Essays in International Macroeconomics

Mitchell Vaughn

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Abstract

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This dissertation studies topics in international macroeconomics.

In the first chapter, I develop a heterogeneous agent model of a small open economy and studies how households differ in their responses to aggregate productivity and interest rate shocks. Poor households display stronger consumption responses to an aggregate productivity shock because they are more likely to be constrained in liquid assets. In contrast, rich households display stronger consumption responses to an interest rate shock because they are more likely to be unconstrained in liquid assets. When the economy experiences a sudden stop, defined as transitory contractionary shocks to productivity and the interest rate, the interest rate effect neutralizes the productivity effect. As a consequence, the sudden stop generates consumption-income elasticities that display little variation along the income distribution, similar to a permanent shock. My finding captures the observed behavior of households in the Mexican Peso Crisis of 1994.

In the second chapter, I study a small open economy subject to a borrowing constraint which experiences stochastic volatility in its output endowment. I find that volatility shocks induce substantial changes in borrowing by households, in excess of the precautionary savings response. Household responses to volatility shocks increases the standard deviation of borrowing, but not the standard deviation of consumption, suggesting small welfare costs. Stochastic volatility increases the frequency of financial crises in a decentralized economy that overborrows
due to a pecuniary externality, but not a socially optimal economy.

In the third chapter, I introduce income heterogeneity into a small open economy model with an occasionally binding collateral constraint. Income heterogeneity generates poor households that borrow up to the constraint to smooth over their income shock. This differs from representative agent models that require a depressed aggregate state for the representative household to interact with the constraint. As a consequence, the model displays a higher average marginal propensity to consume which generates a higher volatility of aggregate consumption. The model with income heterogeneity fails to generate sudden stops. This occurs as the income shock generates rich households that are able to consumption smooth throughout contractions.

In the fourth chapter, I trace the path between a benchmark representative agent model and a benchmark heterogeneous agent model. Heterogeneous agent models typically introduce idiosyncratic income risk, a financial friction in the form of a borrowing or non-negativity constraint, and recalibrate the impatience of households. This paper studies the effect of each term. With the minimal financial friction that households cannot starve, complete markets fail, but income risk has no significant effect on the aggregate response of consumption to an endowment or interest rate shock relative to a representative agent benchmark. Heterogeneity and significant financial frictions generate empirically realistic marginal propensities to consume, but fail to alter the aggregate consumption response. Decreasing the impatience of households is necessary to significantly alter aggregate responses to endowment and interest rate shocks.
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Dedication

To Fiona.
Chapter 1: Sudden Stops with Heterogeneous Agents

1.1 Introduction

Emerging markets frequently experience sudden stops: sharp recessions that coincide with a reversal of the trade balance. An open question is what drives sudden stops. Two candidates are unexpected, transitory shocks that throw the economy off balance and permanent shocks to the fundamentals of the economy. Until recently, these shocks have only been considered in the context of representative agent (RA) models. This paper investigates the ability of transitory shocks and permanent shocks to explain sudden stops through the lens of a heterogeneous agent (HA) model. Using the HA model, I study the ability of each shock to match the data along two dimensions. At the household level, I study the ability of each shock to match the consumption responses of households during the Mexican Peso Crisis of 1994, documented in Guntin et al. (2023). At the aggregate level, I study the ability of each shock to match the signature features of the Mexican Peso Crisis: a sharp drop in investment and a reversal of the trade balance.

To this end, I build a heterogeneous agent small open economy (HASOE) model in which households face idiosyncratic income risk, limited access to financial markets, and store the vast majority of their wealth in illiquid assets. I consider two approaches to generate a sudden stop. Under the first approach, the economy experiences a transitory decline in productivity that coincides with a transitory increase in the interest rate. This captures the procyclical nature of interest rates in emerging markets, documented in Kaminsky et al. (2004): in bad times, emerging markets typically face an increase in the interest rate which increases the cost of smoothing aggregate fluctuations.¹ Under the second approach, the economy experiences a permanent decline in productivity.

¹For further discussion of procyclical interest rates, see Calvo (1998), Calvo and Reinhart (2002), and Calvo et al. (2006). Within my case study of the Mexican Peso Crisis, I explicitly motivate the interest rate increase using the data.
ity, as in Aguiar and Gopinath (2007) and Guntin et al. (2023). I show that each approach is able to recreate the consumption responses observed in the household data. In contrast, the transitory shocks generate a stronger decline in investment and reversal of the trade balance that characterize the Mexican Peso Crisis. This occurs because the increase in the interest rate motivates households to substitute from capital into the external bond of the economy.

For the permanent shock, the intuition of the representative agent benchmark of Aguiar and Gopinath (2007) holds at the household level: conditional on a permanent decline in income, households cannot sustain their current consumption level, so their optimal choice is to immediately decrease their consumption with income. This holds for both low income households that display a large response to transitory income fluctuations, and high income households that display a smaller response to transitory income fluctuations.

The ability of the transitory approach to generate the observed consumption responses is more nuanced. Because they are likely to be financially constrained, low income households display a large consumption response to the transitory decline in labor income and a small consumption response to the increase in the interest rate. Because they are more likely to hold liquid assets, high income households display a smaller consumption response to the transitory decline in income. However, precisely because they hold liquid assets, the increase in the interest rate incentivizes high income households to increase their liquid asset position, financed through a decrease in consumption. When the two effects are added together, the stronger response to the interest rate overcomes the weaker response to income so that high income households display a large decline in consumption, as is observed in the data. This occurs despite substantial heterogeneity in income.

Guntin et al. (2023) differs in that it features a permanent decline in an income endowment. Within my model, a permanent decline in productivity generates a permanent decline in the general equilibrium labor income endowment that households receive. This paper abstracts from more sophisticated models of highly persistent declines in productivity, like that of Queralto (2020).

How strongly consumption decreases depends on an individual household’s financial position. In the model, unconstrained households that behave more like permanent income consumers respond to the long run decline in income. In contrast, constrained households that would like to consume more only respond to the immediate decline in income.

The small response to the interest rate shock holds for both its wealth and substitution effects. The substitution effect is neutralized because constrained households feature a wedge in their Euler equation. In the model, constrained households hold zero wealth and hence lack wealth effects. This is supported in my empirical analysis of the Mexican Family Life Survey, which shows that low income households are less likely to hold both savings and debts.
wealth, and access to financial markets.

The two shocks differ in their implications for aggregate variables. The transitory shocks generate a sudden stop which features a sharp decline in investment and reversal of the trade balance, as is observed in the data. The permanent shock generates a similar decline in consumption, but weaker responses of investment and the trade balance. The transitory shocks generate a stronger investment decline due to the increase in the interest rate, which motivates households to substitute from domestic capital to the external bond of the economy. In the context of the Mexican Peso Crisis, this supports the episode as being driven by the procyclical increase in the interest rate rather than a permanent decline in productivity.

The household problem of the model features a two asset environment similar to that of Kaplan et al. (2014), Kaplan et al. (2018), and Hong (2023a). Households have access to a liquid bond and an illiquid asset that is subject to convex adjustment costs. Both assets are subject to a non-negativity constraint. While the non-negativity constraint prohibits borrowing, it generates a large portion of low income households that hold neither debts nor savings, aligning with the data. I compare the two asset model to a traditional single asset model in the style of Bewley (1977). The two asset model improves on the single asset model along two dimensions. First, the two asset model can match two key empirical targets: the percentage of financially constrained households, and the average marginal propensity to consume (MPC). The former governs how many households display a substitution effect to an interest rate change, and the latter governs how responsive households are to income fluctuations. Secondly, the two asset model features a weaker relation-

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5Within the Mexican Peso Crisis there exists a disconnect between the aggregate and household data. Because I target nondurable consumption, the household data features a consumption to income response of no more than one to one. This contrasts with the aggregate consumption, which features durable consumption and a more than one to one consumption to GDP response. From the perspective of the household data, a more than one to one response of durable consumption relative to income is strongly supported in McKenzie (2006).

6Following the national expenditure decomposition of GDP, the overreaction of aggregate consumption partially supports the large reversal of the trade balance observed during the Mexican Peso Crisis. Because I target the household nondurable consumption response, which does not overreact relative to income, both the transitory and permanent approaches generate weaker trade balance reversals relative to the reversal observed in the data.

7I support this using household data from the Mexican Family Life Survey.

8The marginal propensity to consume measures the consumption response to a small and transitory change in income.
ship between income and the likelihood of being financially constrained, which is overstated in the single asset model. Because the single asset model overstates this relationship, the lowest income decile displays a one to one consumption response for even a temporary decrease in income. This leads the single asset model to overstate heterogeneity in households responses for both a change in income and a change in the interest rate.

The persistence of aggregate shocks plays a critical role in how households differ in their consumption responses. For a small level of persistence, a productivity shock generates small and brief changes in income. This leads to consumption responses that mimic the marginal propensity to consume, which is higher for low income households, as documented empirically in Hong (2023b). As the persistence of a productivity shock increases, the persistence of the general equilibrium fluctuation in labor income increases. This is largely ignored by low income households, who only respond to the initial change in labor income. In contrast, high income households are less likely to be constrained and respond immediately to future changes in income. As the persistence of the labor income fluctuation increases, high income households display a larger response, which reduces heterogeneity in consumption responses across low and high income households. For the two asset household problem, this leads to the implication that low and high income households can display identical consumption responses for a highly persistent income shock without a permanent change in income.\footnote{The primary measure of consumption responses, also used in this paper, is the consumption-income elasticity, the percentage change in consumption divided by the percentage change in income.}

The opposite occurs for a fluctuation to the interest rate. Constrained households respond to neither current or future changes in the interest rate, whereas unconstrained households display significant responses to future changes in the interest rate. In this case, an increase in persistence only amplifies heterogeneity in consumption responses. Changes in the interest rate do generate indirect labor income effects through an increase in investment, as in Kaplan et al. (2018), but in this case households display muted heterogeneity in consumption responses because the majority of wage changes are in the future.

I motivate this paper using four stylized facts from the data. First, I characterize the Mexican
Peso Crisis at the aggregate level. Second, I discuss the procyclical nature of the interest rate in emerging markets, which features most prominently in Uribe and Yue (2006) and Neumeyer and Perri (2005). Third, closely following the empirical work of Guntin et al. (2023), I replicate that high income households display a large decline in consumption during the Mexican Peso Crisis using the Mexican National Survey of Household Income and Expenditure (ENIGH). Lastly, I document that access to financial markets increases with income using the Mexican Family Life (MFL) survey.

**Literature**  This paper contributes to a variety of literatures. First, I contribute to the literature that studies the drivers of emerging markets, which includes Neumeyer and Perri (2005), Uribe and Yue (2006), Aguiar and Gopinath (2007), García-Cicco et al. (2010), and Chang and Fernandez (2013). I contribute to this literature by studying transitory shocks and permanent shocks through the lens of an HA model and comparing their ability to match both the aggregate and household data. My analysis supports interest rates as a driver of emerging markets, although I focus on a crisis episode rather than the business cycle.

I contribute to the literature that studies sudden stops in emerging markets, which includes contributions from Calvo (1998), Mendoza (2002), Mendoza and Smith (2004), Calvo et al. (2006), Mendoza (2010), and Bianchi and Mendoza (2020). Contributions to the nascent HA literature include Cugat (2019), Guntin et al. (2023), and Villalvazo (2023). Similar to Cugat (2019), my model is able to replicate the consumption response of the sudden stop because a significant portion of households lack access to financial markets. My model differs from Cugat (2019) in that the inability to consumption smooth using a bond is generated endogenously through an occasionally binding constraint and idiosyncratic income risk, whereas Cugat (2019) features hand to mouth households like those of Campbell and Mankiw (1989). In addition, I abstract from heterogeneity in household employment in the tradable and nontradable sectors. This paper differs from Villal-

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10. García-Cicco et al. (2010) include both permanent shocks and interest rate shocks in a small open economy model. Their analysis finds a small contribution of permanent shocks.  
11. This differs from the typical collateral constraint and eventual Fisherian deflation that generates a strong consumption decline in RA models.
vazo (2023) in that it focuses on heterogeneity along the income distribution rather than the wealth distribution and focuses on the Mexican Peso Crisis rather than the Global Financial Crisis. In addition, my model lacks the boom bust episodes that Villalvazo (2023) generates through aggregate risk.\footnote{Because Villalvazo (2023) uses a global model, the economy features a buildup preceding the sudden stop similar to the RA model of Mendoza (2010). The use of a global solution method is out of the scope of this paper because this paper needs a rich distribution of income, whereas global HA models typically use binary low and high income states.}

This paper contributes to the literature that studies the distributional effects of aggregate shocks. Contributions, which primarily focus on interest rate fluctuations, include Auclert (2019), Di Maggio et al. (2017), and Amberg et al. (2022). Contributions specific to small open economies include deFerra et al. (2020), Guo et al. (2021), Zhou (2022), Guntin et al. (2023), and Oskolkov (2023). I contribute to this literature by studying the heterogeneous effects of aggregate productivity and interest rate shocks.

While similar in objectives, this paper contrasts most strongly with Guntin et al. (2023), who interpret the weak relationship between consumption-income elasticities and income as supporting a permanent decline in income.\footnote{Within the model, I generate a permanent decline in income by introducing a permanent decline in the productivity of the representative firm.} I show that a two asset model with a realistic combination of transitory productivity and interest rate shocks can match the observed consumption-income elasticities of the data. In addition, my model can comment on the larger dynamics of the sudden stop because it features a production sector.

This paper is similar to Hong (2023b) in that it studies heterogeneous consumption dynamics during a financial crisis. Hong (2023b) focuses on Peru during the Global Financial Crisis whereas this paper focuses on the Mexican Peso Crisis. Similarly, Hong (2023b) rejects the permanent shock as the sole driver of consumption responses and finds that low income households respond primarily to a financial friction shock that scales the magnitude of adjustment costs for the illiquid asset, whereas high income households respond to a trend shock in productivity. In this paper, low income households respond to a stationary productivity shock, and high income households respond to a mix of stationary productivity and interest rate shocks. Both papers are similar in that
they introduce an alternative to the standard stationary income shock to explain the consumption
type behavior of high income households. From the aggregate perspective, this is necessary to replicate
the large consumption declines observed during the Mexican Peso Crisis and the Global Financial
Crisis because high income households take up a disproportionate share of consumption.

The rest of the paper is organized as follows. Section 3.2 presents the model. Section 3.3
presents the calibration. Section 3.4 studies the ability of the transitory and permanent approaches
to match the Mexican Peso Crisis of 1994. Section 1.5 presents stylized facts from the aggregate
and household data. Section 3.5 concludes.

1.2 Model

This section describes the model. Time is discrete and infinite. The model is a small open
economy that saves on the international market. The economy is populated by a representative
firm and a unit continuum of households that are heterogeneous in income and asset holdings. The
consumption good is produced by the representative firm using a combination of capital, managed
by the firm, and labor, hired from a labor union that represents the households. Household het-
erogeneity is generated by idiosyncratic income risk. Households have access to two assets: a
liquid asset that provides a certain return and an illiquid asset that is subject to convex adjustment
costs. Both the liquid and illiquid assets are subject to non-negativity constraints. Aggregate sav-
ings from the liquid asset are saved on the international market. The illiquid asset is shares of the
representative firm.

The rest of this section is organized as follows. Section 1.2.1 describes the household problem.
Section 1.2.2 describes the production side of the economy. Section 1.2.3 defines market clearing,
and Section 1.2.4 defines the perfect foresight equilibrium.

1.2.1 Households

This section describes the household problem. The economy is populated by a unit continuum
of households, indexed by i. Household i has preferences over infinite streams of the consumption
good \( \{c^i_t\}_{t=0}^\infty \) given by

\[
E_0 \sum_{t=0}^\infty \beta^t u(c^i_t)
\]  

(1.1)

where \( 0 < \beta < 1 \) is the discount factor. In each period, household \( i \) must satisfy the budget constraint

\[
c^i_t + b^i_t + a^i_t + \chi(a^i_t, a^i_{t-1}) = (1 + r^b_t)b^i_{t-1} + (1 + r^a_t)a^i_{t-1} + e^i_tw_tL_t
\]

(1.2)

where \( b^i_t \) denotes the liquid asset and \( a^i_t \) denotes the illiquid asset, both in terms of the consumption good, and \( \chi(\cdot, \cdot) \geq 0 \) is a convex adjustment cost function.\(^{14}\) At time \( t \), the returns of the liquid and illiquid assets are given by \( r^b_t \), which is known in period \( t - 1 \), and \( r^a_t \), which is determined in period \( t \). Aggregate labor income is given by \( w_tL_t \), and household \( i \)'s individual labor income is given by \( e^i_tw_tL_t \), where \( e^i_t \) is mean one, exogenous and follows a known stochastic process. In each period, household \( i \)'s asset holdings are subject to the non-negativity constraints

\[
b^i_t \geq 0,
\]

(1.3)

and

\[
a^i_t \geq 0,
\]

(1.4)

respectively.\(^{15}\)

When solving their problem, households take as given the path of aggregate labor income \( \{w_tL_t\}_{t=0}^\infty \), the path of the interest rate \( \{r^b_t\}_{t=0}^\infty \), and the path of the illiquid asset return \( \{r^a_t\}_{t=0}^\infty \). I collect these as \( \Gamma = \{w_t, L_t, r^b_t, r^a_t\}_{t=0}^\infty \), which I refer to as the ‘household inputs’.\(^{16}\) Taking \( \Gamma \) as given, household \( i \) chooses the paths of their consumption \( \{c^i_t\}_{t=0}^\infty \), liquid asset holdings \( \{b^i_t\}_{t=0}^\infty \), and illiquid asset holdings \( \{a^i_t\}_{t=0}^\infty \) to maximize (1.1) subject to the budget constraint (1.2) and the non-negativity constraints (1.3) and (1.4).

When solving their problem, households face idiosyncratic income risk. For a generic house-

\(^{14}\)I use the convention that \( b^i_t \) denotes liquid asset holdings between period \( t \) and \( t + 1 \), and similarly for \( a^i_t \).

\(^{15}\)The use of a non-negativity constraint rather than a more generous constraint that allows for borrowing does not have a material effect on the ability of the model to generate constrained households that display a significant marginal propensity to consume. It does have a significant effect on the heterogeneity in responses to interest rate shocks because there are no constrained debtors that display negative wealth effects to an interest rate increase.

\(^{16}\)In practice, the household problem only depends on \( w_tL_t \) and not \( w_t \) and \( L_t \) separately.
hold, the idiosyncratic income component $e$ satisfies

$$\log e_t = \rho e \log e_{t-1} + \sigma_e \epsilon_t^e, \epsilon_t^e \sim \mathcal{N}(0, 1), \tag{1.5}$$

where $0 \leq \rho_e < 1$ is the autocorrelation of idiosyncratic income risk, $\sigma_e$ is its standard deviation, and $\mathcal{N}(0, 1)$ is the standard normal distribution.

Dropping $i$, we can compute the first order conditions

$$u(c_t) = \mu^b_t + \beta E_t(1 + r^b_{t+1}) u(c_{t+1}), \tag{1.6}$$

$$u(c_t)(1 + \chi_1(a_t, a_{t-1})) = \mu^a_t + \beta E_t(1 + r^a_{t+1} - \chi_2(a_{t+1}, a_t)) u(c_{t+1}), \tag{1.7}$$

where $\mu^b_t$ is the Lagrange multiplier of the liquid asset non-negativity constraint (1.3), and $\mu^a_t$ is the Lagrange multiplier of the illiquid asset non-negativity constraint (1.4). Equation (1.6) denotes the Euler equation of the liquid asset and equation (1.7) denotes the Euler equation of the illiquid asset. In addition, households satisfy the conditions

$$\mu^b_t \geq 0, \mu^a_t \geq 0, \mu^b_t b_t = 0, \text{ and } \mu^a_t a_t = 0. \tag{1.8}$$

Solving the household problem produces a series of policies

$$\{c_t(e, b, a; \Gamma)\}_{t=0}^{\infty}, \{b_t(e, b, a; \Gamma)\}_{t=0}^{\infty}, \{a_t(e, b, a; \Gamma)\}_{t=0}^{\infty}$$

that depend on the entire path of households inputs $\Gamma$.

The distribution of households at time $t$ is described by the cumulative density function (CDF) $\Psi_t$, where

$$\Psi_t(e, a, b; \Gamma) = Pr(e_t \leq e, a_{t-1} \leq a, b_{t-1} \leq b; \Gamma). \tag{1.9}$$
The distribution function $\Psi_t$ satisfies the law of motion

$$\Psi_{t+1}(e', b', a'; \Gamma) = \int_{e,b,a} P_r(e_{t+1} \leq e'| e_t = e) \mathcal{I} \left[ a_t(e, b, a; \Gamma) \leq a'; b_t(e, b, a; \Gamma) \leq b' \right] d\Psi_t(e, b, a; \Gamma),$$

(1.10)

where $\mathcal{I}$ is the indicator function and $b_t(e, b, a; \Gamma)$ and $a_t(e, b, a; \Gamma)$ denote a household’s policies in period $t$ as a function of their idiosyncratic income $e$ and asset positions $b$, and $a$. At time $t$, aggregate consumption, liquid assets, illiquid assets, and adjustment costs are defined as

$$C_t = \int_{e,b,a} c_t(e, b, a; \Gamma) d\Psi_t(e, b, a; \Gamma),$$

(1.11)

$$B_t = \int_{e,b,a} b_t(e, b, a; \Gamma) d\Psi_t(e, b, a; \Gamma),$$

(1.12)

$$A_t = \int_{e,b,a} a_t(e, b, a; \Gamma) d\Psi_t(e, b, a; \Gamma),$$

(1.13)

and

$$\chi_t = \int_{e,b,a} \chi(a_t(e, b, a; \Gamma), a) d\Psi_t(e, b, a; \Gamma),$$

(1.14)

respectively.\(^\text{17}\)

At time $t$, I define the marginal propensity to consume (MPC) out of liquid assets at household position $(e, b, a)$ as

$$\text{MPC}_t(e, b, a; \Gamma) = \frac{c_t(e, b + \epsilon, a; \Gamma) - c_t(e, b, a; \Gamma)}{\epsilon}$$

(1.15)

for a small $\epsilon > 0$.\(^\text{18}\)

\(^{17}\)Because the mass of households is size one, aggregates of household variables coincide with the mean of household variables.

\(^{18}\)When solved in the discretized state space, the linear approximation of the MPC at point $(e, b_j, a)$ is given by $\text{MPC}(e, b_j, a) = \frac{c(e, b_{j+1}, a) - c(e, b_j, a)}{b_{j+1} - b_j}$ where the liquid asset grid takes the form $\{ \ldots, b_{j-1}, b_j, b_{j+1}, \ldots \}$. 

1.2.2 Production

The consumption good is produced by a representative firm using capital and labor. The firm maximizes

\[ E_0 \sum_{t=0}^{\infty} Q_{0,t} \pi_t \]  

where \( \{Q_{0,t}\}_{t=0}^{\infty} \) is a discount factor and \( \{\pi_t\}_{t=0}^{\infty} \) is the path of dividends distributed to equity owners. The discount factor \( Q_{0,t} \) is given by

\[ Q_{0,t} = \begin{cases} 
1 & t = 0 \\
\Pi_{a=0}^{t} \frac{1}{1+r_a} & t > 0,
\end{cases} \]  

where \( \{r_a\}_{t=0}^{\infty} \) is the path of the illiquid asset return. The firm generates the consumption good using the Cobb-Douglass production function

\[ Y_t = z_t K_{t-1}^{\alpha} L_t^{1-\alpha} \]  

where \( z_t \) is the firm’s productivity at time \( t \), \( K_{t-1} \) is the capital stock chosen in period \( t - 1 \), and \( L_t \) is labor. The budget constraint of the firm is given by

\[ \pi_t + I_t + \Phi(K_t, K_{t-1}) = z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - w_t L_t \]  

where \( w_t \) is the market wage for labor, \( \Phi(\cdot, \cdot) \geq 0 \) is a convex adjustment cost function, and \( \delta \) is the depreciation rate of capital.

Given the path of productivity and the discount factor \( \{z_t, Q_{0,t}\}_{t=0}^{\infty} \), the firm chooses dividends, capital, and labor \( \{\pi_t, K_t, L_t\}_{t=0}^{\infty} \), to maximize (1.16) subject to the discount factor (1.17), given the constraints (1.19) and (1.20). The first order condition with respect to capital and labor are
given by

\[(1 + r_t^a)(1 + \Phi_1(K_t, K_{t-1})) = E_t \left( z_{t+1} \alpha K_t^\alpha L_{t+1}^{1-\alpha} + 1 - \delta - \Phi_2(K_{t+1}, K_t) \right), \quad (1.21) \]

and \( w_t = z_t(1 - \alpha)K_{t-1}^\alpha L_t^{-\alpha}, \quad (1.22) \)

respectively.

The quantity of equity shares is normalized to one. Given the path of the price of equity shares \( \{q_t\}_{t=0}^{\infty} \), the gross return on equity is given by

\[1 + r_t^a = \frac{q_t + \pi_t + \chi_t}{q_{t-1}}, \quad (1.23)\]

Here the illiquid asset adjustment costs are reimbursed with firm profits.\(^{20}\)

Following Hong (2023a), labor is supplied at the aggregate level by a labor union. Taking the market wage \( w_t \) as given, the labor union solves

\[ \max L_t w_t - \kappa \frac{1}{1 + \omega} (L_t)^{1+\omega} \quad (1.24) \]

where \( \kappa > 0 \). This provides the labor supply curve\(^{21}\)

\[ w_t = \kappa (L_t)^{\omega}. \quad (1.25) \]

---

\(^{19}\) Starting with \( q_0 = \frac{q_1 + \pi_1 + \chi_1}{1 + r_1} = \frac{\pi_1 + \chi_1}{1 + r_1} + \frac{q_2 + \pi_2 + \chi_2}{1 + r_2} \) and iterating forward, we find that \( q_0 = \sum_{t=0}^{\infty} \frac{\pi_t + \chi_t}{1 + r_t} \). This reveals that the firm maximizes \( \pi_0 + q_0 + \chi_0 \), the present value of the firm.

\(^{20}\) This removes illiquid asset adjustment costs from the resource constraint so that output follows the standard decomposition into consumption, investment, and the trade balance.

\(^{21}\) This is equivalent to individual households having Greenwood-Hercowitz-Huffman preferences \( u(c, l) = \frac{c^{1-\gamma}}{1-\gamma} - \kappa \frac{1}{1+\omega} L_t^{1+\omega} \).
1.2.3 Prices and Clearing

The interest rate on the bond is given by

\[ r_t = r^* + \mu_t - 1, \quad (1.26) \]

where \( r^* > 0 \) is the steady state interest rate and \( \mu_t \) is an exogenous shock that equals one at the steady state. The realized return on liquid asset holdings at time \( t \) is known one period in advance:

\[ r^b_t = r_{t-1}. \quad (1.27) \]

Given a firm share price of \( q_t \), the clearing condition of the illiquid asset market is given by

\[ A_t = q_t, \quad (1.28) \]

which equalizes the value of equity shares held by households and the market value of the firm. The trade balance is given by

\[ TB_t = B_t - (1 + r_t)B_{t-1} \quad (1.29) \]

and the trade balance to output ratio is given by

\[ TBY_t = TB_t / Y_t. \quad (1.30) \]

Integrating over the household budget constraint (1.2) with respect to \( \Psi_t \), applying the illiquid asset clearing condition (1.28), and the definitions of the trade balance (4.20) and investment (1.20) provides the aggregate resource constraint

\[ Y_t = C_t + I_t + \Phi_t + TB_t, \quad (1.31) \]

which decomposes output into consumption, investment, and the trade balance.
1.2.4 Equilibrium

**Decentralized Equilibrium.** Given the path of productivity and the deviation of the interest rate \( \{z_t, \mu_t\}_{t=0}^{\infty} \), a decentralized equilibrium is a path of prices \( \{w_t, r_t^b, r_t^a, q_t\}_{t=0}^{\infty} \), a path of household policies \( \{c_t(e, b, a; \Gamma), b_t(e, b, a; \Gamma), a_t(e, b, a; \Gamma)\}_{t=0}^{\infty} \), a path of the distribution of households \( \{\Psi_t\}_{t=0}^{\infty} \), and a path of quantities \( \{C_t, A_t, B_t, \chi_t, Y_t, \pi_t, K_t, I_t, TB_t, TBY_t\}_{t=0}^{\infty} \) such that, given \( \Gamma = \{w_t, L_t, r_t^b, r_t^a\}_{t=0}^{\infty} \):

1. \( \{c_t(e, b, a; \Gamma), b_t(e, b, a; \Gamma), a_t(e, b, a; \Gamma)\}_{t=0}^{\infty} \) satisfy conditions (1.2), (1.6) - (1.8).

2. The distribution \( \{\Psi_t(e, b, a; \Gamma)\}_{t=0}^{\infty} \) follows the law of motion given by equation (4.17).

3. \( \{Y_t, \pi_t, K_t, w_t, L_t, I_t, TB_t, TBY_t\}_{t=0}^{\infty} \) satisfy (2.1), (1.19), (1.21), (1.22), (1.25), (1.20), (4.20), and (4.21) respectively.

4. \( \{C_t, A_t, B_t, \chi_t\}_{t=0}^{\infty} \) are given by the aggregation equations (1.11) - (1.14).

5. The interest rates \( \{r_t, r_t^b\}_{t=0}^{\infty} \) are given by equations (1.26) and (1.27), respectively, and the illiquid asset return \( \{r_t^a\}_{t=0}^{\infty} \) is given by (1.23).

6. The illiquid asset clearing condition (1.28) is satisfied.

1.2.5 Solution Method

I solve for the solutions to perfect foresight shocks using the methods and toolkit developed in Auclert et al. (2021b). In every exercise, I assume the economy is initially at the stationary steady state that arises with the presence of idiosyncratic income risk. I consider both ‘transitory’ shocks and ‘permanent’ shocks. Under a transitory shock, both \( z_t \) and \( \mu_t \) returns to their initial steady state values. Under a permanent shock, \( z_t \) transitions to a new long run steady state. For the transitory shock, I assume the economy returns to the stationary steady state within \( T = 400 \) periods. For the permanent shock, I compute the new long run steady state and compute the transition path to the new steady state. Given \( \{z_t, \mu_t\}_{t=0}^{T} \), I solve for the equilibrium by iterating.
on the path of the illiquid asset return \( \{r_t^a\}_{t=0}^T \) to satisfy the illiquid asset market clearing condition (1.28). To inspect the household problem, I compute the path of the distribution of households \( \{\Psi_t(e, b, a; \Gamma)\}_{t=0}^T \) and household policies \( \{c_t(e, b, a; \Gamma)\}_{t=0}^T, \{b_t(e, b, a; \Gamma)\}_{t=0}^T, \{a_t(e, b, a; \Gamma)\}_{t=0}^T \) conditional on the path of general equilibrium household inputs \( \Gamma = \{w_t, L_t, r_t^b, r_t^a\}_{t=0}^T \).

1.3 Functional Forms, Calibration, and Steady State

This section describes the model’s functional forms, calibration, and steady state. I use standard household preferences and functional forms. I calibrate the model at the steady state using a combination of parameters from the literature and targets from the household data. I then characterize heterogeneity at the steady state.

1.3.1 Functional Forms

This section describes the functional forms of household preferences and the adjustment costs for the illiquid asset and capital. Households have constant relative risk aversion (CRRA) preferences over consumption given by

\[
u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{1.32}\]

where \( \gamma > 0 \) is the inverse intertemporal elasticity of substitution.

Adjustment costs of the illiquid asset are given by

\[
\chi(a_t, a_{t-1}) = \frac{\chi_1}{2} \left( \frac{a_t - (1 + r_t^a)a_{t-1}}{(1 + r_t^a)a_{t-1} + \chi_0} \right)^2 ((1 + r_t^a)a_{t-1} + \chi_0), \tag{1.33}
\]

where \( \chi_0 > 0 \) and \( \chi_1 > 0 \). Parameter \( \chi_0 \) ensures adjustment costs are well-defined for \( a_t = a_{t-1} = 0 \). Equation (1.33) represents a growing standard in two asset HA models, and originates from Auclert et al. (2021b)’s discrete time implementation of the adjustment costs presented in Kaplan et al. (2018).

---

22Because the interest rate does not vary with debt, like in Schmitt-Grohé and Uribe (2003) and García-Cicco et al. (2010), I can avoid iterating on \( r_t \) because it is effectively exogenous. Secondly, I directly clear the labor market by substituting the labor supply condition (1.25) into the representative firm’s labor demand condition (1.22).
Adjustment costs of the capital stock are given by

\[ \Phi(K_t, K_{t-1}) = \frac{\phi}{2} K_{t-1} \left( \frac{K_t}{K_{t-1}} - 1 \right)^2, \]  

(1.34)

where \( \phi \geq 0 \).

1.3.2 Calibration

This section presents the calibration of the model. The time unit is one year. Parameters can be placed in two groups: parameters that are set externally and internally calibrated parameters.

Table 1.1 displays the set of parameters that are set externally. I set the inverse intertemporal elasticity of substitution \( \gamma \) to a standard value of 2. I set \( r^* \) to 5% per annum, a standard annual value for emerging markets such as Mexico. I draw the idiosyncratic income process from Villavalvoz (2023) estimate for Mexico, which provides \( \rho_e = 0.91, \sigma_e = 0.18 \). The depreciation rate \( \delta \) is set to 10% per year. Following the widely used parameters of García-Cicco et al. (2010), I set the capital share \( \alpha \) to 0.32 and the elasticity of labor supply \( \omega \) to 0.60.

Table 1.2 describes the set of parameters that are internally calibrated. I normalize \( \kappa \) to 1.86 so that the aggregate labor supply \( L_{ss} \) is equal to one at the steady state. Within the household problem, I jointly calibrate \( \beta \) and \( \chi_1 \) so that 60% of households are constrained and households display an average MPC of 0.55.\(^{23}\) This provides \( \beta = 0.89 \) and \( \chi_1 = 2.73 \), respectively.\(^{24}\) Finally, I solve for the steady state illiquid asset return \( r_{ss}^a \) that clears the illiquid market, which provides \( r^a = 0.078 \). In the baseline model, I set capital adjustment costs to zero, \( \phi = 0 \). In RA models such as García-Cicco et al. (2010) or Uribe and Yue (2006) this would lead to a dramatic over-response of investment. Such an over-response does not occur in this model because the firm finances investment through equity owned by the households, which faces significant convex adjustment

\(^{23}\)These two targets govern the aggregate dynamics of the model. The percent of constrained households controls how many households display a direct response to interest rate fluctuations. The MPC governs the average response to labor income fluctuations.

\(^{24}\)As is standard, I calibrate a smaller value of \( \beta \) relative to a representative agent benchmark to rationalize households’ observed liquid asset holdings with the large precautionary savings effect generated by idiosyncratic income risk.
costs. As a consequence, the firm investment response inherits the equity adjustment costs that households face.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source / Target</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>2</td>
<td>Standard</td>
<td>Inverse IES</td>
</tr>
<tr>
<td>( r^* )</td>
<td>0.05</td>
<td>Standard</td>
<td>Steady State Interest Rate</td>
</tr>
<tr>
<td>( \rho_e )</td>
<td>0.91</td>
<td>Villalvazo (2023)</td>
<td>Persistence income risk</td>
</tr>
<tr>
<td>( \sigma_e )</td>
<td>0.18</td>
<td>Villalvazo (2023)</td>
<td>Standard deviation income risk</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.10</td>
<td>Standard</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.32</td>
<td>García-Cicco et al. (2010)</td>
<td>Capital share</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.60</td>
<td>García-Cicco et al. (2010)</td>
<td>Labor supply elasticity</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.0</td>
<td>Varies</td>
<td>Capital adjustment costs</td>
</tr>
</tbody>
</table>

Table 1.1. Externally Calibrated Parameters

Notes: The time unit is one year.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source / Target</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa )</td>
<td>1.86</td>
<td>( L_{ss} = 1 )</td>
<td>Labor disutility</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.89</td>
<td>60% Households Constrained</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>( \chi_1 )</td>
<td>2.73</td>
<td>Average MPC 55%</td>
<td>Illiquid asset adjustment costs</td>
</tr>
<tr>
<td>( r_{ss}^a )</td>
<td>0.078</td>
<td>Illiquid Market Clearing</td>
<td>Illiquid asset return</td>
</tr>
</tbody>
</table>

Table 1.2. Internally Calibrated Parameters

I now characterize the steady state of the model that develops in the presence of idiosyncratic income risk and no aggregate risk. In this case, the exogenous variables \( z_t \) and \( \mu_t \) are set to their steady state values \( z_{ss} = 1 \) and \( \mu_{ss} = 1 \), respectively, which produces static values for labor income \( w_{ss}L_{ss} \), the interest rate \( r_{ss} \), and the illiquid asset return \( r_{ss}^a \). Given \( \Gamma_{ss} = \{ w_{ss}, L_{ss}, r_{ss}, r_{ss}^a \} \), solving the household problem with only idiosyncratic risk produces a stationary distribution
This figure characterizes the stationary steady state of the model. The left panel plots the average MPC within each income decile. The right panel plots the percent of liquidity constrained households within each income decile. The MPC at position $(e, b, a)$ is computed as $\text{MPC}(e, b, a) = \frac{c_{ss}(e, b+\epsilon, a) - c_{ss}(e, b, a)}{\epsilon}$ for small $\epsilon > 0$. A household is constrained if $b_{ss}(e, b, a) = 0$. For each panel, the average within each income decile is computed by interpolating along the idiosyncratic income distribution.
Table 1.3. Moments of Income and Wealth Distributions, Model and Data

<table>
<thead>
<tr>
<th>Moment</th>
<th>Income</th>
<th>Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share Top 5</td>
<td>0.11 0.11</td>
<td>0.22 0.60</td>
</tr>
<tr>
<td>Share Top 10</td>
<td>0.21 0.23</td>
<td>0.36 0.72</td>
</tr>
<tr>
<td>Share Bottom 50</td>
<td>0.33 0.45</td>
<td>0.12 0.02</td>
</tr>
<tr>
<td>Share Bottom 20</td>
<td>0.13 0.17</td>
<td>0.02 -0.01</td>
</tr>
</tbody>
</table>

Notes: This table compares moments of the steady state distribution of the model and the empirical data. ENIGH denotes Mexican National Survey of Household Income and Expenditure. WID denotes the World Inequality Database. In the model, income denotes labor income $wL$. Wealth is defined as the sum of liquid and illiquid asset holdings, $b + a$.

\[ \Psi_{ss}(e, b, a; \Gamma_{ss}) \] and household policies \( c_{ss}(e, b, a; \Gamma_{ss}), b_{ss}(e, b, a; \Gamma_{ss}), a_{ss}(e, b, a; \Gamma_{ss}) \).

I now consider how households differ at the steady state along the income dimension. The left panel of Figure 1.1 plots the percentage of constrained households within each income decile. As is explicitly targeted, 60% of households are constrained. The likelihood of being constrained varies significantly with income. Within the poorest two income deciles, 88% of households are constrained compared to 12% in the highest two income deciles. Relative to the data in Section 1.5.4, the model overstates the relationship between income and the likelihood of being liquidity constrained. The right panel of Figure 1.1 plots the average MPC within each income decile. As is targeted, households display an average MPC of 0.55. The MPC displays significant heterogeneity with respect to income. Households in the bottom two deciles of income display an average MPC of 0.84 whereas the top two deciles of income display an average MPC of 0.25.

Only 7% percent of households are constrained in their illiquid asset holdings. From the perspective of the household data in Section 1.5.4, the measure of households that are constrained in illiquid asset holdings can be viewed as generous or conservative. For the most generous definition of illiquid assets that includes durable goods, the portion of constrained households is accurate. For more stringent measures that require housing or financial assets, the distribution understates the quantity of households that possess no illiquid assets.
I now compare inequality within the steady state distribution relative to the data. Table 1.3 compares inequality in income and wealth. I consider the top five, top ten, bottom fifty, and bottom twenty shares. For income, I draw my empirical counterpart from the residualized distribution of after tax income computed using MFL. For wealth, I draw my empirical counterpart from the World Inequality Database, which uses the methodology described in Bajard et al. (2022). In general, the model overstates inequality at the bottom end of the income distribution. The model predicts bottom 50% and bottom 20% income shares of 0.33 and 0.13, respectively, whereas the data provides shares of 0.45 and 0.17, respectively.

Similar to Hong (2023b) and Villalvazo (2023), who feature the same household problem, the model faces difficulty in capturing wealth inequality. I draw wealth shares from the World Inequality Database (WID). Within the model, I define wealth as \( b + a \), the sum of liquid and illiquid asset holdings. The model predicts a top five and top 10 wealth shares of 0.24 and 0.39, respectively, whereas the data provides shares of 0.60 and 0.72, respectively. The model predicts bottom 50% and bottom 20% income shares of 0.11 and 0.02, respectively, whereas the data provides shares of 0.02 and -0.01, respectively.

### 1.4 Results

In this section I evaluate the ability of the transitory and permanent approaches to replicate the Mexican Peso Crisis of 1994. The focus of each exercise is twofold: to capture the heterogeneous consumption responses of households discussed in Section 1.5.3 and the aggregate responses of the economy discussed in Section 1.5.1.

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25 Bajard et al. (2022) impute measures of wealth inequality in Mexico using a cluster of similar countries.

26 Hong (2023a) matches wealth inequality by introducing entrepreneurs that lack income risk or borrowing constraints. I do not include entrepreneurs because there is no way to place them within the income distribution.

27 Because agents hold the vast majority of their wealth in illiquid assets, inequality in net wealth is largely determined by inequality in illiquid assets.
1.4.1 View I: Transitory Shocks

This section studies the ability of the transitory approach to generate a sudden stop. The transitory approach features simultaneous contractionary shocks to the productivity of the representative firm and the external interest rate of the economy. The path of productivity $z_t$ and the interest rate shock $\mu_t$ are given by

$$\log z_t = \rho^z_t d z_0,$$

$$\log \mu_t = \rho^\mu_t d \mu_0,$$

where $0 \leq \rho_z < 1$, $0 \leq \rho_\mu < 1$.

Building the transitory shocks requires choosing paths for productivity and the interest rate, each determined by their initial fluctuations $d z_0, d \mu_0$ and persistences, $\rho_z, \rho_\mu$. I calibrate the shocks in two steps. First, I introduce aggregate risk to calibrate the persistence of the productivity process. I assume productivity $z_t$ and the interest rate $\mu_t$ follow the processes

$$\log z_t = \rho^z_t \log z_{t-1} + \sigma^z \epsilon^z_t, \epsilon^z_t \sim N(0, 1)$$

and

$$\log \mu_t = \rho^\mu_t \log \mu_{t-1} + \sigma^\mu \epsilon^\mu_t, \epsilon^\mu_t \sim N(0, 1),$$

where $\rho^z = 0.62$ and $\sigma^z = 0.019$, calibrated from the Moody’s Baa corporate bond series as a measure of the world interest rate. Next, I linearize the model and jointly calibrate $\rho_z$ and $\sigma_z$ to match the autocorrelation and standard deviation of Mexican GDP. This provides $\rho_z = 0.53$ and $\sigma_z = 0.0166$.

In the second stage, I calibrate the magnitude of the initial productivity and interest rate shocks to match the sudden stop. Table 1.4 displays the calibrated parameters. Regardless of the path of the interest rate, the initial decline in output is completely determined by the initial decline in productivity. Therefore, I calibrate $d z_0 = -0.054$ to match the decline in the cyclical component of GDP of 8.9%. Calibrating the consumption response requires more nuance. The household data, which feature nondurable consumption, displays an average two-year consumption-income elasticity of 0.86. In contrast, the aggregate data, which features durable consumption goods,

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28I use the annual HP filtered series of GDP with a smoothing parameter of 6.25 from 1965 to 2010.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{z0}$</td>
<td>$-0.054$</td>
<td>$dY_0/Y_{ss} = -8.9%$</td>
<td>Initial decline in productivity</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>$0.53$</td>
<td>Mexico GDP autocorr. 1965-2010</td>
<td>Persistence of decline in productivity</td>
</tr>
<tr>
<td>$d\mu_0$</td>
<td>$0.12$</td>
<td>$dC_0/C_{ss} = dY_0/Y_{ss}$</td>
<td>Initial increase in interest rate</td>
</tr>
<tr>
<td>$\rho_\mu$</td>
<td>$0.62$</td>
<td>Moody’s Baa Yield</td>
<td>Persistence of increase in interest rate</td>
</tr>
</tbody>
</table>

**View II : Permanent Shock**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\zeta_0$</td>
<td>$-0.054$</td>
<td>$dY_0/Y_{ss} = -8.9%$</td>
<td>Initial decline in productivity</td>
</tr>
<tr>
<td>$\rho_P$</td>
<td>$0.00$</td>
<td>$dC_0/C_{ss} = dY_0/Y_{ss}$</td>
<td>Persistence of permanent decline</td>
</tr>
</tbody>
</table>

**Table 1.4. Sudden Stop Drivers: View I (Transitory) and View II (Permanent)**

**Notes:** This table describes the aggregate shocks under the transitory and permanent views.

delivers a much larger two-year consumption-GDP elasticity of 1.31 and a one-year elasticity of 1.33. To remain grounded from the perspective of the household data, I target a one-to-one initial response of aggregate consumption relative to output (GDP). Critically, I only target the aggregate consumption response and leave any heterogeneity in household responses completely untargeted. Given the calibration, this provides $d\mu_0 = 0.12$. Relative to the observed increase in the EMBI+ rate for Mexico discussed in section 1.5.2, I view this increase as reasonable.

**Aggregates** I now characterize the response of the model to the transitory shocks at the aggregate level. Figure 1.2 displays the impulse responses of output, aggregate consumption, investment, and the trade balance to output ratio as percentage deviations from their steady state values. Output and consumption display the calibrated decrease of 8.9%. Investment displays an initial decrease of 26.9%, slightly larger than the decrease in the annual cyclical component of 24.7% observed in the data. The trade balance to output ratio displays an initial increase of 3.5%, whereas the annual data displays a trade balance reversal of 5.0%. The exercise successfully generates the

---

29 I compute the ‘aggregate’ elasticities over the HP filtered cyclical component of the annual series.
30 Because labor income is proportional to output, this implies a one to one consumption to labor income elasticity.
31 I compute the cyclical component using the Hodrick-Prescott Filter with a smoothing parameter of 6.25.
32 Here I refer to the cyclical component, as with investment.
Figure 1.2. View I (Transitory): Impulse Responses

Notes: This figure displays the impulse responses of aggregate variables to the transitory shocks described in Table 1.4. Consumption, output, and investment are expressed as percentage deviations from their respective steadystates. The trade balance to output ratio is expressed as percentage point deviations from its steadystate.
signature features of a sudden stop: a recession that features a sharp decline in consumption and investment that coincides with a reversal of the trade balance.

Relative to the Mexican Peso Crisis, the trade balance displays a weak reversal. This also holds in Mendoza (2010)’s simulated sudden stop, Cugat (2019)’s similar sudden stop exercise, and Villalvazo (2023)’s simulated sudden stop. The exercise predicts a weak trade balance response because it targets the weaker consumption response exhibited in the household data rather than the stronger consumption response observed in the aggregate data. In an exercise that targets the aggregate consumption response, the trade balance would display a sharper reversal.

**Heterogeneity**

I now study how households differ in their consumption responses to the transitory shocks. Figure 1.3 plots the average consumption-income elasticity (the elasticity) within each income decile. Households display an average elasticity of 0.78, close to the average elasticity of 0.86 observed in the household data. The lowest income decile displays an elasticity of 0.68. In contrast, the highest income decile displays a larger elasticity of 0.94. This replicates Guntin et al. (2023)’s empirical finding that income is a poor predictor of consumption-income elasticities during sudden stops.

Figure 1.3 shows that the transitory approach can match the consumption responses observed in the data but does not provide a transparent explanation for why low and high income households display similar responses. To study this, I follow the decomposition exercise of Kaplan et al. (2018) and separately input the general equilibrium fluctuations of labor income \( \{ w_t L_t \}_{t=0}^T \), the interest rate \( \{ r_t \}_{t=0}^T \), and the illiquid asset return \( \{ r^a_t \}_{t=0}^T \). The methodology is described in more detail in

---

33Cugat (2019) is similar in that the sudden stop is triggered by contractionary shocks to productivity and the interest rate. We can view Villalvazo (2023)’s simulated sudden stop as conservative because it only features aggregate fluctuations to the interest rate and excludes productivity or endowment fluctuations.

Broadly speaking, each paper and this paper use the resource constraint \( Y_t = C_t + I_t + TB_t \), with the exception that Cugat (2019), and Villalvazo (2023) lack investment. This paper is more similar to Mendoza (2010) in that the trade balance reversal is financed through an overreaction of investment relative to GDP where as Cugat (2019) and Villalvazo (2023) generate the reversal through an overreaction of consumption relative to GDP. Each paper falls short of the observed trade balance reversal in that it either lacks i) the overreaction of investment or ii) the overreaction of consumption.

34To follow the data, I compute the elasticity using a two-year difference in observed consumption and income. Within the model, this coincides with period \( t = -1 \), where the economy is at the steady state, and period \( t = 1 \), one period after the initial impact of the shocks.
Figure 1.3. View I (Transitory): Consumption-Income Elasticities

Notes: This figure plots the two-year consumption-income elasticities in response to the transitory shocks described in Table 1.4.
Section A.4. Because the household problem is nonlinear, I include a separate nonlinearity term that captures the difference between perfect foresight responses to a collection of simultaneous shocks and the sum of consumption responses to individual shocks.

Figure 1.4 plots the decomposition. The contribution of labor income is flat at nearly 0.65 for the first six deciles, after which it declines to 0.52 for the highest income decile. The interest rate generates a small negative consumption response of about -0.1 for the first four income deciles, after which it increases to 0.41 for the highest income decile. The illiquid asset makes a small average contribution of 0.06 across all income deciles, and displays a weak decline in income. The increasing relationship between income and the response to the interest rate overwhelms the decreasing relationship between income and the consumption response to the decline in income. As a consequence, high income households display weakly larger consumption responses than low income households.

Why are high income households less responsive to the decline in labor income? The driver is the relationship between income and the likelihood of having access to financial markets. Low income households are more likely to be constrained in their asset holdings. As a consequence, low income households consume a larger portion of their immediate increase in labor and illiquid asset income. In contrast, high income households are more likely to hold liquid assets. This allows them to better consumption smooth over the partial equilibrium transitory decline in income. Relative to the MPCs presented in Section 1.3.2, the response to labor income displays less heterogeneity. This occurs because the sudden stop generates a persistent more fluctuation of income whereas the MPC computes the response to a one period cash infusion. While constrained households only respond to the immediate change in income, unconstrained households behave more like a permanent income consumer and respond to future changes in income. Because high income households are more likely to be unconstrained, their consumption response to the labor income shock increases relative to their MPC.

The negative contribution of the interest rate is a feature of the two-year consumption-income elasticities. The decomposed one-year consumption-income elasticity, displayed in figure A.1.5, features a strictly contractionary contribution of the interest rate to consumption across all income deciles.
The contribution of the interest rate to the consumption response increases in income. Perhaps surprisingly, the driver of this relationship is the same as that of the labor income shock. Low income households are more likely to be constrained. This introduces a wedge in their Euler equation so that they lack a direct response to the interest rate increase. In contrast, high income households are more likely to be unconstrained. These households feature a standard Euler equation in which consumption varies directly with the interest rate.\footnote{Because we’ve decomposed the consumption response at the household level, any ‘indirect effects’ of the interest rate, like those discussed in Kaplan et al. (2018), are included in the labor income and illiquid asset return terms.} This relationship is supported in two empirical studies. Vissing-Jørgensen (2002) and Havranek et al. (2015) both show that liquid asset holders display larger responses to interest rate fluctuations.\footnote{The two studies differ in that Vissing-Jørgensen (2002) studies asset holders within the United States whereas Havranek et al. (2015) performs a meta analysis of countries with varying levels of financial development.}

The Role of Illiquid Assets Heterogeneity in consumption responses to the illiquid asset return fluctuation are driven by two effects: First, low income households respond more to changes in income generated by the illiquid asset. Secondly, high income households feature stronger absolute monetary income changes generated by the illiquid asset. Because the model features wealthy hand to mouth households, high income households respond significantly to the higher changes in capital income they experience. Conditional on a change in the illiquid asset return, the first effect generates a declining response of consumption with respect to income, whereas the second effect supports an increasing response. In this calibration, the first effect dominates the second effect, which produces a declining consumption response conditional on an illiquid asset return shock. The dominance of the first effect is small, so that the illiquid asset only makes a weak contribution to heterogeneity in responses. See Section 1.4.5 for an analysis that features a more prominent contribution of the illiquid asset.

Relationship with Guntin et al. (2023)’s Credit Tightening View Unlike Guntin et al. (2023) credit tightening view, I have shown that transitory shocks can replicate the sudden stop at both the household and aggregate level. This paper differs in that it uses two shocks, contractionary
Notes: This figure decomposes the two-year consumption-income elasticities within each income decile with respect to labor income, the interest rate, the illiquid asset return, and a nonlinearity term, conditional on the sudden stop. The decomposed responses are computed by separately inputting in the general equilibrium paths of labor income, the interest rate and the illiquid return. The nonlinearity is computed as the difference between the consumption-income elasticity computed using all inputs and the sum of the consumption responses, weighted by the percentage change in income, conditional on each input.
productivity and interest rate shocks, whereas Guntin et al. (2023) uses a single contractionary endowment shock. Within their household problem, Guntin et al. (2023) implement a borrowing constraint of the form
\[ b_t \geq -\theta Y_t^\nu \] (1.37)
where \( b_t \) is the bond position of a generic household, \( \theta > 0 \) scales the borrowing constraint, \( Y_t \) is an exogenous aggregate income endowment and \( \nu \) determines how strongly the constraint contracts with the income endowment.\(^38\) When aggregate income contracts, the borrowing constraint contracts with it, forcing constrained households to deleverage. At the aggregate level, this is helpful as it generates a stronger contraction of consumption and the deleveraging explicitly forces an increase in household savings. At the household level, the approach is problematic because the contraction disproportionately falls on low income households, since they are more likely to be constrained. This amplifies the consumption response of poor households so that the relationship between income and consumption is even more strongly decreasing than with a static constraint.\(^39\) Lastly, because the constraint depends on the aggregate endowment \( Y_t \) and not the individual endowment \( Y_t e_i^t \), households with the lowest levels of income have the same access to borrowing as households with the highest levels of income. This leads to extreme levels of leverage for poor households.

At the stationary steady state that arises when households borrow subject to equation (1.37), the vast majority of low income households are indebted. From a purely empirical perspective, this is problematic because the data in Section 1.5.4 shows that the vast majority of low income households are financially constrained in that they hold neither savings nor debts. The presence of indebted households also has implications for responses to an interest rate shock. If poor households borrow up to a nontrivial collateral constraint, they display negative wealth effects in response to an interest rate increase. This significantly changes the consumption-income elasticity curve

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\(^38\) Similar to this paper, household \( i \) receives individual income \( e_i^t Y_t \) where \( e_i^t \) is an exogenous idiosyncratic income shock.

\(^39\) Here, a ‘static’ constraint denotes constant borrowing constraints and a non-negativity constraint. See Villalvazo (2023) for an HA model that features a non-static collateral constraint. Villalvazo (2023) differs in that collateral constraint depends on individual levels of asset holdings, so that poor households exhibit reasonable leverage levels.
conditional on an interest rate shock, which provides another motivation to use the non-negativity constraint rather than a borrowing constraint that allows for debt.

Lastly, households make their asset decisions at the stationary steady state without knowledge of an impending contraction. If households could anticipate a contraction, they would be less likely to borrow up to the constraint to begin with. This would, however, require coding a global model like that of Villalvazo (2023). I do not use a global model because I need to build a full distribution of income, rather than using binary low and high income states that the majority of global HA models are bound to.

1.4.2 View II: Permanent Shock

![Figure 1.5. View I (Transitory) and View II (Permanent): Consumption-Income Elasticities](image)

**Notes:** This figure displays the average two-period consumption-income elasticity within each income decile under three approaches: a transitory decline in productivity that coincides with a transitory increase in the interest rate, a permanent decline in productivity, and a permanent decline in productivity that coincides the transitory increase in the interest rate from the first approach. Consumption-income elasticities are computed over two periods.

This section studies the ability of a permanent decline in productivity to generate a sudden stop. As with the transitory shocks, I evaluate the ability of the permanent shock to capture the

---

40Mendoza (2010) uses this line of reasoning to explain the infrequent nature of sudden stops.
features of the sudden stop at the household and aggregate level. I consider long run changes to productivity of the form

\[
\log z_t = \log z_{t-1} + \zeta_t
\]

\[
\zeta_t = \rho_P d\zeta_0,
\]

where \(d\zeta_0\) is the initial change in productivity and \(|\rho_P| < 1\) is its persistence. Given \(d\zeta_0\) and \(\rho_P\), productivity \(z_t\) moves to a new long run steady state value that produces a new and unique stationary steady state of the model. To solve for the new long run steady state, I leave all parameters unchanged and solve for the new illiquid asset return that clears the illiquid asset market, \(r_{ass}\). I then solve the perfect foresight path from the initial steady state to the long run steady state to produce a path of all aggregate variables, the distribution of households \(\{\Psi_t\}_{t=0}^T\), and the series of household policies \(\{c_t(e, b, a; \Gamma), b_t(e, b, a; \Gamma), a_t(e, b, a; \Gamma)\}_{t=0}^T\).

Building the permanent shock requires choosing the initial decline in productivity \(d\zeta_0\) and the persistence term \(\rho_P\). Table 1.4 describes the calibrated parameters. As with the transitory shocks, I set the initial decline in productivity to \(d\zeta_0 = -0.054\) to match the observed cyclical decline in output of 8.9%. I then calibrate \(\rho_P\) to target a one-to-one initial consumption to output response. This provides \(\rho_P = 0.00\), similar to Aguiar and Gopinath (2007)’s quarterly persistence of 0.00.\(^{41}\)

**Heterogeneous Responses** Figure 1.5 displays the average consumption-income elasticity within each income decile for the transitory and permanent approaches. As with the transitory approach, the permanent shock succeeds at recreating the consumption responses observed in the data. This expands the lack of heterogeneity in consumption responses conditional on a permanent shock observed in Guntin et al. (2023)’s single asset model to the two asset model. Households display

\(^{41}\)Here I refer to Aguiar and Gopinath (2007)’s estimate in Column IV, Table 4. The estimate of \(\rho_P\) is smaller than that of Guntin et al. (2023)’s because Guntin et al. (2023) generates a direct permanent decline in income. Within the model of this paper, a permanent decline in productivity with \(\rho_P = 0.00\) still generates a persistent decline in the equilibrium path of labor income. This occurs because the marginal product of labor depends on the capital stock. Following a permanent shock, the firm does not unproductively ‘burn’ any capital on the transition to the new steady state. As the capital stock gradually deteriorates to its long run value, the marginal product of labor, and hence labor income, further declines from its initial decrease to its lower long run value.
large consumption responses for a different reason than under the transitory approach. Broadly, the intuition of Aguiar and Gopinath (2007) holds at the household level for all levels of income. Because the decline in labor income is permanent, households know they cannot maintain their current consumption level in the long run. As a consequence, the optimal choice is to immediately decrease consumption with income. This holds for low income households, who are more likely to be constrained, and high income households, who are less likely to be constrained.

Figure A.1.6 decomposes the consumption responses conditional on the permanent shock with respect to labor income and the illiquid asset return. In this case, the contribution of labor income strictly increases with income. This occurs because the long run decline in labor income is larger than the initial decline in labor income, as seen in Figure A.1.7. Constrained households, who would like to consume more, only decrease their consumption by the initial amount that is forced by the initial income decline. Unconstrained households, that behave more like a permanent income consumer and are more likely to be high income, display a larger response because they respond to the long run decline in labor income.

In addition, I plot the consumption responses that arise from introducing both the permanent decline in productivity and the temporary increase in the interest rate. In this case, high income households display too large of a consumption response relative to the data. This occurs because high income households respond simultaneously to their long run decline in labor income and the temporary increase in the interest rate.

**Aggregates**  Up to this point, I’ve shown that both the transitory and permanent approaches can match the consumption responses observed in the household data. From the household perspective, this leaves it ambiguous which type of shock drove the Mexican Peso Crisis. To further differentiate between the views, I study how they differ in their aggregate responses. Figure 1.6 plots the impulse responses of the exogenous drivers, productivity and the interest rate, along with the aggregate responses of consumption, output, investment, and the trade balance to output ratio. All variables are presented as percentage deviations from the initial steady state except the interest
Figure 1.6. View I (Transitory) and View II (Permanent): Impulse Responses

Notes: This figure compares the aggregate impulse responses of the model to the transitory and permanent approaches described in Table 1.4. The transitory approach features a 5.4% decline in productivity with a persistence of 0.62 and a 12% increase in the interest rate with a persistence of 0.62. The permanent approach features a permanent decline in productivity of 5.4%. All variables are in percentage deviations from the initial steady state.
Figure 1.7. View I (Transitory) and View II (Permanent): Comparison with Mexican Peso Crisis

Notes: This figure compares the responses of the transitory and permanent approaches to generate a sudden stop with the observed cyclical components of the Mexican Peso crisis. Each view of the data is normalized by the period preceding the sudden stop. For the data, this coincides with 1994; for the model, this coincides with the steady state. The annual cyclical component is detrended using an HP Filter with a smoothing parameter 6.25. Source: WDI.
rate and trade balance to output ratio which are expressed in percentage points. In the long run, aggregates in the transitory approach revert to their original steady states while aggregates in the permanent approach revert to their lower long run steady state.

As is explicitly calibrated, each approach delivers identical initial responses of consumption and output. The two models differ in their implications for investment and the trade balance. The transitory approach displays a stronger initial decline in investment of 27%, whereas the permanent approach displays an initial decline of 15%. Similarly, the transitory approach displays a stronger trade balance reversal of 3.5%, compared to 0.9% for the permanent approach. Inspecting the resource constraint in equation (1.31), because the initial responses of output and consumption are matched, the initial stronger trade balance reversal under the transitory approach is financed exclusively through its stronger initial investment decrease.

The transitory and permanent views differ in what drives the decline in investment. Under the transitory approach, the increase in the interest rate motivates households to temporarily substitute away from the illiquid asset to the liquid asset. The selloff of the illiquid asset generates a decline in household financing of equity which forces the firm to scale back investment. Because the increase in the interest rate is sharp and brief, investment displays a large but brief initial decline. While the permanent shock generates a permanent decline in investment, it fails to account for the large initial response.

Figure 1.7 compares each approach to the cyclical components observed in the Mexican Peso Crisis. Specific to the Peso Crisis, we can see that output, consumption, and investment quickly revert to their trend. This is better matched by the transitory approach than the permanent approach. As stated before, the transitory approach delivers a weak trade balance response relative to the data because it undershoots the aggregate consumption response.

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42Because I introduce the cyclical component of the crisis, this approach is slightly biased towards the transitory approach in terms of aggregates reverting to their long run trends.
1.4.3 Productivity and Interest Rate Shocks

Section 1.4.1 features simultaneous productivity and interest rate shocks. To build intuition, this section studies the separate contributions of the productivity and interest rate shocks to the aggregate and household responses.

Figure 1.8. Productivity Shock: IRFs and Heterogeneous Responses

*Notes:* This figure studies the responses of the model to the decline in productivity that contributes to the sudden stop. Productivity features an initial decline of 0.054 that reverts to its steady state value with a persistence of 0.53. The left figure plots the aggregate impulse responses of the model. All variables are in percentage steady state deviations from their initial value. The right figure plots the average two-year consumption-income elasticity within each income decile, along with its decomposition.

**The Productivity Shock** This section studies the response of the model to the transitory productivity shock. I consider the transitory productivity shock used to build the sudden stop: a 5.4% decrease in productivity that reverts to its steady state with a persistence of 0.53. The left panel of Figure 1.8 displays the impulse responses. Output displays an initial decrease of 8.9%, driven by a combination of lower productivity and labor usage. Consumption displays an initial decrease of 4.9% and investment displays an initial decrease of 18.9%. Because households do not absorb the entirety of the output decrease, the trade balance to output ratio depreciates by 1.7%. Because the
Figure 1.9. Interest Rate Shock: IRFs and Heterogeneous Responses

Notes: This figure studies the responses of the model to the increase in the interest rate that contributes to the sudden stop. The interest rate features an initial increase of 0.12 that reverts to the steady state with a persistence of 0.62. The left figure plots the aggregate impulse responses of the model. All variables are in percentage steady state deviations from their initial value. The right figure plots the average two-year percentage change in consumption within each income decile, along with its decomposition.
interest rate is exogenous, it is unchanged throughout the productivity shock.\footnote{Models that follow Schmitt-Grohé and Uribe (2003) can feature feedback between productivity shocks and the interest rate.}

I now study how households differ in their consumption responses, conditional on the productivity shock. The right panel of Figure 1.9 plots the average consumption-income elasticity for each income decile and its decomposition. Households display an average elasticity of 0.71.\footnote{The average elasticity of consumption among households does not coincide with the aggregate consumption elasticity because richer households take up a disproportionate share of consumption.} The poorest income decile displays an elasticity of 0.74. The elasticity declines to 0.56 for the highest income decile.

The transitory productivity shock alone fails to replicate the sudden stop. This is not surprising given that a significant portion of households have some ability to consumption smooth. At the household level, I reject the productivity shock as the driver of the sudden stop because the consumption-income elasticity is declining in income and does not match the high consumption-income elasticity observed for high income households. At the aggregate level, consumption does not display a one to one response with income, and the trade balance to output ratio fails to display a reversal. The household level and aggregate failures are tied to each other through the behavior of high income households. Because high income households take up a larger share of aggregate consumption, it will be challenging to generate a large aggregate consumption response so long as high income households use their assets to consumption smooth.

The Interest Rate Shock

This section studies the responses of the model to the transitory interest rate shock. I consider the interest rate increase introduced in the transitory approach: a 12\% percent increase in the interest rate that reverts to the steady state with a persistence of 0.62. The left panel of Figure 1.9 displays the impulse responses. Output does not display an initial response because productivity is unchanged, capital is determined in the previous period, and the labor supply does not feature wealth effects. The increase in the interest rate increases the relative price of current consumption, which motivates households to save. This leads to an initial consumption decrease of 5.1\%, which supports a 6\% appreciation of the trade balance to output ratio. The interest rate
shock leads to a decline in investment of 10.0%.

I now consider how households differ in their responses to the interest rate shock. The right panel of Figure 1.9 displays the two-year percentage change in consumption for each income decile.\textsuperscript{45} Households display an average percentage change in consumption is 0.86%. The lowest income decile displays a consumption increase of 0.38%. In contrast, the highest income decile displays a consumption decrease of 2.81%.

The interest rate shock makes two important contributions. First, aggregate consumption displays variation that is generated independently of the productivity of the firm and hence the labor income that households receive. This is the driver of papers such as Neumeyer and Perri (2005) and Uribe and Yue (2006) that generates an increase in the volatility of consumption relative to output and a countercyclical trade balance.\textsuperscript{46} Secondly, the increase in consumption variation is generated disproportionately by high income households. Both of these contributions play an important role in successfully generating a realistic sudden stop at the aggregate and household level.

1.4.4 The Role of Persistence

This section studies how the heterogeneous responses of consumption vary with the persistence of a transitory aggregate shock. I first consider how the consumption responses change with the persistence of productivity, $\rho_z$. The left panel of Figure 1.10 plots the average consumption-income elasticity to a 1% increase in productivity for four different values of the persistence, $\rho_z = 0.50, 0.70, 0.80, 0.90$. The consumption-income elasticity is computed using the general equilibrium paths of labor income and the illiquid asset return developed under each value of the persistence. As $\rho_z$ increases, the consumption response increases for all households because the present value of the labor income and illiquid asset value fluctuations is higher. Low income households display a weaker increase in consumption relative to high income households. This occurs because low income households are more likely to be constrained in which case they can only re-

\textsuperscript{45}Because income is unchanged in the initial period, the consumption-income elasticity is undefined.

\textsuperscript{46}The contribution of the interest rate to consumption variation is much more involved in this model because it features a significant contribution of direct effects. In a RA model the contribution of interest rates fluctuations to consumption variation is independent of labor income at the first order.
Figure 1.10. Heterogeneous Consumption Responses: The Role of Persistence

Notes: This figure studies how consumption responses differ with the persistence of transitory aggregate shocks. The left panel plots the average one-period consumption-income elasticity within each income decile conditional on a 1% increase in productivity for four values of the persistence of productivity: $\rho_z = 0.50, 0.70, 0.80, 0.90$. The right panel plots the average one-period percentage change in consumption within each income decile conditional on a 1% decrease in the interest rate for four values of the persistence of the interest rate shock: $\rho_\mu = 0.50, 0.70, 0.80, 0.90$. 


spond to the immediate change in income or the value of illiquid assets. High income households, however, display a stronger increase in consumption with respect to the increase in persistence. This occurs because high income households are less likely to be constrained so that they behave more like a permanent income consumer and display a significant response to future changes in income.

I now consider how consumption responses change with the persistence of the interest rate, $\rho_{\mu}$. The right panel of Figure 1.10 plots the average percentage change in consumption for each income decile to a 1% decrease in the interest rate for four values of the persistence, $\rho_{\mu} = 0.50, 0.70, 0.80, 0.90$. For a low persistence of $\rho_{\mu} = 0.50$, low income households display a small consumption increase relative to high income households. This occurs because low income households are constrained and do not display a direct response to the interest rate. As $\rho_{\mu}$ increases, low income households display a small increase in their consumption response and high income households display a significant increase in their response. This occurs because low income households display a weak direct response to both current and future fluctuations in the interest rate. As with the labor income fluctuation, high income households behave more like a permanent consumer and care about future fluctuations in the interest rate.

1.4.5 Sensitivity Analysis

Stronger Indirect Effects In the calibrated sudden stop, the market value of the illiquid asset displays a muted contribution to household consumption responses. I consider a stronger contribution of the illiquid asset by increasing the capital adjustment costs of the firm. I consider three values of capital adjustment costs, $\phi = 2.0, 5.0, 10.0$. As discussed in Alves et al. (2020), stronger capital adjustment costs increase the decline in the value of the firm conditional on an interest rate increase. As in the baseline model, I calibrate the initial increase in the interest rate to match a one to one initial consumption to output response. Conveniently, capital adjustment costs have no bearing on the steady state, so all initial household consumption and asset positions are identical.

47Because constrained households do not hold any assets, they also do not display any wealth effects to the interest rate fluctuation.
I keep the same path of productivity as in Table 1.4. Figure A.1.8 plots the general equilibrium pathway of the illiquid asset return, the value of the firm, and the calibrated increase in the interest rate for each value of $\phi$. Notably, higher capital adjustment costs induce a stronger decline in the value of the firm but also require a lower interest rate increase to match a one to one consumption to output response. Why does this happen? For a given increase in the interest rate, households display their original direct consumption response to the interest rate and also respond to the stronger decline in the value of their equity shares. The stronger decline in the value of equity amplifies the consumption responses so that a smaller increase in the interest rate is necessary to match the consumption response target. Each value of capital adjustment costs $\phi = 2.0, 5.0, 10.0$ lowers the calibrated interest rate increase to $d\mu_0 = 0.11, 0.10, 0.09$, respectively.

Figure A.1.10 decomposes the consumption responses for the middle level of capital adjustment costs, $\phi = 5.0$. Unlike in Figure 1.4, Similar to the contribution of labor income, the contribution of the illiquid asset declines in income. I emphasize that this model understates illiquid asset inequality along two dimensions. First, only 7% of households are constrained in illiquid asset holdings, so the majority of households display a substitution effect to the illiquid asset return. From the perspective of the household data in Section 1.5.4, this is a reasonable approximation for durable goods but a bad approximation for purely financial assets. Second, the model understates total wealth inequality. A model that better captures the wealth shares of high income households would feature stronger wealth effects for high income households and weaker wealth effects for low income households. Within the decomposed responses, this could present as a flat or even increasing contribution of the illiquid asset to consumption responses.

Figure A.1.9 plots the one and two-period consumption-income elasticities for each value of $\phi$. We can see the two-period elasticities are still able to match the consumption responses, despite a smaller calibrated increase in the interest rate. The stronger contribution of the illiquid asset return leads to a flattening effect of consumption responses. This occurs because the illiquid asset return displays a stronger effect for low income households. In addition, because the calibrated increase in the interest rate decreases, the stronger direct response of high income households decreases.
Combined, the stronger illiquid asset contribution and weaker direct contribution further weaken household heterogeneity in responses.

**Consumption Measurement Timing**  An important difference between the model and the data is that ENIGH surveys households in 1994:Q3 and 1996:Q3, a time difference of two years, whereas the time period of the model is one year. Figure A.1.11 plots the one, two, and three-year consumption-income elasticities.

**Smaller and Larger Interest Rate Increases**  This section studies the sensitivity of the main result to the magnitude of the interest rate increase. Figure A.1.12 plots the two-year consumption-income elasticity conditional on the transitory productivity shock described in Section 1.4. I consider three increases in the interest rate, \( dr_0 = 0.06, 0.12, 0.18 \). As \( dr_0 \) increases, low income households display smaller consumption responses and high income households display larger responses.48

1.4.6  Comparison with a Single Asset Model

This section compares the two asset household problem with a single asset household problem. I consider a **bewley_permanent_1977** model where households have the same consumption preferences and satisfy the constraints

\[
c_i^i + b_i^i = (1 + r_b) b_{i-1}^i + c_i^i w_i L_t \tag{1.40}
\]

\[
b_i^i \geq 0 \tag{1.41}
\]

where \( r_b \) is the interest rate, \( w_i L_t \) is the average labor income, and \( c_i^i \) is the idiosyncratic component of household \( i \)'s income. I assume households face the same interest rate, average income, and idiosyncratic income risk as in the stationary steady state of the two asset model. I consider three

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48The smaller response of low income households conditional on a larger interest rate increase is specific to the two-period elasticity and not the one-period elasticity.
calibrations of $\beta$: leaving $\beta$ fixed to its value in the two asset model, and calibrating $\beta$ to match the percent of constrained households or the average MPC of the two asset model, respectively.

Figure A.1.13 compares the steady states of the two and single asset models. The left panel displays the percent of constrained households within each income decile, and the right panel displays the average MPC within each income decile. The fixed $\beta$ model and MPC matching model feature a sharp change in the percent of constrained households around the sixth decile of income. This implies that all households in the top four deciles of income have access to financial markets. The model that matches the percent of constrained households more closely tracks the two asset model, but features an average MPC of nearly one for the first four income deciles.

Figure A.1.14 plots the one and two-period consumption-income elasticities for the single asset model that targets the average MPC. I input the path of the general equilibrium decline in labor income generated during the calibrated sudden stop and allow the initial increase in the interest rate to vary from 0.01 points to 0.15 points, all of which revert to the steady state with a persistence of 0.62. As seen in the one-period elasticities, the single asset model can recover the consumption responses of high income households, but has a unique consumption-income elasticity of one for the low income households. The lowest income decile displays a nearly fixed consumption-income elasticity because nearly all households are constrained and the average MPC is nearly one. We might presume that moving from a two asset model to a single asset model eliminates wealthy hand to mouth households, so that high income households display a lower consumption-income elasticity. However, because they behave less like a hand to mouth household and more like a permanent income consumer, the high income households of the single asset model display a larger response to the interest rate increase. In this case, the latter effect overwhelms the former effect so that high income households actually display a larger consumption-income elasticity than in the baseline calibration of the two asset model. From this perspective we can view the two asset model as being conservative in how it models high income households.

Figure A.1.15 studies how responses of the single asset model vary with the persistence of aggregate shocks. The upper panels display the responses to labor income fluctuations that fea-
ture increasing persistence. I use the general equilibrium labor income pathways produced for aggregate productivity declines that feature increasing persistences of 0.50, 0.70, 0.80, 0.90. This ensures the responses are driven by the different household problems and are not features of the production side of the two asset economy. I ignore the movement of the illiquid asset return which is not featured in the single asset model. We can see the consumption responses of the single asset model to a labor income shock are always decreasing in income because the lowest income decile always displays an elasticity of nearly one, regardless of the increase in the interest rate. This differs from the two asset model which displays an increasing consumption response to a productivity shock for a persistence of 0.90.

1.5 Data

This section presents the data. I characterize four stylized facts. First, I present the frequently studied aggregate data of the Mexican Peso Crisis. Second, I discuss the procyclical nature of interest rates in emerging markets. Third, using the Mexican National Survey of Household Income and Expenditure (ENIGH) dataset, I compute how household differ in their consumption responses during the Mexican Peso Crisis.\textsuperscript{49} Finally, I use the Mexican Family Life Survey (MFL) to show that the likelihood of having access to financial markets increases with income.\textsuperscript{50}

1.5.1 Aggregates: The Mexican Peso Crisis

This section describes the Mexican Peso Crisis from the perspective of the aggregate data. Leading up until the fourth quarter of 1994, the Mexican economy experienced a boom in borrowing and investment. In late December, the Mexican Peso devalued, triggering an outflow of capital and a recession that reached a trough in the second quarter of 1995.

I collect quarterly consumption, GDP, investment, and the trade balance to GDP ratio from the International Monetary Fund International Financial Statistics dataset (hereafter IMF-IFS). All

\textsuperscript{49}I closely follow Guntin et al. (2023)'s computations for Mexico.

\textsuperscript{50}I use MFL because it features balance sheet data rather whereas ENIGH only features expenditure data.
Figure 1.11. Mexican Peso Crisis: Quarterly Aggregates

Notes: This figure plots the evolution of quarterly aggregate GDP, consumption, investment, and the trade balance to GDP ratio in Mexico during the mid 1990s. All variables excluding the trade balance to GDP ratio are in real, log-level, per capita terms. Quarterly population is computed by linear interpolating the log level of the annual population. All variables are normalized by their 1993:Q1 values. Vertical line on 1995:Q2. Source: IMF-IFS
series are in real, per capita terms and seasonally adjusted.51 Figure 1.11 displays the series in log levels, excluding the trade balance to GDP ratio, with each term normalized by the first quarter of 1993. The crisis displays the characteristic features of a sudden stop: an abrupt decline in consumption and GDP that coincides with a reversal of the trade balance. From peak (1994:Q4) to trough (1995:Q2), GDP and consumption display contractions of 9.2% and 11.2%, respectively. Investment displays a much sharper decline of 40%, and the trade balance displays a reversal of 6.4% from -2.3% to 4.1%. Leading up to the crisis, the economy displays a characteristic buildup in GDP, consumption, and borrowing before the sudden stop.52 Figure A.2.2 and A.2.1 of the appendix depict the crisis in terms of quarterly cyclical components and growth rates.

Figure 1.12 displays the annual series. All variables are in annual, per capita log levels, excluding the trade balance to GDP ratio which is in levels. For the annual data, GDP and consumption displays decrease by 8.4% and 12.7% from 1994 to 1995, respectively. Inspecting the quarterly data, annual consumption displays a larger decline than GDP because it remains depressed throughout 1995. Relative to the quarterly data, investment displays a smaller drop of 25.6% and the trade balance to GDP ratio displays a slightly smaller reversal of 5.9%.

1.5.2 Aggregates: Procyclical Interest Rates in Emerging Markets

In this section, I characterize the cyclicality of the interest rate in emerging and developed markets. Using the replication data available in Neumeyer and Perri (2005), I measure the interest rate in emerging markets using the 90 day treasury bill plus the J.P. Morgan EMBI+ spread.53 Figure 1.13 plots the interest rate and cyclical component for several emerging markets, including Mexico, Argentina, Brazil, and the Philippines. We can observe that when output experiences a decline, the interest rate simultaneously increases. This captures the ‘when it rains it pours’ phe-

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51 I compute quarterly population levels by linearly interpolating the log level of annual population levels.
52 Because I consider perfect foresight shocks from the stationary steady state, my model excludes the ‘boom-bust’ that develops in global models such as Mendoza (2010) and Villalvazo (2023).
53 As discussed in Neumeyer and Perri (2005) and Uribe and Yue (2006), because the EMBI spread is in denominated in US dollars, the real rate is computed by subtracting a measure of US inflation. Using measures of the lending and deposit rate from IMF-IFS deliver negative interest rates with a variety of inflation expectation measures due to the inflation experienced during the Mexican Peso Crisis.
Figure 1.12. Mexican Peso Crisis: Annual Aggregates

Notes: This figure plots the evolution of annual aggregate GDP, consumption, investment, and the trade balance to GDP ratio in Mexico during the mid 1990s. All variables excluding the trade balance to GDP ratio are in real, log-level, per capita terms. All variables are normalized by their 1993 values. Vertical line on 1995. Source: WDI.
Figure 1.13. Neumeyer and Perri (2005): Procyclical Interest Rates in Emerging Economies

Notes: This figure characterizes the cyclical component of output and interest rates in emerging markets. Output is seasonally adjusted and detrended using a log-linear trend. For each country, the interest rate measure is the EMBI index of dollar denominated bonds specific to each country. Source: Neumeyer and Perri (2005) replication files.
Figure 1.14. Neumeyer and Perri (2005): Countercyclical Interest Rates in Developed Economies

Notes: This figure characterizes the cyclical component of output and interest rates in emerging markets. Output is seasonally adjusted and detrended using a log-linear trend. Source: Neumeyer and Perri (2005) replication files.
nomenon described in Kaminsky et al. (2004): during bad times the cost of borrowing to maintain consumption has increased.

While most dramatic for the Argentinian recessions, the increase in the interest rate during the Mexican Peso Crisis is clear. In the first quarter of 1994, the rate was below 5% per annum. By the second quarter of 1995, the rate had increased to a peak of 19%. Afterwards, the rate displayed a trough of 6.5% in the third quarter of 1997.

Figure 1.14 plots the interest rate and cyclical component of output for several developed markets, including Australia, Canada, New Zealand, and the Netherlands. Here we can see that when output decreases, the interest rate closely follows with a decrease, and the reverse occurs during an output increase.\(^5\)

As noted in Kaminsky et al. (2004), the interest rate acts as a buffer for output fluctuations: in bad times it is affordable to borrow to maintain consumption and in good times the interest rate increase puts downward pressure on consumption.

1.5.3 Household: Consumption Responses During the Crisis

This section studies how households differ in their consumption responses during the Mexican Peso Crisis. I replicate Guntin et al. (2023)’s finding that households display little variation in consumption responses across the income distribution during the Mexican Peso Crisis. My primary measure is the consumption-income elasticity: the percentage change in consumption with respect to the percentage change in income. I compute consumption-income elasticities using ENIGH, which surveys Mexico biennially from 1992 to 2014 and documents household income and expenditures. For the income measure I include after tax salaries, business income, and transfers. For the consumption measure I include expenditures on food, personal items, and clothing.

Using the 1994 and 1996 datasets, I characterize the responses of consumption and income using the model

\[
X_{it} = \alpha + \beta Y_{it} + \gamma \text{POST}_t \times Z_{it} + \zeta d_{it} + \delta \text{POST}_t \times d_{it} + \epsilon_{it} \quad (1.42)
\]

\(^\text{5}\)We can interpret the lag in the interest rate change as the endogenous response of the monetary policy setter.
where \( X_{it} \) denotes the log consumption or log income of household \( i \) at time \( t \). Following Blundell et al. (2008), the term \( Y_{it} \) includes controls for household size, locality size, and the sex, education, and a quadratic function of the age of the household head. POST denotes whether an observation occurs during 1996. \( d_{it} \) denotes the household \( i \)'s income decile at time \( t \), and \( POST_t \times d_{it} \) denotes the household \( i \)'s income decile, interacted with \( POST_t \). \( POST_t \times Z_{it} \) includes the sex and education of the household head, interacted with \( POST_t \). Using this method, \( \delta \) measures the percentage change of consumption or income for households within each income decile from 1994 to 1996. I compute the consumption-income elasticities across the income distribution by dividing the estimate of the percentage change in consumption for each income decile with the estimate of the percentage deviation in income for each decile. Finally, I compute bootstrapped errors by taking 2000 samples from the household data with replacement, using the sample weights provided in the data.

Figure 1.15 plots average the consumption-income elasticity within each income decile. Notably, the average consumption-income elasticity is nearly one for one with income and displays little variation across the income distribution. For our purposes, the average response of nearly one to one is not remarkable. In fact, in the aggregate time series, the cyclical component of consumption moved more than one for one with the cyclical component of GDP.\(^{55}\) To the extent that the household data can replicate the aggregate data, we would expect to see a similar average response.

Figure 1.15 motivates this paper from the perspective of the household data. Why do households display little heterogeneity in consumption responses? From a certain perspective, Figure 1.15 supports the representative agent framework: households follow similar consumption policies and can be abstracted into a single representative household. Empirically, this is rebutted by Hong (2023b), who documents heterogeneity in the marginal propensity to consume (MPC) across the income distribution using the methodology presented in Blundell et al. (2008). Hong (2023b)'s computation and Figure 1.15 differ in that Hong (2023b) studies the response to an identified small increase in liquid assets, whereas the aggregate data suggests that Figure 1.15 is almost certainly

\(^{55}\) This holds for both peak (1994:Q4) to trough (1995:Q2) and from the 1994 sample of ENIGH (1994:Q3) to the 1996 sample of ENIGH (1996:Q3).
Figure 1.15. Mexican Peso Crisis: Consumption-Income Elasticities

Notes: This figure plots the average consumption-income elasticity within each income decile from 1994 to 1996. Data is drawn from the 1994 and 1996 samples of ENIGH. The heterogeneous changes of consumption and income for each income decile are identified using equation (1.42) and residualized by household size, locality size, and the sex, education, and a quadratic function of the age of the household head. Standard errors are computed using 2000 bootstrap replications. Figure A.2.4 displays the separate heterogeneous responses of consumption and income.
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*Note:*  \(^p<0.1; \ ^{**}p<0.05; \ ^{***}p<0.01\) Standard Errors in Parentheses

**Table 1.5. MFL: Income and Access to Financial Markets**

**Notes:** This table characterizes the relationship between income and access to financial markets. Data is drawn from the 2005 wave of the Mexican Family Life (MFL) Survey. Access to financial markets is indicated by whether a household possesses a given type of asset. Assets include formal savings, formal debts, informal savings, and informal debts, categorized in Tables (A.3) and (A.4). A household holds formal liquid assets if it holds either formal savings or formal debts. Informal liquid assets is similarly defined.

The relationship between income and access to financial markets is measured using the probit model of equation (1.43), which includes a set of income quartiles and a set of controls. Income includes salaried and business income. The set of controls includes the sex, education, a quadratic function of the age of the household head, and controls for the state of residence.

 driven by multiple large shocks.

1.5.4 Household: Heterogeneity in Access to Financial Markets

This section characterizes the financial environment of households in Mexico using the Mexican Family Life Survey (MFL). Similar to Allen et al. (2016), I show that the likelihood of having access to financial markets is increasing in income.

MFL underwent three waves in 2002, 2005-2006, and 2009-2012 and surveys households on their income, liquid assets, and illiquid assets. I use the 2005-2006 wave which includes 5785 households. I include households that have a household head between 25 and 60 years of age and earn some income over the year. The first restriction reduces the sample to 4363 households;
<table>
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<th>Durable</th>
<th>Property</th>
<th>Animal</th>
<th>Financial</th>
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<td>2nd Income Quartile</td>
<td>0.405***</td>
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<td>2946</td>
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</tbody>
</table>

*Note:* "p<0.1; **p<0.05; ***p<0.01
Standard Errors in Parentheses

Table 1.6. MFL: Income and Illiquid Assets

Notes: This table characterizes how households differ in their possession of illiquid assets. Data is drawn from the 2005 wave of the Mexican Family Life (MFL) Survey. Illiquid assets are classified into durable goods, property, animals, and financial assets. A household only owns property if it owns its home outright.

The relationship between income and illiquid asset possession is measured using the probit model of equation (1.43), which includes a set of income quartiles and a set of controls. Income includes salaried and business income. The set of controls includes the sex, education, a quadratic function of the age of the household head, and controls for the state of residence.
the second restriction further reduces the sample to 2946 households. Income includes after tax salaries, wages, piecework, tips, bonuses, and net business income.\textsuperscript{56} Household income is computed by summing over the income of individuals within the household between 25 and 60 years of age.

MFL surveys households about their savings and debts at both the household and individual level. I use data at the individual level because it includes more detailed information on the type of savings and debts. Households hold savings in banks, cooperatives, the \textit{caja solidaria} program, within the household, \textit{afores} programs, with a friend, and other. Debts originate from banks, savings funds, moneylenders, friends, relatives, work, pawnshops, verbal agreements, and government programs. I define a savings or debt as ‘formal’ if it is likely to charge interest.\textsuperscript{57} Formal savings are held at banks, cooperatives, \textit{afores}, and \textit{caja solidaria}. I define formal debts as those from banks, savings funds, moneylenders, and pawnshops.\textsuperscript{58} I define informal savings as those held at the house, with a friend that is not the household head, and at work, and I define informal debts as those from relatives, friends, pawnshops, and verbal agreements. A household has a formal saving if any of its members between the ages of 25 and 60 possess a formal saving, and similarly for formal debts, informal savings, and informal debts. I define a household as possessing formal liquid assets if it possesses formal savings or formal debts, and I similarly define informal liquid assets.\textsuperscript{59} Lastly, I also consider how households differ in their possession of illiquid assets, which I categorize into durable goods, property, animals, and financial assets.\textsuperscript{60}

I measure the relationship between income and access to financial markets using the probit

\textsuperscript{56}MFL either reports a participants salary or provides a decomposition of their salary into wages, piecework, tips, etc.

\textsuperscript{57}Within MFL, interest rates are explicitly documented for debts. Rates of return are not documented for savings.

\textsuperscript{58}All debts are originated within the previous 12 months.

\textsuperscript{59}Under this definition, a household that has both debts and savings and a zero liquid wealth position is defined as possessing liquid assets.

\textsuperscript{60}These assets make up the vast majority of the illiquid asset stock. I only document a household as owning housing if it owns its house outright, and exclude machinery because it is owned by a small portion of households. I only consider outright home ownership because MFL does not consider net home value for households that are on a mortgage.
model

\[ Y_i = \alpha + \beta Z_i + \kappa D_i + \epsilon_i \]  

(1.43)

\[ X_i = \begin{cases} 
1 & Y_i \geq 0 \\
0 & Y_i < 0 
\end{cases} \]

where \( X_i \) is an indicator for whether a household possesses a certain asset type, \( Z_i \) is an indicator for a household’s income quartile, \( D_i \) is a vector of controls, and \( \epsilon_i \) is an error term. The vector of controls \( D_i \) includes the sex, education, and a quadratic function of the age of the household head, and controls for the state of residence.

Table 1.5 displays the regression results for savings and debts. The most salient result is that access to financial markets is increasing in income. This holds for both savings and debts regardless of whether they are formal or informal. This result also holds without controls, documented in Table A.6. Table 1.6 displays the results for illiquid assets. Relative to the lowest income decile, higher income deciles are more likely to possess durable goods and financial assets. The likelihood of owning one’s home outright displays small but significant variation with income, and low income households are more likely to own animals.

Tables 1.5 and 1.6 consider liquid and illiquid asset holdings from a purely static perspective. However, the observed inequality in access to financial markets has important implications for household responses to aggregate fluctuations of the economy. Following an income shock, we expect unconstrained households to use their liquid assets to maintain their consumption.\(^{61}\) Because high income households are more likely to be unconstrained, we therefore expect them to display smaller consumption responses to a decline in income relative to low income households. Conditional on an interest rate shock, unconstrained households have an incentive to change their consumption because its relative price has changed. Following the same line of logic, we expect

\(^{61}\)This assumes that constrained households hold zero liquid assets due to a constraint rather than coincidentally. Kaplan et al. (2018)’s model implements this by introducing a wedge between the lending and deposit rates. While harder to implement in discrete time, this is supported by the wedge between lending and deposit rates we observe in the IMF-IFS interest rate series.
high income households to display larger consumption responses to interest rate fluctuations relative to low income households.\textsuperscript{62} Empirically, this is documented by Vissing-Jørgensen (2002) and Havranek et al. (2015) who show that asset holders display larger responses to interest rate fluctuations.\textsuperscript{63}

1.6 Conclusion

What drove the Mexican Peso Crisis of 1994? I examine this question using a heterogeneous agent model that characterizes the financial environment of households in emerging markets. I show that a combination of transitory productivity and interest rate shocks captures the Mexican Peso Crisis from the perspective of both the household and aggregate data. A permanent decline in productivity, which generates a permanent decline in income, can capture household consumption responses but fails to account for the responses of aggregates such as investment and the trade balance.

The presence of an illiquid asset allows the model to better capture the household financial environment in Mexico. When calibrated to an emerging market, the single asset model implies an extremely high MPC for the lowest income decile, so that they display a near one to one response for even temporary declines in income. The presence of an illiquid asset weakens this relationship so that low income households do not display a one to one response to income. This reduces the spread between the consumption responses of low and high income households. In terms of the equilibrium dynamics, the contribution of the illiquid asset is conservative in that the general equilibrium movement of the illiquid asset return makes a small contribution to consumption responses. A model that better captures inequality in asset holdings and equilibrium asset price movements could feature a more significant role of the illiquid asset for high income households.

The Mexican Peso Crisis featured a significant increase in the interest rate. This paper takes the

\textsuperscript{62}An exception to this is if interest rate shocks primarily generate consumption fluctuations through indirect effects such as labor income, like in Kaplan et al. (2018). This will be explicitly allowed and studied in the model.

increase as given and studies the consequences on the household and production sides of the economy. This abstracts from the policy setter that had a motivation to combat high levels of inflation and bring stability to the exchange rate. While the interest rate increase is purely contractionary in this model, a richer model could introduce the exchange rate devaluation and hyperinflation observed during the Mexican Peso Crisis to develop and study a richer policy tradeoff.

It is well known that economies can feature permanent declines in productivity and consumption following a crisis, as documented in Cerra and Saxena (2008). This paper provides an explanation for large consumption declines outside of the context of a permanent decline in the economy. The critical feature is the behavior of high income households. If high income households are able to maintain their consumption during a crisis, then we expect the aggregate consumption response to be small because high income households take up a disproportionate share of aggregate consumption. If income is the only driver, and displays a temporary decrease, high income households exploit their asset holdings to maintain their consumption. This paper introduces one of the signature features of a sudden stop, the procyclical increase in the interest rate, which motivates high income households to decrease their consumption and is largely ignored by low income households. This generates consumption responses that align with the data and aggregate responses that align with those observed during the Mexican Peso Crisis.
Chapter 2: Output Volatility and Borrowing Constraints

2.1 Introduction

A constant point of study is whether emerging markets’ status as borrowers affects their economies. This is studied by introducing a borrowing constraint in the model and calibrating the economy to be near the borrowing constraint on average. Another belief is that emerging markets are subject to ‘stochastic volatility’: stochastic changes in the standard deviation of shocks to the economy. This paper studies the interaction of borrowing constraints and stochastic volatility.

I study this problem in an extension of the borrowing model in Bianchi (2011). Households receive a stochastic endowment in output and decide how much to borrow each period, subject to a borrowing constraint. My novel contribution is to expand the stochastic process of output to include a stochastic output-volatility state that determines the standard deviation of output shocks in the next period. Households are able to observe the stochastic volatility state and adjust their borrowing policies accordingly. For simplicity, I restrict the value of nontradable output to a constant and only allow tradable output to vary. I estimate a stochastic volatility process for tradable output over quarterly Brazilian industrial production.

The inclusion of stochastic volatility leads to several changes in the economy. First, households change their borrowing level significantly in response to changes in the volatility level. The peak response to a one standard deviation increase in volatility is 1.25% of borrowing. The large response is driven by the borrowing constraint and not driven by precautionary savings or changes in the level of output. Secondly, stochastic volatility leads to a 16% higher standard deviation of borrowing. This occurs as households have useful information about the distribution of output in the next period that they respond to. Finally, stochastic volatility leads to a 20% increase in the frequency of financial crises.
The model limits borrowing by the ‘collateral constraint’ that originates from Mendoza (2010), in which borrowing is limited by the tradable denominated value of nontradable and tradable output. The collateral constraint is denominated in tradable terms to capture that borrowing is typically financed in foreign currencies, such as the Dollar or the Euro. The exchange rate is relevant through the collateral constraint as it determines the tradable value of nontradable output.

My simulation of the economy captures responses to changes in volatility by using a global solution method. This differs from perturbation methods that need to approximate the model at order three or higher to capture a direct response to changes in volatility, as in Fernández-Villaverde et al. (2014). In practice, the solution method is similar to that of Bianchi (2011), but the set of exogenous state variables is expanded to include the volatility state. When solving for the equilibrium, this only increases the complexity of the state space and the transition probability matrix of the discretized exogenous processes. Real values such as consumption, borrowing, or the value of collateral only depend the original state and control variables of Bianchi (2011).

I solve a decentralized model that exhibits the pecuniary externality of Bianchi (2011): households do not take into account the effect of their consumption decision on the relative price of nontradables. This changes the borrowing opportunities of other households as the collateral constraint is denominated in tradable output and hence depends on the relative price. In a socially optimal economy, or the planner’s economy, households take into account the effect of their consumption decision on the relative price.

I calibrate the economy to match an emerging market in the overborrowing literature. This requires that households are made impatient so that they are highly leveraged on average. A challenge when calibrating the model is setting the parameter which scales the restrictiveness of the collateral constraint, $\kappa$. Converting to a quarterly model implies that $\kappa$ should increase as borrowing ability is determined by quarterly income and not annual income. The slope of the collateral constraint with respect to borrowing increases proportionally with $\kappa$. As studied in Schmitt-Grohé and Uribe (2021), this allows for multiple equilibria of the economy. In a pessimistic equilibrium, agents perceive a restrictive collateral constraint and borrow little. The small amount of borrowing
induces a low equilibrium relative price of nontradable and confirms the low value of the collateral constraint. In an optimistic equilibrium, agents perceive a high collateral constraint and borrow heavily. The high amount of borrowing inflates the relative price and confirms the high value of the collateral constraint.

I avoid the problem of multiple equilibria by using a simplified version of the long term debt setup from Chatterjee and Eyigungor (2012). The borrowing setup assumes only a portion $\lambda$ of bonds mature in each period, rather than all bonds maturing. As only a fraction of bonds mature, the effect of borrowing on tradable consumption is weakened. This lowers the absolute change in the price level that follows from a change in borrowing. The slope of the collateral constraint with respect to borrowing is reduced so that there is a unique point where agents’ borrowing level intersects the value of collateral.

My first finding is that household borrowing responds significantly to changes in volatility, in excess of the precautionary savings response. Following a one standard deviation increase in volatility, borrowing displays a maximum response of 1.25%, while consumption displays a maximum response of 0.4%. The significant response of the constrained economy occurs due to the collateral constraint. When volatility increases, the probability of receiving a low value of output in the next period increases. To smooth other the drop in output, households need to increase their borrowing level in the next period. In the constrained economy the increase in borrowing is bounded above by the collateral constraint. The constraint prohibits households from increasing their borrowing to smooth over the decrease in output. To reduce consumption risk in the next period, households decrease their borrowing today. This increases consumption in the next period with certainty as households have less debt to pay off. I compute the precautionary savings response by solving an unconstrained economy where borrowing is only bounded by the natural debt limit. Households in the unconstrained economy are not limited by the collateral constraint, so they simply increase their borrowing in the next period to smooth over the output drop. The precautionary savings response of the unconstrained economy is near zero, implying the response of households in the constrained economy follows from the collateral constraint.
I compare the economy with stochastic volatility to an economy with constant volatility that is calibrated to generate a similar unconditional output distribution. Several differences arise between the two economies. First, the economy with stochastic volatility displays significantly more variation in borrowing levels. The increased variation in borrowing follows from households responding to both output shocks and volatility shocks, as opposed to just output shocks. Intuitively, the volatility state provides useful information about the distribution of next period output values. When the volatility level shifts, households adjust their borrowing function to take into account the higher or lower risk of output drops in the next period. The borrowing policy becomes more conservative with an increase in volatility and more aggressive with a decrease in volatility.

Secondly, the economy with stochastic volatility experiences a higher frequency of binding borrowing constraints or ‘financial crises’. This follows from the endogenous response of households to decreases in volatility. A decrease in volatility has two effects on the likelihood of a financial crisis. The direct effect is that strong decreases in output are less likely to occur. This reduces the probability of output decreases that households optimally smooth over using the entirety of their borrowing power. Also, as the collateral constraint depends on the value of tradable output, there is less likely to be a severe decrease in the value of collateral that induces a binding constraint. The indirect effect is that households endogenously increase borrowing when volatility decreases. The indirect effect overwhelms the direct effects so that the frequency of financial crises increases from five to six crises for every one hundred quarters.

My paper relates to the literature on time varying volatility in emerging markets. The use of stochastic volatility is most similar to that of Fernández-Villaverde et al. (2014), who introduce a stochastic volatility process for the interest rate and input it into the model as an exogenous process. They find that increases in interest rate volatility induce contractions in output, labor, hours, and investment. As my model takes a given exogenous output process, only responses of borrowing and consumption can be studied. Like Fernández-Villaverde et al. (2014), I find a contractionary response of borrowing and consumption.

My paper relates to the literature on overborrowing and the pecuniary externality. I find that
stochastic volatility increases the frequency of financial crises by about 15%. In the socially optimal economy that does not experience the pecuniary externality, stochastic volatility does not increase the frequency of financial crises. This implies that if an emerging market is able to tax borrowing optimally, the economy can experience stochastic volatility without suffering an increase in financial crises. If emerging markets fail to correct for the pecuniary externality, the frequency of financial crises will expand.

The rest of the paper is organized as follows. Section 2.2 describes the model and defines an equilibrium. Section 2.3 describes the estimation of the stochastic volatility process and the calibration of the model. Section 2.4 computes impulse responses to an increase in volatility. Section 2.5 studies second moments and the incidence of financial crises. Section 2.6 compares the decentralized and socially optimal economies. Section 2.7 concludes.

2.2 Model

This section presents the setup of the output processes and the constrained economy. I present two output processes which drive the economies. Both processes are stochastic in tradable output, but differ in whether the standard deviation of innovations to output is stochastic. The ‘stochastic volatility’ process specifies a stochastic process for the standard deviation of output innovations. The ‘no-stochastic volatility’ process sets the standard deviation of output innovations to a constant.

2.2.1 The Output Process

I model an exogenous output process in which the standard deviation of output innovations is stochastic. The process includes the output state $y_t^T$ and volatility state $\sigma_t$. The volatility state $\sigma_t$ follows an AR(1) process with mean $\bar{\sigma}$, autocorrelation $\rho_\sigma$, and standard deviation of innovations $\sigma_\nu$. Log tradable output follows an AR(1) process with autocorrelation $\rho_y$ and standard deviation
of innovations $\exp(\sigma_t)$. The process can be written as:

$$\log y^T_t = \rho_y \log y^T_{t-1} + \exp(\sigma_{t-1})\epsilon_t \quad (2.1)$$

$$\sigma_t = (1 - \rho_\sigma)\bar{\sigma} + \rho_\sigma \sigma_{t-1} + \sigma_\nu \nu_t \quad (2.2)$$

$$\epsilon_t, \nu_t \sim \mathcal{N}(0, 1)$$

The shocks $\epsilon_t, \nu_t$ are independent of one another. I label $\epsilon_t$ as the ‘output shock’ and $\nu_t$ as the ‘volatility shock’. The output shock directly impacts $y^T_t$ and has no effect on $\sigma_{t-1}$ or $\sigma_t$. The $\nu_t$ shock affects $\sigma_t$ directly and affects $y^T_{t+1}$ by scaling the effect of the output shock $\epsilon_{t+1}$. I denote the parametrization by $\theta = (\rho_y, \bar{\sigma}, \rho_\sigma, \sigma_\nu)$.

Section 2.5 compares an economy which receives an endowment according to equations (2.1) - (2.2) to an economy with a constant standard deviation of output innovations. The stochastic process for the constant volatility economy can be written as

$$\log y^T_t = \rho_y^{NSV} \log y^T_{t-1} + \exp(\sigma^{NSV})\epsilon_t \quad (2.3)$$

$$\epsilon_t^1 \sim \mathcal{N}(0, 1)$$

where ‘NSV’ denotes ‘no-stochastic volatility’. Note that the standard deviation of innovations $\exp(\sigma^{NSV})$ is constant.

2.2.2 Decentralized Economy

I now present the decentralized economy. Consider a small open economy which receives an endowment in nontradable output $y^N$ and tradable output $y^T$. Nontradable output is set to constant $y^N = 1$ and tradable output is given by equations (2.1) - (2.2). A continuum of identical households have preferences over consumption of tradable goods $c^T$ and consumption of nontradable goods $c^N$. The relative price of nontradable goods with respect to tradable is $p^N$. Households can borrow from a long term bond $d_{t+1}$ which matures in the next period with probability $\lambda$. The relative price

---

1The output shocks $\epsilon_t$ of equation (2.3) are independent of the output shocks in equation (2.1).
of the long term bond with respect to tradable is \(q\). The one period interest rate is \(r\). Households borrow subject to the collateral constraint \(d_{t+1} \leq \kappa(p_N^T y_t^N + y_t^N)\) which is proportional to the tradable value of output. The multiplier \(\kappa\) scales the restrictiveness of the collateral constraint. I use the convention that \(d_{t+1} \geq 0\) denotes being indebted or taking on debt.

Households maximize utility

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^T, c_t^N) \tag{2.4}
\]

subject to the intertemporal budget constraint

\[
c_t^T + p_t^N c_t^N + d_t(\lambda + (1 - \lambda)q) = y_t^T + p_t^N y_t^N + d_{t+1}q \tag{2.5}
\]

and the collateral constraint

\[
d_{t+1} \leq \kappa(p_N^T y_t^N + y_t^N) \tag{2.6}
\]

The budget constraint is denominated in tradable goods and equalizes expenditures on current consumption and payment of last period’s debt with income derived from the output process and borrowing through the bond. Expenditures on last period’s debt includes a unit payment for each of the \(d_t\lambda\) bonds that mature, and a remaining indebtedness of \(d_t(1 - \lambda)q\). The household sells \(d_{t+1}\) bonds for income \(d_{t+1}q\).

Households preferences over tradable and nontradable consumption are described by the CES aggregator

\[
c_t = c(c_t^T, c_t^N) = \left[\omega \left( c_t^T \right)^{1-\frac{1}{\xi}} + (1 - \omega) \left( c_t^N \right)^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\xi}}, \tag{2.7}
\]

where \(\omega \in (0, 1)\) and \(\xi > 0\) denotes the elasticity of substitution. Households have CRRA utility over the consumption aggregator as \(u(c) = \frac{c^{1-\gamma}}{1-\gamma}\) where \(\gamma\) is the relative risk aversion.
2.2.3 Optimality Conditions

Households choose nontradable consumption, nontradable consumption, and next period debt to maximize their expected discounted utility subject to the intertemporal budget constraint and the collateral constraint, taking the exogenous process \( \{y_t^T, \sigma_t\} \) as given. We can write the optimality conditions as

\[
\lambda_t = \frac{\partial}{\partial c_t^T} u(c_t^T, c_t^N) \tag{2.8}
\]

\[
p_t^N = \frac{1 - \omega}{\omega} \left( \frac{c_t^N}{c_t^N} \right)^{1/\xi} \tag{2.9}
\]

\[
\lambda_t = \beta \lambda + (1 - \lambda) q E_t \lambda_{t+1} + \mu_t \tag{2.10}
\]

\[
0 = \mu_t \left( \kappa (y_t^T + p_{y_t}^N y_t^N) - d_{t+1} \right) \tag{2.11}
\]

\[
\mu_t \geq 0 \tag{2.12}
\]

Equation (2.8) computes the marginal value of tradable denominated wealth in terms of tradable consumption. Equation (2.9) forms the intratemporal optimality condition between tradable consumption and nontradable consumption as a function of the relative price. Equation (2.10) is the Euler equation which now includes \( \mu_t \), the multiplier of the collateral constraint. When the collateral constraint does not bind and \( \mu_t = 0 \), the traditional Euler equation holds. Further restrictions will lead to \( \frac{\lambda + (1 - \lambda) q}{\xi} = 1 + r \) in equilibrium, so that the Euler equation of the one period bond, \( \lambda_t = \beta (1 + r) E_t \lambda_{t+1} + \mu_t \), also holds. Equations (2.11) and (2.12) require that the collateral constraint is satisfied in equilibrium.

2.2.4 Recursive Formulation

I now present the recursive formulation of the household’s problem. The current state includes the output pair \( y_t = (y_t^T, y_t^N) \), the volatility state \( \sigma_t \), and the current debt level \( d_t \). Note that, as specified in equation (2.1), \( \sigma_t \) affects the standard deviation of \( y_{t+1}^T \) and has no effect on the current output value \( y_t^T \). The state variables can be collected as \( s_t = (y_t, \sigma_t, d_t) \). Household choice
variables are tradable consumption \( c_t^T \), nontradable consumption \( c_t^N \), and borrowing \( d_{t+1} \). The equilibrium is identical to that of Bianchi (2011), but expands the state space to include the output volatility state \( \sigma_t \). The primary technical challenge of the model is the pecuniary externality. In order to capture households not taking into account their individual effect on the relative price \( p^N \), the relative price is determined by debt of other households \( D_t \), rather than individual debt \( d_t \). This imposes a relative price \( p^N(D_t, y_t, \sigma_t) \) which is independent of \( d_t \). The disconnect between individual borrowing \( d_t \) and its equilibrium effect on the price level \( p^N \) is the mechanism that drives the pecuniary externality studied in Bianchi (2011).

The problem can be written as

\[
V(d, D, y, \sigma) = \max_{d', c^T, c^N} u(c(c^T, c^N)) + \beta EV(d', D', y', \sigma')
\]  
(2.13)

subject to

\[
c_t^T + p_t^N(D_t, y_t, \sigma_t)c_t^N + d_t(\lambda + (1-\lambda)q) = y_t^T + p_t^N(D_t, y_t, \sigma_t)y_t^N + d_{t+1}q
\]  
(2.14)

\[
d' \leq \kappa(p^N(D_t, y_t, \sigma_t)y^N + y^T)
\]  
(2.15)

\[
D' = \Gamma(D_t, y_t, \sigma)
\]  
(2.16)

2.2.5 Equilibrium

I now define a decentralized recursive competitive equilibrium for the constrained economy. The equilibrium includes a household optimization condition, a rational expectations condition, and a market clearing condition. The household optimization condition requires that household behave optimally given how they believe other agents will borrow, as determined by the borrowing policy \( \Gamma \). Practically, households must behave optimally given their personal forecast of the price level. The rational expectations condition requires that individual households must actually follow the same borrowing policy as other households, \( d'(d, D_t, y_t, \sigma) = \Gamma(D_t, y_t, \sigma) \).

**Definition 2.2.1. Decentralized Recursive Competitive Equilibrium:**

Given an exogenous process \( \{y_t, \sigma_t\} \) a decentralized recursive competitive equilibrium is a pricing
function \( p^N(D, y) \), a perceived law of motion \( \Gamma(D, y, \sigma) \), and decision rules

\[
\{ \hat{d}(d, D, y, \sigma), \hat{c}^T(d, D, y, \sigma), \hat{c}^N(d, D, y, \sigma) \}
\]

with associated value function \( V(d, D, y, \sigma) \) such that the following conditions hold:

1. Household optimization: \( \{ \hat{d}(d, D, y, \sigma), \hat{c}^T(d, D, y, \sigma), \hat{c}^N(d, D, y, \sigma) \} \) solve the recursive optimization problem described by equations (2.13) - (2.16) given \( p^N(D, y, \sigma) \) and \( \Gamma(D, y, \sigma) \).

2. Rational expectations: the perceived law of motion is consistent with the actual law of motion, \( \Gamma(D, y, \sigma) = \hat{d}(d, D, y, \sigma) \).

3. Market Clearing: \( y^N = \hat{c}^N(d, D, y, \sigma) \) and \( \Gamma(D, y, \sigma) + \hat{c}^T(d, D, y, \sigma) = y^T + D(1 + r) \).

2.3 Calibration

This section describes the calibration of the output processes and the different economies. The output processes include the stochastic volatility process described in equations (2.1) - (2.2) and the constant volatility process described in equation (2.3).

My calibration of the constrained and unconstrained economies follows that of the overborrowing literature. The most challenging parameter to calibrate is the scaling term of the collateral constraint, \( \kappa \), as converting from an annual model to a quarterly model allows for the multiple equilibria studied in Schmitt-Grohé and Uribe (2021). I use a quarterly model despite the possibility of multiple equilibria as stochastic volatility does not appear in annual output data. The multiple equilibria problem will motivate my use of long term debt.

2.3.1 Output Process

I estimate equations (2.1) - (2.2) and (2.3) using quarterly Brazilian Industrial Production, available from the World Bank Global Economic Monitor. The raw industrial production series is monthly, expressed in constant US dollars, and seasonally adjusted. A total of 353 monthly
observations are available from 1991:M1 - 2020:M5. I aggregate the series to the quarterly level as the multiple equilibria problem is worse in monthly calibrations of the model. I exclude the second quarter of 2020 as industrial production displays a sudden drop due to the Covid-19 crisis. The final time series has 117 quarterly observations. The nonlinear nature of the stochastic volatility process requires that it be estimated using a particle filter, like the interest rate process in Fernández-Villaverde et al. (2014). I take 50,000 draws from the particle filter after a burn in period of 25,000 draws. I implement the particle filter using the No-U-Turn algorithm of RSTAN, developed by Hoffman and Gelman (2014).

Table 2.1 lists the prior distribution for each component of $\theta$. For correlation parameters $\rho_y$ and $\rho_\sigma$ I adopt the Beta distribution with mean 0.5 and standard deviation 0.2. For mean volatility $\bar{\sigma}$ I choose a normal distribution centered at $-4.0$ with standard deviation 1.0. It is permissible to use a prior which takes negative values for $\bar{\sigma}$ as the standard deviation of output depends on $\exp \sigma_t$ and not $\sigma_t$.

For $\sigma_\nu$ I pick the Gamma distribution with mean of 0.3 and standard deviation of 1. The prior for standard deviation terms is typically a Gamma-Inverse distribution rather than a Gamma distribution. However, the Gamma-Inverse distribution assumes that $\bar{\sigma}$ is positive. Neither the Gamma or Gamma-Inverse distribution place positive weight on $\sigma_\nu = 0$, but the Gamma Distribution places positive weight on very small values $\sigma_\nu \approx 0$ which represent the case of near constant volatility. Small values of $\sigma_\nu$ would be rejected by the Gamma-Inverse prior, not allowing for the possibility of constant volatility.

The second column of Table 2.1 lists the posterior mean and standard deviation of each parameter. The presence of stochastic volatility is implicitly tested by $\sigma_\nu$ which has mean 0.35 and standard deviation 0.11. The positive mean of $\sigma_\nu$ supports stochastic volatility existing in the data as constant volatility would require no variation in $\sigma_t$, or $\sigma_\nu = 0$. Volatility $\sigma_t$ has a mean of $-3.80$ and standard deviation of 0.14. Auto correlation of volatility has mean of 0.65 and standard deviation of 0.17. The high standard deviation is expected as the model must first estimate the latent variable $\sigma_t$ and then study how it changes over time. Auto correlation of output has mean
0.82 and standard deviation 0.06. Figure B.2.2 of the appendix displays the posterior distributions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>Std.</th>
<th>Mean</th>
<th>Std.</th>
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</thead>
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<td>$\rho_y$</td>
<td>Beta</td>
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<td>0.2</td>
<td>0.82</td>
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<td>$\bar{\sigma}$</td>
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<td>1</td>
<td>-3.80</td>
<td>0.14</td>
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<tr>
<td>$\rho_{\sigma}$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.64</td>
<td>0.17</td>
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<tr>
<td>$\sigma_\nu$</td>
<td>Gamma</td>
<td>0.3</td>
<td>1</td>
<td>0.35</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 2.1. Prior and Posterior Distributions of Stochastic Volatility Process

2.3.2 Model

Table 2.2 presents my calibration of the model. The quarterly interest rate $r$ is set to 0.032. The elasticity of substitution in consumer preferences between tradable and nontradable consumption is set to $\xi = 0.83$. The weight on tradables in the CES aggregator is set to $\omega = 0.31$. The discount factor is set as $\beta = 0.964$ and the collateral constraint multiplier is set to $\kappa = 0.60$. The chance that a bond matures in the next quarter is $\lambda = 1/40$. The calibrations of $\omega$ and $\xi$ are drawn from Bianchi (2011). The calibrations of $\beta$ and $r$ are drawn from quarterly flow-collateral constraint economy described in Chapter 12 of Schmitt-Grohé and Uribe (2017).

The most challenging parameter to calibrate is the multiplier $\kappa$, which determines the restrictiveness of the collateral constraint. For low values of $\kappa$, the level of the collateral constraint is smaller and households are more likely to borrow up to the constraint. By scaling the collateral constraint up and down, the parameter $\kappa$ therefore governs the frequency of a binding collateral constraint, or a ‘financial crisis’. Annual models choose $\kappa = 0.3$ to target an annual crisis frequency of about 5%. 

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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>0.032</td>
<td>Quarterly Interest Rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2</td>
<td>Risk Aversion</td>
</tr>
<tr>
<td>$\xi$</td>
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<td>CES Elasticity of Substitution</td>
</tr>
<tr>
<td>$\omega$</td>
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<td>Parameter of CES Aggregator</td>
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<tr>
<td>$\beta$</td>
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<td>Quarterly Discount Rate</td>
</tr>
<tr>
<td>$\kappa^T$</td>
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<td>Collateral Constraint Multiplier</td>
</tr>
<tr>
<td>$\lambda$</td>
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<td>Long Term Debt Parameter</td>
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<tr>
<td>$y^N$</td>
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<td>Restriction</td>
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</table>

Table 2.2. Calibration

*Notes:* The time period is one quarter.

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<tr>
<td>$\sigma_\nu$</td>
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Table 2.3. Calibration of Exogenous Processes
2.3.3 Multiple Equilibria

Figure 2.1 discusses the different types of equilibriums that can appear in the model. The calibrations vary in their value of $\kappa$ and rate at which debt matures, $\lambda$. I introduce changes in $\lambda$ to capture the rate at which debt matures. Changes in $\kappa$ are motivated by changes in the frequency of the model. Each panel plots borrowing and the value of collateral for a specific calibration of the model and a specific fixed value of tradable output. In the simplest equilibrium, there is a unique level that households can borrow up to given the output state. In a high $\kappa$ calibration, agents must borrow heavily to sustain the value of collateral. Otherwise they violate the collateral constraint or starve. In between is the multiple equilibrium, where the nonlinearity of the relative price leads to multiple points where the collateral constraint can bind.

The top left panel plots the collateral value and borrowing level as a function of the current borrowing state $d_t$ with $\kappa = 0.30$ and $\lambda = 1$. This represents the simple case that arises in annual models where the equilibrium is unique. For low levels of borrowing households have slack in the collateral constraint. As households increase borrowing the value of the collateral constraint increases with the relative price, but by less than the increase in borrowing, so that equilibrium slack is reduced. Eventually households reach an upper limit where the level of borrowing equals the value of the collateral constraint. Households cannot borrow past this point.

In the quarterly model borrowing is determined by quarterly income. If an agent can borrow $\kappa \times Y^{Annual}$ using their annual income, they should be able to borrow $\kappa \times 4 \times Y^{Quarterly}$ using their quarterly income. This implies a quarterly collateral constraint parameter of $\kappa^{Quarterly} = 4 \times \kappa^{Annual}$. This directly increases the slope of the collateral constraint $\kappa(p^N(d_t, d_{t+1}, y^T)y^N + y^T)$ with respect to $d_{t+1}$. The top right panel displays an issue that occurs in the quarterly model with $\kappa = 4 \times 0.30 = 1.20$. The new value of $\kappa$ increases the slope of the collateral constraint four fold. To satisfy the collateral constraint, agents must borrow heavily in equilibrium to keep the
exchange rate sufficiently high. Individual agents who exhibit the pecuniary externality do not consciously borrow more to increase the relative price. The effect still holds when output is high and the collateral constraint is not restrictive. Following a restrictive period where agents borrow heavily, they must still borrow heavily in the next period to pay off their debt from the last period.

Lowering the value of $\kappa$ to somewhere between the extremes of 0.30 and 1.20 can induce multiple equilibria. The bottom left panel illustrates how multiple equilibria can arise in the quarterly model with $\kappa = 0.60$. When combined with the nonlinearity of the relative price $p^N$, it is possible to generate two solutions to the binding constraint equation

$$d_{t+1} = \kappa \left( p^N(y_t^T, d_{t+1}, d_t)y^N + y^T \right). \quad (2.17)$$

This is seen in the borrowing level intersecting with the value of the collateral constraint twice. The multiple equilibriums of Schmitt-Grohé and Uribe (2021) are more apparent as they use $\xi = 0.53$ which leads to a stronger curvature in the value of collateral. Multiple equilibriums of the model exist that depend on household expectations of the price level, as studied in Schmitt-Grohé and Uribe (2021). Households that anticipate a low price level perceive a restrictive borrowing constraint and borrow less. The small level of borrowing, however, only confirms the low price of nontradables and leads to a restrictive equilibrium collateral constraint. In this case households borrow before the first intersection of borrowing with the collateral constraint. In contrast, households may anticipate a high price level and predict a less restrictive collateral constraint. The households feel comfortable to borrow more and confirm the high price level. This increases the value of the collateral constraint, confirming that households can borrow more. In this case, households are past the second intersection of borrowing with the collateral constraint.

The bottom right panel shows the effects of using the long term debt model of Chatterjee and Eyigungor (2012). I simplify the setup by prohibiting default, setting the coupon to zero, and requiring that the long term bond not be state contingent. A long term bond matures in the next period with probability $\lambda$. This differs from the one period bond that matures with certainty.
Figure 2.1. Equilibria Types

Notes: This figure plots equilibria for different values of the collateral constraint restrictiveness $\kappa$ and the portion $\lambda$ of bonds that mature in each period.
in Bianchi (2011). Alternatively, we can think of a single long term bond as representing a unit continuum of bonds, of which \( \lambda \) will mature with certainty in the next period. The motivation is to lower the slope of the relative price

\[
p_t^N = \frac{1 - \omega}{\omega} \left( \frac{y_t^T + q d_{t+1} - d_t (\lambda + (1 - \lambda) q)}{y_t^N} \right)^{1/\xi} \tag{2.18}
\]

with respect to \( d_{t+1} \) so that the collateral constraint only intersects the borrowing level twice. The price of long term debt scales \( \partial p_t^N / \partial d_{t+1} \) by \( q \).

I solve for the price \( q \) of a single long term bond using an arbitrage condition. We can think of an international lender who has access to both the one period bond and the long term debt bond. Given \( \lambda \) and the quarterly one period risk free rate \( r \), the no arbitrage condition is

\[
q(1 + r) = \lambda + q(1 - \lambda). \tag{2.19}
\]

This equates the value of purchasing a single long term bond for price \( q \) with the payoff of purchasing \( q \) of the one period bond that matures with certainty. The equilibrium price is then

\[
q = \frac{\lambda}{r + \lambda} < 1. \tag{2.20}
\]

Intuitively, a single long term bond is less valuable than a one period bond as it is less likely to mature in the next period. I set \( \lambda = 1/40 \) so that the average bond matures in 40 quarters, or ten years. This is supported by the dataset constructed in Broner and Schmukler (2013), where the 75th percentile of Brazilian bond maturity is ten years. This provides a bond price \( q = \frac{\lambda}{r + \lambda} = .4386 \). In the intertemporal budget constraint the scalar which multiplies \( d_{t+1} \) is 0.4386 and the scalar which multiplies \( d_t \) is 0.4526. These differ from the multipliers for the bond that matures each period, which are \( \frac{1}{1 + r} \) and 1, respectively.

The ideal is to choose \( \lambda \) to scale the \( d_t, d_{t+1} \) terms by about one-fourth. This would allow for \( \kappa \) to be scaled by 4 while keeping the same slope of the collateral constraint as in the annual model.
Targeting $q = 0.25$ requires $\lambda = 0.0133$, which corresponds to 75 quarters, or 18 years. This is less than the maximum maturity of Brazilian bonds studied in Broner and Schmukler (2013), but for realism I calibrate long term debt to the 75th percentile of maturity.

The bottom right panel illustrates the unique equilibrium that arises from using long term debt. The slope of the collateral constraint has decreased as a given change in borrowing has a weaker effect on tradable consumption and hence the relative price. Like the annual case, the level of borrowing intersects the value of the collateral constraint once, inducing a unique equilibrium.

2.3.4 Discretization

Table 2.4 presents the discretization of the state space. The economy has two exogenous states $y^T$ and $\sigma_t$, one endogenous state $d_t$, and one control variable $d_{t+1}$. I use 21 equally spaced points to approximate log tradable output on the interval $[\ln y^T, \ln \bar{y}^T] = [-.25, .25]$ and 21 equally spaced points to approximate $\sigma_t$ over the interval $[\sigma, \bar{\sigma}] = [\bar{\sigma} \pm 3\text{std}(\sigma)]$, where $\text{std}(\sigma)$ denotes the unconditional standard deviation of $\sigma_t$, as given by equation (2.2). I discretize the exogenous state $(y^T, \sigma)$ by i) discretizing equation (2.2) over the $\sigma_t$ grid and then ii) discretizing equation (2.1) for each value in the $\sigma_t$ grid. Each discretization of equation (2.1) uses the same output grid. I discretize equations (2.1) and (2.2) using the Tauchen method.

I simulate the economy for one million observations and exclude a burn in period of one hundred thousand observations. Simulating the economy entails generating the state series $\{s_t = (y_t, \sigma_t, d_t)\}_{t=0}^{\infty}$ and policy series $\{d_{t+1}(s_t)\}_{t=0}^{\infty}$. The exogenous output-volatility series $(y_t, \sigma_t)$ can be generated independently of the debt series. I simulate the output-volatility series as follows: at time $t$ collect the output state $(y_t, \sigma_t)$. Draw next period volatility $\sigma_{t+2}$ from the discretization of equation (2.2). Draw next period output $y^T_{t+1}$ from the discretization of equation (2.1) for $\sigma = \sigma_t$. This forms the next period output-volatility state $(y_{t+1}, \sigma_{t+1})$. Start at time $t = 0$ and repeat for $t = 1, 2, 3, \ldots$ to generate the series $\{y_t, \sigma_t\}_{t=0}^{\infty}$. Simulating the debt series requires tracking the current level of debt which is input into the debt policy function. At time $t$ fix current state $s_t = (y_t, \sigma_t, d_t)$ and compute policy $d_{t+1}(s_t)$. The next period output-volatility state $(y_{t+1}, \sigma_{t+1})$ is given
<table>
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<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td>( n_y^T )</td>
<td>21</td>
<td>Grid points for ( \ln y^T ), equally spaced</td>
</tr>
<tr>
<td>( n_\sigma )</td>
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<td>Grid points for ( \sigma_t ), equally spaced</td>
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<td>( n_d )</td>
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<td>([\ln y_{\text{min}}^T, \ln y_{\text{max}}^T])</td>
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</tr>
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<td>([d_{\text{min}}, d_{\text{max}}])</td>
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<td>Range for borrowing</td>
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<td>([\sigma_{\text{min}}, \sigma_{\text{max}}])</td>
<td>(\bar{\sigma} \pm 3\text{std}(\sigma))</td>
<td>Range for volatility</td>
</tr>
</tbody>
</table>

Table 2.4. Discretization

Notes: This table presents the discretization of tradeable output \( y^T \), the log standard deviation of output innovations \( \sigma \), and the debt grid \( d \).

exogenously. The next period debt \( d_{t+1} \) is determined by the current debt policy \( d_{t+1} = d_{t+1}(s_t) \).

This forms the next period state \( s_{t+1} = (y_{t+1}, \sigma_{t+1}, d_{t+1}) \). Begin at time \( t = 0 \) and repeat for \( t = 1, 2, 3, \ldots \). We can then collect the debt series as \( \{d_t\}_{t=0}^\infty \).

The left panel of Figure B.2.3 in the appendix displays the distribution of borrowing for the stochastic and constant volatility economies. The upper bound of the debt grid is never reached in the simulation. The lower bound is reached, but only rarely.

2.3.5 Policy Illustration

In this section I demonstrate the effect of an increase in output volatility on borrowing. Recall that the state is of the form \((y_t, \sigma_t, d_t)\). For each panel in Figure 2.2 I fix tradable output \( y_t^T \) and let the current debt \( d_t \) form the x-axis. On the y-axis, I plot the borrowing policy \( d_{t+1}(y_t, \sigma_t, d_t) \) separately for different volatility states. I choose three volatility states \( \exp \sigma_t = 0.01, 0.05, 0.09 \) to capture a reduced, average, and high volatility level. This provides an easy way to compare borrowing policies in terms of \( \sigma_t \) while conditioning on current indebtedness.

The left panel plots policies for \( y_t^T = 0.90 \), a below average level of output. Borrowing is always highest under the low volatility state \( \exp \sigma_t = 0.01 \). This occurs because households...
know the distribution of next period output will have a small standard deviation. Given the small variance, they can borrow more comfortably. Likewise, the borrowing policy for $\exp \sigma_t = 0.05$ is more aggressive than the borrowing policy for $\exp \sigma_t = 0.09$.

For low values of borrowing, the difference in borrowing policies is small. Beginning around 1.0 points of debt, the difference between borrowing policies begins to expand. Eventually the borrowing policies reach the borrowing constraint where the difference between policies is strongest. As the collateral constraint is independent of the volatility level, all of the policies coincide. The collateral constraint is decreasing in the debt state $d_t$ as it deflates the relative price $p^N$. As the borrowing policy for $\exp \sigma_t = 0.01$ is more aggressive, it reaches the borrowing constraint first, followed by the $\exp \sigma_t = 0.05$ and $\exp \sigma_t = 0.09$ policies.

The right panel plots policies for $y_T = 1.10$ and displays similar results as Panel A. The downward sloping collateral constraint has shifted rightward as the value of $y_T$ has increased. The shift is strong enough that the collateral constraint cannot be reached by the discretized debt grid. Differences between policies still emerge for higher values of indebtedness. The increase in the collateral constraint appears dramatic given a 22% increase in the level of output. However, the value of collateral depends endogenously on household perceptions. As $y_T$ has increased, households anticipate a more generous collateral constraint and borrow more. The high value of borrowing increases the relative price of nontradables. This directly increases the collateral value $\kappa(p^N y^N + y^T)$. 
Notes: This figure illustrates borrowing policies for different values of $\sigma_t$. In the left panel, borrowing policies interact with the collateral constraint. In the right panel, borrowing policies do not interact with the collateral constraint.
2.4 Impulse Responses

The previous section shows that changes in output volatility induce substantial borrowing responses by households. However, this depends on households being close to the collateral constraint. If households are patient and not close to the borrowing constraint on average, then we should expect a small response to changes in volatility.

To better approximate responses to volatility, I compute impulse responses in the simulated time series. Part of the response to a volatility increase is purely precautionary: risk averse households know that the variance of the next period’s endowment has increased and save more. In Figure 2.2 this is seen in small but positive differences between borrowing policies for low levels of the debt state. I substitute for the precautionary savings response by solving the unconstrained model described in section (B.1), where household borrowing is not limited by the collateral constraint.

Figure 2.3. IRF Responses to a Volatility Shock
2.4.1 Computation

I use a simulation approach that searches for windows where there is an increase or decrease in volatility. Recall the stochastic volatility process

\[ \log y_t = \rho y \log y_{t-1} + \exp(\sigma_t) \epsilon_t \]  \hspace{1cm} (2.1)

\[ \sigma_t = (1 - \rho) \bar{\sigma} + \rho \sigma_{t-1} + \sigma_\nu \nu_t \]  \hspace{1cm} (2.2)

Within the time series of the discretized model, this implies a value of \( \nu_t \):

\[ \hat{\nu}_t = \frac{\sigma_t - (1 - \rho) \bar{\sigma} - \rho \sigma_{t-1}}{\sigma_\nu}. \]  \hspace{1cm} (2.21)

I capture positive and negative shocks to volatility by identifying periods \( t \) where \( \hat{\nu}_t \in [0.9, 1.1] \) or \( \hat{\nu}_t \in [-1.1, -0.9] \), respectively. Over the simulated time series, I compute the positive and negative impulse responses as

\[ IRF^+_\nu(X, j) = E_t [X_{t+j} | \hat{\nu}_t \in [0.9, 1.1]] - EX_t, \]  \hspace{1cm} (2.22)

\[ and \ IRF^-\nu(X, j) = E_t [X_{t+j} | \hat{\nu}_t \in [-1.1, -0.9]] - EX_t, \]  \hspace{1cm} (2.23)

respectively, where \( EX \) denotes the unconditional expectation of \( X \).

Figure 2.3 plots responses to a one standard deviation increase in volatility for the constrained and unconstrained economies. The paths of \( y^T \) and \( \sigma_t \) are identical for the both economies as the exogenous series \( \{y^T_t, \sigma_t\} \) is independent of the collateral constraint. Volatility displays an initial increase of \( \sigma_\nu \) and then reverts to its mean, following the AR(1) process of equation (2.2). The mean level of tradable output \( y^T_T \) displays a slight increase. This occurs as equation (2.1) describes the law of motion for output in log terms. The mean of \( y^T_t \) depends on the variance of \( \log y^T_t \) which increases with the volatility shock. The maximum response is small, at less than 0.10 percentage points.
Borrowing in the constrained economy displays an initial decrease of about 0.5% and reaches a maximum decrease of 1.20% at $t + 4$, after which it reverts to its mean. Tradable consumption displays a small initial decrease of 0.36 percentage points. The long run response of tradable consumption is positive. This occurs as households deleverage in early periods without minimal change to the average value of output. Eventually the deleveraging pays off so that households can increase consumption. The change in consumption is small relative to the change in borrowing due to the long term debt setup. From the intertemporal budget constraint, the value of tradable consumption is

$$c_t^T = y_t^T + qd_{t+1} - d_t(\lambda + (1 - \lambda)q).$$

The long term debt price $q \approx 0.43 < 1$ scales down the effect of changes in borrowing on the consumption level.

The borrowing response of the unconstrained economy is negligible. This is important as it shows the borrowing response of the constrained economy is not driven by precautionary savings. When solving the model numerically, the unconstrained economy is subject to a constant natural debt limit that can induce similar responses as the collateral constraint. Traditionally households do now borrow near the collateral constraint. However, overborrowing models calibrate households to be highly impatient so that they borrow heavily. In the unconstrained case, households are impatient enough that they borrow up to the constraint. The small response of borrowing suggests the numerical constraint does not matter much in equilibrium.

Figure B.2.4 of the appendix plots responses to a decrease in volatility. Volatility $\sigma_t$ and output $y_t^T$ display symmetric responses. The constrained economy increases borrowing in response to a volatility decrease.
2.5 Moments and Financial Crises

This section studies the effect of introducing stochastic volatility into the model. I compare the economy with stochastic volatility to the economy with constant volatility. The constant volatility economy serves as a benchmark. Two changes occur when stochastic volatility is introduced in the model. First, the economy displays a higher standard deviation of borrowing as households respond to the volatility state. Secondly, the economy experiences a higher incidence of financial crises.

Recall that under the economy without constant volatility, tradable output follows the AR(1) process

\[ \log y_t^T = \rho_y^\text{NSV} \log y_{t-1}^T + \exp(\sigma^\text{NSV}) \epsilon_t, \]  
\[ \epsilon_t \sim \mathcal{N}(0, 1). \]  

The target of the non-stochastic volatility process is to generate the same distribution of tradable output as the stochastic volatility process. In this way, the stochastic and constant volatility economies only differ in their conditional distributions of output, but not their unconditional distribution of output. I set the economy constant volatility to have the same autocorrelation of output as the economy with stochastic volatility and calibrate the constant volatility term \( \sigma^\text{NSV} \) to generate the same unconditional standard deviation of output. The variance term \( \sigma^\text{NSV} \) needs to increase to match the output standard deviation of the stochastic volatility process. This occurs as the stochastic volatility process inherits more variation through the stochasticity of \( \sigma_t \). I use identical borrowing and output grids to simulate the NSV economy.

Figure B.2.7 displays the histogram of the unconditional distributions of output for the stochastic and constant volatility processes. As the processes use an identical output grid, I make a single histogram bin for each unique output value. The unconditional distributions are similar but do not match perfectly. Notably, the process with stochastic volatility visits the tail ends of the output distribution more often.
Table 2.5. Frequency of Financial Crises

Notes: This table displays the frequency of financial crises in each quarter of the decentralized economy with and without stochastic volatility. A financial crisis is defined as when the borrowing constraint binds.

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<td>$y_T &gt; 0.80$</td>
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<tr>
<td>$y_T &gt; 0.90$</td>
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<tr>
<td>$y_T &gt; 0.95$</td>
<td>2.7%</td>
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</table>

Table 2.6. Frequency of Financial Crises with Restricted Output

Notes: This table displays the frequency of financial crises in the decentralized economy with and without stochastic volatility while conditioning on the level of output. A financial crisis is defined as when the borrowing constraint binds.
Table 2.7. Moments of Stochastic and Constant Volatility Decentralized Economies

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<tr>
<td>$\sigma_{dT}$</td>
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<td>$\sigma_{CA}$</td>
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<tr>
<td>$\sigma_{TB}$</td>
<td>0.022</td>
</tr>
<tr>
<td><strong>Correlation with Output</strong></td>
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<tr>
<td>$\rho_{cT,yT}$</td>
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<tr>
<td>$\rho_{d_{t+1},yT}$</td>
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</tr>
<tr>
<td>$\rho_{TB,yT}$</td>
<td>0.222</td>
</tr>
</tbody>
</table>

**Notes**: This table compares moments of the decentralized economy with and without stochastic volatility.
Figure 2.5 compares the quarterly frequency of financial crises for each economy. The stochastic and constant volatility economies experience crises at a rate of six and five for every one hundred quarters, respectively. This corresponds to a 20% increase of the rate in which financial crises occur. A possibility is that the higher incidence of financial crises is driven by the economy with stochastic volatility visiting low output values more often, as seen in Figure B.2.7. To test this, I compare the frequency of financial crises while controlling for the current value of output. Figure 2.6 displays the incidence of financial crises for three restrictions on the value of tradable output. The economy with stochastic volatility still displays a higher frequency of financial crises when restricting that tradable output be above 0.80, 0.90, or 0.95.

The economy with stochastic volatility experiences a higher incidence of financial crises occurs because of the endogenous response of households to a decrease in volatility. Similar to the finding of Fernández-Villaverde et al. (2014), households deleverage in response to an increase in volatility. However, a reverse effect happens when volatility decreases. A decrease in volatility implies that the distribution of tradable output in the next period will be tighter. This decreases, but does not eliminate, the change of receiving a low endowment of tradable output in the next period. Typically a low value of output will pressure households to deleverage today, so that they can borrow more in the next period to smooth over the output drop. As the change of a low value of output is less likely, households face less pressure to save and increase their borrowing.

A decrease in volatility does not necessarily decrease the incidence of financial crises. Although the probability of a crisis inducing output shock decreases, the household increases their borrowing. The household will need to borrow more in the next period to sustain their consumption, putting them closer to the collateral constraint. Secondly, the decrease in volatility is not permanent. If volatility returns to a high state then households should deleverage to hedge the risk of a consumption drop. However, the household just increased their borrowing in response to a low volatility state. Deleveraging in the next period will decrease the tradable consumption level. This places upwards pressure on the household to continue borrowing at high levels, despite a return to a medium or high volatility state.
2.5.1 Moments of the Economies

Table 2.7 displays second moments for the stochastic and constant volatility economies. As was targeted in the calibration, the economies display a similar standard deviation of output. The standard deviation of borrowing is 0.01 points higher in the stochastic volatility economy, an increase of 14%. This occurs as households are also respond to volatility shocks. The standard deviation of the current account and trade balance display smaller increases as they both include tradable output, which is calibrated to have the same standard deviation across output processes. Stochastic volatility leads to a minimal increase in the standard deviation of consumption. This suggests that the welfare costs of stochastic volatility are not large. This is similar to Bianchi (2011)’s finding that the welfare costs of the pecuniary externality are not large.

The second panel displays the correlation of variables with tradable output. Stochastic volatility slightly decreases the correlation of borrowing and tradable output. This occurs as households are also responding to volatility shocks that are uncorrelated with output shocks. The decrease in correlation is small, as output shocks are still the main driver of the economy. The current account and trade balance become slightly more pro-cyclical with stochastic volatility.

2.6 Stochastic Volatility and Optimal Financial Crises

This section compares financial crises in the decentralized economy and the socially optimal economy. The two economies are generally not equivalent as households in the decentralized economy experience the pecuniary externality of Bianchi (2011). In equilibrium this leads them to overborrow and experience a higher frequency of financial crises. Households in the socially optimal economy take into account the effect of their borrowing decision on the relative price $p^N$. Rather than perceiving an exogenous relative price $p^N(D_{t+1}, D_t, y)$ that depends on aggregate borrowing $D_t, D_{t+1}$, the relative price depends on individual borrowing $p^N(d_{t+1}, dt, y)$. This can be interpreted as the planner inputting the rational expectations condition before making their borrowing choice.
Figure 2.4 plots the typical financial crisis for both the decentralized and socially optimal economies. I first find all periods \( t \) where the collateral constraint is binding. For variable \( x \) I draw the window \([x_{t-15}, \ldots, x_{t-1}, x_0, x_{t+1}, \ldots, x_{t+15}]\). Periods \( t - 15 \) to \( t - 1 \) and \( t + 1 \) to \( t + 15 \) capture periods before and after the financial crisis, respectively. For volatility \( \sigma_t \) I compute the mean level over all windows. For other variables I compute the mean percentage deviation from the unconditional mean. I compute paths for volatility, tradable output, borrowing, and tradable consumption.

The first panel plots the path of \( \sigma_t \) for the decentralized and socially optimal economies. Both economies experience an increase in the volatility state that precedes the crisis. The increase in volatility induces the more volatile output shocks at period \( t \) which instigate the crisis. Tradable output is the main driver of financial crises in both economies. Output decreases up to the financial crisis, and displays a sudden drop at the time of the financial crisis. The sudden drop in output induces households to borrow up to the constraint. Borrowing increases in the lead up to the crisis and drops at the time of the crisis. Tradable consumption displays a stronger drop than tradable output as households deleverage at the time of the crisis.

The novel contribution is to study what type of volatility states precede a financial crisis. The typical volatility state that precedes a financial crisis in the decentralized economy is about average. In the socially optimal economy, the average volatility state is 0.10 points higher. The difference is small, less than one-third of the standard deviation of volatility shocks, but captures the consequences of the pecuniary externality. Decentralized households disproportionately experience financial crises in low volatility states during where they should not need to use all of their borrowing power. Interestingly, the distribution of volatility states that precede financial crises is similar to its unconditional distribution. With the introduction of volatility as an observable state variable, households have the opportunity to avoid financial crises. If households were more proactive about using their volatility state,

Financial crises in the decentralized economy are less dramatic on average due to the pecuniary externality. As households overborrow they disproportionately experience financial crises in low
Table 2.8. Frequency of Financial Crises in Decentralized and Planner’s Economies

<table>
<thead>
<tr>
<th>Stochastic Volatility</th>
<th>Decentralized</th>
<th>Planner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>6.01%</td>
<td>1.00%</td>
</tr>
<tr>
<td>No</td>
<td>5.09%</td>
<td>1.08%</td>
</tr>
</tbody>
</table>

Notes: This table compares the quarterly frequency of financial crises in the decentralized and planner economies with and without stochastic volatility. A financial crisis is defined as when the borrowing constraint binds.

volatility states that the planner avoids. This is seen in the left panel of Figure 2.5, which compares the distribution of volatility states that precede a crisis, conditional on a crisis occurring. The low volatility states precede less severe output drops, so that the average crisis is less severe for the decentralized economy.

This does not imply, however, that the decentralized economy is better off than the socially optimal economy. Households in the decentralized economy experience more financial crises induced by high volatility states due to their overborrowing. This is seen in the right panel of Figure 2.5 which compares the absolute frequency of financial crises over one million observations. So while the average decentralized crisis is less dramatic, the decentralized economy still experiences all of the high volatility induced crises that the socially optimal economy experiences.
Figure 2.4. Typical Financial Crisis

Notes: This figure plots the responses of the decentralized and planner economies during a financial crisis. At time $t = 0$, a financial crisis is defined as when the borrowing constraint binds.

Figure 2.5. Financial Crises
2.7 Conclusion

This paper studies a small open economy that experiences stochastic volatility in its output process. Stochastic volatility interacts with the borrowing constraint to induce several changes in the economy. First, the presence of the borrowing constraint induces significant responses of households borrowing to volatility shocks. Households’ endogenous response to volatility shocks lead to a higher standard deviation of borrowing. Stochastic volatility leads to a higher frequency of financial crises in the decentralized economy as households endogenously increase their borrowing in response to a decrease in volatility. In the socially optimal economy which does not overborrow, stochastic volatility does not increase the frequency of financial crises.

The standard deviation of borrowing increases as households have access to a useful output-volatility state which provides information about the distribution of output in the next period. As shown in the paper, households deleverage in response to an increase in volatility that predicts stronger output shocks. In a model with constant volatility, households do not observe the volatility state and therefore cannot deleverage to prepare for the extreme shock. Stochastic volatility does not induce a significantly higher standard deviation of consumption, despite the higher frequency of financial crises. This occurs as the pecuniary externality is not that costly in welfare terms. While the frequency of financial crises increases substantially, the increase is not that costly to households’ welfare.

The empirical challenge of the paper is to find a compromise between tractable annual models with unique equilibriums and higher frequency models that exhibit stochastic volatility in the data. I choose a quarterly model as stochastic volatility is not displayed in annual output data. The quarterly model experiences multiple equilibria under the one-period debt setup of Bianchi (2011) and Schmitt-Grohé and Uribe (2021), even for the conservative calibration of $\xi = 0.83$. I avoid multiple equilibria by using the long term debt setup of Chatterjee and Eyigungor (2012). As only a fraction of bonds mature each period, the link between borrowing and tradable consumption is weakened. This lowers the change of the relative price with respect to borrowing so that there is
a unique borrowing level where the collateral constraint binds. Multiple equilibria are avoided but the compromise is not very satisfying. Financial crises still occur at an inflated rate of about five for every one hundred quarters rather than five for every one hundred years.
Chapter 3: Income Risk in Emerging Markets

3.1 Introduction

Small open economies are more volatile behavior than developed economies. At the household level, this is expressed in a higher marginal propensity to consume (MPC) and portion of financially constrained households, as documented in Hong (2023b), Cugat (2019). At the aggregate level, this is expressed in a higher standard deviation of aggregate consumption relative to output. Additionally, small open economies display occasional ‘sudden stop’ episodes, sharp recessions where asset inflows rapidly reverse despite a depression in output.

This paper studies the ability of household heterogeneity, generated by idiosyncratic income risk, to explain these phenomena. I show that heterogeneity increases the average marginal propensity to consume and the aggregate volatility of consumption. This occurs as income risk generates impoverished households for whom financial frictions, in the form of borrowing constraints, generate nonlinear behavior regardless of the aggregate state. This differs from representative agent (RA) models that require a depressed aggregate state for financial frictions to have a significant effect on consumption behavior.¹ In contrast, income heterogeneity decreases the intensity of sudden stops at the aggregate level. The occurs because income heterogeneity generates high income households that save their transitory increase in income. This places them far from the borrowing constraint that affects impoverished households, allowing them to effectively consumption smooth throughout both output contractions and sudden stops. Because rich households take up a disproportionate share of consumption and savings, their ability to consumption smooth during a sudden stop episode produces a weaker decrease in aggregate consumption relative to the RA model and

¹Mendoza (2010) notes that this explains why RA models with financial frictions can generate occasional dramatic crisis episodes but have similar long run moments.
I start by building a RA small open economy model. Households receive an exogenous aggregate endowment and can borrow on the international market, subject to a collateral constraint that depends on the aggregate endowment. The collateral constraint represents a financial accelerator in which access to liquidity tightens during recessions, studied in Kiyotaki and Moore (1997) and Bernanke et al. (1999). From an emerging markets perspective, the financial accelerator is studied in Mendoza (2002), Mendoza (2010), and Bianchi (2011).

Within the RA model, the collateral constraint generates characteristic sudden stops events that feature a sharp decrease in consumption and a sudden reversal of the trade balance. The trade balance reverses because the collateral constraint binds, forcing households to save, despite the decrease in their income. Because the constraint only occasionally binds, however, the dramatic behavior of sudden stops does not have a significant effect on the unconditional moments of the model, as noted in Mendoza (2010). This leads the RA model to fail to explain excess consumption volatility and the high average MPC of households.

I then introduce income heterogeneity into the household problem. This leads to several differences between the models. At the household level, the HA model displays a higher portion of constrained households and hence a higher marginal propensity to consume. At the aggregate level, the HA model displays a higher volatility of consumption relative to output. This is the natural result of aggregate consumption incorporating the more volatile consumption responses of constrained households. However, the HA model fails to generate a sudden stop as it displays a weaker drop in consumption and a muted response of the trade balance relative to the RA model.

I leverage two assumptions to solve the model globally. I exploit the assumption, specific to small open economies, of a static interest rate. I additionally assume that households do not care

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2Within the HA model, I define a sudden stop as a series of aggregate shocks that generate a sudden stop within a RA model, which can easily match the observed behavior of sudden stops in the style of Mendoza (2010).

3I define a sudden stop as an episode where the collateral constraint binds and the trade balance to output ratio increases by 2%.

4Because the debt or savings position of the economy can float on the world market, we do not need to solve for a market clearing price as in Krusell and Smith (1998).
about the distribution of earnings and asset holdings. This allows me to solve the model globally by simply appending the aggregate income endowment state to the households’ state space. This differs from Reiter (2009) and Auclert et al. (2020) who linearize in aggregates. The method is adaptable to any model where the relevant aggregate states to the household problem are exogenous.

The improvements of the HA model stem from the more frequent interaction of households with the collateral constraint. In the RA model, the constraint occasionally binds during a contractionary aggregate state. In the HA model, the income shock generates a population of households that are poor irrespective of the aggregate state. No differently than an RA household that would borrow in response to a contractionary aggregate income shock, these poor households take on debt to smooth over their temporary decline in income. This leads to an average of 41% of households being constrained in the HA model, whereas households in the RA model are constrained in 14% of periods. From an empirical perspective, this is closer to Cugat (2019)’s empirical estimate of 58% of households being hand to mouth. Constrained households display a high MPC as they wish to consume more but are prohibited by the collateral constraint. These household contribute to an average annual MPC of 55% in the HA model, close to Hong (2023b) empirical estimate of 54.5% to 59.2%.

The improvements raise a serious question of why the HA model fails to generate a sudden stop. Recall that a canonical RA model with a financial accelerator features that all households are constrained at the time of a crisis. This implies that no household in the economy is able to consumption smooth, and all households are forced to increase their savings. Constrained households in the HA model display similar behavior. The income shock, however, generates a population of households that are rich regardless of the aggregate state. Because the income shock is tran-

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5 Even if the distribution were appended to the household’s state space, their optimal policy functions would be independent of it as it does not provide any information about the (completely static) interest rate in the next period.
6 If we were to calibrate the percentage of financially constrained households in the model to match Cugat (2019)’s estimate for Mexico of 58%, this would lead the model’s average MPC to overshoot Hong (2023b)’s empirical estimate. Kaplan and Violante (2022) documents this phenomenon for single asset models that target developed economies.
sitory, these households save a significant portion of their elevated income into the single liquid asset. This places them far from the collateral constraint and allows them to consumption smooth throughout the sudden stop. Within the population of rich households, this is reflected in a lower percentage decrease in consumption relative to income and an increase in borrowing. The ability of rich households to consumption smooth has a significant affect on aggregate consumption as they make a disproportionate share of consumption relative to poor households. As a consequence, the HA model displays a weaker decrease in aggregate consumption relative to the RA model. In addition, the respective increase and decrease in savings of low and high income households nullify each other so that trade balance to output ratio fails to reverse.

3.1.1 Related Literature

This paper contributes to the literature on sudden stops in emerging markets. In contrast to Cugat (2019), and like Villalvazo (2023), I find that household heterogeneity weakens the size of sudden stops. Cugat (2019)’s HA model generates a stronger sudden stop than her RA model because the RA benchmark never faces a collateral constraint. In contrast, no households in my RA model are able to consumption smooth during a sudden stop due to the collateral constraint. My model is driven by income endowment shocks, while Villalvazo (2023) is driven purely by interest rate shocks. Computationally, my model is more similar to Mendoza (2010). Because I use a nonlinear global solution method, sudden stops are anticipated and not generated by large, unexpected shocks. My model differs from Mendoza (2010) in that prices are completely static, which implies that there is no Fisherian debt deflation to initiate or accelerate sudden stops. I interpret this as making my results conservative.

My paper suggests that single asset HA models may understate the magnitude of financial crises relative to RA models. This applies to single asset heterogenous agent models that attempt to explain dramatic aggregate behavior during contractions. This includes Auclert et al. (2021a), where rich households are able to smooth over their decrease in real income following an exchange

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7The pressure to save in Cugat (2019) is driven by a simultaneous increase in the interest rate.
rate depreciation. Additionally, deFerra et al. (2020) features net wealth rich households are not forced to deleverage after an exchange rate depreciation expands their gross debt holdings. Both of these examples focus on decreases in real income or net asset holdings generated by an exchange rate depreciation. However, the core principle is that rich households have acquired liquidity before the crisis that allows them to consumption smooth throughout its duration.

My paper contributes to the growing literature on the effects of heterogeneity in emerging markets. My implications are most similar to Hong (2023a). Namely, I show that financial frictions drive a higher MPC and excess volatility of consumption. This provides a contrast to the interest rate fluctuations of Neumeyer and Perri (2005), and the growth rate shocks of Aguiar and Gopinath (2007), both of which are beyond the scope of this model.

I replicate Guntin et al. (2023)’s finding that single asset heterogenous agent models fail to capture the nearly one to one consumption and income elasticity of rich households during crises. I further show the single asset structure leads to two additional failures at the aggregate level: a weak response of consumption relative to the RA model, and a failure of the trade balance to reverse. Guntin et al. (2023) argue that this supports permanent income shocks as being the driver of steep drops in consumption during crises, rather than financial frictions. However, I argue this is a weakness of the single asset model and not financial frictions. The single asset model imposes that rich households can only save in a purely liquid asset, and saving mechanically places them far from the collateral constraint. From this perspective, the assumption of a single asset is quite restrictive.

The rest of the paper is organized as follows. Section 3.2 details the quantitative models. Section 3.3 describes the calibration. Section 3.4 describes the results and Section 3.5 concludes.

3.2 Model

This section develops the quantitative RA and HA models. Both models receive an exogenous aggregate income endowment $Y_t$ that follows a known stochastic process. Households have access
to a single risk free bond with a constant interest rate $r$. The RA model features a continuum of identical households. The HA model features a continuum of households that are heterogeneous due to an idiosyncratic, uninsurable income shock. The income shock generates a distribution of households with varying levels of wealth and income. Borrowing is limited by a collateral constraint that depends on the aggregate endowment of income.

Within the HA model I denote household terms using lower case letters $x$ and aggregate terms using upper case letters $X$. Throughout the paper, I denote a generic household in the HA economy as household $i$. Within the RA economy, all households follow the same policy functions as they are identical. This implies household terms coincide with aggregate terms. I therefore cast the RA model directly in terms of aggregates $X$. Section 3.2.1 describes the RA model and Section 3.2.2 describes the HA model.

### 3.2.1 RA Model

The economy is populated by a continuum of identical households. The representative household has preferences over infinite consumption streams $\{C_t\}_{t=0}^{\infty}$ given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t).$$  \hspace{1cm} (3.1)

Households satisfy the sequential budget constraint

$$C_t + (1 + r)D_{t-1} = D_t + Y_t$$  \hspace{1cm} (3.2)

where $C_t$ is the choice of consumption, $D_t$ is the quantity of debt held, and $Y_t$ is the aggregate endowment. Borrowing is permitted, denoted by $D_t > 0$, but is subject to the collateral constraint

$$D_t \leq \kappa Y_t$$  \hspace{1cm} (3.3)
where $\kappa > 0$ is a constant. The collateral constraint works as a financial accelerator, studied in Kiyotaki and Moore (1997) and Bernanke et al. (1999). The role of the collateral constraint in generating sudden stops in emerging markets is studied in Mendoza (2010). The dependence of the collateral constraint on an exogenous variable simplifies the dynamics that it generates. There is no debt deflation cycle, like in Mendoza (2010), that reduces the market value of collateral through prices. The lack of prices removes any possible price externalities, like that discussed in Bianchi (2011), that increase the frequency of financial crises. Finally, the lack of endogenous prices removes the possibility of the multiple equilibria, studied in Schmitt-Grohé and Uribe (2021).

The representative household’s recursive problem can be expressed recursively as

$$V(Y_t, D_{t-1}) = \max_{C_t, D_t} u(C_t) + \beta EV(Y_{t+1}, D_t)$$  \hspace{1cm} (3.4)

$$C_t + (1 + r)D_{t-1} = D_t + Y_t$$  \hspace{1cm} (3.5)

$$D_t \leq \kappa Y_t$$  \hspace{1cm} (3.6)

We can write the first order conditions as

$$u'(C_t) = \mu_t + \beta(1 + r)Eu'(C_{t+1}),$$  \hspace{1cm} (3.7)

$$0 = \mu_t(D_t - \kappa Y_t),$$  \hspace{1cm} (3.8)

and $\mu_t \geq 0$,  \hspace{1cm} (3.9)

where $\mu_t$ denotes the inequality multiplier associated with equation (3.3).

3.2.2 HA Model

A continuum of households is indexed by $i \in [0, 1]$. Household $i$ has preferences over infinite consumption streams $\{c^i_t\}_{t=0}^{\infty}$ given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c^i_t).$$  \hspace{1cm} (3.10)
Household $i$ satisfies the sequential budget constraint

$$c^i_t + (1 + r)d^i_{t-1} = d^i_t + e^i_t Y_t$$  (3.11)

where $d^i_t$ is household $i$’s debt position and $e^i_t$ is their idiosyncratic income shock. Rather than receiving the aggregate income $Y_t$, household $i$ receives the personal income $e^i_t Y_t$, where $e^i_t$ is mean one and follows a known stochastic process. Household $i$ can borrow subject to the collateral constraint

$$d^i_t \leq \kappa Y_t.$$  (3.12)

The recursive formulation of household $i$’s problem can be expressed as

$$V(e^i_t, d^i_{t-1}; Y_t) = \max_{c^i_t, d^i_t} u(c^i_t) + \beta EV(e^i_{t+1}, d^i_t; Y_{t+1})$$  (3.13)

$$c^i_t + (1 + r)d^i_{t-1} = d^i_t + e^i_t Y_t$$  (3.14)

$$d^i_t \leq \kappa Y_t.$$  (3.15)

The first order conditions are given by

$$u'(c^i_t) = \mu^i_t + \beta (1 + r) E u'(e^i_{t+1}),$$  (3.16)

$$0 = \mu^i_t (d^i_t - \kappa Y_t),$$  (3.17)

and $\mu^i_t \geq 0,$  (3.18)

where $\mu^i_t$ is the inequality multiplier associated with equation (3.15). I denote the solutions to the above problem by $c(e_t, d_{t-1}^t; Y_t)$ and $d'(e_t, d_{t-1}^t; Y_t)$.
**Distribution and Aggregates** The distribution of households is defined by the CDF function

\[
\Psi_t(e, d) = Pr(e_t \leq e, d_{t-1} \leq d). \tag{3.19}
\]

The distribution \( \Psi \) satisfies the law of motion

\[
\Psi_t(e^*, d^*) = \int_{e, d} \pi(e' < e^*|e, d') I(d'(e, d; Y) < d^*) d\Psi_{t-1}(e, d). \tag{3.20}
\]

Aggregate consumption and savings are defined as the sum of individual household policies:

\[
C_t = \int c(e_t, d_{t-1}; Y_t) d\Psi_t(e_t, d_{t-1}) \tag{3.21}
\]

\[
D_t = \int d(e_t, d_{t-1}; Y_t) d\Psi_t(e_t, d_{t-1}) \tag{3.22}
\]

### 3.2.3 Aggregates and Household Variables

The trade balance is defined as net exports of the consumption good

\[
TB_t = (1 + r) D_{t-1} - D_t \tag{3.23}
\]

and \( TBY_t = TB_t / Y_t \) denotes the trade balance to output ratio. The term \( \text{BIND}^{RA}_t \) denotes whether the RA household is constrained in period \( t \):

\[
\text{BIND}^{RA}_t = I(D_t = \kappa Y_t) \tag{3.24}
\]

where \( I \) is an indicator function. The term \( \text{BIND}^{HA}_t \) denotes the portion of constrained households in the economy:

\[
\text{BIND}^{HA}_t = \int I(d(e_t, d_{t-1}; Y_t) = \kappa Y_t) d\Psi_t. \tag{3.25}
\]
The marginal propensity to consume (MPC) in the HA and RA models are defined as

\[
\text{mpc}^{HA}(e^i, d^i; Y) = \frac{c(e^i, d^i + \epsilon; Y) - c(e^i, d^i; Y)}{\epsilon},
\]

and

\[
\text{MPC}^{RA}(Y, D) = \frac{C(Y, D + \epsilon) - C(Y, D)}{\epsilon},
\]

respectively, for small \(\epsilon > 0\).

3.2.4 Functional Forms and Risk

Households have CRRA utility given by

\[
u(c) = \frac{c^{1-\gamma}}{1-\gamma},
\]

where \(\gamma > 0\) denotes the inverse intertemporal elasticity of substitution. Aggregate endowment risk and idiosyncratic income risk both follow an AR(1) process in logs:

\[
\log Y_t = \rho_Y \log Y_{t-1} + \epsilon_t^Y, \epsilon_t^Y \sim \mathcal{N}(0, \sigma_Y),
\]

and

\[
\log e_t^i = \rho_e \log e_{t-1}^i + \epsilon_t^e, \epsilon_t^e \sim \mathcal{N}(0, \sigma_e)
\]

where \(\mathcal{N}\) denotes the normal distribution, and \(\sigma_Y > 0\) denotes the standard deviation of aggregate endowment innovations.\(^9\)

3.2.5 Equilibrium Definition

This section defines equilibria for both the RA and HA models.

**RA Decentralized Recursive Competitive Equilibrium.** Given an aggregate output process (3.29), a RA decentralized recursive competitive equilibrium is a value function \(V(D, Y)\) and a set of policies \(C(D, Y), D'(D, Y)\) such that

- \(V(D, Y), C(D, Y), D'(D, Y)\) satisfy equations (3.4) - (3.6).

\(^9\)I reserve \(\sigma_Y\) for the unconditional standard deviation of the endowment.
HA Decentralized Recursive Competitive Equilibrium. Given an aggregate output process (3.29), and an idiosyncratic income process (3.30), an HA decentralized recursive competitive equilibrium is a value function $V(e, d; Y)$, a set of policies $c(e, d; Y)$, $d'(e, d; Y)$, and a distribution $\Psi(e, d)$ such that

- $V(e, d; Y)$, $c(e, d; Y)$, $d'(e, d; Y)$ solve the recursive problem given by equations (3.13) - (3.15).
- The distribution $\Psi$ follows the law of motion given by equation (4.17).
- Aggregate consumption and borrowing are given by equations (3.21) and (3.22).

3.3 Calibration

This section calibrates the model. The time period is one year. The parameters that calibrate the models can be separated into three groups: parameters that are identical across the HA and RA models, parameters that differ across models, and parameters that are exclusive to the HA model.

Table 4.1 lists the set of shared parameters. I set the inverse intertemporal elasticity of substitution $\sigma$ to one. I set the collateral constraint multiplier $\kappa$ to $-1/3$. I arbitrarily set $\rho_Y$ to 0.70 and $\sigma_Y$ to 0.043 to target an unconditional standard deviation of output of 0.06.

Table 4.3 lists the set of parameters that are specific to the HA model. This includes the persistence and standard deviation of the idiosyncratic income shock, $\rho_e$ and $\sigma_e$, respectively. I set $\rho_e = 0.88$ and $\sigma_e = 0.26$ to match Guntin et al. (2023)’s annual calibration for Italy.

Table 4.2 lists the parameter that differs across models, $\beta$. I calibrate $\beta$ to 0.979 in the RA model to target a household net foreign asset position to GDP ratio of -11%. To maintain the same ratio in the HA model, $\beta$ needs to be recalibrated.

The model is solved over a discretized state space, presented in Table 3.4. This requires discretizing the aggregate endowment $Y_t$, the idiosyncratic income shock $e_t$, and the space of the liquid asset position $d_t$. The RA model only features the aggregate and liquid asset states, while the HA model additionally features the idiosyncratic income shock state. I use the same grids and
Parameter | Value | Description | Source  
--- | --- | --- | ---  
$\gamma$ | 1 | Inverse IES | Standard  
$r$ | 0.02 | Interest Rate | Standard  
$\kappa$ | .3 | Collateral Constraint | Standard  
$\rho_Y$ | 0.70 | Persistence of Aggregate Endowment | Arbitrary  
$\sigma_Y$ | 0.043 | Standard Deviation of Aggregate Endowment | Arbitrary  

Table 3.1. Calibration, Shared Parameters

*Notes:* The time unit is one year.

| Parameter | Value | Source  
--- | --- | ---  
$\rho_e$ | 0.88 | Guntin et al. (2022)  
$\sigma_e$ | 0.26 | Guntin et al. (2022)  

Table 3.2. Calibration, HA Parameters

| Parameter | RA | HA | Target  
--- | --- | --- | ---  
$\beta$ | 0.977 | 0.956 | NFA/GDP = $-11\%$  

Table 3.3. Calibration, Differing Parameters

discretizations for the aggregate endowment and liquid asset in both models. Equations (3.29) and (3.30) are discretized using the Rouwenhorst method. I use 11 points for $Y_t$ and 11 points for $e_i^t$. I use 300 points for the liquid asset grid. The liquid asset grid is generated using the method in Auclert et al. (2020)’s Sequence Space Jacobian package, which places more points near the borrowing constraint, where household policies are more nonlinear. I set the highest borrowing position to $\kappa Y$, the maximum possible level of borrowing as allowed by the collateral constraint. I set the maximum savings position to have a magnitude of 30.00, which is never reached by any household in the simulation.
3.3.1 Simulation

I simulate both models for 300,000 periods and remove an initial 10,000 burn in periods. Additionally, I generate an income shock series \( \{e_t\}_{t=0}^N \) to simulate the consumption and savings decisions of an individual household within the HA model.

The simulation methods are as follows:

**RA Simulation Algorithm.** Simulate \( \{Y_t\}_{t=0}^{N+N_{\text{burn}}} \) using equation (3.29). Pick some initial debt level \( D_0 \).

For \( t = 1, \ldots, N + N_{\text{burn}} \):

1. Set \( D_t = D'(D_{t-1}, Y_t) \).
2. Compute \( C_t = C(D_{t-1}, Y_t) \).

Drop burn in periods \( t = 1, \ldots, N_{\text{burn}} \) to form the series \( \{C_t\}_{t=0}^N \), \( \{D_t\}_{t=0}^N \).

**HA Simulation Algorithm.** Simulate \( \{Y_t\}_{t=0}^{N+N_{\text{burn}}} \) using equation (3.29). Pick some initial household distribution \( \Psi_0 \).

For \( t = 1, \ldots, N + N_{\text{burn}} \):

1. Compute the updated distribution of household \( \Psi_{t+1} \) using equation (4.17).
2. Compute \( C_t, D_t \) and any statistics that depend on \( \Psi_t \).

Drop burn in periods \( t = 1, \ldots, N_{\text{burn}} \) to form the series \( \{C_t\}_{t=0}^N \), \( \{D_t\}_{t=0}^N \).

**HA Household Simulation Algorithm.** Simulate \( \{Y_t\}_{t=0}^{N+N_{\text{burn}}} \) and \( \{e_t\}_{t=0}^{N+N_{\text{burn}}} \) using equations (3.29) and (3.30), respectively. Pick some initial position \( e_1, d_0 \).

For \( t = 1, \ldots, N + N_{\text{burn}} \):

1. Set \( d_t = d'(e_t, d_{t-1}; Y_t) \).
2. Compute \( c_t = c(e_t, d_{t-1}; Y_t) \).

Drop burn in periods \( t = 1, \ldots, N_{\text{burn}} \) to form the series \( \{c_t\}_{t=0}^N \), \( \{d_t\}_{t=0}^N \).
<table>
<thead>
<tr>
<th>Object</th>
<th>Count/ Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
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<td>$n_Y$</td>
<td>11</td>
<td>Number of grid points for $Y_t$</td>
</tr>
<tr>
<td>$n_b$</td>
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<td>Number of grid points for $b_t$</td>
</tr>
<tr>
<td>$n_e$</td>
<td>11</td>
<td>Number of grid points for $e_t$</td>
</tr>
<tr>
<td>$[\bar{Y}, \bar{Y}]$</td>
<td>[0.85, 1.17]</td>
<td>Range for aggregate endowment</td>
</tr>
<tr>
<td>$[\bar{b}, \bar{b}]$</td>
<td>[-0.35, 30.0]</td>
<td>Range for savings</td>
</tr>
<tr>
<td>$[\bar{e}, \bar{\tau}]$</td>
<td>[0.23, 3.29]</td>
<td>Range for idiosyncratic income shock</td>
</tr>
</tbody>
</table>

Table 3.4. Discretization

Notes: This table describes the discretization space for the RA and HA models. All parameters are shared except those describing idiosyncratic income risk, which are exclusive to the HA model.

3.4 Results

This section compares the HA and RA models. I first compare the long run moments of each model and their response to a typical aggregate endowment shock. I then compare sudden stops in each model.

3.4.1 Moments

Figure 3.5 displays selected moments for the HA and RA models. I denote the relative volatility of consumption by $\sigma_C / \sigma_Y$ where $\sigma_C$ is the standard deviation of aggregate consumption and $\sigma_Y$ is the standard deviation of aggregate output. The RA model features a relative volatility of consumption of 0.70, whereas the HA model improves on this with a ratio of 0.83. Because output is an exogenous endowment, and hence identical across models, the change in the ratio is driven entirely by the change in $\sigma_C$.

The trade balance to output ratio is procyclical in the RA model, with a correlation to output of 0.69. The HA model offers a weaker correlation of 0.62. The models differ significantly in their standard deviation of the trade balance to output ratio. The RA model exhibits a standard deviation of 4.2%, while the HA economy displays a weaker standard deviation of 2.4%.

Why does the HA economy display a lower standard deviation of the trade balance to output
ratio? Recall the aggregate resource constraint

\[ C_t + TB_t = Y_t. \] (3.31)

Because \( Y_t \) is exogenous, the variance on the right hand side of the equation is fixed. The principle of consumption smoothing is to funnel variation from the right hand side of the equation into the trade balance \( TB_t \), rather than consumption \( C_t \). When unconstrained, the RA model is fairly successful at accomplishing this. There are cases when the RA model is constrained, and the majority of variation will channel into \( C_t \). Because these cases are rare, however, they will not have a significant effect on long run moments, as commented in Mendoza (2010). This produces a less volatile consumption \( C_t \) and more volatile \( TB_t \).

Mendoza (2010)’s comment on the small contribution of financial frictions does not apply in the HA model. This is because the idiosyncratic income shock generates poor households that are near the constraint at any point in time. No differently than they would respond to a transitory aggregate shock, these households take on leverage to smooth over their transitory income shock. This can be seen in Figure 3.1 which plots the long run distributions of income and slack for a generic household in the RA and HA economies. The left figure shows that the HA household experiences much more variance in income relative to the RA household. This highlights that fluctuations in aggregate income mask enormous variation in idiosyncratic income. The right figure displays the empirical cumulative density function (ECDF) of slack for the RA and HA households. I use an ECDF because the distribution of slack features a mass point at zero and is continuous on nonzero values. The typical HA household is constrained in 41% of periods. In contrast, the RA household is constrained in 14% of periods. The HA household is also more likely to be close to but not on the constraint.

The decrease in the standard deviation of the trade balance to output ratio may be viewed as a failure. However, I emphasize that the relationship between endowment shocks and the trade balance to output ratio is weak to begin with. In standard models that feature a production sec-
Figure 3.1. Income and Slack Distributions of HA and RA Households

Notes: This figure displays income distribution and CDF function of slack for the RA and HA models. The HA income distribution includes variation from idiosyncratic idiosyncratic risk $e$.

The majority of variation in aggregate income (output) is generated by stationary technology shocks. García-Cicco et al. (2010)’s representative agent model estimates that stationary technology shocks, account for only 1.3% of the variation in the trade balance to output ratio. They find that the variation in the trade balance is almost entirely driven by preference shocks and interest rate shocks, neither of which are present in this model. Hong (2023a)’s baseline heterogenous agent business cycle model estimates a larger contribution of 37%, however this reduces to 2% in an expanded model.

3.4.2 Output Contraction

This section compares the responses of each economy to a typical output contraction. I define a contraction at time $t$ as when output is one standard deviation below its mean. I then collect windows $[t, \ldots, t + 10]$ of aggregate output, aggregate consumption, the trade balance to output ratio, and the percentage of constrained households.\textsuperscript{10} Figure 3.2 displays the responses. At time $t = 0$, output is 8.8% below its mean, and gradually reverts to its mean. The HA economy displays\textsuperscript{10}Within the RA model, this is more strictly interpreted as the probability of all households being constrained at a given point in time.
<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma_C/\sigma_Y$</th>
<th>$\sigma_C$</th>
<th>$\sigma_Y$</th>
<th>$\rho(TBY,T)$</th>
<th>$\sigma_{TBY}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HA</td>
<td>0.83</td>
<td>0.050</td>
<td>0.06</td>
<td>0.62</td>
<td>0.024</td>
</tr>
<tr>
<td>RA</td>
<td>0.70</td>
<td>0.043</td>
<td>0.06</td>
<td>0.69</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Table 3.5. Moments

Notes: This table collects moments from the simulations of the HA and RA economies. Each economy is simulated for 300,000 periods with 10,000 burn in periods.

<table>
<thead>
<tr>
<th>Model</th>
<th>EMPC</th>
<th>EBIND</th>
</tr>
</thead>
<tbody>
<tr>
<td>HA</td>
<td>55.2%</td>
<td>41.2%</td>
</tr>
<tr>
<td>RA</td>
<td>28.7%</td>
<td>13.9%</td>
</tr>
</tbody>
</table>

Table 3.6. Household Moments

Notes: This table collects household level moments from the simulations of the HA and RA economies. Each economy is simulated for 300,000 periods with 10,000 burn in periods.

an initial decrease in aggregate consumption of 7.6%, larger than the RA model’s response of 5.9%. As dictated by the resource constraint, the RA economy’s weaker decrease in consumption is financed through a stronger deterioration of the trade balance.

Because the HA model features a nondegenerate distribution of income, it is possible to decompose the aggregate consumption response into the consumption of distinct income groups. I separate households by income into the bottom 20%, middle 60%, and top 20%.\textsuperscript{11} Throughout the remainder of the paper, I label these groups as the ‘poor’, the ‘middle class’, and the ‘rich’, respectively. Figure 3.3 displays the responses of each income group. The upper left panel displays the percentage of constrained households within each income group. At time $t = 0$, 97% and 80% of poor and middle class households are constrained, respectively. Notably, never more than 6% of rich households are constrained throughout the output contraction.

The upper right panel of Figure 3.3 displays the percentage change in consumption of each income group, normalized by each income group’s long run mean of consumption. The poor

\textsuperscript{11}A similar decomposition can be done based on asset holdings. This makes computations more complex because the liquid asset distribution changes every period, whereas the normalized distribution of income is completely static. Additionally, income and liquid asset holdings strongly predict one another.
Figure 3.2. Output Contraction

Notes: This figure plots the responses of each model to an output contraction. An output contraction at time $t$ is defined as when the output is one standard deviation below its long run mean.
and middle class households display the largest drops in consumption. This occurs because each income group contains a larger percentage of constrained households. In contrast, the rich display a weaker drop in consumption because none of them are constrained.

The lower left panel of Figure 3.3 displays the absolute contribution of each income group to the change in aggregate consumption. We can see that, while the poor households display the most dramatic drop of consumption in percentage terms, their contribution to the fluctuation of aggregate consumption is small. This occurs because poor households do not consume much to begin with. Rich households, in contrast, consume a large amount relative to their group size. This implies rich households will have a more significant effect on the response of aggregate consumption.

The bottom right panel of Figure 3.3 displays the absolute change in each group’s liquid asset holdings. The poor households display an increase in savings, induced by the contraction of the collateral constraint. Despite their size, middle class households display a muted response as it includes a mix of constrained and unconstrained households. Because the vast majority of them are constrained, rich households significantly increase their borrowing. As with consumption, the aggregate response of poor households’ liquid asset position is overwhelmed by that of rich households, causing the trade balance to decrease.

3.4.3 Sudden Stops

This section compares the response of each economy to a typical sudden stop event. Within the RA economy, I define a sudden stop as when the collateral constraint binds and the trade balance to output ratio increases by two percentage points. I collect windows 10 periods before and 10 periods after the sudden stop and average over them. An immediate challenge in the HA economy is that the percentage of constrained households is continuous and not binary. To take an agnostic approach, I identify sudden stop episodes within the RA model and collect the same sudden stop windows from the HA model. This allows me to study the responses of the HA economy to output.

\[12\text{e.g. the deviation of each income group sums to the deviation of aggregate consumption.}\]
Figure 3.3. Decomposed HA Output Contraction

*Notes:* This figure plots the responses of individual income groups during a standard output contraction.
sequences that generate sudden stops in the RA economy.

Using this method, the RA economy experiences a sudden stop in 3.1% of periods. Figure 3.4 plots the responses of output, aggregate consumption, the trade balance to output ratio, and the percentage of constrained households during a sudden stop, for both the RA and HA economies. Because the same windows are collected, the economies feature identical output series. The RA economy exhibits the characteristic features of a sudden stop. Output decreases preceding the sudden stop, followed by a sudden decrease that triggers it. At time 0, the collateral constraint binds with certainty. This forces households to save, generating a reversal in the trade balance. The increase in savings can only be financed through a decrease in consumption. As a consequence, the RA model displays a 9.9% decline in consumption, just below the 10.0% decline in aggregate income. In addition, the RA model displays a trade balance reversal of 4.4% between periods $t-1$ and $t$. In contrast, the HA model fails to capture the critical features of a sudden stop. Consumption declines by 8.8%, less than the decline in output, and the trade balance to output ratio appreciates by only 0.5%.

To understand why the HA economy does not feature sudden stop episodes, it helps to compare the percentage of constrained households in each economy. In the RA economy, the definition of the sudden stop requires that the representative household, and hence every household, is constrained. This implies that no households are able to consumption smooth, and all households are forced to deleverage in response to the contraction of the collateral constraint. In the HA economy, only a continuous percentage of households are constrained. Theoretically, this proportion can approach 100%, but it never reaches more than 92% in equilibrium. This implies that a nontrivial proportion of households are unconstrained at the time of the sudden stop.

To study the effects of unconstrained households, I decompose the sudden stop by income groups into the bottom 20%, middle 60%, and top 20%. The upper left panel of Figure 3.5 displays the portion of constrained households in each group throughout the sudden stop. The vast majority of poor and middle class households are constrained leading up to the sudden stop, due to the contractionary aggregate income shocks that precede it. On impact this proportion jumps
Figure 3.4. Sudden Stop in RA Economy

Notes: This figure plots the responses of each model to a sudden stop. Within the RA model, a sudden stop at time $t$ is defined as when the collateral constraint binds and the trade balance to output ratio increases by two percentage points. Time series windows for the HA model are drawn from sudden stop windows in the RA model.
Notes: This figure plots the responses of individual income groups during a sudden stop episode. Within the RA model, a sudden stop at time $t$ is defined as when the collateral constraint binds and the trade balance to output ratio increases by two percentage points. Time series windows for the HA model are drawn from sudden stop windows in the RA model.
significantly for middle class households. Throughout the sudden stop event, never more than 9% of rich households are constrained. This implies that the majority of rich households are able to consumption smooth throughout the sudden stop.

The upper right panel of Figure 3.5 displays the percentage change in consumption for each income group, normalized by each group’s long run mean consumption. As with the output contraction, poor households display a strong consumption response relative to the decline in income. This occurs because the majority of poor households are financial constrained, like those of the RA model. Rich households display a weaker percentage change in consumption because they are able to consumption smooth. The response of the middle class households is between that of the poor and rich.

The lower left panel of Figure 3.5 displays the aggregate response of consumption, decomposed in absolute terms by income groups. While poor households experience a more dramatic decrease in consumption in percentage terms, they consume a small amount relative to their group size. This leads their aggregate decline in consumption to be small. In contrast, rich households take up a disproportionate share of consumption relative to their group size. This implies that their consumption smoothing behavior is more strongly reflected in the aggregate consumption response.

The lower right panel of Figure 3.5 displays the aggregate savings of each group throughout the sudden stop. Poor households display a small, positive savings increase at impact. The increase in savings is forced through the contraction of the collateral constraint. Middle class households take on some debt. In contrast, rich households significantly increase their debt position. This nullifies the increase in savings of both the middle class and poor households, driving the muted trade balance response.

3.4.4 Comparisons and Moving Forward

The failure of the HA model to generate a sudden stop differs from the results in Cugat (2019), who finds that heterogeneity amplifies the effects of a sudden stop. Our models differ in both
the presence of financial frictions and the nature of household heterogeneity. Cugat (2019)’s RA benchmark is a perturbed model that does not feature an occasionally binding collateral constraint. Her HA model introduces hand to mouth households that do not have access to any assets. Lastly, the sudden stop is generated through a sudden decrease in tradable productivity and an increase in the external interest rate. During the sudden stop, unconstrained households in Cugat (2019)’s RA benchmark have access to the bond throughout the contraction. This allows them to consumption smooth like the rich households in my HA model. Because her HA model includes a proportion of hand to mouth households, the aggregate response of consumption is larger relative to the RA benchmark. In contrast, the sudden stop definition in my RA benchmark imposes that all households in the economy are constrained. Like in Cugat (2019), only a proportion of households in my HA economy are constrained during the sudden stop. Because all households in my RA economy are constrained, the aggregate response of consumption is more dramatic relative to the HA model.

My model expands on Guntin et al. (2023)’s analysis of the failure of financial frictions to account for financial crises in heterogenous agent models. Guntin et al. (2023)’s model, and mine, predict that richer households have a smaller percentage response of consumption relative to income. This contradicts the data which supports a consumption-income elasticity of nearly one across all income deciles.\(^\text{13}\) My model replicates this result and shows that it occurs because the majority of rich households are financially unconstrained, which allows them to consumption smooth throughout output contractions and sudden stops. I go further by showing the model fails to account for the reversal of the trade balance because rich households increase their borrowing during sudden stop episodes.

Guntin et al. (2023) conclude that the Mexican Peso Crisis of 1994 and the experience of Italy during the Global Financial Crisis were driven by permanent income shocks rather than financial frictions. I argue that the failure of HA stems from the asset structure of the model rather than financial frictions. Because the model features a single asset, households that save more are mechanically further from the financial friction. This produces a well known failure of single asset

\(^\text{13}\)Here I refer to Guntin et al. (2023)’s empirical analysis of the Mexican Peso Crisis of 1994, and the Global Financial Crisis of 2008 and the European Debt Crisis of the early 2010s within Italy and Spain.
3.5 Conclusion

This paper studies the ability of household heterogeneity, in the form of idiosyncratic income risk, to explain well known empirical features of emerging markets. I show that heterogeneity increases the marginal propensity to consume and the percentage of financially constrained households. This occurs because income risk generates poor households that take on leverage to smooth over their temporary income shock. These households interact more frequently with the financial friction, producing nonlinear consumption behavior throughout the business cycle. At the aggregate level, this is reflected in a higher volatility of consumption. Household heterogeneity weakens the ability of the model to generate sudden stops, reflected in a dampened response of aggregate consumption and the trade balance. I show this occurs because the income risk generates rich households that use their asset holdings to better maintain their consumption throughout crisis episodes.
Chapter 4: The Pure Effects of Household Heterogeneity

4.1 Introduction

Within the study of emerging markets, heterogeneous agent (HA) models have received a large increase in attention for their potential to explain economic phenomena at both the household and aggregate level. Models that introduce heterogeneity typically introduce three deviations from a benchmark representative agent (RA) model: heterogeneity in the form of idiosyncratic income risk, substantial financial frictions in the form of a borrowing constraint, and recalibration of household impatience. However, it is unclear which of these features, or their interactions, drive the changes in consumer behavior that we observe across HA and RA models. This paper studies that question.

I consider three calibration exercises that iteratively add heterogeneity, substantial financial frictions, and explicit recalibration of impatience. I find that ‘pure’ heterogeneity, with minimal financial frictions and no recalibration of impatience, has no significant effect on aggregate responses to an endowment or interest rate fluctuation. A combination of heterogeneity and financial frictions can generate a realistic percentage of financially constrained households and marginal propensity to consume (MPC). However, they struggle to change aggregate responses because the consumption share of constrained households is static. I finally consider a prototypical calibration exercise that includes all three. I show that the introduction of income risk leads aggregate consumption to display a stronger initial response to an aggregate income shock as a significant percentage of households are constrained and sensitive to income changes. However, these households do not display a direct response to interest rate changes. As a consequence, models with income risk feature a smaller responsiveness of aggregate consumption to interest rate fluctuations.

I start by building a minimal representative agent (RA) model. The representative household
receives an exogenous income endowment and can save or borrow using a non state-contingent bond. To induce stationarity, the interest rate on the bond increases with the amount of debt held, as in Schmitt-Grohé and Uribe (2003). I then form an HA model by introducing idiosyncratic income risk into the household problem. Like Debortoli and Gali (2022), I first consider a model where the only ‘friction’ that households face is the avoidance of starvation. In this case, the borrowing constraint coincides with an analog of the natural debt limit, which households never borrow up to because starvation is infinitely undesirable. This is less restrictive than the standard Bewley (1977) household block that features a more restrictive borrowing constraint.

I solve for general equilibrium perfect foresight responses to aggregate endowment shocks and interest rate shocks. I find the HA and RA models feature virtually identical responses of consumption and the trade balance to output ratio. The results are robust to increasing the elasticity of the interest rate with respect to debt. In fact, a higher elasticity strengthens the similarities of the two models. My finding replicates Krusell and Smith (1998) and Debortoli and Gali (2022) finding of similarities across RA and HA models in the context of a small open economy.

I then consider the effects of introducing realistic financial frictions at the household level. For a given level of impatience, I study how the models change as income risk increases. I first solve the models with a weak relationship between aggregate debt and the interest rate. In this case, the introduction of income risk leads to enormous precautionary savings by households. An empirically realistic calibration of income risk leads to an aggregate savings to output ratio of more than 200%. The extreme precautionary savings presents an important departure from Krusell and Smith (1998). Because the interest rate displays a weak response to changes in aggregate savings, households attempt to precautionary save away from the entirety of their idiosyncratic income risk. In models that feature a production asset with decreasing return to scale, the rapid decline in the return of the productive asset induces a smaller equilibrium increase in savings.

I find that, despite an increase in aggregate savings, the percentage of constrained households and average MPC both increase with income risk. This occurs because the variance of household level savings increases with income risk as households try to smooth over their stronger individual
income fluctuations, despite an increasing mean. More of the left tail of the savings distribution is caught by the borrowing constraint, which generates financially constrained households that feature a high MPC. I consider an additional calibration that features a more strongly decreasing relationship between aggregate savings and the interest rate. Within the liquid asset environment, this mimics the decreasing returns to scale of the production asset in Krusell and Smith (1998). Moving from minimal to an empirical target of income heterogeneity increases aggregate savings from 0% of income (targeted) to 40% (untargeted).

This brings forward the importance of additionally recalibrating household impatience, the standard method of the literature. I consider a prototypical calibration exercise that introduces heterogeneity, financial frictions, and recalibrates impatience to explicitly target aggregate savings. I find that the required impatience to rationalize a static aggregate savings target is decreasing in idiosyncratic income risk. This occurs because increases in idiosyncratic income uncertainty increase the precautionary savings motive.

The standard calibration experiment introduces several changes at the household and aggregate level. First, the percent of constrained households increases with income risk. This occurs because higher income risk generates poorer households that need to borrow even more. As risk increases, more of these households borrow up to the constraint. Secondly, the average MPC increases to realistic levels for emerging markets.

The new calibration method makes several tradeoffs at the aggregate level. Following an income shock, the HA model displays a stronger aggregate consumption response relative to the RA model. This occurs because constrained households consume the majority of the immediate income increase. However, because the models feature identical permanent incomes, the aggregate response of consumption in the HA model is lower than that of the RA model several periods later. Additionally, the HA model features a weaker aggregate response to interest rate shocks. This occurs because constrained households do not respond directly to changes in the interest rate as they feature a distorted Euler equation.
4.1.1 Relationship to the Literature

This paper relates to the literature that studies the implications of heterogeneity for aggregate dynamics. Like Debortoli and Gali (2022), I find that heterogeneity has little effect on the path of aggregate consumption in the presence of minimal financial frictions. I differ from Debortoli and Gali (2022) by modeling an open economy rather than a closed economy. In addition, I study the intermediate step of introducing heterogeneity and significant financial frictions without recalibration of impatience.

A much older similarity result across RA and HA models is found in Krusell and Smith (1998). I differ from Krusell and Smith (1998) by including a non state-contingent bond rather than a capital asset. Unlike Krusell and Smith (1998), I only focus on ‘liquid wealth’ calibrations that target the external asset position of emerging markets. Calibrations that target ‘total wealth’ would strengthen my equivalence results as households would face a lower risk of starvation.

Kaplan and Violante (2022) studies what drives the differences in the MPC observed across HA and RA models. This paper differs from Kaplan and Violante (2022) by studying how heterogeneity changes the aggregate responses of the economy over time to aggregate shocks.¹ In addition, my calibrations are oriented towards emerging markets, which feature a higher MPC and portion of constrained households than Kaplan and Violante (2022)’s calibrations for developed economies.

This paper contribute to the literature that studies the drivers of emerging markets. I show that heterogeneity increases responsiveness of the economy to income shocks and decreases responsiveness to interest rate shocks. The latter presents a challenge if one believes that interest rates play a major role in driving the consumption and external borrowing position of emerging markets. This is easily explained by the fact that the endowment model lacks a pathway for interest rates to affect household income. In more generalized production economies, such as Kaplan et al. (2018), the interest rate primarily drives consumption through labor income.

¹The MPC is not a sufficient statistic for the aggregate response of consumption, because the current aggregate response depends on future fluctuations in income.
4.2 Model

This section presents the RA and HA models. Time is discrete and infinite. The economy receives an aggregate income endowment $Y_t$ that follows a known stochastic process. Throughout the paper, I use the terms ‘endowment’ and ‘output’ interchangeably. Households can borrow or save in a bond at the market rate $r_t$.

4.2.1 RA Household

The economy is populated by a continuum of identical households. Households have preferences over consumption streams $\{C_t\}_{t=0}^{\infty}$ given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

(4.1)

where

$$u(C) = \frac{C^{1-\gamma} - 1}{1 - \gamma}$$

(4.2)

is the period utility function, and $\gamma > 0$ denotes the inverse intertemporal elasticity of substitution. Households satisfy the sequential budget constraint

$$C_t + B_t = (1 + r_{t-1})B_{t-1} + Y_t,$$

(4.3)

where $B_t$ is the household’s bond holdings. Household must satisfy the no starvation rule

$$C_t > 0,$$

(4.4)

and the borrowing constraint

$$B_t \geq -\kappa,$$

(4.5)
where $\kappa > 0$.\(^2\)

The Euler equation of the household problem can be written as

$$u'(C_t) = \mu_t + \beta (1 + r_t) E_t u'(C_{t+1}),$$

where $\mu_t \geq 0$ denotes the multiplier of the borrowing constraint (4.5). Additionally, the representative household satisfies the slackness condition

$$0 = \mu_t (\kappa + B_t).$$

4.2.2 HA Household

The economy is populated by a continuum of households that are heterogeneous in their income and asset holdings. Given the aggregate income $Y_t$, household $i$ receives $e_i^i Y_t$, where $e_i^i$ is an idiosyncratic income shock with a known stochastic process. Household $i$ has preferences over consumption streams $\{c_i^i\}_{t=0}^\infty$ given by

$$E_0 \sum_{t=0}^\infty \beta^t u(c_i^i),$$

where

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}$$

is the period utility function, and $\gamma > 0$ denotes the inverse intertemporal elasticity of substitution. Household $i$ satisfies the sequential budget constraint

$$c_i^i + b_i^i = (1 + r_{t-1}) b_{i-1}^i + e_i^i Y_t,$$
where $b^i_t$ denote household $i$’s asset holdings decision. Households must satisfy the no starvation condition

$$c^i_t > 0,$$  \hspace{1cm} (4.11)

and the borrowing constraint

$$b^i_t \geq -\kappa.$$  \hspace{1cm} (4.12)

The Euler equation of household $i$ can be written as

$$u'(c^i_t) = \mu^i_t + \beta(1 + r_t)E_t u'(c^i_{t+1}),$$  \hspace{1cm} (4.13)

where $\mu^i_t \geq 0$ denotes the Lagrange multiplier of the borrowing constraint. Households satisfy the slackness condition

$$0 = \mu^i_t(\kappa + b^i_t).$$  \hspace{1cm} (4.14)

The marginal propensity to consume (MPC) is defined as

$$\text{MPC}(e, b) = \frac{c(e, b + \epsilon) - c(e, b)}{\epsilon}$$  \hspace{1cm} (4.15)

where $\epsilon > 0$ is a small change in asset holdings.\(^3\)

**Distribution** The distribution of households is denoted by

$$\Psi_t(e, b) = \text{Pr}(c^i_t \leq e, b^i_{t-1} \leq b),$$  \hspace{1cm} (4.16)

\(^3\)For HA models, or models that feature nontrivial income risk, the MPC is computed empirically using the household consumption policies at each stationary steady state. For the RA model, I compute the MPC using Kaplan and Violante (2022)’s computation $\text{MPC} = 1 - (1 + r)^{-1}[(1 + r)\beta]^{1/\gamma}$. 

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which follows the law of motion

$$\Psi_t(e^*, b^*) = \int_{e_{t-1}, b_{t-1}} Pr(e_t \leq e^* | e_{t-1}) \mathcal{I}(b_t(e_{t-1}, b_{t-1}) \leq b^*) d\Psi_{t-1}(e_{t-1}, b_{t-1}),$$  \hspace{1cm} (4.17)

where $\mathcal{I}$ is an indicator function. Aggregates are denoted by

$$B_t = \int_{e,b-} b(e, b_-) d\Psi_t(e, b_-) \hspace{1cm} (4.18)$$

$$C_t = \int_{e,b-} c(e, b_-) d\Psi_t(e, b_-), \hspace{1cm} (4.19)$$

where a generic household is identified by their idiosyncratic income $e$ and asset position $b_-$ (determined in the previous period).

The trade balance and trade balance to output ratio are given by

$$TB_t = B_t - (1 + r_{t-1})B_{t-1}, \hspace{1cm} (4.20)$$

$$\text{and} \quad TBY_t = TB_t / Y_t, \hspace{1cm} (4.21)$$

respectively.

### 4.2.3 Market Clearing

The presence of starvation risk generated by the income shock is sufficient to induce stationarity in the HA model. The steady state debt allocation of the RA model, however, is indeterminate. I induce stationarity by making the external interest rate $r_t$ decreasing in aggregate savings $B_t$:

$$r_t = r^* + \psi(e^{-(B_t - \underline{B})} - 1) + \mu_t - 1, \hspace{1cm} (4.22)$$

where $\psi > 0$ and $\mu_t$ is an exogenous fluctuation to the interest rate with mean one at the steady state. Parameter $\underline{B}$ is a constant such that, when $B_t = \underline{B}$, then $r_t = r^*$ at the steady state.

Within perturbed RA models, equation (4.22) is the standard method to induce stationarity,
originating in Schmitt-Grohé and Uribe (2003). The parameter $\psi$ can be set to a small positive value, typically 0.01, to induce stationarity. However, later papers such as García-Cicco et al. (2010) directly estimate higher values of $\psi$. I will consider both low and high values of $\psi$.

4.2.4 Resource Constraint

The resource constraint is given by

$$C_t + TB_t = Y_t.$$  \hfill (4.23)

4.2.5 Shocks

The aggregate endowment $Y_t$ and the interest rate shock $\mu_t$ follow the stochastic processes:

$$\log Y_t = \rho_Y \log Y_{t-1} + \epsilon_Y^t, \text{ and}$$

$$\log \mu_t = \rho_{\mu} \log \mu_{t-1} + \epsilon_{\mu}^t,$$  \hfill (4.24)

\hfill (4.25)

respectively. Household income risk follows the stochastic process

$$\log e_i^t = \rho_e \log e_i^{t-1} + \epsilon_e^t, \epsilon_e^t \sim \mathcal{N}(0, \sigma_e),$$  \hfill (4.26)

where $\mathcal{N}$ denotes the normal distribution.

4.2.6 Equilibrium Definitions

This section defines the equilibriums of the RA and HA models.

**RA Decentralized Recursive Competitive Equilibrium.** Given a path of the aggregate endowment $\{Y_t\}_{t=0}^T$ and a path of the interest rate deviation $\{\mu_t\}_{t=0}^T$, an RA Decentralized Recursive Competitive Equilibrium is a series of equilibrium variables $\{C_t, B_t, r_t, TB_t, TBY_t\}$, such that

- $\{C_t, B_t\}_{t=0}^T$ satisfy equations (4.3) and (4.6).
• The trade balance and trade balance to output ratio are given by equations (4.20), (4.21).

• The interest rate $r_t$ satisfies equation (4.22).

**HA Decentralized Recursive Competitive Equilibrium.** Given a path of the aggregate endowment $\{Y_t\}_{t=0}^T$ and a path of the interest rate deviation $\{\mu_t\}_{t=0}^T$, an HA Decentralized Recursive Competitive Equilibrium is a series of household policies $\{c_t(e, b), b_t(e, b)\}_{t=0}^T$, a series of the distribution of households $\{\Psi_t\}_{t=0}^\infty$ and a series of equilibrium variables $\{C_t, B_t, r_t, TB_t, TBY_t\}$, such that

• The household policies satisfy equations (4.10), (4.12), (4.13).

• The distribution of households satisfies equation (4.17).

• The path of aggregates $\{C_t, B_t\}_{t=0}^T$ satisfy equations (4.18), and (4.19).

• The trade balance and trade balance to output ratio are given by equations (4.20), (4.21).

• The interest rate $r_t$ satisfies equation (4.22).

**4.3 Calibration**

This section describes features of the calibration that are common across all models and experiments. The time period is one year. Table 4.1 lists the set of shared parameters. I set the steady state annual interest rate $r^*$ to 0.08 and the inverse intertemporal elasticity of substitution $\gamma$ to 1. I consider two values for $\psi$: a minimal value of 0.01 and a more empirically realistic value of 0.10. For a lower value of $\psi$, the interest rate is less responsive to aggregate bond holdings.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^*$</td>
<td>0.08</td>
<td>Annual Interest Rate</td>
<td>Standard</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
<td>Inverse IES</td>
<td>Standard</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.01, 0.10</td>
<td>Interest Rate Friction</td>
<td>Arbitrary</td>
</tr>
<tr>
<td>$\bar{B}$</td>
<td>0.00</td>
<td>Aggregate Savings</td>
<td>Arbitrary</td>
</tr>
</tbody>
</table>

Table 4.1. Shared Parameters

Notes: The time period is one year.

4.4 Results

This section analyzes models with varying levels of income heterogeneity and financial frictions. Section 4.4.1 studies the model with income risk and minimal financial frictions. Section 4.4.2 studies the model with income risk and significant financial frictions. Section 4.4.3 studies the standard method of introducing income risk, financial frictions and recalibrating impatience to target aggregate bond holdings.

4.4.1 Model with Minimal Financial Frictions

This section considers a version of the model that features minimal financial frictions at the household level. I only require that households satisfy the no starvation condition

$$c_t > 0.$$ (4.27)

At the steady state, this introduces an analog of the natural debt limit:

$$b_t \geq -\frac{eY_{ss}}{1 + r}.$$ (4.28)
<table>
<thead>
<tr>
<th>Model</th>
<th>$\beta$</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA</td>
<td>0.926</td>
<td>Steady State Euler</td>
</tr>
<tr>
<td>HA</td>
<td>0.912</td>
<td>$B_{ss}^{HA} = \bar{B}$</td>
</tr>
</tbody>
</table>

Table 4.2. Differing Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_e$</td>
<td>0.90</td>
<td>Persistence of Income Risk</td>
<td>Arbitrary</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>0.20</td>
<td>Standard Deviation of Income Risk</td>
<td>Arbitrary</td>
</tr>
</tbody>
</table>

Table 4.3. HA Parameters

where $e$ is the minimum value of the discretized process for equation (4.26) and $Y_{ss}$ is the steady state value of $Y$.$^4$

Table 4.3 displays parameters specific to the HA model. I consider a standard annual calibration of idiosyncratic income risk with $\rho_e = 0.90$, $\sigma_e = 0.20$. Table 4.2 displays the set of shared parameters that have different values. The model includes one free parameter to calibrate, $\beta$. I calibrate $\beta$ to 0.912 to target an aggregate bond holdings of zero at the stationary steady state. Within the RA model, $\beta$ is pinned down by the steady state Euler equation (4.6). This produces $\beta^{RA} = 1/(1 + r^*) = 0.926$.

**Steady State of the HA Model**  Figure 4.1 plots the distributions of asset holdings and consumption at the stationary steady state. As is explicitly targeted, the aggregate (mean) of the distribution of bond holdings coincides with the steady state level of aggregate bond holdings in the RA model. Households display significant variation in asset holdings that is induced by the idiosyncratic income shock. This occurs because the income shock is transitory, so households save or borrow to smooth over their income change. The distribution is bounded below by the personal debt limit $-eY_{ss}/r$.\(^5\) The right panel displays the distribution of consumption, which displays significant

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$^4$In response to a perfect foresight aggregate shock, this generalizes to $b_t \geq \frac{\phi_Y}{1+r^*}$ for $t \geq 0$.

$^5$This limit is more restrictive than the representative agent natural debt limit $Y_{ss}/r$. 

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Figure 4.1. Minimal Frictions: Stationary Distributions of Asset Holdings and Consumption

Notes: This figure displays the steady state distribution of savings and consumption in the HA model that lacks financial frictions. Both savings $b$ and consumption $c$ are normalized by the steady state average income of one.

variance. This implies that the non state-contingent bond is insufficient to remove idiosyncratic risk.

Figure 4.2 plots the average MPC within each income decile, defined by equation (4.15). As suggested by the nontrivial distribution of consumption, households display heterogeneity in their marginal propensity to consume. The lowest income decile displays a MPC of 0.16, which decreases to 0.10 for the highest income decile. The cross-sectional average of MPCs is higher than that of the RA model, but is far from Hong (2023b)’s annual MPC estimate of 0.55-0.59.

Responses to Aggregate Shocks This section studies the responses of the HA and RA models to aggregate shocks in the endowment $Y_t$ and the interest rate $r_t$. For the endowment shock, I
Figure 4.2. Minimal Frictions: Average Annual MPCs by Income Decile

Notes: This figure plots the average annual MPC within each income decile. Households are subject to the borrowing limit $b_t^i \geq -\frac{\bar{\epsilon}^Y}{\bar{\tau}_s}$ where $\bar{\epsilon}$ is the minimum value of the discretized income distribution.
consider a 1% increase in the endowment with a persistence of 0.70. For the interest rate shock, I consider a 1% decrease in the interest rate with a persistence of 0.70.

**Low $\psi$** Figure 4.3 displays impulse response functions (IRFs) of each model to the endowment shock. The upper left panel displays the exogenous path of the endowment. The upper right panel displays the endogenous path of consumption for each model. The RA model displays an initial response of 0.33% and is highly persistent. The consumption response is persistent as the interest rate displays a negligible response to the increase in savings. This allows the RA model to evenly consume their increase in income over time. The HA model features a stronger initial response of consumption because its households feature a higher average marginal propensity to consume. These households wish to consume more but face significant starvation risk if they increase their borrowing position. When income increases, they can comfortably increase consumption. While the initial response is stronger, the HA model must satisfy the same resource constraint as the RA economy. As a consequence, the HA model eventually displays a weaker increase in consumption relative to the RA model. As with consumption, the models display similar responses of the trade balance to output ratio. The RA models displays a slightly stronger improvement of the trade balance because its initial response of consumption is smaller. Over time, this is reflected in a higher quantity of aggregate assets in the RA model.

Figure 4.4 displays the responses of each model to a negative interest rate shock. Because $\psi$ is relatively low, the interest rate shock $\mu$ features near perfect pass through to the equilibrium interest rate $r_t$. As the relative price of current consumption has decreased, both economies display an expansion in consumption. Because the endowment is unchanged, the increase in consumption is financed by a deterioration of the trade balance. Unlike with the endowment shock, the responses of the HA and RA economies coincide almost perfectly.

I now consider the effects of increasing $\psi$ to 0.10, a more empirically realistic value, and resolve the models. Because the models do not feature aggregate risk at the steady state, the steady

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6Because the consumption paths are similar, and the endowment is exogenous, the similarity of the trade balance is dictated by the aggregate resource constraint.
Figure 4.3. Minimal Frictions: IRF to 1% Increase in Endowment, $\psi = 0.01$

Figure 4.4. Minimal Frictions: IRF to 1% Decrease in Interest Rate, $\psi = 0.01$
states of the RA and HA models do not change with $\psi$. Figure 4.5 displays the impulse responses of each model to an endowment shock. For each model, the response of aggregate consumption is larger compared to the response when $\psi = 0.01$. This occurs because households initially desire to save part of their windfall, which places downward pressure on the interest rate. The decrease in the interest rate reduces the marginal value of saving, so that in equilibrium households consume more of the increase in endowment. Both models display an initial consumption response of about 0.6%. Notably, the models display even more similar responses of consumption, with a now marginally stronger response of the HA model.

Figure 4.6 displays the impulses responses of each model to the interest rate decline. Because $\psi$ is higher, the interest rate increases endogenously in response to the initial partial equilibrium increase in borrowing. This increases the marginal value of saving so that households display a smaller consumption increase in equilibrium. As with the endowment shock, the consumption and trade balance responses are virtually identical across models.

This section introduces an important theme in the heterogeneous agent model. With the introduction of heterogeneity, consumption poor households desire to consume more immediately. When $\psi$ is low, it is cheap to move consumption forward and consume less in the future. When $\psi$ is high, the market endogenously increases the price of moving consumption forward. This tempers the motivation of poor households to immediately consume more of their increase in income.

4.4.2 Model with Financial Frictions and No Recalibration

This section studies the second step of introducing heterogeneity in an environment with non-trivial financial frictions and no recalibration of household impatience. The household financial friction is a borrowing constraint of the form

$$b_t \geq -\kappa$$  \hspace{1cm} (4.29)
Figure 4.5. Minimal Frictions: IRF to 1% Increase in Endowment, $\psi = 0.10$

Figure 4.6. Minimal Frictions: IRF to 1% Decrease in Interest Rate, $\psi = 0.10$
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>1/3</td>
<td>Borrowing Constraint</td>
<td>Standard (Annual)</td>
</tr>
<tr>
<td>$\bar{B}$</td>
<td>0</td>
<td>Steady State Savings</td>
<td>Arbitrary</td>
</tr>
<tr>
<td>$[\sigma_e, \sigma_e]$</td>
<td>0.00, 0.05, 0.10, 0.15, 0.20</td>
<td>Income Risk Values</td>
<td>Even Spacing</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.92</td>
<td>Impatience</td>
<td>Clearing $\sigma_e=0.05$</td>
</tr>
</tbody>
</table>

Table 4.4. Calibration: Model with Financial Frictions and no Recalibration of Impatience

Notes: This table displays the calibration of the models that feature significant financial frictions and a constant level of impatience $\beta$ across models. Models are indexed by the standard deviation of income risk $\sigma_e$.

where $\kappa > 0$ is a constant. I calibrate $\kappa$ to a typical value for models that feature occasionally binding borrowing constraints.\(^7\) I solve and compare a series of HA models with increasing values of income heterogeneity, indexed by $\sigma_e$. The RA model is included as the case when $\sigma_e = 0.00$.

I compare the models along two dimensions. First, I compare the features of the stationary steady state. Secondly, I compare the response of each economy to aggregate shocks in the endowment and the interest rate at which households can save or borrow.

Table 4.4 details the calibration. I set $\kappa$ to $1/3$, a typical value of annual models that feature an occasionally binding constraint. I include 5 values of $\sigma_e$, evenly spaced on $[0.00, 0.20]$. I calibrate $\beta$ to 0.92 to target a steady state aggregate savings of $\bar{B} = 0$ for the second value of $\sigma_e$, $\sigma_e = 0.05$.

To keep a constant level of impatience, I maintain this value of $\beta$ for the remaining values of $\sigma_e$. For the RA model, the calibration $\beta(1 + r^*) < 1$ leads households to take on debt relative to the $\sigma_e = 0.05$ model. This leads to an endogenous increase in the interest rate, which restores the steady state equilibrium of the RA model’s Euler equation. I consider two cases of financial frictions at the aggregate level. In the first case, $\psi$ is set to a low value of 0.01.\(^8\) In the second case, $\psi$ is set to an empirically more realistic value of 0.10.

\(^7\)RA models with occasionally binding borrowing constraints typically feature a varying constraint of the form $b_t \geq -\kappa Y_t$. I keep a constant borrowing constraint as this is the norm of HA models, with the exception of Villalvazo (2023).

\(^8\)This follows the literature that sets $\psi$ to a small positive value to induce stationarity.
This section compares the models when $\psi$ is at a low value of 0.01. Figure 4.7 displays the aggregate asset holdings, the equilibrium interest rate, the percentage of constrained households, and average MPC for the stationary steady state as a function of $\sigma_e$. The most dramatic change is that the aggregate bond position increases exponentially in $\sigma_e$. Even a conservative annual estimate of income uncertainty, such as $\sigma_e = 0.20$, produces steady state savings to output of more than 200%. The increase in savings produces a decline in the steady state interest rate. For positive values of income risk, the steady state percentage of constrained households and the average MPC across households are both decreasing in $\sigma_e$.

The changing moments can be somewhat contradictory. One of the motivations of the HA literature is to explain the significant portion of liquidity constrained households and the high MPC that RA models fail to rationalize. This reveals an important difference between the capital asset of Krusell and Smith (1998) and the bond asset that is featured in the Bewley (1977) household block of decentralized models. In Krusell and Smith (1998), the capital asset features a naturally declining marginal productivity. The bond asset features a weaker and concave decline in productivity, specified in equation (4.22). Realistic values of income uncertainty introduce an enormous precautionary savings motive, as the scale of income uncertainty is several magnitudes larger than that of aggregate uncertainty. Households in the Krusell and Smith (1998) model do take on precautionary savings, but are quickly faced with an unforgiving decreasing returns to scale. This leads to a small equilibrium quantity of precautionary savings compared to the RA benchmark. If households save in a bond and face a (near) constant interest rate, they can save arbitrary amounts without decreasing the marginal return of the asset.

Figure 4.8 plots impulse responses to an endowment shock for different values of $\sigma_e$. I consider four values of $\sigma_e$, $\sigma_e = 0.00, 0.05, 0.15, 0.20$. For the lowest value, $\sigma_e = 0.00$, the economy consumes about 40% of the initial increase in income. As $\sigma_e$ increases, the magnitude of the consumption response decreases and its persistence increases. With the initial introduction of $\sigma_e = 0.05$, aggregate consumption displays a larger initial response. As income uncertainty increases, households amass more savings and display a smaller and flatter consumption response.
Figure 4.7. No Recalibration: Effects of $\sigma_e$ on Steady State, $\psi = 0.01$

Notes: This figure characterizes the steady state for different values of $\sigma_e$ in the model that features financial frictions and no recalibration of impatience $\beta$. The interest rate friction is set to a low value of $\psi = 0.01$.

Figure 4.9 plots impulse responses to an interest rate shock. The lowest value of $\sigma_e$ displays a standard consumption response. Consumption initially increases because its relative price has decreased. Because permanent income is unchanged, households must later reduce consumption. Higher values of $\sigma_e$ both produce an initial increase in consumption and a much stronger eventual decrease. This occurs because households have a significant negative wealth effect to a decrease in the interest rate given the large stock of assets they have acquired.

High $\psi$ I now increase the elasticity of the interest rate $\psi$ to a higher value of 0.10. Figure 4.10 characterizes the stationary steady state for each value of $\sigma_e$. Equilibrium aggregate savings is still increasing with respect to $\sigma_e$, but at a much lower rate. A realistic value such as $\sigma_e = 0.20$ produces a steady state aggregate savings of 32% of output. The lower increase in aggregate savings is the
Figure 4.8. No Recalibration: IRF to Endowment Shock, $\psi = 0.01$

Notes: This figure displays responses to a 1% increase in the aggregate endowment for different values of income risk $\sigma_e$. The base model features financial frictions and no recalibration of impatience $\beta$. The interest rate friction is set to a low value of $\psi = 0.01$. 


Figure 4.9. No Recalibration: IRF to Interest Rate Shock, $\psi = 0.01$

Notes: This figure displays responses to a 1% decrease in the interest rate for different values of income risk $\sigma_e$. The base model features financial frictions and no recalibration of impatience $\beta$. The interest rate friction is set to a low value of $\psi = 0.01$. 
general equilibrium result of the higher value of $\psi$. As $\sigma_e$ increases, households have an increased incentive to precautionary save. By equation (4.22), the increase in precautionary savings generates a decline in the interest rate that reduces households’ motivation to save.

Perhaps surprisingly, the portion of constrained households and the average MPC across households is initially increasing is $\sigma_e$. Why does this happen? Two forces are at work in the distribution of asset holdings. The mean of asset holdings (which is identical to aggregate savings) increases, which implies that the average household saves more. However, the variance of asset holdings also increases with $\sigma_e$ because impoverished households need to borrow further to smooth over their idiosyncratic income shock. Despite the increasing mean, the increase in variance is strong enough that more of the left tail of asset holdings is caught by the borrowing constraint. These households display a higher MPC as they wish to consume more but cannot take on more debt. As $\sigma_e$ further increases, the effect of the increasing mean in savings eventually overwhelms the increasing variance. This eventually produces a decline in the percentage of constrained households, and a small decline in the average MPC.

Figures 4.11 and 4.12 plot impulse responses with respect to the endowment and interest rate, respectively. The models with non-degenerate income risk display similar impulse responses. Despite the higher portion of financially constrained households, the models with income risk display a smaller initial response of consumption than the RA model for both the endowment and interest rate shocks.

Figure 4.13 displays the effects of income risk on volatility of consumption relative to output. I compute the relative volatility of aggregate consumption by treating the impulse response to an endowment shock as a linearized response and then compute the long run standard deviation of aggregate consumption. Here we can see the increase in income risk decreases the volatility of aggregate consumption relative to aggregate income, despite a higher percentage of constrained households and a borderline empirically realistic average MPC.
Figure 4.10. No Recalibration: Effects of $\sigma_e$ on Steady State, $\psi = 0.10$

*Notes:* This figure characterizes the steady state for different values of $\sigma_e$ in the model that features financial frictions and no recalibration of impatience $\beta$. The interest rate friction is set to a low value of $\psi = 0.10$. 
Figure 4.11. No Recalibration: IRF to Endowment Shock, $\psi = 0.10$

Notes: This figure displays responses to a 1% increase in the aggregate endowment for different values of income risk $\sigma_e$. The base model features financial frictions and no recalibration of impatience $\beta$. The interest rate friction is set to a high value of $\psi = 0.10$. 
Figure 4.12. No Recalibration: IRF to Interest Rate Shock, $\psi = 0.10$

Notes: This figure displays responses to a 1% decrease in the interest rate for different values of income risk $\sigma_e$. The base model features financial frictions and no recalibration of impatience $\beta$. The interest rate friction is set to a low value of $\psi = 0.01$. 

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4.4.3 Model with Financial Frictions and Recalibration

This section studies the effects of introducing heterogeneity, financial frictions, and explicitly recalibrating the impatience of households. For each level of idiosyncratic income risk $\sigma_e$, I recalibrate household impatience $\beta$ to match the aggregate savings target. As dictated by equation (4.22), the steady state interest rate is static across models because they feature the same aggregate savings.

Figure 4.14 compares the steady states for each value of $\sigma_e$. The required $\beta$ to rationalize $B_{ss} = 0$ is decreasing in $\sigma_e$. This occurs because an increase in $\sigma_e$ increases the precautionary savings motive and hence aggregate savings for a given $\beta$ and $r$. To return to the targeted steady state aggregate savings, a lower calibration of $\beta$ is required.\footnote{For the realistic case of $\sigma_e = 0.20$, the model in section 4.4.1 can use a larger $\beta$ because the borrowing constraint is much more generous and induces a smaller precautionary response.} The RA benchmark requires a $\beta$ of 0.926, imposed by the Euler equation of the RA model. The highest value $\sigma_e = 0.20$ requires a $\beta$ of 0.889.

The percentage of constrained households and the average MPC both monotonically increase with income risk. For the RA benchmark of $\sigma_e = 0.00$, 0% of households are constrained and households display an average MPC of 7.4%. For the empirically realistic case of $\sigma_e = 0.20$, 49% of households are constrained and households display an average MPC of 65%. This differs from...
Figure 4.14. With Recalibration: Effects of $\sigma_e$ on Steady State, $\psi = 0.01$

Section 4.4.1 and 4.4.2 because the recalibration of $\beta$ neutralizes aggregate precautionary savings effect. The only distributional effect that remains is an increase in the variance of asset holdings. As the variance increases, more households borrow up to the constraint and display a high MPC. Within the emerging markets HA literature, this presents the standard method of achieving an empirically realistic average MPC.

Figure 4.15 displays impulse responses to a 1% increase in the endowment with a persistence of 0.70. The models differ in their responses of consumption. With recalibration, a clear ordering emerges with respect to model responses. Higher income risk models feature a stronger and less persistent initial response of consumption. As determined by the resource constraint, the models with higher income risk display a weaker appreciation of the trade balance. Because the models feature identical permanent incomes, the medium and high income risk models display a quicker return of consumption to the steady state.

Figure 4.15 displays impulse responses to a 1% decrease in the interest rate with a persistence of 0.70. In this case, the initial consumption response to an interest rate shock decreases with
income risk. As with the endowment shock, the differing responses are driven by the portion of constrained households. In the low income risk model, the majority of households are unconstrained. These households feature a standard Euler equation where consumption varies directly with interest rates. As income risk increases, the portion of constrained households increases. These households do not display direct responses to the interest rate.

I now consider the case when $\psi$ is set to a higher value of 0.10. Figure 4.17 plots the impulse responses to the endowment shock. The ordering remains in which the models that feature nontrivial income risk display a stronger initial consumption response. In addition, the higher value of $\psi$ leads the models that feature nontrivial risk to display similar responses. A similar result occurs for Figure 4.18, which plots impulse responses to the interest rate shock.

Figure 4.19 plots the relative volatility of consumption conditional on an endowment shock for each value of $\sigma_e$. The presence of income risk and financial frictions leads to a 0.10 points increase in the relative volatility of consumption.
Figure 4.15. With Recalibration: IRF to Endowment Shock, $\psi = 0.01$

Notes: This figure displays responses to a 1% increase in the aggregate endowment for different values of income risk $\sigma_e$. The base model features financial frictions and recalibration of impatience $\beta$ to normalize aggregate savings. The interest rate friction is set to a low value of $\psi = 0.01$. 
Figure 4.16. With Recalibration: IRF to Interest Rate Shock, $\psi = 0.01$

Notes: This figure displays responses to a 1% decrease in the interest rate for different values of income risk $\sigma_e$. The base model features financial frictions and recalibration of impatience $\beta$ to normalize aggregate savings. The interest rate friction is set to a low value of $\psi = 0.01$. 

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Figure 4.17. With Recalibration: IRF to Endowment Shock, $\psi = 0.10$

Notes: This figure displays responses to a 1% increase in the aggregate endowment for different values of income risk $\sigma_e$. The base model features financial frictions and recalibration of impatience $\beta$ to normalize aggregate savings. The interest rate friction is set to a high value of $\psi = 0.10$. 
Figure 4.18. With Recalibration: IRF to Interest Rate Shock, $\psi = 0.10$

Notes: This figure displays responses to a 1% decrease in the interest rate for different values of income risk $\sigma_e$. The base model features financial frictions and recalibration of impatience $\beta$ to normalize aggregate savings. The interest rate friction is set to a low value of $\psi = 0.10$. 
Figure 4.19. With Recalibration: Volatility of Consumption Relative to Output and Share of Consumption by Constrained Households
4.5 Conclusion

This paper slowly develops a benchmark RA model into a benchmark HA model. Within the context of an open economy, I replicate Debortoli and Gali (2022)’s finding that pure heterogeneity delivers the same aggregate responses as a RA model. Heterogeneity and financial frictions can explain the percentage of liquidity constrained households and high average MPC that we observe in the data. However, they do not necessarily generate significant changes in the aggregate response of consumption. The standard calibration that includes heterogeneity, financial frictions, and re-calibration of impatience generates significant changes at both the household and aggregate level. The HA model displays a stronger initial response of consumption to an income shock because more households are constrained. In reverse, the HA model displays a weaker initial response to an interest rate shock because more households are constrained and feature a distorted Euler equation.
Bibliography


Sudden Stops with Heterogeneous Agents

A.1 Model

A.1.1 Impulse Responses

Figure A.1.1. Impulse Responses to Productivity Shock

Notes: This figure plots aggregate impulse responses to a 1% increase in productivity with a persistence of 0.53.
Figure A.1.2. Impulse Responses to Interest Rate Shock

Notes: This figure plots aggregate impulse responses to a 1% decrease in the interest rate with a persistence of 0.62.
Figure A.1.3. Impulse Responses to Sudden Stop

Notes: This figure plots impulses responses to the transitory shocks described in Table 1.4.
Figure A.1.4. Impulse Responses to Permanent Shock

Notes: This figure plots impulse responses to the permanent decline in productivity described in Table 1.4. For each variable, the impulse response is relative to its initial steady value.
Figure A.1.5. Decomposed Consumption-Income Elasticities: One-Period Elasticity

Notes: This figure decomposes the consumption responses of households to the transitory shocks described in Table 1.4.
Figure A.1.6. View II (Permanent): Decomposed Consumption Response

Notes: This figure decomposes the consumption responses of households to the permanent shock described in Table 1.4.

Figure A.1.7. View II (Permanent): Long Run Movement of Capital and Labor Income

Notes: This figure plots the responses of capital and labor to the permanent shock described in Table 1.4.
Figure A.1.8. Stronger Illiquid Asset Contribution: Drivers

Notes: This figure plots the general equilibrium paths of the illiquid asset return, the firm value, and the interest rate under the transitory sudden stop exercise for three values of capital adjustment costs, $\phi = 2.0, 5.0, 10.0$. For each value of $\phi$, I input the calibrated productivity shock of Table 1.4 and calibrate the increase in the interest rate $d\mu_0$ to target a one to one initial consumption to output response.
Figure A.1.9. Stronger Illiquid Asset Contribution: Consumption Responses

Notes: This figure plots the one and two period elasticities consumption income elasticities under the transitory sudden stop exercise three values of capital adjustment costs, $\phi = 2.0, 5.0, 10.0$. For each value of $\phi$, I input the calibrated productivity shock of Table 1.4 and calibrate the increase in the interest rate $d\mu_0$ to target a one to one initial consumption to output response.
Figure A.1.10. Stronger Illiquid Asset Contribution: Decomposed Consumption Responses

Notes: This figure decomposes the two year consumption-income elasticities within each income decile with respect to labor income, the interest rate, the illiquid asset return, and a nonlinearity term, conditional on the sudden stop. The sudden stop is recalibrated to included moderate capital adjustment costs, \( \phi = 5.0 \). The decomposed responses are computed by separately inputting in the general equilibrium paths of labor income, the interest rate and the illiquid return. The nonlinearity is computed as the difference between the consumption-income elasticity computed using all inputs and the sum of the consumption responses, weighted by the percentage change in income, conditional on each input.
Figure A.1.11. One and Three Year Consumption-Income Elasticities

Notes: This figure computes consumption-income elasticities while allowing for larger gaps of time between the first and second consumption and income policies that are used to compute the elasticities. For all elasticities, the first observation is given by consumption and income one period before the shock, which is given by the steady state. ‘One year’ computes the elasticities using the immediate consumption and income responses at time 0. ‘Two years’ computes elasticities using the consumption policies in periods 1 and −1, respectively. ‘Three years’ computes elasticities using the consumption policies in periods 2 and −1, respectively.
Notes: This figure plots two year consumption-income elasticities for three values of the initial increase in the interest rate, $dr_0 = 0.06, 0.12, 0.18$. Each elasticity features the same path of productivity as described in Table 1.4.
Figure A.1.13. Comparison with Single Asset Model: Steady State

Notes: This figure compares the steady state of the two asset household problem presented in Section 1.2.1 with three calibrations of the single asset problem presented in Section 1.4.6. ‘\( \beta \) matching’ uses the discount factor of the calibrated two asset model. ‘Constrained matching’ and ‘MPC matching’ recalibrate \( \beta \) to match the percent of constrained household and average MPC of the two asset model, respectively. The left panel plots the percent of constrained households within each income decile under each model. The right panel plots the average MPC within each income decile.
Figure A.1.14. Single Asset Model: Consumption Responses

Notes: This figure plots the consumption responses of the single asset model presented in Section 1.4.6 to transitory shocks in labor income and the interest rate. Labor income $wL$ follows the endogenous labor income path generated by the transitory shocks of Table 1.4. The interest rate features a variable initial increase and reverts to its steady state with a persistence of 0.62. Darker lines correspond to a higher increase in the interest rate, with an initial increase of 0.01 and maximum increase of 0.15. The left panel plots one year consumption-income elasticities. The right panel plots two year consumption-income elasticities.
Figure A.1.15. Single Asset Model: The Role of Persistence

Notes: This figure studies how household responses vary with the persistence of aggregate shocks in the single asset household problem. Average income and idiosyncratic income risk match their counterparts in the steady state of the two asset model.

The upper two plots display the one and two period responses to a 5.4% decline in productivity with persistence values of 0.50, 0.70, 0.80, 0.90. For each value of the productivity persistence, Households respond to the general equilibrium path of labor income in the two asset model. The lower two plots display the one and two period responses to a 12% percent increase in the interest rate with persistence values of 0.50, 0.70, 0.80, 0.90.
A.2 Data

A.2.1 Aggregate

Figure A.2.1. Growth Rates, Mexican Peso Crisis

Notes: This figure plots the year over year growth rates of consumption, GDP, investment, and the trade balance to GDP ratio. All variables are in real, seasonally adjusted, and in per capita terms. Quarterly population is computed by linearly interpolating the log level of the annual population. Vertical line on 1995:Q2. Source: IMF-IFS.
Figure A.2.2. Aggregates, Mexican Peso Crisis

Notes: This figure plots the cyclical components of aggregate consumption, GDP, investment, and the trade balance to GDP ratio in Mexico during the mid 1990s. All variables are in real, seasonally adjusted, in per capita terms and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Quarterly population is computed by linearly interpolating the log level of the annual population. Vertical line on 1995:Q2. Source: IMF-IFS
Figure A.2.3. Annual Cyclical Components, Mexican Peso Crisis

Notes: This figure plots the annual cyclical components of the Mexican Peso Crisis. All variables are in real, per capita log levels and detrended using the HP Filter with a smoothing parameter of 6.25. Vertical line on 1995. Source: WDI
A.2.2 ENIGH: Heterogeneity in Responses

The model is given by

\[ X_{it} = \alpha + Z_{it} + POST_t D_{it} + \gamma_{it} + POST_t \beta_{it} + \epsilon_{it} \]  

(A.1)

where \( Z_{it} \) is a set of controls that includes the sex, education, and a quadratic function of the age of the household head. POST denotes whether the year is 1996. POST\(_t\) \( D_{it} \) includes the sex and education of the household head, interacted with POST\(_t\). \( \gamma_{it} \) denotes the household’s income decile, and POST\(_t\) \( \beta_{it} \) denotes the household’s income decile, interacted with POST\(_t\).

Figure A.2.4. ENIGH: Consumption and Income Fluctuations, by Decile

Notes: This figure plots the percentage change in consumption and income for each income decile using the 1994 and 1996 data of ENIGH. Consumption and income are residualized by household size, locality size, and the sex, education, and a quadratic function of the age of the household head. The average percentage change of consumption or income for each income decile is obtained by interacting the income decile with an indicator for whether the period is 1996.
<table>
<thead>
<tr>
<th>Code</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>tb36a_2</td>
</tr>
<tr>
<td>Wages</td>
<td>tb36aa_2</td>
</tr>
<tr>
<td>Piecework</td>
<td>tb36ab_2</td>
</tr>
<tr>
<td>Tips</td>
<td>tb36ac_2</td>
</tr>
<tr>
<td>Extra Hours</td>
<td>tb36ad_2</td>
</tr>
<tr>
<td>Christmas Bonus</td>
<td>tb36ae_2</td>
</tr>
<tr>
<td>Annual Bonus</td>
<td>tb36af_2</td>
</tr>
<tr>
<td>Profit Distribution</td>
<td>tb36ah_2</td>
</tr>
<tr>
<td>Other</td>
<td>tb36am_21</td>
</tr>
<tr>
<td>Second Job</td>
<td>tb36b_2</td>
</tr>
<tr>
<td>Net Income Main Business</td>
<td>tb36p2_2</td>
</tr>
<tr>
<td>Net Income Second Business</td>
<td>tb36s2_2</td>
</tr>
</tbody>
</table>

Table A.1. MFL: Income Sources

*Notes:* This table documents income sources in MFL. For the majority of households, income is documented under 'income'. Otherwise, income is decomposed under wages, piecework, tips, etc.

A.2.3 MFL: Heterogeneity in Asset Holdings
<table>
<thead>
<tr>
<th>Asset</th>
<th>Possession</th>
<th>Value</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>ah03a</td>
<td>ah04a_2</td>
<td>Property</td>
</tr>
<tr>
<td>Other House</td>
<td>ah03b</td>
<td>ah04b_2</td>
<td>Property</td>
</tr>
<tr>
<td>Bicycle</td>
<td>ah03c</td>
<td>ah04c_2</td>
<td>Durable</td>
</tr>
<tr>
<td>Vehicle</td>
<td>ah03d</td>
<td>ah04d_2</td>
<td>Durable</td>
</tr>
<tr>
<td>Electronics</td>
<td>ah03e</td>
<td>ah04e_2</td>
<td>Durable</td>
</tr>
<tr>
<td>Washing Machine/Stove</td>
<td>ah03f</td>
<td>ah04f_2</td>
<td>Durable</td>
</tr>
<tr>
<td>Domestic Appliance</td>
<td>ah03g</td>
<td>ah04g_2</td>
<td>Durable</td>
</tr>
<tr>
<td>Financial Asset</td>
<td>ah03h</td>
<td>ah04h_2</td>
<td>Financial</td>
</tr>
<tr>
<td>Machinery</td>
<td>ah03i</td>
<td>ah04i_2</td>
<td>Durable</td>
</tr>
<tr>
<td>Bull/Cow</td>
<td>ah03j</td>
<td>ah04j_2</td>
<td>Animal</td>
</tr>
<tr>
<td>Horses/Mules</td>
<td>ah03k</td>
<td>ah04k_2</td>
<td>Animal</td>
</tr>
<tr>
<td>Pigs/Goats</td>
<td>ah03l</td>
<td>ah04l_2</td>
<td>Animal</td>
</tr>
<tr>
<td>Poultry</td>
<td>ah03m</td>
<td>ah04m_2</td>
<td>Animal</td>
</tr>
<tr>
<td>Other</td>
<td>ah03n</td>
<td>ah04n_2</td>
<td></td>
</tr>
</tbody>
</table>

Table A.2. MFL: Illiquid Assets

*Notes:* This table documents how illiquid assets are classified. ‘Possession’ is an indicator for whether a household possesses a type of illiquid asset. ‘Value’ lists the estimated value of the asset. ‘Type’ refers to the asset’s classification.

<table>
<thead>
<tr>
<th>Savings Location</th>
<th>Variable</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Savings</td>
<td>cr29_1a</td>
<td></td>
</tr>
<tr>
<td>Bank</td>
<td>cr29_1b</td>
<td>Formal</td>
</tr>
<tr>
<td>Coop</td>
<td>cr29_1c</td>
<td>Formal</td>
</tr>
<tr>
<td>Savings Bank</td>
<td>cr29_1d</td>
<td>Formal</td>
</tr>
<tr>
<td>Friend (not household head)</td>
<td>cr29_1e</td>
<td>Informal</td>
</tr>
<tr>
<td>afores</td>
<td>cr29_1f</td>
<td>Formal</td>
</tr>
<tr>
<td>caja solidaria</td>
<td>cr29_1g</td>
<td>Formal</td>
</tr>
<tr>
<td>At House</td>
<td>cr29_1h</td>
<td>Informal</td>
</tr>
<tr>
<td>Work</td>
<td>cr29_1i</td>
<td>Informal</td>
</tr>
<tr>
<td>Other</td>
<td>cr29_1j</td>
<td>Informal</td>
</tr>
</tbody>
</table>

Table A.3. MFL: Has Savings

*Notes:* This table documents savings types in MFL and whether they are classified as formal or informal.
<table>
<thead>
<tr>
<th>Loan Location</th>
<th>Value</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank</td>
<td>1</td>
<td>Formal</td>
</tr>
<tr>
<td>Savings Fund</td>
<td>2</td>
<td>Formal</td>
</tr>
<tr>
<td>Moneylender</td>
<td>3</td>
<td>Formal</td>
</tr>
<tr>
<td>Relative</td>
<td>4</td>
<td>Informal</td>
</tr>
<tr>
<td>Friends</td>
<td>5</td>
<td>Informal</td>
</tr>
<tr>
<td>Work</td>
<td>6</td>
<td>Informal</td>
</tr>
<tr>
<td>Pawn Shop</td>
<td>7</td>
<td>Formal</td>
</tr>
<tr>
<td>Verbal Agreement Credit Program</td>
<td>8</td>
<td>Informal</td>
</tr>
<tr>
<td>Other Government</td>
<td>9</td>
<td>Informal</td>
</tr>
<tr>
<td>Other</td>
<td>10</td>
<td>Informal</td>
</tr>
</tbody>
</table>

Table A.4. MFL: Loan Sources

Notes: This table documents loan types in MFL and whether they are classified as formal or informal.

<table>
<thead>
<tr>
<th>Loan Type</th>
<th>Percent of Loans that Charge Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank</td>
<td>62</td>
</tr>
<tr>
<td>Savings Fund</td>
<td>82</td>
</tr>
<tr>
<td>Money Lender</td>
<td>75</td>
</tr>
<tr>
<td>Relative</td>
<td>10</td>
</tr>
<tr>
<td>Friends</td>
<td>36</td>
</tr>
<tr>
<td>Work</td>
<td>22</td>
</tr>
<tr>
<td>Pawnshop</td>
<td>66</td>
</tr>
<tr>
<td>Other Government</td>
<td>32</td>
</tr>
<tr>
<td>Other</td>
<td>27</td>
</tr>
</tbody>
</table>

Table A.5. MFL: Percent of Loans that Charge Interest, by Type

Notes: This table documents the percentage of loans that charge interest for each loan type. Loans are taken from individual level data in MFL. Computations are weighted using the panel weights provided in the data.
### Table A.6. MFL: Income and Financial Inclusion, No Controls

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd Income Quartile</td>
<td>0.079***</td>
<td>-0.036***</td>
<td>0.264***</td>
<td>0.090***</td>
<td>0.123***</td>
<td>0.025***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>3rd Income Quartile</td>
<td>0.286***</td>
<td>0.275***</td>
<td>0.106***</td>
<td>0.112***</td>
<td>0.063***</td>
<td>0.114***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>4th Income Quartile</td>
<td>0.837***</td>
<td>0.831***</td>
<td>0.596***</td>
<td>0.362***</td>
<td>0.173***</td>
<td>0.375***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Controls</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Observations</td>
<td>2946</td>
<td>2946</td>
<td>2946</td>
<td>2946</td>
<td>2946</td>
<td>2946</td>
</tr>
</tbody>
</table>

*Note:* *p<0.1; **p<0.05; ***p<0.01

Table A.6. MFL: Income and Financial Inclusion, No Controls

**Notes:** This table repeats the probit regression of Table 1.5 without controls.

### Table A.7. MFL: Income and Illiquid Assets

<table>
<thead>
<tr>
<th></th>
<th>Durable</th>
<th>Property</th>
<th>Animal</th>
<th>Financial</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd Income Quartile</td>
<td>0.311***</td>
<td>-0.076***</td>
<td>-0.400***</td>
<td>-0.064***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>3rd Income Quartile</td>
<td>0.273***</td>
<td>-0.002**</td>
<td>-0.529***</td>
<td>0.395***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>4th Income Quartile</td>
<td>0.465***</td>
<td>0.109***</td>
<td>-0.632***</td>
<td>0.869***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Controls</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Observations</td>
<td>2946</td>
<td>2946</td>
<td>2946</td>
<td>2946</td>
</tr>
</tbody>
</table>

*Note:* *p<0.1; **p<0.05; ***p<0.01

Table A.7. MFL: Income and Illiquid Assets

**Notes:** This table repeats the probit regression of Table 1.6 without controls.
A.3 Conditions

A.3.1 Steady State Conditions

\[ z_{ss} = 1 \quad (A.3.1.2) \]
\[ \mu_{ss} = 1 \quad (A.3.1.3) \]
\[ r_{ss} = r^* \quad (A.3.1.4) \]
\[ r^b_{ss} = r^* \quad (A.3.1.5) \]

\[ \log(e') = \rho_e \log e + \sigma_e \epsilon_e, \epsilon_e \sim \mathcal{N}(0, 1) \quad (A.3.1.6) \]

\[ u(c_{ss}(e, b, a)) = \mu^b_{ss}(e, b, a) + \beta(1 + r^*)E u(c_{ss}(e', b', a')) \quad (A.3.1.7) \]

\[ u(c_{ss}(e, b, a))(1 + \chi_1(a_{ss}(e, b, a), a)) = \mu^a_{ss}(e, b, a) + \beta E(1 + r^a - \chi_2(a_{ss}(e, b, a), a))u(c_{ss}) \quad (A.3.1.8) \]

\[ c_{ss}(e, b, a) + b_{ss}(e, b, a) + a_{ss}(e, b, a) + \chi(a_{ss}(e, b, a), a) = (1 + r^*)b + (1 + r^a)a + e\omega L \quad (A.3.1.9) \]

\[ \mu^b_{ss}(e, b, a) \geq 0 \quad (A.3.1.10) \]
\[ \mu^a_{ss}(e, b, a) \geq 0 \quad (A.3.1.11) \]
\[ \mu^b_{ss}(e, b, a)b_{ss}(e, b, a) = 0 \quad (A.3.1.12) \]
\[ \mu^a_{ss}(e, b, a)a_{ss}(e, b, a) = 0 \quad (A.3.1.13) \]

\[ \Psi_{ss}(e, b, a) = Pr(e \leq a, a \leq b, b \leq b) \quad (A.3.1.14) \]

\[ C_{ss} = \int_{e,b,a} c_{ss}(e, b, a)d\Psi_{ss}(e, b, a) \quad (A.3.1.15) \]
\[
B_{ss} = \int_{e,b,a} b_{ss}(e, b, a) d\Psi_{ss}(e, b, a)
\]

(A.3.1.16)

\[
A_{ss} = \int_{e,b,a} a_{ss}(e, b, a) d\Psi_{ss}(e, b, a)
\]

(A.3.1.17)

\[
\chi_{ss} = \int_{e,b,a} \chi(a_{ss}(e, b, a), a) d\Psi_{ss}(e, b, a)
\]

(A.3.1.18)

\[
Y_{ss} = z_{ss} K_{ss}^{\alpha} L_{ss}^{1-\alpha}
\]

(A.3.1.19)

\[
\pi_{ss} + K_{ss} = z_{ss} K_{ss}^{\alpha} L_{ss}^{1-\alpha} + (1 - \delta) K_{ss} - w_{ss} L_{ss}
\]

(A.3.1.20)

\[(1 + r_{ss}^\alpha) = z_{ss} \alpha K_{ss}^{\alpha-1} L_{ss}^{1-\alpha} + 1 - \delta\] (A.3.1.21)

\[
w_{ss} = z_{ss}(1 - \alpha) K_{ss}^\alpha L_{ss}^{-\alpha}
\]

(A.3.1.22)

\[
L_{ss} = 1
\]

(A.3.1.23)

\[
w_{ss} = \kappa(L_{ss})^\omega
\]

(A.3.1.24)

\[
I_{ss} = \delta K_{ss}
\]

(A.3.1.25)

\[1 + r_{ss}^\alpha = \frac{q_{ss} + \pi_{ss} + \chi_{ss}}{q_{ss}}\] (A.3.1.26)

\[
A_{ss} = q_{ss}
\]

(A.3.1.27)

\[
T B_{ss} = B_{ss} - (1 + r_{ss}) B_{ss}
\]

(A.3.1.28)

\[
T B Y_{ss} = T B_{ss} / Y_{ss}
\]

(A.3.1.29)
A.3.2 List of Equilibrium Conditions

\[ \log e_t = \rho e \log e_{t-1} + \sigma e \epsilon_t^e, \epsilon_t^e \sim \mathcal{N}(0, 1) \]  
(A.3.2.1)

\[ u(c_t(e, b, a)) = \mu_t^b + \beta E_t(1 + r_t^b)u(c_{t+1}(e, b, a)) \]  
(A.3.2.2)

\[ u(c_t(e, b, a))(1 + \chi_1(a_t(e, b, a), a)) = \]  
(A.3.2.3)

\[ \mu_t^a + \beta E_t(1 + r_{t+1}^a) - \chi_2(a_{t+1}(e', b', a'), a_t(e, b, a))u(c_{t+1}(e', b_t(e, b, a), a_t(e, b, a))) \]  
(A.3.2.4)

\[ c_t(e, b, a) + b_t(e, b, a) + a_t(e, b, a) + \chi(a_t(e, b, a), a) = (1 + r_{t+1}^b)b + (1 + r_{t}^a)a + ew_tL_t \]  
(A.3.2.5)

\[ \Psi_t(e, a, b) = Pr(e_t \leq e, a_{t-1} \leq a, b_{t-1} \leq b) \]  
(A.3.2.6)

\[ \Psi_{t+1}(e', b', a') = \]  
(A.3.2.7)

\[ \int_{e,b,a} Pr(e_{t+1} \leq e'|e_t = e)\mathcal{I}[a_t(e, b, a; \Gamma) \leq a', b_t(e, b, a; \Gamma) \leq b'] d\Psi_t(e, b, a) \]  
(A.3.2.8)

\[ C_t = \int_{e,b,a} c_t(e, b, a)d\Psi_t(e, b, a) \]  
(A.3.2.9)

\[ B_t = \int_{e,b,a} b_t(e, b, a)d\Psi_t(e, b, a) \]  
(A.3.2.10)

\[ A_t = \int_{e,b,a} a_t(e, b, a)d\Psi_t(e, b, a) \]  
(A.3.2.11)
$$\chi_t = \int_{e,b,a} \chi_t(a_t(e,b,a), a) d\Psi_t(e,b,a)$$  
(A.3.2.14)

$$Y_t = z_t K_{t-1}^{\alpha} L_t^{-\alpha}$$  
(A.3.2.15)

$$\pi_t + K_t + \Phi(K_t, K_{t-1}) = z_t K_{t-1}^{\alpha} L_t^{-\alpha} + (1 - \delta) K_{t-1} - w_t L_t$$  
(A.3.2.16)

$$(1 + r_{t+1}^a)(1 + \Phi_1(K_t, K_{t-1})) = E_t \left( z_{t+1} \alpha K_t^{\alpha} L_{t+1}^{-\alpha} + 1 - \delta - \Phi_2(K_{t+1}, K_t) \right)$$  
(A.3.2.17)

$$w_t = z_t (1 - \alpha) K_{t-1}^{\alpha} L_t^{-\alpha}$$  
(A.3.2.18)

$$w_t = \kappa(L_t)^\omega$$  
(A.3.2.19)

$$I_t = K_t - (1 - \delta) K_{t-1}$$  
(A.3.2.20)

$$1 + r_t^a = \frac{q_t + \pi_t + \chi_t}{q_{t-1}}$$  
(A.3.2.21)

$$r_t = r^* + \mu_t - 1$$  
(A.3.2.22)

$$r_t^b = r_{t-1}$$  
(A.3.2.23)

$$A_t = q_t$$  
(A.3.2.24)

$$T B_t = B_t - (1 + r_t) B_{t-1}$$  
(A.3.2.25)

$$T B Y_t = T B_t / Y_t$$  
(A.3.2.26)
A.4 Decomposition Methodology

Elasticities As discussed in Section 3.2, the path of aggregate shocks \( z = \{ z_t \}_{t=0}^{T} \), \( \mu = \{ \mu_t \}_{t=0}^{T} \) produces the general equilibrium path of households inputs \( \Gamma = \{ w_t L_t, r^b_t, r^a_t \}_{t=0}^{T} \). Given \( \Gamma \), I compute the path of the distribution \( \{ \Psi_t(e, b, a; \Gamma) \}_{t=0}^{T} \), and policies \( \{ c_t(e, b, a; \Gamma) \}_{t=0}^{T}, \{ b_t(e, b, a; \Gamma) \}_{t=0}^{T}, \{ a_t(e, b, a; \Gamma) \}_{t=0}^{T} \). I then integrate over the liquid and illiquid assets to form the marginal distributions with respect to income and aggregates policies for each level of income: \( \{ \Psi_t(e; \Gamma) \}_{t=0}^{T}, \{ C_t(e; \Gamma), B_t(e; \Gamma), A_t(e; \Gamma) \}_{t=0}^{T} \) where

\[
\Psi_t(e; \Gamma) = \int_b^a \int_a d\Psi_t(e, b, a; \Gamma) \tag{A.4.1}
\]

\[
C_t(e; \Gamma) = \int_b^a \int_a d\Psi_t(e, b, a; \Gamma) d\Psi_t(e, b, a; \Gamma) \tag{A.4.2}
\]

and similarly for \( B_t(e; \Gamma), A_t(e; \Gamma) \). Finally, I interpolate along the distribution of \( e \psi_e(e; \Gamma) \) to compute the average policy of the \( j \)th income decile, \( \{ C_t(j; \Gamma), B_t(j; \Gamma), A_t(j; \Gamma) \}_{t=0}^{T} \) for \( j = 1, \ldots, 10 \) at each time \( t \).

Dropping \( \Gamma \), the percentage change in consumption of decile \( j \) after \( t \) periods is given by

\[
\mathcal{E}_{C,t}(j) = \frac{C_t(j) - C_{-1}(j)}{C_{-1}(j)} \tag{A.4.3}
\]

where \( C_{-1}(j) \) is the consumption of decile \( j \) before impact. Given \( \{ w_t, L_t \}_{t=0}^{T} \), the percentage change in labor income of decile \( j \) is

\[
\mathcal{E}_{wL,t}(j) = \frac{e_t(j)w_tL_t - e_{-1}(j)w_{-1}L_{-1}}{e_{-1}(j)w_{-1}L_{-1}} = \frac{w_tL_t - w_{-1}L_{-1}}{w_{-1}L_{-1}} \tag{A.4.4}
\]

where I’ve applied that the distribution of idiosyncratic income is exogenous and static, e.g. \( e_t(j) = e_{-1}(j) \) for each income decile \( j \) and time period \( t \). This implies all income deciles experience the

\footnote{After integrating over \( b \) and \( a \), the distribution \( \Psi_t(e) \) is static and given by the stationary distribution of equation (4.26).}

\footnote{In the basic exercise, \( C_{-1}(j) \) coincides with the steady state \( C_{ss}(j) \).}
same percentage change in income. Given the percentage change in consumption over two years \( \mathcal{E}_{C,1}(j) \) and the percentage change in income over two years \( \mathcal{E}_{wL,1}(j) \), I can compute the two-year consumption-income elasticity for decile \( j \) as \( \mathcal{E}_1(j) = \mathcal{E}_{C,1}(j)/\mathcal{E}_{wL,1}(j) \). Similarly, we can compute the one-year elasticity as \( \mathcal{E}_0(j) = \mathcal{E}_{C,0}(j)/\mathcal{E}_{wL,0}(j) \).

**Decomposition** Given \( \{w_tL_t, r^b_t, r^a_t\}_{t=0}^T \), the consumption response of household \((e, b, a)\) at time \( t \) can be decomposed as

\[
c_t(e, b, a; wL, r^b, r^a) = c_t(e, b, a; wL) + c_t(e, b, a; r^b) + c_t(e, b, a; r^a) + \epsilon(e, b, a; wL, r^b, r^a) \tag{A.4.5}
\]

where \( c_t(e, b, a; wL) \) denotes the consumption response when only the path of \( \{w_tL_t\}_{t=0}^T \) deviates from the steady state and similarly for \( r^b_t \) and \( r^a_t \). As before, I integrate with respect to \( b \) and \( a \) and interpolate over \( e \) to decompose the average consumption within each income decile \( j \) as

\[
C_t(j; wL, r^b, r^a) = C_t(j; wL) + C_t(j; r^b) + C_t(j; r^a) + \epsilon_t(j; wL, r^b, r^a). \tag{A.4.6}
\]

We can then compute the percentage change in consumption

\[
\mathcal{E}_{C,t}(j|X) = \frac{C_t(j; X) - C_{-1}(j)}{C_{-1}(j)} \tag{A.4.7}
\]

for \( X = wL, r, r^a \). Given the percentage change in labor income given in equation the time \( t \) consumption-income elasticity for income decile \( j \) is decomposed as

\[
\mathcal{E}_t(j; \Gamma) = \underbrace{\frac{\mathcal{E}_{C,t}(j; wL)}{\mathcal{E}_{wL,t}(j)}}_{\text{Contribution of } wL} + \underbrace{\frac{\mathcal{E}_{C,t}(j; r^b)}{\mathcal{E}_{wL,t}(j)}}_{\text{Contribution of } r^b} + \underbrace{\frac{\mathcal{E}_{C,t}(j; r^a)}{\mathcal{E}_{wL,t}(j)}}_{\text{Contribution of } r^a} + \underbrace{\epsilon_t(j; \Gamma)}_{\text{Nonlinearity}} \tag{A.4.8}
\]
Output Volatility and Borrowing Constraints

B.1 Unconstrained Economy

I compare the constrained economy to an unconstrained economy where borrowing is not limited by a collateral constraint. When computing the response of borrowing to a change in volatility, the response of the unconstrained economy benchmarks the precautionary savings response. The unconstrained household problem can be written as

\[ V(y^T, d, \sigma) = \max_{c^T, d'} u(c(c^T, c^N)) + \beta E \left[ V(y^{T'}, d', \sigma') | y, \sigma \right] \]

\[ d + c^T = y^T + \frac{d'}{1 + r} \]  

(B.1.1)

Several features of the constrained economy have been removed, simplifying the solution method of the unconstrained economy. First, households are no longer subject to the collateral constraint, only the intertemporal budget constraint. Secondly, the unconstrained problem is independent of the relative price \( p^N \). Households cannot use borrowing to exchange between tradable consumption and nontradable consumption, so the relative price is irrelevant. In the constrained economy the relative price is only relevant as it determines the international value of nontradable output.
B.2 Socially Optimal Economy

The social planner solves

\[
V(y^T, d, \sigma) = \max_{c^T, d'} u(c(c^T, c^N)) + \beta E \left[ V(y^{T'}, d', \sigma')|y, \sigma \right]
\]

\[
d + c^T = y^T + \frac{d'}{1+r}
\]

\[
d_{t+1} \leq \kappa \left( y_t^N \left( \frac{y_t^T + q d_{t+1} - d_t (\lambda + (1 - \lambda) q)}{y_t^N} \right)^{1/\xi} + y_t^T \right)
\]

\[(B.2.1)\]

Figure B.2.1. Industrial Production Series
Figure B.2.2. Posterior Distribution of $\theta$
Figure B.2.3. Distribution of Borrowing

Figure B.2.4. Response to a Decrease in Volatility
Figure B.2.5. Response to an Increase in Output

Figure B.2.6. Response to a Decrease in Output
Figure B.2.7. Comparison of Unconditional Output Distributions
Income Risk in Emerging Markets

Figure C.0.1. Decomposed Sudden Stop in HA Economy: by Income Decile

Notes: This figure plots the responses of individual income deciles during a sudden stop episode. Within the RA model, a sudden stop at time $t$ is defined as when the collateral constraint binds and the trade balance to output ratio increases by two percentage points. Time series windows for the HA model are drawn from sudden stop windows in the RA model.