A Structuralist Model of the Small Open Economy in the Short, Medium and Long Run

Hian Teck Hoon
Edmund S. Phelps

Discussion Paper #:0102-25

Department of Economics
Columbia University
New York, NY 10027
March 2002
A Structuralist Model of the Small Open Economy in the Short, Medium and Long Run

Abstract

In open-economy macroeconomics there is a monetary model in the Keynesian tradition that is deemed serviceable for analyzing the short run and there is a nonmonetary neoclassical theory thought capable of handling the long run. But do the Keynesian and neoclassical models meet the challenges thrown out by the main events of the past few decades—the ’80s shock to Europe taking the form of an external jump in real interest rates; the sort of shock experienced in the U.S. and parts of northern Europe in the second half of the ’90s: the emerging prospect of new industries in the future creating increased needs for capital; and what may have been a major shock of the ’60s: the large Kennedy tax cut, mostly the reduction in income taxes, enacted in the U.S. in 1964? We suggest that the effects of these shocks on the open economy are not well portrayed by either the standard Keynesian model or by standard neoclassical theory. We then provide a careful development of a nonmonetary model of the equilibrium path of the real exchange rate, and natural output, employment and interest based on trading frictions in the goods market, and compare its implications not only over the medium run but for the short run and the long run as well. (*JEL* E24; F3; F4)

In open-economy macroeconomics there is a monetary model in the Keynesian tradition that is deemed serviceable for analyzing the short run (Robert Mundell, 1962, 1963; Rudiger Dornbusch 1980) and there is a nonmonetary-
tary neoclassical theory thought capable of handling the long run (Olivier Blanchard and Stanley Fischer, 1989; João Ricardo Faria and Miguel León-Ledesma, 2000).¹ For years the weak point in this arsenal was agreed to be the medium run (Edmond Malinvaud, 1994, 1996). This run, which follows the short-run adjustment of production, hiring and training, advertising and other investment rates, is a period of adjustment for the various business assets, such as customers and trained employees as well as plant, during which nationals’ private wealth holdings and social entitlements are regarded as constant—a period we will think of as emerging in the second year following a shock and running for half a decade or so. By now, several dynamic nonmonetary models of a “structuralist” possessing such a medium run have emerged (Edmund Phelps, 1988a, 1988b; Hian Teck Hoon and Phelps, 1992; Phelps, 1994; Maurice Obstfeld and Kenneth Rogoff, 2000). At present, though, the structuralist models tend to be seen as niche products offering no competition to the established short-run and long-run models.

But do the Keynesian and neoclassical models meet the challenges thrown out by the main events of the past few decades? We suggest that the effects of these shocks on the open economy are not well portrayed by either the standard Keynesian model or by standard neoclassical theory. Consider the

¹Carlos Rodríguez (1979), in addition to studying the short-run effects, also studies the long-run effects of monetary and fiscal policies in an open-economy Keynesian model under flexible exchange rates where the stock of foreign assets has fully adjusted to the stream of induced short-run international capital flows. This approach, however, suffers from the weakness that it assumes nominal wage or price-level stickiness even in the long run.
‘80s shock to Europe: an external jump in real interest rates. The Mundell-Fleming model and its descendants, applied to fluctuating-exchange-rate economies such as the US, EU and Japan, predict that a rise in the overseas interest rate interacts with the home country’s supply of liquidity, or LM curve, to cause a release of liquidity fueling an increase of output and employment; in the usual extension, employment would gradually be forced back to its fixed natural level. Neoclassical theory would depict interaction of the interest rate increase with the supply of labor, which would (or could) cause an increase in the work week and possibly in the labor force. In fact, European employment went into a huge decline in the ‘80s and by 2000, nearly 20 years later, unemployment rates had hardly recovered at all except in those countries that caught the internet boom of the late 1990s or implemented economic reforms.

Consider next the sort of shock experienced in the U.S. and parts of northern Europe in the second half of the ‘90s: the emerging prospect of new industries in the future creating increased needs for capital—as a macroeconomic approximation, an anticipated future shift in the productivity parameter (see Phelps and Glyfi Zoega, 2001). The Mundell-Fleming model could only represent such an event as an increase in the marginal efficiency of investment, thus a shift of the IS curve, but such a disturbance would appreciate the currency by so much as to leave little or no rise in domestic interest rates and consequent release of liquidity, thus little or no rise in gross domestic output and employment. Neoclassical theory would depict a jump in the demand for leisure (and pro tanto a jump in the real wage), in parallel to a jump in consumer demand, and hence a decrease in the work
week and possibly a decrease in labor force participation. In fact, from 1996 to 1999 or 2000, employment zoomed, the labor force clearly rose, and hours per employed person inched up, at least in the U.S. and, it appears, in the other booming economies too.

Finally, consider what may have been a major shock of the '60s: the large Kennedy tax cut, mostly the reduction in income taxes, enacted in the U.S. in 1964.\(^2\) The Keynesian models implied that such a tax cut, in causing a real exchange rate appreciation, might fail to expand output through a net stimulus to aggregate demand but it might still be expansionary by reducing the costs of imported intermediary goods. Neoclassical theory suggested an expansion through the tax cut’s impact on the supply and demand for labor. In fact, the U.S. showed no significant real appreciation—there was at first a mild depreciation and subsequently a recovery of the exchange rate—and the labor force did not appear to rise more than might have resulted from the decline of the unemployment rate. The fact that the unemployment did decline over the decade suggests that other mechanisms or channels may have delivered the famed expansion; aggregate demand cannot be excluded (since there was no appreciation), but the decline of inflation at mid-decade speaks somewhat against heavy reliance on that channel.

We think that this somewhat disappointing performance of the Keynesian and neoclassical approaches should impel a closer look by macroeconomists into the behavior of the relatively new structuralist models to which we have

\(^2\)Personal income taxes were cut by more than 20 percent. The marginal personal income tax rates, which ranged from 20 to 91 percent before the cut, ranged from 14 to 70 percent as a result of the Revenue Act of 1964.
referred. And since this third kind of model is dynamic, or intertemporal, a careful development of such a model provides the opportunity to compare its implications not only over the medium run but for the short run and the long run as well. Here we will develop more fully than has been attempted so far the small open-economy version of the customer-market model, first introduced by Phelps and Sidney Winter (1970) and extended in a general-equilibrium way in several papers since then (Guillermo Calvo and Phelps, 1983; Nils Gottfries, 1991; Paul Krugman, 1987; and Phelps, 1994). The result is a nonmonetary model of the equilibrium path of the real exchange rate, and natural output, employment and interest based on trading frictions in the goods market.3

The paper is organized as follows. In Section I, we develop and study the properties of a model suitable for a short-run and medium-run analysis. For concreteness, we think of a medium run here as a period during which nationals’ holdings of net foreign assets are constant. (The justification is that wealth will not change by enough, and soon enough, to influence greatly the early responses of the jumpy variables and the growth or decline of the customer stock.) More precisely, as we will be examining the three shocks discussed above under the simplifying assumption that the small open economy in question is initially neither a net creditor nor net debtor, the level of nationals’ holdings of net foreign assets held constant in the short- and

3Charles Engel and John Rogers (2000) show empirically that nominal price stickiness, in conjunction with fluctuating nominal exchange rates, alone is not sufficient to explain why real exchange rates fluctuate so dramatically. Trading frictions in goods markets are also empirically important.
medium-run analysis is zero. We will establish conditions under which a unique perfect foresight path exists in a $3 \times 3$ dynamic system with the stock of customers as a slow-moving variable. We then move on to apply the model to analyze three economic shocks: an expectation of a future step-increase in the level of Harrod-neutral productivity parameter; an increase in the exogenously given world real rate of interest; and a permanent balanced-budget cut in the wage income tax rate. Section II then turns to the long run, and studies the steady-state effects of the three economic shocks under the assumption that the levels of net foreign assets held by nationals adjust fully. We also make some observations regarding the current account behavior of the small open economy in response to the shocks. Some concluding remarks are contained in Section III.

I. The Model in the Short and Medium Run

In the product market, firms use domestic labor only to produce for a global customer market. Initially, the firms’ stock of customers currently equals the population of nationals. The size of the domestic labor force, which is also the initial stock of customers at domestic firms, is a positive constant, both belonging to a population in a demographic stationary state. We normalize the size of the domestic labor force to one. The number of domestic firms is also taken to be fixed, and the firms are all in identical (or symmetrical) circumstances. The characteristic of the customer market in the Phelps-Winter model is the informational frictions that would impede a quick flow of customers to a firm were it to choose to post a lower price than
that being charged elsewhere. A firm setting a price always below that set by the other (identically behaving) firms would only gradually drain customers from its competitors. Symmetrically, it is supposed that a firm setting a price always above that set by the others would see an equally gradual erosion of its customer stock, as customers sought alternative suppliers (including foreign suppliers) and required varying lengths of time to find them. This is our way of modeling ‘trading costs,’ which Obstfeld and Rogoff (2000) argued to be essential for explaining a number of puzzles in open economy macroeconomics. The Law of One Price, accordingly, can be violated temporarily in equilibrium even for traded goods though it holds in steady state (Engel, 1999). This is because one country’s producers may price their goods more expensively than those of another at the price of a loss of market share over time.

In the labor market, our firms, while collectively beneficiaries of information imperfections in the product market, are victims of imperfect information in the workplace. At each firm the members of the firm’s work force suffer randomly timed and statistically independent episodes of shirking. Workers are able to shirk without certainty of being caught because monitoring is expensive and the firms cannot afford continuous monitoring. But given that the firm can monitor costlessly at times when the opportunity presents itself, workers know that an act of shirking will be detected with some positive probability. Consequently, a worker of given accumulated assets will succumb to the urge to shirk with less frequency the greater the opportunity cost of being caught and hence dismissed. The effort rate is increasing in the real wage offered by the firm where the worker is currently employed, \( w^i \), decreas-
ing in wage-income prospect of persons in the unemployment pool proxied by the product of the employment rate (that is, one minus the unemployment rate) and the expected market wage, \((1 - u)v^e\), and decreasing in the worker’s average independent income, \(y^w\). (Note that if we conceive of the wage rate as real wage—measured in domestic consumer goods, say—the corresponding average nonwage income \(y^w\) should be thought of as real nonwage income after nonwage transfers and nonwage taxes.) Making the convenient assumption that firms have the identical distribution of employees by age and wealth, so that there is nothing special about one firm’s employees, and assuming that the effort rate is homogeneous of degree zero in the three arguments, we can simply write the effort function as \(e((1 - u)v^e/y^w/v)\). We assume that \(\epsilon_1 < 0, \epsilon_2 < 0, \epsilon_{11} < 0, \epsilon_{22} < 0, \text{ and } \epsilon_{12} < 0.\) The assumptions that \(\epsilon_{11} < 0\) and \(\epsilon_{22} < 0\) ensure that the second-order conditions are satisfied while \(-\epsilon_{12} > 0\) means that the marginal effect of raising the real wage on effort (given \((1 - u)v^e\)) is lower the higher is the nonwage income.

A firm can combat the shirking propensity of its workers by offering an increased wage, raising it above the market-clearing level. As firms generally adopt the strategy of paying above-market-clearing wages, the same wage at the individual firm confers a reduced advantage—labor input and hence output costs more—so unemployment arises. For labor-market equilibrium,

---

3When we later consider a reduction in lump-sum transfer payment to match the permanent wage income tax cut, we will see that as the decrease in lump-sum transfer payment reduces the after-transfer nonwage income, the efficiency wage needed to minimize unit cost is also reduced. The transfer payment adds to nonwage income on the assumption that the worker receives it from the government whether or not he is or will be employed.
\( v^e \) at any given \( 1 - u \) is just large enough that, for all \( i \), \( v^i = v^e \): none of the identical firms sees an advantage in offering a still-higher wage to further reduce shirking. This apparatus delivers the well-known wage curve portraying the equilibrium wage as rising with \( 1 - u \). We would remark that other models of the wage curve exist, for example Christopher Pissarides (2000), which suitably formulated, would permit the same results that we obtain. We now develop the supply side of the model in the form of the labor-market equilibrium before turning to study the product and capital markets.

A. The Labor Market

Let the \( i \)th firm’s output \( z^i \) be given by \( \Lambda n^i \), where \( \Lambda \) is the index of Harrod-neutral productivity level and \( n^i \) is the number of the firm’s employees. For costs to be at a minimum at any given employment level, the wage at the \( i \)th firm must be high enough to minimize the ratio of its wage, \( v^i \), to effort at the firm, \( \epsilon ((1 - u)v^e/v^i, y^v/v^i) \). The minimization problem at such a firm may be described as \( c^i = \min [v^i/(\Lambda \epsilon ((1 - u)v^e/v^i, y^v/v^i))] \) by choosing \( v^i \).

With the labor force a constant, the number of persons supplied to the labor market per firm is a constant, \( n_s \). If \( n \) is the average employment at the other firms, then \( u \) is approximately \((n_s - n)/n_s \). The optimal choice of \( v^i \) yields the generalized Solow elasticity condition:

\[
(1) \quad - [(1 - u)(v^e/v^i)(\epsilon_1 + (y^v/y^v)(\epsilon_2/\epsilon)] = 1.
\]

Labor-market equilibrium and the similarity of firms imply that \( v^i = \)
\( v = v^\varepsilon \). Under these conditions, (1) simplifies to

\[
(2) \quad - \left[ (1 - u)\left(\varepsilon_1 \varepsilon_2 + (\frac{y^w}{v})\varepsilon_2 \varepsilon_3 \right) \right] = 1.
\]

Noting that the first and second arguments in the effort function are now given by \( 1 - u \) and \( y^w/v \), respectively, we infer from (2) that \( v/y^w \) is an increasing function of \( 1 - u \):

\[
\frac{\partial (v/y^w)}{\partial (1 - u)} = \frac{2\varepsilon_1 + (1 - u)\varepsilon_1 + (\frac{y^w}{v})\varepsilon_1 \varepsilon_2 \varepsilon_2 \varepsilon_2}{2\varepsilon_2 + (1 - u)\varepsilon_1 + (\frac{y^w}{v})\varepsilon_2 \varepsilon_2} > 0.
\]

We write

\[
(3) \quad \frac{v}{y^w} = \Phi(1 - u); \quad \Phi'(1 - u) > 0.
\]

In the absence of any taxes and transfers, and recalling that we are holding the level of net foreign assets constant at zero for a short- to medium-term analysis, a suitable normalization of the fixed number of firms at one implies that \( y^w + (1 - u)v = \Lambda e[1 - u] \). Then, using (3), we obtain

\[
y^w = \frac{\Lambda e[1 - u]}{1 + (1 - u)\Phi(1 - u)},
\]

whence \( \zeta = v/(\Lambda e) = (v/y^w)(y^w/(\Lambda e)) = [(1 - u)\Phi(1 - u)][1 + (1 - u)\Phi(1 - u)]^{-1} \). Hence, the unit cost \( \zeta \) is a monotone increasing function of the economy-wide employment rate, \( 1 - u \). Now if we limit ourselves to situations where increased output would require increased employment (and so excluding the case where the effect of reduced unemployment on effort might swamp the effect of having more persons at work), we make \( z \) a monotone increasing function of \( 1 - u \).\(^5\) Noting that \( z = \Lambda e[1 - u] \equiv \varepsilon^ax \), where \( \varepsilon^a \) is the output

\(^5\)The mathematical condition is that \( (1 - u)\varepsilon_1 - (1 - u)e_2\Phi^{-2}\Phi' + \varepsilon > 0.\)
supplied per customer and $x$ is the total stock of customers, we can write $\zeta = \Upsilon(c^*x/\Lambda);$ $\Upsilon'(c^*x/\Lambda) > 0.$ Hence the firm’s unit cost, $\zeta$, is increasing in aggregate output (relative to the measure of productivity), and there may be said to be rising “industry cost” despite constant cost at the individual firm through the wage and shirking effects of increased employment.

B. The Product Market

Our objective here is to develop a model of the small open economy in which all firms, foreign and domestic, operate in a market subject to informational frictions. In the case we are examining here, initially all the relevant customers of national firms—firms that produce only with national labor—are nationals. Although the small open economy is too small to affect perceptibly the world real rate of interest, by definition, disturbances to the demand of its national customers will certainly be felt by national firms, and so will the exchange rate and the real interest rate in terms of the good supplied by national firms and their price.

With regard to the $i$th firm, we let $x^i$, a continuous variable, denote (the size of) its customer stock; let $c^i$ denote the amount of consumer output it supplies per customer; and let $p^i$ denote its price, say, in units of the domestic good.\textsuperscript{7} We will let $p$ denote the price at the other domestic firms and $p^f$

\textsuperscript{6}The intermediate step to prove this is to use the relationship $\epsilon(1 - u, \Phi^{-1})[1 - u] \equiv c^*x/\Lambda$. Since $1 - u$ is increasing in $c^*x/\Lambda$ under our assumption, and $\zeta$ is increasing in $1 - u$, hence $\zeta$ is increasing in $c^*x/\Lambda$.

\textsuperscript{7}Expressing all prices in terms of the foreign good would not substantively alter the analysis.
the price that the firm and its customers expect is being charged by other domestic firms (all measured in units of the domestic good). We introduce a variable \( \hat{p} \), where \( \hat{p}^{-1} \) tells us how many units of the domestic good must be given up in exchange for one unit of the foreign good. Consequently, an increase in \( \hat{p} \) is a real exchange rate appreciation.

In product-market equilibrium, by definition, every firm and its customers have correct expectations about the other firms, that is, \( p = p^e \). With their expectations thus identical in product-market equilibrium, the identically situated domestic firms will then behave alike, so that \( p^i = p = p^e \).

A firm, in maximizing the value of its shares, has to strike a balance between the benefits of a high price, which are increased revenue and reduced cost, thus increased profit, in the present, and the benefits of a low price, which are an increased profit base in the future as customers elsewhere gradually learn of the firm’s price advantage. The key dynamic is therefore the law of motion of the firm’s customer stock,

\[
\frac{dx_i^j}{dt} = g\left(\frac{p_i^j}{\hat{p}_i}, \hat{p}_i\right)x_i^j; \quad g_1 < 0; \quad g_{11} \leq 0; \quad g_2 < 0; \quad g(1, 1) = 0.
\]

The joint assumption that \( g_1 < 0 \) and \( g_{11} \leq 0 \) means that the marginal returns to price concessions are nonincreasing, in the sense that successive price reductions of an equal amount by firm \( i \) yield a nonincreasing sequence of increments to the exponential growth rate of customers. The inequality \( g_2 < 0 \) implies a drain of customers to foreign suppliers when the real exchange rate appreciates. What the sign of \( g_{12} \) is relates to the question of what the effect of foreign competition on domestic firms’ market power is. Suppose that \( \hat{p} < 1 \) so there has been a real exchange rate depreciation,
hence foreign goods are selling at a premium. Then each identically situated
domestic firm is increasing its market share at the expense of foreign suppli-
erers. In such an environment, a reduction in $p_i$, given $p$, can be expected to
generate a smaller increase in the rate of inflow of customers compared to a
situation where $\hat{p} > 1$ (and each identically situated domestic firm is losing
customers to foreign suppliers). Since stiffer foreign competition (higher $\hat{p}$)
confers a higher marginal return to a price concession, firm $i$ is induced to go
further in reducing its markup, holding other things constant. In our theory,
therefore, the assumption that $g_{12} < 0$ implies that an appreciation of the
real exchange rate will lead to lower domestic markups and hence increased
output supplied due to the increased competition that domestic producers
face from foreign suppliers. We will proceed with our analysis under the
assumption that $g_{12} < 0$ but make some observations along the way about
the case where $g_{12} = 0$.

The firm has to choose the price at which to sell to its current customers.
Raising its price causes a decrease, and lowering the price an increase, in
the quantity demanded by its current customers according to a per-customer
demand relationship, $D(p_i/p, c^s)$, where $c^s$ in this context is set equal to the
average expenditure per customer, $c^d$, at the other firms. For simplicity, we
assume that $D(\cdot)$ is homogeneous of degree one in total sales, $c^s$, and so we
write $c^{s*} = \eta(p^i/p)c^s$; $\eta'(p^i/p) < 0$; $\eta(1) = 1$. Each firm chooses the path
of its real price or, equivalently, the path of its supply per customer to its
consumers, to maximize the present discounted value of its cash flows. The
maximum at the $i$th firm is the value of the firm, $V^i$, which depends upon
$x^i$:

$$V_0^i = \max \left \{ \int_0^\infty \left [ \frac{p^i_t}{p_t} - \Upsilon \left ( \frac{c^s x_t}{\Lambda} \right ) \right ] \eta \left ( \frac{p^i_t}{p_t} \right ) c^s_t x^i_t \exp^{-\int_0^t r_s ds} dt \right \}.$$ 

The maximization is subject to the differential equation giving the motion of the stock of customers of the $i$th firm as a function of its relative, or real, price and the real exchange rate given by (4) and an initial $x^i_0$. The current-value Hamiltonian is expressed as

$$\left [ \frac{p^i_t}{p_t} - \Upsilon \left ( \frac{c^s x}{\Lambda} \right ) \right ] \eta \left ( \frac{p^i_t}{p_t} \right ) c^s x^i_t + q^i_m g\left ( \frac{p^i_t}{p_t}, \hat{p} \right ) x^i,$$

where $q^i_m$ is the shadow price, or worth, of an additional customer. The first-order condition for optimal $p^i$ is

$$(5) \quad \eta \left ( \frac{p^i_t}{p_t} \right ) c^s x^i_t + \left [ \frac{p^i_t}{p_t} - \Upsilon \left ( \frac{c^s x}{\Lambda} \right ) \right ] \eta \left ( \frac{p^i_t}{p_t} \right ) c^s x^i_t + q^i_m g\left ( \frac{p^i_t}{p_t}, \hat{p} \right ) x^i = 0.$$ 

Another two other necessary first-order conditions (which are also sufficient under our assumptions) from solving the optimal control problem are:

$$d q^i_{ml} \frac{dt}{dt} = \left [ r_t - g\left ( \frac{p^i_t}{p_t}, \hat{p}_t \right ) q^i_{ml} \right ] q^i_{ml} - \left [ \frac{p^i_t}{p_t} - \Upsilon \left ( \frac{c^s x_t}{\Lambda} \right ) \right ] \eta \left ( \frac{p^i_t}{p_t} \right ) c^s x_t.$$

We now show that “marginal $q$” denoted $q^i_{ml}$ is equal to “average $q$,” which we denote as $q^i_a \equiv V^i / x^i$. Taking the time derivative of the product $q^i_{ml} x^i$, we obtain

$$\frac{d(q^i_{ml} x^i)}{dt} = q^i_{ml} \frac{dx^i}{dt} + x^i \frac{dq^i_{ml}}{dt}$$

$$= g\left ( \frac{p^i_t}{p_t}, \hat{p}_t \right ) q^i_{ml} x^i + \left [ r_t - g\left ( \frac{p^i_t}{p_t}, \hat{p}_t \right ) \right ] q^i_{ml} x^i - \left [ \frac{p^i_t}{p_t} - \Upsilon \left ( \frac{c^s x_t}{\Lambda} \right ) \right ] \eta \left ( \frac{p^i_t}{p_t} \right ) c^s x^i.$$ 

$$= r_t q^i_{ml} x^i - \left [ \frac{p^i_t}{p_t} - \Upsilon \left ( \frac{c^s x_t}{\Lambda} \right ) \right ] \eta \left ( \frac{p^i_t}{p_t} \right ) c^s x^i,$$
which we can integrate and then use (7) to obtain

\[ q^i_{m} x^i_t = \int_t^\infty \left[ \frac{p^i_k}{p_k} - Y \left( \frac{c^i_{k} x^i_{k}}{\Lambda} \right) \right] \eta \left( \frac{p^i_k}{p_k} \right) c^i_{k} x^i_{k} \exp^{-\int_t^k r_{s} ds} dk \equiv V^i_t. \]

Hence, \( q^i_{m} = q^i_a = q^i. \)

Equating \( p^i \) to \( p \), and setting \( q^i = q \), delivers the condition on consumer-good supply per firm for product-market equilibrium:

\[ \left[ 1 + \frac{\eta(1)}{\eta'(1)} - Y \left( \frac{c^i x}{\Lambda} \right) \right] = - \left( \frac{q^i}{c^i} \right) \left( \frac{g_1(1, \hat{p})}{\eta'(1)} \right); \quad \eta(1) = 1. \]

The expression in the square brackets is the algebraic excess of marginal revenue over marginal cost, a negative value in customer-market models as the firm supplies more than called for by the static monopolist’s formula for maximum current profit, giving up some of the maximum current profit for the sake of its longer-term interests. An increase in \( q \) means that profits from future customers are high so that each firm reduces its price (equivalently its markup) in order to increase its customer base. Hence lower prices in the Phelps-Winter model are a form of investment, an investment in market share. Note also the role played by the real exchange rate (\( \hat{p} \)). If stiffer foreign competition leads to reduced market power of domestic firms, then a higher \( \hat{p} \) leads domestic firms to increase their output even further beyond the point where current marginal revenue equals marginal cost as dictated by a static monopolist. This channel is absent if \( g_{12} = 0 \).

From (8), we can express consumer-good supply per customer relative to productivity, \( c^i / \Lambda \), in terms of \( q / \Lambda, \hat{p}, \) and \( x \), that is, \( c^i / \Lambda = \Omega(q / \Lambda, \hat{p}, x) \). It is straightforward to show that \( 0 < e_{q/\Lambda} = d \ln(c^i / \Lambda) / d \ln(q / \Lambda) < 1 \), \( e_{\hat{p}} = d \ln(c^i / \Lambda) / d \ln \hat{p} > 0 \) and \( -1 < e_x = d \ln(c^i / \Lambda) / d \ln x < 0 \), where \( e_j \) denotes
the partial elasticity of $c^e/\Lambda$ with respect to the variable $j$. As explained before, an increase in $q$ makes investments in customers through reducing the markup attractive and so expands output. An increase in $\hat{p}$, that is, a real exchange rate appreciation causes markups to decrease as domestic firms face stiffer competition from foreign suppliers and hence increases output and employment. Finally, with rising marginal costs, an increase in the number of customers at each firm leads to a less than proportionate decline in the amount of output supplied per customer.

C. The Capital Market

Finally we sketch the mechanisms of saving, investment and asset valuation in the capital market. Households have to plan how much of income to save, putting their savings in domestic shares; any excess is invested overseas and any deficiency implies the placement of shares overseas. Firms have to plan their accumulation of customers, issuing (retiring) a share for each customer gained (lost); any excess of customers over the domestic population implies some customers are overseas and any deficiency means that foreign firms have a share of the market. Since the stock of customers, hence shares, is sluggish, the level of the share price must clear the asset market.

In a symmetric situation across firms, (6) simplifies to

\[ \frac{[1 - \gamma(c^e + x)]e^x}{q} + \frac{\hat{q}}{q} + g(1, \hat{p}) = r, \]

where a dot over a variable denotes its time derivative. This equation in the firm’s instantaneous rate of return to investment in its stock of assets, which are customers, is an intertemporal condition of capital-market equilibrium.
it is entailed by correct expectations of $\hat{q}$, $r$ and $\hat{p}$ at all future dates. We will make the assumption that initially the shares issued by domestic firms are all held by nationals.

Drawing upon the Blanchard-Yaari model of finite-lived dynasties subject to exponential mortality, it is argued that the economy here satisfies an Euler-type differential equation in the rate of change of consumption per customer, $c^d$. Consumption growth is governed by the excess of the interest rate over the rate of pure time preference, denoted $\rho$, and by the ratio of (nonhuman) wealth, denoted $W$, to consumption. Upon setting customers’ consumption per customer equal to the output supplied to them per customer, $c^x$, we obtain

$$\frac{dc^d}{dt} = (r_t - \rho)c^d_t - \theta(\theta + \rho)W_t,$$

where $\theta$ denotes the instantaneous probability of death and $W = qx$ here. In requiring here that $q$ at each moment be at such a level as to make the path of planned consumption (its growth as well as its level) consistent with the path of output from (8), we are requiring that the market where goods are exchanged for shares (at price $q$) be in equilibrium. No household will find the prevailing share price different from what is expected.\(^8\)

Finally, for international capital-market equilibrium with perfect capital mobility, we must satisfy the real interest parity condition, which states that any excess of domestic real interest rate, $r$, over the exogenously given world real rate of interest, $r^*$, must be met by an exact amount of expected rate of interest:

$$r = \rho + \theta(\theta + \rho)(qx/c^x) + (c^x/c^x).$$

By noting that $c^x/\Lambda = \Omega(q/\Lambda, \hat{p}, x)$, we can further replace $(c^x/c^x)$ by $\epsilon_{q/\Lambda}(q/\Lambda)$ + $\epsilon_p(\hat{p}/\hat{p}) + \epsilon_x g(1, \hat{p})$.

\(^8\)From (10), we obtain an expression giving us the consumer’s required rate of interest: $r = \rho + \theta(\theta + \rho)(qx/c^x) + (c^x/c^x)$. By noting that $c^x/\Lambda = \Omega(q/\Lambda, \hat{p}, x)$, we can further replace $(c^x/c^x)$ by $\epsilon_{q/\Lambda}(q/\Lambda)$ + $\epsilon_p(\hat{p}/\hat{p}) + \epsilon_x g(1, \hat{p})$. 

17
real exchange depreciation. This equation is:

\begin{equation}
(11) \quad r = r^* - \frac{\dot{p}}{\ddot{p}}.
\end{equation}

Equations (8) to (11) give us four equations in the four variables: $c^s/\Lambda$, $q/\Lambda$, $\dot{p}$, and $x$. However, using the relation $c^s/\Lambda = \Omega(q/\Lambda, \dot{p}, x)$ derived from (8), we can reduce the system to three dynamic equations in the three variables: $q/\Lambda$, $\dot{p}$, and $x$, the last being a slow-moving variable. We proceed to do the necessary substitutions to obtain and analyze the $3 \times 3$ dynamic system, showing the conditions under which a unique perfect foresight path exists.

D. The $3 \times 3$ Dynamic System

The dynamics of the system can be described by the behavior of the endogenous variables $q/\Lambda$, $\dot{p}$, and $x$ after substituting out for $c^s/\Lambda$ using $c^s/\Lambda = \Omega(q/\Lambda, \dot{p}, x)$:

\begin{align*}
\left(\frac{\dot{q}}{q}\right) &= \left[\frac{1 + e_\dot{p}}{(1 - e_{q/\Lambda}) + e_\dot{p}}\right] f\left(\frac{q}{\Lambda}, \dot{p}, x\right) + \left[\frac{e_\dot{p}}{(1 - e_{q/\Lambda}) + e_\dot{p}}\right] h\left(\frac{q}{\Lambda}, \dot{p}, x\right), \\
\left(\frac{\dot{p}}{\ddot{p}}\right) &= \left[\frac{1 - e_{q/\Lambda}}{(1 - e_{q/\Lambda}) + e_\dot{p}}\right] h\left(\frac{q}{\Lambda}, \dot{p}, x\right) - \left[\frac{e_{q/\Lambda}}{(1 - e_{q/\Lambda}) + e_\dot{p}}\right] f\left(\frac{q}{\Lambda}, \dot{p}, x\right), \\
\left(\frac{x}{\dot{x}}\right) &= g(1, \dot{p}),
\end{align*}

where

\begin{align*}
\dot{f}\left(\frac{q}{\Lambda}, \dot{p}, x\right) &= \left[-1 + \Omega(q/\Lambda, \dot{p}, x)\right]\left[\frac{\Omega(q/\Lambda, \dot{p}, x)}{q/\Lambda}\right] + \rho + \frac{\theta(\theta + \rho)qx}{\Lambda \Omega(q/\Lambda, \dot{p}, x)} \\
&\quad - [1 + e_x] g(1, \dot{p}), \\
\dot{h}\left(\frac{q}{\Lambda}, \dot{p}, x\right) &= r^* - \rho - \frac{\theta(\theta + \rho)qx}{\Lambda \Omega(q/\Lambda, \dot{p}, x)} + e_x g(1, \dot{p}).
\end{align*}
The linearized dynamic system around the steady-state \((q/\Lambda)_{ss}, \hat{p}_{ss}, x_{ss}\), where \(\hat{p}_{ss} = 1\) and \(x_{ss} = 1\) is given by:

\[
\begin{bmatrix}
\frac{\dot{q}}{q} \\
\frac{\dot{\hat{p}}}{\hat{p}} \\
\frac{\dot{x}}{x}
\end{bmatrix}^T = A \begin{bmatrix}
\frac{q}{\Lambda} \\
1 - \frac{q}{\Lambda}
\end{bmatrix}_{ss} - \hat{p} - 1 \begin{bmatrix}
1 \\
x - 1
\end{bmatrix}^T,
\]

where \([\cdots]^T\) denotes a column vector, and the \(3 \times 3\) matrix \(A\) contains the following elements:

\[
\begin{align*}
a_{11} &= \left[ \frac{1 + e_{\hat{p}}}{(1 - e_{q/\Lambda}) + e_{\hat{p}}} \right] f_{q/\Lambda} + \left( \frac{e_{\hat{p}}}{1 + e_{\hat{p}}} \right) h_{q/\Lambda} \\
a_{12} &= \left[ \frac{1 + e_{\hat{p}}}{(1 - e_{q/\Lambda}) + e_{\hat{p}}} \right] f_{\hat{p}} + \left( \frac{e_{\hat{p}}}{1 + e_{\hat{p}}} \right) h_{\hat{p}} \\
a_{13} &= \left[ \frac{1 + e_{\hat{p}}}{(1 - e_{q/\Lambda}) + e_{\hat{p}}} \right] f_{x} + \left( \frac{e_{\hat{p}}}{1 + e_{\hat{p}}} \right) h_{x} \\
a_{21} &= \left[ \frac{1 - e_{q/\Lambda}}{(1 - e_{q/\Lambda}) + e_{\hat{p}}} \right] h_{q/\Lambda} - \left[ \frac{e_{q/\Lambda}}{(1 - e_{q/\Lambda}) + e_{\hat{p}}} \right] f_{q/\Lambda} \\
a_{22} &= \left[ \frac{1 - e_{q/\Lambda}}{(1 - e_{q/\Lambda}) + e_{\hat{p}}} \right] h_{\hat{p}} - \left[ \frac{e_{q/\Lambda}}{(1 - e_{q/\Lambda}) + e_{\hat{p}}} \right] f_{\hat{p}} \\
a_{23} &= \left[ \frac{1 - e_{q/\Lambda}}{(1 - e_{q/\Lambda}) + e_{\hat{p}}} \right] h_{x} - \left[ \frac{e_{q/\Lambda}}{(1 - e_{q/\Lambda}) + e_{\hat{p}}} \right] f_{x} \\
a_{31} &= 0 \\
a_{32} &= g_{\hat{p}} \\
a_{33} &= 0.
\end{align*}
\]

We have \(g_{\hat{p}} < 0\) as a real exchange rate appreciation leads to a flow decrease of customers (so \(\dot{x} < 0\) when \(\hat{p} > 1\)), and we can readily check that \(f_{q/\Lambda} > 0\), \(h_{q/\Lambda} < 0\), \(f_{x} > 0\), and \(h_{x} < 0\). In conjunction with the following two assumptions, we obtain signs for \(f_{\hat{p}}\) and \(h_{\hat{p}}\), which provide sufficient conditions for a unique perfect foresight path:
Assumption 1: Ceteris paribus, an increase in $\hat{p}$ raises the rate of return to holding a share in the domestic firm by raising the quasi-rent, $[1 - \Upsilon(c^{s}x/A)]c^{s}$, taken as a ratio to $q$, by more that it decreases the rate at which the customer base shrinks, $g_{p}$.

Assumption 2: Ceteris paribus, an increase in $\hat{p}$ reduces the customer’s required rate of interest through shrinking the (nonhuman) wealth to consumption ratio, $\theta(\theta + \rho)(qx/c^{s})$, by more than it increases the required interest rate through raising the growth rate of consumption, $-e_{x}g_{p}$.

Under Assumptions 1 and 2, we also have $f_{\hat{p}} < 0$ and $h_{\hat{p}} > 0$. We can then sign the elements in the matrix $A$ as follows:

Lemma 1: $a_{11} > 0$, $a_{12} < 0$, $a_{13} > 0$, $a_{21} < 0$, $a_{22} > 0$, $a_{23} < 0$, and $a_{32} < 0$.

To examine stability of the above 3 x 3 dynamical system, we evaluate its Jacobian matrix $A$ at the steady-state equilibrium, $((q/A)_{ss}, \hat{p}_{ss}, x_{ss})$, where $\hat{p}_{ss} = 1$ and $x_{ss} = 1$. The eigenvalues of $A$ are the solutions of its characteristic equation

$$(16) \quad -\gamma^{3} + \text{Tr}(A)\gamma^{2} - \text{R}(A)\gamma + \text{Det}(A) = 0,$$

9To obtain the signs of $a_{11}, a_{12},$ and $a_{13}$, we use the property that $f = r^{*} - g - [1 - \Upsilon](c^{s}/q) - h$ and apply Assumptions 1 and 2 to obtain: $a_{11} = -[(1 + e_{p})/((1 - e_{q}/A)) + e_{p})][\partial((1 - \Upsilon)(c^{s}/q))/\partial(q/A) + [1 - (e_{p}/(1 + e_{p}))]|h_{q}/A| > 0$, $a_{12} = -[(1 + e_{p})/((1 - e_{q}/A)) + e_{p})][\partial((1 - \Upsilon)(c^{s}/q))/\partial p + g_{p} + [1 - (e_{p}/(1 + e_{p}))]|h_{p}| < 0$, and $a_{13} = -[(1 + e_{p})/((1 - e_{q}/A)) + e_{p})][\partial((1 - \Upsilon)(c^{s}/q))/\partial x + [1 - (e_{p}/(1 + e_{p}))]|h_{x}| > 0$. 

20
where the trace of $A$ is $\text{Tr}(A) \equiv a_{11} + a_{22}$, the determinant is $\text{Det}(A) \equiv a_{33}(a_{21}a_{13} - a_{11}a_{23})$, and $R(A) \equiv a_{11}a_{33} - a_{12}a_{21} - a_{23}a_{32}$. Using Lemma 1, we see readily that $\text{Tr}(A) > 0$. We now show that $\text{Det}(A) < 0$. Since $a_{33} < 0$, we need to show that $a_{21}a_{13} - a_{11}a_{23} > 0$ or $[(-a_{23})/(-a_{21})] > [a_{13}/a_{11}]$. We write out explicitly the following expressions:

\[
\begin{align*}
\frac{-a_{23}}{-a_{21}} &= f_x - \frac{1}{e_{q/A}h_x} > 0 \\
a_{13} &= f_x + \frac{1}{e_{q/A}h_x} > 0.
\end{align*}
\]

Since $[1 - e_{q/A}] > 0 > -e_{p}/(1 + e_{q})$, and $-a_{23} > 0$, $-a_{21} > 0$, $a_{13} > 0$ and $a_{11} > 0$ by Lemma 1, we have $[(-a_{23})/(-a_{21})] > [a_{13}/a_{11}].$

The product of the roots of the system is given by the determinant. Since it is negative, this establishes that there are either three negative roots or one. A necessary condition for stability of a $3 \times 3$ system is that the trace of the matrix $A$ be negative. Since $\text{Tr}(A)$ has been shown to be positive, accordingly, the dynamic system represented by (15) is unstable, implying it has at least one positive root. Since we know it has either zero or two

\[10\]

Let $a = f_x > 0$, $b = -h_x > 0$, $c = f_{q/A} > 0$, and $d = -h_{q/A} > 0$. Also, let $F \equiv [(1 - e_{q/A})/e_{q/A}] > 0$ and $G \equiv [e_{p}/(1 + e_{q})] > 0$. It follows that since $a - Gb > 0$ and $c - Gd > 0$, $[a + Fb]/[c + Fd] > [a - Gb]/[c - Gd]$ if and only if $F > -G$ (a condition that is satisfied since $G > 0$) and $bc > ad$. The latter condition can be reduced to $\frac{\mu^{-1} \xi}{\zeta} < \mu - 1$, where $\mu$ is the ratio of price to marginal cost (or markup). In equilibrium, $\mu^{-1} = \zeta$. Hall (1988) uses value added as a measure of output and estimates the value of $\mu$ to range from over 1.8 to a little under 4 for the seven one-digit industries he studies, with an average value of 2.8. Using the average value of $\mu$ as a benchmark, we will require that a one percent increase in $x$ leads to a less than 1.8 percent rise in unit cost.
positive roots, it must have two. There is, therefore, a unique negative root, denoted \( \gamma_1 \), and a unique perfect foresight path that converges to the steady state exists.\(^{11}\)

Of the dynamic variables, the customer stock is constrained to move continuously at all times, while the price of the share and the real exchange rate may both jump instantaneously in response to new information. The solution of the linearized system for the behavior of the stock of customers is:

\[
\begin{align*}
\text{(17)} & \quad x_t = 1 - (1 - x_0) \exp^{\gamma_1 t}, \\
\text{whence} & \quad x_t = -\gamma_1 (1 - x_t).
\end{align*}
\]

Corresponding to any given level of the customer base are an equilibrium level of the real exchange rate and a share price. In particular,

\[
\begin{align*}
\text{(19)} & \quad \hat{p}_t = 1 + \left( \frac{\gamma_1}{\phi} \right) (x_t - 1).
\end{align*}
\]

Note that (19) describes a linear relationship between \( \hat{p} \) and \( x \) that is positively sloped. The relationship linking share price to the stock of customers

\(^{11}\)We could also have shown that there is one negative eigenvalue and two eigenvalues with positive real part by an application of the Routh theorem, which requires that the number of roots of the polynomial in (16) with positive real parts be equal to the number of variations of sign in the scheme 

\[-1 \quad \text{Tr}(A) \quad - R(A) + [\text{Det}(A)/\text{Tr}(A)] \quad \text{Det}(A).\]

With only the sign of \( R(A) \) ambiguous but \( \text{Tr}(A) > 0 \) and \( \text{Det}(A) < 0 \), there are exactly two changes in sign. Accordingly, there are one negative eigenvalue and two eigenvalues with positive real parts.
is given by\(^{12}\)

\[
\frac{q_t}{\Lambda} = \left( \frac{q}{\Lambda} \right)_{ss} + \left[ \left( \frac{\gamma_1}{\gamma_1 - a_{11}} \right) \left( \frac{a_{12}}{a_{32}} \right) + \left( \frac{a_{13}}{\gamma_1 - a_{11}} \right) \right] (x_t - 1),
\]

where the terms \(a_{ij}\) can be identified as being the appropriate elements of the matrix appearing in (15).

We note from (20) or the alternative expression appearing in footnote (12) that the linear relationship between share price \(q\) and customer stock \(x\) can be positively or negatively sloped. The economic intuition is that there are two opposing forces working to affect \(q\) as we slide upwards along the \((\hat{p}, x)\) saddle path. Suppose that, beginning at an initial steady state with \(x_{ss} = 1\) and \(p_{ss} = 1\), there is a helicopter drop of customers, so \(x\) is increased above one. As domestic interest rate is increased as a result (on account of an increased nonhuman wealth to consumption ratio, and an expected flow decrease of customers, which raises the growth rate of consumption per customer by the amount \(-e_x[x/x] = -e_x g(1, \hat{p}) > 0\), and an incipient capital inflow brings about a real exchange rate appreciation, firms are induced to reduce their markups and so increase output as competition with foreign suppliers intensifies. This acts to raise the quasi-rent earned on each customer, thus causing an increase in the share price \(q\). This effect acts to give a positive slope to the \((q/\Lambda, x)\) schedule. An increase in \(x\), however, partially crowds out the consumer supply per customer because of rising marginal costs. This effect tends to reduce the quasi-rent earned on each customer, and tends to impart a negative slope to the \((q/\Lambda, x)\) schedule. The effect of a larger stock

\(^{12}\)We could alternatively express this relationship as: \((q_t/\Lambda) = (q/\Lambda)_{ss} + [(\gamma_1(\gamma_1 - a_{22}))/a_{32a_{21}}] - (a_{23}/a_{21})(x_t - 1)\).
of customers on the value of the stock market is therefore ambiguous and
two cases have to be considered.

Nevertheless, we can ask whether empirical evidence can help us deter-
mine the more relevant case. As we shall see when we examine the shock
taking the form of an increase in the external real interest rate, the stock
market response on impact shows a decrease in share prices in the case of a
positive \((q/\Lambda, x)\) schedule but an increase in share prices in the other case.
Since empirically, we tend to observe a decline in stock market value in, say,
Asian stock markets when the US interest rate rises, it might be that the
case of a positive \((q/\Lambda, x)\) schedule is empirically more relevant.\(^{13}\) The rea-
son that the stock market may \textit{theoretically} rise in the small open economy
when there is an increase in the external real interest rate is that the re-
sulting real depreciation leads to an increased flow of customers gained from
foreign suppliers. Due to rising marginal costs, an increase in the growth
rate of customers gained implies a decline in the growth rate of consumption

\(^{13}\)It should be remarked that for this case to suhvdil, it is nhchshduy thdt
g_1^2 < 3, so
thdt stiftu i ouhign comsht lhdds to uhduchd mdukht sowhu i ou domhstic fiums1 Thh
uhddhu cdn chhck thdt whhn
g_1^2 = 3, wh hdvh
fp > 3, hp < 3 dnd so
a_1^2 > 3 dnd CJ22 < 3.
Li nhvhuthhlhss,
T
+ +
a_1^2 > 3, wh obtdin d uniquh shuih ct iouhsight sdth, dnd
thh +
(q/\Lambda, x),
schhdulh is undmbiguously nhgdtivh +shh +53,,1 Thh intuition iou thh nhgdtivh
slosh is thh i ollowing1 Sussosh thdt stduting iuom thh sthddy stdth, thhuh is d hhlicosthu
duos oi customhus so
x
incuhdshs dbovh onh1 Although thh highhu domhstic inthuh udth
buings dbout uhdl dssuhcidtion, thh highhu
p
hds no diuhct hffhct on thh consumhu sussly shu
customhu undhu thh d ssumstion thdt 512 = 3Lnsthdd thh highhu
cuowds out consumhu
sussly shu customhu, which uhduchs qudsi-uhnt1 Thh highhu discount udth dss lihd to d
uhduchd flow oi qudsi-uhnts undmbiguously uhduchs
q
14
per customer, which acts to lower the domestic rate of interest. The lower
discount rate applied to future cash flows tends to raise stock market value.

E. Analysis of Shocks

We now apply our apparatus to study the short- and medium-run effects
of three shocks: (i) an anticipation of a step-increase in the Harrod-neutral
index of productivity to occur in the future; (ii) an increase in the external
real rate of interest; and (iii) a permanent cut in the wage income tax.

Anticipation of a future step-increase in productivity level.—Consider a
small open economy initially in steady state, which is neither a net creditor
nor a net debtor, so the real exchange rate, \( \bar{p} \), equals one, and the stock
of customers buying from domestic firms include all and only nationals. At
time \( t_0 \), there is news that at some time \( t_1 \) in the future, the index of Harrod-
neutral productivity parameter will experience a permanent increase from \( \Lambda_0 \)
to \( \Lambda_1 \).

To understand the economy’s response to such a shock, it is useful to first
consider the case of an unanticipated permanent increase in \( \Lambda \). As we can
observe from the system of equations given in (12) to (14), or its linearized
version given in (15), expressed in terms of the following three variables:
\( q/\Lambda \), \( \bar{p} \), and \( x \), such a shock is neutral for the real exchange rate, the real
interest rate (which equals the exogenously given world real interest rate, \( r^* \)),
and the stock of customers. If the increase of productivity were 10 percent,
say, share price denoted \( q \) will rise immediately by 10 percent (leaving \( q/\Lambda \)
unchanged) as also will consumer expenditure and supply per customer. The
trade balance, initially equal to zero, remains unchanged.

When the same shock is expected to occur only some time in the future, however, the share price will rise initially by less than 10 percent. With increased financial wealth, domestic consumption demand, and hence aggregate demand, increases. Such increased aggregate demand imposes an increased domestic real interest rate, which brings about an incipient capital inflow that causes the real exchange rate to appreciate. To preserve real interest parity, the extent of current exchange rate appreciation must be high enough to cause an expected rate of real depreciation ($-\hat{p}/\hat{p} > 0$) that is equal to the interest premium, $r - r^*$. The stronger domestic currency increases the extent of foreign competition, resulting in domestic firms shrinking markups and so increasing supplies and expanding employment. As business asset valuation has also increased ($q/\Lambda$ is up), the profits accruing to future customers are higher so that each firm is induced to reduce markup on this score.$^{14}$ In the short run, therefore, the small open economy experiences a structural boom.

To study the medium-term adjustment, it is useful to develop a diagram (see Figure 1 for the case of a positively-sloped $(q/\Lambda, x)$ schedule and Figure 2 for the case of a negatively-sloped $(q/\Lambda, x)$ schedule). We will focus our discussion here by making reference only to Figure 1. It is clear that the final rest point coinciding with the medium run analysis (point $X$ in Figure 1) is also the original point we started the analysis with. Upon receiving the news of a future productivity increase, $\hat{p}$ jumps up from point $X$ to point $Y$ in the upper panel of Figure 1, and $q/\Lambda$ similarly jumps up in the

$^{14}$This result is obtained whether the $(q/\Lambda, x)$ schedule is positively or negatively sloped.
lower panel of Figure 1. At time $t_1$ when the productivity increase actually takes place, the economy must be back on the respective saddle paths, point $Z$ in the lower and upper panels of Figure 1. In the interim before the actual productivity increase takes place, that is, in the pre-surge period, the real exchange rate (which had initially appreciated upon receiving the news) gradually weakens so that the algebraic flow loss of customers (recall $\dot{x} = g(1, \hat{p})x; \ g_p < 0$) also gradually falls. At a time before $t_1$ (when the productivity increase actually takes place), the real exchange rate in fact falls below one and the economy begins to regain customers ahead of the actual productivity increase. Meanwhile, the stock market enjoys a bull run. What happens to employment in the interim before $t_1$? We see that employment, which had jumped up initially, continues to be pulled up by a rising share price (relative to the productivity parameter) but gradually pulled down by a gradual end to the real appreciation, and by the loss of customers.

What is unambiguous is what happens to employment at $t_1$ when the productivity increase actually materializes? Employment at that point must experience an abrupt decline (so the unemployment rate suddenly jumps up) as an unchanged output is now produced by a smaller number of workers. In fact, at point $Z$ in the upper and lower panels of Figure 1, which corresponds to the arrival of the productivity surge, employment must be below the initial level. Consequently, a structural slump takes place in the post-surge period although recovery gradually takes place as market share is regained that was lost to foreign suppliers in the pre-surge period. At the medium-run rest point, employment is back to where it was originally so the recovery is complete. We leave it to the reader to trace the implied path of employment
corresponding to the dynamics shown in Figure 2.

An increase in the external real interest rate.—To analyze what happens to the small open economy in response to the rise of the external real interest rate, it is useful to state the conditions satisfied in the medium-run steady state. The relevant equations are:

\[
(21) \quad \left[ 1 + \frac{\eta(1)}{\eta'(1)} - \Upsilon \left( \frac{c^s x}{\Lambda} \right) \right] = -\left( \frac{q}{c^s} \right) \left( \frac{q_1(1,1)}{\eta'(1)} \right) ; \quad \eta(1) = 1
\]

\[
(22) \quad r^* = \frac{[1 - \Upsilon \left( \frac{c^s x}{\Lambda} \right)] c^s}{q}
\]

\[
(23) \quad r^* = \rho + \theta (\theta + \rho) \left( \frac{q x}{c^s} \right),
\]

where use has been made of the fact that in the steady state, \( r = r^* \) and \( \hat{\rho} = 1 \). Equation (21) gives us our consumer supply equation, (22) states the equality between the rate of return to holding a share and the world real interest rate, and (23) states the equality between the world real interest rate and the consumer’s required rate of interest. This is a system of equations in the three variables, \( q/c^s \), \( c^s/\Lambda \) and \( x \).

From (21), we note that \( c^s x/\Lambda \) is a monotone increasing function of \( q/c^s \) as an increase in the latter induces firms to reduce their markups and hence increase output. Substituting out for \( c^s x/\Lambda \) in (22) using this function, we see that \( q/c^s \) is uniquely pinned down by the external rate of interest, \( r^* \). In particular, an increase in \( r^* \) reduces \( q/c^s \) and causes firms to increase their markups. Using this result in (23), we can infer that in the medium-run steady state, an increase in \( r^* \) reduces \( q/c^s \) and necessitates a rise in \( x \) to equate the consumer’s required rate of interest to the now higher external
interest rate.

The sudden increase in the external real rate of interest above domestic interest rate causes an outflow of capital, which leads to an immediate real depreciation. As domestic firms are shielded from foreign competition, they are induced to raise their markups and shrink supplies. Consequently, employment falls on this account. If we take the empirically relevant case of a decline in business asset valuation in response to a higher external real interest rate, there is a further cutback on firms’ supplies as firms’ reduced valuation of an additional customer induces them to raise their markups, raising profits in the near term. This case is depicted in Figure 3.  

How does employment adjust in the medium term? As market share is gradually increased, employment slowly recovers. Furthermore, the recovery is aided by a gradual appreciation as the real exchange rate slowly increases back to the value of one, and the share price, \( q \) slowly recovers in the empirically relevant case. But will the recovery be complete in the sense that at the medium-run rest point, employment is back to the pre-shock level? The answer is no, so the rise in external real interest rate permanently raises the equilibrium rate of unemployment. To see this, recall that we have shown

\[ q \] 

If the \((q/A, x)\) schedule is negatively sloped (not shown), it is implied that \( q \) initially jumps up. This is because the real depreciation (decline in \( \hat{p} \) from a value of one) causes \( x \), which was originally zero, to turn positive as overseas customers are gained. Since an increase in \( x \) causes the amount of consumer supply per customer to decline due to rising marginal costs, the domestic interest rate is reduced by the amount \( e_x [x/x] \equiv e_x g(1, \hat{p}) > 0 \) as \( \hat{p} \) falls below one. The lower discount rate applied to future profits increases the valuation of a customer and acts to offset the tendency for \( q \) to decrease on account of reduced quasi-rents per customer, \(|1 - \tau| c^a\), brought about by a real depreciation.
above that in the medium-run steady state, \( q/c^s \), is reduced as a result of a higher \( r^* \). This implies that \( c^s x/\Lambda \equiv \epsilon(1-u) \) is decreased as firms raise their markups permanently. A plot of the employment rate time path shows that it is permanently shifted down. We also take note that \( q \) (equivalently \( q/\Lambda \) as \( \Lambda \) here is held fixed) also does not fully recover back to its original level. To see this, we write

\[
\frac{q}{\Lambda} = \left( \frac{1}{x} \right) \left( \frac{q}{c^s} \right) \left( \frac{c^s x}{\Lambda} \right).
\]

Noting that the last bracketed term, \( c^s x/\Lambda \) is monotone increasing in \( q/c^s \), it is clear that a rise in \( r^* \) leads to a fall in \( q/\Lambda \) in the medium-run steady state since \( x \) is increased and \( q/c^s \) is decreased.

**A permanent reduction in wage income tax rate.**—To study the effects of a permanent cut in the wage income tax rate, let us now suppose that initially a wage income tax is applied to finance a lump-sum transfer that the worker receives from the government whether or not he is or will be employed. Let the amount of the lump-sum transfer per member of the labor force be equal to \( y^s \). The real wage in terms of domestic output received by the worker, \( v^h \), is related to the labor cost to the firm of a worker, \( v^f \), by \( v^f = (1+\tau)v^h \), \( \tau \) being the proportional payroll tax rate. Our nonwage income, \( y^w \), now includes a positive level of the government transfer.\(^{16}\) Solving the cost minimization problem of the firm now leads to the supply-wage locus:

\[
\frac{v^h}{y^w} = \Phi(1-u); \, \Phi'(1-u) > 0.
\]

\(^{16}\)In the basic model studied above, we had assumed for the sake of simplicity zero taxes and transfers.
With $y^w$ now including a positive level of government transfer, we now have the accounting relationship $y^w - y^s + (1 - u)v^f = \Lambda \epsilon [1 - u]$. Using (24) in this accounting relationship, we derive

$$y^w = \frac{\Lambda \epsilon [1 - u] + y^s}{1 + (1 - u)(1 + \tau) \Phi(1 - u)},$$

whence the unit cost, $\zeta$, can be written as

(25) \hspace{1cm} \text{unit cost} \equiv \frac{v^f}{\Lambda \epsilon} = \Phi(1 - u) \left[ \frac{(1 - u) + \left( \frac{\epsilon}{\Phi(1 - u)} \right)}{(1 + \tau)^{-1} + (1 - u) \Phi(1 - u)} \right].

With balanced budget, the level of government transfer per member of the labor force is related to the payroll tax rate, $\tau$, by

(26) \hspace{1cm} \tau v^h (1 - u) \equiv \left( \frac{\tau}{1 + \tau} \right) v^f (1 - u) = y^s.

Using the government budget constraint given in (26), we can obtain, after some tedious steps, an expression for $y^s/\Lambda \epsilon$ appearing in (25):

(27) \hspace{1cm} \frac{y^s}{\Lambda \epsilon} = \tau (1 - u) \left[ \frac{(1 - u) \Phi(1 - u)}{1 + (1 - u) \Phi(1 - u)} \right].

Using (27) to substitute out for $y^s/\Lambda \epsilon$ in (25), we see that at given employment rate, $1 - u$, a balanced-budget reduction in the payroll tax rate, $\tau$, reduces the unit cost, $\zeta$. On the proviso made earlier that increased output would require increased employment, we establish that $\zeta = \Upsilon(c^s x / \Lambda, \tau); \ U_1 > 0; \ U_2 > 0$.

The set of conditions representing the medium-run steady state with a positive payroll tax, which finances a government transfer in a balanced budget manner, is represented by

(28) \hspace{1cm} \left[ 1 + \frac{\eta(1)}{\eta'(1)} - \Upsilon\left( \frac{c^s x}{\Lambda}, \tau \right) \right] = -\left( \frac{q}{c^s} \right) \left( \frac{g(1, 1)}{\eta'(1)} \right); \ \eta(1) = 1
Suppose that the small open economy, initially in a steady state with payroll tax rate equal to $\tau_0$, imposes a permanent tax cut with the payroll tax rate reduced to $\tau_1$, $\tau_1 < \tau_0$, and lump-sum transfers are correspondingly reduced to balance the budget. We see from (28) and (29) that such a permanent tax cut leaves $q/c^s$ unchanged from its initial level and hence the equilibrium markup unaffected. The reason is that as a result of the tax cut, the unit cost is reduced at any given level of a firm’s output (equivalently, at a given rate of employment) as workers are encouraged to put in more work effort or to shirk less on account of a higher take-home pay relative to nonwage income. (In effect, the tax cut shifts down the wage curve in the $(v, 1 - u)$ plane.) The increased work incentive, at any employment rate, means that firms’ profits are up, which leads to increased hiring. Normal profits are restored only when the increased employment and consequent increase in each firm’s output pulls up unit cost back to its original level.

From (30), we see that with $q/c^s$ unchanged, the stock of customers, $x$, is also unchanged. Since we know that employment jumps up, consumer supply per customer, $c^s$ must have increased. We want to prove that consumer demand per customer $c^d$, which was equal to consumer supply per customer $c^s$ initially, must jump up by exactly the same amount. Note that in the Blanchard-Yaari set-up, consumer demand per customer is given by $c^d = (\theta + \rho)[H + W]$, where $H$ is human wealth and $W \equiv qx$ is financial wealth. Now, human wealth can, at the initial steady state, be expressed as $H = [(v^h(1 -
Using the government budget constraint, \( \tau v^h (1 - u) = y^s \), and the relation \( v^f \equiv (1 + \tau) v^h \), allows us to write human wealth as \( H = v^f (1 - u) / (r^* + \theta) \). Dividing and multiplying by \( \Lambda \epsilon \) allows us to express \( H \) in terms of unit cost, \( \zeta = \Upsilon(c^s x / \Lambda, \tau) \), that is, \( H = \Upsilon(c^s x / \Lambda, \tau) c^s x / (r^* + \theta) \).

Consequently, we can express consumer demand per customer as

\[
(c^d = (\theta + \rho) \left[ \frac{\Upsilon(c^s x / \Lambda, \tau) c^s x}{r^* + \theta} + qx \right]).
\]

Recall that the tax cut leaves the unit cost unchanged, and \( q \) increases by the same proportion as \( c^s \) increases, since \( q/c^s \) is unchanged. With \( x \) unchanged, if \( c^d \) equals \( c^s \) initially, it must rise by the same proportion as \( c^s \) rises. The end result is that employment is expanded without a need for any real exchange rate adjustment since human wealth and financial wealth rise by the same proportion to cause consumer demand per customer to rise and match the higher consumer supply brought about by increased employment. As Figure 4 shows, \( q/\Lambda \) jumps up with no change in \( x \) and \( \bar{p} \).

II. The Long Run

For a medium-run analysis, we held nationals’ holdings of net foreign assets constant. Now, we introduce explicitly into our analysis another variable, the private holdings of net foreign assets, denoted \( F \), which we allow to adjust to economic conditions. Like the stock of customers, \( x \), the holding of net foreign assets, \( F \), is a slow-moving variable. Around the steady state, where the economy is initially neither a net creditor nor a net debtor so \( F = 0 \), an additional equation to be introduced (in terms of the unit cost
\(\zeta = \Upsilon(c^x/\Lambda)\) and supposing zero taxes and government transfers) is

\[
\dot{W} = rW + \Upsilon\left(\frac{c^x}{\Lambda}\right)c^x - c^d; \ W \equiv F + qx.
\]  

We focus on the steady state and we are interested to ask how the employment rate, \(1 - u\), and the level of net foreign assets, \(F\), behave in the long run. Once we allow explicitly for the level of net foreign assets to adjust, it turns out convenient to use the Blanchard-Yaari equation, which can be expressed as \(r^* = \rho + \theta(\theta + \rho)W/c^d\). Using the consumption function used earlier, and the definition of human wealth, we obtain

\[
r^* = \rho + \frac{\theta}{1 + \left(\frac{v}{y^w}\right)(1 - u)}; \quad y^w \equiv (r^* + \theta)(F + qx).
\]

Note that nonwage income, \(y^w\), includes interest earnings (payments) on net foreign assets. Using (33), we obtain a relationship that is a hyperbola in Figure 5. Along such a schedule, the level of net foreign assets adjusts to achieve capital market equilibrium. The other positively-sloped schedule describes labor-market equilibrium and is given by (3). Their intersection determines the long-run employment rate, \(1-u\). Algebraically, to determine long run employment we can substitute out for \(v/y^w\) in (33) using (3) to obtain:

\[
r^* = \rho + \frac{\theta}{1 + \Phi(1 - u)(1 - u)}.
\]

To determine the long-run level of net foreign assets holding, we use the relationship

\[
\frac{F}{\Lambda} = \left[\frac{r^* - \rho}{(r^* + \theta)(\theta + \rho - r^*)}\right] \left[\frac{c^x}{\Lambda}\right] \Upsilon\left(\frac{c^x}{\Lambda}\right) - \left[\frac{q^x}{\Lambda}\right].
\]
Equation (34) tells us straightforwardly that if the economy starts off initially at the long-run steady-state position, the productivity shock leaves the employment rate unaffected in the long run even after holdings of net foreign assets have fully adjusted. The long-run natural rate of unemployment is neutral to the anticipated surge of productivity even though unemployment adjusts in the short and medium run. From (21) and (22), which continue to apply in the long run, \( q/c^* \) is unchanged. Using this result in (35) shows that with \( F = 0 \) initially, the small open economy returns to that position in the long run. Thus the economy runs a deficit on its trade balance initially as consumption jumps up in anticipation of the future surge in productivity. At some stage, however, the trade balance moves into surplus in such a way that its present value equals the present discounted value of the trade deficits incurred.

Equation (34) also tells us that a higher external rate of real interest leaves the natural rate of unemployment higher in the long run after holdings of net foreign assets have fully adjusted. (In terms of Figure 5, an increase in \( r^* \) shifts the downward-sloping schedule towards the origin.) Hence the employment rate never fully recovers even in the long run. As to the level of net foreign assets holding in the long run, an examination of (35) tells us that it is ambiguous. The increase of \( r^* \) reduces both \( qx \) and \( e[1 - u] = c^*x/\Lambda \).

What about the long-run effects of the permanent wage income tax cut after holdings of net foreign assets have fully adjusted? From (28) and (29), which continue to apply to the long run, \( q/c^* \) is unchanged, so the equilibrium markup is unchanged. This implies that each firm’s output and employment expand. To see that the holding of net foreign assets, which was zero to begin
with remains at zero, we write the Blanchard-Yaari equation as follows:

\[
(36) \quad r^* = \rho + \frac{\theta}{1 + \frac{T(\frac{\epsilon^s}{r} \frac{\epsilon^s}{r})}{(r^* + \theta)(\frac{\epsilon^s}{r} + \frac{\epsilon^s}{r})}}.
\]

We see from (36) that since the permanent tax cut leaves unit cost unchanged and \(q/c^s\) unchanged, if \(F\) were initially zero, it remains at zero. Hence the permanent wage income tax cut leaves the natural rate of unemployment permanently lower in the long run.

III. Concluding Remarks

Much effort in open-economy macroeconomics in the past three decades or so has been directed at providing better microfoundations for the Mundell-Fleming model. Two prominent papers here are the celebrated Dornbusch exchange-rate overshooting model (Dornbusch, 1976), which replaced static expectations of the exchange rate with rational expectations, and the Obstfeld-Rogoff Redux model (Obstfeld and Rogoff, 1995), which provided better microfoundations by introducing imperfect competition and nominal rigidities into a dynamic general equilibrium model.\(^{17}\) As noted by Rose (2000) and Dornbusch (2000), however, there are very small differences between the results of the fortified models and the original model, a point often acknowledged by the authors themselves, who are “at pains to show how little their work diverges from the predictions of Mundell and Fleming” (Rose, 2000, p. 17).

\(^{17}\)The recent survey of open-economy macroeconomics by Lane (2001) suggests that research in the past few years has mainly aimed at bringing further refinements to the Obstfeld-Rogoff Redux model.
Consequently, the effects of three major shocks of the past few decades are no better described by the second-generation models than by the original version.

Structuralist models such as the one here provide an altogether different paradigm for thinking about problems in open-economy macroeconomics. Even if the standard assumptions are made—perfect international capital mobility, rational expectations, prominence of demands and a Q theory of investments—the propagation mechanism through which shocks work their effects requires a fundamental shift in mindset. When we bring in trading frictions in the goods market (Phelps and Winter, 1970 and Obstfeld and Rogoff, 2000), the shifts in demands, such as shifts in consumer demand and shifts in government purchase of labor or the consumption good, have strong employment effects but they work via supply effects of the real interest rate and business asset prices. The path of the natural rate of unemployment is displaced as a result. The shocks also disturb the equilibrium path of the real exchange rate as trading frictions in the goods market lead to a deviation from the Law of One Price in the short to medium run. The structuralist model is analytically more complex, as the dimensionality of the problem increases with the addition of new state variables, and asymmetric information problems plague the labor market, so future research may require carefully devised calibration techniques. Yet, we believe that the gain in economic insight from further development of the structuralist approach is huge.

Notwithstanding the increased complexity of our analysis, we suggest that the essence of the model’s predictions of the responses to the major shocks
of the past few decades can be conveyed intuitively. Consider the shock experienced in the U.S. in the second half of the ’90s, which we liken to the sudden expectation of a future surge in productivity. In our theory, this anticipation causes a boom in the stock market, which drives up consumer demand and domestic interest rate. An incipient capital inflow leads to an appreciation of the real exchange rate and a worsening trade balance. Because domestic firms then face stiffer competition from foreign suppliers, they are induced to reduce their markups, and consequently expand output and employment. On top of this, the increased valuation of business assets means that profits from future customers are high so that each firm lowers its current price and increases supply. The resulting economic boom is non-inflationary. Our theory also suggests that fiscal shocks—in the form of tax changes and welfare entitlements—have first-order effects on the supply side. The Kennedy permanent income tax cut produced an expansion of employment without any real appreciation largely because the wage-incentive effect on workers’ behavior increased employment and hence output to match the increased consumption stimulated by the tax cut. The increase in the external real interest rate in the ’80s facing the European economies caused an outflow of capital and brought about a real exchange rate depreciation there. Being shielded from foreign competition, European firms raised their markups and cut back supplies, consequently contracting employment.
Figure 1: Response to anticipated future step-increase in $\Lambda$

(case of positively-sloped $(q/\Lambda,x)$ schedule)
Figure 2: Response to anticipated future step-increase in $\Lambda$

(case of negatively-sloped $(q/\Lambda , x)$ schedule)

Figure 3: Response to a rise in $r^*$
Figure 4: Response to a permanent wage income tax cut
Figure 5: Long-run determination of employment
References


_____.


_____.


_____.

40


