Analyzing Instructional Practices within Interdisciplinary and Traditional Mathematics Teaching: A Phenomenological Study

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Abstract

Analyzing the Depth of Mathematics within Interdisciplinary and Traditional Mathematics Teaching: A Phenomenological Study

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This study highlighted factors informing instructors’ instructional beliefs and practices and the activities that help students engage in and develop a deep understanding of mathematics. The study also described instructors’ instructional activities and curricular practices when teaching mathematics and an interdisciplinary curriculum that integrates mathematics with other subjects. Through a qualitative phenomenological approach, surveys, semi-structured interviews, and analyses of instructional activities using an adapted version of the Teaching for Robust Understanding in Mathematics (or TRU Math©) framework characterized the experiences of 13 instructors, from elementary through college years, who taught mathematics as a subject and within an interdisciplinary lesson.

The study revealed several factors that informed instructors’ beliefs, practices, and activities (B, P, & A) about teaching mathematics and interdisciplinarity through descriptions and synthesis of meanings and TRU Math analyses of artifacts. Instructors felt strongly about helping students value learning, making mathematics meaningful and joyful, and saw their students as capable problem solvers. They utilized activities to illuminate thinking and understanding of mathematics and used assessments to communicate mathematics. The study also revealed three significant ways that instructors engaged in interdisciplinarity as seen through the practices of the Constructors, Curators, and Connectors, and referred to accordingly as the 3C’s framework. These interdisciplinary characterizations reveal instructors’ practical ways of
using various approaches to practice interdisciplinarity. It also showed how frameworks like TRU Math helped assess an interdisciplinary activity’s potential to foster a deep understanding of mathematics content. The conclusions offer implications for research and practice.
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Dedication

This work is dedicated to my parents, Hayden and Decima Baptiste.

Your kindness to others and love for God will always light my way.
Chapter 1: Introduction

1.1 Need for the Study

Improving mathematics teaching and learning is a multifaceted and ambitious task (Gutstein et al., 2005) that educators can influence from elementary grades through college years. Teachers play a significant role in this process (National Council of Teachers of Mathematics [NCTM], 1989; National Mathematics Advisory Panel [NMAP], 2008), and what they believe and do in the classroom matters. Some studies have found that teachers’ attitudes toward mathematics, and their mathematical content knowledge, can influence their classroom effectiveness and instructional practices (Ball, 1993; Ernest, 1989; Fennema & Franke, 1992; Wilkins, 2008). Other studies have shown that mathematics teachers’ practices are closely linked to what they believe and know about mathematics (Beswick, 2012; Ernest, 1989; Schoenfeld, 2011, 2014; Thompson, 1992; Wilkins, 2008). Based on the apparent interdependence of these areas of knowledge, beliefs, and practices, Bransford et al. (2000) concluded that “teachers’ goals for instruction are a reflection of what they think is important in mathematics and how they think students best learn it” (p. 164). As such, “what matters in teaching is not so much what people say, but what they do” (Schoenfeld, 2011, p. 458).

Unfortunately, not all students have equal access to instructional practices that foster a deep understanding of mathematics content and prioritize their personal academic needs and backgrounds. The National Center for Education Statistics (NCES) reported that over 30% of public school children in the United States attend urban public (city) schools (Taie & Goldring, 2017). However, while this number increases each year, researchers have found that urban students are more likely to be taught by teachers with less experience and lower content knowledge (Battey, 2013). Even when teachers in an urban elementary school received culturally
relevant training in mathematics pedagogy—i.e., training in pedagogy that is attentive to “relational, cultural, and racial aspects of the classroom” (Battey, 2013, p. 126)—they did not always exhibit this behavior in their teaching practices (Brown et al., 2019). Other studies have shown that students from disadvantaged backgrounds have been taught using disconnected concepts, emphasizing following steps, teacher-driven information, and directions, and students providing answers without explanations (Haberman, 2005; Ladson-Billings, 1997; Lubienski, 2002). These techniques stand in stark contrast to best practices observed by Silver and Stein (1996)—problem solving, discussions, and student thinking—that resulted in higher scores on traditional tests and on problem-solving tasks for students in urban schools.

Moreover, while numerous reforms are premised on a need to improve teaching and to learn for all students, they do not adequately consider the lives of the children they wish to serve. (Berry et al., 2013). Considerable debate exists concerning the best ways to teach mathematics to students of all backgrounds and learning levels. Some researchers have advocated learning mathematics through a more traditional approach based on separate mathematics content areas, i.e., Algebra, Geometry, Trigonometry, etc. Others have suggested a more integrated approach through interdisciplinary studies weaving science, technology, engineering, and mathematics (or STEM) throughout elementary and middle schools. In one study conducted by Czerniak et al. (1999), middle grade students reported feeling more confident in their abilities as learners when taught using interdisciplinary curricula. These students reported that the integrated STEM approach made the mathematics subject matter more relevant to real life. In another study conducted at the elementary school level, Becker and Park (2011) found that students taught using interdisciplinary curricula expressed increased interest in learning mathematics by the end of the academic year; however, they showed slight gains in achievement.
In contrast, another interdisciplinary study conducted in the classrooms of 131 teachers across five curricular areas showed statistically significant academic gains in traditional measures of students’ mathematics skills (Stone, 2007). Nevertheless, in Berry et al.’s (2013) summary of the history of failed reforms in education, they observed that teachers who understand their students’ lives, backgrounds, and cultures, are better suited to develop mathematics understanding across contexts. Interdisciplinary teaching can situate mathematics learning across disciplines and various contexts with opportunities for learning content deeply (among other challenges and opportunities). There is a growing need for conceptualizing interdisciplinary learning in practice by illuminating its principles and the reflections of instructors’ (and students’) experiences (McKinney et al., 2009; Williams, 2019).

According to Schoenfeld (2014), much research has discussed what makes good mathematics teaching, but there has not been enough classroom evidence to support such claims. Schoenfeld suggested a framework to measure classroom practices and student performance that is “a) comprehensive; b) focused on key aspects of mathematical sensemaking; c) a relatively small number of important dimensions…and d) used in perhaps twice real-time to code classroom data…so that large-scale data analysis would be feasible” (p. 406). The Teaching for Robust Understanding of Mathematics (or TRU Math) rubric was developed from this framework. The rubric addresses five classroom dimensions: (a) mathematics; (b) cognitive demand; (c) access to mathematical content; (d) agency, authority, and identity; and (e) formative assessment. This approach orients mathematical experiences toward the students’ point of view. It also aligns the mathematics teaching and learning experience with the kinds of classroom practices enacted by teachers to pursue a robust understanding of mathematics (Schoenfeld, 2015).
The TRU Math framework may also be used to analyze interdisciplinary mathematics teaching. Li and Schoenfeld (2019) carefully advocated for TRU Math’s framework to be applied to other disciplines in STEM since it focuses on the development of thoughts and ideas in the context of teaching and learning. Each discipline sees value in centering instruction on students’ learning experience and developing their thinking and understanding of subject matter. Each dimension of the framework and those selected for my study—subject content, cognitive demand, and use of assessments—are applicable across subjects, with careful attention to disciplinary needs. The TRU framework is a helpful guideline to the extent that an interdisciplinary lesson employs instruction that emphasizes making sense of concepts, conceptual connections among the discipline, and a deep understanding of content.

Furthermore, the TRU framework may also shed light on the depth of content within interdisciplinary lessons. Since Li and Schoenfeld (2019) acknowledged that other disciplines in STEM employ learning content deeply, cognitively demanding instructional activities, and soliciting understanding through assessments, TRU Math is applicable across disciplines. When interdisciplinary activities are designed to explore the mathematics involved, they present opportunities for authentic collaboration among teachers across disciplines and students to learn real mathematics within such lessons (Tytler et al., 2019). This study showed how the TRU framework assessed understanding of mathematics when taught as a subject and within an interdisciplinary curriculum. However, more studies are needed to show productive and cohesive ways to structure lessons in this way.

Some researchers have analyzed instructional practices using the TRU Math framework at the undergraduate mathematics classroom (Milman, 2016) and middle school levels (Delaware Mathematics Coalition (DMC)-TRU Math Project, 2018; Schoenfeld et al., 2014). However,
little published documentation exists of studies using the TRU Math framework to analyze the depth of mathematics taught within an interdisciplinary curriculum. A qualitative study (Creswell & Poth, 2017) is a practical methodology that can provide a general understanding of this phenomenon and robustly describe teachers’ instructional activities.

Examining instructional practices for mathematics in a qualitative study may obtain greater insight into how teachers enact mathematical practices that effectively promote understanding (Boaler, 2002b) within interdisciplinary curricula. With this understanding, instructors may be better able to use best practices over different mathematics curricula to promote deep learning and academic achievement. Additionally, results may contribute to the continually growing body of research on STEM and other forms of interdisciplinary education. Administrators may also use these findings to conduct professional development for inservice teachers along these lines.

This study affirmed the use of the TRU framework in other disciplines, notably in exploring mathematics within an interdisciplinary curriculum. Since TRU is a guide to assessing for understanding mathematics, it addressed the need for designing interdisciplinary activities that allow for mathematics to be explored and make connections with other disciplines. Current research provides definitions, methods, and guidelines for interdisciplinarity and integration (Doig & Williams, 2019; Sutaphan & Yuenyong, 2019; Roehrig et al., 2021; Thibaut et al., 2018). The results from this study enable teachers to reimagine ways to use practices and pedagogies to help students explore mathematics through integration and interdisciplinary learning.
1.2 Purpose of the Study

The overall purpose of this study was to describe the kinds of instructional and curricular practices that mathematics teachers use when teaching mathematics separately and within an interdisciplinary curriculum that integrates mathematics into other subjects. I sought to highlight factors informing teachers’ beliefs, practices, and activities and how they influence the design of activities and conversely. Lastly, I highlighted how instructional activities promote engagement and a deep understanding of mathematics or other subjects.

The following research questions guided this study:

1. What factors inform teachers’ beliefs and conceptions towards teaching and learning mathematics and the instructional practices enacted in the classroom? How do these factors differ between mathematics lessons and an integrated or interdisciplinary curriculum such as STEM or STEAM?

2. Are there differences in mathematics content, cognitive demand, and assessments between mathematics and integrated or interdisciplinary activities?

1.3 Procedures for the Study

The participants in my study were instructors from elementary schools through college settings with experience in teaching mathematics as a subject and within an integrated or interdisciplinary curriculum. Instructors completed a survey regarding their attitudes, beliefs, motivations, and dispositions towards teaching and curriculum development, instruction, and assessment practices. The teachers chosen for this study professed to have taught mathematics and an interdisciplinary course integrating mathematics with at least one other subject, such as art, writing, science, etc.
To answer the research questions, I collected data by administering a survey on teaching practices. I also conducted semi-structured open-ended interviews and collected artifacts of instructional activities, which included lesson plans, student work, assessment tasks, student and teacher reflections, and projects.

To address the first research question, I asked each teacher to complete a survey adapted from the 2008, 2013, and 2018 versions of the Teaching and Learning International Survey (TALIS) teacher questionnaire. This questionnaire allowed me to determine the dimensions of instructional quality that they espouse to be active in their classrooms. Rather than forcing teachers to self-identify within a specific model of teaching (which might suggest that beliefs and conceptions remain static), I began this study by affirming what Thompson (1992) analyzed from research studies that “belief systems are dynamic, permeable mental structures, susceptible to change in light of experience” (p. 140). The TALIS survey uses both Likert and frequency scale responses to determine the quality and regularity of these instructional practices. Upon completing the survey, I collected artifacts of each instructors’ instructional activities, followed by an interview to provide each instructor with an opportunity to “elucidate the dialectic between beliefs and practices” (Thompson, 1992, p. 140). Each interview included questions about the teachers’ philosophy on teaching and their goals and intentions as they taught mathematics with a goal of understanding and within an interdisciplinary course. Interview questions also elicited teachers’ beliefs about teaching, students, the nature of mathematics, class norms, and assessments, while artifacts helped characterize teachers’ enacted practices.

I analyzed the surveys by characterizing practices according to the dimensions detailed in the methodology section. Then, I conducted interviews that included questions to help teachers discuss the factors influencing practices as seen through their choices of instructional activities.
Finally, I determined how closely aligned the enacted practices are to the beliefs espoused in the survey, as seen through their choices of activities and their responses to the interview questions. I then described the enacted practices through their choices of activities and compared that with answers to the survey and interview questions.

To answer the second research question, I analyzed each set of the instructional activities using dimensions of the Teaching for Robust Understanding in Mathematics (TRU Math) rubric (Schoenfeld, 2014, 2015; Schoenfeld et al., 2014). I scored assignments on a continuum from 1 to 3, based on an adapted and truncated version of the TRU Math framework’s rubric (Schoenfeld et al., 2014). Each participant provided 2-3 different instructional activities from mathematics lessons and an interdisciplinary course that integrates mathematics with other subjects. These instructional activities included (but were not limited to) homework, classwork, formative or summative assessments, or unit projects. Teachers were allowed to submit these documents online. I analyzed each activity to explore connections between lessons and describe the mathematics being explored. Then, I assigned a TRU Math score for each activity based on the dimensions of the rubric.

After coding each instructional activity, I calculated the mean score for the selected dimensions. I described the lessons for each type of instruction using the chosen dimensions of TRU Math. I used notes from the interviews to provide any necessary additional details to explain instructors’ thought processes and reflections on their reasons for designing and choosing activities and how well each activity fulfilled the learning goals. I assessed the mean scores for an instructional activity using the TRU Math rubric. Lastly, I compared the TRU Math Mathematics Content score between mathematics and interdisciplinary lessons.
In summary, I employed Creswell’s (2013) structure for data analysis by organizing the data, reading and memoing transcripts, comparing a priori and in vivo codes, and contextualizing participants’ experiences to present and discuss the data collected. I used participants’ data from the surveys, interviews, and analysis of artifacts to corroborate themes and codes through a thematic approach to analysis and coding (Charmaz, 2014; Kuckartz, 2014). I followed Charmaz’s (2014) advice in remaining open while analyzing instructors’ interviews to discover new meanings due to my years of teaching mathematics and interdisciplinary courses. Lastly, I used instructors’ descriptions to describe their beliefs about teaching and the design of activities.
Chapter 2: Review of the Literature

Mathematics teachers’ beliefs and conceptions about teaching and learning are important for classroom practice. While the debate continues over consistencies and inconsistencies in beliefs and practices (Raymond, 1997; Barkatsas & Malone, 2005; Thompson, 1992; Ashton, 2015), teachers, particularly mathematics teachers, want students to have positive learning experiences in and out of the classroom. However, while some beliefs can help explain what teachers generally do in the classroom, studies have shown that teachers’ experiences in the classroom affect what they believe and do (Thompson, 1992; Fennema & Franke, 1992). Indeed, if we seek to foster students’ deep understanding of mathematics (NCTM, 1989; Leinwand, 2014), how can teaching experiences improve our knowledge of the dynamic relationship between teachers’ beliefs and practices? What does this mean for mathematics teaching in general? Furthermore, what do practices like teaching mathematics within an integrated or interdisciplinary curriculum mean for teachers and students? Are there any benefits to teaching both types of lessons, and what can such an experience add to research on the complex and intricate relationship between mathematics teachers’ beliefs, knowledge, and practices?

Teacher knowledge, which includes (among others) content, pedagogy, curriculum, and knowledge of learners, has also been deemed essential and influential in teachers’ practices (Hill et al., 2005; Ben-Peretz, 2011; Schwab, 1973; Shulman, 1989). However, studies have shown that trying to improve tangible factors like teacher knowledge and pedagogical strategies can be seen as abstract if teachers cannot translate these principles into action. For example, one study reported that even when teachers were trained in culturally relevant mathematics pedagogy, they did not always exhibit this behavior in their teaching practices (Brown et al., 2019). On the national front, organizations like the National Council of Teachers of Mathematics (NCTM), in
its *Principles to Action* report, have asserted that the combination of teachers’ deep content knowledge, understanding of how students learn mathematics across grade levels, and skilled instructional practices develop learning for all students (Leinwand, 2014). Knowing this, McDonald et al. (2013) called for using instructional activities to teach around core practices and promote purposeful learning for students, among students, and between teacher and students. These authentic experiences, when documented, can serve as a teaching and learning tool to improve classroom practices.

Teaching practices matter in higher education as well. Some studies have spoken to specific practices that have improved student understanding and enjoyment of learning mathematics, especially for non-mathematics majors. Others have reported the benefits of using tools like standards-based grading (SBG) to assess students on course learning objectives and improve transparency and fairness in the grading process. These instructors have reported that assignments are more meaningful by providing specific feedback on the task being assessed. However, this kind of grading allows adjusting instruction for what students know and still need to know (Scriffiny, 2008). In another study, Marbouti et al. (2016) used mathematical models to show how the practice of SBG in homework assignments may help identify at-risk students at an early stage in the course. Nevertheless, these results provided only general claims. More research is needed to document how instructors’ experiences with such practices have informed mathematics teaching, their views on mathematics learning, and their teaching and learning beliefs in general.

Teaching experiences with interdisciplinary curricula like STEM (science, technology, engineering, and mathematics) or STEAM (science, technology, engineering, art, and mathematics) may also affect what teachers believe and do. Some teachers have reported that
with proper training on integrating subjects, they could design instructional activities that engaged students (Lesseig et al., 2016) and saw achievement gains over time (Amaral et al., 2002). The literature strongly affirms the dynamic relationship between teachers’ beliefs about mathematics and classroom practices, establishing a reciprocal relationship between the two entities (Nisbet & Warren, 2000; Brady, 2014). However, some studies have concluded that teachers may change practices without changing beliefs, and a cause-and-effect relationship has not been clearly established (Brady, 2014).

In Ashton’s (2015) historical overview of teachers’ beliefs, “theoretically and empirically grounded” (p. 45) efforts to influence teachers’ ideas for improving students’ motivation, achievement, and relationships with teachers, show promise in enhancing their teaching practices and well-being of their students. Similarly, Doig and Williams (2019) pointed to empirical studies that show that school practices incorporating an interdisciplinary curriculum have an overall positive impact on the attitudes of teachers and students experiencing inquiry-based learning. Could teachers’ experiences with teaching mathematics as a subject and within an interdisciplinary curriculum illuminate how they impact their beliefs, knowledge, and teaching practices? How does teaching mathematics within an interdisciplinary curriculum impact what teachers believe about the nature of mathematics and, ultimately, how their students engage with mathematics in their instructional activities? A qualitative exploration of this phenomenon may add valuable insight into the dynamic role of teaching experiences.

This chapter provides an overview of the literature that connects teachers’ beliefs and practices and how this relates to their views about the nature of mathematics and its integration with other subjects. I provide an in-depth overview of the history of views about the nature of mathematics and its influence on curriculum and instruction, teachers’ beliefs, knowledge, and
practices, and some components of instructional practices connected to student understanding. Also embedded within this review is a discussion of the theoretical framework that guides this study.

2.1 Views about the Nature of Mathematics

What one perceives about the nature of mathematics affects what one believes and practices. A brief historical overview traces the roots of modern-day notions back to two schools of thought attributed to Plato and Aristotle (Beswick, 2012; Li & Schoenfeld, 2019). In short, while Plato believed that mathematics is pursued in an external world beyond the individual’s mind, his student Aristotle believed that the construction of mathematical ideas came through experiencing it through objects (Dossey, 1992). If one sees mathematics as existing outside the human mind, there is a greater chance to believe that it may or may not be understood by all learners. On the other hand, believing that one could learn mathematics through engaging with objects encourages the teachers to create more engagement opportunities for learners. It is easy to see how these thoughts have supported myths about who can and cannot do mathematics (Battey, 2012; Li & Schoenfeld, 2019).

Over time, research increased its focus on improving how mathematics is taught and the influence of beliefs and practices (NCTM, 1989; NMAP, 2008; Boaler 2002a, 2002b; Martin, 2003). Ernest (1989) summarized that teachers’ beliefs are comprised of conceptions, values, and ideologies of two premises: the nature of mathematics and general conceptions of mathematics teaching. As a result, instructors’ views of the nature of mathematics affect how they teach and their goals for student understanding. Their views also govern teachers’ decision-making processes, which will affect their instructional practices and activities. Consequently, teachers’
awareness of their beliefs and their impact on student learning is of continued interest to researchers and practitioners in mathematics education.

Some studies found evidence that mathematics teachers’ practices were closely linked to what they believed and knew about mathematics (Beswick, 2012; Ernest, 1989; Schoenfeld, 2011, 2014; Thompson, 1992; Wilkins, 2008). As in Yates’ (2006) study of 127 primary school teachers, others showed that teachers’ beliefs about the nature of mathematics were not statistically significantly related to their beliefs about teaching and learning, emphasizing the need for more qualitative data to elucidate findings through qualitative research. Still, in Perkkilä’s (2003) mixed-methods study of 140 teachers of students in the second and third grades, she found that teachers’ beliefs about mathematics content were more strongly linked to teaching practices and their past experiences learning mathematics than what they believed about the nature of mathematics or their professed beliefs about teaching and learning. The study implied the need to examine further how teachers’ reflections on beliefs and practices may improve the quality of their instruction. More qualitative research may help explain teachers’ thoughts and their impact on their students.

2.2 Teaching Experiences Impact Teaching and Learning

As indicated previously, teachers’ beliefs are influenced by their experiences. A significant number of studies have suggested that teachers’ experiences as mathematics learners and preservice teachers affected how they taught (Boaler, 2002a; Wilkins, 2008; Superfine, 2009; Beswick, 2012; Schoenfeld, 2015; Yang et al., 2020). Students who experience mathematics in a dynamic way with real-life applications reported positive learning experiences. This was especially crucial for elementary teachers because they were more likely to not see themselves as mathematicians than teachers of other learning levels (Cirillo & Herbel-
Eisenmann, 2011). Additionally, when teachers harbored negative beliefs about themselves as mathematics doers, that tended to impact students’ learning negatively because they were more prone to using teaching methods such as providing procedures with less focus on conceptual understanding, teaching only to the test, not spending enough constructive time on mathematics lessons, etc. Moreover, teachers can transfer this attitude to their students (Michaluk et al., 2018).

Alternatively, teachers’ positive learning experiences also transfer to practice as well. In one in-depth case study, Sawyer (2018) examined the practices of an African American mathematics teacher with over 13 years of experience. She adopted her father’s creative and hands-on approach to teaching her as a student in his ninth grade algebra class while implementing her mother’s empathy in being sensitive to various levels of learning needs. She credited her father for modeling this approach to teaching and believed in the potential of all her students to learn mathematics well. Even though stories of successful mathematics learning abound (Walker, 2014, 2012, 2010; Berry et al., 2013; Varelas et al., 2012; Cooper, 2000), teachers need positive mathematics experiences throughout their years as undergraduates and preservice teachers that can transfer practically in the classroom.

**Teachers’ Experiences as Mathematics Students and Student Teachers**

Teachers’ preservice field experience in methods courses and student teaching are other factors that contribute to their beliefs and attitudes towards teaching and learning mathematics. Spooner et al. (2008) found that preservice teachers in an extended (two-semester versus one-semester) internship reportedly felt better about their relationship with their supervising teacher and had a greater understanding of school policies and procedures than their peers. However, differences in perceptions about teaching abilities were not statistically significant, with
preservice teachers in the extended internship feeling slightly more confident. This did not correlate with a lack of benefit from an extended internship experience but instead was an impetus for further study on the impact and benefits of teaching experience.

Similarly, Hancock and Gallard (2004) concluded that observing K-12 preservice teachers over their undergraduate careers versus a field experience in a semester-long methods course would reveal more about factors influencing a modification in their beliefs about teaching and learning science. While some teachers expressed appreciation for the student-centered socially interactive method of their field experience, as opposed to the teacher-centered lecture and rote memorization approach of their personal experiences, they felt they needed more time to develop students’ critical thinking skills actively before employing this method. Based on these results, researchers agreed that more time was needed to determine how preservice teachers’ personal experiences in learning science and the current ways in which their students (within field experiences) were learning interacted to display slight changes in their prestated beliefs.

Of interest to researchers is how teachers interpret and use their experience. While some teachers admitted to teaching in the way they were taught, others used their “negative” experiences as learners to guide instructional decisions (Bromme, 1994; Swars et al., 2016). In one case study analysis conducted by Oleson and Hora (2013), the instructor modified in-class examples by specific cases, then transitioned to abstract ideas or proofs since beginning with abstract ideas did not make sense. Another instructor looked for cues in student classroom behaviors, recalling their frustrations with making sense of concepts (Oleson & Hora, 2013). Indeed, positive faculty-student interactions can lead to retention and persistence, a phenomenon experienced by many students of color and particularly those attending a Historically Black College or University (HBCU) (Battey, 2013; Kezar & Maxey, 2014). In Borum and Walker’s
(2012) study, Black women reported that supportive faculty and positive learning experiences encouraged them to pursue and earn doctorates in mathematics. This was particularly striking since of the 35,117 total doctorates earned, citizens and permanent residents of Black/African American descent in the United States who majored in Mathematics and Computer Science (separated from the other mathematical sciences) accounted for 53 of those degrees (National Science Foundation, 2016). In summary, high-quality faculty-student interactions, academic support, peer mentors, and positive, nurturing interactions with successful teachers lead to positive mathematics learning experiences for students and faculty (Artzt & Curcio, 2008; Hannula, 2002; Kezar & Maxey, 2014). These experiences, especially for those who will eventually teach students, will assist them in providing quality instruction and strong beliefs about their ability to do mathematics.

**Teachers’ Professional Learning Experience**

Teachers’ professional learning experiences—from preservice to inservice—also influence their beliefs and instructional practices. Systematic and extensive reviews have shown a complex relationship between teachers’ knowledge, beliefs, and practices. Much research has documented the importance of teachers’ professional knowledge, which includes content knowledge (CK), pedagogical content knowledge (PCK), mathematical pedagogical content knowledge (MPCK), and mathematical knowledge for teaching (MKT), along with intra- and interdepartmental collaboration, professional learning communities, and professional development (Depaepe et al., 2013). However, these relationships are intricate, with results showing a varied role that experience plays among variables. While Depaepe et al.’s (2013) review reported a positive correlation between teaching experience and pedagogical content knowledge, other studies have highlighted the complex ways beliefs interact with this result.
In one study of relationships among Chinese preservice teachers’ *stated* beliefs, knowledge, and practices, mathematical content knowledge (MCK) and mathematical pedagogical content knowledge (MPCK) did not significantly correlate with instructional practices (Yang et al., 2020). Instead, there was a strong and significant correlation between dynamic views about the nature of mathematics and constructivist views about teaching and learning mathematics and the self-reported instructional practices of these preservice teachers. The preservice teachers in this study reported experiencing reformed (or inquiry-based) curricula as grade school students and preservice teachers in training. They also reported having an inquiry-based instructional style of teaching and learning before the study began. As such, Yang et al. concluded that while most students in the study acquired inquiry-based skills during a methods course, this did “not necessarily guarantee a significant association between MPCK and their self-reported instructional practices” (p. 291).

Teachers’ beliefs have varied effects on other areas of professional learning. If intervention strategies for improving teaching practices conflict with teachers’ beliefs about mathematics and their approach to teaching and learning, their beliefs may or may not be influenced. Goldsmith et al. (2014) reviewed over 100 articles revealing details of these relationships. Some studies showed that professional development might alter teachers’ views about mathematics and the importance of problem-solving tasks. Nonetheless, teachers’ beliefs about their roles—for example, as a dispenser of knowledge versus one that facilitates opportunities for students’ comprehension—sometimes caused them to lessen the cognitive demands of these tasks. Cognitive demand refers to the extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students’ mathematical development (Schoenfeld, 2015). In one study, a teacher even felt compelled to
alter his approach to fostering student understanding because a colleague disagreed with his strategy. Additionally, more experienced teachers with firmly held beliefs may even selectively implement parts of a reformed curriculum and omit tasks that do not align with their beliefs (Goldsmith et al., 2014).

Professional development interventions also had varying effects on instructional practices. Some studies in Goldsmith et al.’s (2014) summary reported few to no changes, while small-scale studies saw improvements in classroom discourse, mathematics content of lessons, and students’ intellectual autonomy. Notwithstanding, the overall effects of collaborating with colleagues as mentors and peers were most beneficial when they were practiced, challenged, critiqued, and adjusted. It is not enough only to attend professional development on reformed practices or strategies for implementing an interdisciplinary curriculum. When observed by coaches within their disciplines who provided feedback and the opportunity to reflect and re- implement feedback, teachers noticed improvements in their teaching practices (Swars et al., 2018; Podolsky et al., 2019).

**Teaching Experience and Achievement**

Research on the impact of years of teaching experience continues to evolve. In Podolsky et al.’s (2019) extensive analysis of 30 peer-reviewed articles (published since 2003) on teaching experiences in K-12 public schools in the USA, there was a positive correlation between years of teaching and gains in student achievement in mathematics, along with improvements in teacher effectiveness. Findings suggested that teachers are better able to generate student learning as they gain experience, both in the early years of teaching and later on in their careers. Of interest are the contexts surrounding such results.
One study reported that strong professional environments helped mathematics teachers improve their effectiveness more than weaker ones. The strong environments “were characterized as trusting, respectful, safe, and orderly…with collaboration among teachers, school leaders who support teachers, time and resources for teachers to improve their instructional abilities, and teacher evaluation that provides meaningful feedback” (Podolsky et al., 2019, p. 298). The report called for more research on the circumstances of learning environments associated with the extent to which teachers become effective over time (Podolsky et al., 2019; Kraft & Papay, 2014).

Nevertheless, other studies have shown that years of teaching experience do not always correlate with a willingness to change. One study examined the teaching practices of two sixth grade teachers in a suburban school district while implementing a new, innovative, inquiry-based mathematics curriculum. The results showed that the 16-year veteran was more likely to revert to more conventional and comfortable practices when students became frustrated with nontraditional, open-ended tasks and used more teacher-directed discussion with right or wrong answers and worksheets when students felt uncomfortable reasoning through solutions (Superfine, 2009). This teacher relied on the fact that “she knew her students best” (p.12) and felt she needed to pick and choose which tasks would fit within their time constraints. Superfine explained that, on the other hand, the teacher with six years of experience treated the curriculum as a prescription and used it to guide students to collaborate and develop a deeper conceptual understanding of concepts like adding and subtracting decimals. The teacher with less experience relied on the suggestions from the new curriculum to help manage instructional decisions and handle students’ frustrations.
For preservice teachers, students reported that a one-year methods course that taught how to implement pedagogy and afforded more time observing cooperating teachers enabled them to report feeling more confident about their teaching ability than their peers in a semester-long course designed with the same purpose (Spooner et al., 2008). This result spoke to the association between length of preparation and confidence in one’s ability to teach and its implications on teaching reform. These findings are even more crucial as schools are compelled to meet 21st-century career needs through teaching mathematics within an interdisciplinary or integrated curriculum. As such, researchers of interdisciplinary studies in mathematics education are curious to investigate the factors that help or hinder the implementation of interdisciplinary curricula and the continued influence of traditional forms of teaching (Lessie et al., 2016; Williams et al., 2016).

Since teachers’ experiences interact with their beliefs and practices, some teachers mention a lack of expertise in interdisciplinary mathematics teaching or rather a successful implementation of it as a deterring factor. One study cited the “challenge of extracting meaningful mathematics from cross-disciplinary topics in a way that extends mathematical thinking” (p. 63) as a critical aspect of their experience with interdisciplinary teaching (Tytler et al., 2019). Alternatively, the teachers who enjoyed this challenge cited support from other teachers with extensive mathematics experience in their quest to work through this. This suggests that how teachers navigate challenges may influence their teaching practices when implementing an integrated or interdisciplinary curriculum.

Teachers’ and Students’ Experience with Integration and Interdisciplinarity

There are increased calls for reforms that involve interdisciplinary learning to engage students and motivate them toward learning. Since the goals of research, policy, and practice do
not always fully align (Berry et al., 2013), more studies are needed to describe systematic approaches to teaching and research of interdisciplinary learning. What does purposeful engagement in mathematics and interdisciplinary or integrated activities look like? Some teachers are concerned that STEM-related work does not always involve interdisciplinary learning or emphasize mathematics and its relation to other disciplines, especially if a project does not require mathematics to fulfill its tasks (Williams, 2019; Volmert et al., 2013). One of the challenges cited in STEM learning is the portrayal of mathematics as a data representation tool with few opportunities for students to understand mathematical ideas or make connections with other subjects (Coad, 2016; Li & Schoenfeld, 2019). Such approaches only reinforce myths like “math does not make sense,” among other misconceptions. Nonetheless, when studies about interdisciplinary mathematics illustrate cases that offer principles, procedures, and practitioners’ experiences, they show opportunities to learn ‘real’ mathematics and motivate students and teachers to challenge themselves (Tytler et al., 2019).

**Interdisciplinarity**

According to Williams and Roth (2019), interdisciplinary teaching and learning or interdisciplinarity is a “multifaceted but partially nested system of concepts, where different forms of inquiry are situated at one or another level of complexity” (p. 14). It promotes learning content from different perspectives across disciplines (Williams et al., 2016), with STEM or STEAM learning playing significant roles. Other integrated approaches to curricula, such as Culturally Relevant Mathematics Pedagogy (CRMP), attempt to use history, literature, language, and cultural aspects of students’ backgrounds to achieve teaching and learning goals that are not just culturally sensitive but also powerful (Leonard, 2008). Proponents of CRMP claim that it gives both instructors and students a sense of agency that focuses on what matters most—that
learning actually takes place. In one Australian case study, a key feature of its interdisciplinary
mathematics program’s success was that the applications of mathematics were authentic and
meaningful to students, with some developing new content or applying known mathematics in
new ways (Tytler et al., 2019).
There is genuine concern about integrating mathematics into interdisciplinary curricula
(Williams et al., 2016). In one study conducted by Czerniak et al. (1999), middle grade students
reported feeling more confident in their abilities as learners because interdisciplinary curricula
made the subject matter relevant to real life. Moreover, even if confidence contributed to a
positive mathematics identity—defined by Martin (1997) as the belief “about one’s ability to
perform in mathematical contexts, their beliefs about the instrumental importance of
mathematics knowledge, and their resulting motivations and strategies to learn or do
mathematics” (p. 24)—schools, especially K-12 schools, also want to see gains in academic
achievement. For example, Becker and Park (2011) found that students expressed increased
interest in learning mathematics after experiencing a STEM curriculum but only showed slight
gains in achievement by the end of the school year. Indeed, implementing interdisciplinary work
presents challenges as teachers may not be well versed in other disciplines and pedagogy
necessary to integrate mathematics with other subjects competently. They reported needing more
role models, professional development, and administrative support. Alternatively, another
interdisciplinary study conducted by Stone (2007) on 131 teachers across five curricular areas
showed statistically significant gains in traditional measures of mathematics skills.
Similarly, results reported in Dorn et al.’s (2005) mixed methods study showed promise
for improving mathematics skills tested in Arizona’s state exam. In this study, teachers from
“rural and urban, economically well off and disadvantaged, and ethnically homogeneous and

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diverse schools” (p. 94) worked with researchers and professional geographers to design 80 lessons integrating mathematics skills into the teaching of K-8 geography using the GeoMath curriculum. The comparison of pre-and post-test assessments after each lesson for the 3008 students involved revealed statistically significant evidence (at p < 0.001) “where math skills improved as a result of teaching math in the context of an authentic geography lesson” (Dorn et al., 2005, p. 99). Additionally, students improved their conceptual understanding of geography as measured by teachers’ performance-based activities. Lastly, even though 75% of teachers reported no change in their comfort levels in mathematics instructions, the remaining 25% reported a change in attitude ranging from very uncomfortable-to-comfortable through comfortable-to-very comfortable. Such results suggest that teachers’ experiences with instructional activities and gains in student achievement and understanding may alter their attitudes about teaching. Interestingly, can more qualitative studies provide details about teachers’ perceptions of factors involved in changing their attitudes about mathematics instruction through their experiences with an interdisciplinary or integrated mathematics curriculum?

Not all educators advocate for implementing an interdisciplinary course in the classroom. Some have argued that proper integration is complex, and students and teachers may not have enough knowledge of disciplines to reason critically through concepts at deep levels (Doig & Williams, 2019). Others have expressed concern about the actual application of the mathematics presented (Kelly & Knowles, 2016) and authentic ways to assess student work (Coad, 2016). Instructors must see integration as a way to develop a deep understanding of mathematics. Research has shown that mathematics teachers believe students should be given tools to understand concepts, as this is a primary frustration for students K-12 and beyond (Hiebert et al.,
Additionally, Stein et al. (2008) reported that teachers believed developing mathematical thinking is necessary for engaging with instructional activities and can assist learners in making connections—an essential skill in interdisciplinary learning. Learning mathematics and science as disjointed subjects also causes frustration for students who value understanding connections in their learning experiences and underscores the need for skilled mathematics teaching within an integrated or interdisciplinary curriculum (Patel, 2019).

Effective teaching of an interdisciplinary mathematics curriculum relies heavily on a skillful integration of the subjects involved. This focus on thinking and developing ideas in mathematics allows for a better connection of mathematics with other disciplines in STEM (Li & Schoenfeld, 2019). Making sense of mathematics leads to idea generation and design activity that will support teaching and learning STEM and integration into STEM education. Sensemaking supports investigation and discovery, which naturally fits within teaching and learning science, engineering, and technology (Slough & Milam, 2013; Tam, 2000). Some teachers have advocated for project-based learning (PBL) to facilitate interdisciplinary instruction and for students to integrate the learning of 21st-century skills into mathematical behaviors like problem solving, communication, collaboration, and critical thinking (Li & Schoenfeld, 2019). PBL is not just about doing projects. Instead, it promotes making connections among subjects and disciplines; encourages collaboration and discourse among students, teachers, and the community; and preserves rigor and relevance in student-centered instruction (Kwietniewski, 2017). More studies are needed to illuminate how deeply mathematics can be learned within an interdisciplinary curriculum and the kinds of support structures necessary for its success.
2.3 Theoretical Framework

**TRU Math—An Approach to Understanding**

Researchers and instructors continue to grapple with how to teach mathematics for understanding *effectively* (Stylianides & Stylianides, 1996; Hiebert et al., 1997; Howe, 1999; Schoenfeld et al., Li & Schoenfeld, 2019). Seeing mathematics in a dynamic way with real-life applications can affect how students learn, and future teachers teach (Artzt & Curcio, 2008). This may be especially crucial for elementary teachers, who are more likely to *not* see themselves as mathematicians than teachers in middle and high school (Cirillo & Herbel-Eisenmann, 2011). Additionally, when elementary teachers harbored negative beliefs about themselves as mathematics doers, this tended to impact students’ learning negatively because they were more prone to using non-ideal teaching methods (such as providing procedures with less focus on conceptual understanding, teaching only *to* the test, not spending enough constructive time on mathematics lessons, etc.); moreover, this negative attitude could be transferred to their students (Michaluk et al., 2018).

According to Schoenfeld (2014), there is much research on *what* makes good mathematics teaching, but more studies are needed to show *how* this is done. Schoenfeld suggested a framework to measure classroom practices and student performance that was “a) comprehensive, b) focused on key aspects of mathematical sensemaking, c) contained a relatively small number of important dimensions…and, d) could be used perhaps twice in real-time to code classroom data…so that large-scale data analysis would be feasible” (p. 406). The Teaching for Robust Understanding of Mathematics (or TRU Math) rubric was developed from this framework. This rubric addressed five dimensions—*mathematics; cognitive demand; access to mathematical content; agency, authority, and identity; and uses of assessments*—of the
classroom that focus on mathematical experiences from the students’ point of view and the kinds of classroom practices teachers enact in pursuit of a robust understanding of mathematics (Schoenfeld, 2015).

The quality of mathematics taught in the classroom is critical since this affects how much students learn. Furthermore, this kind of mathematics goes beyond lecturing to focus more on providing experiences for students in sensemaking and making sense of problems (Li & Schoenfeld, 2019). In Schoenfeld’s (2015) framework, cognitive demand addresses how activities allow students to engage in sensemaking and making sense of ideas. It also provides opportunities for students to struggle productively through tasks designed for understanding concepts deeply. (Equitable) access to mathematical content ensures that all learners have equal opportunity to engage fully in content and learning practices of the discipline in a way that benefits students individually. Agency, ownership, and identity ensure that all students participate in developing ideas and feel confident in their mathematical abilities to do so. Uses of assessments give students a chance to evaluate their thinking and instruction and provide feedback that instructors will consider when adjusting or improving the learning process (see Appendix Q for an overview of each dimension).

Teaching and Learning Mathematics for Understanding

Mathematical thinking and activity do not occur in a vacuum and require instructional input from students and teachers. Both students and teachers play integral roles in constructing meaning from activities and in the quest to learn mathematics with understanding (Alcock et al., 2016; Hiebert et al., 1997). With students at the heart of teaching mathematics for understanding, Hiebert et al. (1997) cite the nature of tasks, teachers’ role, social culture of the classrooms, and mathematical tools for supporting learning and equity and accessibility as essential factors in
students’ ability to make sense of the mathematics being learned. The comprehensive nature of equity and socio-culturalism is outside the scope of this study. However, this does not prohibit citing Martin’s (2003) caution in measuring equity:

…it is not enough for mathematics educators to work toward equity in mathematics education simply for the sake of equity in mathematics education. Equity discussions and equity-related efforts in mathematics education need to be connected to discussions of equity and in the larger social and structural contexts that impact the lives of underrepresented students. (p. 15)

Additional considerations and cautions are necessary when framing equity within conceptual and theoretical frameworks of mathematics and interdisciplinary education (Tate, 1994; Martin, 2003, 2013, 2015, 2019; Bullock, 2017; Horn, 2018).

Evidently, Schoenfeld’s (2015) framework for classroom activities encompasses Hiebert et al.’s (1997) description of the nature of tasks “to be rich with mathematics to make sense of and explore,” and that leaves behind some mathematical value or “residue” that will serve as insight for future learning and discovery (p. 22). The belief in mathematics as a dynamic activity should be supported by an environment conducive to learning mathematics in this way (NCTM, 1991). As such, McDonald et al. (2013) called for the use of instructional activities [as a] way to construct authentic episodes of teaching around core practices for…learning…. [These] activities guide how teachers and students are expected to interact, how materials are to be used, and how classroom space is to be arranged…. [They] allow teachers to attend to how children’s ideas are given voice in the classroom, and competently and enable children to orient to one another’s ideas and meaningful ideas in the content. They also challenge teachers’ ideas about who can learn and what it means to learn in school. (pp. 382–383)

Hiebert et al. cite teachers’ role in directing mathematical activities and classroom norms to select and sequence tasks that build upon students’ prior knowledge and balances the tension
between respecting mathematics as discipline and students as thinkers. Similarly, Schoenfeld’s TRU Math framework calls for assessments to solicit student thinking and that subsequent instruction responds to those ideas by building on productive beginnings or addressing emerging misunderstandings. Teachers who teach mathematics as a subject and within an interdisciplinary curriculum agree that the ideal use of assessments should reflect student thinking, and grading practices like standards-based grading should provide feedback on the learning process, thus supporting teaching and learning objectives (Guskey & Jung, 2006). This practice is also necessary for facilitating productive mathematical discussions that allow students of all learning levels to reason through cognitively demanding tasks that lead to mathematical understanding (Stein et al., 2008; Manouchehri & John, 2006). These researchers and practitioners, among others (see Walshaw & Anthony, 2008), seem to agree that learning mathematics through careful communication and reflection allows students to make sense of it, leading to understanding.

The TRU Math framework focuses on developing mathematical ideas and thinking through student-centered instruction. It highlights conceptual reasoning or sense-making, and developing, designing, and connecting mathematical ideas or making sense (Li & Schoenfeld, 2019; Schoenfeld, 2020). These practices provide a valuable lens for investigating ideas encapsulating excellent teaching across disciplines. Furthermore, as Li & Schoenfeld explain, many disciplines other than mathematics, but especially those within STEM and STEAM, make use of sense-making and making sense in disciplinary thinking and classroom activities. Thus, the TRU framework applies to any discipline. This study extended the exploration of its validity by examining mathematics within an interdisciplinary curriculum with the goal of understanding and opportunities for deep content learning. Since few qualitative studies use TRU Math to
assess the level of mathematics in an interdisciplinary curriculum, this study also hoped to shed light on this phenomenon through teachers’ experience.

2.4 Conclusion

From the literature, the research clarified that beliefs are consequential, teaching mathematics for understanding influences teachers’ teaching and students’ learning experiences, and exposure to interdisciplinary curriculum affects how students learn multiple subjects. The current study stemmed from a desire to learn more about how factors in these domains bear upon one another to understand better teachers who taught mathematics and an interdisciplinary curriculum that integrated mathematics into other subjects. “Studies suggest that challenging curriculum and cognitively demanding mathematical tasks can play an important role in helping teachers attend more closely to their students’ thinking” (Goldsmith et al., 2014, p. 17). To investigate this phenomenon, more studies are needed to learn about interactions between curriculum materials and instructional tasks and how they affect teachers’ learning and understanding of their students.

The current study focused on mathematics taught for understanding and integrated with other subjects within an interdisciplinary curriculum because both are grounded in evidence about how people learn. In both contexts, critical thinking and problem-solving skills seem to be valued and contributed to “positive attitudes towards helping learners solve problems in some sort of inquiry classroom practice” (Doig & Williams, 2019, p. 2). Gains in mathematics achievement are always desired. However, Doig and Williams (2019) stresses transparency when linking mathematics learning to a specific practice since “traditional measures are not designed to measure what interdisciplinary practices are designed to develop” (p. 2). Accordingly, this review (and corresponding study) has sparked a continued interest in the teaching and learning
process and the points at which beliefs impact what teachers do when striving for mathematics learning with understanding. I was also curious to know the following for its potential to add context for connections between beliefs, practices, and instructional activities:

- How do teachers’ and students’ experiences with instructional activities inform teaching practices and what teachers believe?
- Is there an advantage to teachers who teach mathematics and an interdisciplinary curriculum as part of one’s course load?
Chapter 3: Methodology

3.1 Introduction

The study aimed to understand better the practices and beliefs of instructors who teach mathematics and an interdisciplinary curriculum such as STEM. This interdisciplinary work may also be practiced by integrating mathematics with other subjects such as history, writing, literacy, art, or any other subject or combination thereof. It aims to describe the instructional activities of both types of lessons and their relationship to teachers’ beliefs and practices about teaching and learning mathematics. The instructional activities were examined across three dimensions—mathematics, cognitive demand, and use of assessments—described by an abbreviated Teaching for Robust Understanding in Mathematics (TRU Math) Scoring Rubric (Schoenfeld, 2014, 2015). This study aimed to answer the following research questions:

1. What factors inform teachers’ beliefs and conceptions towards teaching and learning mathematics and the instructional practices enacted in the classroom? How do these factors differ between mathematics lessons and an integrated or interdisciplinary curriculum such as STEM or STEAM?

2. Are there differences in mathematics content, cognitive demand, and assessments between mathematics and integrated or interdisciplinary activities?

This chapter outlines the methods used and describes the data and analyses conducted in completing the research study. The data used for this study were obtained through surveys, interviews, and a collection of instructional activities. It will also provide information on participants, data analysis, limitations, and a summary of the research methodology.

Research Design
Phenomenological studies seek to understand a phenomenon common to the individuals of the study through a rich and detailed description of a lived experience (Creswell & Poth, 2017). Based on my own teaching experiences and from Thompson’s (1992) analysis, my initial hypothesis is that experience in teaching both mathematics and an interdisciplinary curriculum and length of career will influence the design of instructional activities. I also sought to learn more about how this influence occurs over time. Nevertheless, according to Creswell and Poth (2017), a crucial part of phenomenology is the bracketing phase—prior to data collection—during which the researcher must suspend their assumptions to allow participants to define their experience with the phenomenon in question. While suspending assumptions can be challenging, it is necessary as they can impact how data are interpreted. Figure 3.1 shows an outline of my study, including the data collection process and stages of analysis.
Recruitment

I recruited instructors from elementary grades through college years with experience teaching mathematics and a curriculum that integrates mathematics with other subjects or disciplines. This study sought to understand more about the beliefs and practices of these instructors through their description of their mathematics teaching practices and by examining artifacts of their instructional activities. The teaching experience of participants ranged from elementary school to undergraduate students in public and parochial schools. I chose purposive sampling since all of the participants shared the experience of teaching in this way, and I wanted to provide a rich and detailed account of this phenomenon (Creswell & Poth, 2017). Snowball sampling was also used to gain access to additional individuals with this teaching experience. Once participants reached out to me, they received a welcome email (Appendix D) with a unique, five-digit personal identification (PID) number that was used to sign the consent form and survey online. The consent form and survey were administered through Qualtrics, and I used a random number generator to assign the PID to each participant.

The New York City schools in Appendix O were chosen because they all claimed to teach mathematics as a separate subject and within an interdisciplinary or multidisciplinary curriculum throughout the day (based on the description provided via the school’s website). Mathematics teachers who teach at such schools were eligible to participate in my study. I conducted an internet search for New York City public schools that taught subjects within an interdisciplinary or multidisciplinary curriculum to find these schools. From that search, I looked at the description of the school via its website. I also looked at the mission and vision of each school and the kinds of learning promoted through the information posted on the school’s website. I then narrowed this list to the ones that align closer to the purpose of my study.
Appendix O shows the final list of schools and their principals. I shared the principal letter (see Appendix P) and an earlier version of the recruitment flyer (see Appendix E) with principals, asking them for permission to contact and work with their teachers. Of the 33 principal emails, three responded to decline participation. Only one responded with an email to eight potential participants, granting permission for teachers to contact me if they were interested in participating. Two of the eight teachers contacted me, and both completed my study.

At the time of recruitment, we were going through the COVID-19 pandemic. This may have also contributed to the low response rate from the New York City Public schools. As such, I was advised to use other means of recruitment. I posted the recruitment flyer in Appendix E to Twitter, Facebook Education Groups, and community forums managed by the Mathematical Association of America, National Council of Teachers of Mathematics, and Teachers College. Two participants resulted from snowball sampling, as they were recommended by another participant who completed my study. Lastly, one participant responded to an email request sent through the Teachers College, Mathematics, Science, and Technology Colloquium email listserv. A total number of thirteen mathematics instructors participated in this study.

**Researcher’s Role**

My mathematics teaching experience and degrees in bioengineering and mathematics education provided some background knowledge for the survey, interview, and artifact collection. Before enrolling in the doctoral program, I taught for eight years in public and private schools and had teaching experience as a mathematics instructor. As part of my teaching assignment two years before my doctoral program, I collaborated with colleagues in the Art and English departments to create project-based assignments that integrated mathematics, writing, and art. My teaching experiences influenced the selected survey and interview questions with the
lesson planning process, collaboration with colleagues, and interactions and feedback from our students. Consequently, I was able to reference my practices to probe deeper into my participants’ experiences during the semi-structured interview. In doing so, I was able to gain more detailed descriptions of participants’ work in teaching mathematics and an integrated or interdisciplinary course.

3.2 Data Collection

In this research, I used three main methods of data collection: survey, semi-structured interview, and submission of instructional artifacts. The restriction of not being able to observe participants in the field amplified the need for triangulation. Researchers use triangulation to confirm and support phenomena experienced by participants and present in multiple data sources, thus validating the findings (Creswell & Poth, 2017; Merriam, 1998).

Instrument/Protocols

To answer the research questions, data collection for each instructor involved a survey on teaching practices, semi-structured, open-ended interviews, and a collection of mathematics and interdisciplinary instructional activities. These activities included lesson plans, lesson openers, journal prompts, formative and summative assessment tasks, teacher feedback on student work, and class projects or portions thereof.

Survey

I conducted a survey before the interview. This survey was an adapted version of the 2008, 2013, and 2018 versions of the Teaching and Learning International Survey (TALIS) teacher questionnaire. “TALIS is a large-scale international survey providing the perspectives of teachers and school leaders on their teaching and learning environments, as well as contextual information, for schools in participating OECD countries, partner countries, and economies…”
The survey allowed instructors to identify beliefs and practices about teaching mathematics as a subject and within integrated or interdisciplinary lessons before the interview and artifact collection phases. This instrument was also a guide to how their instructional activities would be analyzed using the rubric in Appendix H and precursor to the interview. This was the first step in the triangulation process to see what themes were developing from this instrument. Since experience and other structures tend to influence beliefs, Thompson (1992) reported that systems of beliefs could be permeated and ultimately change with time. The TALIS survey uses both Likert and frequency scale responses to determine the quality and regularity of these instructional practices. Thus, instructors commented on how a practice happened rather than being restricted to binary options of whether it did.

The 2018 TALIS survey in Appendix U can be compared to my truncated and adapted version (see Appendix F) used for this study. Participants began by documenting their teaching preparation and training, daily teaching responsibilities, and professional development. The survey continues by asking participants about their beliefs and practices about teaching mathematics and an interdisciplinary curriculum like STEM or STEAM, along with instructional activities within the two types of lessons. The final sections addressed preliminary views on the development of ideas in mathematics, opportunities for productive struggle in activities, and the use of assessments. These final questions prepared them for the interview, which allowed them
to elucidate their descriptions of what they believe about teaching and what is seen in instructional activities and practiced in the classroom.

**Instructional Activities**

I collected examples of instructors’ instructional activities as a window to the types of assignments encountered by students. Initially, my focus was on how well these artifacts aligned with the beliefs and practices they espoused in the survey. However, as I began to interview instructors, it became apparent that with more teaching experience and student feedback, the activities influenced each other—an observation about which I will expound more in the findings and discussions. These instructional activities included lesson plans, guided practice, homework, formative and summative assessment tasks, and projects or portions thereof.

Initially, I asked each participant to provide 2–3 different lessons for mathematics and interdisciplinary activities where the mathematics overlapped. If the content did not overlap, they could submit 2–3 lessons for both types in which new material was covered. In the early stages of this study, I intended to compare the mathematics taught within each type of lesson. However, as I began the initial assessment of artifacts, it was apparent that some assignments were a combination of mathematics and integration of subjects or disciplines. Consequently, I accepted 1–3 artifacts if the assignments were combinations of mathematics and an integrated or interdisciplinary lesson. I will explain how these data were analyzed in the next section.

Teachers submitted redacted copies of these documents to me via email. In cases where the participants inadvertently included identifiers like school name or district, I redacted this information before saving it to their individual folders. I then analyzed each activity using the adapted TRU Math rubric (seen in Table 5.1) to examine the assignment’s mathematics content,
cognitive demand, and assessment opportunities. Each activity was given a TRU Math score based on the dimensions of the rubric and discussed in the findings.

**Interview**

As mentioned in an earlier section, I conducted semi-structured, open-ended interviews to give teachers a chance to “elucidate the dialectic between beliefs and practices” (Thompson, 1992, p. 140). The interview was influenced by Milman’s (2016) study that compared courses in a community college. Accordingly, as seen in Appendix G, this study’s interview was an adapted version of Milman’s (2016) interview. The interview included questions about each instructor’s philosophy on teaching and their goals and intentions as they teach mathematics and an interdisciplinary or integrated course.

The interview began with instructors discussing their journey towards teaching, followed by their philosophy of teaching mathematics or just teaching in general. Participants were flexible in their responses, with some melding their philosophies of teaching both types of lessons and others discussing how one influenced the other. The remainder of the interview addressed the following:

1. Student engagement with mathematics through activities,
2. how students make sense of mathematics ideas,
3. the use of assessments to build on student thinking, and
4. how one’s beliefs and practices progressed over the teaching experience.

Participants were asked to clarify any responses as necessary through follow-up questions. The interview was audio recorded and transcribed. Interviews lasted between 50 and 120 minutes.
3.3 Data Analysis

The first research question sought to determine what aspects of a teachers’ beliefs and conceptions towards teaching affect their instructional practices—particularly the design and execution of instructional activities—and whether or not this differs between mathematics and interdisciplinary lessons. The second research question investigates the mathematics content, cognitive demand, and assessments of assignments or instructional activities in the mathematics and interdisciplinary courses. Each teacher completed a survey adapted from the 2008, 2013, and 2018 versions of the Teaching and Learning International Survey (TALIS) teacher questionnaire, which sought to determine the dimensions of instructional quality that they espouse to be active in their classrooms. Rather than forcing teachers to self-identify within a specific model of teaching (which might suggest that beliefs and conceptions remain static), this study began by affirming what Thompson (1992) analyzes from research studies that “belief systems are dynamic, permeable mental structures, susceptible to change in light of experience” (p. 140). The TALIS survey uses both Likert and frequency scale responses to determine the quality and regularity of these instructional practices.

Instructor profiles were organized according to years of teaching experience—early (E), \(0 < E \leq 5\), mid-career (M), \(5 < M \leq 15\), and advanced (A), \(15 < A\),—by characterizing responses according to frequencies and opportunities. Recall that Thompson (1992) concluded that beliefs are not static, and I wanted to show the extent to which beliefs and practices aligned with what was espoused. The questions are organized in the following way:

- **Questions 1 – 9**: teaching experience and educational background,
- **Questions 10 – 12**: purported beliefs and practices,
- **Questions 13 and 14**: opportunities for standards-based instructional activities and
• **Questions 15 – 17**: Dimension of the TRU Math rubric (mathematics content, cognitive demand, and uses of assessments).

**Frequency and Opportunity Scores**

Using a similar analysis of the 2018 TALIS, questions were analyzed along three measures: *instructions, student engagement/cognitive action*, and *enhanced activities*. These measures are described as follows:

- *instructions* are the ways in which teachers set up the teaching and learning process;
- *student engagement/cognitive action* are the ways in which teachers design activities to meet students’ individual and collaborative learning needs; and
- *enhanced activities* are student activities designed to extend learning beyond direct instruction.

The frequency scales on these responses are *never or almost never, occasionally, frequently, and always*. Numerical values of one through four were assigned for each response from *never* to *always*, respectively, with the following characterization:

- **High use** of all types, for a frequency score, $u$, $u \geq 3$;
- **Mixed use** of all types, for a frequency score, $2 \leq u < 3$;
- **Low use** of all types, for a frequency score, $u < 2$.

The survey also looked at participants’ orientations towards mathematics, cognitive demand, and the use of assessments regarding instructional activities. Again, the responses, *never or almost never, occasionally, frequently, and always*, were assigned a numerical value from one through four. The scores are described and measured as follows:

- **High opportunity**, $o \geq 3$;
- **Mixed opportunity**, for a score of $2 \leq o < 3$;
• **Low opportunity**, for a score of $o < 2$.

Teacher profiles included a description of the enacted practices through their choices of activities and further categorized practices according to dimensions identified by the TALIS survey.

Each participant submitted instructional activities (or assignments) for mathematics and interdisciplinary lessons upon completing the survey. Initially, I wanted the mathematics content to overlap; however, I found that was not always the case when the instructors of my study practiced their work in real time. In that the mathematics did not always overlap, assignments from the mathematics and interdisciplinary or integrated courses typically covered different material. Instructional activities included lesson plans, homework, classwork, formative and summative assessments, unit projects or portions thereof, and teacher feedback on student reflections. An instructional artifact was analyzed using an adapted version of the *Teaching for Robust Understanding in Mathematics (TRU Math)* rubric (Schoenfeld et al., 2014; Schoenfeld, 2015) and assigned a score from 1 to 3 in 0.5 increments along the three dimensions—mathematics, cognitive demand, and use of assessments. (See Appendix Q for a description of each level along the dimensions of the rubric). A **combined TRU Math** score is the sum across dimensions for mathematics and interdisciplinary activities. It could range from three to nine, with higher scores suggesting higher levels of complexity. Each activity is described for the **extent to which** mathematics is explored and integrated into each type of lesson. Lastly, a **mean** score for each dimension of all instructional activities provided an overall description of assignments for both types of lessons.

I used the interview to allow instructors to describe further the design and execution of these activities, which helped with triangulation. The interview allowed instructors to “elucidate the dialectic between beliefs and practices” (Thompson, 1992, p. 140). It included questions
about teaching philosophy; beliefs about teaching, students, and the nature of mathematics; curriculum goals and intentions when teaching mathematics and interdisciplinary courses; and factors influencing practices as seen through their choices of instructional activities. Participants also described their thought processes and reflections on their reasons for choosing activities and how well each activity fulfilled the learning goals for each lesson. At times, I used my experience to ask probing questions that would elicit more context behind the responses to questions that seem abstract, like one’s philosophy of teaching mathematics or an understanding of how assessments should be used based on the instructors’ experiences.

Lastly, I used a process of decontextualizing and recontextualizing the raw data from the interview to help identify themes and relationships and ultimately draw conclusions (Starks & Brown Trinidad, 2007). Starks and Brown Trinidad summarized that during decontextualization, the raw data from the interview is separated from contexts. Then, units of meaning are developed from the text to which codes are assigned. During the recontextualization process, I examined the codes for patterns, reorganized codes into themes, and then identified relationships among the themes and across the narratives.

Figure 3.2 is a sample of some significant statements, and formulated meanings for the theme “help students value learning.” Initially, the statements in color seemed to support this theme. However, the statements appeared to support other themes upon re-examining, hence the two different colors. The defining and (if necessary) refining of codes were done so that I presented the participants’ perspectives from their actions and points of view (Charmaz, 2014). Once patterns and themes were established, I used them to create a blended story that described what the instructors had in common and what this meant (Bloomberg & Volpe, 2014). The final narrative analysis described the experiences of participants and what this possibly means. I used
a thematic approach to analysis to perform open and axial coding, explained in more detail in section 3.4 of this chapter. Finally, I used the themes to describe the instructors’ experiences conveying the essence of teaching mathematics and interdisciplinary lessons integrating mathematics with other subjects.

To answer research question one, I compared the frequency and opportunity scores from the pre-survey with responses to interview questions and TRU Math Scores of instructional artifacts. Moreover, I compared the frequency scores of the belief and practices, standards-based instructional activities, and TRU Math dimensions (of the survey) with the themes developed from the interview and juxtaposed these observations with the TRU Math Scores of the
instructional activities. I determined how closely aligned the stated beliefs and practices espoused in the survey were with their choices of activities and responses to interview questions. Finally, I described the instructors’ beliefs and practices and their relationships with instructional activities for all participants.

To answer research question two, I compared TRU Math dimension scores of lesson activities for both types of lessons to determine levels of complexity. I used participants’ responses to the interviews to shed light on teaching practices and their experiences preparing and implementing these instructional activities, and how students engaged in mathematics within these assignments.

3.4 Validity and Reliability

Ensuring that data is trustworthy is essential to validity in qualitative studies (Creswell & Poth, 2017; Grossoehme, 2014). Furthermore, Grossoehme (2014) asserts that researchers should avoid using their labels and rely on the participant’s narrative to develop themes and categories. Consistent use of common language among participants yields a final model with more profound and nuanced meaning. The additional step of member checking further enhances validity. During this phase, some or all participants are invited to review the findings and the summative description to ensure that it accurately reflects their collective experiences (Birt et al., 2016). Three primary data sources were collected to allow triangulation: a pre-study survey, submission of instructional activities, and a semi-structured interview. Once the final model was complete, instructors were invited to engage with the data and allowed to provide feedback on its accuracy.

Coding Process

Following Creswell’s (2013) structure for data analysis, I utilized organizing the data, reading and memoing transcripts, comparing a priori and in vivo codes, and contextualizing
participants’ experiences to present and discuss the data collected. Since the survey used an adapted version of the TALIS 2008, 2013, and 2018 questionnaires, organized around beliefs, conceptions, and practices (OECD, 2010), these sections provided a priori codes which opened the analysis process of “investigating, conceptualizing, and categorizing data” (Kuckartz, 2014).

However, since this study was phenomenological, I also wanted to ensure that the participant’s words formed the structure of their teaching experiences (Creswell, 2013). Thus, upon completing the semi-structured interviews, I transcribed and read the transcripts without interview questions to "see what" instructors said. I set aside my personal experience to ensure that I captured the instructors' experiences accurately. I read through each transcript multiple times until I became familiar with each participant’s point of view. I highlighted texts that aligned with the purposes of the study and wrote notes about my impressions of the participants’ narratives. This type of reflection sought to understand themes by asking, “What is this example an example of?” (Creswell, 2013, p. 195, citing van Manen, 1990) to establish codes and categories. Additionally, extracting significant statements from the interviews uses participants’ lived experiences to provide a rich description (Bloomberg & Volpe, 2012).

I utilized Kuckartz’s (2014) open coding step of analyzing interviews through a “line-by-line or paragraph-by-paragraph analysis” (p. 28) for interpreting interviews and creating codes. As I continued reading and rereading transcripts, I looked for emerging patterns across instructors’ stories and then used this as evidence to support the developing categories. I highlighted statements that showed parallel experiences, but I also looked for unique perspectives. I used action words to characterize beliefs and conceptions to avoid type-casting instructors with static labels that ignored other aspects of their lives (Charmaz, 2014). Then, I organized the developing categories into themes and mapped them to participants’ responses. I
interpreted the data in light of the literature and my background in teaching mathematics and interdisciplinary lessons. This process helped in forming the factors that answered the first research question. I then reorganized the categories into larger groups, which helped to form the basis of the three ways instructors practiced interdisciplinarity.

While quantitative analysis relies on intercoder and interrater reliability, it uses a more procedural approach to discuss and resolve differences. Kuckartz (2014) refers to this as consensual coding, where members of a research team rely on each other to agree on analysis. In light of my constraints as the only researcher, I used member checking to confirm that I captured participants’ ideas accurately and elaborated on any categories when necessary (Charmaz, 2014). I emailed selected instructors (who previously volunteered) the excerpt of the findings that described their teaching practices. I asked them to confirm that I captured their viewpoints accurately and provide feedback to adjust accordingly. Instructors responded by corroborating the narrative but were also encouraged to point out anything I may have missed. I also discussed my findings with colleagues to check for bias and make sense of my results.

The pre-study survey was adapted from the 2008, 2013, and 2018 TALIS surveys. According to Ainley and Carstens (2018), the TALIS surveys were designed with a rigorous review of research to ensure validity, reliability, and comparability. Please see Appendices F and R, respectively, to compare the adapted survey used for this study to the 2018 TALIS survey. I maintained (primarily) the constructs of the items specific to answering the research questions of this study. This was necessary to preserve the validity and reliability of the TALIS survey.

Additionally, a close comparison of the truncated and adaptive version of this study’s TRU Math framework resembled the original rubric designed by Schoenfeld (2015), which also reported on the original framework’s validity and reliability. Using the data collected from the
thirteen participants, I juxtaposed beliefs and practices in the survey with participants’ statements from interviews and TRU Math scores of assignments. These findings are reported and discussed in the results and discussion sections. While no researcher can guarantee a bias-free process, I took these steps to reduce the inevitable bias and provide a reasonable description of the participants’ experiences.

3.5 Summary

Qualitative research uses assumptions and interpretive frameworks to understand research problems through the lives of those who experience them (Creswell (2013)). Consequently, my study sought to understand factors informing the beliefs and practices of instructors who taught mathematics as a subject and within an interdisciplinary curriculum. From elementary grades through college level years, the thirteen participants completed a survey and participated in a semi-structured, open-ended interview lasting between 50 and 120 minutes. I analyzed the interview to look for patterns and themes emerging from the analysis process. Instructors also submitted mathematics and interdisciplinary artifacts that I analyzed using an adapted version of the TRU Math rubric. Comparing responses from the survey, interviews, and artifacts, I looked for consistencies and inconsistencies within the instructors’ data and among participants. I then characterized their experiences through their descriptions. I acknowledged my role as the researcher and any issues of trustworthiness or concerns about ethics. The findings, organized in Chapters 4 and 5, provide factors influencing instructors’ beliefs, practices, and activities and artifacts’ potential for helping to understand mathematics, respectively. Chapter 6 offers an analysis of these findings along with conclusions and implications.
Chapter 4: Results: Description of Factors

This chapter presents data collected from the pre-study survey and corroborated by interviews and artifacts of the instructors to answer research question one:

1. *What factors inform teachers’ beliefs and conceptions towards teaching and learning mathematics and the instructional practices enacted in the classroom? How do these factors differ between mathematics lessons and an integrated or interdisciplinary curriculum such as STEM or STEAM?*

It begins with the participants’ backgrounds, followed by graphical summaries of beliefs, practices, and activities (or B, P, & A) reported on the survey. Emerging themes and factors obtained from analyzing the surveys, interviews, and instructional artifacts, are also presented. The section concludes with a summary.

4.1 Participant Demographics

The instructors represent diverse backgrounds in education, training, and experience, akin to their variations of mathematics and interdisciplinary teaching. Table 4.1 presents background information, organized from least to most years of teaching experience. They *all* have degrees beyond their Bachelor’s, and four participants earned PhDs. The instructors taught students from first grade to post-secondary levels and represented public schools, private-parochial schools, community and liberal arts colleges, and universities. Four participants taught post-secondary students, three high school teachers, one middle school teacher, four elementary teachers, and one teacher who taught mathematics to all students in her school in Grades 4 – 8. There were nine instructors with over ten years of teaching experience. They taught a range of mathematics and interdisciplinary courses, from elementary mathematics to a university-mandated interdisciplinary seminar for undergraduate honors students.
<table>
<thead>
<tr>
<th>Name *</th>
<th>State or Country</th>
<th>Experience (Years)</th>
<th>Education</th>
<th>Grade Level</th>
<th>Subjects/Courses</th>
<th>Institution</th>
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<td>New York</td>
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<td>M.A. Mathematics Education; B.A. Math &amp; Elementary Education</td>
<td>Grades 4-8</td>
<td>Mathematics (upper elementary and middle school)</td>
<td>Catholic Boys' School</td>
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<td>New York</td>
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<td>Master's (Teacher Ed. Program)</td>
<td>Grade 2</td>
<td>STEAM</td>
<td>Elementary School</td>
</tr>
<tr>
<td>Selma</td>
<td>New York</td>
<td>8</td>
<td>M.S.Ed., Mathematical Education</td>
<td>Grades 11, 12</td>
<td>STEAM Course; 11th &amp; 12th Grade Mathematics</td>
<td>High School (Int'l Students)</td>
</tr>
<tr>
<td>Paul</td>
<td>West Virginia</td>
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<td>Ph.D., Mathematics; B.S. Mathematics; B.A. Mathematics &amp; Chemistry Education</td>
<td>Undergraduates</td>
<td>Organic Chemistry; General Chemistry; Calculus 1-3; Differential Equations; Science Education</td>
<td>University</td>
</tr>
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<td>Undergraduates</td>
<td>Calculus; Modern Algebra; Number Theory with Applications</td>
<td>College</td>
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<td>Grades 1-4</td>
<td>4th Grade Mathematics; STEM Studies</td>
<td>Elementary School</td>
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<td>New York</td>
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<td>Undergraduates</td>
<td>Calculus 1-3; Mathematical Modeling; Mathematical Design; Mathematical Computing; Combinatorics</td>
<td>Community College</td>
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<tr>
<td>Emily</td>
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<td>Grade 3</td>
<td>Mathematics; Science; STEM Math &amp; Science</td>
<td>Elementary School</td>
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<td>Grades 11, 12</td>
<td>Math for Decision Making; Modern Math</td>
<td>High School (Alternative)</td>
</tr>
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<td>Undergraduates</td>
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<td>Elizabeth</td>
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<td>25</td>
<td>M.S.Ed., Mathematical Leadership</td>
<td>Grade 3/4</td>
<td>Literacy, Mathematics, Inquiry, Art, Drama, &amp; Religious Education</td>
<td>Catholic School</td>
</tr>
</tbody>
</table>

4.2 Pre-Study Survey Results

Before conducting interviews and collecting artifacts, participants completed the survey in Appendix F. This was the first step in the triangulation process to begin seeing preliminary themes about the participants’ beliefs and practices. I used both Likert and frequency scale responses to determine the quality and regularity of instructional practices. As a reminder, frequency scores represent how often a particular practice occurs, and opportunity scores signal the likelihood of students engaging in a teaching practice during an instructional activity.

The scoring is as follows, with an explanation in Chapter 3:

- high use or opportunity: scores ≥ 3
- mixed use or opportunity: 2 ≤ scores < 3
- low use or opportunity: scores < 2

Finally, using a similar analysis of the 2018 TALIS results, I analyzed responses along these measures:

- instructions—the ways in which teachers set up the teaching and learning process;
- student engagement—the ways in which teachers design activities to meet students’ individual and collaborative learning needs;
- cognitive action—the ways in which activities required students to engage in problem solving through evaluation, integration, and knowledge application; and
- enhanced activities—student activities designed to extend learning beyond direct instruction.

The graphs show the instructors’ perceptions of their beliefs, teaching practices, and instructional activities. High scores may signify a tendency to provide more socially desirable answers.
(OECD, 2010). Hence, themes from the interviews and some artifacts are included to add context to the participants’ claims.

**Beliefs and Conceptions about Teaching and Learning**

The first purpose of this study was to highlight factors informing teachers’ beliefs, practices, and activities (or B, P, & A) and how they influence the design of activities and conversely. This section presents beliefs and conceptions, followed by factors that influence them. Some artifacts are presented to explain certain practices of instructors. The instructors shared common beliefs about teaching mathematics related to their students’ learning experiences. Figure 4.1 displays the responses to the question, “To what extent do you agree with the following statements about your beliefs and practices?”

**Figure 4.1**

*Instructors’ Teaching Philosophy from Survey*
All participants *mostly* or *absolutely* agreed with statements about beliefs and conceptions about their teaching, with all 13 participants claiming that they *absolutely* “help students value learning.” In fact, the average score in all but one statement was above 3.5. These scores reflect the instructors’ *agreement* with the statements. Additionally, and due to the nature of this study, these pre-stated beliefs were expected.

Statements like “allowing students to provide alternate explanations,” “presenting students with a variety of strategies for deep learning,” and “designing activities for struggling productively” are part of what Schoenfeld (2020) described as necessary for developing students into “powerful mathematical thinkers” (p. 1170). It also suggested that they believed students are capable of learning mathematics deeply—a belief that indicates a dynamic view of the nature of mathematics, as represented in the literature. Indeed, most instructors indicated beliefs about mathematics and their students’ abilities consistent with these results during the interviews.

At the start of each interview, I asked instructors to share their philosophies about teaching mathematics. Even though they all had various ideas of what this meant, with some claiming to have *no philosophy* on teaching, *each* shared ideas about what they felt characterized a positive mathematics learning environment. Some instructors focused on their own mathematics learning experiences and their desire to reproduce this experience for their students. One instructor recalled not realizing she was “good in math” until her teachers pointed it out. Another instructor commented that regardless of his students’ learning abilities, he felt it was “really important” to give his students what his geometry teacher gave him—“really careful logical thought.”

Nonetheless, others reflected on their experiences as teachers. One teacher talked about reflecting on her mathematics development as an adult. She admitted to becoming more
intentional about her students “seeing themselves as doers of math” by purposefully “seeing herself as a doer of math.” Engaging in this practice provided a lens through which she could help her students and support other elementary school teachers as a mathematics coach.

However, it was another instructor’s explanation that contextualized this best for the group:

My philosophy when I'm teaching in my classroom is to help students learn to love math regardless [of] the content or regardless of how well they understand the subject. I want them to learn how to love math, mostly because I believe that the way you think when trying to solve a math problem is critical [to] learn how to problem-solve. You learn how to approach problems; you learn how to use the tools that you have to solve these problems, to think through problems critically. And I think that it's a really important skill that can be mirrored in other things, [and] a lot of students miss [that] purpose of math.

**Beliefs and Conceptions about Teaching Practices**

The survey organized practices around how instructors set up their lessons and opportunities for students to engage in problem solving. Figure 4.2 displays responses to both mathematics and interdisciplinary lessons. It shows that differences exist, with average scores higher in mathematics than in interdisciplinary lessons. Even though all participants are primarily mathematics instructors, they engaged in various levels of interdisciplinarity. This idea of variation in interdisciplinarity is explored later in this chapter.

These teaching practices reflected what instructors try to do in service of learning mathematics. The high scores in mathematics concerning “justifying answers and critical thinking” and “making connections between topics” not only foster students’ development as mathematical thinkers but also encourage deep understanding (Ainley & Carstens, 2018; Schoenfeld, 2020). Since interdisciplinarity is still a growing field with no known seminal text on pedagogy, these results were not alarming. Also, these instructors are mathematics teachers by education and training. Thus, it makes sense to claim to have higher opportunities in mathematics in the dimensions of instruction and cognitive action.
Again, instructors referred to personal and professional experiences as influencing their practices. One instructor recalled her anxiety about timed tasks in deciding how students learned multiplication facts. She explained, “I do math with teachers, and I do it with children. Moreover, as a ‘doer of math,’ that whole ‘speed thing’ made me feel inadequate because I was not fast.” Another instructor talked about how critical small groups were for learning, as she strongly felt that “children learn more from each other than from the teacher.” As a result, most tasks were group-oriented and focused on knowing what students understood. It seemed like one major factor influencing teaching practices was to gain more insight into how students thought about mathematics.

One instructor graded homework for “completion and not for correctness” because she wanted students to “try all of the problems and show their thinking.” She is not alone. All instructors
found ways to assign work so that students felt comfortable sharing their mathematical knowledge. Another instructor recalled how their teaching style evolved due to this practice. Early in his teaching career, he admitted to “just looking that they showed their work.” He realized that he wanted to “understand more of their thinking process, so...[he] focused more on their work.” Each year, he learned something new and then adjusted his teaching style accordingly. Perhaps these insights may show how instructional activities were designed. The survey results also seemed to suggest an association between instructors’ intentions for their students and the objectives of their activities.

Factors Informing B, P, & A

**Students are Capable Problem Solvers.** All instructors felt strongly about their students’ ability to do mathematics. They felt that their beliefs in students’ abilities influenced students’ perceptions of themselves as doers of mathematics. Paul pointed to his students’ “natural ability” to reason through everyday decisions as a precursor to problem-solving skills:

> …when you talk to some of these students you start to realize that they’re good at logical thought, like in day-to-day situations they have a lot of natural intuition... And it’s the rigor and the symbols, and it’s the culture of mathematics, to me, that sort of keeps people out of it and makes them think they don’t belong there. So when I teach, I try to use their natural decision-making to my advantage. To me, it’s more a game of ‘Can I convince them that they’re capable of this?’ [rather] than ‘Can I make them capable of this?’

Others used this viewpoint as one of the primary reasons students work in groups. They felt strongly about students’ ability to learn from each other and assigned projects or activities where students worked in groups or engaged in the peer review process. Several instructors adopted the peer review process in assessing work to encourage “good mathematical discussions (to help build critical thinking skills) and engagement with mathematics assignments.” However, the instructors had to teach them how to give constructive feedback, collaborate, and learn from each
other by reasoning through the mathematics within tasks. This did not come easily for some students. As such, instructors had to model the peer review process to be an effective practice for students to learn mathematics from each other.

One instructor showed her students samples of reviewers’ comments for research articles that she submitted for publication in mathematics but were not fully accepted. Seeing that her goal was to help students feel comfortable when making mistakes, she shared feedback from one of her papers that needed to be revised and resubmitted. This vulnerability was a necessary component of the culture of empathy and trust that guided mathematical conversations. Additionally, a byproduct of the peer review process was that students realized that mathematics extended beyond solving problems, and they could learn mathematics by engaging in conversation with their peers.

Seeing students as capable learners builds a culture of trust in which both students and teachers learn from mistakes. For example, one instructor tried to figure out “under what circumstances would the problem have been correct” and then gave his feedback. Another instructor felt that trust meant “caring about students’ lives and taking ownership for teaching in a way that they can grasp concepts.” Still, one instructor who taught at an alternative school for students at risk of failing high school saw trust as a critical component of connecting with his student. Against the advice of his colleagues, he used “real” Geometry tools like a sharp-tipped compass, as opposed to a dull version, because they get better results, which then “helps students feel more confident doing mathematics.” He felt strongly that his 23 years of experience helped him learn to do this in authentic ways. Among his accolades for being a national conference speaker and a sought-after leader in his mathematics department was his most prized possession: being “voted Most Helpful Teacher by students, multiple times in a row.” He displayed this
award conspicuously, not to “brag” but “establish trust.” He wanted his students to know that they “can trust that [he’s] not going to sit in [his] chair and tell them [what] to do…, but that [he’ll] come over to [their seats] to show them how to do it.” Even during the COVID-19 pandemic, he felt comfortable continuing this practice. He taught in socially distanced classrooms with a strict mask policy. Still, he continued following his classroom protocol of meeting students at their tables when they had a question because he felt it was the only way to remain engaged with them.

An environment of trust was also fostered by validating students’ ideas. Most teachers moved from student to student, ensuring they were on track. One teacher used the narrative of “What’s the next step?” to help students learn to think critically and mathematically. She also encouraged her students to “notice” and “wonder” out loud when working through “story problems.” One instructor who taught at a community college believed that this practice helped keep students accountable. In his college class, he worked one-on-one with students to ensure they were on track since his students did not usually attend office hours. He shared his reasoning for doing this:

If I don’t do that, then it’s easy to see them get off track. One of the things that my colleagues worry about is cheating. And as I see students making progress over the semester, and doing those little [things] like, “I’m stuck here. How do I get past that?” and I help them get passed that, they can’t be cheating. I see their actual work, and I see their actual thought process, and you can’t fake that. Having that constant dialogue, class-to-class-to-class, where I get to hear what they’re working on, or where they got stuck, I guess it means that they have to be accountable, because every time they come, they’re [going to] feel bad if they didn’t make any progress. But it happens.

**Help Students Value Learning.** As mentioned earlier, all instructors indicated in the survey that they absolutely “help students value learning.” However, the practice of valuing learning is practical and purposeful. Therefore, instructors felt that students value learning when their learning is valued and when they see how much they have learned. For one instructor, she
used student work instead of commercial posters to reference important concepts throughout the year. When questions arose about specific skills or concepts, she pointed students to the “anchor charts” on the wall, assigning credit and ownership to the student who created them. According to another instructor, projects are an excellent opportunity for students to apply what they know and produce a “product that they can share with people outside of the class as well.” The project then becomes “something that students can be proud of” and more valuable to them.

Empathy and care for students’ learning needs denote value. Most instructors felt that giving students enough time to complete tasks shows students that you “care about where they are in their journey.” It helps students to feel comfortable sharing what they know, even if there are mistakes. Creating an environment where students feel comfortable making mistakes and sharing them is challenging in a mathematics classroom where the goal is often “to get the right answer.” Yet, every instructor spoke about the “value of [making mistakes] as a necessary part of doing mathematics.” One instructor summed it this way to her students:

[Being wrong] is really valuable because it helps us know how to get closer to doing it right. I don't care what you put, don’t care if the answers are right. I just want to see what you're thinking. I want to see your thought process because that's what I’m interested in.

The design of instructional activities can provide both students and teachers a window into what students know. As such, there seems to be an association with valuing learning and being student-conscious when designing instructional activities, placing student thinking and comprehension at the center of activities.

**Activities Illuminate Thinking and Understanding.** Instructional activities are a vehicle for teaching around core practices and promote purposeful learning for students, among students, and between teachers and students (McDonald et al., 2013). Instructors seemed to agree that activities that develop mathematical thinking and illuminate students’ understanding
promote learning mathematics deeply. One instructor shared that she designed tasks so that students could “make connections among mathematics skills.” They also provided “discourse to help students move along in their thinking.” When students got stuck, she encouraged them to talk to their classmates to help them get over the “hump” to the following steps, which was more critical than simply struggling to complete a task. She surmised that when students recognized helpful strategies from their peers that helped them arrive at a deeper understanding of mathematics concepts, they were encouraged by learning from each other. She admitted that her design of mathematics tasks improved when she started to “think about how to reach students of different levels of support and ability.” Still, another instructor shared a sentiment that summed it up nicely: “I’m always trying to get [my students] away from, ‘I need someone to tell me how to do this’ and into ‘I need to learn how to think about this.’”

Learning to design tasks, so students learn from each other productively developed over time. Instructors described undergoing a “pedagogical productive struggle” of sorts, whereby they were mindful of why things worked through student work and thus adjusted curriculum, tasks, and classroom behaviors accordingly. One instructor based her activities on “knowing where the children come from…their understanding, …what they know, and [then] building on that.” During the height of the COVID-19 pandemic, she relied more on her practice of watching video recordings of students “explaining what they know” to help students share their work and understanding. She admitted that while the process “was tiring, never in the past had [she observed] so much evidence of recorded learning…which was brilliant!”

All 13 instructors pointed to the development of mathematical thinking as their primary goal for activities. However, “thinking mathematically” has historically had various definitions, which have not all been accurate (Schoenfeld, 2020). In Schoenfeld’s in-depth overview
summarizing his decades-long research in mathematical thinking, he described it as “inquiry, sense making, and exploring how things fit together” (p. 1167). This definition focuses more on the dynamics of the thinking process and is the preferred perspective for my study. For example, several instructors mentioned using Flipgrid to “hear and see students displaying their mathematical knowledge and engaging in their mathematical understanding” since students had to “work problems out and explain their reasoning.” Another instructor incorporated more “comprehension” videos, which were previously reserved for assessments until the COVID-19 pandemic made it necessary to hold students accountable for what they were learning online. Although the videos started as a way for students to demonstrate what they were learning each week, they became a handy tool for displaying what students understood:

By having them explain it, even the students I thought knew what they were doing (and in previous semesters would have blown me away by their ability to do things right) were making conceptual mistakes along the way. And so, this gave me a better opportunity to give them feedback and say, ‘You know, you got the right answer or [you have] the right approach, but when you went from step three to four…let’s reassess that.’ Or, ‘At minute 2, you said this. Let’s think about that for a second.’ And so I was able to give them more nuanced feedback because I could hear their thought processes as they were explaining this to the screen.

**Beliefs and Conceptions about Instructional Activities**

Figure 4.3 is a graphical representation of all responses to the question, “To what extent do instructional activities in mathematics allow students to do the following?” Participants answered a similar question about activities in an interdisciplinary curriculum like STEM or STEAM. Again, most instructors made strong claims about how instructional activities allowed students to make sense of concepts. Concerning the cognitive action within mathematics and interdisciplinary activities, the data suggested slightly more opportunities for making sense of ideas in interdisciplinary work than in mathematics activities. In this instance, the
interdisciplinary activities operated in service of mathematics learning. However, this did not provide insight into the depth of mathematics explored within activities.

Figure 4.3

*Instructional Activities in Mathematics and an Interdisciplinary Curriculum-Survey Scores*

Recall that cognitive action encompasses the extent to which students engage in sensemaking and making sense of concepts, i.e., cognitive demand. The more cognitively demanding the task, the higher the student engagement in activities. Moreover, struggling productively to grasp concepts is necessary for maintaining high cognitive demand. Instructors in this study admitted that obtaining optimal productive struggle can be elusive at times. Nevertheless, the struggle is only productive if “students can maintain the task's initial goals and cognitive demand, support their thinking by acknowledging effort and mathematical understanding, and move forward in solving the task through their actions” (Permatasari, 2016,
p. 97). Thus, productive struggle, cognitive demand, and cognitive action are interrelated and necessary for understanding mathematics deeply.

Instructors scored highest in the “making sense” category of the cognitive action of instructional activities. For these instructors, making sense and struggling productively were necessary tools for developing mathematical thinking. Mathematical engagement is more than excitement; students need to verbalize and communicate their understanding. An instructor described struggling productively as “fruitfully spinning our wheels.” Once students begin to show steps that are “valid but not helpful,” both instructor and student are no longer “fruitfully spinning [their] wheels.” This is the point where the “struggle” is no longer productive—a point at which most instructors in the study admitted they either stepped in too early or, at other times, too late, causing students to give up on the process and miss out on “connecting to bigger ideas.” At this critical point, which is not always explicit, teachers can facilitate persisting through the challenge to overcome a roadblock in conceptual understanding. Some researchers, however, have felt that students could, with enough space and time, “figure it out.” In Hiebert et al.’s (1997) example of a problem-centered classroom, persisting through struggles happens when students are afforded a significant amount of time (at least 40 minutes) with mathematically rich tasks, opportunities to shape and reshape their thinking, and no interruptions from teachers.

Instructors also made strong claims about teaching mathematics for understanding as defined by the adapted TRU Math framework. Recall that I used this framework to assess mathematics understanding in artifacts. Figure 4.4 shows their pre-study survey TRU Math scores. Their average scores as a group were above 3 in every category for the dimension in mathematics content, signifying high opportunities for learning mathematics deeply. The scores for cognitive demand were also interesting, with slightly higher scores in mathematics than in
interdisciplinary activities. Only a careful analysis of activities can help sort out these differences and what they may mean. These issues are addressed in Chapter 5, while some factors informing these scores are shared in the next section.

Figure 4.4

Instructors’ Learning Outcomes for TRU Math-Survey Scores

Factors Informing Mathematics B, P, & A

Mathematics Should Be Meaningful, Joyful, and Accessible. Instructors expressed a desire to make mathematics meaningful, joyful, and accessible to all their students. They also seemed to associate valuing learning with making mathematics meaningful to students through contextual learning relevant to them. For example, an instructor who was both an advanced elementary school teacher and a mathematics coach believed that “mathematics should be meaningful” and presented in a way “relevant to students’” lives. She taught “geometry and
visual-spatial skills through hair braiding…and facilitated teaching mathematics through jewelry making.” Nevertheless, she capitalized on her background in literacy education to help introduce concepts in mathematics:

So, I mentioned that I connect [mathematics] to literacy. I have a book collection that I use. [When] I open a unit on division, I use a book called *The Doorbell Rang*. And [this] is a very old book to the point where the characters do not look like any of the children I serve. So, I just made a PowerPoint slide and *kind of* reinterpreted the story, and I just put my own kids in it, or I use [my students], so they can get to know me, and connect to the mathematics.

Instructors also felt strongly about creating a comfortable environment for learning by getting to know the students’ backgrounds and past hurdles and showing empathy. One instructor at a liberal arts college wanted to make mathematics “accessible and joyful” to students in her classroom. She used a mathematics autobiography to get to know the students better before class and provide them with sample problems that they will see during the first week. Along with being their professor, she considered herself a partner in their academic journeys, describing her intention as follows:

I want to be aware of the hurdles that students face in getting into a math classroom; and understand that [many of] the students in my class have faced racial barriers…language barriers to mathematics, and just be…more of a partner in their learning. So, I try to get to know each student early….to forge a connection with the students in my class so that the learning is actually like a joyful process of discovery instead of a panicked race to assimilate information. And, while they’ll need to know the proof logically…I also want to make room for students’ self-expression which gets into the [mathematics being] integrated with other types of pedagogy….

Figure 4.5 is an excerpt of her feedback on her first writing assignment in a Modern Algebra course. Her style is more conversational so that students can learn how to write proofs, persist through the challenge, and not give up. For the complete writing assignment, see Appendix T.
Another university instructor credited his research on “mathematical habits of mind and identities” for helping him become conscious of issues that could “distract” his students from learning. He recognized that if students “don’t feel like they belong” or have a “workload that is too heavy,” it will not be productive to “push through content.” Thus, he aimed to “remove as many barriers as he can” so that students are effective in learning mathematics. When “particular mathematical language” seemed to be lacking, he met students “halfway” by being clear about what needed to be solved. For instance, students seeing differential equations for the first time may “not know the difference between an explicit and implicit solution.” It is “important that I
tell them, ‘I want an explicit solution, and by that, I mean a solution of the form \( y = X \) stuff.’”

**Assessments Communicate Mathematics to Different Audiences.** A common theme among these instructors is the need for assessments to serve as a learning tool for students. One approach to accomplish this is to use an assessment to “communicate mathematics to different audiences,” as described by the instructors. Sometimes, this mathematics communication is between the instructor and the student or the student and their peers. However, some instructors saw much value in creating assessments that allowed students to communicate mathematics they know with persons outside of the classroom. One instructor shared that part of her teaching philosophy was for students to “like math and like talk[ing] about math with other people.” As such, two-thirds of the final exam was comprised of oral and conversational tasks.

As mentioned, communication on assessments between the instructor and student was crucial. According to Stefanakis (2002), “the word assess comes from the Latin *assidere*, which means to sit beside. Literally then, to assess means to sit beside the learner.” While some may not be literally sitting next to their students, they can communicate this “posture” through the kinds of assigned tasks and even through their assignments' directions. One instructor explained his reasoning for this:

I look at the assessment as a teaching opportunity as well, usually for smaller ideas…. But even if you’re going to end up with a zero on this quiz, I still want you to be able to take something away from it. So, [I] try to help them take something away from this and to be able to gain access to it in the first place. I don’t want them not to answer the question because they didn’t understand what I was asking them to do. If they can do it, they can do it—that’s what I care about.

The grading process is another opportunity to communicate mathematics. The kinds of feedback and the message it sends to students matter whether they are encouraged in the learning
One instructor felt that the onus was on him to “look for opportunities and search for meaning” in students’ responses. He explained,

I like to try to ‘pull out their thinking,’ [to some extent] by asking myself, ‘What were you thinking here? Why did you make that decision? Was it that you were just desperately grabbing for points, or was it something in the way that I worded the question that led you to think that way?’

He admitted that sometimes methods and concepts get “tangled up” in students’ minds, and if there is a way to “pull them apart,” he was determined to find it. He also believed that inserting “his voice” in the instructions of quizzes and tests helped diminish mathematical anxiety and gave students a better chance to show him what they knew and learned from the unit.

Some instructors strongly advocated using standards-based grading (SBG) to organize learning objectives. It provided transparency and flexibility, especially for students who need “multiple attempts” to demonstrate understanding or mastery of concepts. For one instructor, this practice “really addressed a difficulty that [he] had with most math classes, which is ‘what does a grade represent?’” While no process is problem-free, SBG provided more transparency in allowing “grades to represent how well a student understood the material.” Table 4.2 shows a typical scale used for grading a project in one of his courses. His standards focused more on conceptual understanding of concepts and clear communication of students’ comprehension. He explained his approach in the following way:

So, I grade based on saying, ‘Okay, so here are 11 different criteria based on [the project description].’ So they submit their paper, their notebook, and the presentation, and I grade each of these points on a [E-M-R-N] scale. This is a classic standards-based grading scale. So, do you meet the expectations on each of the standards or not? If you do meet the expectations, was it clear and complete? That would be exemplary! Otherwise, maybe they did it, but they could have pushed it to excellent, or maybe it wasn't complete and clear. And then you have to show me; then you can redo it. You need to put in more work [since] you haven’t met the standard. So based on those scores, I assign a grade based on whether they mastered every single standard or not. And so, this is actually my standards-based grading mixed with a rubric from the class so that if
somebody needs to revise something, they don’t get an A. If you need to revise something, you get a C; you have a chance to resubmit if you want to.

Table 4.2

Instructors’ SBG Scale and Explanations

<table>
<thead>
<tr>
<th>Scale</th>
<th>If the work meets expectations, then depending on how complete and clearly communicated your work is, you will receive one of the following scores:</th>
</tr>
</thead>
<tbody>
<tr>
<td>E - Exemplary</td>
<td>The work meets or exceeds the expectations of the assignment. Communication is clear and complete. Mastery of the concepts is evident. There are no non-trivial errors in understanding.</td>
</tr>
<tr>
<td>M - Meets Expectations</td>
<td>Understanding of the concepts is evident through correct work and clear, audience-appropriate explanations. Some revision or expansion is needed, but no significant gaps or errors are present.</td>
</tr>
<tr>
<td>R - Revision Needed</td>
<td>If the work does not meet expectations, then you have not demonstrated understanding of the concept. In this case, I will determine if you show partial understanding, and you will receive one of the following scores:</td>
</tr>
<tr>
<td>N - Not Assessable</td>
<td>Partial understanding of the material is evident, but there are significant gaps that remain. Needs further work, more review, and/or improved explanations.</td>
</tr>
<tr>
<td>Grades</td>
<td>Not enough information is present in the work to determine if there is understanding of the concepts. Work is fragmentary or contains significant omissions. Or, there are too many issues to justify correcting each one.</td>
</tr>
</tbody>
</table>

Your final project grade will be based on the number of scores at each level as follows. If you do not participate equally in the groupwork, your grade will be reduced accordingly.

- **A (95+)**: Earn a score of M or higher on all standards and a score of E on at least four standards.
- **B (85)**: Earn a score of M or higher on all standards and a score of E on at least two standards.
- **C (75)**: Earn a score of M or higher on three standards and at most one N score.
- **D (65)**: Earn a score of M or higher on three standards and at most one N score.
- **F (50-)**: Have fewer than three E or M scores OR earn two or more N scores.

This approach to grading is also part of his philosophy of teaching and learning. In his projects and assignments, He found ways to get “students to share their concepts, share their knowledge, and then create rubrics based on that information to give grades.”

**Beliefs and Conceptions about Interdisciplinarity**

**Interdisciplinarity—Challenges and Opportunities.** The literature cites legitimate concerns about how interdisciplinary learning occurs in schools (Thibaut et al., 2018; Millar, 2020). However, every instructor whose course load included an interdisciplinary course has cited “on-the-job” learning as the primary way of gaining experience. One instructor stated that she “learned…by doing it, and also because my school has been very supportive and pushed me in ways [to] encourage me to think about [my actions]”. This experience summed up the
experience of other instructors as well. Some even admitted that teaching in different ways other than colleagues could be “scary” and requires “courage.” However, once these instructors saw how much their efforts impacted their students’ mathematics learning and understanding, they were committed to refining this teaching method until it was productive.

The “integrated/interdisciplinary” learning space is purposefully messy because the process is iterative. One instructor described the process as “jazz improvisation, where the melody and its counterpoint are inconspicuously layered in a structured way.” This approach may not always work well with traditional curricula that tend to fit a script for various reasons, including, but not limited to, state exam preparations, teaching a “prescribed” set of skills, and even classroom management. Simply put, teachers are afraid to try new curricula because they are afraid to fail. Some teachers are the least likely to integrate because they feel that if they are not experts, they will not execute the lesson plan well. Although this is a natural and understandable hindrance, the fact remains that teachers learn to integrate by actually doing so. While training is valuable, most of the instructors in the study did not undergo special training to prepare them to teach an interdisciplinary course, at least at inception. However, through collaboration, vulnerability, reflection, feedback, and reconfiguration of the curricula, these instructors learned ways to help students engage in authentic interdisciplinary work.

Interdisciplinary learning, by design, is not perfect; but specific tools like project-based activities and standards-based grading can provide the necessary support for achieving learning goals.

**Interdisciplinarity in Practice.** The instructors described how they designed activities for students to engage in interdisciplinary learning by integrating subjects, disciplines, and pedagogies into mathematics and (sometimes) vice versa. While authentic interdisciplinary work benefits from substantial exposure to prior knowledge of the other disciplines, it is also designed
to inspire curiosity about the disciplines involved. This seeming dilemma is also the nature of the work itself and may partly explain why it’s possible to learn mathematics content in interdisciplinary work. Some instructors incorporated “different activities involving subjects with mathematics” and “worked with faculty to incorporate mathematics in non-disciplinary ways.” Others integrated “curriculum areas,” including “STEM-based concepts.” Some participants used pedagogies from English, History, and written and oral communications as part of direct instruction and assessment. As the literature revealed, some instructors’ interdisciplinarity focused more on some disciplines than others. For example, a few instructors claimed they taught “STEM daily through a math and science interdisciplinary curriculum” with not much purposeful attention to the engineering and technology disciplines. Another instructor said that “the STEAM approach to teaching” is used in all subjects. This idea of using the term “STEAM approach” is vague if it is not clear that students are making strong connections among the disciplines (Thibaut et al., 2018).

Although all instructors taught mathematics as a subject, they did not all engage in the same level of interdisciplinary or integrated work. Instead, interdisciplinarity and mathematics learning occurred along a spectrum. In some cases, instructors taught mathematics lessons for understanding, with interdisciplinarity playing a minor role. In other cases, while interdisciplinarity was the intended goal, activities revealed that mathematics was used procedurally with little emphasis on conceptual understanding. However, some activities showed that mathematics was intended to be understood, explored, and extended. Nevertheless, there were some instructors whose mathematics and interdisciplinarity placed understanding mathematics at the core of their activities. Based on responses to the survey, interview questions,
and activities, results showed that instructors engaged in interdisciplinarity in three significant ways, each explained later in this chapter.

Factors Informing Interdisciplinary B, P, & A

Interdisciplinarity Supports Mathematics Teaching. Interdisciplinarity presents opportunities for teaching and learning mathematics in exciting ways, with some instructors claiming significant benefits from teaching other subjects outside of mathematics. In one instructor’s reflection on why he integrated his understanding of other disciplines into mathematics, he paraphrased Neil Postman’s advice in Teaching as a Subversive Activity, “Every teacher should teach outside of his or her subject area because then you'd understand what it's like to be a learner.” With a B.A. in English, political science, and philosophy, a Master’s in education, and a Ph.D. in applied science, his alternative route to earning his certification in mathematics teaching made him feel like “an outsider” since he did not become a teacher “the traditional way.” Nevertheless, he felt like this experience made him empathize with his students, primarily seniors, who get “very discouraged and often feel like giving up” because they are usually “behind in math credits” at his alternative high school. These students, who are at risk of failing high school (altogether), have an alternative route to earning credit for Secondary Math 3 by taking his Math for Decision Making and Modern Math courses. Stan aimed to help his students remain motivated with projects that were meaningful to them. In two of his classes, students gathered statistical data based on their interests and “made sense of it”:

One student looked at aftermarket modifications that make your car go faster…[while] another group of students looked at the correlation between your mood and how much time you spend on social media because a lot of this data is just in your iPhone. And my interest was physical activity and exercise motivation. My students probably picked up on that, but [together], we’ve looked at mood and how it correlates with how many steps you took in the last 24 hours and things of that nature. So, we do our best to gather some data and make some meaning of it.
Although he neglected to call his course “interdisciplinary,” his projects were designed to help students develop “careful, logical, and rigorous” thinking skills.

Another instructor shared that the flexibility of an interdisciplinary curriculum created space and time to teach in a way that was more sensitive to the various ways in which her students learned. She taught high school students who have an “interrupted education from their native country” and often missed several years of elementary and middle school mathematics, alongside students ready for college. She is always thinking of ways to balance her work of preparing students for college mathematics while ensuring that those who need gaps filled are ready for their weekly activities; this can be difficult at times. During a STEAM project, she felt compelled to provide “direct instruction” for prerequisite mathematics concepts that her students had not learned before the course. However, due to time constraints, she was challenged by her co-teacher to take a different approach:

In this example, [the] student did a graphical sketch…but didn’t have data. [This] is where it finally clicked that [my co-teacher] was right, …[where] sometimes doing a sketch might help the student get to the data part and understand the math. I never understood it that way. I was thinking very linear…which tends to be a real thing among math educators….

Her teaching career benefited from collaborating with her co-teacher. It helped her become a better educator, as collaborating with her colleague “improved her pedagogy” and helped her to “develop a growth mindset” as an instructor. Interdisciplinary teaching requires collective, constructive input and feedback from the community—i.e., students, co-teachers, administrators, and other experts—for growth and improvement. Interdisciplinary instructors learn and grow from each other’s disciplines. Explaining your subject matter to other interdisciplinary team members makes you “really think about [how] someone is learning mathematics, which ultimately benefits students. Interdisciplinary instructors work in tandem,
spending time planning, brainstorming, and sometimes teaching together, which can be like in-school, real-time professional development. Their challenges and successes also show students what working together as a team entails.

**Interdisciplinarity Curates Content through Different Lenses.** Participants described interdisciplinarity by describing how they and their students experienced it. One instructor described her approach as “curat[ing] mathematics content through different lenses.” She collaborated with the directors of the writing and oral communication centers at her college for expert support in implementing these pedagogies into the class structure and assignments. She felt strongly that “those amazing resources” made her job “way easier” because they were on campus, and she did not have to be the expert in them. Her interdisciplinary curriculum incorporated writing pedagogy and speaking/oral communication pedagogy into subjects across the curriculum. Another instructor saw mathematics content as the anchor around which mathematical thinking developed. She wanted students “to see” how mathematics fit within other subjects and how other subjects incorporated mathematics.

As mentioned previously, some instructors pointed to project-based learning and standards-based grading as two structural supports. These supports helped instructors design activities that helped students develop a deep understanding of concepts. However, it is not just about completing projects. If the project is too broad, it is difficult for students to learn new mathematics or extend concepts learned in class. Thus, it was necessary to have multiple ways to display their understanding, which benefited students and maximized students’ interests and strengths. The integrated subjects such as art, literature, multi-media, etc., also serve as a way for students to produce work. Additionally, students learned and used techniques from other disciplines for comprehension.
In one instructor’s Number Theory with Applications course, the projects are “almost all cryptography,” including a midterm which is both mathematics computation and conversation:

Students select *any* employee on campus outside of the math or computer science departments…and their goal is, in a 20-minute conversation, to get across the idea of RSA cryptography….and to communicate why we need mathematics for this to work…. Priscilla then met with the employee to see how well the student communicated the mathematics by seeing what they learned. It is an excellent way for different members of the college campus to collaborate. Of importance to interdisciplinarity, the final product—whether an essay or work of art—showed how mathematics was necessary to accomplish the task, which became an opportunity for students “to make sense” of it. The project allowed time for students and instructors to analyze, make conceptual and procedural connections, discuss, and provide feedback, a testament to resources mentioned in the literature that are necessary but not always available for interdisciplinary learning to be successful. I shared an excerpt of the midterm in Figure 4.6, with the complete assignment located in Appendix T.

**Interdisciplinarity Occurs on a Spectrum.** While they all taught mathematics as an *isolated* subject, they engaged in interdisciplinarity in various ways. Instead, interdisciplinarity and mathematics learning occurred along a spectrum. Upon closer analysis of instructors’ interviews and artifacts, the participants engaged in interdisciplinarity in three ways.

*Constructors* focused on learning mathematics with understanding. They utilized context from other disciplines, critical thinking, and understanding connections to help students construct mathematical knowledge. Steven, Stan, and Sally—veteran educators with over 15 years of experience *each*—did not claim to employ an explicit interdisciplinary curriculum in the classroom. However, they all integrated discipline-specific pedagogy in teaching mathematics to
middle and high school students. Since teaching and learning mathematics for understanding is not a “one-size-fits-all” endeavor, they reimagined and integrated practices from other disciplines to help students develop a “strong conceptual understanding.” Sally used discourse and her understanding of students’ brain development to help them think through problems. She told her students that “math is a discussion and search for patterns because our brains are
designed to look for patterns in the environment.” She used a mixture of research and her experience in teaching to guide instructional decisions and activities.

At the time of this study, Steven and Stan worked at an alternative high school for students at risk of failing K-12 altogether. Steven used his training in special education to help students develop a conceptual understanding of mathematics vocabulary:

I built on my special ed training, where when you teach vocabulary, you don't say, ‘Hey, we're learning this word, and then here’s the definition.’ Instead, you say, ‘Let's talk about this idea. Now, what we need is a word to encapsulate this idea, so here’s the word that people use to describe this idea…. ’ But get the idea firmly ‘in the head’ before you introduce the word, and the same thing is true in math. The vocabulary comes at the end and not at the beginning.

Steven and Stan expressed the importance of showing students that they were worthy of learning mathematics regardless of their backgrounds or long-term goals. In Stan’s interview, he spoke genuinely about his intentions to help students think through problems logically. He acknowledged that it is difficult to teach mathematics to students who are at risk of failing high school. However, he learned ways to motivate them for post-high school success:

One of the things that I really push is using spreadsheets because I think they'll see them again; there might be a lot of jobs where they're going to show up. And one of my former students emailed me and said, ‘Hey, I’m the assistant manager at McDonald's, and we're trying to figure out a system to keep track of how much petty cash we need to have.’ And so, I had her come in, and I helped her set up a spreadsheet template where it would do the calculations. But she felt confident enough in her spreadsheets that she was like, ‘Oh, I could use a spreadsheet, and I can track this, and it would do the calculations for me, and I wouldn't have to think about it; it would be like a good job aid for me.’ So that was a success, I think, and they're sometimes few and far between.

I shared excerpts of artifacts from both Sally and Stan in Appendix S.

Figure 4.7 shows Steven’s hand-drawn worksheet reproduced each time his students see exponential functions for the first time. He transitioned from “block model” to “table” to “graph” to “equation,” noting facts like domain, range, etc., along the way. More importantly, was Steven’s motive for this as he believed this was a necessary component of “constructing
Figure 4.7

*Steven’s Hand-Drawn Worksheet—Exponential Equations*
understanding.” He shared the following:

I walk around and look at every paper, every step, and I’m looking for problems. And once we've done this, I say, ‘now set this [worksheet] aside, and you help me change the pattern. Tell me what number you want to start with? Tell me what we should multiply by?’ And we’d make the first three or four steps of the pattern, and then I’d say, ‘Now you take out your second piece of graph paper, and you do all of this yourself. How far can you get?’ And I’d do that for five days in special ed, in regular ed, maybe three, so kids can connect tables to a graph to Algeblocs and then equations.

Curators, the second type, incorporated STEM or STEAM strategies into direct instruction, projects, and assignments. Priscilla, Paul, Elizabeth, Emma Sue, Emily, Evelyn, and Eleonor, span learning levels from elementary school through college years, course loads, and years of teaching experience. However, what they all shared was their focus on mathematics content, implicitly and explicitly taught through the lens of other disciplines. As a curator, they carefully chose when and how a pedagogy, practice, discipline, or subject lent itself well in the service of learning mathematics. Of importance was that this does not always conform to interdisciplinarity as defined in the literature review. Furthermore, mathematics was sometimes used procedurally in projects or may not be explicitly explored through their activities. They taught mathematics and worked with faculty to incorporate mathematics in a non-disciplinary manner. Their interdisciplinary work included integrating other subjects like writing, art, and reading comprehension into mathematics tasks or projects that tended to have a STEM or STEAM focus.

Priscilla’s interdisciplinary strand incorporated writing and communication pedagogy into mathematics courses across an undergraduate curriculum. Projects required students to show why mathematics was necessary for ideas or “products” to work and communicate that idea to different audiences. Paul taught mathematics to non-majors but also engineering students. He admitted that his background in chemistry and mathematics sparked interesting examples and
conversations with his engineering students. However, what he cared about was that all students “understood core ideas” and were able to “use those ideas to complete [mathematical tasks].”

Both Priscilla and Paul used their research as an added lens informing their work in the classroom. Appendix T shows examples of their work.

Elizabeth, Emma Sue, Eleonor, Emily, and Evelyn taught elementary and middle school students from first through eighth grades. The most experienced teacher, Elizabeth, whose background included teaching students in four different languages as a young teacher in Mumbai, incorporated literary techniques into assessing mathematical understanding for her elementary students. In her current post at a private Catholic school, she capitalized on all her students’ experiences. Since most of her students attended the same Catholic mass, she looked for ways to bring this experience into the classroom. For example, in a recent sermon on “the parable of five loaves and two fish,” she used that story to reinforce multiplication, division, measurements skills, and religious history that her students encountered that week. She engaged students with her homemade bread replicating the five loaves and used her literacy lens to analyze the religious and historical significance of the parable. Emma Sue, the sole mathematics teacher at a Catholic boarding school for boys with talent in music, also felt a huge responsibility to make mathematics meaningful for her students. She shared that she was always thinking of creative ways to incorporate her students’ artistic inclinations. Appendix T contains sample artifacts and her reflections. Figure 4.8 is an excerpt of her “Math is Everywhere” hexagonal wall, where students create a tessellation of mathematics in their daily lives.

Emily, Evelyn, and Eleonor all taught students at schools designed for STEM learning. Although Emily and Evelyn taught at a school designated STEM and Magnet, “curriculum standards” were “cookie-cutter” by design, making it more traditional than intended. However,
they both referred to using “STEM approaches” to provide students a way to “think about things.” Emily used a reading comprehension technique from the ELA department to help her students analyze “math story problems” since “all curricula use some form of asking and answering questions with text-based evidence.” She encouraged students to “look at math as not just ‘numbers that I’m afraid of’ but apply a reading lens to look at mathematics problems.” Students begin by “asking the question, restating the question, and providing an answer; then go back to the equation to cite and support their claims.” Evelyn used her background in technology to incorporate technology-based learning for assessments. She used the game-based app Kahoot,
and interactive whiteboards, for her formative assessments in mathematics, which she felt kept students engaged. However, Evelyn balanced this with her version of “error analysis for second grade students.” When students made a mistake on a quiz, she “highlighted it.” She then placed students in pairs to “talk about why a procedure was highlighted” and gave them a chance to rework the problem.

Eleonor also worked at a “STEM studies” elementary school, but they “did not have a curriculum that addressed STEM directly.” Students used an “adapted science curriculum” to conduct scientific experiments. Teachers, indeed, were encouraged to integrate scientific literature for “the reading and writing aspect of instruction,” but many desired more training to do so effectively. They “also had a maker space which allowed [them] to teach and facilitate STEM lessons.” In addition to these efforts, they conducted weekly “STEM stations,” where teachers volunteered to help “create a STEM course where those practices were somehow related to science, technology, engineering, and mathematics.”

The last group of instructors, Connectors, focused on mathematics instruction with explicit and implicit connections to other disciplines and subjects. Pruitt, Selma, and Pearl taught mathematics and an interdisciplinary course in the same semester or consecutively by semesters, thus embodying the core of the study. Their examples of interdisciplinary activities used mathematics to create products like digital art and computer programs, along with projects in which students displayed efforts to extend the learning of mathematics concepts or crystallize understanding. Their interdisciplinary courses were co-planned, co-designed, and co-taught with faculty from other disciplines, as with Selma and Pearl. In Pruitt’s case, he collaborated with members from different disciplines or subjects during planning to ensure that the principles from those disciplines were preserved, acknowledged, and assessed. They strove to maintain the
integrity of interdisciplinarity, even with the challenges inherent in this process. Learning mathematics and making connections were essential learning objectives. Furthermore, they showed evidence of using practices that represented a deeper level of interdisciplinarity, by design, intending to learn mathematics concepts. I used the TRU Math framework as a lens to examine the depth and understanding of mathematics in their activities. The results from this analysis and their stories are highlighted in Chapter 5.

4.3 Summary

The overall purpose of this chapter was to answer research question one:

1. What factors inform teachers’ beliefs and conceptions towards teaching and learning mathematics and the instructional practices enacted in the classroom? How do these factors differ between mathematics lessons and an integrated or interdisciplinary curriculum such as STEM or STEAM?

The instructors cited several factors through their pre-study survey results, interview responses, and artifacts.

These instructors were passionate about helping students value learning and their students’ abilities to do mathematics. They saw their students as capable problem solvers and used instructional activities to illuminate thinking and understanding of mathematics. Through their instructional activities, they sought to make mathematics meaningful, joyful, and accessible and seemed to associate that with valuing learning. Additionally, they used assessments to communicate mathematics and the various ways they integrated disciplines and subjects with mathematics and vice versa.

For these instructors, interdisciplinarity played a role in their instruction because they used it in the service of learning mathematics. Due to their different ways of practicing
interdisciplinarity, they were characterized as Constructors, Curators, and Connectors or referred to as the 3C’s framework. Figure 4.9 offers a visual model with short descriptions of the categories of interdisciplinarity.

Figure 4.9

*Visual Model of the 3C’s Framework*

<table>
<thead>
<tr>
<th>Constructors</th>
<th>Curators</th>
<th>Connectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teach mathematics with an eye towards understanding.</td>
<td>Integrate disciplines, subjects, and pedagogies in teaching mathematics.</td>
<td>Combine mathematics and interdisciplinary teaching experiences to learn mathematics through making connections.</td>
</tr>
</tbody>
</table>

In summary, this study was not meant to advocate for one type of lesson over the other but to examine how teaching *both*—mathematics and interdisciplinary—influenced how mathematics instructors taught. The Connectors came closest to this in their teaching. Their stories are highlighted in chapter 5 and used to answer research question two:

2. Are there differences in mathematics content, cognitive demand, and assessments between mathematics and integrated or interdisciplinary activities?
Chapter 5: Results: Portraits of Connectors

The second purpose of this study was to highlight how instructional activities promoted engagement in mathematics as a subject and when integrated with other disciplines. The TRU Math rubric showed promise in testing for engagement in mathematics and other disciplines (Li & Schoenfeld, 2019), but this study extended its use for examining an interdisciplinary lesson. I used an adapted TRU Math rubric (Table 5.1) as the framework for analyzing engagement in the activities submitted by instructors. Since Connectors taught mathematics as a subject and within an interdisciplinary lesson as defined in the literature, I used their artifacts to answer research question two:

2. *Are there differences in mathematics content, cognitive demand, and assessments between mathematics and integrated or interdisciplinary activities?*

Each instructor’s section incorporates the following categories:

1. Educational and Professional Background
2. Philosophy and Teaching Beliefs
3. Mathematics and Interdisciplinary Course Overviews
4. Analysis of Instructional Activities along TRU Math Dimensions:
   a. Mathematics Content
   b. Cognitive Demand
   c. Use of Assessments
5. Summary of Instructors’ Beliefs and Practices

Data from the surveys, interviews, and instructional artifacts of Pruitt, Selma, and Pearl are presented. Complete replicas of submitted artifacts are in the Appendices referenced throughout the chapter. The chapter concludes with a summary that addresses research question two.
# Table 5.1

<table>
<thead>
<tr>
<th>Score</th>
<th>I. Mathematics</th>
<th>II. Cognitive Demand</th>
<th>III. Use as Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Activity is aimed at &quot;getting an answer&quot; without addressing underlying reasoning.</td>
<td>Activity requires no more that applying formulas or memorized facts.</td>
<td>Assessment is limited to corrective feedback or encouragement, with limited to no indication of pursuing student reasoning.</td>
</tr>
<tr>
<td>2</td>
<td>Activity is at grade level, with few opportunities for students to make connections between procedures &amp; concepts or engage in mathematical practices. Teacher support is minimal and does not exploit them.</td>
<td><em>Activity offers possibilities of rich conceptual ideas or challenging word/story problems, but interventions from the teacher &quot;scaffold&quot; away the challenges, including productive struggle.</em></td>
<td><em>Teacher reflection shows evidence of where students discuss their thoughts on the problem and/or common mistakes; but students' ideas are not built upon or developed. Teacher simply corrected student work without addressing challenges.</em></td>
</tr>
<tr>
<td>3</td>
<td><em>Activity shows intentional opportunity for mathematical connections, engagement, and practices. Teacher reflections show where students are encouraged to support their reasoning with a coherent and connected view of the mathematics.</em></td>
<td><em>The teacher's reflection on the activity shows where a hint or scaffold supported students in &quot;productive struggle&quot; to build understanding and engage in mathematical practices.</em></td>
<td><em>Teacher reflection shows evidence of the solicitation of student thinking and subsequent discussions responding to those ideas, by building on the productive beginnings or addressing emerging misunderstandings.</em></td>
</tr>
</tbody>
</table>

*If the data is insufficient to determine this, the instructor may submit a reflection with additional information about the activity.*
5.1 Pruitt’s Story

*Educational and Professional Background*

Pruitt practiced interdisciplinarity as a mathematician and an artist. He is an advanced career educator with over 16 years of experience, who taught mathematics and interdisciplinary courses at an urban community college. He has a Ph.D. in mathematics and no formal training in education outside of “a one-week TA workshop” in preparation for fulfilling his teaching assistant duties while pursuing his doctorate. Although he felt very confident in mathematical content knowledge, he did not feel that he had enough training to prepare him to teach mathematics. Based on Pruitt’s responses, it seems as if he developed his pedagogy through teaching his courses and from collaborating with colleagues:

…it’s interesting to talk to you about what it is to be a teacher of art because I’ve been reflecting on each of these bits along the way…. When I started having essays and had to be an English teacher or writing teacher, I started to see how the peer review process is super important and worth taking time out of the semester to do that… [I incorporated it]. And then being a computer science teacher and understanding that people learn the material in different ways…. I created tutorials that let people advance at different rates, instead of doing it in class where some people are super bored, and some are just falling behind…[because] a tutorial where people can go at their speed [is] a really useful way for computer scientists. And as an artist, allowing there to be no correct answer...[was] a [considerable] jump for students…. When I go into my mathematical modeling class, and there's no right answer, [students say,] ‘What do you mean there’s no right answer. It’s a math class!’ [I then say] ‘Well, there are multiple ways to approach this, and if you can justify it, then that’s important!’ So, all of these different ways help me be a better educator. I learn new techniques. I guess I always like learning new things too. And so I push myself in this way….

Pruitt implemented interdisciplinarity by incorporating discipline-specific practices into his courses, consulting with experts within departments at his institution, and from his teaching and learning experiences. He organized learning in all his classes around specific standards. Although standards-based grading (SBG) “is not perfect,” it allowed for more transparency, in
his opinion. Thus, “if somebody [got] an ‘A,’ we know that they understand most of the material completely and some of the material, at least at a competent level.” SBG also allowed him to create rubrics around learning, where “students share their [understanding of] concepts and knowledge” and are given grades based on that information. Students then have several projects, usually at least three, over which skills and concepts are developed and explored. This model is Pruitt’s teaching philosophy because “students are more motivated by projects” and prefer not just to memorize skills and procedures. Students in his mathematics and interdisciplinary courses end with a project they share online or with potential employers, depending on their academic and career goals.

Even though Pruitt expressed that he became interested in the study because he wanted an opportunity to reflect and “learn more about interdisciplinary teaching,” based on his artifacts and reflections shared during his interview, he was the instructor with the most experience with interdisciplinary mathematics. In their theoretical description of interdisciplinarity, Williams and Roth (2019) suggest the following definition of interdisciplinary mathematics:

> [It] involves various sorts of conjunctions of mathematics with other knowledge in problem solving and inquiry. This ‘other’ knowledge is generally outside mathematics whether this involves one or more disciplines…or just extra-mathematical or ‘everyday’ knowledge… [such as] traffic flow…. As the relationship between mathematics and other disciplines becomes more interconnected, a genuine ‘inter’ disciplinarity emerges when mathematics interacts with other disciplines to become something new and different…as in…quantitative reasoning…or mathematical physics…. (p.14)

By Pruitt’s admission, he was also not necessarily trying to “convert” students into majoring in mathematics. Instead, he was much more interested in helping people see the beauty in mathematics and not be terrified of it:
We have this **awful** thing in society that ‘you’re either a math person or not a math person.’ And this hatred of math is just out there, and it gets past [on]. You get one teacher when you’re like in first or second grade who is like, ‘Oh, I never was very good at math!’ Or [a teacher] who is really scared about approaching a math lesson, which translates onto the students. So, if we had **more beauty** in math, **more time to play** with math, **and more time to be creative** with math, **maybe** we wouldn’t have such a ‘math phobic’ society. And that, whether it’s people coming into this class...[saying] ‘I don’t **want to** take a math class,’ instead of, ‘Oh my god, I **have to** take a math class’ [in a more ominous tone] .... If we have more math majors or not, I don’t really care. It’s more just maybe you would actually have some people who wouldn’t be so math-phobic, maybe we would have less anti-science in this country.

Additionally, Pruitt is a member of a community of mathematicians and artists whose mission is to “foster research, practice, and new interest in mathematical connections to art, music, architecture, and culture” (Bridges Organization). He creates and sells mathematical jewelry and has presented at the Bridges national conference in the past. In short, Pruitt’s mathematics and interdisciplinary lives intersect inside and outside of the classroom.

*Philosophy and Teaching Beliefs*

As a tenured and advanced career professional, Pruitt’s philosophy and praxis are that of one who is always “evolving” while placing students at the center of lesson planning and execution. He strongly believes in incorporating student reflections about their work as part of his standards-based grading process and implements this in both types of courses. Each year, incoming students have access to thoughts and suggestions from past students, accessed through his course websites under a “Letters to Students” section. Appendix R contains selected notes of past students labeled by course names. He admitted to always “having his ear out” for what is new in education and “bringing his students along for the ride.” He approaches teaching with a “growth mindset” and tries to instill this attitude in his students. His survey responses, interview, and course and project objectives and standards showed this student-centered approach.
Figure 5.1 shows Pruitt’s frequency scores for his teaching *instructions, student engagement/cognition*, and *enhanced activities*. These are explained in Chapter 3 but restated here:

- **instructions**—the ways in which teachers set up the teaching and learning process;

- **student engagement**—the ways in which teachers design activities to meet students’ individual and collaborative learning needs;

- **cognitive action**—the ways in which activities required students to engage in problem solving through evaluation, integration, and knowledge application; and

- **enhanced activities**—student activities designed to extend learning beyond direct instruction.
OECD (2010) summarized that teaching practices with high support for instructions, cognitive action, student engagement, and enhanced activities offer great opportunities for high-quality, conceptually rich instruction. Recall that opportunity scores, \( o \), where \( o \geq 3 \) indicate a high use of such practices, \( 2 \leq o < 3 \) mixed opportunities, and \( o < 2 \) low use. The lowest opportunity score for teaching practice is one, and the highest is four. Pruitt’s B, P, & A opportunities suggested high opportunities for student engagement in both of his courses. A closer analysis of his artifacts using the TRU Math rubric illuminated such claims.

Pruitt tied valuing learning to students’ needs, and as such, his philosophy and corresponding instructional activities connect his teaching experiences to how students learn. He described his evolution as an educator:

> So, in terms of my philosophy, as I’ve done more and more teaching, there [are] two big things that have surfaced, one of which is that I like to do project-based teaching because I feel that students are more motivated to create, to do projects than they are to regurgitate material. Furthermore, it gives the students an opportunity to [apply] the concepts they’ve learned in class. I feel that students get sort of super motivated by that. Furthermore, they have a product that they can then share with people outside of the class at the end of the semester. So, it’s something that’s just made for the class. It’s something that, if they’re proud of it, then they can share with future employers or use as a portfolio of the types of things that they’re able to do.

Pruitt, however, did not feel that he had different teaching and learning goals for students in his mathematics than in his interdisciplinary courses. He thought that the level of cognitive action experienced in both classes was the same. The pre-study survey’s data also seem to reflect and support his claim that opportunities for student engagement are the same in interdisciplinary and mathematics activities when comparing the scores in cognitive action and cognitive demand of both kinds of activities (seen in Figure 5.1). However, an analysis of his activities provided more insight into the depth of mathematics explored within activities.
Mathematics and Interdisciplinary Course Overviews

*Calculus II: Integral Calculus*

This mathematics course was organized around standards (see Appendix I). Pruitt has taught all levels of calculus. Calculus II taught students fundamental techniques from integral calculus that students must “be able to apply to unfamiliar examples.” Other goals of the course included the following:

1. Recall the statements, consequences, and applications of main definitions and theorems.
2. Develop familiarity with theory and evaluation of sums, integrals, and applications of integrals.
3. Develop the ability to work productively on mathematics with others.
4. Develop techniques for succeeding in college classes, especially in mathematics.

After seeing how students experienced project-based learning in his interdisciplinary courses, Pruitt added the goblet project.

*Mathematical Design*

Pruitt planned his Mathematical Design course similar to a Creative Computing course in the Design Program in the Art department. This course was an art class for non-majors that used principles of artistic design along with piecewise-defined functions and parametric equations to create digital art. Teaching art made Pruitt more comfortable designing open-ended projects where the final product highlighted “student’s creativity even though it [may] differ from the instructor’s.” As part of his artifacts, Pruitt also shared student reflections and their progression through the projects. The learning objectives were organized around standards (see Appendix J), and students were expected to do the following:

- Develop familiarity with cartesian and polar coordinates.
- Develop familiarity with a variety of cartesian, trigonometric, polar, and parametric functions.
• Understand geometric objects and the behavior of a variety of transformations on them.
• Develop an appreciation for mathematical constructs and their aesthetics.
• Successfully implement algorithmic techniques, including iteration and randomization.
• Gain an ability to analyze a problem and identify and define the mathematical foundations and computing requirements appropriate to its solution.
• Use mathematics and programming for experimentation and as creative tools.
• Apply the design process and communicate the decisions made therein, including ideation, artistic principles, prototyping, and revisions.
• Advance teamwork skills by collaborating with classmates, discussing and solving problems in a group, and practicing giving and receiving constructive feedback.
• Develop techniques for succeeding in college classes, including instilling a growth mindset.

Analysis of Instructional Activities along TRU Math Dimensions

The complete analyses of Pruitt’s mathematics and interdisciplinary instructional activities are in Appendices I and J, respectively, along with my reasoning behind the TRU Math scores. The title of the activities and combined scores for each dimension is in Table 5.2.

Table 5.2

<table>
<thead>
<tr>
<th>Instructional Activity</th>
<th>Combined TRU Math Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Activity 1: Vocabulary Homework: Understanding Definitions</td>
<td>8</td>
</tr>
<tr>
<td>Mathematics Activity 2: Goblet Project: Overview and Tasks</td>
<td>8.5</td>
</tr>
<tr>
<td>Mathematics Activity 3: Goblet Project – Two-Page Writeup</td>
<td>8.5</td>
</tr>
<tr>
<td>Interdisciplinary Activity 1: Determining Function Transformations using Desmos</td>
<td>7</td>
</tr>
<tr>
<td>Interdisciplinary Activity 2: Take-Home Quiz on Standards 4 - 6</td>
<td>9</td>
</tr>
<tr>
<td>Interdisciplinary Activity 3: Project 1 – Functions and their Families</td>
<td>9</td>
</tr>
</tbody>
</table>

Recall that a combined TRU Math score is the sum across dimensions for each activity and ranges from 3 to 9, with higher scores suggesting higher levels of complexity. Pruitt organized his courses very well, making it easier to analyze along similar structures. Additionally, all courses have an extended project for applying concepts.
Comparison of Calculus II and Mathematical Design

Mathematics Content

Table 5.3 compares the two courses along the three dimensions of the TRU Math Rubric. The highest score for each dimension is 3. Even though there is a slight difference in the *mathematics content* domain, students have high opportunities (in general) to demonstrate a coherent understanding of mathematics content and practices in both mathematics and interdisciplinary activities. Pruitt’s TRU Math opportunity scores from his pre-study survey (in Figure 5.2) and the mean scores of instructional activities assessed using the TRU Math rubric follow a similar pattern.

Table 5.3

**TRU Math Dimension Scores of Artifacts per Courses—Pruitt**

<table>
<thead>
<tr>
<th>Course Title</th>
<th>TRU Math Mean Scores of Artifacts - Pruitt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematics</td>
</tr>
<tr>
<td>Calculus II: Integral Calculus</td>
<td>3</td>
</tr>
<tr>
<td>Mathematical Design</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Figure 5.2

Average Scores-TRU Math Dimensions (on Survey), Mathematics & Interdisciplinary—Pruitt
pattern. However, the average score in the mathematics content dimension is higher in Calculus II than in Mathematical Design for his artifacts than was claimed on the survey. Pruitt’s clear standards and design of activities provide significant context for analyzing them with the TRU Math rubric. As a result, the context needed to test for complexity was available in the artifacts he submitted.

The complexity of Pruitt’s mathematics artifacts was not surprising because he was firstly a mathematics professor. His mathematics and interdisciplinary activities provided students with opportunities to share their understanding. The mathematics content of the activities was appropriately challenging and offered solid opportunities for students to make conceptual and procedural connections. Pruitt’s artifacts required students to show how and why mathematics concepts worked. Besides, Pruitt designed his activities for students to understand mathematics content by making sense of it. For example, in the “Understanding Vocabulary” homework assignment, students must write the definition in their own words and include examples and “non-examples” of an antiderivative of a function and a differential equation. Similarly, students in the Design course must also make sense of mathematics, even though it is used to create art. Students are assessed on their understanding of the mathematics standards throughout each project to hold them accountable and perform well on the final assessment. Students also explored mathematics concepts in the Mathematical Design course based on the standards and requirements in the interdisciplinary activities. One student reflected on his performance in the Design course:

We’ve come to the end of the semester. I’ve had time to reflect on the progress that I’ve made as a mathematician and an artist. The thing I could say is that when I first came to the class, I struggled a lot because I didn’t get the concept. I didn’t even know it was
possible to use mathematical functions to make art. So, it was something new to me. It was those types of conditions that created my first piece of artwork.

Pruitt’s pre-study survey results and rubric scores for mathematics content on activities align. He indicated high opportunities for students to learn mathematical content and make connections in both courses. However, since the nature of my study called for only two to three activities, more activities are necessary to provide more context for the difference in scores. Additionally, the Mathematical Design project showed an intense level of interdisciplinarity as defined in the literature review. It was designed to make explicit, deliberate connections between art, mathematics, and technology.

_Cognitive Demand_

While Pruitt’s TRU Math survey scores (Figure 5.2) indicated that both courses provided an equally challenging environment for “struggling productively” on tasks and students’ mathematical development, a closer analysis of artifacts revealed slight differences. Activities required students to make procedural and conceptual connections and go beyond seeking answers to soliciting evidence of comprehension by asking for “examples and non-examples” of terminology. Both projects received the highest score (of three) in providing rich opportunities for engaging in mathematical practices like reasoning and sensemaking. The projects in each course revealed a high opportunity for cognitive action as students were required to show how their knowledge and understanding contributed to the group project (in Calculus II) and in the peer review process (in Mathematical Design). In my analysis, I wanted more evidence of the conceptual challenge of specific tasks in Calculus, and as such, the scores for cognitive demand are lower than anticipated. However, activities at the early phase of a course or project are
usually geared toward building knowledge and skills. More assignments and possibly observing a lesson may shed light on this. Pruitt, however, provided some insight as he described his approach to helping students think carefully:

…part of what you want the students to get out of the classes is a way to think carefully, analyze definitions, and apply definitions. And there are a number of different things in all of my classes that have that same idea, especially in my mathematical computing class, where the students have such varied backgrounds. Some students have been programming in many different languages, and for some, this is maybe their first computer class. So, when I design my rubrics, I want to take that into account. I’m not going to set the same bar for everybody. I will say, “okay, so you're here; I want you to get to here.” And I want you to engage with the material, and that’s what I want to measure, and that’s what you’re going to be graded on.

Use as Assessment

Pruitt's assessments actively solicited students’ thoughts about their understanding of mathematics. His activities showed medium to high levels of rich opportunities to build on students’ ideas and provide a chance for them to level up in understanding. One of the student reflections in the Mathematical Design course described this very process:

…my first time touching Desmos, I didn’t have a lot of understanding of the concepts that we [would] soon cover in class. So, I [tried] to be creative with the simple things I already know. [By the time] I submitted my first project, I had a little bit more tools to use. Before that, I had no idea I could use sine and cosine instead of just Desmos [to create an image]. So, I [could add] color in my work using the greater-than and less-than functions. There is another part to this art that is hidden that you can’t see because his head [in the picture] was a beautiful shape that if you manipulated it with a slider, it created a beautiful pattern. But I felt like it wasn’t enough to be able to hand it in, so I just kept building upon it.

Interestingly, Pruitt’s pre-study survey results suggested that tasks in his interdisciplinary course scored slightly higher in their productive use as an assessment than in his mathematics course. This is possible because they were heavily project-based in conjunction with his standards, which made it easier to measure its feasibility as an assessment. Overall, Pruitt expected students
to discuss their thoughts “carefully,” providing multiple chances for input from him at critical stages along the process.

**Summary of Pruitt’s Beliefs and Practices**

Pruitt’s artifacts showed slight differences in mathematics content and cognitive demand than was reported in his pre-study survey. Furthermore, the cognitive demand for his interdisciplinary activities was slightly higher than in his mathematics course. Although he claimed these levels were the same during the interview and on his survey, the TRU Math rubric provided more insight. Students in both types of courses were held to expectations of learning, outlined clearly in his standards. As such, his artifacts in both courses worked well as assessments. His experiences teaching both courses also influenced his design of activities.

Pruitt supported his student-centered pedagogy expressed in his survey and interview with his instructional activities. Even though he was personally motivated by his love of learning, he found ways to use that in the service of his students. He used a “tree metaphor” to summarize his style of teaching. Nonetheless, this is understood better in Pruitt’s explanation of how projects enabled him to reach students on different learning levels:

> I present some content, and I almost feel like it’s a tree. I give everybody the same basis…but then based on what you want to study, everybody will go in a slightly different direction. And by two-thirds of the way through the semester, we now have the situation where people’s knowledge bases are different because they are based on different things. Then in the last part of the semester, they’re supposed to create an interactive app using Mathematica. And in that situation, I give them some small building blocks, but then everybody goes in completely different directions. By the end, we’ve got this huge tree happening where everybody has learned different techniques and different parts of the software. [In] the end, you see the diversity of the projects that people have made. And so, it’s not just diversity in terms of their capabilities, but everybody brought some part of themselves into the project. And that’s probably one of the most rewarding things about teaching this way: the students have been able to access in themselves something that they’re interested in and use the content from the class to push it further
and grow. I’m not seeing everybody as having the same background. I’m taking the fact that everybody has a different background and highlighting it and allowing that to show in the projects they come up with.

5.2 Selma’s Story

*Educational and Professional Background*

Selma is the daughter of Latinx immigrant parents who moved to the United States for its opportunities. Due to her mathematics scores on placement tests, she was placed in honors classes through middle and high school and believed, in retrospect, that this afforded her scholarship opportunities in college. She majored in mathematics as an undergraduate student because her teachers had always told her that she was “good in math.” One of the scholarship organizations that aided her in attending college invited her to mentor immigrant students. During this time, she developed a love for working with English Language Learners (ELL) and completed a Master’s in mathematics education. Selma did not have a good student teaching experience, and at times “it was everything [she] didn’t believe in.” As a result, she determined that she would only apply to teach in “schools whose values matched [her] values.” Eventually, she accepted a position at an urban high school for international students and had been there for all eight years of teaching experience at the time of her interview. She shared portions of the interview process, which became an integral part of her teaching philosophy:

When I walked into the interview, eight people were around me, each part of a different subject. The principal was there, and they all took turns asking me questions, and all of the questions were really thinking about *the students that we serve*. They were thinking about my belief[s], and they really wanted to know me, and I was completely fair and honest and [genuine]. And I told them, ‘Look, the fact that there’s eight of [you] sitting around me, and you’re all [of] different content says much about your school. It says how you value, even the drama teacher…. [and] how every person here plays a role in this decision…. [and] I respect that…because schools should be like this; and the fact that you exist gives me hope.’ And so, they hired me.
Selma previously taught 11th and 12th grade mathematics. However, her STEAM curriculum was co-designed and co-taught with the media/arts and computer science teachers on her team, including history and English teachers. The course allowed students to get one credit each in science, mathematics, history, and ELA. The school created a special schedule, so students worked on content for four days and had one day designated for support. The course was organized as follows:

[Seniors taking the course] attend two classes a day for one hour and 15 minutes, and they have some asynchronous work based on their needs. So, every morning we check in with them for about 15 to 20 minutes [during which] they review the schedule for the day. Seniors taking this course have only two or three classes [altogether]. We believe that it’s a nice way to create community among the students and check in with them. So first and second period is usually one of the content classes. [There is] also a college class, a PE class, [and] many support classes embedded in here…. Wednesday is a support day, so we have support classes for students from 9:30 to 11:05, and then we have office hours from 1:30 to 3, which is run very similar to a support class. We have 110 students, with small groups of 10 to 12 students in each support class.

*Philosophy and Teaching Beliefs*

Selma admitted that much of her background and academic experience informed her educational philosophy. She valued heterogeneous classrooms in ability because all students can learn something valuable from each other, regardless of learning levels and capabilities. As seen in Figure 5.3, Selma truly believed her philosophical approach to teaching valued students very well and tried her best to center learning around their needs. However, she is just as passionate about teachers learning from each other. She said in her interview, “teachers should not be alone in the classroom. There should be at least two of us, with different content, so that we can produce some really powerful things that are authentic and related to the world”. She considered herself “quite lucky” to work at a school that aligned with this belief while recognizing that is
not the norm for many K-12 teachers. She and her main co-teacher worked very well together, even if they did not always agree, lauding her creativity and deeming her “a phenomenal teacher.”

Consequently, Selma felt that her best qualities were vulnerability and honesty. She was open to critique from her students and admitted when she was “learning along with them.” She was also open to feedback from her colleagues and used that process to design the best learning experience for her international students. She reflected candidly in our interview on her relationship with her colleagues:

They asked me lots of questions, but then it’s always been in a loving way; it’s never been ‘You’re doing it wrong. It’s always been, like, ‘I wonder, have you thought about this?’…and then they ask themselves those questions. And everybody in my school is
amazing, every single teacher. I would say there’s even a team that I think is very
traditional. I [may not always] get along with them individually; [but] I believe they are
great educators and people. Students love them! But I don’t agree with some things
regarding equity…some of them are blind to it. And so, I’ve had some moments where I
got really annoyed, and I had to call people out, and I don’t know if I got through to them
because I need to figure out how to do that in a loving, more compassionate way. But I
would say, even they are amazing educators.

Mathematics and Interdisciplinary Course Overviews

11th & 12th Grade Mathematics

The projects in this course were designed to address skill gaps in students’ mathematical
knowledge due primarily to interruptions in their education. Selma’s school welcomed
immigrant students who may have missed several years of education in their home countries for
various reasons. It was not uncommon for students to arrive in the 11th or 12th grade without
sufficient knowledge and skills of Algebra because of these interruptions. She created these
projects for students to explore concepts needed to pass the state’s standardized exam for core
high school subjects.

There were students of all learning levels in this course. They all “worked on activities
together” as a heterogeneous group, then “broke off” to work on individual problems based on
high, medium, and independent support levels. Selma admitted that the heterogeneous group
work was difficult at times because students needing low levels of support may not be “as
patient” with those requiring high levels of support. She explained:

…it’s really complicated and [challenging] for me, and it’s taken me eight years. I’ve
not mastered it, but I have some ideas, and the reason I got to this place was that I would
often focus on the students who were struggling, and then the students who were ready to
move on would give me a lot of pushback. And I’d [say]… ‘Can you help each other in
the group?’ And I didn’t know how to have the students collaborate in authentic ways,
and it’s also just a lot for the stronger, …I wouldn’t say stronger—I don’t know what
language to use for [those] who were more independent. I don’t know how to make sure
that they can feel good about helping someone else; it must feel really frustrating. [At times] they’re like, ‘I explained it to them 500 times, they still don’t get it’… and then the other student would feel really bad and embarrassed.

Nevertheless, Selma felt that students were learning important life lessons despite these frustrations. She saw it as “detrimental to the students who were independent” not to have this group work, because…they needed to learn how to work with different people and find the strengths of those people”. Consequently, I chose two to three student exemplars for each activity to showcase how students of varying ability levels approached the assignment. Each activity is further explained in Appendix K.

**STEAM Curriculum**

Selma’s interdisciplinary course was a STEAM curriculum, co-planned and co-taught with the media/arts and computer science teacher. Seniors who took this course received four days of content and one day of support to work on projects and received feedback. The first project was a data graphics project, where students “identified social issues that were relevant to their lives, and then analyzed the data to create graphic illustrations.” For the second project, students designed a political game on Scratch. The third project, called Songs of Protest, analyzed the trigonometry of sound waves. Nonetheless, although very excited to have the flexibility of exploring mathematics within these courses, Selma admitted that her international students with interrupted education had a steep learning curve:

In [retrospect]…each of these projects should be its own class. I wish some [mathematical] skills were taught earlier on, so we can continue to build on [them], and it doesn’t mean that it will be a linear process. It might be that some students [are] able to get to learn about Statistics and others, who might not have had [trigonometry] by the time they get to 12th grade,…my project will cover trig. [But] rather than just one project, we have a course…. And it could also cover other things that they might have covered in statistics [and build on them].
Selma was honest and vulnerable. She admitted: “I still struggle with rubrics; that’s not my best.” Still, she tried to make rubrics “general enough” so that they met students at their point of academic needs to accommodate the mathematics that they were learning. For my study, Selma shared student exemplars from the data graphics project and two activities from the scratch project. These are analyzed in Appendix L.

**Analysis of Instructional Activities along TRU Math Dimensions**

The complete analyses of Selma’s instructional activities are in Appendices K and L, along with my reasoning behind the TRU Math scores. The title of the activities and their *combined* scores are in Table 5.4. Selma’s courses were tailored to the school’s student body and its mission to support the needs of immigrant students. However, this did not mean that there were no standards or grades. As a result, Selma continued to think about ways to “push the students ready for college to the next [phase].” At the same time, she also ensured that students who “needed those gaps filled, felt accomplished and felt like they can step into something further [for the next phase of learning].” Her combined TRU Math scores showed that activities varied between medium to near high levels of complexity.

### Table 5.4

<table>
<thead>
<tr>
<th>Instructional Activity</th>
<th>Combined TRU Math Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Activity 1: Optimization: Linear Programming, Two &amp; Three Variables</td>
<td>8</td>
</tr>
<tr>
<td>Mathematics Activity 2: Geometry: Tessellations in the Real World</td>
<td>8.5</td>
</tr>
<tr>
<td>Mathematics Activity 3: Statistics: Linear Regression and Social Justice</td>
<td>7</td>
</tr>
<tr>
<td>Interdisciplinary Activity 1: Data Graphic Illustration (Project)</td>
<td>8.5</td>
</tr>
<tr>
<td>Interdisciplinary Activity 2: Understanding Vocabulary for Computer Science &amp; Scratch (HW)</td>
<td>7</td>
</tr>
<tr>
<td>Interdisciplinary Activity 3: Binary Numbers—Sending Secret Messages (HW)</td>
<td>5</td>
</tr>
</tbody>
</table>
Comparison of 11\textsuperscript{th} & 12\textsuperscript{th} Grade Mathematics and STEAM Curriculum

\textit{Mathematics Content}

Table 5.5 compares the instructional artifacts of the two courses along the three dimensions of the TRU Math Rubric. When compared with the pre-study survey TRU Math opportunity scores in Figure 5.4, they did not fully align. Selma self-reported higher scores in all three dimensions for her interdisciplinary courses than in her mathematics. Throughout the interview, Selma felt like she should be teaching “more [prerequisite] mathematics” and admitted that her anxiety “kept her up at night” because she didn’t feel like she was a “strong math teacher.”

Table 5.5

\textit{TRU Math Dimension Scores of Artifacts per Courses—Selma}

<table>
<thead>
<tr>
<th>Course Title</th>
<th>Mathematics</th>
<th>Cognitive Demand</th>
<th>Use of Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>11th &amp; 12th Grade Mathematics</td>
<td>2.3</td>
<td>2.3</td>
<td>2.7</td>
</tr>
<tr>
<td>STEAM: Computer Science &amp; Graphics</td>
<td>1.8</td>
<td>2</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Figure 5.4

\textit{Average Scores—TRU Math Dimensions (on Survey), Mathematics & Interdisciplinary—Selma}
Selma felt very passionate about her teaching responsibilities and had even been a guest on panels to discuss equity in the mathematics classroom, especially among ELL students of diverse backgrounds. She saw herself as a representative of her immigrant students and their parents. This self-added pressure may “cloud” the lens she judged herself. It may have also contributed to her claims of lower pre-study survey scores than that analyzed from her artifacts. For example, Selma’s response to the question about the extent to which “mathematics ideas are developed appropriately when taught as an individual subject” was Never or Almost Never. As a result, I looked at student examples over different concepts to see what insight I could gain as an outside observer.

The mathematics essays on optimization, linear regression, and tessellations showed the plus side of giving students projects where they pursued mathematics content congruent with their learning levels. Selma provided templates for students who needed them but still kept high expectations of them. At the same time, her independent students used this project to approach the problems in different ways. For example, one of her students used his independent work time to explore linear equations in two and three variables using Gaussian elimination and row echelon form. Additionally, since they did not reach this far in content during teaching time, he explored much of this independently. These students tried to make sense of optimization, even in their misunderstandings.

Selma provided feedback and opportunities for students to revise their work and addressed errors in conceptual understanding. This activity was engaging and encouraged students to make sense of this topic. Students’ essays on tessellations in the real world seemed to suggest a high opportunity for mathematical engagement. Furthermore, incorporating
Geometer’s Sketchpad helped to enhance the activity. As reflected in her instructional activities, Selma reported a high opportunity for enhanced activities in her survey. The essays on linear regression were relevant to each student’s life. Still, I wanted more evidence of disciplinary statistics, which may have been outside the scope of the overall objective of the assignment.

Selma’s current STEAM course was still in its infant stage. She even commented that she did not have the curriculum “beautifully mapped out” since she and her co-teacher were “creating as we go.” Nevertheless, the school offered as much support by providing a flexible schedule where students worked on content for four days and received academic support and mentorship on the fifth day. Selma used her support time to teach content, providing students with “STEAM journals” to work problems out and reflect on their progress. With the additional humanities strand, seniors who took the course received credit for math, science, ELA, and history, and all counted towards graduation.

In the first project, where students created a graphic of a social issue relevant to them, they initially wanted them to “collect data from each other and understand what different kinds of data look like.” At first, students struggled with constructing questions that matched the graphic they wanted to design. Selma and her co-teacher, “Laura,” the expert in art and computer science, were sometimes at odds. She admitted to learning “so much from her” about STEAM education and being a “phenomenal” teacher. However, when students were “just not getting past the mathematics,” Laura suggested, “maybe, creating the sketch will help them with the math.” Selma disagreed but eventually had an instance where “doing a sketch helped the student
Selma and Laura then completed their own graphic illustration, choosing to collect “real-world data” on policing in their community. Their spreadsheet and examples of student graphic designs are in Appendix L. Despite their “struggles,” Selma felt like the project was an overall success, hoping to teach more “college statistics” in the next iteration because the project “needed more mathematics.” She felt students experienced authentic moments of trying to make sense of the data but lacked the mathematics to really understand ideas conceptually. She explained further:

They looked at actual data in the world for the different topics that they identified as an issue they wanted to [research]. We pre-selected data from the problems they chose because it’s a lot to do on your own. [Earlier in the project] I had them look up data on their own, but that’s been crazy! So, they looked at data and had to isolate which data they wanted to focus on and tap into ‘What do you want to do with that data; what’s the missing piece?’ .... Some of them chose to do it in pairs, and some did it individually. [They] were looking at that data and trying to figure out…what kind of graphs to use to
represent it. But also, I would push them, and I would say, ‘But that graph is misleading. That graph has a problem because it shows that the police have killed more white folks than black and Latino’, which is true. But [they didn’t know] to compare that to the population in the US….

This context elevated the mathematics content of the graphic design activity from 1 to 2 since students grappled with making sense of the data.

The other two activities were related to the Scratch project. The mathematics content was primarily skills-oriented in service of the project. Students were asked to justify their answers in the second activity on loops, a critical mathematical practice. As such, Selma’s TRU Math mathematics content score spoke more to essential disciplinary practices than concepts, which was allowed in that rubric dimension. The actual difference in mathematics content of activities was justified.

Based on my interview, Selma’s self-evaluation of her mathematics skills was at play. She was always “looking for ways to improve” her practice, but she was her “harshest critic.” Selma felt that the STEAM strand, due to its flexible schedule, academic support, and relevance to students, provided more opportunities for students to explore mathematics. However, as she admitted, they may need to be more intentional about how this happened, as she planned to revise the project for use next time.

Cognitive Demand

Selma’s pre-study survey scores claimed that the cognitive demand for interdisciplinary activities was higher than in her mathematics course. However, upon closer analysis, TRU Math scores revealed that activities in mathematics scored higher in cognitive demand. Additionally, the difference in scores between the two courses is not as vast as reported in the survey. One
crucial observation is that Ana’s mathematics course integrated other subjects like writing, art, and technology, which may have contributed to that score. All of the mathematics essays showed students trying to make sense of the concepts and grappling with ideas of what they knew. Students were demonstrating and building on their understanding. This was not always clear in the interdisciplinary activities. The scratch activities, for example, showed possibilities where students could make conceptual connections, but they were not designed to be challenging. In addition, there was not enough evidence from the interview to show that students struggled productively on the graphic illustration project, despite the challenges they experienced, as I explained in the mathematics content dimension.

*Use as Assessment*

Selma’s mathematics activities were good uses as summative and formative assessments. The tessellations essay showed students summarizing what they learned from their research. Selma then expounded in the interview on how students’ ideas were solicited, developed, and responded to in those activities. She shared how one of her independent student’s explorations led him from Gaussian elimination to invertible matrices. When “he got stuck,” she assisted him by helping him find the appropriate videos on YouTube to help him move past his “conceptual hurdle.” Since other students needed high levels of support, she told him that he had to go through the videos “on his own” and explain his experience in his essay. This exploration of concepts is the kind of “residue” or mathematical value that may lead to future learning and discovery (Hiebert et al., 1997).

Selma’s graphic illustration project also made great use as a formative and summative assessment. Teachers solicited students’ reasoning on connecting data with graphs and addressed
students’ misunderstandings of interpretation and statistical analysis. When necessary, students had time to address pre-requisite mathematics content they lacked before the course through conversations with Selma and lecture videos. The scratch homework assignments certainly had the potential for good use as a formative assessment. Nevertheless, I wanted more information to determine the full extent to which student understanding was built upon in the process. Also, even though students needed to “show their work” on the binary numbers activity, the assignment was primarily skills-based and intended for corrective feedback.

Summary of Selma’s Beliefs and Instructional Activities

Selma’s pre-study survey claims did not align with the TRU Math analysis of activities. There was a difference in the cognitive demand and mathematics content of activities, but these scores were higher in the mathematics course than in the STEAM course. Even though this interdisciplinary curriculum is relatively new, the school has always leaned towards integrating subjects and teaching in teams. Selma’s mathematics essays incorporated technology, writing, and art, for learning content, and mathematics was not just a tool for representing data. She may have overestimated (on the pre-study survey) how well this integration manifested in students learning mathematics in her STEAM course. However, she alluded to “more mathematics needed” in the STEAM course during the interview. This became more evident once the STEAM artifacts were analyzed using the TRU Math rubric.

It was evident how vital student feedback was for what they learned and how she designed activities. Due to her students’ backgrounds, it was not surprising that students’ academic needs were diverse, and as such, this feedback was a critical part of meeting their learning needs. She shared passionately her awareness that some students “worry” about
ensuring that they are ready for “college writing,” while other students want “more math.” Still, some wanted to “dive deeper into the computer science.” Selma took all of these concerns into consideration and the subject matter to create her courses. She assured her students that if they “give honest feedback and say what they want and need,” she would ensure to “bring this into the classroom.” The challenge of converging all of these interests was an integral part of her as a Connector in principle and practice.

5.3 Pearl’s Story

Educational and Professional Background

Pearl is a professor of mathematics at a suburban university in the northeast with over 18 years of teaching experience. She earned her Ph.D. in “pure math logic, model theory specifically, and started on the Research 1 track, with a postdoc” at a prestigious university on the west coast. However, due to positive changes in family circumstances, Pearl “landed” at her current institution. By her admission, “teaching was kind of an afterthought.” In graduate school, she took two elective courses in “instructional issues” and was intrigued by Pruitt Mazur’s active learning pedagogy enacted in his physics classes.

Along with some training in online learning, Pearl tried to implement some of the active learning theory and online activities in her current work. But “there was a lot of pushback,”; and the demands of tenure and research “was difficult to do in [such an] environment with a higher teaching load.” She then switched to “mathematics education research,” which presented opportunities to learn new technologies she implemented in her calculus course. This fortunate change led to research opportunities that “opened [her] eyes to the visual aspects of
mathematics,” which encouraged the incorporation of applications from other disciplines into her calculus course.

Pearl’s courses included Calculus III, Statistics, Honors Thesis Design and Development, an interdisciplinary capstone seminar for seniors, and a junior-level Honors Interdisciplinary seminar. She was engaged in administrative and committee work and personnel supervision and is the honors program director, an interdisciplinary program of all the different majors. She further described her duties:

I teach a research methods class for those students before they do their thesis, so I’m hitting all the different majors. I see different projects every semester; it’s crazy! And part of the program is an interdisciplinary seminar that the students take that the topic changes from semester to semester. And I created one course with a Musician—Math, Music, & Art, which was featured on the National Collegiate Honor Council website as a model honors interdisciplinary seminar.

Pearl is a reflective practitioner in principle and practice. Her courses allowed students to reflect on the content and share their progress in their conceptual understanding. Her goal for her mathematics courses was for students to have “a better appreciation for [and improve] their three-dimensional reasoning skills, giving them that extra tool for problem solving.” Pearl wanted her students to learn “content” while encouraging them in “mathematical habits of mind” to help them persist through challenges. I expound more on these goals and intentions in the section on Lesson Overviews.

Philosophy and Teaching Beliefs

Pearl’s philosophy is student-centered—provide students with applications that they’re interested in, and they get excited about learning. During the interview, she explained her reasoning:
“Students are really craving applications [in calculus]; and not just physics applications…. I mean, few of the students do. But they are a little too artificial; they view it more as a story problem, I think”. Pearl’s pre-study survey results in Figure 5.6 showed high opportunities for enhanced activities.

Figure 5.6

Average Scores-B, P, & A (on Survey), Mathematics & Interdisciplinary—Pearl

She began the course with a preview of skills and why they were important. “But within the first week, students engage in an activity on 10-dimensional space” where they see “a vector that represents themselves, and each component is a characteristic of them”—one boring and one fun—that they all share in common with each other.” She then puts “these characteristics in an array in an excel sheet with their names…and used “1’s and 0’s if they have the characteristic or not.” Then, they discussed which students had the most in common as a class. This “may be a simple exercise to the students.” Nonetheless, they were introduced very early in the multivariable calculus course to “cosine similarity, which is the basis of latent semantic indexing
for the search algorithm,” and how they all connect. Thus, students saw the concept of a “basic
vector, multi-dimensional space, and an application they all knew.” She also started
incorporating applications from other disciplines:

…not [just] physics, because that’s the typical one, that’s still there. But I added on
things like machine learning, animation, … how computer vision works, and all of those
are applications that we do throughout the semester of the material.

Recent to the time of our interview and in response to end-of-semester reflections, she
asked students to reflect on something they were “curious to learn more about and why.” One
student mentioned that he wanted to learn more about “the Google algorithm.” With excitement,
she explained how she could find “the original article from 1998 by Larry Page and Sergey Brin”
and discussed it with her student. Pearl felt strongly that these applications had transformed
teaching and learning calculus for her and her students. She remarked:

It’s opening these doors of interest in communication that extend beyond the class I
never had before. Nobody ever said, “I want to learn more about something you’re
teaching.” [All] they wanted to know is what they needed to do to get the A on the exam.

Mathematics and Interdisciplinary Course Overviews

*Calculus III: Multivariable Calculus with Analytic Geometry*

This is the last course in a three-course calculus sequence. Some topics include “conic
sections, plane curves, parametric equations, vectors and curves in the plane, functions of several
variables, partial derivatives gradients,” etc. Pearl taught one of several sections but realized that
students were “craving more applications beyond the typical physics problems.” Over time, she
altered her course to emphasize more “graphing and graphical interpretations” and less on the
“algebraic computational aspects of it that the computers can do.” Along with “typical” physics
applications, students are exposed to “things like machine learning, animation, how Google
search works, how computer vision works, how 3D printing works,” some of which were inspired by teaching her interdisciplinary courses, explained later in this section.

Her goal for this course was for students to “have a better appreciation for three-dimensional [space] and improve their three-dimensional reasoning skills.” Students were required to submit essays and video recordings explaining their reasoning behind the problems they solved. In the past, these videos were occasional; but they became permanent during the COVID-19 pandemic. Pearl felt like videos were a more accurate way to assess students based on course objectives and mathematical practices that she emphasized during the class.

Mathematical and Visual Principles of Computer Animation

This course is an interdisciplinary seminar for students in the honors program at Pearl’s university. It was co-created with a Media Arts professor in computer animation and examined the mathematics and design principles behind computer animation. As described in the syllabus, students “learned some of the algorithms underlying ray-tracing software” and “mathematics necessary to understand these algorithms.” Additionally, students “learned and experienced the creative and technical step-by-step process of producing an animation from inception to final product. They are given opportunities to practice the mathematics and visual principles while programming animations.” During her interview, Pearl expounded more on the rationale behind the course:

…we created a course [where] the mathematics and the visual principles [was] kind of ‘underneath the hood,’ so the students weren’t using one of these drag-and-drop programs; they were…coding the old-fashioned way—it was very retro [both of us laughing]. They were using open-source software from the 90s to create these visualizations, but then they could understand the importance of the parametric curves and that sort of thing. [In addition], that course was [for] all majors, so not everybody had calculus. But, using CalcPlot3D and some templates, the students began to appreciate
how they could edit a formula to get the effect they wanted. And some of them knew how
to code; others learned on the spot as we were going through that class, and they created a
group project at the end. And so, it was a great course. And it was that course that made
me [realize]: ‘Oh wait—this is like multivariable calculus; I can pull these examples in
the multivariable calculus and talk about how Ray-tracing works and the importance of
Easing functions and parametric curves’…like if you have a motion and you want it to
slow down and speed up…you could compose two functions to do that and the role of
Bezier curves in all of this as well.

When asked directly, Pearl confirmed that her work in the interdisciplinary course
influenced her mathematics teaching because it made her “rethink how she taught.” Teaching
and planning with a colleague from another discipline gave her an appreciation for how people in
other disciplines think and what that meant in a mathematics course for someone who had “never
had calculus before.” She recalled making mental notes about her colleague’s presentations and
even how she structured the assignments. Admittedly, she would not have made these changes in
her mathematics courses because “when you’re teaching by yourself, you don’t have to
communicate with anybody, so you just keep it all [in mind] about why you're doing this thing.”
However, co-teaching and co-planning together, they “wrote out the objectives for each
assignment and spelled it out.” She continued, “I would have never done that had I not been in an
interdisciplinary setting—there was no way I would have had those artifacts.”

Analysis of Instructional Activities along TRU Math Dimensions

The complete analyses of Pearl’s mathematics and interdisciplinary instructional
activities are in Appendices M and N, respectively, along with my reasoning behind the TRU
Math scores. The activities' titles and combined scores for each dimension are in Table 5.6. Pearl
is the only Connector whose educational background includes coursework in “instructional
studies” and whose post-doctoral work includes research in mathematics education. She was also
the only instructor to acknowledge how interdisciplinary teaching influenced her mathematics pedagogy. Immediately, what stands out is the difference in the complexity of activities in both courses. Several factors contribute to this, including the kind of assignments submitted. For example, Pearl’s mindmap was a comprehensive activity, with individual and group contributions that focused on students demonstrating how deeply they understood concepts. The animation course, notwithstanding, fulfilled one of the STEAM requirements of the honors program and ran more like a seminar. Students also had a comprehensive STEAM final project where they demonstrated their understanding, but the exemplars showed more about how they built their knowledge of computer science and only some of the mathematics leading up to that point.

**Comparison of Calculus III and Mathematical and Visual Principles of Computer Animation**

**Mathematics Content**

Pearl’s pre-study survey scores (in Table 5.7) and TRU Math mean scores (in Figure 5.7) showed that students had more opportunities for making sense of mathematics in her calculus course than in the STEAM course. However, Pearl’s interview and activities provided the

<table>
<thead>
<tr>
<th>Instructional Activity</th>
<th>Combined TRU Math Score</th>
</tr>
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<tbody>
<tr>
<td>Mathematics Activity 1: Graph Points and Vectors using CalcPlot3D. (Group Activity)</td>
<td>6</td>
</tr>
<tr>
<td>Mathematics Activity 2: Group Project 3: Individual Mindmap</td>
<td>8</td>
</tr>
<tr>
<td>Mathematics Activity 3: Group Project 3: Group Mindmap</td>
<td>9</td>
</tr>
<tr>
<td>Interdisciplinary Activity 1: Coding with the POV-Ray Software (Homework)</td>
<td>5</td>
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<tr>
<td>Interdisciplinary Activity 2: Constructive Solid Geometry (Homework)</td>
<td>4</td>
</tr>
<tr>
<td>Interdisciplinary Activity 3: POV-Ray Animations (Homework)</td>
<td>6</td>
</tr>
</tbody>
</table>
necessary context for understanding how her experiences teaching both courses worked in tandem. The first mathematics activity showed students using CalcPlot3D to graph vectors.

Table 5.7

*TRU Math Dimension Scores of Artifacts per Courses—Pearl*

<table>
<thead>
<tr>
<th>Course Title</th>
<th>TRU Math Mean Scores of Artifacts - Pearl</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculus III: Multivariable Calculus</td>
<td>Mathematics 2.7</td>
</tr>
<tr>
<td>Mathematical &amp; Visual Principles of Computer Animation</td>
<td>Mathematics 1.7</td>
</tr>
</tbody>
</table>

Figure 5.7

*Average Scores-TRU Math Dimensions (on Survey), Mathematics & Interdisciplinary—Pearl*

This activity seemed “skills-based” at first glance. Nevertheless, it came at the end of a lesson on how vectors are connected to search algorithms and 3D printing. Furthermore, technology was more than just a tool in this course. Not only was Pearl “blown away” by the training she
received in using CalcPlot3D in the semester before starting to teach “Calculus 3”, but she was also able to join a team of researchers who received an NSF grant to develop this software further. As the “resident expert,” Pearl used this technology in her courses and prioritized applications of all types.

The mindmap, comprising the second and third mathematics activities, was a comprehensive and conceptually rich review where students got to make sense of concepts learned during the course, both individually and with their peers. The complete assignment is in Appendix M, but even a glance at some of the following requirements shows a high opportunity for students to make meaningful procedural and conceptual connections:

- Pick at least three major ideas but make sure that you describe these ideas
  (a) graphically
  (b) numerically
  (c) symbolically (you can add math symbols from idroo, mathcha.io, or by hand).
  (d) verbally (both how the concept connects to others but also applications of the concept).

- As you create your mindmap, keep in mind the following:
  (a) Provide citations for any information from sources other than our course video lectures and the textbook.
  b) You may include specific worked-out examples to illustrate a concept, but make sure to explain why the example is relevant or how it is connected to the mindmap.
  (c) Be creative. There is no one best style of the mindmap. You can add footnotes or label arrows, whatever helps demonstrate connections.
  (d) Make sure everyone understands the concepts on the mindmap. This will help everyone study for the final exam.

With the mindmap, “they must describe concepts in the four ways,” which reinforced the idea that “just because you know how to take a partial derivative, does not mean you know what a partial derivative is…. The potential for this activity to be of high mathematical value increased once Pearl explained the final exam. It consisted of three parts: a video where
students worked out computation for specific problems, essay questions that were “meta-cognitive in nature,” and a timed set of online problems, “similar to ones in class,” where students analyzed the graphs of functions without equations. As a result, this activity incorporated high reasoning and essential mathematical practices.

The mathematics of computer animation interdisciplinary course was a seminar intended for students to fulfill the requirements of the honors track. Not all students had a background in calculus. However, students were still required to make necessary connections in coding and mathematics to complete assignments adequately. Appendix N provides examples of activities and TRU Math scores for each dimension. Pearl explained how students prepared for the final:

In the last six weeks of the semester, the students worked in kind of a studio environment in the class and created from a storyboard. And then they broke the project up—they had to figure out that project management aspect, like, ‘You, work on this object, and then, once you have that file, I am going to be working simultaneously on the ground and the sea or whatever, and then you are going to stick that object onto the sea.’ So, they had to break down the project into parts that they could be working on simultaneously, to [then] bring back together at the end for the final code and the final animation. And then they displayed these videos, which were very short, they were [about] 30-seconds… but they displayed them publicly, and we had a movie night, and ‘public choice awards,’ and the students gave their videos and explained their artistic choices that they made. [They also] explained some of the technical hurdles that came up and some of the artistic choices made because of a technical hurdle.

In the first and second interdisciplinary activities, students learned how to use the POV-Ray software and then applied it to constructive solid geometry (CSG). There was potential for making mathematical connections, but the activities were designed primarily for making procedural and conceptual connections in graphics coding. The third activity was designed to illuminate the mathematics involved, and there were some rich procedural and conceptual
connections linking elastic functions, easing curves, and animation. This third activity scored higher than the first two activities in the mathematics content dimension.

*Cognitive Demand*

Pearl’s mathematics activities presented *higher* opportunities for students to make sense of the mathematics than in her interdisciplinary seminar. The activity exploring CalcPlot3D came at the end of the teaching session, and it was not clear if time was allotted for students to explore any potential conceptual connections. However, the mindmap was an activity designed for making sense of disciplinary ideas in multivariable calculus. Students were encouraged to “help [their] teammates better understand the material,” indicating a chance for students to “level up” in their understanding. The individual mindmap was no less cognitively demanding. It preceded the group portion, where students met to “discuss the content of their mindmaps” and “worked in a common file in which all team members would contribute to the group’s mega mindmap.” This presented a *very high* opportunity for each student to grapple with what they understood throughout the Calculus III course.

Additionally, students noted concepts they had not yet fully grasped *ahead* of meeting in their groups, and members were responsible for helping each other “sort out” their misunderstandings. Each team member was assigned a role to resemble “professional settings [where] teams were usually made up of individuals with various roles.” Moreover, the *project management* objective, an integral part of Pearl’s *interdisciplinary projects*, was adapted for use in her mathematics group projects. The assignment called for self and peer evaluations to complete the accountability process, adding another level of accountability for students to be mindful of each others’ learning needs. The mega mind map in the Calculus III course
encapsulated struggling productively in mathematics, with students “engaging meaningfully in mathematical practices” (Schoenfeld, 2018, p. 493).

Pearl’s interdisciplinary activities allowed students to make conceptual and procedural connections since students had to document their code with comments, provide the rationale behind aesthetic choices in the scenes and sketches of animations, and show “correctness, clarity, and completeness of mathematics exercises.” The tasks in activities one and two were not designed to be difficult (see Appendix N). However, there were “Bonus” problems, thus encouraging students to challenge themselves. These problems were procedural, focusing more on building skills necessary for animation but not necessarily “scaffolding away” challenges. Still, these tasks were not trivial. Even Pearl admitted during the interview that she sometimes wondered how students “would create an animation.” She often marveled at her colleague’s pedagogy in helping students attempt coding as a non-STEM major. This helped her to grow professionally. Considering that performing these tasks well-prepared students for the final project, it was possible to expect students to actively build upon their understanding from one activity to the other. However, the actual cognitive demand of the interdisciplinary artifacts was clearly understood once analyzed through the TRU Math rubric.

Use as Assessment

All of the mathematics activities worked great as formative assessments, with the mindmap doubling as a summative assessment for its use in preparation for the final exam. The vector calculations using CalcPlot3D were a group assignment with time built in for sharing. It was unclear if students had enough time to discuss insights or misunderstandings productively. The mindmap was designed for students to assess themselves ahead of the final exam, with
students receiving credit and an opportunity to solidify their understanding of concepts in the Calculus III course. This exemplified two ideal uses of an assessment expressed by instructors in my study—a way to communicate mathematics and a learning opportunity for students. The interdisciplinary activities were also good uses of formative assessments because they solicited students’ thoughts through the documentation of codes. Students build upon previous work since they “used the same scene for future assignments, editing it, and adding more features over the semester.” It was implied that students’ ideas and understandings were developed throughout the course.

Summary of Pearl’s Beliefs and Instructional Activities

Pearl’s beliefs about her mathematics and interdisciplinary activities aligned with her submitted artifacts for analysis. Her activities in mathematics scored higher in cognitive demand and mathematics content than her interdisciplinary activities, as reflected in her pre-study survey and assignments. It was interesting to hear how open Pearl was about integrating examples from other disciplines into multivariable calculus. She altered the course significantly over her 18 years of teaching to not focus so much on the computation side, even if it was still crucial. Students in multivariable calculus must submit a video each week of one of the tasks on their homework, where “they go through and set up the problem, explaining their strategies, why they chose them, then working the problem out.” Pearl felt like she provided better feedback through this process since she could “hear” the reasons behind the computation and correct them accordingly.

As some had expressed in evaluations, she also recognized that not all students see the value in her applications. However, she is not discouraged. Much of Pearl’s data on
interdisciplinarity comes from her *nationwide recognized* honors seminar, where she experienced students learning aspects of multivariable calculus but were non-majors and had little to no background in coding and calculus. The key to *their* understanding came through teaching these students applications of calculus. In her mind, if students with little or no integral calculus can understand its applications, then so can her mathematics students.

5.4 Summary

The overall purpose of this chapter was to answer **research question two:**

2. *Are there differences in mathematics content, cognitive demand, and assessments between mathematics and integrated or interdisciplinary activities?*

The *Connectors*—Pruitt, Selma, and Pearl—more closely portrayed the activities that helped answer this question. Overall, there were differences, with more opportunities for sensemaking in the mathematics activities than in their interdisciplinary ones. Furthermore, the activities in mathematics had higher scores for complexity. Through a “cycle” of refining instruction through insight from activities, the instructors showed how their beliefs, practices, and activities were interrelated. I used the model in Figure 5.8, to sum up their pedagogical style:

**Figure 5.8**

*Connectors’ Pedagogical Approach*

![Diagram](image_url)
In their interviews, however, all three instructors implied that their teaching and learning experiences in their interdisciplinary courses impacted the mathematics teaching and design of its activities.

Another interesting finding was that some of their interdisciplinary artifacts suggested that students learned mathematics concepts in these courses, thus showing that students were engaged in both types of courses. Each instructor expressed various ways they encompassed some of what Li and Schoenfeld (2019) described as the four faces of mathematics, i.e., “(1) computation, formal reasoning, and problem solving; (2) a way of knowing; (3) a creative medium; and (4) applications” (p. 3). Their activities helped students “experience mathematics through different practices and ‘own’ mathematics as a human activity” (Li & Schoenfeld, 2019, p. 3). As a result, using the TRU framework to test for engagement and deep understanding in interdisciplinary mathematics learning showed promise.
Chapter 6: Summary, Conclusion, and Recommendations

6.1 Summary

This study aimed to describe and analyze the instructional activities and curricular practices that instructors used when teaching mathematics as a subject and within an interdisciplinary curriculum. This study highlighted factors that gave insight into instructors’ instructional practices, beliefs, and the kinds of activities that help students engage in and develop a deep understanding of mathematics. The analysis also looked at the complexity of activities based on the dimensions of the framework of Teaching for Robust Understanding in Mathematics (or TRU Math©). At the time of this writing, there were no published seminal texts to build an understanding of interdisciplinary mathematics teaching (Doig & Williams, 2019). This research study sought to provide contextual evidence of teachers’ experiences teaching mathematics and interdisciplinary lessons and its challenges and opportunities.

Some studies found that teachers’ attitudes towards mathematics, their mathematical content knowledge, and their beliefs about mathematics influence classroom behavior and instructional practices (Ball, 1993; Wilkins, 2008; Thompson, 1992; Beswick, 2012). One way to examine these factors is by analyzing teachers’ instructional activities, classroom practices, and beliefs. My study used an adapted version of the TRU Math framework to analyze instructional activities. Furthermore, I highlighted any artifacts that showed opportunities to learn mathematics concepts through interdisciplinary activities. With this understanding, instructors may find ways to leverage best practices over different mathematics curricula to promote deep understanding.
The 13 participants in the study were instructors from elementary grades through college years with experience in teaching mathematics as a subject and within an integrated or interdisciplinary curriculum. The other subjects and disciplines included (but were not limited to) science, art, writing, music, technology, engineering, and history. Instructors completed a survey regarding their attitudes, beliefs, motivations, and dispositions towards teaching and their practices regarding curriculum development, instruction, and assessment. I collected one to three artifacts of instructional activities, which included lesson plans, assignments, assessments, guided practice, homework, and projects. Teachers then participated in semi-structured, open-ended interviews to explain beliefs, practices, and activities (or B, P, & A.).

I analyzed the surveys by characterizing practices according to three dimensions: instruction, student engagement/cognitive action, and technology. Then, I conducted interviews that included questions to help teachers discuss the factors influencing practices as seen through their choices of instructional activities. I determined how closely aligned the enacted practices are to the beliefs espoused in the survey. I then described the enacted practices through themes that characterized the participants, in general, to expound on their beliefs, pedagogy, and their choices of activities. I also included instructors’ ideas about student-centered learning in mathematics and interdisciplinary settings.

The study revealed several factors that informed the instructors’ beliefs, practices, and activities (B, P, & A) about teaching mathematics and interdisciplinarity. They felt strongly about helping students value learning and their abilities to do mathematics. They saw their students as capable problem solvers and used instructional activities to illuminate thinking and understanding of mathematics. Through their instructional activities, they sought to
make *mathematics meaningful, joyful, and accessible* and seemed to associate that with valuing learning. Additionally, they used *assessments to communicate mathematics* and the various ways that they integrated disciplines and subjects with mathematics and conversely.

The study also revealed three significant ways that instructors engaged in interdisciplinarity as seen through the work of the *Constructors, Curators, and Connectors*, also referred to succinctly as the “3 C’s framework” throughout the chapter. *Constructors* taught mathematics lessons for understanding, with interdisciplinarity playing a minor role. They utilized contexts from other disciplines, critical thinking, and understanding connections to help students construct mathematical knowledge. *Curators* incorporated STEM or STEAM strategies into direct instruction, projects, and assignments. Sometimes, activities focused exclusively on integration or interdisciplinarity, while other times, mathematics was intended to be understood, explored, and extended. *Connectors* taught mathematics as a subject and within an interdisciplinary lesson. Their activities focused on mathematics instruction with explicit and implicit connections to other disciplines and subjects.

The *Connectors’* artifacts provided more comprehensive insight (as a group) into both types of activities, and their stories were highlighted in chapter 5. It also revealed opportunities for teaching *disciplinary* mathematics within an interdisciplinary curriculum. I analyzed each set of their instructional activities using an adapted version of the TRU Math rubric (Schoenfeld et al., 2014). I scored assignments on a continuum from 1 to 3 and analyzed each activity to explore its mathematics content, cognitive demand, and use as an assessment. Then, I assigned each activity a TRU Math score based on the dimensions of the rubric. Upon completion, I compared the TRU Math scores of both types of lessons with data from the pre-
study survey to determine alignment. Lastly, I compared the TRU Math mean scores to ascertain differences in mathematics content, cognitive demand, and assessments between mathematics and interdisciplinary activities. I used the results of the analysis to answer two research questions.

6.2 Conclusions

Research Question 1

1. What factors inform teachers’ beliefs and conceptions towards teaching and learning mathematics and the instructional practices enacted in the classroom? How do these factors differ between mathematics lessons and an integrated or interdisciplinary curriculum such as STEM or STEAM?

The first purpose of this study was to highlight factors informing instructors’ beliefs and practices and how this influenced the design of activities in mathematics and interdisciplinary lessons. The resulting interpretations and conclusions follow the findings to answer this research question. They address four issues: a) the relationship between student efficacy and instructors’ beliefs and practices; b) the role of understanding in instructional activities; c) the position of mathematics learning in interdisciplinarity and d) leveraging teaching and grade level experience in mathematics and interdisciplinary lessons.

Connecting Beliefs to Student Efficacy

Instructors in this survey shared their thoughts about teaching and learning mathematics as a subject and within an interdisciplinary curriculum like STEM or STEAM based on opportunities provided through instructional activities. The first two factors, believing that students are capable problem solvers and their role in helping students value
learning, may indicate how the instructors position themselves within student-centered learning. Seeing students as capable builds their confidence, which is helpful for persistence in solving problems (Hiebert et al., 1997). This may also signal instructors’ attempts to know their own biases about who can and cannot do mathematics. Relatedly, while instructors believe in helping students value learning, assumptions about who needs this help and why they need it should be clear (Bol & Berry III, 2005). Helping students value learning and helping students see ways in which they already value learning may have different outcomes. As Paul reflected on the students in his College Algebra Course, he said, “students have had 18 years of making decisions, computing, and measuring experiences, from life and grade school. I like to use that to my advantage to convince them that they are already capable of [doing the work]”. Indeed, this sentiment was evident across grade levels, years of experience, and interdisciplinarity practices, with each instructor sharing their interpretation of this belief.

These findings are consistent with the literature on the interconnectedness of student efficacy and achievement (Hill et al., 2005; Hannula, 2002). The literature also cites how positive mathematics learning experiences between instructors and students influence decisions to major in and teach mathematics (Kezar & Maxey, 2014; Borum & Walker, 2012; Artzt & Curcio, 2008). Nonetheless, how do we bridge the gap between what teachers believe and do? While inconsistencies between beliefs and practices are well established in the literature (Raymond, 1993, 1997), we also know they are closely linked (Ernest, 1989; Thompson, 1992; Beswick, 2012). Based on these previous findings, it may seem that it matters how these beliefs are formed.
Towers (2013) discussed that one factor contributing to consistency between beliefs and practices in her beginning teachers was how well their teacher education programs prepared them to reflect on their practice. These preservice teachers used reflections about themselves and their students to help them form their teaching philosophies. The findings in my study are consistent with this research and best practiced by Pruitt. Pruitt’s belief that students prefer projects because they are “more motivated to create, to do projects than they are just to regurgitate material” is affirmed throughout his student reflections (seen in Appendix R). One student in his Calculus II course reflected on the project like this: “…when you get to do your class project and see and hold the math you [learned] in your own hands…. that feeling was magical to me, and I hope you will feel the same”.

One conclusion that can be drawn is that when teachers form beliefs based on students’ experiences, they are likely to be more consistent with what they will do in the classroom. Teachers often use student reflections to improve the learning experiences in future courses. The improvement of such courses over time results in instructors’ success. A further and related conclusion is that the motivation behind these beliefs may be rooted in their effect on both teachers’ and students’ identities. As one instructor plainly stated, “If only five out of 40 kids pass my class at the end of the semester, that makes me look really bad, it is not on [them].”

Another interesting observation is that most instructors in my study (if not all) were less prone to declare themselves traditional versus non-traditional or have a static versus the dynamic view of mathematics. Even though this was surprising to me, it was consistent with results found in the literature. Recall that Perkkilä’s (2003) study found that teachers’ beliefs about
mathematics content were more strongly linked to teaching practices and their past experiences in teaching mathematics than what they believed about the nature of mathematics or their professed beliefs about teaching and learning. However, this does not negate the value of experiencing one pedagogical approach over another as a student. As seen in the literature, Yang et al. (2020)’s study of preservice teachers in China showed that because they experienced inquiry-based learning as grade school students, they were unsure how it interacted with learning to use as a teacher. Therefore, acquiring inquiry-based skills during a methods course did “not necessarily guarantee a significant association between [mathematical pedagogical content knowledge] and their self-reported instructional practices” (p. 291).

**Centering Practices Around Understanding**

The factors informing instructors’ claims about mathematics teaching and understanding are as follows: *mathematics should be meaningful, joyful, and accessible; instructional activities should illuminate understanding; assessments are a way to communicate mathematics to different audiences*. These findings are not surprising as there is an interconnectedness among them.

Instructors in this study believe that meaningful mathematics, accessibility, and joy are all intertwined; the literature also supports this. Schoenfeld (2020) encourages mathematical practices focused on understanding how and why concepts work “in the spirit of Euler and Polya” (p. 1169) and not *just* as a prescribed list of rules for students to follow. Schoenfeld further added that the very act of building understanding *is* part of the joy of learning mathematics. Thus, mathematics classrooms should accommodate all students *and* support those who wish to advance in learning mathematics. This requires in-depth content and pedagogical
knowledge with implications across grade levels and years of experience. This also requires that instructors create mathematics classrooms cognizant of predispositions about achievement gaps and student challenges, a challenge well documented in the literature (Gutiérrez, 2008; Boaler, 2002a; Martin, 2003, Berry III et al., 2014).

Even though instructors associated understanding mathematics with how students think about it, defining mathematical thinking is well debated in the literature (Burton, 1984; Schoenfeld, 2020). Furthermore, as Schoenfeld cautioned, thinking about the subject matter of mathematics is not the same as providing students with tools to make sense of problems. Instructors should be mindful of this not only in activities but also in direct instruction. A recurring theme among the instructors is the iterative aspect of the learning process. Emily, an advanced elementary educator with over 16 years of experience across three states, described her approach to building understanding in the following way:

So my understanding is that when a concept is very new, we definitely need to be able to see it, feel it, experience it, and then eventually a visual model. I’ll start to connect that visual model to that concrete, and last is the abstract. Nevertheless, as the unit progresses, I'm also thinking about moments where they may need to return to that concrete; and I'm always ready. In my lessons, when I think about [the] unit, I always have these [examples] ready to go for them. If I come across a student who was kind of stuck and not seeing that connection and what is happening, I have my arsenal of concrete, or my visuals, to kind of “get them there,” and then I bring them back to the abstract.

Still, instructors also need a deep understanding of mathematics themselves as “limited subject matter knowledge restricts a teacher’s capacity to promote conceptual learning among students’” (Ma, 1999, p. 36). TRU Math then may also serve as a semi-structured guide that positions learning mathematics as less of a set of acquired knowledge and skills and more towards understanding mathematics through reasoning, making sense of concepts, applying
concepts learned, and communicating mathematics creatively (Schoenfeld, 2015). Not only is this transferable across subjects (Li & Schoenfeld, 2019), but helpful in learning subject matter within an interdisciplinary curriculum, as seen through this study. Additionally, as one advances along the levels of each dimension, they are closer to enacting practices that promote a more profound understanding (Schoenfeld 2014, 2015). A conclusion is that TRU Math may serve as a bridge between factor differences in teaching mathematics as a subject and within an interdisciplinary curriculum, to add depth and rigor when learning content.

Therefore, this study highlighted the differences in interdisciplinarity, i.e., the role of content in interdisciplinarity, characterized by the actions of the *Constructors*, *Connectors*, and *Curators*, explained more in the next section. While more studies continue to delineate interdisciplinarity through definitions (Capraro & Slough, 2013; Doig & Williams, 2019; Williams & Roth, 2019), procedures (Bryan, 2015; Sutaphan & Yuenyong, 2019), and methods (Kennedy & Odell, 2014; Zhou et al., 2020), even as research shows trends towards integration (Roehrig et al., 2021; Roberts et al., 2022; Kitty & Burrows, 2022), instructors may not necessarily be looking for more *prescriptions* (though useful for research). Thibaut et al.’s (2018) systematic review of STEM in secondary education *exclusively* organized 49 instructional practices over nine categories of STEM instruction: integration of STEM content, focus on problems, inquiry, design, teamwork, student-centered, hands-on, assessment, and 21st-century skills. The authors further distilled these categories into a STEM integration framework organized around five fundamental principles: *integration of STEM content, problem-centered learning, inquiry-based learning, design-based learning, and cooperative learning.*
The results of my study fit within these findings of the literature, as instructors employed several of the instructional practices described. However, since the 3C’s framework used instructors’ descriptions of the way they practiced interdisciplinarity, it is less of a “toolbox” and more towards practical ways to characterize how teachers are already able to practice this learning. It uses language instructors are comfortable with, which is transferrable across disciplines. Since instructors are already cautious about taking risks if student achievement appears to be compromised and for fear of their teaching practices being criticized or condemned (Superfine, 2009), the 3C’s framework considers these needs, as it may help repurpose pedagogy instructors are already comfortable with to integrate content learning and interdisciplinary practices. The 3C’s also may help contextualize a continually evolving educational field practically and theoretically. More research may show combinations of and further delineation within the 3C’s that can continue to provide context for instructors, administrators, and teacher educators. We turn now to the last section, which explicitly addresses research question one.

**Positioning Mathematics within Interdisciplinarity**

The three factors informing instructors’ understanding and practice of interdisciplinarity are rather interesting. Instructors view interdisciplinarity as a means for supporting mathematics teaching and learning and as a way to curate content through different lenses. Furthermore, it showed that instructors taught mathematics within an interdisciplinarity lesson using various integration methods. Current literature confirmed these results, as more recent research published seeks to systematically describe interdisciplinarity (Ortiz-Revilla et al., 2020) as referenced in the previous section. Similarly, instructors did not have the same level of structure for
interdisciplinary lessons, mainly when used as a *supplement* for learning mathematics. Even in
the cases where teachers claimed that STEM or STEAM is “completely woven into the standard
curriculum,” it fell short of interdisciplinary learning (and interdisciplinary mathematics) as
described in the literature (Volmert et al., 2013).

However, the teachers reveal a *working* understanding of how instructional practices help
students learn mathematics. For example, integrating writing and oral communications
pedagogies provided students in Priscilla’s college mathematics courses the *accessibility* to
mathematics that she desired. Using special education strategies at Steven’s alternative high
school taught his students mathematics so that they could understand and move past introductory
courses. Using the peer review process for error analysis was Emma Sue’s way of instilling
mathematical habits of mind at her Catholic school for boys with musical talent. One conclusion
that can be drawn is that the relationship is *still* one of support, i.e., interdisciplinary learning
supports mathematics learning to build understanding. It flows directly from how instructors
carefully choose to describe their practice. In this regard, it can also be concluded that
instructors’ primary purpose for teaching interdisciplinarity is less about whether they practiced
“STEM or STEAM,” for example, but *how* they integrated subjects, pedagogies, or disciplines.
This is the foundation for the *3C’s* framework. The implications of this are used in answering
research question two.

Overall, instructors reported *slightly higher* opportunities for enacting high-quality
instructional practices when teaching mathematics as a subject than in interdisciplinary lessons.
Baumert et al.’s (2010) study found that teaching practices promoting powerful instruction (like
those in the dimensions of the TRU framework), activities high in critical thinking and deep
conceptual understanding, and enhanced activities such as projects are linked to gains in student achievement and engagement. Based on the answers to survey and interview questions, instructors claimed that they enacted these practices more when teaching mathematics. These results are not surprising as they reflect instructors’ pedagogical practices, which they use daily in their mathematics teaching, the primary subject they are trained to teach (Boaler, 2002b; Smith et al., 2022).

On the other hand, interdisciplinary work takes on many forms—from after-school programs to actual curriculum—where pedagogy may not always fit a specific formula (Vomert et al., 2013; Kitty & Burrows, 2022). Since all the participants were primarily mathematics instructors, they spoke more about instilling disciplinary mathematical practices in their classroom teaching. These practices included thinking through problems critically, communicating a cohesive understanding of mathematics learned, or even persisting through tasks—behaviors they believed helped students make sense of concepts by understanding how they fit together and observable using the TRU Math framework (Schoenfeld, 2020). These practices are also transferrable over any curriculum.

**Research Question 2**

2. Are there differences in mathematics content, cognitive demand, and assessments between *mathematics* and *integrated* or *interdisciplinary activities*?

The second purpose of this study was to highlight how instructional activities promote engagement and a deep understanding of mathematics as taught as a subject and within an interdisciplinary curriculum. Assessing an activity’s mathematics content, cognitive demand, and assessments using the TRU framework shows its potential for rigor and the possibility of
fostering a deep understanding of mathematics. In reverse order of how research question one was addressed, this section begins by answering research question two. It then transitions towards interpreting such differences through the instructional practices of Selma, Pearl, and Pruitt. Lastly, it concludes in light of implications for instructors and the influential role that teaching experience plays in the process.

The resulting interpretations and conclusions follow the work of the Connectors, who taught both types of lessons as separate courses within a term or in consecutive terms. This does not imply that Curators and Constructors do not provide insight into this research question. However, Connectors captured the original description of the participants. Furthermore, as a group, they provided significantly more artifacts for comparison. There were a couple of Curators with a similar number of artifacts to show some delineations. However, there were not enough artifacts for each Constructor and Curator to provide substantial comparisons for analysis within and across groups. Since this level of comparison was not the study's original intent, it lends itself well to suggestions for future research offered later in this chapter.

**Characterization through the 3C’s Framework**

Instructors’ pre-study survey responses suggested higher opportunities for understanding mathematics, engaging in cognitively demanding tasks, and more productive use of activities in interdisciplinary lessons than in mathematics. Upon closer examination of artifacts using the adapted TRU Math rubric (in Appendix H), the scores for instructional artifacts were slightly lower. The survey results and analyses of artifacts did not fully align. It may be essential to note that opportunity scores were still high since they were 2.85 and greater.
This is consistent with literature as instructors’ beliefs about one's teaching practices and the activities they assign are at times misaligned (Raymond, 1997; Milman, 2016). One conclusion is that instructors may have overestimated their ability to design rigorous interdisciplinary activities due to their perceived (and observed) ability to design mathematics activities toward that purpose. This disconnect signaled the importance of frameworks like TRU Math to help classroom practices and activities promote depth of understanding and rigor.

Another interesting finding is the different ways in which instructors engaged in interdisciplinarity. Through practical descriptions of their work, the instructors used everyday language to describe how they practiced interdisciplinarity. The Constructors taught mathematics for understanding, with interdisciplinarity playing a minor role. The Curators incorporated STEM or STEAM strategies into direct instruction, projects, and assignments, in the service of learning mathematics. Some interdisciplinary lessons focused on learning mathematics, while other cases showed them using mathematical skills more procedurally. The Connectors, however, placed understanding mathematics at the core of their mathematics and interdisciplinary activities, which also corresponded well for analysis through the TRU Math framework. These instructors combined different interdisciplinary instructional practices suggested by the literature (Thibaut et al., 2018; Sutaphan & Yuenyong, 2019).

One conclusion that can be drawn from this is that instructors, especially K-12, are seeking more ways to practice interdisciplinarity and not just more definitions and procedures authentically. As pointed out in the previous section, more and more studies provide definitions of what it means to “do STEM”; however, instructors need more focused guidance. Characterizing ways to practice interdisciplinarity and a toolbox of methods may help decrease
frustrations about teaching interdisciplinarity and provide more focus for instructors to incorporate it in their classrooms.

**The Nature of Differences and TRU Math**

In general, the differences in TRU Math scores for mathematics and interdisciplinary activities were more profound when the purpose of the assignment did not align well with the dimensions of the rubric. For example, some concepts used in *forming* understanding were more complex when assessed on an exam. These instructors may set up these activities *purposefully*, but additional support structures like Pruitt’s practice of organizing learning around standards helped to maintain rigor (Permatasari, 2016). Activities can be enjoyable to students, but they can only be a problem-solving challenge or an opportunity to build and develop an understanding of mathematics *by design* (Hiebert, 1997; Tytler et al., 2019). Further examination of their cases explained the meaning behind variations in the artifacts’ mathematics content, cognitive demand, and use as assessments. The differences also spoke to their use of interdisciplinary activities *supporting* mathematics learning.

In Selma’s case, the differences were *misaligned*. After analyzing her activities through the TRU Math rubric and considering *critical* interview and survey responses, the average TRU Math scores were *higher in mathematics activities* than in her STEAM curriculum. Selma may not have recognized that she integrated subjects with mathematics well. Selma’s activities used technology, writing, and art to help students communicate their understanding of mathematics concepts. She also acknowledged that future revisions of the STEAM course *might* include more mathematics. Selma’s case also highlighted additional supports necessary in a K-12 setting that are not always available in most traditional schedules: a revised schedule that balances days for
content, work, and academic support; a team of expert teachers who worked well together to co-plan and co-teach the course; and a supportive administration that tailored academic needs of the community of students they serve, among others. Selma’s years of teaching experience may also have played a role in her practice. Both Pruitt and Priscilla have over 15 years of teaching experience, while Selma was in the middle of her eighth year of teaching. As teachers gain more experience and see students’ gains in achievement, it correlates to improved effectiveness over time (Podolsky & Darling-Hammond, 2019). It may be fascinating to see a revised course and compare the new activities along the TRU Math dimensions with the old ones.

Pearl’s case showed that her pre-study survey claims aligned with her interview and activities. She recorded higher opportunities for making sense of concepts in mathematics than in her interdisciplinary seminar for honors students. However, she was the only instructor who categorically showed how teaching her interdisciplinary course influenced her pedagogy in multivariable calculus. She continually linked pedagogical content knowledge of both courses, including what she observed in her co-teacher, which helped to improve her teaching practices (Depaepe et al., 2013). In the interdisciplinary seminar, Pearl witnessed “non-majors” learning applications of multivariable calculus “without taking a calculus course.” She also admitted that spending time planning and teaching with her co-instructor has validated and improved her mathematics teaching. Pearl taught mathematics in her interdisciplinary seminar; however, she made significant changes in the applications students saw in multivariable calculus. As such, her teaching experiences, at the least, influenced each other.

Pruitt’s case was an example of interdisciplinary mathematics teaching and learning in practice. His courses were not co-taught, but he collaborated with colleagues from other
departments at his college. His pre-study survey scores were similar to his TRU Math mean scores, with interdisciplinary activities scoring slightly higher. Pruitt’s beliefs about mathematics learning were robust and, as such, clearly visible in the objectives of his interdisciplinary courses. Three things worked in his favor: standards-based grading (SBG), student feedback, and project-based learning. His case is consistent with the literature showing strong associations between content and pedagogy (Shulman, 1989; Schoenfeld, 2015; 2020).

Pruitt used SBG to be transparent about what students were learning and how they were assessed (Scriffiny, 2008; Marbouti et al., 2016). Students were afforded opportunities to revise work, and they still received “letter grades.” Even though he continued to improve his grading system, students understood the quality of work it took to obtain the grade they desired (Hill, 2005). Student feedback was an integral part of his pedagogy. Students reflected on their learning throughout projects as part of their final exam. They also advised future students in their end-of-course reflections. This kind of evidence-based learning (Diery, 2020) provided Pruitt with valuable feedback for future courses and with an understanding of the various ways that students of all learning levels can genuinely understand mathematics.

Last but not least, projects were an invaluable method of instructional activities (Capraro & Slough, 2013). It provided time, space, and great use of assessment due to how well they were organized around standards (Slough & Milam, 2013). Pruitt also had “touch-points” throughout the semester to check in with students so that they did not fall behind. Also, as “students engage with [projects], they are…mirroring…the processes used by scientists and engineers to solve real-world issues through the active construction of new knowledge and the development of problem-solving skills” (Siew et al., 2015, p. 2). Even with over 16 years of teaching experience, he
accepted the invitation to participate in the study to “learn more about teaching interdisciplinary courses.” Pruitt’s claim of a common goal for both types of lessons is warranted because students in his mathematics and interdisciplinary courses learn mathematics concepts well. However, he, too, admitted that his interdisciplinary projects influenced his use of projects in his calculus course.

One important conclusion is that there may be an advantage to how one leverages their experiences. Teaching both types of courses had advantages because instructors used their experiences to influence each other. However, if teachers do not have that kind of real-time feedback that Emily and Pearl shared with their co-teachers, they will need to find creative ways to solicit student feedback (like Pruitt) actively. Another conclusion is how student learning experiences mediate course design and teaching decisions. This adds consistency to teachers’ beliefs about their practice and what they do.

The “3C’s framework offered ways to practice interdisciplinarity by characterizing instructors’ practices with terms transferrable across subjects and pedagogies. Even though I analyzed the activities of Connectors more in-depth, it does not mean that we cannot learn from the activity of Curators and Constructors, whose examples were highlighted in chapter 4. They, too, helped in understanding characterizations. They may show a possible trend in practicing interdisciplinarity. The reality of teaching multiple courses and preps may not be sustainable outside of the kind of support structures in place, especially in K-12 settings. Lastly, the differences in the robustness of activities reported in the survey and analyzed in artifacts are not always apparent to the observer. They may be more associated with instructors’ conceptions of
conceptual depth and less about the alignment of beliefs and practice. This further underscores the value of frameworks like TRU Math to help bridge this gap in understanding.

**Significance of this Study**

The 3C’s framework characterized how instructors practice interdisciplinarity, using terms that they are already familiar with and use in their teaching practices. Since current research provides definitions, methods, and guidelines, this type of characterization is a unique contribution to research in interdisciplinary mathematics. This framework enables teachers to reimagine ways to use practices and pedagogies to help students explore mathematics through integration and interdisciplinary learning. I offer future studies to help develop this framework theoretically later in this chapter.

Additionally, the TRU Math framework makes good use of assessing the building and developing a deep understanding of mathematics. Instructors could use it to evaluate the understanding of any STEM discipline. Even though my study focused on mathematics within an interdisciplinary curriculum, it also touched on issues of complexity in activities. Using this framework shows a way to explore mathematics concepts within an interdisciplinary curriculum with understanding, especially with proper support.

Instructors also used culturally responsive practices where students learned mathematics. Even though I only used the TRU Math rubric to assess these artifacts' cognitive demand and mathematics content, they showed evidence of responsiveness to students’ backgrounds and mathematics identities. Thus, there seems to be a connection between culturally responsive pedagogical practices and the TRU framework. This connection affirms using the remaining two
dimensions—agency, authority, and identity, and access to mathematical content—for future studies.

Another significant finding is the illumination of how student learning fuels the formation of beliefs. It is vital to have “methods courses” and professional development that bring awareness to how preservice and inservice teachers’ beliefs about themselves and their students impact teaching and learning. However, teachers who use student-centered, evidenced-based experiences of understanding mathematics have beliefs rooted in serving students' interests. Helping teachers understand why this is important may positively impact their philosophical beliefs and pedagogy.

In my study, teacher supports included establishing mathematics standards for all activities, using formative assessments to help solicit students’ ideas about mathematics, incorporating academic support with flexible schedules (in K-12), and a team of educators who are experts in their discipline. Instructors also organized projects and rubrics around standards. These practices assisted them in practicing interdisciplinarity. Instructors need these reinforcements and support from colleagues and administrators if we want them to feel more comfortable taking risks outside of their teaching comfort zones.

6.3 Recommendations and Suggestions for Research

Limitations

Classroom observations were not allowed or available due to the COVID-19 pandemic. Thus, I had to rely on teachers’ recollections of events in executing these activities. While some instructors were able to provide redacted reflections from students and other individuals associated with projects, other activities were not designed to include such exemplars. An
extension of this study would include observations, which makes space for all five rubric dimensions. A task may have medium cognitive demand but still be highly meaningful to students; this, I believe, is better observed in real time.

Also, researchers need more modes of instruction, including lectures, observing group work, documenting classroom narratives between teachers and students and among students, and individual student reflections. That way, scores represent a more comprehensive profile of instruction and activities, as initially intended (Schoenfeld, 2018). Additional modes of instruction and more activities can add quantitative value that supports qualitative results. It is better to have more than one person scoring activities.

Additional scorers improve interrater reliability and objectivity, which are necessary for research studies. Again, since this study was primarily for my dissertation, I was the only researcher involved. Scoring activities along the TRU Math dimensions rubric are intended to be objective, but the process of qualitative analysis is subjective. I tried to account for this by analyzing activities at least three times and ensuring that I did not analyze the dimensions more than once. It is important to be clear about what dimensions mean to assess activities accurately. Multiple scorers could help mitigate this process in future studies.

**Suggestions for Further Research**

Based on the limitations of this study, researchers might consider a larger sample of interdisciplinary instructors from kindergarten to post-secondary learning to assess whether similar characteristics exist and uncover other practical ways interdisciplinarity is being practiced. Additionally, they could incorporate all five dimensions of the TRU Math rubric for a complete perspective. Consider other studies that focus on Curators and Connectors to see what
else can be learned from their practices and other characterizations and combinations thereof. Further research should be done to analyze student experiences in learning mathematics and interdisciplinarity within a course and as a longitudinal study. Also, researchers may consider using the TRU framework for assessing the complexity of an activity and its potential to understand mathematics deeply within an interdisciplinary curriculum since it applies to different subjects and grade levels.

**Suggestions for Teacher Educators**

Professors should encourage preservice teachers to ground their teaching philosophies in student reflections and in their own learning experiences. Relatedly, preservice teachers should be cautious about ascribing to beliefs formed outside of their students’ learning experiences. It is easy to state a belief, especially when prominent in current research. However, Mensah’s (2021) case study analysis of preservice science teachers showed them developing new insights about multicultural interdisciplinary science after implementing a curriculum in an urban school during their field teaching experience. Also, consider employing research methods that contemplate teachers’ and students’ identities jointly. It is crucial to recognize preservice teachers’ fears and misconceptions to be more comfortable taking risks during the program and in their teaching careers. This may mean partnering with schools that practice authentic interdisciplinarity that could prepare preservice teachers to begin employing these practices early in their teaching careers. When possible, preservice teachers may benefit from incorporating both content and interdisciplinary teaching in field experiences.

**Suggestions for K-12 Teachers**
Partner with teachers from other disciplines or departments to create authentic, integrated learning lessons when lacking professional development. Relatedly, take part in assessing any current programs to ascertain where improvements are needed. Keep up with current research, but do not feel overwhelmed by the volume and different approaches. It is essential to be clear on what is integrated and when disciplinary subject learning may not be the overall goal. Look for peer-reviewed practices approved by your administration and national education organizations that fit well within your school day or after-school programs. Teachers should use guides like TRU but be careful to understand the essence of each dimension to avoid overreliance on the framework. Continue to expand your content and pedagogical knowledge through professional development and partnerships. Be flexible and do not be afraid to fail, as this is a necessary part of teaching outside of one’s comfort zone.

**Suggestions for Post-Secondary Instructors and Program Directors**

More students need access to interdisciplinary courses that fulfill necessary credits for graduation. Consider how the college/university may offer an interdisciplinary seminar like those offered to students in Pearl’s honors program. Piercy (2019) integrated interdisciplinary learning throughout his mathematical journey toward completing a Ph.D. in mathematics, which provided a better theoretical foundation for understanding concepts. His current research integrates mathematics, ethics, inquiry-based learning, and interdisciplinary collaboration. Also, when possible, partner with K-12 teachers to model ways to collaborate and co-teach courses. Such professional development settings have the potential to offer high rewards for learning in *comfortable* spaces where instructors learn from each other (Darragh et al., 2011).

**Suggestions for Administrators**
Provide time and space for teachers to collaborate and assess current interdisciplinary programs and co- and team-teaching assignments. It may even be helpful to partner post-secondary co-teaching practitioners with K-12 teachers to exchange ideas about what works and suggestions for improvement. Become knowledgeable about the evolving trends in interdisciplinary education and consider professional development that combines the toolbox of strategies with practical ways to integrate content and interdisciplinarity. Be supportive and patient with teachers as they learn to teach outside of their grade levels and subject areas. Much like multidisciplinary work in industry, interdisciplinary projects are complex and require time and patience. Be open to suggestions for training, program improvements, and necessary support. Also, maintain realistic expectations regarding district policies, teachers’ workloads, and student demographics.

6.4 Closing Comments

Several instructors mentioned how reflecting during their participation in this study on how their students learn gave them a “renewed focus” and was a “booster shot” of encouragement towards their teaching practice. It was an important reminder of their humanity in my research and analysis. Some challenges of the teaching profession may at times impact teachers’ willingness to adapt and change. We know that “identity and learning are deeply intertwined” (Nasir et al., 2021. p. 560). Research and practice that skillfully considers both teachers’ and students’ identities jointly may help support equitable teaching and learning goals. Reflecting on my experiences as a student from K-12 through doctoral candidacy and my work as a researcher added much value and perspective for analysis. Interdisciplinarity may also add value to these efforts since it influences the identities of teachers and students.
Even so, teachers must be willing to learn and adjust their pedagogy. Students’ learning must affirm and improve their lives if the aim is equity (Nasir et al., 2021). One instructor said, “I want my mathematics legacy to be that my students know that I cared about their learning.” When students are taught in ways that affirm how they learn best, they associate that with how much teachers care about them. Connecting interdisciplinarity to mathematics learning may be one avenue for both teachers and students to learn together.
References


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Walker, E. N. (2010). More than test scores: How teachers’ classroom practice contributes to and


Appendix A

Teachers College, Columbia University IRB Approval

Attachments:
- NYC-DOE-IRB-Adult-Consent-Form-Dyanne Baptiste v4_FINAL.pdf

Teachers College IRB

To: Dyanne Baptiste
From: Myra Luna Lucero, Research Compliance Manager
Subject: IRB Approval: 20-268 Protocol
Date: 05/19/2020

Thank you for submitting your study entitled, “Analyzing Interactions between Mathematics Elementary School Teachers’ Instructional Activities and their Practices and Beliefs about the Teaching and Learning of Mathematics”; the IRB has determined that your study is Exempt from committee review (Category 1) on 05/19/2020.

Please note, due to COVID-19 quarantine, all in-person study activities are suspended. The IRB will announce when in-person research can resume and what steps to take at that time. We will post updates about COVID-19 on TC IRB’s website/Updates.

Please keep in mind that the IRB Committee must be contacted if there are any changes to your research protocol. The number assigned to your protocol is 20-268. Feel free to contact the IRB Office by using the “Messages” option in the electronic Mentor IRB system if you have any questions about this protocol.

Please note that your Consent form bears an official IRB authorization stamp and is attached to this email. Copies of this form with the IRB stamp must be used for your research work. Further, all research recruitment materials must include the study’s IRB-approved protocol number.

As the PI of record for this protocol, you are required to:
- Use current, up-to-date IRB approved documents
- Ensure all study staff and their CITI certifications are on record with the IRB
- Notify the IRB of any changes or modifications to your study procedures
- Alert the IRB of any adverse events

You are also required to respond if the IRB communicates with you directly about any aspect of your protocol. Failure to adhere to your responsibilities as a study PI can result in action by the IRB up to and including suspension of your approval and cessation of your research.

You can retrieve a PDF copy of this approval letter from Mentor IRB.

Best wishes for your research work.

Sincerely,
Dr. Myra Luna Lucero
Research Compliance Manager
IRB@tc.edu

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Appendix B

New York City, Department of Education IRB Approval

Dear Dynanne Baptiste,

I am happy to inform you that the New York City Department of Education Institutional Review Board (NYC DOE IRB) has completed its review of this proposed research and approved your research proposal, "Analyzing Interactions between Mathematics Teachers' Instructional Activities and their Practices and Beliefs about the Teaching and Learning of Mathematics." The Board has determined this research to pose Minimal Risk. The NYC DOE IRB has assigned your study the protocol number of 3304. Please make certain that all correspondence regarding this project references this number. The approval is for a period of one year:

Approval Date: September 29, 2020
Expiration Date: September 28, 2021
Review Level: No answer provided
Funding: No money provided

Approved Study Team Members: Dynanne Baptiste, Dynanne M.A.T., M.B.A.
Walker, Erica E.D.

We want your feedback!

--- COVID-19 Response:

If this study requires in-person interactions or interventions with research subjects, note that these procedures cannot take place without explicit written permission for site access from the NYC DOE IRB and principal or site director.

Unexplicit: If this protocol is reviewed and approved by the NYC DOE IRB, the principle investigator may not take place.

This information is subject to change without notice.***

Responsibilities of Principal Investigators: Please find below a list of responsibilities of Principal Investigators who have DOE IRB approval to conduct research in New York City public schools:

- Prior to connecting individual schools or principals, all designated personnel named in this protocol to conduct research in NYC public schools with NYC DOE staff or students, or using NYC public school student data, must complete the NYC DOE security clearance process. This includes but is not limited to being fingerprinted by the NYC DOE Office of Personnel and Payroll. Each individual must provide a signed copy of the IRB or Ethics Board approval or clearance letter, along with a government issued photo identification and a valid social security card.

- Each must be provided with a stamped copy of the IRB or Ethics Board approval or clearance letter, along with government issued photo identification and a valid social security card. Each must be provided with a stamped copy in accordance with the procedures outlined in this protocol.

- The individual must provide a stamped copy of the IRB or Ethics Board approval or clearance letter, along with government issued photo identification and a valid social security card.

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- The individual must provide a stamped copy of the IRB or Ethics Board approval or clearance letter, along with government issued photo identification and a valid social security card.
In the event that contracts, external approvals, or other documents are pending at the time of this approval, they must be submitted for NYC DOE IRB review by Amendment once obtained.

**Mandatory Reporting to the IRB:** The Principal Investigator must report to the DOE IRB, within 24 hours, any serious problem, adverse effect, or outcome that occurs with frequency or degree of severity greater than that anticipated. In addition, the Principal Investigator must report any event or series of events that prompt the temporary or permanent suspension of a research project involving human subjects or any deviations from the approved protocol. All reports must be submitted using the IRB Manager Protocol Violation, Deviation, Adverse Event, and/or Unanticipated Problem Report form.

**Amendments/Modifications:** All amendments/modification to this protocol require prospective IRB approval, except those involving the prevention of immediate harm to a subject, which must be reported within 24 hours to your IRB of record and to the NYC DOE IRB.

**Continuation of your research:** It is your responsibility to ensure that an application for Continuing Review is submitted 90 days before the expiration date noted above. If you do not receive approval to continue research before the expiration date, all study activities, including, but not limited to, analysis of collected data, must stop until said approval is obtained.

**Research findings/Study Closures:** The NYC DOE IRB requires a copy of the report of findings from this research. Interim reports may also be requested for multi-year studies. Further, you are required to formally close this protocol by submitting a Study Closure form once all research procedures, including, but not limited to, all analysis of coded or identifiable data, have concluded.

**Data Request:** Note that approval of this research does not constitute confirmation of release of data requested in a Data Request form. All data requests are processed and approved by the Data Request Fulfillment Team. Please email rpsgresearch@schools.nyc.gov with any questions you may have regarding this matter.

If you have any questions, please contact Marianna Azar at [redacted]

Good luck with your research.

Sincerely,

Marianna Azar
Director and Chair, Institutional Review Board
Appendix C

Adult Informed Consent Form

Teachers College, Columbia University
525 West 120th Street
New York NY 10027
212 678 3000

Adult Consent Form to Participate the Research Study

1. Title of research study and general information.

**Study title:** Analyzing Interactions between Mathematics Teachers’ Instructional Activities and their Practices and Beliefs about the Teaching and Learning of Mathematics

**IRB Numbers:** Teachers College IRB 20-268 and NYC Department of Education IRB 3504

**Participation duration:** 4 hours

**Anticipated total number of research participants:** 20

**Sponsor/Supporter:** Professor Erica Walker

2. Researchers’ contact information.

**Principal Investigator:** Dyanne Baptiste, M.A.T, M.B.A., Doctoral Student and Research Fellow, Teachers College, Columbia University

**Phone Number:** [Redacted]

**Email Address:** db2933@tc.columbia.edu

**Faculty Advisor:** Professor Erica Walker, Teachers College, Columbia University

**Phone Number:** [Redacted]

**Email Address:** [Redacted]

3. What information is on this form?

You are invited to participate in this research study called “Analyzing Interactions between Mathematics Teachers' Instructional Activities and their Practices and Beliefs about Teaching and Learning Mathematics”. The study aims to describe the instructional and curricular practices and beliefs of mathematics teachers of all levels, who also teach an interdisciplinary curriculum such as science, technology, engineering, and mathematics (STEM) or science, technology, engineering, art, and mathematics (STEAM). The study
seeks to understand the ways in which both kinds of instruction foster positive socialization and spaces around learning mathematics, while promoting a deep understanding of mathematics content. Additionally, results may help inform research that could potentially be used to conduct professional development for in-service teachers along these lines of reform.

Please take the time to read this form before signing it. Once you have read the form, you should ask any questions you have about it and the research study. Please email me with any questions or concerns and I will address them immediately. If you prefer, you may also schedule a phone meeting to discuss these issues. Please know that you do not have to participate if you don't want to.

4. Why is this study being done?

This study seeks to describe connections between teachers’ belief and practices as seen through instructional activities taught in mathematics and in interdisciplinary lessons, aimed at a deep understanding of content. As such, this study seeks to answer the following research questions:

1. What is the relationship between teachers’ beliefs and conceptions towards teaching and learning mathematics and the instructional practices enacted in the classroom? How does this relationship differ between mathematics lessons and an interdisciplinary curriculum such as STEM or STEAM etc.?

2. Are there differences in mathematics content, cognitive demand, and formative assessments between mathematics lessons and an interdisciplinary curriculum such as STEM or STEM etc.?

This study serves as the researcher’s doctoral dissertation in Mathematics Education at Teachers College, Columbia University.

5. Who is being included?

You may qualify to take part in this research study if you are an instructor who teaches mathematics and an interdisciplinary curriculum such as STEM, STEAM, etc. as part of your teaching assignment.

Approximately 20 teachers and professors will be invited to participate in this study. This research study is focused on describing participants’ practices enacted through instructional activities and to highlight the mathematics content, cognitive demand, and their approach to equitable teaching as seen through these activities.
6. What will I be asked to do if I choose to be in this study?

- This study will take about **four hours** and **there are no observations**. You will receive a participant identification number (PID) which will maintain your confidentiality. All identifying information will be redacted and only your PID will be used to identify your artifacts. I will keep the PID list in a secure, password protected file, to which I alone will have access. Any data collected will be stored in a password protected, individual folder accessible only by you and me.

- You will receive a link to sign the consent electronically and then the study will begin.

- You will receive a link to complete a short survey online.

- I will request that you provide 1 – 3 different examples each, of mathematics and interdisciplinary assignments or sample exercises, where the mathematics content **preferably overlaps**. Examples of instructional assignments include homework problem sets or samples, classwork guided practice, formative and/or summative assessments, unit projects or portions thereof, or any other artifact that represents your work with student learning. You will upload blank copies of these to your individual Google folder. These activities will be analyzed using a truncated rubric of Dr. Alan Schoenfeld’s TRU Math framework.

- You may be asked to provide a short reflection on both types of lessons. The prompts will be provided for you.
  
  - Upon analysis of instructional activities, the researcher will ask teachers for a reflection of both a mathematics and interdisciplinary assignment.
  
  - The reflection exercises and the interview will be opportunities for you to share your experiences of students engaging with the lessons, how you perceived students would approach a task, how students actually approached the task, and what you learned about designing activities in the process. This study focuses on your lived experiences creating instructional activities for understanding and your beliefs about teaching and learning mathematics. You will upload the completed reflections to your individual folder.

- I will conduct a semi-structured interview via an online platform that will be digitally recorded. The interview will take no more than 60 minutes. It will be transcribed by either myself or a professional transcriber who will sign a non-disclosure agreement.
7. Are there any risks?

This is a minimal risk study, which means the harms or discomforts that you may experience are not greater than you would ordinarily encounter in daily life while talking to a fellow colleague. However, there are some risks to consider. You do not have to answer any questions or share anything you do not want to talk about. You can stop participating in the study at any time without penalty. Even though your information will be kept confidential, you might feel concerned that things you say might get back to your supervisor. I will take all precautions to keep your information confidential and prevent anyone from discovering or guessing your identity. I will be using a link for the survey instead of a unique identifier and all information will be stored on a password protected computer and kept confidential.

8. Are there any benefits?

There is no direct benefit to you for participating in this study. However, you may help to inform teaching practices surrounding STEM education and have a chance to reflect on your own practice for enhancing future lessons. Participation may also benefit the field of teacher education to better understand the best way to train STEM and Mathematics teachers.

9. What about my privacy?

In order to maintain confidentiality, you will receive a participant ID number (PID), assigned using a random number generator. Any information that identifies the participant or mentions the name of the institution will be redacted and only your PID will be used to identify your artifacts. All materials collected, including the consent form, will be linked to participants using the PID and any biographical and school information will be redacted from the forms. Additionally, only blank copies of assignments will be collected.

I will create individual folders on a secure, password protected Google Drive using this PID. Each participant will have access to their own folders. Except for me, no other individual will have direct access. This folder will contain all the information associated with each individual participant.

Digital Survey & Instructional Activities

I will keep all electronic or digital information on a computer that is password protected. Also, the list that identifies each participant to their PID will be kept in a secure, password protected file, to which I alone will have access. This document will then be stored in a folder on my password protected Google Drive. All data collected will be kept confidential and only the PID will be used on any forms and blank assignments. Your name and school (Continued)
information will be redacted before storing it in your Google folder. The research folder with all data collected will be kept in a password protected computer. Only I will be able to see this file. Every effort will be made to keep your personal information private and confidential. However, total privacy cannot be guaranteed.

**Digital Recording of Interview**

- The interview, audio recorded via a digital recorder, will not include any identifiers. It will be saved on a computer that is password protected.
- The file name will not be linked to your real name. No names or any other identifying information will be mentioned in the interview.
- I will analyze the recording for my research study for educational purposes. It will not be shared with anyone else or used for any other purpose.
- Only I will have access to these recordings. It will be kept until I complete the dissertation process and then be destroyed. The process is complete once the manuscript is approved by my dissertation committee.
- Participants will not be compensated for being audio recorded.

**Publishing Results**

The results of this study will be published in my dissertation, which is a public document. It may also be published in journals, chapters, and books, and presented at academic and professional conferences and meetings. Your name and other features of your identity be removed from all data the researcher includes in such reports before publication and/or use for educational purposes.

For quality assurance, the study sponsor (Professor Erica Walker of Teachers College), and/or members of the Teachers College Institutional Review Board (IRB) may review the data collected from you as part of this study. Otherwise, all information obtained from your participation in this study will be held strictly confidential and will be disclosed only with your permission or as required by U.S. or State law.

**10. Will I get paid or be given anything to take part in this study?**

You will not receive any payment or other reward for taking part in this study.

**11. Will I incur costs if I take part in this study?**

There will be no costs to you for being in this study.

**12. What are my rights if I take part in this study?**
Taking part in this study is voluntary. You can refuse to participate or withdraw participation at any time without penalty. You will not lose any benefits to which you are otherwise entitled.

13. Who can I call if I have questions?

If you have any questions about taking part in this research study, you should contact the primary researcher, Dyanne Baptiste at [redacted] or email db2933@tc.columbia.edu.

If you have any questions about your rights as a research participant, or if you have a concern about this study, you may contact the Institutional Review Boards listed below.

Institutional Review Board
New York City Department of Education
52 Chambers Street, Room 310
New York, NY 10007
Telephone: (212) 374-3913
MAzar@schools.nyc.gov

Institutional Review Board
Teachers College, Columbia University
525 W. 120th ST New York, NY 10027
Telephone: (212) 678-4105
IRB@tc.edu

14. Statement of consent and electronic signatures

Statement of consent

I have read this consent form. The research study has been explained to me. If I choose to participate, I agree to be in the research study as described above.

By agreeing to participate in this study, I have not given up any of the legal rights that I would have if I were not a participant in the study.

By checking “I agree”, I am electronically signing this consent form to participate in this study and to be audio recorded using a digital recorder.

I affirm that an electronic signature has the same effect as a written signature. I also confirm that I am 18 years or older and teach mathematics as a subject and within an
interdisciplinary/multidisciplinary curriculum (e.g. STEM, STEAM, etc.) as part of required weekly tasks that occur during the normal school day.

- I agree to participate in the research study as described in this form.
- I give my consent to be recorded.
- I do not consent to be recorded.

Please type PID # below as this is required. Thank you.

PID
Appendix D

Invitation to Participate Email

Thank you for expressing interest in participating in my dissertation study titled, *Analyzing Interactions between Mathematics Teachers’ Instructional Practices and their Beliefs about Teaching and Learning Mathematics*. My name is Dyanne Baptiste and I am a Ph.D. candidate in the mathematics education program at Teachers College, Columbia University. This study seeks to better understand teachers who teach mathematics as a subject and within interdisciplinary lessons. The results of this study may help to inform researchers and teachers on the level of rigor in these lessons, as well teachers’ beliefs and practices about teaching and learning mathematics.

This study will take about 4 hours, spread over 4 weeks, and there are no observations. You will be assigned a numerical identification number (PID) which will be the only identifier on all documents collected. All biographical and school information will be redacted from data collected before being stored in a password protected folder, accessible by only the participant and me. Before you can begin the study, you must first complete the consent form. You will then take part in a digital survey on your teaching beliefs and practices, provide artifacts of instructional activities, and participate in an interview to talk about your thoughts in the design and execution of these activities. While there will be no class observations, I will request that you submit a short reflection on a mathematics and an interdisciplinary lesson that you felt went well and one that you would love to improve for the following year. All data will be collected digitally, and you will be able to upload all of your instructional activities. Once all the artifacts have been analyzed, we will have the online interview.

Please note that your involvement in this study will not be shared with any of your colleagues or used for any evaluation purposes. Anything shared with me will be confidential, and I will be the only one who will have access to this information. As I mentioned, I will be writing reports and giving presentations resulting from this study; the study is my doctoral dissertation which will also be published. I will never use your real name or the name of your school or of persons associated with you. Your participation in this study is voluntary and you can leave at any time if you are no longer interested in participating.

If you are interested in this study, you will need a participant identification number (PID) which is needed to complete and sign the consent form seen here. The consent form is necessary to begin the study. Please email me at db2933@tc.columbia.edu to receive your PID and complete the consent form online to begin the study. Thank you so much for considering this invitation. I look forward to hearing from you!

Kind regards,

Dyanne Baptiste
Ph.D. Candidate – Mathematics Education Program
Teachers College, Columbia University
Appendix E

Invitation to Participate Recruitment Flyer and Social Media Post

Call for Participants

MATHEMATICS INSTRUCTORS who also teach STEM, STEAM, or another integrated curriculum

Do you teach mathematics? Do you also teach mathematics integrated with science, art, technology, history, engineering, writing, or another subject? I would love to hear from you!

This dissertation study seeks to better understand your beliefs and instructional practices. Your participation includes a short survey and an interview.

Please email me to inquire about participating.
Dyanne Baptiste, Ph.D. Candidate, Primary Researcher

db2933@tc.columbia.edu

TC IRB #20-288
NYC DOE IRB #3504
Appendix F

Pre-Study Survey for Participants with Dimension Highlights

<table>
<thead>
<tr>
<th>Background</th>
<th>Preparation and Training</th>
<th>Professional Development</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How long have you been teaching?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. What subjects and levels do you teach now, and if different, what have you taught in the past?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Describe the interdisciplinary curriculum (such as STEM, STEAM, etc.) used at your institution, how often is it taught and in what way?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. What is your formal education background? Did it include a teacher education or training program?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Were the following elements included in your formal education or training?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inclusion in education or training</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>a) Mathematical Content</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>b) Subject Content of Interdisciplinary/Integrated course (e.g., STEM, STEAM, etc.)</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>c) Pedagogy of teaching mathematics as a subject</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>d) Pedagogy of teaching subjects in an integrated way (e.g., STEM, STEAM, etc.)</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>e) Classroom practices when teaching mathematics as a subject</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>f) Classroom practices when teaching subjects in an integrated way</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>g) Monitoring students’ development and learning of mathematics</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>h) Monitoring students’ development and learning of integrated subjects (e.g., STEM,...)</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>i) Using technology in teaching one or all subjects</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>j) Teaching cross-curricular skills (e.g., creativity, critical thinking, problem solving)</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>

Day-to-Day Teaching Responsibilities

8. Approximately how many hours per week do you routinely spend on the following tasks, over the course of an average week? Include tasks that take place during weekends, evenings or other times outside of class hours.

<table>
<thead>
<tr>
<th>Preparations</th>
<th>Not at all</th>
<th>Somewhat</th>
<th>Well</th>
<th>Very well</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Individual planning or preparation of lessons either at school or out of school</td>
<td>Hours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) Grading/returning of student work</td>
<td>Hours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) Student counseling (including student supervision, mentoring, virtual counseling, career guidance and/ or behavior guidance)</td>
<td>Hours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) Communication and cooperation with parents or guardians</td>
<td>Hours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) Actual teaching time, including direct and guided instruction</td>
<td>Hours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f) Professional development activities</td>
<td>Hours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g) Other work tasks (List some examples: )</td>
<td>Hours</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Professional Development**

9. Of the instructor professional development and training opportunities attended within the past 18 months, were any of the following topics included?
(Continued)

<table>
<thead>
<tr>
<th>Teaching Philosophy</th>
<th>Yes No</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Knowledge and understanding of mathematics</td>
<td>○ ○</td>
</tr>
<tr>
<td>b) Pedagogical competencies in teaching mathematics</td>
<td>○ ○</td>
</tr>
<tr>
<td>c) Analysis and use of assessment in mathematics</td>
<td>○ ○</td>
</tr>
<tr>
<td>d) Approaches to student engagement in learning mathematics</td>
<td>○ ○</td>
</tr>
<tr>
<td>e) Knowledge and understanding of STEM, STEAM, etc.</td>
<td>○ ○</td>
</tr>
<tr>
<td>f) Pedagogical competencies in teaching STEM, STEAM, etc.</td>
<td>○ ○</td>
</tr>
<tr>
<td>g) Approaches to student engagement in learning STEM, STEAM, etc.</td>
<td>○ ○</td>
</tr>
<tr>
<td>h) Analysis and use of assessments in STEM, STEAM, etc.</td>
<td>○ ○</td>
</tr>
<tr>
<td>i) Teaching cross-curricular skills (e.g., creativity, critical thinking, problem-solving)</td>
<td>○ ○</td>
</tr>
<tr>
<td>j) Other topic of importance not listed</td>
<td>○ ○</td>
</tr>
</tbody>
</table>

**Teaching Practices/Mathematics**

<table>
<thead>
<tr>
<th></th>
<th>Never or almost</th>
<th>Somewhat</th>
<th>Mostly</th>
<th>Absolutely</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. Thinking about your teaching, how often do you do the following when teaching Mathematics as a subject?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Present a summary of recently learned content</td>
<td>○ ○ ○ ○</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) Explain what I expect the students to learn</td>
<td>○ ○ ○ ○</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) Explain how new and old topics are related</td>
<td>○ ○ ○ ○</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) Present tasks for which there is no unique solution</td>
<td>○ ○ ○ ○</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) Cover tasks that require students to think critically and justify answers</td>
<td>○ ○ ○ ○</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f) Allow students to work in small groups to come up with a joint solution to a problem or task</td>
<td>○ ○ ○ ○</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g) Ask students to decide on their own procedures for solving complex tasks</td>
<td>○ ○ ○ ○</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h) Refer to a problem from everyday life or work to demonstrate why new knowledge is useful</td>
<td>○ ○ ○ ○</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i) Have students projects that require at least one week to complete</td>
<td>○ ○ ○ ○</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>j) Let students use digital technology for projects or class work</td>
<td>○ ○ ○ ○</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Teaching Practices: Interdisciplinary Curriculum (STEM or STEAM etc.)

12. How often do you do the following when teaching an interdisciplinary curriculum such as STEM or STEAM etc.?

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Never or Almost Never</th>
<th>Occasionally</th>
<th>Frequently</th>
<th>Always</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Present a summary of recently learned content.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>b) Explain what I expect the students to learn.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>c) Explain how new and old topics are related.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>d) Present tasks for which there is no obvious solution.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>e) Give tasks that require students to think critically and justify answers.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>f) Allow students to work in small groups to come up with a joint solution to a problem or task.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>g) Allow students to decide on their own procedures for solving complex tasks.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>h) Bring in a problem from everyday life or work to demonstrate why new knowledge is useful.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>i) Give students projects that require at least one week to complete.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>j) Let students use digital technology for projects or class work.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>

### Instructional Activities

13. To what extent do instructional activities in Mathematics allow students to do the following?

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Never or Almost Never</th>
<th>Occasionally</th>
<th>Frequently</th>
<th>Always</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Develop problem-solving skills through investigations (e.g., scientific, design or theoretical investigations)?</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>b) Work in small groups?</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>c) Make predictions that can be tested?</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>d) Make careful observations or measurements or calculations?</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>
### Learning Outcomes (TRU Math)

To what extent do your instructional activities accomplish the following:

13. **Mathematics** (How do mathematical ideas develop in specific lessons?)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Never</th>
<th>Occasionally</th>
<th>Frequently</th>
<th>Always</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Mathematics ideas are developed appropriately when taught as an individual subject.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) Mathematics ideas are developed appropriately when taught within an interdisciplinary curriculum such as STEM or STEAM etc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) Mathematics lessons allow students to engage in mathematics ideas.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) Lessons taught within an interdisciplinary curriculum such as STEM or STEAM etc. allow students to engage in mathematics ideas.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16. **Cognitive Demand** (How do instructional activities provide opportunities for students to make sense of mathematics ideas?)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Never</th>
<th>Occasionally</th>
<th>Frequently</th>
<th>Always</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Students struggle productively with mathematics ideas when taught as a subject.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) Students struggle productively with mathematics and/or other ideas when taught within an interdisciplinary curriculum such as STEM or STEAM etc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) Students share their struggles with peers to understand and solve problems in mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) Students struggle productively with mathematics and/or other ideas when taught within an interdisciplinary curriculum such as STEM or STEAM etc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

17. **Use of Assessments** (What do you know about each student’s thinking and how do you build on it?)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Never</th>
<th>Occasionally</th>
<th>Frequently</th>
<th>Always</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Assessments in mathematics show how each student is following the lesson.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) Assessments within an interdisciplinary curriculum such as STEM or STEAM etc. show how each student is following the lesson.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) Assessments in mathematics allow students to choose their own methods to promote mastery of the objectives of the lessons.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) Assessments within an interdisciplinary curriculum such as STEM or STEAM etc. allow students to choose their own methods to promote mastery of the objectives of the lessons.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

18. What is your gender? Please write **N/A** if you prefer not to specify.

19. What is your racial and ethnic identification?

- [ ] Multiracial
- [ ] Black/African American
- [ ] Native Indian
- [ ] White/Caucasian
- [ ] Asian/Pacific-Islander
- [ ] Other
- [ ] Prefer not to say

This is the end of the survey! Thank you for your time and cooperation.

* STEM = Science, Technology, Engineering, and Mathematics; STEAM = Science, Technology, Engineering, Art, and Mathematics
Appendix G

Interview Protocol for Participants

Interview
The goal of this interview is to gain insight on the three dimensions of the TRU Math rubric as seen through each teacher’s set of instructional activities for the mathematics and STEM lessons.

Introduction:
1. PID_______________________
2. Describe your philosophy on teaching mathematics:
3. Describe your philosophy on teaching integrated subjects such as STEM, STEAM, etc:
4. Do you feel you have adequate preparation and resources to teach all of your current courses/subjects? Why/Why not? ______________________________

TRU Math Dimensions

1. Dimension # 1: The Mathematics
(Core question to explore: How do mathematical ideas from each lesson—mathematics and STEM—develop in specific set of lessons?)
   a) How do goals for the lessons covered in mathematics compare to the goals for the lessons covered within an interdisciplinary curriculum such as STEM, STEAM, etc.?
   b) How were the instructional activities capable of allowing students to engage with mathematical ideas in mathematics? Within an interdisciplinary curriculum such as STEM or STEAM etc.

2. Dimension # 2: Cognitive Demand
(Core question to explore: What opportunities do students have to make their own sense of math ideas?)
   a) In your experience teaching mathematics and an interdisciplinary curriculum such as STEM or STEAM, etc., how did your instructional activities allow students to have to make their own sense of important mathematical ideas within each type of lesson?
   b) Which instructional activities allowed for students to struggle with mathematical ideas?
   c) Were your students’ struggle productive? How much scaffolding did you have to give to your students in each type lesson?
   c) Did students share their struggle with others in the mathematics? Within an interdisciplinary curriculum such as STEM or STEAM etc.? If so, in what ways?
(Continued)

d) What kinds of instructional activities create opportunities for students to make their own sense of important mathematical ideas?
e) What resources were available for students to use when they encounter struggles?
f) Comment on the class norms around the value of the struggle and mistakes in each of your classes.

Dimension # 3: Uses of Assessment
(Core question to explore: What do we know about each student’s current mathematics thinking, and how can we build on it?)

d) In a typical assessment, how did you identify that a particular student is following the lesson in mathematics? Within an interdisciplinary curriculum such as STEM or STEAM etc.?
e) How did students share their mathematical ideas and reasoning in formative or summative assessments in mathematics? Within an interdisciplinary curriculum such as STEM or STEAM etc.?
a) In between assessments, what type of instructional activities did you use to promote the mastery of the learning outcomes in mathematics and STEM lessons?

Final Questions on teaching and learning:

f) What’s one change that you’d make in designing instructional activities for the next year in mathematics? Within an interdisciplinary curriculum such as STEM or STEAM etc.?
a) Did you learn anything new about your students through this reflection?
b) Did you learn anything new about your beliefs and practices about teaching through this reflection?
   Do you have anything else you’d like to share that may be relevant for this study?
<table>
<thead>
<tr>
<th>Score</th>
<th>Mathematics</th>
<th>Cognitive Demand</th>
<th>Use as Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Activity is aimed at &quot;getting an answer&quot; without addressing underlying reasoning.</td>
<td>Activity requires no more than applying formulas or memorized facts.</td>
<td>Assessment is limited to corrective feedback or encouragement, with limited to no indication of pursuing student reasoning.</td>
</tr>
<tr>
<td>2</td>
<td>Activity is at grade level, with few opportunities for students to make connections between procedures &amp; concepts or engage in mathematical practices. Teacher support is minimal and does not exploit them.</td>
<td><em>Activity offers possibilities of rich conceptual ideas or challenging word/story problems, but interventions from the teacher &quot;scaffold&quot; away the challenges, including productive struggle.</em></td>
<td><em>Teacher reflection shows evidence of where students discuss their thoughts on the problem and/or common mistakes; but students’ ideas are not built upon or developed. Teacher simply corrected student work without addressing challenges.</em></td>
</tr>
<tr>
<td>3</td>
<td><em>Activity shows intentional opportunity for mathematical connections, engagement, and practices. Teacher reflections show where students are encouraged to support their reasoning with a coherent and connected view of the mathematics.</em></td>
<td><em>The teacher’s reflection on the activity shows where a hint or scaffold supported students in “productive struggle” to build understanding and engage in mathematical practices.</em></td>
<td><em>Teacher reflection shows evidence of the solicitation of student thinking and subsequent discussions responding to those ideas, by building on the productive beginnings or addressing emerging misunderstandings.</em></td>
</tr>
</tbody>
</table>

*If the data is insufficient to determine this, the instructor may submit a reflection with additional information about the activity.*
Mathematics Activity 1: Vocabulary Homework: Understanding Definitions (with Example)

Directions:
In your class notebook, complete the following assignment.
For each of the following terms, write down on a piece of paper:

1. Antiderivative of a function
2. Differential equation

(a) The precise definition given in the book
(b) A sentence or two explaining what the definition means to you
(c) An example of something satisfying the definition
(d) An example of something not satisfying the definition (a “non-example”)

Instructor’s Example:
Here is an example of what I am looking for if the terms were "domain and range of a function".

DOMAIN:

- Precise definition: For a function $f$, which is a rule that assigns to each element $x$ in a set $D$ exactly one element, called $f(x)$ in a set $E$, then the domain of $f$ is the set $D$.

- My understanding: The domain is the set of all $x$-values that it makes sense to plug into $f$.

- Example that shows you understand the definition: Consider $f(x) = \sqrt{x - 2}$. The only values that can be plugged in for $x$ are those where $x - 2 \geq 0$. In other words, $D = [2, +\infty)$.

- Non-example that shows you understand the definition: Consider $f(x) = \sqrt{x - 2}$. The possible $y$-values are $y \geq 0$, but $[0, +\infty)$ is the RANGE of the function, not the DOMAIN!
RANGE:

- **Precise definition:** The range of \( f \) is the set of all possible values of \( f(x) \) as \( x \) varies throughout the domain.

- **My understanding:** The range is the set of all \( y \)-values that are output from \( f \) when considering all the inputs to \( f \) in its domain.

- **Example that shows you understand the definition:** Consider \( f(x) = \sin(x) \) on the domain \((-\infty, +\infty)\). This function can (and DOES) output every value from \(-1\) to \(1\) and nothing else. So its range is the interval \([-1, 1]\).

- **Non-example that shows you understand the definition:** Consider \( f(x) = \sin(x) \). The possible \( x \)-values you can plug into the function are \((-\infty, +\infty)\), but that is the DOMAIN of the function, not the RANGE!

**Mathematics Content:** This activity is primarily skills-based and focused on learning disciplinary terminology. There is an attempt to try to make sense by having students describe “what the definition means to them” and provide examples and non-examples. This is a required homework assignment that students must bring for discussion during the next time they meet for class. This “discussion” is more than likely a whole class activity, with opportunities for students to share with classmates. Based on his teaching style, Eric will have some students share, but this does not seem like an activity for which all students will receive individual feedback from him. The search for examples and non-examples presents a strong opportunity for students to engage with content and make sense of it, as they grapple supporting claims for and against examples of the terms. **SCORE: 3**

**Cognitive Demand:** Students are required to go beyond definitions and fact memorization by displaying their own understanding and providing examples. This is important and conceptually rich in solidifying ideas about terms. **SCORE: 2.5**

**Use as an assessment:** This homework is required for discussion during the next class. As such there is evidence to suggest that students will have a chance to discuss their thinking. However, due to the nature of this homework, all students may not have a chance to share thoughts that can be addressed within the time allotted. **SCORE: 2.5**
Mathematics Activity 2: Goblet Project: Overview and Tasks

Overview
Let’s create a goblet using calculus!

- You will work in a team of three people to design a goblet that satisfies desired criteria.
- At the end of the semester, we will print it out on the math department’s 3D printer and you will get to hold your goblet in your hands!

Goblet Project Standards

Grading: This project contributes to three of our class’s standards this semester. You will be given a score as follows:

- **Standard 23:** Project Management. I will assess the extent to which that you are able to produce the deliverables of this project, on time. I will determine if you are able to produce a well-written project summary. I will assess the extent to which you have worked together as a group by reading the group dynamics paragraph you submit.
- **Standard 24:** Computer Skills. I will assess the extent to which your submitted Mathematica file and STL file each satisfies the requirements laid out above.
- **Standard 25:** Disk/Washer Method. By reading the writeup, I will assess the extent to which you have set up the integrals and correctly calculated the integrals necessary for computing the volume of the goblet. I will assess the extent to which your goblet satisfies the desired volume and thickness requirements. You are expected to explain your ideas carefully in your writeup in order to show me that you understand the concept of the Disk/Washer Method.

Goblet Criteria

You will design a goblet that satisfies the following criteria:

- The goblet must be a solid of revolution created by revolving a piecewise-defined function.
- The goblet must hold 150–200 cm$^3$ (cubic centimeters) of liquid.
- The height of the goblet must be no more than 19 cm.
- The stem thickness must be at least 1 cm at its thinnest point.
- The diameter of the base of the goblet must be at least 1/3 of the height of the goblet, so that it does not fall over.

Specifications
The final product of this project will consist of:

- A two-page writeup about your goblet.
- A Mathematica file with a 3D visualization of your goblet.
- An STL-file that is a 3D model of your goblet.
- A short "group dynamics paragraph". Include your interpretation of how each group member contributed to this project and whether they fulfilled their part of your agreed upon expectations.

**Mathematics Content:** The activity is appropriately challenging and requires students to make connections between conceptual and procedural understanding. As is, whether individually or within a group, it provides a strong opportunity for students to make conceptual and procedural connections. **SCORE: 3**

**Cognitive Demand:** This project requires mathematics and computer skills. The goblet must “hold a certain amount of liquid, the height has to be no more than 'this', the stem thickness has to be at least 'that', along with other criteria”, which means that students are building upon their understandings and engaging in mathematical practices. Due to time constraints, Pruitt provides “a notebook that [they] can use so that they don’t get bogged down in the coding part of it and they can just focus on the math part of it.” It is unclear how vital the coding is to the course in that it is a pure math course. The integration of coding and mathematics makes for a conceptually rich task. **SCORE: 2.5**

**Use as an assessment:** Students are “expected to explain your ideas carefully in your writeup in order to show me that you understand the concept of the Disk/Washer Method" (Standard 25). This project is a strong use of an assessment for learning and extending ideas. **SCORE: 3**
Mathematics Activity 3: Goblet Project – Two-Page Writeup Requirements:

- Include your group members’ names, and a name for your goblet at the beginning.
- Discuss the piecewise-defined function: Write down the explicit piecewise-defined function. Write a paragraph explaining why it is a continuous function, and how you went about constructing it.
- Verify that your goblet satisfies the criteria listed above. You will need to explain in words why your goblet satisfies the criteria listed above and supplementing your writing with hand calculations or computer calculations as necessary.
- Be formatted in a clear and organized manner. The paragraphs will be written in correct English and be written in such a way that other students in Math 142 can understand your work. You should use course appropriate language such as "surface of revolution".

Mathematics Content: The task is designed for students to show a coherent and connect view of mathematics. During the interview, Pruitt explained that the goblet has “to be a solid of revolution created by using piece-wise defined functions.” Students must show the how and why for the mathematics involved. SCORE: 3

Cognitive Demand: This task, as is, provides a chance for showing their own understanding. The combination of explaining in words with calculations provides a good opportunity for students to grapple with what they actually know, but also solidify this understanding. Since this is a final product, I don’t know how much time is allotted for students build upon their understanding. SCORE: 2.5

Use as an assessment: This task is a rich use of a summative assessment in the area of soliciting student thinking. It is not designed to address misunderstandings or build upon ideas per se. SCORE: 3

TRU MATH MEAN SCORES for Mathematics Activities

Mathematics: 3
Cognitive Demand: 2.5
Use of Assessments: 2.8
Overview
Let’s create a goblet using calculus! You will work in a team of three people to design a goblet that satisfies desired criteria. At the end of the semester we will print it out on the math department’s 3D printer and you will get to hold your goblet in your hands!

Goblet Criteria
You will design a goblet that satisfies the following criteria:

- The goblet must be a solid of revolution created by revolving a piecewise-defined function.
- The goblet must hold 150–200 cm³ (cubic centimeters) of liquid.
- The height of the goblet must be no more than 19 cm.
- The stem thickness must be at least 1 cm at its thinnest point.
- The diameter of the base of the goblet must be at least 1/3 of the height of the goblet, so that it does not fall over.

Specifications
The final product of this project will consist of:

- A two-page writeup about your goblet.
- A Mathematica file with a 3D visualization of your goblet.
- An STL-file that is a 3D model of your goblet.
- A short “group dynamics paragraph”. Include your interpretation of how each group member contributed to this project and whether they fulfilled their part of your agreed upon expectations.

The two-page writeup will:

- **Include your group members' names, and a name for your goblet at the beginning.**
- **Discuss the piecewise-defined function.**
  Write down the explicit piecewise-defined function. Write a paragraph explaining why it is a continuous function, and how you went about constructing it.
- **Verify that your goblet satisfies the criteria listed above.**
  You will need to explain in words why your goblet satisfies the criteria listed above, and supplementing your writing with hand calculations or computer calculations as necessary.
- **Be formatted in a clear and organized manner.**
  The paragraphs will be written in correct English and be written in such a way that other students in Math 142 can understand your work. You should use course appropriate language such as 'surface of revolution'.
(Continued)

- **Give a plot of the bounding functions.**
  Show the 2D plots of the inside and outside of the functions.

- **Show the 3D visualization of your goblet.**
  Use the relevant functions to create a 3D model of your goblet.

- **Use the correct Mathematica Commands to export your goblet to an STL file.**

You may use [this Mathematica notebook](#) as a template to turn your group work done on paper into a 3D model.

**Timeline**

- **Form Project Groups by [Date]**
  By the end of class on Monday, November 13, you should have chosen your groupmates to form a group of three. Make sure to exchange phone numbers and determine times during the week when you are able to work on this project outside of class. Find Mathematica in a computer lab on campus or register for and download Mathematica to your home computer.

- **Prepare Project Ideas for [Date]**
  For this day, you and your classmates will work to have a firm understanding of the project requirements. Make sure to spend time outside of class researching the concepts that you will need for your project. Start a Mathematica notebook where you play around with some of these concepts. Bring it to class and I will brainstorm with you about your project.

- **Turn in Mathematica and STL files on [Date]**
  By this date, you are expected to have completed the virtual design of your goblet. You will be expected to turn in your final Mathematica file and your STL file so that we can 3D print your goblet by the following week.

- **Final Report and Presentation on [Date]**
  On this day, your two-page writeup is due, as is your group dynamics paragraph. Also, you will present your goblet to the class. We will vote on which one the class likes the best!

**Printing and Critiquing**

3D printing using Fused Deposition Modeling is a very slow process. For this reason, I will only be able to print out one copy of your goblet per group. Once the goblets are all printed, we will bring them to class and display them all for everyone to see. The class will vote on their favorite goblet.

**Grading**

This project contributes to three of our class’s [standards](#) this semester. You will be given a score as follows.

- **Standard 23: Project Management.** I will assess the extent to which that you are able to produce the deliverables of this project, on time. I will determine if you are able to produce a well-written project summary. I will assess the extent to which you have worked together as a group by reading the group dynamics paragraph you submit.

- **Standard 24: Computer Skills.** I will assess the extent to which your submitted Mathematica file and STL file each satisfies the requirements laid out above.
Calculus II Course Standards (Snippet)

As detailed on the syllabus, your assessment grade in this course will be determined by your proficiency on a variety of standards. (This is known as Standards Based Grading.)

Here is the list of standards that form the basis for this class, along with guiding questions that address each standard. You will be assessed on these standards throughout the semester.

What is different from the high-stakes "tests" that you might associate with a math class, there is an opportunity for you to re-assess your knowledge if you did not master the knowledge the first time around.

Mathematical Maturity

Standard M1. Mathematical Collaboration. (core) Have you met with classmates and completed the writing prompts that are assigned at regular intervals in the class?

Standard M2. Definitions. (core) Do you understand what a definition is? Are you able to write an explicit definition statement? Are you able to explain what the definition means in your own words? Are you able to give examples and non-examples of the definition?

Standard M3. Theorems. Do you understand what a hypothesis of a theorem is? Do you understand what a conclusion of a theorem is? For the Squeeze Theorem and the Intermediate Value Theorem: Are you able to determine the hypotheses and conclusions of a given theorem? Are you able to explain what the theorem means in your own words? Are you able to understand the consequences and non-sequences of a theorem?

Standard M4. Mean Value Theorem. Do you understand Rolle's Theorem and the Mean Value Theorem? Are you able to determine the hypotheses and conclusions of these theorems? Are you able to explain what the theorem means in your own words? Are you able to understand the consequences and non-sequences of the theorem? Can you apply the MVT?

Limits

Standard L1. Limit Basics. Do you understand the concept of a limit of a function? A one-sided limit? Are you able to approximate a limit from a given graph of a function? From a table of values that you compute?

Standard L2. Computing Limits. (core) Can you evaluate the limit of an expression at a point, using appropriate justification and notation?

Standard L3. Continuity. (core) Do you know the definition of when a function is continuous and understand it conceptually? Can you prove that a function is discontinuous at a point by using limits? Can you prove that a function is continuous by appealing to its properties or its construction?

Standard L4. Limits Involving Infinity. Can you compute the limit of a function at infinity? Can you determine when a limit is infinite? Can you use these techniques to determine the vertical and horizontal asymptotes of a function's graph?

Standard L5. Limit Definition of a Derivative. Can you write down the formula for the derivative as a limit of difference quotients in both ways? Do you understand where these formulas come from? Can you compute the derivative of a function using the limit definition when the function is a polynomial, root, or rational function?

Derivatives

Standard D1. Differentiability. Can you determine the tangent line to a curve using derivatives? Can you draw the graph of the derivative of a function given the graph of the function? Can you determine where a function is differentiable? Do you understand the consequences of Theorem 2.2.4?
Appendix J

TRU Math Analysis of Interdisciplinary Artifacts—Pruitt

Interdisciplinary Activity 1: Determining Function Transformations using Desmos

Before Class Tasks:
Complete the following activities to learn about function transformations and get practice working with them. In class you will be working on more complex examples so make sure you understand these basics well.

- Watch this video about applying transformations to parent functions. (7 min 51 sec) The transformations are applied to parabolas.
- Watch and follow along with this video I created about understanding simple transformations visually. (6 min 28 sec)
- Optional: If you would like some more examples of how transformations apply to a variety of parent functions, watch this video.
- Required: Complete this Desmos activity to get practice with finding the relationship between a function transformation and the corresponding changes to the graph. Feel free to work with another classmate.
- Go to our Campuswire community. Your task is to contribute to the class feed. Either ask a question about something we’ve discussed in class, something in the videos about transformations, or some part of the technology we’ve been using. OR, reply with an answer to someone else’s question. To ask a question, click on the Blue + sign at the top of the page, choose a relevant category, give a descriptive title, and explain your question.

In-Class Task: Transformation Examples Desmos Worksheet
[Students were given a hyperlink to Desmos with the function, \( f(x) \), below. Students can then click on the Example Functions to see what they would look like. The following excerpt is a screenshot of what a completed worksheet looks like. This task may correspond the required homework above. However, I did not have access to homework hyperlink.]
Mathematics Content: Students use the online graphing tool Desmos to complete this task during class. When you click on the link, they see the original f(x) and are provided with four transformed functions that they can click on and note the changes. Since they already had practice with this as a homework assignment, listed as a required before-class task, there is reason to believe that they were given explicit instructions then. However, when you go to “In-class” link for this assignment, there are no directions for completing the task. Assuming that students are continuing to “get practice with finding the relationship between a function transformation and the corresponding changes to the graph” this task goes beyond getting an answer. Students are certainly making connections, but there is not enough evidence to show if they are required to make a “coherent and connected view of the mathematics” beyond recognizing a transformation as a “shift” or “flip”. **SCORE: 2.5**

Cognitive Demand: This task goes beyond memorization and applying formulas. Due to my own experience teaching this concept, I can certainly see students describing “positional” changes, yet more evidence is needed to determine how useful this task was for them to begin to make the connections between changes in position and transformations. Assuming it continues from an activity in class, it offers a possibility for conceptual richness, but this task is not necessarily challenging, by design. **SCORE: 2**

Use as an assessment: This task is designed to provide students an opportunity to discuss their thinking about transformations, with a possibility for follow-up as a whole class discussion. The
task could lend itself to address misunderstandings, but this is not clear from the instructions.

**SCORE: 2.5**

(Continued)

**Interdisciplinary Activity 2: Take-Home Quiz on Standards 4 - 6**

**Instructions:** Write out solutions using complete sentences to explain your work. Use a separate piece of paper for each standard. You may use the internet as long as it does not solve the problem for you. Remember you are convincing me that you understand the concepts, not simply giving me “the answer”. I know the answer. When you are done, upload your work to Gradescope.

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**Standard 4.**

4-1. Draw the graphs of \( y = |x|, y = \sqrt{x}, \) and \( y = e^x \). What are some defining characteristics of these parent functions? (Some possible characteristics include properties of their shapes, their intercepts, asymptotes, domain, range, and anything else you think is relevant.)

4-2. To the right there are transformed versions of two parent functions. Determine the parent function for each of them and explain how you know you are correct.

4-3. What is the parent function for a parabola? Explain your reasoning.

---

**Standard 5.**

5-1. When the parent function \( f(x) = x^2 \) is reflected across the x-axis, compressed horizontally by a factor of 3, and translated up by four units, what is the equation of the corresponding graph? Explain your work.

5-2. In the second graph on the right (the red curve from Question 4-2), only reflections and translations have been applied. What is the equation of this graph? Justify your answer.

5-3. Consider the equation \( y = \frac{1}{4}|(x - 2)| - 3 \). Identify out the parent function, the transformations being applied to the parent function, and use the transformations to carefully draw the curve of its graph on coordinate axes. Explain your work.
Standard 4. Parent Function Recognition: For linear, parabolic, cubic, square root, and exponential functions—Given the curve of a function, can you determine which type of function it represents? Given the equation of a function, can you determine what shape its curve will have?

Standard 5. Transformations: Can you determine the sequence of transformations that transforms an initial curve into a final curve? Can you determine the equation of the final curve? If you are given an equation of a composition of transformations, can you determine its graph by applying a sequence of transformations from the initial curve?

Standard 6. Trigonometric Functions: Can you identify the graphs of the sine, cosine, and tangent functions? Can you draw the graphs of these functions? Can you apply the basic transformations to these functions?

Mathematics Content: The tasks in this quiz require students to make connections and show a coherent understanding of concepts. Students must be engaged with the material prior to and on this assignment in order for them to make a solid attempt at completing it. This is “real” mathematics in a course with a main objective of creating art. **SCORE: 3**

Cognitive Demand: Mathematical reasoning and making connections are imbedded within each standard assessed. Student support is scaffolded within directions, by being clear about what is a “characteristic” of a parent function, as in section 4-1. The task is designed for students to actively engage with the mathematics. **SCORE: 3**

Use as an assessment: Students’ thoughts are actively solicited. They must provide reasoning and justification for all of their answers. Since we have examples that led up to
this assessment, there is evidence to suggest that the assignment builds upon understandings that developed or began to develop and previous activities. **SCORE: 3**

(Continued)

**Interdisciplinary Activity 3: Project 1 – Functions and their Families**

*Overview:* The first project is to create an image based on a family of related functions.

**Specifications**

**The final product of this project will consist of:**
- One exported SVG file.
- A Desmos notebook.
- A one-page writeup.

**The exported SVG must:**
- Be created by exporting a digital design from your Desmos notebook.
- Not include grid lines or coordinate axes.
- Involve a color palette with color(s) different from the standard Desmos colors.

**The Desmos notebook must:**
- Include the plot of one or more functions.
- Apply mathematical transformations to each function, involving one or more parameters.
- Use lists to specify the values of each parameter.

**The one-page writeup must:**
- Provide key details about your artwork, including the choices you made.
- Convey the mathematics behind the drawing and discuss how you stretched your knowledge.
- Using full sentences, proper English, and flow well.
- Use 1-inch margins, 1.5x spacing, and 11-point Times New Roman font.
- Include a cover page with the title of your artwork, your name, and the date.

**Grading:**

This project represents 10% of your semester grade. You will be graded on each of the following standards.

**Timeliness:**
- Did you make steady progress on your project from start to finish, respecting project deadlines?
- Did you regularly attend the in-class work days, discuss your progress with classmates, and check in with the professor?
- Did you turn in your final project by the deadline?

**Desmos Notebook and Intentionality:**
- Did you create a plot of one or more functions?
- Did you use transformations of functions to create your artwork?
- Did you use the Desmos skills from class?
- Is the Desmos notebook well organized, with related parts grouped together and presented in a logical order?
Discussion of Mathematical and Functional Techniques:
- Have you explained how you arrived at the function(s) that forms the basis for your work?
- Have you explained how you determined which transformations you applied to your functions?
- Have you explained the programming methods in Desmos that you applied?

Discussion of Artistic Qualities:
- Have you explained the artistic qualities you were going for in this piece?
- Have you explained the relationship between the artistic qualities and the mathematics you applied?

Discussion of Process and Revisions:
- Have you explained your artistic process, including a discussion of how your piece changed over time?
- Have you explained how the peer review process impacted your final piece?

Writing style and format:
- Does your artwork have a title?
- Did you use full sentences, use proper English, and do your paragraphs flow well?
- Did you follow the writing format requirements?

Mathematics Content: Students are creating art based on their understanding of functions and their transformations. Students MUST show how the mathematics learned thus far, form the basis of their design. It is designed to make explicit, deliberate connections among the disciplines. **SCORE: 3**

Cognitive Demand: Students can engage in mathematical and artistic practices. Students can also build upon mathematics learned by applying it, on principle to another discipline. This task presents a rich opportunity for productive struggle. **SCORE: 3**

Use as an assessment: Students must explain their artistic process AND they must also explain how they arrived at the functions and subsequent transformation The following is an excerpt from a student’s reflection on his design for project 1, as well as a picture of the final product for Project.

...my first time touching Desmos, I didn't have a lot of understanding on the concepts that we will soon cover in class. So, I [tried] to be creative with the simple things that I already know. [By the time] I submitted my first project, I had a little bit more tools to use. Before that, I had no idea that I could use sine and cosine instead of [just] Desmos [to create an image]. So I was also able to “color in my work using the greater than and less than function. There is another part to this art that is hidden that you can’t really see because his head [in the picture] was a beautiful shape that if you manipulated it with a slider, it created a beautiful pattern. But I felt like it wasn't enough to be able to hand it in, so I just kept building upon it. **SCORE: 3**

TRU MATH MEAN SCORES for Interdisciplinary Activities:

**Mathematics: 2.8**

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Mathematical Design, Standards

As detailed on the syllabus, your assessment grade in this course will be determined by your proficiency on a variety of standards. (This is known as Standards Based Grading.)

Here is the list of standards that form the basis for this class, along with guiding questions that address each standard. You will be assessed on these standards throughout the semester.

What is different from the high-stakes "tests" that you might associate with a math class, there is an opportunity for you to re-assess your knowledge if you did not master the knowledge the first time around.

The Standards

Standard 1. Mathematical Collaboration. Have you met with classmates and completed the writing prompts that are assigned at regular intervals in the class?

Standard 2. Functions. Can you give the definition of a function? Can you determine if a given rule, table, or curve represents a function?

Standard 3. Parent Function Recognition. For linear, parabolic, cubic, square root, and exponential functions: Given the curve of a function, can you determine which type of function it represents? Given the equation of function, can you determine what shape its curve will have?

Standard 4. Transformations. Can you determine the sequence of transformations that transforms an initial curve into a final curve? Can you determine the equation of the final curve? If you are given an equation of a composition of transformations, can you determine its graph by applying a sequence of transformations from the initial curve?

Standard 5. The Unit Circle. Can you convert between radians and degrees? Can you find specified angles on the unit circle? Can you determine the x- and y-coordinates of a point on the unit circle? Do you understand how the unit circle is related to the sine curve and the cosine curve?

Standard 6. Basic: Parametric Functions. Can you convert a function of the form \( y = f(x) \) or \( x = g(y) \) into a basic parametric function of the form \( (p(t), q(t)) \)? Can you draw the graph of a basic parametric function?

Standard 7. Transformations of Parametric Functions. Can you apply reflections, translations, and dilations? How does the graph of \( (a \cdot p(t)+h,c \cdot q(t)+k) \) compare to the graph of \( (p(t), q(t)) \)?

This list will continue to grow over the semester. Check back later.
As detailed on the syllabus, your assessment grade in this course will be determined by your proficiency on a variety of standards. (This is known as Standards Based Grading.)

Here is the list of standards that form the basis for this class, along with guiding questions that address each standard. You will be assessed on these standards throughout the semester.

What is different from the high-stakes "tests" that you might associate with a math class, there is an opportunity for you to re-assess your knowledge if you did not master the knowledge the first time around.

The Standards

**Standard 1. Mathematical Collaboration.** Have you met with classmates and completed the writing prompts that are assigned at regular intervals in the class?

**Standard 2. Functions.** Can you give the definition of a function? Can you determine if a given rule, table, or curve represents a function?

**Standard 3. Parent Function Recognition.** For linear, parabolic, cubic, square root, and exponential functions: Given the curve of a function, can you determine what type of function it represents? Given the equation of a function, can you determine what shape its curve will have?

**Standard 4. Transformations.** Can you determine the sequence of transformations that transforms an initial curve into a final curve? Can you determine the equation of the final curve? If you are given an equation of a composition of transformations, can you determine its graph by applying a sequence of transformations from the initial curve?

**Standard 5. The Unit Circle.** Can you convert between radians and degrees? Can you find specified angles on the unit circle? Can you determine the x- and y-coordinates of a point on the unit circle? Do you understand how the unit circle is related to the sine curve and the cosine curve?

**Standard 6. Basic Parametric Functions.** Can you convert a function of the form $y=f(x)$ or $x=g(y)$ into a basic parametric function of the form $(p(t), q(t))$? Can you draw the graph of a basic parametric function?

**Standard 7. Transformations of Parametric Functions.** Can you apply reflections, translations, and dilations? How does the graph of $(a\cdot p(b\cdot t+h)+c, q(ct+k))$ compare to the graph of $(p(t), q(t))$?

This list will continue to grow over the semester. Check back later.
Appendix K

TRU Math Analysis of Mathematics Artifacts—Selma.

Note: These “essays” are mathematics exemplars for students needing help in passing a regional state exam. Due to interruptions in their education from their home countries, some students enter 11th and 12th grade and needing help passing 9th and 10th grade Algebra requirements. As a result, Selma gives them skills they need to pass this exam, but allowing students to explore the concepts they need help with the most. Each activity features 2 – 3 student examples to showcase how students of varying support levels completed this assignment. She admitted that that course is “traditional,” though not going into complete detail as to what this infers. She describes that process as follows:

[Students] engage in some problems that were related optimization and it was mostly like algebra 1 concepts. You know, like looking at intersections of linear equations and finding out there are optimal points [is] exciting. And so, they did some activities here, then they created their own problems and I gave them different products in their groups that they had to use to design [another] problem...as a group of heterogeneous students [in terms of ability]. They [then] solve that problem, and then they have to create their own individual problem, but they [do so] in homogeneous groups. And so, this is where [they] split up and then the students that needed...high level support, [worked on.... The students that needed medium levels of support, they would review and revise the problems that they had done earlier and some of the work and the problems that they created in their heterogeneous groups and homogeneous groups. [Finally], students needing minimal to no support, [worked independently].

Students also created a presentation to explain what they understood about matrices and how to solve them. I chose three examples optimization from the various support levels to examine using the TRU Math Dimension.

Mathematics Activity 1: Optimization: Linear Programming in Two or Three Variables
Overview: Use linear programming to find optimal solutions to problems, where limited resources are of concern.

This first student needed high levels of support because due to his history with Algebra. As such, Selma, provided additional structure for his project and essay with a template. He wanted to “create [a] business in order so that [he can] buy a pair shoes.” Selma admits that
even with the conceptual challenges, this student produced a good essay for his ability level. She reflects on the process and how it connects to her struggles as an educator:

And then we asked him, like does it make sense, did you actually profit? And a lot of students realize that they had to think about what is profit really...and a lot of those concepts started coming into play. But even with this...the struggle that I have is, [that] it wasn’t a real story. It wasn’t a real product; it was a make-believe product.... I don’t know how much he’ll be able to transfer over the skills that he acquired here in other places. That’s something I struggle with when I’m doing project-based [activities].

**Mathematics Content:** This activity goes provides students with flexibility to understand linear programming by coming with their own problem. Student are trying to make conceptual connections at an ability level most comfortable for them. Students experienced challenges, as reflected in their essays. Some were conceptual in determining constraints and “limiting functions”. Selma’s independent student use this project to explore additional ways to solve linear equations in three variables using and Gaussian elimination and row echelon form. This activity is engaging and encourages students to make sense of this topic. Their struggles coincide with their ability levels, as expressed in their reflections at the end of the assignment. **SCORE: 2.5**

**Cognitive Demand:** This activity is conceptually challenging. Students are grappling with this idea and their struggles are imbedded in the assignment. While Selma admits that students needing high support were provided with a template, she did not “lower her expectations” since they were still required to “put it into words and explain it.” She further explained this in her interview:

This is an example of a student who struggles and had interrupted education [referring to the student needing a high level of support]. And so...I gave them a template that they filled in and they had to choose which products—very straightforward. I gave them this box too, and I said put your inequalities here, your constraints, and explain what they are, and then they would delete my questions, and they would leave the boxes with their responses...But it’s really their work. Students were also challenged to revise and resubmit upon feedback. **SCORE: 2.5**

**Use as an assessment:** This activity certainly shows where Selma is trying to meet students at their point of academic need and at the same time, challenge them to go further. Students’ thoughts are solicited, supported, and responded to. This activity works well as a formative assessment. **SCORE: 3**
Optimization Student 1 Snippet

Background Information

I want to create a business to make money in order for me to buy some shoes, the pair of shoes that I want to buy is Jordans 11 retro, which cost $298. The products that I’m planning to sell in my business is a box of Fruit Snacks, a box of Doritos & Cheetos Mix, and Rice krispies treats.

The reason that I want to sell these snacks in school is that they are healthy products and kids in school usually don’t like school lunch and are hungry, so they would be willing to buy snacks. I plan to use optimization in order to create my business, which I would start by becoming a person that mostly everyone knows that sells in school and that is a nice person. Also I would give a discount for some people which would make them come back to buy from me again. It would be also posted in social media that I would be selling specific days and it would give my friends a heads up for them to bring money, and they would also pass the word to others.
(Continued)

Optimization Student 2 Snippet

Optimization Essay

Proposing Solutions with Limited Resources

Essential Question: How can we use linear programming to find optimal solutions to problems while having limited resources?

Introduction:

The world is filled with mathematics in many aspects, but which of them do we see the most in our daily life? The answer is applications of linear programming. People actually use linear programming everywhere to achieve their expectations of finding best solutions and solving problems based on their ideas of maximization or minimization. Linear programming is the optimization of an outcome based on some set of constraints$^1$ using a linear mathematical model. As we know, Linear programming is used for obtaining the most optimal solution$^2$ for a problem with given constraints. In linear programming, we formulate our real life problem into a mathematical model. It involves an objective function$^3$, linear inequalities with subject to constraints. For example, when a company is creating a business like manufacturing sugar and seek for the maximization of profit made with certain constraints of time, budget and labor. This is one type of linear programming application, and it can only be reached when every constraint is satisfied. Based on the linear programming, people will come up with an optimization problem first for the optimal solution that they try to obtain. Optimization is the process of finding the

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$^1$ The restrictions or limitations on the decision variables. They usually limit the value of the decision variables.

$^2$ The best solution for the optimization problem that matches with objective function.

$^3$ It is defined as the objective of making decisions and use to find the final solution.
Mathematics Activity 2: Geometry: Tessellations in the Real World

Overview: Students used this final essay to research tessellations in the real world and create their own using Geometer’s Sketchpad.

Mathematics Content: In this activity, students try to demonstrate their understanding of the geometry of tessellations and an appreciation for it in nature and applications. They used Geometer's Sketchpad to create tessellations and reflect on transformations of polygons on this process. They try to highlight disciplinary skills in geometry to reason and make connections in their understandings. **SCORE: 2.5**

Cognitive Demand: These essays show students really trying to make sense of the geometry behind tessellations. This activity is a deliberate attempt to build on understanding and engage in mathematical practices. Students are explaining their ideas. The essays, as written, seem to suggest students took ownership for these ideas. **SCORE: 2.5**

Use as an assessment: These essays show that students ideas are solicited in this activity. I do not have sufficient information as to how they were developed. However this is a good use of a summative assessment. **SCORE: 2.5**
Geometry and Tessellations in the Real World

"I think the universe is pure geometry - basically, a beautiful shape twisting around and
dancing over space-time" said by Antony Garrett Liss, an American theoretical physicist famous for
his works on the "E_4 Theory". The quote means that Geometry can be found everywhere in the
universe, may it be on land, in water, on different planets or in any other part of the universe.

Geometry is a branch of Math. It relates with shapes, size, points, lines and planes. So, in this
Geometric project, the main focus is tessellation. Tessellation is a work of art, that consists of one or
more shapes which repeat over and over again creating a pattern and covering a plane without leaving
any gaps or overlapping any other figure. Tiling is another word for tessellation. The purpose of this
project is to understand how does Geometry exist in nature and how can we recreate them using the
different tools of Geometry. This knowledge is essential because we may see Geometry in our
day-to-day life, but we never may have realized that it exists. For example, when you look at the floor
and how the tiles are arranged do you ever think of Geometry? Most people would respond a “No!” as
an answer. But for a fact Geometry plays a big role in determining how to arrange those tiles. These
arranged tiles then create patterns, and hence a tiling, i.e., a tessellation. To understand how are these
tessellations created by the tools of Geometry, certain tasks are required to be performed. Firstly, we
must create an original tessellation using the modern technology called Geometer’s Sketchpad. This
tool helps us create tessellations on the laptop, by applying translation, rotation, reflection, midpoint
and many other such options. Next, we must research and find out a real life tessellation that exists in
the nature, like the honey bee’s honeycomb. Lastly, for understanding this study better, we make
connections with the tessellation that we created and the ones in the nature.

Before commencing the project, there are certain components that one must know about
tessellations. A tessellation can be valid if and only if it satisfies the five conditions: made up of a
(Continued)

GBAT Student Snippet

Geometry PBAT

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**Geometry and Tessellations in the Real World**

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Before commencing the project, there are certain components that one must know about tessellations. A tessellation can be valid if and only if it satisfies the five conditions: made up of a
Mathematics Activity 3: Statistics: Linear Regression and Social Justice

Overview: Students choose a particular phenomenon of a social justice issue that they are passionate about, to examine using regression, in order to make predictions.

Mathematics Content: The activity provides some opportunity to make sense of linear regression by tying it to an issue that students care about. Students are using descriptive statistics. More evidence is needed to show what standards are used in helping students show their understanding of linear regression itself, within statistics as a discipline. **SCORE: 2**

Cognitive Demand: Students are trying to make sense of regression conceptually. They dedicate a portion of the essay to try to explain what variables mean within the context of their particular issues. These attempts show signs of productive struggle, but more evidence is needed to determine how they were supported. **SCORE: 2**

Use as an assessment: Students are studying linear regression within a context relevant to them. The essay shows that they were required to “show” their thinking, but we did not discuss how these ideas were developed. **SCORE: 2**

TRU MATH MEAN SCORES for Mathematics Activities

- Mathematics: 2.3
- Cognitive Demand: 2.3
- Use of Assessments: 2.7
(Continued)

PBAT Student 1 Snippet

Topic: Racial Discrimination in Prison Populations

12th Grade Statistics PBAT

Social justice could be when you get arrested for something but you already know that you did not do anything wrong and you want justice. Another example of a social justice issue that occurred in the past can be the Supreme Court case of Loving v. Virginia. This court case is about a black women and white man who fell in love and married. They lived in Virginia and in this state and this time it was illegal for them to be together since they were of different races. They were arrested but after a time they decided to get justice and they took this case to the Supreme Court in the end they got social justice. We still see these kinds of problems in our society. Racism and religious oppression still exist today, although it may look like it doesn’t. For this project, I focus on youth in U.S prisons. One of the questions that comes to my mind about this topic is: What are the racial inequities that are in the prison system? Racism could be one problem because as in the United States most of the people judge a person by their skin color and their race.

Sub Question 1: Why are young people going into prison?
(Youth -18)

I found this information of the prison policy initiative that shows the reasons why young people are going to jail in the U.S. It is qualitative data because it is counting and has one variable.

<table>
<thead>
<tr>
<th>Young People who are locked up</th>
<th>Crimes</th>
<th>Number of Prisoners</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

206
Drug (Drug Traffic, Other Drug offenses) | 1,900
---|---
Personal Crime | 13,600
Property | 8,100
Public Order | 3,700
Technical Violation | 6,600

This data is related to my research topic because the table shows us the reasons why youth are in prison and the number of youth in prisoners. The bar graph is a good option to make the graph because the data have just one variable. Also it is going to be easy to understand the data shown in this way because each bar in the graph has a reason why youth were put in prison and the bar that is more high, it is the personal crime with more prisoners convicted of it.

A personal crime is when someone does a robbery, sexual assault, aggravated assault, simple assault or other kind of crime. Drug crimes are those crimes that are involved in selling or trafficking illegal substances. Property crimes are those crimes that relate to the theft or destruction of property of another person. Public orders are those kinds of crimes like prostitution, disorderly conduct, public drunkeness, or other alcohol-related crimes. A technical violation is when the police is watching your behavior and if you do something wrong, you get arrested. The people that are watching is because they are suspects for the police.

**Youth Incarceration that are locked for nonviolent offenses**

The graph above shows us what are the reasons why the young people are in prison. For example, something that we can see is that the personal crime has more prisoners than other crimes. The drug crimes show a smaller amount of prisoners than some of the other crimes.
(Continued)

PBAT Student 2 Snippet

We all experience some form of stress some time in our life. What stresses you out?
What do you think stresses out teenagers? My research question is: “How do [REDACTED] students have different perceived level of stress depending on their age?” According to the National Alliance on Mental Illness, “1 in 5 children ages 13 - 18 have or will have a mental illness.” (National Alliance on Mental Health) The problem is that teachers are seeing more and more students who have mental health problems and this affects their work and behavior in school.

Sub Question 1: What are the reasons why students have stress?

According to afterschoolapp.com, stress is a very important part of life and that teen should know about their mental health. This website is the largest teen-only social network in the United States. This social network is dedicated to share the thoughts, opinions, and beliefs of the teens. Thanks to the information collected, American teenagers have a collective voice. More that 35,000 teens share their opinions about the topic of suicide. Suicide is what happens when some teens cannot handle their stress so this is a very important topic to understand. (Afterschool.com) This data (below) represent what things stress teenagers the most.

Table 1: Summary of Information. Responses about what makes students have stress

<table>
<thead>
<tr>
<th>Response</th>
<th>Number of Students</th>
<th>Total Number of Students Surveyed</th>
<th>Fraction</th>
<th>Percent %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relationships</td>
<td>9,494</td>
<td>34,881</td>
<td>0.26494</td>
<td>27.22%</td>
</tr>
<tr>
<td>Teachers</td>
<td>8,562</td>
<td>34,881</td>
<td>0.24352</td>
<td>24.55%</td>
</tr>
<tr>
<td>Other issues</td>
<td>7,358</td>
<td>34,881</td>
<td>0.21090</td>
<td>21.09%</td>
</tr>
<tr>
<td>Parents</td>
<td>4,695</td>
<td>34,881</td>
<td>0.13460</td>
<td>13.46%</td>
</tr>
<tr>
<td>College</td>
<td>3,302</td>
<td>34,881</td>
<td>0.09474</td>
<td>9.47%</td>
</tr>
</tbody>
</table>
This data above represent what things stress teenagers the most. This table represents the consequences that make teenagers get stress, the number of people and the percent that each consequence makes stress. Using a pie graph we can see the results of the percentage of what the 34,881 students answered about what makes them feel stress in the survey that afterschool.com made.

<table>
<thead>
<tr>
<th></th>
<th>1,470</th>
<th>34,811</th>
<th>4,470</th>
<th>4,21%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>34,881</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td></td>
<td>100%</td>
</tr>
</tbody>
</table>

### The Things that Make Teenagers get Stressed

![Pie chart showing stressors]

This pie chart shows that some students choose different responses. Depending on the responses of the student, we make a pie chart to show what percent of the 35,000 students felt stress in each of the areas. In this pie graph, we can see the things that stress students.
Appendix L

TRU Math Analysis of Interdisciplinary Artifacts—Selma

**Interdisciplinary Activity 1: Data Graphic Illustrations Project (Graphic Illustrations—Student Examples; Police Brutality Data—Teacher Example)**

*Overview:* This is a “data graphic project” where students identified social issues that were relevant to their lives and analyzed the data to create illustrations.

*Notes:* Students began by “creating their own questions”. Teachers wanted them to “collect data from each other but also understand what different kinds of data look like”. The graph also needed to represent this data accurately. In Selma’s interview, she explained some of the conceptual and logistical challenges. This included pairing students with “interrupted education” with those who were “ready for college”, among other issues. She explains:

This year I didn't get to cover college statistics, it was too much to do at once. But it is something that if I were to redo this [project], I would bring in all those different levels of Math that students can reach, because we do have both ends [of abilities]…. But me and my coworker created our own data graphic to do what the students were expected to do, to really figure out, what is it that they need to be able to do at the end…. At one point, [my co-teacher] felt like we were focusing a lot on math and she was like, “You know, I think we should get students to create sketches now”. And I’m like “they have to learn the math, though, before they can create the sketches.” *And then she was really hurt by that.* And she was like, “you don't get it. For some students, maybe, creating the sketch will help them with the math” and I didn't get it.

Selma and her co-teacher completed an example to help “move the process along”. When students looked at data, they lacked the tools to know where to begin (and what) to analyze it. Others worked in pairs, but misinterpreted what graphs meant. Eventually, Selma had to supplement classroom support with online videos, with some students going even further to seek out their own.

**Mathematics Content:** This activity is more than just “getting an answer”. Students are using data to illustrate real world issue relevant to them; however, there is tremendous support needed to make the necessary conceptual mathematical connections. The interview provided more context as to why this project has great potential for students to engage in mathematical reasoning. However, it also revealed why students struggled with disciplinary statistical skills required for analysis. **SCORE: 2**

**Cognitive Demand:** There are high opportunities for students to engage in conceptual ideas of statistics. Both the teachers and students gained insight in connecting the disciplines of graphics and statistics. Students had time to build upon their understandings and grapple with how data should be represented, with adequate teacher support. **SCORE: 2.5**

**Use as an assessment:** Teachers solicited students reasoning on connecting data with graphs and addressed misunderstanding of interpretation and analysis. When necessary, students had time to pursue mathematics lacking through conversations with Selma and/or videos. Student reasoning surfaces and is actively pursued. **SCORE: 3**
(Continued)
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer Choices</th>
<th>Type of Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How old are you?</td>
<td>Interval / Ratio</td>
<td></td>
</tr>
<tr>
<td>2. What is your race?</td>
<td>White, Black, Native American, Asian, Hawaiian, Pacific Islander, Mixed Race</td>
<td>Nominal</td>
</tr>
<tr>
<td>3. Are you of Latino or Hispanic heritage?</td>
<td>Yes, No</td>
<td>Nominal</td>
</tr>
<tr>
<td>4. What is your native country?</td>
<td></td>
<td>Nominal</td>
</tr>
<tr>
<td>5. Approximately how many years have you been in the U.S.?</td>
<td></td>
<td>Interval / Ratio</td>
</tr>
<tr>
<td>6. Which borough do you live in?</td>
<td>Queens, Brooklyn, Bronx, Manhattan, Staten Island</td>
<td>Nominal</td>
</tr>
<tr>
<td>7. Do you work?</td>
<td>Yes, No</td>
<td>Nominal</td>
</tr>
<tr>
<td>8. How many hours a week do you work?</td>
<td></td>
<td>Interval / Ratio</td>
</tr>
<tr>
<td>9. How many siblings do you have?</td>
<td></td>
<td>Interval / Ratio</td>
</tr>
<tr>
<td>10. Do you take care of your siblings?</td>
<td>Yes, No</td>
<td>Nominal</td>
</tr>
<tr>
<td>11. How many people live in your household?</td>
<td></td>
<td>Interval / Ratio</td>
</tr>
<tr>
<td>12. How many people work in your household (including you)?</td>
<td></td>
<td>Interval / Ratio</td>
</tr>
</tbody>
</table>

**Biographical Questions**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer Choices</th>
<th>Type of Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. How often should Police arrest people with brutality?</td>
<td>All the time, Almost Always, Occasionally, Never</td>
<td>Nominal</td>
</tr>
<tr>
<td>6. Do you think that there is more Physical Violence towards Black People?</td>
<td>Sometimes, Always, Most of the time, Rarely, None of the time</td>
<td>Ordinal</td>
</tr>
<tr>
<td>7. From the scale of 1-10 how nervous do you get when you see a police officer?</td>
<td></td>
<td>Ordinal</td>
</tr>
<tr>
<td>8. If you had seen some people getting shot by police, how hard is it for you to get out of the house?</td>
<td>Very hard, Hard, Little hard, Not hard, Have not seen any</td>
<td>Ordinal</td>
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<tr>
<td>9. How do you feel when they’re around the police?</td>
<td>Safe, Feel safe, Don’t care</td>
<td>Ordinal</td>
</tr>
<tr>
<td>10. Do you feel overwatched seeing the news of black lives being killed by police officers?</td>
<td>All of the time, Most of the time, Some of the time, None of the time</td>
<td>Ordinal</td>
</tr>
<tr>
<td>11. How many times have you been touched physically by the police officer?</td>
<td></td>
<td>Ordinal / Ratio</td>
</tr>
</tbody>
</table>

**Social Issue Questions**

<table>
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<tr>
<th>Question</th>
<th>Answer Choices</th>
<th>Type of Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. How many times have you run away from a police officer?</td>
<td>A) 9-10, B) 6-8, C) 0-5</td>
<td>Interval / Ratio</td>
</tr>
<tr>
<td>13. How many police officers do you get to see around your neighborhood, per day?</td>
<td>Short Answer</td>
<td>Interval / Ratio</td>
</tr>
<tr>
<td>14. How satisfied are you with the service of the New York Police Department?</td>
<td>A) Very satisfied, B) Somewhat satisfied, C) Not satisfied</td>
<td>Interval / Ratio</td>
</tr>
<tr>
<td>15. How many blacks away is the police station from your house?</td>
<td>Short Answer</td>
<td>Interval / Ratio</td>
</tr>
</tbody>
</table>

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</tbody>
</table>
(Continued)

<table>
<thead>
<tr>
<th>Race</th>
<th>People Shot to death in U.S. by Police 2020</th>
<th>Rate</th>
<th>odds likelihood ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>60%</td>
<td>40%</td>
<td>-20.1 - 1.3</td>
</tr>
<tr>
<td>Black</td>
<td>13.4%</td>
<td>21%</td>
<td>7.8 - 1.3</td>
</tr>
<tr>
<td>LatinX</td>
<td>18.5%</td>
<td>13%</td>
<td>-5.5 - 1.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Race</th>
<th>2017</th>
<th>2018</th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
<th>2022</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>267</td>
<td>208</td>
<td>199</td>
<td>211</td>
<td>202</td>
<td>207</td>
</tr>
<tr>
<td>Black</td>
<td>223</td>
<td>223</td>
<td>235</td>
<td>244</td>
<td>242</td>
<td>247</td>
</tr>
<tr>
<td>Hispanic</td>
<td>178</td>
<td>145</td>
<td>126</td>
<td>127</td>
<td>36</td>
<td>46</td>
</tr>
<tr>
<td>Other</td>
<td>44</td>
<td>45</td>
<td>41</td>
<td>44</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>Unknowns</td>
<td>84</td>
<td>204</td>
<td>202</td>
<td>36</td>
<td>179</td>
<td>721</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Race and Hispanic Origin</th>
<th>Likelihood of Police Killing</th>
</tr>
</thead>
<tbody>
<tr>
<td>White done (not Hispanic or Lat)</td>
<td>80.1%</td>
</tr>
<tr>
<td>Black or African American -</td>
<td>12.4%</td>
</tr>
<tr>
<td>Hispanic or Latino</td>
<td>16.5%</td>
</tr>
<tr>
<td>American Indian and Alaska Native</td>
<td>3.5%</td>
</tr>
<tr>
<td>Asian done</td>
<td>5.9%</td>
</tr>
<tr>
<td>Native Hawaiian and Other Pacific</td>
<td>0.2%</td>
</tr>
<tr>
<td>Two or More Races</td>
<td>2.8%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Race/Ethnicity</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>0.67</td>
</tr>
<tr>
<td>Black</td>
<td>1.64</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Race</th>
<th>U.S. Population (U.S. Census 2010 - 2019)</th>
<th>People Shot to death in U.S. by Police 2020</th>
<th>Rate</th>
<th>odds likelihood ratios</th>
</tr>
</thead>
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<tr>
<td>Black</td>
<td>13.4%</td>
<td>21%</td>
<td>7.8</td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>60%</td>
<td>40%</td>
<td>-20.1</td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Race</th>
<th>2017</th>
<th>2018</th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
<th>2022</th>
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<tr>
<td>White</td>
<td>267</td>
<td>208</td>
<td>199</td>
<td>211</td>
<td>202</td>
<td>207</td>
</tr>
<tr>
<td>Black</td>
<td>223</td>
<td>223</td>
<td>235</td>
<td>244</td>
<td>242</td>
<td>247</td>
</tr>
<tr>
<td>Hispanic</td>
<td>178</td>
<td>145</td>
<td>126</td>
<td>127</td>
<td>36</td>
<td>46</td>
</tr>
<tr>
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<td>45</td>
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<tbody>
<tr>
<td>White</td>
<td>0.67</td>
</tr>
<tr>
<td>Black</td>
<td>1.64</td>
</tr>
</tbody>
</table>
Interdisciplinary Activity 2: Understanding Vocabulary for Computer Science and Scratch-Homework

Overview: In order to gain a deeper understanding of terminology in computer science and scratch, teachers designed a digital activity for students to develop a conceptual understanding. This assignment here is a pdf version of the actual task that is digital and interactive.

Mathematics Content: This assignment introduces students to terminology and gives them practice with coding. Learning new mathematics is not the intention of the design. However, this task goes beyond seeking “an answer,” to students providing reasoning for their answers. They are asked not to justify their comprehension of a loop and provide an example from their own life. The process of justification is a mathematical practice even though there are no disciplinary mathematical concepts learned. SCORE: 1.5

Cognitive Demand: The activity offers some possibility of conceptual richness by asking for examples to demonstrate understanding of vocabulary and by tasking students with creating their own scratch project. SCORE: 2

Use as an assessment: This homework assignment doubles as a formative assessment. Since this is homework, they have time to work on the tasks, with a video included for reference. Their thoughts are solicited throughout the activity and they must also upload their work. More information is necessary to determine the full extent to which student understanding was built upon. SCORE: 2
**STATION #3: Iteration & Loops**

Read the worksheet carefully.

Iteration happens in all subjects. For instance, I can say that I iterated this drawing of a cup 12 times to understand its shape and to find its most beautiful point of view. I prefer my seventh iteration because I used a cool shadow within the drawing.
**Iteration** = repeating a process to achieve a result

Programmers use **loops** to make iterations happen in code.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>For Loop</strong> = a loop that repeats <strong>for</strong> a certain number of times</td>
<td><img src="image" alt="For Loop Diagram" /></td>
</tr>
<tr>
<td>2. <strong>While Loop</strong> = a loop that is repeated <strong>while</strong> an expression is true</td>
<td><img src="image" alt="While Loop Diagram" /></td>
</tr>
<tr>
<td>3. <strong>Forever Loop</strong> = a loop that repeats <strong>forever</strong> because it has no end condition</td>
<td><img src="image" alt="Forever Loop Diagram" /></td>
</tr>
</tbody>
</table>
4. **Nested Loops** - a loop that appears inside of another loop.

**DIRECTIONS:**
1. Create a new Scratch project.
2. Name it “Station #3 - Iteration & Loops”.
3. Watch the video to see examples.
4. Try to code a cool gif while you are designing!
5. Share your Scratch project link below.
6. Answer the questions for this Station below.

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>RESPONSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Put your scratch link here</td>
<td></td>
</tr>
<tr>
<td>Take a screenshot of one of the codes you made AND explain what each block programs the computer to do.</td>
<td>[PUT SCREENSHOT HERE]</td>
</tr>
<tr>
<td>Explain the difference between the 4 types of loops in your own words.</td>
<td></td>
</tr>
<tr>
<td>Based on the drawing example shown above, provide another example of iteration.</td>
<td></td>
</tr>
<tr>
<td>Provide an example of something that is a loop in your own life.</td>
<td></td>
</tr>
</tbody>
</table>
Interdisciplinary Activity 3: Binary Numbers—Sending Secret Messages

Overview: Students will practice writing coded messages using numbers in binary form.

Mathematics Content: There is an alphabet code binary sheet that assigns numbers to each letter. There is some mathematics involved as students must re-write numbers as the sum of base-2 numbers. This activity is at grade level and is intended for practice. There is no evidence to suggest that it should be inherently challenging, even though there is an opportunity to make conceptual connections. SCORE: 2

Cognitive Demand: This activity is not inherently challenging. Students come up with their own message, but the task largely regurgitates the guided example. It is good for practice. SCORE: 1.5

Use as an assessment: Student thinking is not explicitly solicited. While students must “show their work” when they come up with their own message, this activity is intended for encouragement and corrective feedback, but I can also see it as a way to “think about the problem”. SCORE: 2

TRU MATH MEAN SCORES for Interdisciplinary Activities:
Mathematics: 1.8
Cognitive Demand: 2
Use of Assessments: 2.3
BINARY NUMBERS

Sending Secret Messages

DEF: A bit is a digit that is either 0 or 1. A byte is a string of 8 bits.

How to Write a Secret Message in a String of 8 Bits

<table>
<thead>
<tr>
<th>STEPS</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1: Determine the word you want to express</td>
<td>Step 1: I want to write &quot;LOVE&quot; in binary code</td>
</tr>
<tr>
<td>Step 2: Look at the alphabet binary code sheet</td>
<td>Step 2: The number (76) = the letter (L)</td>
</tr>
<tr>
<td>Step 3: Determine which &quot;bits&quot; can add up to the number for your letter.</td>
<td>Step 3: (76 = 2^5 + 2^3 + 2^2)</td>
</tr>
<tr>
<td>Step 4: Place a &quot;1&quot; in the &quot;bit &quot; you used to add in step 3, and place a &quot;0&quot; in the other space.</td>
<td>Step 4: (See table below)</td>
</tr>
</tbody>
</table>

***Remember 1 = TRUE and 0 = FALSE in binary***

1) DIRECTIONS: Read the review of Friday’s lesson, then write the binary code for the word “LOVE” in the table below.
(Continued)

<table>
<thead>
<tr>
<th>Letter</th>
<th>#</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>Binary Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>76</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>01001100</td>
</tr>
</tbody>
</table>

O
V
E

Write the Binary Code for the word “LOVE”

2. **DIRECTIONS** Write another word (of your choice) in binary code below.

What word(s) do you want to write?

Show your work below. You may add more rows to the table below if needed.

<table>
<thead>
<tr>
<th>Letter</th>
<th>#</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>bit</th>
<th>Binary Code</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the Binary Code for your word

Check your work by using this Binary Symbol Table.

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Appendix M

TRU Math Analysis of *Mathematics* Artifacts—Pearl

**Mathematics Activity 1: Graph Points and Vectors using CalcPlot3D. (Group Activity)**

**Overview:** After a lesson on search algorithms and latent semantic indexing, students were placed in groups to practice plotting points and vectors using the CalcPlot3D program.

*Some of the directions that students must follow:*

- As a group decide how you would identify the corner points of a pyramid.
- In CalcPlot3D, delete the default graph and plot these points and verify that they seem to trace out a pyramid.
- Compute the vectors connecting the edges of the pyramids and add these to the graph. (each student might take a pair of adjacent corner points to spread out the work)
- Graph the vectors, placing them at the corner points to form a pyramid.
- When you’re finished go to the menu option…. select “File” then “Encode View in URL”.
- If time permits, compute the lengths of the sides of your pyramid.

**Mathematics Content:** At first, this activity seems “skills-based”; however, it comes at the end of a lesson on how vectors are connected to search algorithms and 3D printing. Since this is multivariable calculus, the applications are necessary, if one aims to teach vectors beyond “computation,” i.e. addition, multiplication, translation etc. Additionally, CalcPlot3D is an integral part of the technology used in this course and students need to know how to use it. Considering this activity comes at the end of the lesson, it certainly has the potential for students to make connections to the applications presented earlier in the session. There is a high opportunity for students to make procedural and conceptual connections, but there is not enough evidence to suggest how much meaning can be made of these connections. **SCORE: 2**

**Cognitive Demand:** The steps outlined in the directions are procedural, *with the intention* of teaching students some of the basics of CalcPlot3D. As such, it would be counterproductive to have this be too much of a challenging activity for time allotted and its objective. It is not clear if time was allotted for students to explore any potential conceptual connections deeply. **SCORE: 2**

**Use as an assessment:** Placing this activity at the end of the session, makes good use of it as a formative assessment. It is a group activity so students can share thoughts with each other. They must also upload their pyramid, so they may receive some feedback from Myrtle. There may not be enough time to begin a *productive* discussion on insights or misunderstandings. **SCORE: 2**
Introduction Breakout (Latent Semantic example)

In your breakout groups

- identify 3 things that you all have in common. One of the things should be boring and one should be fun.
- Record your common characteristic in the Google Sheet.

Google Search - 2 tasks, lots of math

- What sites show up on a search?
  - Latent Semantic Indexing
  - Vectors and dot products

- What order do the sites appear?
  - Google Page Rank
  - Linear algebra eigenvalues and eigenvectors

Student Exercise

- Remind each other of your names.
- Designate one student to share his/her screen.
- At any point to indicate that you need to ask Prof. VanDieren a question, use the breakout feature.
- As a group, find a way to identify the corner points of a pyramid.
- In NetLogo, delete the default graph and plot these points and verify that they seem to trace out a pyramid.
- Compute the vectors connecting the edges of the pyramids and add these to the graph. (each student might take a pair of adjacent corner points to spread out the work)
- Graph the vectors, placing them at the corner points to form a pyramid.
- When you're finished, go to the menu option (the 3 bars in the upper left), select "File" then "Encode View in URL."
- Copy and paste the long URL into a new slide below.
- When you're finished go to the menu option (the 3 bars in the upper left), select "File" then "Save Plot as a PNG" (alternatively take a screenshot of your graph) and insert into as an image in a slide below.
- If time permits, compute the lengths of the sides of your pyramid.
- Return to the main room when instructed.
Mathematics Activity 2: Group Project 3: Individual Mindmap

Overview: Students create an individual mindmap assigned from one of the following three concepts: parametric curves, differentiation of surfaces, or integration of surfaces.

Pearl explained the assignment’s overall purpose during the interview:

... [students] pick which topic at the end of the semester that they want to focus on and they do a mindmap of that topic. They sign up for that on a Google form and then I try to create groups so that the group is diverse in terms of their topics. And they all get together and show each other their mindmaps, and then they have to create a mega mindmap. And, it’s not just putting things together; but seeing where things connect, so they’re drawing connections between those topics [in their individual mind maps] as a group project.

Some of the requirements include:

- In Lucid Charts, Diagram.net, or Matcha.io, individually construct a mind map of the content from your assigned topic using material from the textbook and the corresponding video assignments in WeBWorK.
- Your mindmap should meet the objectives of the group mindmap.
- Share you mindmap with the group and the professor.

Mathematics Content: This assignment is tied directly to the content of the course. Since students are preparing this to contribute to the “mega” mindmap, it is designed for them to make meaningful connections and identify gaps in understanding. This is a rich opportunity for students to try to make sense of concepts for themselves and for a group. SCORE: 3

Cognitive Demand: This activity is conceptually rich, and provides an opportunity build understanding. However, it’s potential increases in a group setting where students will need to justify connections and address any misunderstandings. As such, I can see students really trying to grapple with concepts and engage meaningfully with the content, due to the nature of the assignment. SCORE: 3

Use as an assessment: This is a great use as a formative assessment. Students’ thoughts on concepts form the basis of their mindmaps and any misunderstandings will be addressed in the next phase of the project. SCORE: 3
Mathematics Activity 3: Group Project 3: Group Mindmap

Overview: Students create an individual mindmap assigned from one of the following three concepts: parametric curves, differentiation of surfaces, or integration of surfaces

1. During your first team meeting, everyone should present their individual mindmaps to the group.
2. Your goal during this meeting is to discuss the content on the mindmaps. Are there questions about the material presented? Do you notice omissions or see corrections? Help your teammates better understand the material. This is where the Clarifier can take the lead.
3. Work in a common file in which all team members can contribute to (and perhaps copy and paste items from individual mindmaps).
4. Your team mindmap should describe the most important concepts from the entire semester. There won't be time/room to cover all the concepts.
5. Pick at least 3 major ideas but make sure that you describe these ideas:
   a) graphically
   b) numerically
   c) symbolically (you can add math symbols from idroo, mathcha.io, or by hand.
   Unfortunately, there is not one single (free) app that allows for math symbols and group editing).
   d) verbally (both how the concept connects to others but also applications of the concept).
6. As you create your mindmap, keep in mind the following:
   a) Provide citations for any information that you got from sources other than our course video lectures and the textbook.
   b) You may include specific worked-out examples to illustrate a concept, but make sure to explain why the example is relevant or how it is connected to the mindmap.
   c) Be creative. There isn't one best style of mindmap. You can add footnotes or label arrows, whatever helps demonstrate connections.
   d) Make sure everyone understands the concepts on the mindmap. This will help everyone study for the final exam.
7. When your team is ready to submit the final draft of the mindmap. The Scribe should share the file with [the professor].
8. Each Member will be responsible for separate peer and self-assessments.
Mathematics Content: This is conceptually rich by design. Students are addressing concepts from different lenses to go beyond procedures. They are engaging in key mathematical practices and must justify reasoning. **SCORE: 3**

Cognitive Demand: There is high opportunities for productive struggle embedded in this task. Part of the instructions including making “sure everyone understands the concepts”. This feature, among others, ensures that all students are “leveling up” in understanding. Group dynamics are always of concern; but Pearl addition of peer and self-assessments may help to mitigate this. **SCORE: 3**

Use as an assessment: This is an excellent formative assessment. Students’ thoughts are solicited and developed, and misunderstandings must be addressed. It also prepares the class for the final exam. **SCORE: 3**

TRU MATH MEAN SCORES for Mathematics Activities
Mathematics: **2.7**
Cognitive Demand: **2.7**
Use of Assessments: **2.7**
Group Project 3

Math Section A

Individual portion due Fri
Final group version due Fri
Peer and Self Assessments due Sat

Instructions

This group project involves an individual assignment followed by a group assignment. Both of these assignments involve creating a mindmap. What is a mindmap? Why are we being assigned a mindmap? How do I create a mindmap? Find the answers to these questions here.

This group project not only provides an opportunity for you to practice and develop your mathematical skills but also to prepare for the professional workforce. It is becoming more and more important in business settings for teams to work on projects and meet strict deadlines.

1 Individual Mindmap

1. In Lucid Charts (google add on with educational account) or Diagram.net (google add on) or Matcha.io (website gmail login), individually construct a mindmap of the content from your assigned topic (parametric curves, differentiation of surfaces, or integration of surfaces) using materials from the textbook and the corresponding video assignments in WeBWorK.

2. Your mindmap should meet the objectives of the group mindmap below.

3. When you are finished with your individual mindmap, share the file with your teammates.

4. This mindmap is due on Fri before your first team meeting.

2 Group Mindmap

1. During your first team meeting, everyone should present their individual mindmaps to the group.
2. Your goal during this meeting is to discuss the content on the mindmaps. Are there questions about the material presented? Do you notice omissions or see corrections? Help your teammates better under the material. This is where the Clarifier can take the lead.

3. Work in a common file in which all team members can contribute to (and perhaps copy and paste items from individual mindmaps).

4. Your team mindmap should describe the most important concepts from the entire semester. There won’t be time/room to cover all the concepts. Pick at least 3 major ideas but make sure that you describe these ideas
   (a) graphically
   (b) numerically
   (c) symbolically (you can add math symbols from idroo, mathcha.io, or by hand. Unfortunately there is not one single (free) app that allows for math symbols and group editing).
   (d) verbally (both how the concept connects to others but also applications of the concept).

As you create your mindmap, keep in mind the following:
   (a) Provide citations for any information that you got from sources other than our course video lectures and the textbook.
   (b) You may include specific worked out examples to illustrate a concept, but make sure to explain why the example is relevant or how it is connected to the mindmap.
   (c) Be creative. There isn’t one best style of mindmap. You can add footnotes or label arrows, whatever helps demonstrate connections.
   (d) Make sure everyone understands the concepts on the mindmap. This will help everyone study for the final exam.

5. When your team is ready to submit the final draft of the mindmap, the scribe should share the file with

3 Self and Peer Assessments

Each member of the group will also be responsible for submitting separate self and peer assessments through this form. This is due on Sat

Group Roles and Responsibilities

In professional settings teams are usually made up of individuals with various roles. Therefore in this project each team member will be assigned a role. Your
role for the second project will be different than the one that you are assigned for this first project. The roles for this project are manager, reporter, scribe, illustrator, and clarifier:

Manager The manager is responsible for arranging and running the group meetings. If one of the team members is not present or does not fulfill their duties, the manager is responsible for taking on their role and/or re-assigning this role to another willing team member. These re-assignments should be included in the Reporter’s record. The manager not only sets the agenda of each meeting, but also communicates to [redacted] if there are questions or problems with either mathematics or team dynamics.

Reporter The reporter writes a record of how the team sessions went, how long the team met, what difficulties or successes the team may have had. If there was disagreement about a solution of a problem, the report should include alternative solutions or explanations for the differences of opinion. The report should list dates and times of the group meeting, the members of the team who attended the meeting, comments on how the group worked together, and a brief summary of each of their contributions to the session. These reports should be submitted to [redacted] shortly after each team meeting via email and should cc the other team members.

Scribe The scribe is responsible for writing up the single final version of the project. This is the only set of solutions which will be accepted and graded. The final solutions should be contained in one written document explaining the group’s answers and demonstrating the work. Please use complete sentences. Include images and links from CalcPlot3D where appropriate to graphically verify or to demonstrate your computations. You can create this report in LucidCharts, diagram.net, Google Docs/Slides, or in mathcha.io. The scribe should work with the clarifier to assure agreement with all group members before submitting the final draft.

Illustrator The illustrator makes connections between what people in the group say and think and helps to communicate these things. This person asks questions like "How does this idea connect to that idea?" The illustrator is responsible to create any tables, equations, or graphs using mathcha.io, idroo.com, or CalcPlot3D and give these to the Scribe and Reporter for inclusion in their final reports.

Clarifier During the team meetings the clarifier assists the group by paraphrasing ideas presented by the group members (e.g., "Is this is what you were thinking? What did you mean by that? ...") The clarifier is responsible for making sure that everyone in the group understands the work.

Expectations

Teams should meet at least one time online together. Before the first meeting, you are responsible for completing Part 1 individually. If you do not have your
Appendix N

TRU Math Analysis of Interdisciplinary Artifacts—Pearl

Interdisciplinary Activity 1: Coding with the POV-Ray Software (Homework)

Overview: Use POV-Ray to create a preliminary scene file using the elements of the user interface and objects from the basic-shapes.inc file.

Learning Objectives
- Vocabulary: pixel, raster image, pixelation, compression, vector graphics, ray tracing, polygon projection model
- Programming Skills and Syntax: documenting code with comments, Cartesian coordinate system, POV-syntax including camera, object, light source, .inc files, rotate, scale, and translate.
- Planning: gain insight into how ray-traced scenes are planned and executed, enjoy seeing variations on a theme.

Some tasks to complete:
- Download and install the POV-Ray software
- Download the files from the Google Drive directory to your personal computer or network drive
- Read Chapter 1; Appendix A.1-A.2; Appendix B.1-B.4
- View the images on pages 91, 92 and 104 of the textbook for some inspiration for possible scenes that can be created from basic shapes.
- Create a preliminary scene file using the elements of the user interface and objects from the basic-shapes.inc file.
- Plan a scene and sketch (it can be a very rough) the scene on paper using the Cartesian coordinate system to determine placement of objects and camera in a 3D-view.
- Program the scene that you have sketched. You may want to add one item at a time and render the scene each time you add or edit an object.
- (BONUS): You can read page 94 in the textbook about “for loops” and use this programming tool to generate your scene.

Assessment
- Correctly follow all of the steps
- Create a scene rendered in POV-Ray using the shapes and objects in the included files
- Intentionality and consistency of aesthetic choices
- Clearly commented code

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-Uniqueness within the constraints of the assignment
(Continued)

**Mathematics Content:** This course is a computer animation and mathematics interdisciplinary course for all majors in the honors track. It was designed for students to code “the old-fashioned way” and not using a “drag-and-drop” method. In this activity, students are learning the main software used in the course, and these instructions are procedural by design. It focuses more on the coding, and the majority of connections are made between coding and its applications. However, since the Cartesian coordinate system is foundation for analytic geometry, there are some implied connections here between the mathematics and animation. Student will have another activity (HW # 6) that makes this connection explicit, but this task is not mathematically challenging by design. **SCORE: 1.5**

**Cognitive Demand:** Students need to comment on their codes and since this is the first homework, may also want to be thinking about how their choices show that they are being intentional about developing the programming skills necessary for the course. They need to plan a scene and program it and as such, there is potential to make some procedural and conceptual connections. There is also an option for more challenge with the “BONUS” task. **SCORE: 2**

**Use as an assessment:** Students are explicitly assessed on following steps and being intentional about their aesthetic choices. There is not enough evidence to show how their reasoning is actively pursued, developed and built upon. There is a task requiring them to “comment the code” which will be necessary for tasks in the future. This seems like some way to solicit students’ thoughts but it is not clear. **SCORE: 2**
POV-Ray, Part I

Course: Behind The Curtain
Instructors [redacted]
Lead instructor for this assignment: [redacted]

Learning Objectives

Vocabulary: pixel, raster image, pixelation, compression, vector graphics, ray tracing, polygon projection model

Programming Skills and Syntax: documenting code with comments, Cartesian coordinate system, POV-syntax including camera, object, light source, .inc files, rotate, scale, and translate.

Planning: gain insight into how ray-traced scenes are planned and executed, enjoy seeing variations on a theme.

Instructional Activities

Illustrated lecture and hands-on demo

Assignment

This is an adaptation of Project B.5 on page 105 of the textbook.

Required Software and File
- POV-Ray 3.7 (see page 29 of the textbook for installation instructions).
- basic-shapes.inc file from Google Drive. This file contains shapes that you may choose to add to your scene (e.g., ball, pencil, box, can, book, glass, gumdrop, pinwheel, bucket, ring, heart, sword, die, spring, and star). See page 104 of the textbook for images of these objects. The default position of these objects is centered at (0,0,0) and each object is scaled to fit within a 1x1x1 cube.

Steps to Completion
Download and install the POV-Ray software
Download the files from the Google Drive directory to your personal computer or network drive
1. Read Chapter 1; Appendix A.1-A.2 (beginning on page 29); Appendix B.1-B.4 (beginning on page 96).
2. View the images on pages 91, 92 and 104 of the textbook for some inspiration for possible scenes that can be created from basic shapes.
3. Create a preliminary scene file using the elements of the user interface and objects from the basic-shapes.inc file.
   a. Choose New from the File menu and Save the new file to your computer or drive giving it a meaningful name. This file should be saved to the same directory as the basic-shapes.inc file.
   b. On the toolbar of the new file, click the Templates drop-down and choose Camera. Accept the defaults and click OK to insert code for a camera into your file.
   c. Repeat step b. to insert a Light Source. (You may optionally insert a sky and floor).
   d. Repeat step b. to insert an Object.
(Continued)

e. Place your cursor before the final curly bracket } of the object’s code and then repeat step b. to insert a Pigment. Accept the default.

g. At the beginning of this file add the lines 
   
   ```
   include "basic-shapes.inc"
   include "colors.inc"
   ```
   
   This tells your scene file about the 15 extra shapes like the pencil and gives you several colors to apply to objects.

h. At the end of your scene file, insert the code `object (Pencil)`.

i. Comment the code (this will become important later in the assignment when this file will become longer).

j. Render the scene. The image is saved in the same folder as the .pov file. You will notice that the Pencil is very stubby and the positioning of the pencil is not ideal. The next step will resolve this.

   ```
   object (Pencil) scale<1,1,1> rotate<0,0,90>
   ```

4. Plan a scene and sketch (it can be a very rough) the scene on paper using the Cartesian coordinate system to determine placement of objects and camera in a 3D-view. You will use this sketch to determine location and sizes of objects when you program the scene. This scene should include at least five objects from the built-in objects or the basic shapes file (they may all be the same object). Your scene should include objects that differ from the default pigment, scale, orientation, and position.

5. Save a copy of the source code for the preliminary scene from part 3 under a name such as vandieren-assignment1.pov. Delete the object and Pencil that you added (unless they are part of your planned scene).

6. Using the steps in part 3 as a guide, program the scene that you have sketched. You may want to add one item at a time and render the scene each time you add or edit an object. You may want to change the file name as you make major edits to the scene so that you can easily go back to previous versions of your scene. Your scene should make use of scale, rotate, translate, and pigment options. You may edit the background and cameras.

7. Bonus (optional): You can read page 94 in the textbook about “for loops” and use this programming tool to generate your scene. This is helpful if you would like to include several similar objects in the scene.

You will be using the same scene for future assignments, editing it, and adding more features over the course of the semester!

Instructions for Turning in Assignment

Due in-class at the beginning of class Monday, Please add the source code and the rendered image for part 6 to the Google Drive folder for this class. You do not need to submit part 3.

Assessment

- Correctly follow all of the steps
- Create a scene rendered in POV-Ray using the shapes and objects in the included files
- Intentionality and consistency of aesthetic choices
- Clearly commented code
- Uniqueness within the constraints of the assignment
Interdisciplinary Activity 2: Constructive Solid Geometry—CSG, HW #3 (Homework)

Overview: Create a scene with POV-Ray using tinker toy objects.

Learning Objectives
- **Vocabulary**: affine transformation, rotation, translation, union, intersection, difference, lathe, spline, CSG
- **Programming Skills and Syntax**: documenting code with comments, Cartesian coordinate system, POV-syntax including union, intersection, difference, lathe, linear_spline, spline, #for, #declare, .inc files, macros

Some tasks to complete:
1. Skim through Section 7.4 of the textbook about CSG, read page 155 exercise 6.35 about macros (you don’t have to do the exercise), read Appendix C pages 158-164, read page 223 D.3.3 about .inc files, read page 223-224 section D4 about documenting code for others.
2. Read…class discussion: Sections 6.1 and 6.2.
3. Select one of the tinker toy objects from the box in class. This will be the object that you will be creating using CSG and following the goals listed on page 225 in D.5.1. A model that you may like to follow is D.6 and tinker-toy-project.inc. If you would like to produce more “rounded” shapes consider using the shapes in shapes3.inc which is documented here.
4. Create a scene using your newly created object and .inc file. The scene does not need to be complicated; it should simply demonstrate that your .inc file and newly created object work as intended.
5. Bonus: you may create a macro for your object that allows the user to specify the color or dimensions.

Mathematics Content: The focus of CSG is on its application. Most conceptual and procedural connections are primarily on the graphics side. Though mathematical connections *can* be made, it is not the focus of this task. **SCORE: 1.5**

Cognitive Demand: There is certainly opportunities to make connections between CSG and understanding basic coding applications. While students must document code and create a scene, “this does not need to be complicated” as expressed in Step 4. Since this is not the first time and the steps of the task are similar to the prior assignment, it is not meant to be a challenging assignment. **SCORE: 1.5**

Use as an assessment: The tasks here are procedural and student reasoning is not necessarily being developed. Students are demonstrating that they can follow steps for creating a scene using their object. However, they must still document on their code. **SCORE: 2**
POV-Ray Constructive Solid Geometry (HW 3)

Course: Behind The Curtain
Instructors: [Redacted]
Lead instructor for this assignment: [Redacted]

Learning Objectives

Vocabulary: affine transformation, rotation, translation, union, intersection, difference, lathe, spline, CSG

Programming Skills and Syntax: documenting code with comments, Cartesian coordinate system, POV-syntax including union, intersection, difference, lathe, linear_spline, spline, #for, #declare, .inc files, macros

Instructional Activities

Illustrated lecture and hands-on demo

Assignment

This is an adaptation of Project D.6 on page 228 of the textbook.

Software, Files, Links
- POV-Ray 3.7 (see page 29 of the textbook for installation instructions).
- tinker-toy-project.inc (optional example file) and shapes3.inc (optional .inc file for rounded shapes)
- Optional explanations for CSG and examples: http://www.f.lomueller.de/pov_tut/csg/povcsg1e.htm

Steps to Completion

Download and install the POV-Ray software

1. Reading for this assignment: Skim through Section 7.4 of the textbook about CSG, read page 155 exercise 8.35 about macros (you don’t have to do the exercise), read Appendix C pages 158-164, read page 223 D.3.3 about .inc files, read page 223-224 section D4 about documenting code for others.

2. Reading for October 2 class discussion: Sections 6.1 and 6.2.

3. Select one of the tinker toy objects from the box in class. This will be the object that you will be creating using CSG and following the goals listed on page 225 in D.5.1. A model that you may like to follow is D.6 and tinker-toy-project.inc. If you would like to produce more “rounded” shapes consider using the shapes in shapes3.inc which is documented here.

4. Create a scene using your newly created object and .inc file. The scene does not need to be complicated; it should simply demonstrate that your .inc file and newly created object work as intended.

5. Bonus: you may create a macro for your object that allows the user to specify the color or dimensions.

You will be sharing your .inc file with classmates (and you will receive copies of their .inc files as well) so that we will have several pre-made objects to use in the next assignment.

Instructions for Turning in Assignment

Due in-class at the beginning of class Monday, October 2, 2017. Please add the source code and the rendered image for part 3 and part 4 to the Google Drive folder for this class.

Assessment

Correctly follow all of the steps
Create a scene rendered in POV-Ray using the shapes and objects in the included files
Intentionality and consistency of aesthetic choices
Clearly commented code
Uniqueness within the constraints of the assignment
Interdisciplinary Activity 3: POV-Ray Animations HW #6 (Homework)

Overview: Create an animation by connecting an elastic curve to an easing function.

Learning Objectives:
- **Vocabulary**: Bezier Curve, control points, easing functions, clock, elastic movement, decay, slope, derivative of a function, tangency, curvature
- **Mathematics**: connect cubic Bezier curves smoothly in POV-Ray, find values for an elastic function to smoothly connect to a given function
- **Programming Skills and Syntax**: documenting code with comments, Cartesian coordinate system, POV-syntax including clock, POV-animation file structure, declaring functions and time variables, macros, if-then statements

Some steps to completion:
1. Create a macro for connecting together three easing functions and demonstrate that the macro works by creating a simple animation.
2. Find an elastic function to connect to the easing curve \( e(t) = t^3 \) smoothly. Create a short animation (e.g. a ball rolling across the floor) demonstrating that your elastic function is as desired. Either in a google document or in the commented section of code, describe the process that you used to find the elastic function.
3. Consider the Cubic Bezier Curve
   
   spline{cubic_spline
   -.25, <-1,0,1>
   0.00, <1,1,1>
   1.00, <2,.5,1>
   1.25, <1,0,0> }

4. Find another Bezier Curve that will connect to this curve smoothly, starting at the point (2, 0.5, 1) and end at the point (4,5,6). [See student example included at the end.]
5. Determine whether or not Spline 1 connects smoothly to Spline 2. If the connection is not smooth explain if the continuity fails because of connection, tangency, and/or curvature.

Assessment
-Correctly follow all of the steps
-Correctness, clarity, and completeness of mathematical exercises
-Create an animated scene rendered in POV-Ray
-Intentionality and consistency of aesthetic choices
-Clearly commented code
-Uniqueness within the constraints of the assignment
(Continued)

**Mathematics Content:** This task is designed to illuminate the mathematics involved. There are some rich procedural and conceptual connections among the disciplines. Observing this lesson may provide insight as to the extent to which the mathematics of elastic functions and Bezier curves are explored. That mathematics is skills-based but necessary for creating the animation. **SCORE: 2**

**Cognitive Demand:** This task is more than just a routine exercise must be correct, clear, and complete. Students must explain reasoning using vocabulary (Step 5), which presents an opportunity for understanding these terms. The task has potential for procedural and conceptual connections within the mathematics taught and in conjunction with programming. **SCORE: 2.5**

**Use as an assessment:** This is a good use of a formative assessment because students’ thoughts are solicited. While students are presented with ways to think about these problems, I need more information as to how their ideas or misunderstandings were addressed. **SCORE: 2.5**

**TRU MATH MEAN SCORES for Interdisciplinary Activities:**

- **Mathematics:** 1.7
- **Cognitive Demand:** 2
- **Use of Assessments:** 2.2
POV-Ray Animations (HW 6)

Course: Behind The Curtain
Instructors: [Redacted]
Lead instructor for this assignment: [Redacted]

Learning Objectives

Vocabulary: Bezier Curve, control points, easing functions, clock, elastic movement, decay, slope, derivative of a function, tangency, curvature

Mathematics: connect cubic Bezier curves smoothly in POV-Ray, find values for an elastic function to smoothly connect to a given function

Programming Skills and Syntax: documenting code with comments, Cartesian coordinate system, POV-syntax including clock, POV-animation file structure, declaring functions and time variables, macros, if-then statements

Instructional Activities

Illustrated lecture and hands-on demo

Assignment

Software, Files, Links

- POV-Ray 3.7 (see page 29 of the textbook for installation instructions).
- demonstration files and links sent through email
- Optional program for calculating derivatives and solving math problems: Wolframalpha.com

Steps to Completion

Download and install the POV-Ray software

1. Create a macro for connecting together three easing functions and demonstrate that the macro works by creating a simple animation.

2. Find an elastic function to connect to the easing curve \[ e(t) = t^3 \] smoothly. Create a short animation (e.g. a ball rolling across the floor) demonstrating that your elastic function is as desired. Either in a google document or in the commented section of code, describe the process that you used to find the elastic function.

3. Consider the Cubic Bezier Curve
   
   ```pov
   spline(cubic_spline
      .25, <1,0,1>
      0.00, <1,1,1>
      1.00, <2.5,1>
      1.25, <1,0,0>)
   ```

   Find another Bezier Curve that will connect to this curve smoothly, starting at the point \((2, 0.5, 1)\) and end at the point \((4,5,6)\).
4. Determine whether or not Spline 1 connects smoothly to Spline 2. If the connection is not smooth explain if the continuity fails because of connection, tangency, and/or curvature.

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<th>Spline 2</th>
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Instructions for Turning in Assignment
Due at the beginning of class Monday, October 30, 2017. Please add the source code and the rendered .avi file for parts 1-2 to the Google Drive folder for this class. You may turn in parts 3-4 either electronically in the Google Drive folder or on paper in class.

Assessment
- Correctly follow all of the steps
- Correctness, clarity, and completeness of mathematical exercises
- Create an animated scene rendered in POV-Ray
- Intentionality and consistency of aesthetic choices
- Clearly commented code
- Uniqueness within the constraints of the assignment
Example - CalcPlot3D

Find a Bezier curve that goes through the points (2,0,0) and (-2,1,1) but does not intersect the sphere centered at the origin of radius 1.

Type your solution in the essay box along with a link of your solution in CalcPlot3D.
## Appendix O

### List of New York City Public Schools

<table>
<thead>
<tr>
<th>School</th>
<th>Name</th>
<th>Focus</th>
<th>Principal &amp; Contact</th>
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<tbody>
<tr>
<td>MAN.</td>
<td>PS 152M: The Magnet School of Innovation in a Global Community</td>
<td>STEAM</td>
<td>Julia Pietri <a href="mailto:jpietri@schools.nyc.gov">jpietri@schools.nyc.gov</a></td>
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</tr>
<tr>
<td></td>
<td>P.S. 189M: The Magnet School of Inquiry and Expression</td>
<td>Integrated</td>
<td>Rosalina Perez <a href="mailto:rperez10@schools.nyc.gov">rperez10@schools.nyc.gov</a></td>
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<td></td>
<td>Linden Tree Elementary</td>
<td>Integrated</td>
<td>Ms. Lisa DeBonis <a href="mailto:lisa@lindentree567.com">lisa@lindentree567.com</a></td>
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<tr>
<td></td>
<td><strong>BRONX</strong> PS 178 Magnet School of Multimedia Arts &amp; Design</td>
<td>Integrated</td>
<td>Schwanna Ellman <a href="mailto:sellman@schools.nyc.gov">sellman@schools.nyc.gov</a></td>
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<td></td>
<td>The Jermaine L. Green STEM Institute of Queens, PS 354</td>
<td>STEM</td>
<td>Mrs. Askew <a href="mailto:RAAskew@ps354.org">RAAskew@ps354.org</a></td>
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<tr>
<td></td>
<td><strong>QUEENS</strong> P.S. 123 Queens</td>
<td>STEAM</td>
<td>Mr. Anthony M. Hooks <a href="mailto:AHooks@schools.nyc.gov">AHooks@schools.nyc.gov</a></td>
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<tr>
<td></td>
<td>P.S. 80Q Thurgood Marshall Magnet School of Multimedia and Communication</td>
<td>STEAM thru literacy</td>
<td>Kersandra M. Cox <a href="mailto:kcox4@schools.nyc.gov">kcox4@schools.nyc.gov</a></td>
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<td>PS 349 Magnet School for Leadership and Innovation through STEAM</td>
<td>Magnet, STEAM</td>
<td>Tanya Bates-Howell <a href="mailto:tbateshowell@schools.nyc.gov">tbateshowell@schools.nyc.gov</a></td>
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<td><strong>P.S. 201Q The Discovery School for Inquiry and Research</strong></td>
<td>Magnet, STEAM</td>
<td>Umit Serin <a href="mailto:userin@schools.nyc.gov">userin@schools.nyc.gov</a></td>
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<td>P.S. 92Q, The Harry T. Stewart, Sr. Magnet School for Engineering, Architecture, and the Arts</td>
<td>Magnet-Art, Engineering, Architecture</td>
<td>Mr. Pasquale Baratta; <a href="mailto:pbaratt@schools.nyc.gov">pbaratt@schools.nyc.gov</a></td>
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<tr>
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<td><strong>P.S. 140Q EDWARD K. ELLINGTON ELEMENTARY</strong></td>
<td>Magnet-Science, Dance, Art</td>
<td>Ms. R. Hasberry-Signal, <a href="mailto:Rsignal@schools.nyc.gov">Rsignal@schools.nyc.gov</a></td>
<td>Yes--2 of 8 completed</td>
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<td>P.S. 55Q Maure Magnet School of Communication Arts, Technology and Multimedia</td>
<td>Magnet-Communications, technology</td>
<td>Ralph Honore <a href="mailto:rhonore@schools.nyc.gov">rhonore@schools.nyc.gov</a></td>
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<td><strong>P.S. 254Q THE ROSA PARKS SCHOOL</strong></td>
<td>Magnet-Leadership Dvlpt &amp; the Arts</td>
<td>Pamela Markham <a href="mailto:pmarkha2@schools.nyc.gov">pmarkha2@schools.nyc.gov</a></td>
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<td><strong>P.S. 316Q Queens Explorers Magnet School for Global Conservation and Service Learning</strong></td>
<td>Magnet-Interdisciplinary</td>
<td>Nicole Grant <a href="mailto:ngrant10@schools.nyc.gov">ngrant10@schools.nyc.gov</a></td>
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<td><strong>PS 160Q - Walter Francis Bishop Magnet School of the Arts</strong></td>
<td>Magnet-Art</td>
<td>Tiffany Hicks <a href="mailto:thick2@schools.nyc.gov">thick2@schools.nyc.gov</a></td>
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<tr>
<td>School Name</td>
<td>Program</td>
<td>Contact Person</td>
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<tr>
<td>P.S. 120Q (Flushing Dragons)</td>
<td></td>
<td>Robert Marino</td>
<td><a href="mailto:rmarino4@schools.nyc.gov">rmarino4@schools.nyc.gov</a></td>
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<td>P.S. 307 Daniel Hale Williams Elementary</td>
<td>STEM</td>
<td>Stephanie Carroll</td>
<td><a href="mailto:SCarroll2@schools.nyc.gov">SCarroll2@schools.nyc.gov</a></td>
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<td>P.S. 123K The Suydam Magnet School for STEAM</td>
<td>STEAM</td>
<td>Donna Stalzer</td>
<td><a href="mailto:dstalzer2@schools.nyc.gov">dstalzer2@schools.nyc.gov</a></td>
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<td>P.S. 245 Magnet School of Arts and Science</td>
<td>Magnet-STEAM, Arts &amp; Science</td>
<td>Ms. Erica Williams</td>
<td><a href="mailto:ekelly31@schools.nyc.gov">ekelly31@schools.nyc.gov</a></td>
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<td>P.S. 46 Edward C. Blum, Magnet School of Communications and Media Arts Through Applied Learning</td>
<td>Magnet-Media &amp; Communication</td>
<td>Maria Guzman</td>
<td><a href="mailto:mguzman9@schools.nyc.gov">mguzman9@schools.nyc.gov</a></td>
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<tr>
<td>P.S. 121 Nelson A. Rockefeller</td>
<td>Magnet, Applied Sciences</td>
<td>Mr. Zakariah Haviland, I.A.</td>
<td><a href="mailto:zhaviland@schools.nyc.gov">zhaviland@schools.nyc.gov</a></td>
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<td>P.S. 54 Sam Barnes, Magnet-- Environmental Science, Technology &amp; Community Wellness</td>
<td>Magnet-- Envir. Sci., Tech. &amp; Com. Wellness</td>
<td>Emma Pelaezvelazquez</td>
<td><a href="mailto:epelaezvelazquez@schools.nyc.gov">epelaezvelazquez@schools.nyc.gov</a></td>
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<td>Joy-Ann Morgan</td>
<td><a href="mailto:jmorgan2@schools.nyc.gov">jmorgan2@schools.nyc.gov</a></td>
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<tr>
<td>P.S. 15 Patrick F. Daly Magnet School of the Arts</td>
<td>Magnet--Arts</td>
<td>Peggy Wyrs-Madison</td>
<td><a href="mailto:pwyns@schools.nyc.gov">pwyns@schools.nyc.gov</a></td>
<td></td>
</tr>
<tr>
<td>J.H.S. 123 James M. Kieran: Bronx Urban Community Magnet School</td>
<td>Magnet-STEAM</td>
<td>Richard Hallenbeck</td>
<td><a href="mailto:rhallenbeckjr@schools.nyc.gov">rhallenbeckjr@schools.nyc.gov</a>; <a href="mailto:rhallenbeckjr@is123x.org">rhallenbeckjr@is123x.org</a></td>
<td></td>
</tr>
<tr>
<td>M.S. 358 Magnet School for S.T.E.A.M. Exploration and Experiential Learning</td>
<td>Magnet-STEAM</td>
<td>Brendan Mims</td>
<td><a href="mailto:bmims2@schools.nyc.gov">bmims2@schools.nyc.gov</a></td>
<td></td>
</tr>
<tr>
<td>P.S. 582 Magnet School for Multimedia, Technology, and Urban Planning</td>
<td>Magnet-STEAM</td>
<td>Mr. Merced <a href="mailto:jmerced4@ms582.org">jmerced4@ms582.org</a></td>
<td><a href="mailto:jmerced4@schools.nyc.gov">jmerced4@schools.nyc.gov</a></td>
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<tr>
<td>NYC Middle</td>
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<tr>
<td>I.S. 240: Andries Hudde, STEM Magnet</td>
<td>Magnet-STEM</td>
<td>Anya Munce-Jarrett</td>
<td><a href="mailto:amunce@schools.nyc.gov">amunce@schools.nyc.gov</a></td>
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</tr>
<tr>
<td>P.S./I.S. 686 Brooklyn School of Inquiry (K-8)</td>
<td>STEM</td>
<td>Eric Havlik</td>
<td><a href="mailto:ehavlik@schools.nyc">ehavlik@schools.nyc</a></td>
<td></td>
</tr>
<tr>
<td>P.S./I.S. 157 The Benjamin Franklin Magnet School for Civic Leadership in Health &amp; Science (K-8)</td>
<td>Magnet, Leadership in Health &amp; Science</td>
<td>Kourtney Boyd</td>
<td><a href="mailto:kboyd@schools.nyc.gov">kboyd@schools.nyc.gov</a></td>
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<td>NYC-Combination</td>
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<tr>
<td>Young Women's Leadership School—East Harlem (6-12)</td>
<td></td>
<td>Colleen McGeehan</td>
<td><a href="mailto:cmgee@tywls.org">cmgee@tywls.org</a></td>
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<tr>
<td>The Laboratory School of Finance and Technology (6-12)</td>
<td>Traditional/Integrated</td>
<td>Dr. Ramon M. Gonzalez</td>
<td><a href="mailto:info@mshs223.org">info@mshs223.org</a></td>
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<tr>
<td>Good Shepard School</td>
<td>STEM</td>
<td>Mrs. Geraldine Lavery</td>
<td><a href="mailto:GeraldineLavery@gsschoolnyc.org">GeraldineLavery@gsschoolnyc.org</a></td>
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Appendix P

Letter to School Principals

Dyanne Baptiste

October 7, 2020

New York City Department of Education

Re: NYC DOE IRB Protocol 3504

Dear Principal________:

First of all, I hope that you and your family and loved ones are safe and well. My name is Dyanne Baptiste and I am very interested in working with teachers from your school that are eligible for my dissertation study titled, *Analyzing Interactions between Mathematics Teachers’ Instructional Activities and their Practices and Beliefs about the Teaching and Learning of Mathematics*. I am currently a doctoral candidate in the Mathematics, Science, and Technology department at Teachers College. I am now contacting you since my protocol was approved by Teachers College on May 19, 2020 and the NYC DOE IRB on September 29, 2020.

My study focuses on describing teachers’ mathematics and interdisciplinary instructional practices and beliefs, as enacted through the design and use of activities. It aims to see how the rigor of mathematics is preserved in both kinds of lessons, as teachers reflect upon the purposeful execution of these activities in online and classroom environments. All certified teachers who teach mathematics as a subject and within an interdisciplinary or multidisciplinary curriculum such as STEM or STEAM are eligible to participate in this study. I will be recruiting from schools that promote such dual learning within the normal school day. The use of interdisciplinary or multidisciplinary curriculum through an after-school program is outside of the scope of this study.

This study will take about five hours, spread over six – eight weeks of the semester, and there are no observations. Teachers agreeing to participate will first complete a survey of their beliefs and practices, followed by a collection of artifacts (blank copies) of instructional activities. These activities will be analyzed using Dr. Alan Schoenfeld’s TRU Math framework for their depth of mathematics content and its potential for allowing students to engage in the concepts—from the teacher’s point of view. Teachers will be encouraged to review student work and reflect on the lessons. They will then participate in an online interview to expound on their thoughts and reflections in the design and execution of these activities. Teachers will also be
asked to submit a short reflection on both a STEM/STEAM etc. and traditional mathematics lessons and/or instructional activity. I will not be looking at student work. The reflection exercises and the interviews will be opportunities for teachers to share their experiences of students engaging with the lessons, what they thought students would approach a task, how students actually approached the task, and what they learned about designing activities in the process. This study focuses on teachers’ lived experiences creating and executing instructional activities for understanding mathematics, as experienced in both types of lessons and assignments.

In order to maintain confidentiality, each participant will receive an identification number (PID) using a random number generator, before filling out the consent form. This PID will identify all materials and help maintain confidentiality. All biographical and school information will be redacted from any documents collected and identified only using the PID. Participants will also have sole access to individual folders, stored on a password protected Google Drive folder that will store all materials. The participant and I will be the only ones that have access to their folder. This data will be used for my dissertation study and for academic research only.

I have attached a recruitment flyer with my contact information. If you consent to me working with your teachers, please share it with your mathematics department. Upon your approval, teachers who are interested may contact me directly. Additionally, and upon your consent, I would also like to reach out to teachers who may be eligible, in order to expedite the recruitment process. If possible, please let me know your decision by [redacted]. Thank you very much for the opportunity to work with your teachers and much success on this new school year! It would be an honor to work with your teachers to understand how their beliefs about teaching and learning interact with their instructional practices and activities.

Sincerely,

Dyanne Baptiste, M.A.T., M.Phil.

I, ______________________________, give consent to Dyanne Baptiste to work with teachers at our school for the dissertation study.

I do not give consent to teachers to participate in this study.
### TRU Math: Teaching for Robust Understanding in Mathematics

#### Scoring Rubric

Release Version Alpha | REVISED July 31, 2014

This document provides the summary scoring rubric for the TRU Math (Teaching for Robust Understanding of Mathematics) classroom analysis scheme. TRU Math addresses five general dimensions of mathematics classroom activity, and one dimension that is algebra-specific. Each of these six dimensions is coded separately during whole class discussions, small group work, student presentations, and individual student work.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>The extent to which the mathematics discussed in the observed lesson is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained.</td>
<td>The extent to which classroom interactions create and maintain an environment of productive intellectual challenge that is conducive to students’ mathematical development.</td>
<td>The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class.</td>
<td>The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another’s ideas, in ways that contribute to students’ development of agency, authority, and their identities as doers of mathematics.</td>
<td>The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.</td>
</tr>
</tbody>
</table>

**Content Elaboration for Contextual Algebraic Tasks:** The extent to which students are supported in dealing with complex modeling and applications problems, which typically call for understanding complex problem contexts (most frequently described in text), identifying relevant variables and the relationships between them, representing those variables and relationships symbolically, operating on the symbols, and interpreting the results.

This document is a research tool; it is not intended for use in teacher evaluations. Detailed instructions regarding the use of this scoring rubric are provided in the TRU Math Scoring Guide. Information regarding the genesis, rationale, and applications of the TRU Math scheme can be found in the document An Introduction to Teaching for Robust Understanding in Mathematics (TRU Math). Both documents, along with this scoring rubric and TRU Math coding sheets, are available at [http://jats.berkeley.edu/tools.html](http://jats.berkeley.edu/tools.html).

---

1 This work is a product of The Algebra Teaching Study (NSF Grant DRL 0909815 to Pis Alan Schoenfeld, U.C. Berkeley, and NSF Grant DRL 0909851 to Robert Fleden, Michigan State University), and of The Mathematics Assessment Project (Bill and Melinda Gates Foundation Grant OPP33842 to Pis Alan Schoenfeld, U.C. Berkeley, and Hugh Burkhardt and Malcolm Swan, The University of Nottingham). Suggested Citation: Schoenfeld, A. H., Fleden, R. E., & the Algebra Teaching Study and Mathematics Assessment Project. (2014). The TRU Math Scoring Rubric. Berkeley, CA & E. Lansing, MI: Graduate School of Education, University of California, Berkeley & College of Education, Michigan State University. Retrieved from [http://jats.berkeley.edu/tools.html](http://jats.berkeley.edu/tools.html).
### Summary Rubric

<table>
<thead>
<tr>
<th>The Mathematics</th>
<th>Cognitive Demand</th>
<th>Access to Mathematical Content</th>
<th>Agency, Authority, and Identity</th>
<th>Uses of Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>How accurate, coherent, and well justified is the mathematical content?</td>
<td>To what extent are students supported in grappling with and making sense of mathematical concepts?</td>
<td>To what extent does the teacher support access to the content of the lesson for all students?</td>
<td>To what extent are students’ sources of ideas and discussion of them? How are student contributions framed?</td>
<td>To what extent is students’ mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?</td>
</tr>
</tbody>
</table>

1. Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement with key grade level content (as specified in the Common Core Standards).
   - Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.
   - There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue.
   - The teacher initiates conversations. Students’ speech turns are short (one sentence or less), and constrained by what the teacher says or does.
   - Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.

2. Activities are at grade level but are primarily skills-oriented, with few opportunities for making connections (e.g., between procedures and concepts) or for mathematical coherence (see glossary).
   - Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to “scaffold away” the challenges, removing opportunities for productive struggle.
   - There is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students.
   - Students have a chance to explain some of their thinking, but the teacher is the primary driver of conversations and arbiter of correctness. In class discussions, student ideas are not explored or built upon.
   - The teacher refers to student thinking, perhaps even to common mistakes, but specific students’ ideas are not built on (when potentially valuable) or used to address challenges (when problematic).

3. Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for building a coherent view of mathematics.
   - The teacher’s hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.
   - The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; OR what appear to be established participation structures result in such engagement.
   - Students explain their ideas and reasoning. The teacher may ascribe ownership for students’ ideas in exposition, AND/OR students respond to and build on each other’s ideas.
   - The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.
### Individual Work

Student seat work is coded as N/A unless the teacher is actively circulating through the classroom and consulting with students on an ongoing basis. Note that with a stationary camera it is impossible to see individual student work. Hence, unless there is evidence from the conversation, one cannot discern student errors.

<table>
<thead>
<tr>
<th>The Mathematics</th>
<th>Cognitive Demand</th>
<th>Access to Mathematical Content</th>
<th>Agency, Authority, and Identity</th>
<th>Uses of Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>How accurate, coherent, and well justified is the mathematical content?</td>
<td>To what extent are students supported in grappling with and making sense of mathematical concepts?</td>
<td>To what extent is there equitable access to meaningful participation for all students?</td>
<td>To what extent are students the source of presented ideas; do students respond to presented ideas?</td>
<td>To what degree does the teacher monitor and help students refine their thinking as he or she circulates through the class?</td>
</tr>
<tr>
<td>May be N/A if there are insufficient data; or...</td>
<td>May be N/A if there are insufficient data; or...</td>
<td>May be N/A if there are insufficient data; or...</td>
<td>May be N/A if there are insufficient data; or...</td>
<td>May be N/A if there are insufficient data; or...</td>
</tr>
</tbody>
</table>

1. Materials are aimed at “answer getting” without addressing underlying reasoning. Materials demand no more than applying familiar procedures or memorized facts. A significant number of students appear disengaged and there are no overt mechanisms to support engagement. Teacher shows or tells students how to do the mathematics, possibly correcting student work. Student ideas are not elicited or built upon. Teacher actions are limited to corrective feedback or encouragement.

2. Materials for student work provide some affordances for coherent mathematics, but teacher support is minimal and does not exploit them. Materials offer possibilities of conceptual richness or problem solving challenge, but teaching interventions tend to "scaffold away" the challenges. Students appear to be working, but there are no clear mechanisms for students who want or need support or attention to receive it. One-on-one interactions give students the opportunities to talk about their ideas and/or provide access to varied ways to engage in the mathematics. Individual interactions provide opportunities for students to discuss their thinking, and teacher responses address such thinking explicitly (not simply correcting student work).

3. The teacher’s interventions with individual students support a coherent and connected view of the mathematics. The teacher’s hints or scaffolds support students in “productive struggle” in building understandings and engaging in mathematical practices. Teacher’s and/or surrogates’ attention is clearly and widely available for those students who want it, resulting in access to the mathematics. A score of 3 is not coded unless the student has ample opportunity and agency to develop his/her idea interacting with the teacher, OR the teacher takes the student idea up for class discussion right after individual work ends. The teacher solicits student thinking and subsequent discussions respond to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

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Appendix R

Pruitt's Student Artifacts: Sample of Letters to Future Students

**Mathematical Design**

<table>
<thead>
<tr>
<th>Dear Future Math XXX students,</th>
</tr>
</thead>
<tbody>
<tr>
<td>This class will be unlike any math class you have ever taken. I never thought I would be able to see art in math but after this class my entire view has changed. Math can be used to make beautiful art and you will use the math concepts learned in this class to create your own pieces. Some advice I have for this class is to do all the work given because there is no busy work and it will greatly benefit you and your development in the class. Thinking back to the beginning of the semester, there are a few things I wish I knew about this class before taking it.</td>
</tr>
<tr>
<td>One thing is to save absolutely every art piece you make throughout the semester. This includes drafts, experimental pieces, and absolutely anything you create. It will be very useful for you at the end of the semester. Another thing is to ask a lot of questions if you didn't understand something. Professor _____ has always been very understanding and patient. It is much better to ask and learn than to pay for it later. Even though I never really enjoyed math, I really loved this class and I hope that you will too!</td>
</tr>
</tbody>
</table>

| When I first entered the class, I felt like I was so engaged within the topic just because the class is unique. Learning math and art into one is very original and something that I have not heard of doing better either. So combining them together became very intriguing, and followed my thoughts. I would tell any student that has creativity to take this class. An incoming student could potentially change their views on art after this class, just like it happened to me. You learn to create art in a way you have never, so it is something to look forward to. The math part in this class however, one has to pay attention because if not, they will end up getting lost and will have to do heavy research outside of class. I would also tell someone to look at other pieces of works done by other people because it can [be used] as inspiration for future projects and will definitely help you in creating art for yourself. |

| Hello, I'm a student from Math _____ and I will be giving suggestions for this class and explaining my past experiences. First off, I know the class might seem a little difficult at first since many of you haven't ever used math to create art but I promise after a while you will enjoy it. When I first started the class I was a little confused because of the many equations and formulas we had to use but after you actually try to explore those equations and graphs, I promise it will get easier. Something that I would've liked to know at the beginning is how important your participation is for class and for campus wire as well. It's a part of your grade and using will actually be very beneficial since you can ask as many questions and help others out. |

| Some detailed suggestions I will provide is first getting highly familiar with Desmos. Desmos is very crucial for this class. Everything you will do and present will revolve around it but you also have to get familiar and comfortable with the formulas and equations. Something I did was play around a lot on Desmos and with the formulas because you get to physically see what changing the numbers or variables in an equation does to the design. Another thing is attending office hours, Professor _____ is very understanding and helpful so attending and participating in office hours will really help. And finally, reaching out to other classmates. I know at first it will be hard and trust me I know because it look me sometime to actually reach out, but everyone is in the same boat as you, so don't worry about it. After I reached out to someone, I was actually able to receive and provide help which was very imperative. I hope these suggestions have helped out and I really think you guys will enjoy Math with Professor _____! Good luck guys! |
### Calculus II

You're probably required to take Mathematics for your major if you're taking Calculus II. If you're taking it for fun, I commend you. If you're lucky enough to take Professor____ class, you're in for a good time. Professor_____ cares, understands, communicates, and teaches. He takes extra measures to make sure every student has the opportunity to learn and succeed. Your friends in other math classes wish they registered for his class.

**Assessments.** Professor____ shifts way from the conventional methods of having one or more exams throughout the semester and then a final exam. Instead, he has a list of specific standards that all students should learn and understand throughout the semester. What makes this important and effective is the emphasis he is now able to put on actually understanding why certain rules and methods to answering a problem are what they are. On top of that, if you did poorly on an assessment, you have infinite opportunities to correct the grades you receive for the standards - so long as you communicate with him.

**Communication.** Professor____ is very good at communicating with and understanding each student. If you feel behind or left out, reach out to him and see him during his office hours. I'm certain he loves to chat as well. Besides, communication will ensure success in his class since you'll probably need to do at least one or two reassessments. (I know I'm not the only one in the world who needed to do them.) Aside from himself, Professor____ is very insistent that students work and study together. I've definitely seen a change in my studying and ability to understand the material after understanding group dynamics. Group work is for your own good. It's important to be able to understand teach the material you must know for his class.

**Final Project.** Not only does Professor____ care about you and the material he's covering, he goes as far as to even have a really cool final project for the end of the semester. It's not difficult or extremely time consuming at all, and you'll have a group to do it with.

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Welcome to Math class! This is a very cool class, even though it might not seem like it at the beginning. In the end it will all make sense to you when you get to do your class project and actually see and hold the math you are going to learn in your own hands. That feeling was magical to me, and I hope you will feel the same. You will learn in this class that calculus has a wide range of applications in your daily life. Derivatives and Integrals are so important, they are an essential part [of] what is building our future. Artificial intelligence, Machine Learning, and Automation are big buzzwords we hear every day in the news and the algorithms that make these things work all involves Derivatives and Integrals. This class is cool!

One piece of advice I want to give you is that you should pick more than one person to be in your group for the project. Because when that other person decides not to come to class in the middle of your project, you get stuck with the project all by yourself, ALL of it! I wish I had known that earlier in the beginning, so my advice to you is, be more social, join a larger group, and make new friends! Also spending time practicing homework problems is very important too, otherwise at the end of the semester you will forget what you’ve learned and be overwhelmed by trying to relearn everything the day before your final exam, and it is impossible! I hope you enjoy this class and enjoy learning more calculus!

Best Wishes

Former MATH student
Appendix S

Sample Constructor Artifacts

Stan's Spreadsheet Activity on Statistical Data & Social Justice

Imagine there is a mean of 3 officer involved shootings with a standard deviation of .5 per million people per state in the US.

Draw out the chart and answer the questions
What percent of states have officer involved shootings between 2.5 and 3.5 per million people?
What percent of states have officer involved shootings of less than 2.5 per million people?
What percent of states have officer involved shootings between 2 and 4 per million people?
What percent of states have officer involved shootings of more than 4 per million people?
What percent of states have officer involved shootings between 1.5 and 4.5 per million people?
What percent of states have officer involved shootings of less than 1.5 per million people?

A normal curve means:

Skewed Right:

Skewed Left:
(Continued)

<table>
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<th>Job Title</th>
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<th>Median Income</th>
<th>Fatalities per 100,000</th>
<th>Deviation</th>
<th>Squared Deviation</th>
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</tbody>
</table>

What is the mean number of deaths that happen to workers in America? 

What is the standard deviation?

68% of jobs have between _____ and _____ deaths per 100,000

46 officers were intentionally targeted and died in 2016. Almost as many died from car accidents. Police suffered 28,740 nonfatal injuries which required a median of nine days off to recover. How many jobs in America were more dangerous than police officer this year?

Number of Jobs Between Fatality Rates

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Does this data show?</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Is the mean pulled to the right or left?</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Is the curve right skewed or left skewed?</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

250
(Continued)


<table>
<thead>
<tr>
<th>Race, ethnicity</th>
<th>% of US population</th>
<th>% of US incarcerated population</th>
<th>National incarceration rate (per 100,000 of all ages)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White (non-Latino)</td>
<td>64</td>
<td>94,650 per 100,000</td>
<td></td>
</tr>
<tr>
<td>Latino</td>
<td>16</td>
<td>19,831 per 100,000</td>
<td></td>
</tr>
<tr>
<td>Black</td>
<td>13</td>
<td>44,230 per 100,000</td>
<td></td>
</tr>
</tbody>
</table>

Are White (non-Latino) inmates over or under-represented in the U.S. Jail and Prison systems?

Are Latino inmates over or under-represented in the U.S. Jail and Prison systems?

Are Black inmates over or under-represented in the U.S. Jail and Prison systems?


According to the Washington Post, there were 692 people who died as a result of officer involved shootings in 2018. Below is the ethnic breakdown:

<table>
<thead>
<tr>
<th>Race, ethnicity</th>
<th>% of death as a result of officer involved: Total Fatalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>White (non-Latino)</td>
<td>370</td>
</tr>
<tr>
<td>Latino</td>
<td>158</td>
</tr>
<tr>
<td>Black</td>
<td>235</td>
</tr>
<tr>
<td>Other</td>
<td>39</td>
</tr>
<tr>
<td>Unknowns</td>
<td>202</td>
</tr>
</tbody>
</table>

Are White (non-Latino) people over or under-represented in officer involved shootings?

Are Latino people over or under-represented in officer involved shootings?

Are Black people over or under-represented in officer involved shootings?

Here is what we found:

<table>
<thead>
<tr>
<th>Race, ethnicity</th>
<th>% of US population</th>
<th>% of US incarcerated population</th>
<th>% died as a result of officer involved shooting</th>
</tr>
</thead>
<tbody>
<tr>
<td>White (non-Latino)</td>
<td>64</td>
<td>94,650 per 100,000</td>
<td>27.04%</td>
</tr>
<tr>
<td>Latino</td>
<td>16</td>
<td>19,831 per 100,000</td>
<td>15.82%</td>
</tr>
<tr>
<td>Black</td>
<td>13</td>
<td>44,230 per 100,000</td>
<td>23.52%</td>
</tr>
</tbody>
</table>

What reasons can you think for these different numbers?
Proving the Pythagorean Theorem

Use what you have learned from the tilted squares problem to do the following:

1. Explain clearly and carefully how you know that the two shaded areas are equal.

2. Write down the shaded areas in terms of the lengths $x$ and $y$.

3. If the two shorter sides of a right triangle have lengths $x$ and $y$, what is the length of the longest side? (This is called the hypotenuse).
Square Areas (revisited)

The dots on the grid are all one unit apart.

1. The square shown here can be described as a 7 by 6 square. Find its area.
   Show all your reasoning

2. Draw a 7 by 5 square.
   Find its area.
   Show all your reasoning.

3. Sketch a 7 by \( y \) square.
   Find its area in terms of \( y \).
   Show all your reasoning.
Appendix T

Sample Constructor Artifacts

Priscilla’s Proof Writing Assignment: Student Work and Teacher Feedback

Writing Assignment #1

Either you or your writing partner should login to ShareLaTeX, click this link: Modern Algebra Template, then copy the project to your account (Menu > Copy Project) and share it with the other writing partner. Writing assignments are graded holistically out of 15 points: 5 for logic, 5 for math, and 5 for style.

p. 9  #7, 10, 13(a), 15, 16

For #7, 10 and 13(a), please give a formal induction proof, similar to the one in Example 1.1 on p. 5. In particular, you must explicitly

– define the set \( X \),

– establish the base case (i.e. show that \( 1 \in X \)),

– state the induction hypothesis (“Let \( n \geq 1 \) and assume \( k \in X \) for all \( 1 \leq k < n \)),

– indicate where the induction hypothesis is used to show \( n \in X \).

Your answer to #15 should be a sentence or two, not a formal proof.

For #16, you can start by letting \( Y \) be a nonempty subset of \( N \). Now let \( X = N \setminus Y \). The Principle of Mathematical Induction says that if \( X \) satisfies (1) and (2), then \( X = N \). What is the contrapositive of this implication? (Don’t forget De Morgan’s Laws!)
Proposition (p. 9, #7) Suppose that $a$ and $r$ are real numbers with $r \neq 1$. Then,

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a - ar^n}{1 - r}.$$ 

Proof: We proceed by induction. Let $P(n)$ be the statement that

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a - ar^n}{1 - r}.$$

Let $X = \{n \in \mathbb{N} | P(n) \text{ is true} \}$.

Base Case: When $n = 1$,

$$a + ar^{1-1} = a + a = \frac{a - ar^1}{1 - r} = \frac{a - a}{1 - r} = \frac{0}{1 - r} = 0.$$

Thus, $P(1)$ is true.

Induction Hypothesis: Let $n \in \mathbb{N}$ and $n \geq 1$. Assume $P(k)$ is true for all $k < n$, $k \in \mathbb{N}$. That is,

$$a + ar + ar^2 + \cdots + ar^{k-1} = \frac{a - ar^k}{1 - r},$$

for all $k < n$.

Induction Step: For $n$ as above, observe,

$$a + ar + ar^2 + \cdots + ar^{n-1} = (a + ar + ar^2 + \cdots + ar^{n-2}) + ar^{n-1}.$$

by the associative property of addition. By the induction hypothesis,

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a - ar^{n-1}}{1 - r}.$$

Thus,

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a - ar^{n-1}}{1 - r} + ar^{n-1}.$$

$$= \frac{a(1 - r) + ar^{n-1}(1 - r)}{1 - r}.$$

$$= \frac{a - ar^n + ar^{n-1} - ar^n}{1 - r}.$$

$$= \frac{a - ar^n + ar^{n-1} - ar^n}{1 - r}.$$

$$= \frac{a - ar^n}{1 - r}.$$

Therefore, $X = \mathbb{N}$. \qed
Proposition (p. 18, #10). A complete graph on $n$ points has exactly $\frac{n(n-1)}{2}$ lines. [Let $n \in \mathbb{N}$ and $n \geq 3$.]

Proof. We proceed by induction. Let $P(n)$ be the statement that a complete graph on $n$ points has exactly $\frac{n(n-1)}{2}$ lines. Let $X = \{n \in \mathbb{N} \mid P(n) \text{ is true}\}$.

Base Case: When $n = 3$,

$$\frac{3(3-1)}{2} = 3.$$

Since there are 3 lines that connect the 3 points in a triangle, $P(3)$ is true.

Induction Hypothesis: Let $n \in \mathbb{N}$ and $n > 3$. Assume $P(k)$ is true for all $k < n$, $k \in \mathbb{N}$. That is, for a graph with $k$ points, there are:

$$\frac{k(k-1)}{2}$$

lines.

Induction Step: For $n$ as above, observe that for a graph with $n$ points, there are $(n-1)$ more lines than a graph with $n-1$ points. Thus, by the induction hypothesis, there are:

$$\frac{(n-1)(n-2)}{2} + (n-1).$$

lines for a graph with $n$ points. Observe,

$$\frac{(n-1)(n-2)}{2} + \frac{n-1}{2} = \frac{(n-1)(n-2) + 2(n-1)}{2}$$

$$= \frac{n(n-1)}{2}.$$

Therefore, $X = \{3, 4, 5, \ldots\}$.

\[ \square \]

Proposition (p. 18, #13(a)). When considering the Fibonacci sequence $(a_n)$,

$$(a_{n+1})/(a_{n-1}) = (a_n)^2 + (-1)^n.$$

Proof. We proceed by induction. Let $P(n)$ be the statement that

$$(a_{n+1})/(a_{n-1}) = (a_n)^2 + (-1)^n.$$

Let $X = \{n \in \mathbb{N} \mid P(n) \text{ is true}\}$.

Base Case: When $n = 2$,

$$\frac{a_2}{a_0} = \frac{1}{1} = 1.$$ 

Recall that the Fibonacci sequence is defined by $a_1 = 1, a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$.

This helps your reader know where the sequence starts for you. (The book starts with $a_0 = 0, a_1 = 1$, and the recursion.)
**COMMENTS ON WA #1**

**UGI** Most propositions on this assignment are of the form UGI:

“For all \( n \in N \), if some hypothesis, then formula involving \( n \).”

This is because we are proving (via induction on \( n \)) that some formula/property holds for the natural numbers. The other numbers in the formula—like \( a, r \in R \) with \( r \neq 1 \), or \( a_0 = 1, a_1 = 1, a_n = an \cdot a_{n-2} \) for \( n \geq 2 \), are part of the hypothesis of the implication.

Given a proposition of this form, the first line of your proof should state itself: “Proof. Let \( n \in N \).” Then you may assume your hypothesis:

Assume \( a, r \in R \) with \( r \neq 1 \).

Assume \( \exists n \) such that the Fibonacci sequence and recall that \( a_0 = 1 \), \( a_1 = 1 \), and \( a_n = a_{n-1} + a_{n-2} \) for \( n \geq 2 \).

“Assume that \( L_n \) is the number of lines in a complete graph on \( n \) points.”

**P(n)** In many proofs on this assignment, authors equivalenced on the use of \( P(n) \). Sometimes in a single proof, \( P(n) \) was a statement (“the complete graph on \( n \) lines has \( \frac{n(n-1)}{2} \) points”), an expression (“\( P(n) = \frac{n(n-1)}{2} \)”) and even a graph (“\( P(n) \) is the complete graph on \( n \) points”). This is particularly bad form! If you fall into this category, try rewriting the proof without using \( P(n) \) at all. (Let \( X = \exists n \in N | a_n = a_{n-1} \cdot a_{n-2} \).)

(Continued)
The hand-of-cows induction "proof" fails because P(n) (2) is not true for n=2. Thus, it is not a valid proof by induction.

(There is perhaps even a creative argument that the statement fails for n=1. Is this an empty case? To my way of thinking, it just isn't.)

(Mostly I just wanted to draw a cow... 😊)

Style

• Don't use \times unless you're calculating a cross product or reminiscing about elementary school—the time has come to use \cdot for multiplication (and usually only when juxtaposition—e.g., "ab"—is unclear—e.g., "2 \cdot 6"). Similarly, don't use *.

• Equations—there's a whole page on this. Read on, dear reader—[EQNS] (next page)

• Sentences don't start in math mode ($$ or \[ \right \] ). Sentences end w/ punctuation (even after an eqn). Math that occurs in a sentence should appear in math mode. E.g., "the set $P(n)$ where $n-1 > s$..."

• When in doubt, read it out [loud]. If it sounds awkward, try to restate it out loud a few times.

• Proofread your work. Proofread your partner's work. Proofread twice or thrice.

• Err on the side of formality.
Often in math, we find ourselves with an initial equation that we would like to transform to prove something. For example, in Linear Algebra, we might start with $Ax = 0$ for a nonsingular matrix $A$ and wish to show that $x = 0$.

**WHAT NOT TO DO:**

1. $Ax = 0$
2. $A^{-1}(Ax) = A^{-1}0$
3. $A^{-1}Ax = 0$
4. $Ix = 0$
5. $x = 0$

1. The reason this is poor style is that the equations are changing both line-to-line (e.g., mult by $A^{-1}$) and left-to-right (e.g., manipulating on the LHS of Eqn (2) while using a different property on the RHS of Eqn (1) to obtain (2)). Thus, your reader needs to read up-to-down and left-to-right simultaneously to follow your argument... eek!

**TWO ALTERNATIVES**

1. **[I call this "one side at a time"]**

   Premultiplying both sides of the equation by $A^{-1}$, on the LHS we have
   
   $$A^{-1}(Ax) = (A^{-1}A)x$$
   
   $$= Inx$$
   
   $$= x.$$ 

   On the RHS, we have $A^{-1}0 = 0$.

   Thus $x = 0$.

2. **[I call this "unwinding a 2-column proof"]**

   Thus we have
   
   $$x = Inx$$
   
   $$= (A^{-1}A)x$$
   
   $$= A^{-1}(Ax)$$
   
   $$= A^{-1}0$$
   
   $$= 0.$$
Priscilla’s Conversational Midterm

Math Conversation with 🆘
3 messages

Dear [Name],

I’m writing to request your assistance in a class assignment for a new assignment in a new math course, Number Theory (Math 361). One focus of this course is on applications of number theory to cryptography. As a pilot run for an eventual Speaking Intensive course, we are broadly focusing on the oral communication of mathematical ideas. While talking about math to other mathematicians is obviously important, we also want to give students practice describing math to non-specialists. For this specific assignment, I have asked each student in my class to give an oral “conversation” (an informal presentation using pen and paper) of the RSA encryption system to a single member of the Hamilton College faculty or staff of their choice whose focus is outside math or computer science, and has chosen you.

If you are still interested in participating and would come to my office where he will have a 15 minute discussion with you. Whether you are very active asking questions or more passively listening would be up to you (and somewhat), I would only observe and would not be part of the discussion. After the 15 minutes, I would leave you and I would have a short discussion on your thoughts of his success at communicating the ideas. I would use a combination of your thoughts and my own observations to evaluate the conversation. Your total time commitment would be no more than 20 minutes.

We’ll be scheduling presentations for the week of April 3rd. If you are interested in participating and will be available, please send me a set of 30 minute time slots that work for you, and I will work with you to find a suitable time for us all to meet. The only times I am certain we cannot meet are:

- Monday: 6 am - Noon
- Tuesday: 9-11 am
- Wednesday: 9 am - Noon
- Thursday: 4-11 am
- Friday: 8 am - Noon

If you won’t be able to participate, please let me know so I can find another faculty or staff member.

I’ve included the instructions I gave to the students below. Please let me know your interest and availability at your earliest convenience.

Best,

Courtney

Instructions to Students:

The goal of your presentation is to communicate:
- what an asymmetric key cryptosystem is,
- what RSA encryption is and (broadly) how it works,
- why understanding such a system is a worthwhile number theoretic inquiry, and
- more generally, why such in-depth mathematical explorations are important as part of your liberal arts mathematics education.

In preparing your presentation, you should think carefully about the person you are presenting to. While some aspects of your presentation should be similar to how you might present to a more mathematically knowledgeable audience, you should probably emphasize different portions of RSA depending on your audience.

---

Hi 🆘

I would be happy to participate. I am available Monday from 1:30pm - 5:00pm and Friday from 1:30pm - 5:00pm as well. I will be out of town on Tuesday, Wednesday & Thursday.

Hopefully we can work it out.

---

[Name]
Head Football Coach
(Continued)

Priscilla’s Feedback-Student 1

Name: [Blank]

Instructions to Students:
The goal of your presentation is to communicate:

- what an asymmetric key cryptosystem is,
- what RSA encryption is and (broadly) how it works,
- why understanding such a system is a worthwhile number theoretic inquiry, and
- more generally, why such in-depth mathematical explorations are important as part of your liberal arts mathematics education.

In preparing your presentation, you should think carefully about the person you are presenting to. While some aspects of your presentation should be similar to how you might present to a more mathematically knowledgeable audience, you should probably emphasize different portions of RSA depending on your audience.

- Asymmetric Key: 3/3
  Loved the National Treasure reference!

- RSA: 2.5/3
  You did a fantastic job communicating RSA to a self-described non-math person. Your mom could give a pretty good overview of RSA after your presentation. You also knew your audience well and didn’t push too much of the arithmetic (like multiplication) when you saw that it wasn’t going to be too effective at getting the point across. I also thought you did a nice job describing the vulnerability (that went above and beyond the scope of the presentation). I wasn’t sure if you overstated the vulnerability a little bit, though -- now that it’s known, the implementation is much more carefully done. Your mom wasn’t totally convinced of the security of RSA when we chatted afterwards.

- Why learn this? 2/3
  This was the one part of your presentation that I felt fell really short of the mark. When I asked your mom why this might be a good thing for math students to learn, she had some answers of her own but couldn’t really point to anything in your presentation that helped her answer this question. However, she felt that RSA is important to understand, at least at the big-picture level, to feel okay about how online security can be handled mathematically.

- Answering questions 1/1
  Great job here.

Total: 8.5/10
Priscilla’s Feedback - Student 2

Instructions to Students:
The goal of your presentation is to communicate:
- what an asymmetric key cryptosystem is,
- what RSA encryption is and (broadly) how it works,
- why understanding such a system is a worthwhile number theoretic inquiry, and
- more generally, why such in-depth mathematical explorations are important as part of your liberal arts mathematics education.

In preparing your presentation, you should think carefully about the person you are presenting to. While some aspects of your presentation should be similar to how you might present to a more mathematically knowledgeable audience, you should probably emphasize different portions of RSA depending on your audience.

• Asymmetric Key: 3/3
  Chaise was very into the distinction between symmetric and asymmetric keys and had a good understanding when I quizzed him about it afterwards.

• RSA: 2.5/3
  I liked the internet pirate picture a lot -- it gave a good understanding of what RSA is trying to beat (an observer that knows the encryption protocol but not the private key). I also thought that you addressed Chaise’s questions well and clarified any misunderstandings he had about the public key somehow including enough information to give away the private key. I also thought your example about writing a paper and figuring out the day of the week it would be due was a super way to describe modular arithmetic. Your example went over well with Chaise, too. The only thing that didn’t quite come across was how hard factoring a large number is. Chaise seemed to think it was hard but feasible to crack a public modulus in a few months (it’s more like “basically infinite” for a regular (powerful) computer running even the best factoring algorithms).

• Why learn this? 3/3
  You did a great job here. When I asked Chaise why this is a good thing for math students to learn about and present, he used words like “fascinating” and “feel more secure knowing why it works” and said it got him excited about math. (We definitely spent longer than I expected talking about math in general after you left the room!)

• Answering questions 1/1
  Excellent job here.

Total: 9.5/10
Paul’s College Algebra Activity on Defining Functions

Activity [redacted] Name:

Definition of Function

One of the most important values we hold in mathematics is precision. In other words, we have to be precise with our language to make sure we’re all talking about the same thing.

Most of you are probably familiar with the term function from previous math classes. You also probably remember seeing notations like $f(x)$ and $g(x)$ used to represent functions. But do you remember the precise definition?

Here are a few mathematical expressions. Circle the ones that you think are functions, and write a brief explanation for why you think they are. For the ones you don’t circle, give a quick explanation of why they’re not functions. (Don’t worry, I’ll give you the definition on the next page — but I want you to try and pull it out of your memory first!)

1. $f(x) = 2x + 5$

2. $y = 1 - x$

3. $x^2 + y^2 = 9$

4. $x + 2y = 4$

5. $y^2 - x = 8$

6. $y = x^2 - 8$
Emma Sue’s Reflection on Her Artifacts

**Peer Review Chart and its Creation:** I felt that the students in general understood what was expected of them in the peer review comments but I felt that they were unable to effectively critique their partner’s homework because they lacked the proper vocabulary and questions. Therefore, I decided to create the Peer Review Chart that helped further elucidate the goals of peer review as well as provide students with sentence starters for effectively critiquing their peers. Since the students have had the Peer Review Chart, I have been able to raise my own expectations and feel confident that the students will meet them. There are still improvements to be made with the Peer Review process but we are getting there.

**STEM Activities/Lessons:**

- Creating a 6-feet of distance shape with masking tape (which you know but let me know if you want more info). (I attached the instruction sheet for 4th and 8th so you can see the difference)
- Where do you see STEM in the world? (which you also know about but let me know if you want more info)
- Mathematician diversity board: I am having the students create little bio sheets about famous mathematicians and making sure to include people of all races and gender instead of just the classic white men.
- Math Autobiography Flipgrid: I had the students at the start of the year complete a math autobiography about their relationship with math. This required them to reflect and process about their own experience. My plan is to have the students complete the same activity their 8th grade year and compare the two videos to each other. I have attached the instructions to the email.
Emma Sue's Guidance for Peer Review in Mathematics

It's Time to Peer Review!

To be an effective PeerReviewer, you need to give successful feedback. Successful feedback is Kind, SPECIFIC, and Helpful.

The Goal of Peer Review is to actively engage with and understand your classmate's mathematical process.

1. Discover how they solved the question.
2. You are NOT the grader and it is NOT your role to decide if something is correct or wrong.
3. Comments like "good", "nice job", "not quite right", etc are not useful comments for your peer and should only appear once or twice.
4. There should be one comment for every question that has an answer.

You can use the following sentence starters as a way to more effectively review your peer's work.

Sentence Starters for Glows:

I like how you...
I like the way you included...
You did an excellent job at...
Your greatest strength is...
You were successful at...
The strategy you chose is great because...

Sentence Starters for Grows:

It might be helpful to...
Your response or answer may be more effective if you...
One suggestion would be to...
Be sure you remember to...
Your next steps might be...
I chose a different strategy because...
(Continued)

Emma Sue’s STEM Artifacts

6 Feet of Distance Activity

According to the CDC, “social distancing (or physical distancing) means keeping a safe space between yourself and other people. To practice social distancing, stay at least 6 feet away from other people.”

John is having a hard time thinking about staying 6 feet away from everyone and what that looks like. His friend, Tayshia, suggests thinking about an imaginary shape drawn on the ground whose sides are 6 feet away from him in every direction at all times. What shape do you think will be best to imagine drawn on the ground and why?

For example:

What is your shape?: _____________________

What is your prediction? Will this shape be a good reference for 6 feet from all sides? Will this shape be the best shape? __________

________________________________________

________________________________________

________________________________________
Using masking tape, tape out your shape around your desk/chair making sure each edge is 6 feet away.

What did you discover? What are the positives of using your shape (at least 2)? What are the negatives of using your shape (at least 2)? Was your prediction correct? Why or why not? Looking at the other shapes, which shape do you think is the best shape for thinking about 6 feet of distance? Why is this the best shape? Will there be any other shapes that could be better than the ones we tested today? Why or why not?

You can write out your answer below or submit a Flipgrid Response: Flipgrid info: [https://flipgrid.com/5b55b67c](https://flipgrid.com/5b55b67c)
Appendix U

TALIS 2018 Teacher Questionnaire

Organisation for Economic Co-operation and Development (OECD)
Teaching and Learning International Survey (TALIS) 2018

Teacher Questionnaire

[ISCED 2011 level x] or PISA schools

Main Survey Version
International English, UK Spelling

About TALIS 2018
The TALIS 2018 Survey is an international survey that offers the opportunity for teachers and principals to provide input into education analysis and policy development. TALIS is being conducted by the Organisation for Economic Co-operation and Development (OECD), along with more than 60 other countries, including the United States.

Data analysis for this survey will allow countries to identify other countries facing similar challenges and to learn from other policy approaches. School principals and teachers will provide information about issues such as the professional development they have received, their teaching tasks, and the support, materials, and facilities they perceive they need to perform their work and the feedback and recognition they receive about their work, and various other school leadership, management, and teacher issues.

In the TALIS study, it is our intention to have a picture of the different educational practices in all the participating countries. Countries and individuals may differ in their educational attempts, the way they organise an individual's expertise to describe their work and opinion as accurately as possible.

Being an international survey, it is possible that some questions do not fit very well within your national context. In these cases, please answer as best as you can.

Confidentiality
All information that is collected in this study will be treated confidentially. While results will be made available by country and, for example, by the type of school within a country, you are guaranteed that neither you, this school, nor your personnel will be identified in any report of the results of the study. Participation in this survey is voluntary and any individual may withdraw at any time.

About the Questionnaire
- When questions refer to this school, we mean by school national school definition.
- Every question should take approximately 15 to 60 minutes to complete.
- Guidelines for answering the questions are provided in the survey. Most questions can be answered by marking one or more appropriate options.
- When you have completed this questionnaire, please see the detailed instructions provided in the last section of the questionnaire.
- If you have any questions about the questionnaire or the study, you can reach us by using the following contact details: National centre contact information, phone number, and preferably a web address.

Thank you very much for your participation!
Background and Qualification

These questions are about your prior education and the time you have spent in teaching. In responding to these questions, please mark the appropriate choice(s) or provide figures where necessary.

1. Are you female or male?
   - [ ] Female
   - [ ] Male

2. How old are you?
   Please write a number.
   ______ Years

3. What is the highest level of formal education you have completed?
   Please mark one choice.
   - [ ] Below ISCED 2011 Level 3
   - [ ] ISCED 2011 Level 3
   - [ ] ISCED 2011 Level 4
   - [ ] ISCED 2011 Level 5
   - [ ] ISCED 2011 Level 6
   - [ ] ISCED 2011 Level 7
   - [ ] ISCED 2011 Level 8

4. How did you receive your first teaching qualification?
   - [ ] A "regular concurrent teacher education or training programme" grants future teachers a single credential for studies in subject matter content, pedagogy, and other courses in education during the first period of post-secondary education.
   - [ ] A "regular consecutive teacher education or training programme" requires future teachers to complete two phases of post-secondary education: university education with the focus on subject-matter and a second phase with the focus on pedagogy and practical experience.
   Please mark one choice.
   - [ ] A "regular concurrent teacher education or training programme"
   - [ ] A "regular consecutive teacher education or training programme"
   - [ ] A "dual-track or specialised teacher education or training programme"
   - [ ] Education or training in another pedagogical profession
   - [ ] Subject-specific education or training only
   - [ ] I have no formal qualification related to the subject I am teaching or to any type of pedagogical education. ➔ Please go to Question [7].
   - [ ] Other

5. When did you complete the formal education or training that qualified you to teach?
   An approximate year is sufficient.
   Please write in a number:
   ______
9. What is your employment status as a teacher at this school?
   Please mark one choice.
   □ Permanent employment (an on-going contract with no fixed end point before the age of retirement)
   □ Fixed-term contract for a period of more than 1 school year
   □ Fixed-term contract for a period of 1 school year or less

10. What is your current employment status as a teacher, in terms of working hours?
    Please consider your employment status at the school and for all of your teaching employment together.
    Please mark one choice in each row.
    
    | Full-time (more than 50% full-time) | Part-time (50% to 20% of full-time) | Part-time (20% to 10% of full-time) | Part-time (less than 10% of full-time) |
    |-----------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
    | Yes                               | Yes                                 | Yes                                 | Yes                                 |
    | No                                | No                                  | No                                  | No                                  |

11. How many years of work experience do you have, regardless of whether you worked full-time or part-time?
    Do not include any extended periods of leave such as maternity/paternity leave.
    Please write a number in each row. Write 0 (zero) if none.
    Please round up to whole years.
    a) ______ Years(s) working as a teacher at this school
    b) ______ Years(s) working as a teacher at other school(s)
    c) ______ Years(s) working in other education roles, not as a teacher (e.g. as a university lecturer, nurse)
    d) ______ Years(s) working in other non-education roles

12. Do you currently work as a teacher of [CSEC 2011 level] 15-year-olds at another school?
    Please mark one choice.
    □ Yes
    □ No → Please go to Question [14].

13. If ‘Yes’ in the previous question, please indicate at how many other schools you currently work as a [CSEC 2011 level] teacher/teach to 15-year-old students.
    Please write a number:
    □ □ School(s)

14. Across all years [CSEC 2011 level] classes/classes where most students are 15 years old at this school, how many are special needs students?
    ◐ ‘Special needs’ students are those for whom a special learning need has been identified because they are mentally, physically, or emotionally disadvantaged. Often these will be those for whom additional work on planning, materials, or financial have been provided to support their education.
    Please mark one choice.
    □ None
    □ Some
    □ Most
    □ All
15. Were the following subject categories included in your formal education or training, and do you teach them during the current school year to any 18-year-old student in this school? (Continued)

- Reading, writing and literature
  - Includes reading and writing (oral and written) in the mother tongue, the language of instruction, or a second language (for non-native speakers), foreign languages, public speaking, literature.

- Mathematics
  - Includes general, discrete, statics, statistics, geometry, algebra, etc.

- Science
  - Includes general, discrete, physics, chemistry, biology, environmental science, agriculture, etc.

- Social studies
  - Includes historical, political, social, economic, cultural, environmental, legal, ethical, etc.

- Modern foreign languages
  - Includes language different from the language of instruction.

- Technology
  - Includes computer science, Internet, programming, electronics, robotics, etc.

- Physical education
  - Includes physical education, gymnastics, dance, health, etc.

- Religion and ethics
  - Includes religious education, ethics, moral behavior, etc.

- Practical and vocational skills
  - Includes technical skills (preparation for a specific occupation), business, domestic, agriculture, etc.

16. During your most recent complete calendar week, approximately how many 60-minute hours did you spend in total on tasks related to the day job at this school? (Continued)

- Include time spent on teaching, planning, grading, collaborating with other teachers, participating in staff meetings, participating in professional development and other work tasks. Also include time that took place during evenings, weekends, or other out of class hours.

- A complete calendar week is one that you are scheduled to teach, work, attend meetings, etc.

- Please round to the nearest whole hour.

17. Of this total, how many 60-minute hours did you spend on teaching at this school during your most recent complete calendar week?

- Please only count actual teaching time.

- Total actual teaching time will be recorded in the next question.

- Please round to the nearest whole hour.

18. Approximately how many 60-minute hours did you spend on the following tasks during your most recent complete calendar week, in your role at this school?

- Include tasks that took place during meetings, evenings or other out of class hours. Include all time spent teaching, as this was recorded in the previous question.

- Rough estimates are sufficient.

- If you did not perform the task during the most recent complete calendar week, write 0 hours.

- Please round to the nearest whole hour.

- Individual planning or preparation of lessons either at school or out of school.

- Team work and dialogue with colleagues within this school.

- Marking and grading of student work.

- Counseling students (including student supervision, monitoring, virtual counseling, career guidance, etc).

- Participation in school management.

- General administrative work (including communication, paperwork, and other clerical duties).

- Professional development activities.

- Communication and cooperation with parents or guardians.

- Engaging in extracurricular activities (e.g., sports and cultural activities after school).

- Other tasks
Professional Development

In the section, "professional development" is defined as activities that aim to develop an individual's skills, knowledge, expertise, and other characteristics in a sector.

Please only consider professional development you have undertaken after your initial education or training.

19. Did you take part in any induction activities?

Induction activities are designed to support new teachers' introduction into the teaching profession and to introduce teachers to the work environment and to support experienced teachers who are new to a school, and they are either organized in formal, structured programmes or informally arranged as separate activities.

Please mark as many choices as appropriate in each row.

- [ ] I took part in a formal induction programme
- [ ] I took part in informal induction activities

If you did not answer 'Yes, at this school' to either a) or b) please go to Question 20.

20. When you began work at this school, were the following provisions part of your induction?

Please mark one choice in each row.

- [ ] Counselling/mentoring
- [ ] Online courses/lectures
- [ ] Professional development
- [ ] Planned meetings with principal and other experienced teachers
- [ ] Supervision by principal and experienced teacher
- [ ] Networking/abundance with other new teachers
- [ ] Team teaching with experienced teachers
- [ ] Portfolios/classroom
- [ ] Induction teaching load
- [ ] General administrative introduction

If you answered 'No' to all of the above, please go to Question 21.

21. Are you currently involved in any mentoring activities as part of a formal arrangement at this school?

Mentoring is defined as a support structure in schools where more experienced teachers support less experienced teachers. This structure might involve all teachers in the school or only newer teachers.

Please mark any choice in each row.

- [ ] I currently have an assigned mentor to support me.
- [ ] I am currently an assigned mentor for one or more teachers.

22. During the last 12 months, did you participate in any of the following professional development activities?

Please mark one choice in each row.

- [ ] Counselling/mentoring
- [ ] Online courses/lectures
- [ ] Supervision by principal and experienced teachers
- [ ] Planned meetings with principal and other experienced teachers
- [ ] Professional development
- [ ] Networking/abundance with other new teachers
- [ ] Team teaching with experienced teachers
- [ ] Portfolios/classroom
- [ ] Induction teaching load
- [ ] General administrative introduction

If you answered 'No' to all of the above, please go to Question 23.

23. Were any of the topics listed below included in your professional development activities during the last 12 months?

Please mark any choices in each row.

- [ ] Knowledge and understanding of my subject field(s)
- [ ] Pedagogical competencies in teaching my subject field(s)
- [ ] Knowledge of the curriculum
- [ ] Knowledge of the assessment
- [ ] ICT (information and communication technology) skills for teaching
26. Thinking of the professional development activity that had the greatest positive impact on your teaching during the last 12 months, did it have any of the following characteristics?

Please mark one choice in each row.

- a) It built on my prior knowledge.
- b) It adapted to my personal development needs.
- c) It had a coherent structure.
- d) It appropriately focused on content needed to teach my subjects.
- e) It provided opportunities for active learning.
- f) It provided opportunities for collaborative learning.
- g) It provided opportunities to practically use ideas and knowledge in my own classroom.
- h) It provided follow-up activities.
- i) It took place at my school.
- j) It involved most colleagues from my school.
- k) It took place over an extended period of time (e.g. several weeks or longer).
- l) It focused on innovation in my teaching.

27. For each of the areas listed below, please indicate the extent to which you currently use professional development.

Please mark one choice in each row.

- a) Knowledge and understanding of my subject(s) (e.g. pedagogical content knowledge, knowledge of the curriculum).
- b) Pedagogical competencies in teaching my subject(s) (e.g. effective lesson planning, classroom management).
- c) Knowledge of the curriculum (e.g. content, objectives, assessment).
- d) Student assessment practices.
- e) ICT (information and communication technology) skills for teaching.
- f) Student behaviour and classroom management.
- g) School management and administration.
- h) Approaches to individualised learning.
- i) Teaching students with special needs.
- j) Teaching in a multicultural or multilingual setting.
- k) Teaching cross-curricular skills (e.g. creativity, critical thinking, problem solving).
- l) Analysis and use of student assessments.

28. How strongly do you agree or disagree that the following present barriers to your participation in professional development?

Please mark one choice in each row.

- a) I do not have the pre-requisites (e.g. qualifications, experience, seniors).
- b) Professional development is too expensive.
- c) There is a lack of employer support.
- d) Professional development conflicts with my work schedule.
- e) I do not have time because of family responsibilities.
- f) There is no relevant professional development offered.
- g) There are no incentives for participating in professional development.

No need at all
Low need
Moderate need
High need

(Continued)