Higher Volatility with Lower Credit Spreads:
The Puzzle and Its Solution

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ABSTRACT

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This dissertation explains the puzzling negative relationship between changes in stock volatility and credit spreads of corporate bonds. This relationship has been encountered in some empirical studies but has remained unexplained in the theoretical literature, which unanimously suggests the opposite relationship. This dissertation shows that this negative relationship can be produced by the dynamic endogenous asset composition of borrowing firms. On the one hand, higher asset volatility corresponds to lower future volatility of the firm’s investments and lower credit spreads if the firm can reallocate resources optimally. On the other hand, short-term stock volatility corresponds to the current allocation of resources and thus increases with asset volatility. The combination of these two effects produces the negative relationship between changes in stock volatility and credit spreads.

The empirical part of the dissertation shows that the relationship between changes in stock market volatility and credit spreads of long-term, high-quality corporate bonds (controlling for other variables) is negative, robust, and economically significant. Consistent with the predictions in this dissertation, the corresponding regression coefficient is a U-shaped function of the credit quality of the bonds. In addition, the dissertation shows that the relationship changes its sign in distressed market conditions and that a combination of normal and distressed market conditions can produce erroneous results.
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1 Introduction

The existing theoretical literature uniformly suggests a positive relationship between changes in stock volatility and credit spreads of corporate bonds, in accordance with the perception that higher volatility corresponds to a higher probability of default.\(^1\) Some empirical studies, however, have obtained results indicating a negative relationship (controlling for other variables) in the case of investment grade debt.\(^2\) This relationship remains unexplored and unexplained, since no existing model can produce such a relationship. Using a comprehensive set of bond transaction data, the present study empirically confirms that in the case of long-term, high-quality corporate debt, the negative relationship between changes in stock volatility and credit spreads is robust and economically significant.\(^3\) The paper also provides a model to explain this negative relationship.

This paper extends Leland’s (1994) capital structure framework and incorporates endogenous asset composition using the approach in Merton’s (1969) model. Leland (1994) presents a capital structure model with long-term debt financing and strategic bankruptcy. In that model, the firm chooses the optimal structure of the right-hand side of the balance sheet (liabilities and owners’ equity). One of the assumptions in that model is that the left-hand (asset) side of the firm’s balance sheet is fixed: all funds are invested in the risky project.

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\(^1\) See, for example, Goldstein, Ju, and Leland (2001).

\(^2\) See Table VIII in Collin-Dufresne, Goldstein, and Martin (2001) and the discussion of the relationship between S&P volatility and credit spreads in the benchmark results on p. 2709 in Cremers et al. (2008).

\(^3\) Long-term debt has become especially important in the current ultra-low rate environment according to observations in the Financial Times: "As developed world rates slide, leaving $13tn of bonds trading at negative yields, investors led by insurers and pension funds are fanning out in search of income, encouraging companies and governments to borrow at unprecedented maturities." (Moore, 2016).
The present study relaxes this assumption using Merton’s (1969) model, which incorporates investment decisions but does not consider long-term borrowing. In the extended model, risk-averse agents (i) make investment and payout/consumption decisions as in Merton’s model and (ii) can borrow and declare bankruptcy as in Leland’s model. This model, in particular, predicts that, contrary to the general perception, credit spreads can be lower when asset volatility is higher.

The general perception is that the credit spread increases with volatility because higher asset volatility corresponds to a higher probability of default and associated losses. Essentially, equity holders have a (put) option to default. The value of this option is higher when volatility is higher. Accordingly, risky debt is the same as the combination of the corresponding risk-free debt and a short position in the default option. Since the value of the option increases with asset volatility, the value of the debt should be lower and the credit spread should be higher when volatility is higher.

The above logic is correct in the context of models with static asset composition, but it does not hold true if borrowers take into account the riskiness of projects when they make investment decisions. If the composition of assets is determined by the borrower, the

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4 Several empirical papers show that firms’ investment and financial decisions are related to managers’ risk-aversion. See, for example, Bertrand and Schoar (2003), Panousi and Papanikolaou (2012), and Graham, Harvey, and Puri (2013).

5 I use payout and consumption interchangeably (and together as “payout/consumption”), since in the model, the firm’s payout corresponds to the consumption of the manager/owner who makes policy decisions. Similarly, I use agent to refer to the firm together with the manager/owner and do not distinguish between them before bankruptcy. I also use project and asset interchangeably, as investments in projects are assets on the firm’s balance sheet.
outcome can be very different. Wealthy, risk-averse borrowers act more conservatively when volatility is higher: they choose safer projects to the extent that the total volatility of their investment portfolio is lower. In addition, they reduce the outflow of funds (dividends in the case of firms) when they face higher volatility of assets. The net effect of this behavior is a lower probability of default, and therefore, lower credit spreads.

More conservative behavior in a riskier environment has been observed in various contexts. A salient example of this phenomenon is provided by Sweden. On September 3, 1967, the traffic in Sweden was switched from the left-hand side of the road to the right-hand side. Since people did not have any experience driving on the right-hand side, an increase in the number of car accidents should have been anticipated. On the contrary, the number of deaths on the road decreased and was approximately two times lower than the usual level for several weeks after the switch. Figure 1 shows the drop in the number of road accident deaths in Sweden. This evidence indicates that a riskier environment can actually correspond to less risk undertaken by agents. The present paper extends this idea to the investment decisions of economic agents.

Several empirical papers have documented more conservative investment decisions of

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6 Wealthy firms are firms that have the total value of asset much higher than the amount of debt.

7 This decrease resembles the Peltzman effect. Peltzman (1975) shows that drivers’ behavior is riskier when cars have protective devices required by the safety regulation, to the extent that "there is some evidence that regulation may have increased the share of this [death] toll borne by pedestrians and increased the total number of accidents" (p. 677).


9 The outcome was similar when Iceland switched driving from the left-hand side to the right-hand side of roads in 1968. See Wilde (1982) for references.
firms in more volatile environments. When volatility (measured using different approaches) is higher, firms scale back risky investments, as shown, for example, in Leahy and Whited (1996), Bulan (2005), Bloom, Bond, and Van Reen (2007), and Kellogg (2014). Moreover, firms hold more safe assets when volatility is higher. In particular, Bates, Kahle, and Stulz (2009) study cash holdings of US industrial companies and conclude that “cash ratios increase because firms’ cash flows become riskier” (p. 1985). In the most recent study, Alfaro, Bloom, and Lin (2016) show that higher volatility causes firms to reduce real investments, increase cash holdings, and cut dividends. In addition, Bertrand and Schoar (2003) find that managers matter for the policy decisions of firms, and more recent studies show that managers’ risk-aversion is related to investment decisions (Panousi and Papanikolaou, 2012) and to corporate financial policies (Graham, Harvey, and Puri, 2013). Bloom (2014) suggests that more cautious corporate investment decisions, when uncertainty is high, may be explained by the
incentives of top executives.

The investment decisions of risk-averse economic agents can be examined in the context of a simple two-period model. In this model, the agent has a constant absolute risk-aversion utility function; she invests in the first and consumes in the second period. The investment choice of the agent consists of the risk-free and the normally distributed, risky asset. The agent optimizes the expected utility. This optimization problem corresponds to the return-variance trade-off: the expected excess return versus the variance of the investment portfolio scaled by the risk-aversion coefficient. As the outcome of this trade-off, the optimal investment rule corresponds to a lower volatility of the investment portfolio when the asset volatility is higher. The reason is quite simple. Riskier investments are less attractive to the risk-averse agent. Therefore, when the volatility of the risky asset is higher, the agent scales back risky investments. Less risky investments mean lower expected excess returns. Accordingly, the volatility of the optimal investment portfolio has to be lower, because otherwise the return-variance balance would be violated.

A similar relationship between the volatility of the risky asset and the volatility of the investment portfolio holds in the infinite-horizon Merton (1969) model, in which the agent chooses the consumption rate and the allocation of wealth between risky and risk-free assets. As in the two-period model, the optimal proportion of wealth allocated to the risky asset is inversely proportional to the square of asset volatility. Accordingly, the volatility of the investment portfolio is inversely proportional to asset volatility, and thus, decreases with asset volatility. In addition, the consumption rate in the model also decreases with volatility.
(when the relative risk aversion (RRA) coefficient is greater than one as assumed hereafter).

To investigate the channel from asset volatility to credit spreads, the present study employs Merton’s (1969) investment portfolio allocation model to extend Leland’s (1994) capital structure model as illustrated in Figure 2. In the extended model, borrowers make investment and payout decisions taking into account their wealth (the value of their assets). When their wealth is high, the value of the option to default is low. Accordingly, the optimal investment and payout policies of wealthy borrowers are similar to optimal policies that avoid bankruptcy: to invest a part of the wealth in the risk-free asset to service the debt and to use the remaining wealth according to the optimal policies in Merton’s model. These policies lead to a lower volatility of the investment portfolio when the volatility of risky assets is higher. Additionally, the payout rate also decreases with volatility. The net result is a lower probability of default, and therefore, a lower credit spread of high-quality debt when asset volatility is higher.

The model predicts a negative relationship between asset volatility and credit spreads, but asset volatility is difficult to observe. However, asset volatility affects stock volatility that
Figure 3. Volatility Channels. The figure illustrates how asset volatility affects credit spreads and equity volatility in the model with endogenous asset composition.

can be observed on the market. To produce a testable implication, I examine the channel from asset volatility to equity volatility. If it takes even an infinitesimal time to reallocate resources to changes in asset volatility, then the instantaneous volatility of the investment portfolio increases with asset volatility. Accordingly, equity volatility increases when the volatility of the risky asset increases. Figure 3 illustrates the channels from asset volatility to credit spreads and from asset volatility to equity volatility. The combination of these channels produces a negative relationship between changes in equity volatility and credit spreads. In a nutshell, equity volatility is driven by the volatility of the current investment portfolio, whereas the credit spread depends on the volatility of investments during the lifetime of the bond and thus is less sensitive to adjustment delays.

Some evidence indicating a negative relationship between stock volatility and credit
spreads can be found in empirical papers, but remains unexplained in the finance literature. Collin-Dufresne, Goldstein, and Martin (2001) investigate the determinants of credit spread changes, expecting a positive relationship, but their regression model, estimated using bond transaction data, has a negative coefficient on changes in volatility measured by the VIX. Cremers et al. (2008) follow Campbell and Taksler (2003) using implied volatilities and also expect a positive relationship, but encounter negative regression coefficients. These negative coefficients remain unexplained.\textsuperscript{10,11} On the other hand, this evidence is consistent with implications of the presented model with dynamic endogenous asset composition.

The empirical part of this paper examines the relationship between changes in market volatility and credit spreads using the latest enhanced feed of the Trade Reporting and Compliance Engine (TRACE) data that covers over-the-counter (OTC) bond transactions. The results show that for high-quality (A rated and higher) long-term bonds,\textsuperscript{12} the relationship between changes in market volatility and credit spreads (controlling for other variables) is negative.\textsuperscript{13} This relationship is statistically and economically significant and robust to

\textsuperscript{10} The negative relationship in Collin-Dufresne et al. (2001) is obtained using a relatively small sample of 29 bonds and has been ignored, probably, because of the lack of theoretical explanations although the result is highly statistically significant. Cremers et al. (2008) acknowledge that the significant negative coefficient on S&P volatility is difficult to explain.

\textsuperscript{11} Note that some other empirical studies obtain positive regression coefficients using different bond sets, sample periods, and regression specifications. For example, Campbell and Taksler (2003) find, in most cases, a positive relationship between the standard deviation of daily stock index return and credit spreads. However, in some cases, the relationship is insignificant or significantly negative (see Table V).

\textsuperscript{12} Long-term debt allows for adjustment of asset composition that can be rigid in the short run. In the long run, asset composition can be adjusted through asset depreciation and reinvestment of revenue. In the short run asset disposition can be costly. In addition, as shown in Lamont (2000), implementing investment decisions takes time.

\textsuperscript{13} The control for other variables such as stock returns is important. The correlation between stock volatility and credit spreads is positive. This positive correlation is induced, in particular, by changes in the value
different sample splits. These results confirm the theoretical predictions.

The empirical part of the paper also shows that the relationship between changes in stock volatility and credit spreads can have the opposite sign in distressed conditions when the model assumptions are not satisfied. In the severely distressed market after the collapse of Lehman Brothers, the relationship was positive. It is shown that mixing observations corresponding to distressed and normal market conditions can produce misleading results. The regression coefficient on changes in volatility can be positive if the sample includes observation corresponding to the period of distress; it becomes negative as soon as the regression specification is corrected by interaction terms with the indicator of the distressed period. This may explain a positive relationship between volatility and credit spreads obtained in some empirical studies.

In addition to the negative relationship between changes in equity volatility and credit spreads of long-term, high-quality debt, the model predicts that the regression coefficient is a U-shaped function of the credit quality of bonds. The coefficient is negative for high-quality debt. It decreases in absolute value and becomes positive for low-quality debt, because the option to default becomes more valuable as the value of assets approaches the bankruptcy threshold. The absolute value of the coefficient also decreases as the value of assets increases to infinity, because the debt becomes essentially risk-free and thus insensitive to changes in asset (and therefore, stock) volatility. This U-shaped relationship is corroborated in the of assets: Lower values of assets correspond to higher stock volatility and higher credit spreads. Changes in leverage or stock returns provide controls for changes in the value of assets. The negative relationship between changes in volatility and credit spreads controlling for other variables can be explained by changes in asset volatility as shown in the present paper.
The regression coefficient is negative and very significant for A-rated bonds. For bonds rated A+ and higher, it is negative but smaller in absolute value. The absolute value of the coefficient also declines as the bond rating declines toward lower investment grades. For high-yield bonds rated from BB+ to B-, the coefficient is positive. Thus, consistent with the model's predictions, the regression coefficient becomes more negative as the credit quality of bonds increases till some threshold (from high-yield to A-rated) and becomes less negative thereafter.

1.1 Literature Review

The present paper is related to several intertwined strands in the corporate finance and asset pricing literature. The proposed theoretical model incorporates the dynamic asset composition derived from the literature on optimal portfolio allocation. In addition, this model includes risky debt studied in the capital structure literature. The paper provides a novel approach to structural credit models and explains several phenomena presented in earlier studies. The empirical part of the paper is related to the credit literature, and in particular, to papers that examine credit spreads of corporate bonds. The rest of this section provides a brief review of these strands of literature.

The model in the present paper is founded on the dynamic portfolio allocation and consumption model proposed by Merton (1969, 1971). A rigorous treatment of the optimal portfolio allocation and consumption problem is provided in Karatzas et al. (1986) and in later papers collected in Sethi (1997), which examine investment and consumption decisions
in the presence of bankruptcy at the zero-wealth level. Jeanblanc, Lakner, and Kadam (2004) add continuous debt repayment to the model and find a closed-form solution for the optimal default boundary. Following Magill and Constantinides (1976), several papers consider the portfolio allocation problem with transaction costs. In the case of proportional transaction cost, there is a non-trade region around Merton’s solution, as shown in Davis and Norman (1990), Dumas and Luciano (1991), and Shreve and Soner (1994). Guasoni, Liu, and Muhle-Karbe (2014) solve the optimal consumption and allocation problem for cash-flow generating risky assets. Their results show that it may be optimal never to sell risky assets, but instead to adjust the portfolio using generated cash, thus achieving the optimal asset composition without costly asset disposition.

The asset composition of a company is considered in another strand of literature founded on the work of Miller and Orr (1966), which investigates the dynamics of cash holdings (which are similar to investments in the safe asset in the context of the present paper) and risky investments generating stochastic cash flows. Hennessy and Whited (2005) develop a dynamic model with endogenous choices of leverage, distributions, and real investment. More recent studies extend the model and consider costly external financing. Recent notable papers in this strand of literature include Decamps et al. (2011), Bolton, Chen, and Wang (2011, 2014), Anderson and Carverhill (2012), Babenko and Tserlukevich (2013), Boot and Vladimirov (2015), Hugonnier, Malamud, and Morelec (2014), and Bolton, Wang, and Yang (2014). The main subject in this body of literature is corporate cash holdings that provide a liquidity buffer in the presence of costly disposition of productive assets and expensive
external financing. The present paper also considers investments in the risk-free and risky assets and extends the analysis to debt prices.

Another related strand of studies explores corporate capital structure, using Merton’s (1974) analytical framework for valuing debt contracts. Black and Cox (1976) augment this framework, in particular, with safety covenants and an endogenous default boundary. Leland (1994) proposes a model that incorporates taxes and bankruptcy costs and obtains the optimal capital structure. Leland (1998) combines capital structure and risky investment decisions. Goldstein, Ju, and Leland (2001) add a cash flow generating asset and dynamic borrowing to the model.\textsuperscript{14} This framework is extended in subsequent papers. An extensive review of the related literature is provided in Sundaresan (2013).\textsuperscript{15} The present paper is particularly related to the study of Chen, Miao, and Wang (2010) who consider borrowing decisions of risk-averse agents. Among more recent publications, Sundaresan, Wang, and Yang (2015) investigate dynamic investment, financing, and default decisions of a firm. Most of the papers in this body of literature assume that investments are lumpy and irreversible and that investors are risk neutral or sufficiently diversified. The present paper relaxes these assumptions and investigates implications of endogenous asset composition for equity volatility and debt prices, using the approach considered in Chang and Sundaresan (2005).

A number of empirical papers examine the relationship between uncertainty measured by asset volatility and corporate investment decisions. A review of the early literature is

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\textsuperscript{14}See Mauer and Triantis (1994) for an earlier dynamic model with endogenous investment and financing decisions.

\textsuperscript{15}See also Graham and Leary (2011) for a review of empirical studies.
provided in Carruth, Dickerson, and Henley (2000). The general conclusion in that paper is that higher uncertainty corresponds to lower capital expenditure. A similar conclusion is obtained in several subsequent papers (Bulan, 2005, Bloom, Bond, and Van Reenen, 2007, Stein and Stone, 2013, and Kellogg, 2014). Alfaro, Bloom, and Lin (2016) establish a causal relationship: Higher volatility causes firms to reduce real investments, increase cash holdings, and cut dividends. Most papers in this body of literature explain the lower investment activity by a higher value of the real option in the case of higher uncertainty. The present paper provides an alternative explanation for this phenomenon based on the risk-aversion of economic agents. Such explanation is considered in Bloom (2014). In addition, the present study extends the volatility-investment channel to debt prices, producing a novel prediction that higher asset volatility can correspond to lower credit spreads (controlling for other variables). This relationship produces predictions tested in the empirical part of the paper.

The empirical part of the paper builds on studies that investigate the credit spread of corporate bonds, and in particular, those that consider different factors that explain credit spreads. Among these studies, Elton et al. (2001) explore components of credit spreads; Collin-Dufresne, Goldstein, and Martin (2001) investigate determinants of credit spread changes; Campbell and Taksler (2003) examine the effect of idiosyncratic equity volatility on bond yields; Blanco, Brennan, and Marsh (2005) analyze the relationship between credit spreads and credit default swaps (CDS); Driessen (2005) decomposes bond returns using default, liquidity, and tax factors; Longstaff, Mithal, and Neis (2005) find that the majority
of the credit spread is related to default risk; Chen, Lesmond, and Wei (2007) reveal that liquidity is priced in credit spreads; Cremers et al. (2008) analyze the relationship between firm volatility and credit spreads; and Zhang, Zhou, and Zhu (2009) use high-frequency stock prices to explain the CDS premium. Ericsson, Jacobs, and Oviedo (2009) follow Collin-Dufresne et al. (2001) to study determinants of CDS premiums; Bao, Pan, and Wang (2011) find that liquidity becomes more important for credit spreads during the financial crisis, Acharya, Davydenko, and Strebulaev (2012) highlight the importance of the endogeneity of cash holdings for credit spreads. The present study is closely related to Collin-Dufresne et al. (2001). That study encounters cases of a negative relationship between changes in market volatility and credit spreads but leaves it unexplained. The present study analyzes more recent bond transaction data and confirms that the relationship is indeed negative and statistically and economically significant for long-term, high-quality debt. The present paper provides new insight into the origins of this relationship and shows how the creditworthiness of borrowers affects this relationship. In spirit, this paper is closer to results in Acharya, Davydenko, and Strebulaev (2012) that "suggest that theoretical and empirical studies of credit risk (and likely other areas of asset pricing) should account for endogeneity of corporate financial and investment policies" (p. 3574).

The paper is organized as follows. Section 2 provides a description of the model, which is solved in Section 3. Section 4 considers the implications of debt financing for investment decisions of economic agents and explains such phenomena as “flight to quality,” “gambling for resurrection,” and “bankruptcy for profit.” Section 5 examines how investment and payout
decisions affect debt prices and, in particular, shows that higher asset volatility can correspond to lower credit spreads. Section 6 establishes the relationship between asset volatility and stock volatility. The relationship between changes in stock volatility and credit spreads is tested using bond transaction data in Section 7. Section 8 concludes the paper.

2 Model

This section presents a dynamic partial equilibrium model that synthesizes the capital structure model proposed in Leland (1994) and Merton’s (1969) portfolio allocation model. The present model combines investment and payout/consumption decisions of economic agents with risky borrowing. The model links endogenous asset composition to debt prices. The economic agent in this model can be interpreted as an entrepreneur or a manager of a company who makes investment and payout decisions on behalf of the company, who is the equity holder,\(^\text{16}\) and whose consumption is tied to the company.\(^\text{17}\) In this paper, terms “agent”, “company”, and “firm” are used interchangeably, and “wealth” and “resources” mean the total value of assets when the company is not bankrupt.

\(^{16}\)Since the equityholders-managers agency problem is not considered in the paper, and due to homogeneity of the utility function, all results hold true if the agent owns a portion of equities and receive the corresponding portion of the payout stream.

\(^{17}\)Straight forward extensions of the model can include additional constant income streams from a saving account and/or salary (similar to wages in Merton, 1971). These income streams do not change conclusions and are omitted to keep the model parsimonious.
2.1 Investment Opportunities

As in Merton’s model, the investment opportunity set consists of the risk-free and the risky project/asset. The value of the risk-free asset, \( P^{(1)}_t \), grows at the constant rate \( r \):

\[ dP^{(1)}_t = rP^{(1)}_t \, dt. \]

The value of the risky asset, \( P^{(2)}_t \), follows the geometric Brownian motion:

\[ dP^{(2)}_t = \mu P^{(2)}_t \, dt + \sigma P^{(2)}_t \, dZ_t, \]

where the drift, \( \mu \), and volatility, \( \sigma \), are assumed to be positive constants, and \( Z_t \) is a standard Brownian motion.\(^{18,19}\) Parameters \( r, \mu, \) and \( \sigma \) correspond to after tax dynamics.

2.2 Borrowing

Similar to Leland’s model, the agent establishes a company at time zero. The company is funded by equity \( V^E \) and debt \( D.\(^{20}\) The debt contract delivers amount \( D \) at time zero and

\(^{18}\)The assumption of one risky asset is without loss of generality in the case of a constant opportunity set. If there are risky assets with constant drifts and volatilities, then due to the mutual fund separation theorem, the agent is indifferent between alternative 1, investing in risky assets and the risk-free asset, and alternative 2, a portfolio of risky assets and the risk-free asset.

\(^{19}\)Alternatively, assets can be specified as earning generating projects similar to Goldstein, Ju, and Leland (2001). Since asset composition and the payout rate can be freely adjusted in the current model, the process for value of assets is specified directly to keep the model parsimonious.

\(^{20}\)In this paper, I assume the given initial amount of equity. Similar results can be obtained if the total amount of initial investments (debt plus equity) is fixed.
requires the borrower to pay coupon rate $\bar{c}$ per unit of time thereafter. Coupon payments provide a tax shield on income. Thus, the effective payment rate for the borrower is $c = \bar{c} (1 - \tau)$, where $\tau$ is the tax rate.\footnote{More strictly speaking, since interest and equity incomes are subjects of taxation, $\tau$ corresponds to Miller’s (1977) effective tax benefit of debt $\tau = 1 - \left(1 - \tau_c \right) \left(1 - \tau_e \right) / (1 - \tau_i)$, where $\tau_i$ is the tax on interest income, $\tau_c$ is the corporate income tax, and $\tau_e$ is the tax on equity income.}

The company has the option to default on coupon payments and declare bankruptcy. Once bankruptcy is declared, the agent is free from debt but has lost some wealth and cannot raise risky debt after bankruptcy.\footnote{I assume that the absolute priority rule is violated. As stated in Franks and Torous (1989) “It does appear that deviation from absolute priority are rule rather than the exception” (p. 754). Similar conclusions are made in a number of empirical papers that investigate restructuring in and out of bankruptcy courts. See, for example, Eberhart, Moore, and Roenfeldt (1990), Weiss (1990), Franks and Torous (1994), Betker (1995), Tashjian, Lease, and McConnell (1996). Some more recent papers (for example, Ayotte and Morrison, 2009 and Bris, Welch, and Zhu, 2006) argue that the violation of the absolute priority is not so severe during 1990s and 2000s (see Hotchkiss et al. 2008 for the discussion of the literature). It can be shown that the key findings in the present paper hold if the absolute priority rule is satisfied, the debt has protective covenants on the value of assets, and the agent has an additional fixed income stream.} If the value of assets is $V_{T^B}$ right before bankruptcy at time $T^B$, the agent retains $\alpha V_{T^B} - K$ after bankruptcy, and the share of the borrower’s assets seized by the lender is $(1 - \alpha) V_{T^B}$. Thus, the borrowing contract is described by $(\bar{c}, \alpha, K, D)$, in which the proportion of split $\alpha$ and the fixed dead-weight cost $K$ imposed on the borrower represent the legal environment and bargaining power in the case of bankruptcy and cannot be altered by the agent.\footnote{The fixed cost increases the punishment for bankruptcy such that equityholders can be wiped out completely. The fixed cost is also consistent with observations that the direct cost of restructuring exhibits economies of scale in the case of bankruptcy (Warner, 1977 and Ang et al., 1982) and in the case of out-of-court restructuring (Gilson, John, and Lang, 1990). Note that main results in the present papers are valid when this cost is zero. In the presented model, the fixed cost is imposed on the borrower. Adding fixed cost imposed on the creditor does not change results and is omitted to keep the model parsimonious.} The amount of debt $D$ is determined by the value of future coupon payments $\bar{c}$. At time zero,
the agent chooses the coupon rate that corresponds to the optimal amount of debt for the chosen amount of equity.

Denote \( P^D (V_0, \bar{c}) \) the fair price of the debt contract that pays coupon rate \( \bar{c} \) and is issued by the company with the total value of assets \( V_0 \). This price is given by the risk-neutral valuation

\[
P^D (V_0, \bar{c}) = E^Q \left[ \int_0^{T_B} \bar{c} e^{-rt} dt + e^{-rT_B} (1 - \alpha) V_{T_B^-} \right],
\]

where \( E^Q \) is the expectation taken under the risk-neutral measure and \( T_B \) is the time of bankruptcy.\(^{25} \) The first term in the square brackets corresponds to coupon payments before bankruptcy, and the second term is the value recovered in the case of bankruptcy. Assuming that the lending market is competitive, the amount raised by issuing debt equals the fair price of the debt (i.e., the lender’s participation constraint is binding), and \( D \) satisfies the following fixed point condition

\[
P^D (V^E + D, \bar{c}) = D.
\]

### 2.3 Dynamics of the Value of Assets

Denote \( V_t \) the total value of assets held by the company at time \( t \). The initial value of assets is equal to the initial equity plus debt,

\[
V_0 = V^E + D.
\]

\(^{25} \)Hereafter, I use the convention \( e^{-rT_B} = 0 \) when \( T_B = \infty \).
The agent chooses proportion $\pi_t$ invested in the risky asset and invests the rest in the risk-free asset. The agent also chooses the payout/consumption rate $C_t$. Additionally, before bankruptcy, the agent pays effective coupon rate $c$ on the debt. Therefore, the dynamics of the total value of assets are given by

$$dV_t = ((r(1 - \pi_t) + \mu \pi_t) V_t - C_t - c) \, dt + \sigma \pi_t V_t dZ_t.$$  \hspace{1cm} (6)$$

In this equation, the drift terms correspond to the expected return on investments minus payout to equity holders and coupon payments, and the volatility term is induced by investments in the risky asset. After bankruptcy, the agent makes investments,$^{26}$ but there are no coupon payments. Post bankruptcy, the total value of assets evolves according to

$$dV_t = ((r(1 - \pi_t) + \mu \pi_t) V_t - C_t) \, dt + \sigma \pi_t V_t dZ_t.$$ \hspace{1cm} (7)$$

2.4 Optimization Problem

Ex-post-borrowing, the agent chooses payout $C_t$, allocation $\pi_t$, and bankruptcy time$^{27}$ $T^B$ to maximize the expected utility (given the initial capital $V_0$ and debt coupon $\bar{c}$). Therefore,

---

$^{26}$To keep the model parsimonious, I assume the same opportunity set before and after bankruptcy.

$^{27}$At each time $t$, the agent decide to declare bankruptcy ($T^B = t$) or not.
the ex-post-borrowing indirect utility function is

\[
U (V_0, \bar{c}) = \sup_{A(V_0, \bar{c})} \left\{ E \left[ \int_0^\infty e^{-\delta t} u(C_t) dt \right] \right\},
\]

subject to budget constraints (6) and (7), respectively, before and after bankruptcy, where \( A (V_0, \bar{c}) \) is the set of admissible controls \((C_t, \pi_t, T^B)\) and \( u(C) \) is the utility function. In this paper, I consider the constant relative risk aversion (CRRA) utility function, \( u(C) = \frac{C^{1-\gamma}}{1-\gamma} \) with \( \gamma > 1 \), although many subsequent results hold true in the case of general utility function that is strictly increasing, strictly concave, and twice continuously differentiable. The CRRA utility function allows for an intuitive explanation of obtained results.

Ex-ante, the agent chooses coupon payments that maximize the indirect utility \( U (V^E + D, \bar{c}) \). Although, the agent also has the option to abstain from borrowing, the borrowing participation constraint, \( U (V^E + D, \bar{c}) \geq U (V^E, 0) \), is automatically satisfied for the optimal coupon \( \bar{c} \geq 0 \) because the zero coupon rate corresponds to the zero debt. Thus, the ex-ante optimization problem can be stated as

\[
\sup_{\bar{c} \geq 0} U (V^E + D, \bar{c}),
\]

subject to conditions (4) and (3) on the price of debt, where \( U \) is the ex-post-borrowing indirect utility function given by (8).

In sum, the risk-averse agent maximizes the lifetime utility by borrowing at time zero (described by the coupon payment rate and the corresponding amount of debt), strategically
declaring bankruptcy, and making investment and payout/consumption decisions before and after bankruptcy.

3 Solution

As is common in the real option theory, the model is solved backward in time. First, I solve the stochastic control problem after bankruptcy. Next, for the given coupon \( \bar{c} \), I solve the stochastic control problem with a free boundary before bankruptcy, using boundary conditions imposed by the value function after bankruptcy. Finally, I solve for the optimal coupon rate, using the value function and price of debt as functions of the coupon rate.

Note that due to the time homogeneity of the problem before and after bankruptcy, the only state variable is the total value of assets \( V_t \), and the optimal time to declare bankruptcy corresponds to the first passage time

\[
T^B = \inf \{ t : V_t \leq V^B \}
\]  

for some endogenously determined bankruptcy boundary \( V^B \). To simplify the notation, I drop the time subscript where it does not produce ambiguity.

3.1 After Bankruptcy

After bankruptcy, there is no long-term debt, and the agent invests in the risk-free and risky asset and chooses the consumption rate. Therefore, the problem is the classic Merton’s port-

21
folio allocation problem. In this case, the value function, allocation ratio, and consumption rate, respectively, are given by the following well-known formulas

\[ U^B (V) = U^M (V) = \frac{bV^{1-\gamma}}{1-\gamma}, \quad (11) \]

\[ \pi = -\frac{\mu - r}{\sigma^2} \frac{U^B (V)}{VU^B_{VV} (V)} = \frac{\mu - r}{\sigma^2 \gamma}, \quad (12) \]

and

\[ C = I (U^B_V (V)) = b^{-1/\gamma} V, \quad (13) \]

where

\[ b = \left\{ \frac{1}{\gamma} \left[ \delta - (1 - \gamma) \left( r + \frac{1}{2\gamma} \frac{(\mu - r)^2}{\sigma^2} \right) \right] \right\}^{-\gamma}. \quad (14) \]

Hereafter, \( I (\cdot) \) is the inverse function of \( u' (\cdot) \), and subscripts \( V \) and \( VV \) denote, respectively, the first and the second order derivatives. In this case, the relative risk aversion in wealth,

\[ \xi = \frac{VU_{VV}}{U_V}, \quad (15) \]

is the same as the relative risk aversion in consumption, \( \xi = \gamma \).

### 3.2 Avoiding Bankruptcy

After the debt has been raised and before bankruptcy, there are two cases that depend on the debt burden and severity of the punishment for bankruptcy. In one case, bankruptcy occurs
with some positive probability; in the other case, bankruptcy is avoided with certainty. This subsection derives the solution for the case without bankruptcy; the following subsection provides a necessary and sufficient condition when bankruptcy is avoided; and the case with bankruptcy is considered afterward.

First, note that if the wealth at any time is not high enough to service the debt forever, \( V_t < c/r \), then bankruptcy occurs with positive probability. See Appendix A for the formal proof. Because zero payout/consumption rate is not feasible in the case of the CRRA utility function, \( V_t = c/r \) also leads to bankruptcy (with positive probability); otherwise, the agent can stay out of bankruptcy by investing in the risk-free asset and consuming nothing.

If the value of assets is higher than the price of the risk-free debt paying coupon \( c \), \( V_t > c/r \), then the agent can allocate amount \( c/r \) to service the debt.\(^ {28} \) To pay the coupon rate forever, this amount can be invested in the risk-free asset. The remaining resources, \( V_t - c/r \), can be invested in the risky and risk-free assets and used for payouts to equityholders, as in the case without debt. The maximal expected utility in the case without debt is considered in the previous subsection. Thus, the maximal expected utility that can be obtained without declaring bankruptcy, \( U^{NB}(V_t; \bar{c}) \), satisfies \( U^{NB}(V_t; \bar{c}) \geq U^{M}(V_t - c/r) \).

In addition, for any payout and investment policy \((C_t, \pi_t)\) that does not lead to bankruptcy, \( V_t > c/r \) at any time \( t \) (as discussed, otherwise bankruptcy cannot be avoided with certainty). Therefore, the allocation rule can be reformulated as follows: allocate amount \( c/r \)

\(^{28}\) Note that \( c = (1 - \tau) \bar{p} \) is not the coupon paid to lenders, and \( c/r \) is essentially the cost of the risk-free debt. In this section, the price of the risk-free debt means the price of the risk-free debt paying coupon \( c \), that is \( c/r \).
to pay the coupon and split the rest of the resources, $V_t - c/r$, between the risky and risk-free assets in proportions $\pi_t = \pi_t V_t / (V_t - c/r)$ and $(1 - \pi_t) = ((1 - \pi_t) V_t - c/r) / (V_t - c/r)$, respectively. Since $(C_t, \pi_t)$ is a particular policy corresponding to the value of assets $V_t - c/r$, the expected utility obtained following this policy cannot exceed the maximal expected utility corresponding to $V_t - c/r$, that is, $U^M (V_t - c/r) \geq U^{NB} (V_t; \bar{c})$. Hence,

$$U^{NB} (V_t; \bar{c}) = U^M (V_t - c/r).$$

(16)

Accordingly, optimal investment and payout policies that avoid bankruptcy are

$$\pi^{NB} (V; \bar{c}) = -\frac{\mu - r}{\sigma^2} \frac{U^M_V (V - c/r)}{V U^M_{VV} (V - c/r)} = \frac{\mu - r V - c/r}{\gamma \sigma^2 V},$$

(17)

and

$$C^{NB} (V; \bar{c}) = I \left( U^M_V (V - c/r) \right) = b^{-1/\gamma} (V - c/r).$$

(18)

These policies correspond to the investment of $c/r$ in the risk-free asset and the allocation of the remaining resources as in Merton’s model without debt. The optimal investment and payout policies that avoid bankruptcy will be utilized in later sections to explain the agent’s behavior in the case with bankruptcy.

The left panel in Figure 4 shows the optimal payout and allocation policies that avoid bankruptcy. In the case without bankruptcy, the payout ratio $C/V$ is proportional to the

\[\text{----}^{29}\]These formulas can be derived directly as shown in Appendix B.
allocation ratio $\pi$, following (17) and (18). So, lines corresponding to the payout and allocation ratios coincide in the figure. As the value of assets goes to infinity, these values converge to corresponding values in Merton’s model without debt. Both ratios go to zero as the value of assets decreases toward the cost of debt $c/r$.

The right panel shows the relative risk aversion in wealth and the optimal proportion of resources allocated to the risky investments in the case without bankruptcy. The relative risk aversion in wealth is inversely proportional to the allocation ratio. As the value of assets increases, the relative risk aversion in wealth and the allocation converge to corresponding values in Merton’s model. As the value of assets approaches the cost of debt $c/r$, the
allocation ratio goes to zero, and the relative risk aversion in wealth goes to infinity.

Whether avoiding bankruptcy is optimal depends on the cost of carrying debt and the severity of the punishment for bankruptcy. The cost of paying coupon rate $\bar{c}$ forever is $c/r$. The punishment is more severe if the portion of wealth retained after bankruptcy, $\alpha$, is smaller, and/or the fixed cost, $K$, is larger. Thus, $K/\alpha$ measures the severity of the punishment. Appendix A shows that it is optimal to avoid bankruptcy if and only if the severity of punishment is higher than the cost of carrying debt forever,

$$K/\alpha \geq c/r.$$  

Interestingly, this condition does not depend on any other parameters of the economy and preferences. This solvency condition can be written as a constraint on the coupon rate.

**Solvency Constraint on Debt Payments**

There is no bankruptcy if and only if the coupon rate satisfies

$$\bar{c} \leq \frac{r}{(1 - \tau)} \frac{K}{\alpha}.$$  \hspace{1cm} (19)

As one can see, the firm’s debt is risk-free if and only if debt payments are sufficiently low. The limit on these payments is determined by the severity of the punishment for bankruptcy, $K/\alpha$, and the before-tax risk-free rate, $r / (1 - \tau)$. More severe punishment for bankruptcy and higher risk-free rates correspond to higher possible risk-free coupon rates.

As shown in Appendix A, the solvency condition can also be stated in terms of the
amount of debt.

**Solvency Constraint on the Amount of Debt**

There is no bankruptcy if and only if the amount of debt satisfies

\[ D \leq \frac{K}{(1 - \tau) \alpha}. \]  (20)

Hence, the firm’s debt is risk-free if and only if the amount of debt is sufficiently small. The limit is determined by the punishment for bankruptcy and the tax rate; it does not depend on other parameters of the economy and preferences.

Thus, in an equilibrium without bankruptcies, debt repayments and the amount of borrowing are limited respectively by (19) and (20). These limits are similar to endogenous solvency constraints on debt that are considered a part of equilibrium conditions in Alvarez and Jermann (2000, 2001).\(^{30}\) As in those papers, “these constraints ensure that agents will not default, since they will never owe so much as to make them choose to default” (Alvarez and Jermann, 2001, p. 117). These constraints are also “not too tight” in the sense that they allow the highest possible risk-free lending and the agent will default in some states of the economy if these constraints are violated.\(^{31}\)

Note that if the solvency constraint (19) is satisfied, then the expected utility and op-

\(^{30}\)These papers are based on the model proposed by Kehoe and Levine (1993) who consider analogous solvency constraints. They also provide a justification for solvency constraints by showing that they can emerge endogenously as equilibrium outcomes.

\(^{31}\)Note that there are some differences in models, and, therefore, in constraints. Alvarez and Jermann (2000, 2001) consider state-contingent borrowing, and their constraints are also state contingent. In this paper, lending contracts and constraints are not state contingent.
Figure 5. Value Function in Cases with and without Bankruptcy. The figure shows the indirect utility functions. Both panels use the same notation. The dash-dotted line corresponds to the utility function in Merton’s model without debt, $U^M(V) = bV^{1-\gamma}/(1-\gamma)$. The dashed red line corresponds to the indirect utility obtained by declaring bankruptcy, $U^B(V) = U^M(\alpha V - K)$. Its asymptotic line at $K/\alpha$ is shown by the vertical dashed line. The solid green line corresponds to the maximal indirect utilities achieved with policies that avoid bankruptcy, $U^{NB}(V) = U^M(V - c/r)$. Its asymptotic line at $c/r$ is shown by the vertical dotted line. The left panel corresponds to the case when the solvency constraint is satisfied, $K/\alpha > c/r$. The solvency constraint is violated in the case shown in the right panel.

timal investment and payout rules correspond to the ones that avoid bankruptcy and are given by (16), (17), and (18), respectively. If the solvency constraint is violated, then bankruptcy is possible. In this case, the expected utility values provided by (16) are attainable but not maximal. Accordingly, investment and payout rules (17) and (18) are feasible but not optimal. The next subsection provides the optimal solution for the case with possible bankruptcy.

Figure 5 illustrates the solvency constraint using indirect utility functions. The left panel shows the indirect utility of wealth in the case without bankruptcy, that is, when the debt is risk-free. When the solvency constraint is satisfied, $c/r \leq K/\alpha$, the utility obtained by
declaring bankruptcy is always below the utility that can be obtained without declaring bankruptcy. Thus, it is never optimal to declare bankruptcy. The right panel in the figure shows the case with bankruptcy. In this case, the solvency constraint is violated, $c/r > K/\alpha$. Due to the asymptotic behavior of the indirect utility functions, there is a region of values of assets where the utility obtained by declaring bankruptcy is higher than the utility that can be obtained by avoiding bankruptcy. Thus, it is not optimal to always avoid bankruptcy in this case.

### 3.3 Possible Bankruptcy

This subsection considers the case when the bankruptcy boundary is attainable. This case corresponds to $c/r > K/\alpha$. In this case, there is the optimal default boundary $V^B$. In this section, I use the dynamic programming approach to find the solution for $V > V^B$. Appendix C provides calculation details.

The dynamics of the total value of assets before bankruptcy are given by equation (6). This equation can be written as

$$dV_t = \left( (r + \lambda \pi_t) V_t - \tilde{C}_t \right) dt + \sigma_t \pi_t V_t dZ_t,$$

where $\lambda = \mu - r$ is the risk premium and $\tilde{C}_t = C_t + c$ combines the outflow terms. The Hamilton–Jacobi–Bellman (HJB) equation in terms of new variables is essentially the same.
as in Merton’s model

$$
\delta U = \max_{\tilde{C}, \pi} \left\{ \tilde{u} \left( \tilde{C} \right) + \left( (r + \pi \lambda) V - \tilde{C} \right) U_V + \frac{1}{2} \pi^2 \sigma^2 V^2 U_{VV} \right\}, \tag{22}
$$

where $\tilde{u} \left( \tilde{C} \right) \equiv u \left( \tilde{C} - c \right) = u \left( C \right)$.

The first-order conditions on investment $\pi$ and payout $\tilde{C}$ are, respectively,

$$
\pi = - \frac{U_V}{V U_{VV}} \frac{\lambda}{\sigma^2} \tag{23}
$$

and

$$
\tilde{C} = \tilde{I} \left( U_V \right), \tag{24}
$$

where $\tilde{I} \left( \cdot \right)$ is the inverse function of $\tilde{u}' \left( \cdot \right)$. The substitution of these first-order conditions results in the following differential equation:

$$
\delta U = \tilde{u} \left( \tilde{I} \left( U_V \right) \right) + \left( r V - \tilde{I} \left( U_V \right) \right) U_V - \frac{1}{2} \frac{\lambda^2 \left( U_V \right)^2}{\sigma^2 U_{VV}}. \tag{25}
$$

The value function must satisfy the value-matching and smooth-pasting conditions when the value of assets reaches the bankruptcy boundary:

$$
U \left( V^B \right) = U^B \left( \alpha V^B - K \right) \tag{26}
$$
and
\[ U_V (V^B) = \alpha U_V^B (\alpha V^B - K). \] (27)

where \( U^B (V) \) is the indirect utility function after bankruptcy.

Assuming that the solution is strictly concave (as is the case), the marginal utility of wealth is a strictly decreasing function of wealth and \( v = -\ln (U_V (V)) \) is, accordingly, a strictly increasing function of wealth. Therefore, one can consider the inverse function \( V = V (v) \). This change of variables leads to a linear ODE that admits an analytical solution proposed in Appendix C.\(^{32}\) The value function, the allocation ratio, the payout rate, and the bankruptcy boundary are given, respectively, by equations (127), (128), (129), and (140) in that appendix.

### 3.4 Price of Debt

As shown in Section 3.3, there are two possible cases corresponding to the risk-free and risky debt. If \( \bar{c} \leq rK / (1 - \tau) \alpha \), then the solvency constraint is satisfied, the debt is risk free, and the price of debt is

\[ P^D (V, \bar{c}) = \bar{c} / r. \] (28)

\(^{32}\)This change of variables is proposed by Presman and Sethi (1991) to solve Merton’s model. See also Sethi et al. (1992) for the case with a non-zero consumption boundary.
Let’s consider the case when the solvency constraint is violated. In this case, the price of the debt is given by the risk-neutral valuation formula (3). This formula can be written as

\[ P^D (V, \bar{c}) = \frac{\bar{c}}{r} + \left( (1 - \alpha) V^B - \frac{\bar{c}}{r} \right) P^B, \]  

(29)

where \( P^B = E^Q \left[ e^{-rT^B} \right] \) is the expected present value of the unit payment in the case of bankruptcy.

As noted in the previous subsection, because the marginal utility of wealth is a decreasing function of wealth, \( v = -\ln (U_V (V; \bar{c})) \) is an increasing function of wealth. Therefore, the time of bankruptcy corresponds to the time of reaching the lower boundary \( v^B \):

\[ T^B = \inf \{ t : V_t \leq V^B \} = \inf \{ t : v_t \leq v^B \}, \]  

(30)

where \( v^B \) is determined by the smooth-pasting condition,

\[ v^B = -\log (U_V (V^B; \bar{c})) = -\log \left( \alpha U_V^B (\alpha V^B - K) \right). \]  

(31)

Appendix C shows that under the risk-neutral measure, the marginal utility of wealth corresponding to optimal policies follows a geometric Brownian motion,\(^{33}\) and accordingly, \( v_t \) is a Brownian motion with drift,

\(^{33}\)This is similar to the dynamics of the marginal utility of wealth in Merton’s (1969) model.
\begin{align}
dv_t &= \mu_v^Q dt + \sigma_v^Q dZ_t^Q, \\
\end{align}

where volatility \( \sigma_v = \lambda/\sigma \), and the drift \( \mu_v^Q = r - \psi - \delta \), where \( \psi = \lambda^2/(2\sigma^2) \). Therefore, \( T^B \) is the first passage time of this Brownian motion with drift, corresponding to barrier \( v^B \). The moment generating function (MGF) of the stopping time of the Brownian motion with drift is provided in Appendix C. This MGF gives the expected present value of the unit payment at bankruptcy that can be written in terms of marginal utilities as

\[ P^B = E^Q \left[ e^{-rT^B} \right] = e^{-a(v_0 - v^B)} = \left( \frac{U_V(V; \bar{c})}{\alpha U^B_V (\alpha V^B - K)} \right)^a, \]

where \( a = \frac{1}{\sigma_v^2} \left( \mu_v^Q + \frac{\mu_v^Q}{\sigma_v^2} + 2\sigma_v^2 \right) > 0 \). Thus, the price of the risky debt paying coupon \( \bar{c} \) is

\[ P^D (W; \bar{c}) = \frac{\bar{c}}{r} + \left( \frac{U_V(V; \bar{c})}{\alpha U^B_V (\alpha V^B - K)} \right)^a \left[ (1 - \alpha) V^B - \frac{\bar{c}}{r} \right]. \]

This formula has an intuitive explanation. The first term is the price of the risk-free debt paying coupon \( \bar{c} \). The terms in square brackets correspond to the value received in the case of bankruptcy minus the value lost. Since the marginal utility of wealth \( U_V(V; \bar{c}) \) goes to zero as the value of assets increases to infinity,\(^{34}\) the price of debt \( P^D (V; \bar{c}) \) converges to the value of the risk-free debt, \( \bar{c}/r \), as the value of assets increases. As the value of assets declines toward the bankruptcy threshold \( V^B \), the marginal utility \( U_V(V; \bar{c}) \) converges to

\(^{34}\)Because the indirect utility function is concave and bounded by \( U^{NB}(V) \leq U(V) \leq U^M(V) \).
Figure 6. Price of Debt and Amount of Borrowing. In both panels, the solid red line shows the price of the debt as a function of the value of assets; the horizontal dash-dotted blue line corresponds to the value of the risk-free debt, $c = r$, paying the same coupon rate as the risky debt; and the black star corresponds to bankruptcy. The dotted blue line in the left panel shows the recovery value in the case of bankruptcy, $\alpha W$. The point where the recovery value is equal to the price of debt is marked by the black cross. The dashed green line in the right panel shows the borrowed amount for the corresponding initial value of assets $V_0$. The intersection of this line and the price of debt (marked by the red star) corresponds to the equilibrium amount of debt.

\[ \alpha U_V^B \left( \alpha V^B - K \right) \]
due to the smooth-pasting condition, and the price of the debt approaches the recovery value $(1 - \alpha) V^B$, as expected. This behavior is illustrated in the left panel of Figure 6, which shows the value of debt (for a fixed coupon rate) as a function of the value of assets. Note that in a particular region close to bankruptcy, the price of debt is higher than the recovery value. In this case, it is not optimal for lenders to enforce bankruptcy even if they can do so. Accordingly, debt covenants corresponding to this region of values of assets are irrelevant.

Similar to equation (7) for the price of debt in Leland (1994), equation (34) can be written as

\[
(1 - P^B) \left[ \frac{\bar{c}}{r} \right] + P^B \left[ (1 - \alpha) V^B \right],
\]
where $P^B = \left( U_V (V; \bar{c}) / U_V (V^B; \bar{c}) \right)^{-x}$ is the expected present value of the unit payment in the case of bankruptcy. Note that Leland’s equation
(7) is written in terms of the value of the company’s assets, but equation (34) has marginal utilities. This difference has an intuitive explanation. Leland (1994) considers the fixed asset allocation. In that case, the value of assets follows a geometric Brownian motion process. In the present case, allocation and payout are chosen endogenously. In this case, the marginal utility is the geometric Brownian motion that determines the time of bankruptcy. Marginal utilities seem natural in this context since the agents declare bankruptcy not because their wealth is too low per se (the value of assets at the time of default is sufficient to meet current obligations), but because they expect to suffer so much by bearing debt payments that bankruptcy is a good alternative, despite the associated punishment. The marginal utility of payout/consumption (that is the same as the marginal utilities of wealth) provides a measurement of agents’ suffering.

3.5 Borrowing

The equilibrium amount of debt $D = P^D (V_0, \bar{c})$ corresponds to the (highest) fixed point given by equation (4). Appendix C shows that such a fixed point always exists. The agent chooses the coupon rate $\bar{c}$ that maximizes $U (V^E + D (\bar{c}), \bar{c})$.

The right panel in Figure 6 shows the value of debt $P^D (V_0, \bar{c})$ for a fixed coupon rate as a function of the value of assets along with the amount of borrowing $D$ corresponding to the initial value of assets $V_0 = V^E + D$. The intersection of these lines gives the equilibrium amount of borrowing $P^D (V^E + D) = D$.

When the solvency constraint is satisfied, $\bar{c} \leq rK / (\alpha (1 - \tau))$, the debt is risk-free and
\( P^D = \ddot{c}/r \). This price of debt corresponds to the utility value

\[
U (V_0) = U^M \left( (V^E + \ddot{c}/r) - (1 - \tau) \ddot{c}/r \right) = U^M (V^E + \dddot{c}/r).
\]

Because \( U^M (\cdot) \) is an increasing function, the expected utility increases with the coupon rate (assuming a non-zero tax rate). Therefore, the agent always borrows if \( K > 0 \), and the coupon rate is at least \( \ddot{c} = rK / (\alpha (1 - \tau)) \). This coupon rate corresponds to the case when the solvency constraint is binding. The corresponding expected utility value is \( U^M (V^E + K\tau / (\alpha (1 - \tau))) \), which serves as a lower boundary for the expected utility value. This lower boundary increases with the severity of punishment at bankruptcy \( K/\alpha \). This happens because more severe punishment makes bankruptcy less attractive and, therefore, supports higher risk-free coupon rates.

Figure 7 shows the relative certainty equivalent (without debt) defined as

\[
E = \frac{(U^M)^{-1} (U (V^E + P^D))}{V^E},
\]

(35)

and the corresponding amount of debt as functions of the coupon rate. In the case of the zero coupon rate, there is no debt and the relative certainty equivalent equals one. A higher certainty equivalent \( 1 + \dddot{c}/rV^E \) (marked by stars) corresponds to the highest coupon such that the solvency constraint is satisfied: \( \dddot{c} = rK / ((1 - \tau) \alpha) \). Up to this point, debt is risk free, and the amount of borrowing, \( \ddot{c}/r \), increases linearly with the coupon rate. Higher values of the coupon rate correspond to risky debt. In this case, the amount of borrowing
Figure 7. Amount of Borrowing and Its Certainty Equivalent. The solid blue line (corresponding to the left axis) shows the relative certainty equivalent corresponding to the utility value that is obtained with borrowing; the dashed green line (corresponding to the right axis) shows the borrowed amount; and the dash-dotted line shows the price of the risk-free debt paying the same coupon, \( \bar{c}/r \). The circle, stars, and cross marks correspond to no-borrowing, maximum risk-free borrowing, and optimal borrowing, respectively.
is below the dash-dotted line $\bar{c}/r$, and the certainty equivalent is below $1 + \bar{c}\tau/(rV^E)$. As the coupon rate increases, debt becomes riskier, and the marginal amount of borrowing decreases. At some point, the certainty equivalent and the corresponding utility value start to decline with the coupon rate. This point (shown by cross marks) corresponds to the optimal coupon value.

4 Implications of Debt Financing

This section considers three phenomena associated with economic agents’ investment and payout decisions: flight to quality, gambling for resurrection, and bankruptcy for profit. These phenomena arise endogenously in the presented model. The phenomena show how debt financing affects borrowers’ investment and payout decisions as the value of assets declines and approaches the bankruptcy threshold. Additionally, this section examines payout/consumption changes in the case of bankruptcy.

4.1 Flight to Quality

The upper left panel in Figure 8 shows the allocation of wealth to the risky asset and the relative risk aversion in wealth as functions of wealth. The RRA in wealth and the allocation ratio are not constant, as in Merton’s model without debt. When the level of wealth is high, risk aversion increases as wealth declines. In this case, the agent behaves more conservatively, decreasing the proportion of wealth invested in the risky asset and increasing the portion invested in the risk-free asset, as the value of assets decreases. This shift from risky to safe
Figure 8. Payout, Allocation, and Risk Aversion in the Case with Bankruptcy. The figure shows optimal payout and allocation policies and RRA in wealth as functions of the value of assets (scaled by the amount of debt). In all panels, the dash-dotted line shows the corresponding values in Merton’s model, dotted lines correspond to policies that avoid bankruptcies (as in Figure 4), black stars represent bankruptcy, and minimum and maximum points are marked by black crosses. The upper left panel shows the allocation to the risky asset (the dashed green line corresponding to the left axis) and the RRA in wealth (the thin solid magenta line corresponding to the right axis). The upper right panel shows the payout ratio (the solid red line corresponding to the left axis) and the RRA in wealth (the thin solid magenta line corresponding to the right axis). The lower left panel shows the payout ratio (the solid red line corresponding to the left axis) and the allocation to the risky asset (the dashed green line corresponding to the right axis). The lower right panel shows the payout ratio before bankruptcy (solid red line) and the ratio of payout after bankruptcy to the value of assets before bankruptcy (dashed red line). The thin black line in this panel shows the decrease in the payout rate as the proportion of the value of assets at bankruptcy.
assets (when adverse events negatively affect agents’ wealth) is known as the flight to quality.

The dynamics described by the flight to quality emerge endogenously in the presented model. The intuition behind this effect is as follows. When the agent’s wealth is high, the value of the option to declare bankruptcy is low. In this case, the agent’s wealth allocation is similar to the optimal investment policy that avoids bankruptcy. As shown in Section 3.2, this investment policy is to allocate wealth to the risky asset proportionally to the amount of wealth reduced by the cost of carrying debt forever,

\[ \pi^{NB}(V; c) = \frac{\mu - r}{\sigma^2} \left( 1 - \frac{c/r}{V} \right). \]  

(36)

This is an increasing function of wealth. Thus, the allocation of wealth to the risky asset declines as wealth decreases.\(^{35}\)

### 4.2 Gambling for Resurrection and Bankruptcy for Profit

The agent’s attitude toward risk and resulting investment patterns change as the value of assets approaches the bankruptcy threshold. As shown in the upper left panel in Figure 8, when the agent’s wealth is below a certain point, the RRA in wealth decreases and the proportion of wealth invested in the risky asset increases as wealth declines. This effect is induced by the option to declare bankruptcy. The agent’s risk aversion declines as wealth declines because the bankruptcy protection limits downside risk more when wealth is closer

\[^{35}\text{Note that even though the proportion allocated to the risky asset decreases as the total value of assets declines, the amount allocated to the risk-free asset also decreases if } \frac{1-\pi}{\pi} > \frac{d\pi}{dV}.\]
to the bankruptcy threshold. The agent make more risky investments (gambling for resurrection) rationally anticipating that if the outcome is good, then the wealth can be restored; if the outcome is bad, then creditors will absorb losses. This is the same risk-shifting behavior of borrowers as described in Jensen and Meckling (1976).

A further decline in wealth leads to a higher payout rate as a proportion of wealth. The upper right panel in Figure 8 illustrates these dynamics. As bankruptcy becomes more and more imminent, payout (as a proportion of wealth) increases because the borrower anticipates that, in the case of bankruptcy, debt holders will appropriate wealth that has not been consumed. The borrower rationally increases the payout ratio as a proportion of wealth, even though the higher payout rate makes bankruptcy more probable. Such bankruptcy for profit behavior was exacerbated during the S&L crisis when owners essentially plundered their companies, as discussed in Akerlof and Romer (1993).

The dynamics of the payout-to-wealth ratio are determined by the relationship between the risk aversion in consumption and the risk aversion in wealth. The optimal payout rate is given by the first-order condition

\[ C = (U_V)^{-1/\gamma}. \] \hspace{1cm} (37)

Therefore, the derivative of the payout-to-wealth ratio is

\[ \frac{d}{dV} \left( \frac{C}{V} \right) = \left( -\frac{V U_{VV}}{U_V} - \gamma \right) \frac{(U_V)^{-1/\gamma}}{\gamma V^2} = (\xi - \gamma) \left[ \frac{C}{\gamma V^2} \right]. \] \hspace{1cm} (38)
The term in the square brackets is positive. Thus, the payout-to-wealth ratio increases with wealth when the relative risk aversion in wealth, $\xi$, is higher than the relative risk aversion in consumption, $\gamma$. This seems to be intuitive.

$$\frac{\Delta (C/V)}{\Delta V} = \frac{1}{\Delta V} \left( \frac{C + \Delta C}{V + \Delta V} - \frac{C}{V} \right) = \frac{C}{(V + \Delta V) \Delta V} \left( \frac{\Delta C}{C} - \frac{\Delta V}{V} \right).$$

When the wealth of agents dwindles, they scale back consumption proportionally more than the decline in wealth if they are more risk averse in wealth than in consumption. In this case the consumption to wealth ratio decreases as wealth declines.

At high levels of wealth, the RRA in wealth is approximately the same as in the case without bankruptcy $\xi^{NB}$, which is higher than the RRA in consumption,

$$\xi^{NB} = -\frac{V U^{NB}_{VV}}{U^{NB}_V} = \frac{V \gamma (V - c/r)^{-\gamma - 1}}{(V - c/r)^{-\gamma}} = \frac{V}{V - c/r} \gamma > \gamma. \quad (39)$$

Therefore, in this case, the payout-to-wealth ratio declines as wealth decreases. Close to the bankruptcy threshold, the RRA in wealth is below the relative risk aversion in consumption. In this case, the payout ratio increases as wealth declines. The minimum of the payout ratio is achieved when the RRA in wealth is the same as the RRA in consumption, $\xi = \gamma$.

The minimum of the payout ratio (i.e., the turning point for bankruptcy for profit) corresponds to a lower level of wealth than the minimum of the allocation ratio (the turning point for gambling for resurrection). The reason is as follows. The minimum of the allocation ratio corresponds to the maximum of the RRA in wealth because the allocation ratio and
the RRA in wealth are inversely proportional. Therefore, the RRA in wealth is higher than
the RRA in consumption at that point. Accordingly, the payout ratio decreases as wealth
declines at the minimum of the allocation ratio. Therefore, the minimum of the payout
ratio occurs at a lower level of wealth than the minimum of the allocation ratio, that is,
the gambling for resurrection starts earlier (at a higher level of wealth) than bankruptcy for
profit. The payout ratio and the allocation to the risky asset are shown in the lower left
panel of Figure 8.

4.3 Strategic Bankruptcy

Even though bankruptcy is declared optimally in the model, it is associated with a drop
in consumption. To see that this is the case, note that, by the smooth-pasting condition,
the marginal utility of wealth right before bankruptcy equals the derivative of the value
function after bankruptcy, \( \alpha U^B_B (\alpha V^B - K) \). This value is less than the marginal utility of
wealth right after bankruptcy, \( U^B_B (\alpha V^B - K) \) since \( \alpha < 1 \). Because the marginal utility
of wealth equals the marginal utility of consumption, the marginal utility of consumption
right before bankruptcy is lower than the marginal utility right after. By the concavity of the
utility function, the marginal utility of consumption is a decreasing function of consumption.
Therefore, consumption immediately prior to bankruptcy is higher than consumption right
after, even though bankruptcy is chosen optimally. In the case of the CRRA utility function,
the consumption rate right before and right after bankruptcy can be written as,

\[ C_{\text{before}} = (\alpha b)^{-1/\gamma} (\alpha V^B - K) > b^{-1/\gamma} (\alpha V^B - K) = C_{\text{after}}. \]

Accordingly, the ratio of the consumption rate before and after bankruptcy is \( \alpha^{-1/\gamma} > 1. \)

This observation is relevant to the possibility of defaults. The argument that the borrower will not default because the consequences of a default lead to immediate suffering is not valid in general. It can be optimal for the borrower to declare bankruptcy, even though the immediate outcome is lower consumption, because bankruptcy prevents the depletion of wealth caused by servicing debt obligations.

The lower right panel in Figure 8 illustrates the decrease in consumption in the case of bankruptcy. The solid red line shows the consumption-to-wealth ratio before bankruptcy, and the dashed red line shows the ratio of consumption after bankruptcy to the wealth right before bankruptcy, \( b^{-1/\gamma} (\alpha V - K) / V. \) In the considered case of \( \alpha = 0.25 \) and \( \gamma = 2, \) the consumption rate before bankruptcy is twice as high as the consumption rate after bankruptcy.

In summary, the flight to quality, gambling for resurrection, and bankruptcy for profit describe the sequence of actions of economic agents when wealth deteriorates toward bankruptcy. First, when agents are far from bankruptcy, they reduce payout and risky investments (as a proportion of wealth) as their wealth declines. Then, as their wealth decreases further, they start to increase risky investments, but still reduce their payout ratio. After that, as
their condition deteriorates further, they start to pay out/consume (proportionally) more and make more risky investment bets. Finally, when their wealth reaches the bankruptcy threshold, they declare bankruptcy associated with a drop in consumption.

5 Asset Volatility and Debt Prices

This section considers some implications of endogenous investment and payout decisions of borrowers for debt prices. It shows that, contrary to common beliefs, credit spreads can be lower when asset volatility is higher. The finance literature and the general perception consonantly suggest that higher volatility corresponds to wider credit spreads. The logic behind this suggestion is as follows. Higher asset volatility corresponds to a higher probability of default and associated losses. Therefore, the value of the debt is lower, and the credit spread is higher when asset volatility is higher. Basically, borrowers have a long position in the put option to default, and debt holders have the corresponding short position in this option. Since the value of the option increases with volatility, the value of the debt contract is lower when asset volatility is higher.

This rationale is valid if investments are fixed, and the borrower cannot adjust the composition of assets, but it may become invalid if the assumption of unchangeable asset allocation is relaxed. The outcome can be entirely different if the borrower can choose the composition of assets. Figure 9 shows debt prices as functions of wealth for different values of asset volatility. As one can see, when the value of assets is high, higher volatility corresponds to higher prices and accordingly to lower credit spreads. To explain this effect, this section
Figure 9. Sensitivity of Debt Prices to Asset Volatility. The figure shows the price (scaled by the amount of debt) of the same debt contract as functions of the value of assets (scaled by the amount of debt) for different values of asset volatility. The solid blue, dashed green, and dotted red lines correspond to volatility values of 15, 20, and 30 percent, respectively. Stars correspond to bankruptcies. For the value of assets below the bankruptcy threshold, the debt price corresponds to the recovery value.
investigates the relationship between the volatility of the risky asset and the volatility of the investment portfolio. First, this section considers a simple two-period model without debt. Next, it analyzes Merton’s (1969) model. After that, it extends intuition to the model with debt. Finally, it examines the sensitivity of credit spreads to asset volatility.

5.1 Two-Period Model

Consider a two-period investment problem of the agent with a constant absolute risk aversion (CARA) utility function,

\[ u(C) = 1 - e^{-\gamma C}. \] (40)

The agent has initial wealth \( W_0 \), invests at time zero, and consumes the proceeds at the next period. The opportunity set consists of the risk-free and risky asset. The risk-free asset grows at a constant rate \( r \), and the return on the risky asset is normally distributed with mean \( \mu \) and volatility \( \sigma \).

Denote \( \pi \) the proportion of wealth allocated to the risky asset. The second period wealth is given by

\[ W_1 = \pi W_0 (1 + \mu + \sigma Z) + (1 - \pi) W_0 (1 + r) = [1 + r + \pi (\mu - r)] W_0 + \pi W_0 \sigma Z, \] (41)

and the expected utility is

\[ E[u(W_1)] = 1 - e^{-\gamma (1+r+\pi(\mu-r))W_0 + \gamma^2 \pi^2 \sigma^2 W_0^2 / 2}. \] (42)
Therefore, the optimization of the expected utility is the same as

$$\max_\pi \left\{ \pi W_0(\mu - r) - \gamma \pi^2 W_0^2 \sigma^2 / 2 \right\}$$  \hspace{1cm} (43)$$

Note that $\pi W_0 (\mu - r)$ is the expected excess return, and $\pi^2 W_0^2 \sigma^2$ is the variance of the return. Thus, the agent evaluates the variance-return trade-off: the expected excess return versus the variance of the investment portfolio weighted by $\gamma$. The solution to this optimization problem is

$$\pi = \frac{\mu - r}{\sigma^2 \gamma W_0}$$  \hspace{1cm} (44)$$

Hence, the allocation to the risky asset is inversely proportional to $\sigma^2$ and, as expected, decreases with the volatility of the risky asset. What is more, a higher volatility of the risky asset corresponds to a lower volatility of the investment portfolio,$^{36}$

$$\pi \sigma = \frac{\mu - r}{\sigma \gamma W_0}$$  \hspace{1cm} (45)$$

This relationship has an intuitive explanation. Riskier investments are less attractive to the risk-averse agent if they do not provide higher risk premium. Therefore, the agent scales back risky investments when the volatility of the risky asset is higher. In this case, the

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$^{36}$Here and in the following sections all other modal parameters are kept constant. Note that, in this case, the risk premium is constant. Keeping the risk premium constant is consistent with empirical studies that investigate changes in credit spreads and run the regression of the changes in credit spreads on changes in volatility controlling for market excess returns (or market returns and risk-free rates) and some other variables.
variance of the investment portfolio cannot be higher because the return-variance balance would be violated since less risky investments mean lower expected excess returns. By the same reason, the volatility of the investment portfolio cannot be the same. Thus, a higher volatility of the risky asset corresponds to a lower volatility of the investment portfolio. This result also holds in the context of the Merton (1969) model considered in the following section.

5.2 Merton’s Model

In Merton’s model, the risk-averse agents choose their consumption and allocation of wealth between the risk-free and risky asset. The optimal proportion of wealth allocated to the risky asset is inversely proportional to the square of the volatility value as in the model considered in the previous section,

\[ \pi^M = \frac{1}{\sigma^2} \frac{\mu - r}{\gamma}. \]  

(46)

Therefore, as in the previous model, the volatility of the investment portfolio,

\[ \sigma_{\pi^M} = \frac{1}{\sigma} \frac{\mu - r}{\gamma}, \]  

(47)

is inversely proportional to asset volatility, and thus decreases as asset volatility increases. The rationale behind this effect is also the same as in the previous section: more volatile assets are less attractive to the risk-averse agent, thus the agent invests less in risky assets when volatility is higher. The question is how much less. The same volatility of the portfolio
of more volatile assets corresponds to a lower expected return. In Merton’s model, the agent is not willing to take the same risk (measured by volatility) with a lower expected return, and scales back risky investments so much that the volatility of the portfolio is lower.

The optimal consumption-to-wealth ratio in Merton’s model can be written as

\[
\theta^M = \frac{1}{\gamma} \left\{ \delta - (1 - \gamma) \left[ r + \frac{1}{\sigma^2} \frac{(\mu - r)^2}{2\gamma} \right] \right\}. 
\]  

(48)

Thus, higher volatility corresponds to a lower consumption rate (assuming that \( \gamma > 1 \)). This lower consumption rate can be explained by a lower expected return on the investment portfolio. To show that this is the case, let’s further examine the optimal consumption-to-wealth ratio in Merton’s model.

As shown in Appendix B, the optimal consumption-to-wealth ratio for a given constant allocation ratio \( \pi \) can be written as

\[
\theta^M = \frac{1}{\gamma} \left\{ \delta + (\gamma - 1) \left[ r + (\mu - r) \pi - \frac{1}{2} \gamma (\mu - r) (\sigma \pi)^2 \right] \right\}. 
\]  

(49)

This formula has an intuitive interpretation. The first term in the curly brackets corresponds to the agent’s impatience; higher impatience corresponds to a higher consumption rate. In addition to impatience, there are three terms in the square brackets. The first term corresponds to the risk-free returns; higher returns correspond to a higher consumption rate.\(^{37}\) The second term corresponds to the excess return on the portfolio; a higher expected

\(^{37}\) Assuming that \( \gamma > 1 \), that is the income effect prevails over the substitution effect.
return corresponds to a higher consumption rate. The third term is the precautionary saving that is proportional to the variance of the portfolio; higher portfolio volatility corresponds to a lower consumption rate ceteris paribus.

In the case of the optimal allocation policy (46), the expression for the consumption-to-wealth ratio (49) can be written as

$$\frac{1}{\gamma} \left\{ \delta + (\gamma - 1) \left[ r + \frac{1}{\sigma^2} \frac{(\mu - r)^2}{\gamma} - \frac{1}{2} \frac{1}{\sigma^2} \frac{(\mu - r)^2}{\gamma} \right] \right\}. \tag{50}$$

The second and third terms in square brackets are the only terms that depend on volatility. The second term corresponds to the excess returns on the investment portfolio, and the third term corresponds to precautionary saving. Both terms are inversely related to volatility, but the excess return term dominates the precautionary saving term because it is twice as large. This determines the relationship between the consumption rate and asset volatility: higher volatility corresponds to a lower consumption rate. Note that the precautionary saving term increases consumption when asset volatility increases because the precautionary saving effect depends on the total volatility of the portfolio, which is lower when asset volatility is higher.

### 5.3 Model with Debt

In the model with debt, relationships between asset volatility and optimal (pay-out/consumption and investment) policies that avoid bankruptcy are similar to the ones in Merton’s model. As shown, the optimal allocation policy that avoids bankruptcy is to
invest a portion of wealth in the risk-free asset to service the debt and to allocate the re-
main ing wealth according to Merton’s rule. Accordingly, the amount of wealth allocated to
the risky asset is
\[ \pi_t^{NB} V_t = \pi^M (V_t - c/r) = \frac{1}{\sigma^2} \frac{\mu - r}{\gamma} (V_t - c/r). \] (51)

Therefore, the volatility of the investment portfolio is inversely proportional to asset volatil-
ity,
\[ \sigma \pi_t^{NB} = \frac{1}{\sigma} \frac{\mu - r}{\gamma} (V_t - c/r) \frac{1}{V_t}, \] (52)
and thus decreases with asset volatility. The intuition is the same as in the case of Merton’s
model without debt.

The optimal payout rate corresponding to policies that avoid bankruptcy is given by (18)
and can be written as
\[ C_t^{NB} = \theta^M (V_t - c/r) = \frac{1}{\gamma} \left\{ \delta - (1 - \gamma) \left[ r + \frac{1}{\sigma^2} \frac{(\mu - r)^2}{2\gamma} \right] \right\} (V_t - c/r). \] (53)
Thus, the payout rate declines when asset volatility increases. This formula also has the
same interpretation as in the case of Merton’s model.

When the value of assets is high, optimal policies in the model with debt are similar
to policies that avoid bankruptcy, as discussed above and shown in the lower left panel of
Figure 8. Therefore, higher volatility of the risky asset is associated with lower volatility of
the investment portfolio and a lower payout rate. The net effect of higher asset volatility
is a lower probability of default, a higher price of debt, and a lower credit spread of the
wealthy borrower. This effect is opposite to the relationship between volatility and credit spreads in Leland’s (1994) model (see Table I on page 1224 in that paper). In that model investments cannot be adjusted, and, accordingly, the value of debt declines and the credit spread increases with volatility when the value of assets is high.

When the value of assets is low, the relationship between asset volatility and credit spreads is the opposite of the one when the value of assets is high. In the vicinity of bankruptcy, higher volatility is associated with higher credit spreads. This is again the opposite of the relationship in Leland’s model. In that model, higher volatility increases the value of the default option. As a result, the agent/firm declare bankruptcy at a lower level of the value of assets. In the present model, the agent chooses to declare bankruptcy earlier (at a higher level of the value of assets) when volatility is higher. This can be partially explained by a lower value of the default option due to a lower volatility of the investment portfolio when the value of assets rebounds to a higher level.

Figure 10 shows the allocation of resources to the risky asset, portfolio volatility, payout ratio, and the twenty-year probability of default as functions of the value of assets for different volatility values. As shown, when the value of assets is high, higher volatility corresponds to a lower allocation to the risky asset, and accordingly, to lower portfolio volatility. Along with a lower payout ratio, this leads to a lower probability of default, and consequently, to a higher value of debt. At the same time, higher volatility corresponds to earlier bankruptcy. When the value of assets is low, portfolio volatility and the probability of default are higher, and the price of debt is lower if volatility is higher.
Figure 10. Asset Volatility and Probability of Default. The figure shows the optimal proportion allocated to the risky asset, the investment portfolio volatility, the optimal payout ratio, and twenty-year (risk-neutral) default probability as functions of the value of assets (scaled by the amount of debt) for different values of risky asset volatility. The solid blue, dashed green, and dotted red lines correspond to volatility values of 15, 20, and 30 percent, respectively. Stars correspond to bankruptcies.
5.4 Sensitivity of Credit Spreads to Asset Volatility

The left panel in Figure 11 shows the sensitivity of debt prices to asset volatility. As explained, the sensitivity is positive when the value of assets is high. The sensitivity decreases and becomes negative as the value of assets declines toward the bankruptcy threshold. This effect can be attributed to the default option (as in Merton 1974): the put option to default becomes more valuable when the company approaches bankruptcy. The negative relationship between volatility and debt prices (the positive relationship between volatility and credit spreads) is common for most capital structure models. As one can see, in the cases of endogenous asset composition, this relationship holds only when the company is close to bankruptcy.

The sensitivity of debt prices to asset volatility also declines with the increase in the value
of assets above some level. This effect also has an intuitive explanation. As the value of assets increases further and further, the probability of bankruptcy declines toward zero, and the price of the risky debt approaches the price of the risk-free debt with the same coupon. Since the price of the risk-free debt does not depend on asset volatility, the sensitivity of the risky debt to asset volatility converges to zero as the value of assets goes to infinity.

The credit spread of the risky debt with infinite maturity can be calculated as

$$CS = \tilde{c}/P^D - r.$$  \hspace{1cm} (54)

The right panel in Figure 11 shows the sensitivity of credit spreads to asset volatility. This relationship is opposite to the sensitivity of debt prices to asset volatility and has a U-shaped form. The sensitivity decreases and becomes negative as the value of asset increases to a certain level and increases toward zero (declines in absolute values) afterward. As shown in the following section, the sensitivity of credit spreads to equity volatility inherits this U-shaped relationship.

6 Asset Volatility and Equity Volatility

This section considers equity volatility and its relationship to asset volatility. The theoretical relationship between credit spreads and equity volatility is obtained by combining sensitivities of credit spreads and equity volatility to asset volatility. This relationship is corroborated in the empirical part of the paper.
6.1 Price of Equity

Section 3.3 shows that there are two possible cases: in one case, the debt is risk free, and there is no bankruptcy; in the other case, the debt is risky, and bankruptcy is possible. In the first case, the fair value of equity is given by the total value of assets minus the cost of carrying debt forever,

$$P_E(V; c) = V - \frac{(1 - \tau) \bar{c}}{r}. \quad (55)$$

In the second case, the price of equity has to be adjusted by the cost of bankruptcy,

$$P_E(V; \bar{c}) = V - \frac{(1 - \tau) \bar{c}}{r} + \left[ \frac{(1 - \tau) \bar{c}}{r} - \left( (1 - \alpha) V^B + K \right) \right] P^B, \quad (56)$$

where, $P^B$, is the expected discounted value of unit payment at bankruptcy given by (33). This formula is similar to expression (13) for the price of equity in Leland (1994). The difference is the loss in the case of bankruptcy: $\left( (1 - \alpha) V^B + K \right)$ instead of $V^B$.\(^{38}\)

The left panel in Figure 12 shows the price of equity as a function of the value of assets. The price of equity approaches the price of equity in the case without bankruptcy given by (55) as the value of assets increases. It declines towards the retained value, $\alpha V - K$, as the value of assets approaches the bankruptcy threshold.

\(^{38}\) Of course, formulas for the present value of the expected discounted value of unit payment at bankruptcy are different as explained in Section 3.
The figure shows the price and volatility of equity as functions of the value of assets (scaled by the amount of debt). The solid red line in the left panel corresponds to the price of equity; the dash-dotted blue line corresponds to the value of equity in the company that avoids bankruptcy, $V_c/r$; and the dotted blue line corresponds to the retained value in the case of bankruptcy, $aV - K$. The solid red line in the right panel corresponds to the values of equity volatility. The dash-dotted blue line corresponds to the volatility of Merton’s investment portfolio, $\pi^M\sigma$. Stars correspond to bankruptcy.

### 6.2 Equity Volatility

Using Ito’s formula, the dynamics of equity can be written as

$$dP^E_t = \left[ \frac{\partial P^E_t}{\partial V} \mu_t^V + \frac{1}{2} \frac{\partial^2 P^E_t}{\partial^2 V} \right] dt + \frac{\partial P^E_t}{\partial V} V_t \sigma_t^V dZ_t, \quad (57)$$

where $\mu_t^V$ and $\sigma_t^V$ are the drift and volatility of the total value of assets in equation (6).

From this expression, the equity volatility can be written as the product of the elasticity of the price of equity multiplied by the volatility of the value of assets,

$$\sigma^E_t = \left( \frac{\partial P^E_t}{\partial V} \frac{V_t}{P^E_t} \right) \sigma_t^V = \left( \frac{\partial P^E_t}{\partial V} \frac{V_t}{P^E_t} \right) \pi_t \sigma. \quad (58)$$
The right panel in Figure 12 shows the equity volatility as a function of the value of assets. When the value of assets declines towards the bankruptcy threshold, the price of equity approaches the value retained in the case of bankruptcy. As the value of assets increases, the price of equity approaches the value of assets reduced by the cost of carrying debt forever, $V - c/r$, that is the equity price of the company that implements investment and payout policies that avoid bankruptcy. Accordingly, the elasticity of the equity to the value of assets approaches $V/(V - c/r)$ as the value of assets increases. As previously discussed, when the value of assets is high, the optimal allocation policy is similar to the policy that avoids bankruptcy, $\pi \approx \pi^M (V - c/r) / V$. Thus, when the value of assets is high, equity volatility is approximately $\pi^M \sigma$ as can be seen in the right panel of Figure 12.

6.3 Sensitivity of Credit Spreads to Equity Volatility

This subsection examines the sensitivity of stock volatility to changes in asset volatility. To make the notation clear, the volatility of the risky asset is explicitly specified as a parameter. Consider an unanticipated change in the risky asset volatility from $\sigma = \sigma_1$ to $\sigma = \sigma_2$. As shown in the previous section, the initial equity volatility can be written as

$$\sigma^E (V; \sigma_1) = \left( \frac{\partial P^E (V; \sigma_1)}{\partial V} \frac{V}{P^E (V; \sigma_1)} \right) \pi (V; \sigma_1) \sigma_1. \quad (59)$$

If the company can adjust its investments instantaneously, then the equity volatility after
the changes in asset volatility is

$$\sigma^E (V; \sigma_2) = \left( \frac{\partial P^E (V; \sigma_2)}{\partial V} \frac{V}{P^E (V; \sigma_2)} \right) \pi (V; \sigma_2) \sigma_2. \quad (60)$$

As discussed, when the value of assets is high, the elasticity of the equity price to the value of assets is approximately $V = (V - c/r)$ and the optimal allocation is approximately $\pi^M (V - c/r) / V$. Therefore, the change in equity volatility is

$$\sigma^E (V; \sigma_2) - \sigma^E (V; \sigma_1) \approx \pi^M (\sigma_2) \sigma_2 - \pi^M (\sigma_1) \sigma_1 = \frac{\mu - r}{\gamma} \left( \frac{1}{\sigma_2} - \frac{1}{\sigma_1} \right). \quad (61)$$

Thus, when value of assets is high, equity volatility decreases with asset volatility if the company can adjust investments instantaneously.

Now, suppose that it takes infinitesimal time interval $\Delta t$ to adjust the allocation of resources between the risky and risk-free investments in response to changes in asset volatility. Denote $P^E (V; \sigma_1, \sigma_2)$ and $\sigma^E (V; \sigma_1, \sigma_2)$, correspondingly, the price of equity and equity volatility after the change in the risky asset volatility from $\sigma_1$ to $\sigma_2$. By the continuity of the price of equity as a function of the value of assets, as $\Delta t \to 0$, the price of equity after the change in asset volatility converge to the price of equity corresponding to the new value of asset volatility, $P^E (V; \sigma_1, \sigma_2) \to P^E (V; \sigma_2)$, and therefore,

$$\sigma^E (V; \sigma_1, \sigma_2) \to \left( \frac{\partial P^E (V; \sigma_2)}{\partial V} \frac{V}{P^E (V; \sigma_2)} \right) \pi (V; \sigma_1) \sigma_2. \quad (62)$$
As before, when the value of assets is high, the elasticity of the price of equity is approximately $V/(V - c/r)$, and the allocation is approximately $\pi^M (V - c/r) / V$. Therefore, in the limit, the change in equity volatility is

$$\sigma^E (V; \sigma_1, \sigma_2) - \sigma^E (V; \sigma_1) \approx \pi_1^M \sigma_2 - \pi_1^M \sigma_1 = \pi_1^M \Delta \sigma. \quad (63)$$

where $\pi_1^M = (\mu - r) / (\gamma \sigma_1^2)$ is the Merton allocation corresponding to the original asset volatility and $\Delta \sigma$ is the change in asset volatility. Thus, the equity volatility increases with the volatility of the risky investments if the company cannot adjust investments instantaneously. This positive relationship is shown in Figure 13 that depicts the instantaneous equity volatility before and after the change in asset volatility. As one can see the change from the initial volatility of 15 percent to a higher volatility of assets (to 20 or to 30 percent) corresponds to a higher equity volatility. The figure also shows that the changes in equity volatility can be approximated by equation (63) when the value of assets is high relative to the amount of debt.

The left panel in Figure 14 shows the sensitivity of equity volatility to the volatility of the risky asset. It is positive and, consistent with the approximation (63), approaches the asymptotic line corresponding to $\pi^M = (\mu - r) / (\gamma \sigma^2)$ as the value of assets goes to infinity. The right panel in Figure 14 shows the sensitivity of credit spreads to equity volatility obtained by combining the sensitivity of credit spreads to asset volatility and the
Figure 13. Changes in Asset Volatility and Equity Volatility. The figure shows equity volatility corresponding to changes in asset volatility as functions of the value of assets (scaled by the amount of debt). The solid blue line corresponds to the initial values of equity volatility corresponding to $\sigma_1 = 0.15$. Dashed green and dotted red lines shows the new values of equity volatilities (corresponding to $\sigma_2 = 0.2$ and $0.3$). Corresponding thin dash-dotted lines show the approximated values, $\sigma_1 + \frac{\nu - \tau}{7\sigma_1^2} (\sigma_2 - \sigma_1)$. Stars correspond to bankruptcy.
sensitivity of equity volatility to asset volatility. As one can see the sensitivity is a U-shaped function of the total value of assets. The sensitivity is positive when the company is close to bankruptcy, but it becomes negative as the value of assets increases. When the sensitivity is negative, it initially increases in absolute value with the value of assets. It declines in absolute value after a certain level of the value of assets because very high values of assets correspond to essentially risk-free debt. This U-shaped relationship is essentially inherited from the sensitivity of credit spreads to asset volatility shown in the right panel of Figure 11. This is not very surprising because, when the value of assets is high,

\[
\frac{\Delta s}{\Delta \sigma^E} \approx \frac{1}{\pi^M} \frac{\Delta s}{\Delta \sigma}.
\]

Note that, by the continuity of debt prices, the established relationship between asset volatility and debt prices holds in the limiting case when the adjustment time goes to zero.
Note that the negative sensitivity of credit spreads to equity volatility, in particular, driven by the different response of the long-term debt and equity of wealthy companies to changes in asset volatility. Roughly speaking, the equity volatility is sensitive to short-term asset volatility, but debt prices are driven by the bond’s lifetime volatility of the firm’s assets.

6.4 Regression of Credit Spreads on Equity Volatility

In the case of the delayed allocation adjustments, there are two events corresponding to changes in asset volatility. First, asset volatility changes but the allocation of resources stays the same. Second, the firm adjusts their asset composition. Sensitivities of credit spreads and equity volatility to asset volatility considered in the previous section correspond to the first event. When the firm adjusts their asset composition, the total asset volatility and, accordingly, equity volatility changes again. Appendix D considers the OLS regression of changes in credit spreads on changes in equity volatility corresponding to these two events. It is shown that in the limit when the adjustment delay approaches zero, the slope coefficient of the regression line is given by

$$\beta = \frac{\Delta s}{\Delta_1 \sigma^E - \Delta_2 \sigma^E}. $$

(64)

where $\Delta s$ is the change in credit spreads and $\Delta_1 \sigma^E$ and $\Delta_2 \sigma^E$ are changes in equity volatility corresponding, respectively, to the change in asset volatility and to the change in the allocation policy.
When the value of assets is high relative to the amount of debt, \( \Delta_1 \sigma^E \approx \pi^M_1 \Delta \sigma \) and \( \Delta_2 \sigma^E \approx \sigma_2 \Delta \pi^M \), where \( \Delta \sigma \) is the change in asset volatility and \( \Delta \pi^M \) is the difference in Merton’s allocations corresponding to the original and new asset volatility. In this case, 

\[
\beta \approx \frac{\sigma_2}{\pi^M_1 (\sigma_1 + 2\sigma_2)} \left( \frac{\Delta s}{\Delta \sigma} \right) .
\] 

(65)

Thus, the slope coefficient is proportional to the sensitivity of credit spreads to asset volatility, and therefore, the slope has the same sign as the sensitivity of credit spreads to asset volatility. Hence, a high value of assets corresponds to a negative slope coefficient when the value of assets is high. This negative relationship is illustrated in Figure 15.

Figure 15 shows the slope coefficient of the OLS regression line calculated using equation (64) as a function of credit spreads. The slope is positive for high yield debt and negative for debt with credit spreads below some level. It approaches zero as the credit spread decreases to zero. This U-shaped relationship is analogous to the U-shaped relationship of the sensitivity of credit spreads to asset volatility. The empirical part of the paper shows a similar relationship between credit spreads of long-term debt and stock volatility. The regression coefficient of changes in credit spreads on changes in stock market volatility is positive for high-yield bonds and negative for investment grade bonds. The highest absolute value of the regression coefficient corresponds to A-rated bonds. Bonds rated A+ or higher and bonds with credit ratings below A have lower absolute values of the regression coefficient than A-rated bonds.
Figure 15. Regression Coefficient on Changes in Equity Volatility. The figure shows the slope coefficient as a function of credit spreads for the OLS regression of changes in credit spreads on changes in equity volatility. The solid red and dashed green lines correspond, respectively, to infinitesimal and finite (±5%) changes in asset volatility. The regression lines correspond to two events in the model: changes in asset volatility keeping allocation constant and the adjustment of the allocation to the new value of asset volatility. Changes in credit spreads are measured in basis points, and changes in equity volatility are measured in percentage points.
7 Empirical Analysis

The empirical part of the paper examines the predicted negative relationship between stock volatility and credit spreads of corporate bonds. The regression analysis in this section utilizes a comprehensive set of bond transaction data to estimate the relationship. The regression results show that the relationship between changes in stock market volatility and credit spreads measured by the corresponding regression coefficient is indeed negative for long-term, high-quality bonds. Moreover, it shows that the value of the coefficient is a U-shaped function of the credit quality of bonds, as predicted in the theoretical part of the paper.

The dependent variable in the base regression analysis provided in this section is the change in the credit spreads of long-term, high-quality bonds. The theoretical part of the paper predicts a negative relationship between stock volatility and credit spreads when the borrower’s wealth is high, that is, the borrower is far from the bankruptcy threshold. This high level of wealth corresponds to high-quality (i.e., high credit rating) bonds. The choice of long-term debt can be explained as follows. One of the assumptions in the presented model is the possibility of the reallocation of resources between risky and risk-free assets. In reality, a quick disposition of assets can be costly, and the optimal strategy could be to reinvest earnings into less risky assets or projects when volatility increases.\footnote{See Guasoni et al. (2014) for a justification of this policy.} If this is the case, the reallocation of resources can take time, and the adjustment of investments is more problematic in the short term. Accordingly, the credit spread of long-term and short-term
debt can have different relationships with volatility.

The main explanatory variable of interest is the change in stock market volatility. Stock volatility is considered in several papers in the literature that investigates the relationship between uncertainty about future outcomes and corporate investments. In the base case, I use the Chicago Board of Exchange (CBOE) Volatility Index VIX as the aggregate measure of stock market volatility, as in Collin-Dufresne et al. (2001). The obtained results corroborate the negative relationship between volatility and credit spreads encountered in that paper.

7.1 Data

To test the predicted relationship between volatility and credit spreads, I use the set of factors proposed in Collin-Dufresne et al. (2001) and often considered in the credit literature as determinants of changes in credit spreads. This set consists of returns on the stock market index and changes in the following variables: the level of spot rates, the slope of the yield curve, firms’ leverage, market volatility, and market volatility smirk. I use these factors to explain changes in the credit spreads of long-term, high-quality corporate bonds.

To construct these variables, I use four main sources of panel data. The Trade Reporting and Compliance Engine (TRACE) data covers OTC bond transactions including bond prices

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41 See, for example, Pindyck (1988), Leahy and Whited (1996), Bulan (2005), Bloom, Bond, and Van Reenen (2007), Baum, Caglayan, and Talavera (2008, 2010).

42 The VIX index has several advantages. The theoretical model considers instantaneous volatility, and the VIX measures short-term, thirty-day, volatility. Additionally, the calculation of the VIX does not assume the lognormal distribution of stock returns (see Demeterfi et al., 1999 for details).
and corresponding yields. All broker-dealers registered with the Financial Industry Regulatory Authority (FINRA) are required to report corporate bond transactions using TRACE. The enhanced dataset contains reported transactions from the inception of TRACE in July 2002. The present research uses the available time span of the data, from 07/01/2002 to 12/31/2012. The second set of data is the Mergent Fixed Income Security Database (FISD), which contains an extensive set of characteristics of publicly offered bonds and provides historical records of credit ratings assigned to bonds by the main credit rating agencies. The third dataset is Standard & Poor’s (S&P) Compustat North America database, which includes fundamental information on public companies. I use this database to obtain accounting data related to bond issuers. The fourth dataset is the stock data provided by the Center for Research and Security Prices (CRSP). The CRSP database is used to calculate stock returns and market capitalizations.

Additionally, I use the following time series: S&P 500 and S&P 100 indexes provided in the Compustat database; constant maturity Treasury yields from the Federal Reserve Board H.15 report; the CBOE volatility indexes VIX and VXO; and the volatility surface of S&P 500 options provided by OptionMetrics.

### 7.2 Cleaning and Merging

This subsection describes the data cleaning procedure of the TRACE bond transaction data and the merging of the cleaned data with the FISD, CRSP, and Compustat datasets. Details of the procedure are provided in Appendix E. The resulting dataset is used for regression
analysis in the following subsections.

In the present study, I consider fixed-coupon, dollar-denominated senior corporate debentures and medium-term notes issued by nonfinancial US companies. To be included in the sample, bonds have to be noncallable, non-puttable, non-convertible, and non-exchangeable (i.e., without additional optionality). I exclude debt backed by assets or with credit enhancements (such as guarantees, letter of credit, etc.). I also exclude private placements to mitigate liquidity issues.

Dick-Nielsen (2014) provides a data-cleaning procedure for the Enhanced TRACE dataset that I use for initial cleaning. The main purpose of this procedure is to adjust transaction data in accordance with correction records. As part of the procedure, I exclude transactions under special circumstances, commissioned trades, and trades with nonstandard settlements.

Some remaining transactions have inconsistent price and yield data and have to be excluded. To eliminate transactions with incorrect data, I recalculate bond prices using reported yields and exclude transactions if the recalculated bond price differs from the reported price by more than five basis points (five cents per 100 dollars notional amount).

The remaining transactions are merged with credit rating histories. In some cases, credit ratings assigned by agencies to the same bond issue are very different. To mitigate this inconsistency, I use the lowest credit rating of S&P and Moody’s. In particular, to sort out transactions corresponding to high-quality debt, I select records corresponding to bonds that are rated A or higher by both S&P and Moody’s at the transaction date.

To select long-term debt, I discard transactions when bonds have less than twelve years
until maturity.\textsuperscript{43} Some of the long-term bonds have very long maturities (up to 100 years). Since the maximum maturity of reported Treasury yields is limited to twenty years during the period from February 18, 2002 to February 9, 2006, the calculation of the credit spreads may be not reliable for very long maturities. To address this issue, I exclude transactions with bonds maturing in more than twenty-five years.

I exclude some period after the collapse of Lehman Brothers, which is associated with drastic changes in the market conditions and very severe liquidity problems.\textsuperscript{44} In such circumstances, companies may not have time to adjust their investments. Additionally, credit rating agencies might lag or be reluctant to adjust credit ratings at that time. For example, AIG and Lehman Brothers had triple-A and double-A credit ratings, respectively, before their collapses. In addition, this period is associated with massive government interventions in the capital markets. Since the model does not account for these issues, the period from September 2008 to May 2009 is excluded from the sample.

The remaining transactions are merged with CRSP common stock (share codes 10 and 11) daily data by matching issuers’ CUSIPs. The result is merged with Compustat quarterly data using the CRSP/Compustat Merged (CCM) dataset. Finally, the dataset is merged with the stock-market index, volatility, and Treasury yield time series using transaction dates. The merging procedure excludes dates for which any time series data is not available.

\textsuperscript{43}The same cutoff is used in Collin-Dufresne et al. (2001) for long-term bonds.

\textsuperscript{44}Several empirical studies show that the effect of liquidity on credit spreads became more important at the time of the financial distress. See, for example, Bao, Pan, and Wang (2011), Dick-Nielsen, Feldhutter, and Lando (2012), Friewald, Jankovitsch, and Subrahmanyam (20012), and Acharya, Amihud, and Bharath (2013). In particular, Bao, Pan, and Wang (2011) find that “illiquidity becomes much more important during the 2008 crisis, overshadowing credit risk” (p. 912).
Since most bond issues are traded infrequently, I use monthly frequency to mitigate market microstructure issues. To construct monthly observations, I use the last transaction in each calendar month and exclude transactions if the bond issue had not been traded for more than 100 days.\footnote{The last trade can be several days before the end of the month, and there can be several months between consecutive observations.} To take into account that the last transaction in a month can be not at the end of the month, I use values of explanatory variables corresponding to transaction dates. Because Compustat data is available on a quarterly basis, the corresponding inter-quarter quantities are obtained using linear interpolation.

The detailed description of the data cleaning and merging procedure is provided in Appendix E, and the number of observations remaining after each step is shown in Table IX. The final dataset has 3,351 observations (differences constructed using monthly observations) corresponding to 65 bonds issued by 29 companies. Table X in the appendix shows the list of companies in the final dataset.

### 7.3 Regression Methodology

This subsection provides a description of the base-case regression model that is used to test the relationship between volatility and credit spreads. The dependent variable in the base case regression model is the change in credit spreads of long-term, high-quality corporate bonds; the main explanatory variable is the change in market volatility. Additional explanatory variables are chosen similar to the determinants of credit spreads in Collin-Dufresne et al. (2001).
The panel regression equation is

\[ \Delta CS_{i,t} = \alpha + \beta_1 \Delta Vol_t + \beta_2 \Delta L_{i,t} + \beta_3 R_t^M + \beta_4 \Delta r_t + \beta_5 \Delta (r_t)^2 + \beta_6 \Delta S_t + \beta_7 \Delta K_t + \varepsilon_{i,t}. \] (66)

In this regression, the dependent variable is the change in the credit spread \( \Delta CS_{i,t} \) of bond \( i \) between two consecutive observations. The explanatory variables are the change in market volatility \( \Delta Vol_t \), the change in the market leverage of the issuer \( \Delta L_{i,t} \), the market excess return \( \Delta r_t \), the change in the squared interest rate \( \Delta r_t^2 \), the change in the slope of the yield curve \( \Delta S_t \), and the change in the volatility smirk \( \Delta K_t \). All these changes and returns correspond to the observation dates of credit spreads (i.e., bond transaction dates).

The credit spread is calculated as the difference between the bond yield and the corresponding Treasury yield. Bond yields are provided in the TRACE database. Treasury yields are calculated by linear interpolation using bond equivalent yields of constant maturity Treasuries in the Federal Reserve H.15 report corresponding to the transaction date.

In the base case model, I use the twenty-year constant maturity Treasury yield as the interest rate level \( r_t \); the difference between twenty- and two-year rates as the slope of the yield curve \( S_t \); the excess return on the S&P 500 as the market excess return \( R_t^M \); and the difference between -25 delta and -50 delta six-month put options on the S&P 500 as the volatility smirk \( K_t \). The market leverage \( L_{i,t} \) is calculated as the book value of the debt divided by the market value of common stocks plus the book value of the debt. The book
value of the debt at the time of the bond transaction is calculated using linear interpolation of the long-term and short-term debt (corresponding to codes DLTTQ and DLCQ in the Compustat database) from quarterly reports right before and right after the transaction date. The market capitalization is calculated as the capitalization of stocks using the CRSP database.

I use the following units of measure for regression variables in the base model and in alternative specifications considered later in this section: Changes in credit spreads and interest rates are measured in basis points; changes in squared interest rates are normalized by 10,000; changes in volatility and the smirk are measured in percentage points; returns, excess returns, and changes in leverage are also measured in percentage points.

To mitigate the possible correlation of error terms, I calculate the standard error clustered by two dimensions, the bond’s CUSIP and the observation month. To check the robustness of the results, I also calculate standard errors using different ways of clustering (including clustering by one dimension and no clustering). To be sure that the results are not driven by outliers I winsorize variables at 1 and 99 percent levels and also rerun regressions without winsorizing and with variables winsorized at 5 and 95 percent levels. The results for different model specifications including alternative standard errors are discussed in the following subsection.
7.4 Base-Case Regression Results

The estimation results for the base and reduced specifications are provided in Table I, which shows that the coefficient corresponding to the volatility term is negative and statistically significant in all cases. It is significant at the 0.1 percent level in the case of the base model. The magnitude of the coefficient is also economically significant. The credit spread decreases by approximately one basis point per each percentage point of volatility. The average DV01 of bonds in the sample is 12.8, and the daily standard deviation of the VIX is 1.5 percent in the considered time span. Therefore, one daily standard deviation change in volatility corresponds to 0.2 percent of the face value. This is remarkable, considering that the sample consists of very high-quality bonds (rated A and higher).

Signs of other coefficients are persistent among regression specifications and consistent with the results in Collin-Dufresne et al. (2001). Credit spreads increase with leverage, the slope of the risk-free yield curve, and the volatility smirk. Credit spreads decrease with the market excess return and the level of the risk-free interest rate. Interestingly, the coefficient corresponding to the leverage of the company has the correct sign but is not statistically significant, which is also consistent with observations in Collin-Dufresne et al. (2001). To check the robustness of the results, I re-estimate models excluding the leverage. The coefficients corresponding to the volatility term remain negative and statistically and economically significant in all cases.

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46 DV01 is the dollar value of one percentage point per 100 dollars notional amount. The sample average, 12.81, is a reasonable value for long-term, high-grade bonds.
Table I

Base Case and Reduced Specifications

The first column corresponds to the base case regression with the full set of covariates, $\Delta CS_{i,t} = \alpha + \beta_1 \Delta Vol_t + \beta_2 \Delta L_{i,t} + \beta_3 R^M_t + \beta_4 \Delta r_t + \beta_5 \Delta \left( r^2_t \right) + \beta_6 \Delta S_t + \beta_7 \Delta K_t + \varepsilon_{i,t}$, where $\Delta$ denotes the change in the variables, $CS_{i,t}$ is the credit spread of bond $i$, $Vol_t$ is market volatility measured by the VIX, $\Delta L_{i,t}$ is the issuer’s leverage, $R^M_t$ is the market excess return measured by S&P 500, $r_t$ is the interest rate level measured by the twenty-year Treasury yield, $S_t$ is the yield curve slope measured by the difference between twenty- and two-year Treasury yields, and $K_t$ is the change in the volatility smirk measured by the difference between implied volatilities of -50 delta and -25 delta six-month put options on S&P 500. The remaining columns correspond to reduced specifications. Associated t-statistics are provided in parentheses under the values of coefficients. One, two, and three stars correspond to five, one, and 0.1 percent significance levels.

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<td>$R^2$</td>
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<tr>
<td>adj. $R^2$</td>
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To further corroborate the relationship, I re-estimate the model using subsamples of data. Table II shows the results for different temporal data splits. The financial crisis is a natural time point to split the sample. Columns I and II correspond to splitting the data by observation time, before and after the financial crisis. Longstaff, Mithal, and Neis (2005) consider the time to maturity and the bond’s age as proxies to bond liquidity, arguing that shorter bonds may be more liquid due to “maturity clienteles for corporate bonds,” and older bonds may be less liquid similar to on-the-run and off-the-run Treasuries. Bao, Pan, and Wang (2011) find that illiquidity indeed increases with the maturity and the age of the bond. Columns III and IV correspond to the split by the time to maturity, 12 to 18 years and 18 to 25 years, respectively. Columns V and VI correspond to younger and older bonds with ages below and above 12 years, respectively. In all cases, the coefficient corresponding to the volatility term is negative and statistically significant. It has the same magnitude, approximately minus one, as in the full sample case.

Table III shows the results for the subsamples created based on company and bond characteristics. The outstanding notional amount is considered a liquidity proxy by Longstaff, Mithal, and Neis (2005) since a higher outstanding amount may correspond to a higher availability of bonds. Bao, Pan, and Wang (2011) confirm that illiquidity decreases with the issue size. The size of the bond issue (up to 250 million and more than 250 million) is considered in Columns I and II. Most of the issuers in the sample are manufacturing companies (see Table X in Appendix E). Column III corresponds to manufacturing and Column IV corresponds to other (mostly retail) companies. In all cases, the volatility coefficient is
Table II
Sample Splits by Time, Maturity, and Age

The base model, $\Delta CS_{i,t} = \alpha + \beta_1 \Delta V o_{l,t} + \beta_2 \Delta L_i,t + \beta_3 R^M_t + \beta_4 \Delta r_t + \beta_5 \Delta \left( r^2_t \right) + \beta_6 \Delta S_t + \beta_7 \Delta K_t + \varepsilon_{i,t}$, is estimated using subsamples of the data. Columns I and II correspond to the split of the sample by the observation time, respectively, before and after the financial crisis. Columns III and IV correspond to bonds with 12 to 18 and 18 to 25 years to maturity, respectively. Columns V and VI correspond to subsamples of younger (less than 12 years) and older (more than 12 years) bonds, respectively. Associated t-statistics are provided in parentheses under the values of coefficients. One, two, and three stars correspond to five, one, and 0.1 percent significance levels.

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<td>(-2.32)</td>
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<td>-1.62***</td>
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<td>0.09</td>
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<td>0.07</td>
<td>0.09</td>
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negative and less than one standard error from -1.01 (the value corresponding to the full sample). The coefficient is significant in all except one subsample with a smaller number of observations where the regression coefficient is even more negative than in the full sample (-1.10). In addition, I check that the results are not driven by outliers. Columns V and VI show results obtained without winsorizing and with variables winsorized at the 5 and 95 percent levels. In both cases, the results are similar to the base case.

As an additional robustness check, I re-estimate the model using alternative specifications of explanatory variables. The results are provided in Table IV. In particular, I use the VXO index to measure market volatility, the S&P 100 index to measure market returns, the ten-year instead of twenty-year Treasury yield, and the three-year yield instead of the two-year yield. The results are qualitatively and quantitatively similar to the base case specification. Collin-Dufresne et al. (2001) additionally consider the market return instead of the excess return and the stock return instead of leverage. I also re-estimate the model using these variables. The results are very similar again.

Finally, I use different methods to calculate the standard error.47 Table V provides t-statistics calculated using alternative standard error specifications. The first column corresponds to the base case standard error double-clustered by bond issue (CUSIP) and time (observation month). The remaining columns correspond to the following standard errors: double-clustered by issuer (company) and time, clustered by bond issue, clustered by issuer, clustered by time, heteroscedasticity-consistent error without clustering, and the usual OLS

---

47 Standard errors clustered by firm and time are examined in Petersen (2009) and Thompson (2011).
The base model, $\Delta CS_{i,t} = \alpha + \beta_1 \Delta Vol_t + \beta_2 \Delta L_{i,t} + \beta_3 R^M_t + \beta_4 \Delta r_t + \beta_5 \Delta (r^2_t) + \beta_6 \Delta S_t + \beta_7 \Delta K_t + \varepsilon_{i,t}$, is estimated using subsamples of the data. Columns I and II correspond to the split of the sample by the size of the bond issue, up to and above 250 million, respectively. Column III corresponds to manufacturing and column IV corresponds to all other companies. Columns V and VI correspond to non-winsorized and winsorized at 5/95 percent level variables. Associated t-statistics are provided in parentheses under the values of coefficients. One, two, and three stars correspond to five, one, and 0.1 percent significance levels.

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<td>0.07</td>
<td>0.12</td>
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The base model, $\Delta CS_{i,t} = \alpha + \beta_1 \Delta Vol_t + \beta_2 \Delta L_{i,t} + \beta_3 R^M_t + \beta_4 \Delta R_t + \beta_5 \Delta (r_t)^2 + \beta_6 \Delta S_t + \beta_7 \Delta K_t + \varepsilon_{i,t}$, is estimated using alternative measurements of regressors. Column I corresponds to market volatility, $Vol_t$, measured by the VXO instead of the VIX. Column II corresponds to market excess returns, $R^M_t$, measured by S&P 100 instead of S&P 500. Column III corresponds to marked returns instead of excess returns. Column IV corresponds to calculations using the ten-year instead of twenty-year Treasury yields. Column V corresponds to the yield curve slope calculated using three-year instead of two-year yields. Column VI corresponds to the volatility smirk calculated using one-year instead of six-month options. Columns VII and VIII correspond to, respectively, stock returns and excess returns instead of changes in the market leverage of the company. Associated t-statistics are provided in parentheses under the values of coefficients. One, two, and three stars correspond to five, one, and 0.1 percent significance levels.

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<td>0.08</td>
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<tr>
<td>adj. $R^2$</td>
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<td>0.08</td>
<td>0.08</td>
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standard error. In all cases, the volatility coefficient is statistically significant at the 0.1 percent level. Note that the base case standard error is one of the most conservative. The standard error double clustered by issuer and time produces a higher significance of the volatility coefficient than in the base case. The t-statistic calculated using the usual OLS standard error is the highest and much higher than the base case t-statistic (-4.17 vs. -3.48).

A stronger theoretical prediction is the U-shaped relationship between the credit quality of corporate bonds and the regression coefficient on changes in stock volatility. According to the model, the coefficient should be positive for very low-quality bonds. The value of the coefficient should decrease and become negative for higher quality bonds. The coefficient should diminish for the highest quality bonds. This is exactly what one can see in Table VI, which shows regression results for bonds bracketed by credit ratings. The regression coefficient on changes in stock volatility is positive for high-yield bonds. It is negative for investment grade bonds. The absolute value of the coefficient for A-rated bonds is higher than for bonds rated below A and for bonds rated above A.\(^{48}\) Thus, the regression results reproduce the predicted relationship for bonds of various credit quality.

In summary, the obtained results unanimously show that there is a negative relationship between changes in market volatility and credit spreads. The results are robust with respect to the model specifications. The negative relationship is robust to sample splits and is not driven by outliers or the choice of the standard error. In addition, the empirical results reproduce the predicted U-shaped relationship between the credit quality of corporate bonds

\(^{48}\)Coefficients are statistically significant for A-rated bonds and become less significant as they declines in absolute value towards zero for lower quality bonds and for bonds rated higher than A.
Table V
Alternative Standard Errors

The table shows estimated values of regression coefficients and t-statistics for the base model, $\Delta CS_{i,t} = \alpha + \beta_1 \Delta Vol_t + \beta_2 \Delta L_{i,t} + \beta_3 R^M_t + \beta_4 \Delta r_t + \beta_5 \Delta (r_t^2) + \beta_6 \Delta S_t + \beta_7 \Delta K_t + \varepsilon_{i,t}$. Columns I and II correspond to double clustering by bond issue and observation month and by company (issuer) and observation month. Columns III to V correspond to single clustering by bond issue, company, and observation month, respectively. Column VI corresponds to the heteroscedasticity-consistent error (without clustering), and column VII corresponds to the usual OLS standard error. Associated t-statistics are provided in parentheses under the values of coefficients. One, two, and three stars correspond to 5%, 1%, and 0.1 percent significance levels.

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<td>3.63**</td>
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<tr>
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<td>0.08</td>
<td>0.08</td>
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</table>
The table shows estimated values of regression coefficients and t-statistics for the base model with bonds bracketed by credit ratings. \( \Delta CS_{i,t} = \alpha + \beta_1 \Delta Vol_t + \beta_2 \Delta L_{i,t} + \beta_3 R_t^M + \beta_4 \Delta r_t + \beta_5 \Delta (r_t^2) + \beta_6 \Delta S_t + \beta_7 \Delta K_t + \varepsilon_{i,t} \). Column I corresponds to high-yield bonds rated from BB+ to B-. Column II, III, and IV correspond to investment grade bonds with credit ratings below A, A, and higher than A, respectively. Associated t-statistics are provided in parentheses under the values of coefficients. One, two, and three stars correspond to ten, five, and one percent significance levels.

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<td>1.72***</td>
<td>0.62</td>
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</tr>
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<td>( R_t^M )</td>
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<td>0.12***</td>
<td>0.19***</td>
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<td>( \Delta K_t )</td>
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<tr>
<td>Intercept</td>
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<td>1.35*</td>
<td>0.45</td>
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and the regression coefficient on the changes in stock volatility.

7.5 Cross-Section of Stock Volatility

This subsection shows that the negative relationship between volatility and credit spreads of long-term, high-quality bonds established in previous sections holds if one takes into account the cross-sectional heterogeneity of stock volatility.

To consider the heterogeneity of stock volatility, I merge the dataset with stock volatility data in OptionMetrics Ivy DB and estimate several alternative regression specifications. The results are provided in Table VII. Column I in this table corresponds to the base-case regression specification.

\[
\Delta CS_{i,t} = \alpha + \beta_1 \Delta VIX_t + \beta_2 \Delta L_{i,t} + \beta_3 R_t^M + \beta_4 \Delta r_t + \beta_5 \Delta r_t^2 + \beta_6 \Delta S_t + \Delta K_t + \varepsilon_{i,t}. \quad (67)
\]

This column confirms that the base regression is not affected significantly by the merging procedure. As one can see, the regression coefficients are very similar to the coefficients in the first column of Table I. In particular, the coefficients on changes in the VIX, in the first rows, are very similar (-1.03 vs. -1.01), and the t-statistics are also very similar (-3.54 vs. -3.48).

Column II corresponds to the regression of changes in credit spreads on changes in the implied volatility of thirty-day stock options instead of changes in the VIX.\(^ 49\) The rest of

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\(^49\)In all regressions in this subsection, I use the implied volatility of at-the-money call options. The results are similar if I use put options or the average volatility of call and put options.
The table shows estimated values for several specifications of stock volatility in the regression \( \Delta CS_{i,t} = \alpha + \beta_1 \Delta V_{i,t} + \beta_2 \Delta L_{i,t} + \beta_3 R^M_{i,t} + \beta_4 \Delta r_t + \beta_5 \Delta S_t + \Delta K_t + \varepsilon_{i,t} \). The first column corresponds to the base-case model with changes in the VIX, \( \Delta V_{i,t} = \Delta VIX_t \). The second column corresponds to the implied volatility of stocks, \( \Delta V_{i,t} = \Delta \sigma_{i,t} \). Columns III and IV correspond to the second stage regression with \( \Delta V_{i,t} = \beta_1^{(1)} \Delta VIX_t \), where \( \beta_1^{(1)} \) is the slope of the time-series regressions of changes in stock volatility on the VIX, \( \Delta \sigma_{i,t} = \alpha_i^{(1)} + \beta_1^{(1)} \Delta VIX_t + \varepsilon_i^{(1)} \). Columns III and IV correspond to the implied and the future realized volatility of stocks respectively. Column V corresponds to the implied market variance, \( \Delta V_{i,t} = \Delta (VIX_t^2) \). Columns VI and VII correspond to the second stage regression with \( \Delta V_{i,t} = \beta_1^{(1)} \Delta (VIX_t^2) \), where \( \beta_1^{(1)} \) is the slope of changes in stock variance on the squared VIX, \( \Delta \sigma^2_{i,t} = \alpha_i^{(1)} + \beta_1^{(1)} \Delta (VIX_t^2) + \varepsilon_i^{(1)} \). Columns VI and VII correspond to the implied and the future realized variance of stocks respectively. The last column corresponds to changes in logs, \( \Delta V_{i,t} = \Delta \log VIX_t \). Plus and one, two, and three stars correspond to ten, five, one, and 0.1 percent significance level.

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<tr>
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<td>0.07</td>
<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
<td>0.08</td>
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</table>
the explanatory variables are the same as in the base-case regression. As one can see, the regression coefficient on the change in stock volatility is negative (-0.09) but smaller, and statistically insignificant.\textsuperscript{50,51} This is not very surprising, as implied volatility data is noisy because of asynchronous trading and is very sensitive to calculation assumptions for short-term options. In the sample, the difference between the implied volatilities of at-the-money call and put options is more than half percentage point in most cases and can be as high as nine percentage points. Moreover, the implied volatilities of stock options are affected by earning announcement and one-time firm-specific events unrelated to the long-term volatility of particular assets.

To mitigate these issues, I run two-stage regressions. In the first stage, I run the univariate time-series regression of changes in stock volatilities on changes in the VIX for each stock in the sample using monthly observations,

$$\Delta \sigma_{i,t} = \alpha_i^{(1)} + \beta_i^{(1)} \Delta VIX_t + \varepsilon_{i,t}^{(1)}. \quad (68)$$

In the second-stage panel regression, I use the base-case specification with changes in the VIX replaced by changes in the VIX multiplied by the slope coefficient from the first stage,

$$\Delta CS_{i,t} = \alpha + \beta_1 \beta_i^{(1)} \Delta VIX_t + \beta_2 \Delta L_{i,t} + \beta_3 R_{i,t}^M + \beta_4 \Delta r_t + \beta_5 \Delta r_t^2 + \beta_6 \Delta S_t + \Delta K_t + \varepsilon_{i,t}. \quad (69)$$

\textsuperscript{50}Note that in addition, the coefficient on the volatility smile became insignificant. I rerun the regression omitting the volatility smile variable. All coefficients and significance levels remained essentially the same in this case.

\textsuperscript{51}Running this regression using changes in log implied volatility produces a negative regression coefficient (-3.82) with a slightly higher t-statistic (-1.01). All other regression coefficients are approximately the same.
Column III shows second-stage regression coefficients corresponding to the regression of changes in the implied volatility of thirty-day call options on the VIX in the first stage. In the sample, the mean (median) value of the slope coefficient in the first-stage regression is 0.74 (0.77), the standard deviation is 0.20, and the skewness is -0.07. All slope coefficients are positive, and most of them are statistically significant. The mean (median) value of the t-statistics is 8.81 (8.90), and the standard deviation is 3.61. The results of the second-stage regression are shown in column III. All coefficients are similar to those in the first column. In particular, the coefficient on changes in the VIX multiplied by $\beta_i^{(1)}$ is negative (-1.06). Although the statistical significance of the coefficient declines, it remains significant with the t-statistic of -2.56.

The implied volatility of stock options corresponds to the risk-neutral volatility. In the model considered in the previous sections, the volatility under the risk-neutral and the physical measure are the same; in the real world, they may be different. To consider the physical volatility, I use the realized stock volatility in the first stage (the regression of changes in realized stock volatility on changes in the VIX corresponding to the same time

---

52 Note that the similarity of the regression coefficient on VIX terms in this and the base case regression (-1.06 vs. -1.03) cannot be taken at face value. The first stage slope coefficients are less than one in most cases: the mean (median) value is 0.74 (0.77). Ceteris paribus, the multiplication of the explanatory variable by something less than one should increase the corresponding regression coefficient. If changes in the VIX are multiplied by the mean slope instead of slopes of individual stocks, the regression coefficient is -1.03/0.74=-1.39. Besides, the first stage estimation errors can introduce estimation bias. The effect of the bias is not obvious in this case, although the attenuation bias usually reduces the regression coefficient and makes it less significant. This logic also applies to other two-stage regressions in this section.

53 Strictly speaking, the VIX is the square root of the risk-neutral expectation of the variance of the S&P 500 index. See Demeterfi et al. (1999) for details.

54 See Bekaert and Hoerova (2014) for the equity variance premium.
intervals). The mean (median) value of the slope coefficient is 0.19 (0.22), the standard deviation is 0.29, and the skewness is 0.17. The high proportion of negative coefficients can be explained by a lower predictive power of the VIX for individual stock volatility. Most of the estimated coefficients are not significant. The mean (median) value of t-statistics is 0.63 (0.70), and the standard deviation is 1.15. The results of the second-stage regression are shown in column IV. Although the first-stage results are noisy, the coefficient on changes in the VIX multiplied by the slope coefficient remains negative (-1.12) and is marginally significant with the t-statistic of -1.82. The decline in significance can be explained by the noise introduced by the first-stage regression.

An alternative specification of the base-case regression can use the annualized variance instead of the volatility of the stock market.\(^{55}\)

\[
\Delta CS_{i,t} = \alpha + \beta_1 \Delta VIX_t^2 + \beta_2 \Delta L_{i,t} + \beta_3 R_t^M + \beta_4 \Delta r_t + \beta_5 \Delta r_t^2 + \beta_6 \Delta S_t + \Delta K_t + \varepsilon_{i,t}. \tag{70}
\]

This regression is shown in column V. As could be expected, the coefficient on the changes in variance is negative (-2.12) and highly significant with the t-statistic of -3.48. The rest of the coefficients are very similar to the base-case regression results shown in column I.

Columns VI and VII correspond to two-stage regressions. First, I run regressions of

\(^{55}\)Note that the units of measure for the variance are different from the volatility of stocks. I scale variance by 100 to keep the same order of the magnitude of coefficients.
changes in annualized stock variance on changes in the squared VIX,

$$\Delta \sigma_{i,t}^2 = \alpha_i^{(1)} + \beta_i^{(1)} \Delta VIX_t^2 + \epsilon_i^{(1)}. \tag{71}$$

In the second stage, I multiply changes in the squared VIX by the slope coefficient obtained in the first stage,

$$\Delta CS_{i,t} = \alpha + \beta_1 \beta_i^{(1)} \Delta VIX_t^2 + \beta_2 \Delta L_{i,t} + \beta_3 R_t^M + \beta_4 \Delta r_t + \beta_5 \Delta r_t^2 + \beta_6 \Delta S_t + \Delta K_t + \epsilon_{i,t}. \tag{72}$$

Column VI shows the results of the second-stage regression corresponding to the first-stage regressions based on the implied volatility of stocks (the regression of changes in the squared implied volatility of thirty-day call options on changes in the squared VIX). The mean (median) value of the slope coefficients in the first-stage regressions is 0.95 (0.97), the standard deviation is 0.43, and the skewness is 0.77. All slope coefficients are positive and statistically significant. The mean (median) value of the t-statistics is 9.41 (8.77), and the standard deviation is 4.19. In the second-stage regression, the coefficient on the changes in the squared VIX multiplied by the slope coefficient $\beta_i^{(1)}$ is negative, -1.49, and statistically significant with the t-statistic of -2.25. All other coefficients have the same sign and similar magnitude as in the base-case regression.

Column VII corresponds to the first-stage regression of the future realized variance of individual stocks on changes in the squared VIX. In the first-stage regression, the mean (median) value of the slope coefficient is 0.27 (0.14), and the standard deviation is 0.50.
Similar to the case with realized volatility, some slope coefficients are negative, and most of them are insignificant. The mean (median) of the standard errors is 0.58 (0.45), and the standard deviation is 1.41. Nevertheless, in the second-stage regression, the coefficient on changes in the squared VIX multiplied by the slope coefficient is negative (-1.26) and marginally significant with the t-statistic of -1.71. All other coefficients have the same sign as in the base-case regression, although the coefficient on the volatility smile is smaller and insignificant. Omitting the volatility smile variable does not significantly change other coefficients in this case.

The last column shows the estimation of the regression with changes in the log VIX instead of changes in the VIX. The rationale is that if individual stock volatility is proportional to the VIX, then the coefficient of proportionality is eliminated by the differences in log volatility, and changes in log stock volatility are the same as changes in the log VIX. The corresponding estimation results are shown in column VIII. As one can see, the coefficient on the changes in the log VIX is negative (-15.67) and significant (the t-statistic is -2.83). The rest of the coefficient is similar to the base-case regression.

I also run the second-stage regressions that include the residual volatility term (the difference between changes in implied volatilities (variances) and changes in the VIX (squared VIX) multiplied by the corresponding slope coefficient). In all regressions, the regression coefficient on the residual term is at least five times smaller than the coefficient corresponding to changes in the VIX (squared VIX), it is always insignificant, and all other coefficients stay almost the same.
In summary, the relationship between changes in stock volatility and credit spreads of long-term, high-quality bonds remains negative in all considered cases that take into account the cross-sectional heterogeneity of stock volatility.

7.6 Financial Crisis

This subsection investigates the relationship between changes in stock volatility and credit spreads of long-term, high-quality bonds during the financial crisis. It shows that, during distressed market conditions, this relationship is positive. It also shows that including data from a distressed period can produce an erroneous positive relationship between changes in volatility and credit spreads for the whole sample.

Previous sections consider samples that exclude the period of financial distress after the collapse of Lehman Brothers. In those cases, the relationship between changes in stock market volatility and credit spreads of long-term, high-quality bonds is negative, as predicted by the theoretical model in the present paper. One of the model’s assumptions is that companies can adjust their allocation of resources and that debt is priced according to the expectations for such adjustments. This may not be a good assumption in periods of financial distress, when fluctuations are drastic, and companies may not be able to adjust their investments. Furthermore, several empirical studies have shown that the effect of liquidity on credit spreads became more important during financial distress. See, for example, Bao, Pan, and Wang (2011), Dick-Nielsen, Feldhutter, and Lando (2012), Friewald, Jankovitsch, and Subrahmanyam (2012), and Acharya, Amihud, and Bharath (2013). In particular, Bao,
Pan, and Wang (2011) find that “illiquidity becomes much more important during the 2008 crisis, overshadowing credit risk” (p. 912). In this case, the VIX can be related to credit spreads in a way not described by the model.

Table VIII shows the regression results for samples that include the financial crisis. Column I corresponds to the subsample period from September 2008 until May 2009 (distressed market). The results correspond to long-term, high-quality (rated A or higher) bonds. As one can see, the results are very different from the regression estimation in column I of Table I, which corresponds to the subsample that excludes the period of market distress (normal market). For the distressed market subsample, the relationship between changes in stock market volatility and credit spreads is positive and statistically significant (2.03 with the t-statistic of 2.19). Note that the regression coefficient is significant even though the sample size is relatively small (341 observations). The Wald test shows that regressions coefficients for these two subsamples are significantly different.56

Column II in Table VIII shows the results for the regression model augmented by the distressed market indicator and the interaction terms with this indicator, using the whole sample, including the distressed market period. As expected, the coefficient on changes in the VIX is the same as in the normal market subsample (-1.01 with the t-statistic of -3.48), and the coefficient on the interaction term with changes in the VIX is positive and highly significant.

56To check that this result is not driven by the way of calculations of the covariance matrix, I recalculate the test using different way of clustering and without clustering. In all cases the result is statistically significant at any conventional level; the value of the F-statistics is higher than 9 in all cases.
Columns I, III, IV, and V correspond to the base-case regression, \( \Delta CS_{i,t} = \alpha + \beta_1 \Delta VIX_t + \beta_2 \Delta L_{i,t} + \beta_3 R_t^M + \beta_4 \Delta r_t + \beta_5 \Delta (r_t^2) + \beta_6 \Delta S_t + \Delta K_t + \varepsilon_{i,t} \), and columns II, VI, and VII correspond to the base-case regression augmented by the indicator of the distressed market (from September 2008 till May 2009), \( I_{DM} \), and interaction terms corresponding to this indicator. The table shows the interaction term with the changes in volatility. Other interaction terms are omitted from the table. Column I corresponds to the distressed market period. Column II and III correspond to the full sample including the distressed market period. Column IV and VI correspond to the subsample from the beginning of the sample data (July 2002) till May 2009. Columns V and VII correspond to the subsample from September 2008 till the end of sample data (December 2012). One, two, and three stars correspond to five, one, and 0.1 percent significance level. t-statistics are provided in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
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<td>( \Delta Vol_t )</td>
<td>2.03*</td>
<td>-1.01***</td>
<td>0.65</td>
<td>1.25***</td>
<td>1.22*</td>
<td>-0.89*</td>
<td>-0.99*</td>
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<td></td>
<td>(2.19)</td>
<td>(-3.48)</td>
<td>(1.54)</td>
<td>(3.32)</td>
<td>(2.34)</td>
<td>(-2.32)</td>
<td>(-2.12)</td>
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<tr>
<td>( \Delta L_{i,t} )</td>
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<td>0.88</td>
<td>-0.05</td>
<td>0.13</td>
<td>-1.76</td>
<td>1.27*</td>
<td>-0.11</td>
</tr>
<tr>
<td></td>
<td>(-1.25)</td>
<td>(1.75)</td>
<td>(-0.06)</td>
<td>(0.15)</td>
<td>(-1.12)</td>
<td>(2.39)</td>
<td>(-0.11)</td>
</tr>
<tr>
<td>( R_t^M )</td>
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<td>-1.08***</td>
<td>-0.26</td>
<td>0.17</td>
<td>-0.24</td>
<td>-0.65*</td>
<td>-1.62**</td>
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<td>(-3.96)</td>
<td>(-0.53)</td>
<td>(0.37)</td>
<td>(-0.34)</td>
<td>(-2.13)</td>
<td>(-2.84)</td>
</tr>
<tr>
<td>( \Delta r_t )</td>
<td>-1.12*</td>
<td>-0.75***</td>
<td>-1.18***</td>
<td>-1.73***</td>
<td>-1.22**</td>
<td>-0.24</td>
<td>-0.35</td>
</tr>
<tr>
<td></td>
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<td>(-4.27)</td>
<td>(-3.58)</td>
<td>(-3.95)</td>
<td>(-2.73)</td>
<td>(-0.33)</td>
<td>(-0.84)</td>
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<tr>
<td>( \Delta (r_t^2) )</td>
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<td>5.10**</td>
<td>9.37**</td>
<td>15.14***</td>
<td>6.13</td>
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<td>(0.35)</td>
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<td>( \Delta S_t )</td>
<td>0.48</td>
<td>0.15***</td>
<td>0.19***</td>
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<td>(1.68)</td>
<td>(4.23)</td>
<td>(3.83)</td>
<td>(3.10)</td>
<td>(4.61)</td>
<td>(4.57)</td>
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<tr>
<td>( \Delta K_t )</td>
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<td>3.36</td>
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<td>1.78</td>
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<td></td>
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<td>(2.59)</td>
<td>(1.33)</td>
<td>(1.73)</td>
<td>(0.08)</td>
<td>(2.27)</td>
<td>(0.88)</td>
</tr>
<tr>
<td>( I_{DM} \times \Delta Vol_t )</td>
<td>3.04**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.27)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>Intercept</td>
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<td>-0.01</td>
<td>0.82</td>
<td>-4.30*</td>
<td>1.38**</td>
<td>-0.66</td>
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<tr>
<td></td>
<td>(-0.87)</td>
<td>(1.75)</td>
<td>(-0.02)</td>
<td>(0.94)</td>
<td>(-2.47)</td>
<td>(2.97)</td>
<td>(-0.70)</td>
</tr>
<tr>
<td>N</td>
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<td>3692</td>
<td>3692</td>
<td>2798</td>
<td>1235</td>
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<td>1235</td>
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<tr>
<td>R^2</td>
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<td>0.16</td>
<td>0.12</td>
<td>0.14</td>
<td>0.18</td>
<td>0.18</td>
<td>0.22</td>
</tr>
<tr>
<td>adj. R^2</td>
<td>0.26</td>
<td>0.16</td>
<td>0.12</td>
<td>0.14</td>
<td>0.18</td>
<td>0.17</td>
<td>0.21</td>
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</table>
significant (3.04 with the t-statistic of 3.27).\textsuperscript{57,58,59} Thus, the relationship between changes in volatility and credit spreads of long-term, high-quality bonds are different in the normal and distressed market samples.

The difference in the sign of the regression coefficient for the distressed and normal market subsamples provides a possible explanation for why some empirical studies find a positive relationship between changes in volatility and credit spreads for long-term, high-quality bonds. Column III shows the regression results for the whole sample, from July 2002 until December 2012, including the distressed market period. As one can see, the coefficient is positive although insignificant (0.65 with the t-statistic of 1.54). This result is misleading. The positive coefficient is induced by observations corresponding to the distressed market conditions after the collapse of Lehman Brothers and cannot be interpreted as a general positive relationship between volatility and credit spreads. As shown in column II, the coefficient on changes in volatility is negative and significant if the regression includes interaction terms.

Columns IV and V show two more misleading regressions. Column IV corresponds to the subsample from the beginning of the sample data (July 2002) until May 2009, and column V corresponds to the subsample from September 2008 until the end of the sample data (Decem-

\textsuperscript{57} Note that coefficients in column II are the same as coefficients in column I of Table I. Strictly speaking, this result depends on the winsorizing procedure: I winsorize subsamples before merging to have consistent results. The estimation using winsorizing after merging produces similar results in this case and other cases in this section.

\textsuperscript{58} The value of the interaction coefficient can be calculated as the difference between the regression coefficients on the changes in volatility corresponding to the distressed market subsample and the subsample without the distressed market period.

\textsuperscript{59} Coefficients on other interaction terms and on the indicator variable are insignificant and are omitted from the table. These coefficients are also omitted for subsequent regressions with interaction terms.
ber 2012). Both subsamples include the distressed market period from September 2008 until May 2009. As the result, the regression coefficients on changes in volatility are positive (1.25 and 1.22) and statistically significant (t-statistics are 3.32 and 2.34, respectively). However, the positive sign of the regression coefficients is misleading; the sign is negative if these regressions are corrected by adding interaction terms. The corresponding results are shown in columns VI and VII. In the regression with interaction terms, the coefficients on changes in volatility are, as expected, the same as in columns I and II of Table II corresponding respectively to subsamples before and after the crisis. These coefficients are negative (-0.89 and -0.99) and significant (t-statistics are -2.32 and -2.12, respectively). The coefficients on the interaction term corresponding to changes in volatility are positive and highly significant (2.93 and 3.02 with t-statistics of 2.94 and 3.12, respectively).

In summary, the relationship between changes in stock volatility and credit spreads of long-term, high-quality bonds is negative in normal market conditions but positive in the distressed market after the collapse of Lehman Brothers. Provided examples show that regression results can be misleading if the sample includes data from the distressed market period.

8 Conclusion

This paper explains the puzzling negative relationship between changes in market volatility and credit spreads of corporate bonds, an empirical reality heretofore unexplained in the theoretical literature. To solve this puzzle, the present study extends Leland’s (1994) capital
structure model to employ Merton’s (1969) approach to asset allocation, and explores the interrelationship between corporate investment decisions (the left-hand side of the balance sheet) and debt financing (the right-hand side of the balance sheet). The extended model incorporates borrowing, investment, payout, and bankruptcy decisions of risk-averse agents. It shows how such phenomena as flight to quality, gambling for resurrection, and bankruptcy for profit emerge sequentially as agents’ wealth deteriorates toward bankruptcy.

Investment and payout decisions, in turn, affect debt prices. The paper shows that, contrary to common beliefs, credit spreads can be lower when the volatility of risky assets is higher. This happens when the value of the company’s assets is high relative to the amount of debt. In this case, the payout rate is lower, and the company makes less risky investments when risky assets are more volatile. This conservative behavior decreases the probability of bankruptcy and leads to lower credit spreads. In addition, higher asset volatility leads to higher stock volatility if the company cannot instantaneously adjust its investments to changes in asset volatility. The combination of these two effects produces the negative relationship between changes in stock volatility and credit spreads. This happens because the stocks and long-term bonds of high-quality borrowers react differently to changes in asset volatility. Stock volatility is sensitive to the short-term volatility of the firm’s assets corresponding to fixed asset composition, while credit spreads depend on the bond’s lifetime volatility of the firm’s assets that can be adjusted in the long run.

In addition, the model presented in this paper predicts a U-shaped relationship between the credit quality of corporate bonds and the regression coefficient (in the regression of
changes in credit spreads on changes in stock volatility controlling for other variables). Intu-
itionally, the coefficient is positive when the company is close to bankruptcy because the option
to default is more valuable in this case (as in Merton, 1974). It is negative for high-quality
debt as explained in the present paper. The value of the coefficient approaches zero as the
credit quality increases further, and the debt becomes essentially risk free.

The model considered in the present paper admits an analytical solution and provides
an intuitive explanation of the relationship between changes in stock volatility and credit
spreads. This relationship is robust to model specifications. In particular, assumptions of
continuous reallocation of resources between risk-free and risky assets and constant volatility
of risky assets can be relaxed, though at the expense of analytical tractability. A compli-
mentary study considers an alternative model in which the allocation of resources is fixed for
some period of time and volatility of risky investments follows a two-state Markov process.

The empirical part of the paper tests the negative relationship between changes in credit
spreads and equity volatility using a comprehensive set of bond transaction data. The re-
gression analysis corroborates the negative relationship between changes in market volatility
measured by the VIX and credit spreads of long-term, high-quality corporate bonds. This
relationship is statistically and economically significant, and it is robust to sample splits and
to outliers. Consistent with the theoretical predictions for long-term bonds, the empirical
results show that the regression coefficient is a U-shaped function of bonds’ credit quality: the
coefficient is more negative for A-rated bonds than for bonds rated below and above A.

In summary, endogenous asset composition can explain the negative relationship between
changes in stock volatility and credit spreads and provides insight into the relationship between the stock and bond markets. Further empirical examination of the theoretical predictions induced by endogenous asset composition is the subject of ongoing research.
References


Elton, E.J., M.J. Gruber, D. Agrawal, and C. Mann, 2001, “Explaining the Rate Spread on


Appendices

A Solvency Conditions

A.1 Sufficient Wealth

This section shows that bankruptcy cannot be avoided with certainty if the value of assets is below some threshold.

Proposition

If $V_0 < c/r$, then bankruptcy cannot be avoided with certainty.

Proof

Suppose to the contrary that there are payout and allocation policies, $C_t^*$ and $\pi_t^*$, such that bankruptcy is avoided with certainty. In this case $V_t > 0$ with probability one for any $t$. Under the risk-neutral measure, the dynamics of the value of assets for these payout and allocation policies is given by

$$dV_t = (rV_t - C_t^* - c) dt + \sigma \pi_t^* V_t dZ_t^Q. \quad (73)$$

Therefore, the discounted value, $X_t = e^{-rt}V_t$, follows

$$dX_t = ( -C_t^* - c) e^{-rt} dt + \sigma \pi_t^* X_t dZ_t^Q. \quad (74)$$
The expected value of $X_t$ under the risk-neutral measure can be written as

$$E^Q [X_t] = V_0 + E^Q \left[ \int_0^t (-C_s - c) e^{-rs} ds \right] \leq V_0 - \int_0^t ce^{-rs} ds = V_0 - \frac{c}{r} (1 - e^{-rt}). \quad (75)$$

Thus, $E^Q [X_t^*] < 0$, for any $t^* > -\frac{1}{r} \ln \left( \frac{c/r - V_0}{c/r} \right)$. Therefore, the risk-neutral probability

$$P^Q [V_{t^*} < 0] = P^Q [X_{t^*} < 0] > 0. \quad (76)$$

By the equivalence of the physical and risk-neutral measures on $\mathcal{F}_{t^*}$,

$$P [V_{t^*} < 0] > 0. \quad (77)$$

Contradiction. Q.E.D.

Due to the time homogeneity of the problem, the proposition can be restated as the following obvious corollary.

**Corollary**

If $V_t < c/r$ for any $t$, then bankruptcy cannot be avoided with certainty.

### A.2 Solvency Constraints

This section provides necessary and sufficient conditions on the coupon rate and the amount of debt, which guaranty that bankruptcy never happens.

If $K/\alpha > c/r$, then for any value of assets $V \geq K/\alpha$ before bankruptcy, the amount
retained after bankruptcy, \(\alpha V - K\), is lower than the value of assets reduced by the cost of carrying debt forever, \(c/r\), because

\[
\alpha V - K = \alpha (V - K/\alpha) < \alpha (V - c/r) \leq V - c/r.
\]

Since \(U^B\) is an increasing function of the value of assets,

\[
U^B (\alpha V - K) < U^B (V - c/r) = U^{NB} (V; \tau).
\]

Hence, when \(K/\alpha > c/r\), it is never optimal to declare bankruptcy because a higher expected utility can be obtained if the agent continues to pay the coupon rate forever. Thus, bankruptcy will never occur if \(K/\alpha > c/r\) and \(V_0 > c/r\). Note that if \(K/\alpha > c/r\), then the risk-free debt is a possible equilibrium outcome because in the case of risk-free debt

\[
V_0 = V^E + P^D (V_0, \tilde{c}) > P^D (V_0, \tilde{c}) = \overline{c}/r \geq c/r.
\]

Therefore, assuming that the borrower can always obtain the highest possible price of debt in the competitive lending market, there is no bankruptcy if \(K/\alpha > c/r\), that is, if \(\tilde{c} < Kr / ((1 - \tau) \alpha)\). Similar steps show that in the case of \(K/\alpha = c/r\), the agent cannot achieve higher expected utility by declaring bankruptcy.

\(^{60}\alpha V - K \leq 0\) for \(c/r < V \leq K/\alpha\). I extend \(U^B (V) = -\infty\) for \(V \leq 0\) to have \(U^B (\alpha V - K)\) defined for \(V > c/r\).
In the other case, when $K/\alpha < c/r$,

$$\alpha V - K = \alpha (V - c/r) + \alpha (c/r - K/\alpha) > V - c/r$$

for $V$ sufficiently close to $c/r$. For such values of $V$,

$$U^B (\alpha V - K) > U^B (V - c/r) = U^{NB} (V; \overline{c}).$$

Therefore, in this case, it is more attractive to declare bankruptcy than to continue paying the coupon rate forever. Thus, the bankruptcy state is attainable if $K/\alpha < c/r$, that is, if $\overline{c} > Kr/((1 - \tau) \alpha)$. The following solvency constraint on debt payments combines the considered cases.

**Solvency Constraint on Debt Payments**

There is no bankruptcy if and only if the coupon rate satisfies

$$\overline{c} \leq \frac{r}{(1 - \tau)} \frac{K}{\alpha}.$$ 

To obtain the solvency constraint on the amount of debt, note that the interest rate paid on debt cannot be lower than the risk-free rate. Therefore, either $D = \overline{c}/r$ or $D < \overline{c}/r$. If $D \leq K/((1 - \tau) \alpha)$, then $D = \overline{c}/r$ is a possible outcome because the solvency constraint is satisfied in this case, and $D < \overline{c}/r$ cannot be an equilibrium outcome because the agent would prefer to deviate by choosing the risk-free debt contract with the same amount of borrowing.
at a lower coupon rate. Therefore, if \( D \leq K/(1 - \tau) \alpha \), then there is no bankruptcy. If \( D > K/(1 - \tau) \alpha \), then \( c/r > K/\alpha \) because \( c/r \geq D \). Therefore, bankruptcy cannot be avoided with certainty in this case. This gives the following solvency constraint on the amount of debt.

**Solvency Constraint on the Amount of Debt**

There is no bankruptcy if and only if the amount of debt satisfies

\[
D \leq \frac{K}{(1 - \tau) \alpha}.
\]

### B Investment and Payout Policies without Bankruptcy

This appendix derives optimal investment and payout policies that avoid bankruptcy with certainty in the model with coupon payments (adjusted for the tax shield) at a constant rate \( c \). Results corresponding to the model without debt can be obtained by taking \( c = 0 \). In addition to the case with endogenous investment and payout choices, exogenous investment and/or payout policies are also considered in this appendix.

The optimal investment and payout policies that avoid bankruptcy are feasible if the initial wealth is higher than the cost of debt, \( V_0 > c/r \), as shown in Appendix A. Moreover, the value of assets has to be higher than the cost of debt to avoid bankruptcy with certainty. In the following subsections, I use this fact and consider the value of assets reduced by the cost of debt, \( V_t - c/r \).

Note that, as shown in section 3.2, if the solvency constraint is satisfied, \( c/r \leq K/\alpha \),
then optimal investment and payout policies that avoid bankruptcy are the optimal policies.

**B.1 Endogenous Investment and Payout Policies**

In the notation of the main model, the dynamics of the total value of assets are given by

$$dV_t = ((r + \lambda \pi_t) V_t - C_t - c) dt + \sigma \pi_t V_t dZ_t, \quad (78)$$

where $\lambda = \mu - r$. This equation can be written in terms of the value of assets above the cost of debt $\tilde{V}_t = V_t - c/r$ as

$$d\tilde{V}_t = \left( (r + \lambda \tilde{\pi}_t) \tilde{V}_t - C_t \right) dt + \sigma \tilde{\pi}_t \tilde{V}_t dZ_t, \quad (79)$$

where $\tilde{\pi}_t = V_t \pi_t / \tilde{V}_t$ is the proportion of debt-adjusted wealth allocated to the risky asset. Denote $\bar{U} \left( \tilde{V}_t \right) \equiv U \left( \tilde{V} + c/r \right) = U \left( V_t \right)$. The HJB equation in terms of new variables can be written as

$$\delta \bar{U} = \max_{\tilde{\pi}, C} \left\{ u \left( C \right) + \left( (r + \tilde{\pi} \lambda) \tilde{V} - C \right) \tilde{U}_{\tilde{V}} + \frac{1}{2} \sigma^2 \tilde{\pi}^2 \tilde{V}^2 \tilde{U}_{\tilde{V} \tilde{V}} \right\}. \quad (80)$$

The first-order conditions on investment $\tilde{\pi}$ and payout $\tilde{C}$ are, respectively,

$$\tilde{\pi} = -\frac{\tilde{V}_{\tilde{V}}}{\tilde{W} \tilde{V}} \frac{\lambda}{\sigma^2} \quad (81)$$
and

\[ \check{C} = I \left( \check{U}_{\check{V}} \right), \quad (82) \]

where \( I(\cdot) \) is the inverse function of \( u'(\cdot) \). The substitution of these first-order conditions into the HJB equation gives the following differential equation:

\[ \delta \check{U} = u \left( I \left( \check{U}_{\check{V}} \right) \right) + \left( r\check{V} - I \left( \check{U}_{\check{V}} \right) \right) \check{U}_{\check{V}} - \frac{1}{2} \frac{\lambda^2 \left( \check{U}_{\check{V}} \right)^2}{\check{U}_{\check{V}\check{V}}}. \quad (83) \]

Conjecture

\[ \check{U} = \frac{b \check{V}^{1-\gamma}}{1-\gamma}. \quad (84) \]

This conjecture gives the following equation for \( b \):

\[ \frac{b}{1-\gamma} = \left( \gamma b^{-1/\gamma} + (1 - \gamma) \left( r + \frac{1}{2} \frac{\lambda^2}{\gamma\sigma^2} \right) \right) \frac{b}{1-\gamma}. \quad (85) \]

As in the standard Merton's model, the value of \( b \) is

\[ b = \left\{ \frac{1}{\gamma} \left[ \delta - (1 - \gamma) \left( r + \frac{1}{2} \frac{\lambda^2}{\gamma\sigma^2} \right) \right] \right\}^{-\gamma}. \quad (86) \]

Thus, the value function and optimal investment and payout policies are given, respectively, by

\[ U(V) = \frac{b(V - c/r)^{1-\gamma}}{1-\gamma}, \quad (87) \]
\[ \pi V = \frac{\lambda}{\sigma^2 \gamma} (V - c/r), \quad (88) \]

and

\[ C = \theta (V - c/r), \quad (89) \]

where the payout ratio is

\[ \theta = b^{-1/\gamma} = \frac{1}{\gamma} \left[ \delta - (1 - \gamma) \left( r + \frac{1}{2} \frac{\lambda^2}{\gamma \sigma^2} \right) \right]. \quad (90) \]

### B.2 Exogenous Allocation

Suppose that the allocation policy is an exogenously given constant proportion of wealth adjusted by the cost of debt, \( \bar{\pi} (V - c/r) \). In this case, the dynamics of the value of assets have the same form (79) as in the previous subsection (keeping in mind that \( \bar{\pi} \) is a given constant), and the HJB equation is the same but with one optimization variable,

\[ \delta \bar{U} = \max_C \left\{ u(C) + \left( (r + \bar{\pi} \lambda) \bar{V} - C \right) \bar{U}_W + \frac{1}{2} \sigma^2 \bar{\pi}^2 \bar{V}^2 \bar{U}_{\bar{V} \bar{V}} \right\}. \quad (91) \]

Accordingly, there is just one first order condition given by (82). The substitution of this first-order condition into the HJB equation gives the following ODE:

\[ \delta \bar{U} = \frac{1}{1 - \gamma} \left( \bar{U}_{\bar{V}} \right)^{1-1/\gamma} + \left( (r + \bar{\pi} \lambda) \bar{V} - \bar{U}_{\bar{V}}^{-1/\gamma} \right) \bar{U}_{\bar{V}} + \frac{1}{2} \sigma^2 \bar{\pi}^2 \bar{V}^2 \bar{V}_{\bar{V} \bar{V}}. \quad (92) \]
The same conjecture (84) leads to the value function of the same functional form,

\[ U(V) = \frac{b(V - c/r)^{1-\gamma}}{1 - \gamma}, \]  

(93)

where

\[ b = \left\{ \frac{1}{1 - \gamma} \left[ \delta - (1 - \gamma) \left( r + \tilde{\pi} \lambda - \frac{1}{2} \gamma \sigma^2(\tilde{\pi})^2 \right) \right] \right\}^{-\gamma}. \]  

(94)

In this case, the optimal payout policy is

\[ C = \theta (V - c/r), \]  

(95)

where

\[ \theta = b^{-1/\gamma} = \frac{1}{1 - \gamma} \left[ \delta - (1 - \gamma) \left( r + \tilde{\pi} \lambda - \frac{1}{2} \gamma \sigma^2(\tilde{\pi})^2 \right) \right]. \]  

(96)

Note that, in the case of optimal allocation policy given by (88), equation (94) corresponds to (86). In this case, the value function (93) and the optimal payout policy (95) coincide with the ones in the previous subsection.

### B.3 Exogenous Payout Policy

In the case when the payout rate is an exogenous constant fraction of the debt-adjusted wealth, \( \theta (W - c/r) \), the same approach as in previous subsections leads to the HJB equation
that can be written as

$$\delta \tilde{U}_\pi = \max \left\{ u(C) + \left( (r + \tilde{\pi} \lambda) \tilde{V} - C \right) \tilde{U}_\tilde{V} + \frac{1}{2} \sigma^2 \tilde{\pi}^2 \tilde{V}^2 \tilde{U}_\tilde{V} \tilde{V} \right\}. \quad (97)$$

The substitution of the first order condition and the same conjecture (84) give the following value function,

$$U(V) = \frac{b(V - c/r)^{1-\gamma}}{1-\gamma}, \quad (98)$$

where

$$b = \frac{\theta^{1-\gamma}}{\theta + \left[ -\gamma \theta + \delta - (1-\gamma) \left( r + \frac{1}{2\gamma} \frac{\lambda^2}{\sigma^2} \right) \right]} \quad (99)$$

The optimal allocation is given by

$$\tilde{\pi} = \frac{\lambda}{\gamma \sigma^2}. \quad (100)$$

It does not depend on the payout choice and is the same as in the case with endogenous payout and allocation policies.

Note that in the case of the optimal payout policy given by (90), the term in square brackets in formula (99) is zero, and therefore, (99) corresponds to (86). In this case, the value function (98) is the same as in the case with endogenous payout and allocation policies.
B.4 Exogenous Payout and Allocation Policies

In the case when the payout rate and allocation are exogenous constant proportions of the debt-adjusted wealth, $V - c/r$, the value function satisfies the following ODE:

$$
\delta \hat{U} = u\left(\theta \hat{V}\right) + (r + \pi \lambda - \theta) \hat{V} \hat{U}_\hat{V} + \frac{1}{2} \sigma^2 \hat{V}^2 \hat{U}_{\hat{V} \hat{V}}.
$$

(101)

The solution for this ODE obtained using the same conjecture is

$$
U(V) = \frac{b(V - c/r)^{1-\gamma}}{1 - \gamma},
$$

(102)

where

$$
b = \frac{\theta^{1-\gamma}}{\theta + \left[-\gamma \theta \delta - (1 - \gamma) \left(r + \pi \lambda - \frac{\gamma}{2} \sigma^2 \pi^2\right)\right]}.
$$

(103)

Note that in the case of the optimal payout ratio given by (96), this equation corresponds to (94), and in the case of the optimal allocation rule given by (100), it coincides with (99) as expected.

C Possible Bankruptcy

This appendix provides solution details in the case when the bankruptcy threshold is attainable.
C.1 Payout and Allocation Policies

Before bankruptcy, the dynamics of the total value of assets is given by

\[ dV_t = \left( (r + \lambda \pi_t) V_t - C_t - c \right) dt + \sigma \pi_t V_t dZ_t, \tag{104} \]

where \( \lambda = \mu - r \) is the risk premium. Denote \( \tilde{C}_t \equiv C_t + c \) and \( \tilde{u} \left( \tilde{C} \right) \equiv u \left( \tilde{C} - c \right) = u \left( C \right) \).

In this notation the dynamics are

\[ dV_t = \left( (r + \lambda \pi_t) V_t - \tilde{C}_t \right) dt + \sigma \pi_t V_t dZ_t, \tag{105} \]

and the optimization problem is given by

\[ U (V_0) = \sup_{\tilde{C}, \pi} \left\{ E \left[ \int_0^\infty e^{-\delta t} \tilde{u} \left( \tilde{C}_t \right) dt \right] \right\}. \tag{106} \]

The corresponding HJB equation can be written as

\[ \delta U (V) = \max_{\tilde{C}, \pi} \left\{ \tilde{u} \left( \tilde{C} \right) + \left( (r + \lambda) V - \tilde{C} \right) U_V + \frac{1}{2} \pi^2 \sigma^2 V^2 U_{VV} \right\}. \tag{107} \]

The first order conditions on \( \pi \) and \( \tilde{C} \) are, respectively,

\[ \pi = -\frac{U_V}{V U_{VV}} \cdot \frac{\lambda}{\sigma^2} \tag{108} \]
and

$$\tilde{C} = \tilde{I} (U_V).$$  \hfill (109)$$

where $\tilde{I} (x) = x^{-1/\gamma} + c$ is the inverse function of $\bar{u}' (\cdot)$. The substitution of these first-order conditions results in the following second-order non-linear differential equation:

$$\delta U = \tilde{u} \left( \tilde{I} (U_V) \right) + \left( rV - \tilde{I} (U_V) \right) U_V \frac{1}{2} \frac{\lambda^2 (U_V)^2}{U_{VV}},$$  \hfill (110)$$

The value function has to satisfy the following smooth pasting and value matching conditions at the bankruptcy threshold:

$$U (V^B) = U^B (\alpha V^B - K)$$  \hfill (111)$$

and

$$U_V (V^B) = \alpha U^B_V (\alpha V^B - K),$$  \hfill (112)$$

where $U^B (V)$ is the indirect utility function after bankruptcy.

A similar ODE with a zero-wealth boundary condition is considered in Presman and Sethi (1991). In this section, I extend the solution of ODE (110) provided in that paper to the non-zero bankruptcy boundary determined by (111) and (112).

Assuming that the solution is strictly concave (as it is the case), the marginal utility of wealth is a strictly decreasing function of wealth, and accordingly, $v = - \ln (U_V (V))$ is a strictly increasing function of wealth. Therefore, we can consider the inverse function
$V = V(v)$. Using this change of variables, derivatives of the value function can be written as

$$U_V (V(v)) = e^{-v}$$

(113)

and

$$U_{VV} (V(v)) = -e^{-v}/V'(v).$$

(114)

Using these expressions, ODE (110) can be written as

$$\delta U (V(v)) = \tilde{u}(i(v)) + (rV(v) - i(v)) e^{-v} + \psi e^{-v} V'(v),$$

(115)

where $\psi = \lambda^2/(2\sigma^2)$ and $i(v) = \tilde{I}(e^{-v})$. The differentiation of this equation produces a second-order linear ODE:

$$\psi V''(v) = (\delta + \psi - r) V'(v) + rV(v) - i(v).$$

(116)

This ODE admits a parameterized solution (see the following subsection for verification) that extends the solution proposed in Presman and Sethi (1991) to the case of non-zero bankruptcy barrier $V^B$,

$$V(v; w, V^B) = \frac{i(v)}{r} + A(v) - B(v; w, V^B),$$

(117)
where \( w \leq v \),

\[
A(v) = \frac{e^{y_+v}}{\phi y_+} \int_{i(v)}^{\infty} (\tilde{u}'(x))^{y_+} dx,
\]

(118)

and

\[
B(v, w, V^B) = -\frac{e^{y_-}}{\phi y_-} \int_{i(w)}^{i(v)} (\tilde{u}'(x))^{y_-} dx + e^{(v-w)y_-} \left( \frac{i(w)}{r} + A(w) - V^B \right),
\]

(119)

where \( y_+ \) and \( y_- \) are roots of the characteristic equation,

\[
\chi(y) \equiv \psi y^2 + (r - \delta - \psi) y - r = 0,
\]

(120)

\[
y_{\pm} = \frac{\psi + \delta - r \pm \sqrt{(r - \delta - \psi)^2 + 4\psi r}}{2\psi},
\]

(121)

and \( \phi \equiv \psi (y_+ - y_-) = \sqrt{(r - \delta - \psi)^2 + 4\psi r} > 0 \). In this case, there are two parameters, \( w \) and \( V^B \), corresponding to two boundary conditions (111) and (112).\(^{61}\) Note that \( \chi(0) = -r < 0 \) and \( \chi(1) = -\delta < 0 \). Therefore, \( y_+ > 1 \) and the improper integral in (118) converges (assuming \( \gamma > 1 \)).

The substitution of the solution (117) into (115) gives a parameterized formula for the value function

\[
J(v; w, V^B) \equiv U(V(v; w, V^B)) = \frac{\tilde{u}(i(v))}{\delta} + \frac{r e^{-v}}{\delta} \left[ \frac{\rho_+ B(v; w, V^B) - \rho_-}{y_+} A(v) \right],
\]

(122)

\(^{61}\)The solution considered in Presman and Sethi (1991) corresponds to \( V^B = 0 \).
where \( \rho_\pm = 1 - y_\pm \).

To apply boundary conditions, note that \( V(w; w^B, V^B) = V^B \). The smooth pasting condition (112) gives the value of \( w \) in terms of the bankruptcy boundary,

\[
w^B = - \ln U_V (V(w^B; w^B, V^B)) = - \ln U_V (V^B) = - \ln (\alpha U_V (\alpha V^B - K)).
\]  

(123)

In the case of CRRA utility function, \( U^B_V (V) = b V^{-\gamma} \). Therefore,

\[
w^B = \gamma \ln (\alpha V^B - K) - \ln (\alpha b),
\]  

(124)

where \( b \) is a constant given by

\[
b = \left\{ \frac{1}{\gamma} \left[ \delta - (1 - \gamma) \left( r + \frac{1}{2\gamma} \frac{(\mu - r)^2}{\sigma^2} \right) \right] \right\}^{-\gamma}.
\]  

(125)

The value matching condition provides the equation for the default boundary:

\[
U^B (\alpha V^B - K) = U (V^B) = U (V(w^B; w^B, V^B)) = J(w^B; w^B, V^B),
\]  

(126)

where \( w^B \) is given by (123). The solution to this equation is provided later in this appendix.

To completely specify the solution in terms of the value of assets, note that \( V(v; w^B, V^B) \) is an increasing function of \( v \). Consider the inverse function \( v = Y(V; w^B, V^B) \). The value
function can be calculated as

\[ U (V; \bar{c}) = J \left( Y \left( V; w^B, V^B \right); w^B, V^B \right), \] (127)

where \( V^B \) is the solution to (126), \( w^B \) is given by (123), and \( J(v; w, V^B) \) is given by (122).

From the first order condition (108) and equations (113), and (114) the optimal investment policy can be written as,

\[ \pi (V) = \left( \frac{\mu - r}{V \sigma^2} \right) V' \left( Y \left( V; w^B, V^B \right); w^B, V^B \right) \]

\[ = \left( \frac{\mu - r}{V \sigma^2} \right) \left[ y_+ A \left( Y \left( V; w^B, V^B \right) \right) - y_- B \left( Y \left( V; w^B, V^B \right), w^B, V^B \right) \right]. \] (128)

The optimal payout rate is given by

\[ C = \bar{C} - c = \bar{I} (U_V) - c = I (U_V) = I \left( \exp \left( Y \left( V; w^B, V^B \right) \right) \right). \] (129)

In the case of CRRA utility function, \( I (x) = x^{-1/\gamma} \). Therefore, the non-negative payout condition is satisfied,

\[ C = e^{-Y(V;w^B,V^B)/\gamma} \geq 0. \] (130)

The verification of the optimality of the solution is a standard application of the Ito’s
formula that can be done similar to Karatzas et al. (1986).

C.2 Parameterized Solution

To verify the solution of equation (116) given by (117), (118), and (119), note that $y_+ y_- = -\frac{r}{\psi}$ and $\phi y_+ y_- = -r (y_+ - y_-)$.

$$A' (v) = \frac{e^{y_+ v}}{\phi} \int_{i(v)}^{\infty} (\tilde{u}' (x))^{y_+} dx - \frac{e^{y_+ v}}{y_+ \phi} \tilde{y}' (i (v)) \left( \tilde{u}' (i (v)) \right)^{y_+} = y_+ A (v) - \frac{\tilde{y}' (v)}{y_+ \phi}$$

because by definition $\tilde{u}' (i (v)) = e^{-v}$;

$$B' (v; w, V^B) =$$

$$= -\frac{e^{y_- v}}{\phi} \int_{i(v)}^{\tilde{i}(v)} (\tilde{u}' (x))^{y_-} dx - \frac{e^{y_- v}}{y_- \phi} \tilde{y}' (i (v)) \left( \tilde{u}' (i (v)) \right)^{y_-} + y_- e^{(v-w)y_-} \left( \frac{i (w)}{r} - V^B + A (w) \right) =$$

$$= y_- B (v; w, V^B) - \frac{\tilde{y}' (v)}{y_- \phi} ;$$

$$V' (v; w, V^B) = \frac{i' (v)}{r} + A' (v) - B' (v; w, V^B) =$$

$$= y_+ A (v) - y_- B (v; w, V^B) ;$$

$$V'' (v; w, V^B) = y_+ A' (v) - y_- B' (v; w, V^B) =$$

$$= y_+^2 A (v) - y_-^2 B (v; w, V^B) .$$
Therefore,

\[
\psi V''(v; w, V^B) - (\delta + \psi - r) V'(v; w, V^B) - r V(v; w, V^B) + i(v) = \\
A(v) (\psi y^2_+ - (\delta + \psi - r) y_+ - r) - B(v; w, V^B) (\psi y^2_- - (\delta + \psi - r) y_- - r) = 0.
\]

Q.E.D.

### C.3 Optimal Bankruptcy Boundary

To solve equation (126), note that

\[
B(w, w) = \frac{i(w)}{r} - V^B + A(w).
\]

The substitution of this expression in (122) gives

\[
J(w; w, W^B) = \frac{\tilde{u}(i(w))}{\delta} + \frac{e^{-w}}{\delta} \left[ \frac{\rho_i}{y_+} (w) - \frac{\rho_i}{y_+} rV + \phi A(w) \right].
\]

In the case of CRRA utility function,

\[
\tilde{u}(i(w)) = \frac{e^{w(1-\gamma)/\gamma}}{1 - \gamma}
\]

and

\[
A(v) = \frac{e^{y_+v}}{\phi y_+} \int_{i(v)}^\infty (x - c)^{-\gamma y_+} dx = \frac{1}{\phi y_+ (\gamma y_+ - 1)} e^{v/\gamma}.
\]
Therefore,

\[
J(w; w, V^B) = \frac{e^{w(1-\gamma)/\gamma}}{\delta} \left[ \frac{1}{1-\gamma} + \frac{\rho_+}{y_+} + \frac{1}{y_+ (\gamma y_+ - 1)} \right] + \frac{e^{-w}}{\delta} \frac{\rho_+}{y_+} (c - rV^B). \tag{135}
\]

This expression together with the smooth pasting condition (124) gives

\[
J(w^B; w^B, V^B) = \frac{(ab)^-(1-\gamma)/\gamma (\alpha V^B-K)^{(1-\gamma)}}{\delta} \left[ \frac{1}{1-\gamma} + \frac{\rho_+}{y_+} + \frac{1}{y_+ (\gamma y_+ - 1)} \right] + \frac{\alpha (\alpha V^B - K)^{-\gamma}}{\delta} \frac{\rho_+}{y_+} (c - rV^B). \tag{136}
\]

From this expression and the value matching condition (126),

\[
\frac{b(\alpha V^B - K)^{1-\gamma}}{1-\gamma} = \frac{(ab)^-(1-\gamma)/\gamma (\alpha V^B-K)^{(1-\gamma)}}{\delta} \left[ \frac{1}{1-\gamma} + \frac{\rho_+}{y_+} + \frac{1}{y_+ (\gamma y_+ - 1)} \right] + \frac{\alpha b (\alpha V^B - K)^{-\gamma}}{\delta} \frac{\rho_+}{y_+} (c - rV^B). \tag{137}
\]

This gives a linear equation in \( V^B \),

\[
\frac{(V^B-K/\alpha)}{1-\gamma} = \frac{\alpha^-(1-\gamma)/\gamma b^{-1/\gamma} (V^B-K/\alpha)}{\delta} \left[ \frac{1}{1-\gamma} + \frac{\rho_+}{y_+} + \frac{1}{y_+ (\gamma y_+ - 1)} \right] + \frac{\rho_+}{\delta y_+} (c - rV^B). \tag{138}
\]

Thus, the optimal bankruptcy boundary is given by

\[
V^B = \frac{K}{\alpha} + \left( \frac{c}{r} - \frac{K}{\alpha} \right) \frac{r \rho_+ / y_+}{\delta - \alpha^{-(1-\gamma)/\gamma b^{-1/\gamma}} \left[ \frac{1}{1-\gamma} + \frac{\rho_+}{y_+} + \frac{1}{y_+ (\gamma y_+ - 1)} \right] + \frac{\rho_+}{y_+}. \tag{139}
\]

Note that \( y_+ y_- = -r/\psi \). Therefore, \( r \rho_+ / y_+ = -(r + \psi y_-) \), and after some simplifications
the formula for the optimal bankruptcy boundary can be written as

\[ V^B = \frac{K}{\alpha} + A \left( \frac{c}{r} - \frac{K}{\alpha} \right), \]  

(140)

where

\[ A = \frac{1}{(1 - \alpha^{1-1/\gamma}) \left( 1 + \frac{(1+(\delta+\psi)/r)\sqrt{(1+(\delta+\psi)/r)^2-4\delta/r}}{2(\gamma-1)} \right)} > 0. \]  

(141)

It is easy to see that \( V^B \) is a decreasing function of \( \psi \), and therefore, an increasing function of \( \sigma \).

Not that when the solvency constraint is violated, \( c/r > K/\alpha \), the bankruptcy boundary is higher than \( K/\alpha \), that is \( \alpha V^B - K > 0 \) as expected. Otherwise, bankruptcy is not feasible.

### C.4 Dynamics of Marginal Utility

This section provides the dynamics of the marginal utility of wealth under the risk-neutral and physical measures.\(^{63}\) These dynamics are used to calculate the risk-neutral probability of default and the debt and equity prices.

\(^{62}\) A similar formula is obtained by Jeanblanc et al. (2004) using another approach based on Karatzas et al. (1986).

\(^{63}\) The derivation of the risk-neutral dynamics of marginal utilities follows Presman and Sethi (1996) who considered these dynamics and the probability of bankruptcy under the physical measure in the case of zero-wealth bankruptcy. The primal interest of this appendix is the risk-neutral dynamics and probabilities since they are relevant for debt prices.
Under the risk-neutral measure $Q$, the dynamics of the total value of assets are given by,

$$dV_t = (rV_t - C_t - c) \, dt + \sigma \pi_t V_t dZ^Q_t,$$

where $Z^Q_t$ is the standard Brownian motion under $Q$. These dynamics can be written in terms of variables in equation (105) as

$$dV_t = \left( rV_t - \tilde{C}_t \right) \, dt + \sigma \pi_t V_t dZ^Q_t.$$

The dynamics of $v_t = Y(V_t; w; V^B)$ can be obtained by using Itô’s formula

$$dv_t = \left[ \left( rV_t - \tilde{C}_t \right) Y' + \frac{1}{2} \sigma^2 \pi_t^2 V_t^2 V'' \right] \, dt + \sigma \pi_t V_t Y' dZ^Q_t.$$

Note that by the definition of $Y$,

$$Y'(V) = 1/V'$$

and

$$Y''(V) = -V''/(V')^3.$$

Also note that, from the first order condition (108) and equations (113) and (114), the optimal allocation can be written as

$$\pi = \frac{\lambda}{V \sigma^2} V'.$$

---

To simplify the notation I omit parameters hereafter in this subsection.
The substitution of equations (145), (146), and (147) into the dynamics equation (144) gives

\[ dv_t = \left( r V_t + 2 \psi V'_t - \tilde{C}_t \right) \left\{ \frac{1}{V_t'} - \psi \frac{V''_t}{V'_t} \right\} dt + \frac{\lambda}{\sigma} dZ_t^Q. \] (148)

From (116), the optimal payout rate satisfies

\[ \tilde{C} = V'' + \left( \delta + \psi - r \right) V' + r V. \] (149)

The substitution of this expression into the dynamics equation (148) eliminates dependencies on \( V, V', \text{ and } V'' \):

\[ dv_t = (r - \delta - \psi) + \frac{\lambda}{\sigma} dZ_t^Q. \] (150)

Thus, under the risk-neutral measure \( Q, v_t \) is the Brownian motion with drift \( \mu^Q_v = r - \delta - \psi \) and volatility \( \sigma_v = \frac{\lambda}{\sigma} \). Therefore, the marginal utility of wealthy \( U_V \) (and therefore, the marginal utility of payout/consumption) is a geometric Brownian motion.

The dynamics under the physical measure \( P \) can be obtained using the adjustment of the Brownian motion process by the market price of risk, \( dZ_t^Q = \frac{\lambda}{\sigma} dt + dZ_t \). Therefore, under the physical measure, \( v_t \) is the Brownian motion with drift \( \mu^P_v = r - \delta + \psi \) and the same volatility \( \sigma_v = \frac{\lambda}{\sigma} \).
C.5 Probability of Bankruptcy

By the concavity of the value function, \( v_t = -\ln(U_V(V_t)) \) is an increasing function of wealth \( V_t \). Denote \( v^B = -\ln(U_V(V^B)) \) its value at bankruptcy. The time of bankruptcy corresponds to the first passage time

\[
T^B = \inf \{ t : V_t \leq V^B \} = \inf \{ t : v_t \leq v^B \},
\]

where, by the smooth-pasting condition, \( v^B = -\ln(\alpha U^B_V (\alpha V^B - K)) \).

As shown in the previous section, \( v_t \) is a Brownian motion with drift,

\[
dv_t = \mu^M_v t + \sigma_v dZ^M_t,
\]

where \( M \) is either the physical measure, \( P \), or the risk-neutral measure, \( Q \), volatility \( \sigma_v = \lambda / \sigma \), and the drift \( \mu^P_v = r + \psi - \delta \) under the physical measure and \( \mu^Q_v = r - \psi - \delta \) under the risk-neutral measure. Therefore, the probability to default up to time \( t \) is given by the distribution of the stopping time of the Brownian motion with drift,\(^{65}\)

\[
P^M[T^B \leq t] = \Phi \left( \frac{-(v_0 - v^B) - \mu^M_v t}{\sigma_v \sqrt{t}} \right) + \exp \left( \frac{-2(v_0 - v^B) \mu^M_v t}{(\sigma_v \sqrt{t})^2} \right) \Phi \left( \frac{-(v_0 - v^B) + \mu^M_v t}{\sigma_v \sqrt{t}} \right),
\]

where \( v_0 \) is the initial value and \( \Phi \) is the CDF of the standard normal distribution. The differentiation of this expression with respect to \( t \) gives the density of default that can be

\(^{65}\)See, for example, equation (3.40) in Karatzas and Shreve (1998).
written as

$$f_{TB}(t) = \frac{v_0 - v^B}{t \sqrt{2\pi\sigma^2_t}} \exp\left(-\frac{(v_0 - v^B)^2 + \mu^M_t t}{2\sigma^2_t}\right).$$  \hspace{1cm} (154)$$

The probability of hitting the default boundary in a finite time can be obtained as the limit of (153):

$$P^M[T^B < \infty] = \begin{cases} 
1, & \text{if } \mu^M_v \leq 0, \\
\exp\left(-2(v_0 - v^B) \frac{\mu^M_v}{\sigma_v^2}\right), & \text{if } \mu^M_v > 0.
\end{cases}$$  \hspace{1cm} (155)$$

where $M$ can be $P$ or $Q$. This probability can be written in terms of marginal utilities. When

$$\mu^M_v > 0, \quad P^P = \left(\frac{U_v(V_0)}{U_v(V^B)}\right)^{\frac{\sigma^2}{\sigma^2_v}+1} \quad \text{and} \quad P^Q = \left(\frac{U_v(V_0)}{U_v(V^M)}\right)^{\frac{\sigma^2}{\sigma^2_v}-1}. \quad \text{As expected, the probability of default is higher under the risk-neutral measure.}^{66}$$

Note that if $\mu^M_v < 0$, then the density function (154) is the PDF of the Inverse Gaussian distribution,\(^{67}\)

$$f(x; \lambda, \mu) = \frac{\lambda}{2\pi x^3} \exp\left(-\frac{\lambda(x - \mu)^2}{2\mu^2 x}\right),$$  \hspace{1cm} (156)$$

with parameters $\lambda = (v_0 - v^B)^2/\sigma_v^2$ and $\mu = -(v_0 - v^B)/\mu^M_v$. The expected time to default in this case is

\(^{66}\)Note that if $\mu^Q \leq 0 < \mu^P$, then $P^Q[T^B < \infty] = 1$ and $P^P[T^B < \infty] < 1$, even though $P$ and $Q$ are equivalent measures. This is not a controversy because these measures are equivalent on $\mathcal{F}_t$ for any finite time $t$, but the event $\{T^B < \infty\} \in \mathcal{F}_\infty$ does not belong to any finite-time filtration.

\(^{67}\)See Folks and Chhikara (1978).
$$E[T^B] = (v_0 - v^B) / ( -\mu_v^M) .$$ (157)

If $\mu^M_v > 0$, then a positive probability mass, $1 - e^{-2(v_0 - v^B)\mu^M_v / \sigma^2_v}$, is assigned to $T^B = \infty$ and, accordingly, $E[T^B] = \infty$.

The moment generating function (MGF) of the time of bankruptcy, $M_{T^B}^M (x) \equiv E^Q [e^{xT^B}]$, gives the expected value of a unit payment at bankruptcy when $x = -r$. This MGF for $x < 0$ can be derived as follows.

$$M_{T^B}^M (x) \equiv E^M [e^{xT^B}] = \int_0^\infty \left( \frac{(v_0 - v^B)}{\sigma_v \sqrt{2\pi t^3}} \right) \exp \left( xt - \frac{(v_0 - v^B - \mu^M_v t)^2}{2\sigma^2_v t} \right) dt.$$ 

The expression under the exponent can be written as

$$xt - \frac{(v_0 - v^B + \mu^M_v t)^2}{2\sigma^2_v t} = - \frac{t \sqrt{(\mu^M_v)^2 - 2\sigma^2_v x} - (v_0 - v^B)^2}{2\sigma^2_v t} - \frac{(v_0 - v^B)}{\sigma^2_v} \left( \mu^M_v + \sqrt{(\mu^M_v)^2 - 2\sigma^2_v x} \right).$$

The second term does not depend on $t$, and the first term corresponds to the kernel of the Inverse Gaussian distribution since $(v_0 - v^B) > 0$. Therefore,

$$M_{T^B}^M (x) = \exp \left( - \frac{(v_0 - v^B)}{\sigma^2_v} \left( \mu^M_v + \sqrt{(\mu^M_v)^2 - 2\sigma^2_v x} \right) \right).$$ (158)
Note that this is the MGF of the Inverse Gaussian distribution when \( \mu_v^M < 0 \),

\[
M^M_{T\theta} (x) = \exp \left( \frac{\lambda}{\mu} \left( 1 - \sqrt{1 - \frac{2\mu^2 x}{\lambda}} \right) \right),
\]

(159)

where \( \lambda = (v_0 - v^B)^2 / \sigma_v^2 \) and \( \mu = - (v_0 - v^B) / \mu_v^M \).

### C.6 Borrowing

The equilibrium amount of debt \( D = P^D (V_0, \bar{c}) \) corresponds to the (highest) fixed point given by equation (4),

\[
P^D (V^E + D, \bar{c}) = D.
\]

Such a fixed point obviously exists if the solvency constraint is satisfied, and \( D = P^D = \bar{c} / r \).

Let’s consider the case when the solvency constraint is violated. In this case, the price of debt contract is given by equation (34),

\[
P^D (W, \bar{c}) = \frac{\bar{c}}{r} + \left( \frac{U_V (V; \bar{c})}{\alpha U_{V}^B (\alpha V^B - K)} \right)^{a} \left[ (1 - \alpha) V^B - \frac{\bar{c}}{r} \right].
\]

Consider \( f (x) = P^D (V^E + x, \bar{c}) - x \). Obviously \( f (0) = P^D (V^E, \bar{c}) \geq 0 \) and \( f (\bar{c} / r) = P^D (V^E + \bar{c} / r, \bar{c}) - \bar{c} / r \leq 0 \). Due to the continuity of the first order derivative of the value function, \( P^D (\cdot, \bar{c}) \) is continuous. Therefore, there exists \( x_0 \), such that \( f (x_0) = 0 \).\(^{68}\) This value gives us the equilibrium amount of debt \( D (\bar{c}) \), corresponding to the coupon rate \( \bar{c} \).

---

\(^{68}\)If there is more than one fixed point, then the agent chooses the contract corresponding to the highest amount of debt for the given coupon rate.
Regression Coefficients

This appendix considers the OLS regression of credit spreads on equity volatility corresponding to two events: the change in asset volatility and the subsequent change in asset allocation.

Denote $\Delta_1 s$ and $\Delta_1 \sigma^E$, respectively, the changes in credit spreads and equity volatility before the allocation adjustment, and $\Delta_2 s$ and $\Delta_2 \sigma^E$ the corresponding changes at the time of the adjustment. OLS regression coefficients $\alpha$ and $\beta$ minimize the square deviations from the regression line,

$$\begin{align*}
(\alpha, \beta) &= \min_{(\alpha, \beta)} \left\{ (\Delta_1 s - \alpha - \beta \Delta_1 \sigma^E)^2 + (\Delta_2 s - \alpha - \beta \Delta_2 \sigma^E)^2 \right\}.
\end{align*}$$

(160)

The first order condition with respect to $\alpha$ is

$$\begin{align*}
(\Delta_1 s - \alpha - \beta \Delta_1 \sigma^E) + (\Delta_2 s - \alpha - \beta \Delta_2 \sigma^E) &= 0.
\end{align*}$$

(161)

Therefore,

$$\begin{align*}
\alpha &= \frac{\Delta_1 s + \Delta s_2 - \beta \left( \Delta_1 \sigma^E + \Delta_2 \sigma^E \right)}{2}.
\end{align*}$$

(162)

The first order condition with respect to $\beta$ is

$$\begin{align*}
(\Delta_1 s - \alpha - \beta \Delta_1 \sigma^E) \Delta_1 \sigma^E + (\Delta s_2 - \alpha - \beta \Delta_2 \sigma^E) \Delta_2 \sigma^E &= 0.
\end{align*}$$

(163)
The substitution of the expression for $\alpha$ into this equation gives

$$\beta = \frac{\Delta_1 s - \Delta_2 s}{\Delta_1 \sigma^E - \Delta_2 \sigma^E}. \quad (164)$$

By the continuity of debt prices, when the adjustment time goes to zero, $\Delta_2 s \to 0$ and $\Delta_1 s \to \Delta s$, where $\Delta s$ is the change in credit spreads corresponding to the changes in asset volatility and asset allocation. Therefore, in the limiting case,

$$\beta = \frac{\Delta s}{\Delta_1 \sigma^E - \Delta_2 \sigma^E}. \quad (165)$$

When the value of assets is high relative to the amount of debt, equity volatility is approximately $\pi_1^M \sigma_1$ before the change in the risky asset volatility. After the volatility shock, equity volatility is approximately $\pi_1^M \sigma_2$ before the allocation adjustment, and $\pi_2^M \sigma_2$ after the adjustment. Therefore,

$$\Delta_1 \sigma^E \approx \pi_1^M (\sigma_2 - \sigma_1) = \pi_1^M \Delta \sigma, \quad (166)$$

and

$$\Delta_2 \sigma^E \approx \sigma_2 (\pi_2 - \pi_1) = \sigma_2 \left( \frac{\mu - r}{\gamma \sigma_2^2} - \frac{\mu - r}{\gamma \sigma_1^2} \right) = -\pi_1^M \frac{\sigma_1 + \sigma_2}{\sigma_2} \Delta \sigma \quad (167)$$

in this case. Accordingly,

$$\beta = \frac{\Delta s}{\Delta_1 \sigma^E - \Delta_2 \sigma^E} \approx \frac{\Delta s}{\pi_1^M \Delta \sigma + \pi_1^M \frac{\sigma_1 + \sigma_2}{\sigma_2} \Delta \sigma} = \frac{\sigma_2}{\pi_1^M (\sigma_1 + 2\sigma_2)} \left( \frac{\Delta s}{\Delta \sigma} \right). \quad (168)$$
Thus, the slope coefficient is approximately proportional to the sensitivity of credit spreads to asset volatility when the value of assets is high relative to the amount of debt.

E Data Cleaning and Merging

This appendix provides a detailed description of the data cleaning and merging procedure. Table IX shows numbers of records, security issues, and issuers remaining at each step.

The snapshot of the enhanced TRACE database used in this research contains 114,213,116 transaction records from 07/01/2002 to 12/31/2012. Among these records, there are double counted transaction records entered by both parties of transactions, canceled records, and records corresponding to cancellations, reversions, and adjustments of transactions. A cleaning procedure that removes such records is provided in Dick-Neilsen (2014). This procedure deletes approximately one-third of records. There are 75,523,139 records remaining after applying this procedure.

Some of the remaining trades are non-standard and may have prices deviated from the prevailing market valuation. For example, FINRA Rule 6730 instructs to use “special price” modifier “if a transaction is not executed at a price that reflects the current market price.” To have consistent prices, I delete trades made under special conditions, commissioned trades and trades with non-standard settlements.69 There are 71,029,138 transaction records remaining in the dataset after this step.

To obtain information about issues and issuers and corresponding credit histories, I merge

---

69 The standard bond settlement is three business days.
<table>
<thead>
<tr>
<th>Description</th>
<th>Observation</th>
<th>Issue</th>
<th>Issuer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Enhanced Trace dataset</td>
<td>114,213,116</td>
<td>88,467</td>
<td></td>
</tr>
<tr>
<td>After applying the Dick-Neilsen cleaning procedure</td>
<td>75,523,139</td>
<td>83,142</td>
<td></td>
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<tr>
<td>Excluding non-standard/special transactions</td>
<td>71,029,138</td>
<td>79,493</td>
<td></td>
</tr>
<tr>
<td>TRACE transactions merged with FISD database</td>
<td>69,305,575</td>
<td>69,080</td>
<td>6,195</td>
</tr>
<tr>
<td>USD denominated securities of US issuers</td>
<td>63,908,582</td>
<td>55,030</td>
<td>5,347</td>
</tr>
<tr>
<td>Corporate debentures and notes</td>
<td>50,604,067</td>
<td>26,342</td>
<td>4,228</td>
</tr>
<tr>
<td>Non-financial bonds</td>
<td>27,842,505</td>
<td>14,644</td>
<td>3,334</td>
</tr>
<tr>
<td>Fixed coupon bonds</td>
<td>27,080,456</td>
<td>13,698</td>
<td>3,192</td>
</tr>
<tr>
<td>Excluding credit-enhanced bonds</td>
<td>20,749,572</td>
<td>10,347</td>
<td>2,259</td>
</tr>
<tr>
<td>Excluding callable bonds</td>
<td>4,658,102</td>
<td>3,072</td>
<td>761</td>
</tr>
<tr>
<td>Excluding puttable, convertible or exchangeable bonds</td>
<td>4,538,738</td>
<td>2,917</td>
<td>741</td>
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<tr>
<td>Excluding private placements</td>
<td>4,536,674</td>
<td>2,883</td>
<td>720</td>
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<tr>
<td>Senior debt only</td>
<td>4,464,438</td>
<td>2,501</td>
<td>632</td>
</tr>
<tr>
<td>Excluding bonds backed by assets</td>
<td>4,464,102</td>
<td>2,499</td>
<td>631</td>
</tr>
<tr>
<td>Bonds with valid pricing info</td>
<td>4,144,610</td>
<td>2,464</td>
<td>629</td>
</tr>
<tr>
<td>Transactions with consistent price and yield data</td>
<td>4,099,046</td>
<td>2,437</td>
<td>626</td>
</tr>
<tr>
<td>Daily Observations</td>
<td>769,233</td>
<td>2,437</td>
<td>626</td>
</tr>
<tr>
<td>Monthly observations</td>
<td>90,774</td>
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<tr>
<td>TRACE monthly observations merged with CRSP companies</td>
<td>61,752</td>
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<tr>
<td>Observations with CRSP stock data</td>
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<td>906</td>
<td>303</td>
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<tr>
<td>TRACE merged with Compustat using CCM</td>
<td>44,023</td>
<td>906</td>
<td>303</td>
</tr>
<tr>
<td>Observations with quarterly debt data</td>
<td>42,113</td>
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<td>300</td>
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<tr>
<td>Differences and returns constructed using monthly obs.</td>
<td>41,217</td>
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<td>294</td>
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<tr>
<td>Excluding infrequently traded bonds (100 days)</td>
<td>40,441</td>
<td>865</td>
<td>288</td>
</tr>
<tr>
<td>Excluding crisis time (09/01/08 to 05/31/09)</td>
<td>38,122</td>
<td>865</td>
<td>288</td>
</tr>
<tr>
<td>Excluding observations with bonds rated below A</td>
<td>12,397</td>
<td>286</td>
<td>69</td>
</tr>
<tr>
<td>Final dataset with long-term bonds only</td>
<td>3,351</td>
<td>65</td>
<td>29</td>
</tr>
</tbody>
</table>
the remaining TRACE transactions with FISD database using nine-digit CUSIPs. The merged dataset has 69,305,575 transactions. Among these transactions, there are 63,908,582 records corresponding to dollar-denominated securities issued by US companies.

The TRACE database has transactions corresponding to different debt securities. In addition to corporate bonds, TRACE has agency debt, asset-backed securities (ABS), mortgage-backed securities (MBS), etc. I select records corresponding to corporate debentures and medium-term notes. There are 50,604,067 such transaction records. Among these records, 27,842,505 records correspond to bonds of non-financial companies.\footnote{Financial companies are identified by group code 2 in FISD database.} The majority of these transactions, 27,080,456, correspond to fixed coupon bonds.\footnote{I require that bonds have periodic coupon payments and that the coupon cannot be altered.} Among these transactions, there are 20,749,572 transactions corresponding to bonds without credit enhancements.

Approximately three-quarter of the remaining bonds is callable. There are 4,658,102 transactions corresponding to non-callable bonds. Some of the remaining bonds are puttable, convertible or exchangeable. There are 4,538,738 records corresponding to bonds without these kinds of optionality. To mitigate liquidity issues, I exclude private placement debt. 4,536,674 of transactions correspond to publicly traded bonds. Most of these records, 4,464,438, correspond to senior debt. Several of the remaining bonds are marked in FISD as backed by assets or defeased. The remaining 4,464,102 transactions correspond to bonds satisfying the selection criteria.

Some transactions have the reported yield inconsistent with the reported price. To elim-
inate such transactions, I reprice bonds using SAS function `finance(price, ...`). There are 4,144,610 transaction records with valid pricing information (such as the coupon percent, payment frequency, day count rule, and bond maturity\textsuperscript{72}). I re-price these transactions using reported yields and exclude bonds if the re-calculated price differs from the reported price by more than five basis points (five cents per 100 dollar face value).\textsuperscript{73} This eliminates approximately one percent of transactions. There are 4,099,046 transactions corresponding to 2,437 issues of 626 companies remaining. These transactions correspond to 769,233 daily observations. To construct monthly observations, I use the last observation in each month. There are 90,774 monthly observations corresponding to transactions in the selected dataset.

To obtain the stock market capitalization, remaining monthly observations are merged with CRSP database using the historical and current six-digit CUSIPs. There are 61,752 observations for which the CRSP company identifiers, permco, are found. Stock prices and numbers of shares outstanding are available for 44,037 observations. Almost all these observations, 44,023, can be matched to Compustat (using CCM dataset). Quarterly debt data is available for 42,113 transaction records. These transactions are used to construct 41,217 difference observations corresponding to returns and changes in credit spreads and explanatory variables.

I discard observations with more than 100 days between two consecutive trades. This eliminates approximately two percent of observations; 40,441 observations remain in the dataset.

\textsuperscript{72}I also require the settlement day to be prior to the bond maturity. This excludes defaulted bonds that are traded after the bond maturity.

\textsuperscript{73}Some difference between the recalculated and reported values can be due to the difference in payment schedule calculations in SAS function `finance('price', ...)` utilized to recalculate values.
dataset. 38,122 of these observations do not overlap with the period of the financial distress after the collapse of Lehman Brothers (from September 2008 till May 2009) that was associated with rapid market changes that limited abilities of companies to adjust, a liquidity squeeze, and government interventions that affected prices. There are 115 monthly periods remaining.

I use A as the cut-off credit rating for the high-quality debt. In my sample, there are 12,397 observations corresponding to credit ratings A and above assigned by both S&P and Moody’s. This is approximately one-third of the remaining observations. To select long-term debt, I follow Collin-Dufresne et al. (2001) and consider bonds with more than 12 years to maturity. Some of the long-term bonds have very long maturities (up to 100 years). Since the maximum maturity of available Treasury yields is limited by twenty years during the period from February 18, 2002 to February 9, 2006, the calculation of credit spreads may be not reliable for very long maturities. To address this issue, I exclude observations with bonds maturing in more than twenty-five years. There are 3,351 observations corresponding to 65 bonds of 29 issuers that satisfy all criteria. These observations comprise the final dataset. Table X shows the list of companies in this dataset with the number of bonds, total number of observations and observation periods for bonds of each company. Table XI shows the list of bonds with the number of observations for each bond.
Table X
List of Companies

<table>
<thead>
<tr>
<th>No.</th>
<th>Company Name</th>
<th>Industry</th>
<th>Bonds</th>
<th>Observ.</th>
<th>Months</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AIR PRODS &amp; CHEMS INC</td>
<td>Manufacturing</td>
<td>1</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>ARCHER DANIELS MIDLAND CO</td>
<td>Manufacturing</td>
<td>3</td>
<td>206</td>
<td>108</td>
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<tr>
<td>3</td>
<td>BECTON DICKINSON &amp; CO</td>
<td>Manufacturing</td>
<td>2</td>
<td>161</td>
<td>107</td>
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<tr>
<td>4</td>
<td>BOEING CO</td>
<td>Manufacturing</td>
<td>4</td>
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<td>5</td>
<td>BRISTOL MYERS SQUIBB CO</td>
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<td>7</td>
<td>COCA COLA ENTERPRISES INC</td>
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<td>16</td>
<td>JOHNSON &amp; JOHNSON</td>
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<td>236</td>
<td>115</td>
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<td>PPG INDS INC</td>
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<tr>
<td>23</td>
<td>PROCTER &amp; GAMBLE CO</td>
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<td>UNITED TECHNOLOGIES CORP</td>
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<td>MAY DEPT STORES CO</td>
<td>Retail</td>
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<td>CUSIP</td>
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