NSF FINAL REPORT

Project Participants

Participants
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Activities and Findings
We are pleased to present the National Science Foundation with the final report of our work on Video Interactions for Teaching and Learning (VITAL): A Learning Environment for Courses in Early Childhood Mathematics Education (ESI-0353402). The project ran from June 2004 to November 2009. Its goal was to contribute to the improvement of early childhood mathematics education nationwide by creating model courses for colleges and universities. The content and methodology are based on mathematics education courses taught by Professor Herbert P. Ginsburg (co-PI) at Teachers College, Columbia University, which are designed to support the development of understandings and related skills that early childhood educators need to provide sound instruction in mathematics, a topic seldom taught in the early grades. The courses employ learning activities using video cases to help prospective and practicing teachers understand the development of young children’s mathematical thinking and learning, and analyze the thinking and learning of individual children. These learning activities help prepare teachers to examine early mathematics instruction more critically and to develop pedagogical approaches based on an understanding of children’s mathematical cognition.

The VITAL video analysis software was developed by the Columbia Center for New Media Teaching and Learning (CCNMTL, http://ccnmtl.columbia.edu, Executive Director Frank
Moretti, co-PI). It was designed as a mechanism to provide students with ready access via the web to Ginsburg’s collection of videos on course topics to help them bridge theoretical concepts in the readings with authentic videotaped examples of children’s mathematical thinking and potential instructional methods, and regular opportunities to practice close viewing as a means of developing skills of observation, interpretation, and decision-making with respect to teaching and assessment (Moretti & Pinto, 2006; Preston et al., 2005). Our hope with the VITAL project was to provide faculty at other institutions with a means of offering courses in early childhood mathematics education based on our model.

To visit Ginsburg’s course in VITAL, log in at http://tiny.cc/vitalapp with the username vitaldemo and password vitaldemo. A user guide is available at http://tiny.cc/vitalguide. For more about the development and evaluation of VITAL, see http://tiny.cc/vitaleval.

The products created throughout the course of the grant are:

- **Software**: The VITAL web-based software was created specifically to support Ginsburg’s graduate level course at Teachers College, Columbia University entitled “The Development of Mathematical Thinking.” Students in this course are typically working toward New York State certification in early childhood education (birth through age 8). Subject areas taught align with NCTM recommendations and include coverage of everyday math, early number, shape, space, pattern and measurement, as well as written, symbolic math in the first few years of school. Students also learn methods of assessment, and are introduced to the cognitive processes involved in understanding, problem solving and metacognition. They consider issues of pedagogy and curriculum, including teaching with manipulatives and textbooks, and they learn to understand and analyze the complex strategies and knowledge involved in teaching (for example, pedagogical content knowledge, formative assessment, and creating a synthesis between the formal and the informal). Each assignment in the syllabus is accompanied by videos of teachers working with young children to illustrate the concepts. The VITAL software enables students to access the video content outside of lecture, and gives them opportunities to practice their skills of observation, interpretation, and decision-making with respect to teaching and assessment. These techniques are explicitly taught as well as modeled in class by Ginsburg.

- **VITAL course syllabus**: The course syllabus in VITAL tracks the syllabus of Ginsburg’s Development of Mathematical Thinking course. Students in the course are taught to use VITAL regularly, in conjunction with readings and lectures, to view videotaped examples of children engaging in mathematical activities, and to complete analytical assignments that require them to interpret the videos and develop and defend hypotheses about children’s mathematical thinking and learning.
Video Viewer: VITAL’s “video viewer” (Fig. 2) enables students to select and clip segments from the videos, and attach a note to each clip to help them remember the significance of the content. Clips and notes are saved in a personal workspace where they can be accessed later and used to support an essay.

Multimedia Essay Space: In the “multimedia essay” space (Figure 3) students integrate their clips of teachers working with children into their text. In a typical course, students write essays of 350 words or fewer in response to questions such as, “What does this child understand about cardinality? Please cite from the videos and the readings.” Assignments encourage students to develop their own hypotheses and select evidence from the course material that supports their argument. Completed essays are “published.”

Figure 1: One session in a VITAL course syllabus.

A course syllabus page in VITAL looks like a conventional syllabus, with a list of topics, readings, and assignments, but it also includes a selection of videos for each topic that can be viewed by clicking on the links embedded in the page. Videos open in an interactive viewer (see Figure 2).
within the VITAL environment to be read by the instructor and other students. The instructor can also leave feedback for the student.

• **Guided Lessons in Clinical Interviewing**: Students complete a series of “guided lessons” in clinical interviewing. Assignments are designed to simulate an interview by stepping students through videotaped interviews and prompting them to interpret the child’s behavior and the interviewer’s technique, to anticipate what the child will do next, and to make recommendations for subsequent questions.

• **Reflections**: Students write a weekly reflection in VITAL within 24 hours of class, which is the concluding event for the week and serves as an opportunity to express what they learned, pose questions, and dispute ideas discussed in class.

• **Culminating Project**: In the final month of the course, students design a mathematical lesson or activity, try it out with a child, and interview the child afterward to find out what he or she learned. The student records these events, submits the tape for inclusion in the VITAL library, and writes a research paper, in the form of an extended multimedia essay that details the literature, methods employed, and results obtained. The final project integrates the math content learned in the course with the assessment skills associated with clinical interviewing. The report submitted in VITAL also serves as a demonstration of the students’ ability to think critically about the work they are doing as teachers and what a child might be learning as a result. An evaluation rubric meeting NCATE requirements is used to assess student work.

• **Video library**: The VITAL team created more than 500 new video clips in preschools and elementary schools in New York and New Jersey. Approximately 100 of these clips form the core of the VITAL early childhood math video library. We answer frequent requests for these videos from faculty at other institutions and from professional development.
organizations; for example, we have licensed portions of the library to Wireless Generation, a New York City-based organization that contracted with the Texas Education Agency to conduct statewide math education workshops for early childhood and elementary teachers. Videos in the core VITAL library cover the following topics: everyday/naturalistic math, early concepts such as conservation, understanding of more/less and equivalence, counting/number words, cardinality/enumeration, information addition and subtraction, representation and symbols, assessment, number facts, written procedures, concepts related to understanding such as number sense, making connections, metacognition, explanation, justification and proof, geometry, patterns and early algebra, measurement and data, manipulatives, pedagogy, and examples of curriculum activities. Each semester, in response to assignments, students contribute videos of their own teaching and assessment activities. Usually these are not shared outside of a specific course setting in VITAL.

- **Pilot courses**: VITAL was extensively piloted with graduate and undergraduate teacher education populations at TC, Rutgers, and Hunter. Smaller pilots were conducted at three universities outside the northeast: Arizona State, University of San Diego, and Vanderbilt.

- **Supporting materials**: We created a complete “faculty guide” to teaching the course, including lecture notes, descriptions of the pedagogy, guidelines for teaching with VITAL, assessment, and other topics.

- **Redeveloped software**: The software was completely redeveloped to support a larger number of concurrent users, function outside of Columbia University, and to be easier and more intuitive to use. VITAL now supports approximately 500 students each semester.

- **Open source code**: In order to make VITAL software available to any institution, the VITAL code was released to the open source software development community using popular open source hosting services Google Code: [http://tiny.cc/vitalcode](http://tiny.cc/vitalcode) and Github [http://github.com/ccnmtl/vital](http://github.com/ccnmtl/vital). On the Github site, readme.txt provides an overview of the project and software, and install.txt provides technical information for developers and IT staff.

- **Publications**: The VITAL team published extensively in education journals and conferences on the project throughout the grant period. Some of the articles addressed VITAL as a tool for teacher training, while others address student learning, curriculum development, and integration of technology in the higher education classroom. VITAL was also noted in articles on technology integration authored by others. For a complete list of publications and presentations related to VITAL, visit [http://tiny.cc/vitalpubs](http://tiny.cc/vitalpubs). This list is also included in Appendix C and in the Publications and Products section of this report.
Since the last annual report filed May 2009, we reached the following milestones:

**Continued to offer the course at Teachers College and Rutgers University.**
At Teachers College, Prof. Ginsburg undertook another revision of his “Development of Mathematical Thinking” course and offered it to 44 students, primarily from the Early Childhood program. All participating students at Teachers College complete a final project involving a videotaped lesson and an interview with a child, and research paper-length multimedia essay in VITAL reflecting on their methods and results.

At Rutgers, Prof. Roberta Schorr taught her course “Math & Pedagogy” to a seminar of 12 students. The course is designed to help prepare students teach mathematics at the elementary and middle school levels and to become reflective practitioners who can incorporate state and national standards into classrooms with a high degree of integrity. She drew from VITAL’s model course to cover topics including Free play & mathematical thinking in the early years; Numbers & Counting; Reasoning, Communication, Testing; Rational Numbers; Patterns & Algebra; and Manipulatives.

**Continued to add to and improve the model course.**
From the inception of the grant, Prof. Herbert Ginsburg has offered his course at Teachers College at least once per year in order to meet student demand and to continue the course development process. There are now two working models of the course: an early childhood education focus and a mathematics education focus for students whose primary interest is in teaching at the upper elementary grades and above. After teaching a single section for many years, it was decided that these two groups’ respective needs and expectations were different enough to warrant their own custom-tailored courses. Each course begins with the same foundational material and uses VITAL consistently throughout, but the latter half of the courses diverges to accommodate specialized interests, such as mathematical topics relevant to later grades: algorithms, rational number, and appropriate pedagogies and curriculum. Each course features a syllabus, lecture notes, assigned readings and videos, and one or more exercises for students to complete in VITAL. All previous courses remain archived in VITAL and contribute to the model courses we offer to faculty who are joining the project.

**Developed an open-source version of the VITAL software.**
One of our goals by the conclusion of NSF funding is to make VITAL available widely, along with the model course in early childhood mathematics education. We are currently pursuing two dissemination models: (1) a hosted version at Columbia for partners who wish to participate in the VITAL early childhood mathematics project, which includes both using the materials we have made available and contributing materials and research back to the project; and (2) a distribution of the software that will enable other institutions to install and run VITAL on their own servers, and also to customize it to their needs (and ideally contribute these changes back to the project). As noted above, the VITAL code was released to the open source software development community using Google Code and Github (popular open source hosting services): [http://tiny.cc/vitalcode](http://tiny.cc/vitalcode). Already we have begun collaborating with MIT, where VITAL has been installed and currently supports a course in global performances of Shakespeare. Their modifications to the code base have already made VITAL more readily usable by other institutions.
Completed an evaluation of student learning in VITAL.
Please see under “Research and Education” the final project evaluation conducted by Cornelia Brunner of the Center for Children and Technology and the Close Viewing Study conducted by Brunner and the VITAL team.

Finalized our video collection.
In the past year, our curriculum team at Teachers College continued to review and edit our video collection, which includes trimming and titling each clip and, most importantly, assigning metadata and notes referencing teaching so that the clips can be indexed and searched within a master library in VITAL. Our internal VITAL video database now contains more than 750 clips, of which we plan to distribute about 150 through a library service provider and/or through a partnership with public television stations such as WNET in New York or WNJN in New Jersey.

Completed the VITAL faculty guide.
Prof. Ginsburg and his graduate students have completed a comprehensive faculty guide (attached to this report) to support prospective and new faculty in navigating the innovative features of the course, including the VITAL software, video library, and pedagogical methods. The guide contains sections on the rationale behind the development of the course and technology, the content of the course and the pedagogy of teaching with VITAL, information about assignments and the logistics of using VITAL and evidence of learning through the course and technology, as well as slides and lecture transcripts of the first two class sessions, a list of the most frequently used VITAL videos, and a series of sixteen modules that can be tracked chronologically with the class syllabus or used individually according to individual faculty need. The modules contain descriptions of class topic, topic specific videos with annotations, sample discussion questions and related videos.

Continued to published and present on VITAL.
In the final year of funding we expanded our outreach to new audiences, ranging from instructional technology to child development. The following is a list of papers and presentations from the past year:

- Symposium on pedagogical approaches to VITAL: “A Video-Based Pedagogy for Improving College Students’ Understanding of Development and Education” at the 2009 biennial meeting of the Society for Research in Child Development (SRCD) with papers by faculty partners Lori Custodero and Ann Cami of Teachers College, Noriyuki Inoue of the University of San Diego, and Michael Preston of CCNMTL.
On May 4, 2010 we will offer our third symposium on the various research projects around VITAL at the annual meeting of the American Educational Research Association (AERA).

Research and Education

Research and evaluation over the grant period

The development of an interactive learning space that encourages students to think critically and generate complex learning products creates a challenge for the evaluator. Our underlying hypothesis was that the use of the tool, with its emphasis on close viewing of concrete examples of children’s thinking, interviewing techniques, and teaching practice, aids in the development of the ability to use direct evidence to support theoretical analysis. The further underlying assumption was that the ability to ground theory in evidence increases the teacher’s belief in the usefulness of theory in practice and makes it a more solid foundation for pedagogical decision-making.

We hypothesized that the short term outcomes, changes that might be discernable by instructors and students in the courses, could be the degree of close viewing engaged in by students (i.e., how appropriate the clips are to the argument they are illustrating in essays), any change in the degree or nature of the reflections students share with their instructors and each other (e.g., class discussions), and any change in the depth of the students’ insight. The long term outcomes were hypothesized to be teachers who are better prepared for teaching practice and whose teaching skills have improved as a result of their deeper understanding of the subject matter and their opportunities to conduct more in-depth virtual observations.

The original evaluation of the technology itself, included in Appendix D, had as its focus the extent to which students are able to demonstrate their ability to analyze video segments closely and to use the Web environment to enhance their own learning process. At the beginning of the VITAL project the use of streaming and downloadable video and clipping tools was innovative. During the five years of the project, however, socially shared access to video clips documenting every aspect of our lives has become ubiquitous. As a result, the technical achievement of creating a smoothly functioning instructional system became less notable and users mentioned the technology less directly, starting to consider it standard equipment and integrating it into their response to the course as a whole.

Therefore, the focus of the evaluation shifted to the role VITAL plays in supporting pedagogical practice. In the context of this course, VITAL was most prominently used to provide opportunities for close viewing and easy sharing of carefully chosen, illustrative video material.

The final evaluation analysis and close viewing study analysis follow below. Each addresses data collected from studies conducted after the evaluation focus shifted. We will disseminate these
findings at the May 2010 annual meeting of the American Educational Research Association (AERA).
VITAL Evaluation: Findings and Analysis

Evaluating the role of VITAL in Professional Education

The Course
The purpose of VITAL, as a technology, was to support innovative pedagogical practice by providing teachers and learners with ready access to course videos, tools for close viewing and analysis, and a space for communicating ideas using cited video as evidence.

The instructor had a rich collection of illustrative video material from a long history of documenting clinical interviews with young children to ascertain the nature of their mathematical thinking. The video library includes examples of interviews illustrative of similar aspects of cognitive development conducted by a range of different interviewers with a variety of different children.

The students attending these courses included early childhood teachers and mathematics teachers, developmental and cognitive psychologists, and instructional designers, contributing distinctly different types of background knowledge and professional foci to the use and interpretation of the video material in VITAL. In addition to seeing examples of mathematical thinking in young children in the video archive,

- early childhood teachers can see examples of developmentally appropriate assessment practices to diagnose young children’s mathematical thinking skills.
- mathematics teachers have the opportunity to see and to determine the effectiveness of their teaching strategy in the final project;
- developmental psychologists can examine models of clinical interviewing technique, including their own; and

As part of the course requirement, students prepared a lesson, conducted a clinical interview and documented it via video. The final student projects were designed to integrate theory and practice and required submitting written report and a limited set of video clips illustrating their argument as well as a section evaluating their own performance. Throughout the course, students were asked to post regular reflections on the material covered, submit assignments and access instructor feedback.

The midterm in the course consisted of conducting a video-taped clinical interview with a child and writing a paper about it tying it to theory, analyzing the findings (i.e., diagnosing the child’s mathematical thinking skills), and offering a critique of the interview method shown in the video. The assessment rubric was very specific in describing the structure and purpose of each section of the report and in requiring the use of video to illustrate points in the report. There is a section in the course rubric, which asks students to “be careful not to simply describe what is happening. Instead focus on making an argument…” making to clear that the purpose of the video is to illustrate rather than merely to document.
Students were presented with a template for conducting a clinical interview that asked them to examine several video examples of interviews that illustrated some of the challenges of the technique. They were then asked to select from among three specific mathematical topics (e.g., counting, addition or symbols) that had been covered in depth during the preceding weeks, including readings and video-based lessons. For the interview, they were asked to fill out a template in VITAL which organized the process for them by asking them to consider:

- How will you determine if the child understands your questions?
- What is the possible scaffold you could provide here?
- What else might you ask to encourage the child to share his/her thinking?

The assignment required them to add three more question of their own to the protocol. The protocol template in VITAL provided nine or ten questions specifically designed to elicit the child’s thinking about each of the three topics and included alternative wordings for each question. Students were given a detailed description of how to report on their interview, including an example from a good report.

This kind of intensive scaffolding of the assignment serves as a model for the pedagogy taught in the course. Expectations and process are clear and provide the student with opportunity to discover the connection between theory and practice by through deep analysis, ongoing reflection and as-needed feedback. VITAL became a core ingredient in the teaching of both the development of mathematical thinking, specifically in young children, and of the use of clinical interviewing as a way to diagnose thinking and assess learning. The use of video, both archival and student-made, became increasingly central to the course experience because it supported the key component of the pedagogy, the in-depth analysis of children’s and interviewers’ behavior, interpretation of that behavior in the light of theory, and genuine self-assessment of students’ professional performance.

The Students

<table>
<thead>
<tr>
<th>Professional focus</th>
<th>N</th>
<th>Video assignments</th>
<th>Theory: low</th>
<th>Theory: high</th>
<th>Child thinking: low</th>
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<th>Teaching practice: low</th>
<th>Teaching practice: high</th>
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<td>14</td>
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Table 1: Comparison of final project performance by students in three different classes with different professional foci.

The course was offered to students in different disciplines, including preservice early childhood teachers, students of psychology and preservice math teachers. Each group had a somewhat different professional focus and the syllabus was altered slightly to accommodate their different
Preservice early childhood teachers and psychology students were asked to spend considerably more time studying video clips (10 and 7 assignments) than mathematics teachers (4 assignments), because the archive contains a great deal of relevant material showing young children at play and in clinical interviews. Preservice mathematics teachers were less likely to have early childhood as their professional focus, so they were asked to spend less time watching video from the archives.

Because the samples are relatively small, the courses were not given at the same time and thus had somewhat different syllabi both to accommodate to the needs of different student populations and because the instructor was still shaping the course to take optimal advantage of the learning environment, these data do not lend themselves to statistical analysis. The raw numbers suggest, however, that preservice mathematics educators were more likely to be assessed at a higher level of competence in their use of theoretical constructs than early childhood educators or psychology students, probably because the mathematical ideas discussed in the course are more familiar to them. When it comes to their ability to describe children’s thinking, on the other hand, preservice mathematics educators did less well than the other two groups, which is to be expected, since they spent less time observing it in video clips and probably had far less practice working with young children. The difference between early childhood and psychology students and mathematics educators when it comes to teaching practice may be an artifact of the course as well rather than pointing to any difference in teaching effectiveness. The pedagogy modeled in the course and valued in the project reports was probably more familiar to students who work with younger children and who are less focused on conveying content.

Since these results are what would be expected, we can assume that the coding scheme we applied to the projects, independent of the grading rubric used by the course instructor, has some face validity and measured relevant aspects of the quality of the student performance on the final project.

The Pedagogy

We performed a content analysis of three student reflection assignments interspersed through the course. The first assignment, at the beginning of the course, asked students to reflect on their experience with math and math education (T1). The second assignment, during the middle of the course, asked students to reflect on their experience using VITAL (T2), and the third assignment asked them to reflect on the most important thing they learned in the course (T3).

<table>
<thead>
<tr>
<th></th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
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<tr>
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<tr>
<td>Non-believer</td>
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</table>

*Table 2: The number of students who indicated agreement with the tenants of the course pedagogy at three points during the course.*

Believers were students who showed evidence that they felt sympathetic to the approach of the course and were interested in learning how to apply it in their own practice. Non-believers were
students who either showed no particular interest in pedagogy or showed some evidence of holding alternative beliefs that differed from the pedagogical philosophy of the course.

There were an almost equal number of believers and non-believers at the start of the course, but by the time they were asked to reflect on their experience with VITAL (T2), the majority had become believers. By the end of the course, when they were asked to reflect on what they considered the most important thing they had learned in the course, their adherence to the basic philosophy of the course was almost unanimous. In other words, by the end of the 12-week course, students thought the most important thing they learned was the pedagogical approach it presented.

The Instructors
Five instructors taught the course, including Herbert Ginsburg (co-PI), who developed it. All instructors were very enthusiastic about the use of VITAL and considered it a central component of the course. They agreed that:

- The capacity to track their students’ interpretation of selected video clips helped them make the course “more interactive,” because they could adjust their presentations, choice of material and emphasis based on their more informed sense of what the students were understanding.
- Seeing student work documented in video enabled them to make more informed comments and provide the students with more differentiated feedback.
- Even students who were initially concerned about using technology found the work involved in using VITAL worthwhile because they felt they were learning important and practical information.
- They should put to better use the communications possibilities of the environment by encouraging more online discussion between students.

The four instructors who adopted the course said they altered very little in the syllabus, staying faithful to the intention of the course creator. They did mention, however, that they did not stop the flow of the video when they were watching with the class as frequently as the original instructor did. They found it difficult to get the students to talk about what they were seeing when they interrupted the flow too frequently and they did not think their students liked stopping too often to discuss but wanted to see more of the video before talking about it.

This adaptation of the original instructor’s method could be considered a customization of the original course and be seen as a positive sign of the flexibility of the technology. It may also be the case, however, that the instructors who adopted the course were less persuaded of the pedagogical purpose of the frequent interruption of the video clips. “Close viewing” (i.e., repeated viewing and frequent pausing to assess what one has seen) is one of the main benefits of this technology. It does permit students to observe subtle evidence of children’s thinking; however it also may serve to “break up” the narrative flow of the video. Students may find that disruptive, but it may be the very process that helps dispel misconceptions and highlight untested assumptions about children’s mathematical thinking. The narrative an observer infers from the flow of the videotaped experience is an interpretation based on selective attention to details considered “telling” by the viewer. Disrupting the meaning students spontaneously construct as
they watch is probably an excellent way to persuade them to attend to exactly the kind of detail they would not notice spontaneously because it does not fit into their prior beliefs and assumptions about children’s thinking.

The ability to view the video in very small segments, even single frames, can be considered a core aspect of the use of the technology that supports the pedagogy of this course, its emphasis on constructing meaning based on the close examination of evidence, analyzed in the light of theoretical concepts discussed by experts in the field.

**The Projects**

In one class of 34 students, many of them preservice mathematics teachers, 21 did final projects in which they attempted to teach a mathematical idea to a youngster, while 13 did their project by trying to teach one to an adult.

<table>
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<th>B+</th>
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<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Child Thinking - low</td>
<td>0</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Reflections - high (16 – 19)</td>
<td>1</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Reflections – medium (12 – 15)</td>
<td>1</td>
<td>12</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Reflections – low (8 – 11)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Beliefs (T1) - Believer</td>
<td>1</td>
<td>11</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Beliefs (T1) - Neutral</td>
<td>1</td>
<td>11</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

*Table 3: Comparison of grades for the course and content analysis codes.*

Discussing theory and relating it to their assessment of their own lesson or interview, was a challenge for the majority of these students. Half the class integrated some theoretical reference into their self-assessment. The two students who received the highest grade in the class did not, however, mention theory in their critique. The quality of their practice as documented in the video and report, not their knowledge of theory, was what earned them the high grade. There is no discernable relationship between the extent to which students attempted to integrate theory and practice in their self-assessment and their final grade for the course. This particular class focused more on teaching, on creating effective lessons, than on the mathematical thinking of their project learners, who were often adults.

The class was initially evenly divided between “believers” who started out in sympathy with the basic pedagogical philosophy of the course and “neutrals” or “non-believers” who indicated no particular agreement with the basic tenants of the constructivist pedagogy underlying the course.
The table below shows all students colored to indicate their initial state of adherence to the course philosophy. The yellow dots represent students who were neutral (N), while the purple dots represent students who indicated a prior belief in the approach taken by the course (B).

Table 4: Comparison of “Believers” (purple) and “Neutrals” (yellow) on each code or measure of success in the course.

Relatively more students who did not start out sharing the pedagogical philosophy of the course at the outset scored lower on their knowledge of theory than did their peers who initially subscribed to the course’s philosophy. That may simply mean that the “believers” were already better acquainted with the relevant theory. Conversely, more “believers” scored lower on integrating theory and practice than their non-believing peers did, which may mean that they considered the theoretical basis for their practical projects more of a given.

More “neutrals” scored higher on teacher practice, which might indicate that they interpreted the final project as a way to document their teaching skill rather than seeing it primarily as a way to learn more about diagnosing mathematical thinking. There was no difference between “neutrals” and “believers” in the ability to describe and understand children’s thinking, or in the number of reflections they completed during the course. “Believers” did better on the final paper than the “neutrals” even though by the end of the course, when they submitted their project, almost all the students had become “believers.” The explanation might be that “believers” had an easier time right from the beginning applying theoretical concepts to the practice of observing children’s mathematical thinking and did more of their projects with young children, for which the course content was more directly relevant than it was for adult mathematics learners.
Table 5 shows that the students who completed more reflection assignments also obtained a higher grade for the course than those who completed fewer reflections, even though the reflections were not graded and not counted in the course grade. The reflections provided students with a regular, systematic opportunity to consolidate that week’s learning and are another example of the pedagogical model, by being short and to the point, and focusing on what is of value to the learner rather than on the transmission of information.

In addition to supporting students in regular and systematic reflecting on their learning, VITAL permitted the instructor to keep track of student’s responses to the course material and to calibrate the course presentations and assignments accordingly. The reflections were feedback for the instructor both about what the learners concerns were and how meaningful they found the material.

The potential of students to communicate with each other about what they find of value in the course and to comment on each other’s reflections, seems a natural extension of VITAL in a world in which using Twitter, an application enabling communal communication in the form of short text messages with possible links to images, sound or video, is a core aspect of popular culture.
The Video
We coded the students’ final project independently of the instructor’s evaluation. Essentially we were looking at the way students used the video documentation they produced for their final project. Our code was based on the extent to which students used the self-criticism section of the final report, which was written in the VITAL environment, to interpret their subject’s mathematical thinking, comment on possible improvements in their own teaching (or interviewing) practice, make references to relevant theory, and integrate theory and practice by reflecting on their own practice in light of theory. We found that students who uploaded adequate to high quality video were more likely to engage in critical viewing of their practice, indicating that it is important for instructors to clearly explain the instructions for creating high quality videos, and show exemplars of strong video work during the course. Producing videos did not prove to be merely the acquisition of technical skill, but more importantly, enhanced student ability to comprehend and analyze child behavior and apply relevant theory.

Of the 34 students in one class, two thirds conducted interviews with children and a third conducted them with adults. About a third (13) of the class made good, critical use of the video recording, another third (11) used the video to document their work but made little use or no of the video in their self-critique, and another third (11) submitted video that was technically inadequate and showed little, but served as proof that the interview had been conducted.

Video as record:
About a third of the class submitted video that failed to document anything because the relevant action was not captured. Another third used the video option purely as proof of their work. They showed clips documenting every part of the assignment but offered no analysis of it in their essays. The video clips served solely as a record of the event.

The group of students who submitted essentially useless video either failed to consider lighting or simply placed the camera so that most of the action happened outside the frame. They need a set of basic instructions for thinking about camera placement in relation to the focus of their documentation. Some students also realized that they had selected their video clips based on what they thought was important when they reviewed the tape, but came to realize during the writing of their essay, that it would have been better to illustrate the points they were trying to make and that they had pre-selected the wrong clips and should have waited to clip their own video in conjunction with the writing of the essay.

Video as feedback:
Instructors were clear about the value of video as a teaching tool, as a way to illustrate the points they were trying to make. Some also talked about using the student-made videos as feedback, learning something about the difference between the student’s ability to describe their practice and articulate their thinking and the actual quality of their teaching style. Some students sounded “better” than they acted, meaning that they know how to describe their practice in acceptable theoretical terms, but might not have done they things they implied. They might not have noticed, for instance, that their facial expressions or their timing, the speed with which they move on in the interview or linger to give subjects a chance to self-correct, varied with the correctness of a subject’s answer even though they thought their response is neutral.
Video as evidence:
The students who referred to the video they submitted in their paper used it as evidence for the points they were making in their paper. About a third of the students also used it as evidence for self-assessment and analyzed their interview technique, their teaching method and their use of the technology.

Interview technique:
When critiquing their own performance as clinical interviewers, most students focused on moments when they asked leading or closed rather than open-ended questions. Many of them also noticed times when they did not pick up on responses by the subject and therefore did not provide appropriate feedback or follow-up for the interviewee. A few noticed that their facial expression was not neutral and privileged correct answers, showed confusion when the answers did not correspond to their expectation, or indicated some kind of assessment by where they placed emphasis in their responses.

“I failed to suppress my instinct to just go in and teach…”
“Looking back at the video, I did not pursue every avenue of inquiry.”
“I let Ms. Teacher emerge at various times by validating answers and making leading statements … so I noticed in the video that she often looked at me to read my face to see if she was answering correctly.”
“I failed to ask her how she knew.”
“I wish I had asked her what she meant.”
“I realized I asked a lot of closed questions, trying to direct him to the right answer.”
“I realized I seem to ignore an incorrect answer rather than praising him for both.”

Teaching technique: The students who focused on their teaching performance noticed when they were doing too much “teaching” rather than “guiding”. They also noticed when they had made unwarranted and unexamined assumptions about the similarity or complexity of certain math problems they posed their interviewees and when they seemed to place undue emphasis on the correctness of an answer rather than on the thinking that went into determining it. Quite a few noticed that they had failed to give students an opportunity to show them how they would go about solving a problem before they taught them a particular way to arrive at a solution, thus missing the chance to establish that there are often different ways to perceive and solve math problems.

“Videotaping these sessions allowed me to look critically at my own teaching methods and identify the elements I hope to improve.”
“I did not realize how much I emphasized this until watching the video – poorly done on my part.”
“I need to guide more and teach less, let him find his own answer…”
“I left out a big part by not asking her to create her own example using this method…”
“I had simply assumed that the problems were similar.”
“If I had presented a better problem in a more neutral manner I would have observed
different ways of solving it which would have reflected their thinking.”
“I failed to ask him questions that would help him identify his mistakes.”

Technology: There were a few instances when students’ self-critique contained a reference to
technical issues, particular camera placement. Students expressed a wish to see what the subject
was writing or doing during parts of the interview, but the static camera obscured the action.
More experience with videotaping their practice might have helped because, even when there
was no way to use a single static camera to document everything, greater awareness of what
wouldn’t be visible in the video could have lead to more focused note-taking on the part of the
interviewer.

“I cannot see it on the video, but I think he pointed…”
“I wish I could see on camera what he does with his hands, but I cannot see that on the
video.”
“I couldn’t figure out how to record what we were writing without distracting him or
making him overly self-conscious.”
“Choosing video clips is challenging because it all seems important when you look at it,
but I realized you have to write the paper first so you can select clips based on that.”

Summary:
The purpose of the evaluation was to determine whether VITAL constitutes a digital learning
environment that can successfully support students in learning to observe children’s
mathematical thinking, interpret that thinking in the light of theory and making evidence-based
decisions about appropriate instructional strategies or assessment techniques. The positive
experience students reported of the course, with the technology use it models, and the quality of
the work they produced, implies that VITAL is an excellent tool for preparing students to
understand and practice the progressive pedagogy it supports.

The Close Viewing Study
The evaluation of the project included above was completed in late 2009. At the beginning of the
VITAL project, when the initial evaluation plan was written, it seemed that the most innovative
aspect of this project consisted of allowing students on-demand access to high quality video
material to support them in their study of theory and connect it to educational practice. The
implementation plan called for the gradually increased adoption of the VITAL technology into
analogous courses at different institutions. The final year was therefore intended as a way to
produce a summative evaluation of the technology, comparing the performance of students in
similar courses offered at different institutions, including Hunter College and Rutgers. However,
because the project was so carefully tailored to meet the specific needs of its primary audience at
Teachers College, it was more difficult to create easily validated comparisons among the
participating institutions. As a result, our comparative evaluation of the other settings was
limited, and it has been difficult to separate the effects of the technology from the larger courses
in which it has been embedded.
We designed a separate study to compare the effects of a “guided” method of video-based lessons with a “traditional” method of video analysis apart from any participation in a course taught by one of our faculty partners. We recruited subjects from undergraduate education programs at several major US universities not involved in the VITAL project. Subjects were randomized into groups and completed a series of five online lessons related to number, including counting, enumeration, addition, subtraction, and equivalence. Each lesson comprised a short reading introducing the topic followed by a 2-minute video that captured the interactions between a child and an adult researcher on the given topic. For the Guided condition, videos were broken into a series of brief segments (10-20 seconds), and subjects were prompted at the conclusion of each segment to answer a question interpreting what they saw, e.g., the child’s understanding of the task, possible explanations for the child’s behavior, anticipation of what might happen next, etc. After answering, subjects were shown one or more plausible responses to the question. In the “traditional” Video group, subjects viewed an unsegmented video and answered questions about what the child knew only at the conclusion of the full clip, and they were shown no sample responses. A third group (“Control”) completed only the pre- and post-tests completed by all subjects, which was a response to video clips of young children making and discussing patterns.

The responses to the pre- and post-tests were coded for subjects’ use of claims, evidence, “relational statements” interpreting their evidence, and “modesty” demonstrated by conditional language that acknowledged uncertainty. Each group contained 20 subjects (total N=60), for a total of 120 coded essays. Inter-rater reliability was calculated above .80 for all variables. Using a 3x2 ANOVA to compare the two significant variables among the three conditions, the VITAL group performed significantly better than the Video and Control groups on both relational statements and modesty (see Tables 6 and 7). The data show a significant increase in the guided group for both “relational statements” (p<.01) and “modesty” (p<.01) and no significant differences for any variables in the traditional and control groups; and significant differences between the guided group and the traditional and control groups (p<.01). For relational statements, there was an interaction effect (p < .01) between groups and repeated measures, although the difference between means within subjects was significant only for the VITAL group (p < .01) and not the Video group (p = .106) in spite of its appearance in Table 7, which compares relational statements across the three groups. For modesty, there was again an interaction effect (p < .01) between groups and repeated measures, as expected. As shown in Table 7, only the VITAL group’s change is significant within subjects (p < .01).

These results suggest a relationship between regular, guided practice with video analysis, and subjects’ ability or inclination to (1) interpret what they have directly observed and (2) explicitly acknowledge their uncertainty. While further study is needed to understand the cause of these effects and whether these orientations transfer to teacher behavior in any meaningful way, the results suggest that the guided method of video analysis helps students develop “skeptical practice” in the face of their preconceptions. By extension, an orientation toward interpretation and modesty may better position a teacher to make use new evidence when it becomes available.
Table 6: Pre/post comparison of relational statements for VITAL, Video, and Control groups

Table 7: Pre/post comparison of modesty for VITAL, Video, and Control groups

Educational efforts
Since the start of the grant, VITAL and the early childhood mathematics education curriculum have been used by hundreds of students at Columbia University, Barnard College, Hunter College, the Borough of Manhattan Community College, Rutgers University, the University of San Diego, Vanderbilt University, and Arizona State University.
The curriculum team at Teachers College continues to review and document the videos we have taped for the project. Our internal database now contains more than 500 clips, many of which include notes on subject matter and ideas for teaching. Later this year we intend to create a first release of documentation for faculty that includes a lesson-by-lesson guide to the syllabus which can be used to teach an entire course or modularized for shorter periods of time, and a pedagogical guide to teaching with video—both within VITAL and in the classroom.

**Ongoing writings about teaching and learning with VITAL include:**

- The design of VITAL, which is based on Prof. Ginsburg’s experiences with using video as a teaching manipulative during lectures for courses in developmental psychology and early childhood education.

- The pedagogy of VITAL, which is designed to enable students to practice the techniques they learn about in lecture and provide a certain level of scaffolding.

- Student learning in VITAL, with particular emphasis on critical thinking skills as measured by the structure and complexity of arguments created using the essay workspace, which enables students to write essays that incorporate specific citations from the course videos.

- The challenges of evaluating student performance in a complex learning environment that involves content knowledge, skill building, and critical thinking in the context of a multimedia workspace.

**Opportunities for Training and Development**

The VITAL project has enabled us to work closely with faculty at Columbia as well as personnel at many levels at our partner institutions. For example, at Teachers College, VITAL is currently used in several departments for teaching various aspects of child development; in addition to Prof. Ginsburg’s course on early mathematics education, Prof. Lori Custodero in Arts & Humanities teaches a course on children’s musical development, and Profs. Ann Cami and Jeanne Brooks-Gunn in Developmental Psychology teach courses on personality development. Research from each of these course instances and from our partners at Hunter College and the University of San Diego was featured on a panel at the 2009 biennial meeting of the Society for Research in Child Development (SRCD) in Denver.

VITAL is also extensively in seminars on teaching practice to encourage reflective teaching; videotapes of teaching provide students with opportunities for self-critique and analysis. At Barnard College, Prof. Lee Ann Bell uses VITAL in her “Issues in Urban Education” seminar to view the teaching of and give feedback to all of the members of the course. At Teachers College, Prof. Margaret Crocco uses VITAL in her seminar for preservice social studies teachers to help them learn to reflect on pedagogical goals, instructional methods, and professional experiences.

In addition to VITAL’s use at Teachers College, we have also benefitted from the implementation of VITAL and feedback from its 500-plus users per semester across Columbia University in a wide range of courses including Jazz in Film (GSAS); Film and Television in
Tibet-Inner Asia (School of International and Public Affairs); The Documentary Tradition (School of the Arts); Silent Cinema (School of the Arts); Scientific Inquiry and Decision Making (College of Dental Medicine); Theories of Communication (Teachers College); Intermediate French Conversation (GSAS); Intermediate Spanish (GSAS); Choral Pedagogy (Teachers College); Concepts in Physics, Kinesiology and Biomechanics (Program in Physical Therapy); Motor Development (Teachers College); and numerous clinical practice courses at the Columbia University School of Social Work.

We have also implemented VITAL at several non-partner universities and organizations. These include Georgetown University, where VITAL has been used in courses in English, Film Studies, and Mathematics, and for a multi-disciplinary project on 9/11 recovery called Project Rebirth; WGBH-Boston, which is using VITAL to archive and create assignments around its Vietnam documentary archive; and at MIT, which is using the newly released VITAL code in their Global Shakespeare Project to aid in the study and comparison of videos related to Shakespeare’s works. We are also incorporating the VITAL clipping and annotating and essay writing functions into our next-generation version of this software at CCNMTL.

Outreach Activities
Please see Appendix C for a complete list of publications and presentations given throughout the grant term.

Publications and Products

Publications (chronologically ordered comprehensive list of publications generated throughout the grant)


Web sites
The VITAL project is documented for the public at http://ccnmtl.columbia.edu/vital/nsf/. This site is updated frequently and will soon contain more resources for faculty at other institutions, including the faculty guide attached to this report. To explore VITAL as a guest, log in at http://tiny.cc/vitalapp with the username vitaldemo and password vitaldemo. A user guide for the VITAL software is available at http://tiny.cc/vitalguide.
Other products
The VITAL Web application is the primary “product” of the CCNMTL staff working on the project. Version 3.0 took nearly a year to complete and involved the efforts of multiple programmers, designers, and video personnel. The new version is easier to use and more powerful than its predecessor. The open source code for the VITAL Web application software was released to the development and education communities at Git Hub and Google Code sites in spring 2009. The code is available here: http://tiny.cc/vitalcode.

As described above, the VITAL project has generated an enormous amount of video footage on early childhood mathematics. In addition to courses in VITAL, this footage will likely prove useful for years to come in numerous formats, including teacher development workshops, online courses, and formal curriculum. The videos are relevant to a wide range of disciplines, including pedagogy, mathematics, assessment, psychology, and child development. We are currently investigating methods for disseminating the video library to interested parties.

The VITAL faculty guide, submitted with this report as Appendix E, is a complementary product created by the VITAL team. The guide is designed to help prospective and current faculty users navigate the features of the Development of Mathematical Thinking course and the VITAL technology. In addition to providing the rationale for the course, explanation of its content and the pedagogy of teaching with VITAL, the guide contains the course syllabus, a list of the most often used VITAL videos, teaching modules with annotated videos which can be used in conjunction with the class syllabus or independently (depending on faculty need), slides and transcripts for the first two class sessions, in-depth information on student assignments and projects, a teaching rubric, sample video permission form, and specifications for making a video to be uploaded in VITAL.
APPENDIX A: VITAL Screenshots

Figure 1: Closed view of multimedia syllabus. Clicking on the plus sign reveals links to videos and activities for each assignment.
Figure 2: A single week’s topic from the syllabus in VITAL, including an assignment with one video, plus six recommended videos.
Figure 3: The VITAL video viewer, with editing tools and an annotation space beneath the video, and clips with notes collected in the right-hand column.
Figure 4: The multimedia essay, with the student’s collected video clips on the left side of the screen, and a writing space incorporating text and video on the right. Students click or drag their video clips to add them to their essay.
Figure 5: Open view of VITAL guided lesson assignment, with instructions on the top left, essay writing space on the right, and student progress bar at the top. The video viewer floats above the workspace.
Appendix B: Course Syllabus

HUDK 4027: Development of Mathematical Thinking
Teachers College, Columbia University
Fall 2009 Course Syllabus

This course examines psychological research and theory concerning the development of children’s mathematical thinking, and considers applications of this knowledge to teaching, assessment, and other educational issues. The course makes significant use of a web-based system called VITAL: Video Interactions for Teaching and Learning. VITAL can be accessed via the course homepage on ClassWeb or by bookmarking http://vital.ccnmtl.columbia.edu. Your UNI and password are required to log in.

1. September 2: Introduction and background

Readings are always presented in the order in which they should be read:


National Association for the Education of Young Children and National Council of Teachers of Mathematics. (2002). Position statement. Early childhood mathematics: Promoting good beginnings. This report presents a good overview of the issues we face and the new approach to intentional teaching. Read it to get a general idea of the issues we face and the solutions we need.


2. September 9: Everyday mathematics


streets and in schools (pages13-27). This is an account of some fascinating cross-cultural work on a kind of pragmatic everyday mathematics, very different from the type seen in play.


3. September 16: Early number: Counting, concepts, enumeration

Talbot, M. (2006) The Baby Lab. New Yorker; 9/4/2006, Vol. 82 Issue 27, pp. 90-101. This is an example of good science writing that describes the work of Elizabeth Spelke, a prominent researcher on early number. The main issue to think about is the exact meaning of successful performance on the various tasks she uses to test “knowledge” of mathematics.


Gelman, R. (2000). The epigenesis of mathematical thinking. Journal of Applied Developmental Psychology, 21(1), 27-37. Some of this material is a bit difficult, but it will introduce you to the thinking of Rochel Gelman, who is one of the major researchers in the area of early mathematics.

4. September 23: Informal addition and subtraction

Cross, C. T., Woods, T. A., & Schweingruber, H. (Eds.). (2009). Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity. Washington, DC: National Academy Press. Pages 5-11 to 5-42. This is an overview of how addition grows out of the child’s early knowledge. Focus on the development of relations and operations particularly in children from 4 to grade K. We will review issues of symbolism later. Also see the boxes at the end, particularly Boxes 5-2, 5-10 and 5-11.

Ramani, G. B., & Siegler, R. S. (2008). Promoting Broad and Stable Improvements in Low-Income Children’s Numerical Knowledge Through Playing Number Board Games Child Development, 79(2), 375-394. This is primarily for the psychology students, who can learn from this how a cognitive researcher approaches educational research with an elegant design. On the other hand, consider the limitations of the basic board game task. Educators should think about what children can learn from board games and how to scaffold their learning. Don’t worry so much about the details of these experiments.

Zur, O., & Gelman, R. (2004). Young children can add and subtract by predicting and checking. Early Childhood Research Quarterly, 19(1), 121-137. This is an example of how Gelman’s work can be applied to a classroom setting. What do you think of the use of "prediction" as a central task?

5. September 30: Assessment: observation, test and clinical interview
Piaget, J. (1967). Problems and methods (Introduction), in The child’s conception of the world (1-32). Totowa, NJ: Littlefield Adams. This is the classic—the only thing Piaget ever wrote about clinical interview, as far as I know.

Ginsburg, H. P. (1997). Not a cookbook: Guidelines for conducting a clinical interview. Entering the child’s mind: The clinical interview in psychological research and practice. NY: Cambridge University Press, Ch. 4. The issue now is how you are going to conduct an interview. This is intended to help. Good luck. Piaget said you need a year of daily practice!

Ellemor-Collins, D. L., & Wright, R. J. (2008). Assessing student thinking about arithmetic: Videotaped interviews. Teaching Children Mathematics, 15(2), 106-111. This is an example of using the interview in the classroom. And now even the iPhone is a video camera.

The next two are optional, but a lot of fun.


(According to an impeachable source on the web, the movie story is as follows: In a forest, hunters discover and capture a 12 year old boy who appears to have spent his life living like a wild animal. He is taken to a school for deaf and dumb children in Paris, where is maltreated and used as an object of scientific curiosity. An eminent doctor, Jean Itard, takes an interest in the strange child and resolves to educate him. Little by little, the feral child, whom Itard christens Victor, responds to his benefactor’s patient attempts to civilise him.)

D’Agostino, R. (2005). Low Notes. New Yorker (2005-07-25), 1-3. This is for fun (not that all the others aren’t). What has the musical audition got to do with assessing little children?

6. October 7: Symbols


symbol systems (Miller is fluent in Chinese.). Think about why we have symbols and what influence culture may have on them.


7. October 14: Number facts


Sun, W., & Zhang, J. (2001). Teaching addition and subtraction facts: A Chinese perspective. Teaching Children Mathematics, 8(1), 28-31. This is said to be a Chinese approach-- very different from math blasters.

Trivett, J. (1980) The multiplication table: To be memorized or mastered? For the Learning of Mathematics. 1(1), 21-25. Tables are wonderful ways to teach number facts. This gets a little mathematically complicated but is a good example.

8. October 21: Place value and computing big numbers


Heirdsfield, A. M., & Cooper, T. J. (2004). Factors affecting the process of proficient mental addition and subtraction: Case studies of flexible and inflexible computers. Journal of Mathematical Behavior, 23, 443-463. Using a case study approach, this study proposes that to understand computation, it is important to examine several different processes in combination—in a kind of profile. They include number facts, affect, and other factors too.

Ginsburg, H. P., & Seo, K. H. (1999). The mathematics in children’s thinking. Mathematical Thinking and Learning, 1(2). Read the part from pages 121-125 on blackboard math. If you are interested in the mathematical analysis of a Shakespeare Sonnet, see pages 116 to 118.

9. October 28: The many faces of understanding

This gives an overview of what it means to be proficient in mathematics. Think about how this applies to preschoolers. Does it?

Ginsburg, H. P. Children’s Arithmetic, Chapter 9, pages 183-211. Many specific examples of understanding. Is this account, written before the previous piece, compatible with it?


10. November 4: Shape and space


Recommended

11. November 11. Patterns and Algebra


12. November 18: Teaching: What is it?


13. December 2: Formative assessment


Lee, Y-S, Chiong, C., Pappas, S., & Ginsburg, H. P. How did you know that? - Digging deeper into K-3rd grade students’ mathematical strategy use with in the mCLASS: Math assessment system. Paper to be delivered at AERA

14. December 9: Manipulatives and Curriculum


Appendix C: Papers and Presentations on VITAL throughout the grant period

Papers:


Presentations

March 18, 2005

VITAL at NJEDge.net Conference

Project partner Prof. Rochelle Kaplan of William Paterson University in NJ gave a presentation at the 6th Annual Faculty Best Practices Showcase, organized by NJEDge.net, New Jersey’s higher education network.

April 14, 2005

VITAL at the American Educational Research Association

October 21, 2005
VITAL at the Northeastern Educational Research Association
Prof. Herbert Ginsburg was the invited speaker at the 36th Annual Conference of the Northeastern Educational Research Association (NERA), held October 19-21 in Kerhonkson, NY. The talk was entitled Using Video to Nourish the Teacher’s Intermediary Inventive Mind, a reference to Williams James’ statement that teachers must use their creativity to bridge the science of psychology and the art of teaching (James 1899). Dr. Ginsburg and CCNMTL’s Michael Preston presented the VITAL Web-based learning environment from the perspective of its development history, its theoretical foundations, methods and goals, research agenda, and new insights about the teaching and learning of practitioners.

October 28, 2005
VITAL at the University of Chicago
Prof. Herbert Ginsburg gave a talk called “Rethinking Early Mathematics Education: Children, Curriculum, and Pedagogy” at the University of Chicago symposium Math Education: Promoting Young Children’s Understanding of Math Concepts. He discussed how the VITAL method promotes the meaningful teaching of psychology.

January 24, 2006
VITAL at WGBH
Frank Moretti and John Frankfurt of CCNMTL, along with Peter B. Kaufman of Intelligent Television and Jay Fialkov of the WGBH Educational Foundation, gave a presentation entitled The Economics of Film and Television as part of The Economics of Open Content Symposium, A WGBH lecture hosted by MIT.

March 17, 2006
VITAL in Colombia
Prof. Herbert Ginsburg gave a talk called “El Poder de la Entrevista Clínica para Educadores” (The Power of the Clinical Interview for Educators) at the Congreso Internacional de Matemáticas: Pensamiento, Pedagogía, Tecnología y Competencia de las Matemáticas.

March 31, 2006
VITAL at the National Association of Laboratory Schools
Debbie Marcus and Michael Preston from CCNMTL and Luyen Chou from The School at Columbia gave a talk called “Virtual Fieldwork for Pre-Professional Development” at the National Association of Laboratory Schools (NALS) annual conference, which was co-hosted by the Bank Street College of Education and The School at Columbia. The presenters compared implementations of VITAL at Teachers College and the Columbia School of Social Work, and the research agendas for each.

April 6, 2006
VITAL at the NYU Technology and Learning Symposium
Michael Preston gave a talk on VITAL called “Virtual Fieldwork for Pre-Professional Training” at the Technology and Learning Symposium at New York University. The theme for the conference was “Cognitive Load Theory and Multimedia Learning,” and the subtopic was
Multimedia Learning and Instruction

April 10, 2006
VITAL at the American Educational Research Association
Herb Ginsburg, Debbie Marcus, Ryan Kelsey, and Michael Preston presented a session called “Virtual Fieldwork for Pre-Professional Training” at the 2006 annual meeting of the American Educational Research Association (AERA) in San Francisco on April 10, 2006. The conference theme Education Research in the Public Interest. Our session introduced CCNMTL’s work and methodology, discussed the VITAL learning environment and pedagogy, and explained how VITAL is being used at Teachers College and at the Columbia University School of Social Work, with special focus on our current research framework, data collection, and analysis in both contexts.

June 6, 2006
VITAL at WNET
CCNMTL Executive Director Frank Moretti and Educational Technologist John Frankfurt, along with Peter Kaufman from Intelligent Television, gave a presentation entitled “New Models of Educational Video Production” at the Culture, Commerce, and Public Media conference hosted by WNET.

August 8, 2006
VITAL in Australia
Prof. Herbert Ginsburg gave a seminar entitled “Video Interactions for Teaching and Learning (VITAL): A System for Improving Teaching at the University” at Queensland University of Technology, Brisbane, Australia. He discussed how he uses VITAL to foster the meaningful synthesis of everyday knowledge and academic wisdom.

October 4, 2006
VITAL at NYU Applied Psychology Developmental Colloquium
Prof. Herbert Ginsburg gave a talk entitled “Video Interactions for Teaching and Learning (VITAL): Teaching Psychology With Meaning” at the NYU Applied Psychology Developmental Colloquium. He described his Development of Mathematical Thinking course at Teachers College, and how VITAL contributes to a range of student learning processes.

November 2, 2006
VITAL at Columbia University Seminar on the Evolution of Video in Education
CCNMTL hosted a University Seminar on VITAL and how the use of video has evolved in educational practice. Educational technologists presented an overview of how the application is being used in a wide range of courses and disciplines across Columbia, from Teachers College and the School of Social Work to the School of the Arts.

December 5, 2006
VITAL at CNI
CCNMTL staff members Peter Kaufman and Mark Phillipson attended the Coalition for Networked Information’s Fall 2006 Task Force Meeting in Washington, D.C. During their talk, entitled “Teaching in the New Vernacular: Video as a Participatory Medium,” they provided a
demonstration of VITAL and discussed the implications of educational video and new trends in participatory media such as YouTube and Wikipedia.

December 25, 2006
VITAL in Israel
Prof. Herbert Ginsburg gave a talk entitled “VITAL: A new method for teaching psychology and education at the university level” at Ben Gurion University of the Negev.

February 14, 2007
VITAL at the Illinois Online Conference
CCNMTL staff member Michael Preston gave a presentation entitled VITAL: A Web Environment for Writing with Video at the 2007 Illinois Online Conference, which is conducted entirely on the Web. The presentation was made live using an audio feed and a broadcast of the presenter’s desktop, and audience members were able to ask questions via audio and text.

May 23, 2007
VITAL at Video, Education, and Open Content
Frank Moretti, John Frankfurt, and Michael Preston presented VITAL as an example of best practices for teaching and learning at Video, Education, and Open Content: Best Practices, a conference held at Columbia University and hosted by CCNMTL, Intelligent Television, and Open Educational Resources. The presentation focused on the relationship of the VITAL design to pedagogical goals and student learning outcomes.

June 8, 2007
VITAL as “Teacher 2.0” at EduStat 2007
Frank Moretti and Michael Preston presented VITAL as a Web 2.0 application for teacher development at EduStat Summit 2007, a conference sponsored by SchoolNet and Teachers College. The theme for the conference was “Teacher 2.0: Developing the 21st Century Workforce.”

June 30, 2007
VITAL at the Eastern Conference on the Teaching of Psychology.
Ann Cami, Adjunct Assistant Professor of Psychology and Education at Teachers College, gave a presentation entitled “Multimedia Meaning Making: The VITAL Art of Engagement” at the Eastern Conference on the Teaching of Psychology. The conference, which was hosted by the Department of Psychology at James Madison University, focused on the theme “The Art and Science of Teaching.”

September 11, 2007
VITAL at Discovery Research K-12
Frank Moretti and Michael Preston of CCNMTL attended NSF’s Discovery Research K-12 annual PI meeting in Arlington, VA, and presented a poster on the VITAL Web environment, pedagogy, and evaluation. The conference was attended by representatives of nearly 140 projects to improve teaching, learning, and research in the STEM subject areas (science, technology, engineering, and mathematics). The conference theme was “Transition to the Future,” and the presentations and discussions focused on how new technologies have changed teaching and
November 9, 2007

VITAL at the National Association for the Education of Young Children
Michael Preston, along with Profs. Joon S. Lee and Priscilla Hambrick-Dixon of the Hunter College School of Education, gave a presentation entitled “The use of online video technology in teacher education: A VITAL program” at the 2007 NAEYC Annual Conference and Expo in Chicago. The session was designed to demonstrate how VITAL is used in graduate level child development and early childhood education courses to help teacher candidates learn how to observe carefully and think critically about children’s development and learning.

March 16, 2008

VITAL as a “Multimedia Record of Teaching”
Herb Ginsburg was invited to participate in the Carnegie Foundation’s conference Moving Beyond Parallel Play: A Carnegie Convening about Multimedia Representations of Teaching. MRT is defined as “the documentation and analysis of the richness of teaching at all levels via multimedia representations, using the Web and/or DVD technology for distribution, discussion and exchange.” Prof. Ginsburg writes, “Our goal is to promote learning in which students make meaningful connections among different areas of their experience—what they see, what they assert, and what they read. The hope is that that this kind of rich, critical analysis of children and teaching will transfer to and form a sound basis for classroom practice. VITAL is intended to enrich teachers’ minds.”

March 23, 2008

VITAL at Adult Development
Herb Ginsburg and Michael Preston presented a poster on VITAL as “an innovative Web system for enhancing adults’ critical thinking skills” at the annual symposium of the Society for Research in Adult Development. From the abstract: “We focus in particular on changing adult thinking, and not only on providing instruction in a particular content area or skill set. Specifically, we are developing a method to help prospective teachers (or any university student) connect what they learn from books and lectures with their personal experiences, and to gather real-world data, think critically about it, and make potentially better decisions.”

March 26, 2008

VITAL Symposium at AERA 2008
Members of our VITAL research group gave a symposium at the AERA 2008 Annual Meeting. Entitled “Video as a Manipulative: An Innovative System to Transform University Courses in Psychology and Education” and sponsored by the SIG Technology as an Agent of Change in Teaching and Learning (TACTL), the symposium included four papers:

- The use of video in teaching psychology and education: Theory and a case study (Herb Ginsburg, Ann Cami, Eram Schlegel - Teachers College)
- Adapting pedagogy to a Web-based video analysis system (Joon Lee - Hunter College/CUNY)
- The development of critical thinking skills using a Web-based video analysis system (Michael Preston - CCNMTL)
• Evaluating students’ cognitive outcomes in a Web-based video analysis system (Cornelia Brunner - EDC)

The discussant was Daniel Schwartz, Stanford University School of Education.

June 10, 2008
VITAL at NAEYC professional development institute
Prof. Herbert Ginsburg and CCNMTL’s Michael Preston presented the VITAL project at the National Association for the Education of Young Children (NAEYC)’s 17th National Institute for Early Childhood Professional Development in New Orleans, LA. The theme of the institute was “Technology and Early Childhood Education.” The VITAL presentation focused on the use of video—both in the classroom and online—to engage teachers in close viewing and analysis, with the goal of helping teachers better understand the ideas they learn about in courses and apply them to the assessment and teaching of children. Case studies from Hunter College and Teachers College were used to illustrate teacher learning outcomes.

November 13, 2008
VITAL at NSF DR-K12 Annual PI Meeting
Principal Investigators Frank Moretti of CCNMTL and Herbert P. Ginsburg of Teachers College, Columbia University, presented to their fellow grant recipients at the National Science Foundation’s annual PI meeting of the Discovery Research K-12 program. The presentation, entitled “VITAL: A Web-based Video Analysis System for Teaching University-level Courses in Early Childhood Mathematics Education,” described the evolution of the early mathematics course and VITAL Web environment, as well as what it has taught us about teaching and student learning.

December 16, 2008
Herbert Ginsburg at TERC
Prof. Herbert Ginsburg (Co-PI) of Teachers College, Columbia University, presented on the VITAL project at TERC, a non-profit organization devoted to mathematics and science education curriculum development, training, research and evaluation. The focus of the presentation was on using VITAL to help teachers understand that they are helping children bridge the gap between everyday and scientific thinking, and to better understand the role they can play in their students’ ongoing process of meaning-making. The VITAL video analysis tools are one promising method for developing teachers’ skills of close observation and clinical interviewing, which give them greater access to children’s thinking.

April 2-4, 2009
VITAL at Society for Research in Child Development conference
Partners in our VITAL project presented their research on student learning at the 2009 biennial meeting of the Society for Research in Child Development (SRCD), in Denver, CO. The title of the paper symposium was “A Video-Based Pedagogy for Improving College Students’ Understanding of Development and Education,” and the session was chaired by Prof. Ginsburg of Teachers College and Carol Copple of the National Association for the Education of Young Children (NAEYC). Three papers were presented:
  • Observation as Contexts for Student Interpretation and Critical Thinking (Prof. Lori Custodero and Prof. Ann Cami, Teachers College, Columbia University)
• Teaching Educational Psychology With Video Case Studies: Going Beyond Psychological Theories in Context-Specific Case Analyses (Prof. Noriyuki Inoue, University of San Diego) PDF, 176 KB. Available at http://ccnmtl.columbia.edu/vital/nsf/VITAL_SRCD_2009_INOUE.pdf

• Video-Based Exercises for Developing Early Childhood Educators’ Use of Evidence and Interpretation (Michael Preston, Columbia Center for New Media Teaching and Learning) PDF, 1 MB. Available at: http://ccnmtl.columbia.edu/vital/nsf/VITAL_SRCD_2009_PRESTON.pdf
Appendix D: 2007 Evaluation Overview

Below is a model of the original evaluation design, upon which the 2007 evaluation was based. The evaluation conducted in 2009 is included in the body of the final report. Its results will be disseminated at the 2010 AERA conference.

![Logic Model Diagram]

The resources (inputs) available in VITAL consist of facilitated viewing of some video segments in class, the readings related to assignments available in VITAL, the videos, links to additional articles or books (possibly off-line) and the work done by other students.

The actual process—the activities—consists of clipping, writing assignments and creating or uploading project essays and illustrations (including videos made by students, which are part of the final project).

Outputs, or evidence that these activities have occurred, exist in the form of logs of VITAL sessions and student essays and reflections with clip links embedded.

The short term outcomes, changes that might be discernable by instructors and students in the courses, might be the degree of close viewing engaged in by students (e.g., how appropriate the
clips are to the argument they are illustrating in essays), any change in the degree or nature of the reflections students share with their instructors and each other (e.g., class discussions), and any change in the depth of the students’ insight into the development of early mathematical thinking.

The long term outcomes are hypothesized to be teachers who are better prepared for teaching practice and whose teaching skills have improved as a result of their deeper understanding of the subject matter and their opportunities to conduct more in-depth virtual observations.

The evaluation of the technology itself has as its focus the extent to which students are able to demonstrate their ability to analyze video segments closely and to use the VITAL environment to enhance their own learning process.

During the 2006 school year, previous year we experimented with a video-based subject matter test as one way to investigate students’ ability to view video closely and analytically. During 2007, we experimented with a self-report technique to ascertain whether students’ interpretation of the way in which the VITAL environment is useful changes as they become increasingly familiar with it.

Methods:

**Critical Incident Report (CIT) protocol**

The Critical Incident Technique (CIT) is a method used to collect observations of human behavior in defined situations (Flanagan, 1954). The technique involves collecting structured, open-ended reports that typically include 1) a description of the situation 2) an account of the actions or behavior of the person involved in the incident and 3) the outcome or result. CIT has been widely used to study “what people do” in a variety of fields including industrial psychology, management, health and education (Fivars, 1980; Fivars and Fitzgerald, 2001).

In this study, the CIT method was used to collect structured data about an incident that stands out in students’ minds as a critical moment of recognizing the effect of using the VITAL environment in their course. The protocol we developed asks learners to describe a recent incident during which the VITAL environment was particularly useful in helping them learn. Specifically, we asked them to describe

- the situation - what they were doing in the environment;
- the insight or information they obtained in the environment;
- the specific feature of VITAL that helped them;
- what would have been different without VITAL; and
- how important they consider the VITAL-supported insight.

The CIT was administered twice, in the middle and at the end of the course. We collected CIT reports from 57 students. We obtained two CIT reports from 33 students, of which 28 took the TC course and 5 took the Hunter course.
We coded the 87 CIT reports according to the following scheme:

1 = *features* only, i.e., the VITAL environment made homework more efficient by allowing easy access to the material
1.5 = most of the response is about the efficiency of VITAL features but there is a generic mention of the value of the content.
2 = *content* only, i.e., the video provided students with access to “virtual” children by allowing them to observe children’s thinking, served as a model of professional behavior (interviewing techniques), or enabled them to do better at their assignments by providing them with better illustrations of their points by pointing to the video clips.
2.5 = the major emphasis in the response is about the content of the videos, but there is some general mention of the way in which that content affected their understanding
3 = *process*, i.e., the students reflected on the ways in which the features of the VITAL environment affected their own learning process.

Results:

A content analysis of the responses in the CIT reports yielded the following breakdown:

<table>
<thead>
<tr>
<th>CODE</th>
<th>ASPECT of VITAL</th>
<th>pre</th>
<th>post</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>administrative efficiency</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>content model of profession</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>virtual kids</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>better illustration of readings</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>better argument in assignments</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>good material</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>active learning process</td>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

Figure 1: Frequency of VITAL aspects mentioned in both CIT reports.

The largest number of students mentioned the efficiency features of VITAL, having all the clips available in one place, ease of integrating them into their essays, seeing all the assignments, being able to search through the library of clips and resources, etc. as being most meaningful for them in their CIT.

Among the students who focused on content rather than features, many mentioned that the clips provided them with a good model of how to interrogate kids about their thinking as professionals, both as teachers and as observers or researchers. Some focused more on the way VITAL provided them with access to “virtual kids”, to seeing how kids really think and how beneficial they found it to have such good material for observation.

Some of the students whose focus was content mentioned the benefits of VITAL for their own work, both by providing them with good illustrations of the ideas described in the readings and because the availability of the clips permitted them to create better arguments in their course assignments. There was a shift from the first to the second administration of the CIT, with an
increasing focus on the value of VITAL for producing better assignments, possibly because they
were dealing with their final projects at the time of the second CIT administration.

Students whose focus was on their own learning process, talked about the kind of additional
information one can get from non-verbal clues (visual information), the ability to compare
different children’s responses in different clips dealing with similar problems, and how their own
understanding of children’s thinking changes as a result of multiple viewings of the same clip.
There were more “process” responses during the second administration of the CIT, but since a
number of those students did not participate in the administration of the first CIT, no comparison
can be made.

There was some shift in the codes over the semester. Fewer students focused on purely
administrative use of VITAL and more students focused on its contribution to their own learning
process. The difference is significant according to a Chi-square test (10.075) at alpha = .05

<table>
<thead>
<tr>
<th>CODE</th>
<th>pre</th>
<th>post</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>1.5 - 2</td>
<td>25</td>
<td>22</td>
</tr>
<tr>
<td>2.5 - 3</td>
<td>6</td>
<td>11</td>
</tr>
</tbody>
</table>

Figure 2: Frequency of codes in both CIT administrations.
Fourteen of the students who completed both CIT reports obtained a higher code on the second administration (green area). Eight students obtained a lower code on the second administration (pink area), and eight students obtained the same code on both administrations or completed only one CIT (white area).
There does not appear to be a relationship between students’ performance on the CIT report and the grades they received for the course as a whole, which consists of an average of grades for assignments, reflections and a final paper.

Figure 4: The relationship between the degree of change in CT codes and final grades for the course.

Figures 5 and 6: The relationship between codes on the first CIT report (fig. 5) and the second CIT report (fig. 6) and the final grades for the course.

Figure 4 appears to show a relationship in that the preponderance of students who obtained an increase in CIT codes also seem to have done better in the course overall, but it is not borne out by more systematic analysis. Examination of the relationship between CIT codes and final grade for each administration (figs. 5 and 6) shows that there are as many students who obtained low CIT codes but did very well on the course as there are students who obtained higher CIT codes but did not do as well on the course as a whole.
Implications of 2007 Evaluation:

The findings were very tentative because the sample was so small. If it held in the larger sample, it would imply that the CIT really asks students to think about a very different topic than the course itself. It was the only instance where students are asked to reflect directly on the relationship between the technology and the substance of the course. We asked them to reflect on the benefits of the technology because that is the focus of this evaluation. The empirical question we could consider going forward is: whether students who become increasingly aware of the benefits of the technology not only for administrative purposes and for the content of the video material, but also because engagement with it can positively affect their own learning process, also become more thoughtful about early mathematical thinking in children. At this point, there we found very little basis for linking “learning” in the course with learning about the affordances of technology. The larger study would have to show whether the use of the technology leads to a better understanding of the content of the course. A comparison group would be needed.

Appendix E: VITAL Faculty Guide
VITAL Faculty Guide:

Introduction to a course on the development of mathematical thinking and its technology
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Dear Reader,

Welcome to your introduction to a unique course on the development of mathematical thinking! This guide will navigate you through the many innovative features of the course, including, most importantly, the use of a technology entitled Video Interaction for Teaching and Learning (VITAL). After explaining the rationale behind the development of this course and technology, this guide will then help you learn about the content of the course and the pedagogy of teaching with VITAL. Information about assignments and the logistics of using VITAL are also presented, and evidence of learning through this course and technology is included in appropriate sections.

We hope that you will find this guide helpful and comprehensive, and we look forward to working with you as you implement your first VITAL course!

Sincerely,
The VITAL team
The Course: Development of Mathematical Thinking

Goals

The main goal of Development of Mathematical Thinking is to prepare early childhood teachers for teaching mathematics. To do this, the course helps early childhood teachers learn how to think deeply about the content of early childhood mathematics, and to use this content knowledge in making effective professional decisions about instruction in the classroom. Furthermore, the content is presented within a comprehensive developmental framework, integrating the content with theory on how children think, learn, and interact so that the mathematics becomes relevant to teaching.

Students in the course acquire expertise in several areas, including:

- Knowledge of early childhood mathematics and its interrelations with later math topics
- What children know about mathematics, how they think about it, and how they learn it
- Effective pedagogy for teaching early childhood mathematics
- Tools of assessment and interpretation of children’s knowledge and thinking, including observation and interviews
- Evaluation of early childhood mathematics programs

We have found that, as students progress through the course, they come to think differently about early childhood mathematics instruction in ways that we foster within the course. For example, as they see the extent of mathematical knowledge and thinking possessed by children, they see mathematics as more age-appropriate and child-generated, as well as an appropriate goal of preschool. In addition, students report becoming more comfortable and confident in their mathematics teaching abilities. These attitudes are critical because they are correlated with effort and effectiveness in teaching.

An equally important secondary goal of the course is to help students relate research to practice. To facilitate this, the course utilizes videos of children in rich, diverse learning and teaching situations to illustrate concretely children’s thinking and mathematical development. Students in the course actively engage in observation, interpretation and analysis of children’s behaviors through close viewing of the videos. By practicing such skills repeatedly and reflecting on their work, students learn to make evidence-based pedagogical decisions that are relevant to “real” teaching. Moreover, with the growth of the ability to ground theory in evidence comes an increase in the students’ belief in the utility of theory to the practice of teaching, and therefore the propensity to rely on research on child development and the development of mathematical thinking to guide their teaching. In this way, this “academic” course becomes meaningful, rather than remaining simply “academic” in the pejorative sense.

Again, we have found that students in the course do develop in their ability and willingness to relate research to practice over the semester. Students become more selective with the claims they make about children’s thinking and knowledge from the videos, support their claims with more evidence from the videos and relevant literature, and also grow to realize the limitations of
the claims they make and what additional evidence might help to uncover a child’s understanding.

Finally, it is important to note that, though our focus is on the learning and teaching of mathematics and particularly early childhood mathematics, the methods of the course can work with virtually any academic discipline that can employ videos to illustrate important phenomena. In fact, the video pedagogy that the course relies upon has been successfully applied to areas as diverse as music education, social and emotional development, and couples therapy. Therefore, we are very committed to disseminating this course and the technology of VITAL to others.

**Content**

The content of the course covers research from child development, educational psychology, and mathematics education (Syllabus: Appendix A). In doing so, the course presents a comprehensive developmental approach to early mathematics education that synthesizes early childhood mathematics with psychological ideas, methods for revealing children’s thinking, principles of pedagogy, and examples of promising instructional activities. The course focuses on the following themes:

- Developmentally and educationally appropriate mathematics for young children
- The informal mathematical knowledge of young children, and the formalization of this knowledge
- The application of knowledge of young children’s mathematical development to teaching and learning
- Methods of formative assessment and uses of multiple sources of evidence in assessing children’s learning
- The practice of teaching, focusing on the use of the results of formative assessment to guide further instruction

In developing these themes, each session of the course focuses on a particular aspect of early childhood mathematics teaching and learning that builds upon the prior week’s learning. A session includes a lecture, class discussion, analysis of videos, and other activities that take place in the higher education classroom. Below you will find an outline of the course by week. This outline is based on the 14 weeks from the fall 2009 Teachers College course, and reflects one possible approach to the material. But it is important to note that topics can often be switched in response to the needs of the individual instructor and group of students. The syllabus in the appendix also reflects the fall 2009 course. In exploring this syllabus, notice that in addition to readings, the hallmark of educational method for centuries, we also employ videos, which are becoming increasingly common in many psychology and education courses. Thus, suggested videos are also listed with each topic in the syllabus. You will also find annotations of the videos used in each video in Appendix C as well as lecture slides with transcripts in Appendix D, both of which should further clarify the goals and details of each course section.

The first strand of the course covers material relating to the psychology of the child’s mathematical knowledge, while the second strand examines pedagogy and curriculum. As you
will see, mathematics is typically not presented separately, but instead integrated into the focus on children and teaching.

**Introduction: Session 1**

This session presents the rationale behind and issues surrounding early childhood mathematics education. It spends some time addressing teachers’ hesitancy and, in many cases, anxiety about teaching mathematics, and encourages class discussion about student experiences in learning mathematics. The session then explores what factors need consideration when teaching children mathematics, including children’s capabilities, mathematical content, pedagogy, assessment, and mathematics that children might encounter outside school.

Following each session, students are asked to complete a Reflection in which they comment on issues of interest, discuss key ideas that they learned from the class, offer reasoned disagreements with positions taken in class, and apply the material their own experiences and instructional practice.

**Everyday Mathematics: Session 2**

This session addresses “everyday mathematics,” or mathematics that children encounter outside of formal schooling. The primary goal of the session is for students to learn to recognize and appreciate children’s experiences with everyday mathematics so that they can become more aware of the types of mathematical knowledge and skills with which students enter school. The next goal is to help students think about ways to foster these types of everyday experiences, even in the classroom, and how to mathematize them, that is, make them explicit and formal.

**Early number: Session 3**

This session covers ideas about early number concepts and skills, including ideas about more and less, conservation, cardinality and enumeration. It begins by illustrating the way that young children judge the magnitude of sets based on perceptual rather than empirical evidence (i.e., counting). It then addresses number words and how learning them involves both memorization and recognition of patterns. Finally the session covers enumeration, cardinality, and conservation of number—e.g., what rules must be followed for counting objects accurately, factors that may make some sets of objects more difficult for children to count than others, and challenges that children face in fully grasping cardinality (i.e., understanding that a number word conveys how many objects are in a set and that this value stays the same even if the set is rearranged).

**Enumeration and Addition: Session 4**

This session continues with enumeration and the related ideas of conservation and cardinality. It then addresses children’s early ideas about addition and subtraction, looking at their understanding and the strategies they possess before they begin learning these topics formally in school. Important ideas are detection of change in amount and direction of change, concrete versus abstract representations, and the relationship between enumeration and addition. Different addition strategies (e.g., counting all, counting on, retrieval, derived facts) of increasing sophistication are also presented.
Assessment: Session 5
This session covers issues of assessment and different types of assessment, including testing, observation, and clinical interviewing. The session then focuses on clinical interviewing and shows how it is a powerful method for understanding children’s thinking and learning. Different ways of implementing clinical interviewing in the classroom are also considered. Two long interviews are used to allow for observation and analysis of this method.

Symbols: Session 6
This session focuses on children’s first experiences with formal symbolism. It addresses different types of representations such as tallies, pictures, and finally conventional symbols signifying both numbers and operations, and also addresses the connections among these symbols. A major goal of the session is to help students recognize how foreign these formal symbols can be to children, and the types of challenges they must face when learning to interpret and use them.

Number facts: Session 7
This session addresses number facts and the debate about the importance of mastery versus understanding. The goal is not to take one side, but to deeply examine the issues and learn what is involved in learning number facts. The point is made that both mastery and understanding are important, and that the way they are taught is not always the way they are learned. That is, even if we think we are teaching children to memorize, they might be building deep meaning, or vice versa. Also, the experience of learning the facts is different for different children: while some enjoy learning the facts, for others it is a negative experience fraught with anxiety. Such affective experiences need to be recognized as influencing learning. Finally the session discusses some ways that teaching and learning number facts can be meaningful by exploring children’s invented strategies and how they are formalized into algorithms.

Written Procedures: Session 8
Moving on from counting and combining as a form of addition, students focus on computation by exploring children’s invented strategies and how they are formalized into algorithms. Furthermore, children’s mistakes are used to examine their thinking. The focus is consistently on meaningful strategies rather than rote memorization.

Understanding: Session 9
This session builds on the discussion of the importance of both procedural and conceptual learning of mathematical ideas, examining what it means for students to fully understand them. Understanding is viewed as much more than simply getting correct answers. It involves seeing the big picture—e.g., seeing the structure and purpose of an idea and its connections to other ideas, as well as metacognition and the ability to apply mathematical ideas across appropriate contexts. Understanding also requires an ability to communicate one’s reasoning, justify answers, and connect solution methods and strategies. The challenge of assessing understanding is also considered.

Geometry: Session 10
Children’s understanding of shape and space is explored in this session. Students see that preschoolers and elementary students can discriminate among shapes and sort them by their most obvious attributes, despite lacking the language for describing such differentiation. However,
children often struggle to classify non-prototypical shapes in appropriate categories. A progression of recognizing geometric objects globally to describing objects by their geometric properties to using reasoning to analyze geometric relationships is presented.

**Patterns & Algebra: Session 11**
As algebra is seen as a gatekeeper to more advanced mathematics and even higher education more generally, the National Council of Teachers of Mathematics has included pre-algebra standards in their early childhood mathematics teaching recommendations. For example, pre-K children are expected to recognize and duplicate simple patterns, while Kindergarten should identify, duplicate, and extend simple number patterns in preparation for creating rules to describe relationships. This session explores what these pre-algebraic ideas actually mean for young children and how they can be employed in the classroom. Furthermore, the ability of algebra to strengthen arithmetic understanding is explored.

**Teaching: Session 12**
The challenges of effective teaching are presented. The session explores questions of what teachers need to know in order to teach well, in what ways they need to understand the student, the tricks of the trade they need to know, and how they capture students’ interest and attention. Furthermore, constructivism is deconstructed so that it is a meaningful idea rather than an empty slogan.

**Formative Assessment: Session 13**
This session provides an overview of assessment issues and shows how formative assessment methods like clinical interviewing are a powerful method for understanding children’s performance, thinking and knowledge, and learning potential. Effective formative assessment requires teachers to know the content, the developmental progressions (of both ideas and cognitive processes) that students undergo as they learn that content, and the obstacles that children face in learning the content. The session concludes by addressing how to help teachers approach formative assessment using a new tool being developed by the professor called mCLASS:Math.

**Manipulatives and Curriculum: Session 14**
The session emphasizes that understanding involves connections among various systems of knowledge and skill, and that manipulatives have the power to aid in this process (as well as the potential to misuse manipulatives in ineffective and counterproductive ways). The proposition is made that if a child learns to use manipulatives well in support of his learning, then he should eventually be able to get rid of them. That is, manipulatives allow children to develop a mental model that eliminates the need for external manipulatives.

**Pedagogy**
The use of VITAL, and particularly the access to videos that it provides, is integral to the success of this course in preparing early childhood teachers for teaching mathematics. This is primarily because videos can capture richer, more detailed and complex situations of learning and teaching than can text-based descriptions of such events. As one student noted, “It would be difficult to understand or imagine a lot of the subtle movements or facial expressions [of children] during
reading, but the videos provided these images for us to interpret.” Furthermore, video is succinct, and can be paused and viewed repeatedly, slowing down the processes of interpretation and decision-making that teachers constantly engage in while in the act of teaching. Moreover, these episodes of real classrooms and real children allow for more convenient access to diverse learners and contexts than does live observation. In effect, VITAL acts as a scaffold for students who are learning to carefully observe children in order to interpret their knowledge and understandings and to decide on appropriate teaching moves.

When using the video clips within VITAL in classroom sessions, the course instructor needs to lead his/her students in active discussions about what they have seen. Certain pedagogical techniques may be employed to elicit students’ observations and interpretations of the videos. For example, short clips can be presented repeatedly after an empirical problem has been posed. By focusing student attention on a particular issue and then showing a very short segment several times, students have the opportunity to hone their close observation skills. As the instructor replays these short excerpts, s/he should ask students what occurs, encourage their interpretations of the action, promote follow-up discussions of the interpretations, encourage different viewpoints among students, and press them to lay out the evidence they employed to arrive at a particular interpretation. Then, the instructor might play the video again to clarify the analysis and conclusions, pointing out any weaknesses in the interpretations based on evidence from the clips (e.g., by pointing out ignored evidence or contradictions in the child’s thinking). Ultimately this process can help students to become aware of the dilemmas of evidence-based interpretation and the limits of generalizing from the clips.

The process of examining the video content continuously encourages students to consider the children’s actions and what the evidence of specific actions means for their interpretations, which of course can and should be revised and refined. The video clips we use tend to be brief, sometimes shorter than one minute, but typically no longer than two or three. One minute of rich video can easily generate 10 minutes of fruitful discussion. This type of experience helps students to think critically and creatively when approaching various uncertain and irregular contexts of teaching and learning in an actual classroom.

Perhaps a concrete example will be useful in demonstrating the pedagogy of using video in the classroom. We now present a case study of the analysis of a video involving 18-month old “Baby Hope.” Note that the entire viewing of the video and interchange described lasted less than 2 minutes. The video demonstrates the instructor’s role of selecting the video, framing the questions, and challenging the student’s interpretations by pointing out evidence ignored, contradictions in thinking, or unsound interpretations.

The topic of the week was the origins of mathematical thinking in babies and young children, and readings had addressed research on the ability of babies to perceive key aspects of the physical world. The professor posed the question to students about whether babies begin as “blank slates” or whether they already possess various perceptual and conceptual competencies. After a few initial responses, the class watched the first clip of the Baby Hope video.
The professor introduced the clip by asking a question such as, “What does this teach us about Hope’s idea of number?” This first clip was 8 seconds long and showed the 18-month old baby holding some rings in her hands and referring to them individually as “ring” and then collectively as “rings.” Following the viewing, the professor asks, “What did you see there?” or “What did you notice?” or “Was there anything important there?” Before soliciting responses the professor replayed the clip another time.

Students might respond with observations of what happened in the clip, and the professor must help them to interpret their observations using questions like, “OK, what about that?” For example, in this case study a student reported seeing the baby’s understanding of “one-to-one correspondence.” The professor responded with another question, “But what did the baby do? Never mind the principles of counting. What told you she has one-to-one correspondence?” The professor’s goal was to have the student think through exactly how the theoretical concept she offered might apply (or not) to this particular case. In doing this, the professor also challenges the student’s interpretation with the goal of getting her to make clear its evidentiary foundation. Students are not allowed simply to express an “opinion,” but must make a claim that can be supported by evidence.

Finally, the professor must make explicit the strengths or weaknesses in a student’s comment. Ideally other students would do this, but establishing such a culture can takes weeks, so the professor must often model it initially. This modeling should include telling students that they should attempt to look carefully and buttress claims with relevant evidence.

Most classroom sessions, which are about 90 minutes long, involve somewhere between three to six videos. The instructor offers comments and poses questions to students before, during, and after video clips are shown. Discussion surrounding the video clip ranges from two minutes to twenty minutes, with most discussions lasting less than ten minutes. References to the video clips are made throughout the session, and memorable clips presented in one session are often discussed in later weeks. Students get actively engaged in the process and rate it very highly in their evaluations of the course. Indeed, many students say that analysis of videos is one of the most important features of the course.

We might say that the video clips are the “manipulatives” of the course, analogous to the Cuisenaire rods students use to learn mathematics. In both the case of the videos and rods, the students have a chance to develop abstract ideas through active engagement with the “concrete” materials. Furthermore, just as the effective use of the rods is dependent on the teacher’s knowledge of the content, insight into students’ reasoning, and the ability to respond to the individual student’s insights, so is the effective use of the videos.

Our research indicates that students who study video cases demonstrate an increased ability to apply relevant theoretical concepts about teaching and learning to their understanding of classroom practices, especially when they are given extended time and multiple opportunities to analyze and interpret the cases. Digital technologies provide such opportunities by allowing students, among other things, to instantly access segments of a video clip, annotate those segments, incorporate them into their analytical work, and share their essays with peers and
instructors. Digital analytic and communication tools thus facilitate sophisticated reflection and discourse on video cases. We next elaborate on the technology of VITAL and how it facilitates the pedagogical moves just described.
VITAL

What is it?

“Video Interactions for Teaching and Learning” (VITAL) is a Web-based video analysis system developed by the Columbia Center for New Media Teaching and Learning. The system provides pre-service teachers with ready access to a library of online videos, along with a set of tools for video analysis designed to help them practice and refine their skills of close viewing.

Multimedia Syllabus

A course “home” page in VITAL looks like a conventional syllabus with a list of topics, readings, and assignments, but it also includes a selection of videos for each topic that can be viewed by clicking on the links embedded in the page. (See Fig. 1.) Because the full course library includes more than 100 videos, the careful selection of videos for each topic in the syllabus is critical, particularly when choosing videos for the assignments that students will complete.

Fig. 1: A single week’s topic from the syllabus in VITAL, including an assignment with one required video, followed by six recommended videos.
Digital Video Library

The digital library includes videos of children in classrooms, in naturalistic settings (e.g., playing), and being interviewed. The videos highlight children’s mathematical behaviors during instruction, play, and everyday activities. For example, videos of small group instruction demonstrate children grappling with new ideas and with explaining ideas to others. The interviews display children’s thinking and problem solving across a wide variety of mathematical content.

Video Viewer

VITAL offers several affordances that enable the course’s students to work with video at home via the Web. The first feature is a “video viewer” in which students can select and clip their own segments from the videos. They can also attach a note to each clip to help them remember the significance of the content. (See Fig. 2.) These clips and notes are saved in a personal workspace, where they can be accessed later and used to support an essay. The ability to view videos repeatedly lets students build understanding at their own pace and practice their observation skills in a controlled context.
Multimedia Essay

The second feature is a “multimedia essay” space where students can integrate their clips with text. (See Fig. 3.) In the course, students are asked to write Clipper essays of 350 words or fewer in response to questions such as, “What do the children [in the video] know about number? Please cite from the videos and the readings.” These assignments encourage students to develop their own hypotheses about children’s understandings based upon their observations of a videotape, and then to select evidence from the course material and the video that supports their arguments. Completed essays are “published” within the VITAL environment to be read by the instructor and other students. The instructor can also leave feedback for the student. (See Appendix F for a guide to writing VITAL essays.)

![Fig. 3: The multimedia essay, with the student’s collected video clips on the left side of the screen, and a writing space incorporating text and video on the right. Students click or drag their video clips to add them to their essay.](image)

In addition to essays, students complete a series of “guided lessons” in clinical interviewing. These assignments are linear exercises that step students through videotaped interviews with prompts for them to interpret both the child’s behavior and the interviewer’s technique, as well as to anticipate what the child will do next, and to make recommendations for subsequent interviewer questions. After answering the prompts, students often have the chance to read an expert’s commentary that encourages them to further reflect upon and refine their thinking about the child’s understanding and the interviewer’s technique. All of this serves to simulate a clinical interview for students so that when they proceed to do their own Clinical Interviews they are
more prepared than they would be had they simply read about the process of clinical interviewing.

Students also write weekly Reflections in VITAL within 24 hours of class. This concluding event for the week serves as an opportunity for students to express what they learned, pose questions, and dispute ideas discussed in class.

In the final month of the course, students complete a Final Project that involves designing a mathematical lesson or activity, trying it out with a child, and interviewing the child beforehand and afterwards to find out what he or she learned as a result of the lesson. The student records these events on videotape, submits the tape for inclusion in the VITAL library, and writes a research paper in the form of an extended multimedia essay that details the literature, methods employed, and results obtained. The final project integrates the mathematics content learned in the course with the assessment skills associated with clinical interviewing. The final project report submitted in VITAL also serves as a demonstration of the students’ ability to think critically—even scientifically—about the work they are doing as teachers and what a child might be learning as a result. (See Appendix K for sample permission forms to videotape children and Appendix L for specification of taking video for VITAL.)

In brief, VITAL introduces a sequence of activities—Clipper essays, lessons, Reflections, a Clinical Interview, and a Final Project—that are designed to help students learn to observe, make hypotheses, evaluate interpretations in the light of evidence, use a clinical interview to investigate student thinking, and apply their ideas and skills to teaching. As mentioned, careful study has shown that the use of this technology does indeed improve students’ critical observation and analysis skills. They come to make more careful claims about children’s thinking and understanding, they better support their claims with relevant evidence from clips of the videos, and they better clarify the limits of their claims and evidence in light of children’s inconsistent or ambiguous behaviors.
Using VITAL and Creating a Course

Using VITAL

VITAL and all the videos referenced in this faculty guide can be accessed at http://vital.ccncmtl.columbia.edu. You will need a high-speed connection (e.g., DSL or cable) to use VITAL. Any web browser except Safari will work. (Mozilla Firefox is an excellent browser for both Windows and Mac. It can be downloaded at http://www.mozilla.com/firefox/.) To view the videos in VITAL you will need the current version of QuickTime (at least version 7). It can be downloaded at http://www.apple.com/quicktime/download/.

Students will be assigned an individual log in and will create their own password in order to access their particular class site. This protects the privacy of the work, videos, and ideas that are generated by the students.

Once students reach their course site, they may use the following tips to review the site:

- View a topic by clicking the “+” sign next to it.
- View videos by clicking the video title or thumbnail image, which will launch the video viewer.
- Create video clips by clicking “Set start” and “Set end” while watching a video. Students should title their clips and may also add notes and/or tags, which may make it easier to find them later. Once saved, the clip appears on the right side of the video viewer and is accessible to students to embed in their essays. Clips may be edited and/or deleted at any time.
- Begin essays by clicking the title of the assignment in a particular class session. Students may prefer to write their essays in Word and then paste them into the VITAL text area. This allows them to use the spelling and grammar features of Word, which are not currently available in VITAL.
- Embed video clips by clicking the film icon to place a clip wherever the cursor is located in the essay, or by dragging the icon to where it should appear. The cut/copy/paste commands can be used to move clips within an essay.
- Submit essays by clicking the “Submit” button. Once pressed, the instructor will be able to read the essay, as well as any classmates who have also completed their own essays. You cannot undo a submission.
- Questions/problems? Consult the help documentation link in the upper right hand corner of the VITAL page, or submit a question or problem using the “Report Problems” link that is also in the upper right corner.

Creating Course Assignments

To add assignments in VITAL, you must enter into a course session and click on the tab labeled “Add Assignment.” You then give a title to the assignment, select the type of assignment it is (e.g., essay or guided lesson) and describe the assignment. You can also assign a due date to the assignment. Once the assignment has been created, you can associate a video with the
assignment as well by clicking on the appropriate link. The user-friendly environment of VITAL makes all of these tasks fairly simple.

Aside from the logistics of creating an assignment within VITAL, it is also important to consider the pedagogy of VITAL in developing assignments. With the goal of having students become careful observers of children’s behavior, assignments can scaffold them with targeted prompts for what to look for in a video or what to write about after viewing a video. As an example, you might look at the details of the assignments for the Clinical Interview and Final Project, both of which have attempted to be very clear in what issues students should address and thereby defines the goals of the assignments. For example, students should make hypotheses about what a student knows, back it up with specific clips from videos, and consider alternative explanations for the child’s behaviors.

Managing & Evaluating Student Work

One of the best parts of creating a course in VITAL is that everything is in one place. Students can submit assignments directly to the site and your feedback can also be posted there for their review. This feature also allows students to view others’ assignments once they have posted their own assignment, fostering peer learning and collaboration. Importantly, however, feedback is only visible to individual students.

We provide several rubrics below to help guide you in evaluating student work with the pedagogy of VITAL in mind. See the rubric for Writing VITAL Essays, for the Final Project, and the Teaching Rubric below.

Next Steps

We conclude this guide by mentioning that this course is a continual work in progress. This guide reflects our most recent course that was offered in the fall of 2009. We have included modules of lessons that track the class syllabus; these modules include background data on individual videos, suggested discussion questions and related videos; modules can be used in the same sequence as the syllabus or selected individually to support the needs of differing faculty expertise and curricula. We also provide sample transcripts of the opening sessions of the course, and eventually we hope to provide detailed commentary along with these transcripts that provide the motivations behind the way the course was put together and taught and considerations that were addressed in making choices about course content, video selection, and course assignments.
Appendix A: Syllabus

This course examines psychological research and theory concerning the development of children's mathematical thinking, and considers applications of this knowledge to teaching, assessment, and other educational issues.

Readings are always presented in the order in which they should be read:

Introduction:

Everyday Mathematics:

Early Number:

Enumeration & Addition:
Assessment: Observation, Tests, and Clinical Interviewing


Symbols:


Number Facts:


Written Procedures: Place Value & Computing with Big Numbers

Understanding:
- Ginsburg, H. P. Children’s Arithmetic, Chapter 9, pages 183-211.

Geometry: Shape & Space

Patterns & Algebra:

Teaching: What is it?

Formative Assessment:

Lee, YS et al. How did you know that? - Digging deeper into K-3rd grade students’ mathematical strategy use with in the mCLASS: Math assessment system. Paper to be delivered at AERA.

**Manipulatives & Curriculum:**

## Appendix B: VITAL Video Catalogue

<table>
<thead>
<tr>
<th>Title</th>
<th>Child’s Name</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction &amp; Background</strong></td>
<td></td>
</tr>
<tr>
<td>Sesame Street: Dogs &amp; mud pies</td>
<td>N/A</td>
</tr>
<tr>
<td>Adding bears: invisible to visible</td>
<td>Olivia</td>
</tr>
<tr>
<td>Counting and pointing to numbers chart</td>
<td>Corpus Christi (CC)</td>
</tr>
<tr>
<td>Stanley counting beads during free play</td>
<td>Stanley</td>
</tr>
<tr>
<td>Carrot math-- moving pictures of carrots across mind</td>
<td>Rachel</td>
</tr>
<tr>
<td><strong>Everyday Mathematics</strong></td>
<td></td>
</tr>
<tr>
<td>Working with colored cubes</td>
<td>Corpus Christi</td>
</tr>
<tr>
<td>Playing with building blocks</td>
<td>Armando &amp; Keithly</td>
</tr>
<tr>
<td>Free play clapping and calendar games</td>
<td>Emily and friends</td>
</tr>
<tr>
<td>Infant playing with rings</td>
<td>Hope</td>
</tr>
<tr>
<td>Building a bigger garage</td>
<td>Eduardo &amp; David</td>
</tr>
<tr>
<td>Building a road and a tower</td>
<td>Vienna &amp; HoYoung</td>
</tr>
<tr>
<td>Counting beads during free play</td>
<td>Stanley</td>
</tr>
<tr>
<td>Everyday math examples</td>
<td>Union Theological</td>
</tr>
<tr>
<td><strong>Early number: Counting</strong></td>
<td></td>
</tr>
<tr>
<td>Counting: indicating mistakes</td>
<td>Little Josh</td>
</tr>
<tr>
<td>Counting with fingers</td>
<td>Lateek</td>
</tr>
<tr>
<td>Counting &amp; figuring out decades</td>
<td>Michael</td>
</tr>
<tr>
<td>Lesson: Count Clap &amp; Stomp</td>
<td>Corpus Christi</td>
</tr>
<tr>
<td>Counting in circles</td>
<td>Nick/ Kevin</td>
</tr>
<tr>
<td>Overgeneralization in counting</td>
<td>Tom</td>
</tr>
<tr>
<td>Counting beads during free play</td>
<td>Stanley</td>
</tr>
<tr>
<td><strong>Early number: Conservation</strong></td>
<td></td>
</tr>
<tr>
<td>Cardinality and conservation with bears</td>
<td>Harry</td>
</tr>
<tr>
<td>Conservation of number with bears</td>
<td>Vienna</td>
</tr>
<tr>
<td>Which side has more dots?</td>
<td>Tyquasha</td>
</tr>
<tr>
<td>Conservation of equivalence</td>
<td>Kristin</td>
</tr>
<tr>
<td>Conservation of number with green cubes</td>
<td>Tyquasha</td>
</tr>
<tr>
<td>Which side has more dots?</td>
<td>Vienna</td>
</tr>
<tr>
<td>Conservation of equivalence</td>
<td>Liola</td>
</tr>
<tr>
<td>Seven dots different orientations</td>
<td>Lisa</td>
</tr>
<tr>
<td>Title</td>
<td>Child’s Name</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Conservation of equivalence</td>
<td>Seren</td>
</tr>
<tr>
<td><strong>Early number: Enumeration</strong></td>
<td></td>
</tr>
<tr>
<td>Counting a line of hot dogs</td>
<td>Alexander</td>
</tr>
<tr>
<td>Where is the winner?</td>
<td>Danny</td>
</tr>
<tr>
<td>Counting: configurations</td>
<td>Victoria</td>
</tr>
<tr>
<td>Counting &amp; enumeration with elephants</td>
<td>Ben</td>
</tr>
<tr>
<td>Dice movement game</td>
<td>PS 112</td>
</tr>
<tr>
<td>Cardinality with groups of objects</td>
<td>Talulah</td>
</tr>
<tr>
<td>Cardinality and conservation with bears</td>
<td>Harry</td>
</tr>
<tr>
<td>Enumeration &amp; production of an amount</td>
<td>Chidera</td>
</tr>
<tr>
<td>Production of an amount</td>
<td>Sarah Kate</td>
</tr>
<tr>
<td>Assessing subitizing in small groups</td>
<td>PS 112</td>
</tr>
<tr>
<td>Bag It</td>
<td>Union City</td>
</tr>
<tr>
<td>Making mats the same</td>
<td>Eileen</td>
</tr>
<tr>
<td>Seven dots different orientations</td>
<td>Lisa</td>
</tr>
<tr>
<td>Pushing aside strategy</td>
<td>Julia</td>
</tr>
<tr>
<td><strong>Assessment: observation, test and clinical interview</strong></td>
<td></td>
</tr>
<tr>
<td>Explaining $8+7=15$</td>
<td>Henry</td>
</tr>
<tr>
<td>Whole interview of Tammy</td>
<td>Tammy</td>
</tr>
<tr>
<td>Whole interview of Michael</td>
<td>Michael</td>
</tr>
<tr>
<td>Interview in classroom</td>
<td>Harrison/ Ramaz</td>
</tr>
<tr>
<td>Carrot math-- moving pictures of carrots across mind</td>
<td>Rachel</td>
</tr>
<tr>
<td>Caleb's first interview</td>
<td>Genisis</td>
</tr>
<tr>
<td>Clinician interview lesson: Brush task</td>
<td>Kevin/ Nick</td>
</tr>
<tr>
<td>Animal Parade</td>
<td>Union City (Molina)</td>
</tr>
<tr>
<td><strong>Informal addition and subtraction</strong></td>
<td></td>
</tr>
<tr>
<td>Addition strategy: counting all</td>
<td>Talulah</td>
</tr>
<tr>
<td>Taking away 1 and 2 blocks from box</td>
<td>Santiago</td>
</tr>
<tr>
<td>Adding one: counting and addition</td>
<td>Fausto</td>
</tr>
<tr>
<td>Three sharp toothed buzzards</td>
<td>Various</td>
</tr>
<tr>
<td>Where is the winner?</td>
<td>Julia</td>
</tr>
<tr>
<td>Inverse addition / subtraction</td>
<td>Rachel</td>
</tr>
<tr>
<td>Mental addition of fruit</td>
<td>Harry</td>
</tr>
<tr>
<td>Non-verbal task</td>
<td>Genisis</td>
</tr>
<tr>
<td>Title</td>
<td>Child’s Name</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Prediction game: Zur/Gelman activity</td>
<td>Julia</td>
</tr>
<tr>
<td>Paper towel subtraction</td>
<td>Corpus Christi</td>
</tr>
<tr>
<td>Comparing amounts in cups- Lori Brush task</td>
<td>Michael</td>
</tr>
<tr>
<td>Addition 3 + 1 with rhyme</td>
<td>Michael</td>
</tr>
<tr>
<td>Missing addends with broccoli/ spinach</td>
<td>Santiago</td>
</tr>
<tr>
<td>Where is the winner?</td>
<td>Danny</td>
</tr>
<tr>
<td>Understanding of commutativity</td>
<td>Tanya</td>
</tr>
<tr>
<td>Partial concrete addition</td>
<td>Olivia</td>
</tr>
<tr>
<td>Taking away putting back checkers</td>
<td>Little Josh</td>
</tr>
<tr>
<td>Putting together stories</td>
<td>Union City (Molina)</td>
</tr>
<tr>
<td>Counting all with fingers</td>
<td>Raul</td>
</tr>
<tr>
<td>Sesame Street- Snow White</td>
<td>N/A</td>
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<td>Nonverbal addition</td>
<td>Liola</td>
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**Symbols**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Putting rings around numbers on chart</td>
<td>NJN</td>
</tr>
<tr>
<td>Representing an amount on paper</td>
<td>Tyquasha</td>
</tr>
<tr>
<td>Representing teacup problem</td>
<td>Gabriel</td>
</tr>
<tr>
<td>Explaining equals sign</td>
<td>Brian</td>
</tr>
<tr>
<td>Ordering written numerals</td>
<td>Aiden</td>
</tr>
<tr>
<td>Explaining equals sign in worksheet</td>
<td>Rachel</td>
</tr>
<tr>
<td>Matching tallies to chips</td>
<td>Ben</td>
</tr>
<tr>
<td>Writing multi-digit numbers</td>
<td>Raul</td>
</tr>
<tr>
<td>Representing and operating with tallies</td>
<td>Kenny</td>
</tr>
<tr>
<td>Representing operation with picture</td>
<td>Santiago</td>
</tr>
<tr>
<td>Understanding 3+1 with and without symbols</td>
<td>Kevin</td>
</tr>
<tr>
<td>Connecting equation to manipulatives</td>
<td>Tammy</td>
</tr>
<tr>
<td>Tammy equals</td>
<td>Tammy</td>
</tr>
<tr>
<td>Two meanings of equals sign</td>
<td>Jordan</td>
</tr>
<tr>
<td>Introducing kindergartners to formal equations</td>
<td>Ramaz</td>
</tr>
<tr>
<td>Counting and pointing to numbers in chart</td>
<td>Corpus Christi</td>
</tr>
<tr>
<td>Baby Einstein</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Number facts**

<table>
<thead>
<tr>
<th>Activity</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Number facts: anxiety in memorizing multiplication facts</td>
<td>Beth</td>
</tr>
<tr>
<td>Reciting number facts in preschool</td>
<td>Santiago</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Title</th>
<th>Child’s Name</th>
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<tbody>
<tr>
<td>Cheating: number fact vs. manipulatives</td>
<td>Eddie</td>
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<tr>
<td>Explaining 2 x 4</td>
<td>Zelda</td>
</tr>
<tr>
<td>Addition: Derived facts</td>
<td>Josh</td>
</tr>
<tr>
<td>Number facts anxiety - addition</td>
<td>Sarita</td>
</tr>
<tr>
<td>Reversibility - even with mistakes</td>
<td>Rachel</td>
</tr>
<tr>
<td>Reversibility in addition and subtraction</td>
<td>Rufus</td>
</tr>
<tr>
<td>Sums of 7 with the zoo game</td>
<td>Julia</td>
</tr>
<tr>
<td>Derived fact 8 + 7</td>
<td>Henry</td>
</tr>
<tr>
<td>Explaining 7 x 3</td>
<td>Ian/ Irwin</td>
</tr>
<tr>
<td><strong>Place value and computing big numbers</strong></td>
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<tr>
<td>Explaining place value</td>
<td>Shania</td>
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<tr>
<td>Partial sums method</td>
<td>Molly</td>
</tr>
<tr>
<td>Bug in double digit addition</td>
<td>Julia</td>
</tr>
<tr>
<td>Blackboard Math</td>
<td>N/A</td>
</tr>
<tr>
<td>Regrouping: getting different answers in different ways</td>
<td>Diane</td>
</tr>
<tr>
<td>Invented method for double digit addition</td>
<td>Tara</td>
</tr>
<tr>
<td>Adding double digit numbers mentally</td>
<td>Zoe</td>
</tr>
<tr>
<td>Multiplication different method</td>
<td>Akiko</td>
</tr>
<tr>
<td>Writing multi-digit numbers</td>
<td>Raul</td>
</tr>
<tr>
<td>Different way of figuring out multiplication problem</td>
<td>Akiko</td>
</tr>
<tr>
<td>Adding 37 and 26 with paper and with blocks</td>
<td>Rain</td>
</tr>
<tr>
<td><strong>Shape and space</strong></td>
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<tr>
<td>Infant playing with rings</td>
<td>Hope</td>
</tr>
<tr>
<td>Coloring in triangles</td>
<td>Dillon</td>
</tr>
<tr>
<td>Sorting shapes with vehicles</td>
<td>Jayden</td>
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<tr>
<td>Sorting shapes with princesses</td>
<td>Sarah Kate</td>
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<tr>
<td>Putting shapes away</td>
<td>Eileen &amp; Gabriella</td>
</tr>
<tr>
<td>Big Math: Shape Hunt</td>
<td>N/A</td>
</tr>
<tr>
<td>Identifying shapes</td>
<td>Chidera</td>
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<tr>
<td>Grid task</td>
<td>Ben</td>
</tr>
<tr>
<td>Symmetry Pegboard</td>
<td>Shauna</td>
</tr>
<tr>
<td>Making pictures with Geoblocks</td>
<td>Various</td>
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<tr>
<td>Making shapes with toothpicks and clay</td>
<td>N/A</td>
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<tr>
<td>Identifying shapes tactiley</td>
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</tr>
<tr>
<td>Title</td>
<td>Child’s Name</td>
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<tr>
<td>Sesame Street square game</td>
<td>N/A</td>
</tr>
<tr>
<td>Symmetry lesson with face</td>
<td>N/A</td>
</tr>
<tr>
<td>Triangle sheet</td>
<td>Elsa</td>
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<tr>
<td>Coordinate grid game</td>
<td>Haley</td>
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**Patterns and algebra**

<table>
<thead>
<tr>
<th>Title</th>
<th>Child’s Name</th>
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<tbody>
<tr>
<td>Difficulty extending pattern</td>
<td>Ben</td>
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<tr>
<td>Pattern lesson with blocks</td>
<td>CC</td>
</tr>
<tr>
<td>Filling gaps in patterns</td>
<td>Vienna</td>
</tr>
<tr>
<td>Patterns in the hundred's chart</td>
<td>Rain</td>
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<tr>
<td>Explaining &amp; extending a pattern</td>
<td>Owen</td>
</tr>
<tr>
<td>Transforming a pattern</td>
<td>Seren</td>
</tr>
<tr>
<td>Noticing regularity</td>
<td>Vienna</td>
</tr>
<tr>
<td>Growing patterns</td>
<td>Owen</td>
</tr>
<tr>
<td>Fixing mistakes in patterns</td>
<td>Owen</td>
</tr>
<tr>
<td>Extending &amp; explaining pattern</td>
<td>Seren</td>
</tr>
<tr>
<td>A pattern is a list of colors, extension</td>
<td>Vienna</td>
</tr>
<tr>
<td>AB vs. ABB pattern</td>
<td>Ben</td>
</tr>
<tr>
<td>Duplicating structure of pattern</td>
<td>Morgan</td>
</tr>
<tr>
<td>Pattern lesson: Do re mi</td>
<td>CC</td>
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<tr>
<td>Doubling cubes pattern</td>
<td>Henry</td>
</tr>
<tr>
<td>Pattern in multiplying by five</td>
<td>Ramaz</td>
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**The many faces of understanding**

<table>
<thead>
<tr>
<th>Title</th>
<th>Child’s Name</th>
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<tbody>
<tr>
<td>Experimenting with objects with a pan balance</td>
<td>CC</td>
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<tr>
<td>Pan balance lesson</td>
<td>CC</td>
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<tr>
<td>Ordering children according to height</td>
<td>CC</td>
</tr>
<tr>
<td>Comparing lengths with different materials</td>
<td>Xavier</td>
</tr>
<tr>
<td>Block king</td>
<td>Jordan</td>
</tr>
<tr>
<td>Comparing ribbons indirectly</td>
<td>Dillon</td>
</tr>
<tr>
<td>Talking about animals</td>
<td>Julia</td>
</tr>
<tr>
<td>Graphing hearts activity</td>
<td>Various</td>
</tr>
<tr>
<td>Seriation</td>
<td>Alera</td>
</tr>
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<td>Seriation</td>
<td>Lizbeth</td>
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<tr>
<td>Liquid volume lesson</td>
<td>CC</td>
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<tr>
<td>Comparing ribbons, age 3 &amp; 4</td>
<td>Ben</td>
</tr>
<tr>
<td>Seriation with Cuisenaire rods</td>
<td>Dillon</td>
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<tr>
<th>Title</th>
<th>Child’s Name</th>
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<tbody>
<tr>
<td>Matching horizontal lengths</td>
<td>Dillon</td>
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<tr>
<td>Working with number line</td>
<td>Zelbo kids</td>
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<tr>
<td>Comparing sums</td>
<td>Conrad</td>
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<tr>
<td>Representing 4 x 15 with stars</td>
<td>Akiko</td>
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<tr>
<td>Different ways of representing multiplication</td>
<td>Ami</td>
</tr>
<tr>
<td>Tammy connect chips equation</td>
<td>Tammy</td>
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<tr>
<td>Zoe explaining 10+3 vs 3+10</td>
<td>Zoe</td>
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<tr>
<td>Tanya 3 + 12 can’t equal 3</td>
<td>Tanya</td>
</tr>
<tr>
<td>Reasoning and proof: 5 + 6 = 11</td>
<td>Nicholas</td>
</tr>
<tr>
<td>Subtraction using number line</td>
<td>Ricardo</td>
</tr>
<tr>
<td>Problem solving: informal division (2)</td>
<td>Rufus</td>
</tr>
<tr>
<td>Six pack problem</td>
<td>Various</td>
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<tr>
<td>Fraction blackboard math</td>
<td>The School</td>
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**Teaching: What is it?**

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<th>Title</th>
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<tr>
<td>Liquid volume lesson</td>
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<tr>
<td>Graphing hearts activity</td>
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<td>Six pack problem</td>
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**Teaching: Manipulatives and Curriculum**

<table>
<thead>
<tr>
<th>Title</th>
<th>Child’s Name</th>
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<tbody>
<tr>
<td>4 takeaway 4 = 4</td>
<td>Tanya</td>
</tr>
<tr>
<td>Double digit operations with base ten blocks</td>
<td>Lucretia Simone</td>
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<tr>
<td>Multiplying by nines</td>
<td>Ramaz</td>
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<tr>
<td>Neighbors lesson</td>
<td>Ramaz</td>
</tr>
<tr>
<td>Adding 37 and 26 with paper and with blocks</td>
<td>Rain</td>
</tr>
<tr>
<td>Reviewing base ten homework</td>
<td>The School</td>
</tr>
<tr>
<td>Cheating: number fact vs. manipulatives</td>
<td>Eddie</td>
</tr>
<tr>
<td>Double digit operations with base ten blocks</td>
<td>Megan</td>
</tr>
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Appendix C: Modules and Annotated Videos

Module 1: Introduction to the issues and the course

This module presents the rationale behind and issues surrounding early childhood mathematics education. It spends some time addressing teachers’ hesitancy and in many cases anxiety about teaching mathematics, and encourages class discussion about student experiences in learning mathematics. The module then explores what factors need consideration when teaching children mathematics, early including children’s capabilities, mathematical content, pedagogy, assessment, and mathematics that children might encounter outside school.

The videos below illustrate some of the issues discussed in the lecture. Included are clips of children engaging spontaneously in mathematical thinking during play, learning about mathematics in the classroom, and talking to adults about their mathematics learning. This last type of clip introduces students to the method of clinical (or flexible) interviewing as a way of finding out about children’s mathematical reasoning.

Video: Stanley counting beads
Goal:
- To show that young children are curious about and enjoy mathematics—they often initiate it on their own.

Overview:
- The video takes place during free time in a classroom. A boy and a girl count toys to 14 in unison. Then the boy announces that he has 100 beads. He proceeds to count out 100 of them, one by one, with only a few mistakes. Along the way, other children join in the counting.

Discussion Questions:
- Do you think the children in the video seem to be enjoying mathematics?
- Does their behavior surprise you? Why/why not?
- What implications might this clip have for the argument to teach mathematics in early grades?

Comments:
- The children seem to be enjoying counting. In the second part, they have a goal of counting to 100 and are determined to do so. Mathematics in early grades often receives negative reactions: people may think it is too abstract for kids; that they are not interested; that they should be playing. However here we see children who are interested in mathematical ideas and engaging in mathematical activities spontaneously and with intense attention and effort over a long period of time. This example supports the argument that we should take advantage of children’s interests and bring more mathematics into early childhood classrooms.
**Video: Adding bears: invisible to visible: Olivia, age 4**

**Goals:**
- To show that young children are capable of abstract thinking as well as concrete.
- To have students think about what specific knowledge and skills the child is exhibiting in completing an addition task.

**Overview:**
- The interviewer tests the child’s ability to find the sum of two small collections, specifically to add a visible and invisible set. The interviewer puts three bears on the table one by one, and establishes with the child that there are three. He then asks how many bears there would be if two more were added; however he does not make available extra bears. The child successfully finds the total, using a strategy of counting while pointing to spots (imaginary bears?) on the table. When the interviewer gives her two more bears, she arranges them in a line with the others and checks her answer by counting all five bears, one by one. The video demonstrates that young children are capable of complex, even abstract thinking, and are not limited to the concrete.

**Discussion Questions:**
- What challenge is the interviewer presenting to the child? Be specific.
- What is Olivia doing to solve the problem?
- Is her thinking concrete? Is she dealing with any kind of abstract ideas?

**Comments:**
- The interviewer is challenging the child to add two sets of bears that cannot be physically combined and counted. In order to find the correct answer, she has to first be able to accurately count things (enumerate), but she also has to have some mental representation of the two extra bears in order to add them to the first three. This in turn means that she needs to possess an idea of what “2” is.
- To solve the problem, Olivia did not need concrete objects and was able to represent the imaginary missing bears by the action of pointing to spots on the table. So her solution was partly abstract (imaginary bears) and partly concrete (the physical action of pointing).

**Video: Counting and pointing to number chart; Pre-K**

**Goal:**
- The goal of showing this video is to have students start to think about what type of mathematical instruction, if any, is appropriate for young children, and to bring out any student antipathy towards the idea of deliberate teaching and structured curricula in preschool classrooms.

**Overview:**
- In the video 3-to-5-year-old children are gathered around a 100’s chart and counting aloud. The teacher, audible through a microphone, is leading the counting. A child is pointing to each number on the number chart as the children say it (or try to). When he gets to each number ending in 9 (including 9 itself), the teacher has the children stop
(they say “cut”). She asks them what comes after that number. They continue in this way until they reach 49.

- It is important to note that the lesson takes place in December, and the children have been counting as a group with the teacher since the beginning of the school year. The routine of saying, “cut” after each 9, has also been practiced for the duration of the year.

Discussion Questions:
- What mathematical knowledge and skills were being promoted in this activity?
- What is the goal of having the child point to each number on the number chart?
- Do you think this is an appropriate activity for young children?

Comments:
- The video may strike some students as inappropriate for young children; they may be doubtful that children should be learning to count high numbers and to read written double-digit numerals. However through discussion the argument should come up that if done properly and in the right context, such activities may not only be “appropriate” but engaging and valuable for students. In particular, learning to count to 100 involves learning the base 10 structure of the numbers and is one of the first opportunities children have to explore an important pattern. This activity is intended to help children learn the counting words, including decade numbers and the pattern of appending the numbers 1-9 to the decades (twenty, thirty, and so on). By using the chart the teacher is also encouraging practice matching numerals to counting words, including noticing that the “twenty” numbers start with the numeral 2, the “thirty” numbers with a “3”, etc.

Video: Sesame Street: Dogs & Mud Pies
Goal:
- To demonstrate that more and more, mathematics—and not simply counting—is being incorporated into children’s television shows (and other media too, like web-based activities and games),
- and that it is important for early child educators to be able to make informed judgments about the educational value of such resources.

Overview:
- In this Sesame Street clip starring William Wegman’s dogs, Batty (the dog wearing the dress) is preparing an “old family recipe” of mud pie. The narrator says that they will need four pieces for Batty and the three dogs she is serving. Batty cuts the pie in half to make two pieces (“big pieces”) and then in half again to make four pieces. They then distribute the pieces topped with whipped cream, which the dogs enjoy.

Discussion Questions:
- What mathematical ideas are being presented here?
- Do you think they are presented in an engaging way?
- Do you think they are presented clearly?
- What might children understand and/or not understand?
Comments:
- The clip touches on ideas about counting, addition, and partitioning. The video itself is entertaining and it is likely that children will be engaged. Further, it deals with a real world context with which almost all children are familiar—dividing food fairly among several people.
- Some of the narration is quick and might be missed by children. For example the addition of 1 to 3 to make 4 people and the division of the pie from 2 pieces to 4 pieces could have been carried out more slowly and clearly with careful counting. However it is not unlikely that many children would understand the basic ideas of the clip, and they may even take away some mathematical ideas.
Module 2: Everyday/ Naturalistic Mathematics

This module addresses “everyday mathematics,” or mathematics that children encounter outside of formal schooling. The primary objective is for students to learn to recognize and appreciate children’s experiences with everyday mathematics so that they can become more aware of the types of mathematical knowledge and skills with which students enter school. The next objective is to help students think about ways to foster these types of everyday experiences, even in the classroom, and how to mathematize them, or make them explicit and formal.

Below are several videos of children grappling with mathematical ideas in the everyday world. They range in environment, structure, and artifacts with which children are playing. For each video, students should discuss the environment (where it takes place, with whom the children are interacting, the available materials), the specific mathematical behaviors exhibited (including language spoken), potential motives for these behaviors, the minimal cognitive processes involved, that which is unclear and need further investigation, and ways to encourage and mathematize the experiences.

In the last video, a teacher sits with children who are playing with colored cubes. Some additional discussion questions are suggested that encourage analysis of how the teacher encouraged and mathematized the thinking she observed.

Video: Infant playing with rings; Hope; Age: 18 months
Goal:
➢ To show how even infants have an everyday math.

Overview:
➢ A baby and her mother play with a set of rings, including a hiding game. Of note is the language that Hope uses, the way that she searches for rings, and the different rings that she finds at the end of the clip.

Discussion questions:
➢ What kinds of language does Hope use and what does it mean?
➢ Does she have any mathematical ideas?
➢ What does she know about shapes?
➢ How does the mother interact with her?

Comments:
➢ Some of the language Hope uses indicates a basic understanding of more and less. We hear her say “ring” versus “rings” (understanding the difference between one and many), “more”, “again”, “two”, “all gone”, and potentially some other words (what she actually says can be debated.)
➢ Her reactions during the hiding game suggest that she has some mental representations of the rings, and to some extent, specific amounts of rings. For example when her mother hides the rings in one hand, the child knows to look in the 2nd hand when the 1st one was empty (however on another turn she looks in the 2nd hand even after she finds the rings in...
the 1st hand.) When the mother hides the rings behind her, Hope clearly knows that they are missing, and when the mother recovers just one of them, Hope knows that there are more that are still hidden.

- At the end of the clip Hope finds toys of different shape, size, and color from the initial rings and knows that they are still called “rings”. This indicates an awareness of and generalization of shapes.
- In addition to the child’s behaviors, it is also interesting to note how the mother interacts with the child and encourages and mathematizes the child’s experiences.

**Video: Free play clapping & calendar games; Emily & friends; Kindergarten**

**Goals:**
- To show how ideas about pattern permeate everyday play,
- and also how children play explicitly with symbolic mathematics

**Overview:**
- This video contains two segments wherein kindergarteners are exploring mathematical ideas in everyday play. In the first segment, a child performs a clapping sequence involving repeating and growth patterns. Next, two girls play “teacher and student” and count by twos using a calendar. One points to every other square on the calendar while the other recites the corresponding numbers.

**Discussion questions:**
- What kind of pattern do you see in Emily’s clapping?
- How did she impose mathematical structure on it?
- Why were they engaged in this activity?
- Did the girls really do counting by twos or were they merely reading every other space on the calendar?

**Comments:**
- The children are exploring counting and patterns in their play. The clapping sequence performed by Emily contains a rhythmic pattern that gets repeated increasingly—it’s clapped once, then twice, then three times, etc. At one point she counts along with the rounds to keep track. Her peers appear to appreciate the clapping, calling out “Good music!”
- In the second segment, when the girls are playing “teaching and student”, they are counting by twos with the calendar. It is interesting that when they reach the end of the numbered squares, they keep counting using the empty squares, indicating an understanding of how the pattern continues, both spatially (skipping every other square) and numerically (knowing which numbers are included in the counting-by-twos sequence.
- Why were the children engaged in this activity? We assume that the teacher did not tell them to do it. Perhaps the children were challenging themselves and enjoying the rhythmic pattern of counting by 2s.
**Video: Block architects; Armando & Keithly, Kindergarten**

**Goal:**
- To show the richness of everyday math in play

**Overview:**
- Two boys are building a structure together. In the course of their building they encounter various mathematical ideas such as measurement, number, estimation, shapes and space.

**Discussion questions:**
- What is that “circle thing” that Armando mentions?
- He says “What if it don’t reach?” What does this indicate about his thinking?
- We need two more?” What does that mean?
- What would you do if you were the teacher in the classroom?

**Comments:**
- We see various mathematical ideas touched on in this video. One major one is geometry—working with and communicating about different types of blocks. A salient example is when Armando calls a cylinder a “circle thing,” illustrating his observation that a cylinder is similar to but not the same as a circle, as well as his struggle to accurately describe the object.
- Another major mathematical idea is estimation. The children need to figure out how far apart to set out the two sides of the structure in order to lay a long block across. Later they estimate the amount of blocks needed to complete the top and sides of the structure.

**Video: Small group activity: Working with colored cubes; Pre-K**

**Goal:**
- To show how difficult it is for a teacher to seize upon the teaching moment.

**Overview:**
- In this video a teacher sits with several children working with colored cubes. This started as a sorting activity but was not highly structured: the teacher encouraged the children to do what the wished with the cubes, but the general focus seems to be about mathematics, such as sorting, magnitude, counting, shapes and patterns. We notice several moments in which the children encounter mathematical ideas or the teacher brings such ideas into play.

**Discussion questions:**
- What did the teacher do to set up this activity?
- Are all of the children doing the same thing?
- How does she react to individual children?

**Comments:**
- Note that the teacher has set up the activity by depositing a pile of cubes on the floor and establishing that it will be a sorting activity—that the children should collect different colors. She encourages the children to do whatever they like with the cubes, such as
make a pile or a “long line.” She has also set out a board that has drawn on it a long rectangle made of squares the size of the cubes.

- Students should think about how the teacher encourages mathematical thinking, and what else a teacher might have done to encourage and challenge the children to engage in deeper mathematical thinking.
- Imagine how hard it would be to do this everyday with 20 children and to have such activities form the center of a mathematics program!

**Video: Building a bigger garage; David & Eduardo, Kindergarten**

**Note:**
- This video may be used as a homework assignment where students must make clips in VITAL about the mathematical reasoning they see and think about the types of questions they might ask the children to find out more about the boys’ understanding.

**Overview:**
- Two boys try to construct a garage for their truck. In the process they explore many mathematical ideas, including estimation, balance, stability, number, symmetry, and spatial relations.

**Questions for the clipping assignment:**
- What mathematical thinking do you see? Identify at least 3 different instances of mathematical thinking, using clips to justify your claims.
- What else would you like to know about the boys' mathematical knowledge?

**Comments:**
- After the boys make the initial structure and test it out with the truck, they discuss a plan for continuing. The boy in all red, David, seems to have noticed that the original plan wasn’t working, and is trying to convince this of the boy in the black sleeves, Eduardo. Have students listen carefully to the back and forth and discuss the mathematical ideas David is expressing.
- Next the boys get sidetracked and compete to build a taller tower. They are grappling here with balance and stability, number, and magnitude.
- Next the children revisit the garage idea. Eduardo tries repeatedly to build the original structure. He complains that it is “skinny”, which seems to mean that it is too short for the truck to pass under. You might want to ask him to elaborate on this idea.
- Finally, the boys try to build a garage for an even bigger truck. Note how Eduardo tries to create sides of the same height by counting the blocks in each.
Module 3: Early concepts: More, equivalence, and conservation of number

This module addresses basic concepts in mathematics with which children grapple: ideas about more and less, equivalence, and conservation. Major themes are young children’s tendency to base judgments about amounts on perception (how much space a collection of objects takes up) rather than on empirical evidence such as counting, and early logic that if a set is moved around, the amount will change. When watching the videos students should think about what children’s responses indicate about their conception of the ideas of “how many” and “more”.

Video: Which side has more? Tina; Age 4

Goals:
- To illustrate that young children make judgments about “more” based on perception rather than empirical evidence;
- to have students think about the concept of “more”.

Overview:
- The task in this video was developed by Alfred Binet. In it a child is shown a series of cards on which two collections of dot stickers are separated by a line. Each time, the child asked to point to the side that has more dots. The stickers are arranged in different ways so that in some cases the smaller number of dots is spread out more than the larger number of dots. The child uses a common strategy of using the spatial arrangement of dots rather than space rather than counting to judge relative quantity.

Discussion Questions:
- How does the child judge which side of the boards has more dots? Why do you think she does this?
- Why do you think she hesitates when shown the last board?
- Do you think “more” means the same to this child as it does to adults or even older children?

Comments:
- When showing this video it is important that students first understand the task. They should note it is non-verbal and fast paced. This is because children aren’t expected to count (in fact it can be done before they can count)—they are supposed to quickly evaluate which side has more dots based on perception. It is also important that the students carefully look at each board that the child is shown and how the two arrangements of dots differ.
- The child uses a common strategy of using the spatial arrangement of dots rather than counting to judge relative quantity. This strategy is often effective for children because a larger set usually does take up more space than a smaller set. However, in these items, where the smaller set covers a greater area than the larger set, the child fails.
- When the child sees that last problem of two sets of three, she is clearly unsure what to do. She takes a lot of time and her eyes glance alternately at each collection of dots. This may indicate that the child knows that this problem is different from the others and
perhaps even suspects that the amounts are equivalent. At the same time, she is confused because the interviewer asked which side is more (thus a sort of “trick” question). It may have been informative if the interviewer had asked what she was thinking at this point.

- This child has a different conception of “more” than adults or older children. To her, “more” appears to mean taking up more space. This is common among young children who do not yet have a good grasp of cardinality, or “how many.”

**Related videos:**
- Which side has more? Vienna, age 4

**Video: Conservation of number; Vienna, age 4**

**Goal:**
- To illustrate that young children are often unable to “conserve number” or understand that the number of items in a set will not change even if they are moved around.

**Overview:**
- Interviewer puts 6 bears on the table and rearranges them several times, each time asking how many bears there are. The child counts different numbers, and at the end is not able to say how many there are without counting.

**Discussion questions:**
- Stop after child says there are 5 bears: What do you think it means that she thinks there are now 5 bears after first counting 6?
- Stop after interviewer tells child to say how many without counting: Why is the interviewer telling the child not to count? How do you think the child will respond?
- At end: How do you explain the child’s inability to say how may objects there are?
- What would you conclude about her understanding of counting and number?
- What activities might you encourage for children who display this inability to conserve number?

**Comments:**
- The fact that Vienna needs to count each time the bears are moved around indicates an inability to conserve number. A child who conserves would know that the amount stays the same when they are moved. Further, she does not seem surprised when she counts 6 the first time and 5 the second time.
- This question of how many there are without counting is crucial. A child may think she always needs to count in order to please the interviewer. Yet when explicitly told not to count, she should be able to recall how many there are.
- Although Vienna can count objects pretty well, the evidence above points to the fact that she cannot conserve number—she does not know that when they are moved around, the amount will remain the same.

**Related Videos:**
Video: Conservation of equivalence with blue and red circles; Kristin, age 5

Goals:
- To illustrate another way that young children make judgments of more and less based on perception rather than empirical evidence;
- to encourage students to think about children’s understanding of more, less, and numbers.

Overview:
- Child is shown two lines of circles that are arranged so that each circle in one line is paired with a circle in the other line. When the interviewer spreads out the objects in one line, the child says that that line now has more circles. When he puts them back, she says they have the same amount. The child maintains her position even when she counts both lines and sees that they have the same number.

Discussion questions:
- How does the interviewer line up the circles at the beginning of the task? Why do you think he does this?
- Based on her assessment of which line has more circles, how would you interpret Kristin’s understanding of “more”?
- What does it mean that Kristin again judged that one line had more circles, even if she knew that both had 7? How do you interpret her understanding of number?

Comments:
- This is the classic Piaget conservation of equivalence task. The interviewer lines up the circles so that each circle is paired with one from the other line. To adults and older children, such pairing indicates that the sets are equivalent even without knowing the amount in each line.
- When first asked to compare the lines, Kristin appears to be judging the lines based on the amount of space they take up. This indicates that she cannot yet conserve equivalence, or recognize that if both lines start out with the same amount of objects, they will always have the same amount even if they are rearranged.
- Students might have trouble with the child’s response, suggesting changes in the presentation of the task that might help a child achieve the correct answer. It is important to note that after this task was first published, there were about 10 years of research trying to show that Piaget was wrong. However, after years of debate, the basics of this finding still hold true.
- Even though Kristin can count a set of objects and say how many it has, she does not have a strong sense of what the number means nor how to use it when comparing two sets. It is not obvious to her that if she counts both sets and finds that both have 7 items, the sets necessarily have the same amount. When judging relative magnitude, she is using perceptual rather than numerical evidence. A major Piagetian point is that counting
does not help very much in the conservation task. Rather, the early logic based on appearances dominates, and counting does not help—it is irrelevant.

**Related Videos:**
- Conservation of equivalence; Liola; age 4 (Child behaves similarly to Kristin)
- Conservation of equivalent; Seren; age 5 (Child is successful)
Module 4: Counting & number words

This module addresses children’s learning how to count: not how to count objects (which will be addressed in the next module), rather, how to recite number words in order. This means memorizing the number words one through ten, and to a large extent the words eleven through twenty. After this, children pick up patterns in the number system, such as the rule of appending the numbers one through ten to the decade words when counting numbers beyond this. It is a common belief that children should not or cannot count past 10 or 12. This module aims to challenge that thought, showing that higher numbers are actually easier for children in some ways, and that children are excited about counting and should be encouraged to do so.

Video: Counting and indicating mistakes; Josh, age 3

Goals:
- To show students that for young children learning to count is like learning a song—it is memorizing words that go in a specific order.
- To show that even at a young age children have already abstracted rules about counting.

Overview:
- The interviewer asks Josh to count, and he demonstrates that he can count to 12 without error. Then the interviewer asks the child to point out any mistakes when the interviewer himself counts. Here Josh reveals some beliefs about counting rules.

Discussion questions:
- Stop after initial counting: How would you assess Josh’s ability to count?
- Stop after Josh points out mistakes: What beliefs about counting do his reactions reveal?
- Could this task be used as a classroom activity?

Comments:
- Initially Josh appears to be able to count to 12 very well. It is unclear what he knows about numbers beyond 12.
- The “mistake game” illustrates how versed Josh is in the number sequence—almost like it is a “song” that he has memorized. It also reveals beliefs that he possesses about counting rules. We learn that he believes that counting words must be recited in a certain order and numbers cannot be skipped. He also believes that counting involves a certain type of word—number words, and that you cannot replace them with other types of words. Finally we learn that he believes that in counting you must start with the number “one”. Josh’s beliefs reflect an understanding of counting as a process with certain rules that must not be broken. When Josh congratulates the interviewer on his erroneous counting sequence past 15, he reveals unfamiliarity with the counting sequence past this point along with any of its patterns.
- Children love this game and it is excellent to use in the classroom as a way to help children listen for and articulate mistakes that a teacher makes in counting.
**Video: Counting with fingers; Lateek, Age 4**

**Overview:**
- Child counts a few times, each time using his fingers. His number order is somewhat inconsistent, and he has trouble with the teen numbers. When the interviewer counts past 16, the child stops her and says that you are only supposed to count 15 once (he confuses 15 and 16.) Finally, when the interviewer asks Lateek if he can count higher, he misinterprets the question and raises his hands high in the air as he counts.

**Video: Counting & figuring out decades; Michael, age 5**

**Goals:**
- To show that young children are able to count high numbers using knowledge of patterns in the number sequence.
- To show how one might encourage children to notice even more patterns in the number sequence.

**Overview:**
- Child is asked to count as high as he can. He counts to 69 by himself and to 100 with some scaffolding about the decade numbers.

**Discussion questions:**
- Does counting high numbers in general seem challenging for Michael?
- What does the child know and not yet know about counting?
- What kinds of scaffolding did the interviewer supply?

**Comments:**
- The video demonstrates that counting high numbers, which follow distinct patterns, is actually easier in some ways than counting lower numbers, which are often irregular (e.g., eleven, twelve) and must be memorized. Michael himself can count very high (although he does miss the irregular “fifteen”) and clearly knows the rule that one must append each number one through nine to the end of a series of decade words.
- Michael has some trouble remembering the order of the decades. The interviewer helps him by encouraging him to relate the unit numbers to the decade numbers. When Michael is trying to remember what comes after 69, the interviewer asks “what’s after 6?” When this doesn’t seem to work the interviewer recites just the decades, up to the decade in question, to help the child hear how the decade sequence is related to the units sequence one-nine (by just adding “tee” to many of the numbers one - ten). This running start does seem to help the child, although we can’t be sure if the child does in fact relate the decade order to the units sequence of if he has the decade order memorized.

**Related Video:**
Lesson: Big Math: Count Clap & Stomp A; Various pre-K students

**Overview:**
- A teacher leads her students, ages 3 - 5, in counting numbers to 100. The activity emphasizes patterns in counting by having children do body movements for every number, with a new movement for each decade. The teacher intentionally pauses and
shows the children that it’s helpful to stop and think about what decade word comes next in counting. After 79, she helps the children figure out the next decade by counting 1–8, drawing attention to the pattern in the decades (adding “-ty” to the units words).
Module 5: Enumeration & Cardinality

This module covers enumeration and cardinality: what rules must be followed for counting objects accurately, factors that may make some sets of objects more difficult for children to count than others, and challenges that children face in fully grasping cardinality—or understanding that a number word conveys how many objects are in a set and that this value stays the same even if the set is rearranged (revisiting the idea of conservation a the previous module). For the videos below, students should be encouraged to think carefully about the task itself (what exactly is being asked of the child?) before analyzing the child’s performance. It may be helpful to stop the clip at different points to ask different questions (some stop points are suggested).

Video: Counting small and large sets of objects; Victoria, age 3

Goal:
- To show that young children often can enumerate when objects are arranged neatly or in small sets, but have more trouble with larger sets, especially when they are not arranged in an orderly fashion.

Overview:
- Child counts a line of 7 dots with no difficulty. She then counts two sets of checkers, one with 2 checkers the other with 4, and again has no trouble. When asked to count them altogether, however, she falls apart, seeming to forget all the rules about counting she just was using.

Discussion Questions:
- Stop after child counts first line of dots: How would you assess Victoria’s enumeration ability at this point?
- In what ways could Victoria’s inflection at the end the counting sequence be interpreted?
- Continue until the end: Why is the line of 7 dots and the two sets of checkers easier for Victoria to count than the final pile of 6 checkers?
- How might a teacher use this information to plan counting activities for children?

Comments:
- Victoria can count the first set of items very well. Her inflection is interesting: the high pitched “seven” seems to indicate an understanding that the last number word is important, however we do not necessarily know if she knows that the number seven represents the whole set.
- Young children are often able to count small, orderly sets of objects well, but fall apart when the sets are larger and less organized. They don’t have strategies for doing this yet (such as pushing aside objects already counted), and they get overwhelmed and appear to forget basic rules of enumeration.
- The clip suggests the importance of presenting children with a variety of groups of objects in different arrangements, and to teach strategies such as pushing aside to help them enumerate successfully.
Video: Enumeration and production of an amount; Chidera; age 4

Goal:
- To show that being able to point and count does not necessarily mean that a child understands cardinality, or how many there are in a set.

Overview:
- Child is asked to count sets of 3 and 5 blocks. She counts accurately. Yet after she counts the set of 5, the interviewer pushes her to say how many there are, and the child seems to get confused. Then the interviewer asks her to give a doll 4 blocks, and it is revealed that the child does not seem to understand what that means.

Discussion questions:
- After the child counts 3 blocks and then 5 blocks: How would you assess Chidera’s ability to count objects?
- After the interviewer repeats the question, “How many are there?”: What is the interviewer trying to assess here? Why isn’t it enough for the child to accurately point and count each object? Why do you think the child counts 6 – 10?
- At end: Does the child seem to understand cardinality, or how many objects there are in a set?

Comments:
- At first it seems that Chidera has a good grasp on enumeration. However through the interview it becomes clear that she does not have cardinality. To her, counting is a procedure rather than a goal-oriented task, where the goal is to find out how many objects there are.
- It is a common mistake to just assume a child understands how many objects there are just because he/she carries out the counting procedure accurately. After a child has counted, it is important to push the child to say how many there are after they have counted.
- Note in the second task how the interviewer waits while the child gives the doll blocks. Another common mistake teachers make would be to stop and praise the child after she gave the doll 4 blocks. Doing so would make it difficult to assess whether the child really knew she was giving the doll 4. A criticism is that the child may have continued to give blocks because the adult did not stop her. A better strategy for an interviewer would be to ask the child to give the doll a certain amount and they say, “Tell me when you’re done.”

Video: Counting a line of hot dogs; Alexander, age 4

Goal:
- To explore the child’s ideas about the order in which one may count objects.

Overview:
- The interviewer asks child to count a line of pictures of hot dogs. The child is able to count the pictures accurately, but articulates the belief that objects should be counted in
ord from left to right or right to left, starting from the beginning of a line and
continuing to the end.

Discussion Questions:
- What beliefs does Alexander have about counting?
- What do you think this indicates about his understanding of counting and cardinality?

Comments:
- Alexander believes that counting needs to be performed in order from left to right or right
to left. He also seems to know that the last number counted is important, as evidenced
when he confirms the interviewer’s final number counted yet rejects her counting
procedure: “It’s nine, but you skipped some of the hot dogs.”
- His belief reflects a view of counting as a procedure rather than a goal-oriented task and
may indicate a lack of knowledge that the amount of objects will always be the same no
matter the order in which they are counted.

Video: Dice Movement Game; Lesson; Pre-K
Goals:
- To show a preschool lesson covering enumeration and cardinality.
- To have students analyze the methods that the teachers use in relaying mathematical
ideas.

Overview:
- Two teachers introduce and demonstrate the “Dice Movement Game”, where children
roll a large die, count the number of dots, and then, based the particular card picked, do a
certain movement that many times. Several children take their turns in this game.

Discussion Questions:
- Stop after instructions: How do the teachers instruct the children to figure out the number
on the die? Why do they do it this way?
- Stop after first child takes her turn: Was the first child successful at counting the dots?
How did the teacher help her? Was this a good way to help her?
- Stop after 2nd child takes his turn: Why did the teacher ask how many times they should
clap after a child has just counted 3?
- End: What skills and knowledge do child have to already have in order to participate in
this activity? What skills and knowledge does it intend to teach?

Comments:
- The teachers tell the students to first look at the dots to see if they can figure out how
many there are, and then to count to check. This is a way to encourage children to start to
subitize, or see the amount and not always rely on counting. Even if they do not guess
the correct number, this procedure helps the children to estimate. For the lower numbers,
children often can accurately say the amount before checking.
- The first child had trouble counting the 6 dots. She may not have known that 6 comes
after 5, or she may have had trouble with the rules of enumeration (saying one number
for each item touched). The teacher has her count again, and this time says “6” for the child when she touches the last dot. It is not clear whether the child understood this. The teacher might have had the child do it one more time after helping her.

- After the boy counted 3, the teacher asked him how many times they should clap. This may seem redundant to adults—he has just said 3. However to children it is not always obvious that the procedure of counting yields a cardinal number—the amount of the whole set. Asking children to say how many there are after they count is a good practice because it reinforces the idea that the last number counted tells us how many dots there are total.

- The activity assumes that children already know number words up through 6. It fosters skills in enumeration, subitizing, and cardinal number.

**Related videos:**
- Counting elephants; Ben; age 3
- Pushing aside strategy for enumeration; Julia; age 4
Module 6: Early Operations

Informal addition & subtraction
This module addresses children’s early ideas about addition and subtraction: understanding and strategies they possess before they begin learning these subjects formally in school. Important ideas are detection of change in amount and direction of change, concrete versus abstract representations, and the relationship between enumeration and addition. Different addition strategies of increasing sophistication are also presented.

For each video, students should examine the particular task and think about what it requires, specifically in terms of verbalization of mathematical ideas and its level of abstraction. They should also look at the children’s performance on the task and think about the ideas they are exhibiting: if they succeed at the task, what concepts were necessary for success? If they made a mistake, what does the mistake reveal about their knowledge and abilities? Finally, students should think about how the understanding exhibited in the videos may provide a basis for learning more formal addition and subtraction.

Video: Where is the winner? Julia; age 4
Goals:
- To demonstrate that even without using number words, children can distinguish between small amounts and can detect when they have changed,
- and that in doing so they demonstrate understanding of the basic ideas of addition and subtraction.

Overview:
- In this task the child must distinguish a “winner” and a “loser” from a set of 3 (winner) and 2 (loser) blocks. After teaching the child which is which, the interviewer hides the sets, moves them around, and then asks her to find the winner. The goal is not to see if she can find the winner on the first try; it is to find out if the child can distinguish between the two amounts. Next the interviewer surreptitiously changes the sets to see if the child can detect a change in the amounts and if he knows what happened. Here Julia can distinguish the two sets indicating that she has a mental representation of the amount, even if it is not a verbal representation (like a number word). When the interviewer secretly changes the amount in the winner group, the child notices this change and even indicates an awareness of whether that it has been increased or decreased.

Discussion questions:
- Why do you think the interviewer uses the terms “winner” and “loser” rather than “three” and “two”?
- Does Julia notice that anything has changed after the interviewer secretly modified one of the sets? What does she notice?
- How is this task related to addition and subtraction?
Comments:

- The terms “winner” and “loser” are used instead of numbers because knowing the exact amount is not the point of the task—the point is to see if children can distinguish between sets and can detect when it has changed.
- The task is related to addition and subtraction because these operations are just change to a set—the first step in understanding these operations is being able to detect if a set changes and in which direction it does so.

Related Videos:

- **Winner/loser task; Danny, age 2**: This is the same task as above but with a younger child and smaller amounts. Here the child does not use number words—the video shows that even without number words children can tell the difference between amounts and can tell when a set has changed.

**Video: Non-verbal operations: Genisis age 4**

**Goal:**

- To demonstrate a very concrete form of addition that does not require the use of number words.

**Overview:**

- Both child and interviewer have a mat. Interviewer puts chips on her mat and covers them with a plate. She then adds to or takes away from the chips under the plate and asks the child to make her mat the same. Child successfully figures out the correct amounts.

**Discussion Questions:**

- Does the child need to be able to verbally count in order to succeed at this task?
- In the first two problems (adding 1 to 2 and taking 2 from 3), how might she have figured out the solutions?
- In the third problem (adding 2 to 2), she initially concludes that there are 3 chips. How is this response different from, say, her thinking that there are 2 or even 2 chips? What does this indicate about her basic understanding of addition?
- How does she eventually figure out the correct amount for the third problem?
- How does this task compare to a task in which a child is asked to add two sets or subtract an amount from one set?

**Comments:**

- Important to note in this video is the fact that the child is using concrete objects but needs to mentally represent them in order to figure out the solutions.
- Even though she makes a mistake in the third problem, she has made a guess in the correct direction, indicating that she understands that addition will result in more objects.
**Video: Addition strategies: counting all; Talulah; age 3**

**Goals:**
- To show children’s first strategy when learning to add two sets: they combine them and count all of the objects from the beginning.
- To show how addition stems directly from enumeration.

**Overview:**
- The interviewer gives child a set of blocks and then another set and asks how many there are altogether. Then adds one more block and repeats question. Child exhibits “counting all” strategy both times.

**Discussion Questions:**
- How does the child figure out how many blocks she has altogether?
- In what other ways might a child solve this problem?

**Comments:**
- Note the way the child carefully arranges the blocks, and how this is similar to previous videos of children needing neat arrangements to count successfully.
- Students might point out that another way to solve the problem would be to count on.

**Related:**
- Raul: counting all on fingers

**Video: Adding one: counting and addition; Fausto, age 5**

**Goals:**
- To show that children can figure out simple addition problems using knowledge of counting.
- To illustrate the close relations between enumeration and addition.

**Overview:**
- The interviewer asks the child to count a line of 5 bears. He then adds one block to the collection and asks the child how many are there now. Child successfully says 6. Finally, he adds two more bears to the collection and asks the same question. Child says 7 at first; then with some scaffolding modifies his answer to 8.

**Discussion Questions:**
- Stop after child answers first question correctly: How does Fausto figure out the correct answer? How is this different from the way Talulah figured out the answer in the previous video?
- Continue to end: What does his answer to both of these questions reveal about what he knows and does not yet know about addition?
- Why is adding 2 more difficult than adding 1?
- How is addition similar to and different from enumeration?
Comments:
- The child did not have to count all of the bears from the beginning after 1 was added—he solved it using the explicit knowledge that “six comes after five.” This is not just counting: it is knowing what number comes after another number. Even though he made a small mistake in the second problem, we learn from the task that he knows how to use a sophisticated strategy of counting on from the larger number. The clip illustrates how knowledge about numbers that come after other numbers is relevant for addition.

Video: Taking away blocks from a box; Santiago; age 5
Goal:
- To show how children possess basic ideas of subtraction and can successfully find solutions for simple problems.

Overview:
- Interviewer starts with 6 blocks in a box. She positions the box so the child can’t see inside, takes away 1 block at a time and asks how many are left. Child is successful both times. She then takes away 2 blocks from the remaining 4. Child says there is just 1 left.

Discussion Questions:
- How do you think Santiago figures out the correct amounts?
- Why is subtracting two more difficult than subtracting one?
- How is this similar to the previous video of Fausto adding bears?

Comments:
- Just as knowing what numbers come after other numbers is essential to addition, knowing what numbers come before other numbers is essential to subtraction. Knowing what number comes directly before another number is easier than knowing what number comes two numbers before another number, which makes subtracting 2 more difficult than subtracting one. Even though Santiago gets the second question wrong, he still guessed in the correct direction and was only one number off. He clearly understands the concepts underlying subtraction.

Related
- Informal addition of bears; Olivia (used in lecture 1)

Video: Inverse addition and subtraction; Rachel; age 6 (*can also use in Assessment lecture)
Goals:
- To show a more advanced type of reasoning about operations—using logic, known number facts, and the inverse relationship between addition and subtraction.
- To show that even when a child gives an incorrect answer, through careful questions one can reveal a good understanding of mathematical ideas.
Overview:
- The child is given a word problem involving the operation $6 - 3$. She solves it and articulately explains that since she knows that $3+3 = 6$, $6-3$ must equal $3$. In the next problem, $7 - 2$, she says that the answer is $4$. This time she uses the same logic as the previous example, but her number fact was incorrect (she thought that $4 + 2 = 7$).

Discussion Questions:
- Pause after the first problem: What knowledge and abilities is Rachel exhibiting?
- Pause after child answers $4$ for second problem: How do you think she got this answer?
- Continue to end: What would you conclude about Rachel’s understanding of subtraction?

Comments:
- Compared to children previously seen, Rachel has a very abstract way of solving the problem—she is using the addition-subtraction inverse principle. This is something she most likely developed on her own, rather than through formal teaching.
- The progression from concrete to eventually using principles is the direction in which young children will typically move, and very often they accomplish this on their own.

Video: Prediction task; Julia; age 4
Goal:
- To demonstrate a teaching method that gives children opportunities to explore the ideas behind addition but does not require them to find exact sums.

Overview:
- The child puts several bears in a bag, the interviewer adds a few more, and the child must predict how many there are altogether. The first couple times she predicts unrealistic sums like a million and eighty-eight. After a few tries the child’s predictions become more realistic and closer to the correct amount.

Discussion Questions:
- What is required of this task? How is it different from asking children to add two visible sets?
- How does Julia’s performance change over time?

Comments:
- The task is different than adding two visible sets because without a way to figure out the answer physically, the child is forced to make an estimate. She will likely use knowledge of counting order as Franko did in an earlier video. As you can see in this video, Julia did not seem to know what to do at first; thus she guessed unrealistic numbers like a million. Gradually she figured out a way to make more accurate predictions. One of the virtues of this task is that by letting the child do the counting herself to check her predictions, she is providing feedback for herself.
- Students might note the sophisticated word “predict” used during the task. Although Julia may not have heard this word before, she can likely figure out the meaning of it.
over the course of the task. In teaching we should not expect children to understand everything right away: It is okay for children to fail and work things through.

**Video: Three sharp toothed buzzards; Pre-K classroom**

**Goal:**
- To show one type of addition/subtraction activity that is appropriate for even very young children.

**Overview:**
- A teacher leads 3, 4 and some 5 years olds in the song “Three Sharp-toothed Buzzards”. In the song, three buzzards leave a tree one-by-one and then return one-by-one. Each phase in the song corresponds with a particular motion or pose.

**Discussion questions:**
- What do you think of this activity?
- What might children learn from it?
- What does the teacher do to reinforce the mathematical concepts in the song?

**Comments:**
- Note how at each change of amount, the teacher pauses and reviews the current problem—e.g., “We had 2, and 1 went away…”—at which point the children volunteer how many buzzards they think there are now. This is an excellent way to make sure that the children are paying attention to the story and thinking about the concepts. Here they may connect counting forwards and backwards to addition and subtraction.

**Other teaching videos:**
- Addition stories in a small group
- Paper towel subtraction
Module 7: Assessment & Clinical Interviewing

This module covers issues concerning different types of assessment, including testing, observation, and clinical interviewing. After reviewing the benefits and drawbacks of each, it focuses on the third type, clinical interviewing. The included videos were selected to illustrate the gains of clinical interviewing both individually and in the classroom. Two long videos are also provided that allow for detailed observation and analysis of this method.

Video: Explaining carrot addition; Rachel; age 6
Goal:
- To demonstrate the importance of observing children carefully and asking questions about their thinking.

Overview:
- The interviewer tells child a story about a squirrel and a rabbit who have some carrots. One has 3 and the other has 4, and the interviewer asks how many they have altogether. When figuring out the sum the child first gives an incorrect answer—6. The interviewer asks her to explain her answer. She describes mental images of carrots, where 3 carrots one by one are moving over to a group of 4. In this process she realizes her mistake and corrects her answer to 7. Later she says that the problem was “like a puzzle.”

Discussion Questions:
Stop after child’s initial answer:
- What is your assessment of Rachel’s ability to add?
- What would you do at this point?
Continue to end:
- Does the second part of the video change your mind about her abilities?
- What specifically did the interviewer do that helped him find out more about the child’s thinking?

Comments:
- The video illustrates the fact that children can have interesting and complex approaches to solving problems that educators often miss. Many teachers would simply mark a child wrong for her original answer. Learning techniques to find out about children’s reasoning will ultimately benefit both teachers and students.

Video: Interviewing in the classroom; 2nd grade
Goals:
- To show that interviewing techniques can also be used in the classroom, and that it may benefit other students to hear how a child figured out an answer.
Overview:
- A teacher asks a child how he remembers that 8 + 5 is 13. He explains his strategy, and the teacher mathematizes it on the board using symbols, she then asks another child to explain what he did.

Discussion Questions:
- What does the teacher do to understand the child’s strategy?
- Why do you think she wants the student to explain this?

Comments:
- The teacher makes an effort to get the child to explain his reasoning thoroughly. She pushes him to describe his strategy, saying that she’s a little confused so that he’ll explain it more clearly. She writes up the equation on the board to help give him some scaffolding while he explains it.
- There are several reasons to have a child explain his thinking. First, it helps her to understand his abilities. Second, it helps him to understand it better himself and to remember it. Third, it may benefit classmates who may not have a good strategy for the problem. When she asks another child to summarize Harrison’s reasoning, the teacher is trying to facilitate such student-to-student learning in a direct way.

Video: Whole interviews with Michael and Tammy
Goal:
- The goal of both of the videos below is to have students gain a better understanding of the clinical interviewing process.

Some questions to ask throughout both of these interviews:
- How does the interviewer help the child get comfortable and stay comfortable during the interview?
- What knowledge do you gain about the children’s understanding that you might not have learned through just observation or test?
- What are some good techniques that the interviewer used to tap into the child’s thinking?
- What might you have done differently?

Video: Whole interview of Michael; age 6
Overview:
- Child is interviewed about counting, more and less, equivalence, addition, and subtraction.

To note in this video:
- The interviewer’s ways of probing into the child’s thinking, for example suggesting how other children might think about a problem, or presenting arguments that force the child to defend his answers.
The way the interviewer helps the child to explain his thinking, for example by asking him how he would explain a solution to another child.

**Video: Whole interview of Tammy; age 6**

**Overview:**
- Tammy is interviewed about counting, addition, subtraction, equations, and math that the child is working on in her class.

**Of note:**
- The interviewer’s ways of probing into the child’s thinking, for example asking her to tell him out loud what she is thinking, and persisting in asking these questions.
- The way the interviewer helps the child to explain her thinking, for example, when she isn’t sure, offering certain strategies (e.g., “Did you use your fingers?” or “Were you picturing fingers in your head?”)
- The genuine curiosity the interviewer shows about the child’s thinking and about what she is doing in school.
Module 8: Representation & Symbols

This lecture focuses on children’s first experiences with formal symbolism. It addresses different types of representations such as tallies, pictures, and conventional symbols to signify both numbers and number operations. A major goal of the lecture is to help students recognize how foreign these formal symbols can be to children and the types of challenges they must face when learning to interpret and use them. When watching the videos, students should pay attention to the type of representations that are used and the types of abstractions the children must make in the tasks.

**Video: Matching tallies to chips; Ben, age 3**

Goals:
- To show that even at a young age children can understand some types of abstract representation.
- To illustrate how tallies might be used as a precursor to symbols in representing amounts.

Overview:
- In this video a child is asked to produce the same number of “elephants” (chips) as are shown in lines on paper (three). Footage shown elsewhere demonstrates that this child cannot yet consistently name the cardinal number of a group of more than 2 chips. However, in this clip we see that he is successful at producing the correct number using this tally method.

Discussion Questions:
- What is Ben being asked to do in this clip? Be specific.
- How is this task different from representing the amount with a number, whether it is spoken or written?
- How could a teacher build on this task to create an activity for a class or small group?

Comments:
- The use of an object to systematically stand for something else is a big step for young children. Here there are actually two levels of abstraction: the chips are used to represent elephants and the lines are used to represent the chips. Ben needs to understand that the lines, in turn, represent elephants.

**Video: Representing checkers on paper – Kenny; age 3**

Goals:
- To show that children often use non-traditional symbols such as lines to represent amounts.
- To show that even in doing so, a child may get confused between the concrete objects and the abstract symbols.
Overview:
- The child is shown sets of 3 and 4 checkers and asked to count each. Then the checkers are covered and the child is asked to show how many there are with a marker. The child decides to make lines to show the checkers. He accurately draws sets of 3 and 4 of lines. When asked to figure out how many lines there are altogether, he accurately counts 7. Then when asked how many checkers there are, the child can’t deduce that there must be 7. When he lifts the paper, he miscounts 6 checkers. He concludes that there are more lines than checkers.

Discussion Questions:
- Why does the interviewer keep specifying whether he is asking about the number of lines or checkers?
- How do you explain Kenny’s confusion at the end of the clip about there being more lines than checkers?

Comments:
- The task here is making tallies to represent checkers in each set, and then using them to figure out how many there are altogether. Note that Kenny comes up with the idea to use lines himself. It is significant that he could come up with an abstract way to represent something. Not all children will come up with this on their own.
- The interviewer asks about the lines versus checkers to assess Kenny’s grasp of the idea that the lines stand for the checkers. His confusion at the end indicates a fragile grasp of representation—although he can represent checkers with lines initially, he cannot successfully move to the next step and use those lines as a way to solve a problem about the checkers.

Video: Representation of broccoli subtraction; Santiago; age 5

Goal:
- To show an early, less abstract approach that children might use in representing arithmetic operations—drawing pictures of the actual objects and, in this case, crossing out ones to be subtracted.

Overview:
- Before this episode, the child had been told that a classmate had five “broccolis” and ate two of them, and he was asked to figure out how many were left using a marker and paper. The child decided first to draw a picture of the classmate and a table. In the video we see the child drawing five broccoli stalks and then crossing out two of them. He concludes that there are three left.

Discussion Questions:
- How does Santiago represent the “broccolis”? How does he represent the subtraction of them?
- How is this method of representing an operation different from using numerals and operation symbols? Tallies or dots?
Comments:
- Santiago is showing some abstract and some concrete methods in his representation of the story. The way he signifies the broccoli, with pictures, is concrete compared to lines or dots or even a number. His drawing of the person and the table is also concrete representation. However his method of indicating subtraction is more abstract: he does not need to draw the person eating the broccoli stalks for example, instead he marks them with “X” symbols to indicate that they have been taken away.

Video: Writing multi-digit numbers; Raul, age 6
Goals:
- To show that in learning how to write numbers, children not only need to figure out how to create the actual numerals, but must also understand place value.
- To illustrate that to a child, the way we write numbers does not match with the way we say them.

Overview:
- Before the clip begins, child had accurately written the numerals for 9 and 2. In the video the interviewer asks the child to write various 2- and 3-digit numbers. He has trouble with some, for example, he writes “201” for twenty-one, and “1204” for one hundred twenty-four (writing numbers the way they sound).

Discussion questions:
- What numbers can Raul write and not write correctly? What might be the differences between these numbers?
- What strategies do you think Raul uses for writing multi-digit numbers?
- What does Raul need to understand about writing multi-digit numbers before he can accurately write them correctly?

Comments:
- Writing twenty-one as “201” is a common bug: children write numbers as they sound. In a way what this child is doing is smart– he’s breaking the numbers down by the tens (“twenty” and “one”), however he is not symbolizing them in the conventional way.

Video: Understanding 3+1 with & without symbols; Kevin; age 5
Goal:
- To show that transitioning to symbols is difficult for children: even when children understand operations with concrete objects, the equivalent operation represented formally may seem very different to them.

Overview:
- An interviewer poses the problem 3 + 1 to a child. He responds that the answer is 29, because “my sister told me.” Then he is asked to use chips to show this. The child makes the symbols “3 + 1” with the chips. When the interviewer gives him 3 chips and adds one more, he immediately says there are 4. The interviewer then tries to connect
this to the formal expression “3 + 1”, but the child restates the answer as 29.

**Discussion Questions:**
- How do you explain Kevin’s original statement that 3 plus 1 equals 29?
- How does he represent “3+1”? What does this representation indicate about his understanding of formal equations?
- Are you surprised at his immediate correct answer of 4 when the interviewer shows his 3 chips and 1 more?
- How do you explain his reiteration that 3+1 = 29 at the end of the clip?

**Comments:**
- Notice that the child is trying to show the equation with the chips literally (creating the symbols “3 + 1” with the chips.)
- The problem here is not that Kevin cannot add—he has the everyday knowledge of addition. The problem is his understanding all the symbols: he really understands the basic idea of adding 3 and 1, but the symbols are getting in the way.

**Related Video**
- Representation of a subtraction problem on paper; Gabriel, age 5

**Overview:**
- A child is asked to show on paper an earlier situation wherein 4 teacups were taken away from a set of 7, leaving 3. The child first draws seven circles to represent teacups and crosses out four. When asked how many are left, he says three (notice that when he says this he is glancing back and forth at the real teacups remaining on the table). Next the child is asked to show what happened with the cups using numbers. He writes the numerals 1-7 on the paper and then says that he should circle the “4”. He then proceeds to circle (or as he explains, “take away”) the rest of the numerals, leaving un-circled only the number 3. He announces, “Now the number 3 is there only.” The child’s actions reflect confusion about using numerals to represent cardinal amounts; he sometimes treats numerals as objects.

**Video: Understanding of the equals sign; Tammy; age 6**

**Goal:**
- To show that children often do not possess a deep understanding of the mathematical symbols they use every day. Here we see a common misunderstanding of the equals sign, where children don’t see it as indicating equivalence, but instead see it as something that belongs at the end of an equation, right before the answer.

**Overview:**
- Child is first asked to write equations. When interviewer dictates an equation in which the equals sign comes before the operation sign (e.g., 3 = 2 + 1), she gets confused and says it doesn’t make sense.

**Discussion Questions:**
Why doesn’t Tammy accept the equations in which the equals sign comes before the operation sign?

Why would she say, “Once in a while it could?”

What do you think of Tammy’s explanation of the equals sign as meaning “the end is coming up”?

Comments:

This understanding of the equals sign as “makes” or, as Tammy says, “the end is coming up” is extremely common in children. Even if teachers think they are teaching the equals sign as indicating equivalence, children still might interpret it the way that Tammy does. This is in part attributable to the fact that most equations children see have the equals sign towards the end of the equation.

Notice also that Tammy doesn’t know what the plus sign means; however she can successfully add. Like Kevin, she understands the concepts but is struggling with the quagmire of symbols.

Video: Two meanings of the equals sign; Jordan, age 8

Goals:

To show how even when students do understand the equivalence meaning of the equals sign in one context, they may not understand it in the context of equations.

To show a way to help a child see how the equals sign in an equation can indicate equivalence.

Overview:

The video starts with a child showing that two sets of 6 blocks are equal by placing a piece of paper with the equals sign on it between the two sets. He explains that equals means that they have the same amount. Next, he says that in number sentences, the equals sign means something different—that together with the plus sign, it means “makes”. The interviewer then asks if the symbol can also mean “the same as” in the number sentence context. Although slightly hesitant, he eventually seems to agree that it can.

Discussion Questions:

What are the two ways that Jordan interprets the equals sign? How do the two situations he describes in which the equals sign can be used differ?

How does the interviewer encourage him to rethink his view?

Do you think he is convinced at the end that the equals sign in the equation can mean “the same as”?

Comments:

Note how the interviewer acts naïve when asking about the equals sign. This is a method that Piaget used when he first started interviewing children.

The interview involves teaching as well as assessing—the interviewer uses a probing question to get him to connect his two meanings of the equals sign. Using teaching
methods in clinical interviewing is fine as long as it is done after the interviewer first has a clear understanding of the child’s thinking.
Module 9: Number Facts

This module addresses number facts and the debate about the importance of mastery versus understanding. The goal is not to take one side, but to deeply examine the issues and know what is involved in learning number facts. The point is made that both mastery and understanding are important, and further, that the way they are taught is not always the way they are learned. Even if we think we are teaching children to memorize, they might be building deep meaning, or vice versa. Also, the experience of learning the facts is different for different children: while some kids enjoy learning them, for others it is a negative experience fraught with anxiety. Such negative experiences need to be recognized as being counter-productive for children in the long run. Finally the module covers some ways that teaching and learning number facts can be meaningful.

When watching the videos, students should pay attention to students’ affect, confidence, mastery, and understanding of the problems they are given.

Video: Reciting number facts in preschool; Santiago, age 4
Goal:
➢ To show that number facts are not always a negative experience for kids: many children enjoy learning them.

Overview:
➢ A preschooler shows off his number fact knowledge to a friend during free play.

Discussion Questions:
➢ Where do you think Santiago learned these number facts?
➢ Does it look like he enjoys reciting them?
➢ To what extent do you think he understands what he is saying?

Comments:
➢ Santiago may not really understand what he’s saying – the point here is -that he takes pride in knowing these number facts.

Video: Anxiety in remembering multiplication number facts; Beth; age 8
Goal:
➢ To show that learning number facts can produce anxiety in children;
➢ To show that children often learn the facts by memorization without understanding what they mean.

Overview:
➢ In this video 9-year-old Beth is first quizzed on a series of multiplication facts. She appears anxious and gives some incorrect responses. She is then asked how she can figure out the answers to double check. She doesn’t have a good method for this.
Discussion Questions:
- How confident does Beth appear when answering the multiplication questions?
- What methods does she have for figuring out answers when she doesn’t know them from memory? Are these good methods?
- What kind of methods do you think the interviewer was looking for when he asked about this?
- Do you think learning the multiplication facts has been a positive experience for Beth?

Comments:
- Notice the body language the child is exhibiting. When doing a clinical interview, it is important to pay attention to such cues. This child appears to be stumbling and uncomfortable in the interview.
- This child typical of kids who, when they don’t know the answer, feel bad, are lost, and don’t have any idea of what to do next. It is important that children know how to figure out answers when they don’t know them (without asking the teacher or using a calculator.)

See also:
Anxiety in addition facts; Sarita

Video: Knowing facts versus checking them; Eddie, age 7
Goal:
- To show that children may feel that it is more important to “know” the answer from memory that to make sure it’s correct.

Overview:
- Child is asked to prove that 12 - 9 = 2, a statement he made before the video begins. He counts out 12 chips and then takes away 9. When he sees that 3 are left, he quickly takes one more away to show that 12 – 9 is indeed 2. When asked to double check he does the same thing.

Discussion Questions:
- Why do you think Eddie changes the results of the chips at the last second?
- What is more important to Eddie—having a memorized answer or knowing how to figure out the answer?

Comments:
- Make sure that the students notice what the child did here: He had written that the answer was 2, but when he carefully counted them out with the chips, he found that had 3 chips left. He didn’t like what he came up with when using the manipulatives, so he pushed one chip away.
- This child most likely, before starting school, could deal with problems like this when given the concrete objects. Now, he’s in school and learns the importance of marks on paper. They don’t make sense to him, but he thinks that that’s what the teacher wants.
He places his trust in this school game of writing symbols on paper over his own thinking and experience with manipulatives.

**Video: Series of videos of children figuring out number facts.**
For each, the goal is to understand what it means for a child to master the facts and the following questions can be asked:

- What is the child’s method for solving/proving the fact?
- How does the child’s understanding of the fact compare to Beth’s understanding from the previous video?
- How might simply asking a child how he solves a fact
  - a) benefit a teacher, and
  - b) benefit the student?

**Video: Derived fact 8 + 7; Henry; age 6**

**Overview:**
- Child explains how he knew that 8+7=15. He says that 8+8 is 16, and 7+7 is 14, so if he adds one less to the 16, or he adds one more to the 7+7, he will get 15.

**Comments:**
- This is a meaningful approach to solving the problem– the child is retrieving facts that he knows (doubles facts), and using them to solve this fact. Look how different he is from Beth. You can see as a teacher how clinical interviewing can be informative– Teachers can encourage this type of strategy use.

**Video: Explaining 7 x 3; Irwin; age 7**

**Overview:**
- Child figures out the problem 7x3. He first figures out 15 is 5 threes, and then adds 6, and says 21, because 15+6 is 21. The interviewer asks more about this and the child explains.

**Comments:**
- Here the child is essentially using the distributive property \((a(b+c) = ab + ac, or 3(5+2) = 3(5) + 3(2))\). Clearly he has a good understanding of multiplication. Notice how the child is thinking about the answer– as opposed to trying to retrieve the fact. It’s important when interviewing to wait patiently– not to interrupt the child when he is taking a while to answer.

**Video: Explaining 2 x 4; Zelda, age 7**

**Overview:**
- Child says that 2 x 4 is 8. Interviewer encourages her to explain how she knows this on paper without using numbers.
Comments:

- Notice how the interviewer really challenges the child to explain her reasoning. You’d think that dealing with a small number fact would be easy, but for a child, having to give a detailed explanation is not easy. If the teacher spent time pushing children to explain their thinking when solving a number fact, it might build some deeper understanding.

*Video: Reversibility in addition and subtraction; Rufus; age 6*

**Overview:**
- Child is asked 8-3, he says 5. When asked to explain, he says it’s because 5+3=8 (relating a subtraction fact to its complementary addition fact).

Comments:

- Notice how this interviewer acted naïve so the child would explain things to her. This is a good interviewing technique.
- The idea of relating addition to subtraction was one of the methods discussed in the paper on the Chinese approach to teaching number facts. Teaching addition and subtraction together could be a useful approach.

*Video: Sums of 7 with the zoo game; Julia; age 4*

**Goal:**
- To show an activity in which number facts can be meaningful.

**Overview:**
- Child is challenged to put 7 bears into 2 cages in as many ways as possible. Interviewer helps her to try to find all the ways, writing down the number combinations as they go.

**Discussion Questions:**

- How might this activity help the child learn number facts? Probe: In what ways is the activity different from giving the child a series of pairs of objects to add?
- What are some other ways that number facts can be taught in a meaningful way?

Comments:

- This is an attempt to make the written representations meaningful to relate it to the objects. The child is interested and thinks of it as a game. She is noticing that you can switch the sets of bears to make new facts (6 + 1 = 7 vs. 1 + 6 = 7). With more experiences with the activity, she might realize that by taking one away from one cage and adding it to the other, she can make new facts. She might also figure out how to make sure she has every fact that makes 7.
Module 10: Place value & written procedures

This module addresses challenges children face in learning about place value and written procedures. The major points to convey are that too often, children learn rules and algorithms without really understanding them. Even adults will have a hard time understanding the concepts behind procedures—take for example the division of fractions. When children don’t understand the concepts, they may develop “bugs” or systematic errors, and they may have a difficult time knowing when and how to use the algorithms. When children do understand the concepts, however, they are often able to invent their own procedures. We see in the videos below different children’s abilities with place value and written procedures, and how through careful interviewing we can try to better assess their understanding and help them modify or build on their conceptions.

Video: Explaining place value; Shania; age 6
Goal:
- To demonstrate how place value can be challenging for children to fully understand. Manipulatives can help but may also add a layer of complexity.

Overview:
- Interviewer explores child’s understanding of place value by asking her about the number 13 and then the number 413. They also use base-ten blocks in investigating these questions. Child shows some understanding but also has some difficulty.

Discussion Questions (stop at appropriate points in the interview):
- What does Shania understand and not seem to understand about place value?
- Why did the interviewer ask the questions she did? What else might have been asked?
- Do the blocks seem to help Shania understand the concepts?

Comments:
- Shania’s initial explanation for why the digits belong where they do does not necessarily indicate understanding of the tens and ones places. When asked to show the amount with the base ten blocks, the interviewer is trying to see if she will show the tens and the units of the number. However, she shows 13 with 13 individual units rather than 1 ten block and 3 units. Eventually the interviewer provides some scaffolding to help the child explain, although her explanation is still somewhat lacking in a clear indication of a solid understanding of the place value system.
- This video is a good example of how in interviewing we are not only assessing children’s current understanding but also their learning potential: by providing scaffolding the interviewer is investigating what the child might understand with some help.

Video: Regrouping: getting different answers in different ways; Diane; age 7
Goal:
- To show how learning algorithms is difficult and often takes place without understanding of what the steps mean.
Overview:
- Child carries out an addition problem in two different ways—one with regrouping and one without—and gets different answers for each. She explains that both answers are okay because they were done in different ways.

Discussion Questions:
- What exactly is Diane doing in her first attempt at the problem? What misunderstanding does her mistake reflect?
- What does her explanation of the fact that both answers are right indicate about her understanding of written procedures or mathematics in general?

Comments:
- Note that Diane is carrying out the problem as if it is a subtraction problem: she is confusing the rules for subtraction with addition. When faced with a problem she isn’t sure how to do, she is retrieving rules, possibly from a recent lesson on subtraction, and applying them to the problem at hand without understanding why. The fact that she accepts both answers as correct indicates that the written procedures she is learning have little meaning to her.

Related video
- Bug in double digit addition; Julia, age 7

Video: Invented method for double digit addition; Tara; age 6
Goal:
- To show that when children have a good understanding of the concepts, they can invent their own written procedures for calculation.

Overview:
- Child is given double-digit addition problem to solve that would require regrouping. She has not learned the standard algorithm for regrouping yet, but succeeds in figuring out the answer using her own method of crossing out and combining numbers.

Discussion Questions:
- What exactly is Tara doing? What do her step indicate about her understanding of the problem?
- How is she different from Diane in the previous clip?

Comments:
- Tara’s methods indicate a good understanding of place value and about adding tens and ones. This type of informal calculation abilities that children use outside of school (as she mentions) should be encouraged in school. It is a common misperception that children come to school with little of their own understanding, however we can see with this child that this is far from true. Rather than try to ignore these ideas and teach them
all new ideas that may be meaningless to them, we should try to build on their informal ideas.

**Related video**
- Adding double-digit numbers mentally; Zoe, age 7.

**Video: Partial sums method; Molly; age 8**
**Goal:**
- To show how some curricula are teaching children other written procedures that are based on more intuitive ways of adding the tens before the ones.

**Overview:**
- Molly demonstrates the “partial-sums” method of double-digit addition, where the tens are added before the ones.

**Discussion Questions:**
- Do you think this is a good method to teach children?
- Why might this be easier for some kids than the traditional algorithm (adding the ones first and regrouping if needed)?

**Comments:**
- Note that in this curriculum both methods are taught.
- The partial sums method is actually more intuitive to a lot of kids, because often when people carry out calculations mentally they add the tens first and then the ones.
Module 11: Understanding

This module covers in-depth understanding of mathematics concepts. A major point is that understanding is very difficult to define and to assess, although we must try to get at it as best we can. Some aspects of understanding are grasping “the big picture”, making connections among ideas, awareness of one’s own abilities and knowledge (metacognition), being able to justify and prove ideas, and understanding how to apply knowledge and skills in the real world. Further dissecting understanding, the importance of “number sense” or having a basic feel for numbers (such as having a good mental number line), and being able to use mental models for solving problems are discussed. When watching the videos, students should think about how the skills illustrated might be helpful to a child’s overall understanding, and how in assessment we often miss these insights.

Video: Children working with number line; ages 4, 8 and 10

Goal:
- To examine children’s number sense: In particular this video investigates relative magnitude of numbers/mental number line.

Overview:
- Three children are interviewed about a number line. The interviewer asks where different numbers might be placed on a line given the placement of 0 and 10. The youngest child knows where numbers belong relative to the other numbers, but does not have a good sense of the spacing. For example, he knows that 9 should go before 10, so puts it very close to the 10. He performs similarly on a 0–100 line. The older children have more sophisticated number sense—they are concerned with spacing and know how it relates to position of the numbers. They can talk also talk about negative numbers.

Discussion Questions:
- What do the children know about relative magnitude of the numbers?
- What strategies are they using to position the numbers?
- How are the children’s strategies different?
- Why is mental number line important?

Comments:
- The youngest child has knowledge about what numbers belong before others, and what numbers go between others; however he doesn’t seem to have a good larger picture of the number line and where particular numbers should fall between other numbers. For example he doesn’t know that 5 is halfway between 0 and 10—he thinks that it should be close to 1 because he knows it comes after 1.
- Note how aware the older children are of spacing and proportion when deciding where numbers go—they are thinking about where other numbers would go, demonstrating possession of mental number line.
- Students might note that young children are working with negative numbers. Usually negatives aren’t introduced until later; however it isn’t clear that younger children can’t deal with negative numbers.
Having a mental number line is important for being able to make calculations and detect when a calculation seems right.

**Video: Comparing number combinations; Conrad; age 8**

**Goal:**
- To examine a child’s number sense: In particular this video investigates a child’s ability to compare sums.

**Overview:**
- Child is given pairs of number combinations (e.g., 5+5 and 5+6) and asked which will be more. Child gives explanations for his answers.

**Discussion Questions:**
- How do these problems compare to regular computation problems?
- What knowledge and skills does the child need in order to succeed at this task?

**Comments:**
- The idea of this task is figuring out which sum is more without actually adding up the numbers—this is another type of number sense. The child here actually knows how much more one sum will be than the other without comparing the sums. Notice also how engaged he is in the task.

**Video: Interpreting different models of multiplication; Ami; age 9**

**Goal:**
- To show an interesting problem that gets at children’s understanding of an operation (in this case multiplication).

**Overview:**
- Child is shown 4 different representations of the problem 4 x 3. He then asks the child which one he thinks represents it best. Child explains his opinions.

**Discussion Questions:**
- Which picture would you use for representing this equation?
- What are the benefits of the different ones presented?

**Comments:**
- Students should realize that for many mathematical ideas there are several different types of representations, and some illustrate certain ideas better than others. For example a model using dots works with whole numbers but not rational numbers (e.g., if you are multiplying 1.5 times 2.5.)
- In textbooks you may find 3 or 4 examples of these models used indiscriminately without any explanation.
- If part of understanding is being able to understand and use a mental model, just think about how tests generally miss this.
**Video: Reasoning and proof: 5 + 6 = 11**

**Goals:**
- To illustrate justification and proof (an NCTM standard).
- To emphasize the importance of children not only knowing answers but also being able to justify and prove them.

**Overview:**
- Child says that 6 + 5 is 11 because 5+5 is 10, and one more is 11. Interviewer asks him how he knows that 5 + 5 is 10. Child explains using fingers.

**Comments:**
- To really understand mathematical ideas it is important to be able to explain them. Here we see that Nicholas really does understand why 6+5 is 11 and why 5+5 is 10—he is not just repeating facts he learned in school.
Module 12: Geometry

This module addresses children’s developing understanding of shapes and space. Major themes are children’s abilities to distinguish shapes from a young age even if they don’t know the correct names, their use of visual cues over definitions when identifying shapes by name, and some of the challenges of grasping spatial relations such as orientation and angle. When watching the videos students should pay attention first to the specific tasks used: what are these tasks trying to assess? Second, they should note the questions and probes the interviewers are posing: why was that question asked, and what information does it give the interviewer about the child’s understanding of this content area?

Video: Sorting shapes non-verbally with princesses; Sarah Kate; age 2
Goal:
- To show that even before children know the names of shapes they can recognize and distinguish among them.

Overview:
- Child is given task to sort shapes by giving them to different pictures of princesses. She makes some mistakes at first but eventually seems to understand how to do it.

Discussion Questions:
- Why is the interviewer saying “this kind of shape” rather than saying the names?
- How might the child’s initial mistakes in assigning shapes be explained?
- Why was the small green triangle given to the child at the end?

Comments:
- It is important to note that names of shapes are secondary here—what is important is the shapes.
- At first the child might not understand what is requested of her because she only has one shape example for each princess picture. She might think that the interviewer wants her to sort by size, for example.
- The green triangle is given to the child once she seems to be able to sort the shapes accurately to see if she can transfer her knowledge to a different kind of shape. The child succeeds at this.

Related videos
- Matching shapes during clean up; pre-K.

Video: Sorting shapes verbally with vehicles; Jayden; age 4
Goals:
- To show in learning names of shapes, some names are harder for children than others;
- to show that even if a child confuses the name, he may still have a good understanding of the shape’s criteria.
Overview:
- A child is asked to sort shapes by name (“Put all the circles in the car.”) Child is accurate with circles but confuses triangles with rectangles. Still, in sorting the latter two, he does find all of the correct shapes, just for the wrong name.

Discussion Questions:
- What does the child need to understand in order to succeed at this task?
- Why does the interviewer tell the child to say when he’s finished?
- What are some of the distracter shapes on the cards? Why are they there?

Comments:
- In order to succeed at the task, the child not only needs to be able to discriminate between the shapes on the table, he also needs to know the accurate names.
- The interviewer tells the child to say when he’s finished to make sure he has found all of the correct shapes and that he is not going to choose any of the other ones. Often when assessing children, teachers will stop them right after they have succeeded, thus missing the opportunity to see if the child knows for himself when he has completed the task.
- The distracter shapes, such as the moon shape for the circle task, are there to really test whether the child knows the characteristics of the shape. In the case of the moon shape, the interviewer is testing whether the child will think that any shape with a curve is a circle. To thoroughly test a child’s knowledge, opportunities for mistakes need to be provided.
- Notice how even though the child confuses the names for triangle and rectangle, he is able to accurately find all the correct shapes for the opposite name—even non-canonical forms (like long, thin ones) that are often harder for children.

Video: Identifying and explaining attributes of shapes; Chidera; age 3

Goals:
- To show some of the challenges of recognizing and describing shapes: some names are difficult, orientation of shapes can influence children’s recognition of them,
- and that children’s knowledge of shapes often involves mere appearance- rather than definitions based on analysis (and in fact definitions are difficult for them).

Overview:
- Child is shown several shapes and asked their names and how she knows what they are. She has some knowledge of the names and can recognize some, but has trouble with others.

Discussion Questions:
- How do you interpret Chidera’s explanation of how she knew the first shape was a triangle?
- Why does the interviewer give the child choices of the names of the 2nd shape?
- Why do you think the child says at first that the third shape (the blue diamond) is a square? What does it mean that she changes her answer to a diamond when it is turned?
Comments:

- Chidera knew that the first shape was a triangle and seemed to know how a definition should sound, but didn’t have it right. She is clearly not basing her recognition of the triangle on a definition but rather on the way it looks.
- Giving the child choices of names is a way to scaffold—sometimes the name is hard to remember, but hearing it helps the child remember it.
- The child may think that the parallelogram or diamond is a square because it has similar properties—it has four sides, and in the orientation it was in, it had a horizontal side along the bottom. When it is turned, she recognizes it as a diamond. This further corroborates the hypothesis that her identification of shapes is based on perception rather than a definition. It also indicates that she does not know that a shape remains the same regardless of orientation.

Related video:
Identifying shapes tactiley, Pre-K

Video: Identifying triangles; Dillon; age 4

Goal:
- To show that even if children have an idea about definitions, they may have faulty ones and/or still use perceptual cues to identify shapes.

Overview:
- Dillon is shown a collection of shape figures on a piece of paper and is asked to describe and identify various shapes.

Discussion Questions:
- Why do you think Dillon keeps rotating the page?
- What is Dillon’s definition of a triangle?
- What disqualifies a shape as a triangle, according to Dillon?
- Which triangles does Dillon color immediately and which ones are left toward the end? What about these make them easier or harder to recognize?

Comments:
- The child turns the page to see if he recognizes any shapes as triangles, and it appears here he is turning it so that the base of a shape he is trying to identify is on the bottom.
- Dillon says that a triangle must have 3 points, and 2 long sides and 1 short side. It is not clear why he has this definition, however he does not stick to it the whole time—he is willing to consider some shapes without those characteristics and even decides that some are indeed triangles.
- Notice that the child realizes that some shapes are triangles after working on the sheet—he is probably learning from this activity, more evidence that clinical interviews do not always probe static ability.
- Also notice that the interviewer is asking not only about which shapes are triangles but also which are not triangles. This is important in investigating a child’s understanding of shape.
Related Videos:
- Identifying triangles; Elsa; age 4
- Making shapes with toothpicks and clay; 1st grade

Video: Symmetry with pegboard; Shauna, age 3
Goals:
- To show a task for working with symmetry.
- To show some of the difficulties children have with symmetry.

Overview:
- Interviewer plays a game with the child where she creates a design on one side of a pegboard and the child has to make the symmetrical design on the other side. Child is successful with horizontal lines, but has trouble with diagonal ones.

Discussion Questions:
- What skills and knowledge does the child need in order to succeed at this game?
- Why do you think the diagonal line is more difficult than the other designs?

Comments:
- This task gets at ideas about locations in space, angle, orientation, number, pattern, and symmetry. It could also be a precursor to an activity about coordinate grids.

Related video
- Symmetry lesson with face drawing; Pre-K
Module 13: Pattern

This module covers the content area of patterns and algebraic reasoning. The main ideas are that from a young age children begin to notice rules and regularity, and that work on understanding and articulating these patterns can provide a basis for sophisticated ideas in algebra. The videos illustrate children’s abilities with patterns and some of the tasks that can be implemented with kids beyond the common “What comes next?” task.

*Video: Difficulty in extending a pattern; Ben; age 3*

**Goal:**
- To show a child who does not understand patterns; to demonstrate some types of scaffolding that adults may use to help a child recognize a pattern.

**Overview:**
- Child is shown an alternating pattern and asked to extend it; he has difficulty even after several scaffolding attempts.

**Discussion Questions:**
- What methods does the interviewer use to try to get the child to extend the pattern?

**Comments:**
- The interviewer tried having the child recite the color names, and then recited them herself in a rhythmic fashion. If a child doesn’t see the pattern, he will often hear it when it is recited like this. However this child still does not seem to understand the rule of the pattern.

*Video: Accepting AB but not ABB pattern; Ben, age 4*

**Goal:**
- To show the same child as before, 1 year later, and his understanding of pattern.
- To show how simple AB patterns may be easier for children than those with longer repeating units.

**Overview:**
- Ben has a seemingly good understanding of pattern, especially compared to his understanding a year before. He can extend from both sides and can also fill in missing elements. However he only seems to want to accept AB patterns.

**Discussion Questions:**
- Why do you think Ben rejects the ABB pattern that the interviewer is proposing?

*Video: Noticing regularity; Vienna; age 4*

**Goal:**
- To show that from a very young age, even infancy, children notice patterns and regularity in the world around them, although they may not be able to explain them well.
Overview:
- Child is shown a row of different types of balls that repeat in a pattern. The child immediately starts pointing out how some of the balls are the same.

Discussion Questions:
- What do you think Vienna is trying to explain about the pictures she sees?

Comments:
- Note that the interviewer was not asking about the patterns in the pictures—this was a counting task, yet the child repeatedly points out the regularity in the pictures.

Video: Describing a pattern as a list of colors yet extending correctly; Vienna; age 4
Goal:
- To show that children may understand patterns but not be able to explain them.

Overview:
- Child explains a pattern by listing colors that don’t seem to follow a rule, yet she is successfully able to extend a pattern.

Discussion Questions:
- After child explains what a pattern is: How would you describe Vienna’s understanding of pattern?
- At end: Now how would you describe her understanding of pattern?

Comments:
- We see again in this video that explaining a pattern is difficult for children (and even adults). However poor explanation does not necessarily indicate a lack of understanding of pattern. This is a theme we see in many areas of mathematics.

Videos: Series of activities with patterns
The goal of showing each of the videos below is to have students see different ways that they can encourage thinking about patterns and rules—that patterns do not always have to just be “what comes next” tasks. For each video, have students think about what the children need to understand in order to succeed at the tasks.

Video: Filling in gaps in patterns; Vienna, age 4
Comments:
- Often children learn how to extend patterns by learning the repetitive verbal “chant” of the items (e.g., “Red yellow blue red yellow blue…”). For this task the child is challenged to think more about the core repeating unit and the relationships between the individual units.
**Video: Different pattern, same structure; Morgan, age 5**  
Comments:  
- Algebraic thinking involves stepping back from specific patterns and thinking about common structure. In this task the child must recognize the general format of the pattern (ABBABB) and make a pattern with new colors but using the same rule.

**Video: Growing patterns with bears; Owen, age 5**  
Comments:  
- In this task children must go beyond repetition and think about patterns in growth. Here the child explores skip-counting by twos.

**Video: Growing patterns with cubes; Henry; age 6**  
Goals:  
- To show how patterns are linked to number and operations;  
- to show how more complex pattern activities can help kids think about general rules and solve problems.

Overview:  
- Child works with a doubling pattern that uses green and white cube towers. After child extends the pattern, he is asked to predict a tower further down the line.

Discussion Questions:  
- What patterns do you see in this activity?  
- How does the child figure out the number of cubes in the 8th position?  
- What does he mean when he says that the nine wouldn’t have any “room”?  
- Does the child seem interested in the activity?

Comments:  
- Students should notice how this activity works—the numbers on the papers below the towers tell how many of each color are needed in the corresponding tower and, when doubled, yield the total number of cubes in the tower.  
- The child seems to figure out the answer to the 8th tower not by doubling the 8, but by figuring out what the 10th tower would be (20 cubes tall) and subtracting two sets of two (at first he only subtracts two to get 18, but he then realizes that there wouldn’t be room for the ninth tower).  
- Although this child does not explicitly do it, eventually a child should be able to understand that there is a rule linking the number of cubes in a tower to the position number (2p = c, where p = position number and c = number of cubes). This use of variables, even in an informal way (without letters or symbols), is big idea in algebraic reasoning.
Module 14: Measurement

This module deals with children’s developing understanding of measurement. Big ideas are perceptual knowledge and comparison, seriation, transitive property, conservation of length and volume, common baseline, the inverse relationship between unit size and quantity in measurement, and finally standard units of measurement (e.g., inches, centimeters, or pounds). When watching the interview videos, students should note and discuss the knowledge and skills that the children exhibit and that the interviewers are assessing. For the teaching videos, they should discuss the knowledge and skills necessary for the activities and the concepts that the teachers are trying to convey.

Video: Talking about animals; Julia; age 4
Goal:
To show how children have ideas about measurements—and are especially good at making comparisons—although they may not use sophisticated vocabulary to do so.

Overview:
In this video Julia describes her observations about animals at the zoo. She talks about how the giraffe had a long neck, whereas a panda bear has a “little” neck like hers. She excitedly uses hand gestures when talking about the animals.

Discussion Questions:
- What knowledge does Julie exhibit about length?
- Is she able to compare different lengths?
- What vocabulary does she use?
- What do her hand gestures indicate about her knowledge?

Comments:
- Here we see that Julia is very interested in the length of the giraffe’s neck, and she is skilled at comparing between the giraffe’s neck, her own neck, and the panda bear’s neck.
- She does not yet know the correct vocabulary for some of her concepts, but this is not important at this point.
- Her hand gestures indicate first an excitement about talking about the animal’s necks. They also reveal her knowledge of roughly the size of the animals’ necks, at least in comparison with one another.

Video: Seriation- difficult arranging and difficulty inserting; Alera; age 3 & Lizbeth age 4
Goal:
To show difficulty that very young children have in ordering a series of blocks.

Overview:
Children are shown sets of blocks arranged in height order, and then the blocks are mixed up. One child is not able to put the blocks back in order. The second child is able to do this, but then cannot successfully insert a missing block into the correct place.
Discussion Questions:
Why do the interviewers set up the problem with the blocks in height order?
How do they make sure the children understand how they are arranged?
How do the children attempt to solve the problem?
What is Lizbeth able to do that Alera was not able to do?
Why do you think this task is difficult? What concepts does it involve?

Comments:
This is another classic Piagetian task.
The interviewers set up the problem correctly at the beginning so that the children have a chance to see the goal arrangement. The interviewers further direct the children to observe the height order by showing them how toys can climb the “staircases”.
Piaget attributed the difficulty children have with it to an inability to coordinate relations, or simultaneously compare an object with two different items. In this case the children must understand that each block is longer than one block but shorter than another.

Related video:
Seriation, Dillon, age 4

Video: Comparing ribbons; Ben; age 3 and 4
Goal:
To show how a child compares paper strips/ribbons at two different ages.

Overview:
At age 3, child does not seem to have a good way to check which strip is longer. At age 4, he is able to line up and confidently identify the longer one.

Discussion Questions:
Why does the interviewer ask Ben to “check” which strip is longer?
What strategies are Ben using in the videos to compare the ribbons/strip in both videos?
What do children need to know in order to successfully compare lengths?

Comments:
The interviewer is asking Ben to “check” to see if he can line up the two strips. He does not seem to understand what she is asking, and decides one is longer without comparing them carefully (it is not clear on what his decision is based). In the second video he can line them up and check quickly.
In order to successfully compare two lengths, children need to know how to align the ends so that they have a common baseline.

Video: Comparing heights of children; Lesson; Pre-K
Goal:
To show how a teacher might talk to children about comparing heights and common baseline.
Overview:
Teacher tries to arrange a few students in height order with other children’s feedback.

Discussion Questions:
What concepts is the teacher trying to help the children understand?
What arguments is she encouraging the children to articulate?
What do you think of this lesson?

Comments:
The teacher is trying to help the children understand arranging in order of height (like the seriation task seen earlier) and common baseline.
She encourages the children to explain why standing on tip-toes, standing on a chair, and sitting on a chair doesn’t allow for fair comparison of heights. The children have a difficult time articulating these ideas but seem to understand that it is not a fair way to measure.
Module 15: Pedagogy in teaching mathematics to young children

In this module the goal is for students to think about how teachers can help children understand mathematics concepts in the classroom. For each video, students should discuss the following:

- What are the main concepts the teacher is trying to convey?
- What does she do in order to help the children understand the concepts?
- Do you think the children understood the lesson?
- What else might have been done to help the children understand?

Video: Liquid Volume Lesson; Pre-K

Goal:
To have students assess a teacher’s teaching of mathematical concepts

Overview:
Teacher leads class in experiment comparing volume of two containers.

Comments:
The main concepts that the teacher is trying to convey are that to compare two containers, you need to use identical test containers, and that the container that holds more water will produce a higher level in the test container.

Notice that the teacher has prepared carefully to convey these topics with deliberately labeled containers. She has some of the students predict the container that will hold more water and creates drama about testing it out. She is excited about the activity, which makes the students excited about it as well.

While one child could say that the water level in jar A would be taller if the vase had more water, we don’t know if the other children understood this. This is a difficult concept and it may have been worth more explanation.

Overall it seems that the children were interested in the experiment, and that they will gain from opportunities to continue exploring the ideas in the classroom.

Other videos:
- Graphing lesson; 1st grade
- Reviewing base-ten bl
Module 16: Teaching manipulatives and curriculum

This module addresses manipulatives and curricula in teaching mathematics to young children. It discusses what manipulatives and curricula are, how they might be used, and some cautions when using them. Specifically the point is made that these materials cannot be used blindly. For example it is often assumed that when a child uses a manipulative s/he will automatically understand the corresponding concepts, but that this is not true—children need to learn the correct way to use them and need to be guided to connect the materials to the underlying ideas. Similarly, the use of curricula will be much more effective if the teacher understands what the concepts are that each lesson is intending to convey.

The videos included deal with ideas about manipulatives. Students should consider how the manipulatives involved help or potentially hinder the children’s learning.

Video: Concluding that $4 - 4 = 4$ with manipulatives; Tanya; age 6
Goal: To show that manipulatives do not always make mathematics concepts easier to understand.
Overview:
A child is asked to figure out whether the problem $4 - 4 = 1$ is correct. On a piece of paper she puts two sets of four on either side of a minus sign, and then she takes away one set of four. She concludes that $4 - 4 = 4$.

Discussion Question:
Why do you think Tanya sets up the problem the way she does? Does it make sense? Are the manipulatives helping Tanya here?

Comments:
In this situation the manipulatives are getting in the way of the child solving the problem. It is common for teachers to just assume that if children use manipulatives they will understand the mathematics, but this is not the case: it is important that children know how to use them correctly.

Related Video:
Cheating with manipulatives, Eddie, age 7 (cross-listed with number facts module #9, called “Knowing facts versus checking them”)

Video: Nines rule with Stern Blocks; 3rd grade
Goal: To show how manipulatives can be helpful in conveying ideas
Overview:
A class explores multiplication by nines using Stern blocks. They figure out the rule that \( n \times 9 = (n \times 10) - n \)

Discussion Question:
Why is the teacher having a child write up the equations on the board?
What does the child mean when he says “It keeps going more further away from the next ten”?
Why does the teacher add the green blocks to the number line?

Comments:
Stern blocks are based around rods, 1 – 10, with different colors for each number. At this point the children know that the black rod is nine cubes in length.
The teacher places the black rod—a “nine stick”—on a number line, and it ends at the nine. When she places the second nine stick in line, it ends at the 18. The 3\textsuperscript{rd} stick ends at the 27, and so on.
The video starts out with both symbols and manipulatives. By having the equations written on the board, the teacher is helping the children connect the representations to the formal equations. She is not just letting the children play with the blocks—she is directly connecting the blocks to the ideas.

Related Videos:
Reviewing base ten block homework: 1\textsuperscript{st} grade
Appendix D: Lecture Slides and Transcripts for Two Class Sessions

This appendix includes lecture slides and transcripts for the first two classroom sessions of the Development of Mathematical Thinking course. We are in the process of developing slides and transcripts for each of the 14 class sessions.

Development of Mathematical Thinking

Session 1

Slide 1
Transcript: This lecture presents the rationale behind and issues surrounding early childhood mathematics education. It spends some time addressing teachers’ hesitancy and in many cases anxiety about teaching mathematics, and encourages class discussion about student experiences in learning mathematics. The lecture then explores what factors need consideration when teaching children mathematics, early including children’s capabilities, mathematical content, pedagogy, assessment, and mathematics that children might encounter outside school.

The videos below illustrate some of the issues discussed in the lecture. Included are clips of children engaging spontaneously in mathematical thinking during play, learning about mathematics in the classroom, and talking to adults about their mathematics learning. This last type of clip introduces students to the method of clinical (or flexible) interviewing as a way of finding out about children’s mathematical reasoning.
Welcome to Development of Mathematical Thinking. Today, we are going to talk about some background issues in early childhood math education and then procedural issues involving the course.

There are two major problems we are faced with in this country and maybe in many other countries around the world.

• One problem is that in the United States and other places, academic performance in math is very weak. Children are not doing very well in school mathematics. In the U.S., the math performance of students is lower than students’ math performance in countries like Korea, Japan, and Singapore. One interesting point is that if you break down the data by social class, you’ll find that American middle-class children are doing about as well as some other children in those Asian countries. What is actually happening is that scores of children from lower income classes are pulling down the average for the country. So middle-income children are doing pretty well but low-income children are not doing so well. The issue is not just that low-income children are doing poorly. Their teachers are doing poorly and their schools are doing poorly.

• Hence our second major issue is that quality of education offered to lower-income children is not as good as the quality of education offered to middle-income children. Empirical data show that there is simply not much mathematical teaching going on in early childhood classrooms. If
there is something else teachers can do instead of teaching math, they’ll do it. Furthermore, even if they teach math, empirical data shows that they are not teaching it very well.

The situation is unacceptable

• For the children: The situation gives rise to grave educational and moral concerns (which are not new):

• “Ah, before such dense and willful disregard of the life which is growing within these children, we should hide our heads in shame and cover our guilty faces with our hands!”
  Maria Montessori

Slide 4
Transcript: This situation going on in general is not acceptable. Those are not only educational but moral concerns. This is a quote from Maria Montessori from around 1920s. Here, we are dealing with a very serious situation, children not learning and being bored. This is actually a kind of educational neglect.

Slide 5
Transcript: Here is another quote from Alfred North Whitehead, who was a great philosopher and who with Bertrand Russell wrote one of the key books on the foundations of mathematics (Principia Mathematica)

“When one considers... the importance of this question of the education of a nation’s young, the broken lives, the defeated hopes, the national failures, which result from the frivolous inertia with which it is treated, it is difficult to restrain within oneself a savage rage” Alfred North Whitehead

• And.....
Slide 6

Transcript: Teachers, we said, are not doing very well. They have been placed in an untenable position.

• There are few relevant college level courses for the subject most teachers find hardest to teach! So why are do colleges offer only a few or even no courses relevant to teaching math? This is the topic that teachers find most difficult to teach.

• And also, once teachers get into schools, they receive very poor in-service support. Typically, there is once-a-month relatively useless meeting. There is very little support in showing teachers, for example, how to do a math curriculum or program. So it is not only the children who are being neglected, it is also the teachers. If teachers are not doing a very good job, it is partly because universities are not teaching them very well and because they do not get a good support once they are in schools. And to top it off, salaries are terrible!

Clearly, we need solutions for both children and teachers.

Slide 7

Many responses to the crisis

• Professional reports:
  • Joint position statement of NAEYC/NCTM
  • New NAS report (Cross)
• Agreement:
  • do more early math because it can promote school success, especially for low SES kids [Maeroff: 
    "...it is much easier not to be left behind if you don’t start behind"]
  • and improve teacher training
    .......more
Transcript: There are many responses to a crisis like this. There are official reports, one from National Association for the Education of Young Children/National Council of Teachers of Mathematics and a more recent one from the National Academy of Sciences. Those reports and other professional reports support the idea that doing more early math can promote school success, especially for low income children.

• Also, as Maeroff said, “…it is much easier not to be left behind if you don’t start behind.” This is a reference to the No Child Left Behind legislation, which aims to improve education beginning in the early grades. But if children start school already behind, it is hard for them to catch up. So we need to do early childhood education that prepares children for school. If not, you will have many children who have already been left behind when they enter kindergarten.

• Another major finding of the report is that we need to improve teacher training. This is a new emphasis because in the past in-service early childhood teachers received little training in math education and almost none in how to introduce a systematic curriculum, which as we shall see is a crucial goal. The teacher is at the center of everything. If you had one single reform to do that would be to help teachers do their jobs better because now there are several useful curricula for them to work with.

Slide 8:
Transcript: There is a lot of going on now. We have those reports by NAEYC/NCTM and NAS. There’s a complex political process that takes place. Reports coming out are general professional/policy recommendations. State legislators and people in state education departments realize that something needs to be done such as doing more early childhood math education.

• Currently we have information on 40 states doing some kind of early math education. Some states are revising standards to include more mathematics. However, sometimes, there are policy pronouncements without any particular meaning. For example, in New York State there is universal preschool without any funding for it.

• In Texas, there are statewide educational programs not run by individual districts. They are special statewide programs designed to do more early childhood math education for low income
children in particular. Texas is also doing a lot of work on professional development. They produced a series of video based workshops that are used all over the state schools.

• New York is not necessarily the most innovative state in terms of educational reform.

• We have a new trend of public preschool for children as young as 3 years old where children get the opportunity to do more early childhood math education in the public schools.

• And now Head Start is going to announce major innovations in its programs. Head Start was one of the sponsors of the report by NAS and they are ready to improve their math programs.

• We also have new federal financing. A lot of money will be poured into early childhood math education.

• Unfortunately, the institutions of higher education, colleges and universities, are lagging behind the most. I haven’t seen universities trying to do many new things in the way of math education.

Slide 9

Transcript: So the question is, where do we go from here?

• We want to move ahead, so that we can have better teaching with more support.

• But there is a lot of confusion and resistance, and some of it is emotional.

• So let me ask you: Tell me about your most salient experience in math. Get audience responses.

• Examples of student experiences:

• One student had the experience of doing well in some math classes, and poorly in others; eventually, she realized that it might be the quality of the teacher that determined her success in math.
• Other students experienced low-self esteem during math, trouble understanding math concepts, remembering a lot of memorizing drills, being unsure of the ultimate goal of studying math.

• Another student mentioned a positive experience doing mathematics drills and a powerful mathematics teacher.

• One student mentioned learning about negative numbers at home when she was in Kindergarten; her teacher did not seem to understand this and called her mother saying she should not be teaching her these advanced concepts!

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**Slide 10**

Transcript: Here is something a previous student wrote couple of years ago:

*Possible discussion questions:* Does that sound familiar to you? *Get audience responses.*

[Note: The aim of this discussion is to elicit student fears and negative feelings towards mathematics as a type of T-group experience at the outset to clear the air.]

A lot of people go into the field of early childhood because they don’t want to do too much with math. That’s sad but it’s the reality.
Slide 11
Transcript: This is from another student from Teachers College

Slide 12
Transcript:
• Many people don’t like math.
• It’s embedded in our culture. Fear of math is cultural. For instance, it is very common to hear American adults say, “I hate math, or “I am terrible at math.” Popular culture depicts mathematicians as insane, (i.e. John Nash in “A beautiful mind”) or nerdy (i.e. Steve Urkel in the TV show “Family Matters”). In East Asian countries, it is not unacceptable to like mathematics.
What can we do?

- We means professionals, policy makers, parents and citizens in general.
- We can:
  - get therapy to deal with our anxieties
  - steel ourselves to get over it
  - at least keep an open mind as we proceed

Slide 13
Transcript: What can we do?

- Professional means everybody because this is a cultural phenomenon at least in U.S. We are not supposed to like math and this is not true in all cultures.

We can:

- Get therapy, although not in this class.

- Steel ourselves to get over it

- Try to keep an open mind even if you don’t like math very much and I will try to convince you that math and math teaching can be fun.

Some perplexing questions

- So putting aside emotion, still...
  - ECE professionals are skeptical about teaching math
  - Policy makers may not understand what kind of policies can promote ECME
  - To find solutions, we need to get beyond our fears and vague ideology
  - The goals of the course are to help you deal with critical challenging questions

Slide 14:
Transcript: There are some real questions that we need to address.

- Early childhood professionals are often very skeptical about teaching math. These include teachers who are already teaching in schools and faculty in universities.
• Policy makers might agree that we need to do more early childhood math but it is not clear whether they necessarily understand what this ought to involve. So we need to get beyond our fears and vague ideology floating around early childhood education and begin to discuss basic issues in ways that can have concrete outcomes.

• The goals of the course are to help you deal with critical challenging questions.

<table>
<thead>
<tr>
<th>Are young children capable?</th>
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<tbody>
<tr>
<td>Can they learn an abstract subject like math?</td>
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<tr>
<td>Maybe they can count a bit, but what about mathematical ideas?</td>
</tr>
<tr>
<td>Didn’t Piaget say the little ones are “concrete”?</td>
</tr>
<tr>
<td>Didn’t someone say that learning math is not developmentally appropriate for young children?</td>
</tr>
<tr>
<td>Can somebody like this learn math?</td>
</tr>
</tbody>
</table>

**Slide 15**

**Transcript:** The first most fundamental question is whether young children capable of learning an abstract subject like math?

We have to talk about what an abstract subject means. Some would say yes if all you mean is two plus two equals four. If you ask most people what young children can do they might say count to 10 or 20. But what about mathematical ideas? Can children really deal with them?

• Did not Piaget say that little children cannot deal with abstract ideas? He did say that children’s thinking is different from adults’ but he also pointed out how very young children’s thinking is in some ways often very abstract.

• Some people who did not read Piaget would say that.

• And don’t a lot of people still believe that learning math is not developmentally appropriate for young children? It is a popular ideology in early childhood education.

Sue Bredekamp and Carol Copple, who were the authors of NAEYC statements about developmentally appropriate practice, just came with a new edition of their text in which they tried to clarify what they meant by “developmentally appropriate.” Their new position and the position of NAEYC is that it is developmentally appropriate to teach little children math and abstract ideas. The ideas about developmentally appropriate have changed quite a lot. So we have to rethink what we mean by all this.
Slide 16
Transcript: So can cute, little children learn math?

Slide 17
Transcript: Lesson: Adding bears: invisible to visible: Olivia, age 4

Goals:
To show that young children are capable of abstract thinking as well as concrete. To have students think about what specific knowledge and skills the child is exhibiting in completing an addition task.

Overview:
The interviewer tests the child’s ability to find the sum of two small collections, specifically to add a visible and invisible set. The interviewer puts three bears on the table one by one, and establishes with the child that there are three. He then asks how many bears there would be if two more were added; however he does not make available extra bears. The child successfully finds the total, using a strategy of counting while pointing to spots (imaginary bears?) on the table. When the interviewer gives her two more bears, she arranges them in a line with the others and checks her answer by counting all five bears, one by one. The video demonstrates that young children are capable of complex, even abstract thinking, and are not limited to the concrete.

Discussion Questions:
• What challenge is the interviewer presenting to the child? Be specific.
• What is Olivia doing to solve the problem?
• Is her thinking concrete? Is she dealing with any kind of abstract ideas?

Comments:
• The interviewer is challenging the child to add two sets of bears that cannot be physically combined and counted. In order to find the correct answer, she has to first be able to accurately count things (enumerate), but she also has to have some mental representation of the two extra bears in order to add them to the first three. This in turn means that she needs to possess an idea of what “2” is.

• To solve the problem, Olivia did not need concrete objects and was able to represent the imaginary missing bears by the action of pointing to spots on the table. So her solution was partly abstract (imaginary bears) and partly concrete (the physical action of pointing).

Possible student responses:
• She is visualizing; she counted in her head; she was sometimes dealing with concrete objects

• Her addition was abstract; she counted 3 and then 2 in her head; she pointed with her finger and moved it as she visualized; she estimates or “just knows” there are 3 bears; she could be quickly counting in her head; it’s hard to determine from the video, there are many possible hypotheses; she seems to have ideas about the story or script, for instance, what should happen next in the story

Also think about the interviewing technique. Am I testing this child? [Possible responses: More like a conversation; the interviewer is engaging the child in discussion; might be leading her in some ways by how the questions are phrased].

It’s not a strict test… it’s like a conversation, a back and forth interaction where what I say in part depends on what she says. We will read about Piaget’s method of clinical interviewing later on. This method is much more useful than observing children, which is a major technique taught in early childhood education. Observation is not enough.
What is the content of ECME?

- Suppose we think there should be some kind of ECME. But what?
  - More than "numeracy"
  - Addition? Logic? Solid geometry? Algebra?
  - What about symbols and formulas?
  - What about number facts?
  - So we need to think about what a curriculum should entail at this age level

Slide 18:
Transcript: What is the content of ECME?

- If we say that children can really learn math, then what is the content of the early childhood math that we would like them to learn?

- One of the themes that comes up is that we want to do more than numeracy. Math is not only about number. It is about shapes, patterns, and all kinds of things. We definitely do want to do more than numeracy and we definitely do not want to use that word.

- If we are going to do more than that, what do we do? Addition? Logic? Solid geometry? Algebra? What are the limits?

- We particularly need to think about symbols and formulas. This is a really big and important transition that little children need to make at school: going from informal to formal mathematics. We should think particularly about what should we be teaching little children in formal mathematics.

- I am going to argue that the answer is not that we should avoid teaching symbolism altogether, but that we should teach some of it very carefully so that children understand the meaning of the symbols in context.

- What about number facts? A lot of parents tend to think that knowing math means mastery of number facts, like “how much is two and two?” But, is this what is important in teaching early math? No, children need to engage in meaningful activities and grapple with interesting mathematical ideas. So we need to think about what the content of early childhood math should include at this age level.
Slide 19
Transcript: Another really big issue is: How should we teach?

• We want young children to do more than just play but we also don’t want to treat them as badly as we treat children in elementary school and have them do drill and kill. So what do we do? What is math teaching?

• Some people say that we need to take a constructivist approach? What does that mean?

• How about manipulatives? How do we use them? Are objects on a computer screen manipulatives? Are stones manipulative? We need to think about this topic seriously.

• Should we have planned lessons for little children? When we say that there should be intentional teaching of math, we mean that there should be planned lessons.

• Should we teach in small groups? Large groups?

• Another big issue in teaching math is the role of language. One thing we are going to see is how important is language in learning math. When you are doing math you are doing literacy.

So there are a lot of questions about what is teaching and we will explore them.
Goal:
The goal of showing this video is to have students start to think about what type of mathematical instruction, if any, is appropriate for young children, and to bring out any student antipathy towards the idea of deliberate teaching and structured curricula in preschool classrooms.

Overview:
In the video 3-to-5-year-old children are gathered around a 100’s chart and counting aloud. The teacher, audible through a microphone, is leading the counting. A child is pointing to each number on the number chart as the children say it (or try to). When he gets to each number ending in 9 (including 9 itself), the teacher has the children stop (they say “cut”). She asks them what comes after that number. They continue in this way until they reach 49.

It is important to note that the lesson takes place in December, and the children have been counting as a group with the teacher since the beginning of the school year. The routine of saying, “cut” after each 9, has also been practiced for the duration of the year.

Discussion Questions:
What mathematical knowledge and skills were being promoted in this activity?
What is the goal of having the child point to each number on the number chart?
Do you think this is an appropriate activity for young children?

Comments:
The video may strike some students as inappropriate for young children; they may be doubtful that children should be learning to count high numbers and to read written double-digit numerals. However through discussion the argument should come up that if done properly and in the right context, such activities may not only be “appropriate” but engaging and valuable for students. In particular, learning to count to 100 involves learning the base 10 structure of the numbers and is one of the first opportunities children have to explore an important pattern. This activity is intended to help children learn the counting words, including decade numbers and the pattern of appending the numbers 1-9 to the decades (twenty, thirty, and so on). By using the chart the
teacher is also encouraging practice matching numerals to counting words, including noticing that the “twenty” numbers start with the numeral 2, the “thirty” numbers with a “3”, etc.

Possible student responses to the question, Is this appropriate for 4 year olds?

• No, children aren’t all paying attention;

• There shouldn’t be a focus on written numbers unless they are connected to underlying meaning;

• The activity should be more student-centered: the children should recognize the patterns themselves, not because a teacher points this out.

• Possibly, it depends what else they are doing in the class,

• There may be some place for teaching written numbers;

• It is important to think about how to do this type of activity correctly

• Written numbers and symbols have no place in the preschool classroom

Note that many students will feel at this point that the lesson is inappropriate. The goal is to get all these negative feelings on the table so that they can be explored during the course.

How should we assess?

- Suppose standard testing is not so useful for young kids
- Still, teachers need to know what their students know and how they are learning—“assessment”
- How to get that information in the classroom?
  - Observation
  - Interview
  - Informal test

Slide 21
Transcript: How do we assess these children? This is another major issue.

• Suppose standard testing is not so useful for young children. Certainly we don’t want to use common written tests with young children but still teachers need to know what those children know. If you really want to be child centered teacher you need to understand the child’s mind. You cannot just assume that a four year old is in the concrete operational stage. So assessment is not a bad word, it means finding what children know.
So how do you do it?

• Observation: watch what children are doing. This is the most commonly accepted method in early childhood education.

• Interview: Talk with them in a flexible way. We will explore this quite a bit.

• Informal tests: I will argue some types of informal tests are not so bad. We do this all the time, we hold up a shape and we say “What do you call this, Johnny “and he’s says “It’s a triangle.” Well that’s a little test and there’s nothing so wrong with that.

Slide 22
Transcript: When you are in school you also would want to deal with parents. The issue is how to help parents so that they can help their children. Parents often use popular media like “educational” television

Lesson: Sesame Street: Dogs & Mud Pies

Goal:
To demonstrate that more and more, mathematics—and not simply counting—is being incorporated into children’s television shows (and other media too, like web-based activities and games), and that it is important for early child educators to be able to make informed judgments about the educational value of such resources.

Overview:
In this Sesame Street clip starring William Wegman’s dogs, Batty (the dog wearing the dress) is preparing an “old family recipe” of mud pie. The narrator says that they will need four pieces for Batty and the three dogs she is serving. Batty cuts the pie in half to make two pieces (“big pieces”) and then in half again to make four pieces. They then distribute the pieces topped with whipped cream, which the dogs enjoy.
Discussion Questions:
• What mathematical ideas are being presented here?
• Do you think they are presented in an engaging way?
• Do you think they are presented clearly?
• What might children understand and/or not understand?

Comments:
The clip touches on ideas about counting, addition, and partitioning. The video itself is entertaining and it is likely that children will be engaged. Further, it deals with a real world context with which almost all children are familiar—dividing food fairly among several people. Some of the narration is quick and might be missed by children. For example the addition of 1 to 3 to make 4 people and the division of the pie from 2 pieces to 4 pieces could have been carried out more slowly and clearly with careful counting. However it is not unlikely that many children would understand the basic ideas of the clip, and they may even take away some mathematical ideas.

So you see all the mathematical concepts in this video. We’ll also think about how to analyze videos like and how parents can use them. We want you to realize that there is an educational environment outside the schools: TV, computer software, web materials, etc.

Slide 23
Transcript: A general course goal:

• Think critically and carefully about the issues
• Make useful links between ideas and practice
• Learn to assess so that you can understand deeply and improve your teaching
• Develop your “intermediary inventive minds”

• Think critically and carefully about the issues: We want you to be able to look at a child critically and understand what is going on with his/her mathematical thinking.

• Make useful links between ideas and practice. As a university course, this course deals with ideas, but we want you to be able to make useful links between those ideas and your practice.
Therefore, we need to have ways of thinking about children and teaching that are theoretically sound but also practical for you.

• Learn to assess so that you can understand deeply and improve your teaching: You can’t teach well unless you understand children.

• Develop your “intermediary inventive minds”: William James has a wonderful phrase about what teachers need are “intermediary inventive minds,” which means that you should use the formal, cognitive psychology about children’s thinking to make sense of the children’s learning in your own classroom. You need to connect the theory in meaningful ways to your practice. The teacher is the inventive intermediary mind that takes the abstract ideas and figures out what they mean for the kids in the classroom. The theory needs to make sense to the teacher and needs to be useful.

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**Addressing the issues**

- We draw on
  - The new cognitive psychology (beyond Piaget) – a child centered cognitive approach
  - New work in early math education
  - Everyday experience in classrooms
  - And especially methods of observation and clinical interview

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**Slide 24**

In this course we draw on:

• New cognitive psychology. We are not dealing with the older work of Piaget, which is in many ways outdated although still very important in ways we will describe. The field has gone beyond that now and beyond Vygotsky as well. We now have a child centered cognitive approach which deals in detail with their mathematical thinking.

• New work in early math education.

• Everyday experience in classrooms

And especially methods of observation and clinical interview.
Elements of the course

- Champion TA
- Your photos
- Class web
- Syllabus: using uni, go to class web...

Slide 25

For mechanics of the course please refer to Faculty Guide. The following slides can be used in conjunction with the Guide and should be adapted to meet your class requirements.

Slide 26
Slide 27

Slide 28
Other aspects of the course

- Classroom examination of video is central-- as we did before
- VITAL -- On syllabus page, click on VITAL

Slide 29

Slide 30

Demo

- [VITAL demo]

Slide 31
Then assignments

- Weekly reflections
- Clippers 1 and 2
- Clinical interview lessons
- Clinical interview prospectus
- Clinical interview (40%)
- Final project prospectus
- Final project (50%)
- (10% for participation, reflections, clippers, lessons)

Slide 33

The end!

Slide 34

References
Balfanz, R. (1999). Why do we teach children so little mathematics? Some historical considerations. In J. V. Copley (Ed.), Mathematics in the early years (pp. 3-10). Reston, VA:
National Council of Teachers of Mathematics. *Read this in detail. It’s a really interesting history*

National Association for the Education of Young Children and National Council of Teachers of Mathematics. (2002). Position statement. Early childhood mathematics: Promoting good beginnings. *This report presents a good overview of the issues we face and the new approach to intentional teaching. Read it to get a general idea of the issues we face and the solutions we need.*

Session 2: Everyday math

Slide 1
Transcript:
This module addresses “everyday mathematics,” or mathematics that children encounter outside of formal schooling. The primary objective is for students to learn to recognize and appreciate children’s experiences with everyday mathematics so that they can become more aware of the types of mathematical knowledge and skills with which students enter school. The next objective is to help students think about ways to foster these types of everyday experiences, even in the classroom, and how to mathematize them, or make them explicit and formal.

Below are several videos of children grappling with mathematical ideas in the everyday world. They range in environment, structure, and artifacts with which children are playing. For each video, students should discuss the environment (where it takes place, with whom the children are interacting, the available materials), the specific mathematical behaviors exhibited (including language spoken), potential motives for these behaviors, the minimal cognitive processes involved, that which is unclear and need further investigation, and ways to encourage and mathematize the experiences.

In the last video, a teacher sits with children who are playing with colored cubes. Some additional discussion questions are suggested that encourage analysis of how the teacher encouraged and mathematized the thinking she observed.
Every week, we are going to take comments from previous week and try to elaborate on them and discuss some issues that they raise.

Course requirements posted on class web
Privacy of reflections
Broadcast e-mails
Any issues or questions about the course?
Reflections: first some negatives

- My kindergarten teacher taught me how to count by having me write out numbers on paper. I remember not enjoying this activity.
- I remember my first grade teacher only having me fill out numerous worksheets containing addition and subtraction problems. This was so boring.

The teacher

- As a younger I used to enjoy math; however when I reached 9th grade, all that changed. I was placed with a teacher who herself had no knowledge of math; she didn’t know how to explain clearly to us, and so that effected my performance, and in return effected my self esteem, which lead to my dislike.

Slide 5
Transcript: Many reflections are about teachers. A lot of people report having trouble with their teachers. There really are teachers who don’t understand math. We are going to see how this is possible even in pre-school because some of math that children will be learning is very deep although it may seem very simple. One of themes here will be that if you want to understand children, and if you want to teach well you have to understand math.
Slide 6
Transcript: Part of the problem that people have with math is that issues become internalized as part of self-concept. We don’t know whether this is true for young children but after a certain period in school one can see him/herself as a math person. This is a real identity issue. Some of this is tied up with culture. In some cultures it is desirable to be a math person. In U.S. it has been considered for a long time that a math person is a real nerd, unpopular especially if the student is a girl. It is not a good thing to be.

Slide 7
Transcript: There are some cases that there is shaming in the classroom and children don’t like feeling that they are failing. This is more than getting wrong answer. This is something that affects self-esteem.
But now for some complexity: 
Competition: yes

Math Jeopardy was a competition in our classroom. Mrs. O shouted, “Go!” My opponent and I ran to the chalkboard and sped through the Pythagorean theorem. I slammed the chalk down on the tray first then looked up to Mrs. O for permission to claim victory. “You’ve earned 100 points!” My team cheered as I waved my dusty hands up in air with pride and confidence. That great teacher made math the most fun, social and rewarding part of my day.

Slide 8
Transcript: On the other hand, some people really like competition.

Also: The joy of learning math

...as I sat there absorbed in my own work, I realized that not only did it take 10 white cubes to equal the length of one orange, but that if I thought of the white cube as one, then each rod had a number value. I did not share my discovery with anyone, but sat there figuring out the numerical value of each rod. I could hear the children playing outside, but I did not want to leave the table. More...
Slide 10
Transcript: That’s what math learning ought to be about! We all would love to see some kind of discovery going on, like the spontaneous creation of these essentially important algebraic rules. It doesn’t always happen though; in fact such learning is rare.

Slide 11
Transcript: So we have a student who likes competition, one who doesn’t, one who loves drills. Things are not simple. Children are complicated.
So one question is…

- If you are a constructivist espousing a child-centered point of view, what do you make of the fact that some kids enjoy drill and competition? How do you deal with it?

- If you think that it is really important that children do discovery learning and construct interesting ideas, what do you make of children who enjoy drill and/or competition? How do you deal with this?

- Children have different approaches to learning. One possibility is that we can let them do what they like—lots of drill and competition. Another is that we may not want them to have what they want and we may want to intervene to broaden their horizon. So a “constructivist” teacher may in a sense have to intervene in a non-constructivist way to help children become more constructivist.

- Another possibility is that the teacher may have to intervene to help the child who loves exploration to memorize some facts so as to achieve more accuracy.

- We will explore the meaning of this word “constructivist” more deeply at a later time.
• We want to get a perspective on everyday math. There is a quotation from Vygotsky. It is not only the myopic psychologist that has ignored it. It’s been myopic teachers, parents, legislators, and many others. Everybody has ignored it. Much of this course is about not ignoring it. We need a proper corrective frame.

• It is interesting that Vygotsky and Piaget were born in the same year and Vygotsky lived till he was only 38 [1896-1934]. Piaget lived twice as long. And the things Vygotsky accomplished in his short life can only make us imagine what he would have accomplished if he had lived as long as Piaget.

• In any event, we want to get a broad perspective about what we mean by everyday math. We also want to begin to practice observation and analytic skills. And we want to discuss the teacher’s role in all this.
We are going to examine some examples. In each case, there are some key questions to ask. We are going to be coming back to these all the time.

As you watch these examples think about as you watch children in preschool, on the bus, or on the subway.

• What’s the environment?
• What’s the math activity?

• What’s the motive? Why are they doing this? Possible origins?

• What are the minimum cognitive processes? What is the least they need to know to be able to do this?

• What more do you want to know?

• What could you do to further develop or mathematize?

In your reflections you can explore these questions. What is the big lesson that you learned? Maybe there is a different one from what we discuss.

Baby Hope and the toruses

- Environment
- Activities
- Motive
- Cognition
- What else to learn
- Adult role

Slide 16
Transcript: “Torus” is a mathematical word for ring.
Infant playing with rings; Hope; Age: 18 months

Goal:
To show how even infants have an everyday math.

Overview:
A baby and her mother play with a set of rings, including a hiding game. Of note is the language that Hope uses, the way that she searches for rings, and the different rings that she finds at the end of the clip.
Discussion questions:

What kinds of language does Hope use and what does it mean?
Does she have any mathematical ideas?
What does she know about shapes?
How does the mother interact with her?

Comments:
• Some of the language Hope uses indicates a basic understanding of more and less. We hear her say “ring” versus “rings” (understanding the difference between one and many), “more”, “again”, “two”, “all gone”, and potentially some other words (what she actually says can be debated.)

• Her reactions during the hiding game suggest that she has some mental representations of the rings, and to some extent, specific amounts of rings. For example when her mother hides the rings in one hand, the child knows to look in the 2nd hand when the 1st one was empty (however on another turn she looks in the 2nd hand even after she finds the rings in the 1st hand.) When the mother hides the rings behind her, Hope clearly knows that they are missing, and when the mother recovers just one of them, Hope knows that there are more that are still hidden.

• At the end of the clip Hope finds toys of different shape, size, and color from the initial rings and knows that they are still called “rings”. This indicates an awareness of and generalization of shapes.

• In addition to the child’s behaviors, it is also interesting to note how the mother interacts with the child and encourages and mathematizes the child’s experiences.

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Some lessons and questions

• Even little babies:
  • deal with mathematical issues (more/less/all-gone, shape (torus))
  • develop language around these ideas
  • employ mental representations
    • Generalize
  • The everyday environment offers a lot to work with
  • What should a parent do?

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Slide 17
Transcript: This is what I would have said if you were going to ask me for my conclusions:

Even little babies:
deal with mathematical issues (more/less/all-gone, shape (torus)
develop language around these ideas, like the number words 
employ mental representations – baby Hope had some ideas that some things were not in the 
hand and something was in back. 
Generalize to different shapes

• The everyday environment offers a lot to work with 
• Finally, the question is: What should a parent do? – Maybe you would want to have more of 
similar activities around the house but certainly not drill and practice.

Slide 18
Transcript: Block architects; Armando & Keithly, Kindergarten

Goal:
To show the richness of everyday math in play

Overview:
Two boys are building a structure together. In the course of their building they encounter
various mathematical ideas such as measurement, number, estimation, shapes and space.

Discussion questions:
• What is that “circle thing” that Armando mentions? [Although it is not the right word to
describe a cylinder, he knew what it is.]
• He says “What if it don’t reach?” What does this indicate about his thinking? [He considers a 
hypothetical situation]
• “We need two more?” What was that? [an estimate]
• What would you do if you were the teacher in the classroom?

Comments:
• We see various mathematical ideas touched on in this video. One major idea is geometry—
working with and communicating about different types of blocks. A salient example is when
Armando calls a cylinder a “circle thing,” illustrating his observation that a cylinder is similar to but not the same as a circle, as well as his struggle to accurately describe the object.

• Another major mathematical idea is estimation. The children need to figure out how far apart to set out the two sides of the structure in order to lay a long block across. Later they estimate the amount of blocks needed to complete the top and sides of the structure.

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Slide 19
Transcript: Some lessons and questions

• Kids do many different kinds of math—they were doing numbers, estimation, under/over
• These are low income kids and chances are, according to the statistics, they are not going to do very well in school.
• Someone might comment on how smart Armando and Keithley look. But we cannot know that without normative studies that look at a lot of children and determine whether this is typical or not. It is wonderful to study individuals but we also need group statistics in order to be able to compare the group and the individual.
• The environment is socially constructed (blocks) – we already talked about that
• Observation can’t tell you everything – you want to ask them questions at key times.
• Lots of teaching possibilities emerge as we saw.
Goal:
To show how ideas about pattern permeate everyday play, and also how children play explicitly with symbolic mathematics

Overview:
This video contains two segments wherein kindergarteners are exploring mathematical ideas in everyday play. In the first segment, a child performs a clapping sequence involving repeating and growth patterns. Next, two girls play “teacher and student” and count by twos using a calendar. One points to every other square on the calendar while the other recites the corresponding numbers.

Discussion questions:
• What kind of pattern do you see in Emily’s clapping?
• How did she impose mathematical structure on it?
• Did the girls really do counting by twos or were they merely reading every other space on the calendar?
• Why were they engaged in this activity?

Comments:
• The children are exploring counting and patterns in their play. The clapping sequence performed by Emily contains a rhythmic pattern that gets repeated increasingly—it’s clapped once, then twice, then three times, etc. At one point she counts along with the rounds to keep track. Her peers appear to appreciate the clapping, calling out “Good music!”
• In the second segment, when the girls are playing “teacher and student”, they are counting by twos with the calendar. It is interesting that when they reach the end of the numbered squares, they keep counting using the empty squares, indicating an understanding of how the pattern
continues, both spatially (skipping every other square) and numerically (knowing which numbers are included in the counting-by-twos sequence.

• Why were the children engaged in this activity? We assume that the teacher did not tell them to do it. Perhaps the children were challenging themselves and enjoying the rhythmic pattern of counting by 2s.

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**Some lessons and questions**

- Kids
  - may like to explore formal, symbolic, school math on their own
  - may like to play teacher
  - show mastery
- A new environment: written number
- Need to test and interview?

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**Slide 21**

Transcript:

Here are some lessons:

- In everyday math children may like to explore formal, symbolic, school math on their own.
- They may like to play teacher.
- They show mastery. They like the practice aspect, the mastery.
- And in the calendar episode, we have a different environment— the written number environment.

If you wanted to find out more about those kids knowledge you would have to test them, interview them. By testing them I mean, “Can you go beyond 30?” Interviewing them means “How did you know 32 comes after 30?”
Small group activity: Working with colored cubes; Pre-K

Goal:
To show how difficult it is for a teacher to seize upon the teaching moment.

Overview:
In this video a teacher sits with several children working with colored cubes. This started as a sorting activity but was not highly structured: the teacher encouraged the children to do what they wished with the cubes, but the general focus seems to be about mathematics, such as sorting, magnitude, counting, shapes and patterns. We notice several moments in which the children encounter mathematical ideas or the teacher brings such ideas into play.

Discussion questions:
• What did the teacher do to set up this activity?
• Are all of the children doing the same thing?
• How does she react to individual children?

Comments:
• Note that the teacher has set up the activity by depositing a pile of cubes on the floor and establishing that it will be a sorting activity—that the children should collect different colors. She encourages the children to do whatever they like with the cubes, such as make a pile or a “long line.” She has also set out a board that has drawn on it a long rectangle made of squares the size of the cubes.
• The children were engaged in many different kinds of activities, from classification to pattern.
• Students should think about how the teacher encourages mathematical thinking, and what else a teacher might have done to encourage and challenge the children to engage in deeper mathematical thinking.
• Imagine how hard it would be to do this everyday with 20 children and to have such activities form the center of a mathematics program!
**Slide 23**

Transcript: What does a teacher have to know to exploit the teachable moment?

The teacher really needs to understand what the children are doing and thinking. It’s virtually impossible to do this with a group of children.

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**Some lessons and questions**

- What does a teacher have to know to exploit the teachable moment?
- How hard is it to do?

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**Some big lessons (and add your own)**

- **Environments:**
  - There are many: natural; social; school
  - Each offers possibilities for learning (or “math is all around us”)
- **Questions:**
  - What is universal, cultural, or individual?
  - SES/ethnic differences?
    - Moll: funds of knowledge
    - Gerdes: African indigenous mathematics

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**Slide 24**

Transcript: We can think of environments in several different ways.

- One is a that there is a natural environment like the home, most of which is not deliberately designed to teach math.
- But there are things in the environment that are socially constructed to help children learn math like block and TV programs.
- And then there is school environment and each of these environments is different.
Environments can afford math learning and some people say that “math is all around us.” This could mean that there are many possibilities for learning in these environments. But that does not necessarily mean that children will learn from them. We know for example that many children don’t learn much in school. Children often need help in learning from their environments; the learning does not necessarily occur naturally.

Several questions:
• What is universal, cultural, or individual? – You saw Hope with the rings. It is hard to imagine any culture where you would not have round things, stones of different shapes, etc. So what aspects of the environment are universal so everybody would learn? I think you can argue that all environments have more or less things. But then there are cultural differences: not all societies have Cuisenaire rods. What is individual? Did Hope happen to be a very smart child?
• SES/ethnic differences. – These are very important. We need to consider them carefully as we go along. Luis C. Moll (2005) talks about cultural differences. He worked with Chicano kids in New Mexico and Arizona. He went to their homes and saw that in very poor families there were very interesting mathematical activities going on unique to that culture. Paulus Gerdes studies people in Africa, Mozambique with minimal/no schooling. He found very interesting cultural geometric activities. So some things are clearly linked to culture.

The kids

- The are “...self-monitoring learning machines...” who do
  - many different kinds of math and maybe a lot of it
  - and use many different cognitive processes–
    - non-verbal, perceptual—Grab the bigger group
    - social, cultural—“Three here, only two there”
    - written symbolic—3 > 2
- They are ready to learn math; already doing it
- A key normative question for education: What about SES and ethnic differences?

Slide 25
Transcript: So what do we learn about the kids?

• Gelman says that they are “self-monitoring learning machines” who do a lot of different kinds of math and use a lot of different kinds of processes. Some of them are non-verbal and perceptual like “this is more.” Others are social and cultural, for example when children who say that “There are three here, only two there.” Some are written symbolic.
• They are ready to learn math. The issue is not whether the are ready to it because they are already doing it. Question is whether they are ready to learn school math, which is another issue.
• The key normative question is “What do low income kids really know when they get into school? What do kids from different ethnic backgrounds know?” We need to examine that.

**Slide 26**

Transcript: We saw that there are many different motives ranging from curiosity to drill and practice. Pressure, competition, cooperation, etc. Their learning is not necessarily “romantic,” that is, in the service of a disinterested curiosity. They do love to explore the world, but sometimes they learn for other reasons or even like drill. Often there are combinations of motives, and there are many individual differences among children as well.

**Slide 27**

Transcript: Helping children:

• You need to know, what they are up to, what is going on in their minds, and you need to fight your own egocentrism. Adults, including teachers, are very egocentric; they see things from their point of view. They need to learn to see things from the child’s point of view. So egocentrism is not only a problem of children, it is universal.
• The teachable moment is really hard!
• Most teachers don’t even try (Kontos, 1999)
• Some teachers effectively say “Math is all around us so I don’t need to do anything. Children will learn everything on their own.” This is not true.
• So the question is how to help. If you think that they are not going to learn everything on their own, then what do you do?

### Slide 28
Transcript: Observation is a useful method and may show you what it is possible for an individual to do. From observation you know that Hope can do certain things. Yet these observations don’t let us conclude what is typical for Hope or for other children. We can’t rule out other possibilities. We don’t know how some of those things started in the first place. Did she learn them from her mother?

### Slide 29

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### What we know and don’t know

- **The individual observation can help you**
  - learn what is possible for an individual
  - suggest plausible cognitive processes
  - inspire teachable moments or planned teaching

- **But it cannot**
  - reveal what is typical for this child or others
  - rule out other possible processes
  - determine origins of processes

---

### Therefore

- We also need test and interview
- Normative studies
- Focused study of particular aspects of mathematical knowledge
Transcript: So observation is not really enough. We need tests and observation and interviews. We need normative studies. We also need to focus on very specific knowledge that these children might have, for example regarding number or shape.

The dipping assignment

- In this video we see David (long-sleeved red shirt) and Eduardo (red shirt with short black sleeves) working with blocks during free play. In no more than 300 words, please answer the following:
  1. What mathematical thinking do you see? Identify at least 3 different instances of mathematical thinking, using clips to justify your claims.
  2. What else would you like to know about the boys' mathematical knowledge?

Slide 30
The next several slides relate to an assignment that of course you may use or choose not to.

The purpose of dipping

- Careful observation
- Use of evidence to develop explanations (hypotheses)
- Critical thinking about hypotheses
- This process is the essence of teaching the individual

Slide 31
Everyday mathematics


Appendix E: Reflections

At the end of each class session, students are asked to reflect on the week’s topic. Space is provided within VITAL for students to submit their reflections, which are typically no more than a short paragraph. Students may comment on the videos presented, the relationship among the readings, videos, and their own experiences, questions that arose for them regarding children’s thinking or their own instructional practice, and so on. The reflections should be submitted within 48 hours of the end of class, and at least 12 of the 14 possible reflections must be completed. After missing a class session, students are asked to reflect on the week’s readings.

Within VITAL, the course instructor can comment on each student’s reflection. Additionally, these reflections often serve as the opening conversation to the following week’s session, as they typically allow for a summary of essential points and bring out conflicts within the research and educational communities in regard to best practices. Students report that they find the reflections helpful for consolidating their thinking, understanding how others in the course are reacting to the material, and feeling more connected to the course content and flow.
Appendix F: Writing a VITAL Essay

Students are instructed to:

1. Watch the video clips posted for the topic. Note the most salient elements of the videos and save them as clips.
2. Read the assigned articles/chapters for the topic and find points of convergence, i.e., ideas or findings addressed by the authors that speak to the memorable images.
3. Write your VITAL essay, discussing your observations and relating them to the readings.

Specifically:
Describe what you see in detail. Think about what mathematical skills are being demonstrated and address the mathematical understandings (conceptual information) conveyed in that behavior. Avoid evaluation and try to answer: What is the child telling us about what and how he knows math? Pose any questions that come to mind regarding the context you are viewing.

Check yourself:
- Am I describing what the child is doing or am I making assumptions about what I see?
- Am I asking myself about the child’s experience or am I limiting my observation by adhering to my own definition of math skills and concepts? How might the child be expressing something mathematical in an alternative way?
- Am I avoiding making any grand claims that are not justifiable?
- Am I using a citation from the readings to support any claims of “fact” that are not universally known and agreed upon?
- Link your observations to readings for this particular topic. Links should flow naturally as a result of your description. Once you have described what you see, ask yourself: “What attracted me to this activity?” This will be a first clue to the message you are providing.
# Rubric for VITAL Essays

<table>
<thead>
<tr>
<th></th>
<th>HIGH</th>
<th>MIDDLE</th>
<th>LOW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observations focus on salient issues or themes.</td>
<td>Descriptions are somewhat disorganized.</td>
<td>Aspects of video described seem random.</td>
</tr>
<tr>
<td><strong>Links to Reading</strong></td>
<td>References are used to support points and themes that emanate from observations.</td>
<td>References are less clearly linked to points made regarding observed behaviors.</td>
<td>References chosen do not relate to the discussion of observations.</td>
</tr>
<tr>
<td></td>
<td>Citations include most salient aspects of researchers’ point of view.</td>
<td>Citations emphasize aspects of researchers’ work that are not most relevant.</td>
<td>Citations do not accurately represent the researcher’s work.</td>
</tr>
<tr>
<td></td>
<td>The connections between the references and the observations are clearly stated.</td>
<td>Connections between readings and observations are implied, but not stated.</td>
<td>There is no apparent connection between the references and the observations.</td>
</tr>
<tr>
<td></td>
<td>Direct quotes are used sparingly; when used, they are tied directly to issues discussed.</td>
<td>Quotes are included for ideas that would be more appropriately paraphrased, or they are not tied directly to issues discussed.</td>
<td>Quotes are not tied to discussion.</td>
</tr>
<tr>
<td><strong>Style</strong></td>
<td>No errors in grammar or spelling.</td>
<td>Few errors in grammar or spelling.</td>
<td>Many errors in grammar or spelling.</td>
</tr>
</tbody>
</table>
Appendix G: Clipper Assignment Guidelines

Early in the semester, students gain experience using the tools of VITAL that allow for close viewing and analysis of videos by completing two “clipper” assignments. In response to a prompt about what a student appears to know about a particular mathematical concept, students select (i.e., “clip”) short video segments (usually between 10-20 seconds) to support their arguments. These assignments are short, typically limited to 300 words. For example, the first Clipper assignment from the Fall, 2009 course asked students to select three different examples of mathematical behaviors from a video of a child engaged in block play and to explain why each is “mathematical.”

Clippers can also be used for other pedagogical purposes, such as when students in the course were asked to view a video of a clinical interview and to select two clips of good interview techniques and two clips of not-so-good interview techniques. This activity not only gave students experience with close viewing of video and careful selection of clips to support their claims, but also prepared them for their own clinical interviewing experiences by focusing them on the challenges of conducting such an interview themselves.
Appendix H: Clinical Interview Assignment Guidelines

The course midterm assignment asked students to conduct a clinical interview with a child, and then write up the results. Students had the choice of interviewing a child on one of these three topics:

- Counting and/or enumeration
- Early addition and/or subtraction
- The mathematical symbol "="

In preparation for this assignment, students were encouraged to review the readings from the appropriate week as well as view associated videos on a topic. Once a topic was selected, students were asked to complete one video lesson on that topic. Each video lesson presented a video of a child being interviewed with pauses in the tape prompting the viewer to consider what the child’s responses demonstrated, what follow-up questions might be appropriate, and so on. Doing the video lessons was assumed to help students review the mathematical content from the perspective of a child as well as to help them consider the types of questions to ask during their own interview.

Following this preparation, students submitted an interview protocol in the appropriate section of VITAL. To scaffold students in this endeavor, a basic protocol was provided for each topic that helped students get started with their own protocol. A couple of sample tasks were provided along with several sample follow-up questions that encourage the child to explain his or her thinking, understanding, and strategies. Students were then prompted to generate several of their own tasks and probes within the template in VITAL. Feedback was provided if problems were identified, and again this feedback was given with the VITAL environment.

Finally, the students conducted a clinical interview using the prepared protocol. Students were frequently reminded that the protocol was a roadmap for the interview, but that the key to a successful interview is flexibility in order to pursue the child’s thinking and demonstrated understandings. Students were asked to videotape their interviews, which were then posted within VITAL so that they could clip from the video in writing up their assignments.

Writing Up the Clinical Interview
After completing the clinical interview, students reviewed their video multiple times focusing on what the child appears to understand and how s/he arrived at his or her answers. They then answered the following prompts within VITAL that guided them through the process of reporting on their clinical interview:

- Introduction and goals of the interview: In 200 words or less introduce your topic, explain why it is important for children’s mathematical development, and describe what you wanted to accomplish in the interview.
- **Background literature**: In 400 words or less describe the relevant research in regard to children’s learning of the topic of your interview. This section should justify the questions you asked in your interview in terms of why they are relevant to the development of the mathematical idea and the common ways children engage with the topic.

- **Main findings**: In 750 words or less discuss at least 3 of the most important findings from your interview. Include a description of the evolution of your hypotheses in regard to the child’s understanding and strategy use. Use video clips and/or quotes to support your ideas. Be sure to provide any necessary contextual details for these clips, such as when they occurred in the course of the interview or how they related to other behaviors or responses at other points in the interview. Be careful not to simply describe what is happening. Instead, focus on making an argument, analyzing the child’s responses, and drawing conclusions about what the child knows or is thinking. Also be sure to explain the significance of these findings in relation to the development of mathematical thinking. Please also include basic details about your subject, such as his/her age, as well as relevant details about the interview setting.

- **Self-critique**: In 300 words or less describe what you did differently than you had planned and what you would do differently next time. For example, you might include questions or tasks you wish you had asked, things you wish you hadn’t done or said during your interview, or times when you would provide additional time or resources for the child. Please include at least one clip or quote in which you were dissatisfied with your question, language, or behavior and one clip or quote in which you were satisfied with your question, language, or behavior.

- **Conclusions**: In 200 words or less describe the significance of the interview in terms of what you learned about the math topic, the child, and yourself as an interviewer and teacher. Be sure to describe how these new understandings relate to your future instruction in the classroom.

- **References**: Please provide all references cited in the sections above.
Appendix I: Final Project Guidelines

The final project involves developing a new activity, teaching it in a classroom or one-on-one, observing the student(s) engaged in the activity, and conducting a pre- and post-activity clinical interview designed to investigate what the student(s) learned from the activity. Students videotape this entire process so that they can then clip and discuss the student’s (or students’) learning about an area of mathematics in an essay using VITAL. Students must obtain the relevant permissions from teachers and parents. Sample permission forms are provided in Appendix K.

To get started with the project, students followed these steps:

- Choose a topic from among the following:
  - Symbols
  - Pattern
  - Shape
  - Place value and written computation

- Refer back to the appropriate class sessions, readings, and videos in order to consider what a child might already know about the topic and what might be a next step in their learning of this area. To help with this process, fill in the prospectus template for the chosen topic in VITAL. This prospectus will serve as the basis for the pre- and post-activity clinical interview, as well as the actual lesson activity (see #3).

- Design an activity to teach something new within your topic of choosing to a student or students. This activity should relate directly to what you are assessing in your pre- and post-activity clinical interview. For example, if you evaluate what a student knows about a repeating pattern in your clinical interview, then your activity might involve increasingly difficult repeating patterns as well as a growing pattern. Submit the proposed activity in VITAL within your prospectus (see #2). Be as specific as possible in describing the planned activity, and include a description of any materials that will be using for the activity.

- Conduct the pre-interview, activity, and post-interview.

- As with the midterm assignment, review the videotape multiple times. Focus on what the child appears to understand and what the child still appears not to understand. Additionally, think carefully about how the child arrived at his or her answers, and what this might mean about his or her knowledge and understanding. Finally, make sure to analyze your teaching during the lesson activity, considering what worked, what you would do differently, and how it related to the child.

- Select a maximum of 20 minutes of video to submit for posting in VITAL.
Write up your project in VITAL, adhering strictly to the guidelines below. Also, keep the grading rubric that follows in mind as you complete your write-up.

- **Introduction**: In 300 words or less introduce your topic, explain why it is important for children’s mathematical development, and describe what you wanted to accomplish in the interviews and activity.

- **Background Literature**: In 400 words or less describe the relevant research in regard to children’s learning of the topic of your interview. Be sure to use the literature to justify the questions you asked in your interviews and tasks you introduced in your lesson activity in terms of why they are important to the development of the mathematical idea and the common ways children engage with the topic. You should also discuss your hypotheses for the child’s understanding during the lesson activity based upon the background literature.

- **Main findings**: In 1000 words or less discuss at least 3 of the most important findings from your interviews and lesson activity. Begin with basic details about your subject, such as his/her age, as well as relevant details about the interview setting. Then include a description of the evolution of your hypotheses in regard to the child’s understanding and strategy use. Use video clips and/or quotes to support your ideas. Be sure to provide any necessary contextual details for these clips, such as when they occurred in the course of the interview or how they related to other behaviors or responses. Be careful not to simply describe what is happening. Instead, focus on making an argument, analyzing the child’s responses, and drawing conclusions about what the child knows or is thinking. Also be sure to explain the significance of these findings in relation to the development of mathematical thinking.

- **Self-critique**: In 300 words or less describe what you did differently than you had planned and what you would do differently next time. For example, you might include questions or tasks you wish you had asked, things you wish you hadn’t done or said during your interviews and lesson activity, or times when you would provide additional time or resources for the child. Please include at least one clip or quote in which you were dissatisfied with your question, language, or behavior and one clip or quote in which you were satisfied with your question, language, or behavior.

- **Conclusions**: In 200 words or less describe the significance of the interview in terms of what you learned about the math topic, the child, and yourself as an interviewer and teacher. Be sure to describe how these new understandings relate to your future instruction in the classroom.

- **References**: Please provide all references cited in the sections above.
## Final Project Grading Rubric

<table>
<thead>
<tr>
<th>Project Tasks/Guidelines</th>
<th>Unacceptable/ Poor (C)</th>
<th>Needs Improvement/ Fair (B- or B)</th>
<th>Acceptable/ Good (B+ or A-)</th>
<th>Exceeds Expectations/ Excellent (A or A+)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction</strong></td>
<td>The main idea(s) explored in the project are not thought through, and there is no evidence of an understanding of learning theories or the developmental process of mathematical thinking.</td>
<td>The main idea(s) explored in the project are not well thought through, and demonstrate a superficial understanding of learning theories and the developmental process of mathematical thinking.</td>
<td>The main idea(s) explored in the project are well thought through and demonstrate a good understanding of learning theories and the developmental process of mathematical thinking.</td>
<td>The main idea(s) explored in the project are well thought through and demonstrate a deep understanding of learning theories and the developmental process of mathematical thinking.</td>
</tr>
<tr>
<td><strong>Background Literature</strong></td>
<td>The main idea(s) are not supported by relevant literature.</td>
<td>The use of literature does not seem to relate to or support the main ideas.</td>
<td>The main idea(s) are supported by references to relevant literature.</td>
<td>The main idea(s) are well grounded in relevant literature.</td>
</tr>
</tbody>
</table>

In 300 words or less, introduce your topic, explain why it is important for children’s mathematical development, and describe what you wanted to accomplish in the interviews and activity.

In 400 words or less, describe the relevant research in regard to children’s learning of the topic of your interview. Be sure to use the literature to justify the questions you asked in your interviews and tasks you introduced in your lesson activity in terms of why they are important to the development of the mathematical idea and the common ways children engage with the topic. You should also discuss your hypotheses for the child’s understanding during the lesson activity based on the background literature.
upon the background literature.

| Hypotheses regarding a child’s learning and understanding are not articulated. | Hypotheses regarding a child’s learning and understanding are not clearly articulated or connected to the literature. | Hypotheses regarding a child’s learning and understanding are addressed and partially related to the literature. | Hypotheses regarding a child’s learning and understanding are clear and consistent with the literature. |

### Main findings

In 1000 words or less, discuss at least 3 of the most important findings from your interviews and lesson activity. Begin with basic details about your subject, such as his/her age, as well as relevant details about the interview setting. Then include a description of the evolution of your hypotheses in regard to the child’s understanding and strategy use. Use video clips and/or quotes to support your ideas. Be sure to provide any necessary contextual details for these clips, such as when they occurred in the course of the interview or how they related to other behaviors or responses. Be careful not to simply describe what is happening. Instead, focus on making an argument, analyzing the child’s responses, and drawing conclusions about what the child knows or is thinking. Also be sure to explain

| No information is provided about the child’s background, age, etc. | The description includes sketchy information about the child’s background, age, etc. | The child’s prior and current understandings, and affect/behavior are not adequately discussed. | The child’s prior and current understandings, and affect/behavior are discussed in detail. |

| There is no discussion of the child’s prior and current understandings or affect and behavior. | Findings from the interviews and lesson activity reflect poor observation and interpretation skills. Video is not used to support findings. | Findings from the interviews and lesson activity reflect pure observations without any analysis. Video is sometimes used to support observations, and the | Findings from the interviews and lesson activity involve an interpretation of observations. Video is used to support interpretations, and the |

| Findings from the interviews and lesson activity involve a balanced analysis of observations (e.g., alternative interpretations for observations |
the significance of these findings in relation to the development of mathematical thinking.

| narrative does not provide enough information to supplement video support for conclusions. | narrative provides information that is not in the video. | are presented) that is consistent with theory. Video and narrative are used effectively to support interpretations. |

**Self-critique**

In 300 words or less, describe what you did differently than you had planned and what you would do differently next time. For example, you might include questions or tasks you wish you had asked, things you wish you hadn’t done or said during your interviews and lesson activity, or times when you would provide additional time or resources for the child. Please include at least one clip or quote in which you were dissatisfied with your question, language, or behavior and one clip or quote in which you were satisfied with your question, language, or behavior.

| No analysis of one’s interviewing and teaching is presented. | The analysis of one’s interviewing and teaching is superficial. | A good analysis of one’s interviewing and teaching is presented. |
| The discussion of problems and suggested variations/changes to teaching demonstrates poor critical thinking and reflection skills. | The discussion of problems and suggested variations/changes to teaching demonstrates developing critical thinking and reflection skills. | The discussion of problems and suggested variations/changes to teaching demonstrates good critical thinking and reflection skills. |

**Conclusions**

In 200 words or less, describe the significance of the interview in terms of what you learned about the math topic, the child, and yourself as an interviewer and teacher. Be sure to describe how these new

| No connection to initial hypotheses or the project goals is made. | The connection to initial hypotheses is very general. | The connection to initial hypotheses is made. |
| No connections to instruction are made. | Superficial connections to instruction are | Concrete connections to instruction are |
| Relevant and creative connections to | | |
understandings relate to your future instruction in the classroom.

References
Please provide all references cited in the sections above.

| No references or references are inapplicable. | Few references and they are only superficially applicable | Research is somewhat thorough and number of applicable references are listed | Research is thorough and applied correctly to the case study |
# Appendix J: Teaching Rubric

## Preparation

How prepared was I to teach today’s activity?

<table>
<thead>
<tr>
<th>General Description</th>
<th>LOW</th>
<th>MEDIUM</th>
<th>HIGH</th>
</tr>
</thead>
<tbody>
<tr>
<td>I did not put a lot of thought into this aspect of preparation. Any thought that was involved was minimal or cursory.</td>
<td>I did put some thought into this aspect of preparation. In some respects, my thoughts addressed major pedagogical issues. However, they did not involve an examination of their relationship with the deep mathematical ideas.</td>
<td>I did put quite a bit of thought into this aspect of preparation, including an examination of how the aspect connected with the mathematical ideas and their development.</td>
<td></td>
</tr>
</tbody>
</table>

| My understanding of the math goals... | I know what the math goals are, but I hadn’t really thought about how the activity addresses the goals. | I know what the math goals are, and generally, I know how the activity addresses them. However, I don’t really understand how the various activity components relate to the development of the mathematical ideas in the mind of a child. | I have really thought about the mathematical ideas involved in this activity, including how the ideas develop. I understand how the various activity components relate to the development of the mathematical ideas. |

| My fluency with the lesson ‘plan’... | I had read through the lesson plan once or twice, but had to frequently refer to the plan while teaching to know what to do next. | I was very familiar with the lesson plan, and didn’t really need it while teaching. However, my plan didn’t prepare me for some things that came up during the activity, such as problems with materials, or difficulties that some children had. | I was not only familiar with the lesson plan, but had anticipated possible difficulties or obstacles, and was prepared for them. I had tried out all of the materials and I knew what to expect and how to help children be successful with them. |

| My selection of materials/manipulatives... | I didn’t put a lot of thought into the materials I selected – | I did put some thought into the materials I selected – I initially | I put quite a lot of thought into the materials I selected. |
they were recommended by the curriculum or they were just what I had on hand.

thought of a few different materials that would have been possible, and selected among them. My selection process, though, was not very rigorous.

examined a number of potential materials, and thoughtfully compared the various features and qualities relative to the mathematical goals of the lesson, as well as the children’s physical abilities and general behaviors.

| My selection of group size… | I didn’t really think about the group size I selected. It was just random, or what was recommended by the curriculum. | I did put some thought into the group size I selected. I took into consideration children’s abilities, behavior management, and/or materials. However, any consideration of children’s abilities was only on the level of ‘high’ or ‘low’ achievement. | I put quite a bit of thought into the group size I selected. My consideration involved children’s abilities, pedagogical demands, behavior management, and/or material management. However, any consideration of children’s abilities went beyond ‘high’ or ‘low’ achievement levels. |

Motivation

Did I actively motivate my children’s participation in today’s activity?

<table>
<thead>
<tr>
<th></th>
<th>LOW</th>
<th>MEDIUM</th>
<th>HIGH</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Description</td>
<td>I didn’t really think too much about motivation for today’s activity. I may have thought about them in a cursory way, or simply figured that I would deal with them as needed during the activity.</td>
<td>I used some general strategies to motivate my children’s participation in today’s activity, but had not considered aspects of the activity having differential importance or needs regarding motivation.</td>
<td>I intentionally used planned and spontaneous strategies to motivate my children’s participation in today’s activity, and had specifically considered the differential importance or needs of the different aspects of the activity with regard to motivation.</td>
</tr>
<tr>
<td>I promoted attention…</td>
<td>I didn’t really think about how I would draw children’s</td>
<td>I used some general strategies to help children be attentive</td>
<td>I intentionally used planned and spontaneous strategies</td>
</tr>
<tr>
<td><strong>I promoted engagement</strong>…</td>
<td>I didn’t really think about how I would engage children during the activity. I just figured that I would modify or change my approach if I saw that children were not engaged.</td>
<td>I used some general strategies to help engage children during the activity. Primarily, though, I focused on having an introduction to the activity that was highly engaging and motivating. I didn’t think about my own engagement, though.</td>
<td>I intentionally used planned and spontaneous strategies to engage children during the activity. I thought about the various interests of different children and considered these relative to different aspects of the activity. I also considered ways that made it more engaging for me.</td>
</tr>
<tr>
<td><strong>I promoted interaction</strong>…</td>
<td>I didn’t really think about promoting interactions between the children and myself, or each other. I just figured the different tasks or questions in the activity would promote interaction.</td>
<td>I used some general strategies to promote interactions between the children and myself, and among the children. However, I did not think about how to specifically involve each child relative to his/her personality or needs.</td>
<td>I intentionally used planned and spontaneous strategies to promote interactions between the children and myself, and among the children. I specifically thought about the individual personalities and individual needs and considered how I could involve each of them.</td>
</tr>
<tr>
<td><strong>I promoted enjoyment</strong>…</td>
<td>I didn’t really think about helping children enjoy the activity. Although I hoped that children would enjoy the activity, I just figured that they would or they wouldn’t.</td>
<td>I used some general strategies to help children enjoy the activity, and explicitly intended for the activity to be enjoyable. However, I didn’t consider individual</td>
<td>I intentionally used planned and spontaneous strategies to help children enjoy both the mathematics and the activity. I considered individual personalities, interests,</td>
</tr>
<tr>
<td>I promoted values…</td>
<td>I didn’t really think about how I would promote valuing of others’ ideas. I simply planned to compliment children for the ideas that they shared.</td>
<td>I used some general strategies to promote valuing of others’ ideas, such as encouraging all children to share their ideas. However, I didn’t anticipate how I could use various responses to encourage and model identification of positive attributes.</td>
<td>I intentionally used planned and spontaneous strategies to promote valuing of others’ ideas. I tried to anticipate individual responses, and how I could highlight positive attributes of different responses.</td>
</tr>
</tbody>
</table>

**Instruction**

Did I provide quality instruction in today’s activity?

<table>
<thead>
<tr>
<th>General Description</th>
<th>LOW</th>
<th>MEDIUM</th>
<th>HIGH</th>
</tr>
</thead>
<tbody>
<tr>
<td>I didn’t really think too much about instruction, either in planning or in implementation. I simply did what I typically do.</td>
<td>I generally planned and implemented the instruction. However, in planning, I hadn’t specifically considered individual children’s differences and needs, but generally employed strategies and techniques I feel are effective.</td>
<td>I carefully planned and thoughtfully implemented the instruction. In planning, I had considered individual children’s differences and needs and worked to include my knowledge of each child into this aspect of instruction.</td>
<td></td>
</tr>
</tbody>
</table>

| I introduced tasks and activities… | I didn’t really think about how I was going to introduce the different tasks and activities used today, or the transitions between them. I just did it. | I generally thought about how I was going to introduce the different tasks and activities used today, but didn’t necessarily plan the introductions in detail or think about the transitions between tasks. For example, I may have decided to | I carefully planned the introduction to each task and activity used today, thinking about the specific knowledge and interests I was trying to activate. I thought considered various means of introducing, such as demonstration and |
demonstrate how to do something, but hadn’t actually rehearsed the demonstration.

modeling, and selected those I felt most appropriate. I also carefully considered the thought transitions involved and how to make them flow well.

In introducing the different tasks or activities today, I felt that I struggled to make myself clear to the children. I kept having to re-word or repeat, finding that I didn’t have the instructions clear in my own mind.

In introducing the different tasks or activities today, I felt pretty good about the clarity of my instructions. In general, children seemed to understand, and I felt that I had a pretty clear idea of how to provide the directions. However, in preparing the activity, I hadn’t really thought about alternate means of providing instructions, in anticipation of different difficulties.

In introducing the different tasks or activities today, I felt really good about the clarity of my instructions. In general, children seemed to understand, and I felt prepared to address different possible misunderstandings or difficulties, as I had planned for alternative means of providing the instructions in anticipation of various difficulties or particular children’s challenges.

I used manipulatives…

I used manipulatives, but didn’t really think too much in selecting the manipulatives. I just used something that I’d used before, that was listed in the curriculum, or was simply convenient to use.

I generally thought about the purpose of the manipulatives in today’s activity and tried to select an appropriate manipulative based on my goals. But I didn’t necessarily think about how the manipulative afforded the mathematical purposes, nor did I necessarily compare different manipulatives for the most appropriate one.

I carefully selected the manipulatives used in today’s activity. I thoughtfully considered the mathematical ideas being used in the activity and considered how various manipulatives afforded those ideas through their physical appearance, function, and mathematical representations. After such consideration, I selected the one(s) I felt were most appropriate.

I gave the children the manipulatives for the tasks, but I either

I gave the children the manipulatives for the tasks, and gave them

I gave the children the manipulatives for the tasks and carefully
provided them with rigid step-by-step rules for how to use them to get answers, or I provided them no guidance, feeling that pure discovery is the best way for children to use manipulatives. Neither of these ways, though, really provides children with the appropriate supports for learning with manipulatives.

<table>
<thead>
<tr>
<th>I allowed time…</th>
<th>I didn’t really think about the timing of the activity. I just moved through the various tasks of the activity in a timely manner.</th>
<th>I generally thought about how much time to allow for various tasks. I wanted to make sure that I got through everything so held the activity fairly closely to that time frame to make sure that I finished in the allotted time.</th>
<th>I thought carefully about the timing of the various tasks, but was flexible in the overall timing to allow for unforeseen elements, such as valuable tangents, interesting questions, and other ‘teachable moments’. I recognize that building on such moments can often be more valuable than following the plan.</th>
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<tr>
<td>I allowed children some time to work through the tasks, but there’s never enough time to allow them much time for this. I allowed them enough time to find the answers.</td>
<td>I allowed children some time to think and work through the tasks. Although there’s never enough time to allow much time for this, I tried to build in enough time to allow some exploration and extended thinking. I also monitored the children during their problem solving and exploration.</td>
<td>I allowed children quite a bit of time to think and work through the tasks. Although time is always short, I feel that the children’s time to problem solve and explore is not the best place to cut an activity short. I monitored the children during their problem solving and exploration.</td>
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<td><strong>I used language…</strong></td>
<td>I didn’t really think about the mathematical language included or afforded by this activity.</td>
<td>I generally thought about the mathematical language included in this activity, having reviewed a list of vocabulary words to make sure I understood what they were and why they were included in this activity.</td>
<td>I carefully thought about the mathematical language included in this activity. I reviewed the list of vocabulary words to make sure that I understood what they were and where they fit in with this activity, but I also developed a list of related vocabulary words, considering how I could connect this activity with mathematical ideas taught previously.</td>
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<tr>
<td><strong>I encouraged thinking…</strong></td>
<td>I didn’t explicitly think about the language I used in today’s activity.</td>
<td>I explicitly made sure that I was using the vocabulary included in this activity, and tried to define those words that I thought might cause difficulties for some children.</td>
<td>I explicitly made sure that I was using the vocabulary included in this activity, as well as related vocabulary to review and connect to mathematical ideas previously learned. I also defined the terms or asked children to try to define them, then asked questions to ensure that the children understood the words I was using.</td>
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<td>I didn’t really think about promoting critical thinking. I selected questions/tasks that I thought would be challenging and</td>
<td>I used some general strategies to promote critical thinking. Although I tried to select questions/tasks that would be challenging, I didn’t</td>
<td>I intentionally used planned and spontaneous strategies to promote critical thinking. I tried to select questions/tasks that would be</td>
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<td>I encouraged talking...</td>
<td>I observed behaviors...</td>
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<td>I didn’t really think about encouraging children to talk. I just presented the different tasks or questions and let the children respond.</td>
<td>I watched what the children were doing to ensure that they were staying on task and generally behaving. When I asked a question or gave a task, I would also look to see if children were generally getting the right answers.</td>
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<td>I generally thought about how to encourage children to talk. I know that children need to be able to express their ideas, so I presented tasks and questions that would give children the opportunity to talk about their ideas.</td>
<td>I generally tried to observe children’s behaviors to determine if they understood the activity. I would not only look for wrong or right answers, but also kept in mind that different answers or behaviors may also indicate different levels of understanding. However, I hadn’t really anticipated what answers or behaviors I may see or what they could mean in terms of understanding.</td>
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<td>I carefully planned how to encourage talk around the mathematical ideas included in this lesson. I presented tasks and questions that were open ended and flexible enough that multiple answers were often possible, in hopes that it would encourage open discussion and debate among the children, and promote deeper exploration of the mathematics.</td>
<td>I carefully tried to anticipate various behaviors or answers that I may see in today’s activity. I then tried to think about their different possible interpretations, in terms of what the child may or may not understand. By so doing, I was also able to consider various ways that I could assess the child’s specific understanding and offer support when needed.</td>
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<td><strong>I tried to find out if they could do it...</strong></td>
<td>I didn’t really think too much about assessment, other than to see if the children’s answers were right or wrong.</td>
<td>I generally was interested to find out if children could do the tasks or not. I observed what children were doing, tried to make assessments, and asked occasional direct questions to check their understanding.</td>
<td>I was very interested in finding out if the children could do the tasks, or if not, where their understanding was at. I observed what the children were doing, considered possible interpretations, then asked flexible questions to either present a new task or have the children explain their actions or solutions.</td>
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<tr>
<td><strong>I learned about their thinking...</strong></td>
<td>I didn’t really think too much about what I could learn about the children’s thinking. Although I watched to see whether children were getting right or wrong answers, I didn’t think about their processes or strategies for doing so.</td>
<td>I tried to determine what strategies children were using to solve problems. I watched them use the manipulatives, asked them to describe their strategies, and present their strategies to others.</td>
<td>Through observation and asking children to describe or present their strategies to others, I was generally able to determine what strategies they were using to solve problems. I had in mind a sort of hierarchy of strategies – an understanding of which strategies were more efficient and superior to which others – which provided me a better assessment of where each child was, where they needed to go, and the steps needed to get them there.</td>
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<td><strong>I adjusted my instruction...</strong></td>
<td>I pretty much just followed my lesson plan as written.</td>
<td>I adjusted my lesson plan if I felt that a lot of children didn’t understand part of the activity, but generally stuck to my lesson plan.</td>
<td>I was constantly making little adjustments to my lesson plan based on my assessment of my children. I made adjustments based on engagement, prior knowledge, children’s understanding.</td>
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<td>I repeated question/task types for practice…</td>
<td>I really didn’t repeat things during the activity, or offer any unplanned practice. Although I may have included some planned practice, it was based upon a general plan, not based upon an assessment of the children’s specific needs in my classroom.</td>
<td>I did offer some additional tasks or questions when I felt that some children needed more practice, and may have planned some specific practice problems for the group. But in general, the practice and repetition offered was spontaneous or didn’t necessarily address specific challenges that the children had.</td>
<td>I had carefully planned different practice problems or tasks to address various challenges I anticipated some children would have. Although I may have also planned some specific practice problems for the group, the practice problems offered during the activity were planned to address specific needs and were not necessarily given to the whole group.</td>
</tr>
</tbody>
</table>
Appendix K: Sample Video Permission Form

Date

Dear Parent or Guardian,

I am a student at Teachers College, Columbia University, and I am currently enrolled in a developmental psychology course about the mathematical thinking of young children. As part of the course, we view videotapes of children engaging in math-related activities. These videotapes are extremely valuable instructional tools because they allow us to observe how children think and learn about mathematics.

I am writing to ask for your permission to videotape your child as he or she participates in mathematics activities in his or her classroom [OR as I interview him or her about topics in mathematics.] You do not need to be present for the videotaping; the activities [OR interview] will take place during school hours. The activities [OR interview] should take approximately [X] minutes to complete.

The video of your child and his or her classroom will be used only for teaching and research purposes. The video will be stored in a password-protected Web environment accessibly only by members of my course. Your child’s last name and other personal information will not be shared with anyone, and I will not include any identifying information other than the child’s first name and grade level when showing these videos to others. If at any point your child decides that he or she does not want to be videotaped, I will discontinue the activity [OR interview]. If you have any questions, please do not hesitate to contact me at [phone and/or email].

To give your consent, please sign below and return the form to your child’s teacher. Thank you very much for your cooperation in this research.

Sincerely,

[Name]

A parent or legal guardian must sign below:

I am the [parent]/[legal guardian] of __________________________. I have read the above and understand and agree to its terms on [his]/[her] behalf.

Signature: __________________________ Date: __________________________

Print Name: __________________________

School: __________________________ Teacher: __________________________
Appendix L: Video Specifications for VITAL

The following is an overview of the process for making a video to be uploaded to VITAL. There are many methods for making a video; this document just summarizes the most typical approach.

**The camera**: Digital camcorders are best for making digital video. However, digital cameras often have a big enough memory card to shoot a 30-45 minute video. We highly recommend using a tripod for the recording whenever possible.

**Shooting the video**: Try out the system before you begin the interview. Make sure that the sound recording operates effectively. Try not to shoot into the light (for example a sun lit window behind the child). If possible, place a microphone close to the child. Sit either side by side with the child, or at a right angle to the child, rather than directly opposite, which could suggest an unwarranted degree of separation and authority.

**Selecting a clip**: We will upload only continuous footage, so please do not submit your video with a request for editing of any kind, except for selecting start and end points (if you don't want your entire clip uploaded). You should select the segment that seems most important to you or, if necessary, use software to edit your tape (see below). Remember that you will be able to clip segments of your video in VITAL, so it is not necessary to create exact clips on your submitted video.

**Editing your clip**: If you want to edit your tape, use an application like iMovie (Mac) or Windows Movie Maker (PC). When you have finished editing, you can export your clip to a file. Be sure not to compress too much; if you don't like the way your clip looks, try exporting it again at a higher setting. If you're using iMovie, export using the "Broadband - Medium" setting to create a nicely formatted QuickTime movie file (MOV). Windows Movie Maker will render an AVI or WMV file.

**Submitting your clip**: In addition to mini-DV tape, we can accommodate VHS tape, DVD, and a variety of file formats (MOV, AVI, WMV, etc.). If you edit your clip, you can record it to tape, or, as mentioned above, you can export your edited clip to a file. If you choose the latter option, you can burn a data CD or DVD with your file, or use a free file transfer service like [http://www.yousendit.com](http://www.yousendit.com) or [http://www.dropbox.com](http://www.dropbox.com) to submit it via the Web.

* Be sure to include your name and your course name on whatever you submit, as well as contact information in case we have any questions.

* Uploading a video to VITAL takes anywhere from one to five business days depending on volume, so please be patient.