Essays on International Trade, Welfare and Inequality

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ABSTRACT

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How important are the distributional effects of international trade? This has been one of the most central questions pursued by international economists, particularly because much of the public opposition towards increased openness is due to the belief that welfare changes are unevenly distributed. In this dissertation, I rely on counterfactual analysis and natural experiments to study topics of international trade, welfare and inequality in the context of both developing and developed economies. In particular, I combine theoretical modeling and empirical analysis to examine the effects of international trade on (1) real wages of individuals within and across countries; (2) within-sector wage dispersion caused by heterogeneous responses of firms with different productivity levels to cheaper imported inputs.

In each of the three chapters, I contribute to the existing literature by relaxing simplifying assumptions that have proved to be inconsistent with data and exploring new mechanisms that link international trade to inequality.

Chapter 1, “Trade and Real Wages with Demand and Productivity Heterogeneity,” presents a general equilibrium model that incorporates the effects of trade liberalization on both an individual’s nominal wage and consumer price index. A vast majority of the literature focuses on the income channel, which is its effect on the distribution of nominal wages across workers. A small number of studies consider the expenditure channel, which is its differential impact on consumer price indices. It is well known that the consumption baskets of high-income and low-income consumers look very different. To our knowledge,
there are only three case studies that have looked at these two channels jointly for individual countries, Argentina, Mexico and India. We provide a unified framework incorporating both channels by allowing for non-homothetic preferences and worker heterogeneity across jobs. In spite of its many dimensions of heterogeneity at the individual level, the model remains tractable enough that allows us to estimate its key parameters and perform counterfactuals.

Chapter 2, “Trade and Real Wage Inequality: Cross-Country Evidence,” addresses the following question: what is the impact of trade liberalization on the distribution of real wages in a large cross-section of countries? Trade liberalization affects real-wage inequality through two channels: the distribution of nominal wages across workers and, if the rich and the poor consume different bundles of goods, the distribution of price indices across consumers. Prior work has focused mostly on one or the other of these channels, but no paper has studied both jointly for a large set of countries. Based on the theoretical framework in Chapter 1, I measure the distributional effects of trade liberalization incorporating both channels for a sample of 40 countries. More specifically, I parametrize the model using sector-level trade and production data. Because skill-intensive goods are also high-income elastic in the data, I find an intuitive, previously unexplored, and strong interaction between the two channels. According to my counterfactual analysis, trade cost reductions generate dramatically different results for both nominal wage inequality and price index inequality than what previous research has obtained by focusing on either channel alone. I find that trade cost reductions decrease the relative nominal wage of the poor and the relative price index for the poor in all countries. On net, real-wage inequality falls everywhere.

Chapter 3, “Imported Inputs and Within-Sector Wage Dispersion,” proposes a new mechanism through which trade liberalization affects income inequality within a country: the use
of imported inputs. Intuitively, a firm with higher initial productivity is better at using higher quality foreign inputs. This justifies paying the fixed costs for a larger set of imported inputs when input tariff liberalization decreases their relative price. The firm becomes more import intensive, which enhances its productivity advantage. As a result, the firm hires higher quality workers, produces higher quality products and pays higher wages to its workers, increasing within-sector wage dispersion. We find that both the mean and the dispersion of the distribution of firm productivity, markup and size went up during a period when China reduced its tariffs on imported inputs. More importantly, these results still hold when we consider the subset of firms that survived throughout the sample period, from 1998 to 2007. In addition, we develop a partial-equilibrium, heterogeneous-firm model with endogenous imported inputs and labor quality choice that is consistent with these observations. Finally, we provide empirical evidence that supports the model’s prediction that the differential change in the import intensity of firms with different productivity levels explains these patterns.
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Chapter 1

Trade and Real Wages with Demand and Productivity Heterogeneity

Zheli He and Feiran Zhang
1.1 Introduction

Trade liberalization may impact an individual’s real wage through her nominal wage and her consumer price index. The change in her nominal wage depends on changes in producer prices and the job in which she is employed, where the job of her employment is determined by her characteristics such as age, gender and educational attainment. On the other hand, the change in her consumer price index depends on changes in prices of the basket of goods that she consumes, where her consumption basket is determined by her nominal wage in addition to prices. A vast majority of the literature focuses on the effect of trade on the distribution of nominal wages. A small number of studies consider its differential impact on consumer price indices. In this paper, we provide a unified framework that incorporates both the expenditure channel, i.e., changing consumer price indices, and the income channel, i.e., changing nominal wages, to measure the distributional effects of trade in a large cross-section of countries.\(^1\)

We build a model combining demand heterogeneity across consumers with productivity heterogeneity across workers. On the demand side, we use the Almost Ideal Demand System (AIDS) to capture non-homothetic preferences. This demand specification allows the consumption baskets of high-income and low-income individuals to differ so that price changes resulting from trade liberalization have a differential impact on their consumer price indices. On the supply side, we use an assignment model of the labor market parametrized with a Fréchet distribution to capture heterogeneity of workers across jobs. Individuals have comparative advantage across sectors—based on their age, gender and educational attainment—

\(^1\)We focus on labor earnings, which are the main source of income for most people.
and, therefore, sort into different sectors. Consequently, price changes resulting from trade liberalization have a differential impact on individuals’ nominal wages depending on the sectors in which they work. In addition, we also allow individuals to differ in their absolute advantage such that labor groups differ in their average productivity and, therefore, have different nominal wages regardless of individuals’ sectoral choices.\(^2\) This assumption generates a potential link between the skill distribution and the wage distribution and, as a result, a potential correlation between the change in an individual’s nominal wage and the change in her consumer price index.

A vast body of research has examined the impact of trade on the distribution of earnings across workers. Most recently, Galle et al. (2015) develop the notion of “risk-adjusted gains from trade” to evaluate the full distribution of welfare changes in one measure which generalizes the specific-factors intuition to a setting with endogenous labor allocation. Similarly, we focus on changes in relative nominal wages across labor groups that result from changes in relative demand across sectors driven by international trade.\(^3\) There is a small number of studies that have considered price indices as a channel through which trade liberalization can affect inequality. For example, Fajgelbaum and Khandelwal (2016) develop a methodology to measure the unequal gains from trade through the expenditure channel using only aggregate statistics. We extend this approach to incorporate the differential im-

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\(^2\)Workers in a labor group share the same observable characteristics such as age, gender and educational attainment.

\(^3\)See also Adão (2015), Burstein et al. (2015) and Dix-Carneiro and Rafael (2015). We don’t incorporate some of the mechanisms that have been studied in the literature linking international trade to inequality through the earnings channel. For example, Yeaple (2005), Verhoogen (2008), Bustos (2011), Burstein and Vogel (2016) and Bloom et al. (2015) show that trade liberalization increases the measured skill bias of technology by reallocating resources from less to more skill-intensive firms within industries and/or inducing firms to increase their skill intensity. A major difficulty is the lack of a comprehensive, matched employer-employee dataset in many countries that covers the period of rising inequality which is usually confidential.
Impact of trade liberalization on individuals’ nominal wages. In contrast, Faber (2014) exploits barcode level microdata from the Mexican Consumer Price Index and studies the relative price effect of NAFTA on the differential change in the cost of living between rich and poor households. Fally and Faber (2016) use detailed matched US home and store scanner microdata to explore the implications of firm heterogeneity for household price indices across the income distribution. We complement the existing literature by incorporating both the expenditure and the income channels as well as their interaction in a unified framework to analyze the heterogeneous impact of counterfactual trade shocks across individuals in a large set of countries.

To our knowledge, there are only three case studies that have looked at these two channels jointly. Porto (2006) studies the distributional effects of Mercosur, a regional trade agreement among Argentina, Brazil, Paraguay and Uruguay, during the 1990s. Nicita (2009) extends Porto’s approach by adding a link from trade policy to domestic prices and studies the trade liberalization that took place in Mexico during the period 1990-2000. Marchand (2012) allows the tariff pass-through to differ across geographical regions and studies the trade reforms in India between 1988 and 2000. The structure of our model allows us to estimate the effects for more countries. By looking at a wide range of countries, we are able to identify general patterns across countries with different characteristics. We are also able to conduct model-based counterfactuals of different trade shocks which are important for policymakers. In addition, as critiqued in Goldberg and Pavcnik (2007), the predictions of these studies depend in a crucial way on estimates of the degree of pass-through from trade

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4Atkin et al. (2016) draw on a new collection of Mexican microdata to estimate the effect of foreign supermarket entry on household welfare. They do consider both the price index effect and the income effect, but focus only on the gains from retail FDI.
policy changes to product prices as well as the wage-price elasticities. These are difficult to estimate consistently with time-series data on wages and prices in a setting when many other policies change contemporaneously with trade.

1.2 The Model

1.2.1 The Environment

We study an economy with $N$ countries indexed by $n \in \mathcal{N} = \{1, \ldots, N\}$ and $J$ final good sectors indexed by $j \in \mathcal{J} = \{1, \ldots J\}$. Each good is defined as a sector-country of origin pair. Within any $(j, n) \in \mathcal{J} \times \mathcal{N}$, output is homogeneous, and the market is perfectly competitive. In country $n$, there is a continuum of heterogeneous workers indexed by $z \in \mathcal{Z}^n$ with measure $L^n$. They are grouped into a finite number of types indexed by $\lambda \in \Lambda$ with measure $L^n(\lambda)$ based on observable characteristics: age, gender and education. We assume that types are mutually exclusive: $\mathcal{Z}^n(\lambda) \cap \mathcal{Z}^n(\lambda') = \emptyset$, $\forall \lambda \neq \lambda'$.

1.2.2 Definition of Welfare Change

Consider home country $h$. Trade liberalization induces a set of infinitesimal changes in log-prices, $\{\widehat{p}_{(j,n)}^h\}_{(j,n) \in \mathcal{J} \times \mathcal{N}^h}$, and log-wages, $\{\widehat{w}_z\}_{z \in \mathcal{Z}^h}$.\[^5\] We define the local welfare change of individual $z$ as the equivalent variation associated with this set of changes:\[^6\]

$$
\widehat{u}_z = \sum_j \sum_n s_{(j,n)}^z \left( -\widehat{p}_{(j,n)}^h \right) + \widehat{w}_z
$$

\[^5\] $\widehat{p}_{(j,n)}^h = d \ln(p_{(j,n)}^h)$ is the infinitesimal change in the log of prices and $\widehat{w}_z = d \ln(w_z)$ is the infinitesimal change in the log of wages.

\[^6\] Please see Appendix A.1 for the derivation of the local welfare change as the equivalent variation.
Here, \( s^z_{(j,n)} \) is the initial individual expenditure share on good \((j, n)\). An individual’s welfare is affected in two ways. The first is the change in her cost of living resulting from the change in prices which we refer to as the expenditure effect. Specifically, it is the price change applied to the pre-shock expenditure shares. A decrease in prices reduces the cost of living, and therefore increases an individual’s welfare. The second is the change in her nominal wage which we refer to as the income effect.

We can further decompose the local welfare change of individual \( z \) into three components:

\[
\hat{u}_z = \sum_j \sum_n s^z_{(j,n)} (-p^h_{(j,n)}) + \sum_j \sum_n (s^z_{(j,n)} - s^h_{(j,n)}) (-p^h_{(j,n)}) + \hat{w}_z + \hat{\psi}_z + \hat{\psi}_m
\]

(1.2)

\( \hat{u}_z = \hat{E}^h + \hat{\psi}_z + \hat{w}_z \), that is, the total effect is the sum of the aggregate expenditure effect, \( \hat{E}^h \), the individual expenditure effect, \( \hat{\psi}_z \) and the income effect, \( \hat{w}_z \). \( s^h_{(j,n)} \) is country \( h \)'s aggregate expenditure share on good \((j, n)\). We can think of the aggregate expenditure effect as the impact of trade liberalization on the cost of living under homothetic preferences where the ratios of goods demanded by consumers depend only on relative prices, not on income or scale. This effect is the same across all individuals within a country \( h \). On the other hand, the individual expenditure effect implies that if individual \( z \) spends more on good \((j, n)\), then the price decrease of that good increases her welfare by a larger amount.
1.2.3 Non-homothetic Preferences

We use the Almost-Ideal Demand System (AIDS) to capture the non-homotheticity in consumer preferences. It gives an arbitrary first-order approximation to any demand system and satisfies the axioms of order, aggregates over consumers without invoking parallel linear Engel curves, is consistent with budget constraints, and is simple to estimate. The AIDS allows consumption baskets of high-income and low-income individuals to differ so that price changes resulting from trade liberalization have a differential impact on their consumer price indices. It belongs to the family of Log Price-Independent Generalized Preferences defined by the following indirect utility function:

\[ v(w_z, \mathbf{p}^h) = F \left[ \left( \frac{w_z}{a^h(\mathbf{p}^h)} \right)^{1/b^h(\mathbf{p}^h)} \right] \]  

where \( F [\cdot] \) is a continuous, differentiable, and strictly increasing function. The AIDS is the special case that satisfies:

\[
a^h(\mathbf{p}^h) = \exp \left\{ \alpha + \sum_j \sum_n \alpha^h_{(j,n)} \ln p^h_{(j,n)} + \frac{1}{2} \sum_j \sum_n \sum_{j'} \sum_{n'} \gamma_{(j,n)(j',n')} \ln p^h_{(j,n)} \ln p^h_{(j',n')} \right\} \\
b^h(\mathbf{p}^h) = \exp \left\{ \sum_j \sum_n \beta_{(j,n)} \ln p^h_{(j,n)} \right\}
\]  

where \( a^h(\mathbf{p}^h) \) is a homothetic price aggregator which captures the cost of a subsistence basket of consumption goods. \( \underline{\alpha} \) is the outlay required for a minimal standard of living when prices are unity. \( \alpha^h_{(j,n)} \) is importer \( h \)'s taste for good \( (j,n) \). \( \gamma_{(j,n)(j',n')} \) is the cross elasticity between two goods \( (j,n) \) and \( (j',n') \). \( b^h(\mathbf{p}^h) \) is a non-homothetic price aggregator which captures the
relative price of high-income elastic goods. Goods for which \( \beta_{(j,n)} > 0 \) have positive income elasticity, while goods for which \( \beta_{(j,n)} < 0 \) have negative income elasticity. For AIDS to be a proper demand system, the following parametric restrictions need to be satisfied:\(^7\)

\[
\begin{align*}
\sum_j \sum_n \alpha^{h}_{(j,n)} &= 1 \quad (1.6) \\
\sum_j \sum_n \beta_{(j,n)} &= 0 \quad (1.7) \\
\sum_j \sum_n \gamma_{(j,n)(j',n')} &= 0 \quad \forall (j', n') \quad (1.8) \\
\gamma_{(j,n)(j',n')} &= \gamma_{(j',n')(j,n)} \quad \forall (j, n), (j', n') \quad (1.9)
\end{align*}
\]

Applying Shephard’s Lemma to the indirect utility function, we can derive the individual expenditure shares as follows:

\[
\begin{align*}
 s^z_{(j,n)} &= s_{(j,n)} \left( w_z, \mathbf{p}^h \right) \\
 &= \alpha_{(j,n)}^h + \sum_{j'} \sum_{n'} \gamma_{(j,n)(j',n')} \ln p_{(j',n')}^h + \beta_{(j,n)} \ln \left( \frac{w_z}{a(\mathbf{p}^h)} \right) \quad (1.10)
\end{align*}
\]

According to this equation, if a consumer has relatively low nominal wage, then she spends relatively more on low-income elastic goods. Under the AIDS, we can describe the market by the behavior of a representative consumer with the inequality-adjusted average nominal wage, \( \bar{w}^h = \bar{w}^h e^{\Sigma^h} \), which depends on the average nominal wage in country \( h \), \( \bar{w}^h \), and the Theil index, \( \Sigma^h \equiv \mathbb{E} \left[ \frac{w^h}{\bar{w}^h} \ln \left( \frac{w^h}{\bar{w}^h} \right) \right] \), a measure of inequality within a country. It is therefore straightforward to derive the aggregate expenditure shares in country \( h \):

\(^7\)Under these constraints, the budget shares equations share the properties of a demand function, that is, they are homogeneous of degree 0 in prices and total expenditure, sum of budget shares add up to 1 and they satisfy the symmetry of the Slutsky matrix.
\[ S^h_{(j,n)} = s_{(j,n)}(\tilde{w}^h, \bm{p}^h) \]
\[ = \alpha^h_{(j,n)} + \sum_{j'} \sum_{n'} \gamma_{(j,n)(j',n')} \ln p^h_{(j',n')} + \beta_{(j,n)} \ln \left( \frac{\tilde{w}^h}{a \left( \bm{p}^h \right)} \right) \quad (1.11) \]

Similarly, adjusted for the price level, \( a \left( \bm{p}^h \right) \), if country \( h \) has higher inequality-adjusted average nominal wage, \( \tilde{w}^h \), either because of higher average nominal wage or higher inequality, then it spends relatively more on high-income elastic goods.

It is convenient to rewrite the individual expenditure effect under the AIDS as:

\[ \tilde{\psi}_z = \sum_j \sum_n \left( s^z_{(j,n)} - S^h_{(j,n)} \right) \left( -p^h_{(j,n)} \right) \]
\[ = -\ln \left( \frac{w_z}{\tilde{w}^h} \right) \sum_j \sum_n \beta_{(j,n)} \frac{p^h_{(j,n)}}{\tilde{w}^h} \quad (1.12) \]

Intuitively, for an individual \( z \) who has lower nominal wage relative to the representative consumer in the country, if the price of a low-income elastic good goes down, she’s going to be better off and vice versa. Note that we don’t have to observe each individual \( z \)’s expenditure share on each good \((j,n)\) in order to compute the change in her consumer price index.

Plugging in the above expression for \( \tilde{\psi}_z \), we can write the local welfare change of individual \( z \) under the AIDS that corresponds to an infinitesimal change in prices and nominal wages as follows:

\[ \tilde{u}_z = \tilde{E}^h - \ln \left( \frac{w_z}{\tilde{w}^h} \right) \tilde{b}^h + \tilde{w}_z \quad (1.13) \]
The global welfare change of individual $z$ under the AIDS between an initial scenario under trade and a counterfactual scenario can be derived by integrating each component of the equation above:\(^8\)

$$u_{\text{total effect}}^{tr \rightarrow cf} = \begin{pmatrix} E^h_{cf} / E^h_{tr} \\ w_{z}^{tr} / w_{z}^{h} \\ -\ln(b_{cf}^h / b_{tr}^h) \\ w_{z}^{cf} / w_{z}^{tr} \end{pmatrix}$$ (1.14)

$$E^h_{cf} / E^h_{tr} = \prod_{(j,n)} \left( \frac{p_{h,cf}^{(j,n)}}{p_{h,tr}^{(j,n)}} \right)^{S_{h}^{(j,n)}}$$ (1.15)

$$-\ln\left( \frac{b_{cf}^h}{b_{tr}^h} \right) = - \sum_{j} \sum_{n} \beta_{(j,n)} \ln\left( \frac{p_{h,cf}^{(j,n)}}{p_{h,tr}^{(j,n)}} \right)$$ (1.16)

Note that $E^h_{cf} / E^h_{tr}$ and $-\ln(b_{cf}^h / b_{tr}^h)$ are functions of the set of prices in the two scenarios, the aggregate expenditure shares that are observed in the data and a model parameter, $\beta_{(j,n)}$. If $u_{\text{total effect}}^{tr \rightarrow cf} < 1$, individual $z$ is worse off after the change and vice versa.

1.2.4 Heterogeneous Labor with Comparative Advantage across Sectors

Our supply-side specification allows for heterogeneous labor with comparative advantage across sectors so that different labor types sort into different sectors and prices changes resulting from trade liberalization have a differential impact on their nominal wages. We use an assignment model of the labor market parameterized with a Fréchet distribution. In this environment, workers with different unobservable characteristics but identical observable characteristics may be allocated to different sectors in a competitive equilibrium. In

---

\(^{8}\)Please see Appendix A.2 for the derivation of the aggregate and individual expenditure effects between an initial scenario under trade and a counterfactual scenario.
particular, an arbitrary worker $z$ of type $\lambda$ draws a vector of efficiency units across different sectors from a multivariate Fréchet distribution:\footnote{Fréchet distributions of productivity shocks across factors have been imposed in the recent closed-economy models of Lagakos and Waugh (2013) and Hsieh et al. (2013) as well as the open economy models of Burstein et al. (2015), Costinot et al. (2016), and Fajgelbaum and Redding (2014). Sector and country characteristics are assumed to be perfectly observed by the econometrician, but factor characteristics are not. See Costinot and Vogel (2015) for a detailed discussion.}

\[
G(\epsilon(z); \lambda) = \Pr [\epsilon(z;j) \leq \epsilon(z;j) \forall j] = \exp \left\{ - \left( \sum_j \epsilon(z;j)^{-\theta(\lambda)} \right) \right\}
\]

(1.17)

where $\theta(\lambda) > 1$ governs within-type dispersion of efficiency units. Worker $z$ inelastically supplies $\epsilon(z;j)$ efficiency units of labor if she chooses to work in sector $j$.

Production requires only one factor, labor.\footnote{We do not feature complementarity between different types of equipment and heterogeneous workers across sectors as in Burstein et al. (2015) because we do not have data to compute the share of total hours worked by each labor group that is spent using different equipment types across sectors.}

The production function in country $h$, sector $j$, using $l$ efficiency units of labor type $\lambda$ is:\footnote{Our model does not feature Ricardian-type country-sector productivity. However, we demonstrate in Appendix A.3 that we are not understating the specialization of skill-abundant countries in skill-intensive sectors.}

\[
y^h(l; \lambda, j) = A^h(\lambda)T(\lambda, j)l
\]

(1.18)

$A^h(\lambda)$ is the productivity of type $\lambda$ workers in country $h$ and $T(\lambda, j)$ is the productivity of type $\lambda$ workers who choose to work in sector $j$. $A^h(\lambda)$ captures the absolute advantage of type $\lambda$ workers in country $h$. $T(\lambda, j)$ captures the comparative advantage of type $\lambda$ workers in sector $j$. Consider the partial equilibrium in which output prices, $\{p^h_{(j,h)}\}_{j \in J}$, are given. Perfect competition and free entry entail that the per efficiency unit wage $x^h(\lambda, j)$ of a
worker of labor type $\lambda$ working in sector $j$ in country $h$ is:

$$x^h (\lambda, j) = p^h_{(j,h)}A^h(\lambda)T(\lambda, j)$$  \hspace{1cm} (1.19)

Worker $z \in Z^h(\lambda)$ with realization of the vector of efficiency units $\varepsilon(z) = \{\varepsilon(z;j)\}_{j \in J}$ needs to choose the sector that maximizes her labor earnings which is the product of her draw of efficiency units and per efficiency unit wage:

$$w_z = \max_j w_z(j) = \varepsilon(z;j) \cdot x^h(\lambda,j)$$  \hspace{1cm} (1.20)

The multivariate Fréchet distribution implies that the probability of a type $\lambda$ worker choosing to work in sector $j$ in country $h$ is:

$$\pi^h(\lambda, j) = \frac{\left[p^h_{(j,h)}A^h(\lambda)T(\lambda, j)\right]^\theta(\lambda)}{\sum_{j' \in J} \left[p^h_{(j',h)}A^h(\lambda)T(\lambda, j')\right]^\theta(\lambda)} = \frac{x^h(\lambda, j)^{\theta(\lambda)}}{x^h(\lambda)^{\theta(\lambda)}}$$  \hspace{1cm} (1.21)

where $x^h(\lambda) \equiv \left(\sum_j x^h(\lambda, j)^{\theta(\lambda)}\right)^{\frac{1}{\theta(\lambda)}}$. With a higher $\theta(\lambda)$, which implies that there is less dispersion of efficiency units across sectors, a change in price or a change in productivity affects the factor allocation even more.

As a result, the worker sorting pattern is determined by comparative advantage:

$$\left[\frac{\pi^h(\lambda', j')}{\pi^h(\lambda, j)}\right]^{\frac{1}{\theta(\lambda')}} / \left[\frac{\pi^h(\lambda, j')}{\pi^h(\lambda, j)}\right]^{\frac{1}{\theta(\lambda)}} = \left[\frac{T(\lambda', j')}{T(\lambda', j)}\right] / \left[\frac{T(\lambda, j')}{T(\lambda, j)}\right]$$  \hspace{1cm} (1.22)
If type $\lambda$ workers (relative to type $\lambda'$ workers) have a comparative advantage in sector $j'$ (relative to sector $j$), then they are relatively more likely to sort into sector $j'$, adjusted for potentially different values of $\theta(\lambda)$ and $\theta(\lambda')$. For larger $\theta(\lambda')$ (i.e. less dispersion in efficiency units among type $\lambda'$ workers), it is even more likely for them to sort into sector $j'$, in which they have a comparative advantage.

The distribution for $w_z = \max_j w_z(j)$ conditional on $z \in Z^h(\lambda)$ is:

$$
\Pr(w_z \leq w \mid z \in Z^h(\lambda)) = \exp \left\{ -x^h(\lambda)^{\theta(\lambda)} w^{-\theta(\lambda)} \right\} \quad (1.23)
$$

It is also distributed Fréchet with the scale parameter, $x^h(\lambda)$, the average per efficiency unit wage of labor type $\lambda$ across the sectors, along with the dispersion parameter, $\theta(\lambda)$.

The average nominal wage, $\bar{w}^h$, and the Theil index, $\Sigma^h$, in country $h$ can also be expressed in terms of $x^h(\lambda)$ and $\theta(\lambda)$:

$$
\bar{w}^h = \sum_{\lambda} \frac{L^h(\lambda)}{L^h} \Gamma(\lambda) x^h(\lambda) \quad (1.24)
$$

$$
\Sigma^h = \frac{1}{\bar{w}^h} \sum_{\lambda} \frac{L^h(\lambda)}{L^h} \Gamma(\lambda) \left( x^h(\lambda) \ln x^h(\lambda) - \frac{\Psi(\lambda)}{\theta(\lambda)} x^h(\lambda) \right) - \ln \bar{w}^h \quad (1.25)
$$

where $\Gamma(\lambda) \equiv \Gamma \left( 1 - \frac{1}{\theta(\lambda)} \right)$ is the gamma function and $\Psi(\lambda) \equiv \Psi \left( 1 - \frac{1}{\theta(\lambda)} \right)$ is the digamma function.
1.2.5 General Equilibrium

In the general equilibrium, output prices, \( \{p_{(j,h)}^n\}_{j \in J} \), are determined by the market clearing conditions:

\[
\sum_{\lambda} y^h \left( L^h(\lambda) \pi^h(\lambda, j); \lambda, j \right) = \sum_n \tau^n_{(j,h)} D^n_{(j,h)} \quad \forall j \in J
\]

(1.26)

where \( y^h = A^h(\lambda)T(\lambda, j)\Gamma(\lambda)\pi^h(\lambda, j)^{1 - \frac{1}{\kappa(\lambda)}} \) is the supply of sector \( j \) good by labor type \( \lambda \) in country \( h \).\(^{12}\) \( \tau^n_{(j,h)} \) is the bilateral trade cost between export country \( h \) and import country \( n \) in sector \( j \). \( D^n_{(j,h)} = \left( \frac{S^n_{(j,h)} \bar{w}^n L^n}{p^n_{(j,h)}} \right) \) is country \( n \)'s demand for good \((j,h)\), where \( S^n_{(j,h)} \) is given in equation (11) and \( \bar{w}^n \) in equation (24). It depends on country \( n \)'s wage distribution (in particular \( \bar{w}^n \) and \( \Sigma^n \)) as well as the vector of prices that consumers face in that country \( p^n \) (in particular \( p^n_{(j,h)} = \tau^n_{(j,h)} p^h_{(j,h)} \)). Since these output prices enter both the demand side and the supply side nonlinearly, we apply the Gauss-Jacobi algorithm, an iterative method, to solve the system of market clearing equations numerically.\(^{13}\) We also appeal to the Implicit Function Theorem to show that the price equilibrium that we have found numerically is locally isolated as a function of the parameters.\(^{14}\) That is, in response to a small perturbation, if there exists an equilibrium, then the system stays in the neighborhood of that equilibrium. We find no quantitative evidence of multiple equilibria.\(^{15}\)

\(^{12}\)Please see Appendix A.4 for the derivation of the total supply.
\(^{13}\)We demonstrate existence of an equilibrium numerically.
\(^{14}\)Please refer to Appendix A.5 for a brief discussion of the Gauss-Jacobi Algorithm and the local property of the equilibrium.
\(^{15}\)We have tried multiple starting points and the system always converges to the same equilibrium.
1.3 Conclusion

Trade liberalization may impact an individual’s real wage through her nominal wage and her consumer price index. The change in her nominal wage depends on changes in producer prices and the job in which she is employed, where the job of her employment is determined by her characteristics such as age, gender and educational attainment. On the other hand, the change in her consumer price index depends on changes in prices of the basket of goods that she consumes, where her consumption basket is determined by her nominal wage in addition to prices. A vast majority of the literature focuses on the effect of trade on the distribution of nominal wages. A small number of studies consider its differential impact on consumer price indices. In this paper, we provide a unified framework that incorporates both the expenditure channel, i.e., changing consumer price indices, and the income channel, i.e., changing nominal wages, to measure the distributional effects of trade in a large cross-section of countries.

In order to allow price indices to vary across consumers within a country, we need demand heterogeneity. We use the Almost Ideal Demand System (AIDS) to capture non-homothetic preferences. This demand specification allows the consumption baskets of high-income and low-income individuals to differ so that price changes resulting from trade liberalization have a differential impact on their consumer price indices. In order to allow nominal wages to vary across workers within a country, we need productivity heterogeneity. We use an assignment model of the labor market to capture heterogeneity of workers across jobs. Individuals have comparative advantage across sectors and, therefore, sort into different sectors. Consequently, price changes resulting from trade liberalization have a differential impact on
individuals’ nominal wages depending on the sectors in which they work.

This model with demand heterogeneity across consumers and productivity heterogeneity across workers can be used to quantify the distributional effects of trade liberalization for a wide range of countries. By looking at a large set of countries, we are able to identify general patterns across countries with different characteristics. We are also able to conduct model-based counterfactuals of different trade shocks, which are important for policymakers.
Chapter 2

Trade and Real Wage Inequality: Cross-Country Evidence

Zheli He
2.1 Introduction

Trade liberalization affects real wage inequality through two channels: the distribution of nominal wages across workers and, if the rich and the poor consume different bundles of goods, the distribution of price indices across consumers. Prior work has focused mostly on one or the other of these channels, but no paper has studied both jointly for a large set of countries. Based on the theoretical framework in Chapter 1, I measure the distributional effects of trade liberalization incorporating both channels for a sample of 40 countries. Because skill-intensive goods are also high-income elastic in the data, I find an intuitive, previously unexplored, and strong interaction between these two channels. According to my counterfactual analysis, trade cost reductions generate dramatically different results for both nominal wage inequality and price index inequality than what previous research has obtained by focusing on either channel alone.

In isolation, these two channels have well-understood implications. Shutting down the expenditure channel, I find that the income channel benefits the poor more than the rich in low-income countries and the rich more than the poor in high-income countries. This is consistent with standard factor proportions theory in which a reduction in trade costs raises the relative nominal wage of the abundant factor in every country, benefiting the unskilled (and poor) workers in skill-scarce countries that are low income and the skilled (and rich) workers in skill-abundant countries that are high income. Shutting down the income channel, I find that the expenditure channel benefits the poor more than the rich in every country and more so in high-income countries. Intuitively, lower trade costs increase real incomes and, therefore, decrease the relative demand for and the relative price of low-income elastic goods.
goods. Because low-income consumers spend more on these goods, they benefit relatively more. The expenditure channel benefits the poor relatively more in high-income countries because these countries are net importers of low-income elastic goods.

These two channels do not work in isolation. Studying either channel in the absence of the other leads to profoundly biased results qualitatively and quantitatively. Specifically, their interaction implies that the income channel benefits the rich in every country, which is consistent with a large body of empirical evidence; see e.g. Goldberg and Pavcnik (2007).

Intuitively, when both channels are active, lower trade costs increase real incomes and, therefore, decrease the relative demand for and the relative price of low-income elastic goods as discussed above. Since the poor disproportionately produce unskill-intensive goods, which are low-income elastic, their relative nominal wage falls in every country. This effect is absent when only the income channel is active. Moreover, the interaction of these two mechanisms also implies that the poor’s relative benefit from the expenditure channel is magnified in every country. Intuitively, because nominal wage inequality rises in every country, as just described, the relative demand for and the relative price of low-income elastic goods fall even further, reducing the relative price index for the poor in every country. This effect is absent when only the expenditure channel is active because nominal wage inequality is constant in that case.

I parametrize the model for a sample of 40 countries (27 European countries and 13 other large countries) and 35 sectors using a range of datasets including the World Input-Output Database (WIOD) and the Integrated Public Use Microdata Series, International
(IPUMS-I). WIOD provides information on bilateral trade flows and production data.\footnote{One important feature of the WIOD is that it includes the input-output transactions of a country with itself. Typically, the domestic market accounts for the large majority of demand for most production.} I derive a sectoral non-homothetic gravity equation that allows me to estimate the elasticity of substitution and the income elasticity of goods as follows.\footnote{The sectoral non-homothetic gravity equation based on the AIDS was first derived in Fajgelbaum and Khandelwal (2016). However, their model assumptions imply that the change in income is 0 for all consumers.} First, I estimate the elasticity of substitution by projecting countries’ sectoral expenditure shares on trade costs. Second, I estimate the income elasticity of each good using the following insight: if high-income or more unequal countries spend relatively more on a good, then I infer that this good is high-income elastic. IPUMS-I provides publicly available nationally representative survey data for 82 countries that are coded and documented consistently across countries and over time. It reports individual-level information including age, gender, educational attainment, labor income and sector of work. This rich database enables me to estimate the Fréchet dispersion parameter of the within-group distribution of efficiency units across sectors which determines the extent of worker reallocation and, thus, the responsiveness of group average wages to changes in sectoral output prices. In addition, I am able to estimate the comparative advantage of different labor groups across sectors based on observed worker sorting patterns. Intuitively, if a worker type (relative to another worker type) is more likely to sort into a sector (relative to another sector), then I infer that they are relatively more productive in that sector. Using the estimates of group average wages and other parameters, I can back out the absolute advantage of different labor groups.

With these parameter estimates, I conduct two counterfactual analyses to quantify the distributional effects of trade liberalization. To demonstrate how the model works, I begin with a simple counterfactual exercise in which I consider a 5% reduction in all bilateral trade...
costs. I find that the average welfare gain across the 40 countries is about 1.2%, which is in line with the previous literature that abstracts from relative effects across individuals within countries.\textsuperscript{3} Within each country, as I move up the initial nominal wage distribution, gains decline. Specifically, moving up from one decile to the next reduces gains by 0.1 percentage point: the bottom 10th percentile experiences a real wage gain that is larger than the top 10th percentile in every country, and the difference is 0.8 percentage points in the average country. These results highlight that the distributional effects of trade liberalization are large compared to its average effect. I obtain the result that the poor gain relative to the rich in spite of the fact that I find the opposite result for nominal wages. In the average country, the bottom 10th percentile see their nominal wages decrease by 0.2 percentage points relative to the top 10th percentile. Hence, the reduction in the poor’s relative price index must fall substantially. In the average country, the bottom 10th percentile see their consumer price indices decrease by 1 percentage point more than the top 10th percentile.

The theoretical framework in Chapter 1 also allows me to re-examine the impact of a significant increase in U.S. manufacturing imports from China on U.S. real-wage inequality while accounting for both channels and their interaction.\textsuperscript{4} I consider a uniform reduction in trade costs between the U.S. and China that would yield a $1000 per U.S. worker increase in Chinese manufacturing imports. I find that this reduction in trade costs decreases the consumer price index for a U.S. representative consumer by 0.85%. Individuals whose nominal wages are at the 10th percentile of the initial distribution see a further 0.35 percentage

\textsuperscript{3}Eaton and Kortum (2002) consider a counterfactual where the 19 OECD countries collectively remove the 5 percent tariff on all imports and find that most countries gain around one percent.

\textsuperscript{4}Autor et al. (2013), Autor et al. (2014) and Acemoglu et al. (2016) study the impact of increased Chinese import competition on employment and earnings of U.S. workers by comparing more affected industries and local labor markets to less affected ones but have no implications at the aggregate level.
point reduction in their consumer price indices compared to the representative consumer, while individuals whose nominal wages are at the 90th percentile see their consumer price indices decrease by 0.1 percentage point less than the representative consumer. This result arises because Chinese manufacturing goods are low-income elastic and, consequently, their lower prices benefit more the poor individuals who spend relatively more on these goods. Although the former see a bigger decline in their nominal wages (0.13% vs. 0.11%) because they are more likely to work in manufacturing sectors that are in direct competition with cheaper Chinese imports, this income effect is more than offset by their much lower consumer price indices. Rising Chinese import competition increases the real wage of the poor by 0.43 percentage points more than that of the rich in the U.S.

The remainder of this chapter proceeds as follows. Section 2.2 contains a description of the data, and estimation strategy and results are gathered in Section 2.3. In Section 2.4, I discuss my counterfactual results. Section 2.5 concludes.

2.2 Data

For the demand-side estimation, I use mainly the World Input-Output Database (WIOD), which provides information on bilateral trade flows and production data for 40 countries (27 European countries and 13 other large countries) and 35 sectors in the economy. It also distinguishes between final consumption and intermediate uses.\(^5\)

World Input-Output Table looks like Figure 2.1:

---

\(^5\)I do not use the UN Comtrade Database because it does not have information on the input-output transactions of a country with itself.
For the supply-side estimation, I use mainly the Integrated Public Use Microdata Series, International (IPUMS-I), which provides publicly available nationally representative survey data for 82 countries that are coded and documented consistently across countries and over time and individual-level data with labor incomes and worker characteristics. I divide the workers in IPUMS-I dataset into 18 disjoint groups, $\Lambda$, by age (15-24, 25-49 and 50-74), gender (male and female) and educational attainment (ED0-2, less than primary, primary and lower secondary education; ED3-4, upper secondary and post-secondary non-tertiary education; ED5-8, tertiary education).

2.3 Parametrization

2.3.1 Supply-side Parameters

On the supply side, I need to estimate $\theta(\lambda)$, the worker type specific Fréchet dispersion parameter, $L^h(\lambda)/L^h$, the fraction of type $\lambda$ workers in country $h$, $A^h(\lambda)$, the productivity of type $\lambda$ workers in country $h$ and $T(\lambda, j)$, the productivity of type $\lambda$ workers who choose...
to work in sector $j$.

To estimate the worker type specific Fréchet dispersion parameter $\theta(\lambda)$, I follow the methodology in Lagakos and Waugh (2013) and Hsieh et al. (2013) and match the moments of the empirical distribution of within type worker wages.\textsuperscript{6} In particular, the mean and the variance of nominal wages within a labor group satisfy:

\[
\frac{\text{VAR} \left[ w_z \mid z \in Z^h(\lambda) \right]}{\text{E} \left[ w_z \mid z \in Z^h(\lambda) \right]^2} = \frac{\Gamma \left( 1 - \frac{2}{\theta(\lambda)} \right)}{\Gamma \left( 1 - \frac{1}{\theta(\lambda)} \right)^2} - 1
\]

I restrict my sample in the following way: I drop workers who are younger than 15 years old, are self-employed or work part-time (<30 hours per week), do not report positive labor earnings, or have missing information on age, sex or education. I also drop the top and bottom 1% of earners to remove potential outliers, and to minimize the impact of potential cross-country differences in top-coding procedures. All calculations in my analysis are weighted using the applicable sample weights. I measure $w_z$ as the annual labor earnings; $\epsilon(z; j)$ captures both the hours worked and efficiency units of worker $z$ who chooses to work in sector $j$; $\theta(\lambda)$ reflects dispersion in both hours worked and efficiency units of type $\lambda$ workers; $L^h(\lambda)$ is the headcount of type $\lambda$ workers.

I use IPUMS-I to estimate $\theta(\lambda)$ for 16 countries.\textsuperscript{7} Since the estimates of $\theta(\lambda)$ are very close across the 16 countries for each labor type $\lambda$, I use the average of these estimates for all countries and assume that $\theta(\lambda)$ doesn’t change over time. I back out $x^h(\lambda)$ using

\textsuperscript{6}As a robustness check, I also jointly estimate $\theta(\lambda)$ and $x^h(\lambda)$ for each labor type using maximum likelihood.

\textsuperscript{7}The list of countries can be found in Appendix B.1.
\[ \mathbb{E}[w_z | z \in Z^h(\lambda)] = x^h(\lambda)\Gamma(1 - \frac{1}{\theta(\lambda)}) \] for the 16 countries. Since all earnings data in IPUMS-I are in local currency units, I use the official exchange rate (LCU per US$, period average) from the World Bank to convert all values to US$. I also find that output-side real GDP per capita have strong explanatory power for \( x^h(\lambda) \), so I use the predicted values of \( x^h(\lambda) \) for the rest of the countries.\(^8\)

Since IPUMS-I does not provide information on \( L^h(\lambda)/L^h \) for all of the 40 countries, I use the following complementary datasets. First, I use Eurostat which provides information on the full-time and part-time employment by age, gender and educational attainment; it includes 27 European countries in WIOD. Second, I use UNdata which has information on population 15 years of age and over, also by age, gender and educational attainment, for Russia, Australia, Korea and China. This dataset comes from UNSD Demographic Statistics–United Nations Statistics Division. Third, I use National Statistics, Republic of China (Taiwan) and finally, Population Statistics of Japan.

In order to estimate the sector-level non-homothetic gravity equation, which I explain in detail in the next section, I need to compute the inequality-adjusted average nominal wage of each country, which requires an estimate of its average nominal wage as well as its Theil index. Table 2.1 reports my estimates of the average labor earnings and the Theil index for the 40 countries based on equations (24) and (25). I estimate \( \bar{w}^h \) and \( \sum^h \) for the years 2005, 2006 and 2007, and then take the average.

\(^8\)I get the data on output-side real GDP at chained PPPs (in millions of 2005 US$) and population from the Penn World Tables.
Recall that the Theil index measures the level of inequality within a country, which in my framework is the dispersion in labor incomes. Since my Theil indices are calculated using only the labor earnings of the population aged between 15 and 74, I also use IPUMS-I to construct alternative measures of wage Gini coefficients using three different methods that are widely used in the literature. Let $y_i$ be the labor income of a person indexed in non-decreasing order ($y_i \leq y_{i+1}$), my first two measures of the wage Gini coefficients are calculated as follows: $G_1 = \frac{2}{n} \sum_{i=1}^{n} \frac{iy_i}{n} - \frac{n+1}{n}$ and $G_2 = 1 - \frac{2}{n-1} \left( n - \sum_{i=1}^{n} \frac{iy_i}{\sum_{i=1}^{n} y_i} \right)$. On the other hand, Guillermina Jasso and Angus Deaton independently proposed the following formula: $G_3 = \frac{N+1}{N-1} - \frac{2}{N(N-1)\mu} \left( \sum_{i=1}^{n} P_i X_i \right)$ where $\mu$ is mean income of the population, $P_i$ is the income rank $P$ of person $i$, with income, $X_i$, such that the richest person receives a rank of 1 and
the poorest a rank of $N$. The three methods give me very similar estimates and Figure 2.2 demonstrates that my model-implied Theil indices perform very well against the Jasso and Deaton measure. Their correlation is significantly positive at 0.89.

I plot in Figure 2.3 my model-implied Theil indices for all of the 40 countries against the Gini coefficients reported in the World Income Inequality Database that are computed using all sources of income. The two measures are still positively correlated and the correlation is around 0.61. In the right panel, I exclude the three potential outliers and the positive correlation remains.
In Figure 2.4, I plot my model-implied labor earnings per capita against output-side GDP per capita. My measure of income per capita tracks the data very well. These parameter implications provide evidence that model assumptions on the supply side do well at matching the data.
As discussed above, the worker sorting pattern can be used to parametrize $T(\lambda, j)$. I need $\lambda + J - 1 = 52$ normalization. I pick $\lambda = 1$ such that $T(\lambda, j') = 1 \forall j'$. I also pick $j = 1$ such that $T(\lambda', j) = 1 \forall \lambda'$. Then I have $T(\lambda', j') = \left[ \frac{\pi^h(\lambda', j')}{\pi^h(\lambda', j=1)} \right]^{\frac{1}{\theta h(\lambda', j=1)}} / \left[ \frac{\pi^h(\lambda=1, j')}{\pi^h(\lambda=1, j=1)} \right]^{\frac{1}{\theta h(\lambda=1, j=1)}}$, $\forall \lambda' \neq 1 \forall j' \neq 1$ where $\pi^h(\lambda, j) = L^h(\lambda, j)/L^h(\lambda)$ and $L^h(\lambda, j)$ is the headcount of type $\lambda$ workers in country $h$ that choose to work in sector $j$. Since there is no information on $L^h(\lambda, j)$ in Eurostat or UNdata, I use data on the countries that are available in IPUMS-I to compute $T(\lambda, j)$ and then use the average of the estimates for all of the countries.\(^9\) Given the specified normalization,

$$\frac{\pi(\lambda', j)}{\pi(\lambda, j)} = \frac{T(\lambda', j)^{\theta(\lambda')}}{T(\lambda, j)^{\theta(\lambda)}}$$  \hspace{1cm} (2.2)

I plot in Figure 2.5 this ratio aggregating the 18 labor groups into three broad categories based on educational attainment against an estimate of the skill intensity of each sector which matches the share of hours worked in that sector by workers with a completed tertiary degree in the U.S.\(^{10}\)\(^{11}\) These graphs illustrate that workers with less education are more likely to work in unskill-intensive sectors. This implies that a decline in the relative price of goods in unskill-intensive sectors decreases the relative nominal wage of unskilled workers.\(^{12}\)

To estimate $A^h(\lambda)$, the productivity of type $\lambda$ workers in country $h$, I take a first-order approximation of the following equation at $p = 1$, $T = 1$:

---

\(^9\)This restriction implies, for example, a U.S. and a Chinese female worker who are 25-year-old and college educated are both twice as productive in health care than in mining. Because of data limitations, I cannot estimate $T(\lambda, j)$ for every country. This restriction is reasonable and does well in capturing the systematic relationship between the different labor types and the sectors that they sort into.

\(^{10}\)ED1 corresponds to less than primary, primary and lower secondary education; ED2 corresponds to upper secondary and post-secondary non-tertiary education; ED3 corresponds to tertiary education.

\(^{11}\)I thank Jonathan Vogel for providing me with these estimates.

\(^{12}\)In partial equilibrium, changes in wages are proportional to changes in output prices, where the weight depends on factor allocation in the initial period. An increase in sector $j$’s output price raises the relative wage of labor groups that disproportionately work in sector $j$ in the initial trade equilibrium.
Figure 2.5: Worker Sorting

\[
x^h(\lambda) = \left( \sum_j x^h(\lambda, j)^{\theta(\lambda)} \right)^{\frac{1}{\frac{1}{M}}}
= \sum_{j \in J} \left[ p^h_{(j, h)} A^h(\lambda) T(\lambda, j) \right]^{\theta(\lambda)}
= A^h(\lambda) \left\{ \sum_{j \in J} \left[ p^h_{(j, h)} T(\lambda, j) \right]^{\theta(\lambda)} \right\}^{\frac{1}{\frac{1}{M}}}
\]

which gives me:\(^{13}\)

\[
\log \left( \frac{x^h(\lambda)}{x^h(1)} \right) = \log \left( \frac{A^h(\lambda)}{A^h(1)} \right) + \log J \left( \frac{1}{\theta(\lambda)} - \frac{1}{\theta(1)} \right) + \frac{1}{J} \sum_{j \in J} \log \left( \frac{T(\lambda, j)}{T(1, j)} \right)
\]

I assume that \(A^h(\lambda = 1) = 1 \forall h.\(^ {14}\) Figure 2.6 is a bar chart that plots the average \(A^h(\lambda)\) across countries for each of the 18 labor groups by age, gender and educational attainment.

\(^{13}\)Please see Appendix B.2 for the derivation of equation (2.4).

\(^{14}\)Please refer to Appendix B.3 for a description of the characteristics of each labor group.
As expected, for those who are of the same age and gender, the less education one receives, the lower the average estimate of $A_h(\lambda)$. In addition, for those who are of the same gender and have the same level of education, the younger one is, the lower the average estimate of $A_h(\lambda)$. Finally, a female worker is estimated to have lower average $A_h(\lambda)$ than her male counterpart. Zooming in on education, I aggregate the 18 labor groups into three broad categories. The bar chart on the right illustrates that less educated individuals have lower $A_h(\lambda)$ on average regardless of their age and gender. This implies that less educated workers have lower nominal wages regardless of their sectoral choices.

### 2.3.2 Demand-side Parameters

On the demand side, I need to estimate $\underline{\alpha}$, which can be interpreted as the outlay required for a minimal standard of living when prices are unity. I assign 0 to $\underline{\alpha}$ a priori. I also need to estimate the vector of income elasticities, $\beta = \{\beta_{(j,n)}\}$, and the matrix of cross elasticities,
\[ \Gamma = \{ \gamma_{(j,n)(j',n')} \} \], as well as \( \alpha_{(j,n)}^h \), the overall taste in country \( h \) for the goods exported by country \( n \) in sector \( j \) independently from prices or income of the importer.

On top of the regularity restrictions imposed by the AIDS, I impose additional assumptions on the matrix \( \Gamma \) to reduce the number of parameters I estimate:

\[
\gamma_{(j,n)(j',n')} = \begin{cases} 
\frac{\gamma_j}{N} & j = j', n \neq n' \\
-(1 - \frac{1}{N}) \gamma_j & j = j', n = n' \\
0 & j \neq j'
\end{cases}
\] (2.5)

In words, this implies that within the same sector, cross elasticities are the same between goods produced by different countries and across sectors, there is no substitution.\(^{15}\)

Under these parametric restrictions, the sectoral non-homothetic gravity equation is:\(^{16}\)

\[
S_{(j,n)}^h \equiv \frac{Y_{(j,n)}^h}{Y_h} = \frac{Y_{(j,n)}^h}{Y_W} + K_{(j,n)}^h - \gamma_j M_{(j,n)}^h + \beta_{(j,n)}^h \Omega^h
\] (2.6)

where \( \frac{Y_{(j,n)}^h}{Y_W} \) captures the size of the exporter \( n \) in sector \( j \) in the world economy; \( K_{(j,n)}^h = \alpha_{(j,n)}^h - \sum_{n'} \left( \frac{Y_{n'}^h}{Y_W} \right) \alpha_{(j,n)}^n \) captures the differences in taste across countries for different goods; \( M_{n}^h = \ln \left( \frac{\tau_{(j,n)}^h}{\tau_j^h} \right) - \sum_{n'} \left( \frac{Y_{n'}^h}{Y_W} \right) \ln \left( \frac{\tau_{(j,n')}^h}{\tau_{n'}^h} \right) \) captures bilateral trade costs and multilateral resistance, and \( \Omega^h = y^h - \sum_{n'} \left( \frac{Y_{n'}^h}{Y_W} \right) y_{n'} \) is the non-homothetic component of the gravity equation. For example, a country with a high \( \Omega^h \), either because of its high average nominal wage or its high inequality, is predicted to consume more of the high-income elastic goods.

Following Fajgelbaum and Khandelwal (2016), I proxy \( K_{(j,n)}^h \) with the product of the

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\(^{15}\)Normalization by the number of countries \( N \) is mainly for notational simplicity and is not necessary.

\(^{16}\)Please see Appendix B.4 for the derivation of the sector-level non-homothetic gravity equation.
exporter fixed effect and country $h$’s expenditure share on sector $j$ relative to the world. Since I do not observe directly the trade costs between country pairs, I proxy them with bilateral observables.

To be more specific, I assume importer $h$’s taste for good $(j,n)$, $\alpha_{(j,n)}^h$, can be decomposed into an exporter effect, $a_n$, a sector effect, $a_j$, and an importer taste for that sector, $\varepsilon_j^h$:

$$\alpha_{(j,n)}^h = a_n (a_j + \varepsilon_j^h)$$ \hspace{1cm} (2.7)

Under the additional assumptions on $\Gamma$, aggregate expenditure shares are:

$$S_{(j,n)}^h = \alpha_{(j,n)}^h - \gamma_j \ln p_{(j,n)}^h + \frac{\gamma_j}{N} \sum_{n'=1}^N \ln p_{(j,n')}^h + \beta_{(j,n)} y^h$$ \hspace{1cm} (2.8)

Therefore, the sectoral expenditure shares become:

$$S_j^h = \sum_n S_{(j,n)}^h = \bar{\alpha}_j^h + \bar{\beta}_j y^h$$ \hspace{1cm} (2.9)

where $\bar{\alpha}_j^h = \sum_n \alpha_{(j,n)}^h$ and $\bar{\beta}_j = \sum_n \beta_{(j,n)}$. In the absence of non-homotheticity, $\bar{\beta}_j = 0 \forall j$. In that case, the upper tier is Cobb-Douglas with fixed expenditure shares $\{\bar{\alpha}_j^h\}_{j \in \sigma}$. I further impose the restriction: $\sum_{n=1}^N \alpha_n = 1$. This re-expresses $K_{(j,n)}^h = a_n (S_j^h - S_j^W) - a_n \bar{\beta}_j \Omega^h$.17

I assume that the bilateral trade cost takes the form $\tau_{(j,n)}^h = (d_{n}^h)^{\rho_j} (P_n^h)^{-\delta_j} (b_{n}^h)^{-\delta_j} c_{(j,n)}^h$ where bilateral distance, common language and border information are obtained from CEPII’s Gravity database. This re-expresses $M_{(j,n)}^h = \rho_j \Delta_n^h - \delta_j L_n^{h} - \delta_j B_n^{h} + \varepsilon_{(j,n)}^h$ where $\Delta_n^h \equiv \ln \left( \frac{d_n}{d'} \right) - \sum_{n'} \frac{\varepsilon_{n'}^h}{\varepsilon_{(j,n)}^h} \ln \left( \frac{d_{n'}^{w}}{d^{w'}} \right)$ and $\bar{d}_{n'}^{w'} = \exp \left( \frac{1}{N} \sum_n \ln \bar{d}_{n'}^{w'} \right)$ and $L_n^{h}$ and $B_n^{h}$ are defined in the

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17Please see Appendix B.5 for the derivation of $K_{(j,n)}^h$. 

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same way. To separately identify $\gamma_j$, I again follow Fajgelbaum and Khandelwal (2016) and set the elasticity of trade cost with respect to distance $\rho^j = \rho = 0.177$.\footnote{Alternatively, I can estimate $\gamma_j$ for each non-service sector separately using tariffs as a trade cost shifter as in Caliendo and Parro (2015). Bilateral tariff data at the sector level can be obtained from the UNCTAD-TRAINS.}

Recall that $\Omega^h = y^h - \sum_{n'} \left( \frac{y^{n'}}{Y^{n'}} \right) y^{n'}$ where $y^h = \ln(\frac{\bar{w}^h}{\alpha_{(j,n)}}) + \sum^h$. I proxy the homothetic price aggregator $a(p^h)$ with a Stone index: $a(p^h) = \sum_n S^h_n ln(p_{nn}(d^h)^{\rho_n})$, where $p_{nn}$ are the quality-adjusted prices estimated by Feenstra and Romalis (2014). I obtain estimates of $\bar{w}^h$ and $\sum^h$ from the supply side as reported in the last section.

The estimating equation that I take to the data is the following:

$$\frac{Y_{(j,n)}^h}{Y^h} - \frac{Y_{(j,n)}^W}{Y^W} = a_n (S_{j}^h - S_{j}^W) - (\gamma_j \rho) \Delta^h_n + \left( \gamma_j \delta_j \right) I_n^h + \left( \gamma_j \delta_j \right) B_n^h + \beta_{(j,n)} \Omega^h + \epsilon^h_{(j,n)} \quad (2.10)$$

where $\beta_{(j,n)} = \beta_{(j,n)} - a_n \bar{\beta}_j$. To separately identify $\beta_{(j,n)}$, I need to estimate $a_n$ (in the same equation) and $\bar{\beta}_j = \sum_n \beta_{(j,n)}$ from $S_j^h = \bar{a}_j^h + \bar{\beta}_j y^h = \sum_n \alpha_{(j,n)}^h + \bar{\beta}_j y^h = a_j + \bar{\beta}_j y^h + \epsilon^h_j$. The left-hand side of the equation is computed from WIOD, using average flows between 2005 and 2007 to smooth out any temporary shocks. In the benchmark, I compute expenditure shares as percentages of total expenditure. As a robustness check, I compute expenditure shares as percentages of final consumption expenditure.

Table 2.2 reports my estimates of the cross-substitution elasticities between different suppliers of a good within each sector. Note that the sector-level non-homothetic gravity equations add up to a single-sector gravity equation. The sum of my estimates of $\gamma_j$ across sectors is 0.24. It is very close to the estimate in Fajgelbaum and Khandelwal (2016). Estimating a translog gravity equation, Novy (2013) reports $\gamma = 0.167$ while Feenstra and Weinstein (2010) reports a median $\gamma$ of 0.19.
<table>
<thead>
<tr>
<th>sector</th>
<th>( \gamma_j )-total</th>
<th>( \gamma_j )-final</th>
<th>sector</th>
<th>( \gamma_j )-total</th>
<th>( \gamma_j )-final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>0.0060</td>
<td>0.0048</td>
<td>Sales, Repair of Motor Vehicles</td>
<td>0.0030</td>
<td>0.0030</td>
</tr>
<tr>
<td>Mining</td>
<td>0.0029</td>
<td>0.0008</td>
<td>Wholesale Trade and Commission Trade</td>
<td>0.0115</td>
<td>0.0121</td>
</tr>
<tr>
<td>Food, Beverages and Tobacco</td>
<td>0.0086</td>
<td>0.0102</td>
<td>Retail Trade</td>
<td>0.0104</td>
<td>0.0131</td>
</tr>
<tr>
<td>Textiles</td>
<td>0.0021</td>
<td>0.0017</td>
<td>Hotels and Restaurants</td>
<td>0.0074</td>
<td>0.0109</td>
</tr>
<tr>
<td>Leather and Footwear</td>
<td>0.0004</td>
<td>0.0004</td>
<td>Inland Transport</td>
<td>0.0046</td>
<td>0.0042</td>
</tr>
<tr>
<td>Wood Products</td>
<td>0.0013</td>
<td>0.0003</td>
<td>Water Transport</td>
<td>0.0006</td>
<td>0.0001</td>
</tr>
<tr>
<td>Printing and Publishing</td>
<td>0.0037</td>
<td>0.0017</td>
<td>Air Transport</td>
<td>0.0013</td>
<td>0.0012</td>
</tr>
<tr>
<td>Coke, Refined Petroleum, Nuclear Fule</td>
<td>0.0045</td>
<td>0.0023</td>
<td>Other Auxiliary Transport Activities</td>
<td>0.0025</td>
<td>0.0015</td>
</tr>
<tr>
<td>Chemicals and Chemical Products</td>
<td>0.0068</td>
<td>0.0022</td>
<td>Post and Telecommunications</td>
<td>0.0058</td>
<td>0.0051</td>
</tr>
<tr>
<td>Rubber and Plastics</td>
<td>0.0026</td>
<td>0.0006</td>
<td>Financial Intermediation</td>
<td>0.0180</td>
<td>0.0102</td>
</tr>
<tr>
<td>Other Non-Metallic Minerals</td>
<td>0.0028</td>
<td>0.0007</td>
<td>Real Estate Activities</td>
<td>0.0179</td>
<td>0.0252</td>
</tr>
<tr>
<td>Basic Metals and Fabricated Metal</td>
<td>0.0103</td>
<td>0.0021</td>
<td>Renting of M&amp;Eq</td>
<td>0.0158</td>
<td>0.0058</td>
</tr>
<tr>
<td>Machinery</td>
<td>0.0047</td>
<td>0.0048</td>
<td>Public Admin and Defense</td>
<td>0.0166</td>
<td>0.0317</td>
</tr>
<tr>
<td>Electrical and Optical Equipment</td>
<td>0.0081</td>
<td>0.0048</td>
<td>Education</td>
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<td>0.0133</td>
</tr>
<tr>
<td>Transport Equipment</td>
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<td>0.0052</td>
<td>Health and Social Work</td>
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<td>0.0204</td>
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<td>Manufacturing, nec</td>
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<td>Other Community and Social Services</td>
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<td>0.0143</td>
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<td>Electricity, Gas and Water Supply</td>
<td>0.0072</td>
<td>0.0042</td>
<td>Private Households with Employed Persons</td>
<td>0.0003</td>
<td>0.0006</td>
</tr>
<tr>
<td>Construction</td>
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<td>0.0364</td>
<td>sum</td>
<td>0.2433</td>
<td>0.2580</td>
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</table>

Table 2.2: Cross-substitution between Goods
Table 2.3 reports my estimates of the sectoral income elasticities, $\beta_j = \sum_n \beta_{(j,n)}$. The corresponding elasticities for food, manufacturing and services are -0.022, -0.0051 and 0.0271, respectively. I find that the service sectors have a higher income elasticity as expected.

<table>
<thead>
<tr>
<th>sector</th>
<th>$\beta_{j,\text{total}}$</th>
<th>$\beta_{j,\text{final}}$</th>
<th>sector</th>
<th>$\beta_{j,\text{total}}$</th>
<th>$\beta_{j,\text{final}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>-0.0128</td>
<td>-0.0117</td>
<td>Sales, Repair of Motor Vehicles</td>
<td>0.0020</td>
<td>0.0022</td>
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<tr>
<td>Mining</td>
<td>-0.0052</td>
<td>-0.0002</td>
<td>Wholesale Trade and Commission Trade</td>
<td>-0.0001</td>
<td>-0.0008</td>
</tr>
<tr>
<td>Food, Beverages and Tobacco</td>
<td>-0.0080</td>
<td>-0.0103</td>
<td>Retail Trade</td>
<td>-0.0011</td>
<td>0.0009</td>
</tr>
<tr>
<td>Textiles</td>
<td>-0.0034</td>
<td>-0.0024</td>
<td>Hotels and Restaurants</td>
<td>0.0004</td>
<td>0.0016</td>
</tr>
<tr>
<td>Leather and Footwear</td>
<td>-0.0005</td>
<td>-0.0004</td>
<td>Inland Transport</td>
<td>-0.0041</td>
<td>-0.0044</td>
</tr>
<tr>
<td>Wood Products</td>
<td>-0.0006</td>
<td>0.0002</td>
<td>Water Transport</td>
<td>-0.0008</td>
<td>-0.0012</td>
</tr>
<tr>
<td>Printing and Publishing</td>
<td>0.0007</td>
<td>0.0012</td>
<td>Air Transport</td>
<td>0.0003</td>
<td>0.0002</td>
</tr>
<tr>
<td>Coke, Refined Petroleum, Nuclear Fuel</td>
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<td>0.0004</td>
<td>Other Auxiliary Transport Activities</td>
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<td>0.0011</td>
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<td>Chemicals and Chemical Products</td>
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<td>-0.0009</td>
<td>Post and Telecommunications</td>
<td>0.0005</td>
<td>0.0002</td>
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<tr>
<td>Rubber and Plastics</td>
<td>-0.0005</td>
<td>-0.0003</td>
<td>Financial Intermediation</td>
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<td>0.0032</td>
</tr>
<tr>
<td>Other Non-Metallic Minerals</td>
<td>-0.0009</td>
<td>0.0000</td>
<td>Real Estate Activities</td>
<td>0.0059</td>
<td>0.00106</td>
</tr>
<tr>
<td>Basic Metals and Fabricated Metal</td>
<td>0.0004</td>
<td>0.0004</td>
<td>Renting of M&amp;Eq</td>
<td>0.0131</td>
<td>0.0016</td>
</tr>
<tr>
<td>Machinery</td>
<td>-0.0003</td>
<td>-0.0006</td>
<td>Public Admin and Defense</td>
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<td>0.0051</td>
</tr>
<tr>
<td>Electrical and Optical Equipment</td>
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<td>-0.0014</td>
<td>Education</td>
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<td>0.0026</td>
</tr>
<tr>
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<td>-0.0013</td>
<td>Health and Social Work</td>
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<td>0.0137</td>
</tr>
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<td>0.0002</td>
<td>Other Community and Social Services</td>
<td>0.0005</td>
<td>0.0013</td>
</tr>
<tr>
<td>Electricity, Gas and Water Supply</td>
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<td>0.0010</td>
<td>Private Households with Employed Persons</td>
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<td>0.0004</td>
</tr>
<tr>
<td>Construction</td>
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<td>-0.0111</td>
<td>sum</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 2.3: Sectoral Betas
Figure 2.7: Average Income and Income Elasticity of Production

Figure 2.7 plots the sectoral income elasticity computed from total expenditure and final consumption against the exporter’s log average income. I find a positive relationship which implies that high-income countries specialize in the production of high-income elastic goods, which is consistent with previous findings in Hallak (2006), Khandelwal (2010), Hallak and Schott (2011) and Feenstra and Romalis (2014). The null hypothesis that all income elasticities are zero is rejected.

Figure 2.8 plots the sectoral income elasticity computed from total expenditure and final consumption against the skill intensity of each sector. I find that skill-intensive sectors produce goods that have a high income elasticity. This implies that a decline in the relative price of low-income elastic goods from trade liberalization is correlated with a decline in the relative price of goods in unskill-intensive sectors. This implication, along with the other two mentioned in the last section, suggests that trade liberalization increases the nominal wage inequality within a country.
Finally, to estimate $\alpha_{(j,n)}^h$, I assume that it can be decomposed into an exporter effect, $a_n$, a sector specific effect, $a_j$ and an importer specific taste for that sector, $\varepsilon_j^h$: $\alpha_{(j,n)}^h = a_n(a_j + \varepsilon_j^h)$ as before. I then estimate $a_n$ from the sector-level non-homothetic gravity equation and $a_j + \varepsilon_j^h$ from the sectoral expenditure share: $S_j^h = \bar{\alpha}_j + \bar{\beta}_j y^h = \sum_n \alpha_{(j,n)}^h + \bar{\beta}_j y^h = a_j + \bar{\beta}_j y^h + \varepsilon_j^h$. ¹⁹

2.4 Counterfactuals

Recall that equation (14) can be used to compute the global welfare change of individual $z$ between trade and a counterfactual scenario:

¹⁹Alternatively, note that the aggregate expenditure share in equation (11) is a non-linear function in $\alpha_{(j,n)}^h$ and, $\{p_{(j,h)}^h\}_{j \in J}$, $\forall h$, the output prices in the general equilibrium, given the estimates of $\gamma_j$ and $\beta_{(j,n)}$. I use $S_{(j,n)}^h$ as an initial guess for $\alpha_{(j,n)}^h$ and solve for the prices. Given these prices, I solve for an updated value of $\alpha_{(j,n)}^h$, which is used in the next iteration, and the procedure continues until convergence.
parameterization. Since I am interested in the impact of trade liberalization on different groups of people, in particular, the poor versus the rich, I focus on the difference in welfare change between the 10th percentile and the 90th percentile of the initial nominal wage distribution within each country that comes from each of the components in equation (14). Since the aggregate expenditure effect is the same for every individual within a country, it is differenced out. I define the following terms: \textbf{diff. exp. effect} = \text{ind. exp. effect}_{z=10th} - \text{ind. exp. effect}_{z=90th}.

2.4.1 Five Percent Reduction in Trade Costs

I first consider a simultaneous 5\% reduction in all bilateral trade costs, starting from the baseline parametrization. Since I am interested in the impact of trade liberalization on different groups of people, in particular, the poor versus the rich, I focus on the difference in welfare change between the 10th percentile and the 90th percentile of the initial nominal wage distribution within each country that comes from each of the components in equation (14). Since the aggregate expenditure effect is the same for every individual within a country, it is differenced out. I define the following terms: \textbf{diff. exp. effect} = \text{ind. exp. effect}_{z=10th} - \text{ind. exp. effect}_{z=90th}.

\[ E_{tr}^{cf} = \prod_{(j,n)} \left( \frac{b_{tr}(p_{(j,n)}^{cf})}{b_{tr}(p_{(j,n)}^{tr})} \right) S^{b_{tr}(p_{(j,n)}^{tr})} \] is the aggregate expenditure effect, and it measures the reduction in the price index for a country’s representative consumer. \((\frac{w_{tr}^{cf}}{w_{tr}^{tr}}) - \ln(\frac{b_{tr}^{cf}}{b_{tr}^{tr}})\) is the individual expenditure effect, where \(-\ln(\frac{b_{tr}^{cf}}{b_{tr}^{tr}}) = - \sum_j \sum_n \beta_{(j,n)} \ln(\frac{p_{(j,n)}^{cf}}{p_{(j,n)}^{tr}})\). For individual \(z\) who is richer than the representative consumer, a decrease in the relative price of low-income elastic goods makes her better off. \(\frac{w_{tr}^{cf}}{w_{tr}^{tr}}\) is the income effect, and its change depends on the sector that individual \(z\) works in. An increase in a sector’s output price raises the relative nominal wage of the labor groups that disproportionately work in that sector in the initial trade equilibrium.

\footnote{Please see Appendix B.6 for a discussion about how to implement the counterfactual where each country moves back to autarky.}
Table 2.4: Distributional Effects through Income Channel

<table>
<thead>
<tr>
<th>Active channel(s)</th>
<th>Income</th>
<th>Expenditure</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>diff. exp. effect</td>
<td>0</td>
<td>[0.43, 0.88]</td>
<td>[0.76, 1.36]</td>
</tr>
<tr>
<td>diff. inc. effect</td>
<td>[−0.01, 0.04]</td>
<td>0</td>
<td>[−0.72, −0.04]</td>
</tr>
<tr>
<td>diff. tot. effect</td>
<td>[−0.01, 0.04]</td>
<td>[0.43, 0.88]</td>
<td>[0.24, 1.29]</td>
</tr>
</tbody>
</table>

ind. exp. effect_{z=90th}; diff. inc. effect = income effect_{z=10th} − income effect_{z=90th}; diff. tot. effect = total effect_{z=10th} − total effect_{z=90th}.

2.4.1.1 Income Channel

I first study the distributional effects of trade liberalization through the income channel. The second column of Table 2.4 reports the lower and upper bounds of diff. exp. effect, diff. inc. effect, diff. tot. effect across the 40 countries when only the income channel is active. I shut down the expenditure channel by imposing that $\beta_{(j,n)} = 0 \forall j \in J, n \in N$. This brings us back to a translog demand system which is homothetic. Under these restrictions, the consumer price index for every individual within a country changes by the same amount, i.e. diff. exp. effect=0.

I find that in Estonia, the 10th percentile suffers a decrease in the nominal wage relative to the 90th percentile of 0.01 percentage points. On the other hand, in Portugal, the 10th percentile enjoys an increase in the relative nominal wage by 0.04 percentage points. The change in the relative nominal wage for the rest of the countries lies in between.

Panel A of Figure 2.9 plots diff. inc. effect against the log average income for each country. Panel B plots a country’s skill abundance against its log average income.\(^\text{21}\) The regression lines are based on the weighted least squares with weights equal to the output

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\(^{21}\)I measure a country’s skill abundance, $H_n/(H_n + L_n)$, as the share of workers with a completed tertiary degree (i.e. university graduates and post-graduates).
I next study the distributional effects of trade liberalization through the expenditure channel.

2.4.1.2 Expenditure Channel

I find that the income channel benefits the poor more than the rich in low-income countries that are skill-scarce. These countries have a comparative advantage in unskill-intensive sectors and a reduction in trade costs increases the relative nominal wage of the poor because they are less skilled and more likely to work in unskill-intensive sectors. On the other hand, the income channel benefits the rich more than the poor in high-income countries that are skill-abundant. These countries have a comparative advantage in skill-intensive sectors and a reduction in trade costs increases the relative nominal wage of the rich because they are more skilled and more likely to work in skill-intensive sectors.

**Figure 2.9: Distributional Effects through Income Channel**

The third column of Table 2.4 reports the lower and upper bounds of \( \text{diff. exp. effect} \).
<table>
<thead>
<tr>
<th>Active channel(s)</th>
<th>Income effect</th>
<th>Expenditure effect</th>
<th>Both effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>diff. exp. effect</td>
<td>0</td>
<td>[0.43, 0.88]</td>
<td>[0.76, 1.36]</td>
</tr>
<tr>
<td>diff. inc. effect</td>
<td>[-0.01, 0.04]</td>
<td>0</td>
<td>[-0.72, -0.04]</td>
</tr>
<tr>
<td>diff. tot. effect</td>
<td>[-0.01, 0.04]</td>
<td>[0.43, 0.88]</td>
<td>[0.24, 1.29]</td>
</tr>
</tbody>
</table>

Table 2.4: Distributional Effects through Expenditure Channel

inc. effect, diff. tot. effect across the 40 countries when only the expenditure channel is active. I shut down the income channel by imposing that \( T(\lambda, j) = 1 \forall \lambda \in \Lambda, j \in J \), that is, there is no comparative advantage of different labor types across sectors. Under these restrictions, the nominal wage of every individual within a country changes by the same amount, i.e. \( \text{diff. inc. effect} = 0 \).

I find that the expenditure channel benefits the poor more than the rich in every country. More specifically, in Indonesia, the 10th percentile enjoys a reduction in the consumer price index that is 0.43 percentage points bigger than the 90th percentile. On the other hand, in Taiwan, the 10th percentile enjoys a reduction in the consumer price index that is 0.88 percentage points bigger than the 90th percentile. The poor’s relative benefit from the expenditure channel for the rest of the countries lies in between.

Why does the expenditure channel imply a pro-poor bias in every country? The most direct effect of a reduction in trade costs is to decrease \( a(p^h) \), the homothetic price aggregator, which increases the inequality-adjusted real wage, \( \ln \left( \frac{\tilde{\tilde{w}}} {a(p^h)} \right) \), in every country \( h \), and therefore decreases the expenditure shares on goods with \( \beta_{(j,n)} < 0 \). This is an inward shift in the demand for low-income elastic goods which decreases their relative price. Since low-income consumers spend more on these goods, they benefit more from the expenditure channel.

Figure 2.10 plots the percentage change in the price of each of the \( J \times N = 1400 \) goods.
against its income elasticity, \( \beta_{(j,n)} \). Panel A uses the income elasticity computed from total expenditure while Panel B restricts to final consumption. The correlation is strongly positive regardless of which estimate of \( \beta_{(j,n)} \) I use, that is, there is a decrease in the relative price of low-income elastic goods following trade liberalization.

Across countries, I find that expenditure channel benefits the poor relative to the rich even more in high-income countries that import low-income elastic goods. Panel A of Figure 2.11 plots \textbf{diff. exp. effect} against the log average income for each country. Panel B plots the income elasticity of a country’s imports relative to its production against its log average income.\(^{22}\) The regression lines are based on the weighted least squares with weights equal to the output share of a country in the world economy. Because high-income countries

\[ \left( p_{(j,n)}^{cf} - p_{(j,n)}^{tr} \right) / p_{(j,n)}^{tr} \]

Figure 2.10: Percentage Change in Prices

\(^{22}\)I calculate the income elasticity of a country’s imports relative to its production as \( \bar{\beta}_{imp} - \bar{\beta}_{prod} = \sum_j \sum_{n \neq h} \beta_{(j,n)} S_{(j,n)}^h - \sum_j \beta_{(j,h)} \).
import low-income elastic goods, the decrease in the relative price of low-income elastic goods is magnified by the lower trade costs, which implies a bigger relative benefit from the expenditure channel for the poor.

2.4.1.3 Both Channels

Finally, I study the distributional effects of trade liberalization through both channels. The average gain from a simultaneous 5% reduction in all bilateral trade costs across the countries is 1.2%. As I move up the income distribution, gains decline. More specifically, moving to the next decile reduces gains by about 0.1 percentage points. The third column of Table 2.4 reports the lower and upper bounds of diff. exp. effect, diff. inc. effect, diff. tot. effect across the 40 countries when both the expenditure channel and the income channel are active. Since non-homothetic preferences allow people with different incomes to consume different bundles of goods, price changes resulting from trade liberalization can have a differential
impact on an individual’s consumer price index. I find a pro-poor bias from the expenditure channel in every country, i.e., \( \text{diff. exp. effect} > 0 \). On average, the 10th percentile sees her consumer price index decrease by 1 percentage point more than the 90th percentile.

In addition, since different labor groups sort into different sectors based on comparative advantage, price changes resulting from trade liberalization can have a differential impact on an individual’s nominal wage. I find a pro-rich bias from the income channel in every country, i.e., \( \text{diff. inc. effect} < 0 \). On average, the 10th percentile sees her nominal wage go down by 0.24 percentage points relative to the 90th percentile. Since the expenditure effect dominates the income effect in magnitude, trade liberalization benefits the poor more than the rich in every country, i.e., \( \text{diff. tot. effect} > 0 \). I find that in Luxembourg, the 10th percentile enjoys an increase in the real wage relative to the 90th percentile of 0.24 percentage points. On the other hand, in Taiwan, the 10th percentile enjoys an increase in the relative real wage of 1.29 percentage points. The poor’s relative benefit from both channels in terms of real wages for the rest of the countries lies in between. On average, the difference between the 10th and the 90th percentiles is about 0.8 percentage points.

More interestingly, I find that when both channels are active, the poor enjoy an even bigger relative reduction in consumer price indices in every country compared to the case where only the expenditure channel operates, that is, the range of \( \text{diff. exp. effect} \) across the 40 countries changes from \([0.43, 0.88]\) to \([0.76, 1.36]\). In addition, the poor now suffer a

<table>
<thead>
<tr>
<th>Active channel(s)</th>
<th>Income</th>
<th>Expenditure</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>diff. exp. effect</td>
<td>0</td>
<td>[0.43, 0.88]</td>
<td>[0.76, 1.36]</td>
</tr>
<tr>
<td>diff. inc. effect</td>
<td>[−0.01, 0.04]</td>
<td>0</td>
<td>[−0.72, −0.04]</td>
</tr>
<tr>
<td>diff. tot. effect</td>
<td>[−0.01, 0.04]</td>
<td>[0.43, 0.88]</td>
<td>[0.24, 1.29]</td>
</tr>
</tbody>
</table>

Table 2.4: Distributional Effects through Both Channels
relative decrease in nominal wages in every country. Note that the range of \textbf{diff. inc. effect} across the 40 countries changes from \([-0.01, 0.04]\) to \([-0.72, -0.04]\). That is, the interaction of the two channels quantitatively changes the prediction of the differential impact of trade liberalization on the poor versus the rich through the expenditure channel and qualitatively through the income channel.

To see the comparison visually, Figure 2.12 plots (using blue x) \textbf{diff. inc. effect} when only the income channel is active against \textbf{diff. exp. effect} when only the expenditure channel is active, and then plots (using red diamond) \textbf{diff. inc. effect} against \textbf{diff. exp. effect} when both channels are active and interact. The interaction changes the estimates of both effects significantly. More specifically, each country moves to the right which implies that the poor’s relative benefit from the expenditure channel is bigger. Also, each country moves downward and \textbf{diff. inc. effect} < 0 for all of them, which implies that the rich benefit relative to the poor from the income channel in every country.
Why does the expenditure channel imply a bigger pro-poor bias and the income channel imply a pro-rich bias in every country? When both channels are active, lower trade costs reduce the relative demand for and the relative price of low-income elastic goods as discussed before. However, since the poor disproportionately produce unskill-intensive goods which are low-income elastic, their relative nominal wage goes down in every country. This implies that the income channel benefits the rich everywhere. This effect is absent when only the income channel is active because the income elasticity of every good is 0. On the other hand, as the nominal wage inequality goes up, the relative demand for and the relative price of low-income elastic goods fall even further, reducing the relative price index for the poor in every country. This implies that the expenditure channel benefits the poor even more compared to the case where only the expenditure channel is active. This effect is absent in that case because nominal wage inequality is constant.

How does the poor’s relative benefit from the combined effect of trade liberalization vary across countries? Figure 2.13 plots diff. tot. effect against the log average income for each country. The regression line is based on the weighted least squares with weights equal to the output share of a country in the world economy. Since the expenditure channel benefits more the poor individuals in rich countries and the rich individuals in poor countries, while the income channel benefits more the rich individuals in rich countries and the poor individuals in poor countries, allowing both channels to operate no longer makes income per capita a good predictor of the pro-poor bias of trade liberalization.\(^\text{23}\)

\(^{23}\)Since country characteristics are all correlated and pull in different directions, none in the data that I targeted has significant explanatory power for the variation in the model’s predicted pro-poor bias of trade liberalization across countries.
2.4.1.4 Bias from Considering Two Channels Separately

Table 2.5 reports the bias from considering the two channels separately for each country. In the second column, I add up **diff. inc. effect** when only the income channel is active and **diff. exp. effect** when only the expenditure channel is active, and then compare to **diff. tot. effect** when both channels are active as reported in the third column. I find that estimating the two effects separately and adding them up generates a significant downward bias in the prediction for the poor’s relative benefit from trade liberalization.

In particular, this underestimation is stronger in a country like Japan, which produces high-income elastic goods, compared to a country like Mexico, which produces low-income elastic goods. This pattern generalizes to the entire sample of 40 countries. Figure 2.14 plots the difference in the poor’s relative benefit from trade liberalization between estimating the two effectes jointly and separately against the income elasticity of the country’s production,
\[ \bar{\beta}_{prod}^h = \sum_j \beta_{(j,h)} \]. Panel A uses the income elasticity computed from total expenditure while Panel B is restricted to final consumption. The correlation is strongly positive regardless of which estimate of \( \bar{\beta}_{prod}^h \) I use, that is, the interaction of the two channels benefits more the countries that produce high-income elastic goods.\(^{24}\)

<table>
<thead>
<tr>
<th>Country</th>
<th>Separate</th>
<th>Combined</th>
<th>Country</th>
<th>Separate</th>
<th>Combined</th>
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<tr>
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<td>KOR</td>
<td>0.76</td>
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</tr>
<tr>
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<td>LTU</td>
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<td>0.78</td>
</tr>
<tr>
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<td>LVA</td>
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<td>0.78</td>
</tr>
<tr>
<td>CYP</td>
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<td>1.01</td>
<td>MEX</td>
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<td>0.77</td>
</tr>
<tr>
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<td>0.68</td>
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<td>FRA</td>
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<td>SVN</td>
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</tr>
<tr>
<td>IDN</td>
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<td>0.47</td>
<td>TWN</td>
<td>0.89</td>
<td>1.29</td>
</tr>
<tr>
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<td>0.57</td>
<td>0.57</td>
<td>USA</td>
<td>0.80</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Table 2.5: Bias from Considering Two Channels Separately

\(^{24}\)Luxembourg is an outlier. It is one of the smallest sovereign states in Europe and has the world’s highest GDP per capita. It also has the highest trade share in my sample of countries.
Figure 2.14: Underprediction of Pro-poor Bias of Trade Liberalization

Intuitively, the interaction reallocates workers away from unskill-intensive sectors that produce low-income elastic goods in every country because it decreases the relative price of low-income elastic goods. However, this is already the case in the countries that specialize in the production of high-income elastic goods without the interaction. Therefore, the interaction induces a smaller increase in worker reallocation away from unskill-intensive sectors in these countries, which implies a bigger benefit for the poor who work in these sectors.

2.4.2 Rising Chinese Import Competition

Autor et al. (2013) analyze the effect of rising Chinese import competition between 1990 and 2007 on U.S. local labor markets, and they find that it causes higher unemployment, lower labor force participation, and reduced wages in local labor markets that serve import-
competing manufacturing industries.\textsuperscript{25}\textsuperscript{26} They instrument for the growth in U.S. imports from China using Chinese import growth in other high-income markets to isolate the foreign-supply-driven component of the changes, i.e., China’s productivity growth and falling trade costs. In particular, for their base specifications, they focus on a single channel through which trade with China affects a region: greater import competition in the U.S. market. This ignores the effects of greater U.S. exports to China or greater import competition in the foreign markets that U.S. regions serve. Their main measure of local labor market exposure to import competition is the change in Chinese import exposure per worker in a region, where imports are apportioned to the region according to its share of national industry employment. They also control for the start-of-period manufacturing share within commuting zones so as to focus on variation in exposure to Chinese imports stemming from differences in industry mix within local manufacturing sectors.

Instead of using the variation across local labor markets, I analyze the aggregate effect of a $1K increase in U.S. manufacturing imports from China per worker.\textsuperscript{27} At initial equilibrium, average per capita spending by the U.S. on Chinese manufacturing goods is

$\sum_{j \in M} S_{(j,chn)}^{us} \bar{w}^{us} = 0.0187 \times 22.4128 = 0.42$.\textsuperscript{28} To increase it by $1K is equivalent to an increase in the total expenditure share on these goods of 4.46%.\textsuperscript{29} I shut down the effects of greater U.S. exports to China or greater import competition in the foreign markets that

\textsuperscript{25}Wage changes in \textit{Autor et al. (2013)} are in nominal and not real terms.

\textsuperscript{26}It would be interesting and important to introduce unemployment or search into my framework. There would then be consequences about adjustment to trade shocks in the short- and medium-run. I leave it for future work. Please refer to Joan Monras’ and Matthieu Bellon’s work on these topics.

\textsuperscript{27}1 unit in my framework is approximately $1000.

\textsuperscript{28}Sectors “Agriculture” and “Food, Beverages and Tobacco” are the food sectors; “Mining” and from “Textiles” to “Manufacturing, nec” in the first column in Table 2 and 3 are the manufacturing sectors. The remaining sectors are the service sectors.

\textsuperscript{29}Δ$\sum_{j \in M} S_{(j,chn)}^{us} \bar{w}^{us} = 1 \rightarrow \Delta \sum_{j \in M} S_{(j,chn)}^{us} / \bar{w}^{us} = 1 / \bar{w}^{us} = 0.0446$. Note that this increase in spending on Chinese goods that \textit{Autor et al. (2013)} consider is due to supply and trade-cost-driven changes in China’s export performance, not changes in U.S. import demand as a result of higher income.
the U.S. serves by holding the production prices, $p(j,h) \forall j \in J, \forall h \neq US$, and trade costs, 
$\tau_{(j,h)}^n \forall j \in J, \forall n \neq CHN, \forall n \neq US$, unchanged. To compute the reduction in trade costs in the manufacturing sectors that would lead to this increase in Chinese imports, I apply the expenditure share equation (32), and it follows that 
$$
\Delta S^{us}_{(j,chn)} = \sum_{j \in M} \frac{(1-N)\gamma_j}{N} \log(d\tau)
$$
where $\tau_{(j,chn)}^{us,cf} = \tau_{(j,chn)}^{us,tr} \cdot d\tau$ if $j \in M$.\(^{30}\) Plugging in the estimates of $\gamma_j$, I have $\log(d\tau) = -0.8$.

Applying equation (32) again, I calculate the impact of this reduction in trade costs on U.S. expenditure shares on domestic goods, 
$$
\Delta S^{us}_{(j,us)} = \frac{N}{N} \log(d\tau).
$$
I then solve for U.S. production prices again, $p^{us}_{(j,us)} \forall j \in J$, such that the U.S. market clearing conditions (equation (26)) are still satisfied, taking into account the change in domestic demand.

I find that production prices, $p^{us}_{(j,us)}$, go down in all $j \in J$ in the U.S. as a result of rising Chinese import competition. They decrease in the manufacturing sectors because of the lower demand for the domestically produced goods, and in the non-manufacturing sectors because workers choose to leave manufacturing and work in other sectors in response to lower output prices and wages in manufacturing. This increases the labor supply in the non-manufacturing sectors, putting downward pressure on the output prices in these sectors. The aggregate expenditure effect, $E^{us}_{ct}$, is 0.85%, that is, the reduction in the cost of Chinese manufacturing imports decreases the consumer price index for a U.S. representative consumer by 0.85%\(^{31}\). The individual expenditure effect, 
$$
- \ln(b^{us}_{ct}/b^{us}_{tr})
$$
implies a pro-poor bias of 0.45 percentage points, with individuals whose wages are at the 10th percentile of

\(^{30}\)Note that I attribute this increase in Chinese imports entirely to the reduction in trade costs for simplification. Suppose it is due to China’s improved productivity instead, then its production prices would decrease. Both of these forces have the same effect on US consumer prices, each of which is the product of the production price and the trade cost. Note also that the change in these trade costs also affects $y^{us}$ through its impact on $a(p^{us})$. I ignore it since this effect is negligibly small and does not change the result of the analysis.

\(^{31}\)0.75% of this decrease stems from the lower consumer prices of Chinese imports in the U.S. and the remaining 0.1% comes from the lower production prices of U.S. goods.
the initial distribution see a further 0.35 percentage points reduction in their consumer price indices compared to the representative consumer and individuals whose wages are at the 90th percentile see their consumer price indices decrease by 0.1 percentage points less than the representative consumer. This result comes from the fact that Chinese manufacturing goods are low-income elastic and, consequently, their lower prices benefit more the poor individuals who spend relatively more on these goods.\(^{32}\) The income effect, \(\frac{w_{j}^{\text{f}}}{w_{i}^{\text{f}}}\), implies a pro-rich bias of 0.02 percentage points, while poor and unskilled workers see their nominal wages go down by 0.13\% and rich and skilled workers see their nominal wages go down by 0.11\%. The reason that the former see a bigger decline in their nominal wages is because they are more likely to work in manufacturing sectors that are in direct competition with cheaper Chinese imports. The more pronounced decrease in the output prices in these sectors leads to the bigger decrease in their nominal wages. Combining all three effects, poor individuals gain 0.43 percentage points more compared to rich ones in terms of real wage as a result of the rising Chinese import competition. That is, the pro-rich bias of the income effect is more than offset by the pro-poor bias of the expenditure effect which again underlines the importance of taking both channels into account in assessing the distributional effects of trade liberalization.

\section*{2.5 Conclusion}

This chapter addresses the following question: what is the impact of trade liberalization on the distribution of real wages in a large cross-section of countries? The vast majority of the literature focuses on the effect of trade on the distribution of nominal wages. A small number

\[^{32}\text{11 out of China’s 14 manufacturing sectors have } \beta_{(i,chn)} < 0.\]
of studies consider its differential impact on consumer price indices. To my knowledge, there are only three case studies that have combined both channels to examine how real wages of different groups of people are affected in individual countries, Argentina, Mexico and India.

I use sector-level trade and production data to estimate the parameters of the model in Chapter 1. I find that as a result of a five percent reduction in all bilateral trade costs, the bigger decline in the poor’s consumer price indices more than compensates for their lower relative nominal wage. More specifically, in the average country, real wage of the bottom 10th percentile increases by 0.8 percentage points more than the top 10th percentile. I also find that there is an important interaction between the two channels and, therefore, estimating the two effects separately and adding them up leads to a significant bias. These results highlight the importance of combining both channels in order to measure the distributional effects of trade accurately.

My findings have important policy implications for the distribution of winners and losers from trade reforms. There has been increasing public resistance to freer trade that originates from the belief that the most vulnerable group, i.e., the poor and unskilled, will be hurt the most. This chapter demonstrates that such a belief is misguided.
Chapter 3

Imported Inputs and Within-Sector Wage Dispersion

Mi Dai, Zheli He and Feiran Zhang
3.1 Introduction

The traditional Heckscher-Ohlin model predicts that countries export goods that use intensively the factor they are most abundantly endowed with. According to the Stolper-Samuelson theorem, trade increases the relative return to unskilled labor in developing countries, decreasing wage inequality. However, that prediction is at odds with many empirical findings. Take China as an example, the overall wage inequality, measured as the difference between the 90th and the 10th percentile of the log wage distribution, has been going up consistently in the last two decades, as found in Han et al. (2012). This period of rapid wage inequality increase coincided with China’s implementation of dramatic economic reforms and an open door policy that promoted its trade with the rest of the world. So two important questions arise: did trade liberalization contribute to China’s rising wage inequality? If so, through which channels?

New theoretical developments have been made to provide insights into the effects of trade on wage inequality. Most prominently, Verhoogen (2008) proposes the quality-upgrading mechanism as an explanation. In his model with heterogeneous plants and quality differentiation, an exchange-rate devaluation leads more productive Southern plants to increase exports, upgrade quality, and raise wages relative to less productive ones, increasing within-sector wage dispersion. In this chapter, we propose an alternative mechanism: the use of imported inputs. Intuitively, a firm with higher initial productivity is better at using higher quality foreign inputs. This justifies paying the fixed cost for a larger set of imported inputs when input tariff liberalization decreases their relative price. The firm becomes more import intensive, which enhances its productivity advantage. As a result, the firm hires higher
quality workers, produces higher quality products and pays higher wages to its workers, increas- ing within-sector wage dispersion. We aim to empirically distinguish these mechanisms in the data, as they each have different welfare and policy implications.

First, we use ASIF (Annual Survey of Industrial Firms) from China’s National Bureau of Statistics that report key operational data on Chinese manufacturing firms to document some stylized facts that are both new and interesting. We find that both the mean and the dispersion of the distribution of firm productivity, markup and size went up during a period when China reduced its tariffs on imported inputs. More importantly, these results still hold when we consider the subset of firms that survived throughout the sample period, from 1998 to 2007. Therefore, openness to trade has fundamental effects on the underlying characteristics of firms. Most of recent models of firm heterogeneity assume that these characteristics are fixed and examine the impact of trade on aggregate variables, for example, the average productivity of firms in the economy as a result of change in the composition of surviving firms. On the contrary, we study the differential impact of trade liberalization on heterogeneous firms allowing these characteristics to be endogenous.

We measure firm-level TFP based on OLS, Olley and Pakes, Levinsohn and Petrin, and Ackerberg, Caves and Frazer to ensure that our estimate of firm productivity is as accurate as possible. For firm-level markup calculation, we adopt De Loecker and Warzynski (2012), which is the best available method that we can use given our data limitations. We consider both a Cobb-Douglas gross output production function, and more generally, a translog gross output production function, which matches the data much better. Finally, we measure firm size both in terms of output value and total employment as a robustness check. The empirical patterns are very similar when we use different approaches to measure these three
key firm-level variables.

Second, we develop a partial equilibrium, heterogeneous firm model with endogenous imported inputs and labor quality choice that is consistent with these observations. On the demand side, we adopt the “quality-Melitz” model in Kugler and Verhoogen (2012), where higher price decreases demand but higher quality increases demand. On the supply side, firms differ from each other in the usual dimension of productivity, as in Melitz (2003). In our model, firms combine labor and intermediate inputs to produce physical quantity, in the spirit of Amiti et al. (2014). Output quality, on the other hand, is determined by labor and input qualities, and the advantage of imported inputs over domestic counterparts is augmented by a firm’s own productivity. Since Amiti et al. (2014) focus on exchange rate pass-through, and assume that firms do not foresee fluctuations in exchange rates, they hold the set of imported inputs of each firm fixed. We, on the other hand, study precisely how firms adjust the set of foreign varieties they import in response to input tariff liberalization and changes in firm-level variables that follow. Consequently, our model deviates from theirs in obvious ways, which we explain in more detail in the theory section.

Finally, we use Chinese Customs Data on imports and exports, which provide detailed information on the universe of China’s firm-level trade transactions for the years 2000 to 2006, to highlight firms’ different responses to a dramatic decrease in import tariffs. These observations emphasize the large and growing importance of trade in intermediates, and provide some empirical evidence that supports our hypothesis that the differential change in import intensity of firms with different productivity levels in response to input tariff liberalization explains the increase in both the average and the dispersion of firm-level variables that are observed in the data.
Although the main focus of this chapter is to show that input tariff liberalization affects firms at different levels of productivity in a heterogeneous way, which drives their performance further apart, our results have broader implications. Essentially, we provide a framework in which the differential impact of any element of globalization that leads to a decrease in the marginal cost of production on firm-level characteristics can be analyzed. Our detailed and very disaggregated transaction-level trade data allow us to quantify the impact of such a change during a period when the change was very large in magnitude.

3.2 Related Literature

This chapter is related to several strands of literature. First, there have been studies on the labor market effects of international trade based on recent models of firm heterogeneity, and they ask how trade liberalization affects wages and wage inequality. For example, Amiti and Davis (2012) develop a model, which predicts that a fall in output tariffs lowers wages at import-competing firms but boosts wages at exporting firms, and that a fall in input tariffs raises wages at import-using firms relative to those that only source inputs locally. They find support for the model’s predictions in Indonesian manufacturing census data for the period 1991-2000. Like us, they take explicit account of firm-level heterogeneity and importance of trade in intermediates. Extending the heterogeneous firm model of trade and inequality from Helpman et al. (2010), Helpman et al. (2017) show that much of overall wage inequality arises within sector-occupations and for workers with similar observable characteristics, and wage dispersion between firms is related to firm employment size and trade participation. They again emphasize the importance of employing recent models of
firm heterogeneity in analyzing the contribution of trade to the cross-section dispersion of firms. Frías et al. (2012), on the other hand, offer some empirical evidence that sorting on individual worker ability is not enough to explain the relationship between exporting and wages at the plant level. They use a combination of employer-employee and plant-level data from Mexico, and show that approximately two-thirds of the higher level of wages in larger, more productive plants is explained by higher levels of wage premia, and that nearly all of the differential within-industry wage change is explained by changes in wage premia. They use the late-1994 Mexican peso devaluation as a source of exogenous variation in the incentive to export, while we use import tariff reductions due to China’s accession to the WTO in December 2001 as our exogenous variation. On the contrary, Krishna et al. (2012) find an insignificant differential effect of trade openness on wages at exporting firms relative to domestic firms, using detailed information on worker and firm characteristics to control for compositional effects and allowing for the endogenous assignment of workers to firms. While these papers focus on the effects of trade liberalization on the labor market, we look at other firm-level characteristics, and ask how they are affected, and the resulting implications on wage inequality in China.

Second, our theoretical model borrows insights from a burgeoning research literature on firm import behavior, which has not been extensively studied before. Most importantly, evidence has been found in a wide range of countries that firm productivity rises when a firm imports new input varieties. For example, Kasahara and Rodrigue (2008) conclude that becoming an importer of foreign intermediates improves productivity using plant-level Chilean manufacturing panel data. At the same time, Halpern et al. (2015) find that importing all foreign varieties would increase firm productivity by 12 percent, and that during
1993-2002, one-third of the productivity growth in Hungary was due to imported inputs by estimating a model of importers in Hungarian micro data and conducting counterfactual policy analysis. Bas and Strauss-Kahn (2014), on the other hand, use a firm-level database of imports provided by French Customs for the 1995-2005 period, and find a significant impact of higher diversification and increased number of imported input varieties on firm-level TFP and export scope. They argue that importing more varieties of intermediate inputs increases firm productivity and thereby makes a firm more able to overcome the fixed export costs. Our model predicts that a firm with higher initial productivity has a stronger incentive to expand its set of imported varieties when it faces lower tariff rates, which then makes it even more productive, explaining the empirical patterns observed in the data.

Third, we want to point out that these findings cannot be explained by any previous studies on heterogeneous firms. We discuss briefly a few important papers on the subject. To start with, the workhorse model developed in Melitz (2003) assumes that the preferences of a representative consumer are given by a C.E.S. utility function over a continuum of goods, so each firm chooses the same profit maximizing markup which is constant. After paying fixed entry costs, firms draw their initial productivity parameter, which does not change over time. As a result, the mean and the dispersion of the productivity distribution of a balanced panel of firms remain the same, which is not what we observe in the data. Gains from trade in this model come from expansion in product varieties, and more importantly, the self-selection of more efficient firms into exporting. Relaxing the C.E.S. assumption, Arkolakis et al. (2015) study how variable markups affect the gains from trade liberalization under monopolistic competition, and they show that the welfare effect of a small trade shock is given by $\partial \ln W = -(1 - \eta) \frac{\partial \ln \lambda}{\epsilon}$, where $\lambda$ is the share of expenditure on domestic goods,
and $\epsilon$ is an elasticity of imports with respect to variable trade costs, and $\eta$ is a structural parameter that depends, among other things, on the elasticity of markups with respect to firm production. Although they consider variable markups like us, they assume that firm-level productivity is the realization of a random variable drawn independently across firms from a distribution, which is unbounded Pareto, and it is fixed over time. Instead of a counterfactual analysis that focuses on the welfare effect of a particular shock, Feenstra and Weinstein (2010) use a translog demand system to measure the effects of new varieties and variable markups on the change in the U.S. consumer price index between 1992 and 2005. That is, they use observed trade data to infer changes in particular components of the U.S. price index. Their results highlight the importance of taking into account the implications of pro-competitive effect of trade. However, they ignore the impact of trade on productivity since that is not the main focus of their paper. On the other hand, Feenstra (2014) shows that self-selection of more efficient firms into exporting is the only source of welfare gains when using a Pareto distribution for productivity with a support that is unbounded above. He restores a role for product variety and pro-competitive gains from trade, but still assumes that firms receive a random draw of productivity from a Pareto distribution, which does not change. Finally, borrowing insights from Melitz (2003) that trade openness increases volatility by making the economy more granular since only the largest and most productive firms export, while smaller firms shrink or disappear, Di Giovanni and Levchenko (2013) show that when the distribution of firm sizes follows a power law with an exponent close to -1, the idiosyncratic shocks to large firms have an impact on aggregate output volatility. In their model, these firm-level idiosyncratic shocks may explain the observed increase in dispersion of firm size distribution, but they do not provide a microfoundation to explain why both the
mean and the standard deviation of the distribution go up in a systematic way since they assume i.i.d. transitory shock. Essentially, these theoretical papers consider the effect of a change in the exogenous distribution of firm productivity, while we take firm productivity as an endogenous variable. Therefore, we are able to add something new and interesting to the conversation about the impact of trade based on recent models of firm heterogeneity.

### 3.3 Data

The first dataset, Chinese Customs Data on imports and exports, provides detailed information on the universe of China’s firm-level trade transactions for the years 2000 to 2006. In addition to firm identifiers, this dataset includes information on many important transaction characteristics, including customs regime (e.g. processing trade or ordinary trade), 8-digit HS product code, transaction value, quantity, and source or destination country. Using firm identifiers provided in the dataset, we construct key variables that describe firm-level imports and exports. Figure 3.1 illustrates the customs declaration form that a firm has to fill out if it intends to import from or export to foreign countries.

The second key dataset is from China’s National Bureau of Statistics, which conducts firm-level surveys on manufacturing enterprises. These data collected from Chinese firms include key operational information, such as firm employment, ownership type (e.g. state-owned enterprise, foreign invested firm, or private firm), sales value, R&D expenditure and industry. Merging the firm-level data with the transaction-level data is challenging because firm identifiers used in the two datasets are different. Nevertheless, since both datasets include extensively detailed firm contact information (e.g. company name, telephone number,
zip code, contact person), we merge them using zip codes and the last seven digits of a firm’s phone number, following Yu (2015). In this way, we are able to generate firm-level observations that combine information on the trade with the operational activities of Chinese firms. Table 3.1 compares some of the main characteristics of merged and unmerged firms, and they look very similar on average in terms of employment, sales, value added per worker and TFP, mitigating our concern about sample selection bias.

<table>
<thead>
<tr>
<th></th>
<th>Merged Firms</th>
<th>Unmerged Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Employment</td>
<td>5.37</td>
<td>5.27</td>
</tr>
<tr>
<td></td>
<td>[1.13]</td>
<td>[1.17]</td>
</tr>
<tr>
<td>Log Sales</td>
<td>10.6</td>
<td>10.33</td>
</tr>
<tr>
<td></td>
<td>[1.30]</td>
<td>[1.31]</td>
</tr>
<tr>
<td>Value Added per Worker</td>
<td>87.32</td>
<td>71.58</td>
</tr>
<tr>
<td></td>
<td>[203.32]</td>
<td>[147.69]</td>
</tr>
<tr>
<td>TFP (Olley Pakes)</td>
<td>4.22</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>[1.15]</td>
<td>[1.12]</td>
</tr>
</tbody>
</table>

Table 3.1: Comparison of Merged with Unmerged Firms in the Data
3.4 Stylized Facts

To motivate our theoretical model, we first present some stylized facts about the change in firm-level productivity, markup and size during a period of large scale trade liberalization. We focus on a balanced panel, that is, the set of manufacturing firms that survived the entire sample period, from 2000 to 2006, since we are interested in the within-firm change due to open trade. Unlike most previous literature that only looks at how these variables change on average, we also consider the change in dispersion, and argue that the sample mean is no longer sufficient to explain the impact of trade liberalization on firm performance and the resulted wage inequality within a country. We find that both the mean and the standard deviation of these three variables go up during this period.

3.4.1 Productivity

We measure firm-level TFP based on a few different approaches. Besides simple OLS, we first use Olley and Pakes (OP), a method for robust estimation of the production function allowing for endogeneity of the inputs, selection and unobserved permanent differences across firms. Essentially, they use investment to proxy for firm productivity shock in the first stage, and then use semi-parametric selection correction to correct for endogenous exit. We extend the traditional OP procedure by including an exporter dummy, following Amiti and Konings (2007). Second, we use Levinsohn and Petrin (LP), which instead of investment, use material expenditures as proxy for productivity shock, since investment is zero for many firms. Finally, we adopt Ackerberg, Caves and Frazer, a GMM procedure using orthogonality of lagged labor and productivity shock. They argue that labor and investment in OP, or labor and material
expenditures in LP are likely to be collinear. All of these approaches give us very similar measures of TFP, so we report only the results based on OP and LP. However, we want to point out that these TFP measures are still subject to the usual criticism, that is, they are a residual that lumps together many things: technical efficiency, markups, input and output quality and measurement error. We do not address these issues directly here since that is not the focus of this chapter. With better data, on the other hand, a more robust measure of firm-level TFP is possible.¹ Note that there is an increase in both the mean and the dispersion of firm-level productivity.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Average TFP (Standard Deviation)</th>
<th>Median TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>3.68 (1.09)</td>
<td>3.7</td>
</tr>
<tr>
<td>OP</td>
<td>4.61 (1.02)</td>
<td>4.59</td>
</tr>
</tbody>
</table>

Table 3.2: Productivity, 1999-2007 Pooled

<table>
<thead>
<tr>
<th>Specification</th>
<th>Average TFP (Standard Deviation)</th>
<th>Median TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>3.32 (1.03)</td>
<td>3.37</td>
</tr>
<tr>
<td>OP</td>
<td>4.16 (0.92)</td>
<td>4.18</td>
</tr>
</tbody>
</table>

Table 3.3: Productivity, 1999

<table>
<thead>
<tr>
<th>Specification</th>
<th>Average TFP (Standard Deviation)</th>
<th>Median TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>4.00 (1.18)</td>
<td>4.03</td>
</tr>
<tr>
<td>OP</td>
<td>5.08 (1.09)</td>
<td>5.09</td>
</tr>
</tbody>
</table>

Table 3.4: Productivity, 2007

¹See De Loecker (2013).
3.4.2 Markup

To estimate firm-level markups, we adopt the method in De Loecker and Warzynski (2012). Their approach relies on cost-minimizing producers and the existence of at least one variable input of production. This empirical framework relies on the estimation of a production function and provides estimates of plant-level markups without specifying how firms compete in the product market. There are several advantages in using their method. First, their markup estimates are obtained using standard production data where output, total expenditures on variable inputs, and revenue at the plant level are observed. Second, and more importantly, we are able to relax a few key assumptions maintained in previous empirical work. For example, we do not need to impose constant returns to scale, or to observe and measure the user cost of capital.
Below is a brief summary of their empirical model. Suppose a firm $i$ at time $t$ produces output using the following production technology:

$$Q_{it} = Q_{it}(X_{it}^1, ..., X_{it}^V, K_{it}, \omega_{it})$$ (3.1)

Assuming that producers active in the market are cost minimizing, we can therefore consider the associated Lagrangian function:

$$L(X_{it}^1, ..., X_{it}^V, K_{it}, \lambda_{it}) = \sum_{v=1}^{V} P_{it}^{X_v} X_{it}^v + r_{it} K_{it} + \lambda_{it}(Q_{it} - Q_{it}(\cdot))$$ (3.2)

where $P_{it}^{X_v}$ and $r_{it}$ denote a firm’s input price for a variable input $v$ and capital, respectively.

The first-order condition for any variable input free of any adjustment costs is:

$$\frac{\partial L_{it}}{\partial X_{it}^v} = P_{it}^{X_v} - \lambda_{it} \frac{\partial Q_{it}(\cdot)}{\partial X_{it}^v} = 0$$ (3.3)

where the marginal cost of production at a given level of output is $\lambda_{it}$, since $\frac{\partial L_{it}}{\partial Q_{it}} = \lambda_{it}$.

Rearranging terms and multiplying both sides by $\frac{X_{it}^v}{Q_{it}}$ generates the following expression:

$$\frac{\partial Q_{it}(\cdot)}{\partial X_{it}^v} \frac{X_{it}^v}{Q_{it}} = \frac{1}{\lambda_{it}} \frac{P_{it}^{X_v} X_{it}^v}{Q_{it}}$$ (3.4)

Define the markup, $\mu_{it}$, as $\mu_{it} = \frac{P_{it}}{\lambda_{it}}$, we can rewrite equation (3.4) as:

$$\theta_{it}^X = \mu_{it} \frac{P_{it}^X X_{it}}{P_{it} Q_{it}}$$ (3.5)

where the output elasticity w.r.t an input $X$ is denoted by $\theta_{it}^X$. As a result, we obtain an expression of the markup as follows:
\[ \mu_{it} = \theta_{it}^X (\alpha_{it}^X)^{-1} \]  

(3.6)

where \( \alpha_{it}^X \) is the share of expenditures on input \( X_{it} \) in total sales, \( P_{it} Q_{it} \).

Empirically, we consider two variations of the production function, a Cobb-Douglas gross output production function and a translog gross output production function. In the second case, the production function we take to the data, and estimate for each industry separately, is given by:

\[
y_{it} = \beta_{il} l_{it} + \beta_{im} m_{it} + \beta_{ik} k_{it} + \beta_{il} l_{it}^2 + \beta_{mm} m_{it}^2 + \beta_{kk} k_{it}^2 + \beta_{lm} l_{it} m_{it} + \beta_{lk} l_{it} k_{it} + \beta_{mk} m_{it} k_{it} + \beta_{mkl} m_{it} k_{it} l_{it} + \omega_{it} + \epsilon_{it}
\]  

(3.7)

where \( \epsilon_{it} \) are unanticipated shocks to production and i.i.d. shocks including measurement error.

We follow Levinsohn and Petrin (2003) and rely on material demand,

\[ m_{it} = m_t(k_{it}, \omega_{it}, z_{it}) \]  

(3.8)

to proxy for productivity by inverting \( m_t(.) \), where we collect additional variables potentially affecting optimal input demand choice in the vector \( Z_{it} \). We include a firm’s export status, for instance, in the control function.

In the first stage, we run:

\[ y_{it} = \phi_{it}(l_{it}, k_{it}, m_{it}, z_{it}) + \epsilon_{it} \]  

(3.9)

where we obtain estimates of expected output (\( \hat{\phi}_{it} \)) and \( \epsilon_{it} \). Expected output is given by:
\[
\phi_{it} = \beta_t l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_{lk} l_{it}^2 + \beta_{mm} m_{it}^2 + \beta_{kk} k_{it}^2 + \beta_{lm} l_{it} m_{it} + \beta_{lk} l_{it} k_{it} + \beta_{mk} m_{it} k_{it} \\
+ \beta_{mkl} m_{it} k_{it} l_{it} + h_t(m_{it}, k_{it}, z_{it})
\]  
(3.10)

The second stage provides estimates for all production function coefficients by relying on the law of motion for productivity:

\[
\omega_{it} = g_t(\omega_{i,t-1}) + \zeta_{it}
\]  
(3.11)

After the first stage, we can compute productivity for any value of \( \beta \), where

\[
\beta = (\beta_t, \beta_k, \beta_m, \beta_{lk}, \beta_{kk}, \beta_{lm}, \beta_{lk}, \beta_{mk}, \beta_{mkl})
\]

using \( \omega_{it}(\beta) = \hat{\phi}_{it} - \beta_t l_{it} - \beta_m m_{it} - \beta_k k_{it} - \beta_{lk} l_{it}^2 - \beta_{mm} m_{it}^2 - \beta_{kk} k_{it}^2 - \beta_{lm} l_{it} m_{it} - \beta_{lk} l_{it} k_{it} - \beta_{mk} m_{it} k_{it} - \beta_{mkl} m_{it} k_{it} l_{it} \).

By nonparametrically regressing \( \omega_{it}(\beta) \) on its lag, \( \omega_{i,t-1}(\beta) \), we recover the innovation to productivity given \( \beta, \zeta_{it}(\beta) \).

We can now form moments to obtain our estimates of the production function parameters:
We apply the standard GMM techniques and rely on block bootstrapping for the standard errors.

Output elasticities are computed using the estimated coefficients of the production function. For instance, output elasticity for material is given by:

\[
\hat{\theta}_M = \beta_m + 2\beta_m m_{it} + \beta_{il} l_{it} + \beta_{mk} k_{it} + \beta_{mkl} k_{it} l_{it}
\]  

(3.13)

As mentioned above, we do not observe the correct expenditure share for material, \( m_{it} \), directly since we only observe \( \tilde{Q}_{it} \), which is given by \( Q_{it} \exp(\epsilon_{it}) \). The first stage of our procedure does provide us with an estimate of \( \epsilon_{it} \) and we use it to compute the expenditure share as follows:

\[
\hat{\alpha}_M = \frac{P_m m_{it}}{P_{it} Q_{it} \exp(\epsilon_{it})}
\]  

(3.14)
We obtain an estimate of markup for each firm $i$ at each point in time $t$ using (3.6) as
$$
\mu_{it} = \hat{\theta}^M_{it} (\hat{\alpha}^M_{it})^{-1},
$$
while allowing for considerable flexibility in the production function, consumer demand, and competition. The estimation procedure is essentially the same when we consider a Cobb-Douglas gross output production function. We simply drop higher-order and interaction terms. We assume labor to be either a fully flexible input, that is, a control variable correlated with contemporaneous productivity shock, where we use lagged labor as instrument, or a predetermined variable, that is, a state variable, independent of contemporaneous shock, when we take into account hiring and firing costs, and we use itself as an instrument.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Average Markup (Standard Deviation)</th>
<th>Median Markup</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD (L as control variable)</td>
<td>1.19 (0.27)</td>
<td>1.24</td>
</tr>
<tr>
<td>CD (L as state variable)</td>
<td>1.25 (0.23)</td>
<td>1.27</td>
</tr>
<tr>
<td>TL (L as control variable)</td>
<td>1.21 (0.13)</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Table 3.5: Markup, 1999-2007 Pooled

<table>
<thead>
<tr>
<th>Specification</th>
<th>Average Markup (Standard Deviation)</th>
<th>Median Markup</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD (L as control variable)</td>
<td>1.16 (0.26)</td>
<td>1.20</td>
</tr>
<tr>
<td>CD (L as state variable)</td>
<td>1.22 (0.22)</td>
<td>1.23</td>
</tr>
<tr>
<td>TL (L as control variable)</td>
<td>1.13 (0.09)</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Table 3.6: Markup, 1999

<table>
<thead>
<tr>
<th>Specification</th>
<th>Average Markup (Standard Deviation)</th>
<th>Median Markup</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD (L as control variable)</td>
<td>1.25 (0.30)</td>
<td>1.29</td>
</tr>
<tr>
<td>CD (L as state variable)</td>
<td>1.31 (0.26)</td>
<td>1.32</td>
</tr>
<tr>
<td>TL (L as control variable)</td>
<td>1.30 (0.16)</td>
<td>1.28</td>
</tr>
</tbody>
</table>

Table 3.7: Markup, 2007

We get quite similar estimates of markups across different specifications. When we compare markups across the years, especially at the beginning and at the end of our sample
period, it becomes clear that there is a substantial increase in both the mean and the dispersion of the distribution of markups, most evident in the case of a translog gross value production function.

Figure 3.3: Balanced Panel Markup Estimation I

Figure 3.4: Balanced Panel Markup Estimation II
### 3.4.3 Firm Size

We measure firm size in terms of log output value, both in nominal terms and deflated by 4-digit industry output deflator, and in terms of log employment. Its change follows the same pattern as firm productivity and markups, that is, both the mean and the dispersion of its distribution go up.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Mean (Standard Deviation)</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (nominal output value)</td>
<td>10.62 (1.35)</td>
<td>10.45</td>
</tr>
<tr>
<td>Log (deflated output value)</td>
<td>10.64 (1.34)</td>
<td>10.47</td>
</tr>
<tr>
<td>Log (employment)</td>
<td>5.41 (1.12)</td>
<td>5.32</td>
</tr>
</tbody>
</table>

Table 3.8: Firm Size, 1999-2007 Pooled

<table>
<thead>
<tr>
<th>Specification</th>
<th>Mean (Standard Deviation)</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (nominal output value)</td>
<td>10.18 (1.22)</td>
<td>10</td>
</tr>
<tr>
<td>Log (deflated output value)</td>
<td>10.21 (1.22)</td>
<td>10.03</td>
</tr>
<tr>
<td>Log (employment)</td>
<td>5.36 (1.13)</td>
<td>5.26</td>
</tr>
</tbody>
</table>

Table 3.9: Firm Size, 1999

<table>
<thead>
<tr>
<th>Specification</th>
<th>Mean (Standard Deviation)</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (nominal output value)</td>
<td>11.04 (1.49)</td>
<td>10.91</td>
</tr>
<tr>
<td>Log (deflated output value)</td>
<td>11 (1.48)</td>
<td>10.87</td>
</tr>
<tr>
<td>Log (employment)</td>
<td>5.4 (1.15)</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Table 3.10: Firm Size, 2007
3.5 Tariff Reductions and Imported Inputs

Since China joined the WTO in December 2001, it lowered its average tariff significantly, from 16% to a little above 12% within one year from 2001 to 2002, and the average tariff kept declining steadily over the entire sample period. The last year in our sample, 2006, saw an average tariff rate of only about 10%. It is indeed one of the most dramatic trade liberalization episodes in China’s history. As a commitment to its WTO accession, China also agreed to eliminate all quotas, licenses, tendering requirements and other non-tariff barriers to imports of manufactured goods by 2005.
To motivate our theoretical model, we look at how heterogeneous firms at different productivity levels adjust their set of imported varieties during this period of dramatic input tariff liberalization. We find in the data that firms that belong to a higher quartile of the productivity distribution expanded the number of foreign varieties that they imported by more. This observation leads to an important feature of our model, which we explain in the theory section.

We also see in the data that firms on average increased the number of their imported products and source countries significantly between 2001 and 2002, when they experienced the biggest tariff reductions. They kept expanding their imported varieties (product-country pairs) until 2004, which then tapered off. Firms typically import a large number of products from a number of countries, and therefore, we assume that firms can import a continuum of foreign varieties if they find it beneficial.
<table>
<thead>
<tr>
<th>Year</th>
<th>Productivity Quartile</th>
<th>Mean of Newly Imported Varieties</th>
<th>Productivity Quartile</th>
<th>Mean of Newly Imported Varieties</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>Q1</td>
<td>3.95</td>
<td>Q1</td>
<td>5.35</td>
</tr>
<tr>
<td></td>
<td>Q2</td>
<td>4.21</td>
<td>Q2</td>
<td>5.42</td>
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<td></td>
<td>Q3</td>
<td>4.48</td>
<td>Q3</td>
<td>6.75</td>
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<td></td>
<td>Q4</td>
<td>8.17</td>
<td>Q4</td>
<td>10.72</td>
</tr>
<tr>
<td>2002</td>
<td>Q1</td>
<td>5.21</td>
<td>Q1</td>
<td>4.77</td>
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<td></td>
<td>Q2</td>
<td>6.38</td>
<td>Q2</td>
<td>5.84</td>
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<td></td>
<td>Q3</td>
<td>7.78</td>
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<tr>
<td></td>
<td>Q4</td>
<td>13</td>
<td>Q4</td>
<td>12.79</td>
</tr>
</tbody>
</table>

Table 3.11: Firm Productivity and Newly Imported Varieties

<table>
<thead>
<tr>
<th>Year</th>
<th>Products</th>
<th>Source Countries</th>
<th>Varieties</th>
<th>Products</th>
<th>Source Countries</th>
<th>Varieties</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>Mean</td>
<td>158</td>
<td>16</td>
<td>278</td>
<td>Mean</td>
<td>181</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>92</td>
<td>14</td>
<td>127</td>
<td>Median</td>
<td>105</td>
</tr>
<tr>
<td>2003</td>
<td>Mean</td>
<td>183</td>
<td>19</td>
<td>348</td>
<td>Mean</td>
<td>188</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>108</td>
<td>15</td>
<td>153</td>
<td>Median</td>
<td>108</td>
</tr>
<tr>
<td>2004</td>
<td>Mean</td>
<td>179</td>
<td>19</td>
<td>363</td>
<td>Mean</td>
<td>157</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>102</td>
<td>15</td>
<td>146</td>
<td>Median</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 3.12: Number of Imported Products, Source Countries and Varieties
3.6 Model

In this section, we present a partial equilibrium, heterogeneous firm model with endogenous imported input and labor quality choice to account for the aforementioned empirical findings. On the demand side, we adopt the “quality-Melitz” model in Kugler and Verhoogen (2012), where higher price decreases demand but higher quality increases demand. On the supply side, firms differ from each other in the usual dimension of productivity, as in Melitz (2003). In our model, firms combine labor and intermediate inputs to produce physical quantity, in the spirit of Amiti et al. (2014). Output quality, on the other hand, is determined by labor and input quality, and the advantage of imported inputs over domestic counterparts is augmented by a firm’s own productivity.

3.6.1 Demand

Similar to Kugler and Verhoogen (2012), a representative consumer has the following constant-elasticity-of-substitution (CES) utility function:

\[ U = \left( \sum_{i \in I} (q_i \cdot y_i)^{\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}} \]  

(3.15)

where \( I \) denotes the set of all differentiated varieties available; \( i \in I \) indexes a particular variety; \( \sigma > 1 \) is the constant elasticity of substitution between different varieties; \( y_i \) is the quantity of variety \( i \) consumed; \( q_i \) is the output quality of variety \( i \), chosen by the firm producing variety \( i \) and assumed to be observable to all.
Consumer optimization yields the following demand function for variety $i$:

$$y_i = Y P^\sigma q_i^{\frac{\sigma-1}{\sigma}} p_i^{-\sigma}$$  \hfill (3.16)

where $p_i$ is the price of variety $i$ charged by the firm; $Y = U = \left[ \int_{i \in I} (q_i \cdot y_i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$ is the quality-adjusted aggregate consumption in the economy; $P = \left[ \int_{i \in I} (\frac{y_i}{q_i})^{1-\sigma} di \right]^{\frac{1-\sigma}{1-\sigma}}$ is the quality-adjusted ideal price index. This demand function is increasing in the quality and decreasing in the price.

### 3.6.2 Production

There is a continuum of firms of measure $|I|$, each producing one differentiated variety. Without any ambiguity, we also use $i$ to index the firm producing variety $i$. Firms differ from each other in their productivity, $\varphi_i$, drawn from a known distribution upon entering the market and are thereafter fixed, as in Melitz (2003).\(^2\)

Let us consider a particular firm $i$. Its production can be summarized by two production functions - one characterizing the production of physical quantity, $y_i$, and the other characterizing the production of output quality, $q_i$. The production of physical quantity is summarized by a Cobb-Douglas production function:

$$y_i = \varphi_i L_i^{1-\phi} X_i^\phi$$  \hfill (3.17)

---

\(^2\)Here we assume the support of the distribution is bounded below by 1. This assumption is made mainly to eliminate scenarios in which low productivity firms make high quality foreign inputs less efficient than low quality domestic inputs.
\[ X_i = \exp \left\{ \int_0^1 \gamma_j \log (X_{ij}) \, dj \right\} \quad (3.18) \]

\[ X_{ij} = Z_{ij} + \varphi_i^b a_j M_{ij} \quad (3.19) \]

\( L_i \) is the amount of labor used. \( 0 < 1 - \phi < 1 \) is the labor share of variable costs. \( X_i \) is the intermediate input aggregated from a continuum of inputs of measure 1, indexed by \( j \). \( \gamma_j > 0 \) is the importance of input \( j \) among all intermediate inputs, with \( \int_0^1 \gamma_j \, dj = 1 \). \( X_{ij} \) is the amount of input \( j \) used. For each input \( j \), there are both domestic and foreign varieties denoted by \( Z_{ij} \) and \( M_{ij} \) respectively, which are perfect substitutes. The foreign variety has a natural advantage, \( a_j > 1 \), over its domestic counterpart. However, the actual advantage, \( \varphi_i^b a_j \), is the natural advantage augmented by a firm’s productivity, implying that more productive firms are able to use the same foreign input more efficiently than less productive ones. \( b > 0 \), is a parameter that governs the differential efficiency of foreign input use between firms at different levels of productivity - the larger \( b \) is, the greater the differential efficiency.

The production of output quality is summarized by a constant-returns-to-scale supermodular function in labor quality and intermediate input quality:

\[ q_i = \left[ (1 - \phi) c^\theta + \phi \int_0^1 \gamma_j b_j^\theta \, dj \right]^{\frac{1}{\theta}} \quad (3.20) \]

\[ b_j = \begin{cases} 1 & j \in J_i^Z \\ \varphi_i^b a_j & j \in J_i^M \end{cases} \quad (3.21) \]
$c$ is the labor quality chosen by the firm; $b_j$ is the quality of intermediate input $j$; $J_i^Z$ represents the set of inputs for which domestic varieties are used, and $J_i^M = [0, 1] \setminus J_i^Z$ represents the set of inputs for which foreign varieties are imported; $\theta < 0$ captures the constant degree of complementarity between labor quality and intermediate input quality. A more negative $\theta$ represents a stronger complementarity. With this specification, firms using higher quality foreign inputs also have a greater incentive to use higher quality labor to complement them.

We assume there is only domestic labor market, given the low international mobility of labor relative to capital and intermediate inputs. Workers, $l$, are ex-ante homogeneous with wages normalized to 1. There exists a sector that transforms homogeneous labor into different quality, with the production function: $F(l, c) = \frac{l}{c}$. This implies that the marginal cost of producing one unit of labor with quality $c$ is $c \cdot 1 = c$. Labor market is assumed to be perfectly competitive, hence the price of labor of quality $c$ is $p_L(c) = c$.

For intermediate input $j$, there are domestic market and foreign market. Firms are price takers in both. The equilibrium prices of domestic variety and foreign variety are $p_j^Z$ and $p_j^M$ respectively. However, on top of the price, $p_j^M$, there are variable trade costs, $\tau_j \geq 1$, in the form of iceberg costs. In other words, for a firm to acquire one unit of foreign variety of input $j$, it has to pay for $\tau_j$ units at the costs of $\tau_j p_j^M$.

We assume that there are no fixed costs of importing at each input level. However, each

---

3Labor quality, $c$, is a continuous variable with positive support. It is perfectly observable to firms, so we abstract from any asymmetric information problems.

4Both are expressed in terms of a home currency. Exchange rates are not the focus of this chapter.

5This is mainly to avoid the problem of multiple equilibria. Amiti et al. (2014) have fixed costs of importing at each input level. But they fix the set of imported inputs before the choice of output in equilibrium, because the exchange rate shocks in their paper are assumed to be unforeseen. In this chapter, however, we wish to allow both output and the set of imported inputs to respond to a change in the variable trade costs. The addition of fixed costs of importing at each input level will thus introduce multiple equilibria.
firm has to pay fixed import costs, $f_M$, if it switches from not importing at all to importing some inputs. There are also fixed costs of production, $f$, each period.

### 3.6.3 Equilibrium

In order to solve the firm’s profit maximization problem, we follow the strategy in Amiti et al. (2014). We break down the problem into two stages. In the first stage, we hold the set of imported inputs, $J_i^M$, fixed and solve the optimization problem conditional on $J_i^M$. In the second stage, we allow $J_i^M$ to vary so that we can pin down the optimal set of imported inputs, $J_i^{M*}$.

#### 3.6.3.1 Stage 1: Profit Maximization Conditional on a Fixed Set of Imported Inputs

In this stage, we fix the set of imported inputs, $J_i^M \neq \emptyset$. Then the firm’s profits can be written as:

$$
\Pi_i = p_i \cdot y_i - p_L (c) \cdot L - \int_{J_i^M} (p_j^Z \cdot Z_{ij}) \, dj - \int_{J_i^M} (\tau_j p_j^M \cdot M_{ij}) \, dj - f_M - f 
$$

$$
= Y \frac{1}{\sigma} P q_{i}^{\frac{\sigma - 1}{\sigma}} y_{i}^{\frac{\sigma - 1}{\sigma}} - c \cdot L - \int_{J_i^M} (p_j^Z \cdot Z_{ij}) \, dj - \int_{J_i^M} (\tau_j p_j^M \cdot M_{ij}) \, dj - f_M - f \quad (3.22)
$$

The second equality comes from substituting the price, $p_i = Y \frac{1}{\sigma} P q_{i}^{\frac{\sigma - 1}{\sigma}} y_{i}^{-\frac{1}{\sigma}}$, using the demand function and $p_L (c) = c$, into the first equality.

---

6We restrict attention to firms that import. The determination of the cutoff, $\varphi_{nm}$, below which firms never import is discussed in the next section, by comparing the firm’s profits given its optimal set of imported inputs with those when it does not import any inputs.
The conditional profit maximization problem is thus:

\[
\begin{align*}
\max_{y_i, q_i, c, L_i, X_i, \{Z_{ij}\}, \{M_{ij}\}} \quad &\Pi_i = Y^\frac{1}{\sigma} Y^\frac{1}{\sigma} p_i^{\frac{1}{\sigma}} y_i^\frac{\sigma-1}{\sigma} - c \cdot L - \int_{j^Z} \left( p_{j^Z} \cdot Z_{ij} \right) dj - \int_{j^M} \left( \tau_j p_{j^M}^M \cdot M_{ij} \right) dj \\
&- f_M - f
\end{align*}
\]

s.t.
\[
\begin{align*}
y_i &= \varphi_i L_i^{1-\phi} X_i^\phi \\
X_i &= \exp \left\{ \int_{j^Z} \gamma_j \log (Z_{ij}) dj + \int_{j^M} \gamma_j \log \left( \varphi_i a_j M_{ij} \right) dj \right\} \\
qu_i &= \left[ (1 - \phi) c^\phi + \phi \left( \int_{j^Z} \gamma_j dj + \int_{j^M} \gamma_j \varphi_i a_j dj \right) \right]^{\frac{1}{\phi}}
\end{align*}
\]

Let the Lagrange multipliers associated with (3.24), (3.25) and (3.26) be \(\lambda, \psi, \chi\) respectively.

Firm optimization yields the following first-order conditions:

\[
\begin{align*}
\frac{\sigma - 1}{\sigma} Y^\frac{1}{\sigma} Y^\frac{1}{\sigma} p_i^{\frac{1}{\sigma}} y_i^\frac{1}{\sigma-1} &= \lambda \\
\frac{\sigma - 1}{\sigma} Y^\frac{1}{\sigma} Y^\frac{1}{\sigma} p_i^{\frac{1}{\sigma}} y_i^\frac{1}{\sigma-1} &= \chi \\
L_i &= \chi (1 - \phi) q_i^{1-\phi} c^{\phi-1} \\
c &= \lambda (1 - \phi) \frac{y_i}{L_i} \\
\psi &= \lambda \phi \frac{y_i}{X_i} \\
p_{j^Z}^Z &= \psi X_i \frac{\gamma_j}{Z_{ij}}, \quad \forall j \in J_i^Z \\
\tau_j p_{j^M}^M &= \psi X_i \frac{\gamma_j}{M_{ij}}, \quad \forall j \in J_i^M
\end{align*}
\]

We can solve for the optimal labor quality, output quality, output quantity and profits,
conditional on $J_i^M$, as follows:

$$c = q_i = \left[ \int_{J_i^Z} \gamma_j dj + \int_{J_i^M} \gamma_j \phi^\theta a_j^\theta dj \right]^{\frac{1}{\phi}}$$

(3.34)

$$y_i = \left( \frac{\sigma - 1}{\sigma} \right)^\sigma (1 - \phi)^{(1-\phi)\sigma} \phi^\sigma Y P^\sigma \cdot \varphi_i \cdot q_i^{\phi\sigma - 1} B_i^{\phi\sigma}$$

(3.35)

$$\Pi_i = \frac{1}{\sigma} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} (1 - \phi)^{(1-\phi)(\sigma-1)} \phi^{\phi(\sigma-1)} Y P^\sigma \cdot \varphi_i^{\sigma - 1} \cdot (q_i B_i)^{\phi(\sigma - 1)}$$

$$-f_M - f$$

(3.36)

where $B_i = \exp \left\{ \int_0^1 \gamma_j \log \frac{\tau_j}{\rho_j} dj + \int_{J_i^M} \gamma_j \log \frac{\gamma_j}{\tau_j^\rho} dj \right\}$.

The first equation postulates that, conditional on the same set of imported inputs, firms with higher productivity can make more out of foreign inputs and thus hire higher quality labor to complement them, ultimately producing higher quality outputs. Conditional on the same productivity, firms that import a larger set of inputs also hire higher quality labor due to the increase in the quality of intermediate inputs, and also produce higher quality outputs. As a result, these firms pay higher wages to its workers.

### 3.6.3.2 Stage 2: Determination of the Optimal Set of Imported Inputs

In this stage, we formulate a recursive algorithm that pins down the optimal set of imported inputs. Before that, let us consider a firm with its current set of imported inputs, $J_i^M$, contemplating on whether to import foreign variety for input $j$. In other words, input $j$ is moved from the set $J_i^Z$ to $J_i^M$. After the endogenous adjustment of labor quality, output
quality and quantity, the resulting change in profits is given by:

\[
d\Pi_i = A \varphi_i^{\sigma-1} \cdot d \left[ (q_i B_i)^{\phi(\sigma-1)} \right]
\]

\[
= A \varphi_i^{\sigma-1} \cdot \phi (\sigma - 1) (q_i B_i)^{\phi(\sigma-1)} \cdot \gamma_j \left[ \frac{1}{\theta} q_i^{-\theta} (\varphi_i^{b\theta} a_j^\theta - 1) + \log \frac{\varphi_i^{b\theta} a_j^\theta}{\tau_j p_j^M} \right]
\]  

(3.37)

where \( A = \frac{1}{\theta} \left( \frac{\sigma-1}{\sigma} \right)^{\sigma-1} (1 - \phi)^{(1-\phi)\phi(\sigma-1)} Y P^\sigma \) contains only constants and macroeconomic variables that the firm takes as exogenously given. Under the assumptions on the parameters, the sign of \( d\Pi_i \) is given by:

\[
\text{sign} (d\Pi_i) = \text{sign} \left( \frac{1}{\theta} q_i^{-\theta} (\varphi_i^{b\theta} a_j^\theta - 1) + \log \frac{\varphi_i^{b\theta} a_j^\theta}{\tau_j p_j^M} \right)
\]  

(3.38)

Note that since we do not have fixed costs of importing at each input level, the sign of \( d\Pi_i \) is independent of the scale of production, thus avoiding the problem of multiple equilibria.

The first term, \( \frac{1}{\theta} q_i^{-\theta} (\varphi_i^{b\theta} a_j^\theta - 1) \), is always positive, representing the benefits of importing an extra input \( j \) on output quality, and hence, total revenue. These benefits are increasing in output quality, \( q_i \), due to complementarity between quality of newly imported input \( j \) and quality of existing imported inputs in \( J_i^M \). This implies that there is no crowding out effect - importing more inputs does not make importing other inputs less desirable. In fact, there is crowding in - importing more inputs increases the benefits of importing an extra input, thus increasing the likelihood of importing that input. These benefits are also increasing in firm productivity, \( \varphi_i \), because more productive firms can make more out of the same imported input \( j \).\footnote{Here, we condition on fixed output quality, \( q_i \). This result holds more strongly if \( q_i \) is an increasing function in \( \varphi_i \).}

The second term, \( \log \frac{\varphi_i^{b\theta} a_j^\theta}{\tau_j p_j^M} \), can be either positive or negative, depending on
the advantage-and-trade-cost-adjusted relative price. If foreign variety is relatively cheaper than domestic variety, \( \frac{\gamma_j p_j^M}{\varphi_j a_j} \leq p_j^Z \), then \( \log \frac{\varphi_j a_j p_j^Z}{\gamma_j p_j^M} \geq 0 \). As a result, conditional on already importing some inputs, that is, the firm has already paid the fixed costs of importing at the first unit, \( f_M \), it is always willing to import such input \( j \) because importing is both quality-enhancing and cost-saving.

Next, we characterize the optimal set of imported inputs. Unlike the framework in Amiti et al. (2014), which has a cutoff implicitly defined because the marginal cost of importing is fixed and the marginal benefit of importing is monotonically decreasing, our model does not have a sorting of the inputs nor an implicitly defined cutoff. Instead, we define the optimal set recursively and sequentially using an algorithm. We prove the optimality of the defined set and some of its other desirable properties in the next subsection. For expository simplicity, let us define:

\[
q_i(S) = \left[ \int_{S^c} \gamma_j d\bar{j} + \int_S \gamma_j \varphi_j a_j a_j d\bar{j} \right]^\frac{1}{2}
\] (3.39)

as the conditional optimal output quality for firm \( i \) that imports the set of inputs, \( S \subseteq [0, 1] \), and uses domestic varieties for the complementary set, \( S^c = [0, 1] \setminus S \).

An immediate lemma is:

**Lemma 1** For any two sets, \( S_1 \subseteq S_2 \subseteq [0, 1] \), and some firm \( i \) with productivity \( \varphi_i \), \( q_i(S_1) < q_i(S_2) \). For any two firms \( i \) and \( i' \) with productivity \( \varphi_i < \varphi_{i'} \) and any set \( S \subseteq [0, 1] \), \( q_i(S) < q_{i'}(S) \). Combining the two results, we also have \( q_i(S_1) < q_{i'}(S_1) < q_{i'}(S_2) \).

The proof is trivial once it is noted that \( \varphi_i a_j > 1 \).
Recursive algorithm of computing the optimal set of imported inputs, $J_i^{M^*}$, conditional on having paid $f_M$.

For firm $i$ with productivity $\varphi_i$, its optimal set of imported inputs, $J_i^{M^*}$, can be computed as the following:

Step 1: Define $J_{i0} = \left\{ j \in [0, 1] \mid \tau_i p_j M \varphi_i a_j \leq p_j^Z \right\}$. Based on the argument above, importing these foreign inputs is both quality-enhancing and cost-saving. Therefore, it is always beneficial to import them once $f_M$ is paid. Hence,

$$J_{i0} \subseteq J_i^{M^*}$$

Step 2: Define $J_{i1} = \left\{ j \in [0, 1] \cap J_{i0} \cap J_{i1}^c \mid \frac{1}{\theta} q_i (J_{i0} \cup J_{i1})^{-\theta} (\varphi_i a_j^\theta - 1) + \log \frac{\varphi_i a_j^\theta p_j^Z}{\tau_i p_j^M} \geq 0 \right\}$. Since there is only crowding in effect of importing more inputs and importing inputs in $J_{i1}$ increases profits, it is optimal for the firm to import them.

$$J_{i1} \subseteq J_i^{M^*}$$

Step 3: Define $J_{i2} = \left\{ j \in [0, 1] \cap J_{i0} \cap J_{i1} \cap J_{i2}^c \mid \frac{1}{\theta} q_i (J_{i0} \cup J_{i1} \cup J_{i2})^{-\theta} (\varphi_i a_j^\theta - 1) + \log \frac{\varphi_i a_j^\theta p_j^Z}{\tau_i p_j^M} \geq 0 \right\}$. By the same logic,

$$J_{i2} \subseteq J_i^{M^*}$$

Repeat these steps until for some $N \in \mathbb{N}$ such that,

$$J_{iN} = \left\{ j \in [0, 1] \cap \left( \bigcap_{n=0}^{N-1} J_{in}^c \right) \mid \frac{1}{\theta} q_i \left( \bigcup_{n=0}^{N-1} J_{in} \right)^{-\theta} (\varphi_i a_j^\theta - 1) + \log \frac{\varphi_i a_j^\theta p_j^Z}{\tau_i p_j^M} \geq 0 \right\} = \emptyset$$
implying that there is no input, which the firm has not yet imported, that can increase profits. Then we claim the optimal set of imported inputs is:

$$J_i^{M*} = \bigcup_{n=0}^{N-1} J_{in}$$

### 3.6.3.3 Determination of No-import Cutoff

Due to the existence of the fixed costs of importing, $f_M$, at the first unit, not every firm engages in importing. Firms below a productivity threshold, $\varphi_{nm}$, do not import any input while firms above that threshold do. This threshold is determined by comparing the profits if the firm imports with those if it does not. The marginal firm with productivity $\varphi_{nm}$ is indifferent between importing and not.

The profits of firm $i$ if it imports are given by:

$$\Pi_i (\text{importing}) = A\varphi_i^{\sigma-1} \cdot (q_i (J_i^{M*}) B_i (J_i^{M*}))^{\phi(\sigma-1)} - f_M - f$$

where $J_i^{M*}$ is the optimal set of imported inputs given productivity $\varphi_i$.

The profits of firm $i$ if it does not import are given by:

$$\Pi_i (\text{not importing}) = A\varphi_i^{\sigma-1} \cdot (q_i (\emptyset) B_i (\emptyset))^{\phi(\sigma-1)} - f$$

$$= A\varphi_i^{\sigma-1} \cdot \exp \left\{ \phi (\sigma - 1) \int_0^1 \gamma_j \log \frac{\gamma_j}{P_j} dj \right\} - f$$

Note that $$(q_i (J_i^{M*}) B_i (J_i^{M*}))^{\phi(\sigma-1)} > (q_i (\emptyset) B_i (\emptyset))^{\phi(\sigma-1)} = \exp \left\{ \phi (\sigma - 1) \int_0^1 \gamma_j \log \frac{\gamma_j}{P_j} dj \right\}$$

because of the optimality of $J_i^{M*}$. Even though both $\Pi_i (\text{importing})$ and $\Pi_i (\text{not importing})$
are increasing in $\varphi_i$, the former increases much faster than the latter, but the former starts at a lower value due to the existence of $f_M$. Hence, as illustrated by the figure below, the two profit lines have one intersection, $\varphi_{nm}$, after which importing inputs generates higher profits than not importing. This pins down the no-import cutoff.

![Figure 3.7: No-import Cutoff](image)

**3.6.4 Model Implications**

In this section, we outline some of the properties implied by the model that guide our empirical investigations. Some of these properties come naturally out of the model while others require additional assumptions on the parameters of the model.
3.6.4.1 No-import Cutoff

We first examine how the set of firms that engage in importing respond to trade liberalization in the form of import tariff reductions. Clearly the profit schedule of not importing, $\Pi_i (not\ importing)$, does not change with import tariff reductions. The profit schedule of importing, $\Pi_i (importing)$, does, but this change may or may not affect the no-import cutoff, depending on the nature of import tariff reductions. We summarize our findings in the following proposition.

**Proposition 1**: Suppose there is trade liberalization in the form of import tariff reductions summarized by $\{\tau_j + d\tau_j \mid j \in [0, 1]\}$. The no-import cutoff will either decrease, $\tau_{nm'} < \tau_{nm}$, or remain unchanged, $\tau_{nm'} = \tau_{nm}$. The former is true if and only if the original marginal firm with productivity $\tau_{nm}$ has a strictly larger optimal set of imported inputs, $J_{nm}^{M*}((\tau_j + d\tau_j)_{j=0}^1) \supset J_{nm}^{M*}(\tau_j_{j=0}^1)$, or the same optimal set, $J_{nm}^{M*}(\tau_j + d\tau_j_{j=0}^1) = J_{nm}^{M*}(\tau_j_{j=0}^1)$, but there exists some $j \in J_{nm}^{M*}(\tau_j_{j=0}^1)$ such that $d\tau_j < 0$. The latter is true if and only if the optimal set does not change, $J_{nm}^{M*}(\tau_j + d\tau_j_{j=0}^1) = J_{nm}^{M*}(\tau_j_{j=0}^1)$, and for all $j \in J_{nm}^{M*}(\tau_j_{j=0}^1)$, $d\tau_j = 0$.

An immediate result is that if there is a uniform decrease in import tariffs, the no-import cutoff decreases. More generally, if there is a decrease in tariff on some input in the optimal set of imported inputs of the original marginal firm, the cutoff decreases. This result is consistent with empirical findings that following trade liberalization, previously non-importing firms in the balanced panel start to import higher quality foreign inputs.
3.6.4.2 Optimal Set of Imported Inputs

Since the optimal set of imported inputs is at the center of our model, determining equilibrium labor quality, output quality and quantity, we investigate the properties of this set.

**Proposition 2:** All other things being equal, a more productive firm imports a weakly larger set of foreign inputs. Specifically, if \( \varphi_i < \varphi_{i'} \), then \( J_i^{M*} \subseteq J_{i'}^{M*} \).

**Proof.** Let us consider the recursive algorithms for the two firms. Let \( N_1 \) and \( N_2 \) be defined as in the algorithm for firm \( i \) and \( i' \) respectively. There are three possible cases: \( N_1 = N_2, N_1 < N_2 \) and \( N_1 > N_2 \). We prove the proposition in the three cases separately.

First, suppose \( N_1 = N_2 = N \). To prove \( J_i^{M*} \subseteq J_{i'}^{M*} \), we just need to prove in each step, \( \bigcup_{m=0}^n J_{im} \subseteq \bigcup_{m=0}^n J_{i'm} \) for all \( n \in \{0, 1, \ldots, N - 1\} \). Clearly when \( n = 0 \), \( J_{i0} = \{ j \in [0, 1] | \frac{\tau_j p_j^{M}}{\varphi_j a_j} \leq p_j^{Z} \} \subseteq \{ j \in [0, 1] | \frac{\tau_j p_j^{M}}{\varphi_j a_j} \leq p_j^{Z} \} = J_{i'0} \) because \( \varphi_i < \varphi_{i'} \). Now suppose \( \bigcup_{m=0}^n J_{im} \subseteq \bigcup_{m=0}^n J_{i'm} \) holds for some \( n \in \{0, 1, \ldots, N - 2\} \). Consider in step \( n + 1 \), some arbitrary input \( j \in J_{im+1} \). If \( j \in \bigcup_{m=0}^{n+1} J_{i'm} \), then it becomes trivial that \( j \in \bigcup_{m=0}^{n+1} J_{i'm} \).

So suppose \( j \notin \bigcup_{m=0}^n J_{i'm} \), but we know that \( \frac{1}{\theta} q_i \left( \bigcup_{m=0}^n J_{im} \right)^{-\theta} \left( \varphi_i^{M} a_i^{\theta} - 1 \right) + \log \frac{\varphi_i^{M} a_i^{\theta}}{\tau_j p_j^{M}} \geq 0 \) by definition of \( J_{in+1} \); and \( \bigcup_{m=0}^n J_{im} \subseteq \bigcup_{m=0}^n J_{i'm} \) with \( \varphi_i < \varphi_{i'} \) implies \( q_i \left( \bigcup_{m=0}^n J_{im} \right) < q_{i'} \left( \bigcup_{m=0}^n J_{i'm} \right) \) by Lemma 1. Therefore

\[
\frac{1}{\theta} q_{i'} \left( \bigcup_{m=0}^n J_{i'm} \right)^{-\theta} \left( \varphi_{i'}^{M} a_{i'}^{\theta} - 1 \right) + \log \frac{\varphi_{i'}^{M} a_{i'}^{\theta}}{\tau_j p_j^{M}} \geq 0
\]

Hence \( j \in J_{i'n+1} \subseteq \bigcup_{m=0}^{n+1} J_{i'm} \). This proves \( J_{in+1} \subseteq \bigcup_{m=0}^{n+1} J_{i'm} \), but by assumption, \( \bigcup_{m=0}^n J_{im} \subseteq \bigcup_{m=0}^n J_{i'm} \subseteq \bigcup_{m=0}^{n+1} J_{i'm} \), so we have \( \bigcup_{m=0}^{n+1} J_{im} = J_{in+1} \bigcup_{m=0}^n J_{im} \subseteq \bigcup_{m=0}^{n+1} J_{i'm} \).

By mathematical induction, we have shown that \( \bigcup_{m=0}^n J_{im} \subseteq \bigcup_{m=0}^n J_{i'm} \) for all \( n \in \{0, 1, \ldots, N - 1\} \).
Evaluating this expression at $N - 1$, we have proven $J_{i}^{M*} \subseteq J_{i'}^{M*}$.

The case $N_1 < N_2$ is trivial because we have shown already that $J_{i}^{M*} = \bigcup_{n=0}^{N_1-1} J_{i_n} \subseteq \bigcup_{n=0}^{N_1-1} J_{i'n} \cup \left( \bigcup_{n=N_1}^{N_2-1} J_{i'n} \right) = J_{i'}^{M*}$.

Now, we consider $N_1 > N_2$. We have shown in the first case that $\bigcup_{n=0}^{N_2-1} J_{i_n} \subseteq \bigcup_{n=0}^{N_2-1} J_{i'n} = J_{i'}^{M*}$. Now consider any arbitrary $j \in J_{i,N_2}$. $j$ satisfies $\frac{1}{\theta} q_i \left( \bigcup_{n=0}^{N_2-1} J_{i_n} \right)^{-\theta} (\varphi_{i}^{j\theta} a_{j}^{\theta} - 1) + \log \frac{\varphi_{i}^{j\theta} a_{j}^{\theta} p_{M}^{j}}{\tau_{j} p_{j}} \geq 0$, and we know $q_i \left( \bigcup_{n=0}^{N_2-1} J_{i_n} \right) < q_{i'} \left( \bigcup_{n=0}^{N_2-1} J_{i'n} \right)$ by Lemma 1. So

$$\frac{1}{\theta} q_{i'} \left( \bigcup_{n=0}^{N_2-1} J_{i'n} \right)^{-\theta} (\varphi_{i'}^{j\theta} a_{j}^{\theta} - 1) + \log \frac{\varphi_{i'}^{j\theta} a_{j}^{\theta} p_{M}^{j}}{\tau_{j} p_{j}} \geq 0$$

Hence by definition $j \in J_{i',N_2}$ or $j \notin \bigcup_{n=0}^{N_2-1} J_{i'n}$. The former is impossible because $J_{i',N_2} = \emptyset$, so $j \in \bigcup_{n=0}^{N_2-1} J_{i'n}$. This proves $J_{i,N_2} \subseteq \bigcup_{n=0}^{N_2-1} J_{i'n}$. With $\bigcup_{n=0}^{N_2-1} J_{i_n} \subseteq \bigcup_{n=0}^{N_2-1} J_{i'n}$, we show

$$\bigcup_{n=0}^{N_2} J_{i_n} = \left( \bigcup_{n=0}^{N_2-1} J_{i_n} \right) \cup J_{i,N_2} \subseteq \bigcup_{n=0}^{N_2-1} J_{i'n} = J_{i'}^{M*}.$$ 

Repeat this argument for $n = N_2 + 1, \ldots, N_1 - 1$, we can show that $J_{i}^{M*} = \bigcup_{n=0}^{N_1-1} J_{i_n} \subseteq J_{i'}^{M*}$.

**Proposition 3:** All other things being equal, a reduction in variable trade costs, $\tau_{j}$, for input $j$ increases the likelihood of importing that input for a firm that previously does not import it. Furthermore, it also increases the likelihood of importing other inputs that are not imported before by the firm, if input $j$ is now imported.

Proposition 3 is quite intuitive because a reduction in variable trade costs $\tau_{j}$ decreases the costs of importing $j$ while keeping benefits unchanged. Hence the firm is more likely to import $j$. Conditional on $j$ being imported after the reduction in $\tau_{j}$, the set of imported inputs expands and the output quality increases, further increasing the benefits of importing other inputs due to the crowding in effect. Hence, the likelihood of the firm importing other
inputs increases.

Proposition 3 can be generalized to reductions in multiple/all variable trade costs. The increase in the likelihood is much bigger if the variable trade costs of many inputs that are previously not imported by the firm decrease at the same time.

### 3.6.4.3 Labor Quality, Output Quality, and Wages

Recall that for firms above the no-import threshold, \( \varphi_{nm} \), the optimal labor quality and output quality are given by:

\[
q_i = \left[ \int_{J_i^Z} \gamma_j dj + \int_{J_i^M} \gamma_j \varphi_i^a d_j dj \right]^{\frac{1}{\bar{z}}}
\]

where \( J_i^Z = [0, 1] \backslash J_i^M \) is the equilibrium set of domestic inputs.

By proposition 2, we know that a more productive firm imports a weakly larger set of foreign inputs. Hence, it is obvious from the above expression that this more productive firm uses strictly higher quality labor and produces strictly higher quality output. Since equilibrium labor quality is higher for a more productive firm, it also pays higher wages because \( p_L(c) = c \).

Trade liberalization in the form of tariff reductions increases the set of imported inputs for some, if not all, firms, by proposition 3. These firms switch to higher quality foreign inputs, and hire higher quality labor to complement them. As a result, they produce higher quality output and pay higher wages.

Suppose the tariff reductions induce a decrease in the no-import cutoff as in the first case in proposition 1. Then for those firms with productivities \( \varphi_i \) between \( \varphi_{nm} \) and \( \varphi_{nm} \), they
switch from not importing at all to importing some foreign inputs. Their labor quality and output quality increase from \( c = q_i = \left[ \int_0^1 \gamma_j dj \right]^{1/\theta} = 1 \) to \( \left[ \int_{j^*}^{j_{M^*}} \gamma_j dj + \int_{j_{M^*}}^{M_j} \gamma_j \varphi_i d^j dj \right]^{1/\theta} > 1 \). This increase is larger, the more productive the firm is, because \( \varphi_i \) and \( J_{i}^{M^*} \) are both larger by proposition 2. As a result, they also pay higher wages after trade liberalization.

For those firms that never import - firms with \( c = q_i < \varphi_{nm} \leq \varphi_{nm} \), there is no change in the labor quality, output quality and wages before and after trade liberalization. They are consistently using the low skill labor, \( c = 1 \), producing low quality output, \( q_i = 1 \), and paying low wages, \( p_L (1) = 1 \).

### 3.6.4.4 Firm Profits

In terms of firm profits, we have a set of similar predictions.

Conditional on the same set of imported inputs, a more productive firm has higher profits because of its higher \( \varphi_i \) and higher \( q_i \). By proposition 2, it also imports a weakly larger set of inputs. It chooses to do so because by importing more, its profits increase. Therefore, we have shown that a more productive firm enjoys higher profits. Trade liberalization in the form of input tariff reductions generates higher profits for any firms, provided that they are importing inputs after trade liberalization. This increase in profits comes from two potential sources. After trade liberalization but conditional on the same set of imported inputs, firm profits are as least as large as before. It makes strictly larger profits if there is a reduction in tariff on at least one of its imported inputs. Furthermore, the firm chooses to import a weakly larger set of imported inputs by proposition 3. It chooses to do so only if its profits increase, evident from the recursive algorithm.

However, who enjoys a bigger increase in profits in the face of the same tariff reductions is a
much tougher question to answer. If we restrict our attention to firms that are new importers - those with productivity, \( \varphi_i \in [\varphi_{nm}^*, \varphi_{nm}] \), then it is clear that the more productive firms have a larger increase in profits than the less productive ones. It is not clear, however, if we wish to compare profits of an existing importer with those of a new importer. It is possible that the former increase by more if we impose additional assumptions on \( \{a_j\} \), the natural advantage of foreign varieties over their domestic counterparts, or if the differential efficiency, \( b \) is large enough.

3.6.4.5 Total Factor Productivity

For firm \( i \) with \( \varphi_i \geq \varphi_{nm} \), its total factor productivity (TFP) is defined as:

\[
TFP_i \equiv \frac{y_i}{L_i^{1-\phi}} \exp \left\{ \phi \left( \int_{J_z^*} \gamma_j \log Z_{ij} dj + \int_{J^*_M} \gamma_j \log M_{ij} dj \right) \right\} = \varphi_i \cdot \exp \left\{ \phi \int_{J^*_M} \gamma_j \log a_j dj \right\} = \varphi_i^{1+b\phi \int_{J^*_M} \gamma_j dj} \cdot \exp \left\{ \phi \int_{J^*_M} \gamma_j \log a_j dj \right\}
\]

(3.42)

TFP is greater than \( \varphi_i \) because the use of imported inputs enhances firm productivity by the factor \( \varphi_i^{b\phi \int_{J^*_M} \gamma_j dj} \cdot \exp \left\{ \phi \int_{J^*_M} \gamma_j \log a_j dj \right\} > 1 \). It is also obvious that a firm with higher baseline productivity, \( \varphi_i' = \varphi_i \), ends up with higher \( TFP_i' > TFP_i \). Specifically, from proposition 2, we know that \( J^*_i \supseteq J^*_i \). Let \( H = J^*_i \setminus J^*_i = J^*_i \cap (J^*_i)^c \supseteq \emptyset \) be the difference between the two sets. Then the TFP ratio is given by:
\[
\frac{\text{TFP}_i}{\text{TFP}_i'} = \frac{\varphi_i^{1+b\phi \int_{j_{M^*}}^\gamma_j d_j} \cdot \exp \left\{ \phi \int_{j_{M^*}}^\gamma_j \log a_j d_j \right\}}{\varphi_i^{1+b\phi \int_{j_{M^*}}^\gamma_j d_j} \cdot \exp \left\{ \phi \int_{j_{M^*}}^\gamma_j \log a_j d_j \right\}} = \left( \frac{\varphi_i'}{\varphi_i} \right)^{1+b\phi \int_{j_{M^*}}^\gamma_j d_j} \cdot \varphi_i' \int_H^\gamma_j \log a_j d_j \cdot \exp \left\{ \phi \int_H^\gamma_j \log a_j d_j \right\}
\]

\[
> \frac{\varphi_i'}{\varphi_i}
\]

(3.43)

The first inequality holds with equality when \( H = \emptyset \) and holds strictly otherwise. The second inequality holds strictly because we assume that \( \varphi_i' > \varphi_i \geq \varphi_{nm} \), which results in a non-empty set \( J_{M^*} \).

For firm \( i \) with productivity lower than the no-import cutoff, \( \varphi_i < \varphi_{nm} \), its TFP is:

\[
\text{TFP}_i = \frac{y_i}{L_i^{1-\phi} \exp \left\{ \phi \left( \int_0^1 \gamma_j \log Z_{ij} d_j \right) \right\}} = \varphi_i
\]

(3.44)

Since it does not import any foreign inputs, there is no productivity enhancing effect from imported inputs. Hence, its TFP is the same as its baseline productivity parameter, \( \varphi_i \). It is then trivial to show that the TFP ratio between two firms that are both below the cutoff is

\[
\frac{\text{TFP}_i'}{\text{TFP}_i} = \frac{\varphi_i'}{\varphi_i}.
\]

Trade liberalization increases firm-level TFP through the expansion of the set of imported inputs. Again suppose this trade liberalization induces a reduction in no-import cutoff from \( \varphi_{nm} \) to \( \varphi_{nm'} \), as in the first case in proposition 1. First, consider a firm with productivity
\( \varphi_i \geq \varphi_{nm} \). Before trade liberalization, its optimal set of imported inputs is \( J_i^{M*} \). It expands to \( J_i^{M\prime} \) with \( H \) being the difference between the two sets. Then the expansion in the optimal set of imported inputs induced by trade liberalization increases its TFP by a factor:

\[
\frac{TFP_i'}{TFP_i} = \varphi_i^{b\phi \int_H J_i^{J_i^{M*}}} \cdot \exp \left\{ \phi \int_H \gamma_j \log a_j dj \right\} \geq 1
\]

Equality holds only when \( H = \emptyset \).

For a firm with productivity \( \varphi_i \in [\varphi_{nm'}, \varphi_{nm}) \), its TFP increases by a factor:

\[
\frac{TFP_i'}{TFP_i} = \varphi_i^{b\phi \int_{J_i^{M*}}} \cdot \exp \left\{ \phi \int_{J_i^{M*}} \gamma_j \log a_j dj \right\} > 1
\]

because its set of imported inputs expands from null set to \( J_i^{M\prime} \supset \emptyset \). And further, \( J_i^{M\prime} \) is weakly increasing in \( \varphi_i \) for such a firm by proposition 2. So for a newly importing firm, the increase in TFP is increasing in its baseline productivity.

Trivially, for a firm that never imports, there are no TFP gains from trade liberalization.

However, it is not straightforward to compare the increase in TFP of two arbitrary firms, either both are existing importers, \( \varphi_i, \varphi_i' \geq \varphi_{nm} \), or one is a new importer while the other is an existing importer, \( \varphi_{nm'} \leq \varphi_i < \varphi_{nm}, \varphi_i' \geq \varphi_{nm} \). That depends on the expansion of the set of imported inputs induced by trade liberalization, and on the parameter \( b \), which governs the degree of differential efficiency among firms in using foreign inputs. In general, a more productive firm has smaller room to expand its set of imported inputs. However, this constraint can be alleviated by a large \( b \)- that is, it is also much more efficient in using the higher quality foreign inputs.

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3.7 Empirical Results

3.7.1 First Stage

Since a firm’s decision to import foreign inputs is endogenous, we adopt the fixed effects 2SLS model to estimate the effects of the differential change in import intensity of firms on their productivity and average wage. More specifically, we use the input tariff reductions following China’s accession to the WTO as an instrument for firm import behavior, following Goldberg et al. (2010). Another concern is that the trade reform itself is endogenous in the sense that less efficient industries lobby for higher trade protection. Branstetter and Lardy (2006) discuss the motive for China’s leadership to agree to the conditions required for its WTO accession. They conclude that, “In short, China’s top political leadership made extensive commitments to the WTO in order to advance their domestic reform agenda.” China joined the WTO to speed up the domestic reforms and facilitate the transition into a market economy. The input tariff reductions are unlikely due to the protection pressure from interest groups in less efficient industries.

To examine the effects of tariff reductions on firm import behavior, we have to consider the set of intermediate inputs that a firm may actually import. We construct an industry-level input tariff at the 4-digit Chinese Industry Classification (CIC) level to avoid endogeneity bias following Amiti and Konings (2007), Goldberg et al. (2010) and Ge et al. (2011). We construct an output tariff by taking a simple average of the HS 8-digit codes within each 4-digit CIC code. The difficulty is that our sample period covers a revision of CIC system in 2003 and a major reclassification of the international HS 6-digit codes in 2002. First, we follow Brandt et al. (2012) to create a concordance of standardized 4-digit CIC codes.
consistent before and after 2003. Then we construct a link of 6-digit HS codes before and after 2002. We further construct a crosswalk between standardized 6-digit HS codes and standardized CIC codes based on Brandt et al. (2012). With these links in place, we assign each 8-digit HS product to the 6-digit HS code it belongs to, and then connect this 6-digit HS code with the corresponding 4-digit CIC code, for each year. Next, for each 4-digit industry, we compute an input tariff as a weighted average of the output tariff, where the weights are the cost shares of one industry in the production of a good in another based on the Input-Output Table. We drop output tariff of the industry that a firm belongs to from this calculation to remove the direct effects of output tariffs on wages. Finally, we interact the industry-level input tariff with a firm’s baseline productivity to study their heterogeneous responses. We use the firm-level TFP measured in 1998 to proxy for it, which is independent of a firm’s future import choices.

The first-stage regression that we run is the following:

\[
\text{Import}_{ft} = \alpha_0 + \alpha_1 \text{InputTariff}_{it} \times \text{Productivity}_{f,1998} + \alpha_2 \text{InputTariff}_{it} + Z_{f,t-1} \lambda + D_f + D_t + v_{ft}
\]

(3.45)

where \( \text{Import}_{ft} \) is measured by the total number of newly imported varieties (or products). \( Z \) includes firm age, last year’s log revenue and last year’s export status. \( D_t \) is a set of year dummies that control for possible variation in the macroeconomic environment over time; \( D_f \) is included to control for unobservable individual effects of the firm that could be correlated with its import behavior. We use robust standard errors that are clustered at the firm level.

Table 3.13 reports the regression results at the variety (product-country of origin pair).
level. We find that the total number of varieties that are newly imported goes up as a result of tariff reductions. The coefficient of $Input Tariff_{it} * Productivity_{f,1998}$ is also statistically significant and negative, which suggests that a firm with higher baseline productivity increases its imported inputs by more in response to tariff reductions. The total number of newly imported varieties is also higher for a firm that is bigger, younger and exports. We also look at the corresponding regression at the product level and the results are very similar.

### 3.7.2 Second Stage

Next, we look in the data to see how the expansion of the external margin of imported inputs affects firm-level TFP and average wage.

The second-stage regression that we run is the following:

$$ Y_{ft} = \alpha_0 + \alpha_1 Input Tariff_{ft} * Productivity_{f,1998} + \alpha_2 Input Tariff_{ft} + Z_{f,t-1} \lambda + D_f + D_t + v_{ft} $$

(3.46)
where $Y_{ft}$ is firm-level TFP and its log average wage. Standard errors are robust to heteroscedasticity and clustered at the firm level.

Table 3.14 reports the regression results of TFP measured by Levinsohn-Petrin method and firm log average wage. We use TFP measured by Olley-Pakes method as a robustness check and the two regressions lead to very similar results. Note that imported inputs increase firm productivity and average wage a firm pays to its workers. Since a firm with higher baseline productivity increases its imported inputs by more, its productivity goes up by more as well as its average wage. As discussed in the section of model implications, each unit of newly imported inputs also increases measured TFP and average wage of a more productive firm by more, and this prediction is supported by the statistically significant and positive coefficient of $\hat{import}_{ft} * \hat{Prod}_{f,1998}$ in each of the regressions. Firm productivity and average wage are also higher if a firm is bigger, younger and exports.

<table>
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<th></th>
<th>$TFP_{ft}$</th>
<th>$\log(Wage_{ft})$</th>
</tr>
</thead>
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<tr>
<td>$\hat{import}<em>{ft} * \hat{Prod}</em>{f,1998}$</td>
<td>0.01**</td>
<td>0.004**</td>
</tr>
<tr>
<td></td>
<td>[0.004]</td>
<td>[0.002]</td>
</tr>
<tr>
<td>$\hat{import}_{ft}$</td>
<td>0.054***</td>
<td>0.016**</td>
</tr>
<tr>
<td></td>
<td>[0.018]</td>
<td>[0.007]</td>
</tr>
<tr>
<td>$Export \ Status_{ft-1}$</td>
<td>0.009</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>[0.031]</td>
<td>[0.014]</td>
</tr>
<tr>
<td>$\log(Industrial \ Sale_{ft-1})$</td>
<td>0.308***</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>[0.042]</td>
<td>[0.018]</td>
</tr>
<tr>
<td>$\log(Age_{ft})$</td>
<td>-0.108**</td>
<td>-0.025</td>
</tr>
<tr>
<td></td>
<td>[0.045]</td>
<td>[0.019]</td>
</tr>
<tr>
<td>$Year \ FE$</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

$p < 0.1; \ ** p < 0.05; \ *** p < 0.01$

Table 3.14: Second Stage
3.8 Conclusion and Future Work

In this chapter, we develop a partial equilibrium, heterogeneous firm model with endogenous imported inputs and labor quality choice and establish a link between an improvement in firm performance and the use of imported inputs. The model further predicts that firms that upgrade their intermediate inputs also upgrade their labor quality, resulting in higher wages. Combining Chinese Annual Survey of Industrial Firms and Chinese Customs Data, we are able to test our model’s predictions in the data, and we find considerable support.

There are some issues that we have not yet addressed and are left as future work. First, we wish to empirically test the output quality upgrading hypothesis proposed in Broda and Weinstein (2006). Second, we want to extend our model to accommodate variable markups and then empirically test the model’s prediction about firm markup distribution. Third, we would also like to look into firm ownership structure more carefully so that we can say more about the mechanism that generates the link between firms profits and wage inequality that is observed in the data.
Bibliography


Appendix A

Appendix for Chapter 1

A.1 Welfare Change as Equivalent Variation

Consider the set of changes \( \{ \hat{p}_{(j,n)}^h \} \) and \( \{ \hat{w}_z \} \). The resulting change in the indirect utility is:

\[
\hat{v}_z = \sum_j \sum_n \left( \frac{\partial \ln v(w_z, \hat{p}_j^h)}{\partial \ln \hat{p}_{(j,n)}^h} \hat{p}_{(j,n)}^h \right) + \frac{\partial \ln v(w_z, \hat{p}_j^h)}{\partial w_z} \hat{w}_z \tag{A.1}
\]

The equivalent variation \( \hat{u}_z \) is the proportional change in income at the original prices to induce the same proportional change in indirect utility:

\[
\hat{v}_z = \frac{\partial \ln v(w_z, \hat{p}_j^h)}{\partial \ln w_z} \hat{u}_z \tag{A.2}
\]

They imply, with the help of Roy’s identity,

\[
\hat{u}_z = \hat{w}_z + \sum_j \sum_n \left( \frac{\partial \ln v(w_z, \hat{p}_j^h)}{\partial \ln \hat{p}_{(j,n)}^h} \right) \frac{\partial \ln v(w_z, \hat{p}_j^h)}{\partial \ln w_z} \hat{p}_{(j,n)}^h = \hat{w}_z + \sum_j \sum_n -s_{(j,n)}^z \hat{p}_{(j,n)}^h \tag{A.3}
\]
A.2 Welfare Change

Integrate the aggregate expenditure effect: \( \hat{E}^h = \sum_j \sum_n S^h_{(j,n)} \left( -\frac{p^h_{(j,n)}}{p^h_{(j,n)}} \right) \),

\[
\int d\ln E^h = \sum_j \sum_n S^h_{(j,n)} \left( - \int d\ln p^h_{(j,n)} \right) \rightarrow \ln E^h = - \sum_j \sum_n \ln \left( \left( p^h_{(j,n)} \right)^{S^h_{(j,n)}} \right)
\]

\( E^h = \exp \left(- \sum_j \sum_n \ln \left( \left( p^h_{(j,n)} \right)^{S^h_{(j,n)}} \right) \right) = \prod_{(j,n)} \exp(-\ln \left( \left( p^h_{(j,n)} \right)^{S^h_{(j,n)}} \right)) = \prod_{(j,n)} \left( p^h_{(j,n)} \right)^{-S^h_{(j,n)}} \)

As a result,

\[
\frac{E^h_{cf}}{E^h_{tr}} = \prod_{(j,n)} \left( \frac{p^h_{(j,n)}}{p^h_{(j,n)}} \right)^{S^h_{(j,n)}} \tag{A.4}
\]

Integrate the individual expenditure effect: \( \hat{b}^h = \sum_j \sum_n \beta_{(j,n)} \left( \frac{p^{h,cf}_{(j,n)}}{p^{h,tr}_{(j,n)}} \right) \), \( b^h = \prod_{(j,n)} \left( \frac{p^h_{(j,n)}}{p^h_{(j,n)}} \right)^{\beta_{(j,n)}} \),

\[
\frac{b^h_{cf}}{b^h_{tr}} = \prod_{(j,n)} \left( \frac{p^{h,cf}_{(j,n)}}{p^{h,tr}_{(j,n)}} \right)^{\beta_{(j,n)}} \rightarrow -\ln \left( \frac{b^h_{cf}}{b^h_{tr}} \right) = - \sum_j \sum_n \beta_{(j,n)} \ln \left( \frac{p^{h,cf}_{(j,n)}}{p^{h,tr}_{(j,n)}} \right) \tag{A.5}
\]

A.3 Specialization in Production

We construct an index of a country \( n \)'s relative supply of goods in skill-intensive sectors as the following: \( \frac{\sum_{j=1}^J \alpha_{j,\text{supply}(j,n)}}{\sum_{j=1}^J \text{supply}(j,n)} \), where \( \alpha_j \) denotes the skill intensity of a sector. As expected, skill-abundant countries produce relatively more in skill-intensive sectors in equilibrium.

In addition, we construct an index of a country \( n \)'s relative price increase in skill-intensive sectors as the following: \( \sum_j \left( \frac{p^{cf}_{(j,n)} - p^{tr}_{(j,n)}}{p^{tr}_{(j,n)}} \right) \alpha_j \). As expected, we find that skill-abundant countries see a bigger increase in the relative price of skill-intensive goods after the trade liberalization.
Figure A.1: Specialization in Production and Price Changes

A.4 Total Supply

Output produced by a worker of labor type $\lambda$ who works in sector $j$ in country $h$ is:

$$A^h(\lambda)T(\lambda, j)E(\epsilon z|z \in Z^h(\lambda), w_z(j) \geq w_z(j')) \forall j' \in J$$

$$= A^h(\lambda)T(\lambda, j) \frac{1}{\pi^h(\lambda, j)} \int_0^\infty \epsilon(z, j) Pr\left(\epsilon(z, j) \geq \max_{j' \neq j} \epsilon(z, j') \frac{x^h(\lambda, j')}{x^h(\lambda, j)}\right) dG(\epsilon)$$

$$Pr\left(\epsilon(z, j) \geq \max_{j' \neq j} \epsilon(z, j') \frac{x^h(\lambda, j')}{x^h(\lambda, j)}\right) = \Pi_{j' \neq j} Pr\left(\epsilon(z, j') \leq \epsilon(z, j) \frac{x^h(\lambda, j)}{x^h(\lambda, j')}\right)$$

$$= \Pi_{j' \neq j} \exp\left(-\epsilon(z, j)\theta(\lambda)x^h(\lambda, j)\frac{x^h(\lambda, j')}{x^h(\lambda, j)}\theta(\lambda)\right)$$

$$= \exp\left(-\epsilon(z, j)\theta(\lambda)x^h(\lambda, j)\frac{x^h(\lambda, j')}{x^h(\lambda, j)}\sum_{j' \neq j} \theta(\lambda)\right)$$

$$G\left(\epsilon(z, j), \lambda\right) = \exp\left(-\epsilon(z, j)\theta(\lambda)\right)$$

$$dG\left(\epsilon(z, j), \lambda\right) = \exp\left(-\epsilon(z, j)\theta(\lambda)\right) \theta(\lambda)\epsilon(z, j)^{-\theta(\lambda)-1} d\epsilon$$
\[
A^h(\lambda) T(\lambda, j) \mathbb{E} \left( \epsilon_z | z \in \mathcal{Z}^h(\lambda), w_z(j) \geq w_z(j') \forall j' \in J \right)
\]

\[
= A^h(\lambda) T(\lambda, j) \frac{1}{\pi^h(\lambda, j)} \int_0^\infty e(z, j) \exp \left( -\epsilon(z, j)^{-\theta(\lambda)} x^h(\lambda, j)^{-\theta(\lambda)} \sum_{j' \neq j} x^h(\lambda, j')^{\theta(\lambda)} \right) e(z, j)^{-\theta(\lambda)} - 1 \text{d}\epsilon
\]

\[
= A^h(\lambda) T(\lambda, j) \frac{1}{\pi^h(\lambda, j)} \int_0^\infty \exp \left( -\epsilon(z, j)^{-\theta(\lambda)} x^h(\lambda, j)^{-\theta(\lambda)} \sum_{j' \in J} x^h(\lambda, j')^{\theta(\lambda)} \right) \theta(\lambda) e(z, j)^{-\theta(\lambda)} \text{d}\epsilon
\]

\[
= A^h(\lambda) T(\lambda, j) \frac{1}{\pi^h(\lambda, j)} \int_0^\infty \exp \left( -\epsilon(z, j)^{-\theta(\lambda)} \frac{1}{\pi^h(\lambda, j)} \right) \theta(\lambda) e(z, j)^{-\theta(\lambda)} \text{d}\epsilon
\]

Let \( r = \epsilon(z, j)^{-\theta(\lambda)} \frac{1}{\pi^h(\lambda, j)} \). Then \( \text{d}r = \frac{1}{\pi^h(\lambda, j)} \left( -\theta(\lambda) \right) \epsilon(z, j)^{-\theta(\lambda)} - 1 \text{d}\epsilon \).

Recall that \( \Gamma(t) = \int_0^\infty r^{t-1} e^{-r} \text{d}r \),

\[
\Gamma(1 - \frac{1}{\theta(\lambda)}) = \int_0^\infty \exp \left( -\epsilon(z, j)^{-\theta(\lambda)} \frac{1}{\pi^h(\lambda, j)} \right) \left( \epsilon(z, j)^{-\theta(\lambda)} \frac{1}{\pi^h(\lambda, j)} \right)^{-\frac{1}{\pi^h(\lambda)}} \text{d}r
\]

\[
= - \int_0^\infty \exp \left( -\epsilon(z, j)^{-\theta(\lambda)} \frac{1}{\pi^h(\lambda, j)} \right) \epsilon(z, j)^{-\theta(\lambda)} \frac{1}{\pi^h(\lambda, j)} e(z, j)^{-\theta(\lambda)} - 1 \text{d}\epsilon
\]

\[
A^h(\lambda) T(\lambda, j) \mathbb{E} \left( \epsilon_z | z \in \mathcal{Z}_h(\lambda), w_z(j) \geq w_z(j') \forall j' \in J \right)
\]

\[
= A^h(\lambda) T(\lambda, j) \frac{1}{\pi^h(\lambda, j)} \Gamma(1 - \frac{1}{\theta(\lambda)}) \pi^h(\lambda, j)^{1 - \frac{1}{\pi^h(\lambda)}} = A^h(\lambda) T(\lambda, j) \Gamma(1 - \frac{1}{\theta(\lambda)}) \pi^h(\lambda, j)^{-\frac{1}{\pi^h(\lambda)}}
\]

**Total supply of good \((j, h)\) is:**

\[
\sum_{\lambda} A^h(\lambda) T(\lambda, j) \Gamma(1 - \frac{1}{\theta(\lambda)}) \pi^h(\lambda, j)^{1 - \frac{1}{\pi^h(\lambda)}} L^h(\lambda) \pi^h(\lambda, j)
\]

\[
= \sum_{\lambda} A^h(\lambda) T(\lambda, j) \Gamma(1 - \frac{1}{\theta(\lambda)}) \pi^h(\lambda, j)^{1 - \frac{1}{\pi^h(\lambda)}} L^h(\lambda) \pi^h(\lambda, j)
\quad \text{(A.6)}
\]
A.5 Gauss-Jacobi Algorithm and Property of the Equilibrium

The Gauss-Jacobi algorithm procedure reduces the problem of solving for $n$ unknowns simultaneously in $n$ equations to that of repeatedly solving $n$ equations with one unknown. More specifically, given the known value of the $k$th iterate, $x^k$, one uses the $i$th equation to compute the $i$th component of unknown $x^{k+1}$, the next iterate. Formally $x^{k+1}$ is defined in terms of $x^k$ by the following equations:

$$
\begin{align*}
f_1(x_1^{k+1}, x_2^k, x_3^k, \ldots, x_n^k) &= 0 \\
f_2(x_1^k, x_2^{k+1}, x_3^k, \ldots, x_n^k) &= 0 \\
&\quad \vdots \\
f_n(x_1^k, x_2^k, \ldots, x_{n-1}^k, x_n^{k+1}) &= 0
\end{align*}
$$

(A.7)

The linear Gauss-Jacobi method takes a single Newton step to approximate the components of $x^{k+1}$. The resulting scheme is $x_i^{k+1} = x_i^k - \frac{f'_i(x^k)}{f''_i(x^k)}$, $i = 1, \ldots, n$.

Note that the set of prices enter both the demand side and the supply side nonlinearly. In general, for a system of nonlinear equations, it’s not possible to characterize the conditions under which a solution exists or is unique. We appeal to the Implicit Function Theorem to show that the price equilibrium that we’ve found numerically using the Gauss-Jacobi method is locally isolated as a function of the parameters. It states that if $F$ is continuously differentiable, if $F(x^*) = 0$ and if $DF(x^*)$ has full rank, then the zero set of $F$ is, near $x^*$, an $N$-dimensional surface in $R^L$. Our excess demand functions are continuously differentiable.
and the vector of prices set them to 0. Also, the Jacobian matrix of these functions has full rank ($J \times N = 1400$).

**Total Derivative** Applying the Gauss-Jacobi method to numerically calculate the set of equilibrium prices requires computing the total derive of the market clearing conditions with respect to the prices. This section provides more detail.

**Supply** Recall that the total supply of good $(j, h)$ is:

$$
\sum_{\lambda} A^h(\lambda) T(\lambda, j) \Gamma(1 - \frac{1}{\theta(\lambda)}) \pi^h(\lambda, j)^{1 - \frac{1}{\theta(\lambda)}} L^h(\lambda)
$$

Totally differentiate this term, we have:

$$
\sum_{\lambda} A^h(\lambda) T(\lambda, j) L^h(\lambda) \Gamma \left( 1 - \frac{1}{\theta(\lambda)} \right) \left( 1 - \frac{1}{\theta(\lambda)} \right) \pi^h(\lambda, j)^{-\frac{1}{\theta(\lambda)}} \\
\left[ \frac{d\pi^h(\lambda, j)}{dp^h_{(j,h)}} dp^h_{(j,h)} + \sum_{j' \neq j} \frac{d\pi^h(\lambda, j)}{dp^h_{(j',h)}} dp^h_{(j',h)} \right]
$$

$$
\pi^h(\lambda, j) = \frac{\left[ p^h_{(j,h)} A^h(\lambda) T(\lambda, j) \right]^\theta(\lambda)}{\sum_{j' \in J} \left[ p^h_{(j',h)} A^h(\lambda) T(\lambda, j') \right]^\theta(\lambda)}
$$

$$
\frac{d\pi^h(\lambda, j)}{dp^h_{(j,h)}} = \frac{\theta(\lambda) \left[ p^h_{(j,h)} A^h(\lambda) T(\lambda, j) \right]^\theta(\lambda)-1 A^h(\lambda) T(\lambda, j)}{\sum_{j' \in J} \left[ p^h_{(j',h)} A^h(\lambda) T(\lambda, j') \right]^\theta(\lambda)}
$$
\[
\begin{align*}
&\left[ p^h_{(j,h)} A^h(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \theta(\lambda) \left[ p^h_{(j,h)} A^h(\lambda) T(\lambda, j) \right]^{\theta(\lambda)-1} A^h(\lambda) T(\lambda, j) \\
&\quad \left\{ \sum_{j' \in J} \left[ p^h_{(j',h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \right\}^2 \\
&\quad \left\{ \sum_{j' \in J} \left[ p^h_{(j',h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \right\}^2 \\
&\quad \frac{d\pi^h(\lambda, j)}{dp^h_{(j,h)}} = \theta(\lambda) \left[ A^h(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \left( p^h_{(j,h)} \right)^{\theta(\lambda)-1} \\
&\quad \sum_{j' \in J} \left[ p^h_{(j',h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \\
&\quad \theta(\lambda) \left[ A^h(\lambda) T(\lambda, j) \right]^{2\theta(\lambda)} \left( p^h_{(j,h)} \right)^{2\theta(\lambda)-1} \\
&\quad \left\{ \sum_{j' \in J} \left[ p^h_{(j',h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \right\}^2 = \theta(\lambda) \left[ A^h(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \left( p^h_{(j,h)} \right)^{\theta(\lambda)-1} \right] \\
&\quad \left\{ \sum_{j' \in J} \left[ p^h_{(j',h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \right\}^2 \\
&\quad \theta(\lambda) \left[ A^h(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \left( p^h_{(j,h)} \right)^{\theta(\lambda)-1} \\
&\quad \sum_{j' \neq j} \left[ p^h_{(j',h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \\
&\quad \left\{ \sum_{j' \in J} \left[ p^h_{(j',h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \right\}^2
\end{align*}
\]
\[
\frac{d\pi^h(\lambda, j)}{dp^h_{(j', h)}} = -\left[ p^h_{(j, h)} A^h(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \theta(\lambda) \left[ p^h_{(j', h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda) - 1} \frac{\sum_{j' \in J} \left[ p^h_{(j', h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)}}{\left( \sum_{j' \in J} \left[ p^h_{(j', h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \right)^2} \]

\[
= -\frac{\left[ p^h_{(j, h)} A^h(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \theta(\lambda) \left[ A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \left( p^h_{(j', h)} \right)^{\theta(\lambda) - 1}}{\left( \sum_{j' \in J} \left[ p^h_{(j', h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \right)^2}
\]

\[
\frac{d\pi^h(\lambda, j)}{dp^h_{(j, h)}} dp^h_{(j, h)} + \sum_{j' \neq j} \frac{d\pi^h(\lambda, j)}{dp^h_{(j', h)}} dp^h_{(j', h)} = -\frac{\text{numerator}}{\left( \sum_{j' \in J} \left[ p^h_{(j', h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \right)^2}
\]

\[
\text{numerator} = \theta(\lambda) \left[ p^h_{(j, h)} A^h(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \left\{ \sum_{j' \neq j} \left[ p^h_{(j', h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \left( p^h_{(j', h)} \right)^{\theta(\lambda) - 1} \right\}
\]

\[-\sum_{j' \neq j} \left[ A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \left( p^h_{(j', h)} \right)^{\theta(\lambda) - 1} \left( p^h_{(j, h)} \right)^{\theta(\lambda)} \]

**Demand** The total demand for good \((j, h)\) is \(\sum_n S^h_{(j, h)} \bar{w}^n L^n\). Totally differentiate this term, we have:

\[-\sum_n S^h_{(j, h)} \bar{w}^n L^n \left( p^h_{(j, h)} \right)^2 dp^h_{(j, h)} + \sum_n dS^h_{(j, h)} \bar{w}^n L^n + \sum_n S^h_{(j, h)} L^n d\bar{w}^n\]

where \(dS^h_{(j, h)}\) and \(d\bar{w}^n\) are total differentiation of \(S^h_{(j, h)}\) and \(\bar{w}^n\) with respect to prices.
Under the parametric restrictions, aggregate expenditure shares can be simplified to:

\[ S_{(j,h)}^n = \alpha_{(j,h)}^n - \gamma_j \ln p_{(j,h)}^n + \frac{\gamma_j}{N} \sum_{n'=1}^{N} \ln p_{(j,n')}^n + \beta_{(j,h)} y^n \]

Totally differentiate \( S_{(j,h)}^n \), we obtain:

\[ dS_{(j,h)}^n = -\gamma_j \hat{p}_{(j,h)}^n + \frac{\gamma_j}{N} \sum_{n'=1}^{N} \hat{p}_{(j,n')}^n + \beta_{(j,h)} dy^n \]

The unadjusted average wage \( \bar{w}^n = \sum_{\lambda} \frac{L^n(\lambda)}{L^n} \Gamma (\lambda) x^n (\lambda) \) where

\[ x^n (\lambda) = \left( \sum_j \left[ p_{(j,n)}^n A^n(\lambda) T (\lambda, j) \right]^{\theta(\lambda)} \right)^{\frac{1}{\theta(\lambda)}} \]

\[ d\bar{w}^n = \sum_{\lambda} \frac{L^n(\lambda)}{L^n} \Gamma (\lambda) \frac{1}{\theta(\lambda)} \left( \sum_j \left[ p_{(j,n)}^n A^n(\lambda) T (\lambda, j) \right]^{\theta(\lambda)} \right)^{\frac{1}{\theta(\lambda)} - 1} \sum_j \left\{ \theta(\lambda) \left[ p_{(j,n)}^n A^n(\lambda) T (\lambda, j) \right]^{\theta(\lambda) - 1} \hat{p}_{(j,n)}^n \right\} \]

Alternatively, \( d\bar{w}^n = \sum_{\lambda} \frac{L^n(\lambda)}{L^n} \Gamma (\lambda) dx^n \) where:
\[dx^n = \frac{1}{\theta(\lambda)} \left( \sum_j [p^n_{(j,n)} A^n(\lambda) T(\lambda, j)]^{\theta(\lambda)} \right)^{\frac{1}{\sigma(\lambda)}} \]

\[= x^n(\lambda) \left[ \sum_j \pi^n(\lambda, j) \hat{p}^n_{(j,n)} \right]\]

\[d\bar{u}^n = \sum_{\lambda} \frac{L^n(\lambda)}{L^n(\lambda)} \Gamma(\lambda) x^n(\lambda) \left[ \sum_j \pi^n(\lambda, j) \hat{p}^n_{(j,n)} \right]\]

\[dy^n\]

\[y^n = \ln \left( \frac{\tilde{w}^n}{a(p^n)} \right) = \ln \left( \frac{\tilde{u}^n}{a^n} \right) \rightarrow dy^n = d\ln \tilde{w}^n - d\ln a^n = \hat{w}^n - \hat{a}^n\]

\[lna^n = \alpha + \sum_{(j,h)} \alpha^n_{(j,h)} \ln p^n_{(j,h)} + \sum_{(j,h)} \sum_{(j',h')} \gamma_{(j,h)(j',h')} \ln p^n_{(j,h)} \ln p^n_{(j',h')}\]

\[\hat{a}^n = \frac{dlna^n}{dlnp^n \hat{p}^n} = \sum_j \sum_h \left[ \alpha^n_{(j,h)} + \frac{1}{2} \frac{-2(N-1)}{N} \gamma_{j} \ln p^n_{(j,h)} \right.\]

\[+ \left. \frac{1}{2} \sum_{j' \neq j, h' \neq h} \frac{\gamma_{j'}}{N} \ln p^n_{(j',h')} + \frac{1}{2} \sum_{j' \neq j, h' \neq h} \frac{\gamma_{j'}}{N} \ln p^n_{(j',h')} \right] \hat{p}^n_{(j,h)}\]

Under the restrictions on the matrix \(\Gamma\),

\[\hat{a}^n = \sum_j \sum_h \left[ \alpha^n_{(j,h)} - \frac{N-1}{N} \gamma_{j} \ln p^n_{(j,h)} + \sum_{j' = j, h' \neq h} \frac{\gamma_{j'}}{N} \ln p^n_{(j',h')} \right] \hat{p}^n_{(j,h)}\]

Rewrite, \(\hat{a}^n = \sum_j \sum_h \left[ \alpha^n_{(j,h)} - \gamma_{j} \ln p^n_{(j,h)} + \frac{\gamma_j}{N} \sum_{h'=1}^N \ln p^n_{(j,h')} \right] \hat{p}^n_{(j,h)}\)
The term inside the bracket is $S^n_{(j,h)} - \beta_{(j,h)}y^n$. Therefore,

$$dy^n = \hat{w}^n - \hat{a}^n = \hat{w}^n - \sum_j \sum_h \left[ S^n_{(j,h)} - \beta_{(j,h)}y^n \right] \left( \hat{\tau}_{(j,h)}^n + \hat{p}_{(j,h)}^h \right)$$

The change in the inequality-adjusted average income can be expressed in terms of changes in log-prices:

$$\hat{w}^n = \hat{w}^n + d\Sigma^n$$

$$= - \left( \frac{1}{\hat{w}^n} \right)^2 \left[ \sum_{\lambda} \frac{L^n_{\lambda}}{L^n} \Gamma (\lambda) \left( \ln x^n (\lambda) - \frac{\Psi (\lambda)}{\theta (\lambda)} \right) x^n (\lambda) \right] d\hat{w}^n$$

$$+ \frac{1}{\hat{w}^n} \sum_{\lambda} \frac{L^n_{\lambda}}{L^n} \Gamma (\lambda) \left( 1 + \ln x^n (\lambda) - \frac{\Psi (\lambda)}{\theta (\lambda)} \right) dx^n (\lambda)$$

$$d\hat{w}^n = \sum_{\lambda} \frac{L^n_{\lambda}}{L^n} \Gamma (\lambda) x^n (\lambda) \left[ \sum_j \pi^n (\lambda, j) \hat{p}_{(j,n)}^n \right]$$

$$dx^n (\lambda) = x^n (\lambda) \left[ \sum_j \pi^n (\lambda, j) \hat{p}_{(j,n)}^n \right]$$

The unadjusted average wage and the Theil index can be expressed in terms of $x^n (\lambda)$:

$$\bar{w}^n = \sum_{\lambda} \frac{L^n_{\lambda}}{L^n} \Gamma (\lambda) x^n (\lambda)$$

$$\Sigma^n = \frac{1}{\bar{w}^n} \sum_{\lambda} \frac{L^n_{\lambda}}{L^n} \Gamma (\lambda) \left( x^n (\lambda) \ln x^n (\lambda) - \frac{\Psi (\lambda)}{\theta (\lambda)} x^n (\lambda) \right) - \ln \bar{w}^n$$

where $x^n (\lambda) \equiv \left( \sum_{j' \in J} \left[ \hat{p}_{(j',h)}^h \hat{A}^h (\lambda, j') \right]^{\theta (\lambda)} \right)^{\frac{1}{\theta (\lambda)}}$ and $\Gamma (\lambda) \equiv \Gamma \left( 1 - \frac{1}{\theta (\lambda)} \right)$ is the gamma function, $\Psi (\lambda) \equiv \Psi \left( 1 - \frac{1}{\theta (\lambda)} \right)$ is the digamma function.
Finally, $S^n_{(j,h)}$ is a function of $\{lnp^n_{(j,h)}\}_{h \in N}$, that is, $\{lnp^h_{(j,h)}\}_{h \in N}$ and $\{ln\tau^n_{(j,h)}\}_{h \in N}$. $y^n =$

$ln\tilde{w}^n + \Sigma^n - lna(p^n)$ is a function of $\{p^h_{(j,h)}\}_{j \in J}$, $\{lnp^h_{(j,h)}\}_{h \in N}$ and $\{ln\tau^n_{(j,h)}\}_{h \in N}$. 

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Appendix B

Appendix for Chapter 2

B.1 Countries in IPUMS-I


B.2 Absolute Advantage $A^h(\lambda)$

$$x^h(\lambda) = \left( \sum_j x^h(\lambda, j)^{\theta(\lambda)} \right)^{\frac{1}{\theta(\lambda)}} = \left\{ \sum_{j \in J} [p_j^h(\lambda) A^h(\lambda) T(\lambda, j)]^{\theta(\lambda)} \right\}^{\frac{1}{\theta(\lambda)}} = A^h(\lambda) \left\{ \sum_{j \in J} [p_j^h(\lambda) T(\lambda, j)]^{\theta(\lambda)} \right\}^{\frac{1}{\theta(\lambda)}}$$

$$\log x^h(\lambda) = \log A^h(\lambda) + \frac{1}{\theta(\lambda)} \log \left\{ \sum_{j \in J} [p_j^h(\lambda) T(\lambda, j)]^{\theta(\lambda)} \right\}$$
Take a first-order approximation at \( p = 1, T = 1 \):

\[
\log x^h(\lambda) = \log A^h(\lambda) + \frac{1}{\theta(\lambda)} \left\{ \log J + \frac{1}{J} \sum_{j \in J} \left( [p^h_{(j,h)} T^h(\lambda, j)]^{\theta(\lambda)} - 1 \right) \right\} \\
= \log A^h(\lambda) + \frac{1}{\theta(\lambda)} \left\{ \log J + \frac{1}{J} \sum_{j \in J} \log \left( [p^h_{(j,h)} T^h(\lambda, j)]^{\theta(\lambda)} \right) \right\} \\
= \log A^h(\lambda) + \frac{1}{\theta(\lambda)} \log J + \frac{1}{J} \left[ \sum_{j \in J} \log p^h_{(j,h)} + \sum_{j \in J} \log T^h(\lambda, j) \right] \\
\]

\[
\log x^h(1) = \log A^h(1) + \frac{1}{\theta(1)} \log J + \frac{1}{J} \left[ \sum_{j \in J} \log p^h_{(j,h)} + \sum_{j \in J} \log T(1, j) \right] \\
\]

\[
\log \left( \frac{x^h(\lambda)}{x^h(1)} \right) = \log \left( \frac{A^h(\lambda)}{A^h(1)} \right) + \log J \left( \frac{1}{\theta(\lambda)} - \frac{1}{\theta(1)} \right) + \frac{1}{J} \sum_{j \in J} \log \left( \frac{T^h(\lambda, j)}{T(1, j)} \right) \tag{B.1} \]

### B.3 Labor Groups

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</tr>
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Table B.1: Labor Groups

### B.4 Non-homothetic Gravity Equation

Under the additional assumptions on \( \Gamma \)

\[
\frac{Y^h_{(j,n)}}{Y^h} \equiv S^h_{(j,n)} = \alpha^h_{(j,n)} - \gamma_j \ln \left( \frac{p^h_{(j,n)}}{p^h_j} \right) + \beta_{(j,n)} y^h \tag{B.2} \]
where\( P_h^j = \exp \left( \frac{1}{N} \sum_{n'} \ln p_{(j,n')}^h \right) \). Replacing \( p_{(j,n')}^h = \tau_{(j,n')}^h p_{(j,n')}^{n'} \), I have

\[
\frac{p_{(j,n)}^h}{P_h^j} = \frac{\tau_{(j,n)}^h}{\exp \left( \frac{1}{N} \sum_{n'} \ln \tau_{(j,n')}^h \right)} \cdot \frac{P_h^j}{\exp \left( \frac{1}{N} \sum_{n} \ln p_{(j,n')}^{n'} \right)} \equiv \frac{\tau_{(j,n)}^h}{\tau_{(j,n)}^h} \cdot \frac{p_{(j,n)}^n}{\bar{p}_j} \tag{B.3}
\]

Therefore,

\[
\frac{Y_{(j,n)}}{Y^W} = \sum_{n'} \frac{Y^{n'}}{Y^W} S_{(j,n)}^{n'}
\]

\[
= \sum_{n'} \frac{Y^{n'}}{Y^W} \left( \alpha_{(j,n)}^{n'} - \gamma_j \ln \left( \frac{\tau_{(j,n')}^n}{\tau_{(j,n')}^n} \cdot \frac{p_{(j,n)}^n}{\bar{p}_j} \right) + \beta_{(j,n)} y_{n'} \right) \tag{B.4}
\]

Substracting the second equation from the first:

\[
\frac{Y_{(j,n)}^h}{Y^h} - \frac{Y_{(j,n)}}{Y^W} = \left[ \alpha_{(j,n)}^h - \sum_{n'} \frac{Y^{n'}}{Y^W} \alpha_{(j,n)}^{n'} \right]
\]

\[
\equiv \kappa_{(j,n)}^h
\]

\[
- \gamma_j \left[ \ln \left( \frac{\tau_{(j,n)}^h}{\tau_{(j,n)}^h} \cdot \frac{p_{(j,n)}^n}{\bar{p}_j} \right) - \sum_{n'} \frac{Y^{n'}}{Y^W} \ln \left( \frac{\tau_{(j,n')}^n}{\tau_{(j,n')}^n} \cdot \frac{p_{(j,n)}^n}{\bar{p}_j} \right) \right]
\]

\[
\equiv \mu_{(j,n)}^h
\]

\[
+ \beta_{(j,n)} \left[ y^h - \sum_{n'} \frac{Y^{n'}}{Y^W} y_{n'} \right]
\]

\[
\equiv \Omega^h
\]

\[
\frac{Y_{(j,n)}^h}{Y^h} - \frac{Y_{(j,n)}}{Y^W} = \kappa_{(j,n)}^h - \mu_{(j,n)}^h + \Omega^h \tag{B.5}
\]

\section{B.5 Differences in Tastes across Countries}

Under the additional assumptions on \( \Gamma \) and \( \sum_n \alpha_n = 1 \) combined with the equation \( \alpha_{(j,n)}^h = \alpha_n (\alpha_j + \epsilon_j^h) \),

\[
S_j^h = \sum_n \alpha_{(j,n)}^h + \beta_j y^h = \alpha_j + \beta_j y^h + \epsilon_j^h \tag{B.6}
\]
\[ S_j^W = \frac{Y_j^W}{Y_W} = \sum_{n'=1}^{N} \frac{Y_{n'}^W S_{j}^{n'}}{Y_W} = \sum_{n'=1}^{N} \frac{Y_{n'}^W}{Y_W} \left( \alpha_j + \bar{\beta}_j y^h + \epsilon_j^h \right) \] (B.7)

\[ S_j^h - S_j^W = \alpha_j - \sum_{n'=1}^{N} \left( \frac{Y_{n'}^W}{Y_W} \right) \alpha_j + \epsilon_j^h - \sum_{n'=1}^{N} \left( \frac{Y_{n'}^W}{Y_W} \right) \epsilon_j^h + \bar{\beta}_j \Omega^h \] (B.8)

\[ K_{(j,n)}^h = \alpha_{(j,n)}^h - \sum_{n'=1}^{N} \left( \frac{Y_{n'}^W}{Y_W} \right) \alpha_{(j,n)}^{n'} = \alpha_n (\alpha_j + \epsilon_j^h) - \sum_{n'=1}^{N} \left( \frac{Y_{n'}^W}{Y_W} \right) \alpha_n (\alpha_j + \epsilon_j^{n'}) \]

\[ = \alpha_n \left[ \alpha_j - \sum_{n'=1}^{N} \left( \frac{Y_{n'}^W}{Y_W} \right) \alpha_j + \epsilon_j^h - \sum_{n'=1}^{N} \left( \frac{Y_{n'}^W}{Y_W} \right) \epsilon_j^h \right] = \alpha_n (S_j^h - S_j^W) - \alpha_n \bar{\beta}_j \Omega^h \] (B.9)

### B.6 Counterfactual - Back to Autarky

Recall that the global welfare change of individual \( z \) under the AIDS between an initial scenario under trade and a counterfactual scenario is:

\[ u_{z}^{tr \rightarrow cf} = \left( \frac{E_{tr}^h}{E_{tr}^h} \right) \left( \frac{w_{z}^{tr}}{w_{z}^{cf}} \right)^{-\ln(b_{tr}^h/b_{tr}^h)} \left( \frac{w_{z}^{cf}}{w_{z}^{tr}} \right) \]

where \( E_{tr}^h/E_{tr}^h \) and \(-\ln(b_{tr}^h/b_{tr}^h)\) are functions of the prices in the two scenarios. I need to compute the prices of domestic commodities in autarky \( \{p_{(j,h)}^{h,cf}\} \) as well as the consumer-specific reservation prices of foreign varieties \( \{p_{(j,n),z}^{h,cf}\} \) that are no longer consumed.

The restriction to non-negative individual expenditure shares may bind in the counterfactual. In these cases, I find consumer-specific reservation prices such that the individual shares of dropped varieties are all zero and the remaining individual shares are adjusted using these reservation prices. For each percentile \( z \) in country \( h \), I have that \( p_{(j,h),z}^{h,cf} = p_{(j,h)}^{h,cf} \)
for all \( j \) and \( s_{(j,n),z}^{h,cf} = 0 \) for all \( j \) and \( n \neq h \).

Reservation prices \( p_{(j,n),z}^{h,cf} \) for \( n \neq h \) and individual shares \( s_{(j,h),z}^{h,cf} \) satisfy:

\[
s_{(j,h),z}^{h,cf} = \alpha_{(j,h)}^h - \gamma_j \ln p_{(j,h)}^{h,cf} + \frac{\gamma_j}{N} \left( \ln p_{(j,h)}^{h,cf} + \sum_{n \neq h} \ln p_{(j,n),z}^{h,cf} \right) + \beta_{(j,n)} \ln \left( \frac{w_z^{cf}}{a^{cf}_{h,z}} \right) \tag{B.10}
\]

\[
0 = \alpha_{(j,n)}^h - \gamma_j \ln p_{(j,n),z}^{h,cf} + \frac{\gamma_j}{N} \left( \ln p_{(j,h)}^{h,cf} + \sum_{n \neq h} \ln p_{(j,n),z}^{h,cf} \right) + \beta_{(j,n)} \ln \left( \frac{w_z^{cf}}{a^{cf}_{h,z}} \right) \tag{B.11}
\]

The second equation implies that:

\[
\sum_{n \neq h} \gamma_j \ln p_{(j,n),z}^{h,cf} = (N - 1) \ln p_{(j,h)}^{h,cf} + N \left( \sum_{n \neq h} \alpha_{(j,n)}^h + \sum_{n \neq h} \beta_{(j,n)} \ln \left( \frac{w_z^{cf}}{a^{cf}_{h,z}} \right) \right) \tag{B.12}
\]

Replacing this back into the second equation gives the reservation prices of the foreign varieties in sector \( j \):

\[
\ln p_{(j,n),z}^{h,cf} = \frac{1}{\gamma_j} \left[ \alpha_{(j,n)}^h \sum_{n' \neq h} \alpha_{(j,n')}^h + \left( \beta_{(j,n)} + \sum_{n' \neq h} \beta_{(j,n')} \right) \ln \left( \frac{w_z^{cf}}{a^{cf}_{h,z}} \right) \right] + \ln \left( p_{(j,h)}^{h,cf} \right) \tag{B.13}
\]

where \( a^{cf}_{h,z} = a \left( \left\{ p_{(j,h)}^{h,cf}, p_{(j,n),z}^{h,cf} \right\} \right) \) is the homothetic component of the price index, and \( w_z^{cf} \) is the autarky income of percentile \( z \) from the home country, and it’s a function of \( \left\{ p_{(j,h)}^{h,cf} \right\} \).

For each \( h \), this gives me \( (N - 1) \times J \times Z \) equations in \( \left\{ p_{(j,h)}^{h,cf}, p_{(j,n),z}^{h,cf} \right\} \).

I combine these reservation prices equations with \( J \) market clearing conditions in autarky which set the total supply to equal to the total demand.

The total supply of good \( (j, h) \) is:

\[
\sum_{\lambda} A^h(\lambda)T(\lambda, j) \Gamma(1 - \frac{1}{\theta(\lambda)})\pi^h(\lambda, j)^{1 - \frac{1}{\theta(\lambda)}} L^h(\lambda)
\]
which is a function of \( \{p^{h,cf}_{(j,h)}\} \) in autarky.

Under the parametric restrictions, the expenditure share for consumer \( z \) in goods from country \( n \) in sector \( j \) is:

\[
s^h_{(j,n),z} = \alpha^h_{(j,n)} - \gamma_j \ln p^h_{(j,n)} + \frac{\gamma_j}{N} \sum_{n'=1}^N \ln p^h_{(j,n')} + \beta_{(j,n)} \ln \left( \frac{w_z}{a(p^h)} \right) \quad (B.14)
\]

Adding up across \( n \), the share of sector \( j \) in total expenditures of \( z \) is:

\[
s^h_{j,z} = \sum_n s^h_{(j,n),z} = \bar{\alpha}^h_j + \bar{\beta}_j \ln \left( \frac{w_z}{a(p^h)} \right) \quad (B.15)
\]

where \( \bar{\alpha}^h_j = \sum_n \alpha^h_{(j,n)} \) and \( \bar{\beta}_j = \sum_n \beta_{(j,n)} \).

In autarky, the expenditure shares evaluated at the reservation prices are such that:

\[
s^{h,cf}_{(j,h),z} = \bar{\alpha}^h_j + \bar{\beta}_j \ln \left( \frac{w^{cf}_z}{a^{cf}_{h,z}} \right) \quad (B.16)
\]

\[
s^{h,cf}_{(j,n),z} = 0 \quad , \quad n \neq h \quad (B.17)
\]

The aggregate expenditure shares are thus: \( S^{h,cf}_{(j,h)} = \sum_z \left( \frac{w^{cf}_z}{\sum_{j'} w^{cf}_{j',z}} \right) s^{h,cf}_{(j,h),z} \).

Since \( \bar{w}^{cf}_h \) is a function of \( \{p^{h,cf}_{(j,h)}\} \), the total demand for good \( (j, h) \) in autarky, \( S^{h,cf}_{(j,h)} \bar{w}^{cf}_h L^h/p^{h,cf}_{(j,h)} \), is a function of \( \{p^{h,cf}_{(j,h)}, p^{h,cf}_{(j,n),z}\} \). Market clearing conditions give me \( J \) equations in \( \{p^{h,cf}_{(j,h)}, p^{h,cf}_{(j,n),z}\} \) for each \( h \). Combined with the equations of reservation prices above, I have \( J + (N - 1) * J * Z \) equations in \( J + (N - 1) * J * Z \) unknowns.

Once I have solved for the output prices \( \{p^{h,cf}_{(j,h)}\}_{j \in J} \) in autarky, I can back out \( w^h(\lambda) \) and therefore the wage distribution for each labor type \( \lambda \).
\[ x^h(\lambda) = \left( \sum_j \left[ A^h(\lambda)p^h_{j\lambda} T(\lambda, j) \right]^{\theta(\lambda)} \right)^{-\frac{1}{\theta(\lambda)}} \]

The autarky wage distribution is:

\[ Pr(w_z \leq w) = \sum_{\lambda} Pr \left( w_z \leq w | z \in Z^h(\lambda) \right) Pr \left( z \in Z^h(\lambda) \right) \]

\[ = \sum_{\lambda} \frac{L^h(\lambda)}{L^h} exp \left\{ -x^h(\lambda)^{\theta(\lambda)} w^{-\theta(\lambda)} \right\} = \sum_{\lambda} exp \left\{ -x^h(\lambda)^{\theta(\lambda)} w^{-\theta(\lambda)} + ln \left( \frac{L^h(\lambda)}{L^h} \right) \right\} \]