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Economy with Financial Intermediaries and  
Costly Monitoring**

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**SOCIAL SECURITY AND EQUITY INVESTMENT IN AN ECONOMY  
WITH FINANCIAL INTERMEDIARIES AND COSTLY MONITORING**

**by**

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**Abstract:** This paper aims at extending the analysis of the efficiency of equilibria in an OLG framework with asymmetric information. It focuses on the stationary states of an economy where consumers, firms and financial intermediaries are at work. The process of financial intermediation is affected by ex-post moral hazard due to costly state verification; for this reason, the introduction of social security might be Pareto improving in a market economy even when the economy is dynamically efficient. Moreover, market outcomes are socially inefficient, even when a weaker notion than Pareto optimality is considered. A full characterization of a Constrained Pareto Optimum (CPO) shows that this allocation can never be induced as the result of an optimal policy in a market economy.

**Keywords:** Financial intermediaries, social security, costly state verification.

**J.E.L. classification numbers:** E44, E62.

## **Social security and equity investment in an economy with financial intermediaries and costly monitoring<sup>1</sup>**

### **1. Introduction**

As is well known, competitive equilibria of OLG economies may be inefficient even when markets are complete. The competitive equilibrium of a productive economy may not be Pareto optimal because of overaccumulation of capital (Diamond, 1965): when population growth is higher than the marginal product of capital, the economy is said to be dynamically inefficient. In this case, the introduction of either a tax-transfer scheme, as a pay as you go social security system, or public debt allows for an increase of welfare in the economy. However, if the economy is dynamically efficient, the same policy is not Pareto improving, as it increases welfare of the current old generation at the expense of the young and future ones.

When uncertainty and market incompleteness are present in the economy, intergenerational transfer policies have also the role of providing risk sharing services. Young agents invest in a risky productive activity to be able to consume when old. In absence of a mutual fund, social security or public debt provide a risk-free way to transfer wealth from the first to the second period of life. The same task could as well be undertaken

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by financial intermediaries, which collect deposits and lend to a large number of firms, thereby perfectly diversifying risk.

This paper aims at extending the analysis of the efficiency of equilibria in an Overlapping Generations framework with asymmetric information. More precisely, we study the stationary states of a simple Overlapping Generations Model where consumption, production and financial intermediation are present. Consumers allocate their disposable wealth between equity shares and (bank) deposits, and firms have access to a random technology, whose input can be financed either by selling shares to consumers or by loans. The government regulates the economy by establishing a *pay as you go* social security system.

The economy is affected by ex-post moral hazard due to costly state verification: in other words, output is freely observable only by firms, while lenders have to pay a "monitoring" cost, proportional to the amount lent, to observe its realization. As in Diamond (1984), costly state verification would allow to derive endogenously the existence of financial intermediaries (banks) acting in the economy as delegated monitors of lenders. However, we simply assume that direct lending is not available, and that firms can only borrow from banks. As a result the cost of monitoring affects the deposits interest rate, which in the model is strictly less than the expected return on shares.

Because of the costs associated to financial intermediation, the introduction of social security may be welfare improving even when the economy is dynamically efficient<sup>2</sup>: when monitoring is too costly, in fact, the government should substitute the risk-sharing activity provided by financial intermediaries with a social security scheme. However, when the

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<sup>2</sup> Terminology here could be misleading for those who like to think of a dynamically efficient economy as a Pareto optimum; in my model, the first best solution can not be derived by the market. In the present context, dynamic efficiency means that the marginal product of capital is higher than population growth.

expected net return on capital is high, social security should be optimally set at zero. The reason for this result is that, in a market economy, social security and bank deposits develop essentially the same role. A benevolent government should decide to use only the instrument whose returns allow higher consumption opportunities.

Since the economy is affected by asymmetric information, the inefficiency of competitive equilibria may also be related to factors which can be separated from market incompleteness. For this purpose, we define a notion of Constrained Pareto Optimality, that assumes that market incompleteness cannot be eliminated and that the planner is subject to the same information structure of the individuals in the market economy: We show that, in the case of a productive economy, the constrained Pareto optimum (CPO) and the competitive equilibrium are characterised by different allocations of the instruments. Since the social planner cares only about the expected utility of the representative consumer, he (or she) realizes that, because of the monitoring costs associated with the private financial system, banks' profit maximizing behaviour is privately efficient, but socially inefficient. Competitive banks' best response to firms declaring bankruptcy is to adopt stochastic verification procedures and a severe punishment for dishonest firms. However, the planner internalises the externality associated to the fact that firms are owned by consumers and act in the interest of the latter. From a social point of view, it is never optimal to punish firms too intensively. At a CPO, firms are monitored everytime they declare bankruptcy, but the level of punishment is lower than the one chosen by competitive banks. The banks' profit maximizing behaviour is then responsible for the inefficient outcome of the competitive equilibria.

Previous papers on related topics are Azariadis-Smith (1993) and Reichlin-Siconolfi (1996). They are both dynamic general equilibrium models with asymmetric information; however, they deal with adverse selection issues and discuss equilibria which may involve

credit rationing. Even though the model presented here draws on Reichlin and Siconolfi (1996), we introduce another source of asymmetric information, ex-post moral hazard due to costly monitoring. In this sense, this paper allows to extend the previous literature and to have a full characterization of both market equilibria and the CPO allocation with state verification costs.

The paper is organised as follows. In section 2, we present the model and describe the partial equilibrium financing game played between representative firms and banks. The general equilibrium of the economy is analysed in section 3, while section 4 studies optimal government policies. Section 5 deals with Constrained Pareto Optimality, and 6 concludes.

## **2. The model**

I study the stationary states of a simple general equilibrium OLG model in which consumers, firms, banks and the government are present.

There is a continuum of identical consumers, uniformly distributed on the  $[0,1]$  interval. They live for two periods, and population is assumed to be stationary. Each consumer is endowed with  $W$  units of a non storable physical good when young, but she likes to consume only when old. When young, consumers are taxed by an amount  $T$  by the government, who transfers the revenues to currently old people. In the first period of their lives, consumers face only a portfolio problem: they have to choose how to allocate their after-tax endowment ( $W-T$ ) in two existing assets; more precisely, consumers can buy equity shares by endowing firms with some amount of capital  $K$ , or they can buy deposits ( $D$ ) supplied by financial intermediaries (banks).

Equity shares pay a state contingent rate of return, which depends on the outcome of production; I assume that consumers cannot diversify their equity investment among different firms, but can only buy shares of one single firm. This assumption is an extreme

version of a capital market imperfection which prevents consumers to enter an ideal mutual fund able to completely diversify its portfolio.<sup>3</sup>

A continuum of ex-ante identical firms is uniformly distributed on the  $[0,1]$  interval. Each firm starts with no endowment but has access to a random technology such that  $X$  units of capital input return  $\alpha X$  units of output with probability  $q$ , and  $0$  with probability  $1-q$ , with  $\alpha > 1$ <sup>4</sup>. Investment returns are i.i.d. across firms.

When firms are endowed with  $K$  units of capital from households ( $K$  is then the amount of stocks acquired), they can borrow  $L = X - K$  from banks. Let  $R$  be the gross rate of interest that firms have to pay back when production is positive; with probability  $1-q$ , instead, the firm goes bankrupt and nothing is due to the bank.

There is a finite number of competitive banks. These are assumed not to be able to buy stocks: in other words, there is institutional separation between banks and firms. Banks collect deposits from households and are the only agents in the economy able to perfectly diversify their portfolio by lending to a large number of firms; for this reason, by the law of large numbers, each bank receives revenues equal to  $qRL$ , and bank deposits are a safe asset returning a fixed interest rate to consumers.

The economy is characterised by ex-post moral hazard, in the sense that banks can not freely observe the realization of outcome. However, banks can become perfectly informed about the return of the investment process by accepting to pay a state verification cost, indicated with letter  $\gamma$  and assumed proportional to the loan amount. The lending conditions of the financial contract between banks and firms are studied in the next subsection.

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<sup>3</sup> Excluding that an institution like a mutual fund can endogenously arise and provide perfect equity diversification is a way of creating an incompleteness in markets. See also Reichlin-Siconolfi, 1996.

<sup>4</sup> More generally one can think about  $X$  as including all necessary inputs to production; given the macroeconomic feature of the model, we do not specify the market equilibrium equations for single factors.



### *2.1 The financing game.*

Consider the game between a representative firm and a representative bank. The firm demands a loan, whose cost is set by the bank, and activates its production technology. The outcome of production is private information of the firm, who can decide whether to report it truthfully to the bank or lie. The bank receives the report. If the investment project is reported to have been successful, the loan is repaid and the game ends. Instead, if the firm declares bankruptcy, the bank has to choose whether to trust the report, or pay the monitoring cost and obtain full knowledge of the investment outcome.

Both players are assumed to be able to randomize on their available pure strategies. In this case, the payoffs are given by their expected profits (equations 1 and 2 below). We will look for a Nash equilibrium of this game, where each player chooses his optimal strategy in order to maximise his expected profits, given the opponent's choice.

In a Nash equilibrium, banks will adopt a monitoring strategy. Otherwise firms have an incentive to report the realization of the "bad state" independently of the effective outcome of production. We will show that, as in Bernanke and Gertler (1989), the optimal monitoring strategy of the banks involves stochastic verification procedures, that is banks monitoring firms declaring bankruptcy with probability less than 1. In order to prove this, it is sufficient to show that a deterministic monitoring strategy can never be part of a Nash equilibrium of this game. We start by observing that firms have never an incentive to misreport the outcome of production if banks play their proposed equilibrium strategy, as they will always be monitored. Then, firms do never misreport. Now, it is obvious that banks are not playing a best response to the proposed equilibrium strategy of the firms, because when they costly monitor bad reports there is nothing left to recover. Hence, deterministic monitoring is not optimal, and the financing game does not have any pure-

strategy Nash equilibrium.<sup>5</sup> However, the game has a finite number of players and the set of available strategies is finite: this implies that a Nash equilibrium does always exist if one allows for mixed strategies.

Let  $\mu$  and  $\pi$  be respectively the probability that a firm cheats by reporting the bad state when a positive outcome is realized, and the probability of banks monitoring when bankruptcy is declared. Moreover, let  $V$  denote the punishment to a firm which misreports and is monitored by the bank. Then, we have:

*Proposition 1: A mixed strategy Nash equilibrium of the financing game is:*

$$L^* = D^*, \quad V^* = \alpha(K + L), \quad R^* = \alpha, \quad \mu^* = \gamma L / \alpha q(K+L), \quad \pi^* = L / K+L.$$

*Proof.* Firms will maximise their expected profits by optimally choosing  $\mu$  and the desired amount of loans  $L$ , while banks will try to reach the same target by inelastically supplying their deposits and controlling for  $\pi$ ,  $V$  and  $R$ , the gross interest rate on loans. When choosing the punishment level  $V$ , banks are subject to a limited liability constraint, which implies that  $V \leq \alpha(K+L)$ , meaning that the punishment cannot exceed the outcome of production.

Let  $\Pi$  be profits from production. The firms' problem is formally written as follows:

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<sup>5</sup> Notice that the optimality of stochastic monitoring procedures is quite general, and not limited to our two-state example. This was already stressed in the pioneering article by Townsend (1978), who pointed out that "stochastic verification procedures can dominate deterministic procedures". Moreover, sometimes stochastic monitoring is also observed in the real world. In Townsend's words: "Stochastic verification procedures are not uncommon. The timing of bank audits by government agencies is somewhat random. Similarly corporations use stochastic procedures in monitoring internal divisions, and it is also said that tax audits by the IRS are determined in part at random."

$$1) \underset{L, \mu}{Max} E\Pi = q[\alpha(K + L) - (1 - \mu)RL - \mu\pi V]$$

In the good state, firms realise output, pay back the loan plus the charged interest if they do not cheat, and are punished by an amount  $V$  in the case they cheat but are monitored (this happens with probability  $\mu\pi$ ).

In this game banks are price takers on the deposits side. When closing the general equilibrium model, free entry and perfect competition will determine an interest rate on deposits at a level such that the expected profits of intermediation will be zero.<sup>6</sup> Thus, profit maximization implies that banks have to solve the following optimization problem:

$$2) \underset{\pi, R, V}{Max} EP = qRL(1 - \mu) + q\mu\pi V - \pi\gamma L - ID$$

$$s. t. \quad V \leq \alpha(K + L)$$

$$and \quad L \leq D$$

where  $P$  denotes banks' profits, and  $I$  the gross interest rate due on deposits.

With probability  $q$ , banks receive  $RL$  if firms report truthfully; if these cheat (which happens with probability  $\mu$ ), banks recover  $V$  only when they monitor (with probability  $\pi$ ); in this case, they also have to pay the cost of state verification.

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<sup>6</sup> The assumption on price taking behavior, which implies that banks do not control the interest rate on deposits, is analogous to considering banks as price makers which strategically compete on deposits in a Bertrand sense.

Monotonicity of EP with respect to R and V implies that these variables are set at their maximum levels. Then,  $V^* = \alpha(K+L)$ . This allows to rewrite the objective functions in problems 1) and 2) as

$$3) E\Pi = q[\alpha K + (\alpha - R)L + \mu[RL - \pi\alpha(K+L)]]$$

and

$$4) EP = qRL(1-\mu) + \pi[q\mu\alpha(K+L) - \gamma L] - ID$$

In order to optimally select  $\mu$  and  $\pi$ , maximization of 3) and 4) implies that

$$5) \begin{array}{ll} \mu^* = 0 & \text{if } J < 0 \\ \mu^* = 1 & \text{if } J > 0 \\ \mu^* \in [0,1] & \text{if } J = 0 \end{array}$$

where  $J = RL - \pi\alpha(K+L)$ ,<sup>7</sup> and

$$6) \begin{array}{ll} \pi^* = 0 & \text{if } H < 0 \\ \pi^* \in [0,1] & \text{if } H = 0 \\ \pi^* = 1 & \text{if } H > 0 \end{array}$$

with  $H = q\mu\alpha(K+L) - \gamma L$ .<sup>8</sup>

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<sup>7</sup> J can be interpreted as the difference between the cost of being honest (LR) and the expected cost of cheating ( $\pi\alpha(K+L)$ ).

<sup>8</sup> H is the difference between the expected revenues and the cost of monitoring.

In equilibrium, it is a standard property of any mixed strategy Nash solution that the optimal probability choices of a player are determined by the opponent. Hence,

$$7) \mu^* = \frac{\gamma L}{\alpha q(K+L)}$$

and

$$8) \pi^* = \frac{RL}{\alpha(K+L)}$$

The demand for loans is

$$9) L^* = \begin{cases} 0 & \text{if } \alpha < R \\ \in [0, \infty] & \text{if } \alpha = R \\ \infty & \text{if } \alpha > R \end{cases}$$

Moreover, 9) allows to define the maximum level of  $R$  that banks can set:  $R^* = \alpha$ , also implying

$$8') \pi^* = L / (K+L).$$

Since the demand for loans when  $R^* = \alpha$  is not determined, the model is closed by letting equilibrium loans be equal to the amount supplied. Then,  $L^* = D$ .  $\square$

In equilibrium banks monitor randomly. This creates an opportunity for firms to misreport good outcomes with a positive probability and an incentive for consumers in

investing in the stock market: the yield from holding shares of firms can be really profitable when the latter cheat and are not monitored.

### 3. General Equilibrium.

Consumers save in their first period of life and consume when old. Consumption tomorrow is a stochastic variable depending on the realized returns of the first period saving choice. We assume that preferences of our consumers are continuous and strictly monotone. More precisely, we also require that the following holds:

*H.1* Preferences are represented by a continuous utility function  $u(\cdot)$  which is twice continuously differentiable, strictly increasing and strictly concave, and satisfies

$$\lim_{c \rightarrow 0} u'(c) = \infty, \quad \lim_{c \rightarrow \infty} u'(c) = 0$$

Let  $\tilde{z}$  the stochastic gross return on stocks and  $W-T$  the after tax endowment of units of the non storable good. Our consumers face the standard problem of

$$\begin{aligned} 10) \quad & \underset{K,D}{\text{Max}} Eu(\tilde{c}) \\ \text{s.t.} \quad & K+D = W-T \\ & \tilde{c} = T + I D + \tilde{z} K \\ & K, D \geq 0 \end{aligned}$$

which can be rewritten as

$$11) \quad \underset{D \geq 0}{\text{Max}} Eu[T + \tilde{z}(W - T - D) + ID]$$

The first order condition is

$$12) \quad \begin{array}{l} E[u'(\tilde{c})(I - \tilde{z})]D \leq 0, \\ \text{with } E[u'(\tilde{c})(I - \tilde{z})] = 0 \quad \text{if } D > 0 \end{array}$$

When choosing D optimally, consumers take I and  $\tilde{z}$  as parametric.

$\tilde{z}$  has the following distribution:

$$\tilde{z} = \begin{cases} 0 & \text{w.p. } (1-q) + q\mu\pi \\ \alpha & \text{w.p. } q(1-\mu) \\ \alpha\left(\frac{K+D}{K}\right) & \text{w.p. } q\mu(1-\pi) \end{cases}$$

In fact, buying stocks pays back nothing if the bad state of the world is realized *or* if the firm cheats but gets monitored: the cumulative probability of this event is  $(1-q)+q\mu\pi$ . With probability  $q(1-\mu)$  instead, stocks pay back exactly  $\alpha$ , while the most profitable gross return is  $\alpha(K+D)/K$ , whose probability is equal to  $q\mu(1-\pi)$ . The expected gross return on stocks is  $\alpha q$ . This can be derived after substituting for the optimal probability choices derived in section 2.

Free entry in the banking sector requires the equilibrium level of I being determined by the following zero profit condition.

$$13) \quad EP = qRL(1-\mu) + q\mu\pi\alpha(K+L) - ID - \pi\gamma L = 0$$

In the expression above, substitute the equilibrium conditions  $R^* = \alpha$ ,  $L^* = D$  and plug in the optimal probability choices  $\mu^*$  and  $\pi^*$ . After manipulating and simplifying one obtains

$$14) I = \alpha q \left[ 1 - \frac{\gamma D}{\alpha q (K + D)} \right]$$

Since our consumers preferences satisfy H1,  $E(\bar{z}) > I$  is a necessary condition for an interior solution  $(K, D) \gg 0$ . 14) implies that this is always the case.

*Definition:* A stationary equilibrium for the model economy is an array of rates of return  $(I, \bar{z})$ , probabilities  $(\mu, \pi)$ , portfolio allocation  $(K, D)$  and loans  $L$  such that:

- i) loans are equal to deposits;
- ii) banks have zero expected profits;
- iii) consumers maximise expected utilities and firms expected profits;
- iv) the good market clears.

*Proposition 2:* For any  $T \in [0, W)$ , there exists a stationary equilibrium for the competitive productive economy ( $\alpha q > 1$ ), with  $(K, D) \gg 0$ .

*Proof.* Let  $F(D, \cdot) = E[u'[T + \bar{z}(W - T - D) + ID](I - \bar{z})]$ .  $F$  is continuous in  $D$ . From 10), a competitive equilibrium is either  $D^* = 0$ ,  $F(0, \cdot) \leq 0$ , or  $D^* > 0$  and  $F(D^*, \cdot) = 0$ . Let  $T \in [0, W)$ . If  $D=0$ ,  $K=W-T$ , and future consumption is equal to  $T$  with probability  $(1-q)$ , and to  $T + \alpha(W-T)$  with probability  $q$ . Then,



$$\begin{aligned}
F(0,.) &= [(1-q)] I u'(T) + [q(I -\alpha)] u'(T+ \alpha(W-T)) = \\
&= [(1-q)] I u'(T) - q (\alpha-I) u'(T+ \alpha(W-T)) > \\
&> u'(T) [(1-q) I - q (\alpha-I)] = 0
\end{aligned}$$

If  $D = W-T$ ,  $K=0$  and  $F(W-T,.) = u'[T+I(W-T)] (I-E(\bar{z})) < 0$ .

By continuity of  $F(.)$  there exists a  $(K^*,D^*) >> 0$ , such that  $F(K^*,D^*) = 0$ .

$(K^*,D^*)$  is the competitive equilibrium.  $\square$

Productive equilibria are characterised by the contemporaneous existence of financial intermediation and equity investment. Financial intermediation provides risk sharing services to risk averse consumers. However, costly state verification implies that banks do not monitor with probability one firms reporting bad outcomes. If a firm misreports a positive outcome and it is not monitored, its shareholders receive a high return. Equity investment will then also be positive in productive market economies.

#### 4. Optimal policies in a competitive equilibrium.

In our economy the government can optimally establish the amount of social security  $T$ . Under perfect information (Samuelson, 1958, Diamond, 1965), the relevant conclusions about whether introducing another asset (money, debt) or implementing a tax-transfer scheme as the one we are considering are:

- if the economy is dynamically inefficient, i.e. the rate of return on capital is lower than population growth, it is welfare improving to introduce either public debt (Diamond, 1965), or a tax-transfer scheme such as a pay as you go social security (Blanchard-Fischer, 1989). In a monetary model (Wallace, 1980) it is also true that introducing fiat money allows to reach a Pareto efficient equilibrium where money is positively valued.

- if the economy instead is dynamically efficient, then nothing can be said about the Pareto ranking of different competitive equilibria. In particular, social security is surely beneficial only to the current old generation, but at the expense of all future agents. However, when one considers only the stationary states of the economy, the optimal level of social security is zero.<sup>9</sup>

In our model, with a stochastic return on capital, we have two possible cases.

When the economy is productive,  $E(z) = \alpha q > 1$ . Otherwise,  $\alpha q < 1$  and the economy can not achieve an efficient allocation. In this latter case, it is obvious that the optimal policy for the government is to set  $T = W$ .  $T$  pays a gross return equal to 1, and all other assets are dominated in the returns they offer. Introducing a pure intergenerational transfer of this kind in this economy is then welfare improving, coherently with standard analysis.

When  $\alpha q = 1$ , the optimal level of  $T$  is still  $W$ . This follows from risk aversion: our consumers will always prefer a safe asset to a stochastic one with the same expected return. Although bank deposits are safe too, monitoring costs make them strictly dominated by social security.

When  $\alpha q > 1$ , that is when the economy is dynamically efficient, in the perfect information case  $T = 0$  was optimal in the stationary state. Here, there are cases in which the optimal policy in a market economy implies unambiguously a zero level of social security, and cases in which a role for social security emerges. The reason is that financial intermediation and social security develop essentially the same role, that is they provide risk sharing. When  $\alpha q$  is very high, equation 14) implies that the safe net return on deposits is positive. In this case deposits strictly dominate  $T$ , so that social security should be optimally set to zero; financial intermediation and equity investment will instead be active. However,

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<sup>9</sup> In the monetary model, money is proved not to have a positive value in equilibrium.

if the economy is still dynamically efficient, but  $\alpha q$  is close to one, the optimal policy implies  $T^* = W$  because of the relatively high monitoring costs affecting the financial system. In fact, let  $\alpha q = 1 + \varepsilon$ , with  $\varepsilon$  arbitrarily small. In this case  $I < 1$ , and if consumers are sufficiently risk averse, they will be strictly better-off by consuming  $T^* = W$  with probability equal to one, with respect to any other consumption allocation resulting from a portfolio of two safe assets  $(T, D)$  with returns respectively equal to 1 and  $I < 1$ , and a risky asset  $(K)$  with expected return  $1 + \varepsilon$ .<sup>10</sup>

### 5. Constrained Pareto Optimality.

Competitive equilibria of OLG economies may be subject to different sources of inefficiency: they can be dynamically inefficient because of overaccumulation of capital, or inefficient because of lack of complete portfolio diversification in a context of uncertainty at the individual level. In this second case, once market incompleteness is eliminated, equilibria turn out to be Pareto-optimal.

When economies are also subject to asymmetric information, the inefficiency of market clearing allocations is usually related to the agents' behaviour in a way which can be distinguished from market incompleteness. Hence, even by assuming that market incompleteness cannot be removed, one could try to establish whether market equilibria satisfy a weaker notion of efficiency than Pareto optimality. This notion will be called Constrained Pareto Optimality, and it is derived under the assumption that the planner cannot introduce additional assets and is subject to the same information structure of the individuals. In our case, the planner suffers from an informative disadvantage equal to that

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<sup>10</sup> It remains to be established whether for  $1 + \varepsilon < \alpha q < 1 + (\gamma D / K + D)$ ,  $T^* \in (0, W)$ , or if there is a cut-off value of  $\alpha q$  in this interval (call it  $\xi$ ), such that  $\alpha q < \xi$  implies  $T^* = W$ , while  $\alpha q > \xi$  implies  $T^* = 0$ .

experienced by the competitive banks, that is, she cannot costlessly observe production outcomes. Therefore, we give the following

*Definition:* A CPO is an allocation  $(K,D,T)$  such that the expected utility of the representative consumer is maximized, subject to the resource allocation constraint, individual budget constraints, and the informative constraint that the planner cannot freely observe outcomes of production activity.

Notice that when  $\alpha q \leq 1$ , the Pareto efficient allocation is trivially  $T^* = W$ , which corresponds to the optimal policy in the market economy. Henceforth, we limit our attention to dynamically efficient economies where  $\alpha q > 1$ .

The social planner can try to achieve his (her) target by controlling many different instruments, the portfolio allocation  $(K,D,T)$ , the supply of loans  $(L)$ , the interest rates on loans and deposits  $(R,I)$ , the punishment to the firm when this is caught cheating  $(V)$ , and the probability  $(\pi)$  of monitoring when bad outcomes are reported. Moreover, contrary to the representative consumer, the planner internalizes the effect of his savings and monitoring choices on the assets' returns. The planner's problem can then be written as follows:

$$\begin{aligned}
 & \underset{K,D,T,L,\pi,V,R,I}{\text{Max}} && EU(\tilde{c}) \\
 & \text{s.t.} && W = T + K + L \\
 & && \tilde{c} = \tilde{z}K + IL + T \\
 & && V \leq \alpha(K + L) \\
 & && L \leq D
 \end{aligned}$$

and the relevant nonnegativity constraints.

One variable can be eliminated by understanding that deposits are always equal to loans. In the model there is no other asset in which banks' liabilities can be invested. Hence,

set  $L = D$ . Now, the planner has to consider two additional constraints. The first is due to the fact that  $\mu$  is optimally chosen by firms, that is

$$\mu^* = \begin{cases} 1 & \text{if } RL \geq \pi V \\ \in [0,1] & \text{if } RL = \pi V \\ 0 & \text{if } RL \leq \pi V \end{cases}$$

The second is the resource constraint, establishing that the sum of aggregate (average) investment and consumption has to be equal to aggregate production plus aggregate endowments minus the amount of resources destroyed in the monitoring activity. Since there is a continuum of individuals, by the law of large numbers, the resource constraint is

$$(15) \quad E(c) + (K + L) = W + \alpha q(K+L) - \pi \gamma L$$

Also the interest rate on deposits ( $I$ ) is not an instrument for the planner, because it is determined by (15). In fact, since

$$\begin{aligned} E(c) &= T + IL + E(z)K, \text{ and} \\ E(z)K &= \alpha q(K+L) - q\mu\pi V - q(1-\mu)RL, \end{aligned}$$

it turns out that (15) can be rewritten as:

$$(16) \quad W - (K+L) + IL + \alpha q(K+L) - q\mu\pi V - q(1-\mu)RL = W + \alpha q(K+L) - (K+L) - \pi \gamma L$$

from which it is easy to obtain

$$(17) \quad \Pi L = q(1-\mu)RL + q\mu\pi V - \pi\gamma L$$

The other assets' returns are:

T pays back 1 (gross) for sure, while capital investment returns are

$$\begin{array}{ll} 0 & \text{w.p. } 1-q \\ \alpha(K+L)-V & \text{w.p. } q\mu\pi \\ \tilde{z} K = \alpha(K+L)-RL & \text{w.p. } q(1-\mu) \\ \alpha(K+L) & \text{w.p. } q\mu(1-\pi) \end{array}$$

Hence, consumption opportunities in the four states of nature are:

$$\begin{array}{ll} c_1 = T + \Pi L & \text{w.p. } 1-q \\ c_2 = T + \Pi L + \alpha(K+L) - V & \text{w.p. } q\mu\pi \\ c_3 = T + \Pi L + \alpha(K+L) - RL & \text{w.p. } q(1-\mu) \\ c_4 = T + \Pi L + \alpha(K+L) & \text{w.p. } q\mu(1-\pi) \end{array}$$

*Claim 1: For any  $\mu \in [0,1)$ , the expected utility of consumers is strictly increasing in  $R$ .*

*Proof.* If  $\mu^*=1$ ,  $EU(c)$  is not affected by  $R$ .

Now, assume  $\mu \in [0,1)$ . Then  $\Pi L = qRL - \pi\gamma L$ .

$$d[EU(c)]/dR = qL * EU'(c) - q(1-\mu)L * U'(c_3) =$$

$$\begin{aligned}
&= qL [(1-q)U'(c_1) + q\mu\pi U'(c_2) + q\mu(1-\pi)U'(c_4)] + qL U'(c_3)[q(1-\mu)-(1-\mu)] > \\
&> qL U'(c_1)[1-q + q(1-\mu)-(1-\mu)] + qL [q\mu\pi U'(c_2) + q\mu(1-\pi)U'(c_4)] = \\
&= qL U'(c_1) [\mu(1-q)] + qL [q\mu\pi U'(c_2) + q\mu(1-\pi)U'(c_4)] > 0. \quad \square
\end{aligned}$$

Since  $d[EU(c)]/dR \geq 0$  ( $> 0$  for  $\mu^* \neq 1$ ), the optimal value of  $R$  is greater or equal than  $\alpha$ . However, if  $R > \alpha$ , the demand for loans drops to zero since firms are profit maximizers. Hence, Claim 1 implies  $R^* = \alpha$ . Moreover, at a CPO consumption opportunities satisfy:

$$\begin{aligned}
c_1 &= T + IL && \text{w.p. } 1-q \\
c_2 &= T + IL + \alpha(K+L) - V && \text{w.p. } q\mu\pi \\
c_3 &= T + IL + \alpha K && \text{w.p. } q(1-\mu) \\
c_4 &= T + IL + \alpha(K+L) && \text{w.p. } q\mu(1-\pi)
\end{aligned}$$

*Claim 2: Let  $\gamma$  be arbitrary small. Then, at a CPO,  $V^* = \alpha L$  and  $\pi^* = 1$ .*

*Proof.* Suppose that  $\gamma = 0$ , and let  $\mu^* \in (0,1)^{11}$ . In this case,  $\pi V = \alpha L$ . Moreover, a change in  $V$  only affects consumption opportunities  $c_2$  and  $c_4$ , as well as their probabilities. Hence, the optimal choice of  $V$  should maximise the expected utility of consumption in these two states.

Observe that  $c_2 = c_4 - V$ . The problem can then be written as

$$\text{Max}_V \frac{\alpha L}{V} U(c_4 - V) + (1 - \frac{\alpha L}{V}) U(c_4)$$

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<sup>11</sup>This is without loss of generality. A similar argument can be applied for  $\mu^* = 0$ , or for  $\mu^* = 1$ .

Differentiation with respect to  $V$  gives

$$\begin{aligned} \frac{dEU(\bar{c})}{dV} \Big|_{\gamma=0} &= \frac{\alpha L}{V^2} [U(c_4) - U(c_4 - V)] - \frac{\alpha L}{V} U'(c_4 - V) = \\ &\frac{\alpha L}{V^2} [U(c_4) - U(c_4 - V) - VU'(c_4 - V)] < 0 \text{ by concavity of } U(.). \end{aligned}$$

Hence,  $V$  should be set at its lower feasible bound. Since  $\pi \leq 1$ ,  $V^* = \alpha L$ , and  $\pi^* = 1$ .

Since  $dEU(\bar{c})/dV$  is a continuous map of the cost  $\gamma$ , and since  $dEU(\bar{c})/dV \Big|_{\gamma=0} < 0$ , for  $\gamma$  small enough, the inequality holds true.  $\square$

Notice that, independently of  $\mu$ , consumption opportunities can be rewritten as

$$\begin{aligned} c_1 &= T + \Pi && \text{w.p. } 1-q \\ c_2 &= T + \Pi + \alpha K && \text{w.p. } q \end{aligned}$$

where  $I = \alpha q - \gamma$ . Therefore:

*Proposition 3: If  $\gamma$  is small, and  $\alpha q - \gamma > 1$ , the CPO is  $(K^*, L^*) \gg 0$ ,  $T^* = 0$ ,  $R^* = \alpha$ ,  $V^* = \alpha L^*$  and  $\pi^* = 1$ .*

*Proof:* Obvious.  $\square$

*Corollary: If  $\alpha q - \gamma < 1$ , the CPO is  $L^* = 0$ ,  $(K^*, T^*) \gg 0$ .*

*Remark 1:* A CPO is characterised by deterministic monitoring. The planner understands that the distortion in the market economy is induced by banks behaving as profit maximizers. Competitive banks monitor firms declaring bankruptcy with probability less



than one, and they punish cheating by setting  $V = \alpha(K+L)$ . However, firms are owned by consumers. The punishment subtracts  $V$  from consumers' consumption, while only  $\pi V$  returns from deposit holdings. From the consumers' point of view, deterministic monitoring dominates any stochastic verification procedure, while for the banks only the latter is supported by profit maximization.

*Remark 2:* Even though, when monitoring is cheap, market outcomes and the CPO are both characterised by the active use of the same instruments (L and K), the equilibrium allocations are different. Moreover, there is no optimal policy in a market economy which can induce banks to monitor with probability equal to one when bankruptcy is declared. The source of inefficiency in this economy cannot be eliminated by any standard quantitative policy intervention.

## **6. Conclusions.**

This paper tried to extend the analysis of the efficiency properties of a simple OLG model with production under asymmetric information. The introduction of ex-post moral hazard in the form of costly state verification allows to provide a realistic characterisation of the conflict between profit maximizing firms and financial intermediaries: in equilibrium banks adopt stochastic monitoring of bankruptcy reports, and firms have an incentive to lie. The optimal policy of the government introduces a social security system under more general conditions than in the perfect information case. In particular, even in cases where the economy is productive (or dynamically efficient), the introduction of a tax scheme able

to transfer consumption opportunities over time (as a social security system) might be Pareto improving, provided that consumers are sufficiently risk averse.<sup>12</sup>

Moreover, market outcomes generally fail to achieve even a weaker notion of efficiency than Pareto optimality, which is derived under the hypothesis that market incompleteness cannot be eliminated. We show that this result depends on the role of profit maximizing financial intermediaries: these adopt stochastic verification procedures which are privately optimal, but socially inefficient in an economy with private ownership. This kind of market equilibria inefficiency cannot be eliminated by using traditional quantitative policies. In order to restore the efficient outcome, the policymaker should act as a regulator of the microstructure of private contracts. In our example, the policy maker should require the lending contract to be designed so as to include deterministic monitoring of bankruptcy reports.

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<sup>12</sup> Notice that in a similar model with adverse selection, Reichlin and Siconolfi (1996) found that it is also possible to obtain different conclusions from the ones obtained under perfect information even when the economy is dynamically inefficient. In fact, they can show that in a credit-rationing equilibrium a la Stiglitz-Weiss (1981) of an OLG stationary economy, the introduction of a new asset (debt, social security or outside money) could be welfare reducing when the rate of return on capital is lower than population growth.

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