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Turnover Reflects Specific Training Better Than Wages Do

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Turnover Reflects Specific Training Better Than Wages Do[*]
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Abstract:

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Turnover falls with tenure — this is one of the best established empirical regularities of labor economics — but finding a tenure effect on wages seems to be very hard. Within-job wage cuts do not seem very uncommon either. We reconcile these findings by revisiting an old question: how gains from firm specific training are split between workers and firms. The division is determined by a stationary distribution of outside offers. The model is ex post monopsony: the lower a wage a firm pays to a specifically trained worker, the more profit it makes and the more eager it is to have her stay, but the more likely she is to leave. The optimal time paths of wages and turnover probabilities show that even if marginal product is increasing, wages need not be increasing; but rising marginal product always implies a falling turnover rate.

JEL Classification: J33, J41, J63

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1 Introduction

Most extant theories of the labor market predict that if specific training occurs, increasing tenure on the job will *both* raise wages monotonically and cut turnover. Since current empirical evidence supports very strongly the proposition that tenure cuts turnover but does not support the proposition that tenure raises wages, we are left in a quandary: extant theory cannot tell us whether specific training occurs or not. In this paper we resolve this quandary with a new theory. If specific training occurs, increasing tenure must cut turnover, but it need not raise wages monotonically. Current empirical evidence thus leads us to believe that specific training occurs.

The early empirical support for wage increases with seniority was based on evidence of positive cross-sectional association between seniority and earnings. However, as Abraham and Farber (1987) and Altonji and Shakotko (1987) argue, this evidence is insufficient to establish that earnings increase with seniority. For instance, if high wage jobs (due to say heterogeneity of worker-firm match quality) are more likely to survive than low wage jobs, then seniority will be positively correlated with high wages even though individual wages do not rise with seniority. Using longitudinal data and corrections for various potential sources of heterogeneity bias, both these studies find that the cross-sectional return to tenure is a statistical artifact of heterogeneity bias, and that the true wage return to tenure is small if not negligible. However, a later study by Topel (1991), also using longitudinal data and accounting for selection due to optimal mobility decisions, shows that wages do rise with seniority. A recent reassessment by Altonji and Williams (1997) concludes that Topel over estimates the returns, and that wage returns to tenure, across all these estimation procedures, are modest at best. Another noteworthy study is Ransom (1993) who finds that among university professors higher seniority is associated with lower salaries. Other studies using longitudinal data from personnel records of large companies, where measurement error is much less likely than in survey data, find that within-job wage cuts are not uncommon either.[1] In summary, this newly emerging empirical literature shows that wage dynamics are far more complicated and variegated than the simple presumption that wages monotonically increase with tenure.

We reconcile these findings by revisiting an old question: how the gains from specific training are split between workers and firms. The division is determined by a stationary distribution of outside offers. The model is ex post monopsony: the lower a wage a firm pays to a specifically trained worker, the more profit it makes and the more eager it is to have her stay, but the more likely she is to leave. We solve for the optimal time path of wages and turnover probabilities. Even if marginal product is increasing, wages need not be increasing; but rising marginal product always implies a falling turnover rate. Hence our model resolves the apparent paradox of the weak or nonexistent tenure effect on wages along side of a strong negative tenure effect on turnover. Specific training should always cut turnover, but there is no theoretical reason why it should always raise wages. The model also shows why within-job wage cuts are a real possibility.

Before proceeding to a discussion of related theory, we briefly outline some salient features of our model. First, the wage sequence a firm can offer a worker depends on the commitment ability of the firm – i.e. whether firms can be trusted not to renege on a promised wage schedule since productivity increases on the job are assumed to be firm specific. Our focus in this paper is on the no-commitment-ability case or on self-enforcing contracts, and thus the main conclusions of the paper are immune from charges of dynamic inconsistency. However, we also show that if firms have complete commitment ability, then they will delay payments for as long as they can. This is an argument for pensions quite independent of disciplinary considerations.

Second, we study *ex post* monopsony and not *ex ante* monopsony as well. As

[1] See for example the study by Baker, Gibbs, and Holmstrom (1994).

a consequence the model is silent about where outside offers come from. However, our central assumption that workers receive outside job offers is based on the following considerations. The idea that workers receive outside offers from a stationary distribution rests on the job search assumption (Burdett 1978) that workers have imperfect information about the location of high wage jobs (Stigler 1962). Jovanovic (1979a) gives this job search framework an equilibrium interpretation by claiming that the distribution of outside job offers is supported by heterogeneity of match quality across all worker-firm pairs. The stationarity of the distribution highlights the fact that the skills acquired in an employment relationship are firm specific. We adopt the simplest job search framework in our paper, namely, that workers receive a single job offer from a stationary distribution in each period.[2] The key question is the determination of the value of a job offer. We assume that firms offer a self-enforcing wage schedule, given a productivity profile, such that expected profits are zero. In other words, we assume a competitive labor market for prospective workers. In this paper we do not explicitly discuss how such a competitive market might arise. A more complete discussion about job search and the market for prospective workers when employment relationships generate specific rents can be found in Munasinghe (2001).

Third, we assume that specific training is strictly learning-by-doing. It happens automatically as a worker's tenure increases; neither firm nor worker needs to make either decisions or sacrifices in terms of investments. As a consequence our model is silent about layoffs.

The remainder of the paper is organized as follows. In section 2 we discuss some related theory. Section 3 presents the model and main results. In section 4 we present some numerical examples based on specific distributions of outside offers to highlight the key results of the paper, including the possibility of within-

[2] Endogenizing search effort would add greater realism to the model, but for analytical simplicity we maintain the simple job search assumption.

job wage cuts. A brief summary and appendix concludes the paper.

2 Related theory

The model in this paper is related to a variety of other compensation and turnover models. The problem of wage determination when an employment relationship generates firm specific rents has been well known since Becker's (1962) original idea of sharing the costs and benefits of firm specific investments as a means of providing mutual insurance to each party's investment. Becker recognized the inherent inefficiency entailed in this sharing hypothesis by noting the "diseconomies" resulting from quits and layoffs that do not take into consideration the resulting loss to the other party's investment. Parsons (1972) builds on Becker's work by clarifying the role of specific human capital in the analysis of inter-firm mobility. The key idea developed in his paper is that quit and layoff rates depend on the division of specific human capital into firm-owned and worker-owned components, respectively, as well as on the volume of specific human capital. Parsons does not focus on the sharing rule *per se* and thus his paper is largely silent about wage dynamics. Hashimoto (1981) argues that the basis for precommiting to a sharing rule is the transaction cost associated with *ex post* evaluation of the worker's productivity in the firm and elsewhere. One implication of this two period model is that the wage in the post investment period will be higher than the wage in the investment period, implying an increasing wage profile. However, the restriction to two periods hampers his ability to observe interesting wage dynamics.

These earlier models offer interesting points of comparison with our model. First, a result common to all these models, including ours, is the inherent inefficiency of turnover. Second, the major difference between these earlier models and our model is that in our model wages do not increase monotonically with tenure even if marginal product increases monotonically with tenure. In the earlier sharing literature a standard implication is an increasing wage-tenure profile. Recent empirical research has failed to turn up evidence of such profiles.

Mortensen (1978) focuses on the inefficiency of the sharing hypothesis and thus considers various wage bargaining strategies – counter offers and compensation as a precondition to termination – that might lead to joint wealth maximizing outcomes in the presence of match specific capital.[3] The joint wealth maximizing strategies generally predict lower turnover rates. However, the key result in Mortensen is that although turnover declines with specific capital, turnover is independent of the division of specific rents. Hence the paper is not focused on rent division and its effects on turnover, but rather on joint wealth maximizing strategies that might overcome the inherent inefficiencies of sharing rules. Hashimoto (1981) observes that transactions costs of determining post-investment productivity are likely to be too great for such bargaining strategies to be compelling. We also exclude *ex post* bargaining in our model.

Jovanovic (1979b) is one of the first theoretical articles explicitly to integrate human capital theory and job search theory. In that sense, this paper is similar to his. In Jovanovic's model, match quality determines expected job duration which in turn jointly determines optimal search effort and investment in firm specific human capital. Jovanovic's central result is that turnover declines with tenure. Although wages are endogenously determined in the Jovanovic model, as in the model presented in this paper, his model is not designed to study wage dynamics. The model here is explicitly designed to do so. Also, in Jovanovic's model the employer makes a wage offer to the worker that is equal to marginal product. The justification for such a policy is based on reputation repercussions. As Jovanovic says, "employers offering wages below marginal product will acquire bad repu-

[3] Munasinghe (2001) is also based on a counter offer mechanism. That is, if a worker receives a better outside offer the current firm is allowed to match the outside offer. This *ex post* bargaining strategy leads to an efficient turnover rule, but it also implies inefficiently high search effort. In the model presented here, we do not allow *ex post* bargaining.

tations and will consequently not be sampled by workers" (p. 1249, Jovanovic 1979b). But firms that do not have the requisite reputation will need to offer time consistent wage policies. By contrast, wage determination is dynamically consistent in the model presented here.

Our model also shares some parallel results with Black and Loewenstein (1991). Their model is based on heterogeneity of mobility costs, and employers have monopsony power because it is costly for workers to switch employers. Although the source of monopsony power in our model is specific training, and thus different, our result of signing bonuses in some cases is equivalent to their result of front loaded contracts because in both cases workers in anticipation of future monopsony power of the employer will demand higher wages up front. Another interesting point is Black and Loewenstein's claim that in the absence of specific training wages are a decreasing function of tenure.[4] However, in their model setup specific training tends to increase wages as tenure lengthens. By contrast our model can generate falling wages even when specific training is on going, depending on the distribution of outside offers.

The model presented here is closely related to an earlier paper (Munasinghe 2001) that is also designed to explain a variety of stylized findings related to wage and turnover dynamics, including the following empirical puzzle: past wage growth on a job reduces turnover, but there is no evidence of serial correlation of wage increases. Although the model in this paper is not explicitly designed to explain the above empirical puzzle, it does suggest that serial correlation of within-job wage growth is likely to be a poor test of wage growth heterogeneity if even wage levels sometimes decrease with tenure. This earlier model, like the model here, assumes a stationary distribution of outside offers, but unlike the model here, it also assumes downward wage rigidity and *ex post* bargaining. As

[4]This result is similar to Ransom's (1993) monopsonistic discrimination model, also based on heterogeneity of moving costs, designed to explain the negative association between earnings and seniority among university professors.

a result the model in Munasinghe (2001) cannot theoretically address the finding about within-job wage decreases. The model here explicitly shows why within-job wage cuts can occur.

3 General Propositions

3.1 Notation

Time is discrete, and t designates the number of periods of completed tenure that a worker has accumulated at the beginning of the current period. Let y_t denote the marginal revenue product of a worker who has accumulated t periods on the job; thus y_0 is the marginal product of a new worker. We assume that (y_t) is a monotonically increasing sequence; specific training occurs.

Let w_t denote the wage of a worker who has completed t periods; we refer to this as the wage of an age-t worker. At the beginning of each period (except period 0) the worker receives an outside offer. The outside offer has a present value of θ, where θ is a random variable drawn independently each period from the same distribution, for which $G(.)$ is the cdf. We assume that $G(.)$ is strictly increasing on its support. Stationarity reflects the assumption that it is *specific* training that we are talking about; since the training affects only the worker's productivity within the firm, there is no reason for the distribution of outside offers to change.

Let v_t denote the worker's value of optimal continuation with the firm. This value is evaluated before the outside option becomes known. Assume the discount factor is $\delta < 1$, and that work involves no utility or disutility. Then

$$v_t = E\left[\max\left(\theta, w_t + \delta v_{t+1}\right)\right],$$

for $t \geq 1$, and

$$v_0 = w_0 + \delta v_1,$$

because the worker cannot receive an outside offer before going to work (we take this as a definition of what an "outside" offer is). It will be convenient to write

$$x_t = w_t + \delta v_{t+1}.$$

Then the probability that a worker quits depends on x_t; specifically a worker stays in period t if and only if $\theta \leq x_t$ (in the event of ties we assume the worker stays).

Let h_t, $\forall t \geq 1$, denote the probability that an age-t worker stays on the job. Then

$$h_t = G(x_t).$$

We refer to h_t as the continuation hazard. Hence

$$v_t = h_t x_t + (1 - h_t) E(\theta | \theta > x_t).$$

For future reference, note that

$$v_t = \int^{x_t} x_t dG(\theta) + \int_{x_t} \theta dG(\theta)$$

and so

$$dv_t = G(x_t) dx_t > 0 \tag{1}$$

On the firm side, let V_t denote the optimal continuation value for a firm with an age-t worker. Each period the firm chooses a wage offer w_t to maximize its expected present value. Since the firm has no pre-commitment power, it can assume that its future wages, w_{t+1}, w_{t+2}, etc., will also be set optimally and dynamically consistently, and so can consider them as given when it chooses w_t. Thus we can think of the firm as choosing x_t rather than w_t. But since $G(.)$ is strictly increasing, it is an invertible function, and we can think of the firm as choosing h_t, the continuation hazard. Let $F(.)$ be the inverse of G:

$$h = G(F(h)).$$

Then V_t is given by the fundamental recursion equation

$$V_t = \max_h [h(y_t - w_t + \delta V_{t+1}) + (1-h)V_0],\ \forall t \geq 1,$$

where

$$w_t = F(h_t) - \delta v_{t+1}.$$

We can further simplify by writing R_t as the total rent from the relationship

$$R_t = V_t + v_t.$$

Then the above recursion equation becomes

$$V_t = \max_h [h(y_t - F(h) + \delta R_{t+1}) + (1-h)V_0].$$

Assume $V_0 = 0$. We make this assumption because we want to study *ex post* monopsony – monopsony after the worker has been hired and has acquired some specific skills – not *ex ante* monopsony as well. So we must suppose that there are many firms identical to this one bidding for the worker, and that if V_0 were positive even more firms would enter. The market for prospective workers is competitive. This assumption is not crucial, but simplifies the mathematics, and is consistent with much other theory in labor economics.

Finally, let R_t^* denote expected surplus at time t contingent on the worker staying on the job that period

$$R_t^* = y_t + \delta R_{t+1}$$

Note that R_t^* is independent of the firm's action at t.

Then, the recursion equation for the firm is

$$V_t = \max_h \{h(R_t^* - F(h))\}$$

Let h_t denote the optimal continuation hazard. Then

$$\begin{aligned} R_t &= V_t + v_t \\ &= h_t R_t^* + (1 - h_t) E(\theta | \theta > F(h_t)) \\ &= h_t R_t^* + \int \theta dG(\theta) \end{aligned}$$

Bargaining and wage setting are somewhat different in period 0. Since we assume as a convention that no outside offers are possible in period 0 (since there is no "outside" until period 0 has been completed), we take the "continuation hazard" to be undefined, since there is no relationship to be continued. Wages have to be set to lure the worker in, not keep her from leaving, and our perfect competition assumption implies that wages are high enough that V_0, the firm's value of a new worker, is zero. Hence

$$w_0 = y_0 + \delta V_1.$$

Since wages in period 0 are set by a different process from wages in other other periods, we will not pay much attention to them. They may include such things as signing bonuses and initiation fees which are not our primary interest in this paper.

3.2 The fundamental proposition

Now we prove that (h_t) is a weakly increasing sequence: turnover always falls as tenure increases.

Consider the firm's optimization problem in period t, and suppose that it has an interior solution. Then the first order condition must be satisfied:

$$R_t^* - F(h_t) - h_t F'(h_t) = 0$$

or

$$R_t^* - z(h_t) = 0 \tag{2}$$

where

$$z(h_t) \triangleq F(h_t) + h_t F^{'}(h_t)$$

Then the second-order condition

$$-z'(h_t) < 0 \tag{3}$$

must be satisfied.

We need two lemmas:

Lemma 1: h_t is a weakly increasing function of R_t^*.

Proof: First suppose that the solution to the firm's optimization problem is interior and so (2) holds. Differentiate (2)

$$dR_t^* - z'(h_t)dh_t = 0$$

and so

$$dh_t = \frac{1}{z'(h_t)}dR_t^*$$

which is positive by (3). Second, suppose $h_t = 0$. Then h_t cannot decrease in response to a change in R_t^*. Finally, suppose $h_t = 1$. Then increasing R_t^* does not cause h_t to decrease. This exhausts all the possibilities.

Lemma 2: R_t is a weakly increasing function of R_t^*.

Proof: Since $R_t = V_t + v_t$,

$$dR_t = dV_t + dv_t.$$

From the envelope theorem and $V_t = \max_h\{h(R_t^* - F(h))\}$, we get:

$$dV_t = h_t dR_t^* \geq 0.$$

From (1):

$$dv_t = G(F(h_t))dF(h_t),$$

and so since by lemma 1 h_t is weakly increasing in R_t^* and $F(.)$ is a strictly increasing function

$$dv_t \geq 0.$$

Hence

$$dR_t \geq 0.$$

Lemma 3: An increase in $y_{t+\tau}$, for any $\tau \geq 0$ increases R_t.

Proof: Let $\tau = 0$. If y_t increases, R_t^* increases and by lemma 2, R_t increases.

Let $\tau > 0$. If $y_{t+\tau}$ increases, $R_{t+\tau}^*$ increases. By lemma 2, the increase in $R_{t+\tau}^*$ increases $R_{t+\tau}$. That implies that

$$R_{t+\tau-1}^* = y_{t+\tau-1} + \delta R_{t+\tau}$$

increases. The increase in $R_{t+\tau-1}^*$ in turn increases $R_{t+\tau-1}$, which increases $R_{t+\tau-2}^*$, and so on, until R_t increases.

Thus we can write

$$R_t = \Omega(y_t, y_{t+1}, y_{t+2}, ...y_{t+\tau}, ...),$$

given G and δ, and note that $\Omega(.)$ is a weakly increasing function of each of its arguments.

Lemma 4: (R_t) is a (weakly) increasing sequence.

Proof: Consider R_t and R_{t+1}.

$$\begin{aligned} R_t &= \Omega(y_t, y_{t+1}, ...y_{t+\tau}, ...) \\ R_{t+1} &= \Omega(y_{t+1}, y_{t+2}, ...y_{t+\tau+1}, ...) \end{aligned}$$

Since $y_{t+1} > y_t$, $y_{t+2} > y_{t+1}$, $y_{t+\tau+1} > y_{t+\tau}$, ..., $R_{t+1} \geq R_t$, by repeated application of lemma 3.

Proposition: (h_t) is a (weakly) increasing sequence.

Proof: By assumption (y_t) is a weakly increasing sequence. By lemma 4, R_t is a weakly increasing sequence. Hence since

$$R_t^* = y_t + \delta R_{t+1}$$

is a weakly increasing sequence, by lemma 1, h_t is a weakly increasing sequence also.

Thus no matter what the distribution of outside offers is, turnover decreases as tenure lengthens. The value of the job to the worker $F(h_t)$ also monotonically increases as tenure grows.

This doesn't imply, however, that wages grow monotonically. Current wages are only part of the worker's inducement to reject outside offers. The other part is the continuation value. The continuation value may or may not be growing faster than the total inducement that the firm wants to provide. Only if the total inducement is growing faster than the continuation value will wages rise. This may or may not happen.

We can also prove that turnover will be inefficiently high in this equilibrium. This result, however, is well-known, intuitively obvious (since it's monopsony), and the proof would introduce more notation. We therefore omit it.

Turnover in this model depends on R_t^*, the rent the relationship would generate if the worker turned down this period's offer. Specific training decreases turnover because it increases rent. More specific training implies lower turnover in the sense that a job where marginal productivity is higher every period than it is in another job will have (weakly) lower turnover every period. Rent drives turnover in Mortensen's matching-offers model, too.

The relationship between rent and turnover is first order, however, not second order: bigger increases in R_t^* do not necessarily correspond with bigger increases in h_t. This is because the function that links h_t and R_t^* is in general non-linear. (Specifically, from (2), $h_t = z^{-1}(R_t^*)$, where z^{-1} denotes an inverse function, but z and its inverse are generally non-linear.) Mortensen's model also has this property.

In one case, however, we can derive a result that resembles "more quickly rising marginal product implies more quickly falling turnover rate." Let (y_t') and (y_t'') be two sequences of marginal products, and let (h_t') and (h_t''), respectively, be the associated sequences of retention rates, all other parameters being the same.

Then the following minor proposition follows almost immediately from lemmas 1, 2, and 3.

Proposition: Let $y'_t = y''_t$ for all $t \geq T > 1$ and $y'_t > y''_t$ for all $t < T$. Then $h'_t = h''_t$ for all $t \geq T$ and $h'_t \geq h''_t$ for all $t < T$, with strict inequality if $h''_t < 1$.

In other words, for any t, $0 < t < T$, and τ, $\tau \geq T$, we have:

$$y''_\tau - y''_t > y'_\tau - y'_t,$$

the double-prime marginal product is increasing faster than the single-prime marginal product; and

$$(1 - h''_t) - (1 - h''_\tau) \geq (1 - h'_t) - (1 - h'_\tau),$$

double-prime turnover rate is falling faster than the single-prime turnover rate.

3.3 Full commitment case

The sequence of wages that the firm offers depends on its commitment ability. We have focussed on the case where the firm has no commitment ability. However, if the firm has complete commitment ability, then the solution to the firm's problem is trivial: it postpones payment as long as it can. In the appendix we show this for the uniform case.

4 Specific distributions and numerical examples

In this section we show how the distribution of outside offers affects the sequence of wages and of continuation hazards. We present one example with monotonically rising wages, and one example without.

4.1 Uniform distribution of outside offers

If outside offers are drawn from a uniform distribution, then (except possibly between period 0 and period 1), wages will rise monotonically, and then be constant (if marginal product rises high enough).

Assume that each period's outside offer θ is distributed uniformly on the unit interval. Then $x_t = h_t$ when both are less than or equal to 1, and so the firm's problem is to choose h to maximize

$$h(R_t^* - h)$$

subject to $h \leq 1$.

The solution to this problem has two phases. If $R_t^* \leq 2$, then $h_t = \frac{1}{2}R_t^* \leq 1$. We call this the low-rent phase. If $R_t^* \geq 2$, then $h_t = 1$. We call this the high-rent phase. Since R_t^* is increasing, the low-rent phase (if there is one) always precedes the high-rent phase (if there is one). During the high-rent phase, the worker is so valuable that the firm assures that she never leaves.

We examine the high-rent phase first. Since the worker never leaves, $R_t^* = R_t$, which is simply the present value of future output. For algebraic simplicity, assume that y_t is rising at rate $(g - 1)$

$$y_t = gy_{t-1}.$$

Then

$$R_t = R_t^* = \frac{y_t}{1 - \delta g}$$

where we assume $\delta g < 1$ to assure convergence. Then a necessary and sufficient condition for period t to be in the high-rent phase is

$$y_t \geq 2(1 - \delta g).$$

Let T denote the first period in the high-rent phase, and for algebraic simplicity, assume

$$y_T = 2(1 - \delta g).$$

Then $x_T = h_T = 1$, $R_T = R_T^* = 2$, $v_T = V_T = 1$, and $w_T = 1 - \delta$.

At period T, surplus is split evenly between firm and worker. However, after period T, the relationship continues to grow more valuable, but the firm no longer needs to pay the worker more, since no outside firm can top what the worker is receiving. Thus, after T, V_T continues to grow but v_t remains at one, and wages remain the same.

In the low-rent phase, quitting is possible. Then

$$\begin{aligned} v_t &= h_t h_t + (1-h_t)E(\theta|\theta > h_t) \\ &= h_t^2 + \frac{1}{2}(1-h_t^2) \\ &= \frac{1}{2} + \frac{1}{2}h_t^2 \end{aligned}$$

and

$$V_t = h_t^2.$$

Hence

$$R_t = v_t + V_t = \frac{1}{2} + \frac{3}{2}h_t^2,$$

and

$$R_t^* = 2h_t = y_t + \delta R_{t+1} = y_t + \delta[\frac{1}{2} + \frac{3}{2}h_{t+1}^2].$$

Solving for h_t, we derive a difference equation:

$$h_t = \frac{1}{2}y_t + \frac{1}{4}\delta + \frac{3}{4}\delta h_{t+1}^2,\ t \geq 1.$$

Since

$$w_t = h_t - \delta v_{t+1},$$

we have

$$w_t = \frac{1}{2}y_t - \frac{1}{4}\delta(1-h_{t+1}^2),\ t \geq 1. \tag{4}$$

These are the fundamental equations for the uniform case.

Since h_{t+1} increases as t increases, it is clear from (4) that wages increase during the low-rent phase (for $t \geq 1$), and increase at a faster rate than output. But they stagnate when the high-rent phase is reached, even though output keeps rising.

For a numerical example, we take $g = 1.02$ (2% growth in output per period), $\delta = .8$ (discount rate of 25%), and $y_0 = \frac{1}{3}$. Then the high-rent phase begins at period 5, with $y_5 = .368$. The sequence of wages and hazard rates in the low-rent phase is shown in figure 1.

Figure 1

Period	1	2	3	4	5	6	7	8	9	10
h_t	.8767	.9190	.9536	.9804	1	1	1	1	1	1
w_t	.1389	.1553	.1691	.1804	.2	.2	.2	.2	.2	.2
y_t	.3400	.3468	.3537	.3607	.368	.375	.383	.391	.398	.406

In period 0, in order to bring $V_0 = 0$, we have $w_0 = .9549$, a large signing bonus is provided. Of course, if marginal product were not so smoothly rising, the bonus would not have to be paid. For instance, if the firm had to incur hiring and start-up costs so that net product in period 0 were $-.4760$, zero-period wages would be the same as first period wages.

Although wages are monotonically rising, except for the signing bonus, there is no simple sharing rule. Wages as a proportion of current output rise from .41 in period 1 to .54 in period 5, and then fall throughout the high-rent phase, asymptotically approaching zero.

4.2 A two-point distribution of outside offers

Now suppose that the outside offer can take one of only two values: θ_l with probability p, and θ_h with probability $(1-p)$, $\theta_h > \theta_l$. (Strictly speaking, this distribution violates our assumption of a strictly increasing cdf, but it can be approximated closely by a distribution with an increasing cdf, and the results will be arbitrarily close to the results here.) With this distribution, the continuation hazard is weakly monotonically increasing, but the wage is not.

With the two-point distribution, we have

$$F(1) = \theta_h,\ F(p) = \theta_l,\ F(0) = 0$$

and the firm's problem is to choose h to maximize

$$\begin{aligned} V_t &= R_t^* - \theta_h \text{ if } h = 1 \\ &= (R_t^* - \theta_l)p \text{ if } h = p \\ &= 0 \text{ if } h = 0. \end{aligned}$$

If for any t $R_t^* < \theta_l$, the relationship will never continue or never begin, and so we ignore this possibility.

As with the uniform case, there are two phases; if

$$R_t^* \geq \frac{\theta_h - p\theta_l}{1-p}$$

then $h_t = 1$ and the worker never leaves. This is the high-rent phase. If not, $h_t = p$, and the worker leaves whenever the outside offer is high. This is the low-rent phase.

In the high-rent phase,

$$R_t = R_t^* = \frac{y_t}{1 - \delta g},$$

as before, since the relationship will last forever. Continuing in employment need only be as good as the high outside offer, and so

$$v_t = \theta_h,\ w_t = (1-\delta)\theta_h.$$

Let T denote the first period of the high-rent phase. In the low-rent phase, the value of the relationship to the worker has to be the same as the low outside offer:

$$x_t = w_t + \delta v_t = \theta_l,$$

and

$$v_t = p\theta_l + (1-p)\theta_h \triangleq \overline{\theta}.$$

Thus if $(t+1)$ is also in the low-rent phase

$$w_t = \theta_l - \delta\overline{\theta}.$$

however if $(t+1) = T$ so that the next period is in the high-rent phase,

$$w_{T-1} = \theta_l - \delta\theta_h < \theta_l - \delta\overline{\theta}.$$

Thus wages fall between periods $(T-2)$ and $(T-1)$. As the anticipated high wages of the high-rent phase get closer, the firm needs to pay less in current wages to beat the low outside offers because the future is so bright. So the time path of wages is not monotonic, even though marginal product is rising monotonically.

Even though wages are not weakly monotonic in this example, the continuation hazard is. It is $p < 1$ throughout the low rent phase, and then rises to one.

To return to the numerical example discussed in the previous section, suppose that all output and discounting variables remain the same, but that instead of a uniform distribution of outside offers, the outside offer is either $\theta_l = 1$ with probability .75 or $\theta_h = 1.25$ with probability .25. These values have been chosen so that the high-rent phase begins at period 5, just as before.

Then the wage stays constant for periods 1 through 3, dips in period 4, and rises to a plateau in period 5. Figure 2 illustrates.

Figure 2

Period	1	2	3	4	5	6	7	8	9	10
h_t	.75	.75	.75	.75	1	1	1	1	1	1
w_t	.0625	.0625	.0625	0	.25	.25	.25	.25	.25	.25
y_t	.3400	.3468	.3537	.3607	.368	.375	.383	.391	.398	.406

The signing bonus in period 0 is .7895.

As in the uniform distribution case, there is no constant sharing rule; the ratio of wage to output varies from period to period. The ratio, however, goes down during the low-rent phase rather than up.

In comparing the two, very different wage and turnover profiles, it is important to remember that marginal product follows the same profile in each case.[5] The time profile of wages is thus a very poor indicator of the time profile of underlying productivity. The tenure-turnover profile is also a poor indicator, but not quite so bad, since at least it has the direction right.

5 Conclusion

We present a model to reconcile recent findings on wage and turnover dynamics. Although the evidence of a positive cross-sectional association between earnings and seniority is widespread, the fairly recent use of longitudinal data have allowed researchers to address whether individual wages do rise with seniority. Somewhat surprisingly the evidence on wage changes with tenure appear to be far more variegated than the simple presupposition that wages rise with seniority. Average wage increases with tenure appear to be small if not negligible and within-job wage decreases are not uncommon either. The challenge to theory is to reconcile this complex picture of wage dynamics with the fact that the negative relation between turnover and tenure remains as ubiquitous as ever. Our model, built on the workhorse theories of specific training, job search, and self-enforcing contracts, shows that even if marginal product is increasing due to specific training, wages need not be increasing; but rising marginal product always implies a falling turnover rate.

6 Appendix: Complete commitment case

With complete commitment ability, the solution to the firm's problem is trivial: it postpones payment as long as it can. It is easiest to understand this conclusion if we impose the restriction that no worker can work for more than T^* periods

[5] This divergence remains even if the expected vaue of outside offers is the same. For instance with $\theta_h = .8$, $\theta_l = .4$, $p = .75$, (so $E\theta = .5$ as in the uniform case) the high-rent phase still begins with period 5, but low-rent phase wages are negative. (The low-rent phase is a training period where the worker pays tuition.)

– for biological reasons, say. Then the firm's problem is to choose a sequence of wages (w_t), $t = 0, ..., T^*$, that maximizes the expected present value of its profit. Let V_0 denote this objective function - evaluated, as it must be, just as a worker is hired. We assume outside offers are uniformly distributed between 0 and 1.

It is easiest and most intuitive to consider the case $T^* = 1$ first. Then

$$V_0 = \max_{w_0, w_1} \left[(y_0 - w_0) + \delta \left\{ w_1 (y_1 - w_1) + (1 - w_1) V_0 \right\} \right]$$

for $t \geq 1$. Clearly profit is maximized by setting $w_0 = 0$ (optimal w_1 is more difficult to calculate). Increases in w_0 merely reduce first period profits; they do not increase period 1 retention because they are just irrelevant history when the worker has to make his period 1 decision.

Now consider $T^* = 2$. Clearly $w_0 = 0$. The probability of not quitting at the beginning of period is

$$x_1 = w_1 + \delta v_2$$

where v_2 is an increasing function of w_2, and can be made as large as desired, up to a maximum of one, by making w_2 sufficiently large for any value of w_1. Conditional on not quitting at the beginning of period 1, the value of additional profit is

$$\pi_1 (w_1, w_2) = y_1 - w_1 + \delta \left(w_2 (y_2 - w_2) + (1 - w_2) V_0 \right).$$

But π_1 is clearly maximized by setting $w_1 = 0$ since any desired value of x_1 can be obtained by setting $w_1 = 0$ and making w_2 sufficiently large. Optimal wage scheme has $w_1 = 0$, since profits after period zero depend only on x_1 and π_1.

By repeated arguments like this we can establish that optimal wage schemes when the firm has unlimited commitment power have positive payments only in the last period; these schemes have maximum retentive power for any expected outlay. Clearly this result holds for any distribution of outside offers, not just the uniform.

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