A_2)$).

Proof of Theorem 2:

[22] One can invoke the finite deviation property, and so only needs to check for deviations within a single period.

[23] Simon and Zame work with sharing rules defined on utilities, but these can also be viewed as endogenous outcome choices (war or peace) following Jackson, Simon, Swinkels and Zame (2002).

Let the costs of war not be overwhelmingly high. We note that the single deviation principle holds (Theorem 4.2 in Fudenberg and Tirole (1993)), and so to verify equilibrium we need only examine deviations at any single node.

We start by showing (I): there are Markov equilibria of the overall game that lead to perpetual peace *if and only if* there exists a pure strategy equilibrium A_1, A_2 to the forced-war game such that $U_i^W(A_i, A_j) = U_i^P(A_i)$ for each i. Suppose that there exists a pure strategy equilibrium A_1, A_2 to the forced-war game such that $U_i^W(A_i, A_j) = U_i^P(A_i)$ for each i. Consider Markov strategies that play A_1, A_2 in each period, and such that country i attacks at any node following armaments A_i', A_j' such that $V_i^W(A_i', A_j') > \delta_i U_i^P(A_i)$, and not otherwise. Let us verify that these form an equilibrium. The attack decisions are best replies, so any improving deviation must involve a change in armament. Any deviation that leads to war cannot lead a higher a payoff than $U_i^W(A_i, A_j)$ given that A_1, A_2 is an equilibrium of the forced-war game. So consider a deviation that leads to peace. For it to be improving, it must involve a lowering of armament in some period. However, a lowering of armament in a given period, say by j to $A_j' < A_j$, leads to $V_i^W(A_i, A_j') > V_i^W(A_i, A_j) = \delta_i U^P(A_i)$, which leads to war by i and then this cannot be improving since A_i, A_j was a forced-war equilibrium. A deviation that leads to a mixture of peace and war must have the countries indifferent between war and the continuation, which means that it must involve the starting arms levels.

Next, let us show that a Markov equilibrium profile (σ_1, σ_2) such that peace obtains after any realization of armament actions from their support, must be a pure strategy equilibrium and must satisfy the condition in (I). Under a peaceful equilibrium, best replies must place probability one on the bottom of the support, as this results in the lowest cost of arming and does not change the war probability. This implies that the only possibility for an always peaceful Markov equilibrium is to have a pure strategy armament profile. Since the cost of war is not overwhelmingly high, each must be arming to a positive amount, or else the other would have a better reply to arm to a higher level and attack. Let the pure armament levels be A_i, A_j. It must also be that each country is indifferent between peace and war in the given period, since if i strictly prefers peace, then j could lower its arms slightly and still get a peaceful outcome which contradicts equilibrium. This indifference in one period translates to indifference over all periods, and it is then easily checked that being willing to postpone war by a period implies that $U_i^W(A_i, A_j) = U^P(A_i)$. Also, it must be that there are no improving deviations in terms of another choice of armament and a choice to go to war, and so $U_i^W(A_i, A_j) \geq U_i^W(A_i', A_j)$ for each A_i', and so A_i, A_j was a forced-war equilibrium.

Next, let us show (II): there are Markov equilibria of the overall game that lead to certain war in the first period *only if* there exists a (possibly mixed) strategy equilibrium σ_1, σ_2 to the forced-war game such that $U_i^W(A_i, A_j) \geq X_i - A_i + \delta_i U_i^W(A_i, \sigma_j)$ for at least one i and almost every A_i, A_j. Consider such an equilibrium. The strategies σ_1, σ_2 on armaments must be an equilibrium to the forced-war game, or else a country could change armament levels and attack and be better off. If $U_i^W(A_i, A_j) < X_i - A_i + \delta_i U_i^W(A_i, \sigma_j)$ for both countries for some armament, then $V_i^W(A_i, A_j) < \delta_i U_i^W(A_i, \sigma_j) = \delta_i U_i^W(\sigma_i, \sigma_j)$ and so both countries would prefer not to attack for the realized pair of armament levels. This can occur for at most a measure 0 set of realizations.

Next, let us show (III): there are Markov equilibria of the overall game that lead to certain war in the first period *if* there exists a (possibly mixed) strategy equilibrium σ_1, σ_2 to the forced-war game such that $U_i^W(A_i, A_j) \geq X_i - A_i + \delta_i U_i^W(\sigma_i, \sigma_j)$ for both i and every realization of A_1, A_2. Consider Markov strategies that play σ_1, σ_2 in each period, and such that country i attacks at any node following armaments A_i', A_j' such that $V_i^W(A_i', A_j') \geq \delta_i U_i^P(A_i)$, and not otherwise. Let us verify that these form an equilibrium. Neither country wishes to change its attack decisions, by construction. Lowering arms to a level outside of the support leads the other country to strictly prefer war, and so that cannot be improving given that the armament levels form an equilibrium to the forced-war game. By the same argument, raising arms can only be improving if it leads to peace. However, raising arms will lead a country to strictly prefer war against any arms that it faces, as it weakly preferred war at previously realized arms levels, and so fails to lead to peace.

Next, let us show (IV): there are symmetric pure strategy Markov equilibria of the overall game that lead to certain war in the first period *if and only if* there exists a symmetric pure strategy equilibrium A, A to the forced-war game such that $U_i^W(A, A) \geq U_i^P(A)$ for each i. The sufficiency of the condition follows from (III), and the necessity follows from (II) given symmetry.

To see (V), note that if all equilibria of the forced-war game fail to satisfy the condition in (II), then they also fail to satisfy (I). Thus, it follows directly from (I) and (II) that in any Markov equilibrium of the overall game and any period there is a probability of war that lies strictly between 0 and 1. ▮

Proof of Theorem 3:

Let μ denote the measure describing the armament choices associated with a symmetric Markov equilibrium in a case where (II) does not hold and the costs of war are not overwhelmingly high. Theorem 2 implies that the equilibrium must sometimes, but not always, result in war.

Let us first show that it must involve mixing over at least two armament levels. If it did not, then there would have to be mixing over the war decision, but just one armament level chosen, $A_i = A_j = A$. The mixing on attacking means that countries are indifferent between immediate attack and a delayed attack. Thus

$$V^W(A, A) = \delta \left(X - A + V^W(A, A) \right).$$

Iterating, this implies that

$$V^W(A, A) = \sum_{t=1}^{\infty} \delta^t \left(X - A \right)^t = \delta U^P(A).$$

For this to be an equilibrium, it must also be that there is no better armament level conditional on going to war, as otherwise a country could deviate to choose that arms level and then attack and get a better utility than what is expected from these arms and attacking today which is the equilibrium expected utility. Therefore, these arms levels must be a forced-war equilibrium. These observations combined imply that (II) holds, which is a contradiction. So, there are at least two arms levels chosen.

Next, let us show that min of the support are doves. Clearly, this level of arms is attacked as much as any other arms level as it has a lower probability of winning a war, and it is attacked at least some of the time since war sometimes occurs, thus (ii) and (iii) of the definition of dove hold. Next let us argue that this armament level does not wish to attack any other arms level in the support.

We argue the following stronger statement. When an arms level meets the same arms level from the opponent in the support of μ, it must strictly prefer peace. If the contrary is true, then a country with a probability of 1/2 of winning weakly prefers war, and any country with a probability of more than 1/2 of winning strictly prefers war. Start with the original equilibrium, but modify it to choose war when meeting a country of exactly the same arms level that is in the support of μ. This must always result in war, since one of the countries either has a higher arms level and then strictly prefers war, or two evenly matched countries meet and both choose war. Moreover, the continuation payoff has not changed since the only change was at a point where both countries were indifferent between war and peace. This is a contradiction of the fact that no pure war equilibrium exists.

Next, let us show that the max of the support are hawks. It is clear that this arms level wishes to go to war whenever a lower level does, and thus (i) and (iii) of the definition of hawk follow from the fact that there is a probability of war between 0 and 1. To show (ii), it is enough to show that the max of the support does not wish to attack the max of the support. This is the same argument as above.

Finally, we establish that there are deterrent arms levels. Let S_H be the armament levels in the support which are "hawks", and let S_o be the complementary part of the support. Let A_H be the max of S_H. We need only consider the case in which every armament level in S_H chooses to go to war with every armament level in S_o, since they must prefer to go to war with some levels in the complement (by definition of hawk, they sometimes attack and are never attacked), and otherwise the Theorem is proven as the remaining levels in S_o must be deterrents. Note that if S_H is not a singleton, then the lowest level in this set would deter attacks by higher levels, while arms levels in S_o would be attacked by higher level hawks, and thus the lowest level, A, in S_H would satisfy (ii) of deterrent. Let us check that it also satisfies (i). A is a strictly better reply than A_H against arms levels in the range $[A, A_H]$ since peace occurs in either case and A saves costs of arming. Thus to be indifferent between these arms levels, A must be a strictly worse response against arms levels less than A, and so (i) is satisfed. Thus, consider the remaining case where A_H is a unique hawk armament level.

Let μ_o denote the marginal of μ on S_o. If A_H is not at least as good a reply as μ_o versus μ_o, then A_H satisfies (i) and deters attacks from itself and so is a deterrent. So consider the case where A_H is at least as good a reply as μ_o versus μ_o. Let $v(\mu_o, \mu_o)$ denote the payoff of playing μ_o conditional on the other country playing μ_o given the equilibrium continuation, and similarly define $v(\mu_o, A_H)$, $v(A_H, \mu_o)$, and $v(A_H, A_H)$. Thus,

$$v(A_H, \mu_o) \geq v(\mu_o, \mu_o). \tag{6}$$

Next, note that

$$v(A_H, A_H) + v(\mu_o, \mu_o) > v(A_H, \mu_o) + v(\mu_o, A_H)$$

This follows since the total expected armaments are the same on each side, but the right hand side has strictly more war, since A_H has at least as much war against μ_o as does μ_o, while A_H does not attack A_H. This implies that

$$v(A_H, A_H) - v(\mu_o, A_H) > v(A_H, \mu_o) - v(\mu_o, \mu_o).$$

From (6) it then follows that the right hand side of the above is nonnegative, and so we conclude that

$$v(A_H, A_H) > v(\mu_o, A_H). \tag{7}$$

Together, (6) and (7) imply that A_H is at least as good a reply as μ_o against μ_o and a strictly better reply against A_H, and so a better reply against μ overall. This contradicts equilibrium in a case where A_H is played with positive probability. So, we are left with a case where A_H is played with 0 probability. Thus, since A_H is in the support, there are other arbitrarily close arms levels in the support of μ. Then given that we earlier established that A_H strictly prefers peace against A_H, continuity of the winning probability implies that A_H also prefers peace against some nearby (lower) arms level A. This implies that A satisfies (ii) of the definition of deterrent, since A_H attacks some lower arms levels. Let us check that A also satisfies (i) in the definition of deterrent. A is a strictly better reply than A_H against arms levels in the range $[A, A_H]$ since peace occurs in either case and A saves costs of arming. Thus to be indifferent between these arms levels, A must be a strictly worse response against arms levels less than A, and so (i) is satisfed. ▮

Proof of Propositions 1 and 2: Given the restriction to three actions, we argue directly that there will be an equilibrium in mixed strategies with positive weight on the three strategies and that it will be the unique equilibrium. Then since a symmetric equilibrium always exists in a symmetric finite game, the unique equilibrium must be a symmetric equilibrium.

First, we show that a lower bound on the continuation utility value for either player in any period is $\delta U^P(M)$. We do this by first showing that a lower bound on the continuation utility value for either country in any period is $\delta U^P(H)$. To establish this, we show that if country 1 always played H, then in no continuation equilibrium country 1 would be attacked by the country 2. It is enough to show that 1 would not be attacked by 2 when 2 has already armed at level H.[24] Instead of attacking, if 2 waits and arms at level H in the next period and attacks regardless of 1's next period arms, then 2's utility would be at least $\delta(X - H + V^W(H, H)$ whereas attacking now would lead to $V^W(H, H)$. The first is larger than the second since $\delta/(1-\delta)(X - H) \equiv \delta U^P(H) > V^W(H, H)$ by assumption. Thus, we have shown that a lower bound on the continuation utility value for either player in any period is $\delta U^P(H)$. Then since the continuation utility is at least $\delta U^P(H) > V^W(H, M)$, by

[24] And given the sequential nature of the war decisions, this makes it clear that no H would attack an H.

playing M a player would never be attacked, and so a lower bound on utility can be obtained by always arming at M and not ever attacking. Thus, a better lower bound on the continuation payoff in equilibrium is $\delta U^P(M)$.

Next we show that in any Markovian equilibrium:

(i) The current best responses to any Markovian strategy that mixes over just L and M are H and/or L (but not M).

(ii) The current best responses to any Markovian strategy that mixes over just M and H are either M or L (but not H).

(iii) The current best responses to any Markovian strategy that mixes over just L and H are either H or M (but not L).

To see (i) note that L is better response against both L and M than M. This follows from the fact that no M or L will ever attack an M or L since the continuation utility is at least $\delta U^P(M)$, and $\delta U^P(M) > V^W(M,L) > V^W(L,L) = V^W(M,M) > V^W(L,M)$. Thus, the expected continuation is higher than war for any such combination, and so arming at the lower level saves costs of arms without risking war.

(ii) follows from an analogous argument as (i), as no M or H would ever attack an M or H.

To see (iii), suppose that there is an equilibrium where just H and L are played by one of the countries in a Markovian strategy as part of an equilibrium.

First, we show that an upper bound on the utility of a country that has L as a best response in equilibrium to a Markovian strategy that places just weight on H and L is $\delta U^P(L)$. Note that a country must be willing to play L in every period if the other country's strategy is Markovian, provided it is willing to play L in any period. Also, the best possible outcome in any war would be to go to war against another L, and the assumptions of the proposition imply that $\delta U^P(L) > V^W(L,L)$.[25] This implies that $\delta U^P(L)$ is at least as good as any stream of L and possibly an eventual war for an L, and thus it is an upper bound on the continuation utility for a country willing to play L against a Markovian strategy.

So, suppose to the contrary of (iii) that L is a best response to a Markovian strategy that mixes over just L and H. We show that this implies a contradiction.

In this regard we first argue that in any Markovian equilibrium if an H meets an L, then the H will prefer a war to the equilibrium continuation. Since the costs of war are not overwhelmingly high, it follows that $X - H + V^W(H,L) > X - L + \delta U^P(L)$. This implies that $V^W(H,L) > \delta U^P(L) = \delta(X-L)/(1-\delta)$. This in turn implies

[25] Here $\delta U^P(L) > \delta U^P(M) > V^W(M,L) > V^W(L,L)$.

that $V^W(H,L) > \delta(X-L)+\delta V^W(H,L)$. This tells us that $V^W(H,L)$ is better than delaying with the best possible peace payoff and having a future war in one period at the best possible odds $(\delta(X-L)+\delta V^W(H,L))$. Inductively, $V^W(H,L)$ is better than having peace at the best level for some finite number k of periods and then a war with the best possible odds. Directly from above $V^W(H,L)$ is also better than delaying infinitely $(\delta U^P(L))$, and so it is better than any possible continuation outcome, and so must be better than the expected equilibrium continuation value.

Second, we argue that H is a better response against a realization of H than L. Suppose to the contrary that L would at least as good a response against H as H. From above, we know that in equilibrium an L will be attacked by an H. Then, the supposition that that L would at least as good a response against H as H implies that

$$X - L + V^W(L,H) \geq X - H + V^{Continuation} \geq X - H + \delta U^P(M).$$

Therefore $H - L \geq \delta U^P(M) - V^W(L,H)$. This, together with the costs of war not being overwhelmingly high implies that

$$V^W(H,L) + V^W(L,H) > \delta U^P(L) + \delta U^P(M).$$

Note that

$$V^W(H,L) + V^W(L,H) = V^W(H,M) + V^W(M,H) < V^W(H,M) + V^W(M,L).$$

Thus,

$$V^W(H,M) + V^W(M,L) > \delta U^P(L) + \delta U^P(M).$$

This contradicts the facts that $\delta U^P(L) > \delta U^P(H) > V^W(H,M)$, and $\delta U^P(M) > V^W(M,L)$.

Finally, we argue that H is a better response against a realization of L than L. Given $\delta U^P(L)$ is at least the continuation value, then since the costs of war are not overwhelmingly high implies that $X - H + V^W(H,L) > X - L + \delta U^P(L)$, the result follows.

Thus, we have shown that if one country mixes over just H and L then the other country would prefer to play H instead of L, reaching a contradiction and establishing (iii).

We have shown that a Markov equilibrium must have weight on all three actions by both countries. We also can deduce (following the arguments above) that an H will attack an L, but all other types prefer peace. From this we can determine the continuation payoffs, and remark that $U^P(M)$ is the ex ante equilibrium continuation value, since a player is always indifferent between playing M and the other strategies in a mixed strategy equilibrium.

We can now characterize equilibrium as follows (and its uniqueness follows from the characterization). Let a be the equilibrium weight on $A_i = L$, b on $A_i = M$, and $1 - a - b$ on $A_i = H$. The equilibrium indifference conditions determining a^*, b^* are that each player should be indifferent between playing M and H, as well as M and L:

$$X - H + aV^W(H, L) + (1 - a)\delta U^P(M) = U^P(M), \tag{8}$$

$$X - L + (a + b)\delta U^P(M) + (1 - a - b)V^W(L, H) = U^P(M). \tag{9}$$

We solve for a^* and b^* from (8) and (9). Noting that $(1 - \delta)U^P(M) = X - M$, we write

$$a^* = \frac{H - M}{V^W(H, L) - \delta U^P(M)} \tag{10}$$

and

$$b^* = \frac{U^P(M)(1 - a^*\delta) - (X - L) - (1 - a^*)V^W(L, H)}{\delta U^P(M) - V^W(L, H)} \tag{11}$$

or

$$b^* = 1 - a^* - \frac{M - L}{\delta U^P(M) - V^W(L, H)} \tag{12}$$

Plugging in for a^*, we can solve for b^*.

$$b^* = 1 - \frac{H - M}{V^W(H, L) - \delta U^P(M)} - \frac{M - L}{\delta U^P(M) - V^W(L, H)}. \tag{13}$$

From this, we also deduce that the remaining weight on H is

$$c^* = 1 - a^* - b^* = \frac{M - L}{\delta U^P(M) - V^W(L, H)}. \tag{14}$$

The probability of peace is $1 - 2a^*c^*$.

$$1 - 2a^*c^* = 1 - 2\left(\frac{H - M}{V^W(H, L) - \delta U^P(M)}\right)\left(\frac{M - L}{\delta U^P(M) - V^W(L, H)}\right). \tag{15}$$

Note that

$$V^W(H, L) + V^W(L, H) = \frac{\delta X(1 + v + \ell)}{1 - \delta} - 2c.$$

Thus,

$$\Pr[\text{Peace}] = 1 - 2\left(\frac{H - M}{V^W(H, L) - \delta U^P(M)}\right)\left(\frac{M - L}{V^W(H, L) + 2c - \frac{\delta(X(\ell+v)+M)}{1-\delta}}\right). \tag{16}$$

The comparative statics follow directly.[26] ▮

Proof of Proposition 3:

Following the proof of Proposition 1, we can still argue that a lower bound on the equilibrium continuation payoff is $\delta U^P(M)$, regardless of the level of π.

The proof of the uniqueness of equilibrium in Proposition 1 worked from the following observations.

(i) The best responses to any mixture over just L and M are either H or L.

(ii) The best responses to any mixture over just M and H are either M or L.

(iii) The best responses to any mixture over just L and H are either H or M.

From these, it follows easily that any equilibrium must involve a mixture over all three strategies. The introduction of a positive probability of settlement, increases the payoff of L versus H, but does not change any of the other payoffs. Thus, (i) and (ii) continue to hold. Thus, provided it does not change conclusion (iii), then it does not change the uniqueness result and the rest of the arguments in the proof of Propositions 1 and 2. Since (iii) holds for $\pi = 0$ (and L is never a best reply in that case), then it will hold for some interval of π. Let $\overline{\pi}$ be the level of settlement such that (iii) changes so that best responses to any mixture over just L and H include L.

As long as π is such that M must be played in equilibrium together with H and L, it continues to be true that the symmetric equilibrium payoff must be equal to $U^P(M)$. Thus, the derivation of a^* is identical to the case with $\pi = 0$.[27]

$$a^*(\pi) = \frac{H - M}{V^W(H, L) - \delta U^P(M)}, \tag{17}$$

which is independent of π. Thus, the only effect of an increase in π is an increase in c^* and a decrease in b^*, eventually hitting the point where b^* becomes 0 for some π less than 1, and other equilibria take over.

To see this, note that the second equilibrium condition is now different from (9), because of the transfer made by the L type:

$$X - L + (a+b)\delta U^P(M) + (1-a-b)[\pi(\delta U^P(M) - T^*) + (1-\pi)V^W(L, H)] = U^P(M) \tag{18}$$

[26] To see the comparative statics in the discount factor, note that we can factor out an expression of $\delta/(1-\delta)$ from each expression in the denominators (including $V^W(H, L)$), except for c.

[27] Given that H must be indifferent between war and peace plus $T^* = V^W(H, L) - \delta U^P(M)$ in equilibrium, the equilibrium indifference condition (8) continues to be the first of the two relevant equilibrium conditions here, and hence the expression for a^* continues to be that in (10).

or

$$X-L+(a+b)\delta U^P(M)+(1-a-b)[V^W(L,H)+\pi(2\delta U^P(M)-V^W(H,L)-V^W(L,H))] = U^P(M). \tag{19}$$

From this we have

$$b^* = 1-a^*-\frac{M-L}{\delta U^P(M)-[V^W(L,H)+\pi(2\delta U^P(M)-V^W(H,L)-V^W(L,H))]} \tag{20}$$

which is decreasing in π.

Plugging a^* we have

$$b^* = 1-\frac{H-M}{V^W(H,L)-\delta U^P(M)}-\frac{M-L}{\delta U^P(M)-[V^W(L,H)+\pi(2\delta U^P(M)-V^W(H,L)-V^W(L,H))]} \tag{21}$$

and

$$c^* = 1-a^*-b^* = \frac{M-L}{\delta U^P(M)-V^W(L,H)-\pi(2\delta U^P(M)-V^W(H,L)-V^W(L,H))}. \tag{22}$$

Given that commitment is available a fraction π of the times in which a war might erupt, the probability of peace in any given period is now

$$\Pr[\text{Peace}] = 1-2(1-\pi)a^*c^*. \tag{23}$$

Recall that

$$2\delta U^P(M)-V^W(H,L)-V^W(L,H) \geq 0$$

since it represents the difference in the total utility from having perpetual peace at a cost of perpetually arming at the deterrent level (the equilibrium continuation expected utility) or instead having a war and incurring the costs of war. For transfers to make sense, this must be positive.

The probability of peace, $1-2(1-\pi)a^*c^*$ is then

$$1-2(1-\pi)\left(\frac{H-M}{V^W(H,L)-\delta U^P(M)}\right)\left(\frac{M-L}{\delta U^P(M)-V^W(L,H)-\pi(2\delta U^P(M)-V^W(H,L)-V^W(L,H))]}\right). \tag{24}$$

Noting again that

$$V^W(H,L)+V^W(L,H) = \frac{\delta X(1+v+\ell)}{1-\delta}-2c$$

we see that c^* becomes

$$c^* = \frac{M-L}{V^W(H,L)+\pi\frac{\delta X}{1-\delta}+(1+\pi)\left(2c-\frac{\delta(X(v+\ell)+2M)}{1-\delta}\right)}. \tag{25}$$

Thus,

$$\Pr[\text{Peace}] = 1-2(1-\pi)\left(\frac{H-M}{V^W(H,L)-\delta U^P(M)}\right)\left(\frac{M-L}{V^W(H,L)+\pi\frac{\delta X}{1-\delta}+(1+\pi)\left(2c-\frac{\delta(X(v+\ell)+2M)}{1-\delta}\right)}\right). \tag{26}$$

The comparative statics follow directly, completing the proof of (I) and (II).
(III): The derivative of probability of peace with respect to π is

$$2\left(\frac{H-M}{V^W(H,L)-\delta U^P(M)}\right)\left(\frac{M-L}{\delta U^P(M)-V^W(L,H)-\pi(2\delta U^P(M)-V^W(H,L)-V^W(L,H))}\right) \tag{27}$$

$$\left[1-(1-\pi)\left(\frac{2\delta U^P(M)-V^W(H,L)-V^W(L,H)}{\delta U^P(M)-V^W(L,H)-\pi(2\delta U^P(M)-V^W(H,L)-V^W(L,H))}\right)\right].$$

Simplifying this expression, it is proportional to

$$\pi+(1-\pi)\left(\frac{V^W(H,L)-\delta U^P(M)}{\delta U^P(M)-V^W(L,H)-\pi(2\delta U^P(M)-V^W(H,L)-V^W(L,H))}\right),$$

which is nonnegative. Thus, the probability of peace increases in π.

As the probability of bargaining being possible increases, the probability of doves stays constant and the probability of Hawks increases. This implies that conditional on realizations where it turns out that bargaining is not possible, we now expect a higher probability of war. ∎