Analyzing Mathematics High School State Examinations in Albania in the 1970s and 2006-2015: Two Decades, Two Historical Periods

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ABSTRACT

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This dissertation is devoted to the history of the Albanian system of education in general, its mathematics education program in particular, and, specifically, the Albanian high school mathematics assessment. Historical in terms of its research methodology, and mathematical-pedagogical in terms of the objects of the study, this research explores and compares the Albanian high school mathematics graduate examinations during 1970s and 2006 – 2015: two decades during two different historical periods. It analyzes the general structure of the examinations, their mathematical task design, and the history of their changes under the influence of political and social processes. The units of analysis here are the questions of each examination, which are examined both individually and in context as part of the examination, investigating the examinations’ topic coverage and comparing the latter to the intended national curriculum.

This study was based on multiple primary sources, including documents from the Albanian Ministry of Education, the Central State Archive of the Republic of Albania, the Internet archive (http://www.arsimi.gov.al), memoirs of former teachers, high school textbooks during the respective years, and other sources. The analysis showed that Albanian mathematics education was not immune to political and social change: both its curriculum and assessment were affected. Examination administration, format, topic coverage, and item characteristics, even the pure mathematical problems, represented in some ways the Albanian social, economic, and political views of the time.
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Chapter I
INTRODUCTION

Need for the Study

Academic assessment is considered one major way of analyzing and improving the educational process. Reforms in elementary and secondary education, which are in the forefront of the public mind and on the agenda of elected officials and educators, rely on testing (Barton, 1998). High-stakes examination scores have a direct impact on a student’s life options and opportunities (Lester, 2003; Moses & Nana, 2007), directly influence many education policies, help refine programs, channel funding, and identify the roots of success (Cizek, 2001). Written examinations were historically among the most important tools of assessment. They have been used internationally and over time as a practical way to measure a person’s knowledge of an academic subject.

Although academic testing in various countries originated from different historical roots, they have three main functions: selection, placement, and certification decisions. In most countries, the significance of testing is the greatest at the transition from secondary to postsecondary schooling (U.S. Congress, Office of Technology Assessment, 1992). The American school system has been viewed as the public “thoroughfare” on which all children journey toward productive adulthood; by contrast, however, the role of the school system and of tests in many European countries has remained principally one of “gatekeeper,” especially in the transition from high
school to postsecondary (U.S. Congress, Office of Technology Assessment, 1992).

Thus, it is not surprising that written examinations have been studied nationally and internationally. Key publications on the topic have been written by Azar (2005), Black (1992), Dossey (1996), Gandal et al. (1994), Karp (2003, 2007), and Wu (1993). Because the success of standards-based education systems depends on strong standards and assessments that measure what the standards expect (Rothman et al., 2002, p. 1), many studies have been conducted on the alignment of the examinations with the curriculum, namely, Al-Sadaawi (2007); La Marca, Redfield, Winter, Bailey, & Despriet (2000); Martone and Sireci (2009); Nasser et al. (2014); Polikoff, Porter, and Smithson (2011); and Webb (1997, 1999, 2002).

Indeed, the analysis of national written examinations has provided a deeper understanding of the educational goals not only of the educators themselves, but also of the government and society, and the practical ways in which these goals were meant to be accomplished. However, this has not been the case for Albanian examinations, which have been scarcely considered, much less analyzed, in any available sources. Albania has experienced a dramatic history, both politically and educationally. For most of their history, Albanians have undergone invasions, followed by occupations of foreign armies, during which time they were prevented from developing their own educational system. Even though the earliest known book written in Albanian, Meshari by Gjon Buzuku, dates back to 1555, the first official Albanian school was only opened in 1887, and attempts to democratize education took place only after the country’s declaration of independence in 1912 (Kola, 2014). In 1944, the communist regime came to power in Albania, as it did in many other eastern European countries. The implementation of compulsory primary education began in 1945 as part of the radical reforms that were taking place in the creation of a new communist state with socialist features, which was
achieved in 1952 (Kola, 2014). Assessment, as part of the education system during that time, developed to a national level as well, and high-stakes examinations in general, and mathematics examinations in particular, have become a formal part of the system. One of these examinations is the High School State Graduation Examination; Dobrushi (2013) has called “Matura Shtetërore” (The State Matura—the High School State Graduation Examination) one of the most important procedures of the Albanian education sector, because of its impact in shaping both the path of a student’s future education and the quality of higher education itself. The results of these examinations represent one of the most important indicators of the education system’s performance, particularly in connection with preparing students with the skills they need to face present and future challenges (Golemaj, 2013).

An analysis of these examinations will provide a better understanding of Albanian education in general and mathematics education in particular by describing the routes it has taken and how it has been influenced over time, especially given that, to some degree, challenging, high-stakes examinations drive the educational systems of their respective countries (Dossey, 1996). It will be of national interest to present information on the curriculum and discuss how these examinations measure the knowledge they require, and whether they have evolved in a coherent way, i.e., if they are broad enough to include key concepts and processes that will maximize students’ opportunities for success in the 21st century (English, 2002). Internationally, the examinations can be viewed as experiments based on methods that differ from those typically used in other countries (Dossey, 1996).

This discussion of the historical evolution of these examinations, presented as a comparison of two decades with two different political regimes—examinations from the 1970s, when Albania was under the communist regime, and examinations from 2006-2015, when
Albania became a democracy—offers insight into the character of the Albanian education system. The history of education is of the utmost importance for assessing the general development of the educational systems in which mathematics education occurs. Karp (2014b) states that a historical study of mathematics education becomes meaningful only when it includes social analysis and examines what happened in mathematics education in connection with the processes that were taking place in society around it (p. 10). Since “the effects of political decisions in education are felt first on the sphere of assessment, and only subsequently everywhere else” (Karo & Vogeli, 2011, p. 370), it was very interesting to know if “a conflict between the isolationist and the internationalist traditions has taken place” in the field of mathematics education in Albania as it did in Russia (Karp, 2006).

Moreover, even though a few studies have been conducted on the history of mathematics education in Albania (Dedej, 1982, 2005; Dedja et al., 2003; Mercanaj, 2010), this study specifically aimed to provide a scholarly comparison of the high school examinations in mathematics between the two different historical periods. In this way, the reader can understand Albanian society during two different political systems as reflected in the school, given that educational policies that govern what occurs in school depend on what occurs outside the school (e.g., national history, culture, politics, etc.) (Sadler, 1900).

**The Purpose of the Study**

The purpose of this study was to explore the evolution of the items of the High School State Examinations in Mathematics in Albania. The examinations from the 1970s and those from the years 2006-2015 were examined and compared. The emphasis of this study was on the analysis of the general structure of these examinations, their item (question) characteristics, and
their topic coverage. The alignment of the examination topic coverage with the expectations of
the high school Mathematics national curriculum was also explored.

The following research questions guided the study:

1. How did the Albanian education system evolve over time from its origin in late 19th
century to today’s national system of education?

2. How do the national curriculum expectations in mathematics for high school
graduates in the 1970s, as set by the Albanian Ministry of Education, compare with
those during the years 2006-2015?

3. How did the High School State Examinations of the years 2006-2015 compare with
the examinations during the 1970s in terms of general structure, topic coverage, and
item characteristics?

4. To what extent were the High School State Examinations in Mathematics aligned with
curriculum expectations for high school graduates during the 1970s and 2006-2015?

**Procedure of the Study**

The approach of this study consisted of analyzing the High School State Examinations in
Mathematics in Albania. The general structure of these examinations, their item (question)
characteristics, and topic coverage were examined. The alignment between the topics that the
examination items assess and national curriculum expectations were examined as well. The
primary sources to answer the research questions were documents from the Albanian Ministry of
Education, the Central State Archive of the Republic of Albania, the Internet archive
http://www.arsimi.gov.al, and other sources. To understand the historical context of the evolution
of the examinations and their items, it was important to present the Albanian history of education
in general, mathematics education in Albania in particular, and the history of assessment in
Albania. This part of the research focused on how Albanian education originated and evolved in relation to the country’s history and answered the first research question.

To answer the second research question, the study focused on describing the curriculum’s topic coverage (its requirements) through documents from the Albanian Ministry of Education website and the Central State Archive of the Republic of Albania. High school textbooks during the years covered in this study were used as a supplemental resource to determine the curriculum requirements. These textbooks were published by the Ministry of the Education and the topics included in them strictly followed the curriculum. How the curriculum was developed and what class hours were assigned for each curriculum framework category were explored and used to describe knowledge expectations for the high school graduates. The curriculum framework, which describes the main categories and their respective topics used in the analysis, follows the Mathematics Framework of the Third International Mathematics and Science Study (TIMSS) (Robitaille et al., 1993). These requirements for two decades, the 1970s and the 2006-2015, were compared, with a focus on the most emphasized topics in the curriculum expectations. The factors that possibly influenced the evolution of the curriculum were also discussed.

To answer the third question, the examinations themselves were analyzed, focusing primarily on the general structure of the examinations, their topic coverage, and their item characteristics. Each examination was treated as one object of study and each item (question) in the examination was considered one unit of that object. The topic coverage analysis followed the main categories and their respective topics and subtopics in the TIMSS (Robitaille et al., 1993), as mentioned above. The intended solutions of each examination (solutions written by those who prepared the examinations) were used in the analysis to identify which topics/subtopics were being assessed by the examination. An analysis of the general structure of these examinations
served to identify the length of the exam in hours, total number of questions, number of multiple-choice questions, free-response short answer, free-response extended answer, structured questions, oral questions, and the possibility of choice (item sets from which the students could choose some among several problems or which were all compulsory). Each examination item (question) was classified as a typical question (similar to the questions in the textbooks) or non-typical. Policies for conducting the State Examinations and their assessment procedures (the grading of the examinations) were explored.

To answer the fourth research question, each examination was studied with respect to the curriculum framework categories assessed by the examination and the breadth of each category’s presentation for each examination. Webb’s (1997) methodology, which guided the alignment analysis in this study, explores the alignment between assessment and standards regarding five dimensions: content focus, articulation across grades and ages, equity and fairness, pedagogical implications, and system applicability. Webb’s content focus on the analysis dimension consists of six subcategories: categorical concurrence, depth of knowledge, range of knowledge, balance of representation, structure of knowledge, and dispositional consonance; each explores the relationship between the assessment and the standards in different ways. The subcategories of categorical concurrence and range of knowledge of Webb’s content focus dimension constituted the bases of the alignment methodology for this study. The categorical concurrence looked at the broad content categories in the curriculum and compared them with content categories present in the examination, while the range of knowledge analysis compared the breadth of the curriculum categories with the breadth of the assessment. This analysis was repeated for the two decades which were the focus of this research, the 1970s and 2006-2015; in addition, the evolution of the alignment was explored.
Chapter II

BACKGROUND OF THE STUDY

This chapter focuses on the history of Albanian education within the country’s history in general. It starts with an introduction to Albania as a country, and continues with a discussion of the origin and evolution of Albanian education in connection with the country’s social and political standing. The history of Albanian education is presented in the following sections:

- education during the National Albanian Renaissance (1830s-1912),
- education after the declaration of independence (1912-1928),
- education under King Zog,
- the 1946 Education Reform,
- the reorganization of the education system—the long-term measures, and
- education after Communism.

The history of mathematics education in Albania as well as the history of examinations generally, and mathematics examinations particularly, in Albanian education are represented here as well.

Introduction

Albania, officially the Republic of Albania, is a country in The Balkans in southeastern Europe, bordering on Greece, Montenegro, Kosovo, and the Republic of Macedonia. Albania has an area of 28,750 sq. km. The total length of its border is 1094 km, of which 316 km are
seaboard: the Adriatic Sea to the west and the Ionian Sea to the southwest. It has a 657 km. land border, a 48 km. river border, and a 73 km. lake border. It is a mountainous country, with two thirds of the territory covered by mountains and hills. The capital city of Albania is Tirana. Illyria (the name of the country until the 11th century) was under the Roman Empire for five centuries (2nd BC-4th AD). It then became part of the Byzantine Empire until its fall in the 15th century, at which time the Ottoman Turks invaded and occupied the Albanian territories for the next five centuries. On November 28, 1912, Albania declared its independence from Turkey, a prologue for creating its own government, to govern the Albanian state.

“A national system of education is a living thing, the outcome of forgotten struggles and difficulties, and ‘of battles long ago’. It has in it some of the secret workings of national life” (Sadler, 1900). The Albanian system of education is no different: it reflects the country’s struggle to survive and it has an early tradition closely connected with its entire history with regard to freedom and progress (Musai, Gjermani, Bushati, & Sula, 2006). The political influence on the Albanian education system, as on everywhere in world, has been present throughout history and is ongoing, affecting every aspect of it. Similarly, as in every country (Karp & Vogeli, 2011), the history of mathematics education in Albania is embedded in the country’s history.

As society was developing during ancient times, Albanian education as a social practice consisted of transferring work experience and the techniques of tool production, while wisdom and traditions were passed on through storytelling. The rise of trade and government led to the use of writing and calculations during the 3rd and 2nd centuries BC (Dedja et al., 2003). During the Roman occupation, education became increasingly private and concentrated in the big cities. The Latin language and Roman culture were introduced through education. Rural areas were left
out and writing there showed up only on headstones (Dedja et al., 2003). The main source of how youth were educated in morality, the ways of living, and the importance of knowledge was the people’s pedagogy used by the elderly. One constitution originating from the Middle Ages, which in addition to its legal value also bears an educational value, is Kanuni I Lekë Dukagjinit (The Code of Lekë Dukagjini). This set of traditional Albanian laws, carefully collected and formulated by the Franciscan Shtiefën Gjeçovi (1874-1929), served in the role of the constitution of the land (Trnavci, 2008). The first book of the Code established the church as an institution and clergymen as authorities, thus giving the church the function of educating the population. From the time when the Code was operating (since the 1500s), people had no doubt that the church and its members symbolized goodness, justice, and divine purity, and they preached messages from God for an honest earthly life (Dedja et al., 2003, p. 47).

During the Ottoman invasion, Turkish schools were opened throughout Albania with the aim of spreading the Islamic religion. Islamic schools taught reading, writing, arithmetic, Arabic, and religion. The Albanian language was taught using the Turkish-Arabic alphabet. It is important to mention that all the schools where the Albanian language was being written using foreign alphabets (Latin, Greek or Turkish-Arabic) did not have a genuine education purpose as their aim was to prepare religious leaders who knew Albanian and use them to spread their respective religions (Dedja et al., 2003).

**History of Education in Albania**

**Education During the Albanian National Renaissance (Mid-19th Century-1912)**

The leaders of the Albanian National Renaissance, a nationwide movement during the 19th century, believed that the spread of education should have a national character among all Albanians, regardless of their religious beliefs, with the religion of albanians is albanianhood as
their battle cry. Using its own language was a necessity for the nation’s progress, both socially and economically, on its way to independence. Even though the Ottoman rule would not allow the opening of such schools, many were opened and operated regardless, and became hotbeds of patriotism and places of refuge for freedom fighters (Dedja et al., 2003). This led to the creation of the first known original Albanian alphabet in 1824, with 32 letters (today’s alphabet has 36 letters), and the first Abetare (an Albanian language book for the first grade) in 1844.

Renaissance efforts within Albania were supported and fueled by well-known Albanian academics living in exile all over Europe. They were in exile because the Ottomans intervened with severity and intimidation to stop the Albanian cultural and educational movement. The Society for the Printing of Albanian Writings, a cultural and educational organization, was founded in Constantinople in 1879; its membership was comprised of Muslim, Catholic, and Orthodox Albanians. Albanian émigrés in Bulgaria, Egypt, Italy, Romania, and the United States supported the society’s work. The Greeks joined the Turks in suppressing the Albanians’ culture, especially Albanian-language education. In 1886, the ecumenical patriarch of Constantinople threatened to excommunicate anyone found reading or writing Albanian, and priests taught that God would not understand prayers uttered in Albanian (Zickel & Iwaskiw, 1994, p. 22).

In 1886, the Albanian society called “Drita” (“The Light”) in Bucharest, Romania, published eight textbooks in Albanian, especially for the Albanian national schools, which were distinguished for their simplicity, scientific correctness, and patriotic spirit. The first official Albanian school, called Mesonjëtorja e Parë Shqipe (the First Albanian Classroom), opened on March 7, 1887, in Korça, Albania. The date is now a commemorative day for all Albanians, known as Teachers’ Day. Many other Albanian schools opened with the support of the Society of Albanian Learning, created in 1888, including at the time 160 members of all religions.
The end of the 19th century and the beginning of the 20th century was a time of crisis for the ruling dynasty of the Ottoman Empire and every aspect of Muslim life. At this time, the Young Turks came to power in 1908 with the idea of limiting monarchy and empowering the New Educated Man. During this time, to expand the education system in Albania legally, Albanian intellectuals took advantage of both the education for all atmosphere and the fact that the Young Turks had not yet sufficiently established power. Many intellectuals living outside of Albania returned to better serve education and other national issues. In addition to schools, Albanian newspapers began to be published in Albania.

The Congress for the Unification of the Albanian Alphabet, known as the Congress of Manastir—an academic conference held in the city of Manastir in November, 1908, with 51 delegates from all Albanian territories and from diaspora—met to improve the organization of the movement and establish one standard alphabet for all Albanians. Under the consensus of all delegates, the many alphabets used so far for writing in Albanian were narrowed down to two, the so-called Instambul abc and the new Albanian alphabet simple Latin; moreover, the creation of others was prohibited. A few years later, the new alphabet simple Latin, easy to use and publish in Europe, took priority and became the only alphabet of the Albanian language. November 22 is now a commemorative day for all Albanians, known as Alphabet Day.

The creation of a nation-wide education system needed well-organized efforts, so the National Education Congress of Elbasan was held in September of 1909. The program’s spotlight included the opening of an Albanian Normal School (pedagogical schools, or schools of education were called Normal schools); the creation of a single center for the organization and structure of the Albanian schools; and the adoption of a common program. Three months later, on December 1, 1909, the first Albanian Normal School was opened in Elbasan with students
coming from all Albanian territories, regardless of their religion (Muslim, Catholic or Orthodox). The faculty had graduated from European universities, and the curriculum included: Albanian, Geography, Arithmetic, Geometry, Logistics, Natural and Physical Science, Drawing, Music, Handwriting, Gymnastics, Turkish, French, and Religion (Muslim and Christian—Catholic or Orthodox) (Dedja et al., 2003). This school would provide new teachers while efforts were made to provide textbooks as well, and the Albanian national education started to bloom. This did not fit well with the Ottoman invaders, who opposed the movement, leading to an Albanian rebellion outbreak and the beginning of the armed conflicts of 1910-1912. During this period, most of the Albanian schools were declared illegal and closed.

As the Young Turks came to power in 1908, many Turkish military forces were brought to Albania. These forces headed to the Albanian Highlands, carrying out attacks there with the pretext of disarming the locals. This became the main cause of the 1910 armed uprising in Kosovo. The suppression of the 1910 uprising and the violent behavior of the Ottoman authorities prepared the ground for new, more organized anti-Ottoman movements throughout Albanian territories. The new Ottoman government (after the resignation of the Young Turks in July 1912), did not seek a military solution; instead, it worked towards an agreement which would be achieved through talks with the Albanians. A number of Albanians were appointed in positions of responsibility in the Ottoman Empire and it was agreed to establish an Albanian state within the empire. Many requests were granted, among which were appointing officials who knew the language and customs of the country, enforcing mountain law for certain regions, permitting the opening of schools in Albania with instruction in the Albanian language, declaring forgiveness to all participants of the movement against the Ottoman government, and so on (Albanian Academy of Science [A.Sh.Sh.], 2007).
After Istanbul approved the Albanian demands, the other Balkan nations (Greece, Serbia, Bulgaria, and Montenegro) wanted the same privileges granted to them as well. Although the Ottomans tried to calm the Balkan nations by informing them that privileges given the Albanians would be extended to all, it was not enough to stop the outbreak of the First Balkan War of the allied Balkan states (Greece, Serbia, Bulgaria, and Montenegro) against Turkey in October 1912. Albania’s neutrality during the First Balkan War was intended to keep the country not only from a downhill fall along with the Ottoman Empire, but also from being divided among the other Balkan states. This was a new and decisive phase of the Albanian National Movement, leading to the declaration of independence, since autonomy under the Ottoman Empire no longer had any meaning (A.Sh.Sh., 2007). When Serbia, Montenegro, and Greece laid claim to Albanian lands during the war, the Albanians declared independence on November 28, 1912. The Great Powers Treaty of Bucharest (August 1913) established that Albania was an independent country; however, large areas, notably Kosovo and western Macedonia with majority Albanian populations, were left outside the new state and failed to solve the region’s nationality problems (Zickel & Iwaskiw, 1994, p. 22). This marked a major turning point in Albanian history and a new era for the development of education and Albanian culture. The first national government, the interim government of Vlora, was formed.

**Education After the Declaration of Independence (1912-1928)**

The Ministry of Education created within the government of Vlora laid the legal foundations for a national education system similar to the ones in other European countries. The use of the Albanian language was made compulsory in all state administration and state school activities. In addition to primary school expansion, (with the main subjects being the Albanian language, Reading, Arithmetic, Music, and Gymnastics), fast-paced Albanian literacy courses
were made available to state employees and other adults. Preparatory Normal schools were opened in order to satisfy the increasing demand for teachers. Many students were sent to study abroad. In agreement with the International Commission Committee appointed in Albania by the Conference of Ambassadors in London in July 1913, special foreign schools were allowed only if subjected to state control, and the teaching of the Albanian language was required.

During World War I (1914-1918), Albania kept a neutral stance towards the two major warring blocks, the Central Powers and the Allies. The three forces occupying different regions of Albania were the Austro-Hungarian (part of the Central Powers) in the north; the French (part of the Allies) in a small part of southeast Albania (mainly in Korça); and the Italians (part of the Allies as well) in the rest. These three European cultures exercised their influence on Albania, imposing their program content and organization of education, which had positive and negative impacts (Dedja et al., 2003). The positive impact consisted of the fact that the three occupying countries reflected three advanced European experiences from which Albanian public education would benefit professionally. The negative impact was that the Albanian public schools did not work with the same program (subjects, content, structure), instead using three different programs and preventing their national equalization. In the majority of the elementary schools, the Albanian language was used as the language of instruction, while German, French or Italian were part of the curriculum, depending on the invader, as foreign languages. During World War I, the secondary education system consisted of three Normal (Pedagogical) schools, a lyceum, a gymnasium, two trade schools, and a school of urban agriculture.

In the first years after the World War I, Albania went through a volatile political era. A deal, agreeing to divide Albania territories among Yugoslavia, Italy, and Greece, was done behind the Albanians’ backs and in the absence of the United States at the Paris Peace
conference in January 1920. Also in that same month and year, the Albanian National Assembly, held in Lushnjë, rejected the partition plan and warned to defend the country’s independence.

In March 1920, President Woodrow Wilson intervened to block the Paris agreement. The United States underscored its support for Albania’s independence by recognizing an official Albanian representative to Washington, and in December the League of Nations recognized Albania’s sovereignty by admitting it as a full member. (Zickel & Iwaskiw, 1994, p. 25)

The country’s borders, however, remained unsettled and the claims on Kosovo, which had divided the Albanians and Serbs since the Middle Ages, intensified. The Serbs considered Kosovo their Holy Land, where their ancestors had settled during the 7th century and medieval Serbian kings were crowned. The Albanians claimed Kosovo as descendants of the ancient Illyrians, the indigenous people of the region who had been there before the Serbs came to the Balkans; moreover, they claimed that the vast majority of Kosovo’s population was Albanian since at least the 18th century (Zickel & Iwaskiw, 1994, p. 22).

Between 1912 and 1925, Albania was experimenting with the construction of a democratic state through multiparty democratic elections, and women were given the right to vote. Between 1918 and 1925, many short-lived, self-declared governments were created and several rebellions took place, some of which were serious; the elimination of political opponents was frequent (Erebara, 2012), leading up to the revolution of 1924. “Interwar Albanian governments appeared and disappeared in rapid succession. Between July and December 1921 alone, the premiership changed hands five times” (Zickel & Iwaskiw, 1994, p. 28).

All governments described the national education as a key factor in the existence of the Albanian nation. Different Ministries of Education had good ideas and wrote down educational programs, but little to nothing was actually put into action to improve education, even though education conventions were held and efforts of patriots to continue the work of the Renaissance
never stopped. The Republic of Albania, proclaimed by the Constitutional Assembly on January 1925, elected its first President, Ahmet Zogu, and on October 8, 1928, Albania was proclaimed a constitutional monarchy with King Zogu I, who ruled Albania till 1939, when Italy took over during World War II.

**Education Under King Zogu**

Established in historically difficult conditions, Zogu’s government inherited poverty all over the country and extremely poor conditions in the field of education, which had somehow been expanded but unfortunately was not affirmed under a nationally unified, clear direction. The so-called *Ivanaj reform* in the early 1930s marked the beginning of the unification of the Albanian education in all categories of schools. All private schools operating thus far were closed, and only religious schools for the preparation of the clergy, held by Albanian religious communities, were allowed. Primary education was made compulsory for children from 4 to 14 years of age. For the first time, the Ministry of Education requested all schools to provide statistical data for primary school attendance by gender and area (rural or urban); also for the first time, health service was introduced in schools. In 1933, “Mother Queen,” The Feminine Institute, was opened in Tirana. This marked the beginning of women’s education above the elementary level (Grades 5-8) and set the standard for other secondary programs developed for women around the country.

Because the education reform framework had the teacher as the decisive factor in education progress, quick measures were taken to remove unqualified teachers. The opened positions were filled with young people who had completed secondary school, and pedagogical school students took priority in receiving scholarships. The full implementation of the elementary school program was dramatically affected by the country’s poverty. Private schools
operated in Albania and were mainly funded from abroad, which never shunned the state programs even though law required them to do so (Dedja et al., 2003).

High schools were located in the major cities, classified into General Schools, Gymnasia, Lycée and Professional Schools, Normal and Technical. Gymnasia went from 6 to 7 and 8 years since 1933. For the first two years, the curriculum, called semimatura, was the same, then branched into two: Classical Branch with a broad program in Philosophy, Psychology, Latin, old Greek, and two modern languages (Italian, French or German); and Real Branch with a pronounced scientific program, descriptive geometry, and organic chemistry. Gymnasia consisted of two cycles: the first cycle, Semimatura, and the second cycle, Matura. The formation of Gymnasium students was a broad general culture. The graduates of the classic branch completed 6 years of Latin, 5 years of old Greek, and was fluent in two modern languages. Graduates of the scientific branch took a thorough curriculum in mathematics, which included functions, analytic geometry, derivatives, integrals, and probability.

The National Lycée of Korça was a general school, based entirely on the French model and equivalent to the French Lycée (Dedja et al., 2003). It was a 9-year school; the first 2 years were concentrated on learning French, while from the third year on, French was the instructional language. At the end of the fourth year, the program was divided into two branches: Philosophy, (humanitarian sciences) and Mathematics (natural science). The first part of Matura ended at the end of the eighth year; the ninth year was Matura’s second part and was earned through examinations according to the branch.

The majority of professional schools were the pedagogical ones located within several high schools in the country and on their own. Normal schools were 8 years, divided into two 4-year phases. The first 4 years had the character of the general gymnasium and the second phase
was devoted to Pedagogy, Psychology, Teaching, History of Pedagogy, and Pedagogical Practice. The proficiency diploma for teachers was awarded to graduates who passed the written and oral external examinations in Albanian Language, Mathematics, Foreign Language, and Pedagogical Practice. Four-year technical schools with specializations in trade, agriculture, nursing, metal processing, driving, typography, arts, among others, were considered low-level gymnasias. The specialization started in the third year.

Albania had no universities before World War II, but graduates from Albanian high schools were accepted in all European universities without entrance examinations, which showed the European quality of the Albanian schools. The Albanian government was generous in granting scholarships, despite the difficult economic conditions of the country, in order to meet the needs of schools with qualified teachers and government offices with the qualified staff. Education development performance during Zogu’s regime was interrupted by the Second World War, which began in Albania in April 1939, with the country’s occupation by fascist Italy. King Zogu and his family fled to Greece and later to London. Two of the multiple reasons why King Zogu was not willing to use force to confront the Italian troops were that he did not have the support of the neighboring countries and the Albanians showed little interest in fighting under his leadership (Tase, 2012).

Although there was some resistance, Mussolini’s troops easily invaded Albania, and decided to use Albanian territories as a springboard to invade Greece. Greek resistance fighters halted the Italian forces and advanced into south Albania. Their chauvinism cooled off the fight of the Albanian forces against the Italians, resulting in a stable front of Italian forces in central Albania. A month after the invasion of Greece and Yugoslavia by Germany and its allies, in April 1941, the Axis (Rome-Berlin-Tokyo) gave Albania control of Kosovo (Zickel & Iwaskiw,
1994, p. 33), making the Albanian nationalists’ dream of uniting most of the Albanian-populated lands come true under their occupation.

Meanwhile, with the help of the leader of the communist party of Yugoslavia, Josip Broz Tito, a Yugoslavian-dominated Albanian Communist Party was established with 130 members under the leadership of Enver Hoxha in November 1941. The communists appealed to the masses by calling for national liberation (instead of Marxist-Leninist propaganda), launching the National Liberation Movement (September 1942) which included different resistance groups, even anticommunist ones. The National Liberation Army, with communist-dominated partisans, was one of the resistance forces. There were two other major anticommunist nationalist resistance forces: Balli Kombetar (National Union), a western-oriented organization (consisting of landowners and peasantry); and Legaliteti (Legality), which consisted of guerrillas from the north who withdrew support from the National Liberation Movement after the communists renounced Albania’s claims on Kosova. These three forces fought the Italian and German occupation forces, but more often each other, in order to gain control of Albania. The communist partisans, which were backed by Yugoslavia and armed with British and United States weaponry, defeated the nationalists. Zickel and Iwaskiw (1994) stated that it was a military victory, not the lure of the Marxism, that brought the Albanian communists to the center of Albanian politics.

Although Albanian writers never tired of pointing out that the communists had ‘liberated’ Albania without a single Soviet soldier setting foot on its territory, they often neglected to mention that the communist forces in Albania were organized by the Yugoslavs and armed by the West or that the Axis retreat from Albania was in response to military defeats outside the country. (Zickel & Iwaskiw, 1994, p. 33)

The war came to a close at the end of 1944, with communists gaining absolute power.
The United States and Britain tried to overthrow Albania’s communist regime by backing anticommmunist and royalist forces within the country after the war. By 1949, American and British intelligence were working with King Zogu and his supporters. They recruited and trained Albanian refugees and émigrés, and guerrilla units entered Albania in 1950 and 1952, but Albanian security forces killed or captured all infiltrators (about 300) with the help of a Soviet double agent, Kim Philby. After this, legislation was enacted in Albania in September 1952, through a penal code that required the death penalty for anyone over 11 years old who was found guilty of conspiring against the state, damaging state property, or committing economic sabotage (Zickel & Iwaskiw, 1994, p. 44).

The 1946 Education Reform

From 1944 to 1992, Albania was ruled by a communist regime. The communist constitution, first adopted in March 1946, created, as it was called, the state of workers and laboring peasants and abolished all ranks and privileges based on heredity, position, wealth, and cultural standing. According to the constitution, all citizens were equal, regardless of nationality, race, or religion. The bourgeoisie was destroyed by the nationalization of industry, transportation, mining, and banking, and the state resumed control over everything. At the political level, the power was centralized, and all schools were soon placed under state management. The regime’s objectives for the new school system were to wipe out illiteracy in the country as soon as possible and eradicate bourgeois survivals in the country’s culture, in order to transmit to Albanian youth the ideas and principles of communism as interpreted by the party and to educate the children of all social classes based on these principles (Kola, 2014).

Educational reform law, which describes the character of the new school, focused on the democratization of school (the school of the masses and for the masses), the school as associated
with the entire life of the country ("school out of politics" is amiss), as well as a fundamental change of programs, texts, subjects, and methods of teaching (Kambo, 2005). All general education schools were secular and the same throughout the country: elementary schools, 7-year schools, and high schools. All private schools were closed. At this stage, the basic category of educational development was primary education, which was made compulsory for children ages 7-14 years and given free to all. In order to eradicate illiteracy, by law, all citizens between the ages of 7 to 40 were forced to attend schools or literacy classes, so that by 1956, more than 90% of the population under 40 were literate (Kambo, 2005).

There was a big gap between the teaching staff inherited from the past (small in number and not politically trained) and the communist government’s ambitious educational plans, which required the provision of a large number of teachers who accepted the party’s policy and defended it. One of the decisive factors in the selection of candidates for teachers (and any state official for that matter) was their political stance and credibility to the system. Teacher training was conducted through seminars aimed at preparing teachers in political and ideological ways (Marxist-Leninist) and methodical seminars aimed at building their methodical and professional capacity (Kola, 2014).

Indeed, the preparation of teachers was a major challenge during this rapidly spreading education. During the early years, the majority of elementary school teachers, especially in rural areas, had only primary education with some accelerated educational courses, a condition which led to a weakening in the level of quality. Due to the opening of new Normal (pedagogical) schools, stimulation of scholarship, and pressured enrollment, the number of graduates from the pedagogical schools began to increase, even though the preference for pedagogical studies was low and the number of specialized teaching staff was scarce. To meet the requirements for
qualified teachers, the Pedagogical Institute, the first higher education institution in the country, opened in Tirana in October 1946 as a 2-year program at first and a 3-4-year program later. It had three departments—Literature and History; Mathematics, Physics, and Chemistry; and Natural Sciences and Geography—all which prepared teachers in the respective fields for 7-year (middle) schools and high schools. To improve the qualification of in-service teachers, a decision of the Council of Ministers in August 1950 required teachers younger than the age of 45 years, who had not completed the necessary studies, to meet qualification requirements by attending a correspondence school (Duro, 2012); this marked the beginning of distance learning. The fact that most of the teaching staff were not at the appropriate professional level created significant differences between the quantitative and qualitative development of the education system, a price which had to be paid to ensure the rapid changes (Kambo, 2005).

The 100% implementation of compulsory primary education in the 1951-1952 school year (Kola, 2014)—8 years after the liberation of the country—paved the way for the implementation of the 7-year compulsory education. The law which made 7-year education compulsory came in June 1952, but its fulfillment was slow for many reasons: the financial condition of the country, the lack of teaching staff, the difficult economic conditions of the population, and the strong patriarchal mentality (particularly for girls), to name some. As a result, during the 1960-1961 school year, approximately 78% of those who finished elementary school continued further (Kambo, 2005). Expansion of the 7-year education led to an increase in the number of students who could attend high schools, a need for more qualified staff, and the expansion of secondary education. Along with an increase in Normal schools, providing more staff with higher education led to the tendency to open more gymnasium then vocational schools at
several of the 7-year schools with at least 60 graduates per year, since 90% of gymnasium graduates continued to higher education, compared to only 40% of vocational graduates.

Two problems needed to be addressed during the spread of secondary education: (a) the social composition of students who would attend, namely students must come from the working class; and (b) contradictions between trends in youth (who preferred gymnasium instead of vocational schools) and the plan of the communist state. “The society’s interest won over small personal desires of young people”; the majority of the students, finding no other way out, subjected to planning and attended the assigned schools, since “the spontaneous preparation of cadres would be an absurd action in a centralized planned economy” (Kambo, 2005, p. 138). It was the same for other educational levels: the tendency was and remained that of quantitative growth, followed by poor qualitative indicators and the lack of teachers with proper education.

Many high school graduates who were trained through vocational schools entered the work force and did not continue to higher education; this led to only a quarter of working specialists had higher education degrees in 1960—the lowest among the eastern European countries. Higher education for the communist leadership represented the key to solving the problem of compiling a suitable framework for the development of a socialist economy. Before World War II, higher education graduates studied mostly in western European countries, so they were not suitable for the new political orientation of the country. After World War II, the preparation of specialists with higher education was occurring abroad in the Soviet Union and other eastern European countries, and in the newly established Higher Education Institutions in Albania. The development of higher education underwent the same problems as other educational levels, and the preparation of new intelligence that necessarily came from the working class or peasantry determined the policies for its development. In 1952, higher
education expanded with the opening of two new institutes, Economics and Medical, and also with the introduction of distance learning in some of them. The opening of new faculties within the existing Higher Education Institutions was an ongoing policy. The creation of these new faculties (stated in the report of the Ministry of Education of 1951-1952) corresponded to the division that was similar to the pedagogical institutes in the Soviet Union and complied with Albanian needs (Duro, 2012). During the 1953-1954 school year, the Economic Institute was offering majors in general economics, industrial, planning, finance, agriculture, trade and transports, while the Pedagogical Institute was offering majors in Albanian Language and Literature, History-Geography, Russian Language and Literature, Mathematics-Physics, and the like (Kambo, 2005). In August of 1954, the Institute of Law opened in Tirana. In the 1955-1956 school year, the system of higher education would include six 4- and 5-year institutes, with 22 faculties and 1,595 students, thus reducing the number of students studying abroad from 80% during the first years to 20% (Kambo, 2005), especially in massive specialties for important branches of industry and agriculture.

Higher education activity thus far—with gained experience in the fields of organization, teaching, and science—created the possibility for transitioning to a more developed form. It had established a university as the leading management center at the national level. In June of 1957, the Presidium of the National Assembly enacted the establishment of the university, named the State University of Tirana, that depended on the Ministry of Education and Culture. Indeed, Tirana State University opened its doors on September 16, 1957, constituting one of the most important events at that time in the field of education and science. It opened with six faculties: Faculty of History and Philology with two branches—history and philology; Faculty of Law; Faculty of Economics with two branches—economy and accounting; Faculty of Engineering
with four branches—mechanical, electrical, construction, and geology; Faculty of Natural Sciences with two branches—mathematics-physics and biology-chemistry; and Faculty of Medicine. It would prepare specialists in 15 fields and had approximately 46 departments. Work began with 125 Albanian academic experts and administrative staff, and 24 experts—high scientific and pedagogical personalities—were brought from the Soviet Union who were thought to contribute to the progress of the first Albanian university for 1 to 2 years. The university faculty and staff would increase with the gradual inclusion of the most qualified specialists that it would produce (Kambo, 2005). The Institute of Science was included in the university, together with ancillary institutions such as the Ethnographic, Archaeology, Biological, and Geological museums, clinics at the Medical Faculty, and the Scientific Library. Initial contributions in the science field were mainly in the preparation of textbooks and simple studies. Attempts to pass from the stage of statistical studies to those with clinical character were made. Comprehensive studies were conducted in the Albanology and Historiography fields where tradition existed. Individual and team research studies in several disciplines began taking place and laid the foundation for future research. The university was gradually becoming the most important educational and scientific institution in the country.

Undoubtedly, the post-World War II educational development signaled the rapid expansion and gradual creation of a broader educational system from preschool up to higher education. The determining factor that led the transformation of the Albanian school system was the political orientation, so that educational reform was accompanied by significant changes in its ideological basis. The use of the same textbooks and programs across the country “must first and foremost ensure the education of the younger generation with the Marxist-Leninist ideology” (Kambo, 2005, p. 183). This process required cleaning up existing textbooks of excessive and
useless chapters (that is, chapters containing bourgeois and reactionary ideology) and developing new textbooks. Works of authors worthy of national literature that played an important role in the Albanian National Renaissance were removed from the program because of their conservative ideas. Subjects such as Logic, Psychology, Latin, and Ancient Greek were removed from the program as *not necessary*, and *essential* subjects such as National Liberation War History (considered by communist leaders as the most heroic and glorious in Albanian history), The 1946 Constitution, Knowledge of Marxism-Leninism, and the like were introduced.

Offers from representatives of western countries (including the U.S. Office of Education) to help with textbooks that could serve as a model were not accepted by the country’s leadership, and the Council of Ministers of 1948 decided to allow only Soviet textbooks for future translations (Kambo, 2005, p. 179). Albanian authors under the supervision of party officials prepared the original texts of national history and culture. The sovietizing of schools took place in every aspect of education; Soviet pedagogy and the methodology of particular subjects were included in plans, guidelines, and proceedings for the organization of the teaching process and extracurricular activities. There were Soviet advisors in the Ministry of Education, in higher education, and in most high schools.

The fact that the Soviet school experience was transmitted mechanically from an industrialized power with high technological development to the traditional schools of an underdeveloped agrarian country led to loaded programs with voluminous textbooks, and did not give the expected results. The excessive workload was a problem even for the Soviet schools (Kambo, 2005, p. 182), but in Albanian schools, this overload became a disturbing phenomenon, especially in political subjects, History, and Literature. Indeed, this was not a reform based on previous studies; the main intent was a combination of political and social factors, including the
requirement to introduce and apply Marxist-Leninist socialist norms to the school system and zealously create a semblance of the Soviet schools as much as possible. Some partial corrections in form and simplifying, concretizing, and making the guidelines more understandable for teachers accompanied the Soviet methods. Even though the fetishism of rules and methodical proceedings seriously impeded the teachers’ initiative and creativity, the methods and experiences of Soviet schools continued to be used even after relations with the Soviet Union had terminated in the early 1960s (Kambo, 2005). The 1946 educational reforms laid the foundation for the new education system, which was secular, unified throughout the country, and based on Marxist-Leninist ideology.

**The Reorganization of the Education System—The Long-term Measures**

In February 1961, the closing of the first stage and the beginning of the second in constructing the Stalinist model of socialism was declared during the IV Congress of the Albanian Labour Party. By the end of 1961, Albania maintained good relations with North Korea, North Vietnam, and China, entered the path to full isolation from the rest of the eastern European communist countries and terminated all connection with the Soviet Union (Kambo, 2014). The termination of loans given to Albania by the Soviet Union and other countries of the communist bloc undermined the realization of important investments in the economy, including education, despite leadership’s efforts to mitigate these negative effects. As the education system was entering this second stage, then, its reforms continued adhering to unchanged general principles—those of the existing Soviet experience. The reorganization had to be done in a unique way since Albanian school opportunities, due to lagging inherited from the past (as party officials put it), were not the same as those of other communist countries (Kambo, 2014). The new programs necessarily had to continue being traversed by Marxist-Leninist ideology and
party politics in every detail.

The education goals of the reorganization were to strengthen the establishment at the theoretical level, link theory with practice, and introduce elements of polytechnic education in order to understand life better and equip students with necessary work habits. The continued spread of compulsory education all over the country to ensure that every school-age child would attend school remained the focus of government officials. The 1960s deepened the process of further expansion and consolidation of preschool education for children 3-7 years of age as the first link in the educational system. The goal was to cultivate correct speech in children’s native language, as well as counting, drawing, and light physical exercises in the form of games. This goal would be reached through carefully designed programs that aimed “to form a revolutionary class education and work ethic for preschool children” (Kambo, 2014, p. 150). Elementary education had extended almost everywhere, in both cities and villages, so the focus now was on improving it.

Serious shortcomings in the implementation of the 7-year education, which represented the base of the educational pyramid, were called alarming and required the intervention of state institutions for the strict enforcement of the law; thus, its expansion sped up. Starting with the 1963-1964 school year, the 7-year existing system became 8-year education, implemented across the country. Measures for preparing lists of students on time, on-site verification, and the opening of new schools—together with the fact that school and local officials were held responsible—positively impacted and consolidated the 8-year education plan as compulsory basic education. The education law that took effect in January 1970 defined the elementary school as an inseparable link of the 8-year education and changed the age of admission from 7 to 6 years (Kambo, 2014). The curriculum in general for the 8-year school included subjects that
were similar to the previous (with a different structure, spread over a longer period of time), and added work hours as part of the weekly program (I-II grades, 2 hours; III-IV grades, 4 hours; and V-VIII grades, 5 hours), aiming to create a close connection between school and life. The work would take place in the school’s workshops (gardening, woodwork, etc.), and its purpose was not to give students a profession, but to create respect for production workers and exercise positive sentiments among the youth.

The country’s circumstances following the termination of its relations with the Soviet Union—which deepened the issue of meeting the demand for qualified staff, especially for the main branches of the economy—pushed the government towards measures with high social and economic costs. The existing network of schools could provide less than 50% of the estimated needs in the social and economic development draft plan for 1961-1965 (Kambo, 2014). The process of expanding general high schools, especially in rural areas, continued rapidly until the mid-1960s; then the consolidation of the existing network and better utilization of its capacity took priority. With the spread of secondary education in the countryside, the Party of Labour fulfilled the economy’s needs and simultaneously improved the social composition of the official staff. Professional secondary education—which not only trained students in a particular specialty, but gave them general culture at the secondary school level with a possible option to continue their higher education—was extended as well, always complying with the requirements of the economy. The government continued to expand vocational schools, which prepared skilled workers for different branches of the economy at a lower cost, in accordance with the needs of new technologies and increasing production. Attempts to liquidate illiteracy continued through the organization of evening primary education for adults, particularly in the countryside and especially for women (Kambo, 2014). The first special boarding school for visually and hearing
impaired children was opened in 1963, followed by one for the mentally challenged in 1970, but neither one of these institutions were considered part of the school system (Hörner, Döbert, Von Kopp, & Mitter, 2007).

Higher education was expanding the existing faculties, reorganizing their branches, opening new faculties, as well as improving programs, textbooks, and necessary materials and laboratories. In the early 1960s, the reorganization resulted in the opening of the Physical Education Institute, the Senior State Institute of Agriculture, the Military Academy of the Republic of Albania, the State Conservatory of Tirana, the Higher Institute of Fine Arts, and the Higher Education School of Actors. To ensure the preparation of the intellectual elite of the communist state, many of whom were devoted to the implementation of its policies, the admission to higher education was granted to high school graduates who had a good political position. Excluded from this opportunity were the children of traitors, collaborationists, and those with the wrong political stance (Kambo, 2014). There was also a Party’s School, founded on November 8, 1945, named the Party’s School “V. I. Lenin” in 1955; in 1968, it was renamed the Higher Education Institute of the Party “V. I. Lenin.” The graduates from the Party’s School were called the instructors of the party, who were equipped with deep Marxist-Leninist preparation to work in important sectors. Their jobs would be as chiefs of the party’s local sections, and as party’s representatives to every institution or organization in the country.

The establishment and expansion of higher education increased the need for qualified specialists in both the teaching and scientific fields. But once again (even though the termination of relations with the eastern communist bloc made it harder to satisfy these needs), the communist leadership would see the better utilization of natural and human reserves within the country as the main way to improve this new reality. Organizational changes were undertaken to
strengthen and expand links to coordinate scientific work in order to accelerate the qualification process of the staff and accommodate graduate students who were returning from abroad without graduating because of the terminated relations. The qualification system levels had been borrowed from Soviet institutions and implemented since 1955. The role of a scientific research center belonged to the system of higher education because Albania lacked an academy of science until 1972.

The first attempts to organize scientific activity were made in 1940 with the creation of the Albanian Institute of Studies, which involved renowned Albanian and Italian scholars (A.Sh.Sh.). In 1948, it was reorganized as the Institute of Science and became the center of organized research and scientific activities. When the State University of Tirana was founded in 1957, the Institute of Science was included in it and laid the basis for the preparation of the scientific staff in all areas of the country. The beginning of the 1970s found the country with more than 20 active research institutions, which called for an organized national scientific-research institution. On October 10, 1972, the Presidium of the National Assembly decreed the creation of the Academy of Sciences of Albania, with 17 regular members and five correspondents who worked with a range of social science institutes as well as natural and technical sciences.

The expansion of education from preschool up to the highest categories continued to remain a priority of the ambitious communist state because the rapid development of the country was a path to a centralized economy. Linking theory with practice was a phenomenon that began in the 1960s, and resulted in the increasing involvement of youth and students in production work as part of the curricula. This was a volunteer activity, intended to familiarize students with the working class, and it was named as one of the most efficient forms of tempering communist
youth education (Kambo, 2005).

In order to improve the quality of teaching, attempts were made to research different teaching methods. This was not an easy task because studies and generalizations in this area were almost absent in Albanian. The use of experience from other countries (communist, of course) was not enough, and it was very important to create an Albanian school experience based on its specific conditions and inherited traditions. Different educational seminars were advising the replacement of forms of instruction—from formal methods of teaching, which led to student passivity and reduced teaching and learning efficiency, to methods that increased student involvement which used examples from their daily lives and relied on their prior knowledge and culture. The preparation of teachers was going fairly well during the 1960s, and gradually, the majority of teachers were graduating from the regular secondary and higher pedagogical education system instead of fast training courses and distance learning. These teachers were professionally prepared and especially framed as missionaries of communist education for the new generations, and this remained the two basic directions of teacher preparation in Albanian politics until the fall of the communist regime.

In 1970, the Institute of Pedagogical Studies was established in Tirana. School politicization reached extreme limits with the implementation of the law “Further Revolutionization of School,” approved on December 24, 1969. The law stated that the entire content of the school must be permeated by the thread of the Marxist-Leninist ideology and the state-party. Moreover, all teaching and educational work must be built on three basic components: teaching, productive labor, and physical and military training (Musai et al., 2006).

The accomplishment of the communist government in creating the first Albanian national system of education throughout the country cannot be ignored. Despite its identified flaws, the
educational system was spread successfully all over the country. During the 1980s, over 700,000 Albanians (of the nation’s population of three million people) were enrolled in school, and over 40,000 teachers were employed across the country (Mercanaj, 2010). By 1990, there was full enrollment in basic education; more than 80% in high school and more than 45% of the population aged 25-35 had at least completed secondary education—better results than any non-OECD member (OECD, 1999). However, the education system, which was narrowly targeted to the production needs of a closed economy and the ideological needs of a particularly isolated socialist state, had limited ability to provide youth with skills appropriate for the radical new world in which Albania found itself at the beginning of the 1990s. At this time, communism had broken down and Albania was opening to the world (Dudwick & Shahriari, 2000).

**Education After Communism**

Communism in Albania broke down in the early 1990s. The democratically elected government in April 1992 launched an ambitious economic reform to put the country on a path toward a free market economy. The fact that Albania was the most isolated country in Central and Eastern Europe and the latest to open itself to democratic and economic changes made its transition crises deeper than in other countries. The Albanian communist regime has been described by the Human Rights Watch (1996) as follows:

> For nearly half a century Albania experienced a brand of communism unknown to the rest of Eastern Europe. A fateful blend of isolationism and dictatorship kept this tiny Balkan country the poorest and most repressive in all of Europe. During his forty-year reign, the Albanian leader Enver Hoxha banned religion, forbade travel and outlawed private property. Any resistance to his rule was met with severe retribution, including internal exile, long-term imprisonment and execution. His domination of Albania’s political, economic and social life was absolute. (p. 1)

> The economic crisis was characterized by severe unemployment, poverty and instability, and an institutional crisis where the collapse of existing institutions was not smoothly
counterbalanced by the establishment of new ones. The deep crisis that Albania was undergoing was reflected in almost every aspect of the education system. Where once the education system had been characterized by security, stability, and order, and teachers felt respected and appreciated by society, this was no longer the case. Education lost prestige because people saw that a good education did not constitute a condition for good income; indeed, those who had a university degree earned a smaller income than the small businessmen (Dudwick & Shahriari, 2000, p. xi). The creation of the free market economy and the opening of the borders led to a massive exodus of teachers abroad or to other jobs; qualified or not, they were better paid, and so the education system lost many of its qualified and experienced teachers, especially in rural areas. Schools fell prey to destruction, vandalism, and theft during these turbulent times as a manifestation of the population’s accumulated hatred against the dictatorship. This feeling festered because schools were seen more as part of the state than of the community; because they did come with the regime and were used as a tool to maintain social control; and because the schools served, among other things, as a place where the Labour Party organized meetings (Dudwick & Shahriari, 2000). Great migrations from rural to urban areas overflowed urban schools, with often 40 to 50 pupils per class, and many schools had to close in deep rural areas.

As the country was trying to find its way to democracy, immediate measures were taken to put the damaged network of education institutions into operation as a prerequisite for other reformation changes. In 1994 and 1995, laws for pre-university and higher education in the Republic of Albania were designed based on democratic principles and helped by foreign institutional assistance and aid programs to rehabilitate the whole education system, its physical infrastructure, and its content (Musai et al., 2006). Albanian education now passed from the emergent phase of assistance to a phase of developmental reforms.
During the past two decades, the pre-university education system underwent a cycle of reforms with three main stages. The first one (until 1995) was intended to cleanse the system of the inheritance and influence of communist ideology. The objectives and content of social studies were reviewed, and some new subjects (information technology, civic education, etc.) were introduced. During the second stage, the legal basis that legalized the general need for change was prepared in collaboration with foreign organizations and the World Bank. Changes affected the structure of the education system, curriculum, and management systems. In the third stage, efforts were underway to provide democratic education, taking into account the requirements of the basic principles of equality of opportunities for quality education and respect for individual differences (Ministry of Education and Science [MoES], 2014a). Until 2004, the Albanian education system was using a single unique textbook for each different subject and grade. The state publishing house produced these textbooks, and authors were selected carefully from professors of subject content and pedagogical knowledge. In 2004, with recommendations from important foreign institutions like the World Bank, the liberalization of textbooks began.

These reforms aimed to achieve international benchmarks, preparing students to travel abroad for higher education and to become citizens and workers in a democratic market-based society, cast in the wider context of European membership rather than a narrow focus on national citizenship. The National Strategy of Education (developed in cooperation with the World Bank) was approved in 2004, and the levels and years of schooling were restructured according to the practices of other member countries of the European Union and the Organization for Economic Cooperation and Development (OECD) (Musai et al., 2006). The Constitution of Albania guarantees each citizen the right of education—in the public schools, compulsory education, and in general high schools, vocational and social-cultural education. The right of education is free.
The MoES is the highest public administrative organ in education. This institution is in charge of the administration of the education system at a national level, according to the basic principles of the education policy defined by the Government and the Parliament of the Republic of Albania.

The public (pre-university) education system consists of: preschool education, basic education, secondary education, and special education. Public basic education schools include two cycles: primary cycle and high cycle, with the latter being a natural and logical continuation of the primary cycle (MoES, 2010). Public education is secular and ideological, and religious indoctrination is strictly prohibited. All children age 6 years in the Republic of Albania should start basic education, which lasts no less than 9 years. The student is required to attend compulsory education up to the age of 16. A distance learning school is opened in each district according to the requests of students whose age is beyond that of compulsory education. The education of minorities (mainly Greek and Roma) conforms with European standards. The teaching plan and the curricula are unified and approved by the MoES, whereas the textbooks have been liberalized since the 2006-2007 school year. Textbook design is done in compliance with the curricula, whereas the MoES determines the standards of the design and evaluation of the textbooks (Musai et al., 2006). The public Special Education of disabled children is implemented through common schools right beside their peers, with some special courses and in separate (special) schools according to their specific needs. Specific programs are developed for special needs students by adapting ordinary school programs according to their needs.

Secondary education is provided as full-time and part-time as general secondary education (gymnasium), professional high schools, and social-culture education. The new gymnasium curriculum structure, implemented for the first time in academic year 2009-2010, includes the arts, physical education and sports, foreign languages, Albanian Language and
Literature, career and training for life, Mathematics, Technology and ICT, the natural sciences and the social sciences, all of which compose the core curriculum and elective curriculum. The duration of the studies in full-time gymnasia is 3 years; in part-time gymnasia, it is 4 years. Professional education offers long-term professional education lasting 2, 3, and 4 years. Social-culture education includes foreign language high schools and high schools in arts and sports areas, which are confined to talented pupils or special gifted ones.

Adult education in Albania is realized through special programs provided from the compulsory and high education system on a part-time basis. In addition to the formal system, a number of training activities operate in Albania; this makes up the informal system such as a series of schools and centers which provide different courses like foreign languages, computer knowledge, and professional trainings like Hospitality, Tailoring, Hairdressing, Customer Care, among others. These schools and centers are registered and award diplomas or certificates at the end of the courses. Informal education is the responsibility of the MoES and the Ministry of Labor and Social Assistance.

The first legal framework for private education was provided through the law dated 06.21.1995, and has been re-formulated (improved) many times since. Private education is allowed at all levels of public education. Their curricula, teaching programs, and conditions for implementation comply with national interests and the Albanian legislation. Private education institutions where religious subjects are taught and/or classes are held in foreign languages are allowed as well. The Albanian literature and language, the history of the Albanian nation, and the geography of Albania must be taught in Albanian, even if other subjects are not.

Teacher education is accomplished in the teacher education branches of different universities. The system of pedagogical practices for preservice teachers implemented in the
universities is named the mentors’ system; this provides a partnership between university and school regarding preservice teacher training. The mentor is an experienced teacher who has constantly positive results and experiences with preservice students, whereas the tutor is a university staff member with scientific authority in his/her area, not only in university but also in pre-university education. As a rule, tutors are preferred as specialists of didactics and education sciences (Musai et al., 2006).

The development of the higher education system has established its path to European Integration. Albania joined the Bologna process in September 2003 when the MoES signed the Bologna Declaration in Berlin’s follow-up meeting of all European Ministers of Education, officially pledging the country to the adaptation of reforms in Higher Education along Bologna principles and guidelines. To Albania at the time, the Bologna Declaration and the policy of reforms devised around it were a ready-made framework to reform a system that was largely perceived to be in need of restructuring. The objectives were to develop: easily readable and comparable degrees, European co-operation in quality assurance, a European dimension in higher education, and mobility for students, teachers, and researchers. They constituted the guidelines for the legislation of the Higher Education Law, effective May 2007. All institutions of higher education, except for the Military University, are governed by this law which implements the usual Bologna degree sequencing. Academies and universities, which are authorized to offer the three principal Bologna cycles, provide both instruction and research. Higher education schools and professional colleges, which offer mainly first-cycle programs,
focus on instruction in applied fields, although some second-cycle programs are available. Programs are provided on a full-time, part-time, and distance basis.

Starting with the Law for Science and Technological Development in 1994, the continuous development and improvement of legislation to contemporary models of organization and cooperation through the application of national and international projects have aimed toward the final integration of Albanian research activity into the European research area. The transformation of Albania’s educational system into a European one is a long-term project, which began soon after the fall of communism. In 2001, Albania adopted the United Nations framework of the Millennium Development Goals for poverty reduction and education sector reform. The National Strategy of Education, approved in 2004, followed that of European Union countries. In 2006, the World Bank’s “Educational Excellence and Equity Project” was adopted at a value of USD 75 million, and foreign expertise and “best practices” served as the guiding principles for the design and implementation of education reform policies.

Reform in Albanian education is an ongoing process aiming at preparing students to solve the complex problems of modern life on a personal level as well as at national and global levels and to strive for equal and quality education for every member of society, regardless of ethnicity, gender, social status, and other changes (Instituti I Zhvillimit te Arsimit [IZHA], n.d.; MoES, 2004). While the government indisputably recognizes commitment to the importance of the process, particularly aiming to develop key European competences for education in Albania, the tradition of strong control from central state and regional/local education directorates still persists, leading to politicization. By contrast, the influence of “let the market decide” attitudes toward the development of private education, especially in higher education, has resulted in some cases of poor quality private educational provision (Project Against Corruption in Albania
The adoption of uniform global policies has not always resulted in a predictable convergence of outcomes. Indeed, gaps have been observed between the design, implementation, and local adoption of new educational ideas and practices, i.e., national education policy and everyday practice. Albanian teachers in particular continue to struggle with the hierarchical structure of their schools, the authoritarian policies of their school directors, the skepticism of their students’ parents, and the absence of adequate resources (Gardinier, 2009).

**History of Mathematics Education in Albania**

While there is not much explicit historical evidence of how mathematics education evolved in Albania, its development has followed the same staggering road as the creation of the national system of education. Indeed, until the 1970s, not much was going on to perfect teaching methods, but teaching plans do show that mathematics has always been one of the main subjects taught at any school level, and consumes about 15% of class time. The mathematics curriculum for the elementary schools during the 1920s included Arithmetic (up to proportions and interest rates) and Geometry (areas and volumes) (Dedja et al., 2003); the mathematics curriculum for the gymnasium included functions in their full extent, analytical geometry, derivatives, integrals, and probability. Moreover, most high school teachers have graduated abroad (France, Austria, Italy, etc.) and mathematics textbooks have been translated mainly from textbooks in Austria.

After World War II, with Albania becoming part of the eastern bloc, the communist government’s ambitious educational reform included a fundamental change of programs, texts, subjects, and methods of teaching. The decision of the Council of Ministers in 1948 restricted future translations to be done only from (same-level) Soviet school documents and modern textbooks, resulting in improved Algebra instruction (Mercanaj, 2010). When preparing new programs and new textbooks, among the subjects that took priority, such as Albanian language
and History (as they wrote it), was Mathematics, which as the foundation of scientific subjects had a 50% increase in teaching hours (Kambo, 2005). Under the idea of close connection between theory and practice, so-called practice (field hours) was added to the mathematics curriculum, during which students would do measurements outside for different geometric shapes.

During the 1980s, the Ministry of Education and Culture conducted a series of studies (Mathematics 1, Mathematics 2, Mathematics 3, and Mathematics 4), focusing on the outcomes of integration of new content areas and teaching practices within elementary school mathematics classrooms around the country, through classroom observations. The experiment included a minimum of two schools in every region of the country and involved a total population of 1,800 students who were followed from Grades 1 to 4 (Mercanaj, 2010). The experiment demonstrated the use of experiential learning as a progressive strategy of instruction. It raised awareness of the nation’s need for qualified teachers who were capable of adjusting to rigorous changes within the math curriculum. It also provided practical advice to elementary math teachers. As well, the study provided critical insight into the limitation of the texts being used within the standard system and the need to invest in books of superior quality. This was the first study done before conducting a national reform, as opposed to the previous reforms based on mechanically experienced transfers from other countries. The reform marked a departure from Russian texts which dominated until the 1980s. However, there is no evidence of a similar study for higher levels of pre-university mathematics education during the communist regime. With the fall of the communist regime and the implementation of democratic education, the politics of mathematics education began to shift to taking into account the requirements of the basic principles of equality of opportunities for quality education (MoES, 2014).
The democratic educational reforms (as mentioned earlier) aimed to achieve international benchmarks, which guided mathematics education as well. The mathematics curriculum has been revised during the last decade, as it has in the vast majority of European countries, intending the inclusion of the learning outcomes approach. This was the response to the declining PISA (Programme for International Student Assessment) results in most European Union countries and led to the establishment of an EU-wide benchmark in basic skills, to be achieved by 2020: “The share of 15-years olds with insufficient abilities in reading, mathematics and science should be less than 15 percent” (Education, Audiovisual and Culture Executive Agency [EACEA], 2011).

In the recommendation of the European Parliament and the Council of April 23, 2008 on the establishment of the European qualifications framework for lifelong learning (Official Journal of the European Union C 111, May 6, 2008, pp. 1-7), learning outcomes are defined as statements of what a learner knows, understands, and is able to do on completion of a learning process; these are described in terms of knowledge, skills, and competences. The Albanian mathematics education goals at the pre-university level as outlined in the curriculum framework are: to possess theoretical knowledge and concrete mathematical skills relevant to application in practical situations of everyday life and the study of similar disciplines; and to value mathematics as a whole, as a form of description and recognition of reality and as part of human culture and human progress (MoES, 2014b).

**History of Examination in Albanian Education**

If we wish to discover the truth about an educational system, we must first look to its assessment procedures. (Rowntree, 1987, p. 1)

Assessment in education in Albania has evolved with the society itself in trying to assess the readiness of its members to enter a profession or another education level, and to compete in and be part of the larger society. During the 1920s and 1930s, the assessment of pupils was done
on academic subjects, behavior, and hygiene. In order to graduate with a primary school diploma, students after finishing the program underwent an examination in major subjects (Reading, Writing, Mathematics, History, and Geography) before a committee composed of the school principal, the classroom teacher, and a representative from the Ministry of Education. The gymnasium examinations were required at the end of the low cycle in order to receive a semi- Matura diploma or to be able to continue for the upper cycle. To earn a Maturity Diploma, the students underwent the demanding maturity examinations (written and oral) in Albanian language, Mathematics, Foreign Language, Latin, and Physics, and an oral only in History. The examination commission here was composed of the school principal, the classroom teacher, and a representative from the Ministry of Education. Before a similar commission, a student graduating from the Normal schools was examined (in writing and orally) in Albanian language, Mathematics, Foreign Language, Pedagogy, and Pedagogical Practice. All of the above examinations were conducted within the school, and everyone, because of the overpowering silence, presumed their importance.

The school year of 1948-1949 marked the establishment of a knowledge control system at the end of the year, which spread to higher education with state diploma exams (Kambo, 2005). Moreover, as the education system spread all over the country and became consolidated nationally as the result of communist reforms, two major pre-university national examinations were conducted. Students were required to demonstrate Mathematics and Albanian Language & Literature proficiency through the examinations called Release Examinations (Provimet e Lirimit) in order to graduate from eighth grade, and the High School exit examinations in the main subjects (also called Maturity Examinations [Provimet e Pjekurise]). These examinations took place in the schools, at the same day and time nationally, at the end of each academic year.
A representative from the district of education directorate would deliver the exams to the schools in a sealed envelope, to be opened at the start time. The appointed commissions graded the examinations for each particular subject, and then they were sent back to the district education directorate to be archived (Kambo, 2014). For every class taken at the higher education level, the students underwent a final examination (written and oral) in front of an appointed commission, and the performance in this examination determined their grade. After passing all the required classes in any particular program, students in order to graduate had to successfully pass the State Diploma Examination, which was focused in their field of studies. Assessment grades were (and are now) as follows: 10-Excellent, 9-Very Good, 8-Good, 7-Average, 6-Satisfactory, 5-Weak but Passing, and 4-1-Failing.

Evaluation as a decision-making procedure (the assessment results would guide curriculum reforms) in Albania started with the creation of the National Center for Education Evaluation and Examination (QKAVP) in December 2001, whose main responsibility was the development of standardized national examination tests. This was followed by the Central Agency for the Evaluation of Student Achievement (AVA) in March 2007, with its main responsibility the State Matura and providing methodical guidelines on the continuous evaluation of student achievement. The Agency of Admissions in Higher Education Institutions (APRIAL) was created in April 2008. The National Examination Agency (AKP), created by the decision of Council of Ministers in December 2010, is a special institution in charge of the technical and professional organization of national examinations in the pre-university system, university admissions, and state examinations for regulated professions. It analyzes and interprets the results of national examinations and assessments by contemporary standards, providing important data on various indicators of school development or the education system as
a whole.

In February 2006, the State *Matura* Examination (which replaced the Maturity Exams) was established. It was conducted at the end of secondary education, through external evaluation. The State *Matura* consisted of compulsory and optional examinations, and secondary education was considered completed when the student passed all of the compulsory examinations. As a rule, admissions to public universities depended 20% on the average grade of all secondary school years and 80% on the average state examination grades. This changed in 2008 to 35% and 65%, respectively (MoES, n.d.). Starting from 2015-2016, the State *Matura* will be only a high school exit examination. Higher education institutions will have the responsibility to establish criteria for the selection of students for admission. The National Examination Agency will no longer rank students under certain university profiles. The Ministry of Education and Sports determines the floor-level high school average grade needed to enter a university.

During the last 10 years, the administrative regulations for the Release Examinations (conducted at the end of the ninth grade) were similar to those of the State *Matura*. These examinations took place only in public schools, one per region, with distance permitted. Together with the exams, the National Examination Agency sends to the committee of examinations in each regional school board the test evaluation scheme. The examination assessment is done in areas selected by the regional school board in appropriate and safe conditions. Two examiners evaluate each test and both sign it. If there are differences between the two evaluators’ points on the same question, the chairman of the assessors for the subject assigns a third one, whose decision is final.

Assessment nowadays is becoming increasingly similar to that of other European countries, following the path of education in general. Besides its own examinations, Albania
(since 2000) has participated in PISA, an internationally standardized assessment that is jointly
developed by participating countries and administered to 15-year-olds in schools (end of
compulsory education). It is repeated every 3 years, with the primary focus shifting between
Reading, Mathematics, and Science. The need to develop national and international
benchmarking and monitoring systems with integrity and public confidence is stressed in the

**History of Examinations in Mathematics in Albania**

Examinations in mathematics have been part of every education-level assessment, written
and oral. The national examinations conducted at the end of any education pre-university level as
a requirement for graduation include mathematics examinations. In order to enter a higher
education institution and graduate with a bachelor’s degree in any major, at least one
mathematical examination has to be passed. The need to assess someone’s mathematical
knowledge appears to be essential in many settings.

One of the most important and most high-stakes mathematical examinations in Albanian
education, as mentioned earlier, is the High School State *Matura*, which has accompanied high
school graduation since the early 1930s. Research on the evolution of the structure of
mathematics examinations and their curricular coverage is lacking in Albania, which legitimizes
the need for this study. Mathematics examinations from the 1970s represent the requirements for
mathematical knowledge needed to become a mathematically *literate* member of a socialist
society; they mirror the evolution that occurred over three decades of communist education.
Those from 2006-2015 represent how the requirements for mathematical knowledge have shifted
towards preparing students to be successful members of today’s *learning* society. These
requirements are reflected in the guidelines from the Institute for Education Development
(IZHA, n.d.). The guidelines for the graduation examination of mathematics (core) in 2014 stated that: mathematical modeling; the ability to solve problems; the ability to use mathematical knowledge in real-life situations and problems with content from other sciences; the ability to think critically; the ability to argue, to judge, to prove; and cross-curricular skills should be the focus during preparation for the mathematics examination.

**Summary**

Albanian education as a social practice originated during ancient times with the transfer of wisdom as storytelling and the work experience. During the 3rd and 2nd centuries BC, the use of writing and calculations in education was a response to the economic and political changes resulting from the rise of trade and government. Education was used by both Romans and Ottomans to spread and consolidate their respective ideologies. During the Roman occupation (2nd BC-4th AD), the Latin language and Roman culture were introduced into education, while during the Ottoman invasion (15th century-1912), Turkish schools were opened all over Albania with the aim of spreading the Islamic religion.

The political nationwide movement during the 19th century, known as the Albanian National Renaissance, had leaders who believed that education should have a national character among all Albanians, and the use of Albanian language was a necessity for the nation’s progress socially and economically on its way to independence. This movement led to the creation of the first known original Albanian alphabet in 1824, the first *Abetare* (Albanian language book for the first grade) in 1844, the first official Albanian school in 1887, the creation of the Society of Albanian Learning in 1888, the first Albanian Normal School in 1909; in short, Albanian education started to bloom. However, because these education developments did not fit well with the Ottoman invaders, they opposed them and Albanian schools were declared illegal and closed.
in 1910, leading to armed conflicts. The major turning point in Albanian history—its declaration of independence from Turkey in November 1912—was accompanied by the creation of the Albanian Ministry of Education, which laid the legal foundation for a national education system similar to those in other European countries. The volatile political situation in Albania following World War I dramatically affected the development of education. Little to nothing from the programs of different Ministries of Education was actually put into action, even though the efforts of patriots to further the work of the Albanian Renaissance, which started in the mid-19th century, never stopped.

Albania was declared a constitutional monarchy in 1928. During his first days, King Zogu universally acknowledged he did not harbor illusions about the rapid expansion of a national education since Albania had lacked any development for centuries. But he was determined to focus attention on building a more effective education. This led to building a European education for the Albanian elite society. The so-called Ivanaj reform during the 1930s marked the beginning of the unification of the Albanian education in all categories of schools. During this period, the quality of the education provided was thought more important than spreading education all over the country, and graduates from Albanian high schools were accepted in all European universities without entrance examinations. Zogu’s regime was interrupted by World War II, with the country's occupation by fascist Italy in 1939. The war ended in 1944 in Albania, with communists gaining absolute power.

The communist constitution, first adopted in March 1946, created, as it was called, the state of workers and laboring peasants and abolished all ranks and privileges based on heredity, position, wealth, and cultural standing. The communists’ goal was to wipe out illiteracy in the country and bring Albanian education and pedagogy in line with the Soviet conception. The
educational reform law described the character of the new school as the school of the masses and for the masses, while the school was associated with the entire life of the country: “school out of politics” was amiss. This political standing of the country resulted in the following developments in the education system.

- Two alternatives were debated about how to proceed (at what pace) with the development of the national education system: first preparing the conditions for a normal development of education and then expanding it, (quality comes before quantity); or, expending it first with existing opportunities and working to improve it later in the future (quantity before quality). The second option was put into place because, according to the Albanian Communist Party, a large intelligence (that supported its policy) could only be created in an accelerated “revolutionary” way. This led to a rapid expansion of education all over the country with sensitive differences between quantitative and qualitative development.

- The social composition of the students who continued education to high school and further came from the working class (one of the regime’s objectives was to eradicate bourgeois survivals in the country’s culture) and their field of study was subjected to planning because of a centralized planned economy.

- The preparation of the same textbooks and programs across the country was accompanied by significant changes in their ideological basis. The schools must first and foremost ensure the education of the younger generation with a Marxist-Leninist ideology. Works of authors worthy of national literature that played an important role in the Albanian National Renaissance were removed from the program because of their conservative ideas. Subjects such as Logic, Psychology, Latin, and Ancient
Greek were removed from the program as *not necessary*, and *essential* subjects such as National Liberation War History (considered by the communist leaders as the most heroic and glorious in Albanian history), the 1946 Constitution, Knowledge of Marxism-Leninism, and the like were introduced. Only Soviet textbooks and programs were allowed for future translations, and offers from representatives of western countries (including the U.S. Office of Education) to help with textbooks that could serve as a model were not accepted.

- All teaching and educational work was built on the basis of three basic components: teaching, productive labor, and physical and military training. With the implementation of the 1969 law “Further Revolutionization of School,” school politicization reached extreme limits.

Communism in Albania broke down in the early 1990s. The change of the political system, the openness to democratic and economic reforms, and the transition of a centralized economy into a market economy were all accompanied by an economic and institutional crisis during the first stages. This destabilized social, economic and political situation was reflected in the education system in many ways. Education lost prestige (a good education did not constitute a condition for good income), and so school attendance (especially at the high school level and beyond) fell dramatically and schools fell prey to destruction and vandalism. The opening of the borders led to a massive exodus of teachers abroad or to other better paid jobs, and migration from rural to urban areas overflowed urban schools, with many schools having to close in deep rural areas. Under the new (democratic) political system, the national education system underwent a cycle of reforms which were all aided by many foreign western organizations, including the World Bank.
Changes have affected the structure of the education system and its curriculum and management systems, aiming to provide democratic education and prepare youth as citizens and workers in a democratic market-based society, who are cast in the wider context of European membership rather than a narrow focus on national citizenship. The objectives and content of social studies were reviewed (intended to cleanse the system from the inheritance and influence of communist ideology), some new subjects were introduced, and the liberalization of textbooks began. Private education become legal at all levels of public education; their curricula, teaching programs, and conditions must comply with the national interests and the Albanian legislation (for example, religious subjects and classes held in foreign languages are allowed). The Albanian National Strategy of Education follows that of the European Union countries and the development of a higher education system has established its path to European integration as well, with Albania joining the Bologna process.
Chapter III

LITERATURE REVIEW

The literature review is organized into four main parts. The first part is about the field of historical research. The other three parts describe how social, political, and economic environmental factors are represented in education development in general, and mathematics education and mathematics assessment in particular. The focus overall is on countries with past communist regimes since Albania is one of them, and the historical period under investigation is both the 1970s, when Albania was under the communist regime, and the years of 2006-2015, the (post-communism transition) democratic regime. The second part presents a review of the literature related to how the national education system in general was conceived and organized in order to fit the social, political, and economic communist regimes and the post-communist political system of government. The third part presents a review of the literature on how mathematics education developed within the education system. The fourth part is related to high-stakes examinations in general and mathematics examinations in particular, emphasizing the changes on the examinations’ format, their item characteristics, and the alignment of the examinations’ topic coverage with the intended curriculum.

Historical Research

Walliman (2011) described research as an activity that involves finding out, in a more or less systematic way, things one did not know and things that no one else knew either, a process
that advances the frontiers of knowledge. The methods used by a researcher during a research study include theoretical procedures, experimental studies, numerical schemes, statistical approaches, among others, while research methodology is the systematic way research is to be carried out, i.e., the procedures by which researchers go about their work of describing, explaining, and predicting phenomena (Rajasekar et al., 2013). Research methods depend on the nature of the research project, and each type of research design has a range of research methods used to collect and analyze the type of data that is generated by the investigations. “Historical Research aims at a systematic and objective evaluation and synthesis of evidence in order to establish facts and draw conclusions about past events” (Walliman, 2011, p. 9). Karp and Schubring (2014) stated that any topic on mathematics education research may be studied from a historical point of view, and this research constitutes an interdisciplinary activity, where “History proves to be most relevant with regard to methodology of research; one has to rely on its methods for all issues of searching for sources and interpreting and evaluating them, for determining the historical contexts of given situations” (p. 5). The science of history, the history of mathematics, the history of education, and sociology are essential in the study of the history of the teaching and learning of mathematics (Schubring, 2006). Mathematics education is also solidly grounded in psychology and philosophy, among other fields, which gives to it a long and distinguished lineage (Schoenfeld, 2002). While the discipline of research in mathematics education is itself quite young, the research on the history of teaching and learning mathematics has a considerable tradition, with recent rapid developments (Schubring, 2014).

“Historians examine the past so that we may have a better understanding of the content of past events and the context in which they took place. This can help us appreciate the similarities and differences between the circumstances and conditions which govern both past and present
events” (McDowell, 2002, p. 4). Historians do this by selecting, evaluating, and interpreting the historical evidence. Historical research (which communicates an understanding of past events) concerns the study and understanding of the past; it involves finding, using, and correlating information within primary and secondary sources (Elena, Katifori, Vassilakis, Lepouras, & Halatsis, 2010). The most important issue here (including the history of mathematics education) consists precisely of determining which sources may be used and how to use them (Karp & Schubring, 2014). The study’s hypotheses guide the researchers’ decision on prospective sources, including primary, secondary, interview subjects, and collection of realia, even though they are “encouraged to cast the widest possible net, including internet searches, oral history interviews, original documents and three dimensional objects along with archival research to make the historical study feel fresh, democratic and inviting” (Danto, 2008, p. 61). The history of mathematics education is part of social history, and the research study in the field will not be reduced to reprinting tables of contents from textbooks if the researcher will understand the twofold nature of the field: historical in terms of methodologies and mathematical-pedagogical in terms of the objects of study and aim in reconstructing a realistic picture based on all available information (Karp, 2014b, p. 23).

**Education and Social and Political Change**

Many different statements are made and widely accepted about the purpose of education in a society; to mention a few: to develop the intellect, to serve social needs, to contribute to the economy, to create an effective work force, to promote a particular social or political system, and so on. Historically, the purpose of education has changed from that of producing a *literate society* to that of producing a *learning society* (Ammons, 1964). Dewey (1934) described it as follows:
The purpose of education has always been to everyone, in essence, the same—to give the young the things they need in order to develop in an orderly, sequential way into members of society . . . But to develop into a member of society in the Australian bush had nothing in common with developing into a member of society in ancient Greece, and still less with what is needed today. Any education is, in its forms and methods, an outgrowth of the needs of the society in which it exists.

The literature describing the purpose of education in society to promote a particular social or political system and the impact of the political system on the education system pertains directly to this study. The relationship between cultural, political, and economic power and the way the education is thought about, organized, and evaluated has been addressed by Apple (2004) as a complicated connection between and among “legitimate” knowledge, teaching, and power. In authoritarian and ideologically homogeneous regimes, one of which is the communist regime, where the freedom is limited and the population is indoctrinated, the “legitimate” knowledge taught in schools is carefully regulated, controlled, and subjected to the demands of the central planned national economies, with a nationally unified curriculum for all school subjects (Birzea, 1995; Cox, 2011; Kola, 2014; Pachocinski, 1993; Parker, 2003; Perdal, 2011; Phillips, 2010). V.I. Lenin, the founder of the Russian Communist Party, leader of the Bolshevik Revolution, and architect and first head of the Soviet state, stated: “it is hypocritical to say that the school is outside of life, outside of politics” (as quoted in Cox, 2011, p. 4). Mandatory textbooks across the country, as well as monitoring and control over the work of the schools by government agencies and over the work of teachers by both school administrators and general and specialized subject supervisors, were part of the increased rigidity of all aspects of education in the Soviet schools under Stalin (Karp, 2014a). In the Soviet Union, the education system was used as a tool for managing social perceptions, as the most effective and universal form of social influence maintained by the state, as a relationship between the government and the people, and as vital tool for the construction and maintenance of the regime (Cox, 2011, p. 3). Romania’s
education system during the communist regime was used as “a platform by the leaders to manipulate the system in an attempt to influence both current and future generations by limiting teachers, controlling curriculum and propagating inaccurate information” (Phillips, 2010, p. 32). Political connections with Russia after World War II gave rise to a strong influence of Russian culture and education to other socialist states within the USSR’s sphere of influence, like Poland, Hungary, Cuba, and so on (Karp & Vogeli, 2011).

The post-World War II Albanian educational system experienced a rapid expansion from preschool up to higher education under a strict political orientation. The communist government’s objectives for the new school system were to wipe out illiteracy in the country as soon as possible (Kola, 2014) and bring the Albanian education and pedagogy in line with the Soviet conception (Hörner et al., 2007). The sovietizing of schools took place in every aspect of the system; Soviet pedagogy and the methodology of particular subjects were included in plans, guidelines, and proceedings for the organization of the teaching process and extracurricular activity (Hörner et al., 2007; Kambo, 2005; Kola, 2014).

The communist ideology which exercised ultimate power over its people for around half a century had developed deep roots, and it would take time to weed out. The sudden collapse of East Germany (the fall of the Berlin Wall on November 9, 1989), which followed political upheavals in Poland and Hungary and accelerated the deterioration of the Soviet empire, caught both political and academic worlds by surprise (Hertle, 2001). Indeed, even the Czech Republic, one of the strongest economies with the lowest poverty rates of former Soviet-bloc countries in Europe, despite significant reforms aimed at moving away from communist-style central management, is struggling to change “old school” pedagogy (National Center on Education and the Economy [NCEE], 2006). The elimination of that ideology from the newly post-communist
education of the early 1990s left a vacuum in the direction and meaning of schools (Birzea, 1995; Cox, 2011; Dudwick & Shahriari, 2000; Parker, 2003; Phillips, 2010; Terzian, 2010). The official forbidding of the communist model of education, which Birzea (1995) described with the equation “one party → one ideology → one nation → one education system → one curriculum → one text book → new man,” was the primary goal for all post-communist governments. It was to be replaced by the western European model, where “in some cases the content of and principles guiding the new educational model were defined solely in terms of their opposition to the previous regime” (Parker, 2003, p. 45). Birzea described the education reforms during the post-communist transition in Romania as going through four stages: deconstruction (the denying of the old regime, public interest for education decreased); stabilization (defining the new legislative framework); restructuring (relaunching public interest by a coherent education policy); and counter-reform (a systematic reform strategy against residual communism).

The democratically elected governments in the early 1990s launched ambitious political and economic reforms toward free-market economies. The education reforms that took (and are continuously taking) place in post-communist countries were guided by international organizations and often funded by the World Bank, the International Monetary Fund, and others (Birzea, 1995). These reforms aim to achieve international benchmarks, preparing students to travel abroad for higher education and for their roles as citizens and workers in a democratic market-based society. The effect of the global environment on local education determines the future of schooling (Baker & LeTendre, 2005). Baker and LeTendre’s research “leaves an astonishing historical and cross-national record of the use of an increasingly similar organizational form of schooling throughout the world . . . the world culture presenting the rules
of the game, but within these basic rules how the game is played in each nation can become quite elaborated” (p. 170).

While the two world wars followed by the Cold War defined the 20th century and international educational endeavors were designed to meet the challenges specific to that era, the increase in global interconnectedness which characterizes the 21st century requires education to match the realities that are currently defining and shaping the world (Balistreri, Di Giacomo, Ptak, & Noisette, 2012; Biddle, 2002).

Mathematics Education and Social and Political Change

Mathematics education is not immune to the developments taking place in an education system in general as a result of changes in the social, political, and economic environment in which it resides. Povey and Zevenbergen (2008), on their review of the field of mathematics education and society, stated that “researchers [of the field] understand mathematics education as being a profoundly political activity—political in the sense of being intimately bound up with issues of power, authority and the legitimization of knowledge” (p. 1). While investigating the prevalent attitudes toward foreign influences and methodologies in Russian mathematics education at different periods in Russian history, Karp (2006) stated that even though during some periods of change “mathematics was considered politically safe” (p. 618), the official ideological position of both “isolationist” and “internationalist” traditions in Russia has affected the development of mathematics education. Karp’s (2014a) research on Soviet mathematics education concluded that schools under Stalin were characterized by the provision of a uniform education to all students (education became accessible to a much greater number of students), standard mandatory textbooks across the country, and the prohibition of deviation from curricula. The study of mathematics was seen as necessary and important (mathematics was almost always
the queen of subjects in the school curriculum), while the humanities were pushed to the periphery of Soviet consciousness. As Karp (2014a) stated, “Normative pedagogy and methodology presupposed comprehensive monitoring and control over the work of the schools by government agencies and over the work of teachers by both school administrators and general and specialized subject supervisors” (p. 317).

The political, economic, and social changes that took place after World War II were radical for the countries that entered the path of a communist regime. Under the Soviet influence, the sovietization of schools in these countries took place in every aspect of it, and the main goal of national education became to, first and foremost, ensure the education of the younger generation with the Marxist-Leninist ideology. Changes in Poland’s system of education, and consequently mathematics education, were exerted through organizational structures, textbooks, other books, teacher training, and the like (Perdala, 2011). Fundamental changes took place in the Hungarian school system beginning in 1946 with the creation of an 8-year “general school” that provided equal opportunities for every student (which was thought to be with much less knowledge to every student than the previous system had provided to a highly selected population) (Fried, 2011, p. 341). During this period, Hungarian mathematics education suffered a shortage of qualified mathematics teachers and textbooks, among other shortages. Fried (2011) described the Russian influence in Hungarian mathematics education as twofold: direct methodological (textbooks, ideas, and didactical materials) and political in the sense that the Hungarian authorities had to verify that their policies were aligned with the soviet ones. After World War II, with Albania becoming part of the European eastern bloc, the communist government’s ambitious educational reform included a fundamental change of programs, texts, subjects, and methods of teaching. The decision of the Council of Ministers in 1948 restricted
future translations to be done only from (same-level) Soviet school documents and modern textbooks; this did result in improved Algebra instruction (Mercanaj, 2010).

The democratic educational reforms that took place after communism broke down aimed to achieve international benchmarks and guide mathematics education. Aiming at developing key European competences (IZHA, n.d.; MoES, 2004), PISA (the most important international assessment institution) participation made a contribution to, and remains part of, Albanian education system reforms (Harizaj, 2011). The mathematics curriculum has been revised during the last decade in the vast majority of European countries, including Albania, intending the inclusion of the *learning outcomes approach*, a EU-wide benchmark. These revisions were (in part) a response to declining PISA results in most European Union countries. One of the EU-wide benchmarks in basic skills to be achieved by 2020 is: *The share of 15-years olds with insufficient abilities in reading, mathematics and science should be less than 15 percent* (EACEA, 2011).

**High-stakes Examinations and Social and Political Change**

The intercorrelation between high-stakes examination results in education and social and political change is widely accepted by education researchers. Education assessment, measuring if the purpose of education is being reached, has evolved and guided the evolution of education that adapts to society’s ever-changing needs, with consideration for the past and imagination of the future (Pellegrino, 1999). Assessment drives change at different levels of an education system, informs the public and those who make decisions, and allocates resources, rewards or sanctions schools by assessing if the purpose of the education has been reached. That is, examination results directly influence many education policies (Cizek, 2001) and also have a direct impact on a person’s life options and opportunities (Moses & Nana, 2007). Because of the far-reaching
impact of these examinations, the alignment of what these tests are measuring with the expectations set by the curriculum must be appropriate (Ananda, 2003; La Marca et al., 2000; Martone & Sireci, 2009; Polikoff et al., 2011; Webb, 1997). The formal written examination, as a form of assessment, started replacing the oral examination in American schools at roughly the same time as schools changed their mission from servicing the elite to educating the masses during the mid-19th century, while public examination systems in Europe developed when mass secondary schooling expanded following World War II and became the principal selection tool setting students on their educational trajectories (U.S. Congress, Office of Technology Assessment, 1992). Because high-stakes examinations serve the masses, their results are in the forefront of the public’s mind and remain high on the agenda of elected officials and educators in general (Bennett, 1998).

Morgan and Sfard’s (2016) research study attempted to provide an improved basis for the evaluation of examinations, so policymakers know more than just the statistically processed results of testing. They developed a method and investigated the evolution of the high-stakes examinations taken by students in England at the end of compulsory schooling during the last three decades using examinations from 1980, 1987, 1991, 1995, 1999, 2004, 2010, and 2011. Morgan and Sfard focused on change that happens over time. They considered mathematics to be a discourse with implications on how change in examinations is investigated, how informative and useful the results are, and what is the process of learning and its outcomes. They looked “at every examination in its entirety, that is, translate characteristics of separate questions, such as complexity of utterances or specific properties of the layout, into a feature of the examination as a whole” (p. 109). Among their findings were the following: the examinee’s freedom in deciding on the problem-solving trajectory and on the format of expected answers declined over time;
also, the percentage of tasks requiring one or two steps grew, whereas the percentage of those requiring three or more steps decreased.

Lawrence, Rigol, Essen, and Jackson (2003) described the SAT evolution path going from measuring mental quickness (100 free-response questions in 80 minutes during the 1930s) to multiple-choice only questions starting in 1942. In 1959, data sufficiency questions were introduced, which in 1974 were replaced by quantitative comparisons questions. The 1994 changes were responsive to the suggestions of the National Council of Teachers of Mathematics (NCTM) to move away from an exclusively multiple-choice test, reduce the impact of speed on test performance, introduce the application of learning to new contexts, and solve problems that have more than one answer. The later redesign of the SAT attempted to enhance its alignment with current high school curricula and emphasize the skills needed for success in college. They stated that the reconfiguration of the SAT over the years has been taking into consideration fairness issues, scaling issues, cost, public perception, face validity, changes in the test-taking population, changes in patterns of test preparation, and changes in the college admissions process.

Britten and Raizen (1996) provided an examination of 1991 and 1992 mathematics, biology, chemistry, and physics examinations taken by college-bound students in seven countries—England, Wales, Germany, Israel, Japan, Sweden, and the United States. The study concentrated on comparing these examinations by looking at every question on the examination and also considering the examination as a whole. One of their findings was that in most systems, the balance of questions which can be answered by routine procedures using learned algorithms and those which require thoughtful translations and applications of principles and procedures was very much in favor of the former (Britten & Raizen, 1996, p. 20). Another result from this analysis was that while the countries’ secondary school curricula generally were aligned with
there were some problems in the correspondence between university curricula and secondary preparation and examinations.

Karp’s (2007) analysis of the history of Russian graduation examinations in mathematics (end of the 19th century-first half of the 20th century) showed that the routes these examinations took can be seen as expressions of social changes. Karp, indeed, is a great proponent that even pure mathematical problems incorporate this political sense—that it is not just about numbers and equations. These problems represent in some specific way the political and social environment. In his own words: “The purely mathematical problems offered on exams can be interpreted as manifestation of specific social views, and the change in their types and structures can be seen as expressions of—or at least wishes for—social change” (p. 39). While before the Soviet times educational districts had some freedom in the preparation of examinations, the central control over examination content and administration increased under Soviet rule. All examinations, viewed as meaningless and harmful (not accurate in knowledge measurement), were canceled after the 1917 revolution. However, in August 1932, the Communist Party found them necessary for knowledge measurement, so graduation examinations, including those in mathematics, were reinstated. After World War II, the Ministry of Education in Moscow was in charge of writing a single examination and distributing it around the entire country. Guidelines for the content of the examination solutions were distributed as well.

On the subject matter and structure of the examination problems, Karp (2007) stated that the typical graduation examination in Algebra before the revolution called for only one problem that contained several sub-problems and required the knowledge of many topics. Karp called such problems “composite problems.” Later on, in order to make matters more convenient for the students, the graduation examination consisted of three problems, with solutions not depending
on each other. After World War II, the number of topics included in the graduation examinations was reduced and the number of problems in the 1948 graduation examination was four. The problems included on the examination were carefully selected since “a standard examination prepared for students across the entire country had no room in it for problem types that were not specifically recommended for inclusion (such as composite problems)” (p. 48). Karp (2007) concluded that mathematics educators (even centuries ago) used the examination preparation (complex constructions of the problems) and the requirements for explanation of every step in the solutions to assess more than just the ability of the students to follow a fixed pattern.

The ability to read (untangle) a problem and to write out every single premise in its solution was equated with understanding, but these methods in the struggle against fixed patterns themselves turned into fixed patterns that teachers taught their students, instead of developing their comprehension and cultivating their creativity. (p. 52)

**Summary**

This literature review showed that the use of education to promote a particular social and political system was one of the main purposes of education in the communist regimes, an authoritarian and ideologically homogeneous political system. The communist regimes used the education system to indoctrinate the population, to manage social perceptions, and to determine the “legitimate” knowledge to be taught in schools. The sovietization of schools in the countries that entered the path of the communist regime after World War II took place in every aspect of it; the rapid expansion of education was accompanied by the reduction of quality education. The education reforms that took place after the breakdown of communism aimed to achieve international benchmarks and were guided, and often founded, by international organizations. The system of education in post-communist countries is becoming part of an increasingly similar organizational form of schooling throughout the world, even though the struggle to weed out the residuals of the communist perception of education accompanies the transition.
Mathematics education resides within education in general, and the literature has shown that the political and social environment has a say in the way it develops. Mathematics education in countries under the Soviet influence was political in the sense that not only mathematics textbooks, programs and didactical materials, but also policies must be aligned with Soviet ones. Nowadays, education reforms in post-communist countries are aiming to achieve international benchmarks in mathematics.

The literature review on high-stakes examinations has shown the connection between these examinations and the social and political environment. Examination results influence education policies, have a direct impact on a person’s life, and so on; mathematics examinations are sometimes not just about numbers and equations, but are a representation of the political and social standing of a country. The literature represented here shows that the development of high-stakes examinations has followed the changes in the social and economic environment and in the test-taking population, considering fairness issues and their alignment with the school curriculum.
Chapter IV

METHODOLOGY

Historical Research Methodology and Major Sources

The research design for this study followed the methodology of historical research in selecting, evaluating, analyzing, and interpreting the sources. The study attempted to use (in combination) most common modes of writing that historians use: description, narration, exposition, and argument. It followed Walliman’s (2011) historical research design in combination with Karp and Schubring’s (2014) approach of historical research in mathematics education. Walliman (2011) stated that “Historical Research aims at a systematic and objective evaluation and synthesis of evidence in order to establish facts and draw conclusions about past events” (p. 9), while Karp (2014b) stressed the importance of the researcher’s understanding of the twofold nature of the history of the mathematics education field. In aiming to reconstruct a realistic picture based on all available information, the history of mathematics education should be studied as part of social history because it is historical in terms of methodologies and it is mathematical-pedagogical in terms of the objects of study (Karp, 2014b).

As mentioned earlier, the purpose of this study was to explore the evolution of the High School State Examinations in Mathematics in Albania during two decades, the 1970s and 2006-2015, and their alignment with the curriculum. These examinations and the curriculum documents constituted the main objects of the study; they were examined in terms of their
mathematical-pedagogical content, the sociopolitical context in which they took place, the similarities and differences between them, and the circumstances and conditions which governed both decades. The historical research here involves finding, using, and correlating information within primary and secondary sources.

Throughout the research, the researcher reviewed various documents. The primary sources consisted of documents published by the Albanian Ministry of Education (listed below), the Central State Archive of the Republic of Albania, the Internet archive http://www.arsimi.gov.al, and memoirs from two high school mathematics teachers who taught in different parts of Albania during the 1970s. The high school textbooks during the respective decades were used as a resource to determine the curriculum requirements because they were published by the Ministry of Education, and the topics included in them strictly followed the curriculum. The most pertinent sources included official documents pertaining to mathematics education:


- High school mathematics curriculum documents published by the Albanian Ministry of Education.


- High school mathematics textbooks from 2006-2015 approved by the Ministry of Education and Science and published by different publishing companies.

- Interviews with two high school mathematics teachers from the 1970s.
The high school exit mathematics examinations found in the primary sources for the 2006-2015 period were of multiple levels which assessed different levels of mathematical knowledge, depending on the academic path students had chosen—the natural sciences and general or social sciences. The high school mathematics developed in two programs—the core mathematics program and the advanced mathematics program—so that the students in the general path had two options from which to choose for their mathematics graduation examination. Two different levels were available: the mathematics core curriculum examination called “The Required Examination,” which assessed core curriculum mathematics for general high school graduates; and the optional mathematics graduation examination, which assessed a higher mathematical level. In order to graduate, students were required to take one of these, but not both. Only one type of examination was administered to general high school graduate candidates during the school years of the 1970s, which consisted of two parts—written and oral.

The objects of study for this research were the core curriculum ones: the Required Examinations for 2006-2015 (see appendix C) and the written examinations from the 1970s (see appendix A). The examinations’ contemporary solutions were used during the analysis to draw a complete picture of what was expected from the students. The source for the 1970s examinations and their solutions was a book published by the Textbook Publishing House (Tirana) in the 1980s entitled Mathematics Exercises and Problems for High School, while the examinations and their solutions for 2006-2015 were downloaded from the internet webpage (public domain) of the National Agency of the Examinations (Agjencia Kombetare e Provimeve, http://akp.gov.al), which (as mentioned in previous chapters) is a special institution in charge of the technical and professional organization of national examinations in the pre-university system, university admissions, and state examinations for regulated professions.
Part of this analysis was examination assessment based on documents from the National Agency of the Examinations for the 2006-2015 and on interviews with two mathematics high school teachers from the 1970s. The analysis focused on the methods used during the examinations’ evaluation and compared these methods between the two periods.

Examination Analysis Methodology

The analysis of the High School State Examinations in Mathematics in Albania during two decades, the 1970s and 2006-2015, included their general structure, the item characteristics, and the topic coverage.

Similar studies can be found that have analyzed high school standardized mathematical examinations. The ones that guided this study are: Karp’s (2003) “Mathematics Examination: Russian Experiments”; Karp’s (2007) “Exams in Algebra in Russia: Toward a History of High-stakes Testing,” and Dossey’s (1996) “Examining Mathematics Examinations.” Karp’s (2003) analysis of high school algebra examinations given in St. Petersburg, Russia, concluded that the evolution of the examinations, in order to satisfy modern demands for greater flexibility with an emphasis on evaluating reasoning in addition to skills, can be described through three main ideas: the use of structured questions, the oversupply of tasks, and the principle of multiple levels. His 2007 study provided a historical analysis of graduation examinations in pre-Revolutionary (before the Revolution of 1917) gymnasia and Soviet high schools (described in the Literature Review chapter). The fact that Russian education expertise, as was mentioned in the previous chapters, had a strong influence on the Albanian education as a whole after World War II, and on mathematics education in particular (Hörner et al, 2007, Mercanaj, 2010), makes Karp’s approach suitable for exploring the characteristics of the examinations and their items, because it was applied on somewhat similar grounds, examinations, and examination questions.
Dossey’s study provided insight into international examinations, and hence into the curricula associated with them, and focused on topics covered, types of questions used, and performances expectations. Indeed, Dossey’s analysis concentrated on comparing the examinations given at the end of secondary education to college-bound students and illustrating their similarities and differences among seven developed countries (England, Wales, France, Germany, Japan, Sweden, and the United States). Thus, it is applicable to an analysis of the Albanian examinations, given that the transformation of Albania’s educational system into a European one (similar programs and evaluation process) started from the beginning of the 1990s.

Dossey’s analysis of the general structure of the examinations consisted of the length of the examination, the total number of scorables (which were the unit of analysis—the smallest, discrete questions in each examination [p. 7]), the number of multiple-choice questions, the number of free-response questions, and the possibility of choice (or oversupply of tasks, as Karp called it). Dossey’s analysis of item characteristics was based on item types: multiple-choice, free-response short answer, free-response extended answer, and practical activity items, as well as the use of diagrams, graphs and tables. Dossey concluded that many of the free-response items had a “building” structure, in which the latter answers depended on the previous parts, indicating the in-depth focus on particular topics (p. 174). The “building” structure of the items, together with the dependence of later answers on the previous ones, is similar to Karp’s approach of defining the examination item as a structured question: a group of questions about one object, some providing hints for the most difficult questions and involving more complex linkages—those that lead from general to specific or the other way around.

This research’s methodology was based on the above approaches (adapted in combination), and included an analysis of the general structure of the examinations, an analysis
of the examinations’ item characteristics, and their curriculum coverage, as explained separately below.

**General Structure of the Examinations**

Each examination was analyzed with respect to its general structure as the object of study and each item (question) in the examination was analyzed on its own as the unit of analysis. The item analysis, which is explained next, was the basis of the following basic features that constitute the general structure of the examinations:

1. the length of the examination in hours,
2. the total number of questions,
3. the number of written questions,
4. the number of oral questions,
5. the number of multiple-choice questions,
6. the number of free-response short-answer questions,
7. the number of free-response extended-answer questions,
8. the number of structured questions,
9. the oversupply of tasks, i.e., the possibility of choice (not all questions are required),
10. the number of typical problems, and
11. the number of non-typical problems.

**Item Characteristics**

As mentioned above, the examination items (questions) constituted the units of analysis; each item was analyzed with respect to its characteristics, identified as follows:

1. Written, if the answer was to be presented in writing, or oral, if the answer was to be presented orally.
2. Multiple-choice, if the students were asked to choose the best possible answer out of a list of options.

3. Free-response short-answer, items that require a short solution (1-2 steps) which consisted of recalling at most two mathematical concepts. A step consists of an operation needed to determine some value, recalling a theorem, a formula, a rule, and the like, or drawing a figure.

4. Free-response extended-answer, items that require a longer solution (more than two steps), which consisted of recalling more than two mathematical concepts, but only one question was asked.

5. Structured question, if the item is comprised of a group of mini-questions about one object, where the previous questions provided some hints and ground for answering the later ones.

6. Typical (if similar problems are present in contemporary textbooks) or non-typical.

The analysis of each examination item included the identification of the mathematical concepts needed to be recalled (the needed mathematical knowledge) in order to answer the question. This part of analysis concluded with the topic coverage of the item in particular and the examination in general, i.e., the assessed mathematical concepts on a particular examination.

**Topic Coverage**

While the analysis of the examinations with respect to their general structure and item characteristic analysis is mainly self-evident, the analysis of topic coverage can be complicated. This complication is attributed primarily to the relatively large number of topics required to answer one item, i.e., one problem covers several topics, and this is not always clearly stated in the question. Most of the items assess a combination of topics, and the determination of the topic
coverage of an item is done through an analysis of its solution, because an item may sound as if it is a problem about functions when it is actually more about equations. For example, in the item: Determine where the function \( y = \sqrt{x^2 - 2x} + \log(1 - x) \) is defined (Question 19 in the 2013 Examination), the question is about functions and their domain, but in order to answer the question the student needs to know:

1. the concept of the domain of a function in general and the domain for the radical and logarithmic functions in particular;
2. how to solve quadratic inequalities, \( x^2 - 2x \geq 0 \) for \((-\infty, 0]\) and \([2, \infty)\);
3. how to solve linear inequalities, \( 1 - x > 0 \) for \( x < 1 \);
4. how to solve a system of inequalities: \( \begin{cases} x^2 - 2x \geq 0 \\ 1 - x > 0 \end{cases} \), and set operations;

thus reaching the solution that the function is defined in the interval \((-\infty, 0]\).

Another important point in topic coverage analysis is considering the intended solutions of the examination item because, given many ways to answer a question, one may solve a problem using, for example, trigonometry when it was actually intended to assess something else. In order to present the intended topic coverage for each item, the typical solutions found in the contemporary textbooks and examinations solutions were used during the analysis to establish which approaches were most typical.

The analysis of each examination’s topic coverage was done with respect to the mathematics concepts and themes that were identified as being present in the curriculum and/or examinations. The curriculum framework, which describes the main categories and their respective topics used in the analysis, followed the Mathematics Framework of the Third International Mathematics and Science Study (TIMSS) (Robitaille et al., 1993) and is shown in Table 1 below (see Appendix D for more details). In order to permit a specific description of the
examinations’ topic coverage, the topics that were never present in either curriculum or 

examinations were excluded. For example, in the TIMSS Mathematics Framework, the category 
of Numbers consists of five subcategories, each one including several topics and subtopics, but 
only the integer and rational exponents subtopic was relevant to this study and it is part of the 
study’s curriculum framework. The other number topics (whole numbers and their operations, 
fractions and decimals, etc.) were excluded.

Table 1

Curriculum Framework

<table>
<thead>
<tr>
<th>Category</th>
<th>Topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and Measurement</td>
<td>Integer &amp; Rational Exponents, Perimeter, Area, Volume and Angles Systematic counting</td>
</tr>
<tr>
<td>Geometry—Form &amp; Relation</td>
<td>Two-dimensional Coordinate Geometry Polygons and Circles Three-dimensional Geometry Relation (congruence and similarity) Vectors</td>
</tr>
<tr>
<td>Functions and Equations</td>
<td>Functions Equations</td>
</tr>
<tr>
<td>Probability and Statistics</td>
<td>Data Representation Uncertainty and Probability</td>
</tr>
<tr>
<td>Elementary Analysis</td>
<td>Infinite Processes Limits &amp; Function Continuity Differentiation Integrations</td>
</tr>
</tbody>
</table>

In the example above: Determine where the function \( y = \sqrt{x^2 - 2x + \log(1 - x)} \) is 
defined, the topics covered are: functions (logarithmic and radical), equations (linear and quadratic), inequalities (linear and quadratic), system of inequalities, and set operation. Thus, 
this item belongs to the Functions and Equations category.
Alignment Analysis Methodology

This study included an analysis of the alignment of the examination topic coverage with the curriculum topics, addressing the forth research question: To what extent were the High School State Examination in Mathematics aligned with curriculum expectations for high school graduates during the 1970s and 2006-2015?

The three most common alignment methods (Martone & Sireci, 2009) are the Webb Methodology (Webb, 1997, 1999, 2002); the Achieve Model (Rothman et al., 2002); and the Surveys of Enacted Curriculum method (Porter & Smithson, 2001). Webb’s methodology explored the alignment between assessment and standards with respect to five dimensions: content focus, articulation across grades and ages, equity and fairness, pedagogical implications, and system applicability (Webb, 1997). The content focus analysis dimension, which is the only one that has been applied in alignment studies (Martone & Sireci, 2009), consists of six subcategories: categorical concurrence, depth of knowledge, range of knowledge, balance of representation, structure of knowledge, and dispositional consonance; each explores the relationship between assessment and standards in different ways.

The subcategories of categorical concurrence and range of knowledge in Webb’s methodology were the basis of the alignment methodology for this study. Categorical concurrence looks at the broad content categories in the curriculum and compares them with content categories present in the assessment items. The match item-objective is called a hit, and Webb (2002) suggested that at least six hits would be needed (i.e., a category should be matched with at least six items) in order for the assessment to result in a reliable alignment. However, Webb stated that the number of hits (six above) may vary and should be adopted according to different situations. In Webb’s analysis, the six-item hits were assumed to be the minimum for an
assessment measuring content knowledge related to a standard and as a basis for making some decisions about students’ knowledge of that standard (p. 4).

This study’s analysis used the number two (50% of the examination’s questions) instead of six for the 1970s written examinations since they consisted of four questions each. Thus, if two questions out of four assessed a particular category, then the examination would result in a reliable assessment for it. In the analysis of the 2006-2015 examinations, which consisted of 25 questions each, if five questions (20% of the examination’s questions) assessed a particular category, then the examination assessment for that category would be called reliable. The range of knowledge subcategory analyzes the breadth of the curriculum objectives as compared to the breadth of the assessment. It looks at the number of topics within a category measured by at least one assessment item. Webb (2002) suggested that at least 50% of the objectives within a category need to be measured by at least one assessment item to have sufficient alignment with respect to the range of knowledge. If 50% of a particular category’s topics is assessed by the examination, then this examination has sufficient alignment with the category’s range of knowledge.

The curriculum framework in this research, as mentioned above, followed the Mathematics Framework of the TIMSS (Robitaille et al., 1993). Its categories are: Elementary Analysis, Functions and Equations, Geometry (Form and Relation), Probability and Statistics, and Number and Measurement. Each category on its own consists of a certain number of topics; for example, the category of Elementary Analysis consists of the following topics: infinite processes (sequences), limits and function continuity, differentiation, and integration. The analysis of the alignment with respect to categorical concurrence counts the number of hits (item-category matches) identified in each examination, while the analysis with respect to the range of knowledge determines what percent of the topics within each category are present in
each examination. Each examination and each item of the examination were examined with respect to their topic coverage; the categorical concurrence analysis described whether the examination covered all curriculum categories, while the range of knowledge subcategory analysis showed the breadth to which each examination assessed each curriculum category. An example of this analysis follows.

**Examination Analysis—An Example**

To illustrate the methodology of examination analysis used here, an item-by-item analysis for the 1975 Examination is given below. The examination consisted of four questions. After each step in the solution, a note (*in bold italics*) is given to identify the topic that it covered (i.e., the knowledge required to answer). The item analysis is given after each question, while at the end, the examination analysis follows.

Primary Examination, May 26, 1975

1. Given the ellipse \( \frac{x^2}{8} + \frac{y^2}{2} = 1 \).
   a) Find the coordinates of the point A on the ellipse, in the first quadrant, such that the tangent to the ellipse at A is parallel to \( x + 2y = 0 \).
   b) Write the equation of the parabola, symmetric with respect to y-axis, with vertex at the origin and passing through A.
   c) Find the area of the figure surrounded by the parabola and the line that passes through the origin and point A.

   **Solution:**
   a) Let \((x_1, y_1)\) be the coordinates of point A. Then the equation of the tangent line to the ellipse at \( A = (x_1, y_1) \) is \( \frac{xx_1}{8} + \frac{yy_1}{2} = 1 \). (*ellipse*)

   Since this line is parallel with \( x + 2y = 0 \) we have \( \frac{x_1}{1} = \frac{y_1}{2} \implies y_1 = \frac{x_1}{2} \) (*lines*)
and A is on the ellipse, in order to find its coordinates we solve the system of the

equations:
\[
\begin{align*}
y_1 &= \frac{x_1}{2} \\
\left(\frac{x_1}{2}\right)^2 + \left(\frac{y_1}{2}\right)^2 &= 1
\end{align*}
\]  \Rightarrow A = (2,1) \ (\text{systems of equations})

Substituting in \(\frac{x \cdot x_1}{8} + \frac{y \cdot y_1}{2} = 1\) we get the equation of the tangent line to the ellipse at point A to be \(x + 2y - 4 = 0\) \ (\text{lines})

b) The equation of the required parabola has form \(x^2 = 2py\) \ (\text{parabola}). Since parabola passes through point \(A = (2,1)\), \(2,1\) satisfy its equation, so we get \(2^2 = 2p \cdot 1 \Rightarrow p = 2\) and the equation of the parabola is: \(y = \frac{1}{4}x^2\) \ (\text{equations}).

c) The equation of the line that passes through \(A = (2,1)\) and the origin \(O = (2,1)\) is \(y = \frac{x-0}{2-0} = \frac{y-0}{1-0}\) or \(y = \frac{x}{2}\) \ (\text{lines}).

To find the required shaded area \(S\), we find the area \(S_1\) of the triangle AOB \ (\text{triangle area}), and the area \(S_2\), under the curve of the parabola and above x-axis, then
\[
S = S_1 - S_2 = \frac{1}{2} \cdot 2 \cdot 1 - \int_0^2 \frac{1}{4}x^2 \, dx = 1 - \frac{2}{3} = \frac{1}{3}
\]
\ (application of definite integrals)

**Item Analysis**

Item characteristics: Identified as structured questions and typical question.

Topics coverage: Geometry – Coordinate Geometry (lines, conic sections), Polygons

Measurement – Area (triangle)

Functions and Equations – Linear equations and system of equations

Elementary Analysis – Integrals (application of definite integrals)
The item’s alignment analysis follows from the topic coverage; this item contributes one hit (item-category match) to the category of Elementary Analysis, one hit to Geometry, one hit to Functions and Equations, and one hit to Measurement, which concludes the categorical concurrence analysis. The range of knowledge analysis results indicates that there is one topic, integration, out of four from the Elementary Analysis category represented in this item, two objectives out of five for the Geometry category (Coordinate geometry and Polygons), and one objective out of two for the Functions and Equations category (Equations).

2. Given the function \( y = \frac{ax - 1}{x^2} \), find the parameter \( a \) such that the tangent line to the function at \( x = 1 \) is parallel to \( x \)-axis. Then using the value of the parameter \( a \), examine the behavior of the function and graph it.

Solution

Finding the derivative of \( y = \frac{ax - 1}{x^2} \) we get \( y' = \frac{ax^2 - 2x(ax - 1)}{x^4} = \frac{-ax + 2}{x^3} \)

\( y'(1) = 0 \implies a = 2 \) which gives us the function \( y = \frac{2x - 1}{x^2} \)

(knowledge required: geometric meaning of derivative and parallel lines property)

Examining the behavior of the function.

1) The function is continuous in the interval \( (-\infty, 0) \cup (0, \infty) \). (domain, continuity)

2) \( y' = \frac{-2x + 2}{x^3} \) is zero at \( x = 1 \).

Since \( y' < 0 \) for \( x < 0 \) and \( x > 1 \) the function is decreasing on the interval \( (-\infty, 0) \cup (1, \infty) \) and increasing on \( (0, 1) \) since \( y' > 0 \) there.

The function takes its maximum of \( y = 1 \) value \( x = 1 \).

(equations, derivative and function behavior)

3) \( y'' = \frac{-2x^2 - 2x(x - 2) + 2}{x^6} = \frac{5x - 6}{x^2}, \) and \( y'' = 0 \) at \( x = \frac{3}{2} \).

\( y'' < 0 \) on \( (-\infty, 0) \cup \left(0, \frac{2}{3}\right) \) and the graph is concave down and \( y'' > 0 \) on \( (0, \infty) \) and the graph is concave up.

The point \( I \left(\frac{3}{2}, \frac{8}{9}\right) \) is its inflection point.
(higher order derivatives, derivative behavior, concavity and inflection points)

4) When \( x \to \pm \infty \), \( y \to 0 \) because \( \lim_{x \to \pm \infty} \frac{2x-1}{x^2} = 0 \)

When \( x \to 0 \), \( y \to -\infty \) because \( \lim_{x \to 0} \frac{2x-1}{x^2} = -\infty \)

\( y = 0 \) when \( x = \frac{1}{2} \)

The graph has horizontal asymptote \( y = 0 \) and vertical asymptote \( x = 0 \)

(limits and functions)

Item Analysis

Item characteristics: Identified as free-response extended answer and typical question.

Topics coverage: Geometry – Coordinate Geometry (lines)

Functions and Equations – Function domain and equations

Elementary Analysis – Limits and Functions, Differentiation, Continuity

The item’s alignment analysis follows from the topic coverage; this item contributes one hit (item-category match) to the category of Elementary Analysis, one hit to Geometry, and one hit to Functions and Equations, which concludes the categorical concurrence analysis. The range of knowledge analysis identified that there are two topics (limits and function continuity, and differentiation) out of four from the Elementary Analysis category represented in this item, two
out of two for Function and Equations category, and one objective out of five for Geometry (lines).

3. A tree trunk in the form of a right circular cone has a height of 180 dm and diameters for the bases are 12 dm and 6 dm. A beam with a rectangular parallelepiped shape with square base is to be cut from the above trunk in such way that the axis of the trunk complies with the central axis of the beam. Find the height of the beam with the largest volume.

Solution

We have \( OO_2 = 180 \text{ dm} \), \( EF = 12 \text{ dm} \) and \( PQ = 6 \text{ dm} \). Let \( x \) be the height of the beam, \( OO_1 = A_1A = B_1B = C_1C = D_1D = x \).

\[
\text{(three dimensional shapes and surfaces and their properties)}
\]

The axial cutting EFPQ of the trunk has the shape of an isosceles trapezoid. We draw \( QK \) and \( PL \) perpendicular to \( EF \), so \( EK = LF = 3 \).

\( EFC_1A_1 \) is an isosceles trapezoid as well, and we get \( (polygons, \ trapezoid) \)

\[
EA = CF = \frac{EF - A_1C_1}{2} = 6 - OA, \text{ and since}
\]

triangles \( A_1AE \) and \( QKE \) are similar we get \( \frac{EA}{EK} = \frac{A_1A}{QK} \)

\[
\text{(similarity of triangles)}
\]

Substituting we have \( \frac{6 - OA}{3} = \frac{x}{180} \) \( \Rightarrow \) \( OA = 6 - \frac{x}{60} \) and since the base of the beam is square we get \( S_{ABCD} = \frac{Ac^2}{2} = \frac{(2 \cdot OA)^2}{2} = 2 \cdot OA^2 = 2 \cdot \left( 6 - \frac{x}{60} \right)^2 \) \( (area) \)

The volume of the of the beam is: \( V = S_{ABCD} = 2 \cdot A_1A = 2 \left( 6 - \frac{x}{60} \right)^2 \cdot x \) \( (volume) \)

Or \( V = 2x \left( 6 - \frac{x}{60} \right)^2 \) where \( 0 < x < 180 \).

\[
V' = 2 \left( 6 - \frac{x}{60} \right)^2 + 4x \left( 6 - \frac{x}{60} \right) \cdot \left( \frac{1}{60} \right) = 2 \left( 6 - \frac{x}{60} \right) \left( 6 - \frac{x}{20} \right) \]

\( V' = 0 \) per \( x = 120 \) since the other is not acceptable because \( 0 < x < 180 \)
Analyzing the sign of the derivative gives us \( x = 120 \, dm \) is the height that maximizes the volume of the beam.

*(derivative and the relationship between derivative behavior and maxima and minima)*

**Item Analysis**

Item characteristics: Identified as free-response extended answer and typical question.

Topic coverage: Geometry – Three-dimensional Geometry, Polygons, Similarity

Measurement – Area and Volume

Functions and Equations – Function domain

Elementary Analysis – Differentiation

The item’s alignment analysis follows from the topic coverage; this item contributes one hit (item-category match) to the category of Elementary Analysis, one hit to Geometry, one hit to Functions and Equations, and one hit to Numbers and Measurement, which concludes the categorical concurrence analysis. The range of knowledge analysis identified that there is one topic (differentiation) out of four from the Elementary Analysis category represented in this item, three out of five (three-dimensional shapes, polygons, similarity) for Geometry, and one objective out of two for Functions and Equations.

4. A regular pyramid has a triangular base with side \( a \), and the angle between two sides is \( \alpha \). Find the side area of the pyramid.

**Solution**

In the regular pyramid SABC the side of the base ABC is \( a \) and the point O in the height segment SO is the center of the base.

*(three dimensional shapes and surfaces and their properties)*
$AD \perp SB$, $\triangle CBD = \triangle ABD$ since they share BD and $CB = AB = a$ and $\angle CBD = \angle ABD$ (SAS) which implies that $\angle CDB = \angle ADB = 90^\circ \implies CD \perp SB$. $CD = AD$ so the triangle ADC is isosceles. Since $AD \perp SB$ and $CD \perp SB$ we have $\angle ADC = \alpha$.

(geometry relation – congruence)
The point E is the midpoint of AC so $DE \perp AC$ and $\angle ADE = \angle EDC = \frac{\alpha}{2}$. In the $\triangle ADB$ we have:

$$AD = \frac{AE}{\sin \frac{\alpha}{2}} = \frac{\frac{a}{2}}{\sin \frac{\alpha}{2}} = \frac{a}{2 \sin \frac{\alpha}{2}}$$

Let $\angle SBA = x$ and in the $\triangle ABD$ we have:

$$AD = AB \sin x = a \sin x$$

from where we get $a \sin x = \frac{a}{2 \sin \frac{\alpha}{2}} \Rightarrow \sin x = \frac{1}{2 \sin \frac{\alpha}{2}}$

In $\triangle SBF$ we find $SF = BF \tan x = \frac{a}{2} \tan x$. Expressing $\tan x$ as a function of $\alpha$, we get:

$$\tan x = \frac{\sin \alpha}{\cos \alpha} = \frac{1}{2 \sin} \frac{\alpha}{2} \Rightarrow \tan x = \sqrt{\frac{1}{1 - \left(\frac{1}{2 \sin \frac{\alpha}{2}}\right)^2}} = \frac{1}{2 \cdot \sqrt{\sin \left(\frac{\alpha}{2} + 30^\circ\right) \sin \left(\frac{\alpha}{2} - 30^\circ\right)}}$$

And the area of the sides of the pyramid is:

$$S = \frac{1}{2} \cdot 3AB \cdot SF = \frac{3a}{2} \cdot \frac{a}{2} \tan x = \frac{3a^2}{8 \cdot \sqrt{\sin \left(\frac{\alpha}{2} + 30^\circ\right) \sin \left(\frac{\alpha}{2} - 30^\circ\right)}}$$

**Item Analysis**

Item characteristics: Identified as free-response extended answer and typical question.

Topics coverage: Geometry – Three-dimensional Geometry, Polygons, Relation

Measurement – Area

Functions and Equations – function (trigonometric) and trigonometric identities

The item’s alignment analysis follows from the topic coverage; this item contributes one hit (item-category match) to the category of Geometry, one hit to Functions and Equations, and one hit to Numbers and Measurement, which concludes the categorical concurrence analysis.

The range of knowledge analysis identified that there is one topic (area) out of three from the
Number and Measurement category represented in this item, three out of five (three-dimensional shapes, polygons, congruence) for Geometry, and two topics out of two for Functions and Equations.

The Examination Alignment Analysis

The categorical concurrence analysis of the examination concluded that out of four items that constituted the examination, there were three hits for the Elementary analysis category (Elementary Analysis knowledge was required to answer three out of four questions), three hits for the Number and Measurement category, four hits for the Functions and Equations category, and four hits for the Geometry category.

The range of knowledge analysis for the examination resulted as follows.

- Elementary Analysis: 75% representation (three out of four topics of this category were present in at least one item);
- Geometry: 80% representation (four out of five topics of Geometry were present);
- Functions and Equations: 100% representation (both topics were present);
- Number and measurement: 33% representation (one out of three topics);
- Probability and Statistics: no representation.

Summary

This chapter established the methodology applied in this study as follows. The historical research methodology was applied to primary and secondary sources in researching the history of Albanian education in connection with the social and political standing of the country. Two high school teachers from the 1970s were interviewed as well. The mathematics national curriculum for both decades, the 1970s and 2006-2015, was described through the study of historical documents as mathematical-pedagogical in terms of the objects of study and historical
in terms of the research methodology. The curriculum framework in this research followed the Mathematics Framework of the TIMSS (Robitaille et al., 1993). The design of the examination analysis methodology for this study (general structure of the examinations, item analysis, and topic coverage) was guided by three approaches previously applied to analyze mathematics examinations. It consisted of a combination of elements from the examination analyses of Dossey’s (1996) “Examining Mathematics Examination,” Karp’s (2003) “Mathematics Examination: Russian Experiments,” and Karp’s (2007) “Exams in Algebra in Russia: Toward a History of High-stakes Testing.” The methodology used to analyze the alignment of the examinations’ topic coverage with the curriculum topics was based on the content focus analysis dimension of Webb’s (1997, 1999, 2002) methodology.
Chapter V

ANALYSIS

The National Curriculum During the 1970s

The school years of the 1970s in Albania started (as mentioned in Chapter II) with the implementation of the law “Further Revolutionization of School” approved on December 24, 1969 (Kambo, 2005). The entire content of the school (as mentioned in the Background) was permeated by the thread of the Marxist-Leninist ideology of the state party, and all teaching and educational work was built on the basis of three basic components: teaching, productive labor, and physical and military training (Musai, 2006). The mathematics programs and textbooks (which mirrored those of Soviet schools,) were part of this revolutionization as well. One interesting way of stressing the importance of learning mathematics pertaining to Albania of the 1970s was publishing the sayings of the communist leaders throughout the textbooks and on the school walls. For example, the Algebra 2 textbook for high school (Hoxha & Boshku, 1976) includes a saying from Karl Marx: “No science reaches perfection, if it is not able to use Mathematics” (p. 3) and Enver Hoxha’s (Albania’s dictator from 1944 to 1985) saying, “Mathematics has its own romanticism, its own momentum, always youthful, so connected with the young generation” (p. 43).

The basic education of the 1970s consisted of two cycles: the Elementary cycle (school classes I-IV) and the Secondary low cycle (school classes V-VIII). The General Secondary
Education, gymnasium, was 4 years and it was completed with the high school graduation examinations, called maturity examinations. The mathematics curriculum requirements for the 4 years of high school pertain to this study and the description of this curriculum is based on mathematics textbooks from the 1970s published by the Textbook Publishing House (Shtëpia Botuese e Librit Shkollor) which strictly followed the curriculum. The school was in session Monday through Saturday, 6 days per week, 4 to 6 hours per day, and the school year started on September 1st and ended at the beginning of June. The mathematics curriculum developed as follows: 9th grade, 5 hours per week, three Algebra and two Geometry; 10th grade, 2 hours Geometry and two Algebra; 11th grade, 3 hours Trigonometry and two Geometry (Stereometry); and 12th grade, 3 hours Analysis and two Analytical Geometry (Muka, 2016; Shehu, 2016). The curriculum for high school ranged from the set of real numbers, \( \mathbb{R} \), its subsets \( \mathbb{N}, \mathbb{Z}, \mathbb{Q} \), and operations with them to Differential and Integral Calculus (see Appendix E for more details). It was developed nationally following the pedagogical guidelines given by the Ministry of Education: stressing the importance of students’ deep and clear understanding of the theory of mathematics and its connection with practice; harmonizing mathematics with production work and military training; and using inter-subject links for an in-depth understanding of teaching materials. Equipping students with different habits and skills on mental calculations, as well as the ability to use different manuals and different instruments to make measurements and geometric constructions was also required. The following subsections describe the emphasized teaching strategies used to execute the curriculum; the use of active learning and problem solving as a tool to foster a deep understanding of the theory and developing critical skills; and teaching through real-life applications in order to accomplish the connection between theory and reality,
and so on. These strategies were recommended through the methodology books and embedded in national textbooks.

**The Teaching of Mathematical Content (Active Learning and Problem Solving)**

Bridging the students’ prior knowledge with the new concepts was emphasized in the teachers’ books. Guga (1976, p. 4) stated that the first stage of the lesson should awaken the intuition and prepare the ground for the new knowledge. The development of the lesson that has both an informative and a formative character—establishing a fair ratio between them in favor of the latter—was recommended to promote the independent activity of the pupils and enable them to learn how to learn (Sula, 1978, p. 9).

Throughout the textbooks, the sections on theory, exercises, problems, and application required a deep understanding of the concepts, not just a reproduction of algorithms. The teaching of limits starts by reviewing the functional and graphical representation of absolute value, the introduction of $\epsilon$ and $\delta$, infinitely small quantities and their properties. Then the definition of limit is introduced and questions like the following example are presented.

Example: (Bujari, Fundo, Kongoli, & Bahima, 1972, p. 38)

Determine the constants $a$ and $b$ such that

$$\lim_{x \to \pm \infty} \left( \frac{x^2+1}{x+1} - ax - b \right) = 0. \quad \text{(Answer: } a = 1, b = -1)$$

The chapter on the application of derivatives starts with a geometric (intuitive) proof of Lagrange’s Theorem and continues with its use in proving the sufficient conditions on the monotony of functions and the existence of the extremums of the functions in an interval. Geometry was taught at every grade of high school and was combined with Algebra, Analysis, and so on. The optimization problems using Geometry constituted the majority of applications of derivatives and integrals in the Analysis textbook and also in the books for teachers published by
the Institute of Pedagogical Studies, one example of which is the following: What should the radius of a straight cone with slant $a$ be so that its volume is at the maximum? (Bujari et al., 1972, p. 89) (Answer: $r = a\sqrt{6}/3$).

The problem-solving approach of teaching was recommended (through the teachers’ books), aiming at the consolidation of the introduced new knowledge and done through special chapters. The chapters covering the Algebra topics of equations, systems of equations of first degree and higher, second degree inequalities, system of inequalities, linear programing and its application in economics, and their algebraic and graphic representations, (Hoxha & Boshku, 1976) were followed by the chapter called “Discussion of Equations.” This chapter focused on analyzing different solution outcomes for equations/inequalities or a system of equations/inequalities with respect to their coefficients or a given parameter illustrated with real-life problem situations. The following example:

For what values of $m$ the equation $(m - 1)x^2 - 2mx + m + 3 = 0$ has two positive roots? (Hoxha & Boshku, 1976, p. 60)

requires a deep-level understanding to answer it. The students need to know that a quadratic equation $ax^2 + bx + c = 0$ has two positive roots if the following conditions: discriminant, $b^2 - 4ac > 0$, $\frac{c}{a} > 0$ and $-\frac{b}{a} > 0$ are all satisfied, to end up solving the system of inequalities

\[
\begin{cases}
-2m + 3 > 0 \\
(m + 3)(m - 1) > 0 \\
2m(m - 1) > 0
\end{cases}
\]

reaching the solution: $m \in (-\infty, -3) \cup \left(1, \frac{3}{2}\right]$.

The problem-solving strategy was implemented through the use of problems requiring a combined knowledge of different mathematical topics to answer as well. These type of problems
were recommended to be used during review sessions that aimed at the mastery of the content. An example of such a problem is the following:

Find the acute angle of a right triangle, if its sides form:

a) Arithmetic progression, and b) Geometric Progression. (Bujari & Dedej, 1979, p. 29)

In order to answer this question, the students need to recall knowledge from Geometry, infinite progression, Algebra, and Trigonometry, as the solution shows.

Solution: a) Let $a < b < c$ be the sides of the right triangle, then we have $b = \frac{a+c}{2}$. From the Pythagorean theorem we have $c^2 = b^2 + a^2$, and from trigonometry we have $\sin \alpha = \frac{a}{c}$.

\[
c^2 = b^2 + a^2 \implies a^2 + \left(\frac{a+c}{2}\right)^2 = c^2 \implies \cdots \implies 5 \left(\frac{a}{c}\right)^2 + 2 \left(\frac{a}{c}\right) - 3 = 0
\]

This implies that $5 \sin^2 \alpha + 2 \sin \alpha - 3 = 0 \implies \sin \alpha = \frac{3}{5} \implies \alpha = 36.89^\circ$

b) Let $a < b < c$ be the sides of the right triangle, then we have $b^2 = a \cdot c$, together with $c^2 = b^2 + a^2$ and $\sin \alpha = \frac{a}{c}$ we get $\sin \alpha = \sqrt{2} - 1 \implies \alpha = 24.47^\circ$.

Most of the textbook chapters were followed by historical knowledge sections, which included elements from the history of mathematics associated with the content being taught. The regular polygons chapter (Paparisto & Çami, 1973, p. 74) concluded with such a section in which the students learned that the 2,000 years of effort to construct regular polygons with 5, 9 or 11 sides ended in 1796 when (then 19-year-old) Gauss proved the Theorem “A regular n-gon can be constructed with compass and straightedge if $n$ is the product of a power of 2 and any number of the form $2^{2k} + 1$.” This was followed by exercises, including construction ones. Many sections include remarks that introduce the students to topics beyond the boundaries of the course curriculum. For example, the chapter on the functions of one variable introduces the multivariable functions using simple volume formulas of the cylinder and parallelepiped.
Teaching Through Real-Life Applications (Modeling)

The teaching of mathematics during the 1970s followed three basic components: teaching, productive labor, and physical and military training, as mentioned above. Moreover, the 1970s teachers’ books, prepared by the Institute of Pedagogical Studies, stressed the importance of equipping the students with the skills and strength needed to do calculations in writing and orally, and to use the different manuals and tables to solve real-life problems in production and military training (Sula, 1978). Balancing real-life problems with more abstract ones (generalization problems) was recommended. For example, Sula (1978, p. 6) suggested that the abstract form of the question: “The points A and B are on one side of the straight line d. Find the point M on the line d, such that the sum of the distances from A and B to M is the shortest” may be continued with the real-life application:

Let d be the shore of the river (consider it a straight line) from which two wheat plots, A and B, on one side of the river can get watering. Where should the motor pump be placed so that the least amount of tubing may be used?

The military training example was:

Between the first echelon and the rear one of a battalion there is a river. Where should the bridge be built so that the ammunition supply and withdrawal of wounded is done through the shortest path?

Real-life application examples were present in all exercise groups following each textbook section. One of the problems from the chapter “Discussion of Equations,” mentioned above, was the following:

A type of one unit of a commodity previously was brought from abroad. In the framework of the scientific-technical revolution, the workers and technical engineering
staff of an enterprise had managed to produce it on their own. The price of this commodity decreased by 30 leke [Albanian monetary unit] for each unit. So, now with $a$ leke, one can buy 4 units more than before. Find how many units of this commodity can be purchased now with $a$ leke? (Hoxha & Boshku, 1976, p. 62)

Solution: Let $x$ be the number of units that can be bought now, then $\frac{a}{x-4}$ the price of one unit when imported, and $\frac{a}{x}$ is the price of one unit now. Need to solve the equation $\frac{a}{x} = \frac{a}{x-4} - 30$. The students need to put restriction on $a$ such that the equation must have a natural number as its root. One can buy $2 + \sqrt{4 + \frac{2a}{15}}$ units, where $a = 7.5n^2 - 30$, (for $n = 2, 3, 4, \ldots$)

The connection of theory with practice was required throughout the curriculum as well and was implemented through field hours. An example of one practical work is shown below (Paparisto & Çami, 1973, p. 28).

The layout of a piece of land. The use of menzula or astrolabe and the similarity of polygons in drawing the layout (planimetry) of a piece of land.

The use of different manuals and tables was required because of the absence of calculators. The students were taught to use of logarithmic tables, as the following example shows.

Use the logarithmic tables to determine the following:

a) Find $\log 396,423$, b) Find $N$ if $\log N = 2.59737$ and c) Find $N$ if $\log N = 2.59737$

Solution: a) Round the number so only its first four digits are non-zero:

$396,423 \approx 396,400$. Since $10^5 < 396,423 < 10^6$ we have the characteristic of the log $396,423 = 5$. (Hoxha & Boshku, 1976, p. 90)
The mantis of \( \log 396,423 \) (i.e., the decimal part) is found in the table to be 0.59813.

<table>
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So \( \log 396,423 \approx 5.59813 \) (using today’s calculators we get \( \log 396,423 \approx 5.5981588 \))

b) Looking at the table, the mantis in \( \log N = 2.59737 \) is \( 59737 \) and belongs to the number 3957 and since the characteristic is 2, we get \( N = 395.7 \) (using today’s calculators we get \( \log 395,7 \approx 2.59736605 \)).

c) Looking at the table, the mantis in \( \log N = 2.59737 \) is \( 59737 \) and belongs to the number 3957, and since the characteristic is \( 2 \), \( N \) starts with two zero digits, so \( N = 0.03957 \) (using today’s calculators we get \( \log 0.03957 \approx -1.4026339 \). Note: \(-1.4026339 = -2 + 0.59737\).

The use of tables of approximated values of trigonometric functions is illustrated by the following example:

a) Find \( \cos 57^\circ \) and b) \( \sin 31^\circ 45' \).
Solutions: a) \( \cos 57^\circ = 0.5446 \), reading it directly from the table.

(Using today’s calculators we get \( \cos 57^\circ = 0.544639035 \)

b) \( \sin 31^\circ 45' \) is not read directly on the table, but we know

\[ \sin 31^\circ < \sin 31^\circ 45' < \sin 32^\circ. \] \( \sin 31^\circ = 0.5150 \) and \( \sin 32^\circ = 0.5299 \).

so as the angle increases from \( 31^\circ \) to \( 32^\circ \) the value of sine increases by

\[ \sin 32^\circ - \sin 31^\circ = 0.5299 - 0.5150 = 0.0149. \] 

So the increase for each minute (1 degree = 60 minutes) is \( \frac{0.0149}{60} \approx 0.000248 \) implying that the increase for 45’ is \( 0.000248 \times 45 = 0.0112 \), so \( \sin 31^\circ 45' = \sin 31^\circ + 0.0112 = 0.5262 \).

(Using today’s calculators we get \( \sin 31^\circ 45' = 0.52621392 \))

(Note: It the angle was measured in degrees, minutes and seconds, then the seconds are rounded to the nearest minute in order to use the table.) (Paparisto & Çami, 1973, p. 90)

**Assessment**

Assessment was done using oral and written questions (Muka, 2016; Shehu, 2016). For oral questions, the students would come to the board (in front of the class) to answer theoretical questions, demonstrate the proof of a theorem, explain a formula or definition, and do short-answer exercises. The written questions came from homework and exams. The grades ranged from 1 to 10, and the cut-off passing level was 45%, which is the same for examination assessment (Muka, 2016; Shehu, 2016).

**Analysis of Examinations From the 1970s**

The High School State Graduation Examination in Mathematics during the 1970s, called the maturity examination, took place at the school and was developed in two parts, written and oral, with equal weight on the final grade. The assessment committee consisted of three members and was chaired either by the school principal or the vice principal in charge of natural sciences. The committee members jointly decided on the grade for the written part of the examination.
The oral part of the examination took place 3 or 4 days after. The questions for the oral examination part were written on slips of papers, each of which had three questions—two theoretical and one exercise. These slips of papers were put in a basket and the students would draw one, prepare their answers for about 30 minutes, and then present it orally before the committee. The theoretical questions were mainly proofs of theorems or the explanation of a formula. The committee members had the right to ask extra questions if they were not sure of the grade they were assigning. The grade on the oral part was (as well) jointly agreed upon by the three committee members. Most of the time, a member of the examination committee was an observer from the Board of Education (Directory of Education) who did not have the right to participate in the assessment.

**General structure of the examination and item characteristics**

The general structure was similar across all of the 1970-1979 examinations. They all consisted of four written questions and three oral questions, except the 1979 examination which consisted of three written and three oral.

As Table 2 (which summarizes general structure) shows, most of the written questions were identified as free-response extended answers. The use of structured questions was rare during the first examinations, while for the last 2 years, all written questions were structured. The written examination lasted 4 hours. There was no oversupply of tasks, all questions were required, there were no multiple-choice questions, and all were identified as typical (similar to the textbook problems). The questions contained in the 1970s written examinations were answered by using multiple algorithms and, in almost all problems, illustration by a geometric figure or graph was required.
The standard composition of problems on the 1970s written examinations were:

1. Equations of circle, ellipse, parabola, hyperbola, their properties and their relations (usually one of the above and the equations of tangent lines to their graphs).

2. Analyzing the function’s behavior (1st and 2nd derivative tests) and graphing.
3. Optimization problems.

4. Analytical Geometry: Measurements, Stereometry, and Planimetry, which are solved by the use of Trigonometry and/or integration.

The oral part of the examination consisted of three questions—two theoretical, which required either discussion of a theorem or a formula, and an exercise, as mentioned above. These questions were identified as free-response short answers, some of which are the following (see Appendix B for more).

- Discussion about the first and second degree equation with one unknown.
- Theorems about the logarithm of the product, quotient, power and radical functions.
- The tangent at one point of a curve, the geometric meaning of the derivative.
- Similarity of the triangles. The three similarity cases. The similarity of right triangles.
- The definition of the maximum of a function. Corresponding theorems.
- Prove that \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \), etc. (Polovina, 1980)

**Topic coverage.** This subsection gives an overview of the topics on each written examination. There are 10 examinations in total, each consisted of four questions, except the last one in 1979 which consisted of three questions. Table 3 contains detailed information on the topic representation for each written examination analyzed.

For example, the number 3 in the highlighted cell belonging to the subtopic of perimeter/area on the 1974 examination indicates that three questions in that examination required knowledge of area and/or perimeter.
Table 3

Mathematics Topics Detailed, Categorical Concurrence, and Range of Knowledge for Each Examination, 1970-1979

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Categorical Concurrence (in percent) 50 50 50 50 50 75 50 50 50 33

Range of Knowledge (in percent) 50 50 50 50 75 75 50 75 50 50
Table 3 contains information on the examination’s categorical concurrence (the percentage of questions on the examination containing some topic from a particular category) and the examination’s range of knowledge (the percentage of a particular category’s topics present in the examination). For example, the highlighted cells belonging to the 1974 examination and the categorical concurrence for the category of number and measurement mean that 75% of the questions (three out of four) required this category’s knowledge. The 33% range of knowledge means that of three topics in the numbers and measurement category, only one was present in the 1974 examination. Table 3 shows only four out of five categories of the curriculum framework to which the examination analysis refers because the probability and statistics category was excluded here since it was not part of the 1970s curriculum. The last column of the table, the subtopic frequency, indicates the number of examinations having the subtopic, so number 10 in the cell belonging to the column of the subtopic of frequency and the subtopic of perimeter/area indicates that this subtopic was present in all 10 examinations.

**Subtopic representation.** The most emphasized subtopics across all 10 written examinations were those from functions. The subtopics domain and range, function properties, and functions in relation to equations were present in all examinations. This is shown in both categorical concurrence values and range of knowledge values: both are 100% for the category of functions and equations. The subtopics of equations with the highest representation were linear and trigonometric equations, followed by quadratic/polynomial equations and system of equations. The equations subtopics with no presentation in the written 1970s examinations were logarithmic & exponential equations, inequalities, system of inequalities, and expressions.

Differentiation was present in all examinations as well. Its subtopics (derivative as related to function variation, derivative as related to minima and maxima, higher degree derivatives)
were present on all 10 examinations, while the geometric concept of derivative and rules of
derivation were present on eight examinations. It is important to mention that differentiation was
assessed through applications, not through direct questions such as “Find the derivative of the
function . . .” Integration was assessed through applications as well on five out of 10
examinations (examinations from 1970 to 1973 did not assess integration) and the limit as
$x \rightarrow \infty$ was present on nine examinations.

The categorical congruence values for the geometry topics show that the percentage of
the questions of the examination containing some topic of geometry was 100% for eight
examinations and 75% for the first two. The range of knowledge values of more than 60% for all
examinations helped to differentiate among the geometry subtopics representation. The most
popular geometry subtopics during the 1970s were lines in two-dimensional space, triangle and
quadrilateral, three-dimensional shapes and surfaces, followed by lines and planes in three-
dimensional spaces. The least popular geometry subtopics were polygons other than triangle and
quadrilateral, vector operations, congruence and similarity. Either parabola or ellipse was
assessed on most examinations, and hyperbola was present on the 1972 examination only.

The only topic from the numbers and measurement category that was present on the
examinations was area/perimeter volume and angle; the other two (exponents/roots/radicals and
systematic counting) had no representation. The subtopics of arithmetic and geometric
sequences, limit as $x \rightarrow a$, and limit theorems had no representation either.

It is important to mention that the oral question banks (which were part of the
examination package) consisted of questions that covered curriculum topics with the least
presentation on the written part of the examination, i.e., the oral question bank was the
complement of the written examination topic coverage with respect to the intended national
curriculum topics. For example, items such as: The theorem of the logarithm of the product, quotient, power and the radicals of functions and Arithmetic/Geometric progression, determining the n-th term and the sum of its n terms, were part of the 1970s oral item bank, but not part of the written examination.

Alignment of the Examinations’ Topic Coverage
With the Curriculum During the 1970s

As described previously, the 1970s curriculum mainly consisted of topics from Algebra, Geometry (plane, space and relations), Trigonometry, Elementary Analysis, and Analytical Geometry (see Appendix … for more details). The curriculum development allocated 2 class hours per week to Geometry throughout the 4 years of high school. The curriculum algebra topics together with the trigonometry topics, which correspond to the category of functions and equations in the study’s curriculum framework, were allocated 2 class hours per week during the 9th and 10th grades and 3 hours per week during the 11th grade. The Elementary Analysis topics were taught during the last year of high school with 3 class hours per week. The fact that Geometry takes first place with respect to class time allocation, followed by the Functions and Equations category, was matched by the examinations topic coverage analysis, in which the functions and equations topics had the highest presentations throughout the 10 1970s examinations (all range of knowledge values were 100%), followed by Geometry with the next highest topics presentation.

As described in the methodology chapter, if a category is matched with at least two items, (50% of the examinations questions), the examination’s assessment for that particular category is reliable, while having sufficient alignment with respect to the range of knowledge, at least 50% of the topics within a category need to be measured by at least one assessment item. The results from the written examinations analysis shown in Table 3 (see section above) for categorical
concurrency and range of knowledge showed that for the all curriculum framework categories (Elementary Analysis, Geometry, Functions and Equations, and Number and Measurement), the assessment was reliable given that the categorical concurrency values were at least 50%, except for the 1979 examination. The 1979 written examination consisted of three questions, and the Elementary Analysis category was assessed by one question, resulting in 33% categorical congruence. The range of knowledge values for the Number and Measurement category was 33% for all 10 examinations, which methodologically resulted in insufficient alignment with respect to range of knowledge within this category. However, it is important to point out that the only topic of the Number and Measurement category which was part of the curriculum was Area/Perimeter, Volume and Angle, which was assessed by all examinations. The range of knowledge values for the other three categories were all at least 50%, suggesting there was sufficient alignment with respect to their breadth representation.

Another way to look at the alignment of the examinations’ assessment with the curriculum expectation is by comparing item characteristics with the way the curriculum was developed. The analysis of the examinations’ general structure showed that all examination questions were identified as identical questions, so all examination items were similar to the problems in the textbooks. The problem-solving approach of the high school mathematics curriculum development carried over to the graduation written examination items, most of which were identified as free-response extended answers (requiring a solution strategy), and the use of multiple algorithms was needed to answer them. The structured questions, which guided the students through the solution process, were present only in the last 2 years of the 1970s decade. At least one of the questions on the examinations was a real-life application, matching teaching
through a real-life application approach of curriculum development, and the theoretical understanding of the content was assessed by the oral part of the examination.

**The National Curriculum During 2006-2015**

Part of paving Albania’s path to becoming a European Union member was the National Strategy of Education (developed in cooperation with the World Bank), approved in 2004. The mathematics curriculum framework outlines (as mentioned in Chapter II) are: to possess the theoretical knowledge and concrete mathematical skills relevant to application in practical situations of everyday life and the study of similar disciplines; and to value mathematics as a whole, as a form of description and recognition of the reality and as part of human culture and human progress (MoES, 2014b).

The levels and years of schooling were restructured according to the practices of other member countries of the European Union and the Organization for Economic Cooperation and Development (Musai et al., 2006). While there has been continuous revision during the last 2 decades, the structure of the pre-university system that dominated the 2006-2015 school years was as follows. The basic education consisted of two cycles: Elementary cycle (school classes I-V), and Secondary low cycle (school classes VI-IX). The General Secondary Education, gymnasium, was 3 years and completed with the high school *matura* examinations.

The mathematics curriculum was revised during the last two decades as well, following the democratic educational reforms (as mentioned earlier) that were aimed to achieve international benchmarks in order to prepare students to travel abroad for higher education and for roles as citizens and workers in a democratic market-based economies cast within the wider context of European membership. The essential role of the learning of mathematics in meeting the goals of high school was stated in all curriculum objectives prepared by the Curriculum and
High school mathematics (as mentioned earlier) was developed in two programs: the core mathematics program and the advanced mathematics program. The knowledge and skills of the core curriculum program were essentials needed for a student who finished high school, regardless of his or her future career. The advanced mathematics program was a more advanced (broader and deeper) program, enriching the core curriculum knowledge needed to succeed in some careers; it was part of the high school curriculum choices. A student graduating from the advanced mathematics program took the advanced mathematics graduation examination (optional) and was bound (equipped with the mathematics knowledge needed) to continue higher education in STEM majors. The mathematics core curriculum is explored here.

In the first 2 years of high school (10th and 11th grade), the mathematics core curriculum was developed with 3 hours per week, total: 36 weeks x 3hrs/week = 108 hours per year. The 12th grade mathematics program was aimed at expanding and deepening the knowledge learned in 10th and 11th grade, closing the cycle of mathematical concepts and skills and preparing the students for the High School Graduation Examination. In the 12th grade, the mathematics core curriculum was developed with 4 hours per week, 34 weeks x 4 hours/week = 136 hours per year. About 16 hours were spent on preparation for the matriculation examination. The advanced curriculum was developed with 72 hours per year during 10th and 11th grade and 34 hours in the 12th grade. The high school core curriculum topics ranged from the set of real numbers, \( \mathbb{R} \), its subsets \( \mathbb{N}, \mathbb{Z}, \mathbb{Q} \), and operations with them to Differential and Integral Calculus, including
Geometry (form and relation), Discrete Mathematics, Combinatorics, Statistics and Probability (see Appendix F).

**Curriculum Objectives**

Mathematics programs, starting from the first grade primary school through high school, were unified and evolved along curriculum lines, not along textbook chapters. Each grade’s curriculum program was structured in lines (topics). For each topic, several objectives were designed which were included in the program. A line spread into several textbook chapters and certain chapters contain parts of several different lines. This distribution as well as their combination was conducted with the aim of reaching a whole conceptualization of the subject, applying one of the basic requirements of the mathematics programs (Lulja, Babamusta, & Bozdo, 2010, p. 5). In order to facilitate the teacher’s annual planning, in the teachers’ books (Lulja & Babamusta, 2012; Lulja, Babamusta, & Bozdo, 2011; Lulja et al., 2010), which were published with the approval of the Ministry of Education and Science, each topic’s objectives were designed in three levels: I (basic), II (average), and III (advanced), with respect to three main categories: mathematical reasoning, problem solving, and mathematical communication (level III contains level II, which contains level I). The basic level intended to be achieved by all required the students to carry out routine procedures that were often encountered in class; define concepts, rules, and main theorems; solve simple exercises imitating different models; reproduce some of the theoretical material; and use traditional methods of reasoning and problem solving. The average level required the students to undertake more complex task solution, combining the knowledge they possess. These students not only reproduced the material thoroughly, but also identified problems, distinguished between essential knowledge, drew their own conclusions, and demonstrated effective skills in communication and interaction. The advanced level aimed
not only to understand or reproduce the material, but also to process and apply it independently in new (unseen) situations. These students should be able to synthesize knowledge and skills, define pathways and modes of action, predict consequences, and evaluate different perspectives. The objectives for each level were illustrated with the following structured question with different levels recommended in the teachers’ book, and showed what the student should know (Lulja et al., 2011, p. 61):

(10th grade) The number of diagonals in a convex n-gon is \( \frac{n(n-3)}{2} \), where \( n > 3 \).

a) What is the number of diagonals of a hexagon? (Level I)

b) In which polygon the number of the diagonals equals the number of its sides? (Level II)

c) Is there a convex polygon with three diagonals? (Level II)

d) Prove that the number of diagonals in a convex n-gon is \( \frac{n(n-3)}{2} \), where \( n > 3 \). (Level III)

The guidelines for lesson planning emphasized the inclusion of the following principles (regardless of the format):

- The aim of the lesson in accordance with the subject’s (and line’s) objectives.
- Each lesson objective aims at a learning achievement.
- Lesson plan to be feasible.
- Learning activities to support the objectives.
- Plan enough time for each activity.

Lessons were partitioned into two major types: the elaboration of new knowledge and knowledge processing (including review lessons, curricular projects, etc.).
New Knowledge Introduction

The elaboration of a new knowledge lesson type was advised to follow the advocacy-implementation-reflection structure. The advocacy stage of the lesson included: recalling prior knowledge of the topic, student motivation for what will happen next, serving as a bridge between the students’ prior knowledge and the lesson’s new knowledge. In the implementation stage, the teacher guided the students towards learning through activities related to the understanding of new knowledge, while the students observed, experimented, and asked questions. During the reflection stage, the students expressed ideas, opinions, and content in their own words. Activities here were creative, analytical, generalist, and reflective, consolidating the new information. An example of the above structure is shown in the guidelines for the (11th grade) lesson:

*Angle between two straight lines given their equations* (Lulja et al., 2010, p. 45), with the basic learning objective:

*Students should be able to find the tangent of the acute angle between two lines given their equations in slope-intercept form and apply their knowledge in simple applications.*

The lesson’s method was: *The use of Trigonometry in solving Geometry problems*. The advocacy stage consisted of vector multiplication, expressing vectors with coordinates that enable the determination of the cosine of the angle between two vectors and the properties of the angles between respective pairs of parallel lines. The implementation stage (which should follow from the structured questions) consisted of finding the cosine and tangent of the acute angle between two lines with equations $y = m_1x + b_1$ and $y = m_2x + b_2$. The lesson continued with the students working in groups on exercises based on their level.
Basic level – Find the angle between the lines $y = \sqrt{3}x + 7$ and $y = -\sqrt{3}x - 4$.

Average level – Given the lines $d_1: y = mx - 3$, $d_2: y = 2x + 3$, $d_3: y = -x - 1$ and $d_4: y = 4x + 1$. The angle between $d_1$ and $d_2$ is equal to the angle between $d_3$ and $d_4$. Find $m$.

Advanced level – Work on exercises with line equations not in slope intercept form and may be asked to independently derive the formula $\tan^2 \varphi = \left(\frac{m_2-m_1}{1+m_1m_2}\right)^2$ (Lulja et al., 2014, Exercise 4, p. 12).

The results of the independent work of students or groups should be discussed and analyzed with the class. An example of a recommended application is the following (Lulja et al., 2010, p. 46):

In a right isosceles triangle, the coordinates of one of the acute angle vertexes are $(5, 7)$ and the equation of the leg across from it is $6x + 4y - 9 = 0$. Find the equations of the other two sides of the triangle.

**Knowledge Processing**

The last line (topic) in each year’s curriculum was called *Mathematical processes* and it was recommended to be integrated into other topics. This approach of consolidating the knowledge aimed to fulfill similar mathematical practice standards in mathematics Common Core State Standards, which are part of each unit, and stated the following:

- Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning.

(corestandards.org)
This curriculum line developed through the Knowledge processing lesson type and set the expectations for the students to become able to communicate mathematically, to judge, argue, reason, prove, and solve problems.

1) The ability to communicate mathematically – Explain orally and in writing, practical activities, assumptions and solution process, use correct mathematical symbols, interpret information from simple geometric two- and three-dimensional shapes and information from graphs, charts, and diagrams.

2) The ability to judge, argue, reason and prove – The ability to use some basic rules of logic, reasoning, argument, justify a conclusion, and judge the authenticity of a given result by the application of known formulas or with the use of technology. Prove simple theorems on all topics and the use of counterexample.

3) The ability to solve problems – Solve a problem using different ways. Mathematicize and solve the problem situations (not complex), with (or without) the help of technology, simulated or real-life examples from the other sciences.

Throughout the high school mathematics textbooks, many sections had a block of exercises called “Exercises for Knowledge Processing,” followed by other blocks of the section exercises. In the “Exercises for Knowledge Processing” block, step-by-step solutions were given for at least one exercise that stressed the understanding of fundamental concepts covered in the section. The example below followed the section on the composition function derivative.

A spherical balloon is being filled with air. At the moment when its radius is 10 cm, the radius is increasing at rate of 0.2 cm/second. At what rate is the area of the balloon increasing at that same moment? (Bozdo, Lulja, & Babamusta, 2014, p. 61)

Solution: We know that the area of the balloon is given by the formula $S = 4\pi r^2$. At the given moment of $t_1$ we have $r'(t_1) = 0.2 \text{ cm/s}$. The rate of the increase of the area with
respect to $r$ is $S'(r) = 8\pi r \Rightarrow S'(r_1) = 8\pi \cdot 10 \approx 251 \text{ cm}^2$. Since we want to find the rate of change of the area with respect to time, not knowing the relation between them, or the relation between $r$ and $t$, we use the chain rule of derivation on the composition function $S(r(t))$ because we know the $r'(t_1) = 0.2 \text{ cm/s}$. So we get: $S'(t_1) = S'(r_1) \cdot r'(t_1) = 251 \cdot 0.2 \approx 50.2 \text{ cm}^2/\text{s}.$

Requirements present in the high school curriculum that followed every topic were: (a) to be familiar with some elements from the history of mathematics associated with the content being taught, and (b) to model mathematically and solve problematic situations, with or without the help of technology, simulated or real-life examples applied to other sciences. The ability to explain how mathematical discoveries came about as the result of real-life phenomena and enriched the students’ mathematical culture with clear information on the evolution of mathematics over the years. The use of technology in teaching and learning mathematics was part of the curriculum throughout the high school years as well. The ability to implement mathematical knowledge was gained during the high school years to solve problems and analyze phenomena from Physics, Economics, Chemistry, Biology, Social Sciences, Health, and so on.

Assessment

During the curriculum development, three types of evaluations—diagnostic, formative, and summative—were recommended. The summative assessment, which measures the required standards, was recommended to be designed in a way that the number of points coming from basic-level questions would be sufficient to pass (Lulja et al., 2010, 2011, 2012) with a recommended passing limit of 25% (Lulja et al., 2011, p. 63). The evaluation should be based on answers to oral questions on the board (20%), participation and group work (curriculum projects) (20%), homework (10%), and written tests (50%) (tests at the end of chapter, semester and year).
In order to have alignment between teaching and assessment, the use of questions structured in the same way as the structure of the information given during lessons was strongly recommended. The following example illustrates the recommended structured questions:

(11th grade) Given points \( A = (2, 2), B = (1, 1) \) and \( C = (3, 5) \), answer the following:

a) Show that they are not collinear.

b) Find the distance between points A and B.

c) Find the equation of \( \overline{AB} \) (the line passing through points A and B).

d) Find the distance of point C from \( \overline{AB} \).

e) Find the area of the triangle ABC. (Compare with the unstructured question: Find the area of the triangle ABC.)

f) Find the equation of the line that point C crosses when the area of the triangle ABC is 10 square units. (Lulja et al, 2011, p. 60)

Short-answer written and oral questions which asked for a determined, exact answer and for more than a simple approval or routine memory, were recommended as well. The use of multiple-choice questions was advised to be present in tests.

As mentioned earlier, the preparation for the Matura Examination consumed about 12% of the class time. It was comprised of a thorough review of the curriculum developed during the 3 years of high school, paying more attention to knowledge of the following concepts and skills:

1. Real numbers, real exponents, roots and logarithms.

2. Algebraic manipulations of expressions with variables (main formulas).

3. The congruence and similarity of triangles.

4. Area of plane figures.

5. Function and their graphs.
6. Number sentences.
7. Solving inequalities of first and second degree.
8. Solving equation of first and second degree,
10. Solution of systems inequalities.
11. Definition and main properties of trigonometric functions of the acute angle (right triangle trigonometry) and of any angle on the unit circle.
12. Main trigonometric formulas (identities), the laws of sine and cosine.
13. Operation with vectors, addition, subtraction, and product.
14. The equation of the line in the plan.
15. The area and volume of three-dimensional objects.
16. The Theorem of three perpendiculars.
17. The second degree curves (parabola, hyperbola, circle and ellipse) properties, shape, equations, and their tangent lines.
18. The limit of a function at a point or at infinity, the limit rules, indeterminate forms and finding horizontal and vertical asymptotes.
19. The derivative of a function and derivation rules, the geometric meaning of the derivative.
20. The study of the monotony of function and its concavity.
21. The domain of a function, finding the extremes of a continuous function.
22. Indefinite integral.
23. Definite integral.
25. The probability of an event, exclusive events, independent events.

**Analysis of 2006-2015 Examinations**

The High School State Graduation Examinations in Mathematics during 2006-2015, known as the state *Matura*, had two purposes: it served as the high school graduation as well as the higher education entrance examination, and was developed in writing only. The assessment was done with respect to the evaluation scheme prepared by the National Agency of the Examinations (AKP), by trained evaluators in assigned evaluation centers. The evaluation scheme contained detailed information on scores pertaining to each question or part of the question. The multiple-choice questions were worth one point each, the open-ended questions were worth two or three points depending on the length of the answer, and the structured questions were assigned points to each of their parts. The examination’s total score was 50. The examination papers were coded, the assessment for each examination was done by two independent graders, and the reconciliation of the two evaluations was completed at the end of each evaluation day. In cases of discrepancies in evaluation between the two evaluators, a third evaluator was assigned to carry out the final assessment and the results were announced no later than 15 days from the examination day.

**General Structure of the Examinations and Item Characteristics**

The 2006-2015 examinations took place in examination centers which accommodated at least 50 students (usually district-wide), on the same day across the country. The examination started at 10 a.m. and the available time for it was 2 hours, 30 minutes. All examinations consisted of a total of 25 written questions. There was no oversupply of tasks, and all questions were required. Thirteen of the 25 were multiple-choice questions, and 12 were workout problems.
Table 4 summarizes the information about the examinations’ general structure. All of the examination questions were identified as typical questions (similar to textbook problems). The 12 open-ended questions on each examination were identified as free-response extended/short-answer or structured questions and, in general, more than one mathematical concept was assessed in one open-ended question. As Table 4 shows, the general structure was steady through all 10 examinations; there was a slight variation with respect to the number of free-response short or extended answers.

Table 4

*General Structure of the 2006-2015 Examinations*

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**Topic coverage.** This subsection gives an overview of the topics in each written examination during 2006-2015. There were 10 examinations in total, each comprised of 25 questions. The multiple-choice questions consisted of solving equations/inequalities, systems of equations, finding the value of a definite integral, finding the derivative of a polynomial function, set operation, volume and areas, the concept of a function, and so on, in general assessing one mathematical concept. In all 10 examinations, one of the multiple-choice questions assessed the concept of the set, to find either the union or the intersection of given two sets. The 2007 examination contained one question that assessed the polar coordinates of a complex number. Neither set operation nor polar coordinates are included in Table 5 which contains detailed information on the topics’ representation for each examination analyzed.

Table 5 shows the five categories, their topics and subtopics, of the curriculum framework to which the examination analysis. The last column in the table gives the number of the examinations (out of 10) assessing a particular subtopic. The table contains information on the examination’s categorical concurrence (the percentage of the questions of the examination containing some topic from a particular category) and the examination’s range of knowledge (the percentage of a particular category topic present in the examination). For example, the number 3 in highlighted cells belonging to the 2011 examination and categorical concurrence for the category of number and measurement means that 12% of the questions (three out of 25) required this category’s knowledge. The 100% range of knowledge means that of the three topics in the Number and Measurement category, all three were present in the 2011 examination.
Table 5

Mathematics Topics Detailed, Categorical Concurrence, and Range of Knowledge for Each Examination, 2006-2015

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Table 5 (continued)

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**Subtopic representation.** The most emphasized subtopic from the Number and Measurement category was perimeter/area and the least emphasized was angle measurement. The other topics, integer or rational exponents, were assessed by at least one question, as were tree diagrams and combinations, resulting in 100% values for the range of knowledge for this category.

Geometry topics were present in all examinations and assessed by 20-40% of the items in the examination. The most popular geometry subtopics were lines and triangle/quadrilaterals, assessed by all examinations with up to 20% of the questions, followed by circle (4-12% of the questions on 9 examinations). All except the 2006 examination assessed shapes, surfaces, lines, and planes in three-dimensional geometry with one question (4% of the examination). Parabola or ellipse were assessed on eight examinations while hyperbola was assessed on three examinations. The subtopic of other polygons (other than triangle/quadrilaterals) was assessed on the 2009 examination only. Either congruence or similarity was assessed on five examinations (2006, 2007, 2008, 2009, and 2013) by one question, and the vector operation was on eight examinations.

The function and equation category, with 100% of the topics, was present in all 2006-2015 examinations. The function’s properties and domain and range were assessed by at least one question on all examinations, while function in relation to equations was on seven of them.
The subtopics of equations and inequality that were present in all examinations were polynomial and quadratic equations (in up to six questions, 24% of the examination); radical equations and rational equations (in up to 20%, 5 questions); linear equations and inequalities (up to 16%, 4 questions); and logarithmic and exponential equations (up to 12%, 3 questions). Expressions and trigonometry equations and identities were present in nine examinations, with up to three questions (12%). Systems of equations were present in eight examinations, with up to two questions, while systems of inequalities were present in six examinations, with one question.

Systematic counting, data representation (central tendency), and numerical probability were present in most of the examinations. The three 4s and seven 8s in the categorical concurrence row that belong to the Probability and Statistics category indicate that this category was assessed by at most two questions out of the 25 on each examination.

The Elementary Analysis category was assessed by all 2006-2015 examinations with the following order of subtopic representation. Geometric concept of derivative/rules of derivation had the highest representation, assessed by all examinations by up to four questions. Antiderivative was assessed by all examinations (up to 2 questions) as was the application of integrals (areas) by one question. Definite integral and derivative in relation to function variation was assessed by nine of 10 examinations, by one or two questions, and so was limit as \( x \to a \). Arithmetic sequences, limit theorems, derivatives as related to minima and maxima, and higher-order derivatives showed up on seven, six, five, and two examinations, respectively, while geometric sequence and limit as \( x \to \infty \) were on one examination only.

Alignment of the Examinations’ Topic Coverage With the Curriculum During 2006-2015

The high school mathematics core curriculum during 2006-2015 included topics from the set of real numbers, \( \mathbb{R} \), its subsets \( \mathbb{N}, \mathbb{Z}, \mathbb{Q} \), and operations with them to Differential and Integral
Calculus, Geometry (form and relation), Discrete Mathematics, Combinatorics, Statistics and Probability. The curriculum developed through a structured design on three levels: I (basic), II (average), and III (advanced) of topic objectives with respect to three main categories: mathematical reasoning, problem solving, and mathematical communication (level III contains level II, which contains level I). The basic level was intended to be achieved by all, and requires the students to carry out routine procedures that are often encountered in class; define concepts, rules, and main theorems; solve simple exercises imitating different models; reproduce some of the theoretical material; and use traditional methods of reasoning and problem solving. The time devoted to preparing students for the graduation examination was 12% of the class time during the last year of high school (16 hours). The review exercises (present in the textbooks) repeated many times were very similar to those on the actual examinations. All questions on all the examinations were identified as typical questions.

The study’s alignment methodology stated that for the 2006-2015 examinations, consisting of 25 questions in total, if a category is matched with at least five items (20% of the examinations questions), the examination’s assessment for that particular category is reliable, while, to have sufficient alignment with respect to range of knowledge, at least 50% of the topics within a category need to be measured by at least one assessment item. The results from the written examinations analysis shown in Table 5 (see section above) for categorical concurrence and range of knowledge indicated that for the curriculum framework categories Elementary Analysis, Geometry (form and relation), and Functions and Equations, the assessment was reliable since the categorical concurrence values were at least 20%. Functions and Equations topics had the highest presentation, followed by Geometry topics in second place and Elementary Analysis in third. Number and Measurement category concurrence values indicated that six of
the 10 examinations had reliable representation, while the other four assessed it by four or fewer questions. The Probability and Statistics category (which was developed with 44 hours of class time—the least compared with the other categories) had the least presentation assessed by at most two questions, resulting in a non-reliable assessment for this category.

The range of knowledge values for all categories were at least 50%, suggesting that there was sufficient alignment with respect to their breadth representation. All topics from the categories of Functions and Equations and Number of Measurement were assessed by at least one question on all 10 examinations, resulting in range of knowledge values of 100%. Elementary Analysis had the next highest breadth representation, followed by the Geometry and Probability and Statistics topics.

**Comparing Subtopic Representation Between the 1970s Examinations and 2006-2015 Examinations**

As mentioned earlier, the most emphasized topic in both sets of examinations was functions and equations. The categorical congruence (the percentage of questions assessing at least one the topics) was 100% for the 1970s and 52-64% for the 2006-2015 examinations, with 100% range of knowledge (all topics were assessed by the examination) for both periods. While the functions subtopics were all assessed, this was not the case for the equations subtopics. The 1970s examinations assessed only 55% of the equations subtopics (expressions, inequalities, system of inequalities, and logarithmic/exponential equation had no representation), whereas the 2006-2015 examinations assessed all of them. The equations subtopics during the 1970s were used to solve application problems, i.e., they were steps of some multistep solution. There were no direct questions such as “solve the equation . . . ” or “evaluate the expression . . . ” during 1970s, which occurred often in the 2006-2015 examinations.
The categorical congruence (the percentage of questions assessing at least one of the topics) for Elementary Analysis was 50% for the majority (eight out of 10) of the 1970s examinations and 20-32% for the 2006-2015 examinations, with 50-75% range of knowledge (the percentage of topics assessed by the examination) for the 1970s examinations and 75-100% for the 2006-2015 examinations. The elementary analysis subtopics, also during the 1970s, were assessed as part of a multistep solution, not through direct questions and the differentiation subtopics had uniform representation in the examination. The most emphasized differentiation subtopic in the 2006-2015 examinations was the geometric concept of derivative/rules of derivation and the least was the higher-order derivatives. Limit as \( x \to \infty \) (asymptotes) was more popular on the 1970s examinations: nine out of 10 examinations assessed limit as \( x \to \infty \) and one examination assessed limit as \( x \to a \). During 2006-2015 questions to evaluate limits as \( x \to a \) were present in all examinations, while limit as \( x \to \infty \) (asymptotes) was rarely assessed.

Geometry had a categorical congruence of 20-36% on the 2006-2015 examinations and 75-100% on the 1970s examinations, with range of knowledge 80-100% and 60-80%, respectively; this means that more geometry topics were assessed during 2006-2015. During the 1970s, vectors had no representation, which was not the case for the 2006-2015 examinations. The most popular geometry topics were triangle/quadrilateral and lines, which remained popular during 2006-2015 as well. While circle and hyperbola were assessed by more examinations in 2006-2015 as compared to the 1970s, the representation of parabola and/or ellipse and three-dimensional geometry subtopics did not change much.

The subtopics of exponents/roots/radicals and systematic counting were not present on the 1970s written examinations, but present on all 2006-2015 examinations. Probability and statistics was assessed by most of the 2006-2015 examination, but it was not present on the
1970s written examinations. This broader subtopic representation came as a result of the increase in the number of questions.

The written examinations of 2006-2015 had more questions than the 1970s, 25 versus 4, respectively. This increased the possibility that the 2006-2015 examinations assessed a broader subtopic range as compared to the 1970s, and more possible ways to assess the same subtopic. The proportion of questions requiring one to two steps was 60-76% on the 2006-2015 examinations, while during 1970s it was 43%. The three oral questions of the 1970s examinations were considered free-response short-answer, which, as mentioned earlier in this chapter, assessed the topics with the least representation in the written part of the examinations.

The following examples describe the difference between the ways assessments took place during the two periods as well. The question below was given on the 1974 examination which assessed the following:

- Functions in relation to equations.
- Linear, quadratic and rational equation.
- Systems of equations.
- Lines (tangent), parabola and hyperbola.
- Geometric concept of derivative/ rules of derivation.
- Derivative as related to function variation and higher order derivatives.
- Limit as $x \to \infty$, asymptotes, and limit as $x \to a$.
- Rectangular area.
- Definite Integral and Applications of Integrals (areas), Antiderivatives.
Given functions $P$: $y = 2x^2 + x$ and $H$: $y = \frac{x}{1+2x}$

a) Examine their behavior and graph the graphs of both functions $P$ and $H$, respectively.

b) Determine the coordinates of the intersection point for $P$ and $H$ and prove the $P$ and $H$ have the same tangent line at the origin $(0, 0)$.

c) Find the area of the figure surrounded by x-axis, the part of the parabola, $P$, in the first quadrant and the line $y = 3$.

**Solution**

a) Analyze the variation and graph the function $P$: $y = 2x^2 + x$

1) This function is a polynomial, so is continuous on $(-\infty, \infty)$.

2) $y' = 4x + 1 \implies y' = 0$ for $x = -\frac{1}{4}$

   $y' < 0$ for $x < -\frac{1}{4} \implies$ the function is decreasing on $(-\infty, -\frac{1}{4})$

   $y' > 0$ for $x > -\frac{1}{4} \implies$ the function is increasing on $(-\frac{1}{4}, \infty)$

3) $y'' = 4 > 0 \implies$ the graph $P$ is concave up.

   The graph of $P$ has no inflection points.

4) When $x \to \pm \infty, y \to +\infty$, per $x = 0, y = 0$

   $y = 0$ for $x = 0$ and $x = -\frac{1}{2}$

5) The graph $P$ has no asymptotes

Analyze the variation and graph the function $H$: $y = \frac{x}{1+2x}$

1) This function is continuous on $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$.

2) $y' = \frac{1}{(1+2x)^2} \implies y' \neq 0$ for $x \in \left[\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)\right]$

   No maximum or minimum values

   $y' > 0$ for $x \in \left[\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)\right] \implies$ the function is always increasing on its domain

3) $y'' = -\frac{4}{(1+2x)^3}$

   $y'' > 0$ for $x < -\frac{1}{2} \implies$ the function is concave up on $(-\infty, -\frac{1}{2})$

   $y'' < 0$ for $x > -\frac{1}{2} \implies$ the function is concave down on $(-\frac{1}{2}, \infty)$

   The graph of $P$ has no inflection points.
4) When \( x \to \pm \infty, y \to \frac{1}{2} \), per \( x = 0, y = 0 \),

because \( \lim_{x \to \pm \infty} \left[ \frac{x}{1+2x} \right] = \frac{1}{2} \)

When \( x \to -\frac{1}{2}, y \to \pm \infty \), per \( x = 0, y = 0 \), since

\( \lim_{x \to -\frac{1}{2}} \left[ \frac{x}{1+2x} \right] = \infty \) and \( \lim_{x \to \frac{1}{2}} \left[ \frac{x}{1+2x} \right] = -\infty \)

Per \( y = 0 \) for \( x = 0 \)

5) The graph of \( H \) has horizontal asymptote the line \( y = \frac{1}{2} \) and vertical asymptote \( x = -\frac{1}{2} \).

b) To find the intersection points of \( P \) and \( H \) we need to solve the system of equations:

\[
\begin{align*}
y &= 2x^2 + x \\
y &= \frac{x}{1+2x}
\end{align*}
\]

\( \Rightarrow \)

\( x_1 = 0 \) and \( y_1 = 0 \)

So their intersection points are: \( O = (0, 0) \) and \( A = (-1, 1) \).
The tangent line to the parabola, $P$, at $(0,0)$ has slope of $y'(0) = 4 \cdot 0 + 1 = 1$, and equation $y = x$.

The tangent line to the graph $H$, at $(0,0)$ has slope of $y'(0) = \frac{1}{(1+2 \cdot 0)^2} = 1$, and equation $y = x$.

So graphs $P$ and $H$ have the same tangent line at $(0,0)$.

c) To find the intersection points of parabola $P$ and the line $y=3$ are the solutions of the system of equations:

$$\begin{align*}
y &= 2x^2 + x \\
y &= 3
\end{align*}$$

$\Rightarrow$ \(\begin{cases} x_1 = 1 \\
y_1 = 3 \end{cases} \quad \text{and} \quad \begin{cases} x_2 = -\frac{2}{3} \\
y_2 = 3 \end{cases}$$

So their intersection points are: $B = (1,3)$ and $C = (-\frac{2}{3},3)$.

So the shaded area is:

$$S = 1 \cdot 3 - \int_{0}^{1} (2x^2 + x) \, dx = 3 - \frac{2}{3} x^3 + \frac{1}{2} x^2 \bigg|_{0}^{1} = 1 \frac{5}{6}$$

The following questions were given on the 2014 examination, and the definite integral and antiderivative were assessed in both.

1. Find the value of $\int_{-1}^{1} x \, dx$ (assessing definite integral, antiderivative)

2. Find the area of the figure surrounded by the graph of $y = \sqrt{2x}$ and $y = x$.

(Assessing system of equations, definite integral and applications of integrals (areas), antiderivatives, triangle area)

The following items are from the 2009 examination and both assessed circle.

1. Find the equation of the circle centered at $(1,6)$ and is tangent with the line with equation $4x - 3y - 1 = 0$. (assessing circle and lines)

2. The graph of the equation $9x^2 + 4y^2 = 36$ represents a

A. *parabola*  
B. *ellipse*  
C. *circle* (assessing circle)  
D. *hyperbola*
The broader range of topic representation came from direct questions like these below, given on the 2012 examination.

1. The value of $\sqrt[3]{27}$ is:
   A. 2
   B. 4
   C. 8 \leftarrow \text{(assessing radical expressions)}
   D. 16

2. The inequality $3x - 2 > x + 4$ is equivalent to:
   A. $x > 3$ \leftarrow \text{(assessing linear inequalities)}
   B. $x < 3$
   C. $x \geq 6$
   D. $x \geq 2$

3. The number of the teams with 4 players out of 6 people is:
   A. 30
   B. 20
   C. 15 \leftarrow \text{(assessing combinations)}
   D. 10

Albania’s Social and Political Standing Represented Through Mathematical Problems and Examinations

The researcher could not find evidence of politicized mathematical problems in the 2006-2015 examinations or in the textbooks analyzed in this study. The questions were more abstract (more mathematically pure) and the applications used in problems were from physics, biology, business, and similar fields. However, this was not the case during the communist regime. The atmosphere of the country’s political standing during the communist regime penetrated mathematics education through the use of different terminologies in mathematics problems and exercises as well. Indeed, the mathematics problems were used as a tool for managing social perceptions. The following is part of an arithmetic and geometric progression lesson, implying that theories coming from religious people are inhumane and Marxism-Leninism is the way to go. In Albania during the communist regime, religious practices were forbidden by law and beautifully printed expressions like: “Religion is opium for the people” or “Glory to Marxism-Leninism” decorated the classroom walls.
Priest Malthus tried to exploit the knowledge of mathematical progressions to argue his reactionary theory according to which the population grows in geometric progression, 1, 2, 4, 8, 16, …, while the material goods grow in arithmetic progression, 1, 2, 3, 4, 5, …, he stated this to make the workers believe that their poverty comes from unquestionable laws of nature, and that natural disasters are needed to reset the proportion. This false and inhuman theory of Priest Malthus was exposed by the classics of Marxism-Leninism which showed that the development of the population does not depend on the laws of nature but on the laws of social development. (Hoxha & Boshku, 1976, p. 112)

Mathematical problems did not escape the communist party’s propaganda that the productivity of goods and services and the well-being of the country were always on the rise. The problems below are from the Algebra 2 textbook (Hoxha & Boshku, 1976):

In 1970 an agricultural cooperative received 600 quintals of corn from the high yield plots. In 1975 the number of these plots tripled and the productivity increased by 25 quintals / ha … (p. 10).

In 1950 we had 129 medical doctors. This number increased with about 31.5 each year till 1960 and in 1965 we had 900 medical doctors. Answer the following … (p. 105)

The regime’s doctrine that the country was under constant threat from the rest of the world (military training was, after all, one of the three basic components of education) infiltrated the mathematics problems using military terminology. Problems such as the ones below were common:

A military tent in the shape of a regular rectangular pyramid is to be built with side length \( a \) … (High School State Mathematics Examination 1976).

Between the first echelon and the rear one of a battalion there is a river. Where should the bridge be built, so that the ammunition supply and withdrawal of wounded is done through the shortest path? (Sula, 1978, p. 6)

The shell (projectile) with which our coastal forces hit an enemy plane flew vertically upwards. During the first second it flew 300m, each second after that flew 10m less. What was the height of enemy plane, if it was crushed after 6 seconds? (Hoxha & Boshku, 1976, p. 109)
To make a military suit, the model shown in the figure is used. Given that the dimensions are given in centimeters, find the area of the model (Baxhaku et al, 1978, p. 33).

This figure taken from the Geometry 2 textbook (Boxhaku et al., 1978) was part of the axial symmetry lesson. The daily life motto during the communist regime was “With Pickaxe and Rifle we build and defend socialism.”

The evolution of the general structure of the high school state examinations and their administration from the 1970s to the 2006-2015 describes in some ways the social and economic development of the country. The number of high school graduates during the 1970s was fairly evenly distribution across the country, allowing for the possibility that the examinations would be administered in the high schools (where everyone knew everyone), with the allocation of a longer time and the use of oral questions. During 2006-2015, the population was concentrated in the big cities, which called for more concentrated management. The examination administration took place in big centers, assigned by the Ministry of Education, under the surveillance of trained proctors, and assessment was done by trained assessors as well. Students would have to register
for the examination and travel to the closest center. The 1970s written examination consisted of only four open-ended questions (format: one single page; stamped papers for the solutions were distributed as needed), while the 2006-2015 examinations consisted of 25 questions in total, 13 multiple-choice and 12 open-ended questions (format: many pages with a barcode, with space for a solution after every question). This change could be attributed to society’s change in attitudes towards the democratic view to make matters more convenient for the examinees, the European direction of national education as a whole, and the development of technology.

Summary

This study’s analysis showed that students’ deep and clear understanding of the theory of mathematics and its connection with practice was an important part of curriculum development in both periods. Among the emphasized teaching strategies used to execute the curriculum during the 1970s were the harmonization of mathematics with production work and military training, the use of inter-subject links for an in-depth understanding, the equipment of students with different habits and skills of mental calculations, and the ability to use various manuals and instruments (because of the absence of the calculators). During the 2006-2015 period, the use of technology in teaching and learning of mathematics was prioritized, and teaching strategies emphasized the importance of enabling the students to implement mathematical knowledge gained during their high school years to solve problems and analyze phenomena from Physics, Economics, Chemistry, Biology, Social Sciences, and other fields.

The graduation examination format and item characteristics evolved as well. While the 1970s examinations consisted of four open-ended written questions and three oral questions, and lasted for 5 hours, the 2006-2015 examinations were 2.5 hours long, written only, with 13 multiple-choice and 12 open-ended questions. All questions were compulsory. Even though the
number of questions on the 2006-2015 examinations increased compared to the 1970s, the proportion of items requiring one or two steps grew, whereas the proportion of those requiring three or four steps decreased, and the examinee’s different possible solution options declined. Another finding was that in all examinations (both periods), all questions could be answered by routine procedures using learned algorithms. Even though some items required thoughtful translations and applications of principles and procedures, similar questions (typical to those on the examinations) were present in contemporary textbooks.

There was central control over examination administration and content in both periods. The graduation examinations were unified; one unique examination was prepared each year by the Ministry of Education (or other institutions) and distributed across the country, administered on the same day and at the same time nationally. Guidelines concerning examination assessment were distributed during both periods as well. During the 1970s, schools had more freedom in evaluating their students’ knowledge, compared to the 2006-2015 period. This freedom resulted from the fact that the evaluation took place in schools and was done by the school’s faculty members; moreover, the oral part of the examination allowed the examiners to have another confirmation of their assessment. By contrast, during the 2006-2015 period, assessment took place in areas selected by the regional school board in appropriate and safe conditions by professional examiners.

In both periods, the alignment of the examinations’ topic coverage with the intended curriculum was considered to be sufficient, according to the study’s methodology. The changes in the examinations’ topic coverage between the 1970s and 2006-2015 mirrored the curriculum changes. The change in the social and political atmosphere between the periods also found its way into the mathematics curriculum, mathematical problems, and examinations.
Chapter VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The study contributes to a better understanding of the history of Albanian education, the history of Albania in general, and the history of Albanian mathematics education in particular. The study provided a historical analysis of the evolution of high school state graduation examinations in mathematics in Albania, emphasizing their general structure, their item (question) characteristics, and their topic coverage. The analysis was done for two decades in two different historical periods—in two different centuries, in fact: the 1970s, when Albania was under the communist regime and isolated from the rest of the world, and 2006-2015, when democratic Albania was open (and welcoming) to western ways. The examinations of these two time periods were analyzed and compared. The national curriculum expectations in mathematics for high school graduates, as set by the Ministry of Education, were also described and compared. The alignment of examination topic coverage with the high school mathematics national curriculum was explored as well. The historical context of the evolution of these examinations, the Albanian history of education in general, mathematics education in particular, and the history of assessment was presented in the background chapter.

This study offered an opportunity to explore connections among changes in education, mathematics education and mathematics examinations, and the political and social life of the
country. The background chapter made clear that the evolution of Albanian education is closely connected with the country’s history. The roots of an Albanian education system sprung from the efforts of the Albanian Renaissance (mid-19th century-1912) leaders to spread education with a national character among all Albanians, in their own language and as a necessity of national survival.

The drastic political change that arrived when the communists gained power over the country (1944-1992) was reflected in the rapid expansion of the national system of education from preschool up to higher education under a strict political orientation. This research showed that in Albania, as in other communist countries (according to the literature), education was developed under a Soviet style, provided uniformly to all students, and used as a tool to manage social perceptions to construct and maintain the communist regime.

The historical analysis given in the Background chapter also showed that Albanian education reforms during the post-communist transition, which aimed to transform Albania’s educational system into a European one, underwent four stages: deconstruction (the denying of the old regime, decreased public interest for education); stabilization (defining the new legislative framework); restructuring (relaunching public interest through coherent education policy), and the counter-reform, a systematic reform strategy against residual communism, still continues. This is in line with Birzea’s (1995) description of the post-communist educational reforms in Romania.

This study described how the purpose of Albanian education changed from ensuring the education of the younger generation with Marxist-Leninist ideology during communism to preparing students to succeed at national and global levels in the post-communist, democratic era. As the country is working on becoming an EU member, educational reforms now aim to
provide democratic education and prepare youth as citizens and workers in a democratic market-based society, to be cast in the wider context of European membership rather than a narrow focus on national citizenship. This change in the purpose of the Albanian education in response to the change in the country’s social and political standing harmonizes with Dewey’s (1934) statement: “Any education is, in its forms and methods, an outgrowth of the needs of the society in which it exists”.

The analysis chapter showed that Albanian mathematics education was not immune to political and social change: both its curriculum and assessment were affected. Examination administration, format, topic coverage, and item characteristics, even the pure mathematical problems, represented in some ways the Albanian social, economic, and political views of the time. The inflexibility of the examination, represented in longer required time (4 hours) and distribution of grades over fewer questions, during the 1970s represented the rigidity of the political system, whereas the 25-item examination of 2006-2015, with its 13 multiple-choice questions and requiring less time, represents the democratic system, which tends to be less rigid and muchfairer. The national curriculum during the 1970s developed with one program only and offered one graduate examination; during 2006-2015, the national curriculum developed with two programs, core and advanced, followed by two options for graduation examinations. This is a representation of the fact that in more rigid societies, especially where communism reigns, individualities are less recognized. These observations appeared to be in line with findings of Karp (2003, 2006), Fried, (2011), and Perdala (2011), who suggested that somehow the political and the ideological standing of a country finds its way into mathematics education and into mathematics examinations.
One of the item characteristics used to analyze the examination questions in this research was the number of steps required to answer it. Considering that items requiring one to two steps are more straightforward questions that address a specific concept, the ways the examinee can approach the solution is somehow dictated by the discourse in the question; by contrast, in assessing mathematical concepts through items requiring a multistep solution, the trajectory of how to approach a solution is not necessary dictated. This research showed that the proportion of questions on the examination requiring one or two steps was higher (60-76%) for the 2006-2015 examinations than it was for the 1970s (43%).

Morgan and Sfard’s (2016) research found that the percentage of tasks requiring one or two steps grew over time, whereas the percentage of those requiring three or more steps decreased and the examinee’s freedom in deciding about the problem-solving trajectory declined over time; thus, a general trend is evident here. Morgan and Sfard investigated the evolution of high-stakes examinations taken by students in England at the end of compulsory schooling during the last three decades, using examinations from 1980, 1987, 1991, 1995, 1999, 2004, 2010, and 2011.

The analysis of examination items with respect to type of question, typical if similar problems are present in the contemporary textbooks or non-typical otherwise, showed that all examinations, during both periods, consisted of typical questions. Questions very similar to those on the examinations were present in the textbooks, and extensive reviews for the examination were part of the teaching programs. Therefore, even though some of the problems required thoughtful translations and applications of principles and procedures, students had practiced with similar questions. This supports Karp’s (2007) conclusion that methods used in the examinations
against solutions requiring fixed patterns were taught in the classrooms and became fixed patterns that teachers taught their students.

The alignment of the examination topic coverage with the high school mathematics curriculum was found to be sufficient during both periods—a conclusion similar to Britten and Raizen’s (1996) results that the examinations’ topic coverage of 1991 and 1992 in mathematics, biology, chemistry, and physics taken by college-bound students in England, Wales, Germany, Israel, Japan, Sweden, and the United States was generally aligned with the high school curricula.

One interesting finding of this study’s analysis was that during the 1970s, the schools had more freedom in evaluating their students’ knowledge, compared to the 2006-2015 period, and the examination administration had become more centralized in democratic Albania than during the communist regime. This freedom resulted from the fact that the evaluation took place in schools, conducted by the school’s faculty members, and the oral part of the examination allowed the evaluators to have further confirmation of their assessment. During the 2006-2015 period (aiming to have a more independent examination evaluation), the assessment took place in areas selected by regional school boards in appropriate and safe conditions by professional examiners.

Conclusions

Answers to the Research Questions

The answers for the research questions which guided this study are given below.

1. How did the Albanian formal education system evolve over time from its origination in late 19th century to today’s national system of education?
The Albanian system of education reflects the country’s struggle to survive: it has an early tradition and is closely connected with the entire history of the country. The first known original Albanian alphabet was created in 1824, the first Abetare (Albanian language book for the first grade) was written in 1844, the first official Albanian school was opened in 1887, the Society of Albanian Learning was established in 1888, and the first Albanian Normal (pedagogical) School was opened in 1909. All of these were achievements of the Albanian National Renaissance, a nationwide political movement during the 19th century. The efforts to create an education with national character among all Albanians were interrupted by the Ottoman invaders, and even though Albania declared independence in 1912, the volatile political situation that accompanied Albania for the next 25 years did not foster a steady development of an Albanian system of education.

Albania was declared a constitutional monarchy in 1928 under King Zogu. During this time, the best known education reform was the Ivanaj reform of the 1930s, which marked the beginning of the unification of Albanian education in all categories of schools. During this period, the quality of the education provided was thought more important than spreading education all over the country. Zogu’s regime was interrupted by World War II and the country's occupation by fascist Italy in 1939. The war ended in 1944 in Albania, with communists gaining absolute power.

The communist regime’s educational reform law (adopted in 1946) described the character of the new school as the school of the masses and for the masses, associated with the entire life of the country: “school out of politics” was amiss. During this period, the spread of education all over the country was thought more important than the quality of education. The movement wiped out illiteracy and brought Albanian education and pedagogy in line with Soviet
conceptions. With the implementation of the 1969 law, “Further Revolutionization of School,” school politicization reached extreme limits. All teaching and educational work was built on three basic components: teaching, productive labor, and physical and military training.

Communism in Albania broke down in the early 1990s. Under the new (democratic) political system, the national education system underwent a cycle of reforms which were all aided by many foreign western organizations, including the World Bank. The current Albanian National Strategy of Education follows that of the European Union countries.

2. What were the national curriculum expectations in mathematics for high school graduates as set by the Ministry of Education during the 1970s and 2006-2015?

The mathematics curriculum for high school for both periods was unified nationally, and ranged from the set of real numbers, $\mathbb{R}$, its subsets $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$, and operations with them to differential and integral calculus; it was also developed nationally following the pedagogical guidelines given by the Ministry of Education. Elements from the history of mathematics associated with the content being taught were part of the curriculum for both periods. Data representation, uncertainty, and probability were part of the required curriculum during 2006-2015, not during the 1970s.

During the 1970s, the high school mathematics curriculum was developed as one unique program through the 4 years of high school, regardless of the careers that graduates would follow; during 2006-2015, the curriculum developed through two programs: a core mathematics program (which pertained to this study) and an advanced mathematics program, depending on the student’s future career path. As described in Chapter V, the emphasized teaching strategies used to execute the curriculum for both periods were the use of active learning and problem solving as a tool to foster a deep understanding of the theory and developing critical skills and
teaching through real-life applications in order to accomplish the connection between the theory of mathematics and real life. During the 1970s, “real life” meant production work and military training, and the importance of mental calculations, the ability to use different manuals and instruments to make measurements and geometric constructions. By contrast, during 2006-2015, “real life” meant the use of technology and mathematical modeling to solve simulated or real-life examples applied to other sciences.

3. How did the High School State Examinations evolve over time in terms of general structure, topic coverage, and item characteristics?

The examination analysis of this study was applied to 20 High School State Examinations papers, 10 of which were administered during the 1970s and 10 during the 2006-2015 period. The analysis shows that while the examinations were very similar with respect to their general structure, topic coverage, and item characteristics within each period, significant differences were present. The 4-hour-long 1970s written examination consisting of four free-response questions, and evolved to a 2½-hour-long examination consisting of 25 written questions (13 multiple-choice and 12 open-ended questions) during 2006-2015. The 1970s examinations had an oral part, while the 2006-2015 examination did not. In all examinations (both periods), there was no oversupply of tasks (all questions were compulsory), and all questions were identified as typical (could be answered by routine procedures using learned algorithms). The proportion of structured written questions increased during the 1970s (to 100% in 1978 and 1979) and was around 16% on the 2006-2015 examinations. The proportion of items requiring one or two steps was higher on the 2006-2015 examinations than on the 1970s examinations. Two graduation examinations were available during 2006-2015 period (the core curriculum and advanced) while only one was available during the 1970s.
4. To what extent were the High School State Examinations in Mathematics aligned with curriculum expectations for high school graduates during the 1970s and 2006-2015?

As described in the Methodology section, if a category is matched with at least two items for the 1970s examinations and at least five items for the 2006-2015 period, the examination’s assessment for that particular category would be considered reliable. For the alignment to be considered sufficient with respect to the range of knowledge, at least 50% of the topics within a category need to be measured by at least one assessment item. The alignment analysis for both periods resulted in a sufficient alignment of the topic coverage of the examinations with the intended curriculum, with the following exceptions. The 1979 written examination consisted of three questions and an Elementary Analysis category was assessed by one question, resulting in 33% categorical congruence. The range of knowledge values for the Number and Measurement category were 33% for all 10 1970s examinations (the Number and Measurement category as part of the curriculum was Area/Perimeter, Volume and Angle, which was assessed by all examinations). The Probability and Statistics category (which was part of the 2006-2015 curriculum only and developed with 44 hours of class time—the least compared with the other categories) had the least presentation assessed by at most two questions, resulting in a non-reliable assessment for this category. Topics with the highest class time allocation during curriculum development were matched by high representations in the examinations topic coverage.

**Limitations of This Study and Recommendations for Further Studies**

One limitation of this study was the difficulty of obtaining original documents on the administration of the examinations as well as curriculum documents from the Ministry of
Education for the 1970s period. The 1970s curriculum description was obtained from textbooks of the time and teachers’ books, while the examination administration information came from interviews from two 1970s high school mathematics teachers. It would be very interesting to continue this study with more original documents from the Ministry of Education. Another limitation related to the alignment methodology. Webb’s alignment analysis, on which this study’s methodology was based, determines the number of hits for a standard as the average number of hits to a standard that each reviewer codes, which accounts for the variability on determining the item-standard match between different reviewers. This study involved only one coder, so the number of hits was based on the researcher’s judgment and reasoning only.

This research may be continued in many different ways. One could explore more examinations during a longer and continuous period of time, not focusing on only two different decades, investigating what happens over time and if there will be a cycle, i.e., if the 1970s style of mathematical assessment will come back. It would be interesting to interview more teachers and talk about educational practices in general and mathematics education practices in particular. Getting student’s perspective on what they think about the examinations by interviewing students from 1970s and from 2006 – 2015 school year would be interesting as well. A more compelling research study would be to explore the evolution of mathematics education not separately but in connection with other school subjects such as Physics, Chemistry, History, or Literature.
REFERENCES


Mercanaj, M. (2010). *The reforms in mathematics education for grades 1 through 12 in Albania from 1945 to 2000*. Dissertation Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy under the Executive Committee of the Graduate School of Arts and Sciences, Columbia University.


Musai, B., Gjermani, L., Bushati, A., & Sula, L. (2006). Education in Albania—National dossier. Project developed by Center For Democratic Education (CDE) and supported by UNICEF. Tiranë, Albania.


APPENDICIES

Appendix A – The 1970s Written Examinations (Polovina, 1980)

People's Socialist Republic of Albania
Ministry of Education and Culture
Department of secondary education

Maturity examinations in secondary schools of general education

Mathematics
(1969-1970 school year)

Primary Exam

1. Solve the equation
\[
\frac{1}{\cos x} + 1 = \sin(\pi - x) - \cos x \tan\frac{\pi + x}{2}
\]

2. For the given function \( y = -x^3 + (m + 1)x - m \) where \( x \) is independent of the parameter \( m \),

a) Determine the value of the parameter \( m \), such that the graph of the function passes through the point \((-1, -4)\).
b) For \( m \) equal the value found in part a) graph the function and examine its behavior.
c) Find the equation of the tangent line to the graph at the point with \( x = 1 \).

3. On the half-circle with diameter \( AB = 2R \) a segment \( CD \) is drawn parallel with the diameter \( AB \). Where should the segment \( CD \) be drawn such that the area of the triangle \( ACD \) is at the maximum?

4. The diagonal of a right-angled parallelepiped with height \( h = 17.77cm \) forms the angle \( \alpha = 51^\circ 25' \) with the base and the angle \( \beta = 32^\circ 17' \) with the side. Find its volume.
Primary Exam

1. For the given function \( y = x(x^2 + mx + 4) \) where \( x \) is independent of the parameter \( m \),

   d) Determine the value of the parameter \( m \), such that the graph of the function passes through the point \((1, 1)\).

   e) For \( m \) equal the value found in part a) graph the function and examine its behavior

2. Find the simplest equation of the ellipse and the coordinates of its foci, \((c, 0)\) and \((-c, 0)\), knowing that half of the length of the minor axis is \( b = 4 \) and \( \frac{a}{c} = \frac{\sqrt{21}}{5} \), where \( a \) is half of the major axis. Write the equation of the tangent lines to the ellipse and perpendicular to the line with the equation \( x - 2y - 2 = 0 \).

3. A straight angled parallelepiped with square base will be constructed from an iron rod with length 12 m. (The rod will serve as its sides.) what should the base side and its height in order for the volume to be at the maximum?

4. The base of the pyramid is an equilateral triangle. Two of its sides are perpendicular to the base, while the third one forms the angle \( \alpha \) with the base. Its height is \( h \). Find the area of its sides and the volume, if \( h = 8.85 \text{ cm} \) and \( \alpha = 46^\circ 10' \).
Maturity examinations in secondary schools of general education

Mathematics
(1971-1972 school year)

Primary Exam

1. For the given function \( y = (x - 1)^2 (x + p) \) determine the value of the parameter \( p \) such that the slope of the tangent line to the graph of the function at the point of intersection with \( y \)-axis is \(-3\). Graph the function using the value of \( p \) you found, and examine its behavior.

2. Find the equation of the hyperbola that passes through the foci of the ellipse \( \frac{x^2}{169} + \frac{y^2}{144} = 1 \). The foci of the hyperbola are the intersection points of the ellipse and \( x \)-axis. Find the equations of the tangent lines to the hyperbola and parallel to \( 5x + 13y - 18 = 0 \).

3. In a sector of a circle with radius \( R \) and 90° central angle is inscribed a rectangle with one vertex at the center of the circle. Find the dimensions of the largest such rectangle.

4. The base of the pyramid is a trapezoid in which the two sides are equal with its small base. Its larger base is \( a \) and the smaller angle is \( \alpha \). All sides form the angle \( \beta \) with the base plane. Find the volume of the pyramid.
Maturity examinations in secondary schools of general education

Mathematics
(1972-1973 school year)

Primary Exam

1. Given functions \( y = x(x^2 - 6x + n) \). Find the value of the parameter \( n \) such that the tangent line to the graph at \( x = 3 \) is parallel to the x-axis. Graph the function using the value of \( n \) you found and examine its behavior.

2. Find the equation of the parabola with vertex the origin, \((0, 0)\), symmetric with respect to x-axis and with its focus in the center of the circle \( x^2 + 2x + y^2 = 7 \). Write the equations of the tangent lines to the parabola passing through the intersection point of parabola’s directrix and the line \( 3x - y - 3 = 0 \).

3. An electric substation, A, is build on one side of a 92 meters wide river. On the other side of the river there is a factory, B, 750 meters away from the projection of the substation across the river, C. (Assume the sides of the river are parallel.) How should the cable be extended through the river and land, in order for the factory to get electricity from the substation so that the cost will be at the minimum, knowing that the water cable costs $100/meter and the land cable costs $60/meter?

4. A circle with radius \( R \) is inscribed in a cone in such a way that the radius of the sphere tangent to the cone forms an \( \alpha \) angle with the plane of the big circle of the sphere which is parallel with the base of the cone. Find the volume of the cone.
Primary Exam

1. Given functions $P: y = 2x^2 + x$ and $H: y = \frac{x}{1+2x}$.
   d) Examine their behavior and graph the graphs of both functions $P$ and $H$ respectively.
   e) Determine the coordinates of the intersection point for $P$ and $H$ and prove the $P$ and $H$ have the same tangent line at the origin $(0, 0)$.
   f) Find the area of the figure surrounded by x-axis, the part of the parabola, $P$, in the first quadrant and the line $y = 3$.

2. Find the equations for the sides of the triangle ABC, knowing the coordinates of the vertex $A = (2, -7)$, equation of the height of the triangle from vertex $B: 3x + y + 11 = 0$ and the equation of the line passing through vertex $C$ and the midpoint of $AB$, $C: x + 2y + 7 = 0$.

3. In the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is inscribed an isosceles triangle with one vertex at point $B = (0, b)$. Find the coordinates of the other vertexes of the triangle that has the largest area. (The area of the isosceles triangle should be expressed in terms of the y-coordinate of the vertexes.)

4. The height of a triangular regular pyramid is $h$. through one of the base sides a perpendicular plane with the pyramid’s side across is drawn. The angle of triangle formed from the cut of the plane with the pyramid is $2\alpha$. Find the volume of the pyramid.
Maturity examinations in secondary schools of general education

Mathematics
(1974-1975 school year)

Primary Exam, May 26, 1975

1. Given the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.
   
   g) Find the coordinates of the point A on the ellipse, in the first quadrant, such that the tangent to the ellipse at A is parallel to $x + 2y = 0$.
   
   h) Write the equation of the parabola, symmetric with respect to y-axis, with vertex at the origin and passing through A.
   
   i) Find the area of the figure surrounded by the parabola and the line that passes through the origin and point A.

2. Given the function $y = \frac{ax - 1}{x^2}$, find the parameter a such that the tangent line to the function at $x = 1$ is parallel to x-axis. Then using the value of the parameter a, examine the behavior of the function and graph it.

3. A tree trunk in the form of a right circular cone has a height of 180 dm and diameters for the bases are 12 dm and 6 dm. A beam with a rectangular parallelepiped shape with square base is to be cut from the above trunk in such way that the axis of the trunk complies with the central axis of the beam. Find the height of the beam with the largest volume.

4. A regular pyramid with a triangular base with side a, the angle between two sides is $\alpha$. Find the area of the side of the pyramid.
1. Find the equation of the ellipse with foci the intersection points of the circle $x^2 + y^2 + 2y = 8$ and x-axis, passing through the center of the circle. Find the equations of the tangent lines to the ellipse which are parallel with the line passing through the center of the circle and the left most corner of the vertex.

2. Given the function $y = \frac{p}{1-x^2}$, find the parameter p such that the tangent line to the function at $x = \frac{1}{2}$ and has slope $k = \frac{16}{9}$. Then using the value of the parameter p, examine the behavior of the function and graph it.

3. A military tent in the shape of a regular rectangular pyramid is to be built with side length a. What should the length of the base side and the height of the pyramid be so that the volume of the pyramid be the largest possible. Find the volume of the air the pyramid contains and the amount of oilcloth needed to build the tent. (Do not consider the amount of oilcloth that get lost during the cutting.)

4. In a regular right circular cone two segments are drawn connecting two points of the perimeter of the base with the cone’s vertex. These two segments form the angle $\alpha$ between them. The plan determined by these two segments form an angle $\beta$ with the base and triangular cutting with area Q. Find the volume of the cone.
1. Given the function $y = -x^2(x + a)$
   a) Find the parameter $a$ such that the tangent line to the function at $x = -1$ is parallel to the line $3x - y + 1 = 0$.
   b) Then using the value of the parameter $a$, examine the behavior of the function and graph it.
   c) Find the area of the figure surrounded by x-axis, the lines $x = -2$ and $x = 0$, and the graph of the function.

2. An ellipse, the axis of which comply with the coordinative axis is tangent with two lines: $x - 4y - 10 = 0$ and $x + y - 5 = 0$ at the points M and N.
   a) Find the equation of the ellipse
   b) The area of the triangle whose vertices are the points M and N and the right most point of the ellipse.

3. The base of a pyramid is the triangle ABC, in which the angle between AB an AC is $\alpha$ and $AB = AC = a$. The pyramid side SBC is perpendicular with the base of the pyramid, while the sides SAB and SAC form with the plan of the base the angle $\varphi$. Find the area of the sides of the pyramid.

4. From reservoir A to the village B a straight irrigation channel is to be build. At one point, P, the channel will branch out to reach the village C, which is located in the perpendicular BC with the channel AB. $AB = a$ and $BC = b$. The main channel, AP, is wider than its branches PB and PC. Knowing that one meter of PB costs $\frac{2}{3}$ of one meter of AP, and one meter of PC costs $\frac{3}{4}$ of one meter of AP, where should the branching point P be in order for the cost of building the whole channel to be at the minimum?
Primary Exam

1. Given the function \( y = (2 + ax - x^2)(x + 2) \)
   d) Find the parameter \( a \) such that the tangent line to the function at \( x = 0 \) is parallel to x-axis.
e) Then using the value of the parameter \( a \), examine the behavior of the function and graph it.
f) Find the area of the figure surrounded by x-axis, the lines \( x = 2 \) and \( x = 0 \), and the graph of the function.

2. Given the points \( A = \left( \frac{1}{2}, 2 \right) \) and \( P = (-2, 0) \).
c) Find the equation of the parabola with axis of symmetry x-axis, vertex at the origin and passing through A.
d) Find the equations of the tangent lines to the parabola passing through P, \( PT_1 \) and \( PT_2 \).
e) Find the equation of the circle tangent to \( PT_1 \) and \( PT_2 \) at the points they touch parabola.

3. In a right circular cone with slant height \( a \), which forms angle \( \alpha \) with the base plane is inscribed the pyramid SABC, with base the right triangle ABC. The side SAC of the pyramid forms angle \( \beta \) (\( \beta > \alpha \)).
a) Find the volume of the pyramid.
b) Find the tangent of the angle between the side SBC and the base plane.

4. From a metal flat sheet shaped as a ring region defined by two concentric circles with radius a and b, a sector with central angle \( x \) is cut along the radiuses used to form a funnel.
a) Find the volume of the funnel.
b) For what value of \( x \) this volume is the largest and what is the largest volume?
Primary Exam

1. Given the parabola \( y = -\frac{1}{4}x^2 + 1 \). Let A and B be the intersection points of the parabola with x-axis, with A having negative x-coordinate.
   g) Find the area of the figure surrounded by the arc \( AB \) of the parabola and x-axis.
   h) The trapezoid ABCD is inscribed in the figure described in a). Taking as an input variable the x-coordinate of the point C, determine analytically the function \( S(x) \), the area of the trapezoid ABCD. For what values of \( x \), \( S(x) \) is defined in our case.
   i) Graph \( S(x) \) on its domain. What is the value of \( x \) that maximizes \( S(x) \)? What is the geometric meaning of the maximum value?

2. A mechanism consists of two rods \( OA = OB = 2 \text{ dm} \), connected in fluidtight manner at point A. The end O of the mechanism is at the origin, while B moves along x-axis and A moves on the coordinate system plane.
   a) Prove that the midpoint M of the segment AB moves along the ellipse \( \frac{x^2}{9} + y^2 = 1 \).
   b) Find the perimeter of the rectangle formed by the tangent lines to the ellipse at the points where the ellipse intersects the lines \( y = x \) and \( y = -x \).

3. For a pyramid with triangular base we know that two angles of the base are \( \alpha \) and \( \beta \), its height is \( h \) and the sides of the pyramid for same angle \( \gamma \) with the base.
   c) Find the projections of the sides of the pyramid.
   d) Find the volume of the pyramid.

1) Second degree equation in one variable, finding the formula for the roots, the discriminant and the properties of the roots.

2) The theorem of the logarithm of the product, quotient, power and the radicals of functions.

3) Arithmetic progression, determining the n-th term and the sum of its n terms.

4) Geometric progression, determining the n-th term and the sum of its n terms.

5) The study of the second degree trinomial sing

6) Prove that \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \)

7) The continuous function

8) The definition of the derivative. Use the definition of the derivative to evaluate the derivative of the function \( y = \sin x \)

9) The geometric meaning of the derivative

10) The derivative of the functions \( y = uv, y = \frac{u}{v} \) and \( y = \frac{a}{v} \), where u and v are functions and a is constant.

11) The derivative of the function \( y = u^m \), u is a function.

12) Monotone functions. What happens to the derivative of an increasing, decreasing or constant function in a given interval? What can you say about a function if its derivative is always positive, negative or zero in a given interval?

13) The definition of the maximum of a function. Corresponding theorems.

14) The definition of the minimum of a function. Corresponding theorems.

15) Similarity of the triangles. The three similarity cases. The similarity of right triangles.

16) Matric relations on the right triangles. Euclid and Pythagoras’s Theorems.

17) The meaning of the area and its measure. The area of a parallelogram, triangle and trapezoid.

18) The direct and inverse theorem of the three perpendiculars
19) The understanding of the volume, pyramid’s volume with triangle base and otherwise

20) The side area of the cylinder and the cone

21) The volume of the cylinder and the cone

22) The definition of the sine function of any angle, its representation on the trigonometric circle and its variation. The solution of the equation \( \sin x = a \).

23) The definition of the cosine function of any angle, its representation on the trigonometric circle and its variation. The solution of the equation \( \cos x = b \).

24) The definition of the tangent function of any angle, its representation on the trigonometric circle and its variation. The solution of the equation \( \tan x = c \).

25) The relationship between the trigonometric functions of: a) angles with difference 180°, b) complementary angles, c) supplementary angles and ç) opposite angles.
The State Matura Exam

SESSION I

(Required)

Friday, June 23, 2006

Time 10:00 to 12:30

Subject: Mathematics - General High School

Instructions for students

The test has 25 requests in total.
Thirteen are multiple-choice questions, for which you will surround the letter that represent the correct answer.
The others are work out questions with space available for you to show the solution and reasoning.
Also at the end of the test is left space for you to perform the operations when answering the questions.
The available time for the exam is 2 hours and 30 minutes.
Points are given for each question.

For use by the evaluation committee

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Points in Total: ____________________

The evaluation commission

1. ..................................................Member

2. ..................................................Member

MATHEMATICS - Session I
Part 1

Questions 1 through 13 are multiple-choice, for which you will surround the letter that represent the correct answer.

4. Given \( A = \{2, 3, 4\} \) and \( B = [1, 5] \), the number of elements in \( A \cap B \) is:

   1 point _______
   
   A. zero
   B. one
   C. two
   D. three

5. The value of \( \frac{3^5}{3^7} \) is equal to:

   1 point _______
   
   E. \( 3^{-2} \)
   F. \( 3^2 \)
   G. \( 3^{12} \)
   H. \( 3^{35} \)

6. Which one of the following inequalities is equivalent to \(-3x \geq 6\):

   1 point _______
   
   A. \( x \geq -2 \)
   B. \( x \geq 2 \)
   C. \( x \leq -2 \)
   D. \( x < -2 \)

7. The solution to the system \( \begin{cases} y = x^2 \\ x + y = 6 \end{cases} \) is:

   1 point _______
   
   A. \( (1,1) \)
   B. \( (2,4) \)
   C. \( (0,0) \)
   D. \( (4,2) \)

8. The product of the real roots of the equation \( x^2 - 5x + 6 = 0 \) is:

   1 point _______
   
   E. \(-5\)
   F. \(5\)
   G. \(6\)
   H. \(30\)
9. Which is the solution set to the inequality \( x^2 - 4x + 3 < 0 \):  
   A. \((-\infty, 1)\)  
   B. \((3, \infty)\)  
   C. \((-\infty, \infty)\)  
   D. \((1,3)\)  

10. The midpoint of the line segment AB, where \( A = (3, 5) \) and \( B = (7, 11) \) is the point:  
   A. \((5,8)\)  
   B. \((7,6)\)  
   C. \((0,0)\)  
   D. \((10,16)\)  

11. Find the exact value of the expression \( \sin^2 110^\circ + \cos^2 110^\circ \).  
   A. \(-2\)  
   B. \(-1\)  
   C. 0  
   D. 1  

12. The graph of the function \( y = \sqrt{x - 3} \) passes through the point:  
   E. \((4,-1)\)  
   F. \((4,1)\)  
   G. \((3,1)\)  
   H. \((0,0)\)  

13. The derivative of the function \( y = x^3 + 3x^2 + 3x - 4 \) at \( x = 1 \) is:  
   A. 1  
   B. 10  
   C. 12  
   D. 16  

14. If the volume of the cube is \( 8 \text{ m}^3 \) then the area in \( \text{m}^2 \) of one side is:  
   A. 2  
   B. 4  
   C. 8  
   D. 64
15. If for every \( x \in \mathbb{R} \) we have \( f(x) = x^2 - 5x \), then \( f(-x) \) is:

- E. \( x^2 - 5x \)
- F. \( x^2 + 5x \)
- G. \(-x^2 + 5x\)
- H. \(-x^2 - 5x\)

16. The definite integral \( \int_{0}^{1} 3x^2 \, dx \) is equal to:

- E. 0
- F. 1
- G. 3
- H. 6

Part 2

Questions 14 through 25 are work out questions with space available for you to show the solution and reasoning.

17. Given the inequality \( (x - 2)(x^2 + 2x + 1) > 0 \)

a) Check if \( x = 3 \) makes the inequality true

1 point _______

b) Find the solution set

2 points_______

18. Determine the interval for which the function \( y = \log\sqrt{x - 1} + \log\sqrt{x + 1} \) is defined.

2 points_______

19. Given the points with coordinates: \( A = (-3, -4) \), \( B = (3, 4) \) and \( C = (5, 0) \)

a) Prove that the points do not lie on the same straight line

2 points_______

b) Prove that the angle \( \angle ACB \) is a straight angle.

2 points_______
20. Two squares with side 2cm and 5cm are positioned as shown in the figure. Find the area of the shaded triangle.  
3 points

21. Given the function \( f(x) = \begin{cases} kx^2 & x \leq 1 \\ 5x - 1 & x > 1 \end{cases} \), find the value of \( k \) for which the function is continuous at \( x = 1 \).  
2 points

22. Find the limit  
   a) \( \lim_{x \to 4} \frac{x-3}{x^2-2x} \)  
   1 point
   b) \( \lim_{x \to 0} \frac{2 \sin x - \sin 2x}{x^3} \)  
   3 points

23. Given the function \( y = 2x^2 - 4x - 5, \ x \in \mathbb{R} \)  
   a) In what interval is the function increasing and in what interval is the function decreasing?  
   1 point
   b) Does the function have any inflection points?  
   1 point
   c) Find the x-coordinate for the points of the graph such that the tangent to the graph at these points passes through \( M = (0,7) \).  
   2 points

24. Given the function \( y = x + \frac{4}{x} \)  
   a) Find the minimum value for the function in \((0,5)\).  
   2 points
   b) Find (if there exist) the vertical and horizontal asymptotes for the graph of the function.  
   2 points

25. Given the sentence \( y_n = \frac{n-1}{n}, \ n \in \mathbb{N} \)  
   c) Is \( \frac{25}{26} \) a term of the sequence?  
   1 point
   d) Is this an arithmetic sequence?  
   2 points
26. For the following figure we know that \( \int_0^b x \, dx = \int_0^b x^2 \, dx \) and \( b > 0 \).
Find the shaded area.

3 points

27. Given the set \( S = \{1, 2, 3, 4, 5, 6\} \), randomly choose two elements from \( S \) and find the probability that their sum is equal seven.

2 points

28. Given the points with coordinates: \( A = (2, 3) \) and \( B = (4, 5) \).

a) Find the angle that the line passing through points \( A \) and \( B \) forms with the x-axis.

2 points

b) Find the equation of the circle centered at the origin and tangent to the line passing through points \( A \) and \( B \).

2 points
The State Matura Exam
SESSION I
(Required)

Friday, June 22, 2007
Time 10:00 to 12:30

Subject: Mathematics - General High School

Instructions for students

The test has 25 requests in total. Thirteen are multiple-choice questions, for which you will circle the letter that represent the correct answer. The others are work out questions with space available for you to show the solution and reasoning. Also at the end of the test is left space for you to perform the operations when answering the questions. The available time for the exam is 2 hours and 30 minutes. Points are given for each question.

For use by the evaluation committee

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Points in Total: ________________ The evaluation commission

1………………………………………..Member

2………………………………………..Member
1. Given \( A = [1, 3] \) and \( B = [0, 4] \), then \( A \cup B \) is equal to:

A. \([-1, 4]\)
B. \([0, 4]\)
C. \([0, 3]\)
D. \([3, 4]\)

2. The expression \( \frac{(2ab)^3}{a^2b^3} \) is equivalent to:

A. \(8a\)
B. \(4a^2\)
C. \(8a^4\)
D. \(8b\)

3. The number of real roots for the equation \( \sqrt{x^2 - 8} = 1 \) is:

A. 0
B. 1
C. 2
D. 3

4. The function \( y = x^2 - 4x + 6 \) attains its minimum at \( x \) equal to:

A. 1
B. 2
C. 3
D. 4

5. If \( 2^{-x} = \frac{1}{\sqrt{x}} \), then the value of \( x \) is:

A. \(\frac{1}{4}\)
B. \(\frac{1}{2}\)
C. 1
D. 2
6. The angle \( \alpha \) lies in quadrant II and \( \sin \alpha = \frac{\sqrt{3}}{2} \). \( \cos \alpha \) is:

A. \( -\frac{1}{2} \)
B. 0
C. \( \frac{1}{2} \)
D. \( \frac{\sqrt{2}}{2} \)

7. Vectors \( \vec{a} = \left( \frac{2}{3} \right) \) and \( \vec{b} = \left( \frac{x}{4} \right) \) are perpendicular. The value of \( x \) is:

A. \( -6 \)
B. \( -2 \)
C. 0
D. 3

8. The graph of the equation \( x^2 + y^2 = 9 \) passes through the point:

A. \((3,3)\)
B. \((9,0)\)
C. \((0,3)\)
D. \((1,1)\)

9. The derivative of the function \( y = \sin^2 x \) is:

A. \(2 \sin x\)
B. \(\cos^2 x\)
C. \(2 \cos 2x\)
D. \(2 \sin x \cos x\)

10. The antiderivative of the function \( y = \frac{1}{x^3} \) is the function:

A. \( y = \frac{1}{x^2} \)
B. \( y = \frac{1}{x^3} \)
C. \( y = -\frac{1}{2x^2} \)
D. \( y = \frac{1}{2x^2} \)
11. If \( \log x = 2 \log 3 - 3 \log 5 \) then \( x \) is equal to:

\[
\begin{align*}
A. & \quad \frac{9}{25} \\
B. & \quad \frac{3}{5} \\
C. & \quad 1 \\
D. & \quad \frac{9}{5}
\end{align*}
\]

12. For what value of \( x \) the expression \( \frac{1}{e^{x-1}} \) is undefined:

\[
\begin{align*}
A. & \quad 0 \\
B. & \quad 1 \\
C. & \quad 2 \\
D. & \quad e
\end{align*}
\]

13. If \( f(x) = x^3 \) and \( g(x) = \sin x \) then \( f(g(x)) \) is:

\[
\begin{align*}
A. & \quad x^2 \sin x \\
B. & \quad (\sin x)^3 \\
C. & \quad \sin(x)^3 \\
D. & \quad \sin 3x
\end{align*}
\]

---

**Part 2**

Questions 14 through 25 are work out questions with space available for you to show the solution and reasoning.

14. Given the function \( y = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \)
   
   c) Graph the function
   
   d) What is its derivative at \( x = 1 \)?
15. Find the limit
   c) \( \lim_{{x \to 2}} \frac{{2x-4}}{{x^2-4}} \)  
   1 points ________

d) \( \lim_{{x \to 0}} \frac{{\sin(x^2)}}{{x\tan x}} \)  
   2 points ________

16. For which values of \( x \in \mathbb{R} \) the function : \( y = 3\sqrt{x} + \sqrt{\frac{{x-4}}{{9-x}}} \) is defined?  
   3 points ________

17. Find the equation of the tangent line to the graph of the function in question 16 at \( x = 8 \).  
   4 points ________

18. Given \( \sin x - \cos x = \sqrt{2} \), find \( \sin 2x \).  
   2 points ________

19. In the isosceles triangle \( ABC \), where \( AB = AC \), let \( M \) be the midpoint of the base \( BC \). 
   Express the vector \( \overrightarrow{AM} \) in terms of the vectors \( \overrightarrow{AM} = \overrightarrow{a} \) and \( \overrightarrow{AC} = \overrightarrow{b} \) and show that \( \overrightarrow{AM} \) is perpendicular to \( \overrightarrow{BC} \).  
   3 points ________

20. The arithmetic mean of five consecutive whole numbers is 7. Find the smallest number.  
   2 points ________

21. How many three-digit even numbers, without repetition of the digits, may be formed using the following sets?
   d) \{1, 2, 3, 4\}  
   2 point ________

e) \{0, 1, 2, 3, 4\}  
   1 point ________
22. Given that the hyperbola satisfies: \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \), \( 2a = 4 \) and the equation of one of the asymptotes is \( y = \frac{1}{2}x \)

e) Find the equation of the hyperbola

f) Find the equations of the tangent lines to the parabola that are parallel to the line \( y = x - 2 \).

23. Given the function \( y = \sin 2x \cos 2x \)

g) Find its minimum value

h) Find the area of the figure between the graph of the function and the x-axis from \( x = 0 \) to \( x = \frac{\pi}{4} \).

24. Given that the base of the pyramid \( SABC \) is a trapezoid (with \( AB \parallel CD \)) and all sides of the pyramid form the same angle with its base, prove that \( AB = CD \) in the trapezoid \( ABCD \).

25. Bring the expression \( (1 + i)^{10} \) in the form of \( a + bi \), where \( a \) and \( b \) are real numbers.
The State Matura Exam

SESSION I

Tuesday, June 17, 2008

Time 10:00

Subject: Mathematics - General High School

Instructions for students

The test has 25 requests in total. Thirteen are multiple-choice questions, for which you will circle the letter that represent the correct answer. The others are work out questions with space available for you to show the solution and reasoning. Also at the end of the test is left space for you to perform the operations when answering the questions. The available time for the exam is 2 hours and 30 minutes. Points are given for each question.

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Points in Total: ____________________  The evaluation commission

1…………………………………………Member

2…………………………………………Member
1. The value of the expression \( \log_2(2c) \) when \( \log_2 c = 3 \) is equal to:
   A. 1
   B. 2
   C. 3
   D. 4

2. The number \( 25^{\frac{1}{2}} \) is equal to:
   A. 5
   B. 25
   C. 125
   D. 625

3. The value of the expression \( 3 \cos^2 x + 3 \sin^2 x - 3 \) is equivalent to:
   A. \( 6 \cos^2 x \)
   B. \( 6 \sin^2 x \)
   C. 0
   D. \( -3 \)

4. The intersection of \( E = [-3, 2] \) and \( F = [0, 1] \) is:
   A. Empty
   B. \( E \)
   C. \( F \)
   D. \( \mathbb{R} \)

5. Among the numbers \( p = 0.12, q = 10^{-1}, \frac{13}{100} \) and \( s = 0.21 \), which is the smallest?
   A. \( s \)
   B. \( p \)
   C. \( q \)
   D. \( r \)
6. The root for the equation $\sqrt{x + 2} = 2$ is:
   A. 0
   B. 1
   C. 2
   D. 3

7. The derivative of the function $y = \sin x - 2x$ at $x = 0$ is:
   A. 1
   B. 0
   C. $-1$
   D. $-3$

8. The function $y = -2x^2 + 8x + 5$ attains its maximum at $x$ equal to:
   A. 0
   B. 1
   C. 2
   D. 5

9. The definite integral $\int_{1}^{e} \frac{dx}{x}$ is equal to:
   A. 0
   B. 1
   C. 2
   D. $e$

10. The slope of the tangent line to the graph of the function $y = \frac{1}{3} x^3$ at $x = 2$ is:
    A. 2
    B. 3
    C. 4
    D. 8

11. In the arithmetic sequence with difference equal 3 and second term 4, the seventh term is:
    A. 15
    B. 17
    C. 19
    D. 21
12. The height of the isosceles triangle with base 16 cm and side 10 cm is:
   1 point _______
   A. 10 cm
   B. 8 cm
   C. 6 cm
   D. 4 cm

13. If in a right triangle the hypotenuse is 10 cm and one of the legs is 6 cm, then the cosine of the angle across from the other leg is:
   1 point _______
   A. 1
   B. 0.8
   C. 0.6
   D. 0.5

Part 2

Questions 14 through 25 are work out questions with space available for you to show the solution and reasoning.

14. Solve the inequality $3^{4x+5} > 81$
   2 points_______

15. Given the function $y = \begin{cases} 2x^2 + 1 & x \leq 1 \\ ax & x > 1 \end{cases}$ what is the value of a for which the function is continuous at $x = 1$?
   2 points_______

16. Given the function $y = 6x - x^2$
   a) Where is the function increasing and where is decreasing?
      2 points_______
   b) At what points is the graph of the function crossing x-axis?
      2 points_______
   c) Find the area of the figure between the graph of the function and x-axis.
      2 points_______
17. Where is the function \( y = \sqrt{x^2 - 3x + 2} \) defined?  
2 points

18. Solve the equation \( \log(x^2) = \log 3x \).  
2 points

19. Given the points \( A = (1, 3) \) and \( B = (1, 7) \)
   
a) Find the equation of the straight line passing through the points  
2 points

   b) Find the equation that represents the graph which satisfies the condition: If \( C \) is a point on this graph then the angle \( \angle ACB \) is a straight angle.  
3 points

20. Given the set of the numbers \( \{4, 5, 6, 5, 4, 7, 7, 8\} \) what number should we add to the set in order to get an arithmetic meant of 6?  
2 point

21. There are 5 spheres in a box, numbered 1 through 5. If we randomly choose two out of the five spheres from the box, what is the probability that one of them is the sphere with number 1?  
2 point

22. Given the hyperbola \( \frac{x^2}{3} - y^2 = 1 \)
   
i) Find the coordinates of its foci  
1 points

   j) Find the equation of the ellipse with the same foci and is tangent with the line \( y = x + \sqrt{3} \).  
2 points

23. Given the trapezoid \( ABCD \) where \( AB \parallel CD, AD = BC, AB = 12 \text{ cm}, CD = 6 \text{ cm} \) and \( \angle DAB = 60^\circ \)
   
a) Find the height of the trapezoid  
2 points

   b) Find the diagonals of the trapezoid  
2 points
24. Solve the equation \( \frac{\sin 2x}{x-\pi} = 0 \)  

2 points

25. The point B is on the perimeter of the top base of a straight cylinder with circular bases and the point C is on the perimeter of the bottom base. The angle between the line segment BC and the bottom base plane is \( 45^\circ \). The radius of the base is 25 cm and the length of BC is \( 14\sqrt{2} \) cm. Find the shortest distance between the segment BC and the axis passing through the two origins of the bases.

4 points
The State Matura Exam
SESSION I
(Required)

Thursday, June 11, 2009
Time 10:00

Subject: Mathematics - General High School

Instructions for students

The test has 25 requests in total. Thirteen are multiple-choice questions, for which you will circle the letter that represents the correct answer. The others are work out questions with space available for you to show the solution and reasoning. Also at the end of the test is left space for you to perform the operations when answering the questions. The available time for the exam is 2 hours and 30 minutes. Points are given for each question.

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Points in Total: _______________________

The evaluation commission

1…………………………………………..Member

2…………………………………………..Member
Questions 1 through 13 are multiple-choice, for which you will circle the letter that represents the correct answer.

1. Given $A = [-2, 3]$ and $B = [1, 4]$ find $A \cup B$.
   
   1 point _______
   
   A. $[1, 3]$
   B. $[-2, 3]$
   C. $[-2, 4]$
   D. $[-2, 4)$

2. The number $\sqrt{8^2}$ is equal to:
   
   1 point _______
   
   A. 2
   B. 4
   C. 6
   D. 8

3. The intersection point for the functions $y = 2x$ and $y = -x$ is:
   
   1 point _______
   
   A. $(0, 0)$
   B. $(0, 1)$
   C. $(2, -1)$
   D. $(-2, 1)$

4. The sides of a triangle are $4 \, \text{cm}$, $5 \, \text{cm}$ and $8 \, \text{cm}$. The sides of a similar triangle are $12 \, \text{cm}$, $x \, \text{cm}$ and $24 \, \text{cm}$. Find the value of $x$.
   
   1 point _______
   
   A. 9
   B. 12
   C. 15
   D. 20

5. If $f(x) = \ln x$ and $g(x) = 3x$, what is $g(f(x))$?
   
   1 point _______
   
   A. $3 \ln x$
   B. $3x \ln x$
   C. $\ln 3x$
   D. $3x + \ln x$
6. Find the value of \( \lim_{x \to 2} \frac{x^2 - 4}{x - 2} \)  
   1 point _______
   A. 0  
   B. 2  
   C. 4  
   D. 8 

7. Given two terms of a geometric sentence \( y_3 = 9 \) and \( y_2 = 3 \) what is \( y_1 \)?  
   1 point _______
   A. 6  
   B. 3  
   C. 1  
   D. 3\(^{-1}\) 

8. If \( \cos x = -0.6 \) and \( \pi < x < \frac{3\pi}{2} \), then \( \sin x \) is equal to:  
   1 point _______
   A. -0.8  
   B. -0.6  
   C. 0.8  
   D. 1 

9. The number of different groups of three books formed from five books is:  
   1 point _______
   A. 3  
   B. 5  
   C. 10  
   D. 20 

10. Given the inequality \( (x - 2)(x + 5) \leq 0, x \in \mathbb{R} \), determine which one of the following values is not part of the solution set.  
    1 point _______
    A. -4  
    B. -2  
    C. 2  
    D. 3 

11. The derivative of the function \( y = \cos 2x \) is:  
    1 point _______
    A. 2 \cos 2x  
    B. -2 \cos 2x  
    C. -2 \sin 2x  
    D. 2 \sin 2x
12. The graph of the equation \(9x^2 + 4y^2 = 36\) represents a

A. parabola  
B. ellipse  
C. circle  
D. hyperbola

13. The value of the definite integral \(\int_0^1 3x^2 \, dx\) is equal to:

A. 1  
B. 2  
C. 3  
D. 4

**Part 2**

Questions 14 through 25 are work out questions with space available for you to show the solution and reasoning.

14. The arithmetic mean of five numbers is 16. What would the mean be if we increase three numbers by five and decrease the other two by two?

2 points

15. Given the inequality \((x - 3)(x^2 + 6x + 9) > 0\).

a) Explain if \(x = -3\) is a solution

1 points

b) Solve the inequality

2 points

16. For what real numbers the function \(y = \log(4 - 3x - x^2) + \sqrt{x}\) is defined?

3 points

17. Given the parallelogram \(OABC\) where \(O = (0, 0),\ \ A = (10, 0)\), the line segment OC has slope \(k = \frac{3}{4}\) and the y-coordinate of point C is 6.

a) Find the area of the parallelogram

1 point

b) Find the coordinates of the other corners.

2 points
18. Given the function $y = x^3 - 3x^2$

a) Find the intervals where the function is concave up and where is concave down 1 point________

b) Find the equation of the tangent line to the graph and parallel to the line with equation $y = -3x + 5$. 2 points_______

19. If we throw two cubic dice, what is the probability that the outcome is two different numbers? 2 points_______

20. Given that a square with perimeter 24 cm has the same area of a regular hexagon, what is the side of the hexagon? 3 point_______

21. Find the equation of the circle centered at (1, 6) and is tangent with the line with equation $4x - 3y - 1 = 0$. 2 point_______

22. Given the function $y = \begin{cases} ax^2 & x \leq 1 \\ 2x - 1 & x > 1 \end{cases}$

k) Show that the function is continuous for $a = 1$ 1 points_________

l) Find the area of the figure surrounded by the graph of the function and the line $y = 1$. 3 points_________

23. Find the value of the parameter $a$ for which the value of the function $y = x^2 - ax - (a - 3)$ is positive for all $x \in \mathbb{R}$. 3 points_________

24. The perimeter of a rhombus is 20 cm, one of the diagonals is 8 cm.

c) Find its area 2 points_______

d) Find the area of the circle inscribed in the rhombus 2 points_______
25. Given a regular pyramid with triangular base SABC. The height of the triangular side in 6 cm and forms an angle of 60° with the base. Find the volume of the pyramid.

3 points
The State Matura Exam

SESSION I
(Required)

Tuesday, June 15, 2010
Time 10:00

Subject: Mathematics - General High School

Instructions for students

The test has 25 requests in total. Thirteen are multiple-choice questions, for which you will circle the letter that represent the correct answer. The others are work out questions with space available for you to show the solution and reasoning. Also at the end of the test is left space for you to perform the operations when answering the questions. The available time for the exam is 2 hours and 30 minutes. Points are given for each question.

For use by the evaluation committee

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Points in Total: ________________

The evaluation commission

1………………………………………..Member
2………………………………………..Member

193
Questions 1 through 13 are multiple-choice, for which you will circle the letter that represents the correct answer.

1. The number \((3^{-2})^{0.5}\) is equal to:
   1 point _______
   A. 3
   B. 1
   C. \(\frac{1}{3}\)
   D. \(\frac{1}{9}\)

2. Using the segments with length 2 cm, 2 cm and 4 cm you can construct
   1 point _______
   A. A right triangle
   B. Equilateral triangle
   C. Isosceles triangle
   D. None

3. The intersection of the letters in “AGRON” and “DRIN” has
   1 point _______
   A. 1 element
   B. 2 elements
   C. 3 elements
   D. 4 elements

4. The value of \(2 \log 3 + \log \left(\frac{1}{9}\right)\) is equal to:
   1 point _______
   A. 0
   B. 1
   C. 3
   D. 9
5. For what values of $x \in \mathbb{R}$ is the expression $\sqrt{4 - 2x}$ defined?  

A. $\mathbb{R}$  
B. $(-\infty, 2]$  
C. $[2, \infty)$  
D. $[-2, 2]$  

1 point  _______

6. If $x - 2 = 5$ then $x^2 - 4$ is:  

A. 15  
B. 25  
C. 35  
D. 45  

1 point  _______

7. The product of the real roots of the equation $x^2 - 3x + 2 = 0$ is:  

A. 0  
B. 1  
C. 2  
D. 3  

1 point  _______

8. The equation $4 - x^2 = 0$ is equivalent to:  

A. $x = -2$  
B. $x = 2$  
C. $(x - 2)(x + 2) = 0$  
D. $x + 2 = 2 - x$  

1 point  _______

9. In an arithmetic sequence we have the first term equal to 2 and the second term equal to 7. What is its sixth term?  

A. 23  
B. 25  
C. 27  
D. 29  

1 point  _______
10. Which one of the following equations is equivalent to \( y = x \).

A. \( y = \frac{x^2}{x} \)
B. \( y = (\sqrt{x})^2 \)
C. \( y = \sqrt[3]{x^3} \)
D. \( y = |x| \)

11. The slope of the tangent line to the graph of the function \( y = x^2 - x \) at \( x = 2 \) is:

A. 1
B. 2
C. 3
D. 4

12. The y-intercept for the line with equation \( 2x - y = 4 \) is:

A. \((-4, 0)\)
B. \((4, 0)\)
C. \((0, 4)\)
D. \((0, -4)\)

13. The vectors \( \vec{a} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \) and \( \vec{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \) are:

A. equal
B. te kundert
C. bashkevizore
D. perpendicular

---

**Part 2**

Questions 14 through 25 are work out questions with space available for you to show the solution and reasoning.

14. Find the value of the expression \( 5\sqrt{18} - 3\sqrt{50} \).
15. Five consecutive even whole numbers have an arithmetic mean of 8. What is the smallest number? 

2 points

16. Solve the system of inequalities \[ \begin{align*} 2x - 3 & \geq 3 \\
\frac{7-x}{5} & > -1 \end{align*} \]

2 points

17. Given the function \( y = 4x - x^2 \).

   c) Find the intervals where the function is increasing, where is decreasing and its extrema

   2 point

   d) Find the area of the figure surrounded by the graph of the function and y-axis

   3 points

18. If we throw two cubic dice, what is the probability that the sum of the outcomes is a multiple of 5?

2 points

19. Given the points \( A = (-5, 0) \), \( B = (5, 0) \) and \( C = (3, 4) \).

   c) Show that the triangle \( ABC \) is a right triangle, \( \angle C = 90^\circ \).

3 points

20. Find where the function \( y = \sqrt{x - \frac{1}{x}} \) is defined.

3 points

21. Sketch the graph of the function \( y = (\sqrt{x})^4 \)

2 point
22. Given the equation of the circle \( x^2 + y^2 - 4x + 6y = 3 \)

   m) Find the coordinates of its center  
      2 points________

   n) Write the equation of the circle, which is symmetric with the above circle with respect to the origin.  
      2 points________

23. The sides of a squared base pyramid form an angle of 60° with the plane of the base. Knowing that the area of the base is 100 cm² find the volume of the pyramid.  

   4 points________

24. Answer the following

   e) Find the integral \( \int_0^\pi \sin 2x \, dx \)  
      2 points________

   f) Find the maximum value of \( y = \sin \frac{x}{2} \cos \frac{x}{2} \)  
      2 points________

25. Given the function \( y = \sqrt{5 - x^2} \).

   a) Find the x-coordinates of the points where the graph of the function crosses the line \( y = 2 \).  
      2 points________

   b) Find the maximum value of the function  
      2 points________
The State Matura Exam
SESSION I
(Required)

Wednesday, June 15, 2011
Time 10:00

Subject: Mathematics - General High School

Instructions for students

The test has 25 requests in total.
Thirteen are multiple-choice questions, for which you will circle the letter that represent the correct answer.
The others are work out questions with space available for you to show the solution and reasoning.
Also at the end of the test is left space for you to perform the operations when answering the questions.
The available time for the exam is 2 hours and 30 minutes.
Points are given for each question.

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Points in Total: ___________________ The evaluation commission

1……………………………………………Member

2……………………………………………Member
Part 1

Questions 1 through 13 are multiple-choice, for which you will circle the letter that represents the correct answer.

1. Given $A = \{n \in \mathbb{N}|n > 1\}$ and $B = \{n \in \mathbb{N}|n < 1\}$ the number of elements in $A \cap B$ is:
   1 point________
   
   A. 9  
   B. 10  
   C. 11  
   D. 12

2. The value of $\frac{5^{-2}}{5^{-3}}$
   1 point________
   
   A. $-2$  
   B. $-3$  
   C. $-5$  
   D. 5

3. $\sqrt{18} - 3\sqrt{2} =$
   1 point ______
   
   A. 0  
   B. $\sqrt{2}$  
   C. $2\sqrt{2}$  
   D. 9

4. $\log_8 8^2 =$
   1 point ______
   
   A. 2  
   B. 6  
   C. 8  
   D. 16
5. The value of $2 \sin 15^\circ \cdot \cos 15^\circ$ is equal to:  
   A. 2  
   B. 1  
   C. $\frac{1}{2}$  
   D. 0  
   1 point _______

6. The first term of an arithmetic sequence is 11 and its difference is $-2$, what is the sum of the first two terms?  
   A. 9  
   B. 11  
   C. 13  
   D. 20  
   1 point _______

7. The graph of the function $y = x^5 - 3x + 1$ intersects y-axis at:  
   E. $y = 5$  
   F. $y = 3$  
   G. $y = 1$  
   H. $y = 0$  
   1 point _______

8. The circle with $x^2 + y^2 = 4$ is tangent to the line with equation:  
   A. $x = 1$  
   B. $x = 2$  
   C. $x = 3$  
   D. $x = 4$  
   1 point _______

9. Diagonals of a rhombus are 4 cm and 8 cm, what is its area?  
   A. $4 \ cm^2$  
   B. $8 \ cm^2$  
   C. $16 \ cm^2$  
   D. $32 \ cm^2$  
   1 point _______
10. If the lines with equations $3x + 2y - 1 = 0$ and $ax + 3y + 2 = 0$ are parallel, what is the value of $a$?
   
   A. 9
   
   B. $\frac{9}{2}$
   
   C. $\frac{7}{2}$
   
   D. 2

11. One of the roots for the equation $x^2 - mx + 3 = 0$ is $x = 1$. Find $m$.
   
   E. 1
   
   F. 2
   
   G. 3
   
   H. 4

12. The derivative of $y = \frac{x^4}{4}$ at $x = -1$ is:
   
   A. $-1$
   
   B. 0
   
   C. 1
   
   D. 2

13. $\int_{0}^{3} x^2 \, dx =$
   
   A. 0
   
   B. 2
   
   C. 3
   
   D. 9

---

Part 2

Questions 14 through 25 are work out questions with space available for you to show the solution and reasoning.

14. Four consecutive whole numbers have an arithmetic mean of 10. What is the smallest number?
   
   2 points________
15. Solve the inequality $3x + 1 < 4x^2$. What is the smallest natural number that satisfies the inequality?  

3 points

16. Given $f(x) = 2x^2 - \frac{17}{9}$, find $f(a) - f(a + 2)$  

2 points

17. Given the vectors $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
   
e) Find the sum $\vec{a} + \vec{b}$  

3 points

f) Prove that the vectors are perpendicular  

2 points

18. Find the set of real numbers for which the function $y = \sqrt{3 - \log_2 x}$ is defined.  

3 points

19. Given the function $y = \begin{cases} 2x + a & x \geq 3 \\ ax - 2 & x < 3 \end{cases}$ Find the value of $a$ that makes the function continuous in $\mathbb{R}$.  

2 points

20. Given the function $y = 2x^3 - 24x$.
   
a) Find the intervals where the function is increasing, where is decreasing and its extrema  

3 points

b) Find the equation of the tangent line to the graph of the function at $x = 1$.  

2 points

21. Given the points $A = (2, 3)$ and $B = (4, 1)$
   
a) Find the equation of the line passing through the points  

2 points

b) Find the equation of the line passing through the midpoint of the segment AB and is perpendicular to it.  

2 points
22. Given the functions $y = x^2 + 2$ and $y = |x|$. Find the area of the figure surrounded by the two graphs.

2 points_______

23. Given the triangle ABC with one side equal to 12 cm and the angle across it 30°. Outside of the triangle ABC plane there is a point P equally distant, 13 cm, from the vertexes. Find the distance of the point P from the plane determined by ABC.

3 points_______

24. In a box we have 5 white spheres and 3 blue spheres. If two of them are randomly selected, what is the probability that both are white?

2 points_______

25. Given the ellipse with equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

   c) Find its foci

      2 points_______

   d) Find the equation of the tangent to the ellipse and parallel to $y = x + 6$

      2 points_______
The State Matura Exam
SESSION I
(Required)

Saturday, June 16, 2012
Time 10:00

Subject: Mathematics - General High School

Instructions for students

The test has 25 requests in total.
Thirteen are multiple-choice questions, for which you will circle the letter that represent the correct answer.
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Also at the end of the test is left space for you to perform the operations when answering the questions.
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Points in Total: ________________
The evaluation commission

1………………………………………………….Member

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Part 1

Questions 1 through 13 are multiple-choice, for which you will circle the letter that represents the correct answer.

1. The value of \( \sqrt[3]{2^9} \) is:
   - A. 2
   - B. 4
   - C. 8
   - D. 16

2. Given the interval \((-4, 3]\), the number of integer elements is:
   - A. 9
   - B. 8
   - C. 7
   - D. 6

3. If the perimeter of a circle is \(8\pi\), then its area is
   - A. \(4\pi\)
   - B. \(8\pi\)
   - C. \(9\pi\)
   - D. \(16\pi\)

4. If point \(M = (2, 4)\) is the midpoint of the segment \(AB\), where the coordinates of \(B = (3, 6)\), then the coordinates of \(A\) are:
   - E. \((2, 2)\)
   - F. \((2, 1)\)
   - G. \((3, 1)\)
   - H. \((1, 2)\)

5. The number of the teams with 4 players out of 6 people is:
   - A. 30
   - B. 20
   - C. 15
   - D. 10
6. If \( x^3 - 8 = 0 \) then the value of \( x^2 - 1 \) is:

\[ \begin{align*}
A. & \quad 4 \\
B. & \quad 3 \\
C. & \quad 2 \\
D. & \quad 1
\end{align*} \]

1 point _______

7. Which one of the equations below has no real solutions?

\[ \begin{align*}
A. & \quad x^2 = 3 \\
B. & \quad x^3 = -3 \\
C. & \quad x^4 = -1 \\
D. & \quad x^3 = 0
\end{align*} \]

1 point _______

8. The inequality \( 3x - 2 > x + 4 \) is equivalent to:

\[ \begin{align*}
A. & \quad x > 3 \\
B. & \quad x < 3 \\
C. & \quad x \geq 6 \\
D. & \quad x \geq 2
\end{align*} \]

1 point _______

9. If the base angle of an isosceles triangle is \( 40^\circ \) then the angle across the base is:

\[ \begin{align*}
A. & \quad 18^\circ \\
B. & \quad 36^\circ \\
C. & \quad 80^\circ \\
D. & \quad 100^\circ
\end{align*} \]

1 point _______

10. \( \lim_{x \to 0} \left( 3x - \frac{\sin x}{x} \right) = \)

\[ \begin{align*}
A. & \quad -1 \\
B. & \quad 0 \\
C. & \quad 1 \\
D. & \quad 3
\end{align*} \]

1 point _______

11. The value of \( \log_3 9 + \log_3 \frac{1}{3} \) is:

\[ \begin{align*}
A. & \quad -3 \\
B. & \quad -1 \\
C. & \quad 1 \\
D. & \quad 3
\end{align*} \]

1 point _______
12. The slope of the tangent line to the graph of the function \( y = \frac{1}{3}x^3 - x^2 + 3 \) at \( x = 2 \) is:

A. 0  
B. 1  
C. 2  
D. 3

13. The value of \( \int_0^1 3 \, dx = \)

A. 3  
B. 2  
C. 1  
D. 0

Part 2

Questions 14 through 25 are work out questions with space available for you to show the solution and reasoning.

14. Solve the system of inequalities \( \begin{cases} x - 3 \geq 0 \\ 5 - x > 0 \end{cases} \) for \( x \in \mathbb{Z} \).

3 points

15. Given the function \( y = x^2 - 8x \).

a) Find the intervals in which the function is increasing and the once the function is decreasing.

2 points

b) Find the equation of the tangent line to the above graph, which is parallel to the line \( y = 10x + 2 \).

2 points

16. Determine where the function \( y = \sqrt{\log(4 - 2x)} \) is defined.

3 points

17. Given the vectors \( \vec{a} = \begin{pmatrix} x - 2 \\ y + 4 \end{pmatrix} \) and \( \vec{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \) such that \( \vec{a} = 2\vec{b} \), find \( x \) and \( y \).

2 points
18. Given the coordinates of the triangle $ABC, A = (-1, 2), B = (2, 3), C = (1, 4)$.
   
   a) Find the equation of the line passing through point A and the midpoint of the segment BC.  
       2 points_______
   
   b) Find the equation of the line passing through point C and is perpendicular to the segment AB.  
       2 points_______

19. Given the function $f(x) = x^2 - 4$ and $g(x) = 2^x$.
   
   a) Find $f \circ g(x)$.  
       1 points_______
   
   b) Solve the equation $f \circ g(x) = 0$.  
       2 points_______

20. Two dice are thrown at the same time. Find the probability that the sum of the two outcomes is less than 7.  
       2 points_______

21. The arithmetic mean of five consecutive even numbers is 14. Find the smallest number.  
       2 point________

22. Given the points $A = (-8, 0)$ and $B = (8, 0)$.
   
   a) Find the equation of the ellipse with foci the above points and that passes through the point $C = (10, 0)$.  
       3 points_______
   
   a) The point $M = (-8, y)$ is in the ellipse. Find the area of the triangle ABM.  
       2 points_______

23. Find the total area of a pyramid with square base knowing that the length of the base side is 8 cm and the four sides of the pyramid form an angle of 60° with the plane of the base.  
       3 points_________
24. Given the function \( y = -x^2 + 4x \). Find the area of the figure formed between the graph of the function and x-axis.

3 points

25. Given the function \( y = \begin{cases} \frac{kx}{3x^2 - 9} & x < 0 \\ \frac{x}{x^2 - 9} & x \geq 2 \end{cases} \). For what values of \( k \) the function is continuous on \( \mathbb{R} \).

2 points
The State Matura Exam
SESSION I
(Required)

Friday, June 14, 2013

Time 10:00

Subject: Mathematics - General High School

Instructions for students

The test has 25 requests in total.
Thirteen are multiple-choice questions, for which you will circle the letter that represent the correct answer.
The others are work out questions with space available for you to show the solution and reasoning.
Also at the end of the test is left space for you to perform the operations when answering the questions.
The available time for the exam is 2 hours and 30 minutes.
Points are given for each question.

For use by the evaluation committee

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Points in Total: ____________________

The evaluation commission

1………………………………………..Member

2………………………………………..Member

211
1. The value of $x$ that makes the expression $\frac{3}{2x-4}$ is:
   
   A. 0  
   B. 1  
   C. 2  
   D. 4

2. Given the equation $x^2 - bx + 4 = 0$. It is has two equal roots that the value of $b$ is:
   
   A. 4  
   B. 8  
   C. 2  
   D. 1

3. If one side of a rectangle is 5 $cm$ and its diagonal is 13 $cm$, then its perimeter is:
   
   A. 18  
   B. 24  
   C. 28  
   D. 34

4. The value of the expression $3^4 \cdot 3^{-3}$ is:
   
   A. $3^{-2}$  
   B. $3^{-1}$  
   C. 3  
   D. $3^0$

5. The line $3x - 2y + 6 = 0$ crosses y-axis at $y =$
   
   A. 2  
   B. 3  
   C. $-2$  
   D. $-3$
6. If the vectors \( \vec{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \) and \( \vec{b} = \begin{pmatrix} 3 \\ x \end{pmatrix} \) are perpendicular \( x = \) 

A. \(-6\)  
B. \(-4\)  
C. \(4\)  
D. \(6\)

7. Given the intervals \( A = (0, 3) \) and \( B = (-1, 4) \). Which one of the numbers is an element of \( A \cap B \)?

A. 5  
B. 4  
C. 3  
D. 2

8. The inequality \(-2x < -6\) is equivalent to:

A. \(x < 3\)  
B. \(x > 3\)  
C. \(x < -3\)  
D. \(x > -3\)

9. The derivative of the function \( y = e^{2x-1} \) at \( x = 0.5 \) is:

A. \(e\)  
B. 2  
C. \(2e\)  
D. \(e^{-1}\)

10. The equation \( \frac{3x-1}{2} = x \) is equivalent to:

A. \(x = 0\)  
B. \(x = 1\)  
C. \(x = 2\)  
D. \(x = 3\)

11. The value of \( \log_3 12 - \log_3 4 \) is equal to:

A. \(\log_3 48\)  
B. 2  
C. 1  
D. 0
12. The formula for the $n^{th}$ term of an arithmetic sequence is $y_n = 3n + 1$. Its difference is:  

1 point _______

A. 1  
B. 2  
C. 3  
D. 4

13. The value of $\int_0^1 (4x^2 + 1) \, dx =$  

1 point _______

A. 1  
B. 8  
C. 6  
D. 2

---

Part 2

Questions 14 through 25 are work out questions with space available for you to show the solution and reasoning.

14. There are five white spheres and four red spheres in one box. If we randomly choose two spheres, what is the probability that both are same color?  

2 points________

15. Given the function $y = 2x^3 - 3x^2$.  

c) Find the intervals in which the function is increasing, the once the function is decreasing and its extrema.  

3 points________

d) Prove that the equation $2x^3 - 3x^2 = 1$ has at least one root on $[0, 2]$.  

1 points________

16. Given the function $y = \begin{cases} 
2x - m & x > 1 \\
mx - 2 & x \leq 1 
\end{cases}$. For what values of $m$ the function is continuous on $\mathbb{R}$.  

3 points________
17. The diagonals of a parallelogram are 6 cm and 8 cm and the angle formed by them is 120°.
   a) Find the sides of the parallelogram. 2 points
   a) Find its area. 2 points

18. Given the \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) which crosses x-axis at (2,0).
   a) Find \( a \) 2 point
   b) Find the equation of the tangent line to the ellipse which is parallel to the line \( y - 2x + 1 = 0 \) 2 point

19. Determine where the function \( f(x) = \sqrt{x^2 - 2x} + \log(1 - x) \) is defined. 3 points

20. The arithmetic mean of 5 numbers is 32. How will the mean change if we increase 3 of them by 4 and decrease the other two by 1? 2 points

21. Find the total area of a pyramid with square base knowing that the length of the base side is 8 cm and the four sides of the pyramid form and angle of 60° with the plane of the base. 3 points

22. Find the area of the figure surrounded by the graph of \( y = x^2 + 2 \) and \( y = 3x \). 3 points

23. Given the points \( A = (-1, 3) \) and \( B = (3, 3) \).
   a) Find the coordinates of the midpoint. 1 point
   b) Find the equation of the line passing through the points A and B. 2 point
   c) Find the equation of the line perpendicular to the segment AB and passing through its midpoint 1 point
24. Solve the equation \( \frac{2}{3} \sin x \cdot \frac{3}{2} \cos x = 1 \) for \( x \in [0, 2\pi] \).

3 points________

25. Given the function \( y = x^2 + ax + b \). The equation of the tangent line to the graph of the function at \( x = 2 \) is \( y = 2x - 1 \). Find \( a \) and \( b \).

3 points________
The State Matura Exam
SESSION I
(Required)

Monday, June 9, 2014 Time 10:00

Subject: Mathematics - General High School

Instructions for students

The test has 25 requests in total.
Thirteen are multiple-choice questions, for which you will circle the letter that represent the correct answer.
The others are work out questions with space available for you to show the solution and reasoning.
Also at the end of the test is left space for you to perform the operations when answering the questions.
The available time for the exam is 2 hours and 30 minutes.
Points are given for each question.

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Points in Total: ________________ The evaluation commission

1………………………………………..Member

2………………………………………..Member
Questions 1 through 13 are multiple-choice, for which you will circle the letter that represent the correct answer.

1. The value the expression $\log_3 9$ is:
   
   A. $-3$
   B. $-2$
   C. 2
   D. 3

2. The value of $x^{\frac{1}{2}}$ when $x = 9$ is:
   
   A. 9
   B. 3
   C. 1
   D. $3^{-1}$

3. The maximum value of the function $y = \sqrt{3} - \cos x$ is:
   
   A. 4
   B. $\sqrt{3}$
   C. $\sqrt{2}$
   D. 2

4. The set $A = \{x \in \mathbb{R} \mid x \leq 0\}$ can be written as:
   
   A. $(-\infty, 0]$
   B. $(0, \infty)$
   C. $(-\infty, 0)$
   D. $[0, \infty]$

5. The diagonals of a rhombus are 6 cm and 8 cm, its perimeter is:
   
   A. 48
   B. 20
   C. 16
   D. 10
6. The point O is the midpoint for the segment AB. For the statement $\overline{AB} = k \cdot \overline{AO}$ to be true $k =$

A. $-2$
B. $-1$
C. $-\frac{1}{2}$
D. 2

7. If $\sin \alpha < 0$ and $\cos \alpha > 0$ then $\alpha$ is in quadrant

A. I
B. II
C. III
D. IV

8. The number of values for $x$ that make the expression $\frac{x+1}{x^2-9}$ undefined is:

A. 3
B. 2
C. 1
D. 0

9. Given the function $y = 1 + x^2$. Which one of the following points is in its graph?

A. $(1, 1)$
B. $(1, 0)$
C. $(0, 1)$
D. $(0, -1)$

10. An arithmetic sequence has difference of 2 and its second term is 5. What is its fifth term?

A. 15
B. 13
C. 11
D. 9
11. The point $A = (x, -3)$ is on the line $2x - 3y + 1 = 0$. The value of $x$ is:

\[ 2x - 3y + 1 = 0 \]

A. $-5$
B. $-3$
C. $-2$
D. $-1$

1 point _______

12. Given the parabola $y = x^2 - 2x + 4$. The x-coordinate for its vertex is:

\[ y = x^2 - 2x + 4 \]

A. 2
B. $-2$
C. $-1$
D. 1

1 point _______

13. The value of $\int_{-1}^{1} x \, dx =$

\[ \int_{-1}^{1} x \, dx = \]

A. $-1$
B. 0
C. $\frac{1}{2}$
D. 2

1 point _______

Part 2

Questions 14 through 25 are work out questions with space available for you to show the solution and reasoning.

14. The arithmetic mean of four consecutive even numbers is 7. What is the largest number?

2 points_______

15. On the circle with diameter AB there is a point C such that $AC = 8 \text{ cm}$.

\[ AC = 8 \text{ cm} \]

e) Find CB if the radius of the circle is $5 \text{ cm}$.

2 points_______

f) Find the sine of the smallest angle of the triangle $ABC$.

1 points_______
16. For what values of \( m \) the trinomial \(-x^2 + 3x + (m - 1)\) is negative for all \( x \in \mathbb{R} \)?

3 points

17. Given the function 
\[ y = \begin{cases} 
\frac{x^2 - 4}{x - 2} & x < 2 \\
2m & x \geq 2 
\end{cases} \]
For what values of \( m \) the function is continuous on \( \mathbb{R} \).

3 points

18. Find the projection of the point \( M = (-5, 1) \) on the line that passes through the points \( A = (0, -4) \) and \( B = (3, 2) \).

2 points

19. Given the ellipse with axis 8 and 10.

a) Write the equation of the ellipse and find the distance between the two foci.

2 points

b) Write the equations of the tangent lines to the ellipse and parallel to \( x + y = 0 \).

2 points

20. Find the derivative of the function \( y = x^3 - \sin 2x \) at \( x = 0 \).

2 point

21. Given the function \( y = 3 + 12x - x^3 \) is defined.

a) Find the intervals where the function is increasing and the intervals where the function is decreasing.

2 points

b) Find the equation of the tangent line to the graph at the point where the graph intercepts y-axis.

3 points
22. Find the area of the figure surrounded by the graph of $y = \sqrt{2x}$ and $y = x$.  

3 points________

23. Given a straight prism with a right angle base with legs $3 \text{ cm}$ and $4 \text{ cm}$. Its side with the largest area is square.

a) Find its total surface.  

3 points________

b) Find its volume.  

1 points________

24. Determine where the function $y = \sqrt{9 - x^2} + \ln(2 - x)$  

3 points________

25. Two identical dice are thrown at the same time. What is the probability that the sum of the two numbers is less than 6?  

2 points________
REPUBLIC OF ALBANIA  
MINISTRY OF EDUCATION AND SCIENCE  
NATIONAL EDUCATIONAL CENTER OF ASSESSMENT AND EXAMS  

The State Matura Exam  
SESSION I  
(Required)  

Tuesday, June 9, 2015  
Time 10:00  

Subject: Mathematics - General High School  

Instructions for students  

The test has 25 requests in total.  
Thirteen are multiple-choice questions, for which you will circle the letter that represent the correct answer.  
The others are work out questions with space available for you to show the solution and reasoning.  
Also at the end of the test is left space for you to perform the operations when answering the questions.  
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Points in Total: ________________  
The evaluation commission  

1………………………………………..Member  

2………………………………………..Member  

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MATHEMATICS - Session I

General High School

Part 1

Questions 1 through 13 are multiple-choice, for which you will circle the letter that represent the correct answer.

1. Given \( A = \{ x \in \mathbb{Z} \mid -3 \leq x < 2 \} \). Which of the following is true?

   \[ \begin{align*}
   &\text{A. } 3 \notin A \\
   &\text{B. } -2 \notin A \\
   &\text{C. } 0 \notin A \\
   &\text{D. } 2 \in A
   \end{align*} \]

   1 point _______

2. The value of \( \left( \frac{1}{9} \right)^{\frac{1}{2}} \) is:

   \[ \begin{align*}
   &\text{A. } -3 \\
   &\text{B. } -\frac{1}{3} \\
   &\text{C. } \frac{1}{3} \\
   &\text{D. } 3
   \end{align*} \]

   1 point _______

3. The value of the expression \( \log 20 - \log 2 \) is:

   \[ \begin{align*}
   &\text{A. } 0 \\
   &\text{B. } 1 \\
   &\text{C. } 2 \\
   &\text{D. } 4
   \end{align*} \]

   1 point _______

4. The value of \( \sqrt{32} - \sqrt{2} \) is:

   \[ \begin{align*}
   &\text{A. } \sqrt{2} \\
   &\text{B. } 2\sqrt{2} \\
   &\text{C. } 3\sqrt{2} \\
   &\text{D. } 4
   \end{align*} \]

   1 point _______

5. The diagonal of a rectangular with perimeter 48 cm and one side 6 cm is:

   \[ \begin{align*}
   &\text{A. } 6 \\
   &\text{B. } 8 \\
   &\text{C. } 10 \\
   &\text{D. } 12
   \end{align*} \]

   1 point _______
6. The equation $\frac{3x-1}{2} = x$ is equivalent to
   A. $x = 0$
   B. $x = 1$
   C. $x = 2$
   D. $x = 3$

7. The vectors $\vec{a} = \begin{pmatrix} 6 \\ m \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ are perpendicular. The value of $m$ is:
   A. $-2$
   B. 1
   C. 2
   D. 3

8. The area of a circle (in $cm^2$) with perimeter $6\pi \text{ cm}$ is:
   A. 9
   B. $6\pi$
   C. $9\pi$
   D. $3\pi^2$

9. The slope of the line with equation $6x - 2y + 1 = 0$ is:
   A. $-2$
   B. 1
   C. 2
   D. 3

10. If $f(x) = \log_3 x$ and $g(x) = 3x$ then $g \circ f (9) =$
    A. 6
    B. 3
    C. 1
    D. 0

11. The minimum value of the function $y = \sqrt{2 + \cos x}$ is:
    A. 0
    B. 1
    C. 2
    D. 3
12. The y-coordinate of the intersection point of the functions \( x = 3 \) and \( y = x^2 - 8 \) is:

A. 1
B. 2
C. 4
D. 8

13. The equation \( x^2 + 2x + m = 0 \) has no real roots for:

A. \( m = 0 \)
B. \( m < 1 \)
C. \( m > 1 \)
D. \( m = 1 \)

Part 2

Questions 14 through 25 are work out questions with space available for you to show the solution and reasoning.

14. For what values of \( m \) the numbers \( m + 2, 3m - 1 \) and \( 4m - 2 \) are consecutive terms of an arithmetic sequence?

15. Determine where the function \( y = \sqrt{x} + \log(2 - x^2) \).

16. Given the function \( y = x^3 - 3x^2 + 3, \ x \in \mathbb{R} \).

a) Determine the intervals where the function is increasing and where the function is decreasing.

b) Find the equations of the tangent lines to the graph of the function at \( x = 1 \).
17. Given the function \( y = 1 - x^2, \ x \in \mathbb{R} \).

a) Find the points where the graph of the function crosses x-axis.  
1 points________

b) Find the area surrounded by the graph of the function and x-axis.  
2 points________

18. Solve the inequality \( \left( \frac{1}{3} \right)^{x^2} > 3^{2-3x} \).  
2 points________

19. Given the points \( A = (2, 1) \) and \( B = (5, 2) \)

   c) Write the equation of the line passing through the two points.  
2 points________

   d) Find the coordinates of point M on x-axis such that the angle \( \angle AMB = 90^\circ \).  
2 points________

20. Given the function \( y = \begin{cases} \frac{x^2 + 2x}{x} & \text{if } x \neq 0 \\ \frac{1}{a+1} & \text{if } x = 0 \end{cases} \). For what values of \( a \) the function is continuous on \( \mathbb{R} \).  
3 point________

21. For what value of \( a \) the tangent to the graph of the function \( y = \ln(ax - 5) \) at \( x = 2 \) forms an angle of \( 45^\circ \) with the x-axis.  
3 points________

22. Given the hyperbola \( x^2 - 4y^2 = 20 \).

   c) Find the halves of the transverse axis (a and b).  
1 points________

   d) Find the equation of the tangent line to the graph of the hyperbola and parallel to \( y = x - 7 \).  
2 points________
23. Given the triangle with two sides 8 cm and 5 cm and the angle between them 60° find the other side and the area of the triangle.

3 points________

24. The diagonal of square base of a pyramid is $8\sqrt{2}$. Find the area of the sides of the pyramid knowing that its sides form a 60° with its base.

3 points________

25. Given the digits 2, 0, 3, 7, 5.

c) How many 4-digit numbers can be formed with them with no repeating digits?
2 points________

d) From the 4-digit numbers in a) we randomly select one. What is the probability that the chosen number is even
2 points________
## Appendix D – Curriculum Framework (Robitaille et al., 1993)
(Categories and their respective topics and subtopics)

### Number and Measurement

<table>
<thead>
<tr>
<th>Topic</th>
<th>Subtopics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exponents, Roots and Radicals</strong></td>
<td>Integer exponents and their properties</td>
</tr>
<tr>
<td></td>
<td>Rational exponents and their properties</td>
</tr>
<tr>
<td></td>
<td>Roots and Radicals</td>
</tr>
<tr>
<td></td>
<td>Real exponents</td>
</tr>
<tr>
<td><strong>Systematic counting</strong></td>
<td>Tree diagram, listing and other forms</td>
</tr>
<tr>
<td></td>
<td>Permutations</td>
</tr>
<tr>
<td></td>
<td>Combinations</td>
</tr>
<tr>
<td><strong>Perimeter, Area, Volume and Angles</strong></td>
<td>Computation, formulas and properties of length and perimeter</td>
</tr>
<tr>
<td></td>
<td>Computation, formulas and properties of surface areas</td>
</tr>
<tr>
<td>**Computation, formulas and properties of length</td>
<td>Use of appropriate instruments</td>
</tr>
<tr>
<td></td>
<td>and perimeter</td>
</tr>
<tr>
<td></td>
<td>Use of appropriate instruments</td>
</tr>
<tr>
<td></td>
<td>(ruler, compass, etc)</td>
</tr>
</tbody>
</table>

### Geometry – Form & Relation

<table>
<thead>
<tr>
<th>Topic</th>
<th>Subtopics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Two – Dimensional Coordinate Geometry</strong></td>
<td>Equation of lines in plane</td>
</tr>
<tr>
<td></td>
<td>Conic section and their equations</td>
</tr>
<tr>
<td></td>
<td>Parabola</td>
</tr>
<tr>
<td></td>
<td>Ellipse</td>
</tr>
<tr>
<td></td>
<td>Hyperbola (including asymptotes)</td>
</tr>
<tr>
<td></td>
<td>Parallelism and perpendicularity</td>
</tr>
<tr>
<td></td>
<td>Basic compass/straightedge constructions</td>
</tr>
<tr>
<td><strong>Polygons and Circles</strong></td>
<td>Triangles and Quadrilaterals: classifications and properties</td>
</tr>
<tr>
<td></td>
<td>Pythagorean theorem and its application</td>
</tr>
<tr>
<td></td>
<td>Other polygons/ properties</td>
</tr>
<tr>
<td></td>
<td>Circles, their properties and equation</td>
</tr>
<tr>
<td><strong>Three – Dimensional Geometry</strong></td>
<td>Three dimensional shapes and surfaces and their properties</td>
</tr>
<tr>
<td></td>
<td>Planes and lines in space</td>
</tr>
<tr>
<td><strong>Vectors</strong></td>
<td>Concept of vectors</td>
</tr>
<tr>
<td></td>
<td>Vector operations (addition/subtraction)</td>
</tr>
<tr>
<td></td>
<td>Vector dot and cross product</td>
</tr>
<tr>
<td><strong>Relation</strong></td>
<td>Congruence</td>
</tr>
<tr>
<td></td>
<td>Concept of congruence (segment, angles, …)</td>
</tr>
<tr>
<td></td>
<td>Triangles (SSS, SAS, …)</td>
</tr>
<tr>
<td></td>
<td>Quadrilaterals, Polygons and Solids</td>
</tr>
<tr>
<td><strong>Similarity</strong></td>
<td>Concept of similarity (proportionality)</td>
</tr>
<tr>
<td></td>
<td>Triangles (AA, SSS, SAS, …)</td>
</tr>
<tr>
<td></td>
<td>Quadrilaterals, Polygons and Solids</td>
</tr>
</tbody>
</table>

### Functions and Equations

<table>
<thead>
<tr>
<th>Topic</th>
<th>Subtopics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Functions</strong></td>
<td>Functions and their properties (range /domain, …)</td>
</tr>
<tr>
<td></td>
<td>Families of functions and their properties</td>
</tr>
<tr>
<td></td>
<td>Related functions (inverse, exp/log, derivative, …)</td>
</tr>
<tr>
<td></td>
<td>Relation of functions and equations</td>
</tr>
</tbody>
</table>

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Equations and Inequalities
Operations with expressions
Equivalent expression (factorization, simplification and solution tests)
Linear equation
Quadratic equations
Polynomial equations
Radical equations
Trigonometric equations and identities (including law of sine and cosine)
Logarithmic and exponential equations
Inequalities and their solutions
System of equation and their solutions
System of inequities and their solution

Probability and Statistics
Data Representation
Collecting Data
Representing Data
Interpreting tables, charts, plots …
Kinds of scales
Measures of central tendency
Measures of dispersion
Uncertainty and Probability
Informal likelihoods and vocabulary
Numerical Probability
Counting as it appears to probability
Mutually exclusive events
Independent events
Conditional probability

Elementary Analysis
Infinite Processes
Sequences
Arithmetic Sequences: n\textsuperscript{th} terms and sums
Geometric Sequences: n\textsuperscript{th} terms and sums
Limits and Function Continuity
Concept and definition
Limit of function as \(x \to a\)
Limits at infinity
Theorems about limits

Differentiation
Concept and definition (algebraic & geometric)
Derivative of power functions
Derivatives of sums, products and quotients
Derivatives of composite functions (chain rule)
Derivatives of implicitly defined functions
Derivatives of higher degree
Relationship between derivative behavior and maxima and minima
Relationship between derivative behavior and concavity and inflection points
Applications of the derivative
Integration
Concept and definition
Antiderivatives
Basic integration formulas
Integration by substitution
Integration by parts
Definite integrals
Application of definite integral
Appendix E – The required curriculum during 1970s

The mathematics curriculum requirements for the four years of high school pertain to this study and the description given below is based on mathematics textbooks from 1970s published from Textbook Publishing House (Shtëpia Botuese e Librit Shkollor) which followed strictly the required curriculum.

Algebra (Hoxha & Boshku, 1976) curriculum for high school started with the review of the knowledge acquired up to the eighth grade and continued with the new topics.

**Topic 1. Equations and systems of equations**

1) The system of equations of first degree and higher, their graphical solutions and applications.

2) Trinomials of 2\textsuperscript{nd} degree, \( f(x) = ax^2 + bx + c \), factoring, discriminant, its sign over \( \mathbb{R} \) with graphical illustration.

3) Second degree inequalities, product, quotient, system of inequalities and their graphical illustrations and applications

4) Linear programming and its application in economics, algebraic and graphic representations

5) Discussion of Equations

**Topic 2. Exponential and Logarithmic functions**

1) The meaning of irrational powers (irrational exponents).

2) Exponential function, its graph and its properties.

3) Logarithmic function, its graph and its properties.

4) Rules of logarithms.

5) Evaluation of the common logarithm of a number and other way around using tables
6) Exponential and Logarithmic equations (algebraic and graphical solutions).

**Topic 3.** Progressions (Sentences)

1) Arithmetic Sentence, formula for the nth term, the sum of the terms for a finite arithmetic sentence.

2) Geometric Sentence, formula for the nth term, the sum of the terms for a finite geometric sentence.

3) Comparison between arithmetic and geometric sequences.

**Geometry 1** (Paparisto & Çami, 1973) started with the review of the knowledge acquired up to the eighth grade: Geometry of the triangle, rhombus, parallelogram, trapezoid, circle etc. and followed by the new topics.

**Topic 1.** Proportionate Segments

1) Theorem of proportionate segments and construction Problems

2) Properties of angle bisector in a triangle and applications

3) Segments obtained from two parallel and lines intersecting at a point

**Topic 2.** Similarity of polygons

1) Definition of similar polygons and their existence

2) Partition of similar polygons into similar triangles and applications

**Topic 3.** Metric relations in polygons (Metric relations between their side (segments) lengths.

   Mainly construction problems.

1) Metric relations in the right triangle and applications

2) The square of the one side of any triangle

3) Metric relations in a circle and applications

4) Historical knowledge
**Topic 3.** Algebra applications in Geometry

1) Construction of given segments using simple formulas
2) Construction problems solved with algebraic method

**Topic 4.** Regular polygons

1) Regular polygons inscribed and circumscribed to a circle and their properties
2) Similarity of regular polygons
3) Construction of polygons inscribed in a circle and their side length
4) Historical knowledge

**Topic 4.** Trigonometric functions of and acute angle

1) Definition of trigonometric functions
2) Construction of an angle given the value of one of the trigonometric functions
3) The values of trigonometric functions for the complementary angles.
4) Using the table of the values of the trigonometric functions
5) Solving the right triangle and applications.

**Geometry 2** (Baxhaku et al, 1978)

**Topic 1.** The length of the circle

1) The limit of the number sentence
2) Determining of the length of the circle

**Topic 2.** Measurement of the area

1) The area of the polygons and applications
2) The area of the circle and its parts

**Topic 3.** Vectors and Operations with them
**Topic 4. Geometric Transformations**

1) The symmetry of a point and a figure with respect to a straight line  
2) Figures with axes of symmetry  
3) Applications  
4) The symmetry of a point and a figure with respect to a point  
5) The center of the symmetry of the parallelogram  
6) Applications  
7) Rotation of a point and a figure, applications  
8) Parallel shifts and Homomorphism  
9) The Generalization of the concept of similarity  
10) Geometric Transformations

**Geometry 3** (Bino et al, 1971)

**Topic 1. The plane and the straight line**

1) The plane and parallel lines  
2) Lines parallel to a plane and parallel planes  
3) Perpendicular lines and planes  
4) Angles between planes  
5) Orthogonal projections on the plan  
6) Corners

**Topic 2. Polyhedrons**

1) General knowledge on prism, parallelepiped, pyramid and pyramid trunk  
2) The area and volume of the prism and applications  
3) The area and volume of the parallelepiped and applications
4) The area and volume of the pyramid and pyramid trunk and applications

**Topic 3.** The round three – dimensional shapes

1) Rotation areas

2) The cylinder, its area, volume and applications

3) The cone, its area, volume and applications

4) The sphere, its area, volume and applications

**Elements of Mathematical Analysis** (Bujari et al, 1973)

**Topic 1.** Functions of one variable

1) Constant and variable quantities and definition of a function

2) Different representations of a function

3) Basic Functions

4) Composition of functions

**Topic 2.** The Limit

1) Absolute value, a functional and graphical approach

2) Infinitely small quantities and the introduction of $\varepsilon$ and $\delta$

3) Properties of the infinitely small quantities as $x$ goes to infinity

4) The limit of the function and the fundamental theorems about it

5) Infinitely small quantities and their properties

6) Undefined forms and the natural number e.

**Topic 3.** Continuous Functions

1) The continuity of a function

2) Operations with continuous functions

3) The continuity of elementary functions
**Topic 4. The Derivative**

1) The velocity of a moving object

2) The definition of the derivative of a function

3) Continuity and differentiability

4) Geometric meaning of the derivative

5) The derivative of a constant, the derivative of the sum, the product, the quotient and composite functions

6) The derivative of the logarithmic, power, exponential and trigonometric functions

7) The second derivative and higher order derivatives

**Topic 5. Applications of the Derivative**

1) Lagrange’s Theorem (Geometric proof) and its implications

2) The monotone functions

3) Functions’ local extremums and absolute minimum and maximum

4) Application of derivative in optimization problems

5) Analyzing the graph of the function; extremums, inflection points, asymptotes etc.

**Topic 5. Integrals**

1) Indefinite Integral
   - The antiderivative of a function and the table of basic integrals
   - Properties of indefinite integral
   - Substitution method, integration by parts and the integration of rational function

2) Definite Integral
   - The definition of the definite integral
   - Application of definite integrals in evaluating areas, volume and Physics
Appendix F – The required curriculum during 2006-2015

- **In the 10th grade** (first year of high school) mathematics core curriculum
  developed with 3 hours per week, total: 36 weeks x 3hrs / week = 108 hours per year. Topics with their sub topics included were:

1. **Topic 1. Number and operations with numbers** (Hours suggested: 12)
   Knowledge about the set of real numbers, \( \mathbb{R} \), and its subsets \( \mathbb{N}, \mathbb{Z}, \mathbb{Q} \), and operations with them.
   The use of mathematical notations in describing the relationships between sets and intervals of real numbers. The properties of rational exponents, and simple rules of logarithms. Predict and judge the results obtained by calculations (with and without the help of technology).

2. **Topic 2. Measurements** (Hours suggested: 9)
   Focusing mainly on indirect measurements, this topic includes knowledge of the triangle trigonometry to solve and describe the scalene triangle and finding areas of plane figures, measurement approximation, operations with vectors on plane and the distance between two points.

3. **Topic 3. Algebra** (Hours suggested: 23)
   It includes knowledge of basic rules of multiplication, division and factoring polynomials, interpretation and solving equations and inequalities of first and second degree with one variable and their systems and solving systems of equations with two variables.

4. **Topic 4. The function** (Hours suggested: 21)
   Describes the relation between the elements of two sets, different ways (verbal, table, formula and graph) of describing and interpreting functions, linear, exponential, logarithmic, square root, their domain, range, the value at a point. Description of the meaning of the arithmetic and geometric sequences as function with domain the natural numbers.
5. **Topic 5. Geometry** (Hours suggested: 29)

1) Plane Geometry: Knowledge on interpretation and application of the congruency and similarity of triangles and the properties of regular polygons

2) Geometric transformations and coordinates, (translating Geometry into Algebra), equation of a straight line (knowing two points) and equation of the circle knowing the center and its radius.

3) Trigonometry: Knowledge of sine and cosine of the angle in a triangle, use of table or calculator, formulate the laws of sine and cosine, the formulas\(\sin(180° - \alpha)\), \(\cos(180° - \alpha)\) and apply them in problem situations


Statistical concepts of population, individual, variable and data (qualitative, quantitative, discrete and continuous) presentation of data in tables and graphically, distribution characteristics. Meaning of random variable, events and the probability of the union of events and some of the laws of logic.

- **In the 11th grade** (second year of high school) mathematics core curriculum aimed at expanding and deepening the knowledge taken in 10th grade and developed with 3 hours per week, total: 36 weeks x 3hrs / week = 108 hours per year. Topics with their sub topics included were:

1. **Topic 1. Number and operations with numbers** (Hours suggested: 7)

Mathematics and Finance in everyday life. The use of mental calculations in reasoning about the results involving real numbers applied in everyday financial activities. Financial operation, equity, period, initial value, accrued interest and future value, interest rates (simple and compound interest formulas), debt and savings.
2. **Topic 2. Measurements** (Hours suggested: 24)

Angle measurement, unit circle trigonometry, the reference angle, the trigonometric functions of any angle, the reduction of angles in the first quadrant, and analytical trigonometry. The volume and area of three dimensional objects: prism, pyramid, trunk of the pyramid, cylinder, cone and sphere.

3. **Topic 5. Geometry** (Hours suggested: 28)

Assertions, implications, equivalent assertions, the meaning for the axioms and theorems, some axioms of geometry of the line and plane. The properties of parallel and perpendicular lines. The mutual condition of a straight line and a plane. The properties of parallel planes. The distance of a straight line from a plane parallel to it. The theorem of three perpendiculars. Description of the shapes of cuts of three dimensional objects by a plane and their properties.

4. **Topic 4. Algebra: Functions and the Limit** (Hours suggested: 38)

Operations with functions (sum, difference, product and quotient), study of variation function with simple methods. Graphing of $-f(x)$, $|f(x)|$, $f(-x)$, $f(x) + a$, $f(x + a)$, given $f(x)$. Variation of basic trigonometric functions. The definition of the limit of a function at a point or infinity. The intuitive understanding of the limit, geometric and table interpretation. The infinite limits and vertical asymptotes, horizontal asymptote. The limit the sum, product and quotient of two functions. The indeterminate forms of limits, and the limits using the squeeze principle (simple cases), and $\lim_{x \to 0} \frac{\sin x}{x}$.

Simple trigonometric equations.
5. **Topic 5. Statistics, Combinatorics, Probability** (Hours suggested: 11)

Analysis of statistical information provided graphically, presentation of statistical data with diagrams, the probability of a simple event, algebra of events, the probability of the union of events, exclusive events, addition rule of probabilities, the probability of the intersection of events, independent events, multiplication rule of probabilities, permutations, combinations and multiplication rule of counting.

- **The 12th grade** mathematics program aimed at expanding and deepening the knowledge taken in 10th and 11th grade, closing the cycle of mathematical concepts and skills and prepare the students for High School Graduation Exam. In the 12th grade mathematics core curriculum developed with 4 hours per week, 34 weeks x 4 hours / week = 136 hours per year. About 16 hours will be spent on preparation for the matriculation exam.

The mathematics curriculum for the last year of high school consisted on five main topics with subtopics as follows.

1. **Topic 1. Geometry – The second degree curves** (Hours suggested: 30)

   1) Circle, its equation, the properties of its graph (center, x and y intercepts, position on the coordinate plane, shape, symmetries), relations between the circle’s equation and the tangent and perpendicular lines to its graph.

   2) Ellipse, its equation, the properties of its graph (foci, eccentricity, x and y intercepts, position on the coordinate plane, shape, symmetries), relations between the ellipse’s equation and the tangent lines to its graph.
3) Hyperbola, its equation, the properties of its graph (foci, x and y intercepts (vertexes), position on the coordinate plane, shape, symmetries, asymptotes), relations between the hyperbola’s equation and the tangent lines to its graph.

4) Parabola, its equation, the properties of its graph (focus, x and y intercepts, position on the coordinate plane, shape, symmetries, eccentricity, directrix), relations between the parabola’s equation and the tangent lines to its graph.

2. Topic 2. Differential and Integral Calculus (Hours suggested: 60)

1) Derivatives

One-sided Limits and the continuity of a function at one point. Continuity of the elementary functions (constant: \( y = c \), power: \( y = x^a \), \( y = \sqrt{x} \), exponential: \( y = a^x, y = e^x \), logarithmic: \( y = \log_a x, \ln x \) and trigonometric functions: \( y = \sin x \) etc.). The definition of the derivative of a function at a point; derivation rules (sum/difference, chain, product and quotient with simple examples), geometric and physical meaning of the derivative, continuity connection with differentiability through examples. The second order derivative, the study of the monotony of function, finding the extremums and inflection points. Optimization problems. Applications (for simple functions) of the derivatives in Physics, Chemistry, Biology, Economics etc.

2) Integrals

The definition of the primitive of a function, the indefinite integral and its properties.

Integration with the use of a basic integral table. Integration using substitution method (simple cases), integration by parts (only once), integration of rational fractions with first degree polynomial denominators. The definition of definite integral, the use of its simple properties and Newton-Leibniz formula. Computation the simple areas in plane using definite integrals.

The definition of the discrete random variable, simple version of its distribution function, expected value. Counting techniques, the use of tree, the connection between the binomial coefficients $C^n_k$ and $C^n_{n-k}$ in simple cases. The construction of a table with two entries, two discrete variables, showing the frequency of all possible outcomes, illustrated with simple real life examples. Interpretation of the classical definition of probability of an event and its use in the calculation of the probability.

4. **Topic 4. Applications of Mathematics in other areas and knowledge of mathematics evolution** (Hours suggested: 11)

The ability to implement mathematical knowledge, gained during the years of high school to solve problems and analyze phenomena from Physics, Economics, Chemistry, Biology, Social Sciences, Health, etc. The ability to explain how mathematical discoveries have come as a result of real life phenomena and enrich the students’ Mathematical culture by clear information on the evolution of mathematics over the years.

5. **Topic 5. Mathematical processes** (Hours suggested: Integrated into other topics)

1) The ability to communicate mathematically

Explain orally and in writing, practical activities, assumptions and solution process, use correct mathematical symbols, interpret information from simple geometric two and three dimensional shapes and information from graphs, charts and diagrams.

2) The ability to judge, argue, reason and prove

The ability to use some basic rules of logic, reasoning, argument, justify a conclusion and judge the authenticity of a given result by the application of known formulas or with the use of technology. Prove simple theorems on all topics and the use of counterexample.
3) The ability to solve problems

Solve a problem using different ways. Mathematicise and solve the problem situations (not complex), with (or without) the help of technology, simulated or real-life examples from the other sciences.