Biological learning in key-value memory networks

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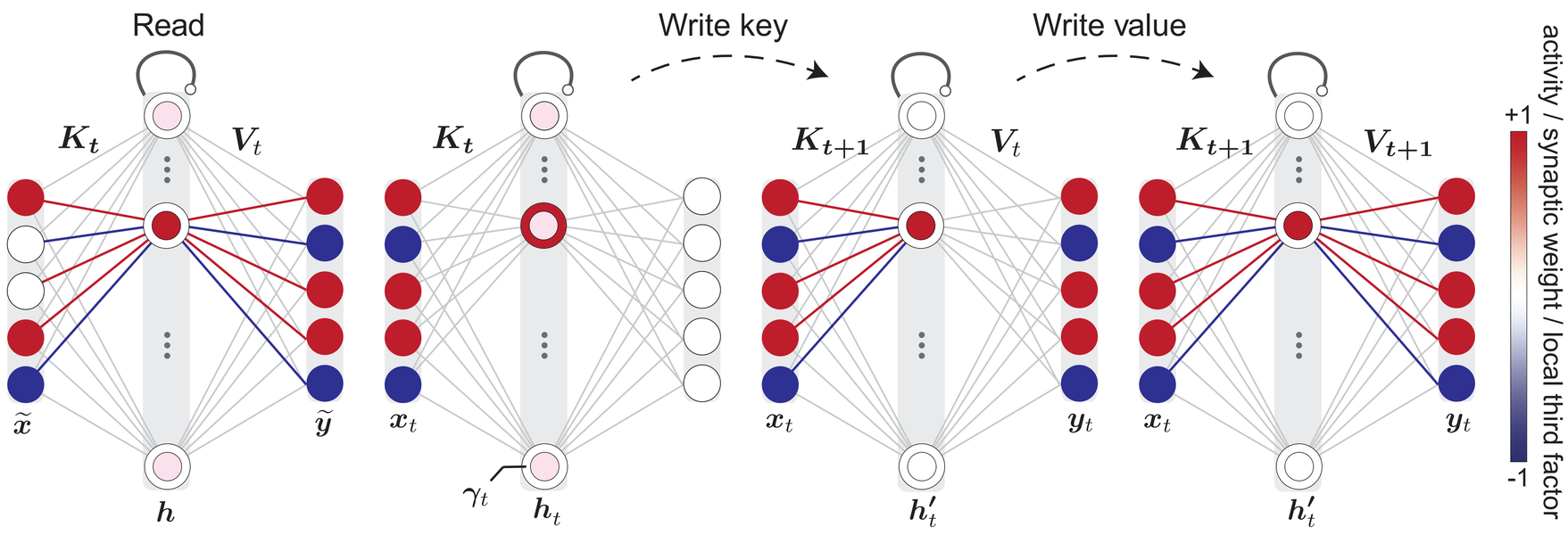
In neuroscience, Hopfield networks [1] are the classical biologically plausible model of long-term memory, relying on Hebbian plasticity for storage and attractor dynamics for recall. In contrast, memory networks in machine learning commonly use a key-value mechanism [2], but it has remained unclear whether it can be implemented by biological systems. Here, we propose a set of of **biologically plausible three-factor plasticity rules** [3] for a simple **key-value memory network** [4] which performs on par with classical Hopfield networks, and can be naturally extended to more complex tasks -- including manipulation of indivudual stored memories -- or embedded in a larger neural system. Overall, our results suggest a compelling alternative to the classical Hopfield network as a **model of biological long-term memory**.

Key-value memory storage and retrieval



$$\boldsymbol{h} = \mathrm{softmax}(\boldsymbol{K}_t \tilde{\boldsymbol{x}})$$

$$\tilde{\boldsymbol{y}} = \boldsymbol{V}_t \boldsymbol{h} = \sum_{i=1}^{N} h_i \boldsymbol{y}_i$$

$$\boldsymbol{K}_{t+1} = (1 - \boldsymbol{\eta}_t^{\mathrm{K}}) \odot \boldsymbol{K}_t + \boldsymbol{\eta}_t^{\mathrm{K}} \odot [\mathbf{1} \boldsymbol{x}_t^T]$$

$$[\boldsymbol{\eta}_t^{\mathrm{K}}]_{ij} = q_t [\boldsymbol{\gamma}_t]_i$$

$$q_t \in \{0, 1\} \qquad [\boldsymbol{\gamma}_t]_i \in \{0, 1\}$$

global third factor (external input)

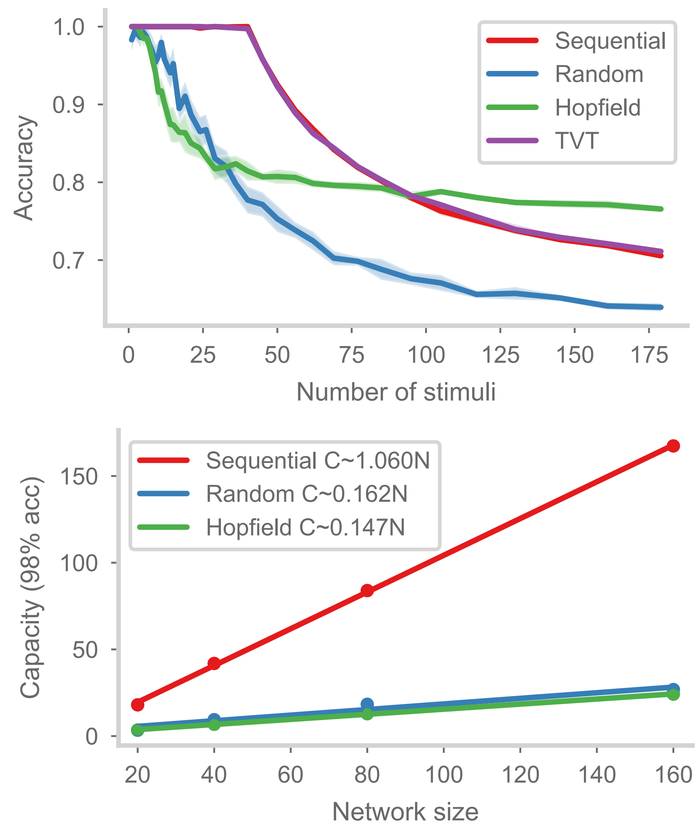
local third factor (**sequential** or **random**)

$$\boldsymbol{V}_{t+1} = (1 - \boldsymbol{\eta}_t^{\mathrm{V}}) \odot \boldsymbol{V}_t + \boldsymbol{\eta}_t^{\mathrm{V}} \odot \boldsymbol{y}_t (\boldsymbol{h}'_t)^T$$

$$\boldsymbol{h}'_t = \mathrm{softmax}(\boldsymbol{K}_{t+1} \boldsymbol{x}_t)$$

$$[\boldsymbol{\eta}_t^{\mathrm{V}}]_{ki} = q_t [\boldsymbol{\gamma}_t]_i$$

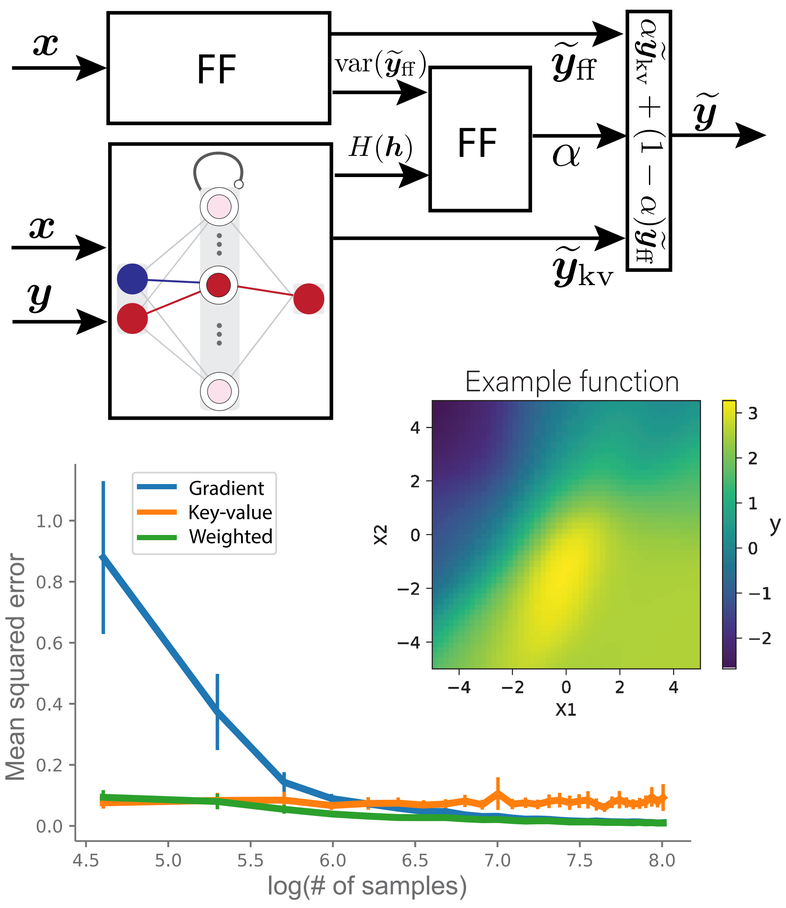
Associative memory



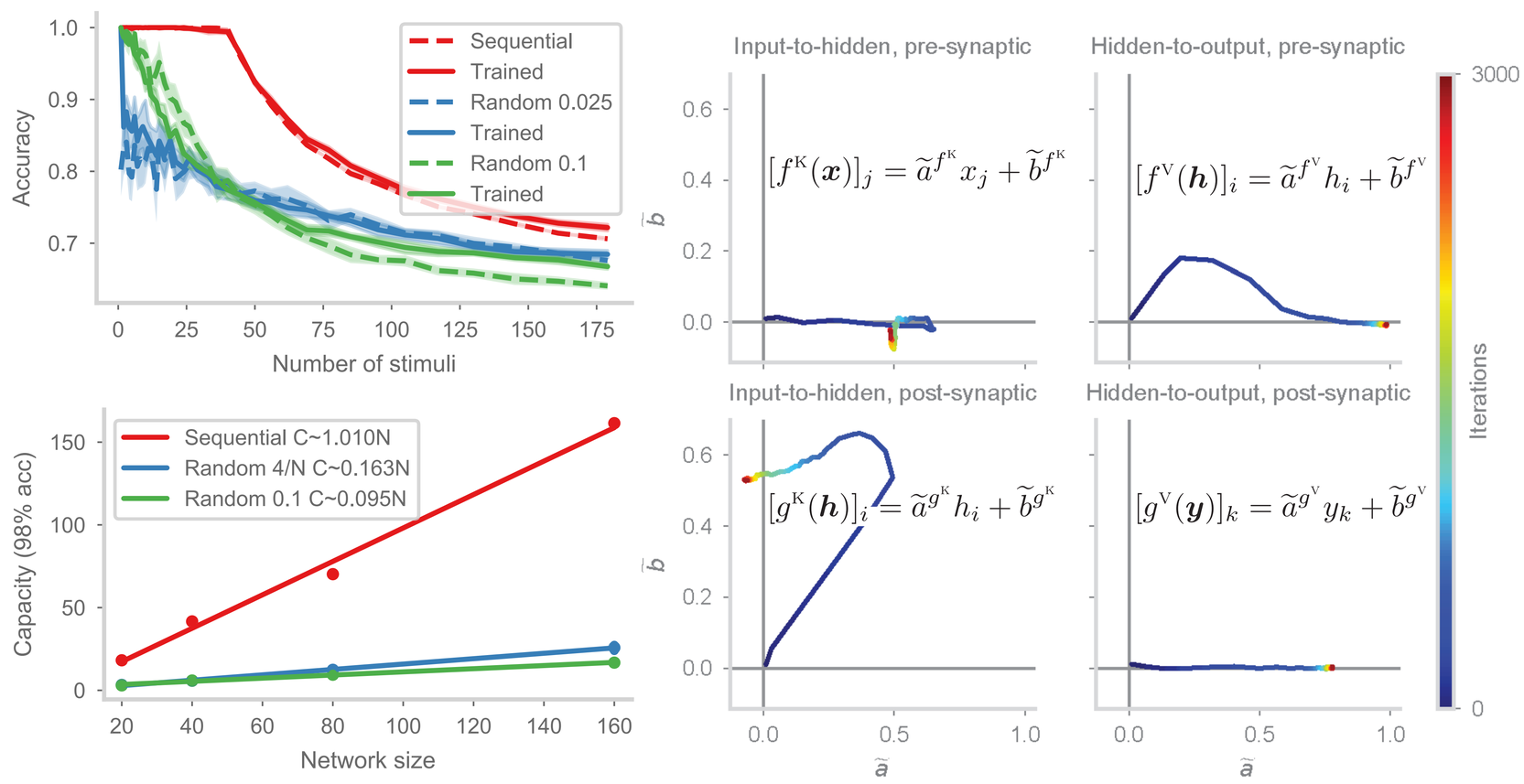
Performance for both sequential and random local third factors is on par with Hopfield networks, and identical to a machine learning key-value memory network (TVT) [5]. Capacity **scales linearly** with the number of hidden neurons, as with the Hopfield network, but with the advantage of **decoupled input dimension and memory capacity**.

Fast memory sub-network

Using our network in conjunction with a gradient-based network to learn smooth functions. With few samples, the **fast-learning key-value network** outperforms the gradient network, but with more samples the **slower gradient network** generalizes. Weighting the sub-network outputs based on their confidence allows inference of the output better than either one alone.



Meta-learned plasticity rules

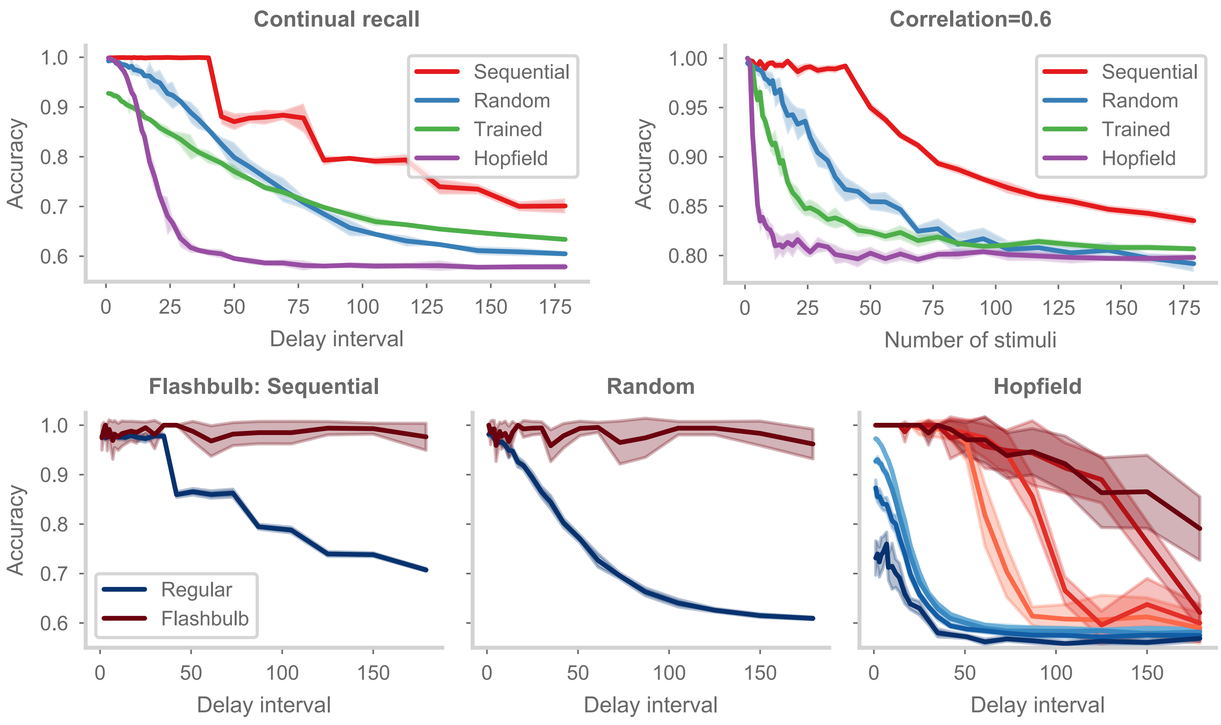


Plasticity rules meta-learned via gradient descent are qualitatively similar to the hand-designed network: **presynaptic non-Hebbian** for writing keys, and **Hebbian** for writing values. Performance and capacity of the meta-learned network remain unchanged.

$$\boldsymbol{K}_{t+1} = (1 - \boldsymbol{\eta}_t^{\mathrm{K}}) \odot \boldsymbol{K}_t + \boldsymbol{\eta}_t^{\mathrm{K}} \odot [g^{\mathrm{K}}(\boldsymbol{h}_t) f^{\mathrm{K}}(\boldsymbol{x}_t)^T]$$

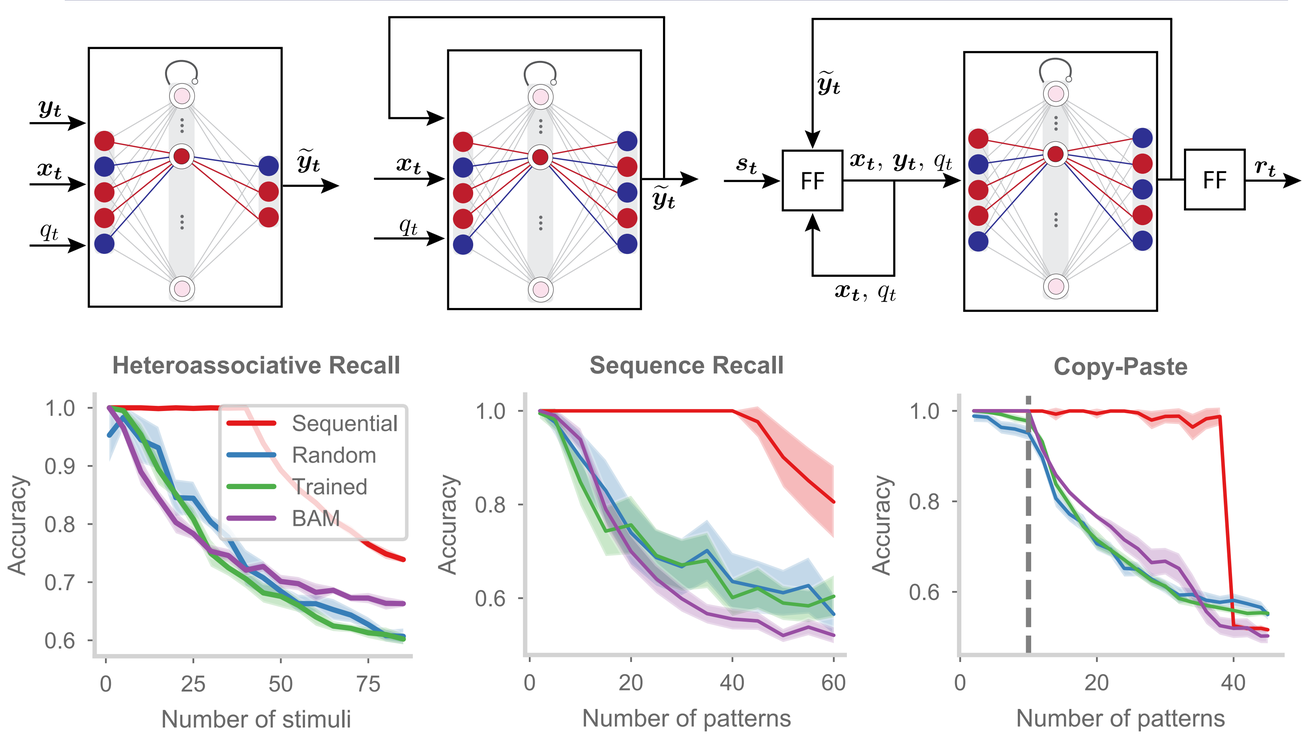
$$\boldsymbol{V}_{t+1} = \boldsymbol{\lambda}_t \odot \boldsymbol{V}_t + q_t \tilde{\boldsymbol{\eta}}^{\mathrm{V}} \odot [g^{\mathrm{V}}(\boldsymbol{y}_t) f^{\mathrm{V}}(\boldsymbol{h}'_t)^T]$$

Complex stimuli



Unlike the Hopfield network's fully distributed storage, our network's factorized architecture **avoids catastrophic forgetting** during continual recall, **reduces interference** between correlated memories, and **allows manipulating individual memories** through simple meta-plasticity for extended storage of "flashbulb memories".

Heteroassociative tasks



With distinct input and output layers, our network can be naturally extended to **heteroassociatve memory**. Including recurrent connections from the output to the input, the network performs **sequence learning**. Embedding our network in a larger system enables storing and recalling **arbitrary input sequences**.

[1] Hopfield 1982, PNAS [2] Graves et al. 2014, arXiv [3] Fremaux & Gerstner 2016, Front. Neural Circuits [4] Krotov & Hopfield 2021, ICLR [5] Hung et al. 2019, Nat. Comm.