Tax Cuts, Employment and Asset Prices: 
A Real Intertemporal Model

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Abstract

We determine the effects of a delayed or immediate tax cut with or without a “sunset” feature in a real customer-market, non-Ricardian economy. Our model incorporates both the supply-sider channel as well as the Feldstein-Rubin-Summers channel. We show that a tax cut may depress both the real asset price and employment. The presence of the paradoxical result of employment contraction does not work through nor imply an immediate increase of the long real interest rate, contrary to financial commentators. A similar analysis applies to the huge pension problem burdening several continental European economies so part of their high unemployment of late may be ascribed to their looming pension outlays. (JEL: E24, E43, E62, F41)

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An anti-cyclical policy of regulating tax rates to moderate swings in economic activity, once advanced by Keynesians and supply-siders alike (Abba Lerner, 1946; Robert Mundell, 1971), faces objections going beyond fine tuning. A familiar drawback is that a tax-rate cut, in widening the budgetary deficit (or reducing the surplus) jeopardizes other goals, such as national wealth accumulation and thus economic growth. A decisive objection, if sustained, is that, by raising long-term interest rates and real exchange rates, thus curtailing domestic investment and driving customers abroad, a tax cut operates perversely to contract employment—at least early on, precisely when the aim is an employment boost. The same objections apply to the claims made in the present debate over secular fiscal policy. Those who would dispose of looming budgetary surpluses by the occasional tax cut lest public spending go on rising in relative terms claim it will have the beneficial side effect of boosting employment. Those who would instead hold tax rates steady and wipe out surpluses with the occasional new public program appear to believe that their approach would also be a boon to employment—at least not a drag. But these objections have not been developed very far.

The first argument against the supposition that a cut in tax rates is expansionary for employment, at least initially, is that by Olivier Blanchard (1981). He takes up the effect of an “anticipated” fiscal expansion—one that is announced at $t_0$ and to be implemented at subsequent $t_1$. Such a premise does not accurately describe tax cuts legislated promptly in response to an emergency but it does seem to fit very well the more structural kind of tax cut enacted in the U.S. in the summer of 2001. (This bill, the Economic Growth and Tax Relief Reconciliation Act (EGTRRA), schedules the largest rate reductions in the years from about 2005 to 2010, with a “sunset” for these cuts scheduled for 2011.) Constructing a dynamized version of the Keynesian closed-economy model, Blanchard shows that in the “bad news” case—and only in that case—the sudden expectation of the future stimulus drives up
the long rate of interest and causes asset prices to drop correspondingly, with
the result that investment expenditure and output contract in the first phase
of the scenario. However, it would be anomalous to use a model lacking a
natural, or structural, path of unemployment to analyze the effects of a tax
cut reaching full strength some 9 years in the future, like the recent Bush
tax cut, or even 3 years, such as the first Reagan tax cut. We would not
know whether, in a model with a natural rate path, an initial fall of output
might be accompanied by a rise of the natural output path, which would
put a different complexion on the first result. Therefore, when the interval of
study may run for a decade, a model of the effects on the natural rate path
is required. Furthermore, a non-monetary perspective on the matter seems
particularly appropriate for evaluating the claims for back-loaded tax cuts
made by adherents to supply-side analysis, since a natural rate model will
incorporate some or all of the supply-sider effects.

Our analysis incorporates both the supply-sider channel, through which
reduced wage income taxes stimulate work effort, as well as a channel revived
by Rubin and Summers in the 90’s and earlier championed by Feldstein in
the early 80’s when he was the chairman of the Reagan Council of Economic
Advisors.¹ The Feldstein-Rubin-Summers contention was that cutting rates
of income tax, in widening the deficit and thus driving up future short rates
of (real) interest, has a chilling effect on present and future investment, so
reducing growth and employment on that account. They suggested that
this countervailing effect was strong enough to make such tax cutting utterly
perverse—the lower tax rates notwithstanding. We examine this financial hy-
pothesis of a perverse effect from tax cuts in a framework in the structuralist
spirit of Edmund Phelps (1994)—hence non-monetary and inter-generational
though, for simplicity, we exclude unemployment arising from incentive wages
and use instead a variable supply of “hours” that is modeled much like the

¹For an account of Feldstein’s views, see Feldstein (1993).
demand for consumption.\textsuperscript{2}

Our model describes a closed economy. It is not the Ricardian type of economy favored by RBC theorists and some public finance theorists. Instead, we follow the treatment by Blanchard (1985) in which worker-savers toil throughout life, save by buying annuities invested in the shares of the firms, and die off exponentially. In order to provide a business asset to back the shares of firms, and in order to give a role to the variation of price-marginal cost markups in explaining the big changes in the wedge between the value marginal product of labor and the marginal rate of substitution in consumption and leisure (the "marginal value of time" measured in consumption units), we use the customer-market model set up by Phelps and Sidney Winter (1970) and placed in a general-equilibrium setting by Guillermo Calvo and Phelps (1983).\textsuperscript{3} Owing to frictions in the transmission of price information, the competition of firms for market share will fail to wipe out all pure profit, and so leave the optimal price charged by firms hanging above the average and marginal cost, provided that the current short-term real interest rate is positive. Firms set mark-ups below the monopolist’s level but

\textsuperscript{2}See Robert Hall (1997) for an empirical effort to explain movements of employment in terms of such a model of labor supply. Our paper makes two main departures from the Hall framework: We introduce a dynamic theory of variable markups, and we also introduce a public finance distortion as in Casey Mulligan (2002). We return to a detailed discussion of the behavior of the markup and the public finance distortion in understanding key historical episodes in the U.S. in the paper’s concluding section. We argue that a theory of dynamic markup variation is needed, on top of changes in marginal tax rates, to quantitatively account for labor-leisure distortions in the U.S. at medium-term frequencies.

\textsuperscript{3}Rotemberg and Woodford (1992) argued convincingly that a model featuring imperfect competition in the product market is required in order to explain how aggregate demand changes, such as increases in government purchases, can increase output while at the same time raise the real wage. Our paper goes further to argue that fully accounting for variations in the wedge between the value marginal product of labor and the marginal rate of substitution in consumption and leisure requires tracking not only changes in the tax rate but also changes in the price-marginal cost markup.
above the pure competitor’s level—how high depending upon the value per unit placed on the average and marginal customer. The output supply and thus also employment is an increasing function of this per-unit asset value normalized by dividing it by the exponentially rising productivity parameter and a decreasing function of the tax rate. Aggregate investment and national saving here are always zero but we study employment and the other variables.

This paper will study cuts in the tax rate on wage income that are immediate or delayed and possess or lack a sunset feature. Of course, this focus is inspired by the Bush tax cut, which would see income tax rates in 2011 go back to their 2000 levels. We find conditions under which fiscal policy is sustainable—so the debt-income ratio neither explodes nor implodes—despite a return of income tax rates, from sunset onwards, to their original rather than higher levels or despite a permanent tax cut. (Despite the pile-up of public debt due to widening budget deficits as a result of income tax cutting, we show that depressed asset prices drive down the short-term natural rate of interest thus alleviating the interest burden of debt.) We show in that case that a delayed tax cut may depress both the real asset price and employment in the period running up to the implementation of the tax cut; so this paradox is not restricted to Keynesian or other monetary models, such as Blanchard’s. Furthermore, we show that the same ambiguity may result even if the tax cut takes effect immediately. The presence of the paradoxical result of employment contraction does not work through nor imply an immediate increase of the long real interest rate, contrary to financial commentators. Finally, we show in the sunset case that the post-sunset period is characterized unambiguously by depressed asset prices and decreased employment before they recover to their original levels.

Our work here relates to some papers other than those already cited. Mankiw and Summers (1986) argue on theoretical and empirical grounds
that a tax cut, in increasing households’ disposable income, could raise the
demand for money and, given an unchanged money supply, contract output in
a Keynesian model. As with our comment on the Blanchard (1981) analysis,
a full understanding of the effects of back-loaded tax cuts that kick in in
full force only towards the latter half of the decade requires a treatment of
the natural path of output and employment. Perotti (1999) examines both
theoretically and empirically how drastic cuts in government deficits—fiscal
consolidation—in countries with exceptionally high levels of the debt-GDP
ratio could lead to consumption booms. His paper differs from ours in
neglecting employment responses by assuming perfectly inelastic supplies of
labor, and relies on a competitive framework. Phelps (1992) develops a
closed-economy customer market model and examines a public debt shock
but it does not incorporate the distortionary effect of the wage income tax
that plays a crucial role in this paper and it neither analyzes the effects of a
back-loaded tax cut nor studies the endogenous evolution of the debt-income
ratio in a fully specified general-equilibrium system.

The paper is organized as follows. In Section 1, we study the optimization
problems solved by individual agents and set up the basic model. In Section
2, we study the response of asset prices, real interest rates and employment to
an announced future helicopter drop of public debt. We contrast these results
to those obtained under an equivalently-sized “entitlement bomb” scenario.
Section 3 treats the debt-income ratio as an endogenous variable and uses
the full general-equilibrium system to study the economy’s response to shocks
to the wage-income tax rate. In the concluding section, we discuss the role
played by the markup and the public finance distortion in understanding
key historical episodes in the U.S. We also point out that a similar analysis

\footnote{Perotti (1999) also provides a review of the related papers on “expansionary fiscal consolidations” but these papers do not rely on the supply-sider channels and asset price channels emphasized in our paper.}
applies to the huge pension problem burdening several continental European economies. So part of their high unemployment of late may be ascribed to their looming pension outlays.

1. The Model

Let us consider a closed economy where investment takes place in “customers.” There is one homogeneous good and four marketable assets—shares, which are titles to the stream of profits, private short- and long-term bonds issued and held by individuals, and government bonds. These non-monetary assets are assumed to be perfect substitutes so arbitrage among them implies that they have the same expected short-term rate of return. It will be innocuous but convenient to suppose that all (nonhuman) wealth, in equilibrium, is held in the form of shares and public debt. The economy is initially in a steady-growth state, growing at the rate $\lambda$, with a nonnegative public debt-to-productivity ratio (called normalized debt). A proportional wage income tax is imposed to finance government expenditure.

Agents derive utility from consumption and leisure, have finite lives and face an instantaneous probability of death $\theta$ that is constant throughout life. Let $c(s, t)$ denote consumption at time $t$ of an agent born at time $s$, $l(s, t)$ labor supply, $w(s, t)$ nonhuman wealth, and $h(s, t)$ human wealth. Also let $y^a(t)$ be government entitlement per agent and $v^h(t)$ be the real hourly household wage (in terms of output, our numeraire good), which is related to the hourly labor cost to the firm, $v^f$, by $v^f \equiv (1 + \tau)v^h$, $\tau$ being the proportional wage income tax rate. We let $r(t)$ denote the real instantaneous short-term interest rate, $\rho$ the pure rate of time preference, and $\bar{L}$ the total time available per worker.

The agent maximizes

$$\int_t^\infty [\log c(s, \mu) + B \log(\bar{L} - l(s, \mu))] \exp^{-(\theta + \rho)(\mu - t)} d\mu, \quad B > 0$$
subject to
\[
\frac{dw(s, t)}{dt} = [r(t) + \theta]w(s, t) + v^h(t)l(s, t) + y^s(t) - c(s, t)
\]
and a transversality condition that prevents agents from going indefinitely into debt. The solution to the agent’s problem is given by
\[
c(s, t) = (\theta + \rho)[h(s, t) + w(s, t)],
\]
\[
\frac{\bar{L} - l(s, t)}{c(s, t)} = \frac{B}{v^h(t)},
\]
where human wealth is given by
\[
h(s, t) = \int_t^\infty [l(s, \mu)v^h(\mu) + y^s(\mu)] \exp^{-\int_\mu^t [r(k) + \theta]dk} \; d\mu.
\]
Normalizing the population (equal to labor force) to unity, dropping the time index \(t\) and denoting aggregate variables by capital letters, we obtain
\[
C = (\theta + \rho)[H + W],
\]
\[
\frac{\bar{L} - L}{C} = \frac{B}{v^h},
\]
\[
\dot{H} = (r + \theta)H - (Lv^h + y^s),
\]
\[
\dot{W} = rW + Lv^h + y^s - C,
\]
where a dot over a variable denotes its time derivative. We note from (3) and (4) that whereas the rate of interest used to discount after-tax wage income and entitlement is \((r + \theta)\), aggregate nonhuman wealth accumulates at rate \(r\). It is this difference in discount rates that results in the non-neutrality of debt and deficits.

The government’s budget constraint can, in general, be expressed as
\[
\dot{D} = rD + G + y^s - \tau Lv^h,
\]
where \(D\) is the level of government debt, \(G\) is the amount of government purchases, \(y^s\) is government entitlement, and tax revenue collected is entirely
from wage income taxation. For simplicity, we will throughout set $G = 0$. Assuming that, in equilibrium, agents have zero holdings of private bonds, $W ≡ V + D$, where $V$ is the total value of shares held by individuals. Taking the time derivative of (1), and using (3) and (4), we obtain

$$\dot{C} = (\theta + \rho)[rW + (r + \theta)H - C].$$

Using (1) in (6), we obtain, after re-arrangement of terms,

$$\frac{\dot{C}}{C} = (r - \rho) - \frac{\theta(\theta + \rho)[V + D]}{C}.\quad (7)$$

Next, defining $\tilde{C} ≡ C/\Lambda$, $\tilde{D} ≡ D/\Lambda$, $\tilde{V} ≡ V/\Lambda$, $\tilde{y}^s ≡ y^s/\Lambda$, and $\tilde{v}^h ≡ v^h/\Lambda$, where $\Lambda$ is the measure of Harrod-neutral productivity level, we can transform (5) and (7) to obtain

$$\dot{\tilde{D}} = (r - \lambda)\tilde{D} + \tilde{y}^s - \tau L\tilde{v}^h, \quad (8)$$

$$\dot{\tilde{C}} = (r - \lambda - \rho)\tilde{C} - \theta(\theta + \rho)[\tilde{V} + \tilde{D}]. \quad (9)$$

We can now turn to the firms. With labor as the only factor of production, individual firms minimize costs taking wages as given. In the general equilibrium, however, wages must adjust to equate aggregate labor demand to aggregate labor supply. Consequently, using (2), unit cost, $\zeta ≡ v^f/\Lambda$, can be expressed as

$$\zeta = \frac{(1 + \tau)B(C^s/\Lambda)}{L - (C^s/\Lambda)x}, \quad (10)$$

where $x$ is the stock of customers, and the consumption demand appearing in (2) has further been equated to the consumption supplied per customer, $C^s$, since in the closed economy, the number of customers is equal to the population, which we have set equal to the size of the labor force. Under our normalization of the population size to one, $x = 1$. We can check from (10) that the partial elasticity of unit cost with respect to $C^s$ is greater than one. The partial elasticity of the unit cost with respect to $x$ is equal to
\(L/(L - L)\). (If we think of a typical eight-hour work day, this is a number like one-half.) Note also that, ceteris paribus, an increase in the wage income tax rate increases unit cost, while a rise in the Harrod-neutral productivity parameter decreases unit cost. We express the unit cost in reduced form as 
\(\zeta = \Upsilon(C^s, x; \tau, \Lambda); \ \Upsilon_{C^s} > 0, \ \Upsilon_x > 0, \ \Upsilon_{\tau} > 0, \ \Upsilon_{\Lambda} < 0.\)

The firm has to choose the price at which to sell to its current customers. Raising its price causes a decrease, and lowering the price an increase, in the quantity demanded by its current customers according to a per-customer demand relationship, \(D(p^i/p, C^s)\). For simplicity, we assume that \(D(\cdot)\) is homogeneous of degree one in total sales, \(C^s\), and so we write \(C^{si} = \eta(p^i/p)C^s; \ \eta'(p^i/p) < 0; \ \eta(1) = 1.\) Each firm chooses the path of its real price or, equivalently, the path of its supply per customer to its consumers, to maximize the present discounted value of its cash flows. The maximum at the \(i^{th}\) firm is the value of the firm, \(V^i\), which depends upon \(x^i:\)

\[V_0^i \equiv \max \int_0^\infty \left[ \left( \frac{p^i}{p_t} \right) C^s x^i \exp^{-\int_0^t r_s ds} \right] dt.\]

The maximization is subject to the differential equation giving the motion of the stock of customers of the \(i^{th}\) firm as a function of its relative, or real, price given by (11) below and an initial \(x^i_0:\)

\[\dot{x}^i = g^i(p^i/p)x^i; \ \ g' < 0, \ g'' \leq 0; \ \ g(1) = 0. \quad (11)\]

The first-order condition for optimal \(p^i\) is

\[\eta(p^i/p) \frac{C^s x^i}{p} + \left[ (\frac{p^i}{p}) - \Upsilon(C^s, x; \tau, \Lambda) \right] \eta'(\frac{p^i}{p}) \frac{C^s x^i}{p} + q^i_m g'(\frac{p^i}{p}) \frac{x^i}{p} = 0, \quad (12)\]

where \(q^i_m\) is the shadow price, or worth, of an additional customer. Another two other necessary first-order conditions (which are also sufficient under our
assumptions) from solving the optimal control problem are:

\[
\begin{align*}
\dot{q}_m^i & = [r - g(p^i_m/p)]q_m^i - \left[ \frac{p^i_m}{p} - \Upsilon(C^s, x; \tau, \Lambda) \right] \eta(p^i_m/p)C^s, \quad (13) \\
\lim_{t \to \infty} \exp^{-\int_0^t r_s ds} q_m^i x_t^i & = 0. \quad (14)
\end{align*}
\]

We note that “marginal q” denoted \(q_m^i\) is equal to “average q,” which we denote as \(q_a^i \equiv V_i^t/\tilde{x}_i^t\), so \(q_m^i = q_a^i = q^i.\)

Equating \(p^i\) to \(p\), and setting \(q^i = q\), delivers the condition on consumer-good supply per firm for product-market equilibrium:

\[
\left[ 1 + \frac{\eta(1)}{\eta'(1)} - \Upsilon(C^s, x; \tau, \Lambda) \right] = -\left( \frac{q}{C^s} \right) \left( \frac{q'(1)}{\eta'(1)} \right); \quad \eta(1) = 1. \quad (15)
\]

The expression in the square brackets is the algebraic excess of marginal revenue over marginal cost, a negative value in customer-market models as the firm supplies more than called for by the static monopolist’s formula for maximum current profit, giving up some of the maximum current profit for the sake of its longer-term interests. An increase in \(q\) means that profits from future customers are high so that each firm reduces its price (equivalently its markup) in order to increase its customer base. To handle economic growth, it is useful to note from (10) that we can express unit cost as \(\tilde{c} = \Upsilon(\tilde{C}_s, \tilde{x}; \tau, \Lambda)\), where \(\tilde{C}_s \equiv C^s/\Lambda\), so (15) can be transformed to

\[
\left[ 1 + \frac{\eta(1)}{\eta'(1)} - \Upsilon(\tilde{C}^s, x; \tau) \right] = -\left( \frac{\tilde{q}}{\tilde{C}^s} \right) \left( \frac{\tilde{q}'(1)}{\eta'(1)} \right); \quad \eta(1) = 1, \quad (16)
\]

where \(\tilde{q} \equiv q/\Lambda\) is normalized Q.

From (16), we can express normalized consumer-good supply per customer, \(\tilde{C}_s\), which equals employment, \(L\), in terms of \(\tilde{q}\), \(x\), and \(\tau\), that

5The proof is as follows: Taking the time derivative of the product \(q_m^i x_t^i\), we obtain
\[
\frac{d(q_m^i x_t^i)}{dt} = q_m^i [dx_t^i/dt] + x_t^i [dq_m^i/dt] = r_t q_m^i x_t^i - [(p_t/p_t) - \Upsilon(C_t^s, x_t; \tau, \Lambda_t)] \eta(p_t^i/p_t) C_t^s x_t^i, \quad \text{after using (11) and (13).}
\]
Integrating, and using (14), we obtain
\[
q_m^i x_t^i = \int_0^\infty [(p_k^i/p_k) - \Upsilon(C_k^s, x_k^i; \tau, \Lambda_k)] \eta(p_k^i/p_k) C_k^s x_k^i \exp^{-\int_0^t r_s ds} dk \equiv V_i^t.
\]
is, $\tilde{C}^s = L = \Omega(\tilde{q}, x; \tau)$. It is straightforward to show that $0 < e_{\tilde{q}} \equiv d\ln \tilde{C}^s/d\ln \tilde{q} < 1$, and $-1 < e_x \equiv d\ln \tilde{C}^s/d\ln x < 0$, where $e_j$ denotes the partial elasticity of $\tilde{C}^s$ with respect to the variable $j$. Also, $\Omega_\tau < 0$. An increase in $\tilde{q}$ makes investment in customers through reducing the markup attractive and so expands current output and employment. With rising marginal cost, an increase in the number of customers at each firm leads to a less than proportionate decline in the amount of (normalized) output supplied per customer.\(^6\) An increase in wage income tax contracts each firm’s supply. Writing $\tilde{q} \equiv \tilde{q}/\tilde{C}^s \equiv q/C^s$, we also note that the markup, $m \equiv \zeta^{-1}$, is a unique decreasing function of $\tilde{q}$. We express this relationship as: $m \equiv \zeta^{-1} = \phi(\tilde{q}); \phi'(\tilde{q}) < 0$. An increase in investment in customers through lowering current markups is attractive when the present discounted value of the future returns from investment ($q$) are high relative to its cost, which depends on the current level of sales ($C$).

In a symmetric situation across firms, (13) simplifies to

$$\frac{[1 - \Upsilon(\Omega(\tilde{q}, 1; \tau), 1; \tau)]\Omega(\tilde{q}, 1; \tau)}{\tilde{q}} + \frac{\dot{\tilde{q}}}{\tilde{q}} + \lambda + g(1) = r; \; g(1) = 0, \; (17)$$

after using $x = 1$ and $\tilde{C}^s = \Omega(\tilde{q}, x; \tau)$. Equation (17) in the firm’s instantaneous rate of return to investment in its stock of assets, which are customers, is an inter-temporal condition of capital-market equilibrium: it is entailed by correct expectations of $\dot{\tilde{q}}$ and $r$ at all future dates. It can be seen from the unit cost function, $\Upsilon(\cdots)$, in (17) that, given $\tilde{q}$ and $x = 1$, a decrease in $\tau$ lowers the unit cost directly, and increases it indirectly through raising $\tilde{C}^s$. Using $\tilde{C}^s = \Omega(\tilde{q}, x; \tau)$ in (16), however, shows that the direct effect dominates so, given $\tilde{q}$, a decrease in $\tau$ unambiguously lowers unit cost. In other words, given that the elasticity of (normalized) output supplied per customer with respect to $\tilde{q}$ is less than one, ceteris paribus, an increase in $\tilde{q}$ lowers the

\(^6\)We use the condition that the partial elasticity of the unit cost with respect to $x$ is equal to $L/(\bar{L} - L)$, a number typically less than one.
markup while a decrease in $\tau$ increases the markup. An alternative way of expressing the markup function will be useful: $m \equiv q^{-1} = \psi(\tilde{q}; \tau); \psi_{\tilde{q}} < 0$ and $\psi_{\tau} < 0$.

Equating consumption demand to consumption supply in (9), and noting $\tilde{V} \equiv \tilde{q}x$, we obtain an expression for $r$:

$$r = \lambda + \rho + \frac{\theta(\theta + \rho)(\tilde{q} + \tilde{D})}{\Omega(q, 1; \tau)} + e_{\tilde{q}} \left( \frac{\dot{\tilde{q}}}{\tilde{q}} \right); 0 < e_{\tilde{q}} < 1,$$

(18)

where we have used $x = 1$. (If we define the long-term (real) interest rate as the yield on consols paying a constant coupon flow of unity, and let $R$ be their yield and hence $R^{-1}$ be their price, arbitrage between short and long bonds gives the condition $R = r + (\dot{R}/R).$) Substituting (18) into (17), and noting that $g(1) = 0$, we obtain

$$[1 - e_{\tilde{q}}] \frac{\ddot{q}}{\tilde{q}} = \rho + \frac{\theta(\theta + \rho)(\tilde{q} + \tilde{D})}{\Omega(q, 1; \tau)}$$

$$- [1 - \Upsilon(\Omega(\tilde{q}, 1; \tau), 1; \tau)] \Omega(\tilde{q}, 1; \tau).$$

(19)

It is clear that the full-fledged model makes both $\tilde{q}$ and $\tilde{D}$ endogenous variables. Before we study the full-fledged model, however, it will be useful, for purposes of developing intuition, to study (19) treating normalized debt, $\tilde{D}$, first as a parameter. We proceed with our analysis in this order in the next two sections.

2. The Analysis Treating Normalized Debt as a Parameter

A. The Debt Bomb Scenario

Suppose that we start off in an initial equilibrium where $\tilde{D} = 0$, $\tilde{y}^s = 0$, and $\tau = 0$. Then, we have the following lemma:

**Lemma 1:** With $\tilde{D} = 0$, $\tilde{y}^s = 0$, and $\tau = 0$, the rational expectations
equilibrium is given by a unique value of $\tilde{q}$ that makes the righthand side of (19) equal to zero.

Since the elasticity of $C_s$ with respect to $\tilde{q}$ is less than one, the righthand side (RHS) of (19) is increasing in $\tilde{q}$. Applying the transversality condition, 

$$\lim_{t \to \infty} [\exp^{-\int_0^t (r_s - \lambda)ds} \tilde{q}_t x_t] = 0,$$

the unique perfect foresight path of $\tilde{q}$ requires that it be stationary at the value that makes $\dot{\tilde{q}} = 0$.

Let us now suppose that at $t_0$, it is announced that at $t_1$, there will be a helicopter drop of public debt, $\Delta > 0$, which will be financed by raising wage income taxes to cover interest payments on the new constant level of (normalized) public debt after $t_1$ so that $\tau L^h = (r - \lambda)\Delta$. To analyze the effects on asset prices, interest rates, and employment, it is convenient to refer to the first panel of Figure 1. Initially, $\tilde{q}$ is equal to $\tilde{q}_A$. Working backwards, let us ask, “What is the value of $\tilde{q}$ in the new stationary state?” (Note that given the instability of the system, the new stationary state must be attained precisely at $t_1$.) As we noted above, the unit cost, $\zeta \equiv \Upsilon(\cdots)$, is a monotone increasing function of $\tilde{q}/\tilde{C}^s$. Substituting that relation into (17), noting that $g(1) = 0$, and setting $\tilde{q} = 0$, we obtain a negatively-sloping relationship relating the LHS of (17) to $\tilde{q}/\tilde{C}^s$ (see Figure 2). Next, turning to (18), setting $\dot{\tilde{q}} = 0$ and letting $\tilde{D} = 0$ initially, gives a positively-sloping relationship relating the RHS of (18) to $\tilde{q}/\tilde{C}^s$. Now with $\tilde{D} = \Delta > 0$, the positively-sloping relationship shifts upwards, hence leftwards. The result is that the value of $\tilde{q}/\tilde{C}^s$ corresponding to the intersection declines. Consequently, the equilibrium markup is increased, and labor demand is decreased on that account, at $t_1$.

We have just shown that $\tilde{q}/\tilde{C}^s \equiv \tilde{q}/\Omega(\tilde{q}, 1, \tau)$ must fall at $t_1$.\footnote{The value of $\tau$ that must be raised to finance the interest on debt is given implicitly by the following equation: $\tau(1 + \tau)^{-1} \Upsilon(\Omega(\tilde{q}_C, 1; \tau), 1; \tau)\Omega(\tilde{q}_C, 1; \tau) = [\lambda + \rho + \theta(\theta + \rho)(\tilde{q}_C + \Delta)(\Omega(\tilde{q}_C, 1; \tau))^{-1}]\Delta$, where $\tilde{q}_C$ is the stock market value attained at $t_1$ and shown in Figure 1.} Given $\tilde{q}$,
an increase in $\tau$ reduces $\hat{C}^s$, which would raise $\hat{q}/\hat{C}^s$. Since the elasticity of $\hat{C}^s$ with respect to $\hat{q}$ is between zero and one, it must be that $\hat{q}$ falls by more than proportionately to achieve the implied drop in $\hat{q}/\hat{C}^s$. The new schedule in Figure 1 relating $\hat{q}/\hat{q}$ to $\hat{q}$ must, therefore, be to the left of the old schedule. As there can only be one initial jump in $\hat{q}$ at the moment the announcement is made, i.e. at $t_0$, and the economy must be at the value of $\hat{q}$ that makes $\hat{q} = 0$ at $t_1$ (namely, at $\hat{q}_C$ in Figure 1), the path taken by the economy after the initial jump is given by $BC$ along the old schedule in Figure 1. Upon receiving the announcement of future fiscal expansion, therefore, asset prices fall immediately from $\hat{q}_A$ to $\hat{q}_B$, and the expected rate of change of $\hat{q}$, i.e., the expected capital gains term, goes from zero to a negative value as market participants form a rational expectation of further asset price declines. In fact, asset prices continue to decline at an increasing rate until $t_1$ when $\hat{q}$ jumps up from a negative value (on the old schedule) to zero (on the new schedule). The stock market value cannot jump at the time of implementation, $t_1$, to avoid the possibility of making anticipated infinite rates of capital gain or loss. The implied paths taken by the current short-term real interest rate and employment, which equals normalized output, are given in the following proposition:

**PROPOSITION I:** At $t_0$, the short-term real interest rate drops, and it continues to fall steadily between $t_0$ and $t_1$. Then at $t_1$, it jumps up permanently to a level higher than the original level. Employment, which equals normalized output, drops at $t_0$, and continues its decline to reach a permanently lower level at $t_1$.\(^8\)

\(^8\)Notice from the third panel in Figure 1 that there is a further discontinuous drop in normalized output, and hence employment, at $t_1$. This is because $\hat{q}$ does not jump at $t_1$ but wage income tax is increased at that point to finance the interest on debt. The negative supply-siders' effect leads to the further decline in employment at $t_1$. 

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To understand how asset prices affect the natural rate of interest, it is useful to develop an analytical framework that is very close in spirit to that used by Lloyd Metzler (1951) in his classic article, "Wealth, saving, and the rate of interest." In that paper, Metzler developed a diagram in the (wealth, interest rate) plane (his Figure 1, p. 101), consisting of an upward-sloping $W$ schedule and a downward-sloping $A$ schedule. The $W$ schedule, which Metzler called the wealth-requirements schedule, gives the combinations of the interest rate and the real value of private wealth that would make "the community’s demand for goods and services as a whole equal [to] its capacity to produce." (p. 101) The reasoning underlying his $A$ schedule is the following: "In the short run the income earned by the common stock is a given amount, determined by the fixed supplies of the various agents of production; and this means that the yield, or the rate of interest, varies inversely with the real value of the stock." (p. 101)

We develop in Figure 3 the Metzler diagram in our non-monetary model. The upward-sloping schedule, $W$, gives the amounts of (normalized) private wealth required, at different interest rates and at given $\dot{\tilde{q}}/\tilde{q}$, to make aggregate demand (here, simply $C_d$) equal to aggregate supply, $C_s$, that is, to attain product-market equilibrium. Since the condition that consumption demand equals consumption supply, $C_d = C_s$, holds at each moment, it is also true that $\dot{C}_t = \dot{C}_s$ for all $t$. Moreover, since $\dot{C}_s = \Omega(\tilde{q}, 1; \tau)$, we can write $\dot{C}_s = e_{\tilde{q}}[\dot{q}/\tilde{q}]\Omega(\tilde{q}, 1; \tau)$. From (18), we can express the wealth-requirements condition as:

$$\tilde{W} = \left[ (r - \lambda - \rho) - e_{\tilde{q}} \left( \frac{\dot{q}}{\tilde{q}} \right) \right] \frac{\Omega(\tilde{q}, 1; \tau)}{\theta(\theta + \rho)}. \quad (20)$$

We note that, initially with $\tau = 0$ and $\tilde{D} = 0$, $\tilde{W} \equiv \tilde{q}$ since $x = 1$. We can, therefore, re-express (20) as:

$$\theta(\theta + \rho) \left[ \frac{\tilde{q}}{\Omega(\tilde{q}, 1; \tau)} \right] = (r - \lambda - \rho) - e_{\tilde{q}} \left( \frac{\dot{q}}{\tilde{q}} \right),$$
from which, since the elasticity of $\hat{C}$ with respect to $\hat{q}$ is less than one, we have, for given $\hat{q}/\bar{q}$, a positive relationship between $r$ and $\bar{q}$. In Figure 3, the $WW$ schedule drawn in bold is the initial schedule with $\tau = 0$, $\bar{D} = 0$ and $\hat{q}/\bar{q} = 0$. The $AA$ schedule, which we can call the assets-market equilibrium locus, is given in our non-monetary model by (17), which we rewrite here as:

$$
    r = \lambda + \frac{[1 - \Upsilon(\Omega(\hat{q}, 1; \tau), 1; \tau)] \Omega(\hat{q}, 1; \tau) + \hat{q}}{\bar{q}}.
$$

(21)

Since the elasticity of $\hat{C}$ with respect to $\hat{q}$ is less than one, we have, for given $\hat{q}/\bar{q}$, a negative relationship between $r$ and $\bar{q}$. The $AA$ schedule drawn in bold is the initial schedule with $\tau = 0$, $\bar{D} = 0$ and $\hat{q}/\bar{q} = 0$.

The initial equilibrium corresponds to point 1 in Figure 3, with $\bar{q}$ equal to the corresponding $\bar{q}_A$ in Figure 1. The sudden announcement at $t_0$ of a helicopter drop of public debt at $t_1$ causes current $r$ to fall for the following reason: The expectation of capital loss on the holding of the common stock requires that current share price, $\bar{q}$, falls in order to bring the total rate of return to holding a share equal to what it was before. Hence, there is a leftward shift of the $AA$ schedule. The expectation of a capital loss implies that there is an expectation of a decline in output supplied as well since firms will be expected to raise their markups on account of the expected fall in $\bar{q}$. This serves to raise the wealth requirement for product-market equilibrium, implying a rightward shift of the $WW$ schedule. The short rate, therefore, unambiguously falls (see Figure 3). Moreover, the expectation of capital loss necessarily reduces the equilibrium value of current $\bar{q}$. To see this, notice that at a given level of $\bar{q}$, say at $\bar{q}_A$, a one percentage point decline of $\hat{q}/\bar{q}$ requires that $r$ falls by $e_{\bar{q}}$ percent to satisfy the product-market equilibrium condition, so the $WW$ schedule shifts down by $0 < e_{\bar{q}} < 1$ percent. To satisfy assets-market equilibrium, however, requires that $r$ falls by exactly one percent. Since the $AA$ schedule shifts down by more than the $WW$ schedule, the fall in $r$ corresponding to the movement from point 1 to point
2 in Figure 3 involves a fall of $\tilde{q}$ from $\tilde{q}_A$ to $\tilde{q}_B$.

As the rate of expected capital loss further rises between $t_0$ and $t_1$ (see the first panel of Figure 1), both the WW and AA schedules continue to shift down, with the latter curve shifting by more than the former, thereby generating further declines in $r$ and $\tilde{q}$. At $t_1$, with $\tilde{q}/\tilde{q}$ going from a negative value to zero, the AA schedule shifts back to its original position (the bold line). The wealth-requirements schedule, however, does not go back to its original position, as wealth now includes also the holding of public debt, $\Delta > 0$. A horizontal wedge, of size $\Delta > 0$, is now driven between the AA and WW schedules with the consequence that the short rate at $t_1$ onwards (corresponding to point 3 in Figure 3) is above the original short rate. We plot the time path of $r$ in the second panel of Figure 1, where we see that the short-term real interest rate first drops at $t_0$ and continues a path of steady decline between $t_0$ and $t_1$ before it jumps up to a plateau that is higher than the level attained before the shock.

The path of normalized output, hence employment, can be inferred from the output supply function, $\tilde{C}^* = \Omega(\tilde{q}, 1; \tau)$. Since $\tilde{q}$ drops at $t_0$ and continues its decline until $t_1$, industry markups first increase and continue to increase further until $t_1$. As a result, employment first contracts and continues to decline until $t_1$. At $t_1$, although $\tilde{q}$ does not jump, the imposition of the wage income tax causes a further contraction in normalized output. Hence employment, though falling steadily since $t_0$ as asset prices steadily decline, suffers a discontinuous drop at $t_1$ due to the supply-sider effect of higher wage income taxes imposed at that time.

It is worth pointing out that since the short rate, $r$, initially drops and continues to fall (until $t_1$) in response to the news of a future helicopter drop of public debt, the depressed stock market could be accompanied by an initial decline in the long rate, $R$. Recall that arbitrage ensures $R = r + (\hat{R}/R)$. If the term structure is downward sloping at announcement, $R$ unambiguously
falls below \( r \) since \( \dot{R}/R < 0 \). We depict an illustrative path (dotted line) in the second panel in Figure 1 denoted \( R^I \). If the term structure is upward sloping at announcement so \( \dot{R}/R > 0 \), it is still possible that \( R \) initially drops if \( r \) falls by more than the rate of capital loss on holding a long bond, where \( R^{-1} \) is the price of the long-term bond. We depict another illustrative path (broken line) in the second panel in Figure 1 denoted \( R^{II} \). Hence observing an initial fall in the long rate is not prima facie evidence against the Feldstein-Rubin-Summers channel, going from prospective debt to higher future short rates and a decline in asset prices and employment, contrary to financial commentators.\(^9\)

Intuitively, the slump in the stock market that follows the announcement of future fiscal expansion can be explained as follows: A lower value of equity in the future is required to make the rate of return in equity investment equal to the higher short rate in the future. Under rational expectation, this anticipation drives down equity prices today. The current drop in stock market value reduces nonhuman wealth, which reduces current consumption demand, and puts downward pressure on the current short rate. Despite lower short rates, and notwithstanding lower long rates, economic activity is not stimulated since the decline in stock market reflects the reduced worth of a future customer and that is decisive in driving firms to raise their current markups. Current output and employment are, therefore, depressed.\(^{10}\)


\(^{10}\)We have demonstrated \textit{theoretically} that the decline in asset prices coincides with an initial decline in the short-term natural rate of interest. Is this relationship observed empirically? Do we tend to see, say, in the past fifteen years, a decline in the stock market being associated with a decline in the short-term real interest rate, and a booming stock market associated with an increase in the short-term real interest rate. We suggest that the answer is in the affirmative. A number of authors who have examined the behavior of the Federal Reserve System since the mid-eighties onwards have argued that implicit inflation targeting by the Fed has involved raising the short-term real interest rate as a
The expectation of the higher short rate from $t_1$ onwards is, however, only one channel causing the slump in asset prices. Since the debt bomb explosion occurring at $t_1$ has the effect thereafter of requiring an increase of tax rates to re-balance the budget, now swollen by the increased debt burden, the $v^J$ curve, hence the cost curve, is pushed up with the result that quasi-rent is reduced beginning at $t_1$. The anticipation of this reduced level of aggregate quasi-rents on the customer stock is another factor causing an immediate drop in the stock market at $t_0$.\footnote{Mundell (1960) showed that when the corporate income tax is raised to finance interest on debt, the capitalization of the tax, hence a drop of the current real asset price, acts to reduce wealth even as increased private holding of public debt increases wealth. He pointed out that the Metzler (1951) analysis regarding public debt failed to take into account this offsetting capitalization effect. Metzler (1973) objected that corporate taxes were not a substantial part of total taxes collected in the U.S. Our analysis, however, shows that the asset market also capitalizes the effects of higher expected \textit{wage} income taxes required to finance the interest on debt.}

B. The Entitlement Bomb Scenario

Republicans may complain that the Democrats’ alternative to the debt bomb, a rain of increased entitlement spending, which in our stylized model also commences in the medium-future, has the same effect. They are right up to a point: The increased spending, in requiring thereafter an increase of tax rates for budget balance, also reduces quasi-rents and as a result the stock market also drops in anticipation of that. (In fact, the Republicans would justify their debt bomb as a second-best move serving to ward off the worse outcome of increased entitlements.)

The economics is more complicated, however, since the debt bomb implies of dampening aggregate demand when asset prices increased (see Ben Bernanke and Mark Gertler, 1999; Richard Clarida, Jordi Gali and Gertler, 2000). When stock prices declined in 2001, the Fed lowered the short-term real interest rate in a series of cuts to stimulate aggregate demand.
pacts on the stock market and hence investment and employment through a second channel—higher expected future short rates from $t_1$ onwards—which the rain of entitlements does not. With the Blanchard-Yaari demographics, the debt bomb is converted to an annuity by individuals so that in expected value sense the whole stream of interest in perpetuity is expected to be consumed. So there is a bigger stimulus to consumption than that imparted by an equal-sized “entitlement bomb” in our non-Ricardian setup.\(^{12}\) (There is also a larger implied rise in demand for leisure under the debt bomb scenario. Using (2), and noting that $v^h \equiv (1 + \tau)^{-1} v^f$ and $m \equiv \zeta^{-1} = (v^f / \Lambda)^{-1}$, we have $(L - L) / \tilde{C} = B(1 + \tau) m$. The greater stimulus to consumption, $\tilde{C}$, carries with it a greater increase in demand for leisure, given the terms on the righthand side. Additionally, there is a greater increase in the current markup as well, so there is also a larger increase in the demand for leisure relative to consumption under the debt bomb scenario.) Consequently, from $t_1$ onwards, the short rate path corresponding to the debt bomb scenario lies everywhere above the short rate path corresponding to the entitlement bomb scenario. (Under the entitlement bomb scenario, the short rate from $t_1$ onwards is back to its $t_0$ level attained before receiving the news.\(^{13}\) The

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\(^{12}\)Suppose that the government announces that it will introduce a steady stream of welfare entitlements at $t_1$ onwards such that $\tilde{y}^s = (r^D_1 - \lambda) \tilde{\Delta}$, where $r^D_1$ is the short rate that prevails from $t_1$ onwards in the debt bomb scenario. Human wealth at $t_0$ is not changed by the welfare entitlement program and is given simply by $f_{t_0}^{\infty} \tilde{v}^f L_t \exp \left[ \int_{t_0}^{r} (r^k - \lambda + \theta) dk \right] \, d\mu$ since what the government gives in quasi-human wealth, viz. the entitlement benefit (the normalized entitlement’s value doesn’t grow at $r(t) - \lambda$, but at $r(t) - \lambda + \theta$), is taken away in taxes on labor. The human wealth under the debt bomb scenario, however, has an additional term, which disappears only under Ricardian Equivalence: $H_{t_0} = f_{t_0}^{\infty} \tilde{v}^f L_t \exp \left[ \int_{t_0}^{r} (r^k - \lambda + \theta) dk \right] \, d\mu + \left[ \frac{\theta \tilde{q}^*}{(r^2_1 - \lambda)(r^2_1 - \lambda + \theta)} \right] \exp^{- \int_{t_0}^{t_1} (r^k - \lambda + \theta) dk}$. Therefore, in our non-Ricardian economy, consumers feel more wealthy under the debt bomb scenario than under an equal-sized “entitlement bomb”.

\(^{13}\)Under the entitlement bomb scenario, use of Figure 2 with $\tilde{D} = 0$ shows that the value $\tilde{q} / \tilde{C}^* \equiv \tilde{q} / \Omega(\tilde{q}, 1; \tau)$ is unaffected in the new stationary state, which the economy must
result is that there is a bigger drop in the stock market at $t_0$ under the debt bomb scenario, and accordingly, a larger increase in industry markup, and a sharper decline in employment at $t_0$. The stronger contractionary effect of the debt bomb does not vanish in the long run. We have the following proposition:

PROPOSITION II: Suppose that an announcement is made at $t_0$ that at $t_1$ onwards, there will be a constant flow of welfare entitlements (equal in value to the interest on debt in the alternative scenario) given to all workers alike, and financed by wage income taxation. Between $t_0$ and $t_1$, the short-term interest rate declines. At $t_1$, it jumps back up to the original level. Although normalized output, which equals employment, jumps down at $t_0$, and continues its decline to reach a permanently lower level at $t_1$, the employment path lies everywhere above the path under the debt bomb scenario.

3. The Analysis Treating Normalized Debt as Endogenous

A. Conditions for Fiscal Sustainability

The preceding analysis showed how an anticipated helicopter drop of public debt, accompanied by an expected increase in wage income taxes required to finance the interest on debt, leads to a drop in asset prices today through two channels: An increase in future short rates of real interest as well as an expectation of reduced future aggregate quasi-rents on the stock of customers due to the anticipated higher wage income taxes, which raise unit costs. However, the tax bill passed in the summer of 2001 (the EGTRRA), in fact, sets the post-sunset tax rate back to its original rather than to a higher level without any planned spending cuts. Indeed, the Republicans are proposing making the tax cuts permanent. This immediately raises the attain at $t_1$. Using (18) and noting that $\dot{q} = 0$ at $t_1$ imply that $r_1 = r_0$. 

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question of whether the proposed fiscal changes are sustainable in the sense that the debt-income ratio will neither explode nor implode when account is taken of all the general-equilibrium effects (see Blanchard, et al. (1990) for the concept of the sustainability of fiscal policy). We will show that, in our model, fiscal sustainability requires that, for a given tax rate, the primary (non-interest) surplus must be made a sufficiently responsive positive function of the debt-income ratio. Conversely, the primary deficit must be reduced sufficiently as the debt-income ratio rises in order to achieve fiscal sustainability. The extent to which policy-makers must reduce the primary (non-interest) deficit, such as through cutting government entitlement programs, depends upon how much a decrease in asset prices decreases tax revenue (relative to income) compared to how much it lowers the real interest rate (net of GDP growth) and the resultant public debt service burden. In a recent study, Bohn [1998], in fact, found significant evidence that for the U.S. the primary surplus (taken as a ratio to GDP) is indeed an increasing function of the debt-GDP ratio (after controlling for war-time spending and cyclical fluctuations) for 1916-1995 and various subperiods. His estimates of the increase in (non-interest) primary surplus in response to a unit increase in the debt-income ratio range from 0.028 to 0.054 for various sub-periods, with 0.054 for the whole period.

The full-fledged general-equilibrium model in \( \bar{q} \) and \( \bar{D} \) can be summarized by a pair of equations, namely, (19) and an additional equation describing the evolution of (normalized) debt:

\[
\dot{\bar{D}} = \left\{ \frac{1}{1 - e_{\bar{q}}} \left[ \rho + \frac{\theta(\theta + \rho)(\bar{q} + \bar{D})}{\Omega(\bar{q}, 1; \tau)} \right] \right. \\
\left. - \left( \frac{e_{\bar{q}}}{1 - e_{\bar{q}}} \right) \left[ 1 - \Upsilon(\Omega(\bar{q}, 1; \tau), 1; \tau) \right] \Omega(\bar{q}, 1; \tau) \right\} \dot{\bar{D}} \\

+ \ \bar{g}'' - \left( \frac{\tau}{1 + \tau} \right) \Upsilon(\Omega(\bar{q}, 1; \tau), 1; \tau) \Omega(\bar{q}, 1; \tau),
\] (22)
where the terms \( r - \lambda \) and \( \tau L \tilde{v} \) in (8) have been replaced using
\[
\begin{align*}
  r - \lambda &= \frac{1}{1 - e_{\tilde{q}}} \left[ \rho + \frac{\theta(\theta + \rho)(\tilde{q} + \tilde{D})}{\Omega(\tilde{q}, 1; \tau)} \right] \\
  \tau L \tilde{v} &= \left( \frac{\tau}{1 + \tau} \right) \Upsilon(\Omega(\tilde{q}, 1; \tau), 1; \tau) \Omega(\tilde{q}, 1; \tau),
\end{align*}
\]

Instead of studying the system described by (19) and (22), it turns out to be convenient to examine a related system where asset price, \( q \), and debt, \( D \), are normalized by GDP rather than by the productivity measure. Defining then \( \hat{D} \equiv \tilde{D}/\tilde{C} \) and \( \hat{y} \equiv \tilde{y}/\tilde{C} \), and using our notation \( \hat{q} \equiv \tilde{q}/\tilde{C} \) introduced earlier, we modify (19) and (22) to yield
\[
\begin{align*}
  \dot{\hat{q}} &= \mu(\hat{q}, \hat{D}) - \left( \frac{1 - (\phi(\hat{q}))^{-1}}{\hat{q}} \right), \\
  \dot{\hat{D}} &= \mu(\hat{q}, \hat{D}) \hat{D} + \hat{y} - \hat{T}(\hat{q}; \tau),
\end{align*}
\]
where \( r - \lambda - e_{\tilde{q}}[\hat{q}/\tilde{q}] = \rho + \theta(\theta + \rho)[\hat{q} + \hat{D}] \equiv \mu(\hat{q}, \hat{D}) \), which gives the interest net of GDP growth. We see that it is increasing in \( \hat{q} \) and \( \hat{D} \) so \( \mu_{\hat{q}} > 0 \) and \( \mu_{\hat{D}} > 0 \). Note also that the tax revenue to GDP ratio is given by \( \tau(1 + \tau)^{-1} \Upsilon(\Omega(\tilde{q}, 1; \tau), 1; \tau) = \tau(1 + \tau)^{-1}(\phi(\hat{q}))^{-1} \equiv \hat{T}(\hat{q}; \tau) \); \( \hat{T}_{\tau} > 0 \), \( \hat{T}_{\tau} > 0 \). To obtain (23) and (24), we have also used the relationships \( \hat{q}/\hat{q} \equiv [1 - e_{\tilde{q}}][\hat{q}/\tilde{q}] \), \( m \equiv \xi^{-1} = \phi(\hat{q}) \) and \( \hat{q} \equiv \tilde{q}/\Omega(\tilde{q}, 1; \tau) \).

To find the condition for fiscal sustainability, we first take note that if the primary surplus (normalized by GDP) given by \( \hat{T} - \hat{y} \), where \( \hat{T} \equiv \tau(1 + \tau)^{-1}(\phi(\hat{q}))^{-1} \) is independent of the debt-GDP ratio, \( \hat{D} \), the steady state of the linearized system given by (23) and (24) is globally unstable so any deviation from the steady state will cause the debt-income ratio to either explode or implode. Technically, the trace of the \( 2 \times 2 \) matrix associated with the linearized dynamic system given below is positive, and the determinant is also positive:
\[
[
\begin{matrix}
\hat{q} & \dot{\hat{D}} \\
\end{matrix}
\right] = A[
\begin{matrix}
\hat{q} - \hat{q}_{ss} & \hat{D} - \hat{D}_{ss} \\
\end{matrix}
\right],
\]

\[24\]
where \([\cdots]'\) denotes a column vector, the system (23) and (24) is linearized around the steady-state values, \(\hat{q}_{ss}\) and \(\hat{D}_{ss}\), and the \(2 \times 2\) matrix \(A\) contains the following elements:

\[
\begin{align*}
a_{11} &\equiv \mu + \mu \hat{q}_{ss} - (\phi(\hat{q}_{ss}))^{-2} \phi'(\hat{q}_{ss}), \\
a_{12} &\equiv \mu \hat{D}_{ss}, \\
a_{21} &\equiv \mu \hat{D}_{ss} - \hat{T}_{\hat{q}}, \\
a_{22} &\equiv \mu + \mu \hat{D}_{ss}.
\end{align*}
\]

We can readily check that \(a_{11} > 0\), \(a_{12} > 0\) and \(a_{22} > 0\) while \(a_{21}\) can either be positive or negative. (Consequently, the trace of \(A\) (\(\text{tr}(A)\)) is clearly positive.) The sign of \(a_{21}\) depends upon the relative influence of a change in \(\hat{q}\) on the tax-GDP ratio on the one hand, and the interest debt burden on the other hand. If a rise in the asset price relative to GDP raises the tax-GDP ratio by more than it raises the interest burden (normalized by GDP) so a booming stock market leads to declining debt-income ratios or, conversely, a depressed stock market leads to rising debt-income ratios, then \(a_{21}\) is negative, and the determinant of \(A\) (\(\text{det}(A)\)), equal to \(a_{11}a_{22} - a_{21}a_{12}\), is clearly positive. In the alternative case when \(a_{21}\) is positive, we can check that \((a_{22}/a_{21}) > (a_{12}/a_{11})\) so once again the determinant of \(A\) is positive. More concretely, we can show that, whether \(a_{21} > 0\) or \(a_{21} < 0\), we obtain \(\text{det}(A)\) \(\equiv \mu \mu_{\hat{q}_{ss}} + \mu \hat{D}_{ss} \hat{T}_{\hat{q}} + [\mu - (\phi'/\phi^2)][\mu \hat{D}_{ss} + \mu] > 0\) since \(\mu > 0\), \(\mu \hat{D}_{ss} > 0\), \(\hat{T}_{\hat{q}} > 0\) and \(\phi' < 0\). Therefore, if the entitlement spending (taken as a ratio to GDP) is held invariant to changes in the debt-GDP ratio, the system is globally unstable. Any deviation from the steady-state of the system is bound to lead to either imploding or exploding debt (as a ratio to GDP).

To achieve fiscal sustainability, it is necessary that we make the (normalized) primary surplus an increasing function of \(\hat{D}\), as suggested by the empirical work of Bohn [1998], so we let entitlement spending as a ratio to GDP decline as the debt-income ratio increases and write \(\hat{y}^s = \Phi(\hat{D}); \Phi'(\hat{D}) < 0\)
since we want to keep $\tau$ as a policy parameter. With entitlement spending as a ratio to GDP made a negative function of the debt-income ratio, we have to modify the original value of $a_{22}$ to get $a_{22} \equiv \mu + \mu \hat{D}_{ss} + \Phi \hat{D}$. In the arguably empirically relevant case where a depressed stock market leads to rising debt-income ratios so $a_{21} < 0$, we find that in order to achieve saddle-path stability, it will be necessary though not sufficient for $\hat{y}^s$ to fall in response to an increase in $\hat{D}$ so that $a_{22}$ is negative. In other words, a unit increase in the debt-income ratio necessitates a cut in entitlement spending (relative to GDP) that more than offsets the rise in interest burden so that the debt-income ratio actually declines. The necessary and sufficient condition for saddle-path stability, and hence fiscal sustainability in response to a tax cut, in the case when $a_{21} < 0$ is for $-a_{22} > -a_{21}(a_{12}/a_{11}) > 0$. Noting that we can write down the respective slopes of $\hat{q}$ and $\hat{D}$ as:

$$\frac{d\hat{q}}{d\hat{D}} \bigg|_{\dot{D}} = \frac{-[\mu \hat{D} + (\mu/\hat{D}_{ss}) + \hat{D}_{ss}^{-1}\Phi \hat{D}]}{\mu \hat{q} - (T_{\hat{q}}/\hat{D}_{ss})} \equiv \frac{-a_{22}}{a_{21}}, \quad (26)$$

$$\frac{d\hat{q}}{d\hat{D}} \bigg|_{\hat{q}} = \frac{-\mu \hat{D}}{\mu \hat{q} + \hat{q}_{ss}^{-1}[\mu - (\phi(\hat{q}_{ss}))^{-2}\phi'(\hat{q}_{ss})]} \equiv \frac{-a_{12}}{a_{11}} < 0, \quad (27)$$

we obtain saddle-path stability only if both stationary loci are negatively sloped in such a way that

$$\left| \frac{d\hat{q}}{d\hat{D}} \bigg|_{\dot{D}} \right| > \left| \frac{d\hat{q}}{d\hat{D}} \bigg|_{\hat{q}} \right|.$$

We show this case in Figure 4.

If a decline in asset prices (relative to GDP) leads to bigger cost savings for the government (as a result of a huge drop in interest debt service burden) than its loss of tax revenue (relative to GDP) so $a_{21} > 0$, then the condition for fiscal sustainability is immediately satisfied by a fiscal rule that makes $\hat{y}^s$ fall sufficiently in response to a rise in $\hat{D}$ to make $a_{22}$ negative. (Referring to (26) and (27), this condition says that when $a_{21} > 0$ and $a_{22} < 0$, we
are assured of saddle-path stability and the stationary locus for $\dot{D} = 0$ is positively sloped in the $(\dot{D}, \dot{q})$ plane.) If $a_{21} > 0$, the condition that $a_{22} < 0$ is sufficient for fiscal sustainability but it is not necessary. If declining asset prices lead to a smaller loss in tax revenue (relative to GDP) than the government can save from a decline in interest burden so the debt-income ratio actually falls, then, in order to attain fiscal sustainability, big cuts in entitlement spending may not be required when the debt-income ratio rises so $a_{22}$ remains positive so long as the condition $0 < a_{22} < a_{21}(a_{12}/a_{11})$ is satisfied. Referring to (26) and (27), this condition says that when $a_{21} > 0$ and $a_{22} > 0$, we obtain saddle-path stability if both stationary loci are negatively sloped in such a way that

$$\left| \frac{d\tilde{q}}{d\tilde{D}} \right| < \left| \frac{d\tilde{q}}{d\hat{D}} \right|.$$

In summary, there are three cases where we obtain saddle-path stability. If a drop in $\dot{q}$ leads to a greater loss in tax revenue (relative to GDP) than cost savings from a lower interest debt service burden so $a_{21} < 0$, the only way to achieve saddle-path stability is to cut entitlement spending (relative to GDP) sharply enough to make not only $a_{22} < 0$ but also to satisfy the condition: $-a_{22} > -a_{21}(a_{12}/a_{11}) > 0$. However, if a drop in $\dot{q}$ leads to greater interest cost savings for the government than the amount of tax revenue lost (relative to GDP), saddle-path stability is guaranteed for a government that cuts entitlement spending (relative to GDP) sufficiently to make $a_{22} < 0$. In this case, we have $a_{21} > 0$ and $a_{22} < 0$ so $\det(A)$ is unambiguously negative. If $a_{21} > 0$, the government can, in fact, attain fiscal sustainability without sharp cuts to entitlement spending (relative to GDP) so long as $0 < a_{22} < a_{21}(a_{12}/a_{11})$. Letting $\gamma_1 = [\text{tr}(A) - \sqrt{\text{tr}(A)^2 - 4\det(A)})]/2$ be the negative root, the slope of the saddle path is given by $(\gamma_1 - a_{11})/a_{12}$, which is unambiguously negative in all the three cases summarized here. The interested reader can proceed to draw the relevant phase diagrams corresponding
to the two cases where a drop in $\hat{q}$ leads to larger interest cost savings for the government than the tax revenue lost (relative to GDP). It is readily checked that the qualitative results regarding the effects on asset prices and employment of the tax shocks we study are similar in all three cases. The differences occur in the short-term movement of the debt-income ratio in response to asset price changes since, at any given $\hat{D}$, a fall of $\hat{q}$ leads to a gradual buildup of the debt-income ratio when $a_{21} \equiv \mu_q \hat{D}_{ss} - \hat{T}_{\hat{q}} < 0$ but to a gradual decrease of the debt-income ratio when $a_{21} \equiv \mu_q \hat{D}_{ss} - \hat{T}_{\hat{q}} > 0$. We will, however, proceed to conduct our analysis with the aid of Figure 4 and so focus on the case where $a_{21} \equiv \mu_q \hat{D}_{ss} - \hat{T}_{\hat{q}} < 0$.

B. Effects of Tax Cuts

We now establish three propositions in the case of sustainable fiscal expansion.

PROPOSITION III: Suppose that the economy is initially in a steady state with $(\hat{D}_0, \hat{q}_0)$. At $t_0$, there is an announcement that at $t_1$, the wage income tax rate will be cut from $\tau$ to $\tau'$ until $t_2$, at which time the tax rate reverts back to $\tau$. Output and employment can either expand ("good news" case) or contract ("bad news" case) from $t_0$ to $t_2$. The same ambiguity may result even if the tax cut takes place immediately. From $t_2$ onwards, however, employment and output are unambiguously depressed relative to the original steady state, to which they gradually recover in both scenarios, whether the tax cut is delayed or immediate.

In Figure 5, we depict the "bad news" case, where the prospective tax cut leads to an immediate decline in $\hat{q}$ from point $A$ to point $B$ at $t_0$, and continues to fall further from point $B$ to point $C$, which it reaches at $t_1$, the
time of implementation.\textsuperscript{14} At $t_1$, both the asset price as well as the level of debt cannot jump but the tax cut itself leads to an increase in output supply, which causes $\hat{q} \equiv \bar{q}/\Omega(\bar{q}, 1; \tau)$ and $\hat{D} \equiv \bar{D}/\Omega(\bar{q}, 1; \tau)$ to drop equiproportionately so moving from point $C$ to point $D$ along the ray $OX$. Between $t_1$ and $t_2$, $\hat{q}$ continues to fall along the path $DE$. At sunset, that is, at $t_2$, once again the asset price and the level of debt cannot jump but now, the restoration of the tax rate back to its original level prompts a fall in output supply, which raises $\hat{q}$ and $\hat{D}$ equiproportionately so moving from point $E$ to point $F$ along the ray $OY$. Point $F$ lies at the intersection of ray $OY$ and the saddle path associated with the initial steady-state point $A$. The bad-news case is one where the Feldstein-Rubin-Summers effect dominates so normalized output, which equals employment, declines during the interval $t_0$ to $t_1$. At $t_1$, the wage income tax rate is cut, and that has a positive supply-sider effect. What is clear is that at $t_2$, $Q$ normalized by GDP is lower than the initial $\hat{q}_0$, and there is no offsetting reduced wage income tax so employment, and hence normalized output, is unambiguously depressed below its pre-announcement level. Gradually, employment and (normalized) asset prices recover back to their original steady-state levels. Figure 6 depicts the “good news” case where the economy initially experiences an expansion. Even in this case, however, employment is depressed from sunset onwards (traveling along $FA$) until the economy finally recovers back to its original steady state.

Notice that in the “bad news” case, the debt-GDP ratio rises between the announcement and implementation as depressed asset prices lead to a larger loss in tax revenues than the cost saving from lower interest brought about by a depressed stock market. This is not so in the “good news” case, \textsuperscript{14}Note that at $t_0$, the level of debt does not jump while the fall in asset price in the “bad news” case leads to a fall in output so the debt-GDP ratio rises. Consequently, point $B$ lies south-east of point $A$. \textsuperscript{29}
where a booming stock market increases tax revenues by more than enough to offset increased debt service burden resulting from upward pressure on the real interest rate.

Bringing forward the tax cut so that $t_1$ coincides with $t_0$, that is, an immediate tax cut, would, in Figure 5, bring about a sharper drop in $\hat{q}$ (to point $B'$) but the contractionary effect on employment on that account would immediately be offset by the positive supply-sider effect of reduced wage income taxes. This does not alter our result that from $t_2$ onwards, asset prices are depressed with correspondingly reduced economic activity. A similar argument applies to the “good news” case.

We can also readily establish the following proposition:

PROPOSITION IV: Making the tax cut permanent, whether it is delayed or immediate, leaves the real asset price (normalized by GDP) permanently depressed notwithstanding the positive supply-sider effect so employment could either contract or expand in the long run.

The new steady state is represented by point $G$ in Figure 5 (the “bad news” case) and Figure 6 (the “good news” case) exhibiting lower $Q$ (normalized by GDP) and higher debt-GDP ratio so it is possible in both cases for employment to contract in the long run despite lower tax rates. The interested reader can infer the whole path of employment in response to this shock using the phase-diagram analysis.

Finally, tax cut advocates might argue that the Feldstein-Rubin-Summers effect of depressed asset prices on account of higher future short rates would be weakened by a higher trend growth in output. The next proposition shows that the equilibrium of our closed-economy model is independent of the trend growth rate, $\lambda$.

PROPOSITION V: Suppose that the economy is initially in a steady state
with \((\hat{D}_0, \hat{q}_0)\), and a trend growth rate of \(\lambda\). Let the trend growth rate suddenly increase to \(\lambda'\). The equilibrium described by \((\hat{D}_0, \hat{q}_0)\) is unaffected.

The result is evident from inspecting (23) and (24), where we see that the general equilibrium system is independent of \(\lambda\). In effect, in our closed economy, a sudden rise in the trend growth rate is immediately translated into an equal increase in the natural rate of interest so the interest burden of debt is not reduced as a result of higher growth.

4. Concluding Remarks

The supply-siders’ thesis that employment activity is predominantly driven by changes in the tax wedge is empirically not the great success that is widely supposed. Mulligan (2002) attempts to establish the part played by public finance distortion in the movements of the supply of labor of American workers over nearly a century, 1889-1996, using the familiar neoclassical model of labor-leisure choice. This leads to the first-order condition

\[ MRS(C, \bar{L} - L) = v^h, \]

where \(MRS\) is the marginal value of time, which is a function of consumption \(C\) and hours worked \(L\), and \(v^h\) is the after-tax hourly wage. The latter is related to the firms’ demand wage \(v^f\) and to the wage income tax rate \(\tau\) by

\[ v^h \equiv (1 + \tau)^{-1}v^f \]

and, invoking pure competition, \(v^f\) is equated to the marginal product of labor, \(MPL\). Consequently,

\[ MRS(C, \bar{L} - L) = (1 + \tau)^{-1}MPL. \]

Mulligan argues from his empirical exercise that marginal tax rates are well correlated with labor-leisure distortions at low frequencies, but they cannot explain the distortions during the

\[ ^{15} \]

In principle, the consumption tax rate, say \(\tau_c\), also appears on the right-hand side, so

\[ MRS(C, \bar{L} - L) = [(1 - \tau_c)/(1 + \tau)]MPL. \]

However, with the assumed functional form in Mulligan (2002) as well as in our paper, it is possible to write

\[ MRS(C/(1 - \tau_c), \bar{L} - L) = (1 + \tau)^{-1}MPL, \]

so that when the measure of consumption used is inclusive of consumption taxes, we do not expect consumption taxes to drive a wedge between measured \(MRS\) and \(MPL\).
Great Depression, the Second World War and the 1980s. He concludes that the within-decade aggregate fluctuations in consumption, wages, and labor supply are hard to reconcile with this simple competitive equilibrium model of labor supply and demand.

The present paper brings in a product-market distortion arising from the gradual diffusion of information about prices, which gives firms some monopoly power, at least transiently. The firms’ inter-temporal perspective makes their current markup \( m \) inversely related to \( \tilde{q} \), the (normalized) shadow price that firms attach to a customer, and also an inverse function of the wage income tax rate \( \tau \), \( m = \psi(\tilde{q}; \tau) \); \( \psi_{\tilde{q}} < 0 \) and \( \psi_{\tau} < 0 \). In this imperfectly competitive framework the analogue to Mulligan’s labor-equilibrium relationship is \( MRS(C, \bar{L} - L) = (1 + \tau)^{-1}MPL \), in which an increase of \( \tilde{q} \) pulls up the right-hand side (i.e., \( v^h \)) and thus induces an increase in hours supplied. Expressing the wedge between the \( MPL \) and \( MRS \) as \( MPL/MRS = (1 + \tau)[\psi(\tilde{q}; \tau)] \), notice that, given \( \tilde{q} \), an increase of \( \tau \) increases the wedge through the \((1 + \tau)\) term but decreases the wedge through the markup term. Because of these two offsetting effects of a change in \( \tau \) on the wedge, one cannot expect to understand well the medium-term responses of employment (here hours) to wage income tax changes without considering the asset price responses to such shocks.\(^{16}\) For example, in our framework, an increase in the tax rates introduced in the mid-1990s under the Clink-}

\(^{16}\)With a utility function such as \( \log C + \frac{B}{(1 - \eta^{-1})}(\bar{L} - L)^{1-\eta^{-1}} \), where \( \eta^{-1} \) gives the constant inter-temporal elasticity of substitution of leisure, an increase in the wedge given by \((1 + \tau)m\) brings about a smaller increase in the demand for leisure at any given level of consumption demand the smaller \( \eta^{-1} \) is. Hall (1997) uses the value \( \eta^{-1} = 0.6 \) in his numerical simulation while Rotemberg and Woodford (1992) use \( \eta^{-1} = 1.3 \) in their baseline simulation. The latter cite studies showing that estimated values of the inter-temporal elasticity of substitution of leisure for males are typically near zero while many studies obtain estimates for female workers that fall within the range 0.5-1.5 with two being the upper bound. Our theoretical model in the text assumes \( \eta^{-1} = 1 \) as also is done by Prescott (2002) in his Ely lecture.
ton administration may have helped to boost employment, contrary to what would be predicted by Mulligan’s competitive equilibrium framework, precisely because the expectation of a decline in the debt-GDP ratio boosted asset prices and thus reduced firms’ markups. The Feldstein-Rubin-Summers channel, from tax increase to the demand for labor, through which a pay-down of the public debt (relative to income) lowers future short rates and elevates asset prices, including the shadow price of customers, $\tilde{q}$, could have pulled up $v^h$ and $L$ more than the contractionary supply-sider effect from the increase of $\tau$ pushed them down. Reduced nonhuman wealth, on account of the shrinking public debt, would further act to depress consumption demand, and along with it, demand for leisure.

We believe that our framework, by introducing a role for asset prices in the fundamental labor-equilibrium condition, also helps to throw light on some puzzles found by Mulligan in his study of labor-leisure distortions at medium-term frequencies within the competitive equilibrium framework. For example, he found that tax distortions alone could not quantitatively explain the wedge between $\text{MRS}$ and $\text{MPL}$ during the Great Depression. “What drove a 40% wedge between marginal product and value of time?” he asks. In our model, $\text{MPL}/\text{MRS}(C, \bar{L} - L) = (1 + \tau)[\psi(\tilde{q}; \tau)]$, so we conjecture that the increase of the wedge during 1929-33 that cannot be explained by an increase in tax rate is attributable to a decline in asset prices, such as a depressed value placed on a customer, which increases firms’ markups. (The recent paper by Chari, et al. (2002) similarly fails to incorporate a role for asset prices in explaining the wedge between $\text{MRS}$ and $\text{MPL}$ during the Great Depression. By using a perfectly competitive framework, hence giving no role for markup adjustments, any shifts in expectations that change current asset prices cannot affect this particular wedge. In their model, as well as in the models of Mulligan (2002) and Prescott (2002), this wedge is termed an intra-temporal wedge. Our framework, however, makes this an
inter-temporal wedge because shifts in expectations can change it even when current marginal tax rates are unchanged.)

Mulligan also found that despite an increase in federal tax rates from practically zero to more than 20% during World War II, leisure during the second world war is lower than implied by the labor-equilibrium condition given by the competitive equilibrium model. Our model suggests that this may be attributable to the fact, highlighted by Mankiw (1985), that the real interest rate was low during the war. Theoretically, the low wartime real interest rate can be explained either by Mankiw’s own introduction of consumer durables into the standard neoclassical growth model or by the introduction of the differences in relative labor intensiveness in the consumer and capital-good producing sectors (see Phelps, 1994). In the former case, an increase in government spending on the aggregative good, which drives capital used in the domestic sector into the commercial sector so reducing the marginal product of capital, and in the latter case, an increase in government spending on the relatively labor-intensive capital good, reduces the real interest rate, and raises asset prices, including the shadow prices firms place on their operating business assets, such as their customers. This counteracts the distortionary effects of increased federal income tax rates.\footnote{Rotemberg and Woodford (1992) adduce evidence in support of a decline in markups when government purchases increase, including during the two world wars, but they use a different model of dynamic markups from ours. They also acknowledge that the imposition of price controls during World War II places a limitation on one’s interpretation of the data.}

Finally, Mulligan pointed out that the falling wedge during the Reagan years could not be fully explained by the decrease in federal labor income tax rates in the 80s. Although the Feldstein-Rubin-Summers channel would imply that the stock market should decline if agents formed expectations of a build-up of public debt, authors such as Blanchard and Summers (1984) have argued that in the early eighties, the fiscal expansion in the U.S. was
offset by the fiscal contraction in the other major OECD countries so that the aggregate inflation-adjusted deficit as a percent of the group’s GNP did not change significantly. They pointed to the strong stock market performance in the 1980s and the strong behavior of investment in the face of increased real interest rates as evidence of a favorable shift in expected profitability. If that inference is correct, the consequent rise in the value placed on customers would cause markups to fall, and thus reduce the wedge beyond what was brought about by reduced wage income tax rates.

While the focus in our paper has been on tax cuts, it is easy to see how we can apply a similar framework to the study of the effects on asset prices and employment of the looming pension problem burdening several continental European economies. For example, a burgeoning gap between future pension outlays and tax revenues resulting from increasing life expectancy of the population, so the number of retirees rises relative to the number of workers, would require an increase in the payroll tax rate at some point in the future if public debt is not to grow unboundedly. The expectation of a build-up of public debt as pension liabilities increase would have a Feldstein-Rubin-Summers effect of depressing current asset prices and contracting employment. When payroll taxes are ultimately increased to ensure fiscal solvency, the $v^f$ curve, hence the cost curve, will shift up and reduce quasi-rent from that time onwards. The anticipation of reduced aggregate quasi-rents on the future stock of customers will lead to a decline of asset prices today. For this reason as well, firms are led to raise their current markups and contract employment.

REFERENCES


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FIGURE 1. ASSET PRICES, INTEREST RATES AND EMPLOYMENT
LHS of eq.(17),
RHS of eq.(18)

RHS of eq.(18)

LHS of eq.(17)

\[ \ddot{q} \equiv \frac{\ddot{q}}{\ddot{C}} \]

FIGURE 2. DETERMINATION OF $\ddot{q} / \ddot{C}$

\[ \Delta \]

3

1

2

\[ q \]

\[ r \]

Private Wealth

FIGURE 3. METZLER DIAGRAM OF WEALTH AND INTEREST IN A NON-MONETARY MODEL

\[ \dot{q} = 0 \]

\[ \dot{D} = 0 \]

\[ \dot{q} \]

\[ D \]

FIGURE 4. SADDLE-PATH STABILITY
FIGURE 5. "BAD NEWS" CASE

FIGURE 6. "GOOD NEWS" CASE