Three Manifold Mutations Detected by Heegaard Floer Homology

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ABSTRACT

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Given a self-diffeomorphism φ of a closed, orientable surface S with genus greater than one and an embedding f of S into a three-manifold M, we construct a mutant manifold by cutting M along $f(S)$ and regluing by $f\varphi f^{-1}$. We will consider whether there exist nontrivial gluings such that for any embedding, the manifold M and its mutant have isomorphic Heegaard Floer homology. In particular, we will demonstrate that if φ is not isotopic to the identity map, then there exists an embedding of S into a three-manifold M such that the rank of the non-torsion summands of $\widehat{HF}$ of M differs from that of its mutant. We will also show that if the gluing map is isotopic to neither the identity nor the genus-two hyperelliptic involution, then there exists an embedding of S into a three-manifold M such that the total rank of $\widehat{HF}$ of M differs from that of its mutant.

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For Elisabeth Knibbe, Tim Clarkson and Lorne Clarkson

Chapter 1

Introduction

Heegaard Floer homology is a topological invariant that assigns a collection of abelian groups to each closed, oriented three-manifold equipped with a Spin^c-structure [36]. Given a topological invariant, it is natural to ask which topological operations it detects. In this thesis, we will consider whether or not Heegaard Floer homology detects *mutation*, the operation of cutting a three-manifold along an embedded surface and regluing by a surface diffeomorphism. In particular, we will show that the version of Heegaard Floer homology denoted by $\widehat{HF}$ can detect mutation by any nontrivial diffeomorphisms of a closed, orientable surface of genus greater than one.

In order to make this statement more precise, we introduce the following terminology and notation. Let $g \geq 2$ be a natural number and let S_g be a genus-g smooth, orientable, closed, connected surface. By a *manifold-surface pair*, we will mean a pair (M, f) where M is a closed, connected, smooth 3-manifold and $f\colon S_g \to M$ is a smooth embedding of S_g into M such that $f(S_g)$ separates M. To an orientation preserving diffeomorphism $\varphi\colon S_g \to S_g$ and a manifold-surface pair (M, f), we associate the *mutant manifold* M_f^φ that results from cutting M along $f(S_g)$ and regluing by $f\varphi f^{-1}$.

Theorem 1.0.1. *Let φ be an orientation preserving self-diffeomorphism of S_g that is not* *isotopic to the identity map. Then, there exists a manifold-surface pair (M, f) such that*

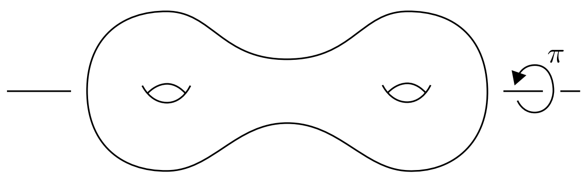
$$\operatorname{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M, \mathfrak{s}) \neq \operatorname{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M_f^{\varphi}, \mathfrak{s}).$$

Here, $c_1(\mathfrak{s})$ is the first Chern class of the Spin^c*-structure $\mathfrak{s}$.*

We will refer to the subrank of $\widehat{HF}$ from Theorem 1.0.1 as the rank of its non-torsion summands. The total rank of $\widehat{HF}$ can also detect mutations by most gluing maps. In order to make this statement more precise, we need one more definition:

Definition 1.0.2. The *genus-two hyperelliptic involution* is the unique order two element of the mapping class group $\mathrm{Mod}(S_2)$ that acts by $-\,\mathrm{id}$ on the homology $H_1(S_2)$. See Figure 1. We will denote this element by τ.

Figure 1: Genus-two hyperelliptic involution τ



Theorem 1.0.3. *Let φ be an orientation preserving self-diffeomorphism of S_g that is isotopic to neither the identity nor the genus-2 hyperelliptic involution. Then there exists a manifold-surface pair (M, f) such that*

$$\operatorname{rk} \widehat{HF}(M) \neq \operatorname{rk} \widehat{HF}(M_f^{\varphi}).$$

The effect of mutating by the genus-2 hyperelliptic involution has been considered for invariants related to $\widehat{HF}$. In particular, Ozsváth and Szabó showed that the Heegaard Floer knot invariant $\widehat{HFK}$ can detect mutations of this form [33, Thm. 1.2]. Conversely, there is computational evidence that the total rank of $\widehat{HFK}$ is preserved by mutation by the genus-2 hyperelliptic involution [31]. Finally, Ruberman showed that the instanton

Floer homology with $\mathbb{Z}/2\mathbb{Z}$ coefficients of an oriented homology 3-sphere is preserved by mutations of this form [38, Thm. 1][1].

The remainder of this thesis is broken into five chapters. In Chapter 2, we outline the properties of Heegaard Floer homology that are used in our proofs of Theorems 1.0.1 and 1.0.3. Then in Chapter 3, we reformulate the theorem statements into statements about normal subgroups of the mapping class group of S_g. This reformulation allows us to focus on mutations by pseudo-Anosov maps. We explore mutations of this form in Chapter 4. After completing this exploration, we will be ready to prove our main results. Chapter 5 contains those proofs. In Chapter 6, we consider some of the implications of Theorems 1.0.1 and 1.0.3. In particular, we discuss how these results can be used to obtain new results about actions of mapping class groups on triangulated categories.

[1] In private communication, Ruberman indicated that there is an issue with the signs in this paper due to a particular moduli space not being orientable. However, this is not relevant when one considers $\mathbb{Z}/2\mathbb{Z}$ coefficients.

Chapter 2

Heegaard Floer homology

Heegaard Floer homology has been defined both for closed, orientable three-manifolds and for knots in closed, orientable three-manifolds. Our goal is to understand how the closed manifold version behaves under the operation of mutation. In the pursuit of this goal, we will consider a three manifold that results from zero-surgery on a particular knot. The knot version of Heegaard Floer homology will be a useful tool for studying this manifold. The remainder of this chapter is devoted to briefly describing these two versions of Heegaard Floer homology.

2.1 Closed three-manifolds

Around the beginning of the century, Ozváth and Szabó introduced a family of topological invariants that associate abelian groups to closed, oriented, three-manifolds, equipped with a Spin^c-structure [35,36]. These invariants have come to be known as Heegaard Floer homology, because the homology groups are constructed by taking the Floer homology of a pair of Lagrangians in the g-fold symmetric product $\mathrm{Sym}^g(\Sigma - z)$ associated to a pointed Heegaard diagram $(\Sigma, \alpha, \beta, z)$.

There are multiple version of Heegaard Floer homology for closed three-manifolds. We will only be concerned with the version known as "HF-hat" and denoted by $\widehat{HF}$.

What follows is a brief discussion of its properties.

2.1.1 Choice of orientation and Spinˆ C-structure

In Chapter 1, we defined mutation as an operation on orientable manifolds whereas Heegaard Floer homology is defined for oriented manifolds equipped with Spin^c-structures. The ambiguity that results from having to choose both an orientation and a Spin^c-structure is resolved by the behavior of $\widehat{HF}$ under changes in orientation and conjugation of Spin^c-structures. In this section, we will use these behaviors to show that both the rank of the non-torsion summands and the total rank of $\widehat{HF}$ are well defined for orientable manifolds.

We begin by recalling two results from the original papers on Heegaard Floer homology. The first result states that $\widehat{HF}$ is preserved by conjugation of Spin^c-structures and the second result relates the Heegaard Floer homology of an oriented manifold M to that of the manifold with the opposite orientation.

Theorem 2.1.1 (Ozsváth-Szabó [35, Thm. 2.4]). *The Heegaard Floer homology groups are symmetric under conjugation of* Spin^c*-structures.*

$$\widehat{HF}(M, \mathfrak{s}) \cong \widehat{HF}(M, \bar{\mathfrak{s}}).$$

Theorem 2.1.2 (Ozsváth-Szabó [35, Prop. 2.5]). *Let M be a closed, oriented three-manifold with a* Spin^c*-structure $\mathfrak{s}$ and let $-M$ denote M with the opposite orientation. Then, there is a natural isomorphism:*

$$\widehat{HF}^*(M, \mathfrak{s}) \cong \widehat{HF}_*(-M, \mathfrak{s})$$

Here, $\widehat{HF}_*$ denotes the usual homology group and $\widehat{HF}^*$ denotes the homology of the dual chain complex, $\mathrm{Hom}(\widehat{CF}(M, \mathfrak{s}), \mathbb{Z})$.

With these results in hand, we are now ready to prove the following corollary:

Corollary 2.1.3. *Both the rank of the non-torsion summands and the total rank of $\widehat{HF}$ are well defined for orientable manifolds.*

Proof. Let M be an oriented manifold. Recall that the rank of the non-torsion summands and the total rank of $\widehat{HF}$ are defined to be respectively

$$\mathrm{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M,\mathfrak{s}) \text{ and } \mathrm{rk}\,\widehat{HF}(M).$$

Now let $\mathfrak{s}$ be a Spin^c-structure on M, $-M$ be M with the opposite orientation and $\bar{\mathfrak{s}}$ be the Spin^c-structure conjugate to $\mathfrak{s}$.

It follows from Theorem 2.1.1, Theorem 2.1.2 and the universal coefficients theorem for cohomology that the following Heegaard Floer homologies are isomorphic as ungraded groups:

$$\widehat{HF}(M,\bar{\mathfrak{s}}) \cong \widehat{HF}(M,\mathfrak{s}) \cong \widehat{HF}(-M,\mathfrak{s}) \cong \widehat{HF}(-M,\bar{\mathfrak{s}}).$$

Furthermore, the set of Spin^c-structures with non-zero first Chern class is preserved by conjugation. Thus, neither the total rank nor the rank of the non-torsion summands of $\widehat{HF}$ are dependent on the choice of orientation. □

2.1.2 Thurston semi-norm

As we discussed in Chapter 1, our strategy for proving Theorem 1.0.1 is rooted in the fact that Heegaard Floer homology detects the Thurston semi-norm on homology. What follows is a review of both the results that underlie that fact and the definition of the semi-norm itself.

Let M be a closed, orientable, three-manifold. The Thurston semi-norm measures the complexity of a surface representing a homology class in $H_2(M;\mathbb{Z})$ and is defined as follows:

Definition 2.1.4 (Thurston [41])**.** For a closed surface $F = \cup_{i=1}^k F_i$ with k connected components, we define

$$\chi_-(F) = \sum_{i=1}^{k} \max(0, -\chi(F)).$$

Then, the *Thurston semi-norm* of a homology class $\omega \in H_2(M;\mathbb{Z})$ is the infimum

$$\theta(\omega) = \inf\{\chi_-(F) \mid F \subset M,\ [F] = \omega\}.$$

In particular, θ is constantly zero if the homology $H_2(M;\mathbb{Z})$ is generated by spheres. Conversely, if M is irreducible and atoroidal, then θ is non-zero on all non-trivial homology classes.

Given a homology class $\omega \in H_2(M;\mathbb{Z})$. We can group the Spin^c-structures on M according to how their first Chern classes evaluate on ω. This grouping gives a decomposition of $\widehat{HF}(M)$ as a $\mathbb{Z}$ graded group. The breadth of the support of $\widehat{HF}(M)$ in this grading determines the Thurston semi-norm of ω. This fact follows from two results. The first, known as the adjunction inequality, states that the breadth of the support is bounded above by the $\theta(\omega)$.

Theorem 2.1.5 (Ozsváth-Szabó [35, Cor. 7.2])**.** *If* $\widehat{HF}(M,\mathfrak{s}) \neq 0$*, then* $|\langle c_1(\mathfrak{s}), \xi\rangle| \leq \theta(\xi)$ *for all* $\xi \in H_2(M;\mathbb{Z})$*.*

The second result states that $\widehat{HF}$ does not vanish in the extremal grading. This result was proven in two stages. First, Ozsváth and Szabó showed that if one uses twisted coefficients, the adjunction inequality is tight [33, Thm. 1.1]. Then, Hedden and Ni observed that applying the universal coefficients theorem to the twisted coefficients result produces a similar result for $\widehat{HF}$ with $\mathbb{Z}$ coefficients [16, Thm. 2.2].

Theorem 2.1.6 (Ozsváth-Szabó [33, Thm. 1.1] see also [16, Thm. 2.2])**.** *Let* $\xi \in H_2(M;\mathbb{Z})$*. Then, there exists a* Spin^c*-structure* $\mathfrak{s}$ *on* M *such that* $\widehat{HF}(M,\mathfrak{s}) \neq 0$ *and* $|\langle c_1(\mathfrak{s}), \xi\rangle| = \theta(\xi)$*.*

2.1.3 Sample computation

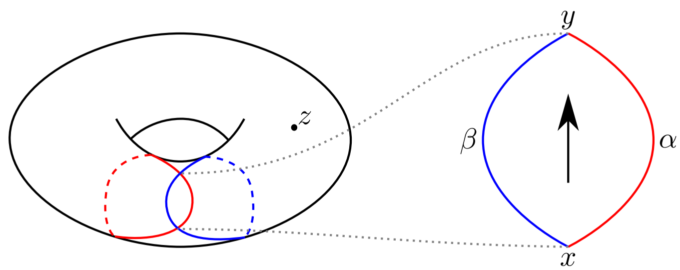
Our proof that Heegaard Floer homology detects mutation, requires an understanding of the Heegaard Floer homology groups of $S^1 \times S^2$. These groups were computed by Ozsváth and Szabó in [35, §3.1]. We include the computation here for the sake of completeness.

Proposition 2.1.7 (Ozsváth-Szabó [35, §3.1])**.** *Let* $\mathfrak{s}_0$ *be the unique torsion* Spin^c-*structure of* $S^1 \times S^2$*. Then, the Heegaard Floer homology groups of* $S^1 \times S^2$ *are*

$$\widehat{HF}(S^1 \times S^2, \mathfrak{s}) = \begin{cases} \mathbb{Z} \oplus \mathbb{Z} & \text{if } \mathfrak{s} = \mathfrak{s}_0 \\ 0 & \text{otherwise} \end{cases}$$

Proof. We begin by computing $\widehat{HF}(S^1 \times S^2, \mathfrak{s}_0)$. We construct a genus-one Heegaard splitting of $S^1 \times S^2$ by taking α and β to be two parallel copies of the same essential simple closed curve on the torus Σ. We then place the base point z in one of the annular components of $\Sigma - \alpha - \beta$. Finally, we perturb the curves slightly to make the other annular component into the disjoint union of two disks. See Figure 2. The resulting pointed Heegaard diagram $(\Sigma, \alpha, \beta, z)$ is weakly admissible for $\mathfrak{s}_0$.

Figure 2: Weakly admissible Heegaard Diagram for $S^1 \times S^2$



The curves α and β intersect in two points x and y both of which represent $\mathfrak{s}_0$. These points are connected by two holomorphic disks from x to y with opposite orientations. (One of these disks is highlighted in Figure 2.) Thus, the chain complex $\widehat{CF}(S^1 \times S^2, \mathfrak{s}_0)$ is generated by x and y and the differential is given by $dx = y - y = 0$ and $dy = 0$. It follows that the homology of this complex is $\widehat{HF}(S^1 \times S^2, \mathfrak{s}_0) = \mathbb{Z} \otimes \mathbb{Z}$.

The diagram that we constructed is also weakly admissible for the other Spin^c-structures of $S^1 \times S^2$. However, none of the other Spin^c-structures are represented by intersection points, so the corresponding chain complexes must be trivial. □

Thus, the total rank of $\widehat{HF}(S^1 \times S^2)$ is two. The class of manifolds for which this is the case is quite limited:

Theorem 2.1.8 (Hedden-Ni [16, Thm. 1.1])**.** *Let M be a closed orientable, irreducible 3-manifold with non-zero first Betti number. If* $\operatorname{rk} \widehat{HF}(M) = 2$*, then M is homeomorphic to the manifold obtained by performing zero-surgery on the trefoil.*

2.2 Knots

In addition to their closed manifold invariants, Ozsváth and Szabó also developed a version of Heegaard Floer homology for knots in oriented three-manifolds [34]. Knot Floer homology was also developed independently by Rasmussen [37]. One version of this invariant associates a bigraded, abelian group $\widehat{HFK}$ to each oriented knot in the three sphere.

Knot Floer homology has number of characteristics in common with the Heegaard Floer homology of closed manifolds. Reversing a knot's orientation does not change the rank of its knot Floer homology groups and thus the ranks of these groups are well defined invariants of an unoriented knot. Furthermore, these ranks can be used to determine the genus of a knot.

Theorem 2.2.1 (Ozsváth-Szabó [Thm. 1.2, 33])**.** *If K is a knot in S^3, then the Seifert genus of K is the largest integer n for which the group* $\widehat{HFK}_*(K, n) \neq 0$.

Knot Floer homology can be computed algorithmically using grid diagrams [3, 27]. Using Python, Droz implemented the variant of this algorithm developed by Beliakova [6]. In Section 5.1.1, we will use calculations performed by Moore and Starkston [31] using Droz's software to study a particular example of mutation by the genus-2 hyperelliptic involution.

Chapter 3

Mapping class group

Recall that the *mapping class group* of a surface is the group of orientation preserving self-diffeomorphisms modulo the maps that are isotopic to the identity map and let $\mathrm{Mod}(S_g)$ denote the mapping class group of S_g. In this Chapter, we will reformulate Theorems 1.0.1 and 1.0.3 as statements about two particular subgroups of $\mathrm{Mod}(S_g)$.

In Chapter 1, we used manifold-surface pairs to give a precise definition of mutating a three-manifold by a surface diffeomorphism. From this definition, it is easy to see that mutating a given manifold-surface pair by isotopic diffeomorphisms produces diffeomorphic mutant manifolds. Thus, there is a well defined notion of mutating a manifold-surface pair by a mapping class. Moreover, mutation behaves well with respect to the mapping class group product:

Lemma 3.0.2. *Let (M, f) be a manifold-surface pair and let α and β be arbitrary self-diffeomorphisms of S_g. Then, mutating M along $f(S_g)$ by the composition $\alpha \circ \beta$ creates a manifold that is diffeomorphic to the mutant that results from mutating (M_f^α, f) by β. In short, mutation respects composition of gluing maps.*

Proof. Let M_1 and M_2 be the closures of the two connected components of $M \setminus f(S_g)$. The mutant manifold M_f^α can be made into a manifold-surface pair by composing the embedding $f|_{M_1}\colon\ S_g \to M_1$ with the inclusion of M_1 into M_f^α. Let (N, h) denote this pair. Mutating (N,h) by β gives the mutant N_h^β which is constructed by using $(f\alpha)\beta f^{-1}$ to glue M_1 to M_2. Therefore, N_h^β is diffeomorphic to $M_f^{\alpha\beta}$ by construction. □

Thus, we can view mutation by a product of mapping classes as a sequence of mutations. Now that we've established a relationship between mutation and the mapping class group $\mathrm{Mod}(S_g)$, we are ready to define the subgroups that are relevant to Theorems 1.0.1 and 1.0.3.

Definition 3.0.3. A mapping class $[\varphi] \in \mathrm{Mod}(S_g)$ is $\widehat{HF}_{\mathrm{nT}}$*-invisible* if for all manifold-surface pairs (M,f) we have that

$$\mathrm{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M,\mathfrak{s}) = \mathrm{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M_f^\varphi,\mathfrak{s}).$$

Similarly, a mapping class is $\widehat{HF}$*-invisible* if for all manifold-surface pairs (M,f) we have that

$$\mathrm{rk}\,\widehat{HF}(M) = \mathrm{rk}\,\widehat{HF}(M_f^\varphi).$$

Proposition 3.0.4. *Both the set of $\widehat{HF}_{\mathrm{nT}}$-invisible mapping classes and the set of $\widehat{HF}$-invisible mapping classes are normal subgroups of* $\mathrm{Mod}(S_g)$.

Proof. This follows from Lemma 3.0.2 and the fact that these sets are essentially kernels of group actions. Here the actions are on equivalence classes of manifold-surface pairs where two pairs are equivalent if the specified ranks of their Heegaard Floer homologies are the same. □

Theorem 1.0.1 is equivalent to the statement that the normal subgroup of $\widehat{HF}_{\mathrm{nT}}$-invisible mapping classes is trivial. Similarly, Theorem 1.0.3 is equivalent to the statement that the normal subgroup of $\widehat{HF}$-invisible mapping classes is either trivial or the order two subgroup generated by the genus-two hyperelliptic involution. Reformulating the theorem statements in this way allows us to leverage the group structure of $\mathrm{Mod}(S_g)$. The remainder of this chapter is devoted to proving a useful proposition about normal subgroups of $\mathrm{Mod}(S_g)$.

3.1 Normal subgroups

The goal of this section is to develop enough of the theory of normal subgroups of mapping class groups to prove the following proposition:

Proposition 3.1.1. *If a normal subgroup* $G \lhd \mathrm{Mod}(S_g)$ *contains no pseudo-Anosov elements of the Torelli group, then it is either the trivial subgroup or the order two subgroup generated by the genus-2 hyperelliptic involution.*

In order to understand this proposition, we need to define the terms *pseudo-Anosov* and *Torelli group.* The former is defined as follows:

Definition 3.1.2. A homeomorphism $\varphi\colon\ S_g \to S_g$ is called *pseudo-Anosov* if there exists a pair of transverse measured foliations $(\mathcal{F}_u, \mu_u)$ and $(\mathcal{F}_s, \mu_s)$ on S_g and a number $\lambda > 1$ such that

$$\varphi(\mathcal{F}_u, \mu_u) = (\mathcal{F}_u, \lambda\mu_u) \text{ and } \varphi(\mathcal{F}_s, \mu_s) = (\mathcal{F}_s, \lambda^{-1}\mu_s)$$

Here u labels the *unstable* foliation and s labels the *stable* foliation. A mapping class in $\mathrm{Mod}(S_g)$ is called *pseudo-Anosov* if its representatives are isotopic to a pseudo-Anosov map.

The Torelli group will be defined in Section 3.1.2 where we will also show that it is torsion free. But for now, we will focus on the fact that it is a normal subgroup of $\mathrm{Mod}(S_g)$. Notice that the condition laid out Proposition 3.1.1 is inherently a statement about the intersection of a normal subgroup of $\mathrm{Mod}(S_g)$ with the Torelli group. Thus, it will be useful to gather information about the intersections of normal subgroups of the mapping class group.

3.1.1 Intersections

Theorem 3.1.3 (Long [26, Lem. 2.1])**.** *If* G *and* H *are two normal subgroups of* $\mathrm{Mod}(S_g)$ *that are neither central nor trivial, then their intersection* $G \cap H$ *is non-trivial.*

In order to make use of this theorem, we need to understand the center of the mapping class group. Happily, this is a well understood and relatively simple group:

Theorem 3.1.4 (See [7, §3.4; 19, Thm. 7.5.D])**.** *The center of* $\mathrm{Mod}(S_g)$ *is trivial if* $g \geq 3$ *and is the order two subgroup* $\langle \tau \rangle$ *if* $g = 2$*.*

Now, we return to the topic of the Torelli group.

3.1.2 Torelli group

Definition 3.1.5. The *Torelli group* is the normal subgroup consisting of those mapping classes whose representatives induce the identity map on homology, and is denoted by $\mathcal{I}(S_g)$.

The fact that the Torelli group is torsion free follows from the following two results:

Theorem 3.1.6 (Nielsen [32] see also [8, Thm. 11.8])**.** *Let* $[\varphi] \in \mathrm{Mod}(S_g)$ *be a mapping class of order* n*. Then, one of the representatives of* $[\varphi]$ *is a periodic diffeomorphism of order* n*.*

Theorem 3.1.7 (Ivanov [18, Thm. 1.3])**.** *Let* $\varphi\colon S_g \to S_g$ *be a periodic diffeomorphism. If* φ *is non-trivial, then the induced automorphism* $\varphi_*\colon H_1(S_g) \to H_1(S_g)$ *is also non-trivial.*

Corollary 3.1.8 (Ivanov [18, Cor. 1.5])**.** *The Torelli group is torsion free.*

In addition to the Torelli group, we also need to understand irreducible subgroups.

3.1.3 Irreducible subgroups

Definition 3.1.9. A subgroup $G \leq \mathrm{Mod}(S_g)$ is called *irreducible* if for any simple closed curve C on S_g there exists an element $[\varphi] \in G$ such that $\varphi(C)$ is not isotopic to C.

Irreducible subgroups are of interest to us, because they often contain pseudo-Anosov elements:

Theorem 3.1.10 (Ivanov [18, Thm. 1])**.** *Every infinite irreducible subgroup of* $\mathrm{Mod}(S_g)$ *contains a pseudo-Anosov element.*

Moreover, infinite normal subgroups are irreducible:

Theorem 3.1.11 (Ivanov [18, Cor. 7.13])**.** *Let* $G \lhd \mathrm{Mod}(S_g)$ *be a normal subgroup of the mapping class group. If* G *is infinite, then* G *is also irreducible.*

We are now ready to prove the proposition from the beginning of Section 3.1.

3.1.4 Proof of proposition 3.1.1

Proposition 3.1.1. *If a normal subgroup* $G \lhd \mathrm{Mod}(S_g)$ *contains no pseudo-Anosov elements of the Torelli group, then it is either the trivial subgroup or the order two subgroup generated by the genus-2 hyperelliptic involution.*

Proof. Let $G \lhd \mathrm{Mod}(S_g)$ be a normal subgroup of the mapping class group that contains no pseudo-Anosov elements of the Torelli group. Also let $H = G \cap \mathcal{I}(S_G)$ be the intersection of G with the Torelli group. Thus, H is also a normal subgroup that contains no pseudo-Anosov elements.

It follows from Theorem 3.1.10 that H is either finite or reducible. Furthermore, the Torelli group is torsion free and thus H must be either trivial or infinite and reducible (Corollary 3.1.8). However, Ivanov also showed that $\mathrm{Mod}(S_g)$ has no infinite, reducible, normal subgroups (Theorem 3.1.11). Therefore, H must be trivial.

Long showed that if the intersection of two normal subgroups of $\mathrm{Mod}(S_g)$ is trivial, then one of those groups must either be central or trivial (Theorem 3.1.3). The Torelli group is neither central nor trivial, so we must conclude that G is either central or trivial. As the center of $\mathrm{Mod}(S_g)$ is either trivial or $\langle \tau \rangle$, it follows G is either trivial or the order two subgroup generated by the genus-2 hyperelliptic involution (Theorem 3.1.4). □

By combining Propositions 3.0.4 and 3.1.1, we see that Theorem 1.0.1 is equivalent to the statement that neither the genus-2 hyperelliptic involution nor any pseudo-Anosov elements of the Torelli group are $\widehat{HF}_{\mathrm{nT}}$-invisible. Similarly, Theorem 1.0.3 is equivalent to the statement that no pseudo-Anosov element is $\widehat{HF}$-invisible. In the next Chapter, we will consider mutations by pseudo-Anosov maps.

Chapter 4

Mutations by pseudo-Anosov maps

The goal of this chapter is to prove the following proposition:

Proposition 4.0.12. *Let* $[\varphi] \in \mathcal{I}(S_g)$ *be a pseudo-Anosov element of the Torelli group. Then, there exists a natural number* N *and a manifold-surface pair* (M, f) *such that* $M = S^1 \times S^2$ *and the mutant manifold* $M_f^{\varphi^N}$ *has a homology class with nonzero Thurston semi-norm.*

In order to determine the effect of mutation on the Thurston semi-norm, we must first establish a relationship between the homology of a three-manifold and that of its mutants. In the case of mutation by elements of the Torelli group, this is achieved by the following lemma.

Lemma 4.0.13. *If* $[\psi] \in \mathcal{I}(S_g)$ *is an element of the Torelli group and* (M, f) *is a manifold-surface pair, then* M *and its mutant* M_f^{ψ} *have isomorphic homology groups*

$$H_i(M) \cong H_i(M_f^{\psi}) \text{ for all } i.$$

Proof. Because M and its mutant M_f^{ψ} are closed three-manifolds, it suffices to show that the first homology groups are isomorphic. In order to do this, we decompose M into two open sets that overlap in a tubular neighborhood of the separating surface $f(S_g)$. A comparison of the Mayer-Vietoris sequence coming from this decomposition to that coming from a similar decomposition of the mutant M_f^ψ shows that the first homology groups are indeed isomorphic. □

Our inquiry will focus on mutating $S^1 \times S^2$ along Heegaard surfaces. We proceed by considering the relationship between the complexity of the Heegaard splittings of a three-manifold and the minimal genera of its homology classes.

4.1 Curve complex

A genus-g *Heegaard splitting* is a decomposition of a three-manifold into two genus-g handlebodies glued together along their boundaries. Such a splitting is determined by two handlebodies with parameterized boundaries. A handlebody with parameterized boundary is in turn determined by the curves on the boundary that bound disks in the handlebody.

Definition 4.1.1. For a genus-g handlebody X with boundary parameterized by a map to S_g, let $\mathcal{V}_X$ be the set of isotopy classes of essential simple closed curves in S_g whose preimages bound disks in X. We will refer to the elements of $\mathcal{V}_X$ as *compression curves* of X.

Given two genus-g handlebodies X and Y with boundaries parameterized respectively by maps a and b to S_g, we can construct a 3-manifold M by using $b^{-1}a\colon \partial X \to \partial Y$ to glue X to Y. We will write $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ for the corresponding Heegaard splitting of M.

The compression curves of a genus-g handlebody can be viewed as points in the curve complex, $C(S_g)$ [15]. The *curve complex* is a simplicial complex with 0-simplices corresponding to isotopy classes of essential closed curves and n-simplices corresponding to $(n+1)$-tuples of isotopy classes that can be realized disjointly. There is a natural distance function d on the 0-simplicies of the curve complex given by viewing the 1-skeleton as a graph with edge length one. Applying this distance function to the sets of compression curves in a Heegaard splitting can provide information about the minimal genera of homology classes of the corresponding three-manifold:

Lemma 4.1.2. *If $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ is a Heegaard splitting of a manifold M and the distance $d(\mathcal{V}_X, \mathcal{V}_Y)$ is greater than two, then M is irreducible and has no essential tori.*

Proof. Haken showed that if M were reducible, then $\mathcal{V}_X$ and $\mathcal{V}_Y$ would have a point in common and thus $d(\mathcal{V}_X, \mathcal{V}_Y)$ would be zero [12, pg. 84].Furthermore, Hempel demonstrated that if M had an essential torus, then $d(\mathcal{V}_X, \mathcal{V}_Y)$ would be ≤ 2 [17, Cor. 3.7]. Thus, $d(\mathcal{V}_X, \mathcal{V}_Y) > 2$ implies that M is irreducible and has no essential tori. □

The distance between the two sets of compression curves in a Heegaard splitting is called the *Hempel distance* of that splitting. Combining this language with the definition of Thurston's semi-norm gives the following corollary to Lemma 4.1.2.

Corollary 4.1.3. *If a three-manifold M has a Heegaard splitting with Hempel distance greater than two, then the Thurston semi-norm is in fact a norm on $H_2(M; \mathbb{Z})$.*

Now that we have established a relationship between the Thurston semi-norm and Hempel distance, we turn our attention to the effect of mutating by a pseudo-Anosov map on the Hempel distance of a Heegaard splitting.

4.2 Laminations

Recall that a pseudo-Anosov map has two associated measured foliations. There is a one to one correspondence between measured foliations and measured laminations [20, §11.8; 22]. Thus, a pseudo-Anosov map also has two associated laminations. Like the associated foliations these laminations are known as the *stable* and *unstable laminations* of the pseudo-Anosov map. It is also possible to define these laminations independently of the associated foliations (See [5, Thm. 5.5]). The stable and unstable laminations of a pseudo-Anosov map are points in the space of projective measured laminations.

The space of *projective measured laminations* $PML(S_g)$ is constructed from the set of measured laminations $ML(S_g)$ in two steps. First, $ML(S_g)$ is endowed with the weak*-topology. Then, the resulting space is quotiented by the scaling action of $\mathbb{R}^+$.

A set of compression curves $\mathcal{V}_X$ can be viewed as a subset of $PML(S_g)$ by simply applying the counting measure to each curve [14, §2]. We will use $\overline{\mathcal{V}_X}$ to denote the closure of $\mathcal{V}_X$ in $PML(S_g)$. Thus, for a given pseudo-Anosov map φ and a given genus-g handlebody X we can ask whether or not the (un)stable lamination of φ is in $\overline{\mathcal{V}_X}$. Hempel showed that repeatedly twisting a Heegaard splitting by a pseudo-Anosov map will increase the Hempel distance if neither the stable nor the unstable lamination of the pseudo-Anosov map is in the closure of the compression curves two handlebodies:

Theorem 4.2.1 (Hempel [17, p. 640] See also [1, §2])**.** *Let X and Y be genus-g handlebodies with their boundaries parametrized by maps to S_g and let $\varphi\colon S_g \to S_g$ be a pseudo-Anosov map with stable lamination s and unstable lamination u. If s and u are not in $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$, then the distance between $\mathcal{V}_X$ and $\varphi^n(\mathcal{V}_Y)$ tends to infinity,*

$$\lim_{n\to\infty} d\left(\mathcal{V}_X, \varphi^n(\mathcal{V}_Y)\right) = \infty.$$

It is worth noting that $(S^g, \mathcal{V}_X, \varphi^n(\mathcal{V}_Y))$ is a the Heegaard splitting of the mutant manifold that results from mutating $X \cup Y$ by φ^n along the Heegaard surface ∂X. We would like to use Hempel's theorem to make statements about mutations of $S^1 \times S^2$ by pseudo-Anosov maps. However, we must first verify that $S^1 \times S^2$ admits Heegaard splittings of the appropriate form.

4.3 Heegaard splittings

The goal of this section is to prove the following lemma:

Lemma 4.3.1. *Let $\varphi\colon S_g \to S_g$ be a pseudo-Anosov map with stable lamination s and unstable lamination u. Then there exists a genus-g Heegaard splitting $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ of $S^1 \times S^2$ such that s and u are not in $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$.*

In order to prove this lemma, we need to further explore both the geometry of the curve complex and the topology of the space of projective measured laminations. The next two sections outline the relevant facts about these two subjects.

4.3.1 Curve complex revisited

In Section 4.1, we defined the curve complex and its metric. In this section, we will take a closer look at the geometry of this complex. We begin with the fact that that $C(S_g)$ is δ-hyperbolic:

Theorem 4.3.2 (Masur-Minsky [29, Thm. 1.1])**.** *The curve complex $C(S_g)$ is a δ-*hyperbolic *space* i.e. *there exists a $\delta \geq 0$ (depending on the genus g) such that for any geodesic triangle in $C(S_g)$ each side is contained in a δ-neighborhood of the other two.*

The fact that the curve complex is δ-hyperbolic allows us to use techniques of Gromov to construct a boundary at infinity:

Definition 4.3.3 (Gromov boundary [11])**.** Choose a base point $c_0 \in C(S_g)$ and define the *Gromov product* on $C(S_g)$ as follows

$$(x \cdot y) = \frac{1}{2}(d(c_0, y) + d(c_0, y) - d(x, y))$$

A sequence $\{x_n\}_{n=1}^{\infty}$ of points in $C(S_g)$ is said to *converge at infinity* if

$$\lim_{n,m\to\infty}(x_n \cdot x_m) = \infty.$$

This property is independent of the choice of base point. We call two such sequences of points $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ *equivalent* if the limit $\lim_{n,m\to\infty}(y_n \cdot x_m)$ is infinite. The *Gromov boundary* of $C(S_g)$ is the set of equivalence classes of sequences that converge at infinity and is denoted by $\partial_\infty C(S_g)$.

Recall that the compression curves of a genus-g handlebody correspond to points in the curve complex. These sets of curves are quasi-convex subsets of $C(S_g)$:

Theorem 4.3.4 (Masur-Minsky [30, Thm. 1.1]). *Let X be a genus-g handlebody with boundary parametrized by S_g. Then the compression curves of X, $\mathcal{V}_X$, form a K-quasi-convex subset of $C(S_g)$* i.e. *any geodesic arc between two points in $\mathcal{V}_X$ stays within a K-neighborhood of $\mathcal{V}_X$. Furthermore, the constant K depends only on the genus g.*

Moreover, we have the following theorem about quasi-convex subsets of δ-hyperbolic spaces:

Theorem 4.3.5 (Abrams-Schleimer [1, Lem. 9.2]). *Let C be a δ-hyperbolic space and V and W be K-quasi-convex subsets. Then, there is a constant R, depending only on δ and K, such that: if $\{v_i\} \subset V$ and $\{w_j\} \subset W$ converge to the same point of $\partial_\infty C$, then $d(V, W) < R$.*

Thus, if two sets of compression curves share a limit point in $\partial_\infty C(S_g)$, then they must be close together:

Corollary 4.3.6. *Let X and Y be genus-g handlebodies with boundary parametrized by S_g. There exists a constant R depending only on g such that: if $\{x_i\} \subset \mathcal{V}_X$ and $\{y_j\} \subset \mathcal{V}_Y$ converge to the same point of $\partial_\infty C$, then $d(\mathcal{V}_X, \mathcal{V}_Y) < R$.*

4.3.2 Laminations revisited

We now return to the space of projective measured laminations $PML(S_g)$. There is a natural action of the mapping class group on this space (See [20, §11.10]). Moreover, this action is minimal:

Theorem 4.3.7 (Thurston [9, Thm. 6.19]). *The action of the mapping class group* $\mathrm{Mod}(S_g)$ *on $PML(S_g)$ is minimal i.e. the orbit of each point is dense.*

Corollary 4.3.8. *The set of stable laminations of pseudo-Anosov maps on S_g is dense in $PML(S_g)$.*

Proof. This follows from Theorem 4.3.7 and the fact that conjugating a pseudo-Anosov map by a representative of another mapping class produces a new pseudo-Anosov map whose stable lamination is simply the translation of the old lamination by the conjugating map. □

By contrast, the set of compression curves of a genus-g handlebody is nowhere dense in $PML(S_g)$:

Theorem 4.3.9 (Masur [28, Thm. 1.2]). *Let X be a genus-g handlebody. Then, the closure $\overline{\mathcal{V}_X} \subset PML(S_g)$ is connected and has empty interior.*

The stable and unstable laminations of a pseudo-Anosov map φ are not only paired through their association with φ, but through their behavior with respect to sets of compression curves:

Theorem 4.3.10 (Biringer-Johnson-Minsky [4, Thm. 1.1]). *Let $\varphi\colon S_g \to S_g$ be a pseudo-Anosov map and let X be a genus-g handlebody with boundary parametrized by a map to S_g. Then the stable lamination (respectively the unstable lamination) of φ lies in $\overline{\mathcal{V}_X}$ if and only if φ has a power that partially extends to X.*

For our purpose, it doesn't matter what is means for a surface diffeomorphism to partially extend to a handlebody. What matters is the fact that the same condition determines whether or not the stable and the unstable lamination of a given pseudo-Anosov map are in the closure of a set of compression curves. Thus, we have the following corollary:

Corollary 4.3.11. *The stable lamination of pseudo-Anosov map lies in the closure of a set of compression curves if and only if the unstable lamination also lies in the closure.*

The connection between the space of projective measured laminations and the curve complex runs much deeper than the fact that isotopy classes of curves can be viewed as points in either space. In particular, their is a correspondence between sequences of curves that converge in $PML(S_g)$ and those that converge to points in $\partial_\infty C(S_g)$:

Theorem 4.3.12 (Klarreich [21, Thm. 1.4]. See also [13, Thm. 1])**.** *Let $\ell \in PML(S_g)$ be both minimal and filling. Thus, we require that each of ℓ's half leaves is dense and that every simple closed geodesic on S_g intersect ℓ transversely. If a sequence of isotopy classes of curves $\{c_1\}_{i=1}^{\infty}$ converges in $PML(S_g)$ to ℓ, then the sequence converges at infinity to a point in $\partial_\infty C(S_g)$.*

Corollary 4.3.13. *If a sequence of isotopy classes of curves converges in $PML(S_g)$ to the (un)stable lamination of a pseudo-Anosov map, then the sequence converges at infinity to a point in $\partial_\infty C(S_g)$.*

Proof. Clearly, it is enough to show that the (un)stable lamination of a pseudo-Anosov map is both minimal and filling. Casson and Bleiler show that these laminations are minimal in their book on the Nielsen Thurston classification of surfaces[7, Lem. 14.11]. The fact that these laminations are filling follows from the fact that pseudo-Anosov maps are irreducible [42]. See also [7, Thm. 13.2; 9, Thm. 9.16]. □

We are now ready to prove the lemma from the beginning of this section.

4.3.3 Proof of Lemma 4.3.1

Lemma 4.3.1. *Let $\varphi\colon S_g \to S_g$ be a pseudo-Anosov map with stable lamination s and unstable lamination u. Then there exists a genus-g Heegaard splitting $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ of $S^1 \times S^2$ such that s and u are not in $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$.*

Proof. For an arbitrary handlebody X, the stable lamination s is in $\overline{\mathcal{V}_X}$ if and only if the unstable lamination u is also in $\overline{\mathcal{V}_X}$ (Cor. 4.3.11). Thus, it is enough to find a Heegaard splitting of $S^1 \times S^2$ such that s is not in the closure of either set of compression curves.

Let $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ be a genus-g Heegaard splitting of $S^1 \times S^2$. The union $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$ is nowhere dense in $PML(S_g)$ (Thm. 4.3.9). Furthermore, The stable laminations of pseudo-Anosov elements of $\mathrm{Mod}(S_g)$ form a dense subset of $PML(S_g)$ (Cor. 4.3.8).

Thus, there exists a pseudo-Anosov map $\psi : S_g \to S_g$ with stable lamination t such that t is not in $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$ and t is not equal to s or u.

We will now show that translating the set $\mathcal{V}_X$ by a high power of ψ will move it away from s. By Theorem 4.2.1, we have that for any n the distance $d(\psi^n(\mathcal{V}_X), \psi^{n+m}(\mathcal{V}_X))$ goes to infinity as m grows. Thus, it is enough to show that if s is a limit point of both $\psi^n(\mathcal{V}_X)$ and $\psi^{n+m}(\mathcal{V}_X)$ in $PML(S_g)$, then these sets must be close together in the curve complex.

Suppose s is an element of both $\overline{\psi^n(\mathcal{V}_X)}$ and $\overline{\psi^{n+m}(\mathcal{V}_X)}$. Let (a_i) and (b_i) be sequences of points in $\psi^n(\mathcal{V}_X)$ and $\psi^{n+m}(\mathcal{V}_X)$ respectively that converge to s in $PML(S_g)$. It follows from work of Klarreich that the sequences (a_i) and (b_i) converge to the same point in the Gromov boundary of the curve complex $C(S_g)$ (Cor. 4.3.13). This in turn implies that the Hempel distance between $\psi^n(\mathcal{V}_X)$ and $\psi^{n+m}(\mathcal{V}_X)$ is bounded above by a constant K which depends only on the genus g (Cor. 4.3.6).

Therefore, there exists an $M \in \mathbb{N}$ such that s is not in $\overline{\psi^n(\mathcal{V}_X)}$ for all $n > M$. Similarly, translating $\mathcal{V}_Y$ by a high power of ψ will move it away from s. Thus, there exists an N such that s is not in $\overline{\psi^N(\mathcal{V}_X)} \cup \overline{\psi^N(\mathcal{V}_Y)}$. By construction, $(S_g, \psi^N(\mathcal{V}_X), \psi^N(\mathcal{V}_Y))$ is a Heegaard splitting for $S^1 \times S^2$. $\square$

We are now ready to prove the proposition from the beginning of this chapter.

4.4 Proof of Proposition 4.0.12

Proposition 4.0.12. *Let $[\varphi] \in \mathcal{I}(S_g)$ be a pseudo-Anosov element of the Torelli group. Then, there exists a natural number N and a manifold-surface pair (M, f) such that $M = S^1 \times S^2$ and the mutant manifold $M_f^{\varphi^N}$ has a homology class with nonzero Thurston semi-norm.*

Proof. Let $s, u \in PML(S_g)$ be respectively the stable and unstable laminations of φ. Also, let $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$ be a genus-g Heegaard splitting of $S^1 \times S^2$ such that s and u are not in $\overline{\mathcal{V}_X} \cup \overline{\mathcal{V}_Y}$. The existence of such a splitting is guaranteed by Lemma 4.3.1. Finally, let (M, f) be the manifold-surface pair where $M = S^1 \times S^2$ and f is the embedding of S_g as the Heegaard surface ∂X from the splitting $(S_g, \mathcal{V}_X, \mathcal{V}_Y)$.

By Theorem 4.2.1, we have that

$$\lim_{n\to\infty} d\left(\mathcal{V}_X, \varphi^n(\mathcal{V}_Y)\right) = \infty.$$

Thus, there exists a natural number N such that $d\big(\mathcal{V}_X, \varphi^N(\mathcal{V}_Y)\big) > 2$. Furthermore, $\big(S_g, \mathcal{V}_X, \varphi^N(\mathcal{V}_Y)\big)$ is a Heegaard splitting for the mutant $M_f^{\varphi^N}$. This implies that $M_f^{\varphi^N}$ is irreducible and has no essential tori (Lem. 4.1.2).

A simple calculation shows that $H_2(M;\mathbb{Z}) = H_2(S^1 \times S^2;\mathbb{Z}) \cong \mathbb{Z}$. It follows that $H_2(M_f^{\varphi^N};\mathbb{Z}) \cong \mathbb{Z}$, because $[\varphi]$ is in the Torelli group (Lem. 4.0.13). Let ω be a nonzero element of $H_2(M_f^{\varphi^N};\mathbb{Z}) \cong \mathbb{Z}$ and let $F \subseteq M_f^{\varphi^N}$ be a surface that represents ω. Because $M_f^{\varphi^N}$ is irreducible and has no essential tori, the genus of F must be at least 2. It follows that the Thurston semi-norm of $\omega = [F] \in H_2(M_f^{\varphi^N};\mathbb{R})$ is nonzero. $\square$

Chapter 5

Detection results

We are now ready to prove Theorems 1.0.1 and 1.0.3. The former is proved in Section 5.1 and the latter is proved in Section 5.2.

5.1 Nontorsion Spin^c-structures

There are two steps remaining in the proof of Theorem 1.0.1. The first is to show that the genus-2 hyperelliptic involution is not $\widehat{HF}_{\mathrm{nT}}$-invisible and the second is to combine Propositions 3.1.1 and 4.0.12 to show that no non-central element of $\mathrm{Mod}(S_g)$ is $\widehat{HF}_{\mathrm{nT}}$-invisible.

5.1.1 Genus-two hyperelliptic involution

In this section, we will show that mutating by the genus-2 hyperelliptic involution, τ can change the rank of the non-torsion summands of $\widehat{HF}$. To accomplish this, we will show that mutations of this form can change the Thurston semi-norm of a manifold obtained by performing zero-surgery on a knot in S^3. The Thurston semi-norm of such a manifold is determined by the Seifert genus of the knot:

Theorem 5.1.1 (Gabai [10, Cor. 8.3])**.** *If M is obtained by performing zero-surgery on a knot K in S^3, then*

$$\text{genus}(K) = \min\{\text{genus}(F) \mid F \textit{ is a nonseparating, oriented, embedded surface in } M\}.$$

With this in mind, we are ready to prove the following proposition:

Proposition 5.1.2. *The genus-2 hyperelliptic involution is not $\widehat{HF}_{\text{nT}}$-invisible.*

Proof. We consider the pair of mutant knots that form the basis of Moore and Starkston's examples of mutations by the genus-2 hyperelliptic involution [31]. Let K and K^τ be the knots denoted respectively by 14^n_{22185} and 14^n_{22589} in Knotscape notation (Fig. 3). These two knots are related by a mutation of S^3 by the genus-2 hyperelliptic involution along the surfaces depicted in Figure 3. From the computations of $\widehat{HFK}$ in Table 1, we see that K has genus two and K^τ has genus one (Thm. 2.2.1).

Now, let M and M^τ be the results of zero-surgery on K and K^τ respectively. Because the mutation of S^3 that transforms K into K^τ involves a surface that is disjoint from the knot, there is a corresponding surface in M. Moreover, mutating M along that corresponding surface by the genus-2 hyperelliptic involution will result in an manifold diffeomorphic to M^τ.

A Mayer-Vietoris argument shows that both $H_2(M;\mathbb{Z})$ and $H_2(M^\tau;\mathbb{Z})$ are isomorphic to $\mathbb{Z}$. Furthermore, it follows from the work of Gabai that the genera of the knots K and K^τ determine the Thurston semi-norm on these homology groups (Thm. 5.1.1). In particular, the semi-norm is constantly zero on $H_2(M^\tau;\mathbb{Z})$ and nonzero on $H_2(M;\mathbb{Z})\backslash\{0\}$. This implies that $\widehat{HF}(M^\tau)$ is supported entirely in the Spin^c-structure whose first Chern class is zero (Thm. 2.1.5) and $\widehat{HF}(M)$ is nontrivial in at least one Spin^c-structure with nonzero first Chern class (Thm. 2.1.6). □

Figure 3: Pair of mutant knots $K_0 = 14^n_{22185}$ and $K_1 = 14^n_{22589}$

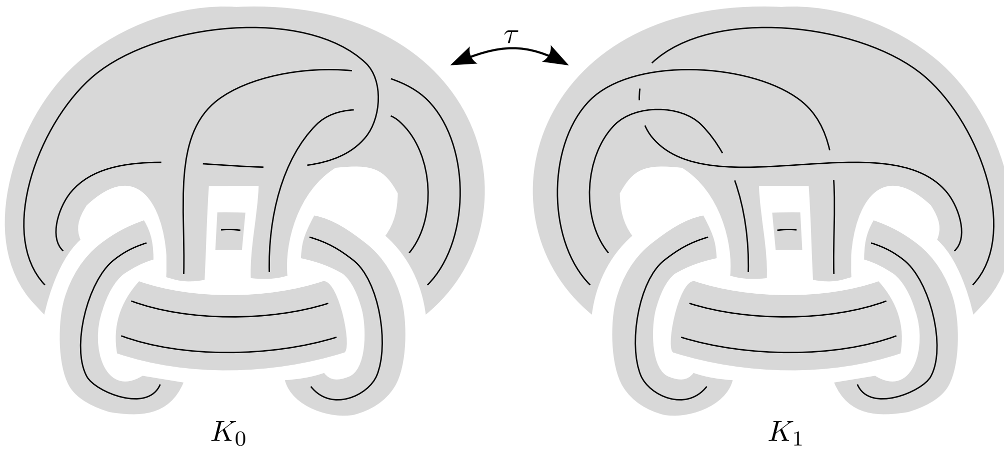


Table 1: Knot Floer groups for $K_0 = 14^n_{22185}$ and $K_1 = 14^n_{22589}$.

$\widehat{HFK}_*(K, n) \otimes \mathbb{F}$

n	-2	-1	0	1	2
K_0	$\mathbb{F}^2$	$\mathbb{F}^4$	$\mathbb{F}^5$	$\mathbb{F}^4$	$\mathbb{F}^2$
K_1		$\mathbb{F}^4$	$\mathbb{F}^9$	$\mathbb{F}^4$	

Here $\mathbb{F}$ is the field with two elements and n is the Alexander grading. This computation was completed by Moore and Starkston in [Table 1, 31] using software developed by Droz [6].

5.1.2 General mutations

Theorem 1.0.1. *Let φ be an orientation preserving self-diffeomorphism of S_g that is not isotopic to the identity map. Then, there exists a manifold-surface pair (M, f) such that*

$$\operatorname{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M, \mathfrak{s}) \neq \operatorname{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M_f^{\varphi}, \mathfrak{s})$$

Proof. Let $G \lhd \operatorname{Mod}(S_g)$ be the set of $\widehat{HF}_{\mathrm{nT}}$-invisible mapping classes. We begin by showing that G contains no pseudo-Anosov element of the Torelli group. Let $[\varphi] \in \mathcal{I}(S_g)$ be a pseudo-Anosov element of the Torelli group. Also, let (M, f) be a manifold-surface pair such that $M = S^1 \times S^2$ and for some $N \in \mathbb{N}$ the mutant manifold $M_f^{\varphi^N}$ has a homology class with nonzero Thurston semi-norm. The existence of such a pair is guaranteed by Proposition 4.0.12.

A simple computation shows that the Heegaard Floer homology of $M = S^1 \times S^2$ is isomorphic to $\mathbb{Z} \oplus \mathbb{Z}$ and is supported entirely in the Spin^c-structure whose first Chern class is zero (Prop. 2.1.7). Thus, the rank of the non-torsion summands of $\widehat{HF}(M)$ is zero

$$\operatorname{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M, \mathfrak{s}) = 0.$$

By construction, $M_f^{\varphi^N}$ has a homology class with nonzero Thurston semi-norm. It follows that $\widehat{HF}(M_f^{\varphi^N})$ is nontrivial in at least one Spin^c-structure with nonzero first Chern class (Thm. 2.1.6). In particular, the rank of the non-torsion summands is positive

$$\operatorname{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M_f^{\varphi^N}, \mathfrak{s}) > 0$$

and therefore

$$\operatorname{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M, \mathfrak{s}) \neq \operatorname{rk} \bigoplus_{c_1(\mathfrak{s})\neq 0} \widehat{HF}(M_f^{\varphi^N}, \mathfrak{s}).$$

Thus, the mapping class $[\varphi^N] = [\varphi]^N$ is not $\widehat{HF}_{\mathrm{nT}}$-invisible. Because the $\widehat{HF}_{\mathrm{nT}}$-invisible mapping classes form a subgroup of $\operatorname{Mod}(S_g)$, we concluded that $[\varphi]$ is also not

$\widehat{HF}$-invisible (Prop. 3.0.4). Therefore, no pseudo-Anosov element of the Torelli group is an element of G.

Furthermore, we showed in Propositions 3.0.4 and 5.1.2 respectively that G is normal and does not contain the genus-2 hyperelliptic involution. Hence, G is trivial by Proposition 3.1.1. □

5.2 Total rank

Theorem 1.0.3. *Let φ be an orientation preserving self-diffeomorphism of S_g that is isotopic to neither the identity nor the genus-2 hyperelliptic involution. Then there exists a manifold-surface pair (M, f) such that*

$$\text{rk } \widehat{HF}(M) \neq \text{rk } \widehat{HF}(M_f^{\varphi}).$$

Proof. Let $G \lhd \text{Mod}(S_g)$ be the normal subgroup of $\widehat{HF}_{\text{nT}}$-invisible mapping classes. By Proposition 3.1.1, it suffices to show that G contains no pseudo-Anosov elements of the Torelli group.

Let $[\varphi] \in \mathcal{I}(S_g)$ be a pseudo-Anosov element of the Torelli group. Also let (M, f) be a manifold-surface pair such that $M = S^1 \times S^2$ and for some $N \in \mathbb{N}$ the mutant manifold $M_f^{\varphi^N}$ has a homology class with nonzero Thurston semi-norm. The existence of such a pair is guaranteed by Proposition 4.0.12.

Let T be the result of zero-surgery on the trefoil. Hedden and Ni showed that T is the only closed, orientable, irreducible three-manifolds with nonzero first Betti number and $\text{rk}\,\widehat{HF} = 2$ (2.1.8). In the proof of Proposition 4.0.12, we showed that the mutant $M_f^{\varphi^N}$ is closed, orientable and irreducible, and its first Betti number is nonzero. Thus, it is enough to show that $M_f^{\varphi^N}$ is not diffeomorphic to T.

A Mayer-Vietoris argument shows that $H_2(T; \mathbb{Z}) \cong \mathbb{Z}$. The Thurston semi-norm is constantly zero on $H_2(T; \mathbb{Z})$, because the trefoil is a genus-1 knot (Thm. 5.1.1). Therefore, the Thurston semi-norm differentiates $M_f^{\varphi^N}$ from T. □

Chapter 6

Implications

There are two ways to interpret Theorems 1.0.1 and 1.0.3 as statements about actions of mapping class groups of surfaces on categories. The first uses bordered Heegaard Floer homology and results in a statement about an action on a category of $\mathcal{A}_\infty$-modules. The second uses the definition of $\widehat{HF}$ and results in a statement about an action on a Fukaya category.

6.1 Bordered Heegaard Floer homology

In [25] and [23], Lipshitz, Ozsváth and Thurston developed a variant of Heegaard Floer homology for three-manifolds with parametrized boundary called *bordered Heegaard Floer homology*. These bordered invariants are related to $\widehat{HF}$ by pairing theorems [23, Thm. 1.3; 25, Thm. 11]. The pairing theorems provide a method for computing $\widehat{HF}(M)$ by cutting M along separating surfaces and computing the bordered Heegaard Floer homology of the resulting components. By applying this method to manifold-surface pairs and their mutants, we can use Theorem 1.0.1 to infer information about the bordered Heegaard Floer homology of mapping cylinders of surface diffeomorphisms.

Let $\mathrm{Mod}_0(S_g)$ denote the *strongly based mapping class group* of S_g, that is the isotopy classes of diffeomorphisms that fix a given disk in S_g. There is a canonical projection

$$p\colon \operatorname{Mod}_0(S_g) \to \operatorname{Mod}(S_g)$$

given by quotienting out by the copy of $\pi_1(S_g)$ that corresponds to pushing the disk around closed curves in S_g as well as by the Dehn twist around the boundary of the disk. Following [23, §8], we assign to each strongly based mapping class $[\varphi] \in \operatorname{Mod}_0(S_g)$ the bimodule $\widehat{CFDA}(\varphi, 0)$ associated to its mapping cylinder equipped with the middle Spin^c-structure. By considering Theorem 1.0.1 from the perspective of bordered Heegaard Floer homology, we get the following result about these bimodules:

Corollary 6.1.1. *If $[\varphi] \in \operatorname{Mod}_0(S_g)$ is a strongly based mapping class such that $[\varphi]$ is not in the kernel of p, then the action of $[\varphi]$ on the category of $\mathcal{G}(Z)$-graded $\mathcal{A}(Z)$-modules given by tensoring with $\widehat{CFDA}(\varphi, 0)$ is not the trivial action. In particular, $\widehat{CFDA}(\varphi, 0)$ is not homotopy equivalent to $\widehat{CFDA}(\mathrm{id}, 0) = \mathcal{A}(Z)$.*

Proof. Let $[\varphi] \in \operatorname{Mod}_0(S_g)$ such that $[\varphi]$ is not in the kernel of p. Also, let (M, f) be a manifold-surface pair such that the rank of the non-torsion summands of $\widehat{HF}(M)$ differs from that of $\widehat{HF}(M_f^{\varphi})$. The existence of such a pair is guaranteed by Theorem 1.0.1. Finally, let M_1 and M_2 be the connected components of $M \setminus f(S_g)$.

The Heegaard Floer homology of M can be computed from the bordered invariants of M_1 and M_2 as follows

$$\widehat{HF}(M) \cong H_*\left(\widehat{CFA}(M_1) \,\widetilde{\otimes}\, \widehat{CFD}(M_2)\right)$$

where $\widetilde{\otimes}$ is the $\mathcal{A}_\infty$-tensor product over $\mathcal{A}(Z)$.

Similarly, decomposing the mutant manifold M_f^{φ} as the union $M_1 \cup C_\varphi \cup M_2$ where C_φ is the mapping cylinder of φ corresponds to the following module decomposition of $\widehat{HF}(M_f^{\varphi})$.

$$\widehat{HF}(M_f^{\varphi}) \cong H_*\left(\widehat{CFA}(M_1) \,\widetilde{\otimes}\, \widehat{CFDA}(\varphi, 0) \,\widetilde{\otimes}\, \widehat{CFD}(M_2)\right)$$

Thus, the difference between $\widehat{HF}(M)$ and $\widehat{HF}(M_f^\varphi)$ must result from the effect of tensoring with $\widehat{CFDA}(\varphi, 0)$. Therefore, the action of $[\varphi]$ on $\mathcal{A}(Z)$-modules given by tensoring with $\widehat{CFDA}(\varphi, 0)$ must not be the trivial action. □

A similar reformulation of Theorem 1.0.3 gives the following result about the action of $\mathrm{Mod}_0(S_g)$ on the category of ungraded $\mathcal{A}(Z)$-modules.

Corollary 6.1.2. *If $[\varphi] \in \mathrm{Mod}_0(S_g)$ is a strongly based mapping class such that $p([\varphi])$ is neither the identity nor the genus-2 hyperelliptic involution, then the action of $[\varphi]$ on the category of ungraded $\mathcal{A}(Z)$-modules given by tensoring with $\widehat{CFDA}(\varphi, 0)$ is not the trivial action.*

Lipshitz, Ozsváth and Thurston proved a similar result:

Theorem 6.1.3 (Lipshitz-Ozsváth-Thurston [24, Thm. 1])**.** *If $[\varphi] \in \mathrm{Mod}_0(S_g)$ is a nontrivial strongly based mapping class, then the action of $[\varphi]$ on the category of ungraded $\mathcal{A}(Z)$-modules given by tensoring with $\widehat{CFDA}(\varphi, \pm(g - 1))$ is not the trivial action.*

There are two main differences between Theorem 6.1.3 and the results of this section. The first is that the bordered Heegaard Floer modules used to define the actions correspond to different Spin^c-structures. Corollaries 6.1.1 and 6.1.2 pertain to the middle Spin^c-structure whereas Theorem 6.1.3 pertains to the second to extremal Spin^c-sturcutes. The second difference is that Theorem 6.1.3 establishes that the action is faithful, whereas Corollaries 6.1.1 and 6.1.2 only limit the mapping classes that can be in the kernel of the action.

6.2 Fukaya categories

When viewed from another perspective, the work of Lipshitz, Ozsváth and Thurston shows that the strongly based mapping class group $\mathrm{Mod}_0(S_g)$ acts freely on a version of the Fukaya category of S_g with a disk removed as well as on a version of the Fukaya category of the $(2g-1)$-fold symmetric product $\mathrm{Sym}^{2g-1}(S_g - D)$ [2].

Theorem 1.0.3 is also related to mapping class group actions on Fukaya categories. In particular, the chain complex that underlies $\widehat{HF}$ of a three-manifold with a genus-g Heegaard splitting corresponds to a morphism group in the Fukaya category of the g-fold symmetric product of S_g with a point removed. Furthermore, the action of the based mapping class group of S_g on the symmetric product $\mathrm{Sym}^g(S_g - z)$ induces a strict action on the Fukaya category $\mathrm{Fuk}(\mathrm{Sym}^g(S_g - z))$ [39, §10b].

Corollary 6.2.1. *If* $[\varphi] \in \mathrm{Mod}(S_g - z)$ *is a based mapping class such that the corresponding element of* $\mathrm{Mod}(S_g)$ *is neither the identity nor the genus-2 hyperelliptic involution, then the action of* $[\varphi]$ *on the Fukaya category* $\mathrm{Fuk}(\mathrm{Sym}^g(S_g - z))$ *is not the trivial action. In particular, the map induced by* φ *on* $\mathrm{Sym}^g(S_g - z)$ *is not Hamiltonian isotopic to the identity.*

Proof. Let $[\varphi] \in \mathrm{Mod}(S_g - z)$ be a based mapping class such that the corresponding element of $\mathrm{Mod}(S_g)$ is neither the identity nor the genus-2 hyperelliptic involution. Also, let (M, f) be a manifold-surface pair such that $f(S_g)$ is a Heegaard surface and

$$\mathrm{rk}\ \widehat{HF}(M) \neq \mathrm{rk}\ \widehat{HF}(M_f^\varphi).$$

The existence of such a manifold is guaranteed by the fact that the proof of Theorem 1.0.3 only uses manifold-surface pairs where the embedded surface is a Heegaard surface. Finally, let T_α and T_β be the corresponding Heegaard tori in $\mathrm{Sym}^g(S_g - z)$.

The action of $[\varphi]$ on $\mathrm{Fuk}(\mathrm{Sym}^g(S_g - z))$ sends T_β to $T_{\varphi(\beta)}$, the Heegaard torus that results from translating the curves of β by φ. Furthermore, T_α and $T_{\varphi(\beta)}$ are the Heegaard tori of a splitting of the mutant manifold M_f^φ. It then follows from the definitions that

$$\widehat{CF}(M) = \mathrm{Mor}(T_\alpha, T_\beta) \text{ and } \widehat{CF}(M_f^\varphi) = \mathrm{Mor}(T_\alpha, T_{\varphi(\beta)}).$$

Because $\widehat{HF}(M)$ and $\widehat{HF}(M_f^\varphi)$ do not have the same rank, we concluded that their underlying chain complexes $\widehat{CF}(M)$ and $\widehat{CF}(M_f^\varphi)$ are not quasi-isomorphic. Thus, the morphism groups $\mathrm{Mor}(T_\alpha, T_\beta)$ and $\mathrm{Mor}(T_\alpha, T_{\varphi(\beta)})$ are not quasi-isomorphic. Therefore, T_β is not isomorphic to $T_{\varphi(\beta)}$. □

It should also be possible to reformulate Theorem 1.0.1 as a statement about an action of the based mapping class group of S_g on a version of the Fukaya category of $\mathrm{Sym}^g(S_g - z)$. Such a reformulation would likely require working with grading data like that described in [40]. We will return to this in a future paper.

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