Monetary Policy and Transmission of Bubbles

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Abstract

The aim of this paper is to investigate the optimal monetary policy when bubbles boost and burst. The monetary policy in the form of open market operation has real effects, that is, influences the growth in investment and the size of bubbles. The central bank faces a trade-off between stimulating investment and appreciating bubbles. The optimal policy is contingent on the state of bubbles. When bubbles arise, the central bank may maintain or gives up easing, depending on how it puts weight on the state of the bursting of bubbles, while when bubbles burst, the central bank takes an easing policy. The optimal policy is the same irrespective of whether foreign capital inflows are allowed for unless capital markets are severely restricted.

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1. Introduction

A widespread view is that the FRB was too late to pull up the interest rate to repress bubbles. For example, Taylor (2007) criticized the FRB that the target federal funds rate using Taylor rule should have been higher by 2 percent than the actual rate for 2003-2007.

The former governor of the FRB, Greenspan objected to this view by arguing that the central bank has no superior information to detect bubbles until bubbles burst, and that killing bubbles might also kill the economic boom. He further commented that a question is if the central bank has any useful tool of controlling bubbles. In 1999, the FRB raised the federal funds rate to repress dot-com bubbles, but failed.

A great concern is what the optimal monetary policy is for the central bank when bubbles boost and burst. The traditional Fed view is against the role of the policy reacting to bubbles. For example, Bernanke and Gerlter (1999) argue that it is difficult for the policy to react simultaneously to bubbles and inflation. They further state that the policy using Taylor rule that reacts to asset prices can destabilize the economy. On the other hand, some argument suggests a positive role of the policy from the prospect of minimizing the cost of financial crisis that would occur if bubbles burst.¹

Does optimal monetary policy exist to control bubbles? If it exists, should it lean on or against bubbles? We answer this question by analyzing the model of rational bubbles, where bubbles can arise if the interest rate is less than the growth rate (e.g., Tirole 1985, and Santos and Woodford 1997).

Figure 1A and 1B illustrate several interest rates and the economic growth rate (in real terms) in the US and Japan, two countries that experienced large bubbles in the past decades.² The choice of the appropriate interest rates is not an easy job, and we use five interest rates. They include the money market rate, Treasury bill rate, the deposit interest rate, the interest rate on long-term bonds, and the lending rate. Interestingly, all

¹ See for example, Adrian and Shin (2008).
² The source of data is International Financial Statistics.
the interest rates except for the lending rate are below the economic growth rate over the periods of bubble episodes in both countries, 2003-2007 in the US and 1986-1991 in Japan. These observations are roughly consistent with the prediction of the theory of rational bubbles.

We provide a model of multiple liquidities where the key ingredient of the model is the asset substitution among the security backed by firm’s value, bubbly liquidity, and liquidity backed by fundamentals. We model the monetary policy in the form of open market operation that indirectly changes the return on the price of fundamental-backed liquidity. Once bubbles arise, maintaining easing stimulates investment and at the same time leads to the boost in bubbles that makes the damages of the bursting excessively severe. Once bubbles burst, easing can lessen the magnitude of the recession. The optimal monetary policy is contingent on the state of bubbles.

Contributions of this paper are as follows. First, the monetary policy in the form of open market operation has real effects, that is, influences the growth in investment and the size of bubbles. If the central bank delays giving up monetary easing, the economy should have excessive bubbles that would magnify the recession on the onset of the bursting of bubbles. On the other hand, if the central bank conducts the excessive monetary tightening, the boom as well as bubbles could collapse.

Second, the optimal monetary policy is contingent on the state of bubbles. When bubbles arise, the central bank will tighten the stance, and when bubbles burst, it conducts easing.

Third, we evaluate monetary reactions of the central bank when bubbles boost and burst in the US and Japan. Following theoretical findings, the central bank in the US did a pretty good job, but the central bank in Japan committed several mistakes.
Forth, the optimal policy is the same irrespective of whether foreign capital inflows are allowed for. Foreign capital appreciates bubbles, but does not contribute to stimulating domestic investment. Despite this, the optimal policy is to lean on bubbles unless capital markets are severely restricted.

This paper is related to several contributions that investigate the role of store of values in macroeconomics, including Tirole (1985), Woodford (1990), Kiyotaki and Moore (1997), and Holmstrom and Tirole (1998). Distinguishable is the fact that this paper introduces the monetary policy into the literature, showing that the monetary policy has real effects in the flexible price model.

This paper is closely related to the literature on the debate as to whether the government should lean on or against bubbles. Some argue that the central bank should not pay attention to asset prices. The literature includes Bernanke and Gertler (1999, 2001), Bernanke (2002), Greenspan (2002), Mishkin (2008) among others. Others are in favor of leaning against bubbles, including Blanchard (2000), Bordo and Jeanne (2002), Borio and Lowe (2002), Borio and White (2003), Cecchetti, Genberg, Lipsky and Wadhwani (2000), Cecchetti, Genberg and Wadhwani (2002), among others. Models of this literature have no theoretical foundation for bubbles, and the debate hinges on the simple premise that (not) raising the interest rate implies leaning against (on) bubbles. Our policy rule is simple but more sophisticated in that it involves leaning on bubbles and raising the interest rate simultaneously.

2. The Model

Consider a model of rational bubbles developed by Farhi and Tirole (2012). In each period a unit mass of agents are born, and live for three periods. The economy lasts to
infinity. There is no population growth or technological progress. Individual agents are endowed with \( A \) units of goods when young, and with \( l \) units of the Lucas tree when old.\(^4\) Each of the date-t trees pays one unit of good as dividend at date \( t+1 \). Their preference is risk neutral in consumption when old. Each generation is indexed by the period in which it is middle-aged.

As owners of the firm, middle-aged agents have access to one linear investment opportunity that transforms one unit of good into \( R^f (>1) \) units of good after one period. To motivate financial market imperfections, we assume that only part of the return, \( R(< R^f) \), is pledgeable to creditors. Debtors are protected by their limited liability. Assume that \( R < 0.5 \) which is necessary for the existence of the bubbly economy that satisfies the binding borrowing constraint, as will be obvious below.

We model monetary policy as controlling the return on trees. As argued later, changing the price of trees requires the market operation by the central bank to affect the market condition. As will be clear below, a tree is fundamental-backed liquidity that competes with bubbly liquidity and the security issued by firms in the investors’ portfolio. Thus monetary policy is aimed to influence asset substitution through the change in the price of fundamental-backed liquidity. This specification of the monetary policy captures reducing costs of finance for firms, and asset allocation between liquidities, but is not conventional in that it does not capture the cost and benefit of holding fiat money.

Suppose that bubbles burst with probability \( \varepsilon \to 0 \). Investors act as if bubbles do not burst, but the government takes a weight \( 1 - \lambda (>0) \) on the state of bubbles that

\(^4\) An alternative specification is, as in Farhi and Tirole, to assume another type of agents who live for one period and supply trees in the market. The drawback of this is the complexity of the welfare function.
occurs with a probability of measure zero, and behaves as if it believes that bubbles burst with probability $1 - \lambda$. Once bubbles burst, the economy returns to the bubbleless economy forever. Investors in periods of bubbles are very often in the state of euphoria and overconfident in the valuation of assets. The government is supposed to be relatively indifferent to the boom but more risk averse to the downward risk of the bursting of bubbles than investors because financial crisis or the increase in the unemployment triggered by the bursting becomes the reason for the alternation of power.

3. Analysis

The wealth of young agents is used to purchase three assets, securities issued by the previous generation to run the firm, trees, and bubbles. The demand for securities is limited to the present value of the pledgeable asset $R_{i-1}/(1 + r)$ when $i_{-1}$ is invested by the firm of the date $t-1$ generation. The demand for trees is equal to the presented value $l/(1 + r)$. The demand for bubbles is $b_{t-1}$. The asset demand function is given by

$$A = \frac{R_{i-1}}{1 + r} + \frac{l}{1 + r} + b_{t-1}. \tag{1}$$

Middle-aged agents finance investment by the internal wealth and issuing securities. The asset supply function is given by

$$i_t = \frac{R_i}{1 + r_{t+1}} + R_{i-1} + l + b_t. \tag{2}$$

Bubbles evolve as

$$b_{t+1} = (1 + r_{t+1})b_t. \tag{3}$$

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See for example Scheinkman and Xiong (2003) and Hong et al (2006).
We define the competitive equilibrium of a bubbly economy that satisfies the binding borrowing constraint, as a sequence \( \{i_t, b_t, r_t\}_{t=0}^\infty \) that satisfies (1), (2), and (3).

Before introducing bubbles into the model, we study the competitive equilibrium of a bubbleless economy that satisfies the binding borrowing constraint. The steady state is described as

\[
(4) \quad i^D = \frac{R^D + l}{1 - R/(1 + r^D)}, \quad \text{and} \quad i^D = \frac{A(1 + r^D) - l}{R},
\]

**Proposition 1:** The competitive equilibrium of a bubbleless economy that satisfies the binding borrowing constraint is dynamically stable.

**Proof:** Equations (1) and (2) yields the dynamic evolution of investment \( (b_{t+1} = 0) \);

\[
\Sigma(i_t) = i_t(1 - \frac{AR}{R_i + l}) = R_{i+1} + l. \quad \text{The RHS is positive and increasing for} \quad i_{t+1} > 0, \quad \text{while}
\]

the LHS satisfies \( \Sigma(0) = \Sigma(i) = 0 \) and is increasing and convex for \( i_i > i = A - l/R > 0 \), and for \( i < 0 \) satisfies \( \Sigma(0) = 0 \) and is increasing and convex for \( i, i > 0 \). There exists the unique steady state \( i^D > 0 \) that satisfies \( \Sigma(i^D) = R^D + 1 \). For \( i_{t-1} < i^D \),

\[
\Sigma(i_t) = R_{i-1} + l > \Sigma(i_{t-1}), \quad \text{and} \quad i_t \text{ is increasing, while for} \quad i_{t-1} = i^D, \]

\[
\Sigma(i_t) = R_{i-1} + l < \Sigma(i_{t-1}), \quad \text{and} \quad i_t \text{ is decreasing. Investment is stable. Q.E.D.}
\]

**Monetary policy**

The central bank controls the return on trees through market operation. At date \( t \) the central bank buys \( \pi(0 \leq \tau \leq 1) \) units of trees at price \( 1 + r_{t+1} \) by using the revenue.
raised from taxing old consumers. At period t+1 the dividend is transferred to old consumers. In this environment, young investors purchase \((1-\tau)l\) units of trees.

Accordingly, (1) and (2) \((b_i = 0)\) are replaced by

\[
A = \frac{Ri_t}{1+r_{t+1}^L} + \frac{(1-\tau)l}{1+r_{t+1}^L}, \text{ and } i_t = \frac{Ri_t}{1+r_{t+1}^L} + Ri_{t-1} + (1-\tau)l. 
\]

Investment is

\[
i_t = \frac{(1+r_t^L)A}{1-R/(1+r_t^L)} \equiv i(r_t^L) \text{ at the steady state. The function } i(r_t^L) \text{ is defined over } [r, r^D], \text{ and increasing for } 1+r^L > 2R, \text{ and decreasing for } 1+r^L < 2R, \]

where \(r^D\) corresponds to \(\tau = 0\) and \(r\) corresponds to \(\tau = 1\).

**Proposition 2:** The monetary easing stimulates investment for \(1+r^L < 2R\), and represses investment for \(1+r^L > 2R\).

We define the objective function of the central bank that decides the monetary stance. The welfare of the date-t generation is composed of the profit from investment

\((R^f - R)i_t\) and the revenue from selling trees \(l/(1+r_{t+1}^L)\), and the transfer net of taxation \(\tau d - \tau l/(1+r_{t+1}^L)\). The central bank evaluates the welfare of all generations with a social discount rate \(\beta\),

\[
E_0 \sum_{t=0}^{\infty} \beta^t \{ (R^f - R)i_t + \frac{(1-\tau)l}{1+r_{t+1}^L} + \tau d \},
\]

We use (1) to rewrite the date-t agent’s welfare only as a function of investment;

\[
(R^f - R)i_t + \frac{A(1-\tau)l}{Ri_t + (1-\tau)l} \equiv V^D(i_t). \text{ We have } \frac{\partial V^D(i_t)}{\partial i_t} = R^f - R - \frac{A(1-\tau)lR}{(Ri_t + (1-\tau)l)^2}. \text{ We}
\]
focus on the case when agents are better off by investing more. We impose the following assumption to guarantee that \( V^D(\cdot) \) is strictly increasing.

**Assumption 1** \( R^f \) is sufficiently high and \( l \) is sufficiently small.

**Proposition 3:** Suppose that Assumption 1 holds. In the bubbleless economy, the central bank conducts monetary easing by purchasing trees at the maximum size \((\tau = 1)\) if \( r^D < 1 \), while the central bank does not conduct monetary easing if \( r^D > 1 \).

**Proof:** Under the assumption of \( R<0.5 \), \( R/(1-R) < 2R < 1 \) and we have

\[
i(r) = i(1) = R/(1-R).
\]

Since investment realizes the minimum at \( 2R \) over \([1+l,1+r^D]\), investment is maximized at \( 1+r(1+r^D) \) if \( r^D < (>)1 \). Q.E.D.

**Bubbly Equilibrium**

We turn to the competitive equilibrium of a bubbly economy that satisfies the binding borrowing constraint. The steady state \((\tau = 0)\) is represented as \(\{i^b, b^b, r^b\}\), satisfying

\[
(5) \quad i^b = \frac{A}{1-R}, \quad b^b = \frac{A(1-2R)}{1-R} - l, \text{ and } r^b = 0.
\]

Note that investment is positive and finite only if \( R<1 \). Bubbles are positively valued if and only if

\[
(#1) \quad \frac{A(1-2R)}{1-R} > l,
\]

which also implies that the interest rate is less than the growth rate, zero in this model.

Thus in Proposition 3, the case for \( r^D > 1 \) is eliminated when \(#1\) holds.
Equation (5) states that when bubbles arise, investment is independent of the market evaluation of trees \( l \), but bubbles are negatively related to them.

**Proposition 4:** Suppose that Condition (\#1) holds. Given \( i_{t-1} > 0 \), there exists maximum feasible bubbles \( b(i_{t-1}) \), for which the competitive equilibrium converges to the bubbly steady state that satisfies the binding borrowing constraint. On the convergent path to the steady state, investment is monotone increasing (decreasing) and bubbles are monotone decreasing (increasing) for \( i_{t-1} < i^b \) (\( i_{t-1} > i^b \)).

Proof: See Farhi and Tirole (2012).

Figure 2 illustrates the phase diagram of the dynamics of the bubbly economy. There exist a unique stable manifold \( b_{t+1} = \Omega(i_t) \) that approaches monotonically the bubbly steady state.

**Proposition 5:** Suppose that Condition (\#1) holds. Bubbles crowd investment in \( l \), that is, \( i^D < i^b \) when \( l > 0 \).

Proof: We eliminate \( r^D \) from (4) and obtain a quadratic equation;

\[
(1-R)i^2 - [A+l-l(\frac{1}{R}-1)]i - \frac{l^2}{R} = 0,
\]

which has two solutions. When \( l > 0 \), one of these solutions is negative, and the other is positive. Letting \( \Gamma(i) \) denote the LHS, \( \Gamma(i) \) is increasing around the positive solution \( i = i^D > 0 \). We see

\[
\Gamma(i^b) = \frac{(1-2R)A-(1-R)i}{(1-R)l} > 0 \quad \text{under (\#1). We find } i^D < i^b. \quad \text{Q.E.D.}
\]
Bubbles crowd investment in. When bubbles arise, the released resource from saving trees stimulates investment. There are three liquidities. The security issued by firms and trees are inside liquidity, and bubbles are outside liquidity.

A central question is if the central bank keeps or gives up monetary easing once bubbles arise. The central bank is more conservative than people on the riskiness of bubbles, and evaluates the welfare by

\[ \sum_{t=0}^{\infty} \beta^t (1-\lambda)^t \{ (R^f - R) i_t + \frac{l}{1 + r_{t+1}} \} + \beta \lambda W^D(i_{t+1}^c) + \beta(1-\lambda) W^D(i_{t+1}^c) + \beta^2(1-\lambda)^2 W^D(i_{t+1}^c) + \ldots \]

The first line represents the welfare given that bubbles persist, and the second line given that bubbles burst. The function \( W^D(i_{t+1}^c) \) is the value function if the bursting of bubbles at date \( t+1 \) represses investment at \( i_{t+1}^c \). Then investment converges monotonically to \( i^D \) and \( W^D(i_{t+1}^c) \) is increasing. As for the first line, we have the following.

**Result 1:** Suppose that Assumption 1 and (#1) hold. Then the welfare of the date-\( t \) generation, \((R^f - R)i_t + l/(1 + r_{t+1})\), is increasing in \( i_t \).

Proof: We define \( (R^f - R)i_t + \frac{Al}{Ri_t + l + \Omega(i_t)} \equiv V^b(i_t) \), where we use the fact the equilibrium satisfies the stable manifold \( b_{t+1} = \Omega(i_t) \), with \( \Omega(i) \) being continuous and...
decreasing. We have 
\[
R^f - R - \frac{A[RI + d\Omega(i)/di]}{(RI_i + l + \Omega(i))^2} > R^f - R - \frac{AIR}{(RI_i + l)^2}
\]

\[
> R^f - R - AR/l > 0.
\]
The final inequality comes from Assumption 1. Q.E.D.

Three policy regimes that we compare are monetary easing, benign neglect, and tightening. We first compare between easing and benign neglect. The central bank conducts easing by buying trees, with \( \tau > 0 \). The steady state equilibrium under easing is described as

\[
i^b = \frac{A}{1-R}, \text{ and } b^b = \frac{A(1-2R)}{1-R} - (1-\tau)l.
\]

On the other hand, the one under benign neglect is described as

\[
i^b = \frac{A}{1-R}, \text{ and } b^b = \frac{A(1-2R)}{1-R} - l.
\]

The difference lies in the size of bubbles, not in the magnitude of investment. We have to analyze the properties of the equilibrium out of the steady state in order to evaluate welfare.

**Proposition 6**: Maximum easing \((\tau = 1)\) realizes the fastest growth in investment, given the same investment level.

**Proof**: Investment is expressed as \(i_t = R_i + A + (1-\tau)l(1 - \frac{1}{1+r_{i+1}})\). Bubbles are monotone decreasing and thus \(1 + r_{i+1} < 1\) for \(i_{i-1} < i^b\), and then the interest rate is less than zero, and thus the last term is maximized when \(\tau = 1\). Investment is the highest when \(\tau = 1\), given \(i_{i-1}\). Q.E.D.
Result 2: The sum of bubbly and non-bubbly liquidities is greater as the central bank conducts easing (as $\tau$ is higher), given the same investment level.

Proof: Without loss of generality, we compare between two cases, $\tau = 0$ and $\tau = 1$.

We prove this in two steps. First we prove that, letting $b_i(b'_i)$ denote bubbles in the tree (no-tree) regime, $b'_i > b_i + l$ for any $i_{t-1} < i^B$. Second, we prove that $i_i$ is decreasing in $b'_i$ for any given $i_{t-1} < i^B$.

In order to prove the first step, we prove that the manifold under the no-tree regime should be above $b_{t+1} = \Omega(i_i)$ more than by $l$ for any given $i < i^B$. We introduce the hypothetical manifold that is proportional shift of $b_{t+1} = \Omega(i_i)$ by $l$, satisfying

$$b_{t+1} = \Omega(i_i + l),$$

and goes through A and B. If the real manifold is above it for $i_{t-1} < i^B$, any point on the path should traverse across the manifold from above. Suppose that, starting from the same $i_{t-1}$, the manifold $\Omega(i_i)$ dictates $b_T$, while $\Omega(i_i + l)$ dictates

$$b'_T = b_T + l \text{ at A. From (1) and (3), } \frac{b'_{t+1}}{b'_i} = \frac{A}{R_i + b'_i} = \frac{A}{R_i + b_i + l} = \frac{b_{t+1}}{b_i}. \text{ Letting}$$

$$b'_{T+1} = b'_{T+1} + \epsilon l, \text{ we have } b_{t+1}/b_i = \epsilon < 1 \text{ since } b_i \text{ is decreasing for } i_{t-1} < i^B$$

(Proposition 4). As Figure 3 illustrates, A goes to C, which should lie below $\Omega(i_i + l)$.

Therefore, the manifold should be above $b_{t+1} = \Omega(i_i)$ more than by $l$ for any given $i < i^B$. Comparing between two manifolds, we find $b'_i > b_i + l$ for any $i_{t-1} < i^B$. 

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In the second step, we arrange (1), (2), and (3) to obtain
\[ i_t = \frac{A - b'_t}{R} + R_{t-1} + b', \]
For \( R < 1 \), we obtain \( \partial i_t / \partial b'_t < 0 \) given \( i_{t-1} \). Q.E.D.

Figure 3 illustrates the saddle path of the economy with trees, \( b_{t+1} = \Omega(i_t) \), and the one without it, \( b_{t+1} = \Omega''(i_t) \). In transition, bubbles under easing are more than the sum of bubbles and the valuation of trees \( l \) under no intervention. The abundant bubbles contribute to enhancing investment.

Starting from the same initial condition, easing realizes the higher investment than no intervention at any point in time. Result 1 and Proposition 6 jointly states that easing realizes the higher first term than no intervention.

We turn to the second line. Suppose that bubbles burst at \( T \) when \( i^D < i_{T-1} \). The equilibrium in the tree regime \( \{\hat{i}_T^c, r_{T+1}^c\} \) is determined by the two equations,

\[ A = \frac{R_{i_T} + l}{1 + r_{T+1}^c}, \]  

\[ \hat{i}_T^c = \frac{R_{i_T}^c}{1 + r_{T+1}^c} + R_{t-1} + l, \]

which are the same as the bubbleless competitive equilibrium. The economy converges monotonically to \( i^D \), satisfying \( i^D < \ldots < \hat{i}_{T+1}^c < \hat{i}_T^c < i_{T-1} \). Note that \( i^D \) depends on the monetary stance after bubbles burst. On the other hand, the equilibrium in the no-tree regime \( \{\hat{i}_T^n, r_{T+1}^n\} \) is determined by

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\[ A = \frac{R_i^n + l}{1 + r_{T+1}^c} \text{, and} \]

\[ i_T^c = \frac{R_i^n}{1 + r_{T+1}^c} + R_{T-1}. \]

This economy also converges to \( i^D \), satisfying \( i^D < ... < i_{T+1}^c < i_T^c < i_{T-1} \). The difference between the two lies in (7) and (9). The term of \( l \) disappears from (9), reflecting that agents that had only bubbles as liquidity lose more internal wealth than when they had both bubbles and trees. Easing boost more bubbles, and thus depresses investment more strongly. We should have \( i_T^c < i_T^l \) for any given \( i_{T-1} \).

**Proposition 7:** Once bubbles burst given the same investment, easing runs the larger repression of investment than no intervention.

Starting from the same investment, before bubbles burst, easing realizes the higher investment at any point in time (Proposition 6), while once bubbles burst, easing depresses investment more severely. Which policy realizes the higher welfare is in general ambiguous, depending on to what degree the government takes weight on the bursting of bubbles. If \( \lambda \) is small, the central bank chooses monetary easing, but if \( \lambda \) is high, the central bank tends to choose to give up easing.

We turn to compare between bursting and bursting of bubbles. We specify tightening as keeping the return on trees with \( r^L > 0 \). Then investors would hold trees

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6 In the bubbly economy, market operation cannot effectively affect the interest rate in the long run, and we have to alternative policy tool to tightening the stance. One way is to issue one-period public debt that matures at the next period, and make the condition for the existence
first, and next bubbles. The interest rate would be raised up to 
\[ 1 + r_{t+1} = 1 + r^f > 1, \]
which makes it impossible to support bubbles. When bubbles occurs given \( i^D \), bursting bubbles makes the economy back to \( i^D \). On the other hand, if the central bank chooses not bursting, investment evolves over \((i^D, i^B)\). The welfare when bursting is
\[
(R^f - R)i^D + \frac{l}{1 + r^D}
\]
for agents of any generation, while the welfare when not bursting is
\[
(R^f - R)i_t + \frac{l}{1 + r_{t+1}}
\]
for date-\( t \) agents. Since \( i_t \) ranges over \((i^D, i^B)\) and \( r_{t+1} \) ranges over \((r^D, 0)\), the first term of the welfare is high when not bursting and the second term is high when bursting. Increasing \( R^f \) can make the welfare when not bursting arbitrarily higher than the former. Additionally, when \( \lambda \) is high, bubbles are less likely to burst, and then investment is on average high. When \( R^f \) and \( \lambda \) are high, the welfare when not bursting tends to be higher than the one when bursting. The government chooses not bursting. We impose the following assumption.

**Assumption 2** \( R^f \) and \( \lambda \) are sufficiently high.

**Proposition 8:** Suppose that Assumptions 1 and 2 hold. Once bubbles arise, the central bank chooses to **lean on bubbles**, and conducts neither monetary easing nor tightening.

The central bank pursues the narrow road to keep the boom (crowding-in) and stability (small bubbles). If the central bank delays stopping the monetary easing, the economy of bubbles (\#1) to be violated. If \( d \) units of debt are issued to meet 
\[
\frac{A(1 - 2R)}{1 - R} < l + d,
\]
bubbles are unsustainable in the long run, and have to burst.
should have excessive bubbles that would magnify the recession on the onset of the bursting of bubbles. On the other hand, if the central bank tightens the monetary stance excessively, the central bank could collapse the boom as well as bubbles.

We finally argue on the monetary stance once bubbles burst. Obviously the optimal policy is the same as the one before bubbles arise.

**Proposition 9:** Suppose that Assumption 1 holds. Once bubbles burst, the central bank conducts monetary easing by controlling the return on trees at $1 + r$.

*Is public debt a perfect substitute of bubbles?*

In an environment where bubbles arise, public debt can be a form of outside liquidity. Suppose that the government issues one-period bonds that are claims on the next period’s tax proceed $d$ levied on old agents. Each of $d$ units of bonds yields one unit at maturity and sells at $1/(1 + r_{t+1})$.

The asset demand function is given by

$$A = \frac{R_{t-1} + l + d}{1 + r_t} + b_{t-1}.$$  

(10)

The steady state is described by

$$i^b = \frac{A}{1 - R}, \quad b^u = \frac{A(1 - 2R)}{1 - R} - l - d, \quad \text{and} \quad r^u = 0.$$  

(11)

Bubbles are positively valued if and only if

$$\frac{A(1 - 2R)}{1 - R} > l + d.$$  

(#2)

By issuing debt and redeeming it by taxation old agents, the government is able to increase the supply of outside liquidity and induces bubbly liquidity to decrease.

Actually, as long as $1 + r_{t+1} < 1$, the government can issue debt and roll it over without
raising taxation. The government has to tax $d/(1 + r_{s1})$, but issues debt $d$.

Interestingly, government debt is more than a perfect substitute of bubbles when it is backed by taxation. Government debt plays a role of enhancing growth in investment (Proposition 6), and of weakening the cost of the bursting of bubbles (Proposition 7). When bubbles arise, the government should not advance the fiscal restoration for retiring debt.

\textit{Capital inflows and monetary stance}

As Greenspan tells from the unsuccessful experience in 1999, whether tightening monetary stance can effectively repress bubbles is a question. We provide one mechanism where monetary tightening rather appreciates bubbles by using the story of capital inflows.

In doing so, assume another country where agents, with a unit mass, are born every period, and live for two periods. Individual agents are endowed with $A'$ units when young, and consume when old. Between periods, they have access to one linear investment opportunity that transforms their wealth $A'$ into $A' (1 + r')$, where we assume $1 + r < 1 + r' < 1$. The former inequality says that agents find it beneficial to invest their wealth domestically in the absence of bubbles, and the latter inequality says that agents find it beneficial to invest their wealth abroad when bubbles arise.

Given this modification, the asset demand function includes foreign as well as domestic assets, and becomes

\begin{equation}
A + A' = \frac{R_{t-1}}{1 + r_t} + \frac{l}{1 + r} + b_{t-1}.
\end{equation}

On the other hand, the asset supply function is given by
The steady state is described as
\[
    i^{b} = \frac{A}{1-R}, \quad b^{b} = \frac{A(1-2R)}{1-R} + A' - I, \quad \text{and} \quad r^{b} = 0.
\]

Bubbles are positively valued if and only if
\[
    \frac{A(1-2R)}{1-R} + A' > I
\]

Capital inflows appreciate bubbles but do not stimulate investment. Foreign capital does not contribute to enhancing internal wealth of domestic firms, and so investment does not change.

In this environment it is interesting to study the optimal monetary policy. A primary concern is whether the central bank can prevent foreign capital from flowing in by maintaining monetary easing. In order to obtain meaningful implications, we restrict the behavior of foreign investors in two ways. Suppose first that foreign agents believe that bubbles burst with probability one so that they do not hold bubbles, and Secondly that part of the return, \( R'(<R) \), is pledgeable to foreigners.

In this environment, the effectiveness of monetary easing on preventing capital inflows depends on whether foreign investors can invest in firms. Firms have an incentive to issue securities to foreigners only if the maximum leverage when securities are issued to them \( l/[1-R/(1+r^{f})] \) is higher than the leverage when issued to domestic investors \( l/[1-R/(1+r_{s+1})] \). If \( R'/(1+r^{f}) < R \), the central bank can effectively prevent capital inflows by cutting the interest rate at \( r^{L} < r^{f} \), where the steady state is
\[
    i^{b} = \frac{A}{1-R}, \quad b^{b} = \frac{A(1-2R)}{1-R}, \quad \text{and} \quad r^{b} = 0.
\]
Note that the equilibrium is the same as the no-tree equilibrium.

**Proposition 10:** (a) Suppose that Assumptions 1, 2, and $R / (1 + r^f) < R$ hold. The central bank conducts monetary easing by controlling the return on trees at $1 + r^f < 1 + r^f$ in the bubbly economy if and only if $A^f > I$.

(b) Suppose that Assumptions 1, 2, and $R / (1 + r^f) > R$ hold. The central bank conducts neither monetary easing nor tightening in the bubbly economy.

Only if foreigners’ access to capital markets is severely constrained, the central bank has a room for trading off repressing foreign bubbles against stimulating domestic bubbles. Otherwise, the central bank is better off by raising the interest rate and accept foreign bubbles. If the central bank keeps monetary easing, the resource released from fundamental-backed liquidity as well as capital inflows would exacerbate bubbles. Then the optimal policy is the same irrespective of capital inflows.

A difficulty arises because then domestic investors hold all bubbles. Suppose that bubbles burst at $T$ when $i_T^D < i_{T-1}$. The asset supply function is then described as

$$i_T = \frac{R_i}{1 + r_{T+1}} + \frac{A}{A + A^f} (R_i^{T-1} + I) - \frac{A^f}{A + A^f} b_T.$$

When there is heterogeneity in belief in bubbles, the size of bubbles directly affects the cost of the bursting of bubbles. Larger bubbles that arise from foreign capital inflows repress investment of the domestic country more strongly.

**4. Some Evidence on Bubbles**
This analysis suggests the following optimal interest rate policy when bubbles boost and burst. When bubbles arise, the central bank raises the interest rate as quickly as possible to be close to the growth rate of the nominal GDP, and when bubbles burst, pulls down the interest rate as quickly as possible at less than the declining growth rate. This policy rule is different from the Taylor’s rule, where the nominal interest rate reacts to the inflation rate. This difference in the policy rule comes from the fact that Taylor rule is intended to work in the economy where the interest rate is higher than the growth rate, but our rule targets the bubbly economy where the interest rate is less than the growth rate. We evaluate monetary policies of the US and Japan from this perspective.

We first have to decide the interval of the bubbly economy. Figure 4A illustrates net assets as a fraction of GDP in the US. The economy is supposed to be stable along the balanced growth path in the ordinal state without bubbles. Actually, this ratio is stable at five and around except for 1998-2008 when the IT dot-com bubbles started and housing bubbles ended. In national income accounts, the stock data accounts for capital gains (losses) of assets, but the flow data does not. This ratio tends to increase in the period of bubble episode when assets grow faster than GDP, and is supposed to represent a good indicator of bubbles. This figure increased persistently from 2003Q1, and peaks out at 6.3 in 2007Q1.

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7 The source of data is the flow of funds accounts of the US (FRB’s). Net worth is defined as total assets less total liabilities of three sectors, households and nonprofit organizations, nonfarm nonfinancial corporate business, and nonfarm nonfinancial noncorporate business. Total assets include tangible assets and financial assets. Note that tangible assets include real estate at market value.

8 Lettau and Ludvigson (2004) investigate the consumption-wealth link using US time-series data, finding that the majority of fluctuations in asset values are attributable to transitory innovations that display virtually no association with consumption.
Figure 4B illustrates net assets as a fraction of GDP in Japan. The ratio started to increase since 1986, and peaks out at 8 in 1990, and except for that interval it is stable around 5.5. We could safely judge that the periods for bubbles are 2003Q1-2007Q1 in the US and 1986-1990 in Japan. The behavior of interest rates and the growth rate that are illustrated in Figures 1A and 1B also supports this judgment.

If the size of bubbles at peak is measured in terms of GDP, it is about 1.3 times GDP in the US, whereas it is 2.4 times in Japan. Real estate assets relative to GDP rose by 0.9 from 2002 to 2007 in the US, while they rose by 2.5 from 1986 to 1990 in Japan, suggesting that housing and land bubbles were significant components of bubbles.

Figure 5A illustrates the central bank’s reaction of the federal funds rate to the nominal GDP growth rate in the US. The FRB raised the federal funds rate from the bottom of 1.0 percent in 2004Q2 to an eventual level of 5.25 percent over 2006Q3 to catch up with the growth rate. Since bubbles started in 2003Q1, the FRB was late one year to react. Looking at the figure, if the speed for catching up is sufficiently quick is a question. The slowly rising interest rate may have contributed to stimulating bubbles. Once bubbles burst in 2007Q1, the FRB reacted quickly and lowered the federal funds rate to be less than the growth rate. The FRB was doing a pretty good job.

Figure 5B illustrates the reaction in Japan. There are two policy mistakes from the optimal policy. First, the Bank of Japan (BOJ) was so late to raise the interest rate (uncollateralized overnight call rate). Since bubbles started in 1986, it took almost two year and a half to start raising the interest rate. However, once the central bank started tightening, it raised the interest rate too quickly, which may have contributed to bursting bubbles. Finally, once bubbles burst, the BOJ was too late to cut the interest rate. It was

9 In Japan the source of the data is the National Income Accounts (Cabinet Office). Net worth is available from “Closing Stocks of Assets/Liabilities for the Nation” in “Supporting Tables”.
not until 1995 when the interest rate is less than the growth rate. The BOJ was doing a poor job! The differential reaction of the monetary authorities may be one source of different size of bubbles across countries.

It is interesting to evaluate the effects of target federal funds rate using Taylor rule, which should have been higher by 2 percent than the actual rate for 2003-2007 (e.g., Taylor, 2007). There are two important features. The simple application of the aggregate Taylor rule might have contributed to repressing bubbles for 2003-2007. On the other hand, it raises the interest rate too much beyond the growth rate, suggesting that it might have made the impacts of the bursting bubbles more serious.

We next provide some evidence where raising the federal funds rate called for capital inflows that are supposed to have contributed to the US bubbles. The US-Japan capital flows are a good example. Hattori and Shin (2008) highlight the role of yen carry trade, where the interbank lending between the Japanese branch of the US banks and headquarter is closely related to the deleveraging of the US financial sector. We show that yen carry trade occurred more systematically through the interbank lending.

In March 2001, the BOJ started a quantitative easing policy by maintaining the uncollateralized overnight call rate at the bank at near zero percent (Zero-Interest Rate Policy), and continued this policy until July 2006.

Following the bursting of dot-com bubbles, the U.S. Federal Reserve Board lowered the federal funds rate from 6.5 at maximum in January 2001, gradually to 1.0 percent in June 2003. However, in June 2004, following the boost of housing bubbles, the FRB turned to rate hiking. The FRB raised the federal funds rates from the bottom of 1.0 percent to an eventual level of 5.25 percent in 2006.
The change in the monetary policy stance in 2004 predicts differentials in both short-term and long-term interest rates between U.S. and Japan.

Figure 6 describes interest rate differentials of assets with different maturities, 2, 5, 10, and 20 years. The differential in the 2-year rate sharply widened in 2004, which also affected the middle-term, 5-year rate differential expanding. Those differentials continued to widening until 2005-2006. On the other hand, the differentials in 10- and 20-year rates do not have an upward trend. This is the situation where the longer-term yields did not rise regardless of the Fed’s rate hikes, Greenspan called ‘conundrum’.

Figure 7 describes the behavior of exchange rate of yen /dollar. Until the last of 2003, the Japanese yen appreciated, but since the first of 2004 when the FRB changed the monetary stance, it started depreciating. The time-series behavior for the exchange rate is consistent with the prediction of capital flow theory, where capital flew from Japan with low interest rates to US with high interest rates.

Figure 8 shows the behavior of the annual net capital flow, based on the net balance of capital and financial accounts. The solid line depicts the net capital flow that corresponds to the current account deficit. Capital flew out from Japan to foreign countries except for 2003, and did strongly from 2005 to 2008 when the US economy was booming. The current account deficit was the highest in 2007, about 4 percent in terms of GDP.

This figure also depicts net investments by type. Among them, what exhibits a remarkable behavior is the “other investment”. Other investment that is distinguished from portfolio investment and FDI includes loans, trade credits, and transfer of other

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10 The source of data is Bloomberg.
11 The source of data is Bloomberg.
12 The source of data is "Balance of Payments"(Bank of Japan).
currency and deposits. They also include the repo transaction, the short-term borrowing that takes the form of the repurchase agreement of safe assets, such as JGB. In addition, these loans include the inter-office account of Japanese banks, that is, in-house loans from domestic headquarters to overseas branches, and the inter-office account of foreign banks, in-house loans from Japanese branches to overseas headquarters.

The net flow of other investment” was inward from 2001 to 2004, but turned outward from 2005 to 2007. Short-term loans mainly contributed to such large outflow. In addition, while in 2001-2002 there were a large amount of loans in inter-office accounts of Japanese banks, since 2004 loans other than that increased. Such loans, including repo, are mainly made at the short term, and are supposed to be used for purchasing longer-term assets through leveraged financing.

Figure 9 describes the chart for fund flows.\textsuperscript{13} Over 2004-2008, loans supplied by headquarters of domestic banks to their overseas branches account for 9 trillion yen. Loans supplied by Japanese branches of foreign banks to their overseas headquarters and branches account for 1.9 trillion yen. In addition, foreigners’ short-term borrowings such as repo from banks and securities houses account for 16.6 trillion yen. The fund flow channeled to the US security investment over 2004-2008 at least accounts for 27.5 trillion yen.

\textbf{References}

\textsuperscript{13} The sources of data are "Flow of Funds", and "Balance of Payments,"(Bank of Japan), and "Flow of Funds Accounts of the United States" (Federal Reserve).


Figure 1A  Interest and Growth Rates in the US

Figure 1B  Interest and Growth Rates in Japan
Figure 2: Phase Diagram of the Bubbly Economy

Figure 3: Dynamics of Bubbles
**Figure 4A**  Net worth/ GDP in US

**Figure 4B**  Net Worth/GDP in Japan
Figure 5A: Monetary Stance in the US

Figure 5B: Monetary stance in Japan
Figure 6. US and Japan Government Bond Yield Differences

Figure 7. USD/JPY exchange rate
Figure 8. Capital and Financial Account

- Direct investment
- Portfolio (securities) investment
- Financial derivatives
- Other investment
- Net financial account
Figure 9: Fund Flow over 2003-2008

- Call market
  - Domestic headquarters of Japanese banks
    - Inter-office
      - US branches of Japanese banks
        - US loans, security investment
          - Fed - Flow of Funds
          - BOJ - Flow of Funds
          - BOJ - Balance of Payments

- Depository institutions except banks, insurance, etc
  - Domestic security dealers, etc
    - Repo/Gen-saki
      - US security investors
        - +17.3USD (Financial sector - Commercial banks-GSE-ABS issuers, etc)
        - +10.1USD (US commercial banks – US branches of foreign banks)

- BOJ’s fund-supplying
  - Domestic branches of foreign banks
    - Inter-office
      - US headquarters of foreign banks
        - US loans, security investment
          - Fed - Flow of Funds
          - BOJ - Flow of Funds
          - BOJ - Balance of Payments