Stellar and gas dynamics in galactic nuclei

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Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2018
ABSTRACT

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Galactic nuclei are important for studies of galaxy evolution, stellar dynamics and general relativity. Many have Supermassive Black Holes (SMBHs) with masses of $10^6 - 10^{10} M_\odot$ that affect the large scale properties of their hosts. They are also the densest known stellar systems, and produce unique electromagnetic and gravitational wave sources via close encounters between stars and compact objects. For example, stars that wander too close to an SMBH are tidally disrupted, producing a bright flare known as a TDE. This thesis investigates the gas and stellar environments in galactic nuclei.

In Chapters 2 and 3, we develop an analytic model for the gas environment around quiescent SMBHs. In the absence of large scale inflows, winds from the local stellar population will supply most of the gas. The gas density on parsec scales depends strongly on the star formation history, and can plausibly vary by four orders of magnitude. In Chapter 3, we use this model to constrain the presence of jets in a large sample of TDE candidates.

In Chapter 4 we construct observationally motivated models for the distributions of stars and stellar remnants in our Galactic Center. We then calculate rates of various collisional stellar interactions, including the tidal capture of stars by stellar mass black holes. This process produces $\sim 100$ black hole LMXBs in the central parsec of the Galaxy (comparable to the number inferred from recent X-ray studies).
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3.2 Contours of $n_{18}$, the CNM density at $r = 10^{18}$ cm (blue lines), as a function of the stellar population in the galactic nucleus. The star formation is parameterized assuming that a fraction $f_{\text{burst}}$ of the stars form in a burst of age $t_{\text{burst}}$, while the remaining stars formed a Hubble time ago. We have assumed a black hole mass of $10^7 M_\odot$ and that both the young and old stars possess a cusp-like density profile, with a corresponding gas density profile $n \propto r^{-1}$. Hatched areas indicate regions of parameter space where massive stars ($\gtrsim 15 M_\odot$) dominate the gas heating rate, but less than one (doubly hatched) or less than ten (singly hatched) massive stars are present on average inside the nominal stagnation radius (eq. 3.9). In these regions discreteness effects not captured by our formalism are potentially important. The red line shows the approximate location of the Galactic Center in this parameter space (see text for details).

3.3 Initial geometry of the jet used for our hydrodynamic simulations. We note that for 1D- two component jet models, we perform separate models for the inner fast core and for the outer sheath, which are later combined to provide the resulting emission. For our 1D simulation we take a slow component extending from $0-\pi/2$ radians to account for the effects of jet spreading.

3.4 Comparison of light curves from 1D and 2D simulations for an on-axis observer ($\theta_j = 0$). We assume an $n \propto r^{-1}$ density profile.

3.5 Contours of the fraction of the kinetic energy of the slow component of the jet ($\Gamma = 2$) which is dissipated at the reverse shock in the parameter space of jet energy, $E_j$, and CNM density, $n_{18}$. The parameters of the suite of jet simulations presented in this chapter are shown as red squares. The approximate location of SwJ1644 in the parameter space is also labeled. Blue lines delineate the parameter space where the slow component of the jet is optically thin/thick at the deceleration time at 1 GHz (left line) and 30 GHz (right line).

3.6 Left: Radio light curves as viewed on axis ($\theta_{\text{obs}} = 0$) for jet energies of $5 \times 10^{53}$ erg (darker-shaded lines) and $5 \times 10^{51}$ erg (lighter-shaded lines), for values of $n_{18} = 2$ (blue), 60 (red), and 2000 (green) cm$^{-3}$. Solid lines show the result of 1D simulations, while 2D light curves are shown as dashed lines (when available). Thick lines show the results of our numerical calculation, while thin lines are power law extrapolations. A gas density profile of $n \propto r^{-1}$ is used for all of the light curves. Radio upper limits and detections are shown as triangles and squares, respectively. The single upper limit in the top panel is for D3-13 at 1.4 GHz from Bower (2011a). Figure continue on next page.
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3.8 Top: Comparison between on-axis light curves for our fiducial $n \propto r^{-1}$ gas density profile, corresponding to a cusp-like galaxy, and the core galaxy profile defined by (C.1) with $r_s = 10^{18}$ cm. Bottom: Comparison between on-axis light curves calculated from 1D simulations with $n \propto r^{-1}$ (solid) and $n \propto r^{-1.5}$ (dashed) gas density profiles. The dash-dotted line shows the on-axis light curve for a 2D simulation with an $n \propto r^{-1.5}$ gas density profile.

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3.12 Upper limits and 5 GHz analytic light curves (eqn. 3.20 with $s = 1$, $a_1 = 1.7$, and $a_2 = -2$) for different jet energies. We use the peak time and luminosity from our numerical $n_{18}=11 \text{ cm}^{-3}$ light curve for the highest energy light curve, as our analytic fits (eqns. 3.18 and 3.19) underestimate the peak luminosity a factor of $\sim 2$ for this density. Then we use our analytic results to scale this light curve to lower energies.

3.13 Histogram of jet energies consistent with existing radio detections (ASSASN-14li, SwJ1644, and SwJ2058) and upper limits (Table 1 of Mimica et al. 2015 and Arcavi et al. 2014), as summarized in Table 3.2.
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ACKNOWLEDGMENTS

This thesis would not have been possible without the support and guidance of many.

First of all I would like to thank my adviser Brian Metzger and Nicholas Stone for their support and contributions. None of the science presented here would have happened without them. They also provided invaluable advice and support on scientific writing, presentation, and career development.

I would also like to thank my other collaborators both inside and outside of Columbia including Jerry Ostriker, Zoltan Haiman, Petar (Pere) Mimica, MA Aloy, Dimitrios Giannios, and Eugene Vasiliev. The Chapter on TDE jets would not have been possible without Pere’s numerical simulations, and the chapter on X-ray binaries would not have been possible without Eugene Vasiliev’s PHASEFLOW code (and Eugene’s advice). I am grateful to Charles Hailey for extensive discussions of his observational results.

Throughout my time at Columbia, I was also influenced by discussions with many of the faculty, post-docs, and students at journal clubs, coffees, wine and cheese, and other department events.

I am grateful to all of the administrative staff at Columbia: Millie, Ayoune, Francisco, and Samantha for their help in getting through sundry bureaucratic hurdles. I thank the HPC staff at Columbia: some of the work presented here made use of the Yeti and Habanero clusters.

I was fortunate to have strong mentors as an undergraduate. In particular, I would like to thank Omer Blaes and Sathya Guruswamy. I would not be where I am today without you.

Finally I would like to thank my family for their love and support, without which this would have been impossible. So this dissertation is really yours as well.
Chapter 1

Introduction

1.1 Overview

Despite their small size galactic nuclei play an important role in galaxy evolution. Many contain supermassive black holes (SMBHs) with masses of $\sim 10^6 - 10^{10}\text{M}_\odot$ (see reviews by, e.g. Kormendy & Richstone 1995; Ferrarese & Ford 2005) that can release a substantial fraction ($\sim 10\%$) of the rest-mass energy they accrete as outflows or light. In fact, the energy released by an accreting SMBH can dramatically alter the evolution of its host galaxy by ejecting or ionizing gas and suppressing star formation (Silk & Rees 1998). The importance of accretion feedback within galaxies is evinced by tight correlations between SMBH mass and large scale properties of their host galaxies (Ferrarese & Merritt 2000; Tremaine et al. 2002). Also, adding AGN feedback in simulations solves the “cooling flow problem.” Without AGN feedback, gas in galaxy clusters would be cooler than observed and would form too many stars (Fabian 1994; Peterson 2003; Gaspari et al. 2012, 2013; Li & Bryan 2014a).

Galactic nuclei also have large stellar densities that produce unique electromagnetic transients and gravitational wave (GW) sources via close encounters. For example, stars that wander too close
to an SMBH are tidally disrupted, producing a bright flare (Hills 1975; Carter & Luminet 1982; Rees 1988). To date, dozens of candidate tidal disruption events (TDEs) have been identified.\footnote{See \url{https://tde.space/} for a compilation.} Stellar mass BHs in a galactic nucleus can tidally capture or disrupt stars, possibly explaining the large concentration of X-ray sources in the Galactic Center (GC). BHs and NSs can also form close binaries with each other by three-body exchange interactions or two-body GW capture. Such close binaries are eventual sources for LIGO and other GW detectors (Antonini & Rasio 2016). Finally, collisional run-aways in galactic nuclei can form SMBHs (Stone et al. 2017b).

This thesis is a study of the environment of SMBHs. Chapters 2 and 3 develop a model for the gas within the gravitational sphere of influence of an SMBH and explore its implications for SMBH growth and TDE observations. In Chapter 4 we calculate observationally calibrated rates of collisional stellar interactions in the center of our Galaxy.

### 1.2 Supermassive Black Holes

The first evidence for SMBHs came from observations of quasars in the early 1960s (Schmidt 1963): their luminosities and short time-scale variability indicated quasars are powered by accretion onto massive compact objects (e.g. SMBHs). Most quasars are at high redshift: their number density peaked between redshifts 2 and 3 (Richards et al. 2006). However, even though quasars are not active today, the SMBHs that powered them still quietly lurk in the centers of their host galaxies (Lynden-Bell 1969; Soltan 1982).

Since the 1980s, observations of stars and gas in galactic nuclei provided direct evidence for the presence of SMBHs, including

1. Measurements of the orbits of resolved stars in the immediate vicinity of the SMBH. So far this is only possible within our own Galactic Center, where the orbits of stars within $\sim 0.01$
pc of the center reveal the presence of a $4 \times 10^6 M_\odot$ SMBH (Genzel et al. 1997; Ghez et al. 1998, 2000; Eckart et al. 2002; Schödel et al. 2002; Ghez et al. 2003; Boehle et al. 2016).

2. Measurements of gas in Keplerian motion around an SMBH. This includes water masers, as in NGC 4258 (Moran et al. 1999), or ionized gas as in M87 (Macchetto et al. 1997). Recently, the Atacama Large Millimeter/Submillimeter Array (ALMA) has allowed SMBH mass measurements from molecular gas dynamics (Davis et al. 2017).

3. Reverberation mapping. Actively accreting SMBHs sometimes exhibit broad emission lines with velocity widths of $10^3 - 10^4$ km s$^{-1}$ (Krolik 1998). These lines come from clouds (or outflows) photoionized by continuum emission from a central accretion disk. Variability in the lines follows variability in the continuum with a lag of a few days. This delay is a measurement of light-travel time (and thus the distance) between the clouds and continuum source. Combining this distance with the observed width of the lines gives a measurement of the central SMBH mass via the virial theorem (see review by Bentz 2015).

4. Fitting orbit models to observed stellar kinematic data. This determines the central SMBH mass in galaxies for which the gravitational sphere of influence of the SMBH is well resolved (e.g. Walsh et al. 2017). Outside of the our own Galaxy, stellar dynamical measurements of SMBH masses rely on measurements of integrated light, which are subject to systematic uncertainties. For example, integrated light measurements of the Galactic Center underestimated the central SMBH-mass by a factor of $\sim 2$ (Feldmeier et al. 2014; Feldmeier-Krause et al. 2017).

Currently there are of order 100 SMBH masses measured directly via stellar or gas kinematic tracers (Saglia et al. 2016; van den Bosch 2016).

In the early 2000s, it was discovered that SMBH mass correlates with its host galaxy’s velocity
dispersion well outside of the SMBH’s gravitational sphere of influence (the $M_\bullet - \sigma$ relation – see Ferrarese & Merritt 2000; Gebhardt et al. 2000). At face value, the observed correlation can be explained via feedback from momentum driven outflows (King 2003). However, it can also be explained by the averaging of galaxy properties in mergers (Peng 2007). Also, it was recently discovered that the $M_\bullet - \sigma$ correlation breaks down for lower mass disk galaxies, suggesting SMBH feedback is less important in these systems (Kormendy & Ho 2013).

1.2.1 SMBH Luminosities

Observed distributions of SMBH luminosity are an important constraint for models of feedback. It is convenient to normalize the luminosity of an SMBH by the Eddington Luminosity. At this luminosity, the outward force of radiation pressure on the surrounding gas exceeds the inward pull of gravity. Quantitatively, the Eddington luminosity is

$$L_{\text{edd}} = \frac{4\pi GM_\bullet m_p c}{\sigma_T} = 1.26 \times 10^{46} \left( \frac{M_\bullet}{10^8 M_\odot} \right) \text{erg s}^{-1},$$

(1.1)

where $G$ is the gravitational constant, $m_p$ is the proton mass, $c$ is the speed of light, $M_\bullet$ is the mass of the SMBH, and $\sigma_T$ is the Thompson scattering cross-section for an electron. The luminosity of an accretion flow is $\epsilon \dot{M}_\bullet c^2$, where $\epsilon$ is the radiative efficiency and $\dot{M}_\bullet$ is the accretion rate onto the SMBH. The structure and efficiency of the flow depend on the ratio of the accretion rate to the critical accretion rate ($\dot{M}_{\text{crit}} \equiv L_{\text{edd}}/c^2$). When $0.1 \lesssim \dot{M}_\bullet/\dot{M}_{\text{crit}} \lesssim 10$ the flow is expected to be a thin disk with $\epsilon$ between between 3.8 and 42% (Shakura & Sunyaev 1973; Novikov & Thorne 1973). When $\dot{M}_\bullet$ is much less than the $\dot{M}_{\text{crit}}$, coulomb collisions can no longer transfer dissipated energy from ions to electrons and the flow necessarily becomes radiatively inefficient, as the gas inflow time is shorter than the cooling time. Dissipated energy would either be advected
into the SMBH or lost in outflows (Rees et al. 1982; Narayan & Yi 1995; Blandford & Begelman 1999). For $10^{-4} \lesssim \dot{M}/\dot{M}_{\text{crit}} \lesssim 0.1$, collective plasma effects can still transfer a substantial fraction of the dissipated energy to the electrons, maintaining a radiative efficiency of $\sim 1\%$, but at still lower accretion rates the efficiency falls off as $\dot{M}/\dot{M}_{\text{crit}}$ (Sharma et al. 2007). A super-critical accretion flow ($\dot{M} \gg \dot{M}_{\text{crit}}$) may also be inefficient due to photon trapping (Begelman 1978; Ohsuga et al. 2005). However, the radiative efficiency of such flows remains an open problem. Recent radiation magnetohydrodynamic simulations of super-critical accretion disks actually find large radiative efficiencies of 5-7% for accretion rates up to $500 \dot{M}_{\text{crit}}$ (Jiang et al. 2014, 2017). However, other simulations (with better treatments of relativistic effects but less exact radiation transport algorithms) find radiative efficiencies of $\sim 1\%$ (McKinney et al. 2014; Sądowski et al. 2014).

Most of the energy released by an SMBH is radiated during short-lived active phases (Active Galactic Nuclei or AGN) with $L/L_{\text{edd}} \gtrsim 0.01$ (Soltan 1982; Gan et al. 2014). However, most SMBHs are quiescent with much lower Eddington ratios. X-ray and UV studies find that at most a few percent of SMBHs at low redshift ($z \lesssim 1$) are active (Greene & Ho 2007; Haggard et al. 2010).\(^2\) Notably, the SMBH in our Galactic Center is inactive—it has a mass of $4 \times 10^6 M_\odot$ and a bolometric luminosity of $\sim 100 L_\odot$ (Yusef-Zadeh & Wardle 2010), corresponding to an Eddington ratio of $10^{-9}$.

### 1.3 The circumnuclear medium

Understanding why most SMBHs appear to be inactive requires characterizing their gaseous environments. Gas near the SMBH sphere of influence, hereafter denoted the ‘circumnuclear medium’ (CNM), controls the mass accretion rate, $\dot{M}_*$. The accretion rate in turn determines the SMBH luminosity and the feedback of its energy and momentum output on larger scales. Dense gas in the nucleus may lead to runaway cooling, resulting in bursty episodes of star formation and AGN

\(^2\)Haggard et al. (2010) consider galaxies to be active if their X-ray luminosities exceed $10^{42}$ erg s$^{-1}$. 
activity (e.g. Ciotti & Ostriker 2007).

Knowledge of how $\dot{M}_\bullet$ depends on the SMBH mass, $M_\bullet$, and other properties of the nucleus informs key questions related to the co-evolution of SMBHs and their host galaxies with cosmic time (e.g. Kormendy & Ho 2013; Heckman & Best 2014). In the low redshift Universe, SMBH growth is dominated by low mass black holes, $M_\bullet \lesssim 10^8 M_\odot$ (e.g. Heckman et al. 2004), a fact often attributed to the trend of ‘cosmic down-sizing’ resulting from hierarchical structure growth (e.g. Gallo et al. 2008). However, the physical processes by which typical low mass black holes accrete could in principle be distinct from those operating at higher SMBH masses, or those in AGN. Of key importance is whether SMBHs grow primarily by the accretion of gas fed in directly from galactic or extragalactic scales, or whether significant growth can result also from local stellar mass loss in the nuclear region.

A better understanding of what mechanisms regulate accretion onto quiescent SMBHs would shed new light on a variety of observations, such as the occupation fraction of SMBHs in low mass galaxies. Miller et al. (2015) use the average relationship between the nuclear X-ray luminosities, $L_X$, of a sample of early type galaxies and their associated SMBH masses to tentatively infer that the SMBH occupation fraction becomes less than unity for galaxies with stellar masses $M_\star \lesssim 10^{10} M_\odot$ ($M_\bullet \lesssim 10^7 M_\odot$). This method relies on extrapolating a power-law fit of the $L_X - M_\bullet$ distribution to low values of $L_X$ below the instrument detection threshold, an assumption that would fail if different physical processes control the accretion rates onto the lowest mass SMBHs.

Gas comprising the CNM of quiescent (non-AGN) galaxies can in principle originate from several sources including wind mass loss from predominantly evolved stars and stellar binary collisions. Stellar wind mass loss is probably the dominant source insofar as collisions are relevant only in extremely dense stellar environments for very young stellar populations (Rubin & Loeb 2011).

The gas inflow rate on large scales is much easier to constrain both observationally and theo-
retically than the black hole (horizon scale) accretion rate. Ho (2009) determines the inflow rates in a sample of early-type galaxies by using X-ray observations to determine the Bondi accretion rate, and also by using estimated mass loss rates of evolved stars. Both methods lead him to conclude that the available gas reservoir is more than sufficient to power the observed low-luminosity AGN, assuming the standard \( \sim 10 \) per cent radiative efficiency for thin disk accretion. Several lines of evidence now suggest that low-luminosity AGN result from accretion proceeding in a radiatively inefficient mode (Yuan & Narayan 2014), due either to the advection of gravitationally-released energy across the SMBH horizon (e.g. Narayan & Yi 1995) or due to disk outflows, which reduce the efficiency with which the inflowing gas ultimately reaches the SMBH (e.g. Blandford & Begelman 1999; Li et al. 2013).

Another approach to determine the inflow rates, which we adopt, is to directly calculate the density, velocity and temperature profiles of the CNM using a physically motivated hydrodynamic model. Mass is injected into the nuclear environment via stellar winds, while energy is input from several sources including stellar winds, supernovae (SNe), and AGN feedback (Quataert 2004; De Colle et al. 2012; Shcherbakov et al. 2014). Unlike previous works, which focused primarily on modeling individual galaxies, here we model the CNM properties across a representative range of galaxy properties, including different SMBH masses, stellar density profiles, and star formation histories (SFHs).

Previous studies, employing multi-dimensional numerical hydrodynamics and including variety of (parametrized) physical effects, have focused on massive elliptical galaxies (e.g. Ciotti & Ostriker 2007; Ciotti et al. 2010). These works show the periodic development of cooling instabilities on galactic scales, which temporarily increase the gas inflow rate towards the nucleus until feedback becomes strong enough to shut off the flow and halt SMBH growth.

In Chapter 2 we present time-independent models, in which the nuclear gas receives sufficient
heating to render radiative cooling negligible. This approach allows us to systematically explore the relevant parameter space and to derive analytic expressions that prove useful in determining under what conditions cooling instabilities manifest in the nuclear region across the expected range of galaxy properties, and whether other (non-AGN) forms of feedback can produce a prolonged state of steady, thermally stable accretion. Even if cooling instabilities develop on galactic scales over longer $\sim$Gyr time-scales, we aim to explore whether a quasi steady state may exist between these inflow events on smaller radial scales comparable to the sphere of influence.

In the presence of strong heating, one-dimensional steady state flow is characterized by an inflow-outflow structure, with a critical radius known as the “stagnation radius” $r_s$, where the radial velocity passes through zero. Mass loss from stars interior to the stagnation radius is accreted, while that outside $r_s$ is unbound in an outflow from the nucleus. The stagnation radius, rather than the Bondi radius, thus controls the inflow rate (although we will show that $r_s$ often resides near the nominal Bondi radius). When heating is sufficiently weak, however, the stagnation radius may move to much larger radii or not exist at all, significantly increasing the inflow rate the SMBH, i.e. a “cooling flow.” However, the hydrostatic nature of gas near the stagnation radius also renders the CNM at this location particularly susceptible to local thermal instabilities, the outcome of which could be distinct from the development of a global cooling flow.

1.3.1 Application: Constraining jets in tidal disruption events

Several dozen thermal TDE flare candidates have now been identified in UV/optical (Gezari et al. 2006, 2008; van Velzen et al. 2011; Gezari et al. 2012; Chornock et al. 2014; Holoien et al. 2014; Arcavi et al. 2014; Vinkö et al. 2015; Holoien et al. 2016a,b; Blagorodnova et al. 2017) and soft x-ray wavelengths (Komossa 2015; Auchettl et al. 2017 and the references therein). Beginning with Swift J1644+57, three TDEs were discovered by their non-thermal x-ray emission (Bloom et al. 2011;
Cenko et al. 2012; Brown et al. 2015). In all three cases the non-thermal x-rays were followed by a radio synchrotron afterglow (Berger et al. 2012; Cenko et al. 2012; Zauderer et al. 2013). Both the radio and x-ray emission can be explained by a relativistic jet pointed along the line of sight, with the x-rays coming from the base of the jet and the radio coming from the interface of the jet and the surrounding circumnuclear medium.

Afterglows from jets have been extensively studied in GRBs after their discovery in the late 1990s (Costa et al. 1997; Groot et al. 1997). The qualitative behavior of the afterglow can be explained by a relativistic blast wave interacting with ambient gas. The blast wave produces a forward shock in the gas that accelerates relativistic electrons. These electrons spiral about magnetic field lines and produce synchrotron emission (Rees & Meszaros 1992; Mészáros & Rees 1997). Models generally assume that a fixed fraction of the thermal energy behind the shock is converted into magnetic fields and relativistic electrons (with a power law distribution of energies). Combined with prescriptions for the expansion of the blast-wave, this gives predictions for radio light-curves and spectra (Wijers et al. 1997; Sari et al. 1998; Panaitescu & Kumar 2000; Frail et al. 2000; Granot & Sari 2002; Piran 2004; Mészáros 2006; Granot 2007). Early analytic models assumed spherical symmetry and self-similar expansion, which is a good approximation at early and late times. At early times, the highly relativistic jet is well described by the Blandford-McKee solution (Blandford & McKee 1976), as strong beaming effects make the jet indistinguishable from a patch on a spherical outflow. At late times, the jet would become spherically symmetric and Newtonian, following the Sedov-Taylor solution (Taylor 1950; Sedov 1959). Early works (Rhoads 1999; Sari et al. 1999) found that the jet would undergo an exponential lateral expansion, so that the transition between these two phases is abrupt. However, recent 2D relativistic hydrodynamic simulations (Zhang &

\( ^3 \)Swift J1112.2 8238 was not promptly followed up in the radio, but subsequent follow-up with ATCA shows radio emission at a much higher level than expected given the galaxies UV/emission line luminosities (Andrew Levan, private communication).
MacFadyen 2009; van Eerten et al. 2010; Wygoda et al. 2011; van Eerten et al. 2012) have shown that the lateral expansion of the jet is more gradual, and that there is a significant intermediate phase in which the jet is non-spherical and its expansion is not self-similar. Simulations have also shown that viewing angle effects make a quantitative (factor of ∼4) difference in the jet energies inferred from GRB observations (van Eerten et al. 2010). Hydrodynamic simulations can also be coupled to radiative transfer codes that account for advection of the accelerated electrons within the jet and synchrotron self-absorption (Mimica et al. 2009b).

Many of the results of afterglow theory apply to TDE jets, but there are a few notable differences. The initial Lorentz factor (∼10) is closer to that of an AGN jet (GRB jets have Lorentz factors of ∼100) (Metzger et al. 2012). As in a GRB, the jet is decelerated by interactions with the surrounding gas. In contrast, AGN jets are quasi-steady structures that propagate through an evacuated cavity. However, fall-back from the stellar disruption can supply energy to a TDE jet at late times, whereas the energy injection in GRB jets is impulsive. Overall, TDE jets represent an intermediate regime between impulsive, ultra-relativistic GRB jets and steady, moderate Lorentz factor AGN jets (Giannios & Metzger 2011).

Although a handful of jetted TDE flares have been observed, their volumetric rate is a very small fraction, ∼10^{-4}, of the rate of observed thermally selected TDE flares (e.g. Burrows et al. 2011), and an even smaller fraction of the theoretically predicted TDE rate (Stone & Metzger 2016). It could be that the majority of tidal disruption events produce powerful jets, but the hard x-rays are relativistically beamed into a small angle, θ_b ∼ 0.01. However, this would require highly relativistic jets with Lorentz factor, Γ ∼ 100. This is much larger than the Lorentz factor typically inferred for Swift J1644+57 (Metzger et al. 2012), or what is typically found in AGN jets.

An alternative way to constrain the rates of TDE jets is to search for non-thermal synchrotron emission. The radio is expected to be quasi isotropic at late times (Giannios & Metzger 2011;
Mimica et al. 2015) and thus is much less sensitive to beaming corrections. Bower et al. (2013) and van Velzen et al. (2013) observed a set thermally selected TDE flares at radio wavelengths on timescales of months to decades after the outburst (see Table 1 of Mimica et al. 2015 for a compilation). Neither of these studies found any radio afterglows definitively associated with the host galaxies of strong thermal TDE candidates.4

More recently, a candidate TDE flare (ASSASN-14li) was observed to have transient radio emission, consistent with either a weak relativistic jet (van Velzen et al. 2015) or a sub-relativistic outflow (Alexander et al. 2015) with overall energy in the range of $10^{48} - 10^{49}$ erg. ASSASN-14li occurred in a very nearby galaxy, and if other (typically more distant) TDE candidates launched similar outflows, their radio afterglows would be below existing upper limits. The lack of strong radio emission in most TDEs may either be due to (i) weak/no outflows or (ii) a low nuclear gas density. If the external density is low, there will be little nuclear gas to produce radio emission and shocks inside the jet would be weak, making the jet unobservable in such cases.

In Chapter 3 we calculate radio light-curves for Swift J1644-like jets across a plausible of circumnuclear gas densities. In particular, we post-process 1D and 2D hydrodynamic simulations of such jets with the SPEV radiative transfer code (including synchrotron self-absorption) (Mimica et al. 2009b). We find that most TDEs cannot have jets as powerful as those in the Swift events (with energies of $E > 10^{53}$ erg s$^{-1}$). This indicates that special conditions are needed to launch powerful jets: e.g. a highly super-Eddington accretion rate (De Colle et al. 2012), a TDE from a deeply plunging stellar orbit (Metzger & Stone 2016), or a particularly strong magnetic flux threading the star (Tchekhovskoy et al. 2014; Kelley et al. 2014).

4There were radio detections for two ROSAT flares: RX J1420.4+5334 and IC 3599. However, for RX J1420.4+5334 the radio emission was observed in a different galaxy than was originally associated with the flare. IC 3599 has shown multiple outbursts, calling into question whether it is a true TDE at all (Campana et al. 2015).
1.4 Stellar dynamics in galactic nuclei

Most galaxies of comparable or lower mass than our own contain a concentration of stars within a few parsecs of their centers (Côté et al. 2006). These Nuclear Star Clusters (NSCs) have large mean stellar densities ($\sim 10 - 10^7 \, M_\odot \, pc^{-3}$; Georgiev & Böker 2014a), and correspondingly high rates of collisional stellar interactions with associated electromagnetic or gravitational wave transients (Leigh et al. 2016).

The rate of such encounters depends on the distribution of stars in a galactic nucleus. In most regions of the galaxy, the gravitational potential can be approximated as smooth, and the time evolution of the stellar system is described by the collisionless Boltzmann equation (modulo issues of star formation and evolution). This is not the case in a galactic nucleus, where scatterings between individual stars become important. This process is known as relaxation and can be divided into resonant and non-resonant (or two-body) relaxation (Binney & Tremaine 1987).

Resonant relaxation occurs in potentials with orbits of reduced dimensionality. For example, highly symmetric potentials (such as the Kepler potential of an SMBH, or the simple harmonic oscillator potential of a globular cluster core) feature closed, 1D elliptical orbits, while any spherical potential will confine its orbits to a 2D plane (Rauch & Tremaine 1996; Hopman & Alexander 2006a). In such cases residual torques from the surrounding stellar system add coherently and change a star’s angular momentum over many orbital periods (although the star’s energy is conserved). Resonant relaxation is sub-divided into vector resonant relaxation (VRR; this affects only the direction of the angular momentum) and scalar resonant relaxation (SRR; this affects the magnitude of the angular momentum, and only occurs in potentials that lack apsidal precession like the Kepler potential). Two-body relaxation will occur in any star cluster. It changes the stars’ energy and angular momentum due to impulsive encounters (on a time-scale shorter than an orbital period).
The time-scales for these processes are

\[ t_{rx} \approx \frac{\sigma^3}{G^2 n m^2 \ln \Lambda}, \]  
\[ t_{rx,SRR} \approx \frac{N_\star \ln \Lambda}{\Lambda} t_{rx}, \]  
\[ t_{rx,VRR} \approx \frac{\sqrt{N_\star \ln \Lambda}}{\Lambda} t_{rx}, \]  

where \( \sigma \) is the one-dimensional velocity dispersion, \( n \) is the number density of stars, \( N_\star \) is the number of stars, \( m_\star \) is the mass of each star, and \( \ln \Lambda \) is the Coulomb logarithm, approximately given by the logarithm of the ratio of the SMBH and stellar masses (Alexander 2017). The two-body relaxation time is \( \sim 10^{10} \) years at 1 pc from the Galactic Center for \( 1M_\odot \) stars (Merritt 2013). In a multi-species stellar population, \( m^2_\star \) in eq. (1.4), should be replaced with the second moment of the present-day mass function. This is typically dominated by stellar mass black holes, which can reduce the two-body relaxation time by up to an order of magnitude.

The SRR time-scale is shorter than the two-body relaxation time inside of \( \sim 0.1 \) pc. In the absence of general relativity, SRR would flatten stellar density profile on small scales, as stars diffuse rapidly into a loss cone of highly eccentric orbits that bring them close enough to the SMBH to be consumed. In practice, general relativistic effects quench SRR before stars diffuse into the loss-cone (Alexander 2017). The VRR time-scale is \( \lesssim 10^8 \) years within 1 pc of the Galactic Center. Therefore, VRR can rapidly isotropize stellar orbits in this region. However, recent work has shown that while light stars isotropize, the steady state configuration of the heaviest stars (or BHs) is a disk (Kocsis & Tremaine 2011; Szölgyén & Kocsis 2018).

The steady state solution for a single mass stellar population evolving in a Keplerian potential due to two-body relaxation is an \( r^{-7/4} \) density profile (the so-called “Bahcall-Wolf cusp” Bahcall &
Wolf 1976). This solution has an outward energy flux, but no net mass flux. The other possible steady state solution is a steeper $r^{-9/4} - r^{-5/2}$ profile. This solution has an outward mass flux, and can arise when star formation or other source terms are present (see Peebles 1972; Bahcall & Wolf 1976; Fragione & Sari 2017 and Chapter 4).

In a population with stars of different masses and compact objects, each species will have a different steady state distribution. An evolved stellar population can be roughly divided into two mass bins: (i) $\sim 10M_\odot$ BHs and (ii) $\sim 1M_\odot$ stars and remnants. If the light stars dominate the diffusion coefficients, the steady state density profile for BHs (light stars) is between $r^{-2}$ and $r^{-11/4}$ ($r^{-3/2}$ and $r^{-7/4}$). If the BHs dominate the diffusion coefficients, the steady state density profile for the BHs (light stars) is $r^{-7/4}$ ($r^{-3/2} - m_\ast/(4m_h)$), where $m_\ast$ and $m_h$ are the masses of the light stars and BHs respectively (Bahcall & Wolf 1977; Keshet et al. 2009; Alexander & Hopman 2009).

1.4.1 X-ray binaries in the Galactic Center

Stellar-mass compact objects, particularly black holes (BHs) and neutron stars (NSs), play an important role in these environments; for example, they form sources of LIGO and LISA-band gravitational waves (e.g. Quinlan & Shapiro 1987; O’Leary et al. 2009; Tsang 2013; Bar-Or & Alexander 2016; Antonini & Rasio 2016; Stone et al. 2017a; Bartos et al. 2017), serve as probes of the relativistic spacetime near the central SMBH (Paczynski & Trimble 1979; Pfahl & Loeb 2004), and potentially contribute to the $\gamma$-ray excess observed in our own Galactic Center (GC; Brandt & Kocsis 2015). Compact objects in NSCs will also induce strong tidal interactions during close flybys with stars. A sequence of weak tidal encounters will stochastically spin up GC stars (Alexander & Kumar 2001; Sazonov et al. 2012), while a single very strong tidal encounter may disrupt the victim star and produce a luminous transient (Perets et al. 2016), but a tidal encounter of intermediate strength will bind the star to the compact object in a “tidal capture” (Fabian et al. 1975). A
sequence of run-away tidal captures by smaller stellar mass BHs can result in the formation of an SMBH (Stone et al. 2017b).

There is strong evidence of a population of NSs and stellar-mass BHs in the Milky Way (MW) GC. The hundreds of O/B stars currently located in the central parsec indicate a high rate of in situ NS/BH formation in this region (e.g. Levin & Beloborodov 2003; Genzel et al. 2003a). The discovery of even a single magnetar within $\lesssim 0.1$ pc of Sgr A* (Mori et al. 2013), given their short active lifetimes, also demands a high current rate of NS formation. The X-ray point sources in the GC also directly indicate a population of binaries containing compact objects. There are a total of six known X-ray transients in the central parsec (Muno et al. 2005; Hailey & Mori 2017). Of these six, three are strong BH X-ray binary (BH-XRB) candidates based on their spectral properties and the long time-scale (> 10 years) between their outbursts. The identity of the remaining transients is unknown, but they may be NS-XRBs. In addition to these transient sources, Hailey et al. (2018) recently discovered 12 quiescent non-thermal X-ray sources within the central parsec. These sources are spectrally consistent with quiescent XRBs and distinct from the magnetic CV that make up most of the X-ray sources outside of the central parsec. Additionally, the luminosity function of these sources is consistent with the luminosity function of dynamically confirmed BH XRBs in the field, while NSs are on average brighter in quiescence (Armas Padilla et al. 2014; Hailey et al. 2018). Other confirmed NS XRBs with similar luminosities in the Galactic Center region (though outside the central parsec) have bright outbursts with a cadence of 5-10 years. Such outbursts are not seen in the quiescent population, also pointing to BH XRBs rather than NS XRBs (Degenaar & Wijnands 2010). (However, there are quiescent NS XRBs in the globular cluster 47 Tuc that have not outburst for decades Bahramian et al. 2014). Overall, the most likely interpretation for this new population is quiescent BH XRBs (though an admixture of up to six MSPs cannot be ruled out). Reasonable extrapolation of the point source luminosity function below the instrumental detection
threshold implies a true number of BH-XRBs inside the central parsec in the hundreds.

The number of BH-XRBs in the GC per unit stellar mass exceeds that of the field population by a factor of $\gtrsim 100 - 1000$. This suggests that the unusual environment of the GC dynamically - and efficiently - assembles BH-XRBs, in a manner analogous to the dynamical overproduction of NS-XRBs in globular clusters (Katz 1975; Benacquista & Downing 2013). Although a high concentration of compact objects in the GC is itself unsurprising (Alexander & Hopman 2009), an overabundance of mass-transferring binaries is more challenging to understand. In other dense stellar systems like globular clusters, exchange interactions that swap compact objects into binaries can explain the overabundance of NS-XRBs and their MSP progeny (e.g. Ivanova et al. 2008). However, this channel is strongly suppressed in NSCs because nearly all primordial stellar binaries would be evaporated by three-body encounters, and those that survive would be so hard as to present a minimal cross-section for exchange interactions (see Leigh et al. 2017 and appendix H).

In Chapter 4 we explore an alternative mechanism for the formation of X-ray binaries: the tidal capture of main sequence stars by compact objects (Press & Teukolsky 1977; Lee & Ostriker 1986). Stars which pass sufficiently close to a compact object—approximately, within its tidal radius $r_t$—are completely torn apart by tidal forces (e.g. Rees 1988). However, for pericenter radii somewhat larger than $r_t$, tidal forces are not necessarily destructive; instead, they transfer orbital energy into internal oscillations of the star, binding it to the compact object. Following a complex and potentially violent process of circularization, the newly-created binary settles into a tight orbit. The necessarily small orbital separation of the tidal capture binary guarantees that subsequent gravitational wave emission will drive the star into Roche Lobe overflow in less than a Hubble time, forming a mass-transferring X-ray source. The high density of compact objects and stars in the GC inevitably lead to a significant rate of tidal captures, representing a promising explanation for the

\[5 \text{Note that the maximum semi-major axis above which binaries are evaporated scales as velocity dispersion } \sigma^{-2}, \text{ whereas the maximum pericenter for tidal capture scales } \sigma^{-5/2}; \text{ see appendix H for more details.}\]
observed overabundance of BH- and NS-XRBs.
Chapter 2

The circumnuclear medium

2.1 Introduction

Supermassive black holes (SMBHs) lurk in the centres of most, if not all nearby galaxies (see reviews by, e.g. Kormendy & Richstone 1995; Ferrarese & Ford 2005). However, only a few percent of these manifest themselves as luminous active galactic nuclei (AGN). Nearly quiescent SMBHs, such as those hosting low luminosity AGN, constitute a silent majority (e.g. Ho 2009).

Understanding why most SMBHs appear to be inactive requires characterizing their gaseous environments. Gas near the SMBH sphere of influence, hereafter denoted the ‘circumnuclear medium’ (CNM), controls the mass accretion rate, $\dot{M}_\bullet$. The accretion rate in turn determines the SMBH luminosity and the feedback of its energy and momentum output on larger scales. Dense gas in the nucleus may lead to runaway cooling, resulting in bursty episodes of star formation and AGN activity (e.g. Ciotti & Ostriker 2007).

Knowledge of how $\dot{M}_\bullet$ depends on the SMBH mass, $M_\bullet$, and other properties of the nucleus informs key questions related to the co-evolution of SMBHs and their host galaxies with cosmic time (e.g. Kormendy & Ho 2013; Heckman & Best 2014). In the low redshift Universe, SMBH
growth is dominated by low mass black holes, $M_\bullet \lesssim 10^8 M_\odot$ (e.g. Heckman et al. 2004), a fact often attributed to the trend of ‘cosmic down-sizing’ resulting from hierarchical structure growth (e.g, Gallo et al. 2008). However, the physical processes by which typical low mass black holes accrete could in principle be distinct from those operating at higher SMBH masses, or those in AGN. Of key importance is whether SMBHs grow primarily by the accretion of gas fed in directly from galactic or extragalactic scales, or whether significant growth can result also from local stellar mass loss in the nuclear region.

A better understanding of what mechanisms regulate accretion onto quiescent SMBHs would shed new light on a variety of observations, such as the occupation fraction of SMBHs in low mass galaxies. Miller et al. (2015) use the average relationship between the nuclear X-ray luminosities, $L_X$, of a sample of early type galaxies and their associated SMBH masses to tentatively infer that the SMBH occupation fraction becomes less than unity for galaxies with stellar masses $M_\star \lesssim 10^{10} M_\odot$ ($M_\bullet \lesssim 10^7 M_\odot$). This method relies on extrapolating a power-law fit of the $L_X - M_\bullet$ distribution to low values of $L_X$ below the instrument detection threshold, an assumption which is questionable if different physical processes control the accretion rates onto the lowest mass SMBHs.

The gas density in galactic nuclei also influences the emission from stellar tidal disruption events (TDEs), such as the high energy transient Swift J1644+57 (Bloom et al. 2011, Burrows et al. 2011; Levan et al. 2011; Zauderer et al. 2011). This event was powered by an impulsive relativistic jet, which produced synchrotron radio emission as the jet material decelerated from shock interaction with the CNM of the previously quiescent SMBH (Giannios & Metzger 2011; Zauderer et al. 2011). Modeling of J1644+57 showed that the CNM density was much lower than that measured surrounding Sgr A* on a similar radial scale (Berger et al. 2012; Metzger et al. 2012). However, a TDE jet which encounters a denser CNM would be decelerated more rapidly, producing different radio emission than in J1644+57. Variations in the properties of the CNM could help
explain why most TDEs appear to be radio quiet (e.g. Bower et al. 2013; van Velzen et al. 2013).

Gas comprising the CNM of quiescent (non-AGN) galaxies can in principle originate from several sources including wind mass loss from predominantly evolved stars and stellar binary collisions. Stellar wind mass loss is probably the dominant source insofar as collisions are relevant only in extremely dense stellar environments for very young stellar populations (Rubin & Loeb 2011).

The gas inflow rate on large scales is much easier to constrain both observationally and theoretically than the black hole (horizon scale) accretion rate. Ho (2009) determines the inflow rates in a sample of early-type galaxies by using X-ray observations to determine the Bondi accretion rate, and also by using estimated mass loss rates of evolved stars. Both methods lead him to conclude that the available gas reservoir is more than sufficient to power the observed low-luminosity AGN, assuming the standard \( \sim 10 \) per cent radiative efficiency for thin disc accretion. Several lines of evidence now suggest that low-luminosity AGN result from accretion proceeding in a radiatively inefficient mode (Yuan & Narayan 2014), due either to the advection of gravitationally-released energy across the SMBH horizon (e.g. Narayan & Yi 1995) or due to disc outflows, which reduce the efficiency with which the inflowing gas ultimately reaches the SMBH (e.g. Blandford & Begelman 1999; Li et al. 2013).

Another approach to determine the inflow rates, which we adopt in this chapter, is to directly calculate the density, velocity and temperature profiles of the CNM using a physically motivated hydrodynamic model. Mass is injected into the nuclear environment via stellar winds, while energy is input from several sources including stellar winds, supernovae (SNe), and AGN feedback (Quataert 2004; De Colle et al. 2012; Shcherbakov et al. 2014). Unlike previous works, which focused primarily on modeling individual galaxies, here we model the CNM properties across a representative range of galaxy properties, including different SMBH masses, stellar density profiles, and star formation histories (SFHs).
Previous studies, employing multi-dimensional numerical hydrodynamics and including variety of (parametrized) physical effects, have focused on massive elliptical galaxies (e.g. Ciotti & Ostriker 2007; Ciotti et al. 2010). These works show the periodic development of cooling instabilities on galactic scales, which temporarily increase the gas inflow rate towards the nucleus until feedback becomes strong enough to shut off the flow and halt SMBH growth.

In this chapter we focus on time-independent models, in which the nuclear gas receives sufficient heating to render radiative cooling negligible. This approach allows us to systematically explore the relevant parameter space and to derive analytic expressions that prove useful in determining under what conditions cooling instabilities manifest in the nuclear region across the expected range of galaxy properties, or whether other (non-AGN) forms of feedback can produce a prolonged state of steady, thermally stable accretion. Even if cooling instabilities develop on galactic scales over longer $\sim$ Gyr time-scales, we aim to explore whether a quasi steady-state may exist between these inflow events on smaller radial scales comparable to the sphere of influence.

In the presence of strong heating, one-dimensional steady-state flow is characterized by an inflow-outflow structure, with a critical radius known as the "stagnation radius" $r_s$ where the radial velocity passes through zero. Mass loss from stars interior to the stagnation radius is accreted, while that outside $r_s$ is unbound in an outflow from the nucleus. The stagnation radius, rather than the Bondi radius, thus controls the inflow rate (although we will show that $r_s$ often resides near the nominal Bondi radius). When heating is sufficiently weak, however, the stagnation radius may move to much larger radii or not exist at all, significantly increasing the inflow rate the SMBH, i.e. a "cooling flow". However, the hydrostatic nature of gas near the stagnation radius also renders the CNM at this location particularly susceptible to local thermal instabilities, the outcome of which could well be distinct from the development of a cooling flow.

This chapter is organized as follows. In §2.2 we describe our model, including the sample
of galaxy properties used in our analysis (§2.2.1) and our numerical procedure for calculating the steady-state hydrodynamic profile of the CNM (§2.2.2). In §2.3 we describe our analytic results, which are justified via the numerical solutions we present in §2.4. We move from a general and parametrized treatment of heating to a physically motivated one in §2.5, where we consider a range of physical processes that can inject energy into the CNM. In §2.6 we discuss the implications of our results for topics which include the nuclear X-ray luminosities of quiescent black holes, jetted TDEs, and the growth of SMBHs in the local Universe. In §2.7 we summarize our conclusions. Table 2.1 provides the definitions of commonly used variables. Appendix A provides useful analytic results for the stagnation radius, while Appendix B provides the details of our method for calculating stellar wind heating and mass input.

2.2 Model

2.2.1 Galaxy models

Lauer et al. (2007) use Hubble Space Telescope WFPC2 imaging to measure the radial surface brightness profiles for hundreds of nearby early type galaxies. The measured profile is well fit by a "Nuker" law parameterization:

\[ I(\xi) = I_b \xi^{2(\beta - \Gamma)/\alpha} (1 + \xi^\alpha)^{- (\beta - \Gamma)/\alpha}, \quad \xi \equiv \frac{r}{r_b}, \]

(2.1)

i.e., a broken power law that transitions from an inner power law slope, \( \Gamma \), to an outer power law slope, \( \beta \), at a break radius, \( r_b \). If the stellar population is spherically symmetric, then this corresponds to a 3D stellar density \( \rho_* \propto r^{-1-\Gamma} \) for \( r \ll r_b \) and \( \rho_* \propto r^{-1-\beta} \) for \( r \gg r_b \). We can
Table 2.1: Definitions of commonly used variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\bullet$</td>
<td>Black hole mass</td>
</tr>
<tr>
<td>$\tilde{v}_w$</td>
<td>Total heating parameter, including minimum heating rate from stellar and black hole velocity dispersion</td>
</tr>
<tr>
<td>$v_w$</td>
<td>Total heating parameter, excluding minimum heating rate from velocity dispersion</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>Stellar velocity dispersion (assumed to be radially constant)</td>
</tr>
<tr>
<td>$t_{\text{dyn}}$</td>
<td>$r/\sigma_0$: Dynamical time at large radii (where stellar potential dominates)</td>
</tr>
<tr>
<td>$t_f$</td>
<td>$(r^3/(GM_\bullet))^1/2$: Free fall time (where the black hole potential dominates)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Alternative heating parameter, $\zeta \equiv \sqrt{1 + (v_w/\sigma_0)^2}$ (eq. [2.16])</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Stagnation radius, where gas radial velocity goes to zero</td>
</tr>
<tr>
<td>$r_{\text{inf}}$</td>
<td>Radius of sphere of influence (eq. [2.3])</td>
</tr>
<tr>
<td>$r_b$</td>
<td>Outer break radius of stellar density profile</td>
</tr>
<tr>
<td>$r_{\text{IA}}$</td>
<td>Radius interior to which SN Ia are infrequent compared to the dynamical time-scale (eq. [2.36])</td>
</tr>
<tr>
<td>$\rho_\star (r)$</td>
<td>3D radial stellar density profile</td>
</tr>
<tr>
<td>$\rho (r)$</td>
<td>Gas density of CNM</td>
</tr>
<tr>
<td>$M_\star (r)$</td>
<td>Total enclosed stellar mass inside radius $r$</td>
</tr>
<tr>
<td>$M_{\text{enc}} (r)$</td>
<td>Total enclosed mass inside radius $r$ (SMBH + stars)</td>
</tr>
<tr>
<td>$q(r)$</td>
<td>Mass source term due to stellar winds, $q \propto \rho_\star$ (eq. [2.8])</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Parameter setting normalization of mass input from stellar winds (eq. [2.8])</td>
</tr>
<tr>
<td>$\tau_\star$</td>
<td>Age of stellar population, in case of a single burst of star formation</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Power-law slope of radial stellar surface brightness profile interior to the break radius</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Power-law slope of the 3D stellar density profile inside of the break radius, $\delta \equiv \Gamma + 1$.</td>
</tr>
<tr>
<td>$M$</td>
<td>Large scale inflow rate (not necessarily equal to the SMBH accretion rate)</td>
</tr>
<tr>
<td>$M_\bullet$</td>
<td>SMBH accretion rate, $M_\bullet = f_{\text{in}} M$, where $f_{\text{in}} &lt; 1$ accounts for outflows from the accretion disc on small scales.</td>
</tr>
<tr>
<td>$M_{\text{IA}}$</td>
<td>Maximum accretion rate as limited by SN Ia (eq. [2.39])</td>
</tr>
<tr>
<td>$M_{\text{C}}$</td>
<td>Maximum accretion rate for thermally stable accretion (eq. [2.29])</td>
</tr>
<tr>
<td>$\dot{q}_{\text{heat}} /</td>
<td>\dot{q}_{\text{rad}}</td>
</tr>
<tr>
<td>$t_h$</td>
<td>Hubble time-scale</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Gas density power-law slope at the stagnation radius (eq. [2.13])</td>
</tr>
</tbody>
</table>

write the deprojected stellar density approximately (formally, this is the $\alpha \to \infty$ limit) as

$$
\rho_\star = \begin{cases} 
\rho_\star |r_{\text{inf}}| (r/r_{\text{inf}})^{-1-\Gamma} & r \leq r_b \\
\rho_\star |r_b| (r/r_b)^{-1-\beta} & r > r_b,
\end{cases}
$$

(2.2)

where $\rho_\star |r_{\text{inf}}|$ is the stellar density at the radius of the black hole sphere of influence$^1$,

$$
|r_{\text{inf}}| \approx GM_\bullet / \sigma_0^2 \approx 14M_\bullet^{0.6} \text{ pc},
$$

(2.3)

$^1$This approximate expression agrees surprisingly well with the mean empirical scaling relation for $r_{\text{inf}}(M_\bullet)$ calculated in appendix C of Stone & Metzger (2016), although we note that this relation has significant scatter. Also, we note that the scaling is somewhat different for cores and cusps. Fixing the power law slope, $r_{\text{inf}} \approx 25(8)M_\bullet^{0.6}$ for core (cusp) galaxies. We will use separate scaling relations for $r_{\text{inf}}$ for cores and cusps unless otherwise noted.
where \( M_{\bullet,8} \equiv M_{\bullet}/10^8 M_\odot \) and the second equality in (2.3) employs the \( M_{\bullet} - \sigma \) relationship of McConnell et al. (2011),

\[
M_{\bullet} \simeq 2 \times 10^8 \left( \frac{\sigma_{\bullet}}{200 \text{ km s}^{-1}} \right)^{5.1} M_\odot.
\]  

(2.4)

This may be of questionable validity for low mass black holes (e.g., Greene et al. 2010; Kormendy & Ho 2013). Also, several of the black hole masses used in McConnell et al. (2011) were underestimated (Kormendy & Ho 2013). However, our results are not overly sensitive to the exact form of the \( M_{\bullet} - \sigma \) relationship that we use.

A galaxy model is fully specified by four parameters: \( M_{\bullet}, \Gamma, r_b, \) and \( \beta \). We compute models for three different black hole masses, \( M_{\bullet} = 10^6, 10^7, 10^8 M_\odot \). The distribution of \( \Gamma \) in the Lauer et al. (2007) sample is bimodal, with a concentration of “core” galaxies with \( \Gamma < 0.3 \) and a concentration of “cusp” galaxies with \( \Gamma > 0.5 \). We bracket these possibilities by considering models with \( \Gamma = 0.8 \) and \( \Gamma = 0.1 \).

We fix \( \beta = 2 \) but find that our results are not overly sensitive to the properties of the gas flow on radial scales \( \gtrsim r_b \). The presence of the break radius \( r_b \) is, however, necessary to obtain a converged steady state for some regions of our parameter space\(^2\). We consider solutions calculated for up to four values of \( r_b \): 50 pc, 100 pc, 200 pc, and 400 pc, motivated by the range of break radii from the Lauer et al. (2007) sample.\(^3\) We neglect values of \( r_b \) which would give unphysically large bulges for a given \( M_{\bullet} \).

Finally, we note that Lauer et al. 2005 find that \( \sim 60\% \) of cusp galaxies and \( \sim 29\% \) of core galaxies have (generally unresolved) emission in excess of the inward extrapolation of the Nuker law.

\(^2\)In addition, for \( \beta \leq 2 \) the stellar density must steepen at still larger radii to avoid the mass enclosed diverging to infinity. However, we do not find it necessary to include this outer break.

\(^3\)For the core galaxies the break radius follows the scaling relationship \( r_b \sim 90 (M_{\bullet,8})^{0.5} \) pc, with scatter of approximately one dex. Most of the cusp galaxies have \( r_b \) between 100 and 1000 pc, and lack a clear trend with \( M_{\bullet} \). The mean \( r_b \) for cusp galaxies in the Lauer et al. (2007) sample is \( \sim 240 \) pc.
Indeed, some low mass galaxies with $M_* \lesssim 10^8 M_\odot$ possess nuclear star clusters (Graham & Spitler 2009), which are not accounted for by our simple parametrization of the stellar density. In such cases gas and energy injection could be dominated by the cluster itself, i.e. concentrated within its own pc-scale “break radius” which is much smaller than the outer break in the older stellar population on much larger scales. Although our analysis does not account for such an inner break, we note that for high heating rates the stagnation radius and concomitant inflow rate are not sensitive to the break radius.

2.2.2 Hydrodynamic Equations

Following Quataert (2004, see also Holzer & Axford 1970; De Colle et al. 2012; Shcherbakov et al. 2014), we calculate the density $\rho$, temperature $T$, and radial velocity $v$ of the CNM for each galaxy model by solving the equations of one-dimensional, time-dependent hydrodynamics,

\begin{equation}
\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho r^2 v) = q \tag{2.5}
\end{equation}

\begin{equation}
\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) = -\frac{\partial p}{\partial r} - \rho \frac{GM_{\text{enc}}}{r^2} - qv \tag{2.6}
\end{equation}

\begin{equation}
\rho T \left( \frac{\partial s}{\partial t} + v \frac{\partial s}{\partial r} \right) = q \left[ \frac{v^2}{2} + \frac{\bar{v}_w^2}{2} - \frac{\gamma - 1}{\gamma - 1} \rho \right], \tag{2.7}
\end{equation}

where $p$ and $s$ are the pressure and specific entropy, respectively, and $M_{\text{enc}} = M_*(r) + M_*$ is the enclosed mass (we neglect dark matter contributions). We adopt an ideal gas equation of state with $p = \rho k T/\mu m_p$ with $\mu = 0.62$ and $\gamma = 5/3$. The source term in equation (2.5),

\begin{equation}
q = \frac{\eta \rho_*}{t_h}, \tag{2.8}
\end{equation}

25
represents mass input from stellar winds, which we parametrize in terms of the fraction \( \eta \) of the stellar density \( \rho^* \) being recycled into gas on the Hubble time \( t_h = 1.4 \times 10^{10} \) yr. To good approximation \( \eta \simeq 0.02(\tau^*/t_h)^{-1.3} \) at time \( \tau^* \) following an impulsive starburst (e.g., Ciotti et al. 1991)\(^4\), although \( \eta \) is significantly higher for continuous SFHs (bottom panel of Fig. 2.7).

Source terms \( \propto q \) also appear in the momentum and entropy equations (eqs. [2.6] and [2.7]) because the isotropic injection of mass represents, in the SMBH rest frame, a source of momentum and energy relative to the mean flow. Physically, these result from the mismatch between the properties of virialized gas injected by stellar winds and the mean background flow. The term \( \propto p/\rho = c_s^2 \) is important because it acts to stabilize the flow against runaway cooling (§2.3.4).

The term \( \propto \tilde{v}_w^2 = \sigma(r)^2 + v^2_w \) in the entropy equation accounts for external heating sources (e.g., Shcherbakov et al. 2014), where

\[
\sigma \approx \sqrt{\frac{3GM_*}{(\Gamma + 2)r} + \sigma_0^2}, \tag{2.9}
\]

is the stellar velocity dispersion. This accounts for the minimal amount of shock heating from stellar winds due to the random motion of stars in the SMBH potential. We take \( \sigma_0 \) to be constant and use \( \sigma_0^2 \approx 3\sigma_r^2 \), where \( \sigma_* \simeq 170M_*^{0.2} \) km/s is the one-dimensional, large scale velocity dispersion from the McConnell et al. (2011) \( M_* - \sigma \) relation. The second term, \( v^2_w \), parametrizes additional sources of energy input, including faster winds from young stars, millisecond pulsars (MSPs), supernovae, AGN feedback, etc (§2.5). We assume that \( v_w \) is constant with radius, i.e. that the volumetric heating rate is proportional to the local stellar density.

Our model does not take into account complications such as more complicated geometries or the discrete nature of real stars (Cuadra et al. 2006, 2008). In the case of Sgr A* these effects reduce

\(^4\)Ciotti et al. (1991) give the mass return rate from evolved stars as a function of B-band luminosity instead of volumetrically, but our expressions are equivalent.
the time-averaged inflow rate to $\sim 10^{-6}M_\odot\text{yr}^{-1}$ – an order of magnitude less than a 1D spherical model (Cuadra et al. 2006). Motions of individual stars can produce order of magnitude spikes in the accretion rate (Cuadra et al. 2008).

To isolate the physics of interest, our baseline calculations neglects three potentially important effects: heat conduction, radiative cooling, and rotation. Heat conduction results in an additional heating term in equation (2.7),

$$\dot{q}_{\text{cond}} = \nabla \cdot (\kappa \nabla T),$$

where $\kappa = \kappa_{\text{spitz}}/(1 + \psi)$ (Dalton & Balbus 1993) is the conductivity and $\kappa_{\text{spitz}} = \kappa_0 T^{5/2}$ is the classical Spitzer (1962) value ($\kappa_0 \approx 2 \times 10^{-6}$ in cgs units). The flux limiter $\psi = \kappa_{\text{spitz}} \nabla T/(5\phi \rho c_s^3)$ saturates the conductive flux if the mean free path for electron coulomb scattering exceeds the temperature length scale, where $c_s \equiv (kT/\mu m_p)^{1/2}$ is the isothermal sound speed and $\phi \lesssim 1$ is an uncertain dimensionless constant (we adopt $\phi = 0.1$). Even a weak magnetic field that is oriented perpendicular to the flow could suppress the conductivity by reducing the electron mean free path. However, for radially-decreasing temperature profiles of interest, the flow is susceptible to the magneto-thermal instability (Balbus 2001), the non-linear evolution of which results in a radially-directed field geometry (Parrish & Stone 2007). In §2.3.3 we show that neglecting conductivity results in at most order-unity errors in the key properties of the solutions.

Radiative cooling contributes an additional term to equation (2.7), of the form

$$\dot{q}_{\text{rad}} = -\Lambda(T)n^2,$$

where $n \equiv \rho/\mu m_p$ and $\Lambda(T)$ is the cooling function. We neglect radiative cooling in our baseline calculations, despite the fact that this is not justified when the wind heating $v_w$ is low or if the mass return rate $\eta$ is high. Once radiative cooling becomes comparable to other sources of heating
and cooling, its presence can lead to thermal instability (e.g. Gaspari et al. 2012, McCourt et al. 2012, Li & Bryan 2014a) that cannot be accounted for by our 1D time-independent model. Our goal is to use solutions which neglect radiation to determine over what range of conditions cooling instabilities will develop (§2.3.4).

Equations (2.5)-(2.7) are solved using a sixth order finite difference scheme with a third order Runge-Kutta scheme for time integration and artificial viscosity terms in the velocity and entropy equations for numerical stability (Brandenburg 2003). We assume different choices of $v_w = 300, 600, 1200 \text{ km s}^{-1}$ spanning a physically plausible range of thermally stable heating rates. Although we are interested in the steady-state inflow/outflow solution (assuming one exists), we solve the time-dependent equations to avoid numerical issues that arise near the critical sonic points.

Our solutions can be scaled to any value of the mass input parameter, $\eta$, since the mass and energy source terms scale linearly with $\rho$ or $\rho*,$; however, the precise value of $\eta$ must be specified when cooling or thermal conduction are included. We check the accuracy of our numerical solutions by confirming that mass is conserved across the grid, in addition to the integral constraint on the energy (Bernoulli integral).

### 2.3 Analytic Results

We first describe analytic estimates of physical quantities, such as the stagnation radius $r_s$ and the mass inflow rate, the detailed derivation of which are given in Appendix A.

\textsuperscript{5}code is available at https://github.com/alekseygenerozov/hydro
2.3.1 Flow Properties Near the Stagnation Radius

Continuity of the entropy derivative at the stagnation radius where \( v = 0 \) requires that the temperature at this location be given by (eq. [A.2])

\[
T|_{r_s} = \left( \frac{\gamma - 1}{\mu m_p} \frac{\nu}{\gamma} \right) \frac{v^2_w}{k} \approx \frac{\gamma - 1}{\mu m_p} \frac{13 + 8\Gamma}{13 + 8\Gamma - 6\nu}
\]

\[
\approx \begin{cases} 
5.0 \times 10^6 \, v_{500}^2 \text{ K core} \\
5.5 \times 10^6 \, v_{500}^2 \text{ K cusp,}
\end{cases}
\]

(2.12)

where \( v_{500} \equiv v_w/(500 \, \text{km s}^{-1}) \) and \( \nu \equiv -d\ln\rho/d\ln r|_{r_s} \) is the density power-law slope at \( r = r_s \).

Empirically, we find from our numerical solutions that

\[
\nu \simeq \frac{1}{6} (4\Gamma + 3)
\]

(2.13)

Hydrostatic equilibrium likewise determines the value of the stagnation radius (Appendix A, eq. A.6)

\[
r_s = \frac{GM_\bullet}{\nu v^2_w} \left[ \frac{13 + 8\Gamma}{4 + 2\Gamma} \right] = \frac{GM_\bullet}{\nu(v^2_w + \sigma^2_0)} \left[ \frac{M_\bullet |_{r_s} + 13 + 8\Gamma}{4 + 2\Gamma} - \frac{3\nu}{2 + \Gamma} \right]
\]

(2.14)

For high heating rates \( v_w \gg \sigma_0 \), the stagnation radius resides well inside the SMBH sphere of
influence. In this case $M_\star r_s/M_\bullet \ll 1$, such that equation (2.14) simplifies to

$$r_s \approx \left( \frac{13 + 8 \Gamma}{4 + 2 \Gamma} - \frac{3 \nu}{2 + \Gamma} \right) \frac{G M_\bullet}{\nu v_w^2} \approx \begin{cases} 8 \text{ pc } M_\star s v_{500}^{-2} & \text{core} \\ 4 \text{ pc } M_\star s v_{500}^{-2} & \text{cusp}, \end{cases}$$  \hfill (2.15)

where we have used equation (2.13) to estimate $\nu$ separately for core ($\Gamma = 0.1; \nu \approx 1$) and cusp ($\Gamma = 0.8; \nu \approx 0.6$) galaxies. This expression is similar to that obtained by Volonteri et al. (2011) on more heuristic grounds (their eq. 6).

In the opposite limit of weak heating ($v_w \lesssim \sigma_0$) the stagnation radius moves to large radii, approaching the break radius $r_b$ in the stellar density profile, implying that all of the interstellar medium (ISM) inside of $r_b$ is inflowing. In particular, we find that $r_s$ approaches $r_b$ for heating below a critical threshold

$$\zeta \equiv \sqrt{1 + \left( \frac{v_w}{\sigma_0} \right)^2} < \zeta_c \approx \sqrt{\left( \frac{r_b}{r_{\text{inf}}} \right)^{(1-\Gamma)}} + 1$$  \hfill (2.16)

Condition (2.16) approximately corresponds to the requirement that the heating rate exceed the local escape speed at the break radius, $r_b$. This result makes intuitive sense: gas is supplied to the nucleus by stars which are gravitationally bound to the black hole, so outflows are possible only if the specific heating rate $\sim v_w^2$ significantly exceeds the specific gravitational binding energy.

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2.3.2 Inflow Rate

The large scale inflow rate towards the SMBH is given by the total mass loss rate interior to the stagnation radius (eq. [2.8]),

\[ \dot{M} = 4\pi \int_0^{r_s} q r^2 dr = \frac{\eta M_* |v_\infty|}{t_h} = \frac{\eta M_*(r_s/r_{\text{inf}})^{2-\Gamma}}{t_h} \]

\[ \approx \begin{cases} 
4.5 \times 10^{-5} M_{\odot}^{1.76} v_5^{-3.8} \eta_{0.02} \text{M}_{\odot} \text{yr}^{-1} & \text{core} \\
3.2 \times 10^{-5} M_{\odot}^{1.48} v_5^{-2.4} \eta_{0.02} \text{M}_{\odot} \text{yr}^{-1} & \text{cusp},
\end{cases} \]  

(2.17)

where we have assumed \( v_w \gg \sigma_0 \) by adopting equation (2.15) for \( r_s \). The resulting Eddington ratio is given by

\[ \frac{\dot{M}}{M_{\text{edd}}} \approx \begin{cases} 
2.0 \times 10^{-5} M_{\odot}^{0.76} v_5^{-3.8} \eta_{0.02} & \text{core}, \\
1.4 \times 10^{-5} M_{\odot}^{0.48} v_5^{-2.4} \eta_{0.02} & \text{cusp},
\end{cases} \]

(2.18)

where \( \dot{M}_{\text{edd}} = 2.2 M_{\odot} \text{M}_{\odot} \text{yr}^{-1} \) is the Eddington accretion rate, assuming a radiative efficiency of ten per cent. Note the sensitive dependence of the inflow rate on the wind heating rate. Equation (2.18) is the radial mass inflow rate on relatively large scales and does not account for outflows from the SMBH accretion disc (e.g. Blandford & Begelman 1999; Li et al. 2013), which may significantly reduce the fraction of \( \dot{M} \) that actually reaches the SMBH. Thus we distinguish between the large scale inflow rate, \( \dot{M} \), and the accretion rate onto the black hole, \( \dot{M}_* \).

The gas density at the stagnation radius, \( \rho |_{r_s} \), is more challenging to estimate accurately. By using an alternative estimate of \( \dot{M} \) as the gaseous mass within the stagnation radius divided by the
free-fall time \( t_{\text{ff}}|_{r_s} = (r_s^3/GM_\bullet)^{1/2} \),

\[
\dot{M} \sim \frac{(4\pi/3)r_s^3\rho|_{r_s}}{t_{\text{ff}}|_{r_s}},
\] (2.19)

in conjunction with eqs. (2.17) and (2.15), we find that

\[
\rho|_{r_s} \approx \begin{cases} 
5.2 \times 10^{-26}M_{\bullet,8}^{-0.2}v_{500}^{-0.8}\eta_{0.02} \text{ g cm}^{-3} & \text{core}, \\
1.0 \times 10^{-25}M_{\bullet,8}^{-0.5}v_{500}^{0.6}\eta_{0.02} \text{ g cm}^{-3} & \text{cusp}
\end{cases}
\] (2.20)

It is useful to compare our expression for \( \dot{M} \) (eq. [2.17]) to the standard Bondi rate for accretion onto a point source from an external medium of specified density and temperature (Bondi 1952):

\[
\dot{M}_B = 4\pi\lambda r_B^2 \rho|_{r_B} v_{\parallel}|_{r_B},
\] (2.21)

where \( r_B \equiv GM/c_{s,\text{ad}}^2 \) is the Bondi radius, \( c_{s,\text{ad}} = \sqrt{\gamma kT/\mu m_p} \) is the adiabatic sound speed, \( v_{\parallel}|_{r_B} = r_B/t_{\text{ff}}|_{r_B} = (GM/\rho r_B)^{1/2} \) and \( \lambda \) is a parameter of order unity.

Equation (2.19) closely resembles the Bondi formula (eq. [2.21]) provided that \( r_B \) is replaced by \( r_s \). Indeed, for \( r_s < r_{\text{inf}} \) we have that (eq. [2.14])

\[
r_s \approx \frac{13 + 8\Gamma GM_\bullet}{4 + 2\Gamma} \frac{\nu v_w^2}{\nu v_w^2} \approx \frac{13 + 8\Gamma}{(2 + \Gamma)(3 + 4\Gamma)} r_B,
\] (2.22)

where the second equality makes use of equation (2.12).

### 2.3.3 Heat Conduction

Our analytic derivations neglect the effects of heat conduction, an assumption we now check. The ratio of the magnitude of the conductive heating rate (eq. [2.10]) to the external heating rate at the
stagnation radius is given by

\[
\nabla \cdot \left( \kappa \nabla T \right) \bigg|_{r_s} \sim \frac{2t_h \kappa_0 T_{7/2}^{7/2}}{r_s^2 \eta \rho_* |r_s| v_w^2} \left( 1 + \frac{\kappa_0 T_{7/2}^{7/2}}{5r_s \phi \rho c_s^2} \right)^{-1} \\
\]

\[
\sim \min \left\{ \begin{array}{ll}
20\eta_{0.02}^{-1} M_{\ast,8}^{-0.8} v_{500}^{6.8} & \text{unsaturated (core)} \\
30\eta_{0.02}^{-1} M_{\ast,8}^{-0.5} v_{500}^{5.4} & \text{unsaturated (cusp)} \\
2\phi & \text{saturated},
\end{array} \right. 
\]

(2.23)

where the second equality makes use of equations (2.12), (2.15), and we have made the approximations \( \nabla^2 \sim 1/r_s^2, \nabla \sim 1/r_s \). The stellar density profile is approximated as (eq. [2.2])

\[
\rho_* |_{r_s} \approx \frac{M_\ast (2 - \Gamma)}{4\pi r_{\text{inf}}^3} \left( \frac{r_s}{r_{\text{inf}}} \right)^{-1-\Gamma} \\
\approx \left\{ \begin{array}{ll}
7.9 \times 10^{-19} M_{\ast,8}^{-1.2} v_{500}^{2.2} \text{ g cm}^{-3} & \text{core} \\
2.3 \times 10^{-18} M_{\ast,8}^{-1.5} v_{500}^{3.6} \text{ g cm}^{-3} & \text{cusp},
\end{array} \right. 
\]

(2.24)

where the stagnation radius is assume to reside well inside the Nuker break radius.

Equation (2.23) shows that, even when conduction is saturated, our neglecting of heat conduction near the stagnation radius results in at most an order unity correction for causal values of the saturation parameter \( \phi < 0.1 \). Our numerical experiments which include conductive heating confirm this (§2.4). We do not consider the possibility that the conduction of heat from the inner accretion flow can affect the flow on much larger scales (Johnson & Quataert 2007).

### 2.3.4 Thermal Instability

Radiative cooling usually has its greatest impact near or external to the stagnation radius, where the gas resides in near hydrostatic balance. If radiative cooling becomes important, it can qualitatively
Figure 2.1: Minimum effective wind heating parameter required for thermal stability as a function of SMBH mass. Black lines show $v_{\text{TI}}$ (eq. [2.27]), the heating rate required for $(q_{\text{heat}}/|q_{\text{rad}}|)_{r_s} > 10$ in the high-heating limit when the stagnation radius lies interior to the influence radius, for different values of the mass loss parameter $\eta$ as marked. Blue lines show the minimum heating parameter required to have $\zeta > \zeta_c = (r_b/r_{\text{inf}})^{0.5(1-\Gamma)}$ (eq. [2.16]), separately for cusp (solid) and core (dashed) galaxies. Based on the Lauer et al. (2007) sample we take $r_{\text{inf}} = 25(8)M_\odot^{0.6}$ pc and $r_b = 90M_\odot^{0.5}$ pc (240 pc) for cores (cusps). For $\zeta < \zeta_c$ the stagnation radius moves from inside the influence radius, out to the stellar break radius $r_b$. This renders the flow susceptible to thermal runaway, even if $v_w > v_{\text{TI}}$. 
alter key features of the accretion flow. Initially hydrostatic gas is thermally unstable if the cooling time is much less than the free-fall time, potentially resulting in the formation of a multi-phase medium (Gaspari et al. 2012, 2013, 2015; Li & Bryan 2014b).

Even if the hot plasma of the CNM does not condense into cold clouds, the loss of pressure can temporarily increase the inflow and accretion rates by producing a large-scale cooling flow. When coupled to feedback processes which result from such enhanced accretion, this can lead to time-dependent limit cycle behavior (e.g. Ciotti & Ostriker 2007; Ciotti et al. 2010; Yuan & Li 2011, Gan et al. 2014), which is also inconsistent with our assumption of a steady, single-phase flow.

Cooling instability can, however, be prevented if destabilizing radiative cooling ($\dot{q}_{\text{rad}} \propto T^{-2.7}$) is overwhelmed by other sources of cooling, namely the stabilizing term $-q_{s}^{2} \propto -T$ in the entropy equation (eq. [2.7]). Neglecting radiative cooling to first order, this term is balanced at the stagnation radius by the external heating term, $\dot{q}_{\text{heat}} = q_{w}^{2}/2$. One can therefore assess thermal stability by comparing the ratio of external heating to radiative cooling $|\dot{q}_{\text{rad}}|$ (eq. [2.11]),

$$\frac{|\dot{q}_{\text{heat}}|}{|\dot{q}_{\text{rad}}|} \bigg|_{rs} \approx \begin{cases} 630\eta_{0.02}M_{\odot,8}^{-0.76}v_{500}^{7.2}, \text{core} \\ 660\eta_{0.02}M_{\odot,8}^{-0.48}v_{500}^{5.8}, \text{cusp}, \end{cases}$$

where we have used equations (2.20) and (2.24) for the gas and stellar densities at the stagnation radius, respectively. We have approximated the cooling function for $T < 2 \times 10^{7}$ K as $\Lambda(T) = 1.1 \times 10^{-22} (T/10^{6} \text{K})^{-0.7}$ erg cm$^{3}$s$^{-1}$, which assumes solar metallicity gas (Draine 2011; his Fig. 34.1).

To within a constant of order unity, equation (2.25) also equals the ratio of the gas cooling time-scale $t_{\text{cool}} \equiv (3nkT/2\mu)/|\dot{q}_{\text{rad}}|$ to the free-fall time $t_{\text{ff}}$ at the stagnation radius. This equivalence
can be derived using the equality
\[
\rho|_{r_s} = \frac{3q_{\text{ff}}|_{r_s}}{2 - \Gamma},
\]  
that results by combining equations (2.17), (2.19), and (2.24). Sharma et al. (2012) argue cooling instability develops in an initially hydrostatic atmosphere if \( t_{\text{cool}} \lesssim 10t_{\text{ff}} \), so equation (2.25) represents a good proxy for instability in this case as well.

Based on our numerical results (§2.4) and the work of Sharma et al. (2012) we define thermally stable flows according to the criterion \( \dot{q}_{\text{heat}} > 10|\dot{q}_{\text{rad}}| \) \( (t_{\text{cool}} > 10t_{\text{ff}}) \) being satisfied near the stagnation radius. This condition translates into a critical minimum heating rate
\[
v_w > v_{\text{TI}} \simeq \begin{cases}
280\eta^{0.14}M_\bullet^{0.11} \text{ km s}^{-1}, & \text{core} \\
240\eta^{0.17}M_\bullet^{0.08} \text{ km s}^{-1}, & \text{cusp}
\end{cases}
\]  

Equations (2.25) and (2.27) are derived using expressions for the stagnation radius and gas density in the high heating limit of \( \zeta > \zeta_c = \left(\frac{r_b}{r_{\inf}}\right)^{0.5(1-\Gamma)} \) (eq. [2.16]). However, for \( \zeta < \zeta_c \), the stagnation radius diverges to the break radius \( r_b \). The quasi-hydrostatic structure that results in this case greatly increases the gas density, which in practice renders the flow susceptible to thermal runaway, even if \( v_w > v_{\text{TI}} \) according to equation (2.27). In other words, the true condition for thermal instability can be written
\[
v_w^2 > \max[v_{\text{TI}}^2, \sigma_0(r_b/r_{\inf})^{(1-\Gamma)}]
\]  

Figure 2.1 shows \( v_{\text{TI}}(M_\bullet) \) for different mass input parameters \( \eta \), as well as the [\( \eta \)-independent] \( \zeta > \zeta_c \) criterion, shown separately for cusp and core galaxies. Based on the Lauer et al. (2007)
sample we take $r_{\text{inf}} = 25(8) M_{\odot}^{0.6}$ pc and $r_b = 90 M_{\odot}^{0.5}$ (240 pc) and thus $r_b/r_{\text{inf}} \simeq 4 M_{\odot}^{-0.1} (30 M_{\odot}^{-0.6})$ for cores (cusps). We see that for high $\eta$ and low $M_*$ the $v_w \gtrsim v_{\text{TI}}$ criterion is more stringent, while for low $\eta$ and high $M_*$, the $\zeta > \zeta_c$ criterion is more stringent.

The minimum heating rate for thermal stability corresponds to the maximum thermally-stable inflow rate. From equations (2.27) and (2.28) this is

$$\frac{\dot{M}_{\text{TI}}}{\dot{M}_{\text{edd}}} \simeq \begin{cases} \min & 2 \times 10^{-4} \eta_0^{0.47} M_{\odot}^{0.36} \\
\text{core,} & 2 \times 10^{-5} \eta_0^{0.02} M_{\odot}^{0.19} \\
\min & 8 \times 10^{-5} \eta_0^{0.59} M_{\odot}^{0.28} \\
\text{cusps.} & 2 \times 10^{-5} \eta_0^{0.02} M_{\odot}^{0.15} \end{cases} \quad (2.29)$$

Note that since the SMBH accretion rate cannot exceed the large scale inflow rate, $\dot{M}_{\text{TI}}$ also represents the maximum thermally stable accretion rate.

What we describe above as “thermal instability” may in practice simply indicate an abrupt transition from a steady inflow-outflow solution to a global cooling flow, as opposed a true thermal instability. In the former case the stagnation radius diverges to large radii, increasing the density in the inner parts of the flow, which increases cooling and creates a large inflow of cold gas towards the nucleus. A true thermal instability would likely result in a portion of the hot ISM condensing into cold clouds, a situation which may or may not be present in a cooling flow. In this chapter we do not distinguish between these possibilities, although both are likely present at some level. Finally, note that if the CNM were to “regulate” itself to a state of local marginal thermal instability (as has previously been invoked on cluster scales; e.g., Voit et al. 2015), then equation (2.29) might naturally reflect the characteristic mass fall-out rate and concomitant star formation rate.
Figure 2.2: Radial profiles of the CNM density (top), temperature (middle), and velocity (bottom), calculated for a representative sample of galaxies. Colors denote values of the effective wind heating rate, $v_w = 1200$ km s$^{-1}$ (blue), 600 km s$^{-1}$ (orange), and 300 km s$^{-1}$ (green). Line styles denote different black hole masses: $M_\bullet = 10^6M_\odot$ (dot-dashed), $10^7M_\odot$ (solid), and $10^8M_\odot$ (dashed). Thin and thick lines denote cusp galaxies ($\Gamma=0.8$) and core galaxies ($\Gamma=0.1$), respectively. Squares mark the locations of the stagnation radius.
2.3.5 Angular Momentum

Our spherically symmetric model neglects the effects of angular momentum on the gas evolution. However, all galaxies possess some net rotation, resulting in centrifugal forces becoming important at some radius $r_{\text{circ}} = l_s^2/(GM_*)$. Here $l_s = \langle rV_\phi \rangle \big|_{r_s}$ is the stellar specific angular momentum near the stagnation radius, from which most of the accreted mass originates, where $V_\phi$ is the stellar azimuthal velocity.

Emsellem et al. (2007) use two-dimensional kinematic data to measure the ratio of ordered to random motion in a sample of early type galaxies, which they quantify at each galactic radius $R$ by the parameter

$$\lambda_R \equiv \frac{\langle |rV| \rangle}{(R\sqrt{V^2 + \sigma^2})} \bigg|_{R \ll r_{\text{inf}}} \frac{V_\phi}{\sigma},$$

(2.30)

where $\sigma$ is the velocity dispersion and the brackets indicate a luminosity-weighted average. The circularization radius of the accretion flow can be written in terms of $\lambda_R$ as

$$\frac{r_{\text{circ}}}{r_s} \approx \frac{r_s}{r_{\text{inf}}} \lambda_R^2 \lesssim \lambda_R^2,$$

(2.31)

where we have used the definition $r_{\text{inf}} \equiv GM_*/\sigma^2$ and in the second inequality have assumed that $r_s \lesssim r_{\text{inf}}$, a condition which is satisfied for the thermally-stable solutions of interest.

Emsellem et al. (2007) (their Fig. 2) find that $\lambda_R$ is generally $< 0.1$ on radial scales $< 10$ percent of the galaxy half-light radius and that $\lambda_R$ decreases with decreasing $R$ interior to this point. From equation (2.31) we thus conclude that $r_{\text{circ}} \lesssim 0.01r_s$. For the low inflow rates considered the gas ($\dot{M}/\dot{M}_{\text{Edd}} < 0.01$) would be unable to cool on a dynamical time and would likely drive equatorial and polar outflows (Li et al. 2013). Our model cannot capture such two dimensional
structures, but would still be relevant for intermediate polar angles where the gas is inflowing.

2.4 Numerical Results

Our numerical results, summarized in Table 2.2, allow us to study a range of CNM properties and to assess the validity of the analytic estimates from the previous section.

Figure 2.2 shows profiles of the density $\rho(r)$, temperature $T(r)$, and radial velocity $|v(r)|/c_s$, for the cusp ($\Gamma = 0.8$) solutions within our grid. As expected, the gas density increases towards the SMBH $\rho \propto r^{-\nu}$ with $\nu \simeq 1$, i.e. shallower than the $-3/2$ power law for Bondi accretion. This power law behavior does not extend through all radii, however, as the gas density profile has a break coincident with the location of the break in the stellar light profile ($r_b = 100$ pc). The temperature profile is relatively flat at large radii, but increases as $\propto 1/r^k$ interior to the sphere of influence, where $k \lesssim 1$, somewhat shallower than expected for virialized gas within the black hole sphere of influence. The inwardly directed velocity increases towards the hole with a profile that is somewhat steeper than the local free-fall velocity $v \propto v_{ff} \propto r^{-1/2}$. The flow near the stagnation radius is subsonic, but becomes supersonic at two critical points. The inner one at $r \simeq 0.1r_s$ is artificially imposed for numerical stability, although we have verified that moving the inner boundary has a small effect on the solution properties near the stagnation radius. The outer one is located near the break radius, $r \simeq r_b$, and is caused by the transition to a steeper stellar density profile exterior to the outer Nuker break radius.

Figure 2.3 shows our calculation of the stagnation radius $r_s/r_{inf}$ as a function of the wind heating parameter $\zeta = \sqrt{1+(v_w/\sigma_0)^2}$, with different colors showing different values of $v_w$. Cusp and core galaxies are marked with square and triangles, respectively. Shown for comparison are our analytic results (eq. [A.7]) with solid and dashed lines for cusp and core galaxies, respectively,
Table 2.2: Summary of Numerical Solutions

<table>
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<th>$M_\ast$</th>
<th>$v_w$</th>
<th>$r_b^{(a)}$</th>
<th>$\frac{r_s}{r_{\text{inf}}}^{(b)}$</th>
<th>$\frac{M}{\dot{M}_{\text{Edd}}}^{(c)}(\eta = 0.2)$</th>
<th>$\frac{\dot{M}<em>{\text{cool}}}{\dot{M}</em>{\text{Edd}}}^{(d)}(\eta = 0.2)$</th>
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<td>(km s$^{-1}$)</td>
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<td>-</td>
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(a) Break radius of stellar density profile. (b)$r_{\text{inf}} = 14M_\ast \rho_\odot^6$ as fixed in our numerical runs. (c)Inflow rate in Eddington units, normalized to a stellar mass input parameter $\eta = 0.2$. (d)Ratio of wind heating rate to radiative cooling rate at the stagnation radius. †Calculated with radiative cooling and conductivity included, assuming mass loss parameter $\eta = 0.2$. ‡Calculated with radiative cooling and conductivity included, assuming mass loss parameter $\eta = 0.2$ and conductivity saturation parameter $\phi = 0.1$. Solutions including radiative cooling were performed for cusp galaxies with $v_w = 300$ km s$^{-1}$ and core galaxies with $v_w = 600$ km s$^{-1}$ for $\eta = 0.02, 0.2$, and 0.6. Values of $\eta$ resulting in thermally unstable solutions are marked in the final column. Solutions found to be thermally unstable for all $\eta \geq 0.02$ are denoted as $\text{TII}$.
calculated assuming $\nu = 1$ and $\nu = 0.6$, respectively.

Our analytic estimates accurately reproduce the numerical results in the high heating limit $\zeta \gg 1$ ($v_w \gg \sigma_0$; $r_s \lesssim r_{\text{inf}}$). However, for low heating the stagnation radius diverges above the analytic estimate, approaching the stellar break radius $r_b \gg r_{\text{inf}}$. This divergence occurs approximately when $\zeta < \zeta_c \propto (r_b/r_{\text{soi}})^{0.5(\Gamma - 1)}$ (eq. [2.16]). Physically this occurs because the heating rate is insufficient to unbind the gas from the stellar potential. Thus, for small values of the heating rate (small $\zeta$) the location of the stagnation radius will vary strongly with the break radius. This explains the behavior of the three vertically aligned green squares. These are three cusp ($\Gamma = 0.8$) galaxies with $M_\bullet = 10^8 M_\odot$ and $v_w = 300 \text{ km s}^{-1}$ but with different break radii ($r_b = 25, 50, \text{ and } 100 \text{ pc from top to bottom}$). This divergence of the stagnation radius to large radii occurs at a higher value of $\zeta$ in core galaxies ($\Gamma = 0.1$), explaining the behavior of the two core galaxies shown in Fig. 2.3 as vertically aligned orange triangles.

Figure 2.4 shows the inflow rate for a sample of our numerical solutions for different values of $v_w = 300, 600 \text{ and } 1200 \text{ km s}^{-1}$, and for both core ($\Gamma = 0.1$) and cusp galaxies ($\Gamma = 0.8$). Shown for comparison is our simple analytic estimate of $\dot{M}/\dot{M}_{\text{edd}}$ from equation (2.18). For high wind velocities ($v_w \gg \sigma_0$) the stagnation radius lies well inside the black hole sphere of influence and our analytic estimate provides a good fit to the numerical results. However, for low wind velocities and/or high $M_\bullet$ (large $\sigma_0$), the numerical accretion rate considerably exceeds the simple analytic estimate as the stagnation radius diverges to large radii (Fig. 2.3).

Fig. 2.5 shows the ratio of wind heating to radiative cooling, $\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}|$ (eq. [2.25] and surrounding discussion) as a function of radius for $v_w = 300, 600, \text{ and } 1200 \text{ km/s}, M_\bullet = 10^7 M_\odot, \text{ and } \Gamma = 0.8$. Radiative cooling is calculated using the cooling function of Draine (2011) for solar metallicity. For high heating rates of $v_w = 600$ and $1200 \text{ km s}^{-1}$ cooling is unimportant across all radii, while for $v_w = 300 \text{ km s}^{-1}$ we see that $\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}|$ and $t_{\text{cool}}/t_{\text{ff}}$ can be less than unity, depending
Figure 2.3: Stagnation radius $r_s$ in units of the sphere of influence radius $r_{\text{inf}}$ (eq. [2.3]) for galaxies in our sample as a function of the stellar wind heating parameter $\zeta \equiv \sqrt{1 + (v_w/\sigma_0)^2}$. Green, orange, and blue symbols correspond to different values of $v_w = 300$, 600, and 1200 km s$^{-1}$, respectively. Squares correspond to cusp galaxies ($\Gamma = 0.8$), while triangles correspond to cores ($\Gamma = 0.1$). Green circles correspond to cusp solutions which would be thermally unstable. The black curves correspond to the analytic prediction from equation (A.7), with thick solid and dot-dashed curves calculated for parameters ($\Gamma = 0.8, \nu \simeq 1$) and ($\Gamma = 0.1, \nu \simeq 0.6$), respectively. The thin black solid line corresponds to the simplified analytic result for $r_s$ from equation (2.15) (recall that $r_{\text{inf}} \simeq GM/\sigma_*^2$).
Figure 2.4: Inflow rate $\dot{M}/\dot{M}_{\text{edd}}$ versus SMBH mass for galaxies in our sample, calculated for different values of the wind heating parameter $v_w = 300 \text{ km s}^{-1}$ (green), $600 \text{ km s}^{-1}$ (orange), and $1200 \text{ km s}^{-1}$ (blue). Squares correspond to cusp galaxies ($\Gamma = 0.8$), while triangles correspond to cores ($\Gamma = 0.1$). The green circle corresponds to a cusp solution which would be thermally unstable. Thin solid and dot-dashed curves correspond to our simple analytic estimates of $\dot{M}/\dot{M}_{\text{edd}}$ (eq. [2.18]) for cusp and core galaxies, respectively. Thick curves correspond to the more accurate implicit analytic expression given by equation (A.6).
Figure 2.5: Ratio of the rates of heating to radiative cooling, $\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}|$, as a function of radius (solid lines) for a $M_\bullet = 10^7 M_\odot$ cusp galaxy ($\Gamma = 0.8$). Dashed lines show the ratio of the cooling time-scale to the free-fall time-scale $t_{\text{cool}}/t_{\text{ff}}$. For high heating rates both ratios are approximately equal at the stagnation radius (squares). When $\dot{q}_{\text{heat}}/\dot{q}_{\text{rad}} \lesssim 10$ (or, equivalently, $t_{\text{cool}}/t_{\text{ff}} \lesssim 10$ near the stagnation radius), then the flow is susceptible to thermal instabilities.
on the wind mass loss parameter $\eta$. Because at the stagnation radius $\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}|$ is within a factor of two of its minimum across the entire grid, the value of $(\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}|)_{r_s}$ is a global diagnostic of thermal instability for cusp galaxies. For core galaxies, the minimum value for the ratio of wind heating to radiative cooling may be orders of magnitude less than the value at the stagnation radius. However, we find that the $\zeta/\zeta_c$ criterion (see eq. (2.16)) combined with the ratio of wind heating to radiative cooling at $r_s$ still gives a reasonable diagnostic of thermal stability for core galaxies.

Also note that for $v_w = 600$ and $1200$ km s$^{-1}$ we have $\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}| \sim t_{\text{cool}}/t_{\text{ff}}$ near the stagnation radius, but these ratios diverge from each other at low heating ($v_w \lesssim 300$ km s$^{-1}$). This results because equations (2.19), (2.20) underestimate the true gas density in the case of subsonic flow (weak heating).

Although most of our solutions neglect thermal conduction and radiative cooling, these effects are explored explicitly in a subset of simulations. Dagger symbols in Table 2.2 correspond to solutions for which we turned on radiative cooling. For cases which are far from being thermal instability when cooling is neglected (e.g., $M_* = 10^8 M_\odot$, $v_w = 600$ km s$^{-1}$, $r_b = 100$ pc, $\eta = 0.2$, $\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}| = 76$), including radiative cooling has little effect on the key properties of the solution, such as the stagnation radius, mass inflow rate, and the cooling ratio $\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}|$. However, for solutions that are marginally thermally stable, including radiative losses acts to significantly decrease $\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}|$. We calculated solutions with radiative cooling for all cusp galaxies with $v_w = 300$ km s$^{-1}$ and for all core galaxies with $v_w = 600$ km s$^{-1}$, each for three different values of $\eta = 0.02, 0.2, 0.6$, spanning the physical range of expected mass loss for continuous star formation (Fig. 2.7). Solutions found to be unstable for all values of $\eta$ are marked in boldface as “TI” in Table 2.2, with the unstable values of $\eta$ provided in the final column.

Including thermal conduction, by contrast, results in only order unity changes to our solutions for our fiducial value of the saturation parameter $\phi = 0.1$, consistent with our analytic expectations.
(eq. [2.23]). However, we found that for runs which included conductivity, the stagnation radius varied with the location of the inner grid boundary: as the boundary is moved inwards, more heat can be conducted outwards from deeper in the gravitational potential well of the black hole. We expect conductivity would begin to have a significant effect for an inner boundary $\lesssim 0.01 r_s$ (the conductive run in Table 2.2 has an inner grid boundary at $\sim 0.03 r_s$). However, it is not clear physically if the magnetic field could remain coherent over so many decades in radius in the face of turbulence from stellar winds.

### 2.5 Heating Sources

Key properties of the flow, such as the mass inflow rate and the likelihood of thermal instability, depend sensitively on the assumed heating rate $\propto q v_w^2$. In this section we estimate the heating rate, $v_w$, taking into account contribution from stellar winds, supernovae, millisecond pulsars and SMBH feedback. We make the simplifying assumption that all sources of energy injection are efficiently mixed into the bulk of CNM gas. In fact, slower stellar wind material may cool and form high density structures as in Cuadra et al. (2005). Additionally, some of the injected energy may leak away from the bulk of the CNM material through low density holes, as described by Harper-Clark & Murray (2009) for stellar feedback and by Zubovas & Nayakshin (2014) for AGN feedback.

We summarize the individual heating sources below, providing estimates of the value of the effective wind velocity

$$v_w = \sqrt{\frac{2 \dot{h} \dot{e}}{\eta \rho_*}}$$

for each, where $\dot{e}$ is volumetric heating rate.
2.5.1 Stellar winds

Energy and mass input to the CNM by stellar winds is the sum of contributions from main sequence and post-main sequence populations. At early times following star formation, energy input is dominated by the fast line-driven outflows from massive stars (e.g., Voss et al. 2009). At later times energy input is dominated by main sequence winds (e.g., Naiman et al. 2013). Mass input is also dominated by the massive stars for very young stellar populations, but for most stellar ages the slow AGB winds of evolved low-mass stars dominate the mass budget.

In Appendix B we calculate the wind heating rate from stellar winds, $v_{\star w}$, and the mass loss parameter, $\eta$ (eq. [2.8]), as a function of age, $\tau_\star$, of a stellar population which is formed impulsively (Fig. B.1). At the earliest times ($\tau_\star \lesssim 10^7$ yr), the wind heating rate exceeds 1000 km s$^{-1}$, while much later ($\tau_\star \sim t_h$) stellar wind heating is dominated by main sequence winds is much lower, $v_{\star w} \sim 50 - 100$ km s$^{-1}$. As will be shown in §2.5.5, for the case of quasi-continuous star formation representative of the average SFHs of low mass galaxies, the heating rate from stellar winds can also be significant, $v_{\star w} \sim 1000$ km s$^{-1}$.

Stellar winds thus contribute a potentially important source of both energy and mass to the CNM. However, two additional uncertainties are (1) the efficiency with which massive stellar winds thermalize their energy and (2) heating from core collapse supernovae, which is potentially comparable to that provided by stellar winds. We neglect both effects, but expect they will act in different directions in changing the total heating rate.

2.5.2 Type Ia Supernovae

Type Ia SNe represent a source of heating, which unlike core collapse SNe is present even in an evolved stellar population. If each SN Ia injects thermal energy $E_{Ia}$ into the interstellar medium, and the SN rate per stellar mass is given by $R_{Ia}$, then the resulting volumetric heating rate of $E_{Ia}R_{Ia}$
produces an effective heating parameter (eq. [2.32]) of

\[ r_{w}^{\Delta} = \sqrt{\frac{2t_{h}R_{\text{Ia}}E_{\text{Ia}}}{\eta}}. \] (2.33)

The thermal energy injected by each SN Ia, \( E_{\text{Ia}} \simeq \epsilon_{\text{Ia}}10^{51} \) erg, depends on the efficiency \( \epsilon_{\text{Ia}} \) with which the initial blast wave energy is converted into bulk or turbulent motion instead of being lost to radiation. Thornton et al. (1998) estimate a radiative efficiency \( \epsilon_{\text{Ia}} \sim 0.1 \), depending weakly on surrounding density, but Sharma et al. (2014) argues that \( \epsilon_{\text{Ia}} \) can be considerably higher, \( \sim 0.4 \), if the SNe occur in a hot dilute medium, as may characterize the CNM. Hereafter we adopt \( \epsilon_{\text{Ia}} = 0.4 \) as fiducial.

The SN Ia rate, \( R_{\text{Ia}} \), depends on the age of the stellar population, as it represents the convolution of the star formation rate and the Ia delay time distribution (DTD) divided by the present stellar mass. In the limit of impulsive star formation, \( R_{\text{Ia}} \) is the DTD evaluated at the time since the star formation episode. The observationally-inferred DTD (Fig. 1 of Maoz et al. 2012) has the approximate functional form

\[ R_{\text{Ia}} = 1.7 \times 10^{-14} (\tau_{*}/t_{h})^{-1.12} M_{\odot}^{-1} \text{yr}^{-1} \] (2.34)

where \( \tau_{*} \) is the time since star formation. This is consistent with the theoretically expected DTD from double white dwarf mergers. The Ia rate will go to zero for \( \tau_{*} \lesssim 40 \) Myr (as this is the minimum time-scale to form a white dwarf). We also assume a flat DTD between \( 4 \times 10^7 \) and \( 3 \times 10^8 \) years, as expected from SNe Ia from a single white dwarf and a non-degenerate star. This channel would likely dominate at these early times; see Claeys et al. 2014.
From equations (2.33), (2.34) we thus estimate that

\[
v_{Ia}^{Ia} \approx 700(\epsilon_{Ia}/0.4)^{0.5}(\tau_*/t_h)^{-0.56} \eta_{0.02}^{-1/2} \text{ km s}^{-1}
\]

\[
\approx 700(\epsilon_{Ia}/0.4)^{0.5}(\tau_*/t_h)^{0.09} \text{ km s}^{-1},
\]

(2.35)

where the second line assumes \( \eta \approx 0.02(\tau_*/t_h)^{-1.3} \) for a single burst of star formation (e.g., Ciotti et al. 1991).

The high value of \( v_{Ia}^{Ia} \) implies that Ia SNe represent an important source of CNM heating. However, SNe can only be approximated as supplying heating which is spatially and temporally homogeneous if the rate of SNe is rapid compared to the characteristic evolution time of the flow at the radius of interest (Shcherbakov et al. 2014). We define the “Ia radius”

\[
r_{Ia} \sim \left( \frac{G}{R_{Ia} \sigma_0} \right)^{1/2} \sim 35M_{*8}^{-0.1}(\tau_*/t_h)^{0.56} \text{ pc}
\]

(2.36)

as the location exterior to which the time interval between subsequent supernovae \( \tau_{Ia} \sim (M_{\text{enc}}R_{Ia})^{-1} \sim G/(r\sigma_0^2R_{Ia}) \) exceeds the local dynamical time-scale \( t_{\text{dyn}} \sim r/\sigma_0 \), where we again adopt the Ia rate for an old stellar population and in the final equality we estimate \( \sigma_0 \approx \sqrt{3}\sigma_* \) using \( M_* - \sigma \) (eq. [2.4]).

Assuming that \( v_{Ia}^{Ia} \gg \sigma_0 \) then by substituting \( v_{Ia}^{Ia} \) (eq. 2.35) into equation (2.15) for the stagnation radius, we find that

\[
\left. \frac{r_{Ia}}{r_s} \right|_{v_{Ia}^{Ia}} \approx \begin{cases} 7 \eta_{0.02}^{-1} M_{*8}^{-1.1}(\epsilon_{Ia}/0.4)(\tau_*/t_h)^{-0.56} & \text{core} \\ 14 \eta_{0.02}^{-1} M_{*8}^{-1.1}(\epsilon_{Ia}/0.4)(\tau_*/t_h)^{-0.56} & \text{cusp} \end{cases}
\]

\[
\approx \begin{cases} 7 M_{*8}^{-1.1}(\epsilon_{Ia}/0.4)(\tau_*/t_h)^{0.74} & \text{core} \\ 14 M_{*8}^{-1.1}(\epsilon_{Ia}/0.4)(\tau_*/t_h)^{0.74} & \text{cusp} \end{cases}
\]

(2.37)
SN Ia can only be approximated as a steady heating source near the stagnation radius for extremely massive SMBHs with $M \gtrsim 10^9 M_\odot$ or for a very young stellar population with $\tau \ll t_h$.

Even if SN Ia are rare near the stagnation radius, they may cap the inflow rate (and thus the SMBH accretion rate) by periodically blowing gas out of the nucleus of low mass galaxies. Between successive SNe, stars release a gaseous mass $M_g \approx \eta M_\star \tau_{\text{Ia}}/t_h$ interior to the Ia radius, which is locally gravitationally bound to the SMBH by an energy $E_{\text{bind}} \gtrsim M_g \sigma_0^2$. From the above definitions it follows that

$$\frac{E_{\text{Ia}}}{E_{\text{bind}}} \lesssim \left(\frac{v_{\text{Ia}}}{\sigma_0}\right)^2 \frac{1}{2\sigma_0^2}. \quad (2.38)$$

Hence, for low mass BHs with $\sigma_0 \ll v_{\text{Ia}}$, SN Ia are capable of dynamically clearing out gas from radii $\sim r_{\text{Ia}} \gtrsim r_s$. Thus even when heating is sufficiently weak that the stagnation radius formally exceeds $r_{\text{Ia}}$, the SMBH accretion rate is still limited to a value

$$\frac{\dot{M}_{\text{Ia}}}{M_{\text{edd}}} \approx \frac{\eta M_\star}{M_{\text{edd}} t_h} \left(\frac{r_{\text{Ia}}}{r_{\text{inf}}}\right)^{2-\Gamma} \approx \begin{cases} 4.5 \times 10^{-4} M_\star^{-1.33} (\tau_*/t_h)^{-0.2} & \text{core} \\ 2.2 \times 10^{-4} M_\star^{-0.84} (\tau_*/t_h)^{-0.6} & \text{cusp,} \end{cases} \quad (2.39)$$

obtained substituting the Ia radius $r_{\text{Ia}}$ (eq. [2.36]) for $r_s$ in the derivation leading to our estimate of $\dot{M}$ (eq. [2.18]).

The deep gravitational potential wells of high mass galaxies prevent SN Ia from dynamically clearing out gas in these systems ($\sigma_0 \gg v_{\text{Ia}}$). Equation (2.39) nevertheless still represents a cap on the accretion rate in practice because the Ia heating rate (eq. [2.35]) is usually high enough to prevent the stagnation radius (calculated including the Ia heating) from substantially exceeding
If the stagnation radius moves inwards from \( r_{Ia} \), then decreased heating will force it outwards again. On the other hand, if the stagnation radius moves well outside of \( r_{Ia} \), then the high level of Ia heating will force it inwards.

### 2.5.3 Millisecond Pulsars

Energy injection from the magnetic braking of millisecond pulsars (MSPs) is a potentially important heating source. If the number of MSPs per unit stellar mass is \( n_{\text{msp}} \) and each contributes on average a spin-down luminosity \( \bar{L}_{sd} \), then the resulting heating per unit volume \( \dot{e} \approx \bar{L}_{sd} n_{\text{msp}} \epsilon_{\text{msp}} \) results in an effective heating rate (eq. [2.32]) of

\[
v_{w}^{\text{MSP}} \sim 30 \left( \frac{\epsilon_{\text{msp}}}{0.1} \right)^{1/2} \left( \frac{\bar{L}_{sd}}{10^{34} \text{ erg s}^{-1}} \right)^{1/2} \eta_{0.02}^{-1/2} \text{ km s}^{-1},
\]

where \( \epsilon_{\text{msp}} \) is the thermalization efficiency of the wind, normalized to a value \( \lesssim 0.1 \) based on that inferred by modeling the interstellar media of globular clusters (Naiman et al. 2013). Our numerical estimate assumes a pulsar density \( n_{\text{msp}} \sim 3 \times 10^{-40} \) MSPs g\(^{-1}\), calculated from the estimated \( \sim 30,000 \) MSPs in the Milky Way (Lorimer 2013) of stellar mass \( \approx 6 \times 10^{10} M_{\odot} \).

Based on the ATNF radio pulsar catalog (Manchester et al. 2005), we estimate the average spin-down luminosity of millisecond pulsars in the field to be \( \bar{L}_{sd} \sim 10^{34} \text{ erg s}^{-1} \), resulting in \( v_{w}^{\text{MSP}} \lesssim 30 \text{ km s}^{-1} \) for \( \eta \gtrsim 0.02 \). For higher spin-down luminosities, \( L_{sd} \sim 10^{35} \text{ ergs s}^{-1} \) characteristic of some Fermi-detected pulsars, then the higher value of \( v_{w}^{\text{MSP}} \lesssim 300 \text{ km s}^{-1} \) makes MSP heating in principle important under the most optimistic assumptions \( \epsilon_{\text{msp}} = 1 \) and \( \eta = 0.02 \).
2.5.4 SMBH Feedback

Feedback from accretion onto the SMBH represents an important source of heating which, however, is also the most difficult to quantify (e.g., Brighenti & Mathews 2003, Di Matteo et al. 2005; Kurosawa & Proga 2009; Fabian 2012 for a recent review). A key difference between AGN heating and the other sources discussed thus far is its dependence on the SMBH accretion rate \( \dot{M}_* \), which is itself a function of the heating rate (eq. [2.17]).

2.5.4.1 Compton Heating

There are two types of SMBH feedback: kinetic and radiative. Radiative feedback is potentially effective even in low luminosity AGN via Compton heating (e.g., Sazonov et al. 2004, Ciotti et al. 2010), which provides a volumetric heating rate (Gan et al. 2014)

\[
\dot{e} = 4.1 \times 10^{-35} n^2 \xi T_C \text{erg cm}^{-3} \text{s}^{-1},
\]

(2.41)

where \( \xi = L/nr^2 \) is the ionization parameter and \( L \) is the SMBH luminosity with Compton temperature \( T_C \sim 10^9 \text{K} \gg T \) (e.g., Ho 1999, Eracleous et al. 2010).

The importance of Compton heating can be estimated by assuming the SMBH radiates with a luminosity \( L = \epsilon \dot{M} c^2 \), where \( \epsilon \) is the radiative efficiency and where \( \dot{M} \) is estimated from equation (2.17). Then using equations (2.15), (2.17), (2.20), (2.24) we calculate from equation (2.32) that the effective heating rate at the stagnation radius is given by

\[
\nu = \frac{24 \nu_{0.02} T_{C,9}^{0.5} \dot{M}_{0.38}^{0.5} v_{500}^{1.4} \text{km s}^{-1}}{10^9}, \text{core}
\]

(2.42)

\[
30 \nu_{0.02} T_{C,9}^{0.5} \dot{M}_{0.24}^{0.7} v_{500}^{1.7} \text{km s}^{-1}, \text{cusp}
\]

where \( T_{C,9} = T_C/10^9 \text{K} \) and \( \epsilon_{-2} = \epsilon/0.01 \sim 1 \). We caveat that, unlike stellar wind heating,
Compton heating depends on radius, scaling as \( v_w^C(r) \propto \left(n^2\xi/\rho_\star\right)^{1/2} \propto r^{(\Gamma-\nu-1)/2} \), i.e. \( \propto r^{-0.6} \) and \( \propto r^{-0.75} \) for core (\( \Gamma = 0.8; \nu \simeq 1 \)) and cusp (\( \Gamma = 0.1; \nu \simeq 0.6 \)) galaxies, respectively. Although our model’s assumption that the heating parameter be radially constant is not satisfied, this variation is sufficiently weak that it should not significantly alter our conclusions.

If Compton heating acts alone, i.e. \( \tilde{v}_w = v_w^C \), then solving equation (2.42) for \( \tilde{v}_w \) yields

\[
\begin{align*}
v_w^C &\simeq \begin{cases} 140n_{0.02}^{0.21}T_{C,9}^{0.21}L_{\star,8}^{0.16} \text{ km s}^{-1} & \text{, core} \\ 95n_{0.02}^{0.34}T_{C,9}^{0.29}L_{\star,8}^{0.14} \text{ km s}^{-1} & \text{, cusp.} \end{cases}
\end{align*}
\tag{2.43}
\]

Compton heating is thus significant in young stellar populations with relatively high mass loss rates, e.g. \( v_w^C \gtrsim 300 \text{ km s}^{-1} \) for \( \eta \gtrsim 1 \).

The inflow rate corresponding to a state in which Compton heating self-regulates the accretion flow is given by substituting equation (2.43) into equation (2.18):

\[
\frac{\dot{M}_C}{\dot{M}_{\text{edd}}} \approx \begin{cases} 2 \times 10^{-3}n_{0.02}^{0.2}T_{C,9}^{-0.8}L_{\star,8}^{0.8} & \text{, core} \\ 8 \times 10^{-4}n_{0.02}^{0.3}T_{C,9}^{-0.7}L_{\star,8}^{0.7} & \text{, cusp.} \end{cases}
\tag{2.44}
\]

Sharma et al. (2007) estimate the value of the radiative efficiency of low luminosity AGN, \( \epsilon_{\text{rad}} \), based on MHD shearing box simulations of collisionless plasmas. For Eddington ratios of relevance, their results (shown in their Fig. 6) are well approximated by

\[
\epsilon_{\text{rad}} \simeq \begin{cases} 0.03 \left( \frac{\dot{M}_\star}{10^{-4}\dot{M}_{\text{edd}}} \right)^{0.9} \frac{\dot{M}_\star}{\dot{M}_{\text{edd}}} & \lesssim 10^{-4} \\ 0.03 & 10^{-2} \gtrsim \frac{\dot{M}_\star}{\dot{M}_{\text{edd}}} \gtrsim 10^{-4} \end{cases}
\tag{2.45}
\]

where \( \dot{M}_\star \) is the BH accretion rate. In general \( \dot{M}_\star \) will be smaller than the inflow rate \( \dot{M} \) calculated
thus far by a factor $f_{\text{in}} < 1$ due to outflows from the accretion disc on small scales. Thus, the full efficiency relating the mass inflow rate to the radiative output is $\epsilon = f_{\text{in}} \epsilon_{\text{rad}}$, where we take $f_{\text{in}} = 0.1$ following Li et al. (2013), who find that the fraction of the inflowing matter lost to outflows equals the Shakura-Sunyaev viscosity parameter of the disc.

Substituting $\epsilon$ into equation (2.44) and solving for the inflow rate results in

$$\frac{\dot{M}_C}{\dot{M}_{\text{edd}}} \approx \begin{cases} 4 \times 10^{-2} \eta_0^{0.2} T_{C,9}^{-0.8} M_{8}^{0.16}, & \text{core} \\ 9 \times 10^{-3} \eta_0^{0.3} T_{C,9}^{-0.7} M_{8}^{0.14}, & \text{cusp}. \end{cases}$$

Equation (2.46) may nevertheless represent a characteristic average value for the inflow rate if a quasi-equilibrium is achieved over many cycles.

### 2.5.4.2 Kinetic Feedback

Kinetic feedback results from outflows of energy or momentum from close to the black hole in the form of a disc wind or jet, which deposits its energy as heat, e.g. via shocks or wave dissipation, over much larger radial scales (e.g. McNamara & Nulsen 2007; Novak et al. 2011; Gaspari et al. 2012).

Assume that the outflow power is proportional to the SMBH accretion rate, $L_j = \epsilon_j \dot{M}_c c^2 = 0.1 \epsilon_j \dot{M} c^2$, where $\epsilon_j < 1$ is an outflow efficiency factor and we have again assumed a fraction $f_{\text{in}} = 0.1$ of the infall rate reaches the SMBH. Further assume that this energy is deposited as heat uniformly interior to a radius $r_{\text{heat}}$ and volume $V_{\text{heat}} \propto r_{\text{heat}}^3$. The resulting volumetric heating rate $\epsilon = \ldots$
$0.1\epsilon_j M c^2 / V_{\text{heat}}$ near the stagnation radius results in a heating parameter given by

$$v^* \approx \left( \frac{0.2 t_h \epsilon_j M c^2}{V_{\text{heat}} \eta \rho_* |_{r_s}} \right)^{1/2} \approx \frac{700}{\sqrt{2 - \Gamma}} \text{km s}^{-1} \left( \frac{\epsilon_j}{10^{-5}} \right)^{1/2} \left( \frac{r_s}{r_{\text{heat}}} \right)^{3/2},$$

where we have used the facts that $\dot{M} = \eta M_* |_{r_s}/t_h$ and $\rho_*(r_s) = M_*(r_s)/(2 - \Gamma)/(4\pi r_s^3)$. If the bulk of the energy from kinetic feedback is released near the stagnation radius, then even a small heating efficiency $\epsilon_j \gtrsim 10^{-4}$ is sufficient for $v^*$ to exceed other sources of non-accretion powered heating. However, if this energy is instead deposited over much larger physical scales comparable to the size of the galaxy, i.e. $r_{\text{heat}} \gtrsim 10$ kpc $\sim 10^4 r_s$, then kinetic feedback is unimportant, even for a powerful outflow with $\epsilon_j \sim 0.1$.

The time required for a jet of luminosity $L_j$ and half opening angle $\theta_j = 0.1$ (characteristic of AGN jets) to propagate through a gaseous mass $M_g$ of radius $r$ is estimated from Bromberg et al. (2011) to be

$$t_{\text{jet}} \sim 4000 \text{yr} \left( \frac{L_j}{10^{40} \text{ erg s}^{-1}} \right)^{-1/3} \left( \frac{r}{\text{pc}} \right)^{2/3} \left( \frac{M_g}{10^8 M_\odot} \right)^{1/3},$$

Approximating $M_g \sim \dot{M} t_{\text{ff}}$, the ratio of the jet escape time-scale to the dynamical time-scale $t_{\text{dyn}} \sim r/\sigma$ is given by

$$\frac{t_{\text{jet}}}{t_{\text{dyn}}} \sim 7 \times 10^{-3} M_*^{0.13} \left( \frac{\epsilon_j}{10^{-6}} \right)^{-1/3},$$

independent of $r$. A jet with power sufficient to appreciably heat the CNM on radial scales $\sim r_s$ ($\epsilon_j \gtrsim 10^{-5}$) also necessarily has sufficient power to escape the nuclear region and propagate to much larger radii. Slower outflows from the accretion disc, instead of a collimated relativistic jet, provide a potentially more promising source of feedback in these systems.
As in the case of Compton heating, kinetic heating could in principle ‘self-regulate’ the accretion flow insofar as a lower heating rate results in a higher accretion rate (eq. [2.48]), which in turn may create stronger kinetic feedback. However, given the uncertainty in the efficiency of kinetic heating, we hereafter neglect its effect and defer further discussion to §2.6.6.

2.5.5 Combined Heating Rate

The total external gas heating rate,

$$v_w = \sqrt{(v_w^*)^2 + (v_{w}^{\text{MSP}})^2 + (v_{w}^{\text{Ia}})^2 + (v_{w}^\bullet)^2},$$

(2.51)

includes contributions from stellar winds, supernovae, pulsars, and radiative SMBH feedback. We implicitly assume the different sources of energy injection mix efficiently. In reality, this may not be the case as slower velocity sources cool and form high density structure due to high pressure from the environment (Cuadra et al. 2005).

The strength of each heating source depends explicitly on the SMBH mass and the stellar population in the galactic nuclear region. The latter could best be described by a single starburst episode in the past, or by a more continuous SFH that itself varies systematically with the galaxy mass and hence $M_\bullet$.

2.5.5.1 Single Starburst

Figure 2.6 shows the contributions of heating sources as a function of time $\tau_*$ after a burst of star formation for black holes of mass $M_\bullet = 10^6 M_\odot$ (top) and $M_\bullet = 10^8 M_\odot$ (bottom). The heating and mass input parameters due to stellar winds, $v_w^*(\tau_*)$ and $\eta(\tau_*)$, are calculated as described in Appendix B (Fig. B.1). The SN Ia and Compton heating rates, $v_{w}^{\text{Ia}}$ and $v_{w}^{\text{C}}$, are calculated from
Figure 2.6: Sources contributing to the gas heating rate at time $\tau_*$ after a single burst of star formation. Top and bottom panels show black hole masses of $M_\bullet = 10^6 M_\odot$ and $M_\bullet = 10^8 M_\odot$ (both cusps with $\Gamma = 0.8$). Solid and dashed lines show the ranges of $\tau_*$ for which the accretion flow is thermally stable and unstable, respectively, according to the ratio of $\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}|$ near the stagnation radius (eq. [2.25]). Shown with horizontal gray lines are the stellar velocity dispersion for each SMBH mass, estimated as $\sigma_0 = \sqrt{3}\sigma_\bullet$ (eq. [2.4]).
Figure 2.7: Top Panel: Sources contributing to the total heating rate of the CNM, $v_w$ (black): stellar wind heating (orange), Ia supernovae (green), millisecond pulsars (blue), and compton heating (pink). Each heating source varies with black hole mass $M_\bullet$ calculated for average SFHs from Moster et al. (2013); see Appendix B for details. Bottom Panel: The ratio of the total heating rate ($\dot{q}_{\text{heat}} \propto v_w^2$) from the top panel to the radiative cooling rate ($|\dot{q}_{\text{rad}}|$) at the stagnation radius (red), parameter $\eta$ characterizing stellar mass loss rate (eq. [2.8]) as a function of black hole mass $M_\bullet$, calculated for average star SFHs from Moster et al. 2013 (blue), the ratio of Ia radius to stagnation radius as a function of $M_\bullet$ (green), and the ratio of $\zeta/\zeta_c$ (purple), where $\zeta_c$ (eq. [2.16]) is the critical heating parameter $\zeta \equiv \sqrt{1 + (v_w/\sigma_0)^2}$ below which outflows are impossible. All quantities are calculated for core ($\Gamma = 0.1$) galaxies. The results for cusp galaxies are qualitatively similar.
equations (2.35) and (2.42), respectively, which also make use of $\eta(\tau_*)$ as set by stellar winds.\(^6\) SN Ia heating depends also on a convolution of the DTD (eq. [2.34]) and the SFH, which reduces to the DTD itself for a single star burst. We account for the expected suppression of SN Ia heating resulting from its non-steady nature by setting $v_w^{\text{Ia}} = 0$ when the stagnation radius (calculated excluding Ia heating) exceeds the Ia radius $r_{\text{Ia}}$ (eq. [2.36]).

Figure 2.6 shows that stellar winds are the most important heating source at early times ($\tau_* \lesssim 10^7$ years). Compton heating becomes more important at later times due to (1) the higher accretion rates that accompany the overall decrease in all sources of heating, coupled with (2) the persistently high mass loss rates and gas densities associated with the still relatively young stellar population. SN Ia dominate the heating rate after $\sim 10^8$ yr. Core collapse supernovae will be an important heating source for times $\tau_* \leq 40$ Myr, albeit a non-steady one for $M_* \lesssim 10^8 M_\odot$. For simplicity we neglect supernova heating for $\tau_* \leq 40$ Myr, keeping in mind that it could raise the overall heating by a factor of a few.

As the total heating rate declines with time, the flow inevitably becomes thermally unstable according to the criterion $(\dot{q}_{\text{heat}}/|\dot{q}_{\text{rad}}|)_{rs} \lesssim 10$ (eq. [2.27]), as shown by dashed lines in Fig. 2.6. Compton heating is neglected in calculating thermal stability because—unlike local stellar feedback mechanisms—it is not clear that SMBH feedback is capable of stabilizing the flow given its inability to respond instantaneously to local changes in gas properties. Thermally unstable flow is present between a few $\times 10^7$ and a few $\times 10^8$ years.

Finally, MSP heating is negligibly small for fiducial parameters and hence is not shown, while kinetic feedback is neglected given its uncertain efficiency (§2.6.6).

\(^6\)Both $v_w^C$ and $v_w^{\text{Ia}}$ (through $r_s/r_{\text{Ia}}$) depend on the total heating rate (eq. [2.51]), requiring us to simultaneously solve a series of implicit equations to determine each.
2.5.5.2 Continuous SFH

A single burst of star formation does not describe the typical star formation history of most galaxies. Qualitatively, smaller galaxies will on average experience more recent star formation, resulting in energetic young stellar winds and supernovae which dominate the gas heating budget. Massive galaxies, on the other hand, will on average possess older stellar populations, with their heating rates dominated by SN Ia (on large radial scales) and SMBH feedback. We estimate the average value of $v_w$ as a function of SMBH mass for each heating source by calculating its value using the average cosmic star formation histories of Moster et al. (2013). Appendix B describes how the average SFH is used to determine the stellar wind heating $v_w^*$ and mass return parameter $\eta$ as a function of $M_\bullet$ (Fig. 2.7; top panel). SFHs are also convolved with the Ia DTD distribution to determine the SN Ia heating.

Figure 2.7 shows $v_w(M_\bullet)$ from each heating source (stars, MSPs, SNe Ia, and black hole feedback) calculated for the average SFH of galaxies containing a given black hole mass. The younger stellar populations characterizing low mass galaxies with $M_\bullet \lesssim 3 \times 10^8 M_\odot$ are dominated by stellar winds, with $v_w^* \gtrsim 700 \text{ km s}^{-1}$. Only for $M_\bullet \gtrsim 3 \times 10^8 M_\odot$ does the lack of young stellar populations significantly reduce the role of stellar wind heating. For these massive galaxies, however, the Ia radius is sufficiently small that SN Ia contribute a comparable level of heating, $v_{w,Ia} \gtrsim 500 \text{ km s}^{-1}$ (SN Ia do not contribute to the heating in low mass galaxies because $r_{Ia} > r_s$; bottom panel).

Strikingly, we find that galactic nuclei that experience the same average SFH as their host galaxies possess thermally stable flows across all black hole masses. An important caveat, however, is that $\tilde{v}_w$ is generally only a few times higher than the stellar velocity dispersion, i.e. $\zeta \sim \zeta_c$ (eq. [2.16]). This implies that the true stagnation radius and the inflow rate could be larger than our analytic estimates, potentially resulting in thermal instability in a significant fraction of galaxies. The bottom panel of Figure 2.7 also shows $\zeta/\zeta_c$, where $\zeta_c$ is estimated using fits for $r_b$, $r_{\inf}$, and
Γ derived from the Lauer et al. (2007) sample. In other words, the CNM of a significant fraction of massive ellipticals may be thermally unstable, not necessarily because they are receiving lower stellar feedback than the average for their galaxy mass, but due to the high heating required for stability given the structure of their stellar potential.

Realistic variations (“burstiness”) in the SFH can also produce lower stellar wind heating rates (Appendix B). Figure B.2 shows the heating rate as a function of average black hole mass for non-fiducial cases in which the current \((z = 0)\) star formation is suppressed by a factor \(\iota\) from its average \(z = 0\) value, for a characteristic time-scale \(\delta t_\star\). For \(\delta t_\star \lesssim 10^7\) yr, we see that \(v_w^\star\) is reduced by at most a factor of \(\approx 2\) from its average value, even for huge drops in the star formation rate \((\iota \sim 10^{-3})\). However, burstiness in the SFH over longer time-scales can suppress heating more significantly. When \(\delta t_\star \gtrsim 10^8\) yr, \(v_w^\star\) becomes very sensitive to \(\iota\): at \(\iota = 0.1\), \(v_w^\star \approx 400\) km s\(^{-1}\) in most galaxies, but for smaller \(\iota\), \(v_w^\star \approx 200\) km s\(^{-1}\) (an exception to this is in galactic nuclei with \(M_\bullet \gtrsim 10^8\) M\(_{\odot}\), where heating is stabilized by SN Ia). Our use of average cosmic SFHs is appropriate provided local variations are either on short time-scales \((\delta t_\star \lesssim 10^7\) yr), or limited in magnitude \((\iota \gtrsim 0.1)\). If both of these conditions are violated, then the effective heating rates for all but the most massive galaxies (where Ia explosions dominate) will fall by a factor \(\gtrsim 4\) from our fiducial volumetric averages, calculated using SFHs from Moster et al. (2013).

Another potential complication is the discreteness of local sources. We assume heating and mass injection are smooth, but energy and mass injection may be dominated by a handful of stars, particularly for small \(M_\bullet\). For example, for \(M_\bullet \lesssim 10^8\) M\(_{\odot}\), O stars dominate the energy injection in the average star formation histories of Moster et al. (2013), but the expected number of O stars inside the stagnation radius is 0.01 for \(M_\bullet = 10^6\) M\(_{\odot}\) and only exceeds 1 for \(M_\bullet \gtrsim 10^7\) M\(_{\odot}\), assuming circular stellar orbits\(^7\).

\(^7\)If the stars providing the heating reside on elliptical orbits, a greater number will contribute at any time due to the portion of their orbital phase spent at small radii.
2.6 Implications and Discussion

2.6.1 Inflow and Black Hole Accretion Rates

Figure 2.7 shows that the heating associated with the average SFH of a galaxy is approximately constant with SMBH mass, except for the largest black holes with $M_\bullet \gtrsim 5 \times 10^8 M_\odot$. For a fixed wind heating parameter, the Eddington ratio $\dot{M}/\dot{M}_{\text{edd}}$ increases $\propto M_\bullet^{0.5(0.8)}$ for core(cusp) galaxies, respectively (Fig. 2.4; eq. [2.18]).

Figure 2.8 (top panel) shows with solid lines the inflow rate as a function of black hole mass for the average star formation heating, calculated from equation (2.18) using our results for the total wind heating (Fig. 2.4). This average inflow rate increases from $\dot{M} \sim 10^{-6} - 10^{-5} M_\odot \text{ yr}^{-1}$ for low-mass black holes ($M_\bullet \sim 10^6 M_\odot$) to $\dot{M} \sim 10^{-4} M_\odot \text{ yr}^{-1}$ for $M_\bullet \sim 10^8 M_\odot$. These fall below the maximum thermally stable inflow rate, $\dot{M}_{\text{Tl}}$ (yellow lines).

For low mass black holes, blow out from SN Ia caps the SMBH accretion rate at a value $\dot{M}_{\text{In}} \sim 10^{-6} - 10^{-5} M_\odot \text{ yr}^{-1}$ (green lines; eq. [2.39]), which is however not low enough to prevent otherwise thermally-unstable flow at smaller radii. For higher $M_\bullet$ the stagnation radius resides close to the Ia radius so Ia heating contributes to the steady gas heating rate, even if the energy released by a single Ia is insufficient to unbind gas from the stellar bulge (eq. [2.38]). This explains why the black and teal lines meet at high $M_\bullet$.

Shown also for comparison in Figure 2.8 (top panel) is the inflow rate, $\dot{M}$, onto SgrA* calculated by Quataert (2004) and by Cuadra et al. (2008). Also shown is the range of $\dot{M}$ for the low-luminosity AGN NGC3115 derived through detailed modeling by Shcherbakov et al. (2014), who find a range of inflow rates depending on the assumed model for thermal conductivity. The inflow rate SgrA* estimated from Quataert (2004)\(^8\) exceeds our estimate of $\dot{M}$ due to the average SFH for the same

\(^8\)There is a typo in Quataert (2004): the inflow rate corresponds total stellar mass loss rate of $5 \times 10^{-4} M_\odot \text{ yr}^{-1}$ not $10^{-3} M_\odot \text{ yr}^{-1}$, as pointed out by Cuadra et al. (2006).
Figure 2.8:  

Top panel: Gas inflow rate $\dot{M}/\dot{M}_{\text{edd}}$ as a function of black hole mass $M_*$, shown for core ($\Gamma = 0.1$, dot-dashed) and cusp galaxies ($\Gamma = 0.8$, solid). Black lines show the inflow rate calculated from equation (2.18) using the heating rate provided at $z = 0$ by the average star formation histories for each galaxy mass (Fig. 2.7). Teal lines show the maximum accretion set by Ia supernovae blow-out or heating, $\dot{M}_{\text{Ia}}$ (2.39). Red lines show the inflow rate obtained if Compton heating acts alone, $\dot{M}_C$ (eq. [2.46]). Yellow lines show the maximum inflow rate for a thermally stable flow near the stagnation radius, $\dot{M}_{\text{Tl}}$ (eq. [2.29]). Also shown are the inferred inflow rates of SgrA* from Quataert (2004) and Cuadra et al. (2008) (gray circles) and for NGC3115 from Shcherbakov et al. (2014) (gray diamonds).

Bottom panel: Growth times $t_{\text{grow}} = M_*/\dot{M}_*$ in units of the Hubble time $t_h$ for each of the accretion rates shown in the top panel, where $\dot{M}_* = f_{\text{in}} \dot{M}$, where $f_{\text{in}} = 0.1$, to account for the fraction of inflowing mass lost to disc outflows on small scales.
$M_\bullet$ by at least an order of magnitude. This is because the mass source term in Quataert (2004) exceeds ours by a factor of $\sim 50$ (for cusp galaxies). The star formation in the central parsec of the Milky Way cannot be described as steady-state: feedback is instead dominated by the stellar winds from the ring of young massive stars of mass $\sim 10^4 M_\odot$ and estimated age $\sim 10$ Myr (e.g., Schödel et al. 2007). Such a star formation history is better described by our impulsive (starburst) scenario, for which the value of $\eta \sim 30$ at $\tau_\star = 10^7$ yr (Fig. B.1) is a factor of $\sim 100$ times higher than the value of $\eta \sim 0.4$ predicted in the average SFH case (Fig. 2.7, bottom panel). Note that the maximum thermally stable accretion rate is dependent on the $\eta$ parameter and would be higher in the case of an impulsive burst of star formation.

However, simulations by Cuadra et al. (2006) find that the inflow rate for Sgr A* is reduced by up to an order of magnitude compared to the result in Quataert (2004) when the discreteness and disc geometry of stellar sources is taken into account, and recent results from Yusef-Zadeh et al. (2015) suggest that the inflow rate could be further reduced to $\dot{M}/\dot{M}_{\text{Edd}} \sim 3 \times 10^{-6}$ once clumping of stellar winds is taken into account.

### 2.6.2 Nuclear X-ray Luminosities

The unresolved core X-ray luminosities of nearby galactic nuclei provide a powerful diagnostic of the SMBH accretion rates and how they vary with SMBH mass and other galaxy properties (e.g., Ho 2008 and references therein). Figure 2.9 shows our predictions for $\langle L_X \rangle = \epsilon_X \dot{M} c^2$ as a function of black hole mass, where $\epsilon_X = f_{\text{in}} \epsilon_{\text{rad}} \epsilon_{\text{bol}}$, $f_{\text{in}} = 0.1$ accounts for the fraction of the inflowing matter loss to outflows from the accretion disc, $\epsilon_{\text{rad}}$ is the radiative efficiency of the accreted matter, and $\epsilon_{\text{bol}} = 0.1$ is the assumed bolometric correction into the measured X-ray band for low luminosity AGN (Ho 2008). In all cases the value of $\langle L_X \rangle$ is calculated using the accretion rate from the top line of equation (2.17) with $\Gamma = 0.7$ for $M_\bullet < 4 \times 10^7 M_\odot$, and $\Gamma = -0.3 \log_{10} \left( \frac{M_\bullet}{4 \times 10^7 M_\odot} \right) + 0.7$ for
$M_\bullet \geq 4 \times 10^7 M_\odot$. This functional form is designed to approximately reproduce the behavior of $\Gamma(M_\bullet)$ in the Lauer et al. (2007) sample.

The left panel of Figure 2.9 is calculated assuming a constant low value for the radiative efficiency of $\epsilon_X = 10^{-4}$, typical of those estimated for low luminosity AGN (e.g., Ho 2009). The right panel luminosities are calculated instead assuming an efficiency of $\epsilon_X = \epsilon_{\text{bol}}\epsilon_{\text{rad}}$ that depends on the Eddington ratio as predicted by MHD shearing box simulations by Sharma et al. (2007, see eq. [2.45] and surrounding discussion). Shown for comparison are the X-ray measurements (black stars) and upper limits (gray triangles) from the sample of early-type galaxies compiled by Miller et al. (2015, cf. Gallo et al. 2010). A black line shows the best power-law fit to the X-ray luminosity from Miller et al. (2015), given by $\langle L_X/L_{\text{edd}} \rangle \sim M_\bullet^\alpha$ with $\alpha = -0.2$ (see also Zhang et al. 2009; Pellegrini 2010; Gallo et al. 2010). Also shown are the maximum accretion rates, respectively, for thermally stable accretion (eq. [2.29]; yellow), as set by SN Ia blow-out/heating (eq. [2.39]; teal), and as allowed by Compton heating feedback (eq. [2.46]; pink).

If the nuclei of elliptical galaxies are heated as expected for the average SFH of galaxies with similar mass, then to first order we predict that $\langle L_X/L_{\text{edd}} \rangle$ should be an increasing function of BH mass. This result is in tension with the observed down-sizing trend: the average Eddington ratio is seen to decrease (albeit weakly) with $M_\bullet$, especially in the model where $\epsilon_X$ increases with the Eddington ratio. However, one must keep in mind the enormous uncertainty in calculating the luminosity of the accretion flow close to the black hole in a single waveband based on the feeding rate on larger scales. Also, if disc outflows indeed carry away most of the infalling mass before it reaches X-ray producing radii (e.g. Blandford & Begelman 1999; Li et al. 2013), then the angular momentum of the infalling gas might also influence the X-ray luminosity indirectly through the inflow efficiency $f_{\text{in}}$, in a way that could depend systematically on the stellar population and hence $M_\bullet$. However, in general the efficiency with which inflowing matter reaches the SMBH (and hence
Figure 2.9: Average nuclear X-ray luminosity, \( \langle L_X \rangle = \epsilon_X \dot{M} c^2 \), as a function of SMBH mass. Green lines show our prediction in the case of heating due to the average SFH for galaxies corresponding to each black hole mass (Fig. 2.8), calculated using \( \Gamma = 0.7 \) for \( M_\bullet < 4 \times 10^7 M_\odot \), and \( \Gamma = -0.3 \log_{10}(M_\bullet/4 \times 10^7 M_\odot) + 0.7 \) for \( M_\bullet \geq 4 \times 10^7 M_\odot \), as derived from the Lauer et al. (2007) sample. Shown for comparison are the measurements (black stars) and upper limits (gray triangles) for \( L_X/L_{edd} \) values, from the Miller et al. (2015) sample of early-type galaxies (a black line shows the best power-law fit \( \langle L_X/L_{edd} \rangle \propto M_\bullet^\alpha \), with \( \alpha = -0.2 \). The gray line shows the typical sensitivity limit for this sample. The top panel is calculated assuming a constant radiative efficiency \( \epsilon_X = 10^{-4}(\epsilon_{bol}/0.1) \), while the bottom panel assumes \( \epsilon_X = f_{in}\epsilon_{bol}\epsilon_{rad} \), where \( \epsilon_{rad} \) is the radiative efficiency of low luminosity accretion discs calculated by Sharma et al. (2007) (eq. [2.45]) and \( f_{in} = 0.1 \) is the fraction of the mass inflowing on large scales that reaches the SMBH. Shown for comparison are the X-ray luminosities calculated for the maximum thermally-stable accretion rate (dashed orange line; eq. [2.29]); the SN Ia-regulated accretion rate (dashed teal line; eq. [2.39]), and the Compton heating-regulated accretion rate (dashed pink line; eq. [2.44])\(^9\), again all calculated for the average SFH corresponding to each SMBH mass.
X-ray luminosity) would naively be expected to decrease with decreasing $M_\bullet$ due to the increasing angular momentum of low mass galaxies, exacerbating the tension between our findings and the observed downsizing trend.

Perhaps a more readily addressable test is whether a steady-state accretion picture developed in this work can account for the the large scatter, typically of $2 \rightarrow 3$ orders of magnitude, in $L_X/L_{edd}$ at fixed $M_\bullet$. Scatter could result from the strong sensitivity of the accretion rate to the stellar wind velocity and mass loss parameter, $\dot{M}/\dot{M}_{edd} \propto v_w^{-3.8(-2.4)}$ for core(cusp) galaxies, respectively (Fig. 2.4; eq. [2.18]). When combined with the significant dependence of $v_w$ on stochastic intermittency in the star formation history (Appendix B.2), this can lead to order of magnitude differences in $\dot{M}$. However, we note that $v_w$ is not expected to vary by more than a factor of a few for a thermally-stable flow, limiting the allowed variation of $\dot{M}/\dot{M}_{edd}$. The discrete nature of stellar wind sources and their motions could result in an additional order of magnitude variation $\dot{M}$ (Cuadra et al. 2008).

Variations in $L_X/L_{edd}$ could also result from differences in angular momentum of the infalling gas from galaxy to galaxy, resulting in differences in the fraction of the gas lost to outflows. Also potentially contributing is the order of magnitude difference, at fixed $v_w$ and $M_\bullet$, between $\dot{M}$ for core and cusp galaxies (Fig. 2.8). Differences in $\dot{M}$ are augmented by the theoretical expectation that $L_X \propto \dot{M}^2$ for radiatively inefficient flows.

### 2.6.3 Steady Accretion versus outbursts

It is also possible, and indeed likely, that many of the Miller et al. (2015) sample of galactic nuclei are not accreting in steady state. This is supported by the fact that many of the X-ray luminosities in Figure 2.9 lie above the predictions for stable accretion, i.e., $L_X \gtrsim \epsilon_X \dot{M}_{TIc}^2$ (yellow line), at least for our choice of radiative efficiency.
Within our steady state solutions, gas outside the stagnation radius $r_s$ is blown out in wind. However, for very massive galaxies ($M_* \gtrsim 10^8 M_\odot$), the energy injected into the gas is not enough to truly eject it from the potential well of the galaxy, even if the gas is blown out of the stellar bulge (e.g. even if the $\zeta > \zeta_c$ criterion is satisfied—see equation (2.16) and surrounding discussion). This gas is likely to build up on the outskirts of the galaxy until a thermal instability develops, causing the CNM to undergo cyclic oscillations of rapid cooling and high accretion rates, followed by quiescent periods once gas has been consumed and/or AGN feedback becomes effective. Such cycles are seen in numerical simulations of elliptical galaxies on large scales and long time-scales (Ciotti et al. 2010). Evidence for such periodic outbursts includes observations showing that a fraction of early-type galaxies in the local Universe have undergone recent ($< 1 - 2$ Gyr old) star formation episodes (Donas et al. 2007).

If the actual radiative efficiency is lower than our fiducial assumption, then an even larger fraction of the X-ray detections would lie above the predictions for stable accretion. For example, we assume that a fraction $f_{\text{in}} = 0.1$ of the mass inflow rate reaches the SMBH, but this fraction could be considerably smaller. In such a case the X-ray detected galaxies will experience less heating than we calculate for the average SFH, and may not be stably accreting at all. This is plausible given that the Miller et al. (2015) sample includes only early-type galaxies with older stellar populations than the average for their stellar mass.

Figure 2.8 (top panel) also shows that $\dot{M}_C > \dot{M}_{\text{T1}}$ across all black hole masses. This indicates that Compton heating is not sufficient to produce a thermally stable accretion rate, at least for conditions corresponding to the average SFH.

Similar cyclic AGN activity could occur on the smaller radial scale of the sphere of influence (e.g. Yuan & Li 2011, Cuadra et al. 2015). In this case the large scatter in the X-ray luminosities shown in Fig 2.9 could result from the wide range of inflow rates experienced over the course of a
cyclic episode between instabilities and periodic nuclear outbursts.

If the galaxies with measured $L_X$ at low $M_\bullet$ in Figure 2.9 are indeed in a state of thermally-unstable outburst, then some of the galaxies with X-ray upper limits could be contributing to a separate population of thermally-stable, steadily accreting nuclei. Such a population could include SgrA*, which has a much lower X-ray luminosity for its SMBH mass than predicted by the Miller et al. (2015) trend. A potentially bimodal population of steady and outbursting galactic nuclei calls into question the practice of using simple extrapolations of power-law fits to the $L_X(M_\bullet)$ relationship to low $M_\bullet$ to constrain the occupation fraction of SMBHs in the nuclei of low mass galaxies.

2.6.4 SMBH Growth Times

Low-mass AGN in the local Universe with $M_\bullet \lesssim 3 \times 10^7 M_\odot$ are observed to be growing on a time-scale comparable to the age of the Universe, while the most massive SMBHs with $M_\bullet \gtrsim 10^9 M_\odot$ possess local growth times which are more than 2 orders of magnitude longer (Heckman et al. 2004; Kauffmann & Heckman 2009).

The bottom panel of Figure 2.8 shows the SMBH growth time, $t_{\text{grow}} \equiv M_\bullet / \dot{M}_\bullet$, for each accretion rate shown in the top panel, recalling that $\dot{M}_\bullet = f_{\text{in}} \dot{M}$, where $f_{\text{in}} = 0.1$. For SMBHs which accrete steadily at the rate set by stellar wind heating due to the average star formation history of their host galaxies, we see that $t_{\text{grow}}$ exceeds the Hubble time by 2–4 orders of magnitude across all $M_\bullet$. Steady accretion therefore cannot explain the growth of low mass black holes, a fact which is not surprising given that approximately half of this growth occurs in AGN radiating within 10 per cent of their Eddington rate (Heckman et al. 2004). Such high accretion rates likely instead require a source of gas external to the nuclear region, triggered either by galaxy mergers associated with the hierarchical growth of structure or thermal instabilities on larger, galactic scales (e.g. Ciotti et al. 2010, Voit et al. 2015).
That the growth time associated with thermally-unstable accretion (yellow line in Fig. 2.8) exceeds the Hubble time across all SMBH masses highlights the fact that significant black hole growth in the local Universe cannot result from thermally-stable steady accretion of gas lost from the surrounding stellar population studied in this chapter. Gas blow-out by SN Ia cannot alone prevent the growth of low-mass black holes, as indicated by the low growth times \( \ll t_h \) allowed by Ia heating (green lines), although Ia heating could play in principle a role in capping the growth rate of \( \gtrsim 10^8 M_\odot \) black holes, again depending on the efficiency of Ia heating.

### 2.6.5 TDE Jets

Our results also have implications for the environments encountered by relativistic jets from TDEs. For low-mass SMBHs with \( M_\bullet \lesssim 10^7 M_\odot \), such as that responsible for powering the transient Swift J1644+57 (Bloom et al. 2011), we predict gas densities of a \( n \sim \text{few cm}^{-3} \) on radial scales of 0.3 pc for wind heating rates within the physically expected parameters \( v_w \sim 500 - 1000 \text{ km s}^{-1} \) and \( \eta = 0.4 \) (see Fig. 2.2). This is comparable to the density \( \sim 0.3-10 \text{ cm}^{-3} \) obtained by modeling the radio afterglow of Swift J1644+57 (Berger et al. 2012, Metzger et al. 2012). However, there is considerable uncertainty in the afterglow modeling. For example, Mimica et al. (2015) find a much higher density of 60 cm\(^{-3}\) at 0.3 pc.

Berger et al. (2012) infer a flattening of the gas density profile of the host of J1644+57 on radial scales \( r \gtrsim 0.3 \text{ pc} \) (their Fig. 6) which looks qualitatively similar to the shape of the density profile we predict for core galaxies (Fig. 2.2, solid line). To obtain such a flattening on the inferred radial scales would require a black hole of \( M_\bullet \sim 10^7 M_\odot \). One somewhat larger scales (\( \sim 0.6 \text{ pc} \)), the observationally inferred gas density profile has an inflection point and begins to steepen. This may be due to a break in the stellar density profile. In particular, this steepening could result from the outer edge of a sub-parsec nuclear star cluster (e.g., Carson et al. 2015). The high stellar densities
of such a compact star cluster could greatly enhance the TDE rate (e.g. Stone & Metzger 2016), possibly making this association uncoincidental. On the other hand, we note that some models for J1644+57 (e.g., Tchekhovskoy et al. 2014) favor a much smaller black hole ($10^5 - 10^6 M_\odot$) than we estimate would be required to produce the observed inner break.

2.6.6 Regulation by SMBH Kinetic Feedback

Our analysis of SMBH feedback has focused on Compton heating instead of kinetic feedback, since the effects of radiative feedback are relatively straightforward to calculate from the properties of the accretion flow. However, it is possible kinetic feedback from SMBH outflows could play an equal or greater role in regulating accretion, even in low luminosity AGN.

Our simple parameterization of kinetic feedback (eq. [2.48]) assumes a uniform volumetric heating and predicts an effective wind velocity which increases with SMBH mass as $v_w \propto M_\bullet^{0.38}$ for core and cusp galaxies. Coupled with the dependence of the SMBH accretion rate on the wind heating, $\dot{M}/\dot{M}_{edd} \propto M_\bullet^{0.76(0.48)} v_w^{-3.8(-2.4)}$ (Fig. 2.4; eq. [2.18]), a dominant source of kinetic feedback of the form we have adopted leads to an Eddington ratio dependence on SMBH mass of $\dot{M}/\dot{M}_{edd} \propto M_\bullet^{-0.7(-0.4)}$.

This prediction is more in line with the observed dependence of $\langle L_X/L_{edd}\rangle$ in elliptical galaxies (Fig. 2.9, green line), suggesting that accretion regulation by kinetic feedback could play a role in determining the X-ray luminosities of elliptical galaxies. However, our assumption that kinetic outflows heat the gas uniformly in volume is rather arbitrary and would need to be more rigorously justified by numerical studies of how jets or disc outflows couple energy to their gaseous environment. As already discussed, it is furthermore unclear whether a steady-state accretion model is at all relevant in the case when SMBH feedback dominates due to the time delay between the small scale

\footnote{These equation correspond to the specific cases of $\Gamma = 0.1$ and $\Gamma = 0.8$, but it is straightforward to generalize these for arbitrary $\Gamma$ as we do for this figure.}
accretion flow and the thermodynamic response of the CNM on larger scales.

2.7 Conclusions

We have calculated steady-state models for the hot gaseous circumnuclear media of quiescent galaxies, under the assumption that gas is supplied exclusively by stellar wind mass loss and heated by shocked stellar winds, supernovae and black hole feedback. We numerically compute solutions for a range of different black hole masses, heating rates, and observationally-motivated stellar density profiles. Then we use our numerical results (Table 2.2) to verify and calibrate analytic relationships (Appendix A). We use the latter to explore systematically how the SMBH accretion rate varies with black hole mass and the galaxy’s SFH. Our results for $\dot{M}(M_\bullet)$ are compared with observed trends of the nuclear X-ray luminosities of quiescent SMBHs and low luminosity AGN. Our conclusions are summarized as follows.

1. A stagnation radius, $r_s$, divides the nuclear gas between an accretion flow and an outgoing wind. In steady-state the gas inflow rate towards the black hole is proportional to the stellar mass enclosed inside of $r_s$. In the limit of strong heating, the stagnation radius resides interior to the SMBH influence radius and coincides with the Bondi radius (eq. [2.22]). In the limit of weak heating ($\zeta < \zeta_c$; eq. [2.16]), the stagnation radius moves to large radii, near or exceeding the stellar break radius $r_b \gg r_{\text{inf}}$, greatly increasing the density of gas on smaller radial scales.

2. In the vicinity of the stagnation radius, including the effects of heat transport by electron conduction results in at most order unity changes to the key properties of the flow (e.g. stagnation radius, mass accretion rate, and thermal stability) for causal values of the conduction saturation parameter $\phi < 0.1$ (eq. [2.23]). However, in principle heat conduction of heat from the inner accretion flow can affect the solution properties on much larger scales (Johnson &
For example, Tanaka & Menou (2006) find that conductivity can contribute to driving bipolar outflows.

Angular momentum of the gas will become important on small scales, where gas stopped by a centrifugal barrier and unable to cool could drive outflows near the equatorial and polar regions of the flow.

3. Radiative cooling has a more pronounced influence on the flow structure when the radiative cooling rate exceeds the gas heating rate near the stagnation radius, where the gas is in nearly hydrostatic equilibrium (Fig. 2.1). This condition is approximately equivalent to the $t_{\text{cool}} \lesssim 10 t_{\text{ff}}$ criterion for thermal instability advocated by e.g. Sharma et al. (2012), Gaspari et al. (2012), Li & Bryan (2014b), and Voit et al. (2015). The transition in the flow properties that occurs as heating is reduced may represent a true “thermal instability” (hot ISM condensing into a cooler clouds), or it may simply represent an abrupt transition from a steady inflow-outflow solution to a global cooling flow. We leave distinguishing between these possibilities to future work.

4. The location of the stagnation radius, and hence the inflow rate, is a sensitive function of the gas heating rate $\propto v_w^2$. Quantitatively, $r_s \propto v_w^{-2}$, implying that $\dot{M} \propto v_w^{-3.8(2.4)}$ for fiducial core (cusp) galaxies. However, $\dot{M}$ can increase even more rapidly with decreasing $v_w$ for two reasons: (1) when $v_w$ becomes comparable to the stellar velocity dispersion ($\zeta < \zeta_c$; eq. [2.16]), then gas remains bound to the stellar bulge and cannot produce an outflow interior to the stellar break radius (2) For low $v_w$, gas near the stagnation radius becomes thermally unstable, which based on the results of previous numerical studies (e.g., Ciotti et al. 2010) is instead likely to result in a burst of high accretion ($\S 2.6.3$).

5. Stellar wind heating, supernovae, and AGN feedback depend explicitly on the SMBH mass as
well as the SFH in the galactic nucleus. A young starburst of age \( \lesssim 10 \) Myr produces mass and energy injection dominated by young stellar winds (\( v_w \approx 1000 \) km s\(^{-1}\)) and supernovae, while for exclusively old stellar populations mass input is dominated by AGB winds and heating is dominated by AGN feedback and supernovae. Ongoing, continuous star formation presents a hybrid situation, with energy input usually dominated by young stars and mass input dominated by old stars (Appendix B). Galactic nuclei with \( M_\bullet \lesssim 10^8 M_\odot \) that are heated according to their average SFHs (as derived from cosmological simulations) receive a stellar wind heating of \( v_w \approx 700 \) km s\(^{-1}\). This heating can be suppressed if the star formation is sufficiently intermittent (Fig. B.2).

6. Type Ia SNe only provide a continuous heating source exterior to the Ia radius \( r_{\text{Ia}} \) (eq. [2.36]) where the time between subsequent SNe exceeds the dynamical time-scale. Non-Ia heating due to the average SFH results in \( r_s < r_{\text{Ia}} \), except for the most massive galaxies (Fig. 2.7). However, if \( r_s \) does approach \( r_{\text{Ia}} \), then SN Ia heating is usually large enough to prevent \( r_s \) from greatly exceeding \( r_{\text{Ia}} \). SNe Ia thus regulate the SMBH accretion rate below the value \( \dot{M}_{\text{Ia}} \) (eq. [2.39]), except possibly in high mass elliptical galaxies with large break radii and \( \zeta(v_{\text{Ia}}) \gtrsim \zeta_c \).

7. Unlike stellar heating, heating from SMBH feedback depends on the accretion rate, which itself depends on the heating. Compton heating is generally unimportant compared to stellar wind heating for the average SFH, but it can be significant in the case of an impulsive starburst (Fig. 2.6). However, SMBH feedback may not be capable of truly stabilizing the flow given its inability to respond instantaneously to local changes in the properties of the gas. Kinetic feedback could also be important in determining \( \dot{M}(M_\bullet) \) when stellar heating is weak (§2.6.6) but is more challenging to quantify.

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8. Stellar wind heating from the average SFH may be sufficient to permit thermally-stable, steady-state accretion, depending on the accretion efficiency of the SMBH. However, the resulting average Eddington-normalized accretion rate is predicted to increase with \( M_\bullet \) (Fig. 2.9), in tension with the (also weak) downsizing trend of measured X-ray luminosities in early-type galactic nuclei (e.g., Miller et al. 2015). Thermally stable accretion models can reproduce the observed scatter in nuclear X-ray luminosities at fixed \( M_\bullet \) (two to four orders of magnitude) due to the combination of (1) differences in the stellar wind heating rate due to stochastic variation in SFH between galaxies, or within one galaxy due to burstiness in the SFH, as in Fig. B.2; (2) variations in the amount of inflowing mass which is lost to small scale outflows from the SMBH accretion disc, likely due to variations in the angular momentum of the accreting gas (cf. Pellegrini 2010).

9. However, for lower X-ray radiative efficiencies, the accretion rates of early-type galaxies are above the maximum value for thermally stable flow, \( \dot{M}_{TI} \) (Fig. 2.9). This implies that current X-ray detections could instead be comprised mostly of nuclei undergoing outburst due to thermal instabilities. In such a case, there may exist a separate (usually undetected) branch of low-\( L_X \) nuclei accreting stably. The existence of such a bimodal population would call into question constraints on the SMBH occupation fraction in low mass galaxies (Miller et al. 2015) derived by extrapolating the \( L_X(M_\bullet) \) relationship to small \( M_\bullet \).

10. Low mass black holes grow in the low redshift Universe over time-scales comparable to the Hubble time (Heckman et al. 2004). The accretion rates so required are too high to be consistent with either the value predicted for the average SFH history or the maximum allowed for steady-state, thermally stable accretion (Fig. 2.8). Perhaps unsurprisingly, low-\( M_\bullet \) growth and AGN activity must instead be driven by a supply of gas external to the nucleus, such as
galaxy mergers or thermal instabilities on larger, galactic scales (e.g. Voit et al. 2015).

We conclude by drawing attention to a few limitations of our model. First, we have neglected any large scale inflow of gas to the nucleus by assuming the only source of gas feeding the black hole is stellar wind mass loss from the local stellar population. Our estimates represent a lower bound on the time averaged gas density of the CNM. Although our model can, under certain assumptions, describe quiescent galactic nuclei, it cannot account for AGN.

Predictions for the SMBH accretion rate are strongly tied to the age and radial distribution of the stellar population within the sphere of influence. Our model therefore cannot make definitive predictions for the accretion mode of individual galaxies without knowledge of their specific SFH. We adopt halo-averaged star formation rates measured using multi-epoch abundance matching models by Moster et al. (2013), and neglect any spatial variation in the star formation rate for any given galaxy. However, galaxies of mass $\gtrsim 7 \times 10^{10}\text{M}_\odot$ assemble their stars from the inside out (Pérez et al. 2013), resulting in galactic nuclei which may be systematically older than assumed in our model and thus more prone to thermal instability.

Finally, we do not account for the effects of non-spherical geometry or discreteness of stellar wind sources (Cuadra et al. 2006, 2008). We assume all of the mass and energy sources are efficiently mixed, and do not take into account the possibility of slower wind material condensing into high density structures (as described in Cuadra et al. 2005) or some of the energy injection leaking away in 3-D (Harper-Clark & Murray 2009; Zubovas & Nayakshin 2014).
Chapter 3

Constraining jetted tidal disruption events

3.1 Introduction

When a star in a galactic nucleus is deflected too close to the central supermassive black hole (BH), it can be torn apart by tidal forces. During this tidal disruption event (TDE), roughly half of the stellar debris remains bound to the BH, while the other half is flung outwards and unbound from the system. The bound material, following a potentially complex process of debris circularization (Kochanek 1994; Guillochon & Ramirez-Ruiz 2013; Hayasaki et al. 2013, 2016; Shiokawa et al. 2015; Bonnerot et al. 2016), accretes onto the BH, creating a luminous flare lasting months to years (Hills 1975; Carter & Luminet 1982; Rees 1988).

Many TDE flares have now been identified at optical/ultraviolet (UV) (Gezari et al. 2008, 2009; van Velzen et al. 2011; Gezari et al. 2012; Arcavi et al. 2014; Chornock et al. 2014; Holoien et al. 2014; Vinkó et al. 2015; Holoien et al. 2016a) and soft X-ray wavelengths (Bade et al. 1996;
Beginning with the discovery of Swift J1644+57 (hereafter SwJ1644) in 2011, three additional TDEs have been discovered by their hard X-ray emission (Bloom et al. 2011; Levan et al. 2011; Burrows et al. 2011; Zauderer et al. 2011; Cenko et al. 2012; Pasham et al. 2015; Brown et al. 2015). Unlike the optical/UV/soft X-ray flares, these events are characterized by non-thermal emission from a transient relativistic jet beamed along our line of sight, similar to the blazar geometry of active galactic nuclei (AGN). In addition to their highly variable X-ray emission, which likely originates from the base of the jet (see e.g. Bloom et al. 2011; Crumley et al. 2016), these events are characterized by radio synchrotron emission Berger et al. 2012; Zauderer et al. 2013; Cenko et al. 2012. The latter, more slowly evolving, is powered by shocks formed at the interface between the jet and surrounding circumnuclear medium (CNM) (Bloom et al. 2011; Giannios & Metzger 2011; Metzger et al. 2012; De Colle et al. 2012; Kumar et al. 2013; Mimica et al. 2015), analogous to the afterglow of a gamma-ray burst.

Although a handful of jetted TDE flares have been observed, the apparent volumetric rate is a very small fraction ($\sim 10^{-5} - 10^{-4}$) of the observed TDE flare rate (e.g., Burrows et al. 2011, Brown et al. 2015), and an even smaller fraction of the theoretically predicted TDE rate (Wang & Merritt 2004; Stone & Metzger 2016). One explanation for this discrepancy is that the majority of TDEs produce powerful jets, but their hard X-ray emission is relativistically beamed into a small angle $\theta_b$ by the motion of the jet, making them visible to only a small fraction of observers. However, the inferred beaming fraction $f_b \approx \theta_b^2/2 \sim 10^{-5} - 10^{-4}$ would require $\theta_b \sim 0.01$ and hence a jet with a bulk Lorentz factor of $\Gamma \gtrsim 1/\theta_b \sim 100$, much higher than inferred for AGN jets or by modeling SwJ1644 (Metzger et al. 2012). This scenario would also require an unphysically low jet half opening.
angle $\theta_j \lesssim 0.01$.

The low detection rate of hard X-ray TDEs may instead indicate that powerful jet production is intrinsically rare, or that the conditions in the surrounding environment are unfavorable for producing bright emission. Jets could be rare if they require, for instance, a highly super-Eddington accretion rate (De Colle et al. 2012), a TDE from a deeply plunging stellar orbit (Metzger & Stone 2016), a TDE in a retrograde and equatorial orbit with respect to the spin of the black hole (Parfrey et al. 2015), or a particularly strong magnetic flux threading the star (Tchekhovskoy et al. 2014; Kelley et al. 2014). Alternatively, jet formation or its X-ray emission could be suppressed if the disk undergoes Lens-Thirring precession due to a misalignment between the angular momentum of the BH and that of the disrupted star (Stone & Loeb 2012). In the latter case, however, even a ‘dirty’ jet could still be generated, which would produce luminous radio emission from CNM interaction.

Bower et al. (2013) and van Velzen et al. (2013) performed radio follow-up of optical/UV and soft X-ray TDE flares on timescales of months to decades after the outburst (see also Arcavi et al. 2014). They detected no radio afterglows definitively associated with the host galaxy of a convincing TDE candidate.\(^2\) Bower et al. (2013) and van Velzen et al. (2013) use a Sedov blast wave model for the late-time radio emission to conclude that $\lesssim 10\%$ of TDEs produce jetted emission at a level similar to that in SwJ1644. Mimica et al. (2015) use two-dimensional (axisymmetric) hydrodynamical simulations, coupled with synchrotron radiation transport, to model the radio emission from SwJ1644 as a jet viewed on-axis. By extending the same calculation to off-axis viewing angles, they showed that, regardless of viewing angle, the majority of thermal TDE flares should have been detected if their jets were as powerful as SwJ1644, which had a total energy of

\(^2\)There were radio detections for two ROSAT flares: RX J1420.4+5334 and IC 3599. However, for RX J1420.4+5334 the radio emission was observed in a different galaxy than was originally associated with the flare. IC 3599 has shown multiple outbursts in the recent years, calling into question whether it is a true TDE at all (Campana et al. 2015). The optical transient CSS100217 (see Drake et al. 2011) had a weak radio afterglow, but its peak luminosity is more consistent with a superluminous supernova than a TDE.
The recent TDE flare ASSASN-14li (Holoien et al. 2016b) was accompanied by transient radio emission, consistent with either a weak relativistic jet (van Velzen et al. 2016) or a sub-relativistic outflow (Alexander et al. 2016; Krolik et al. 2016) of total energy $\sim 10^{48} - 10^{49}$ erg. The 90 Mpc distance of ASSASN-14li, a few times closer than most previous TDE flares, implies that even if other TDEs were accompanied by similar emission, their radio afterglows would fall below existing upper limits. The extreme contrast between the radio emission of SwJ1644 and ASSASN-14li indicates that the energy distribution of TDE jets is very broad.

Previous works (Bower et al. 2013; van Velzen et al. 2013; Mimica et al. 2015) have generally assumed that all TDE jets encounter a similar gaseous environment as SwJ1644. However, the density of the circumnuclear medium (CNM) depends sensitively on the input of mass from stellar winds and the processes responsible for heating the gas (Quataert 2004; Generozov et al. 2015).

The first goal of this chapter is to constrain the range of gas densities encountered by jetted TDEs using the semi-analytic model for the CNM (§3.2) developed in 2. With this information in hand, in §3.3 we present hydrodynamical simulations of the jet-CNM shock interaction which determine the radio synchrotron emission across the allowed range of gaseous environments, for different jet energies and viewing angles. In §3.3.4 we show how the dependence of our results for the peak luminosity, and time to radio maximum, on the jet energy and CNM density can be reasonably understood using a simple analytic blast wave model (§3.3.2, Appendix D), calibrated to the simulation data. Then, using extant radio detections and upper limits, we systemically constrain the energy distribution of TDE jets. One of our primary conclusions is that TDE jets as energetic as SwJ1644 are intrinsically rare, a result with important implications for the physics of jet launching in TDEs and other accretion flows. Our work also lays the groundwork for collecting and employing future, larger samples of TDEs with radio follow-up, to better constrain the shape
of the energy distribution. We summarize and conclude in §3.4.

3.2 Diversity of CNM Densities

3.2.1 Analytic Constraints

Jet radio emission is primarily sensitive to the density of ambient gas near the Sedov radius, \( r_{\text{sed}} \), outside of which the jet has swept up a gaseous mass exceeding its own. For a power law gas density profile, \( n = n_{18} (r/10^{18}\text{cm})^{-k} \),

\[
r_{\text{sed}} = 10^{18}\text{ cm} \left( \frac{E(3-k)}{4\pi n_{18} m_p c^2 (10^{18}\text{ cm})^3} \right)^{1/(3-k)} \\
\approx 3E_{54}^{1/2} n_{18}^{-1/2}\text{ pc.} \quad (3.1)
\]

where \( E = E_{54}10^{54}\text{ erg} \) is the isotropic equivalent energy and in the final equality we have taken \( k = 1 \), typical of our results described later in this section. For a powerful jet similar to SwJ1644, the deceleration radius is typically of order a parsec, but it can be as small as \( 10^{16}\text{ cm} \) for a weak jet/outflow, such as that in ASASSN-14li.

Although an initially relativistic jet will slow to sub-relativistic speeds at \( r \sim r_{\text{sed}} \), significant deceleration already sets in at the deceleration radius (where the jet has swept up a fraction \( \sim 1/\Gamma \) of its rest mass\(^3\)),

\[
r_{\text{dec}} = \frac{r_{\text{sed}}}{\Gamma^{2/(3-k)}}. \quad (3.2)
\]

According to an observer within the opening angle of the jet, the jet reaches the Sedov and decel-

\(^3\)This is really the Lorentz factor of the shock (see Hascoët et al. 2014). For simplicity, we instead use the Lorentz factor of the ejecta. This means our estimate for the deceleration time is too small by a factor of \( \sim 2 \)
eration radii, respectively, at times given by

\[ t_{\text{sed}} \sim \frac{r_{\text{sed}}}{c} \approx 10E_{54}^{1/2}n_{18}^{-1/2} \text{ year} \]  \hspace{1cm} (3.3)

\[ t_{\text{dec}} \sim \frac{r_{\text{dec}}}{2\Gamma c} = \frac{t_{\text{sed}}}{2\Gamma^2(4-k)/(3-k)} = \frac{t_{\text{sed}}}{2\Gamma^3}, \]  \hspace{1cm} (3.4)

where in the final equality we have again taken \( k = 1 \).

### 3.2.1.1 Dynamical Model of CNM

In the absence of large scale inflows, the dominant source of gas in the CNM of quiescent galaxies
is winds from stars in the galactic nucleus. We bracket the range of possible nuclear gas densities
using a simple steady-state, spherically symmetric, hydrodynamic model including mass and energy
injection from stellar winds. The relevant equations are (e.g. Holzer & Axford 1970; Quataert 2004)

\[ \frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v) = q \]  \hspace{1cm} (3.5)

\[ \rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) = -\frac{\partial p}{\partial r} - \rho \frac{GM_{\text{enc}}}{r^2} - qv \]  \hspace{1cm} (3.6)

\[ \rho T \left( \frac{\partial s}{\partial t} + v \frac{\partial s}{\partial r} \right) = q \left[ \frac{v^2}{2} + \frac{\tilde{v}_{\text{w}}^2}{2} - \frac{\gamma_{\text{ad}}}{\gamma_{\text{ad}} - 1} \frac{p}{\rho} \right], \]  \hspace{1cm} (3.7)

where \( \rho, v, p, \) and \( s \) are the density, velocity, pressure (we assume an ideal gas with a mean
molecular weight of 0.62 and adiabatic index \( \gamma_{\text{ad}}=5/3 \)), and specific entropy of the gas, respectively.

The enclosed mass \( M_{\text{enc}} = M_\bullet + M_\star \) includes both the black hole mass \( M_\bullet \) and enclosed stellar mass
\( M_\star \propto \int \rho_\star r^2 \, dr \), where \( \rho_\star \) is the stellar density. At the radius of the sphere of influence, \( r_{\text{inf}} \), the
enclosed stellar and black masses are equal, \( M_\star (r_{\text{inf}}) = M_\bullet \). We take \( r_{\text{inf}} = 3.5M_\bullet n_6^{1/6} \text{ pc} \) (Chapter 2),
where \( M_\bullet, 7 = M_\bullet / 10^7 M_\odot \).

The source term \( q \) is the mass injection rate per unit volume per unit time. We take \( q = \)
\[ \frac{\eta \rho_s}{t_h}, \] where \( \eta \) is a dimensionless efficiency parameter that depends on the properties of the stellar population and \( t_h \) is the Hubble time. The \( \tilde{v}_w^2 = \sigma(r)^2 + v_w^2 \) term in the entropy equation is the specific heating rate of the gas per unit volume, where

\[
\sigma \approx \sqrt{\frac{3GM_\bullet}{(\Gamma + 2)r} + \sigma_0^2}, \tag{3.8}
\]

is the stellar velocity dispersion, which approaches the constant value of \( \sigma_0 \) outside of the influence radius. We have taken \( \sigma_0 = 190M_\bullet^{0.2}\text{kms}^{-1} \) (based on the \( M_\bullet - \sigma \) relation from McConnell et al. 2011).

\( v_w^2 \) is the specific heating rate of the gas from other sources including stellar wind kinetic energy, supernovae, and black hole feedback. We take \( v_w \) to be independent of radius.

Chapter 2 present analytic approximations for the densities and temperatures of steady state solutions to equation (3.7). We apply these results across the physically allowed range of heating \( (v_w) \) and mass injection rates \( (\eta) \), and obtain the corresponding range of gas densities.

3.2.1.2 Stellar density profiles

We assume a broken power law for the stellar density profile, \( \rho_\star \), motivated by Hubble measurements of the radial surface brightness profiles for hundreds of nearby early type galaxies (Lauer et al. 2007). The measured profile is well fit by the so-called “Nuker” law parameterization, i.e. a piece-wise power law that smoothly transitions from an inner power law slope, \( \gamma \), to an outer power law slope, \( \beta \), at a break radius, \( r_b \).

Most galaxies have \( 0 < \gamma < 1 \), and are classified into two broad categories: “core” galaxies with \( \gamma < 0.3 \) and “cusp” galaxies with \( \gamma > 0.5 \). Assuming spherical symmetry and a constant mass-to-light ratio, the inner stellar profile translates to a stellar density of \( \rho_\star \propto r^{-1-\gamma} = r^{-\delta} \).

\footnote{This may be of questionable validity for low mass black holes (e.g. Greene et al. 2010; Kormendy & Ho 2013). Also, several of the black hole masses used in McConnell et al. (2011) were underestimated (Kormendy & Ho 2013). However, the precise form of the \( M_\bullet - \sigma \) relationship has minimal impact on our results.}
Cusp-like stellar density profiles are the most relevant to TDEs, since as described in Stone & Metzger (2016), a cuspy stellar density profile results in a higher TDE rate per galaxy. We adopt a fiducial value of $\gamma = 0.7 (\delta = 1.7)$, motivated by the rate-weighted average value of the inner stellar density profile for the galaxies in Stone & Metzger (2016) (their Table C).

### 3.2.1.3 Gas density profiles

Given sufficiently strong heating, a one-dimensional steady-state model for the CNM is characterized by an inflow-outflow structure. The velocity passes through zero at the “stagnation radius”, $r_s$. Mass loss from stars interior to the stagnation radius flows inwards, while that outside of $r_s$ is unbound in an outflow from the nucleus. Fig. 3.1 shows example radial profiles of the steady-state gas density calculated for a core and a cusp stellar density profile. The stagnation radius is marked as a blue dot on each profile.

As long as the heating parameter, $v_w$, is greater than the stellar velocity dispersion,

$$r_s \approx f(\delta) \frac{G M \bullet}{v_w^2} \simeq 0.4 M_{\bullet,7} v_{500}^{-2} \text{pc},$$

where $v_{500} \equiv v_w/500 \text{ km s}^{-1}$ and $f(\delta)$ is a constant of order unity, which in the second equality we take equal to its fiducial value of $f(\delta = 1.7)=2.5$ (see Chapter 2). The gas density at the stagnation radius, $n(r_s)$, is determined by the rate at which stellar winds inject mass interior to it,

$$\dot{M} = \frac{\eta M \bullet(r_s)}{t_h} \approx 2.8 \times 10^{-6} M_{\bullet,7}^{0.22} \eta_{0.02} \left( \frac{r_s}{\text{pc}} \right) \frac{1.3}{M_{\odot} \text{ yr}^{-1}},$$

where $M \bullet(r_s)$ is the total stellar mass enclosed within the stagnation radius, $\eta_{0.02} = \eta/0.02$ is normalized to a value characteristic of an old stellar population, and the second equality again assumes our fiducial value of $\delta = 1.7$. 

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The density at the stagnation radius, \( n(r_s) \), is estimated by equating the gas injected by stellar winds over a dynamical time at the stagnation radius, \( t_{\text{dyn}}(r_s) \), to the gas mass enclosed at this location.

\[
\frac{4\pi}{3} r_s^3 m_p n(r_s) \simeq \dot{M} t_{\text{dyn}}(r_s) \quad (3.11)
\]

For \( r_s < r_{\text{inf}} \), \( t_{\text{dyn}} = (r_s^3/GM_\bullet)^{1/2} \), while for \( r_s > r_{\text{inf}} \), \( t_{\text{dyn}} = (\sqrt{3}r_s/\sigma_0) \). Thus,

\[
n(r_s) \simeq \begin{cases} 
0.1 \eta_{0.02} M_\bullet^{0.28} \left( \frac{r_s}{\text{pc}} \right)^{-0.2} \text{cm}^{-3} & r_s < r_{\text{inf}} \\
0.1 \eta_{0.02} M_\bullet^{0.02} \left( \frac{r_s}{\text{pc}} \right)^{-0.7} \text{cm}^{-3} & r_s > r_{\text{inf}},
\end{cases} \quad (3.12)
\]

For sufficiently strong heating, the stagnation radius will lie inside the SMBH’s sphere of influence and will be given by equation (3.9). In this case,

\[
n(r_s) \simeq 0.2 v_{5000}^{0.4} \eta_{0.02} M_\bullet^{-0.48} \text{cm}^{-3}, \quad (3.13)
\]

Near the stagnation radius, the radial gas profile has a power-law slope of \( k \approx (4\delta - 1)/6 \), which for our fiducial value of \( \delta = 1.7 \) gives \( n \propto r^{-1} \). The gas density steepens towards smaller radii, approaching \( n \propto r^{-1.5} \), for radii well inside of both the stagnation radius of the flow and the SMBH’s sphere of influence. The gas profile flattens to \( n \propto r^{1-\delta} \) between the stagnation radius and the stellar break radius; however, for our fiducial value of \( \delta = 1.7 \), the resulting profile \( n \propto r^{1-\delta} \approx r^{-0.7} \) is only moderately changed. We expect at the deceleration radius of most jets is bracketed by \( r^{-0.7} \) and \( r^{-1.5} \). For simplicity we adopt

\[
n(r) = n_{18} \left( \frac{r}{10^{18}\text{cm}} \right)^{-1}, \quad (3.14)
\]
as our fiducial density profile, where \( n_{18} \) is the density at \( r = 10^{18} \) cm. We explore the effects of the density slope on jet radio emission in § 3.3.3.

Alexander et al. (2016) use radio observations of the ASSASN-14li flare to infer a nuclear gas density profile of \( n \propto r^{-2.6} \) for its host galaxy on scales of \( \sim 10^{16} \) cm—much steeper than our fiducial density profile. However, we note that this galaxy was active before the flare, possibly explaining the unusually steep density profile.

Combining equations (3.12) and (3.14), we obtain

\[
\begin{align*}
n_{18} & \simeq \begin{cases} 
0.4 \left( \frac{r_s}{\text{pc}} \right)^{0.8} M_7^{-0.28} \eta_{0.02} \text{ cm}^{-3} & r_s < r_{\text{inf}} \\
0.4 \left( \frac{r_s}{\text{pc}} \right)^{0.3} M_7^{0.02} \eta_{0.02} \text{ cm}^{-3} & r_s > r_{\text{inf}}.
\end{cases}
\end{align*}
\]  

(3.15)

For sufficiently strong heating, the stagnation radius will lie inside the sphere of influence and will be given by equation (3.9). In this case,

\[
n_{18} \simeq 0.2 M_7^{0.52} v_{500}^{-1.6} \eta_{0.02} \text{ cm}^{-3}.
\]  

(3.16)

As shown in Fig. 3.1, the gas density profile steepens outside the break radius \( r_b \) of the stellar density profile. However, this will only impact the radio emission near its maximum if \( r_b \) lies inside of the Sedov radius, \( r_{\text{sed}} \) (eq. 3.1). The lines in Fig. 3.1 are colored according to the combination of jet energy and CNM density \( n_{18} \) which results in \( r = r_{\text{sed}} \) at each radius. The measured break radii of all but four of the Lauer et al. (2007) galaxies exceed 10 parsecs, which greatly exceeds \( r_{\text{sed}} \) even in the case of a very energetic jet \( (E = 4 \times 10^{54} \text{ erg}) \) in a low density CNM of \( n_{18} \sim 1 \) cm\(^{-3}\). The presence of a nuclear star cluster (NSC) in the galactic center could produce another break in the stellar density profile near the outer edge of the cluster, which is typically located at \( r_{\text{nsc}} \sim 1 - 5 \) pc (Georgiev & Böker 2014b). But even in this case, only particular combinations of high \( E/\text{low} n_{18} \)
result in $r_{\text{sed}} > r_{\text{msc}}$. We therefore neglect the effects of an outer break in the stellar density profile in our analysis.

### 3.2.1.4 Allowed Density Range

We now estimate the allowed range in the normalization of the CNM gas profile, $n_{18}$. We assume that star formation occurs in two bursts, an old burst of age comparable to the Hubble time $t_h = 10^{10}$ yr, and a “young” burst of variable age $t_{\text{burst}} \ll t_h$ which contributes a fraction $f_{\text{burst}}$ of the stellar mass. We assume a Salpeter IMF for both stellar populations. For a sufficiently large burst of age $\lesssim 40$ Myr, gas heating is dominated by the energetic winds of massive stars.\(^5\) In this case the mass return ($\eta$) and heating parameters ($v_w$) are calculated as described in Appendix B. Given $\eta(t_{\text{burst}}, f_{\text{burst}})$ and $v_w(t_{\text{burst}}, f_{\text{burst}})$, we calculate $n_{18}$ following equation (3.16).

For an older stellar population, a few different sources contribute to gas heating, including Type Ia Supernovae (SNe)\(^6\) and AGN feedback. We focus on quiescent phases, during which SNe Ia dominate. As discussed in , SNe Ia clear out the gas external to a critical radius, $r_{\text{Ia}}$, where the interval between successive Ia SNe equals the dynamical (gas inflow) timescale. For an old stellar population, $n_{18}$ is estimated by equating $r_{\text{Ia}}$ with the stagnation radius in equation (3.15). The Ia radius is calculated as described in Chapter 2.

Fig. 3.2 shows how $n_{18}$ varies with the young starburst properties, $f_{\text{burst}}$ and $t_{\text{burst}}$. We find a maximum density of $n_{18} \sim 1,300 M_\odot^{-0.5} \text{ cm}^{-3}$ is achieved for a burst of age $t_{\text{burst}} \sim 4$ Myr which forms most of the stars in the nucleus ($f_{\text{burst}} \sim 1$). In this case, both the energy and mass budgets

\(^5\)Core-collapse SNe are also an important heating source. In a young stellar population, the power from core-collapse supernovae exceeds that from massive stellar winds after $\sim 6$ Myr (Voss et al. 2009). However, due to discreteness effects the heating from massive star winds will be more important on small scales.

\(^6\)Unbound debris streams from TDEs potentially provide another source of heating localized in the galactic center (Guillochon et al. 2016), which we neglect.

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Figure 3.1: Steady-state radial profiles of the CNM gas density, normalized to its value at $10^{18}$ cm, $n_{18}$. The profiles are calculated for a black hole mass of $10^7 M_\odot$ and a gas heating parameter of $v_w = 600$ km s$^{-1}$. Cusp and core stellar density profiles are shown with solid and dashed lines, respectively. The line colors denote the ratio of isotropic equivalent jet energy to $n_{18}$ which results in $r = r_{sed}$ at each radius.
of the CNM are dominated by fast winds from massive stars. Although a large gas density is present immediately after a starburst, the density will decline with the wind mass loss rate, approximately $\propto t^{-3}$, i.e. by an order of magnitude within just a few Myr.

By contrast, the lowest allowed density $\sim 0.02 M_{\odot}^{0.5} \, \text{cm}^{-3}$ is achieved for a relatively modest burst of young stars $t_{\text{burst}} \approx 10^6$ Myr, which forms a fraction $f_{\text{burst}} = 4 \times 10^{-4}$ of the total stellar mass. In this case the young massive stars provide a high heating rate, while the mass injection rate is comparatively low and receives contributions from both young and old stars.

The lowest allowed $n_{18}$ may be an underestimate as we do not include the effects of discreteness on the assumed stellar population. In particular, we assume that stars provide a spatially homogeneous heating source and mass source, even on small radial scales where the number of massive stars present may be very small. The doubly hatched region in Fig. 3.2 denotes the region where less than one massive star ($\gtrsim 15 M_{\odot}$) is on average present inside of the nominal stagnation radius (eq. 3.9). Discreteness effects are thus important for relatively small bursts of star formation, including the case described above which gives the minimum $n_{18}$. If we instead equate the stagnation radius to the radius enclosing a single star of mass $\gtrsim 15 M_{\odot}$, we find a larger value of $n_{18} \sim 0.3 M_{\odot}^{-0.4} \, \text{cm}^{-3}$. The true minimum density therefore likely lies closer to $0.3 M_{\odot}^{-0.4} \, \text{cm}^{-3}$. However, we caution that this is a very crude estimate, and the low number of mass and heat sources means could there be considerable scatter about this value from stochastic variations in the stellar population. Additionally, stellar angular momentum could reduce the density (see e.g. Cuadra et al. 2006).

Finally, French et al. (2016) find that most optical/uv TDEs have evidence of recent star formation. French et al. (2017) find that these star-bursts typically form $\sim 1\%$ of the stars in the galaxy and ended a few $\times 10^8$ years ago. In this region of parameter space corresponding to the right side of Fig. 3.2, gas heating rate is dominated by SN Ia and $n_{18} \sim 2 \, \text{cm}^{-3}$. However, the density

---

\footnote{Though this may be partially explained by selection effects; see Law-Smith et al. (2017).}
would be enhanced if the star-burst is centrally concentrated, as observed in nearby post-starburst
galaxies (Pracy et al. 2012). For example, if the star-burst forms $\sim 10\%$ of the stars in the stars in
the galactic nucleus $n_{18}$ would increase to $\sim 10$ cm$^{-3}$.

In summary, the CNM densities of quiescent galaxies vary from $\min(n_{18}) \sim 0.3M_{\odot, 7}^{-0.4}$ cm$^{-3}$ to
$\max(n_{18}) \sim 1.3 \times 10^3 M_{\odot, 7}^{0.5}$ cm$^{-3}$, with a characteristic value of $n_{18} \sim 10$ cm$^{-3}$ expected for TDE
host galaxies.

### 3.2.1.5 Mass drop-out from star formation?

Our CNM model predicts the total gas density as sourced by stellar winds, including both hot
and cold phases. For the first few Myr after a starburst, the injected stellar wind material is hot
($T \gtrsim 10^7$ K) due to the thermalized wind kinetic energy. At later times, SNe Ia provide intermittent
heating, but the stellar wind material that accumulates on small radial scales between successive
SNe Ia may be much cooler, with at most the virial temperature $\sim 2 \times 10^5 M_{\odot, 7}^{0.4}$ K. This means the
gas could condense into cold clumps.

The propagation of jets through a medium containing clumps, clouds or stars has been studied
in the context of AGNs (e.g., Wang et al. 2000; Choi et al. 2007) and microquasars (e.g., Araudo
et al. 2009; Perucho & Bosch-Ramon 2012). It was found that the presence of these obstacles has an
effect on the long-term jet stability, as well as observational signatures at high energies. However,
the situation is different in the case of either a very wide or ultra-relativistic outflow (such as a
GRB) for which the emission is expected to be similar for a clumpy and a smooth medium with the
same average density (e.g. Nakar & Granot 2007; van Eerten et al. 2009; Mimica & Giannios 2011).
In the case of SwJ1644, the inferred angular width of the jet (especially of the slow component)
is much larger than in the case of AGNs and microquasars (see discussion in Mimica et al. 2015).
In fact, it is large enough to make the overall effect of the presence of any inhomogeneities in the
external medium minor. An analogous effect is found in case of SN remnants sweeping a clumpy medium (Obergaulinger et al. 2015). We note that we call the “slow component of the jet,” may in fact be an unrelated mildly relativistic outflow.

On the other hand, a fraction of the cold gas may also condense into stars. However, once the density of the hot phase is sufficiently reduced, the cooling time will become much longer than the dynamical time and the gas will become thermally stable, causing the condensation process to stop. For gas at the virial temperature of $\sim 2 \times 10^5 M^0_{\odot}/K$, we find that thermal stability would be achieved for $n_{18} \sim 0.6 M_{\odot}/cm^3$ (where we have defined thermal stability as the cooling time being ten times longer than the dynamical time-scale McCourt et al. 2012). In fact this estimate is conservative. If a fraction of the gas condenses into stars, then feedback from stellar winds would suppress further fragmentation. More realistically, the CNM density may be reduced by less than a factor of $\sim 2$ by star formation.

3.2.1.6 Constraints from the Galactic Center

Due to its close proximity, it is possible to directly observe the gas density distribution on parsec scales in the Galactic Center (GC). Baganoff et al. (2003) find that the hot, diffuse plasma within 10 arcseconds ($\sim 10^{18}$ cm) of Sgr A* has a root mean square electron density of $\sim 26$ cm$^{-3}$. In Fig. 3.2 we show two sets of two-burst star formation models which produce heating and mass return parameters comparable to those derived from the full star formation history of the GC from Pfuhl et al. (2011) (their Fig. 14). Our formalism gives values of $n_{18} \sim 3 - 5$ cm$^{-3}$, too low compared to observations. Discrepancy at this level is not surprising because our model is spherically symmetric, while in reality many of the massive stars in the GC are concentrated in two counter-rotating disks (Genzel et al. 2003b) with a top heavy IMF (Bartko et al. 2010; Lu et al. 2013). The disk stars extend from $\sim 10^{17} - 10^{18}$ cm and inject $\sim 10^{-3} M_{\odot}/yr$ of stellar wind.
Figure 3.2: Contours of $n_{18}$, the CNM density at $r = 10^{18}$ cm (blue lines), as a function of the stellar population in the galactic nucleus. The star formation is parameterized assuming that a fraction $f_{\text{burst}}$ of the stars form in a burst of age $t_{\text{burst}}$, while the remaining stars formed a Hubble time ago. We have assumed a black hole mass of $10^7 M_\odot$ and that both the young and old stars possess a cusp-like density profile, with a corresponding gas density profile $n \propto r^{-1}$. Hatched areas indicate regions of parameter space where massive stars ($\gtrsim 15 M_\odot$) dominate the gas heating rate, but less than one (doubly hatched) or less than ten (singly hatched) massive stars are present on average inside the nominal stagnation radius (eq. 3.9). In these regions discreteness effects not captured by our formalism are potentially important. The red line shows the approximate location of the Galactic Center in this parameter space (see text for details).
Table 3.1: Parameters for on-axis jet simulations.

<table>
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<th>Fiducial value</th>
<th>Other values</th>
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<tr>
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<td>E/10^{54} erg</td>
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<td></td>
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<td><strong>Microphysical parameters</strong></td>
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<tr>
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<td>p</td>
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<td><strong>Nuclear gas density</strong></td>
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<tr>
<td>n_{18}/cm^{-3}</td>
<td>60</td>
<td>2, 11, 345, 2000</td>
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material, much more than the \( \sim 4 \times 10^{-5} \text{M}_\odot \text{ yr}^{-1} \) expected for the global star formation history, explaining the large density of hot gas.

In short, accurate modeling of the gas distribution in a particular galactic nucleus, requires detailed knowledge of the distribution of stars. Our goal here has been to bracket the range of possible nuclear gas densities, by considering a broad range of stellar populations.

The Galactic Center also contains a cold circumnuclear ring (e.g. Becklin et al. 1982) with an opening angle of \( \sim 12 \pm 3^\circ \) (Lau et al. 2013) and a spatially averaged density of \( \sim 10^5 \text{ cm}^{-3} \) (although this varies by a few orders of magnitude throughout the ring–see Ferrière 2012 and references therein). Additionally, the volume from \( \sim 0.4-2.5 \text{ pc} \) is filled with warm, ionized atomic gas with density of \( \sim 900 \text{ cm}^{-3} \) (Ferrière 2012). This gas cannot be accounted for in our model, and may originate from larger scale inflows or a disrupted giant molecular cloud.
3.3 Synchrotron Radio Emission

3.3.1 Numerical Set-Up

We calculate the synchrotron radio emission from the jet-CNM shock interaction across the physically plausible range of nuclear gas densities. We perform both one- and two-dimensional (axisymmetric) relativistic hydrodynamical simulations using the numerical code MRGENESIS (Mimica et al. 2009a). MRGENESIS periodically outputs snapshots with the state of the fluid in its numerical grid. These snapshots are then used as an input to the radiative transfer code SPEV (Mimica et al. 2009b). SPEV detects the forward shock at the jet-CNM interface, accelerates non-thermal electrons behind the shock front, evolves the electron energy and spatial distribution in time, and computes the non-thermal emission taking into account the synchrotron self-absorption (interested readers can find many more technical details in Mimica et al. 2016). We use the same numerical grid resolution as in Mimica et al. (2015).

For the jet angular structure, we adopt the preferred two-component model for SwJ1644 from Mimica et al. (2015), corresponding to a fast, inner core with Lorentz factor $\Gamma = 10$, surrounded by a slower, $\Gamma = 2$ outer sheath. The ratio of the beaming-corrected energy of the fast component is fixed to be 4% of that of the slow sheath. A schematic depiction of the jet geometry is shown in Fig. 3.3. In our 2D simulations the fast inner core spans an angular interval $0 - 0.1$ radians, while the slow outer sheath extends from 0.1 radians to 0.5 rad. The time dependence of the jet kinetic luminosity is given by (Mimica et al. 2015)

$$L_{j,ISO}(t) = L_{j,0} \max[1, (t/t_0)]^{-5/3},$$

(3.17)

where $t_0 = 5 \times 10^5$ s is the duration of peak jet power. This is assumed to match that of the
period of the most luminous X-ray emission of SwJ1644. Integrating equation (3.17) from $t = 0$ to $\infty$ gives the isotropic equivalent energy of the jet, $E_{\text{ISO}}$, where $L_{j,0} = 0.4 E_{\text{ISO}}/t_0$. For the microphysical parameters characterizing the fraction of the post-shock thermal energy placed into relativistic electrons ($\epsilon_e$) and magnetic field ($\epsilon_B$), and the power-law slope of the electron energy distribution $p$, we adopt the values from the best fit model in Mimica et al. (2015) (see Table 3.1).

For our 1D simulations, we modify the geometry of the slow sheath to better mimic the results of the 2D simulations. In our 2D models the sheath is injected within a relatively narrow angular interval; however, at late stages of evolution the bow shock created by the jet-CNM interaction spans a much larger angular range due to lateral spreading. To account for the slow component becoming more isotropic near peak emission in our 2D simulations (bottom two panels of Fig. 8 in Mimica et al. 2015), we instead take the slow component to extend from 0.1 to $\pi/2$ radians in our 1D models. We keep the true energy of the slow component fixed so that the isotropic equivalent
energy of the slow component is a factor of \[\frac{\cos(0.1) - \cos(0.5)}{\cos(0.1) - \cos(\pi/2)}\] \(\approx 0.12\) smaller than in the corresponding 2D simulations.

Figure 3.4 compares light curves calculated from this modified 1D approach to the results of the full 2D simulations. Despite the slow sheath being initially much broader in the 1D simulations than in 2D, the resulting light curves agree surprisingly well. The agreement is particularly good at the highest densities \((n_{18} = 2000 \text{ cm}^{-3})\) because the slow component rapidly isotropizes in 2D. At lower densities \((n_{18} = 60 \text{ cm}^{-3})\), the agreement with the 1D simulations is not as good, particularly at 30 GHz. At high densities, the jet is quickly isotropized and its morphology is closer to that of the wedge we assume in our 1D model. Hence, the late time evolution of the light curve at high CNM densities is well captured by the 1D model. At lower densities, the optically thin emission shows a strongly perturbed axially symmetric jet, with an intricate morphology (Mimica et al. 2015). Thus, the 1D model is not optimal for capturing the slope of the light curve, especially at the highest frequencies (since the ejecta becomes optically thin earlier). However, the 1D model reproduces the peak luminosity from the 2D results within a factor of \(\sim 2\) for \(n_{18}=60 \text{ cm}^{-3}\) across all frequencies.

### 3.3.2 Analytic Estimates

The dependence of the synchrotron peak luminosity, peak time, and late time luminosity power law slope on the ambient gas density and jet parameters can be estimated analytically using a simple model for the emission from a homogenous, shocked slab of gas behind a self-similarly expanding blast wave (e.g., Sari et al. 1998; Granot & Sari 2002). The relevant results, as presented by Leventis et al. (2012), are summarized in Appendix D. The peak luminosity of the slow component of the
Figure 3.4: Comparison of light curves from 1D and 2D simulations for an on-axis observer ($\theta_j = 0$). We assume an $n \propto r^{-1}$ density profile.
jet can be estimated from equation (D.6),

\[ \nu L_{\nu,p} = \begin{cases} 
\left( \frac{2.7 \times 10^{40}}{10^{54} \text{ergs}} \right)^{0.59} \left( \frac{\epsilon_e}{0.1} \right)^{1.3} \times \\
\left( \frac{\epsilon_b}{0.002} \right)^{0.825} \left( \frac{\nu_{\text{obs}}}{5 \text{GHz}} \right)^{0.35} n_{18}^{1.24} \text{erg s}^{-1} & \text{Opt. Thin} \\
\left( \frac{1.1 \times 10^{42}}{10^{54} \text{ergs}} \right)^{0.87} \left( \frac{\epsilon_e}{0.1} \right)^{0.61} \times \\
\left( \frac{\epsilon_b}{0.002} \right)^{0.26} \left( \frac{\nu_{\text{obs}}}{5 \text{GHz}} \right)^{2.01} n_{18}^{-0.14} \text{erg s}^{-1} & \text{Opt. Thick} 
\end{cases} \] (3.18)

where we have adopted fiducial values for the power-law slope of the gas density profile, \( k = 1 \), and the electron energy distribution, \( p = 2.3 \). The top and bottom lines apply, respectively, to the shocked CNM being optically thin and optically thick at the deceleration time (as delineated by blue lines in Fig. 3.5).

The peak luminosity in the optically thin case depends sensitively on \( n_{18} \), while in the optically thick regime the dependence on density is much weaker. The peak fluxes in equation (3.18) are normalized to match those derived from our numerical results.

The time of maximum flux, for the same fiducial values \(( k = 1, p = 2.3 \)), is given by equation
where again the normalizations are chosen to match our numerical results. Note that for the optically thin case the peak time is within a factor of two of the deceleration time (eq. 3.4).

In general, more energetic jets produce emission which peaks later in time. However, the scaling of \( t_p \) with \( n_{18} \) is more complicated: if the emitting region is optically thick at the deceleration time, then the peak time increases with CNM density. In this case the peak flux occurs when the self-absorption frequency passes through the observing band, and this happens later if the nuclear gas density is higher. Otherwise, peak flux is achieved near the deceleration time, which is a decreasing function of \( n_{18} \) (eq. 3.4). Fig. 3.5 shows the division between the optically-thick and optically-thin regimes at 1 and 30 GHz in the parameter space of jet energy and \( n_{18} \).

### 3.3.3 Numerical Light Curves

As summarized in Table 3.1 (and shown in Fig 3.5), we calculate light curves for a grid of on-axis jet simulations for five different values of \( n_{18} \) (2, 11, 60, 345, and 2000 cm\(^{-3}\)) and three different values of the (beaming-corrected) jet energy \( E \) (\( 5 \times 10^{51} \), \( 5 \times 10^{52} \), \( 5 \times 10^{53} \) erg).

The left panels of Fig. 3.6 show example light curves for different jet energies and nuclear gas densities. The peak luminosity is roughly linearly proportional to the jet energy and is virtually
Figure 3.5: Contours of the fraction of the kinetic energy of the slow component of the jet ($\Gamma = 2$) which is dissipated at the reverse shock in the parameter space of jet energy, $E_j$, and CNM density, $n_{18}$. The parameters of the suite of jet simulations presented in this chapter are shown as red squares. The approximate location of SwJ1644 in the parameter space is also labeled. Blue lines delineate the parameter space where the slow component of the jet is optically thin/thick at the deceleration time at 1 GHz (left line) and 30 GHz (right line).
independent of the ambient density. For high CNM densities and low frequencies this is to be expected because the emission is dominated by the slow component, which is optically thick at the deceleration time. However, for high frequencies and small CNM densities, the peak luminosity of the slow component falls off, as shown by the lighter shaded lines in the right panels of Fig. 3.6. Coincidentally, the fast component just compensates for this decline, resulting in the total (fast + slow) on-axis peak luminosity being weakly dependent on $n_{18}$ across the entire parameter space. A good approximation to this universal peak luminosity is given by equation 3.18 for $n_{18} = 2000$ cm$^{-3}$ in the optically-thick case.

Fig. 3.6 also makes clear that the peak time increases with the ambient gas density. Across most of the parameter space the peak occurs after the deceleration time, when the emitting region transitions from optically thick to optically thin, as occurs later for larger $n_{18}$. However, at high frequencies and low densities the slow component is optically thin at the deceleration time, and thus its peak time is a decreasing function of $n_{18}$. For example, at 30 GHz, the slow component peaks later for $n_{18}=2$ cm$^{-3}$ than for $n_{18}=60$ cm$^{-3}$.

The numerical light curves are well fit by a broken power law (see e.g. Leventis et al. 2012),

$$L_{\nu}(t) = \frac{L_{\nu,p}}{2^{-1/s}} \left[ \left( \frac{t}{t_p} \right)^{-s a_1} + \left( \frac{t}{t_p} \right)^{-s a_2} \right]^{-1/s},$$

(3.20)

where $L_{\nu,p}$ and $t_p$ are the peak luminosity and time given by equations (3.18) and (3.19), respectively. The parameter $s$ controls the sharpness of the transition between the early-time power-law slope $a_1$ and the late-time slope $a_2$. Fitting to the numerical light curves, we find that $s \sim 1.0$, $a_1 \sim 1.7$, and $a_2 \sim -1.4$, the latter approximately agreeing with the analytic estimate in equation (D.7). These parameters generally reproduce our numerical light curves to within a factor of a few throughout our parameter space. However, the highest density/lowest energy light curve diverges from the power
Figure 3.6: Left: Radio light curves as viewed on axis ($\theta_{\text{obs}} = 0$) for jet energies of $5 \times 10^{53}$ erg (darker-shaded lines) and $5 \times 10^{51}$ erg (lighter-shaded lines), for values of $n_{18} = 2$ (blue), 60 (red), and 2000 (green) cm$^{-3}$. Solid lines show the result of 1D simulations, while 2D light curves are shown as dashed lines (when available). Thick lines show the results of our numerical calculation, while thin lines are power law extrapolations. A gas density profile of $n \propto r^{-1}$ is used for all of the light curves. Radio upper limits and detections are shown as triangles and squares, respectively. The single upper limit in the top panel is for D3-13 at 1.4 GHz from Bower (2011a). Figure continue on next page.
Figure 3.6: Gray triangles and squares in the second panel indicate upper limits and detections and detections at 3.0 GHz from Bower et al. (2013), while the red triangle is the 3.5 GHz upper limit for PTF-09axc from Arcavi et al. (2014). Black triangles in the third panel indicate upper limits at 5.0 GHz from van Velzen et al. (2013). The red triangle shows the 6.1 GHz upper limit for PTF-09axc from Arcavi et al. (2014). The connected black stars show early time data for SwJ1644 taken with EVLA (Berger et al. 2012; Zauderer et al. 2013), while the connected black squares show late time measurement with the European VLBI network (Yang et al. 2016). Connected blue squares show 5 GHz data for ASSASN-14li (Alexander et al. 2016). Note that we have subtracted the observed quiescent radio emission for ASSASN-14li. We have labeled events which have upper limits across multiple frequencies.

Right: $5 \times 10^{53}$ erg on-axis light curves from left column (darker-shaded lines) and corresponding slow component light curves (lighter-shaded lines). Simulation results at 8 and 30 GHz. Top left panel includes 8.4 GHz and 7.9 GHz upper limits for TDE2 and SDSSJ1201+30 respectively (see Table 3.2).
law fit at late times as the outflow enters into the deep Newtonian regime (see Sironi & Giannios 2013). Also, the 2d, $n_{18} = 60 \text{ cm}^{-3}$ light curve has a somewhat steeper late time light curve that declines as $t^{-2}$.

Fig. 3.7 compares the light curves for observers aligned with the jet axis (on-axis) with those at an angle of 0.8 radians from the jet axis (off-axis). While the on- and off-axis light curves agree well for $n_{18} = 2000 \text{ cm}^{-3}$, the off-axis luminosity for $n_{18} = 2 \text{ cm}^{-3}$ is smaller by an order of magnitude at peak. This is because the peak of the on-axis light curve is dominated by the fast component of the jet, which would not be visible for significantly off-axis observers. However, we find that the late time light curve is nearly independent of viewing angle.

The top panel of Fig. 3.8 shows 1D on-axis radio light curves for our fiducial gas density profile, $n \propto r^{-1}$, and a core galaxy profile (equation C.1), both with $n_{18} = 2 \text{ cm}^{-3}$. The light curves differ by at most a factor of a few. The core and cusp light curves are even closer at higher densities, and virtually indistinguishable at $n_{18} = 2000 \text{ cm}^{-3}$. This is because for larger ambient densities, the jet only samples small radii, where the core and cusp profiles are similar (see Fig. 3.1). It is only at lower densities, for which the Sedov radius lies outside of the flattening of the core density profile, that noticeable differences emerge.

The bottom panel of Fig. 3.8 compares the 1D on-axis light curves for $n \propto r^{-1}$ and $n \propto r^{-1.5}$ gas density profiles with $n_{18} = 60 \text{ cm}^{-3}$. For most times the light curves agree well, which is perhaps not surprising because the density in these two models agrees at $10^{18} \text{ cm}$, which is close to the Sedov radius for these density profiles. However, in 2D hydrodynamical simulations, a jet propagating through an $r^{-1.5}$ density profile develops a more prolate structure than a jet propagating through an $r^{-1}$ profile. This results in a light curve with a much steeper late time slope (see dash-dotted line in Fig. 3.8), although we note that the peak luminosity is nearly the same for the $n \propto r^{-1}$ and $n \propto r^{-1.5}$ density profiles.
Figure 3.7: Comparison between on-axis (solid line) and off-axis (dashed line) light curves from our 1D simulations. The off-axis light curves are calculated for an observer viewing angle of $\theta_{\text{obs}}=0.8$. We adopt a density profile of $n \propto r^{-1}$. We note that the steepening of the $n_{18} = 2 \text{ cm}^{-3}$ light curves after 2 years is not physical and is due to limited angular resolution (see Mimica et al. 2016).
Figure 3.8:  
Top: Comparison between on-axis light curves for our fiducial $n \propto r^{-1}$ gas density profile, corresponding to a cusp-like galaxy, and the core galaxy profile defined by (C.1) with $r_s = 10^{18}$ cm.  
Bottom: Comparison between on-axis light curves calculated from 1D simulations with $n \propto r^{-1}$ (solid) and $n \propto r^{-1.5}$ (dashed) gas density profiles. The dash-dotted line shows the on-axis light curve for a 2D simulation with an $n \propto r^{-1.5}$ gas density profile.
3.3.3.1 Reverse Shock Emission?

Our calculations shown in Figs. 3.4 and 3.6-3.8 include only emission from the forward shock (shocked CNM), while in principle the reverse shock (shocked jet) also contributes to the radio light curve.

The fraction of the initial kinetic energy of the jet which is dissipated by the reverse shock provides a first-order estimate of its maximum contribution to the radio light curve. Fig. 3.5 shows contours of the fraction of the kinetic energy of the slow component dissipated by the reverse shock as a function of the jet energy and CNM density, \(n_{18}\). This is estimated by integrating the shock evolution determined from the jump conditions (see Appendix E for details), approximating the jet as a constant source of duration \(t_0 = 5 \times 10^5\) s and Lorentz factor \(\Gamma = 2\). The parameters defining our grid of numerical solutions are shown in Fig. 3.5 as red squares.

Fig. 3.5 shows that for high ambient densities and/or low energy jets, the reverse shock dissipates an order unity fraction of the kinetic energy of the jet. Even for our highest energy/lowest density model \((n_{18} = 2\) cm\(^{-3}\) and \(E_j = 5 \times 10^{53}\) erg) the reverse shock will dissipate of order 20% of the jet energy. Fig. 3.9 shows the 5 GHz and 30 GHz light curve for this case, separated into contributions from the forward and reverse shocks. The reverse shock emission is comparable to that from the forward shock for the first month. However, this overstates the true contribution of the reverse shock to the observed emission because the latter is strongly attenuated by absorption from the front of the jet, which has not been included in the reverse shock light curve in Fig. 3.9. For 5 GHz the contribution of the reverse shock to the total light curve is negligible at all times. For the 30 GHz, the peak luminosity increases by a factor of 1.5 after reverse shock emission is taken into account. While the reverse shock dissipates an even larger fraction of the jet energy for higher ambient density, its emission will be even more heavily absorbed. We conclude that the reverse shock emission can be neglected for the high energy jets with \(E \gtrsim 10^{53}\) erg, consistent with the
reverse shock not contributing appreciably to SwJ1644 (Metzger et al. 2012).

For low energy jets, we find that the jet is crushed at early times, even for low values of $n_{18}$. In the case of very low power jets the reverse shock structure is replaced by a number of recollimation shocks (shocks driven laterally into the jet by the ambient medium; see e.g. Mimica et al. 2009b). While this is potentially a very interesting case since the emitting volume from recollimation shocks can be larger than from a single reverse shock, because of a much more complex structure we defer a more detailed study of the emission from the reverse/recollimation shocks in the the low energy case to future work.

As a final note of caution, even if the reverse shock dissipates most of the bulk kinetic energy into thermal energy, the latter can be converted back to kinetic energy through adiabatic expansion. However, we expect that the re-expansion will be relatively isotropic compared to the original jet, because the matter is first slowed to mildly relativistic speeds. The net result of a ultra-strong reverse shock (due to a weak jet, and/or an unusually high CNM density) is therefore likely to be the production of two quasi-spherical lobes on either side of the black hole, centered about the deceleration radius (Giannios & Metzger 2011).

### 3.3.4 Parameter Space of Jet-CNM Interaction

The left column of Fig. 3.10 shows contours of the peak luminosity (thick lines) as derived from our grid of numerical on-axis models, covering the parameter space of jet energy $E$ and density $n_{18}$. Also shown with thin lines is the luminosity arising from just the slow, wide angle component. The fast, narrow component of the jet dominates at high frequencies and low densities, while the slow, wide component dominates for large $n_{18}$ and low frequencies. Remarkably, the total peak luminosity is nearly independent of the ambient gas density; this is in part coincidental, as the fast and slow peak fluxes individually vary across the parameter space. For off-axis jets, the peak luminosity is
Figure 3.9: Radio light curve from the forward shock (red line), reverse shock (blue), and the total light curve (black) for a jet of energy $5 \times 10^{53}$ erg and CNM density $n \propto r^{-1}$ with $n_{18} = 2 \text{ cm}^{-3}$. The reverse shock light curve excludes absorption from the front of the jet, which when included in the full calculation results in large attenuation of the emission, such that the total light curve is dominated by the forward shock.
dominated by just that of the slow component, and thus would be a decreasing function of the ambient density above 1 GHz.

The right column of Fig. 3.10 compares our numerical results for the slow component to the analytic estimate given in equation (3.18). For large $n_{18}$, the optically thick case reproduces the peak luminosity to within a factor of a few. By contrast, for 30 GHz and low $n_{18}$, the numerical results are closer to the optically thin limit.

The left column of Fig. 3.11 shows contours of the time of peak flux in days, separately for the slow component (thin lines) and the total light curve (thick lines). Shown for comparison in the panels in the right column is the peak time as estimated from equation (3.19). At 30 GHz, the peak time decreases with $n_{18}$ at small values of the latter, because in this regime the jet is optically thin prior to the deceleration time.

3.3.4.1 Comparison with radio detections and upper limits.

Fig. 3.6 compares our fiducial $5 \times 10^{53}$ erg on-axis jet model to radio detections and upper limits derived from follow-up observations of TDE flares (including SwJ1644\textsuperscript{8}), as compiled in Table 3.2. All of the 5 GHz light curves, corresponding CNM densities, $n_{18}$, of 2, 60, and 2000 cm$^{-3}$, fall above the upper limits. In agreement with the results of previous work, we conclude that most TDEs discovered by their optical/UV or soft X-ray emission do not produce jets as powerful as that responsible for SwJ1644 (Bower et al. 2013; van Velzen et al. 2013; Mimica et al. 2015), a result which is now found to hold for a broad range of CNM environments.

The peak radio luminosity at frequencies $\lesssim 1$ GHz is weakly dependent on the ambient gas density. Radio observations conducted from several months to years after a tidal disruption flare, which tightly constrain the peak flux of a putative jet, can therefore be used to constrain the jet

\textsuperscript{8}Detailed comparison of our model with radio data from SwJ1644 data is given in Mimica et al. 2015.
Figure 3.10: Left: Thick lines show the peak radio luminosity in the parameter space of jet energy and ambient gas density at $10^{18}$ cm, calculated from the grid of on-axis jet simulations in Table 3.1. Thin lines show contours of peak luminosity for the slow component light curve (§ 3.3.1). Right: Analytic estimate for the peak luminosity (dashed lines; eq. 3.18) compared to the numerical results for the slow component (solid lines).
Figure 3.11:  Left: Thick lines show peak time in days in the parameter space of jet energy and ambient gas density at $10^{18}$ cm, calculated from the grid of on-axis jet simulations in Table 3.1. Thin lines show contours of peak time for the slow component light curve (see 3.3.1). Right: Analytic scaling for the peak time (dashed, see equation 3.19) compared to the numerical results for the slow component (solid)
energy. Equation (3.18) shows that an upper limit of $F_{ul}$ on the flux density at 1 GHz of a source at distance $d_L$ results in an upper limit on the jet energy of

$$E \lesssim 4.3 \times 10^{49} \left( \frac{F_{ul}}{50 \mu \text{Jy}} \right)^{1.1} \left( \frac{d_L}{200 \text{ Mpc}} \right)^{2.3} \text{erg},$$

(3.21)

where we have taken $n_{18} = 2000 \text{ cm}^{-3}$ (but the constraint is not overly sensitive to this choice for $n_{18} \geq 2 \text{ cm}^{-3}$). Radio measurements of the peak flux following a TDE therefore serve as calorimeters of the total energy released in a relativistic jet (or spherical outflow).

If the peak flux is missed, late time measurements can still be used to constrain the jet energy. In fact, with late time measurements it is possible to place constraints on the energy of the jet/outflow using higher frequency radio data. Fig. 3.12 compares our analytic fit to the on-axis 5 GHz synchrotron light curve (eq. 3.20) for different jet energies and existing radio upper limits for $n_{18}=10 \text{ cm}^{-3}$, the minimum expected density for stellar populations observed in TDE host galaxies. An increase in $n_{18}$ would simply shift the light curves to the right. Thus, for times after peak each light curve in Fig. 3.12 gives smallest plausible radio luminosity for the corresponding jet energy. As the upper limits are all taken at late times, the $n_{18}=10 \text{ cm}^{-3}$ light curve which passes through each upper limit corresponds to the maximum jet energy consistent with it. We note that that in this case, the deceleration radius is inside both the influence radius and the stagnation radius, and thus we would expect the density profile there to be closer to $r^{-1.5}$, rather than $r^{-1}$. A steeper density profile would cause a steeper late time decline in the light curve, and would make the upper limits less constraining. However, the steeper profile would imply a larger density at $n_{18}$, which would compensate for this.

Fig. 3.13 shows a histogram of the maximum jet energies consistent with the existing radio

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9The peak luminosity will decrease approximately linearly with $n_{18}$ for $n_{18} \leq 2 \text{ cm}^{-3}$. For the smallest plausible value of $n_{18}$, 0.3 cm$^{-3}$, the normalization in equation (3.21) would increase by a factor of 7.
Figure 3.12: Upper limits and 5 GHz analytic light curves (eqn. 3.20 with $s = 1$, $a_1 = 1.7$, and $a_2 = -2$) for different jet energies. We use the peak time and luminosity from our numerical $n_{18}=11 \text{ cm}^{-3}$ light curve for the highest energy light curve, as our analytic fits (eqns. 3.18 and 3.19) underestimate the peak luminosity a factor of $\sim 2$ for this density. Then we use our analytic results to scale this light curve to lower energies.
upper limits and detections of TDE flares with radio follow-up (see also Table 3.2). The detected
events include ASSASN-14li, SwJ1644, and SwJ2058. For ASSASN-14li and SwJ1644 the lightcurves
are well sampled, and the energy of the jet is relatively well constrained to be \( \approx 10^{48} - 10^{49} \) erg
for ASSASN-14li (van Velzen et al. 2016; Alexander et al. 2016) and \( 5 \times 10^{53} \) erg for SwJ1644
(Mimica et al. 2015). For SwJ2058, we take the jet energy to be \( 5 \times 10^{53} \) erg, the same as its “twin”
SwJ1644 (Cenko et al. 2012; Pasham et al. 2015). There are several radio detection in XMMSL1
J0740–85, but no break frequencies are unambiguously detected in the spectrum leaving the total
energy poorly constrained (Alexander et al. 2017). We obtain an upper bound of \( 4 \times 10^{51} \) erg for
\( n_{18} = 10 \text{ cm}^{-3} \).

### 3.4 Summary and Conclusions

We calculate the radio emission from tidal disruption event jets propagating through a range of
plausible circumnuclear gas densities. The latter are motivated by analytic estimates of the gas
supply from stellar winds from Chapter 2. We simulate the jet propagation using both 1D and
2D hydrodynamic simulations, which we then post-process using a radiative transfer calculation to
produce synchrotron light curves. To isolate the effects of the density profile and jet energy we
employ a fixed two component jet model from Mimica et al. (2015), which produces an acceptable
fit to the observed radio data of the on-axis jetted TDE SwJ1644. Our conclusions are summarized
as follows.

1. The radio emission is most sensitive to the density at the jet deceleration radius, which is
typically \( r_{\text{dec}} \approx 0.1 - 1 \text{ pc} \) (Fig. 3.1). We estimate the radial profile of nuclear gas densities
expected from injection of stellar wind material for different star formation histories, and find
that the gas density at \( 10^{18} \text{ cm} \) lies in the range \( n_{18} \approx 0.3M_{\odot,7}^{-0.4} - 1,300M_{\odot,7}^{0.5} \text{ cm}^{-3} \), with
Table 3.2: Inferred jet/outflow energies (and bounds) from radio detections and upper limits of optical/UV and soft X-ray TDE candidates. For each event detected in the radio there are multiple observations at different times/frequencies. Thus, we leave a dash in the time frequency, and luminosity columns and simply to refer to reference in column “Ref.”

<table>
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<th>t (yr)</th>
<th>$\nu$ (GHz)</th>
<th>$\nu L_\nu$ $(10^{36}$ erg s$^{-1}$)</th>
<th>Ref.</th>
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<td>-</td>
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<td>SwJ1644</td>
<td>1900</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>$5 \times 10^{53}$</td>
</tr>
<tr>
<td>SwJ2058</td>
<td>8400</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3</td>
<td>$5 \times 10^{53}$</td>
</tr>
<tr>
<td>XMMSLI-J0740</td>
<td>75</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>$&lt; 4.3 \times 10^{51}$</td>
</tr>
<tr>
<td><strong>Upper limits</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RXJ1624+7554</td>
<td>290</td>
<td>21.67</td>
<td>3.0</td>
<td>27</td>
<td>5</td>
<td>$&lt; 1.4 \times 10^{53}$</td>
</tr>
<tr>
<td>RXJ1242-1119</td>
<td>230</td>
<td>19.89</td>
<td>3.0</td>
<td>17</td>
<td>5</td>
<td>$&lt; 9.6 \times 10^{52}$</td>
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<tr>
<td>SDSSJ1323+48</td>
<td>410</td>
<td>8.61</td>
<td>3.0</td>
<td>100</td>
<td>5</td>
<td>$&lt; 1.0 \times 10^{53}$</td>
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<tr>
<td>SDSSJ1311-01</td>
<td>900</td>
<td>8.21</td>
<td>3.0</td>
<td>280</td>
<td>5</td>
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<tr>
<td>D1-9</td>
<td>1800</td>
<td>8.0</td>
<td>5.0</td>
<td>840</td>
<td>6</td>
<td>$&lt; 4.1 \times 10^{53}$</td>
</tr>
<tr>
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<td>5.0</td>
<td>130</td>
<td>6</td>
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<tr>
<td>D23H-1</td>
<td>930</td>
<td>4.8</td>
<td>5.0</td>
<td>210</td>
<td>6</td>
<td>$&lt; 8.2 \times 10^{52}$</td>
</tr>
<tr>
<td>PTF10iya</td>
<td>1100</td>
<td>1.6</td>
<td>5.0</td>
<td>320</td>
<td>6</td>
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<td>8.6</td>
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<td>4.3</td>
<td>5.0</td>
<td>610</td>
<td>6</td>
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</tr>
<tr>
<td>TDE2</td>
<td>1300</td>
<td>1.1</td>
<td>8.4</td>
<td>1700</td>
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</tr>
<tr>
<td>SDSSJ1201+30</td>
<td>710</td>
<td>1.4</td>
<td>7.9</td>
<td>1100</td>
<td>8</td>
<td>$&lt; 5.0 \times 10^{52}$</td>
</tr>
<tr>
<td>PTF09axc</td>
<td>550</td>
<td>5.0</td>
<td>3.5</td>
<td>700</td>
<td>11</td>
<td>$&lt; 1.8 \times 10^{53}$</td>
</tr>
<tr>
<td>PTF09axc</td>
<td>550</td>
<td>5.0</td>
<td>6.1</td>
<td>550</td>
<td>11</td>
<td>$&lt; 1.7 \times 10^{53}$</td>
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<td>67</td>
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<td>6.1</td>
<td>0.39</td>
<td>12</td>
<td>$&lt; 3.1 \times 10^{48}$</td>
</tr>
</tbody>
</table>


All upper limits are $5 \sigma$. Luminosity distances are calculated using the identified host galaxy redshift and the best fitting Planck 2013 cosmology ($\Omega_M = 0.307$ and $H_0 = 67.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$), as implemented in the Astropy cosmology package.
Figure 3.13: Histogram of jet energies consistent with existing radio detections (ASSASN-14li, SwJ1644, and SwJ2058) and upper limits (Table 1 of Mimica et al. 2015 and Arcavi et al. 2014), as summarized in Table 3.2.

\[ n_{18} \sim 10 \text{ cm}^{-3} \] for star formation histories typical of TDE host galaxies (excluding a possible factor of \( \sim 2 \) reduction from mass drop out from star formation).

2. The slope of the CNM gas density profile depends on the slope of the stellar density profile.

A TDE host galaxy likely possesses a cuspy stellar density profile inside of a few pc, with \( \rho_\star \propto r^{-1.7} \). This translates into a gas density profile ranging from \( n \propto r^{-0.7} \) on large scales to \( n \propto r^{-1.5} \) on very small scales, well inside the stagnation radius, \( r_s \) and influence radius
In general, we expect a density profile bracketed by $n \propto r^{-0.7}$ and $n \propto r^{-1.5}$ near the Sedov/deceleration radius. For simplicity we adopt a single power law $n \propto r^{-1}$ as our fiducial density profile.

3. We perform hydrodynamical simulations of our two component jet model for a range of plausible density profiles and normalizations $n_{18} = 2, 11, 60, 345, \text{ or } 2000 \text{ cm}^{-3}$. We find bright radio emission at a few GHz across this entire range of densities. The peak luminosity is only weakly dependent on the chosen density profile for on-axis jets. For off-axis jets, the peak luminosity at 1 GHz is insensitive to the CNM density profile and viewing angle for $n_{18} \geq 2 \text{ cm}^{-3}$, although it will be a stronger function of density at higher frequencies. While the peak radio flux is largely insensitive to the radial power-law slope for fixed $n_{18}$, a steeper profile $n \propto r^{-1.5}$ (e.g., as expected at radii $\ll r_s, r_{\text{inf}}$) alters the 2D dynamical evolution of the jet in a non-trivial way, resulting in a steeper post maximum decline of the radio light curve.

4. The time of the peak radio luminosity depends more sensitively on the density and can be as early as months, or as late as one decade, after the TDE. By comparing our calculated light curves with upper limits from a set of optical/UV and soft X-ray selected TDE, we show that most of these sources cannot have jets as powerful as SwJ1644.

5. In general, we only calculate the synchrotron radio emission from the forward shock, and neglect reverse shock emission. For high energy jets ($E \gtrsim 10^{53} \text{ erg}$), and frequencies $\lesssim 30$ GHz, we find that the reverse shock has minimal impact on the total light curve. For low energy jets the reverse shock structure may be replaced by a series of recollimation shocks with a large emitting volume, which could contribute significantly to the total emission.

Prompt radio follow-up, as well as regular monitoring, of future TDE flares would provide tighter constraints on the presence of jets. Radio afterglows can serve as calorimeters for
off-axis jets launched by TDEs, and future observational efforts that capture the peak radio
flux in thermally detected TDEs will add to the diversity of jet energies observed in TDE
flares. The broad range of energies (both detections and upper limits) already seen in TDE
jets presents an interesting puzzle for theoretical models of jet launching.
Chapter 4

Collisional stellar interactions in the Galactic Center

4.1 Introduction

A large fraction of low- to moderate-mass galaxies host nuclear star clusters (NSCs). The large mean stellar densities in these clusters, typically $\sim 10 - 10^7 \text{M}_\odot \text{pc}^{-3}$ (e.g. Georgiev & Böker 2014b), result in correspondingly high rates of collisional interactions (Leigh et al. 2016). Stellar-mass compact objects, particularly black holes (BHs) and neutron stars (NSs), play an important role in these environments; for example, they form sources of LIGO and LISA-band gravitational wave (e.g. Quinlan & Shapiro 1987; O’Leary et al. 2009; Tsang 2013; Bar-Or & Alexander 2016; Antonini & Rasio 2016; Stone et al. 2017a; Bartos et al. 2017), serve as probes of the relativistic spacetime near the central supermassive BH (SMBH; Paczynski & Trimble 1979; Pfahl & Loeb 2004), and potentially contribute to the $\gamma$-ray excess observed in our own Galactic Center (GC; Brandt & Kocsis 2015). Compact objects in NSCs will also induce strong tidal interactions during close flybys with stars. A sequence of weak tidal encounters will stochastically spin up GC stars (Alexander &
Kumar 2001; Sazonov et al. 2012), while a single very strong tidal encounter may disrupt the victim star and produce a luminous transient (Perets et al. 2016), but a tidal encounter of intermediate strength will bind the star to the compact object in a “tidal capture” (Fabian et al. 1975), as is the focus of this chapter.

There is strong evidence of a population of NSs and stellar-mass BHs in the Milky Way (MW) GC. The hundreds of O/B stars currently located in the central parsec indicate a high rate of in situ NS/BH formation in this region (e.g. Levin & Beloborodov 2003; Genzel et al. 2003a). The discovery of even a single magnetar within $\lesssim 0.1$ pc of Sgr A* (Mori et al. 2013), given their short active lifetimes, also demands a high current rate of NS formation. The X-ray point sources in the GC also directly indicate a population of binaries containing compact objects. There are a total of six known X-ray transients in the central parsec (Muno et al. 2005; Hailey & Mori 2017). Of these six, three are strong BH X-ray binary (BH-XRB) candidates based on their spectral properties and the long time-scale ($> 10$ years) between their outbursts. The identity of the remaining transients is unknown, but they may be NS-XRBs. In addition to these transient sources, Hailey et al. (2018) recently discovered 12 quiescent non-thermal X-ray sources within the central parsec. These sources are spectrally consistent with quiescent XRBs and distinct from the magnetic CV that make up most of the X-ray sources outside of the central parsec. Additionally, their luminosity function is consistent with that of dynamically confirmed BH XRBs in the field, while NSs are on average brighter in quiescence (Armas Padilla et al. 2014; Hailey et al. 2018). Other confirmed NS XRBs with similar luminosities in the Galactic Center region (though outside the central parsec) show bright outbursts with a characteristic cadence 5-10 years. Such outbursts are not seen in the quiescent population, also pointing to BH XRBs rather than NS XRBs (Degenaar & Wijnands 2010). (However, there are quiescent NS XRBs in the globular cluster 47 Tuc that have not outburst for decades Bahramian et al. 2014). Overall, the most likely identification for this new population is quiescent BH XRBs,
though an admixture of up to six milisecond pulsars cannot be ruled out. Reasonable extrapolation of the point source luminosity function below the instrumental detection threshold implies a true number of BH-XRBs inside the central parsec in the hundreds.

The number of NS XRBs per stellar mass in the Galactic Center is three orders of magnitude greater than in the field, and comparable to the number in Globular clusters (see Table 4.1). The number of BH XRBs per stellar mass in the Galactic Center is also three order of magnitude greater than in the field, and an order of magnitude greater than in any globular, suggesting the Galactic Center BH XRBs are not brought in via globular cluster in-fall. Also, any BH XRBs brought in by globulars are unlikely to survive to the present day, as the lifetime of BH XRBs is at most a few $\times 10^9$ years (see Fig. 4.10).

This indicates that the unusual environment of the GC dynamically - and efficiently - assembles BH-XRBs, in a manner analogous to the dynamical overproduction of NS-XRBs in globular clusters (Katz 1975; Benacquista & Downing 2013). Although a high concentration of compact objects in the GC is itself unsurprising (Alexander & Hopman 2009), an overabundance of mass-transferring binaries is more challenging to understand. In other dense stellar systems like globular clusters, exchange interactions that swap compact objects into binaries can explain the overabundance of NS-XRBs and their MSP progeny (e.g. Ivanova et al. 2008). However, this channel is strongly suppressed in NSCs because nearly all primordial stellar binaries would be evaporated by three-body encounters, and those that survive would be so hard as to present a minimal cross-section for exchange interactions (see Leigh et al. 2017 and appendix H).  

This chapter instead focuses on an alternative channel of XRB formation: the tidal capture of main sequence stars by compact objects (Press & Teukolsky 1977; Lee & Ostriker 1986). Stars that pass sufficiently close to a compact object—approximately, within its tidal radius $r_t$—are completely

$^1$Note that the maximum semi-major axis above which binaries are evaporated scales as velocity dispersion $\sigma^{-2}$, whereas the maximum pericenter for tidal capture scales $\sigma^{-0.2}$; see appendix H for more details.
torn apart by tidal forces (e.g. Rees 1988). However, for pericenter radii somewhat larger than $r_t$, tidal forces are not necessarily destructive; instead, they transfer orbital energy into internal oscillations of the star, binding it to the compact object. Following a complex and potentially violent process of circularization, the newly-created binary settles into a tight orbit. The necessarily small orbital separation of the tidal capture binary guarantees that subsequent gravitational wave emission will drive the star into Roche Lobe overflow in less than a Hubble time, forming a mass-transferring X-ray source. The high density of compact objects and stars in the GC inevitably lead to a significant rate of tidal captures, representing a promising explanation for the observed overabundance of BH- and NS-XRBs.

This chapter is organized as follows. In §4.2 we describe our model for the dynamical evolution of stars and compact remnants in the GC. In §4.3 we use the time-dependent density profiles of the stars and compact objects from our NSC models to calculate the rates of collisions and tidal capture of stars by compact objects, and make predictions for the present-day BH and NS-XRB population. In §4.4 we compare our predictions to observations of the XRB populations in the GC measured by Hailey et al. (2018). In §4.5 we describe several auxiliary predictions of our model for the rates of stellar interactions and exotic transients. In §4.6 we briefly summarize our results and conclude.

## 4.2 Galactic NSC Model

The number of BH-XRBs which form in the GC clearly depends on the number of stellar-mass BHs that reside there. Previous works have predicted that $\gtrsim 10^3 - 10^4$ BHs accumulate within the central parsec over timescales of several Gyr due to radial mass segregation from the stellar population on larger scales (e.g. Morris 1993; Miralda-Escudé & Gould 2000; Freitag et al. 2006;
Table 4.1: Estimated abundances of NS-XRB/BH-XRB/MSP per unit stellar mass in different environments. Q=Quiescent LMXBs ($L_x \lesssim 10^{33}$ erg s$^{-1}$); T=Transient LMXBs; B=Persistently bright or transient LMXBs (only persistently bright sources are NSs). No outbursts have been seen from globular BH XRB candidates, while only one quiescent BH XRB candidate has been identified in the field (Tetarenko et al. 2016). The numbers in the "transient" BH-XRB column is an estimate for the total BH XRB abundance, assuming that bright outbursts occur with a characteristic time-scale of 100 years.

<table>
<thead>
<tr>
<th>Environment</th>
<th>$N_{\text{NS-XRB}}$ (B) [$M_\odot^{-1}$]</th>
<th>$N_{\text{NS-XRB}}$ (Q) [$M_\odot^{-1}$]</th>
<th>$N_{\text{BH-XRB}}$ (T) [$M_\odot^{-1}$]</th>
<th>$N_{\text{BH-XRB}}$ (Q) [$M_\odot^{-1}$]</th>
<th>$N_{\text{MSP}}$</th>
<th>References</th>
</tr>
</thead>
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<tr>
<td>Field</td>
<td>$2 \times 10^{-9}$</td>
<td>-</td>
<td>$2 \times 10^{-8}$</td>
<td>-</td>
<td>$5 \times 10^{-9}$</td>
<td>1-3</td>
</tr>
<tr>
<td>Globular clusters (all)</td>
<td>$3 \times 10^{-7}$</td>
<td>$6 \times 10^{-7}$</td>
<td>-</td>
<td>$10^{-7}$</td>
<td>$7 \times 10^{-9}$</td>
<td>5-7</td>
</tr>
<tr>
<td>47 Tuc</td>
<td>-</td>
<td>$7 \times 10^{-6}$</td>
<td>-</td>
<td>$10^{-6}$</td>
<td>$3 \times 10^{-2}$</td>
<td>8-11</td>
</tr>
<tr>
<td>Terzan 5</td>
<td>$1.5 \times 10^{-6}$</td>
<td>$6 \times 10^{-6}$</td>
<td>-</td>
<td>-</td>
<td>$2 \times 10^{-5}$</td>
<td>12-13</td>
</tr>
<tr>
<td>Galactic Center (central parsec)</td>
<td>$1 - 3 \times 10^{-6}$</td>
<td>-</td>
<td>$2 \times 10^{-5}$</td>
<td>$10^{-5}$</td>
<td>$\lesssim 1.3 \times 10^{-4}$</td>
<td>14-18</td>
</tr>
</tbody>
</table>


Hopman & Alexander 2006b; O’Leary et al. 2009; Dale et al. 2009; Merritt 2010). Most previous models assume that the BHs are distributed at birth in the same way as the lower mass stars, and neglect ongoing star formation (though see Aharon & Perets 2015; Baumgardt et al. 2017).

In fact, much of our NSC’s total stellar population was likely deposited by the infall of globular clusters early in its history (Tremaine et al. 1975; Antonini et al. 2012; Gnedin et al. 2014; Arca-Sedda & Capuzzo-Dolcetta 2014; Abbate et al. 2018). Historically, globular clusters (at least of the kind which survive to the present day) were predicted to lose all but a few of their BHs due to strong kicks in multi-body interactions during a core collapse or Spitzer instability phase (e.g. Spitzer 1987; Kulkarni et al. 1993; Banerjee et al. 2010). Such lossiness would limit the ability of globular infall to seed the GC with BHs, although Morscher et al. (2015) challenged this conventional wisdom by showing that $\sim 10^2 - 10^3$ BHs could be retained in globulars due to three-body processes reversing core collapse (see also Askar et al. 2018). While a few candidate globular cluster BH-XRB have been identified (Strader et al. 2012b; Bahramian et al. 2017) in the MW, and other BH-XRBs have been seen in extragalactic globulars (Maccarone et al. 2007), the total inventory of globular BHs is challenging to infer from observations because the number of BH-XRBs is likely a weak function of

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2Physically, this is because the thermodynamics of the subcluster is regulated by the longer relaxation time of the bulk cluster (Breen & Heggie 2012, 2013).
the number of retained BHs (Kremer et al. 2017).

A potentially much larger population of BHs is formed in the GC by \textit{in situ} star formation. A disk of young stars of age \(\approx 4\) Myr is observed to extend between \(\sim 0.03 - 0.3\) pc of SgrA* (Krabbe et al. 1995; Paumard et al. 2006; Lu et al. 2013), containing a total of \(\sim 100\) WR/O stars with a top-heavy initial mass function (IMF; e.g. Bartko et al. 2010; Lu et al. 2013). If the average formation rate of massive stars is comparable to the rate in the last few million years, a total of \(\sim 10^5\) compact objects would be injected into the Galactic Center over 10 Gyr. However, there is no evidence for multiple bursts of star formation over the last \(\times 10^7\) years (Habibi et al. 2017)\(^3\), and feedback from stellar winds and Supernovae would suppress star formation on this time-scale. This suggests star-bursts occur with a cadence of at least \(4 \times 10^7\) years. In principle, the cadence may be much longer, but this means we are observing the GC at a very atypical time. In this chapter, we focus on models in which bursts of star formation occur every \(4 \times 10^6 - 4 \times 10^7\) years.

This section describes our model for how the 1D radial density profiles of stars and compact remnants in the GC evolve in time. Our goal is to create a small set of simple but physically-motivated models for building up the NSC, which are consistent with both the present-day stellar density profile and the observed rate of compact object formation. Motivated by the above discussion, our model consists of two stellar populations: (1) stars injected in the distant past, near the formation time of the NSC; and (2) a continuously forming \textit{in situ} population with a top heavy IMF concentrated within the central parsec, as is motivated by the observed disks of young stars (Lu et al. 2013). All of our models assume spherical symmetry and isotropic velocities.

In §4.2.1 we motivate the parameters of our models using the observed stellar populations and constraints on the star formation history in the GC. In §4.2.2 we describe our numerical procedure for evolving the density profiles of stars and compact objects through two-body relaxation. To

\(^3\)The existing stellar population would not probe the star formation history on longer time-scales if the IMF is truncated below \(\sim 10M_\odot\).
build up physical intuition, we first calculate how the compact objects evolve in isolation in § 4.2.3, before adding the effects of the stellar background in § 4.2.4. In § 4.2.5 we discuss several additional hypothetical scenarios for building the NSC, in order to assess the range of uncertainties in our work and to make contact with previous work on this topic in the literature (which generally neglect centrally-concentrated compact object formation).

### 4.2.1 Stellar and Compact Object Populations

Eighty percent of the stars in the GC are older than 5 Gyr (Pfuhl et al. 2011), consistent with the bulk of the NSC’s growth being due to the infall of globular clusters via dynamical friction over a period of $\sim 1$ Gyr in the early history of the Galaxy (Gnedin et al. 2014). The observed diffuse stellar light\(^4\) is well fit by the parameterization (Schödel et al. 2018; their Tables 2 and 3),

$$\rho_\star(r) = 2^{(\beta-\gamma)/\alpha}\rho_0 \left(\frac{r}{r_0}\right)^{-\gamma} \left(1 + \left(\frac{r}{r_0}\right)^{\alpha}(\gamma-\beta)/\alpha\right),$$

(4.1)

with best-fit parameters of $\gamma = 1.16 \pm 0.02$, $\beta = 3.2 \pm 0.3$, $r_0 = 3.2 \pm 0.2$ pc for fixed $\alpha = 10$. The density normalization at 1 pc is $0.8 - 1.7 \times 10^5$ pc$^{-3}$.\(^5\)

Compact objects are also deposited at early times if they arrive with the globular clusters. Ivanova et al. (2008) estimate that a typical globular cluster retains 1 NS per $10^3 M_\odot$ of other stars. Although globulars may also bring in a sizeable BH population, this is less certain because, as discussed above, BHs may be ejected from globulars by binary-single interactions (e.g. Kulkarni et al. 1993; Banerjee et al. 2010; see, however, Morscher et al. 2015). Given this uncertainty, and

\(^4\)This profile differs from the observed giant density profile, which has a core inside of $\sim 0.5$ pc (the so-called “missing giants” problem; e.g. Buchholz et al. 2009; Do et al. 2009). The diffuse light tracks emission from early G and late F main sequence and sub-giant stars, and is likely a better probe of the underlying stellar density. One solution to the missing giants problem is mass-stripping by collisions between the giants with other stars and compact objects (Dale et al. 2009) or with a clumpy gas disc (Amaro-Seoane & Chen 2014; Kieffer & Bogdanović 2016).

\(^5\)We use the values from the first preprint version of Schödel et al. (2018). The best fit parameters are slightly different in the published version, but consistent within uncertainties.
because the current BH population is anyways likely to be dominated by \textit{in situ} star formation (see below), we neglect BHs deposited with the old stellar population.

Our fiducial models include a population of compact objects formed \textit{in situ}, motivated by the sub-parsec disk of young stars observed in the GC. The K-band luminosity function of the young stars is consistent with a single starburst that occurred \(2.5 - 5.8\) Myr ago (Lu et al. 2013; Habibi et al. 2017). The burst produced a total of \(~250\) stars of mass \(\gtrsim 8\text{M}_\odot\) with an IMF of the form \(dN/dm \propto m^{-\beta}\) with \(\beta \approx 1.7\). If stars with masses in the range \(m \approx 8 - 25\text{M}_\odot\) form NSs, while those with \(m \gtrsim 25\text{M}_\odot\) form BHs, then a total of \(N_{\text{ns}} \sim 160\) NSs and \(N_{\text{bh}} \sim 90\) BHs were, or will be, formed from the disk stars. If the time since the last star formation episode of \(~4\) Myr is comparable to the typical interval between starbursts, then the implied average formation rates of NSs and BHs in the central parsec are \(\dot{N}_{\text{ns}} \sim 4 \times 10^{-5}\) yr\(^{-1}\) and \(\dot{N}_{\text{bh}} \sim 2 \times 10^{-5}\) yr\(^{-1}\), respectively.

The above estimates assume that the current epoch is a representative snapshot of the central parsec’s average star formation history. In possible tension with this, Pfuhl et al. (2011) find that the star formation rate \(~1 - 5\) Gyr ago was \(~1 - 2\) orders magnitude smaller than the present-day rate (their Figure 14), in which case the average star formation rate is \(\lesssim 10\%\) of its recent value. However these observations probe only low mass stars \(\lesssim 2\text{M}_\odot\), and thus do not constrain the rate of NS/BH formation within the star-forming disks if the top-heavy disk IMF is truncated below a few solar masses. Other nearby galactic nuclei such as M31 possess disks of A stars, but no O and B stars (Leigh et al. 2016); in these NSCs at least, the last major episode of star formation occurred \(\gtrsim 100\) Myr ago.

Motivated by the above, we construct our fiducial models for the GC using the following three populations:

a. “Primordial” stars, which are assumed to form impulsively at \(t = 0\) (10 Gyr ago) with an initial density profile following eq. (4.1). We model all the stars as being of a single mass \(0.3\text{M}_\odot\),
which represents the root-mean-square mass of the main sequence in an evolved Kroupa IMF. For simplicity, the parameters of the stellar profile \( (\alpha, \beta, \gamma) \) are fixed to the best-fit values from Schödel et al. (2018), except for the scale radius \( r_o \) and normalization \( \rho_o \). The cluster expands radially over time, so we chose smaller initial values of \( r_o = 0.5, 1.5 \) pc in order to match the present-day stellar density at 1 pc (though we note that the functional form of the density profile is not exactly preserved in the evolution). A normalization of \( \rho_{\star}(1 \text{ pc}) = 1.1 \times 10^5 \ M_\odot \text{ pc}^{-3} \) is chosen to fix the total stellar mass at a value of \( 5.7 \times 10^7 \ M_\odot \).\footnote{The true total mass \( 2.5 \pm 0.4 \times 10^7 \ M_\odot \) is somewhat lower. The difference comes from the fact that we assume the stellar density profile extends to infinity, but in reality the stellar density steepens at 10 pc.}

b. “Primordial” NSs of mass \( 1.5 M_\odot \), which are deposited impulsively at \( t = 0 \) with the same density profile as the stars. The total number of NSs is normalized to a fraction \( 10^{-3} \) of the number of stars, motivated by their expected abundance in globular clusters (Ivanova et al. 2008).

c. Compact objects from in situ star formation (NSs and BHs of masses 1.5 and \( 10 M_\odot \), respectively) that are continuously injected near the present-day disk of young stars. The source term is narrowly peaked at the potential energy at 0.3 pc (the outer edge of the disk). In physical space, star formation is concentrated inside of this radius with an \( r^{-0.5} \) density profile. We found our results do not change if the star formation is instead concentrated inside of 0.03 pc (the inner edge of the star forming disks). In our “Fiducial” model, we adopt conservative formation rates of \( \dot{N}_{\text{ns}} = 4 \times 10^{-6} \ \text{yr}^{-1} \) and \( \dot{N}_{\text{bh}} = 2 \times 10^{-6} \ \text{yr}^{-1} \), respectively. We also consider a model (“Fiducial \times 10^9”) in which \( \dot{N}_{\text{ns}} \) and \( \dot{N}_{\text{bh}} \) are ten times larger, corresponding to the present day formation rate of massive stars.

The parameters of our fiducial models are summarized in Table 4.2. Several hypothetical (non-fiducial) models are introduced in §4.2.5 in order to assess the robustness of our conclusions.
4.2.2 Numerical Method: Fokker-Planck

The radial distribution of stars and compact objects evolves over time due to two-body relaxation. We follow this evolution using the PhaseFlow code (Vasiliev 2017), which solves the time-dependent, isotropic Fokker-Planck equation for the energy-space distribution function \( f(\epsilon, t) \). This equation can be written in flux-conserving form as

\[
\frac{\partial f(\epsilon, t)}{\partial t} = -\frac{\partial}{\partial \epsilon} \left[ D_\epsilon \frac{\partial f(\epsilon, t)}{\partial \epsilon} + D_{\epsilon \epsilon} f(\epsilon, t) \right] \frac{F(\epsilon)}{\tau_{LC}(\epsilon, t)} + S(\epsilon, t),
\]

(4.2)

where \( \epsilon \) is the binding energy, \( D_\epsilon \) and \( D_{\epsilon \epsilon} \) are the first and second order energy diffusion coefficients, \( F(\epsilon) \) is the mass flux, and the last two terms account for the draining of stars into the loss cone of the SMBH (see eq. 13 in Vasiliev 2017), and injection of stars due to star formation. The diffusion coefficients can be expressed as integrals over the distribution function, which for a single species mass \( m \) are given by

\[
D_{\epsilon \epsilon} = 16\pi^2 G^2 m \ln \Lambda \left[ h(\epsilon) \int_0^\epsilon f(\epsilon')d\epsilon' + \int_\epsilon^{\epsilon_{\text{max}}} f(\epsilon')h(\epsilon')d\epsilon' \right]
\]

(4.3)

\[
D_\epsilon = -16\pi^2 G^2 m \ln \Lambda \int_\epsilon^{\epsilon_{\text{max}}} f(\epsilon')g(\epsilon')d\epsilon',
\]

(4.4)

where \( h(\epsilon) \) is the phase volume and \( g(\epsilon) = dh(\epsilon)/d\epsilon \) is the density of states (see e.g. Merritt 2013). For a Keplerian potential, \( h \propto \epsilon^{-3/2} \) and \( g \propto \epsilon^{-5/2} \).

The one-dimensional Fokker-Planck approach is computationally efficient and reproduces the results from two-dimensional Fokker-Planck (Cohn 1985; Merritt 2015) as well as Monte-Carlo and N-body calculations (Vasiliev 2017) reasonably well. A key assumption of this equation is spherical symmetry, which is in tension with the physical motivation for our source term \( S(\epsilon, t) \): disk-mode star formation. However, it is reasonable to assume that compact remnants will become isotropic.
over time. First of all, there is likely no preferred plane for disk mode star formation in the GC. The current disk of young stars is not aligned with either the Galactic disk or the circumnuclear ring of molecular gas (McCourt & Madigan 2016). Thus, the injected remnants from many different episodes of \textit{in situ} star formation would naturally form with a quasi-isotropic angular distribution. Recent work has found that vector resonant relaxation can lead to a disk configuration for BHs and heavy stars even if they are drawn from sixteen randomly oriented disks (Szölgyén & Kocsis 2018). However, as the number of star formation episodes increases, the disk would thicken and would approach an isotropic distribution (Bence Kocsis, personal communication). In principle, resonant relaxation (Rauch & Tremaine 1996; Hopman & Alexander 2006a) may flatten the stellar density profile by causing stars to diffuse more rapidly into the loss cone. In practice, however, this effect only becomes important on small radial scales \( \lesssim 0.1 \) pc (Bar-Or & Alexander 2016), interior to where most tidal captures occur.

Finally, strong gravitational scatterings by BHs can lead to significant evaporation of low mass stars and remnants from the cusp. This effect is not included in our models, but a post-hoc calculation shows it changes the stellar density profile by \( \lesssim 40\% \) (see §4.2.6).

### 4.2.3 Evolution with Compact Remnants Only

Compact objects which are injected near the present disk of massive stars at \( \sim 0.3 \) pc will diffuse outwards via two-body scattering. To study this process, we solve eq. (4.2) with a constant source function of injected BHs, \( \dot{N}_{\text{bh}} = 2 \times 10^{-5} \) yr\(^{-1} \), corresponding to our “Fiducial×10” model. To whet our intuition in a controlled setting, we initially neglect contributions to the gravitational potential or diffusion coefficients from the background of NSs and stars.

Figure 4.1 (top panel) shows the resulting BH number density profile \( n(r) \) after 10 Gyr of evolution, over which a quasi-steady state is achieved on small radial scales. For comparison, a
dashed line shows how little the solution changes if one neglects the gravitational potential of the stellar mass BHs and the loss cone sink (the final term in eq. 4.2). The steady-state BH profile is well described by a broken power law, with \( n \propto r^{-7/4} \) at small radii \( r \ll r_i \) and \( n \propto r^{-5/2} \) at \( r \gg r_i \). As we now describe, these power-law slopes and the normalization of the BH profile can be understood through basic analytic arguments.

Compact objects injected at \( r_i \) diffuse outwards on the two-body relaxation timescale, which for a single mass population \( m_c \) is approximately given by \(^7\),

\[
\tau_{rx} \approx 0.34 \frac{\sigma^3(r)}{\log \Lambda G^2 n(r) m_c^2}, \tag{4.5}
\]

where \( \sigma(r) \) is the one dimensional velocity dispersion, \( n(r) \) is the number density profile, and \( \log \Lambda \approx 15 \) is the Coulomb logarithm.

In steady state, the formation rate of compact objects per unit volume equals the rate of outwards diffusion, i.e.

\[
\frac{\dot{N}}{r_i^3} \sim \frac{n_i}{\tau_{rx}(r_i)}.
\tag{4.6}
\]

This implies a steady-state density at the injection radius of

\[
n(r_i) \propto r_i^{-9/4} N^{1/2} M^{3/4} m_c^{-1}, \tag{4.7}
\]

where in taking \( \sigma \propto (GM/r)^{1/2} \) we have assumed that the SMBH of mass \( M \) dominates the gravita-

\(^7\)For a Keplerian potential, the pre-factor of eq. (4.5) varies from 0.2−0.4, depending on the density profile power-law slope \( \gamma = 0.5 − 3 \), where \( n \propto r^{-\gamma} \).
tational potential. Normalizing equation (4.7) using results from our numerical solutions, we find

\[ n_i = 1.6 \times 10^4 \left( \frac{r_i}{0.3 \text{pc}} \right)^{-9/4} \left( \frac{\dot{N}}{2 \times 10^{-5} \text{yr}^{-1}} \right)^{1/2} \left( \frac{M}{4 \times 10^6 \text{M}_\odot} \right)^{3/4} \left( \frac{m_c}{10 \text{M}_\odot} \right)^{-1} \text{pc}^{-3}, \]

(4.8)

What sets the power-law slopes of the BH density profile? For a distribution function \( f \propto \epsilon^p \) which extends to a maximum energy \( \epsilon_{\text{max}} \), the flux of mass through energy space (see eq. 4.2) is

\[ F(\epsilon) \propto \epsilon^{2p-3/2} \left[ a_o(p) + a_1(p) \left( \frac{\epsilon_{\text{max}}}{\epsilon} \right)^{p-3/2} + a_2(p) \left( \frac{\epsilon_{\text{max}}}{\epsilon} \right)^{p-1/2} \right], \]

(4.9)

where \( a_1, a_2, \) and \( a_3 \) are dimensionless functions of \( p \).

In steady state, the flux through energy space is constant. For \( p < 1/2 \), one finds \( F \approx a_o(p)\epsilon^{2p-3/2} \) in the limit that \( \epsilon_{\text{max}} \to \infty \) and thus \( F \) will be zero for \( p = 1/4 \); this is the classical “Bahcall-Wolf” (BW) solution (Bahcall & Wolf 1976). If \( p > 1/2 \), then the \( a_2 \) term in equation (4.9) dominates over the first two terms. The \( p = 1 \) profile corresponds to the steady-state solution for a constant, non-zero (outwards) flux. In this case \( \epsilon_{\text{max}} \) must have a finite value, as otherwise the flux would diverge; in our case, this maximum energy corresponds to the location of the source function of injected BHs at \( r_i \). These two steady state solutions (zero flux at small radii and constant outward flux at large radii) correspond to density profiles \( n \propto r^{-7/4} \) and \( \propto r^{-5/2} \), respectively. In this solution energy is transferred from the injection radius to larger scales by stars on eccentric orbits. However, Fragione & Sari (2017) have argued that these stars cannot effectively transfer energy to the bulk of the stellar population and evolve in a decoupled way (an effect which the isotropic Fokker-Planck solver in PHASEFLOW cannot capture). In this case, energy relaxation
Figure 4.1: Top panel: Number density of BHs in the GC as a function of radius $r$ after 10 Gyr of evolution, in the case of continuous BH injection (solid line). BHs are injected at a constant rate $\dot{N}_{bh} = 2 \times 10^{-5}$ yr$^{-1}$ at $r_{in} \approx 0.3$ pc. Bottom panel: Density profiles of NSs and BHs after 10 Gyr of evolution for injection rates at $r_i$ corresponding to our Fiducial model ($\dot{N}_{ns} = 4 \times 10^{-5}$, $\dot{N}_{bh} = 2 \times 10^{-5}$ yr$^{-1}$). Dashed lines show how the results change if the gravitational potential of the compact objects, and the sink term due to SMBH loss cone, are neglected.
Figure 4.2: \textit{Top panel}: Density profiles of stars and compact remnants at $t = 10$ Gyr (solid lines), in the case that compact remnants are continually injected near $\sim 0.3$ pc at the rates corresponding to the Fiducial model. The initial profile of stars is shown as a dashed black line, while the present-day distribution of low mass stars from Schödel et al. (2018) is shown as the shaded region (including uncertainties). \textit{Bottom panel}: Density profiles of stars and compact objects in our Fiducial×10 model. For comparison, dash-dotted blue and green lines show, respectively the profiles of BHs and NSs, neglecting the pre-existing background of low mass stars/NSs (Fig. 4.1).
at any radius would be dominated by the local stellar population, and by mass conservation the
density has a slightly shallower $r^{-9/4}$ profile (see also Peebles 1972). Ultimately, the two-dimensional
Fokker-Planck simulations are necessary to determine the correct outer density profile.

The enclosed mass $\propto nr^3 \propto r^{1/2}$ is dominated by the largest radius out to which the BHs have
had time to diffuse over the system age $t$. The half-mass radius $r_{1/2}$, interior to which the above
steady-state profile is established, can be estimated by equating $t = \pi x$; using equation (4.5), this
gives

$$r_{1/2} \approx 3.3 \text{ pc} \left( \frac{t}{3 \text{ Gyr}} \right)^2 \left( \frac{r_i}{0.3 \text{ pc}} \right)^{-1/2} \left( \frac{\dot{N}}{2 \times 10^{-5} \text{ yr}^{-1}} \right) \left( \frac{M}{4 \times 10^6 M_\odot} \right)^{-3/2} \left( \frac{m_c}{10 M_\odot} \right)^2.$$  (4.10)

BHs injected in the GC therefore have sufficient time to establish a steady-state profile within the
central parsec by the present age ($t = 10$ Gyr).

The bottom panel of Fig. 4.1 shows the present-day density profiles for a calculation otherwise
identical to the BH-only case, but including the evolution of both the BHs and NSs, assuming each
are injected at $r_i$ at rates of $\dot{N}_{\text{bh}} = 2 \times 10^{-5} \text{ yr}^{-1}$ and $\dot{N}_{\text{ns}} = 4 \times 10^{-5} \text{ yr}^{-1}$, respectively (the old
population of stars and their associated NSs are still neglected). The addition of NSs has little
effect on the BH profile compared to the BH-only case. Outside of the injection radius, the slope of
the NS density profile is similar to the BH one, but with a greater overall normalization reflecting
the relatively higher NS injection rate. At radii $\lesssim r_i$, the BH density profile approaches the BW
shape $n \propto r^{-7/4}$ (as in the BH-only case), while the NSs achieve a shallower profile $\propto r^{-3/2}$. A
shallower profile for the lighter species is expected for a two-component model in which the heavy
species dominates the diffusion coefficients (Bahcall & Wolf 1977; Alexander & Hopman 2009).

The analytic arguments presented above are readily extended to the multi-species case. In
particular, the BH density at $r_i$ can again be estimated by replacing the single remnant mass $m_c$ in eq. (4.7) with a weighted generalization

$$\tilde{m} = \left( \frac{N_{bh} m_{bh}^2 + N_{ns} m_{ns}^2}{N_{bh} + N_{ns}} \right)^{1/2}. \tag{4.11}$$

4.2.4 Effects of stellar background and potential

We now explore the effects of also including the old population of low mass stars and NSs on the NSC evolution, the final step in constructing our NSC models. Fig. 4.2 shows the profile of stars, NSs, and BHs at $t = 10$ Gyr in our Fiducial (top panel) and Fiducial×10 (bottom panel) models. BHs and NSs dominate the mass density inside of 0.03 (0.4) pc in the Fiducial (Fiducial×10) model.

The cusp of compact remnants causes the star cluster to expand radially over time, motivating our choice of a more compact initial stellar profile (black dashed line) than the currently-observed one (shaded gray region; Schödel et al. 2018). By contrast, the compact objects become slightly more centrally concentrated than in the previous models where the stars were neglected (Fig. 4.1). There are two reasons for this: (i) stars tend to scatter the higher mass compact objects to larger binding energies (ii) the gravitational potential of the stars suppresses outward diffusions of compact objects.

4.2.5 Non-fiducial NSC Models

This section explores other (“non-fiducial”) scenarios for creating the GC’s NSC, in which all of the stars and compact objects instead form as a single population with a common density profile and standard IMF. For one set of models, we assume all stars formed impulsively 10 Gyr ago. Such models allow us to compare our results to those of past work (e.g. Morris 1993; Miralda-Escudé & Gould 2000; Freitag et al. 2006), and are a useful limiting case if bursts of massive star formation
Table 4.2: Summary of models for assembling the populations of stars and compact remnants in the GC. The top two rows summarize our “fiducial” models, in which low mass stars and NSs are initialized at $t = 0$ following a radial profile given by eq. (4.1) with the scale radius $r_0$ and NS-to-star number ratio $N_{ns}/N_\star$. BHs and NSs are also continuously injected inside of $\sim 0.3$ pc, near the outer edge of the observed young stellar disks, with rates $\dot{N}_{bh}$ and $\dot{N}_{ns}$, respectively. The bottom three rows summarize other, non-fiducial scenarios, in which all compact remnants and stars form with the same radial distribution, either impulsively in the distant past or continuously. The masses of the stars, NSs, and BHs, are taken to be $0.33 M_\odot$, $1.5 M_\odot$, and $10 M_\odot$, respectively.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$r_0$ [pc]</th>
<th>$N_\star(t=0)$</th>
<th>$N_{bh}/N_\star$ (t=0)</th>
<th>$N_{ns}/N_\star$ (t=0)</th>
<th>$\dot{N}_{bh}$ [yr$^{-1}$]</th>
<th>$\dot{N}<em>{ns}/\dot{N}</em>{bh}$</th>
<th>$M_\star$</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiducial</td>
<td>1.5</td>
<td>$1.9 \times 10^8$</td>
<td>$3 \times 10^{-4}, 10^{-3}, 10^{-2}$</td>
<td>$4 N_{bh}/N_\star$</td>
<td>$2 \times 10^{-6}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$4 \times 10^7 M_\odot$</td>
<td>4.2</td>
</tr>
<tr>
<td>Fiducial $\times 10$</td>
<td>0.5</td>
<td>$1.9 \times 10^8$</td>
<td>$3 \times 10^{-4}, 10^{-3}, 10^{-2}$</td>
<td>$4 N_{bh}/N_\star$</td>
<td>$2 \times 10^{-6}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$4 \times 10^7 M_\odot$</td>
<td>4.2</td>
</tr>
<tr>
<td>Impulsive</td>
<td>1</td>
<td>$1.9 \times 10^8$</td>
<td>$3 \times 10^{-4}, 10^{-3}, 10^{-2}$</td>
<td>$4 N_{bh}/N_\star$</td>
<td>$2 \times 10^{-6}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$4 \times 10^7 M_\odot$</td>
<td>4.3</td>
</tr>
<tr>
<td>Continuous SF, Existing SMBH</td>
<td>3</td>
<td>$1.9 \times 10^{-2}$</td>
<td>$3 \times 10^{-4}, 10^{-3}, 10^{-2}$</td>
<td>$4 N_{bh}/N_\star$</td>
<td>$2 \times 10^{-6}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$4 \times 10^6 M_\odot$</td>
<td>4.4</td>
</tr>
<tr>
<td>Continuous SF, Growing SMBH</td>
<td>3</td>
<td>$1.9 \times 10^{-2}$</td>
<td>$3 \times 10^{-4}, 10^{-3}, 10^{-2}$</td>
<td>$4 N_{bh}/N_\star$</td>
<td>$2 \times 10^{-6}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$7% M_{tot}$</td>
<td>4.5</td>
</tr>
</tbody>
</table>

occur with a $\gtrsim 100$ Myr cadence. We also consider models in which stars form at a constant rate with the present day observed profile. This model is unrealistic for the GC, but may be useful for other galactic nuclei with different star formation histories (Leigh et al. 2016). These models are summarized in Table 4.2.

Our models assume that stars are accreted in the distant past or form in-situ, neglecting on-going exchange of stars with the surrounding galaxy. We expect the exchange of low mass stars to be negligible as the energy relaxation time becomes longer than a Hubble time outside of a few parsecs. Ten solar mass BHs within ten parsecs of the center would sink to smaller radii within a Hubble time. Sinking from this scale would be captured by our impulsive models. On-going star cluster in-fall can bring additional stars to the center. However, the majority of the stellar mass brought in via globulars is accreted within $\sim 1$ Gyr (Arca-Sedda & Capuzzo-Dolcetta 2014; Gnedin et al. 2014).

Fig. 4.3 shows the BH profile at $t = 10$ Gyr which results if both stars and compact remnants are formed impulsively at $t = 0$ (initial scale radius of $r_o = 1$ pc) and assuming no subsequent star formation. We show results for a range of models which assume different ratios for the number of stars to compact objects, $N_{bh}/N_\star$. For a Kroupa IMF in which BHs originate from stars of mass
\[ 25M_\odot, \] we expect \( \frac{N_{\text{bh}}}{N_*} \sim 10^{-3} \), which (coincidentally) coincides with the number of BHs injected by the present day in our Fiducial\( \times 10 \) model. However, because the BHs in this case are injected directly at small radii, their density at radii \( \lesssim 1 \) pc in our Fiducial\( \times 10 \) models exceeds the primordial model with \( \frac{N_{\text{bh}}}{N_*} = 10^{-3} \) by a factor of a few.

Fig. 4.4 shows the BH density profile under the assumption that they form continuously at a constant rate over the age of the NSC, with a spatial profile identical to the present-day stellar population. The BH density at small radii \( \lesssim 1 \) pc evolves significantly over time, taking several Gyr to reach a quasi-steady state.

Our previous scenarios assumed the central SMBH possesses a fixed mass, \( 4 \times 10^6 M_\odot \). However, if the NSC is built up by continuous star formation, then the SMBH may grow in concert with the cluster through gaseous accretion or star capture. Fig. 4.5 shows the density profiles of stars and compact objects at \( t = 10 \) Gyr if the SMBH mass is artificially fixed at all times to be 7% of the mass of the NSC. This speeds up the evolution because the velocity dispersion, and hence the cluster relaxation timescale (eq. 4.5) is smaller at early times. Nevertheless, the final distribution of BHs is similar to the previous cases (cf. Fig. 4.4).

Overall, we find that the final distribution of remnants after \( \sim 10 \) Gyr is mostly sensitive to the overall rate of production of BHs versus stars, and is rather insensitive to the details of the star formation history, or its precise radial distribution within the NSC.

### 4.2.6 Effects of strong scattering

So far we have neglected BHs ejecting stars via strong scatterings in our models. In this section we quantify this effect, which under some circumstances can be important for bulk cluster evolution
Figure 4.3: *Top panel:* Density profile of BHs at 10 Gyr for different star formation histories (Table 4.2). Solid lines show non-fiducial models in which the BHs form implausively at $t = 0$ with the same profile as the stars (eq. 4.1), with colors labeling the ratio of BHs to stars. For comparison, dashed lines show our Fiducial and Fiducial×10 models, in which the BHs are instead injected continuously at small radii (see Fig. 4.2 and surrounding discussion). *Bottom panel:* Time evolution of the BH density at $r = 1$ pc for each of the formation histories shown in the top panel.
Figure 4.4: *Top panel*: Density profile of BHs at 10 Gyr under the assumption that the NSC is built up by continuous star formation at a constant rate with a spatial profile identical to the present-day stellar population; colors denote different ratios of BHs to stars, \( N_{\text{bh}}/N_\star \). For comparison, dashed lines show the BH profile in our Fiducial and Fiducial×10 scenarios (Fig. 4.2). *Bottom panel*: Time evolution of the BH density at 1 pc, for each of the formation histories in the top panel.
Figure 4.5: Same as Fig. 4.4, except the central SMBH is fixed to be 7% of the total cluster mass at all times, so that it grows with the cluster.
The volumetric ejection rate at radius \( r \) is

\[
\dot{n}_{ej}(r) = n_{bh}(r)n_*(r) \langle \Sigma(v_\infty)v_\infty \rangle
\]

where \( n_{bh}(r) \) is the number density of BHs, \( n_*(r) \) is the stellar density, \( v_\infty \) is relative velocity at infinity, \( \Sigma(v_\infty) \) is the cross-section for ejection, and the angle brackets denote an average over the relative velocity distribution. This expression may be rewritten as

\[
\dot{n}_{ej} = I(m_c/m_*)n_{bh}(r)n_*(r)\sigma(r)\pi b_o^2
\]

\[
b_o = \frac{G(m_c + m_*)}{\sigma(r)^2}
\]

where \( m_c \) and \( m_* \) are the masses of the compact object and star respectively, \( \sigma(r) \) is the (1D) velocity dispersion of the compact objects. The likelihood of ejection increases with the mass of the compact object, as quantified by the dimensionless number \( I \). In an encounter, the change in the star’s velocity is given by

\[
\Delta v_\parallel = \frac{-2v_\infty}{1 + x^2} \frac{m_c}{m_c + m_*}
\]

\[
\Delta v_\perp = \frac{2v_\infty x}{1 + x^2} \frac{m_c}{m_c + m_*}
\]

\[
x = \frac{b v_\infty^2}{G(m_c + m_*)}
\]
where the first and second lines are the components parallel and perpendicular to the initial relative velocity. The change in the star’s specific energy is

\[ \Delta E = \frac{1}{2} \Delta v^2 + \Delta \mathbf{v} \cdot \mathbf{v}_\star \]  

(4.17)

where \( \mathbf{v}_\star \) is the star’s initial velocity. For a star to be ejected \( \Delta E \) should at least exceed the specific binding energy of the central SMBH, viz.

\[ \Delta E \geq \frac{GM}{2r} = \frac{(1 + \delta)}{2} \sigma(r)^2, \]  

(4.18)

where \( \delta \) is the logarithmic BH density slope. To determine the normalization of the ejection rate, we compute a Monte Carlo ensemble of encounters with different relative velocities, approach angles, and impact parameters. Assuming a Maxwellian velocity distribution for the stars and black holes, a uniform distribution of the cosine of the approach angle, and \( \delta = 1.75 \), the numerical pre-factor \( (I) \) in equation (4.13) is 0.1, 1, and 1.3 for \( m_c/m_\star = 1, 10, \) and 50 respectively (see also Henon 1969).

The total ejection rate may be dominated by stars on eccentric orbits. Considering a thermal eccentricity distribution increases the ejection rate by a factor \( \sim 3.7 \) (relative to purely circular orbits, and assuming that the BH perturbers have an \( r^{-1.75} \) profile). Then, the ejection rate from strong scatterings is

\[ \dot{n}_{ej} \approx 3.7 \pi I(m_c/m_\star)n_\star(r)n_{bh}(r)\sigma(r)^{-3}G^2m_c^2 \]

\[ \approx 4n_\star(r)\ln \Lambda^{-1}\tau_{rx,bh}(r)^{-1}, \]  

(4.19)

where \( \ln \Lambda \approx 15 \) is the Coulomb logarithm and we take \( m_c/m_\star = 10 \). At any radius the time-scale
for a star to be unbound from the central SMBH is approximately four times the local relaxation
time of the BHs. To test how this effect would modify the stellar density, we add an additional
sink term into PHASEFLOW. We find that the stellar density is modified by \( \lesssim 25\% \) (40\%) outside
of 0.01 pc in our Fiducial (Fiducial \( \times 10 \)) models. The total number of stars ejected from the cusp
is \( \sim 2.7 \times 10^6 \) in the Fiducial model and \( 7 \times 10^6 \) in the Fiducial \( \times 10 \) model. These likely represent
upper limits on the uncertainty caused by our neglect of strong scatterings, as Eq. 4.18 represents
a generous ejection criterion.

### 4.3 Tidal Capture Binary Formation

A close encounter between a star of mass \( m_\star \) and a compact object of mass \( m_c \) can lead to the
formation of an XRB through tidal capture. During pericenter passage, tidal forces transfer orbital
energy into stellar oscillations, capturing the star into an elliptical orbit.

The maximum initial pericenter distance which results in tidal capture, \( r_{\text{capt}} \), can be estimated
by equating the hyperbolic orbital energy with the energy deposited in tides (see Appendix D of
Stone et al. 2017b and Appendix F). This condition can be expressed as

\[
\mu \frac{\upsilon_{\infty}^2}{2} = \frac{G m_\star^2}{r_\star} \left( \frac{m_c}{m_\star} \right)^2 \left( \frac{r_\star}{r_{\text{capt}}} \right)^6 T_2(r_{\text{capt}}, m_\star/m_c),
\]

where \( r_\star \) is the stellar radius, \( \mu \) is the reduced mass, and \( T_2 \) is the tidal coupling constant (we
only include the dominant \( l = 2 \) modes; we find that even for an equal mass binary including the
\( l = 3 \) modes only increases the maximum pericenter resulting in tidal capture by 5\%). For distant
pericenters this may be estimated using the linear theory (see Appendix F and Lee & Ostriker
1986). However, for the closest pericenters relevant for capture, linear theory underestimates the
tidal coupling constant by a factor of a few. The magnitude of non-linear effects has been estimated
for polytropic models by Ivanov & Novikov (2001), and we adopt their prescriptions for close pericenters, as discussed in Appendix G.

Fig. 4.6 shows the maximum pericenter distance for tidal capture as a function of the relative velocity at infinity, $v_\infty$, normalized to the stellar escape speed, $v_{\text{esc}} = \sqrt{2Gm_*/r_*}$. The capture radius $r_{\text{capt}}$ is typically $\lesssim 2$ times greater than the characteristic tidal radius $r_t \equiv r_*(m_c/m_*)^{1/3}$. Note that tidal capture cannot occur if $v_\infty \gtrsim v_{\text{esc}}$ and thus is suppressed at small radii $r \lesssim 0.1-1$ pc where the velocity dispersion is large; the same considerations virtually prohibit the tidal capture of giant stars in the GC. For even closer pericenter passages, inside of the so-called disruption radius $r_{\text{dis}} \approx 0.5-1.1r_t$ (depending on stellar structure; Guillochon & Ramirez-Ruiz 2013), stars are tidally disrupted rather than captured.

The combined volumetric rate of tidal captures ($r_{\text{dis}} \leq r_p \leq r_{\text{capt}}$) and disruptions ($r_p \leq r_{\text{dis}}$) at Galactocentric radius $r$ is given by

$$
\Gamma(r,t) = \int_0^{v_{\text{max}}(m_*)} n_c(r,t)n_*(r,t)v_\infty \pi r_o^2
\times \left[ 1 + \frac{2G(m_c + m_*)}{r_o v_\infty^2} \right] f(v_\infty,r,t)dv_\infty,
$$

$$
r_o = \max[r_{\text{capt}}(v_\infty, m_c/m_*)_r, r_{\text{dis}}(m_*)]
$$

where $n_c(r,t)$ is the number density of compact objects, $n_*(r,t)$ is the number density of stars, and $f(v_\infty)$ is the distribution of relative velocities. A hard upper limit to the value of $v_{\text{max}}(m_*)$ is the stellar escape velocity (for faster relative velocities most of star would remain unbound from the compact object in any tidal interaction), but in practice $v_{\text{max}}(m_*)$ is the relative velocity such that $r_{\text{capt}} = r_{\text{dis}}$ in eq. (4.20). This may be smaller than stellar escape speed by a factor of $\sim 2$. We approximate the velocity distribution as a Maxwellian, with a scale parameter equal to the
Figure 4.6: Maximum pericenter distance $r_{\text{capt}}$ (normalized to the BH tidal radius $r_t$) at which a main sequence star can be tidally captured by a BH of mass $10M_\odot$, as a function of the stellar escape speed (normalized to the relative velocity at infinity, $v_\infty$). Results are shown for two stellar masses, $0.3 \, M_\odot$ (solid line) and $1 \, M_\odot$ (dashed line). The former is modeled as an $n = 3/2$ polytrope and the latter is modeled as an $n = 3$ polytrope.
local velocity dispersions of the two species added in quadrature.\(^8\) The term in brackets allows for gravitational focusing, which exceeds the geometric cross section for \(r \gtrsim 0.01\) pc.

Fig. 4.7 shows our calculation of the present-day rate of total stellar tidal disruptions (dashed lines) and tidal captures (solid lines) by BHs and NSs, as calculated using the predictions of our Fiducial model for \(n_*\) and \(n_c\). The capture/disruption rate by BHs exceeds that of NSs by a factor of \(\gtrsim 3 - 10\) across most radii of interest; this is partially because in the limit of gravitationally-focused collisions, the rate of captures/disruptions obeys \(\Gamma \propto m_4^{1/3}\). The rate of tidal captures is somewhat smaller than the rate of disruptions, since for typical relative velocities capture occurs over a narrower range of pericenter distance than does disruption (e.g. for a relative velocity of 100 km s\(^{-1}\) low mass stars would be disrupted for pericenters inside of 1.1 \(r_t\), and would be tidally captured for pericenters between 1.1 and 1.9 \(r_t\)). Some tidal captures may lead to a series of partial disruptions instead of the formation of a stable binary, even if the initial pericenter is outside of the disruption radius. Specifically, significant mass loss from the star is likely to lead to run-away heating that disrupts the star. In this chapter we assume the star is eventually disrupted if it loses more than \(~10\%\) of its mass after its first pericenter passage (see the discussion in § 4.3.2).

Fig. 4.8 shows the tidal capture and disruption rate of stars by BHs as a function of time for different star formation histories corresponding to our Fiducial (§4.2.4) and non-fiducial scenarios (§4.2.5). In the Fiducial scenario, with compact remnant injection inside of \(\approx 0.3\) pc, the encounter rate increases for the first \(\sim 3\) Gyr, as the number of compact objects increases. The rate then declines slightly as the compact objects reach a steady state on small scales, while the population of pre-existing low mass stars are pushed outwards to larger radii. The non-fiducial scenario with impulsive injection of compact remnants and stars shows qualitatively similar behavior, but with the encounter rate peaking much earlier in time. Finally, in the non-fiducial scenario of continuous

\(^8\)In detail the velocity distribution in the Keplerian potential of the SMBH is not Maxwellian, but this is a good approximation for our model stellar density profiles (see e.g. Alexander & Kumar 2001).
Figure 4.7: Present-day cumulative rate of tidal disruptions (dashed lines) and tidal captures (solid lines) inside of radius $r$ for our Fiducial scenario. A total of $\approx 6 \times 10^{-7}$ strong tidal encounters occur per year.
Figure 4.8: Rate of strong tidal encounters (disruptions plus captures) as a function of time in our Fiducial model (§ 4.2.4) as well as two of the non-fiducial scenarios (§ 4.2.5).

star formation, the encounter rate monotonically increases.

### 4.3.1 Tidal Capture and Circularization

As described by Stone et al. (2017b), there are three possible outcomes of a tidal capture: (1) the star continues to lose energy at each pericenter passage, until its orbit is circular; (2) the binary is perturbed by another star or compact object before circularization is complete; (3) the star inflates due to tidal heating, and is destroyed in a series of partial tidal disruptions.
Circularization of the binary can be interrupted (option 2) if the initial pericenter of the encounter is sufficiently large, in which case the tidal energy transfer is weak and the star barely captures into a highly elliptical orbit. In the limit of very large post-capture apocenter, an encounter with another star will perturb the orbital angular momentum faster than circularization can occur. Such encounters generally increase the angular momentum of the binary (since there is more phase volume at larger angular momenta), derailing the circularization process. However, comparing the time-scales for circularization and angular momentum diffusion (Stone et al. 2017b; their eqs. 21, 25), we find that circularization is slower than the outwards angular momentum diffusion time from stellar interactions for only a extremely narrow range of pericenters, within $10^{-3}$ of the maximum value for capture. Only a tiny fraction of tidally captured binaries will be perturbed by a third star before they circularize.

Another hazard for a tidally captured star is a string of partial disruptions due to the energy deposited by tides and tidal stripping near pericenter. Complete destruction of the star is energetically allowed if the energy released during circularization $E_{\text{circ}}$ exceeds the total (internal + gravitational binding) energy of the star $E_\star$. As shown in Fig. 4.9, a star captured by a black hole necessarily has $E_{\text{circ}} \gtrsim E_\star$ (e.g. Kochanek 1992; Alexander & Morris 2003). The energy required for a star to circularize around a NS is smaller than the BH case, but still can be comparable to the energy of a low mass star.

However, even if $E_{\text{circ}} \gtrsim E_\star$, this does not necessarily mean the star will be destroyed. If a significant fraction of the mode energy is deposited near the stellar surface, then it could be radiated away or carried outwards by a wind (Fuller & Lai 2011, 2012; Wu 2017). Whatever remains of the star following this process would then still circularize, albeit with a potentially lower mass and higher entropy than its original state prior to being captured.

The star will lose mass during the circularization process (either due to mode dissipation or
Figure 4.9: Ratio of the required to circularize the star into a binary with a NS or BH ($E_{\text{circ}}$) to the total (internal + gravitational binding) energy of the star ($E_*$) as a function of stellar mass. Although $E_{\text{circ}} \gtrsim E_*$ across much of the parameter space, tidal capture is not necessarily fatal for the star because the circularization energy can be deposited by modes primarily in the outer layers of the star, where it is likely to drive non-destructive mass loss.

direct dynamical stripping at pericenter). The time-scale for mass loss is shorter than the thermal time-scale of the star and its radius will grow adiabatically (Linial & Sari 2017). As the star grows tidal dissipation becomes stronger, potentially leading to run-away heating and disruption of the star (Kochanek 1992). If the mass loss occurs primarily from the side of star closer to the compact object, the pericenter can grow faster than the stellar radius averting the run-away. However, this effect would only become important in nearly equal mass binaries in which the $l=3$ mode enhances (reduces) the displacement on the near (far) side of the star (Manukian et al. 2013).

Another important issue is the time-scale over which the mode energy deposited into the star is
dissipated. Mardling (1995) has argued that a tidally captured star necessarily undergoes a random walk in eccentricity, since mode oscillations from consecutive pericenter passages would interfere with each other, leading to chaotic exchange of energy between the orbit and the star that would likely lead to the latter’s disruption. However, this will not occur if the mode energy is dissipated over the course of a single orbit. This can plausibly occur via non-linear mode-mode couplings (Kumar & Goodman 1996), especially for the large amplitude modes that will be excited by tidal capture in the GC, where the energy deposited into the star on the first pericenter passage is \( \gtrsim 10^{47} \) erg. For example, Kumar & Goodman (1996) show that the \( f \)-modes excited in low mass stars can dissipate energy on a time-scale of 30 \( (E/10^{45} \text{erg}) \) days, where \( E \) is the energy deposited into the star. This would be shorter than the orbital period of a captured low mass star (\( \gtrsim 5 \) days). Higher mass stars can dissipate energy even more efficiently by resonantly exciting g-modes, and nonlinear oscillations may dissipate their energy even faster by steepening into shocks.

The long-term evolution of a highly eccentric tidal capture binary remains an open question, and its solution is beyond the scope of this work. For the remainder of this chapter, we assume that tidal capture binaries are in fact able to circularize without being destroyed, but this assumption must be examined in future modeling.

4.3.2 X-ray Binary Formation and Evolution

Once the star circularizes into an orbit around the compact object, the binary semi-major axis \( a \) will be roughly twice the pericenter radius of the captured star.\(^9\) The orbit will then decay over

\(^9\)The relation \( a = 2r_p \) will be exact if angular momentum and mass are conserved during circularization, except for a correction for stellar spin (Lee & Ostriker 1986).
long timescales due to gravitational wave emission\textsuperscript{10}, such that $a$ decreases according to

$$\frac{\dot{a}}{a} = 2 \frac{\dot{J}_{GW}}{J} = -\frac{64 G^3 m_\star m_c (m_\star + m_c)}{5 c^5 a^4}. \quad (4.22)$$

Here $J$ is the circular orbit angular momentum, and $\dot{J}_{GW}$ is the quadrupole-order rate of angular momentum radiation (Peters 1964). Once the system enters Roche-lobe contact, the subsequent evolution of the semi-major axis and mass accretion rate onto the compact object obey (e.g. Frank et al. 2002)

$$\frac{\dot{m}_\star}{m_\star} = \frac{\dot{J}_{GW}}{J} \left[ 1.2 - \frac{m_\star}{m_c} \right]^{-1}$$

$$\frac{\dot{a}}{a} = 2 \frac{\dot{J}_{GW}}{J} - 2 \frac{\dot{m}_\star}{m_\star} \left( 1 - \frac{m_\star}{m_c} \right), \quad (4.23)$$

where we have assumed the star maintains thermal equilibrium, i.e. that its radius follows the main-sequence, $r_\star \propto m_\star^{0.8}$.

Fig. 4.10 shows the binary lifetime after the star enters Roche-Lobe contact as a function of the masses of the star and compact remnant. The lifetime is defined as the interval over which (1) the star has not yet evolved off the main sequence and (2) the star’s mass still exceeds 0.1$M_\odot$. The last condition is motivated by the fact that once $m_\star \lesssim 0.1M_\odot$ the star’s equation of state changes, resulting in a one to two order of magnitude reduction in the mass-transfer rate (and an undetectably dim X-ray source). Likewise, if the star evolves off the main sequence, the star and compact object may undergo a common envelope phase, with an outcome that is uncertain theoretically.

\textsuperscript{10}In principle spin-down by magnetic braking also contributes to angular momentum losses from the star. However, there is considerable uncertainty in the spin-down rate for high rotation speeds in contact binaries. Empirically, magnetic braking is sub-dominant to gravitational wave emission in BH binaries, as otherwise one predicts a population of bright, persistent short period BH LMXBs that are not observed (see Yungelson et al. 2006; Ivanova & Kalogera 2006). By analogy with cataclysmic variables, magnetic braking is likely also sub-dominant in NS systems with periods $\lesssim 3$ hours.
Fig. 4.10 shows that the binary lifetime decreases for larger compact object masses, due to more rapid evolution through gravitational wave emission. For low-mass stars ($m_\star \lesssim 1M_\odot$), the binary lifetime also increases with $m_\star$ because the tidal radius (and thus the initial separation) is larger for higher mass stars. For massive stars ($m_\star \gtrsim 1M_\odot$), the binary lifetime is instead limited by the main-sequence lifetime.

The present-day ($t = t_h = 10$ Gyr) density of XRBs at radius $r$ is approximately given by

$$n_x(r) = \int_0^{t_h} \int_{r_{\text{min}}(t)}^{r_{\text{max}}(t)} d\Gamma(r_p, r, t) dr_p dt,$$

where

$$d\Gamma(r_p, r, t) = n_c(r, t) n_\star(r, t) \sigma^2 r_p \times \left[ I_0(r_p, \sigma) + \frac{G(m_c + m_\star)}{\sigma^2 r_p} I_1(r_p, \sigma) \right]$$

$$I_0(r_p, \sigma) = \int_0^{v_\infty(r_p)} \sigma \frac{f(v_\infty)}{v_\infty} dv_\infty$$

$$I_1(r_p, \sigma) = \int_0^{v_\infty(r_p)} \sigma \frac{f(v_\infty)}{v_\infty} dv_\infty,$$

is the capture rate per unit pericenter, $v_\infty(r_p)$ is the maximum relative velocity that would result in a capture (the second term in the brackets dominates). The limits of integration in eq. 4.24 are the minimum and maximum initial pericenters for which the binary would be active today.

For close pericenters, the star loses a significant fraction of its mass via direct tidal stripping, leading to the star’s destruction in a series of partial disruptions. Quantitatively, Ivanov & Novikov (2001) find that an $n = 3/2$ (3) polytrope would lose 10% of its mass for a pericenter of 1.5 (1) $r_t$. 

---

11Eq. (4.24) implicitly assumes that binaries are visible as XRB at the radii where they are formed. In reality, binaries radially diffuse over time after forming, an effect we quantify in Fig. 4.12.
Based on these results we also require

\[
 r_p \begin{cases} 
 1.5 r_t, & m_* \leq 0.7 M_\odot \\
 r_t, & m_* > 0.7 M_\odot 
\end{cases}.
\] (4.26)

Modern hydrodynamic simulations (Mainetti et al. 2017) find comparable results with ten percent mass loss at \( r_p/r_t \approx 1.6 \) (\( r_p/r_t \approx 0.95 \)) for \( n=3/2 \) (\( n=3 \)) polytropes. As we use the tidal coupling constants from Ivanov & Novikov (2001), we also use their prescription for stellar mass loss.

To accurately calculate the tidal capture rate at small Galactocentric radii (where the rate becomes zero for stellar velocities equal to the local velocity dispersion \( \sigma \)), we must integrate over the velocity distribution. For a Maxwellian velocity distribution, the integrals over relative velocity in the rate can be evaluated analytically:

\[
 I_0(r_p, \sigma) = \sqrt{\frac{2}{\pi}} \left[ e^{-\frac{v_\infty(r_p)^2}{2\sigma^2}} \left( -\frac{v_\infty(r_p)^2}{\sigma^2} - 2 \right) + 2 \right] \\
 I_1(r_p, \sigma) = \sqrt{\frac{2}{\pi}} \left( 1 - e^{-v_\infty(r_p)^2/2\sigma^2} \right). 
\] (4.27)

Fig. 4.11 shows our calculation of the cumulative tidal capture rate inside radius \( r \), using our Fiducial model for the time-dependent density profiles of BHs, NSs, and stars (Fig. 4.2). We explore the dependence of the capture rate on stellar mass by fixing the number density of the stars, but varying their mass \( m_* \). The per star capture rate is larger for higher mass stars due to their larger tidal radii; however, lower mass stars are more numerous for any realistic mass function and thus dominate the total number of formed binaries.

Fig. 4.12 shows our fiducial model predictions for the present-day total number of accreting BH and NS XRBs interior to a given radius. Dashed lines show the initial radial distribution of the binaries just after forming, while the solid lines show the distribution they would achieve if given
Figure 4.10: Binary lifetime after Roche-Lobe contact is reached as a function of the stellar mass ($m_*$) and compact object mass ($m_c$). The binary lifetime is defined as the interval over which the following criteria are met: (1) the companion mass still exceeds 0.1 $M_\odot$ and (2) the star has not evolved off the main sequence. We use equation 5 from Hurley et al. (2000) for the main sequence lifetime (and assume a solar metallicity star).

sufficient time to relax in the cluster potential. To calculate the latter, we first find the previous time snapshot with a look-back time equal to the mean binary lifetime. Then, we insert a “tracer” population of binaries with the expected initial distribution, and evolve the system forward in time.

Table 4.3 summarizes the predictions of our fiducial models for the number of tidally-captured XRBs in the central parsec of our GC. The average accretion rate for BH (NS) binaries is $10^{-10}$ ($3 \times 10^{-11}$) $M_\odot$ yr$^{-1}$, corresponding to $5 \times 10^{-4}$ ($10^{-3}$) of the Eddington rate $\dot{M}_{\text{Edd}} = L_{\text{Edd}}/0.1c^2$, where $L_{\text{Edd}} = 1.3 \times 10^{38}(m_c/M_\odot)$ erg s$^{-1}$ is the Eddington luminosity. These accretion rates are
Figure 4.11: Present-day binary formation rate from tidal captures of low mass stars by BHs (black lines) and NSs (blue lines) interior to a given Galactocentric radius $r$, as calculated using our Fiducial model for the population of stars and compact remnants in the GC (Fig. 4.2). The thin lines show models in which we have fixed the stellar density but consider a single-mass population of stars with $m_\star = 0.2M_\odot$ or $1M_\odot$. The thick lines show the capture rate assuming a more realistic Kroupa mass function which extends from 0.2 -1 $M_\odot$.

generally less than the theoretical critical threshold value below which the disk is thermally unstable,

$$
\dot{M}_{\text{crit}} \approx 3.2 \times 10^{-11} M_\odot \text{yr}^{-1} \left( \frac{m_c}{M_\odot} \right)^{0.5} \left( \frac{m_\star}{M_\odot} \right)^{-0.2} \\
\times \left( \frac{P}{1 \text{ hour}} \right)^{1.4},
$$

(4.28)

where $m_c$, $m_\star$, and $P$ are the mass of the compact object, mass of the donor star, and period of the orbit, respectively (Dubus et al. 1999). Thus, we expect XRBs formed by tidal capture to be transient sources, with long quiescent periods interspersed with bright outbursts.
Figure 4.12: Cumulative number of tidal capture BH-XRB (top panel) and NS-XRB (bottom panel) predicted inside Galactocentric radius $r$ for our Fiducial and Fiducial×10 scenarios. Dashed lines show the distribution of initially-formed binaries, while solid lines show the final distribution after allowing for dynamical relaxation of the binary population (these are calculated by inserting a tracer population with formed distribution of the binaries into the model snapshot corresponding the mean binary age).
4.4 X-ray observations

Hailey et al. (2018) discovered twelve new non-thermal X-ray sources in the central parsec of our galaxy. Of these, six are solid BH-XRB candidates, while the identity of the remainder is less certain (they may be either additional XRBs or MSPs). In principle, many more sources may be present with luminosities below the Chandra detection threshold of $L_x \approx 4 \times 10^{31}$ erg s$^{-1}$. Indeed, field BH-XRBs are known with luminosities as low as $L_x \approx 2 \times 10^{30}$ erg s$^{-1}$ (Armas Padilla et al. 2014). To estimate the total number of unobserved XRBs lurking in the central parsec, Hailey et al. (2018) first estimate what the flux of the minimum luminosity source from Armas Padilla et al. (2014) would be if it were in the GC (accounting for absorption and instrumental response). Then, extrapolating the observed luminosity function ($N(> F) \propto F^{-\alpha}$, $\alpha = 1.4 \pm 0.1$) to this flux, they conclude that the total number of XRBs could be as high as $300–1000$.

4.4.1 Comparison to Tidal Capture Model

For our fiducial models, we predict a total of $60–200$ BH XRBs, which is comparable to the total number inferred from observations (Table 4.3). These numbers would require the luminosity function to extend a factor of $\sim 3–15$ below the detection threshold. Fig. 4.13 shows that cumulative radial distribution of XRBs from our models at radii $\gtrsim 0.2$ pc also agrees well with the distribution measured by Hailey et al. (2018). (We only consider those binaries outside of 0.2 pc, as observational limits prevent the identification of individual sources inside this radius). Specifically, we predict average XRB surface density profiles $\Sigma \propto r^{-1.4}$ and $\propto r^{-0.9}$ in our Fiducial and Fiducial $\times 10$ models, respectively. These slopes are consistent with the measured surface density profile of high S/N sources, which Hailey et al. (2018) find obey $\Sigma \propto r^{-1.5 \pm 0.3}$ in the radial range $0.2$ pc $\lesssim r \lesssim 1$ pc. There are no strong detections ($> 100$ counts) outside of the central parsec, but there are an additional 40 lower significance detections ($> 50$ counts) between 1 and 3.5 pc (though some of
Table 4.3: Number of tidally-captured BH- and NS-XRBs in the GC predicted for our fiducial scenarios as compared to the observed population. The “Observed” XRBs corresponds to the population detected by Hailey et al. (2018), while the “Extrapolated” sources account for an (uncertain) extrapolation of the X-ray luminosity function below the Chandra detection threshold (see text for details).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>BH-XRB $(r \leq 1 \text{ pc})$</th>
<th>BH-XRB $(r \leq 3.5 \text{ pc})$</th>
<th>NS-XRB $(r \leq 1 \text{ pc})$</th>
<th>NS-XRB $(r \leq 3.5 \text{ pc})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiducial</td>
<td>64</td>
<td>110</td>
<td>29</td>
<td>110</td>
</tr>
<tr>
<td>Fiducialx10</td>
<td>210</td>
<td>640</td>
<td>67</td>
<td>370</td>
</tr>
<tr>
<td>Observed</td>
<td>6–12</td>
<td>$\lesssim 50$</td>
<td>1–3 (LMXB), $\leq 6$ (MSP)</td>
<td>3–6 (LMXBs), $\lesssim 50$ (MSP)</td>
</tr>
<tr>
<td>Extrapolated</td>
<td>300–1000</td>
<td>$\lesssim 200$</td>
<td>$\lesssim 1000$ (MSP)</td>
<td>$\lesssim 1000$ (MSP)</td>
</tr>
</tbody>
</table>

these may be due background contamination).

4.4.2 Neutron Stars

The number of NS-XRBs formed in the central parsec for our Fiducial and Fiducial$\times 10$ models are $N_{\text{ns}} \approx 30$ and $\approx 70$, respectively. These numbers, which are a factor of $\approx 2 – 3$ times lower than the predicted number of tidal capture BH-XRBs in this region, and significantly exceeds the $\leq 3$ NS-XRBs observed thus far.

What might suppress the NS population? First, as in the BH case, not all NS binaries manifest as luminous XRBs. Furthermore, some NS-XRBs may evolve into millisecond pulsars later in their evolution. Only 3% of the known population of MSP have properties which would make them detectable in the GC (Perez et al. 2015); given that up to six of the observed X-ray sources in the central parsec could be MSPs, as many as $\sim 200$ MSPs could exist in this region.

The relative number of tidally captured BHs versus NSs binaries also depends on the fate of massive stars. Although we have assumed that stars of ZAMS mass $\gtrsim 25M_\odot$ become BHs, in reality there is not a single mass separating BH and NS progenitors (Sukhbold et al. 2016), and the fraction of O/B stars that evolve into NSs (as opposed to BHs) may differ between the field and the GC.
The NS population could also be reduced by supernova kicks, which would eject \( \sim 40\% \) of isolated NS formed in the central disk for a Maxwellian kick velocity distribution, with \( \sigma = 265 \text{ km s}^{-1} \) (Hobbs et al. 2005). However, \( \sim 40\% \) (10\%) of the NS binaries in our Fiducial (Fiducial \( \times 10 \)) model come from NSs associated with the old stellar population, calibrated to the Ivanova et al. (2008) model of globular clusters (and this already accounts for supernova kicks). Overall supernova kicks would reduce the number of neutron star binaries by \( \sim 25\% \) (\( \sim 40\% \)) in our Fiducial (Fiducial \( \times 10 \)) model.

Finally, we note the population of NS XRBs is more sensitive to the initial conditions than BH XRBs. The progenitors of NS XRBs in our models typically formed \( \sim 7-8 \text{ Gyr} \) ago (in comparison to \( \sim 4 \text{ Gyr} \) ago for BH XRBs). The cluster expands over time, so our models assume the NSC was initially more compact than it is today. However, a small uncertainty in the present day density translates into a large uncertainty in the initial density. We redid our calculation for the number of NS XRBs holding the stellar density fixed to the present day profile. We find that the number of NS XRBs is reduced by a factor of \( \sim 2 \), while the number of BH XRBs is only reduced by 30\%.

### 4.5 Predictions and implications of our models

In this section we summarize various implications of our models, including properties of binaries and rates of various electromagnetic transients (including tidal disruption events and stellar collisions). We also estimate the formation rate of BH-BH binaries due to bound-free gravitational wave emission. Table 4.4 summarizes the rates of these processes in our GC models.

#### 4.5.1 Properties of binaries

XRBs formed by tidal capture are necessarily short period systems. The binaries in our models have main sequence companions with periods of \( \lesssim 10 \text{ hours} \) (with a median period of \( \sim 3.6 \text{ hours} \)). Any
Figure 4.13: Cumulative number of BH XRBs inside projected Galactocentric radius $r$ from our fiducial models compared with the non-thermal sources identified by Hailey et al. (2018) (black line). We have included the six sources that may be MSPs instead of BH-XRBs in the latter. The dashed blue line shows the distribution of sources scaled up to match the normalization of the Fiducial model. The region inside of 0.2 pc is not included as the population of non-thermal sources is not observationally constrained there.
Figure 4.14: Histogram of companion masses of present-day BH XRBs (*blue*) and companion masses of their progenitors (*red*).

Future periodicity identified in the quiescent population would be a powerful discriminant between tidal capture and other channels (e.g., binary exchange) that can form long period XRBs. We show a histogram of the companion masses of present day BH XRBs in our model in Fig. 4.14.

In the field such short-period XRBs possess low luminosities of $\lesssim 10^{31}$ erg s$^{-1}$ (Armas Padilla et al. 2014) which are below the detection threshold of Hailey et al. (2018) and thus could not be
contributes to the observed population. However, the current sample of short period BH-XRBs is small (only four are known with a period of less than six hours).

### 4.5.2 Tidal disruptions by the central SMBH

Stars may also be tidally disrupted by the central SMBH (Hills 1975). Such tidal disruption events (TDEs) can produce bright electromagnetic flares (Rees 1988). Many candidate flares have now been detected in optical/UV (Gezari et al. 2006, 2008; van Velzen et al. 2011; Gezari et al. 2012; Chornock et al. 2014; Holoien et al. 2014; Arcavi et al. 2014; Vinkó et al. 2015; Holoien et al. 2016a,b; Blagorodnova et al. 2017), and X-ray wavelengths (see Auchettl et al. 2017 and the references therein).

The total TDE rate due to two-body relaxation has been estimated for a large *Hubble Space Telescope* (*HST*) sample of nearby galactic nuclei (Wang & Merritt 2004; Stone & Metzger 2016). These authors find that the average per-galaxy disruption rate is $\sim 1 - 10 \times 10^{-4}$ per year. This range appears discrepant with observationally inferred TDE rate estimates, which are often $\sim 10^{-5}$ galaxy$^{-1}$ yr$^{-1}$ (Donley et al. 2002; van Velzen & Farrar 2014). While recent work has suggested that properly accounting for the broad TDE luminosity function (van Velzen 2017) may bring observational TDE rates into agreement with theory, it is worth considering one limitation of the theoretical estimates: in the smallest galaxies, even *HST* observations underresolve the SMBH influence radius (from which most TDEs are sourced), and moderate inward extrapolation is needed to calibrate theoretical models (Stone & Metzger 2016). The TDE rates predicted by our Fokker-Planck models have been calibrated off scales far smaller than the Sgr A* influence radius, and are thus a useful sanity check on TDE rate calculations in general.

Fig. 4.15 shows the TDE rate for a few different models for the GC. The present-day TDE rate in each is $\sim 10^{-4}$ stars per year ($3 \times 10^{-5} M_\odot$ yr$^{-1}$), similar to previous theoretical estimates
for SMBHs of similar size (Wang & Merritt 2004; Stone & Metzger 2016). Unsurprisingly, the present-day disruption rate is similar for different models as they are all tuned to reproduce the present-day observed stellar density profile. However, different star formation histories lead to very different temporal behavior in TDE rates (see also Aharon et al. 2016). In our models of the GC (Fiducial and Fiducial×10), all of the lower main sequence stars formed impulsively in the distant past, and the star cluster expands over time. Therefore, the TDE rate decreases at late times. In contrast, the TDE rate monotonically increases in a galactic nucleus which is continuously forming stars (see the dashed gray line in Fig. 4.15).

Our current sample of (thermal) TDEs is limited to the low-redshift universe, but LSST and eROSITA are expected to find TDEs out to $z \approx 1$. The rates of high-$z$ tidal disruption that these surveys find will therefore carry information on the growth history of nuclear star clusters (Aharon et al. 2016).\(^\text{12}\)

The SMBH can accumulate a substantial fraction of its mass by disrupting stars and accreting compact objects. After a TDE half of the disrupted star is bound to the SMBH. If the SMBH consumed half of each disrupted star it would grow by $3 \times 10^5$ $(1.4 \times 10^6) M_\odot$ ($\sim 8 - 40\%$ of its present day mass). However, a significant fraction of the initially bound debris may be lost in outflows, so the mass accreted in a TDE may be $\lesssim 10\%$ of the disrupted star’s mass (Metzger & Stone 2016). If the SMBH accretes ten percent of each disrupted star it would grow by $10^5$ $(8 \times 10^5)$ $M_\odot$. For simplicity, we fix the mass of the SMBH to $4 \times 10^6 M_\odot$ in our fiducial models.

### 4.5.3 Tidal disruptions by stellar mass compact objects

Stars that enter the tidal radius of a stellar compact object are also tidally disrupted, powering a transient flare of electromagnetic emission. We calculate the total rate of such “micro-TDEs” in

\(^{12}\text{Although other factors, such as the evolution of the SMBH mass function, will also contribute - see e.g. Kochanek 2016.}\)
Figure 4.15: Rate of tidal disruption by the central SMBH as a function of time for our GC models (Fiducial and Fiducial×10). We also show hypothetical models with continuous star formation (green line), and a single population formed $10^{10}$ years ago (red lines) (see § 4.2.5). The dashed lines show what the disruption rate would be without stellar mass BHs (calculated by excluding the compact objects from the angular momentum diffusion coefficients).
our Fiducial model to be $\sim 3 \times 10^{-7}$ per year (see Fig. 4.7). Thus, the micro-TDE rate in the GC is comparable to the rate from globular clusters, perturbations of wide binaries in the field, and disruptions induced by natal kicks (Perets et al. 2016). Because the resulting flare is short-lived (Perets et al. 2016 estimate the the viscous time-scale of the debris to be less than a day), it is highly unlikely any such disruption events would be observable in our own GC today. However, such events in other galactic nuclei might produce rare short-lived transients detectable at cosmological distances - for example, “ultra long” gamma ray bursts (GRBs; Levan et al. 2014). Taking into account selection effects, the total rate ultra-long GRBs may be comparable to the rate of classic long GRBs: $\sim 10^{-6}$ per galaxy per year (at $z = 0$) after beaming corrections (Guetta et al. 2005). Interestingly, this is comparable to the micro-TDE rate. However, we note that ultra long GRBs can also be explained by the core collapse of massive stars (Greiner et al. 2015).

At very small Galactocentric radii, these micro-TDEs may occur without producing observable accretion flares. This will occur if the relative velocity $v_\infty$ between the star and the compact object is too large for any of the tidal debris to remain bound, i.e. if $v_\infty^2 / 2 > (m_c / m_\star)^{1/3} G m_\star / r_\star$ (Hayasaki et al. in prep). We have excluded such hyperbolic micro-TDEs from our rate estimates.

The small mass ratio between NSs and main sequence stars means that many “micro-TDEs” involving NSs will actually be direct physical collisions, where a Thorne-Zytkow object may be formed (although the stability of such objects remains uncertain).

### 4.5.4 Red giant depletion

As pointed out by Genzel et al. (1996), there is a dearth of bright red giants ($K < 10.5$) within $\sim 0.2$ pc of the GC. There is a similar dearth of intermediate luminosity ($10.5 < K < 12$) giants within $\sim 0.08$ pc. The distribution of fainter stars, on the other hand, is smooth, and has no holes on small scales.
Table 4.4: Present-day rates of various “exotic” collisional stellar interactions in our GC models. From top to bottom: physical collisions between ordinary stars, close encounters between BHs and red giants that would remove a significant fraction of the latter’s envelope \((r_p \lesssim 15R_\odot)\), disruptions of ordinary stars by the central SMBH and by smaller mass remnants, BH-BH binary formation by bound-free gravitational wave emission, ejection of stars from the GC in strong scatterings with BHs.

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Fiducial</th>
<th>Fiducial × 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star-star collisions [yr(^{-1})]</td>
<td>7 × 10(^{-6})</td>
<td>7 × 10(^{-6})</td>
</tr>
<tr>
<td>BH-Red giant collisions [yr(^{-1}) giant(^{-1}) at 0.1 pc]</td>
<td>5 × 10(^{-11})</td>
<td>1.5 × 10(^{-10})</td>
</tr>
<tr>
<td>micro-TDEs (BH) [yr(^{-1})]</td>
<td>3 × 10(^{-7})</td>
<td>10(^{-6})</td>
</tr>
<tr>
<td>micro-TDEs (NS) [yr(^{-1})]</td>
<td>6 × 10(^{-8})</td>
<td>2 × 10(^{-7})</td>
</tr>
<tr>
<td>TDEs (SMBH) [yr(^{-1})]</td>
<td>10(^{-4})</td>
<td>2 × 10(^{-4})</td>
</tr>
<tr>
<td>GW bound-free captures (BH-BH) [yr(^{-1})]</td>
<td>4 × 10(^{-11})</td>
<td>3 × 10(^{-10})</td>
</tr>
<tr>
<td>Ejection of stars by strong scattering [yr(^{-1})]</td>
<td>3 × 10(^{-4})</td>
<td>10(^{-3})</td>
</tr>
</tbody>
</table>

It has been suggested that collisions of red giants with main sequence stars and BHs (Dale et al. 2009, D09 hereafter) could cause the observed holes in the red giant population. D09 find that stripping is only effective in reducing the brightness of giants in the RGB phase (and has little effect on AGB and horizontal branch stars). Furthermore, only close pericenters \((r_p \lesssim 15R_\odot\) for a solar type giant) will remove enough material to significantly alter the evolution of the giant (see also Leigh et al. 2016). They conclude that \(2 \times 10^4\) BHs inside of 0.1 pc are required to explain the observed dearth of intermediate luminosity giants. The gap in the bright giants is harder to explain, as it would require even larger numbers of BHs that would make the gap in the intermediate luminosity giants too large.

In our Fiducial (Fiducial × 10) model the number of BHs inside 0.1 pc is 1200 (3600), much smaller than the number required to explain the depleted giants. The intermediate luminosity giants are \(\sim 2-3\) solar mass stars that spend \(\lesssim 100\) Myr on the giant branch. The time-scale for close encounters only becomes comparable to the giant lifetime inside of \(\sim 0.01\) pc. We conclude that it is difficult to account for the depletion of red giants by collisions with BHs alone. However, there
are many alternative explanations for the dearth of red giants in the literature. For example, red
giants may be destroyed by collision with a clumpy gas disk (Amaro-Seoane & Chen 2014; Kieffer
& Bogdanović 2016).

Ordinary stars may also collide with each other. We calculate the present-day rate of star-star
collisions outside of 0.1 pc to be $7 \times 10^{-6}$ per year.

4.5.5 Two body BH-BH binary formation

Close encounters between BHs can result in the formation of close binaries, either via three-body
interactions or two-body gravitational wave bound-free emission (Antonini & Rasio 2016). GW
capture is generally sub-dominant to three-body processes, but it is one of the few ways to produce
LIGO sources with a non-negligible eccentricity. All else being equal, eccentric sources are louder
and would be detectable to larger distances. Additionally, eccentric sources can sometimes provide
more stringent tests of strong field gravity, as a larger fraction of the energy is emitted when the
source is moving at high velocities (Loutrel et al. 2014). The maximum impact parameter that
results in binary formation is

$$b_{gw} = \left( \frac{340\pi}{3} \right)^{1/7} \frac{Gm_{tot}}{c^2} \eta^{1/7} \left( \frac{v_{\infty}}{c} \right)^{-9/7},$$

$$\eta = \frac{m_1m_2}{(m_1 + m_2)^2},$$

(4.29)

as in equation 17 of O’Leary et al. (2009). The total rate of GW captures in our Fiducial (Fiducial x
10) model is $4 \times 10^{-11}$ yr$^{-1}$ (3 x 10$^{-10}$ yr$^{-1}$; see also Table 4.4), similar to lower end rate estimates
in O’Leary et al. (2009).

An estimate of the total rate of double compact object binary formation, including three-body
processes, is beyond the scope of this paper, and we leave this to future work.
4.6 Summary and Conclusions

Hailey et al. (2018) have recently identified 6-12 quiescent BH-LMXB candidates within one parsec of the Galactic Center, and infer that there may be hundreds of fainter systems in the same region. This means that the GC is three orders of magnitude more efficient than the field at producing BH-XRBs, recalling the analogous massive overproduction of NS-XRBs in a different dense environment (globular clusters). While suggestive, this analogy is incomplete: NS-XRBs are dynamically manufactured in globulars by exchange interactions (e.g. binary-single scatterings), but this mechanism is disfavored in the GC’s high velocity dispersion environment, which only permits the survival of the hardest main sequence binaries.

We instead propose that the observed LMXBs are formed via tidal capture of low mass stars by BHs. We estimated the distribution of stars and compact remnants in the GC using time-dependent Fokker-Planck models that predict close encounter rates. Taken at face value, tidal capture can explain the observed (and extrapolated) inventory of BH-XRBs in the GC. Our primary results are summarized as follows:

1. We calculated the rate at which low mass stars are tidally captured by BHs and NS as a function of time, and used this to predict that there should be \( \sim 60-200 \) accreting BH-XRBs in the central parsec today. The number and radial distribution of these binaries is consistent with the quiescent BH-XRB population identified by Hailey et al. (2018), given reasonable extrapolation below the Chandra detection threshold.

2. Our models also produced a substantial number of NS-XRBs (far more than are currently observed). However, there are several candidate mechanisms for suppressing our predicted NS-XRB population. Alternatively, evolved NS binaries may also manifest as MSPs, whose population is poorly constrained in the GC.
3. The compact object source terms in our Fokker-Planck models were calibrated from the observed number of massive stars in the GC. Most of the stellar mass BHs in the GC may originate in star forming disks with a top heavy IMF, like the one currently observed at $\sim 10^{18}$ cm (Krabbe et al. 1995; Paumard et al. 2006; Bartko et al. 2010; Lu et al. 2013). In our models, *in situ* star formation in these disks has left between $10^4$ and $4 \times 10^4$ BHs within the central parsec, at $z = 0$. Much smaller numbers of BHs would fail to explain the observed BH-XRB population, yielding the first quantitative constraints on the long-theorized “dark cusp” in the GC.

4. We also estimated the rates of other exotic dynamical interactions between stars and compact objects. For example, we found that the rate of disruption of stars by stellar mass BHs (“micro-tidal disruption”) in the Galactic Center is $\sim 10^{-6}$ per year–comparable to previous estimates of the total rate in the field and globular clusters (Perets et al. 2016), as well as the rate of ultra-long GRBs (Levan et al. 2014). The present-day TDE rate from Sgr A* is $\sim 1 - 3 \times 10^{-4}$ yr$^{-1}$, similar to other SMBHs of its mass.

The largest theoretical uncertainty in our model is the assumption that main sequence stars tidally captured by stellar mass BHs are able to circularize and settle into stable Roche-lobe overflow. Such an outcome is not energetically guaranteed, and it is likely that BHs above a certain mass will rapidly destroy tidally captured stars by thermalizing too much mode energy inside them, leading to super-Eddington accretion in a string of partial tidal disruptions. The precise BH mass threshold above which tidal capture becomes catastrophic is an open question that we hope to address in future work. A second concern is that tidal capture binaries have periods $\lesssim 10$ hours. In the field, such systems have low X-ray luminosities, and, if placed in the GC, would fall below the Chandra detection threshold. However, short period field XRBs likely have a different formation mechanism,
and it is not clear if tidal capture XRBs would inherit their luminosity function.

While we have focused on tidal capture in an isotropized population of stars and compact objects, it may be fruitful to examine high mass XRB formation within star-forming disks. Tidal capture rates could be enhanced by an order of magnitude by the larger stellar densities and smaller relative velocities within the disk. By itself, this enhancement is insufficient to overcome the small number and short lifetimes of high mass stars within the disk, so that the expected number HMXBs from tidal capture alone is less than unity. However, it is also possible that stellar mass objects migrating within a gaseous disk may smoothly capture into binaries due to gas dissipation alone, even in the absence of strong tidal coupling. In any case there are no HMXBs present in the GC today (Hailey et al. 2018).

Our tidal capture model was motivated by surprising discoveries in the MW Center, but it carries major implications for extragalactic NSCs as well. If the GC’s inventory of XRBs is representative, it may complicate X-ray searches for low-luminosity intermediate mass black hole AGN in dwarf galaxies. The unresolved, integrated X-ray luminosity from a large XRB population represents a durable if dim contaminant; a single BH-XRB in outburst would represent a more dangerous contaminant for single-epoch searches. The existence of dark cusps in galactic nuclei also carries major implications for the highly uncertain rates of extreme mass ratio inspirals, one of the primary scientific targets for future space-based GW laser interferometers (e.g. eLISA). Future dynamical modeling of XRB formation in the GC may yield more sophisticated constraints on the radial profile of our dark cusp, a local laboratory with which we may calibrate our expectations for stellar dynamics in distant galactic nuclei.
Chapter 5

Conclusions

5.1 Summary of results

This thesis has explored a few different aspects of gas and stellar dynamics in the immediate vicinity of SMBHs. Chapters 2 and 3 developed a model for the gas density $\sim$0.1-10 parsecs from an SMBH. Furthermore, they explore the implications for SMBH growth and astrophysical transients like TDEs.

In Chapter 2 (based on Generozov et al. 2015), we solved for the steady-state gas density profile around an SMBH, assuming gas is supplied by the winds from the surrounding stellar population. The density strongly depends on the heating rate, which is determined by the kinetic energy of the stellar winds as well as Supernova and AGN feedback. In our steady-state, spherically symmetric model, gas inside of the stagnation radius, $r_s \sim GM_\bullet/v_w^2$, will flow towards the SMBH (where $M_\bullet$ and $v_w^2$ are the SMBH mass and heating rate per unit mass respectively). The mass inflow rate is equal to the rate of stellar wind mass loss rate inside of the stagnation radius. Appendix B provides a prescription for the latter for different star formation histories. Thus, the gas density can be estimated analytically for any stellar population.
We also explored the development of cooling instabilities within the flow. The flow will be thermally unstable if the heating time is longer than the cooling time, and if the the cooling time is longer than the dynamical time. In practice, for a young stellar population the kinetic energy of the fast stellar winds ($\sim 1000 \text{ km s}^{-1}$) provides sufficient energy to thermally stabilize the gas. For stellar populations older than $4 \times 10^7$ years, the gas will be susceptible to thermal instability as the mass budget is dominated by slow AGB winds. Heating at these later epochs is dominated by Ia supernovae and SMBH feedback (in particular Compton heating). An important result from Chapter 2 is that any flow that would result in significant SMBH growth is necessarily thermally unstable.

In Chapter 3, we applied the above framework to constraining jets and other outflows in observed tidal disruption events (TDEs). There are a handful of TDE candidates that were detected by the Swift satellite. These events also have observed radio after-glow, consistent with a jet interacting circumnuclear gas. However, the majority of TDEs do not have observed radio emission. As discussed in Chapter 3, this is unlikely to be due to viewing angle effects as the radio emission would be isotropic at times radio follow-up of TDE candidates is performed. We find that for gas densities expected in TDE hosts, jets with energies $\gtrsim 10^{53}$ erg (as seen in the Swift events) can be ruled out. This result suggests the conditions necessary to launch such powerful jets are rarely met in TDEs.

The high stellar densities of galactic nuclei result in high rates of close encounters between stars and compact objects and associated electromagnetic transients. In particular Hailey et al. (2018) recently discovered a population of several hundred X-ray binaries within the central parsec of the galaxy. In Chapter 4 (based on Generozov et al. 2018) we use observationally calibrated models for the stellar population in the Galactic Center to calculate empirically calibrated rates of X-ray binary formation and other other astrophysical transients. We have shown that tidal capture
of low mass stars by stellar mass BHs can account for the recently discovered population of X-ray binaries in the central parsec (Hailey et al. 2018).

5.2 Future directions

Chapter 2 neglects the angular momentum of the gas. This necessarily becomes important on small radial scales. At small accretion rates, the presence of angular momentum will result in bi-conical outflows (Li et al. 2013; Roberts et al. 2017). We will carry our multi-dimensional hydrodynamic simulations to account for this effect.

An important question in Chapter 4 is whether stars can survive the tidal capture process intact. As shown in Chapter 4, the total energy deposited into a sub-solar mass star during a tidal capture by a BH is comparable to or greater than the star’s binding energy. Thus, the star can be destroyed during the tidal capture process. However, if the majority of the energy is deposited in the star’s outer layers the bulk of the star would survive. In this case the outer layers would be lost in a high velocity outflow, but the remainder of the star would remain intact. We argued this scenario was plausible based on the spectrum of modes excited within the star. We will study this question in more detail using simulations that capture non-linear dissipation and mode-mode coupling effects within the star (Kumar & Goodman 1996; Weinberg et al. 2013).

Finally, we can use the our empirically calibrated models for the formation of Galactic Center (Chapter 4) to calculate rates of LIGO binary formation and EMRI formation. In addition, we will explore whether a significant number of hypervelocity stars can be produced in the Galactic Center via scatterings with stellar mass BHs (in particular we will compare to the rate to that expected from other channels like the disruption of binary stars by the central SMBH; e.g. Hills 1988).
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Appendices
Appendix A

Derivation of stagnation radius

Here we derive an analytic expression for the time-independent stagnation radius, \( r_s \). In steady-state, the requirement that the net heating rate on the right hand side of the entropy equation (eq. [2.7]) must equal zero at the stagnation radius (where \( v = 0 \)) fixes the temperature at \( r_s \):

\[
\frac{\gamma}{\gamma - 1} \frac{p}{\rho} \bigg|_{r_s} = \frac{\bar{v}_w^2}{2} \Rightarrow \frac{k_b T}{\mu m_p} = \gamma - 1 \frac{\bar{v}_w^2}{2} \frac{\gamma}{\gamma - 1} \frac{p}{\rho} \bigg|_{r_s} \tag{A.1}
\]

Combining (eq. [2.7]) with the first law of thermodynamics,

\[
T \frac{ds}{dr} \bigg|_{r_s} = \frac{1}{\gamma - 1} \frac{d(p/\rho)}{dr} \bigg|_{r_s} + \frac{\nu}{\rho} \frac{p}{\rho} \bigg|_{r_s} = \frac{q}{\rho \nu} \frac{\bar{v}_w^2}{2} - \frac{\gamma - 1}{\gamma - 1} \frac{p}{\rho} \bigg|_{r_s} \tag{A.2}
\]

where \( \nu \equiv -d\ln \rho/d\ln r \bigg|_{r_s} \). The right side can be evaluated using L’Hopital’s rule, yielding
\[
\lim_{r \to r_s} A = \frac{d}{dr} \left[ \left( \frac{v^2}{2} - \frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \frac{\tilde{v}_w^2}{2} \right) \right]_{r_s} \\
\approx -\frac{\gamma}{\gamma - 1} \left. \frac{d(p/\rho)}{dr} \right|_{r_s} - \frac{3}{2(2+\Gamma)} \frac{GM_{\bullet}}{r_s^2},
\]

where we have used the definition \( \tilde{v}_w^2 = v_w^2 + \frac{3}{2+\Gamma} \frac{GM_{\bullet}}{r} + \sigma_0^2 \) (eq. [2.9]). Substituting this expression back into equation (A.2) and using equation (A.1) gives

\[
\frac{\gamma + 1}{\gamma - 1} \left. \frac{d(p/\rho)}{dr} \right|_{r_s} + \frac{\gamma - 1}{\gamma} \frac{\tilde{v}_w^2}{r_s^2} \left( \frac{d(p/\rho)}{dr} \right)_{r_s} = -\frac{GM_{\bullet}|_{r_s}}{2r_s^2} - \frac{3}{2 + \Gamma} \frac{GM_{\bullet}}{2r_s^2}.
\]

Combining this with condition of hydrostatic equilibrium (eq. [2.6]) at the stagnation point,

\[
\frac{1}{\rho} \left. \frac{dp}{dr} \right|_{r_s} = -\frac{GM_{\text{enc}}}{r_s^2} \Rightarrow \left. \frac{d(p/\rho)}{dr} \right|_{r_s} = -\frac{GM_{\text{enc}}}{r_s^2} - \frac{p}{\rho} \left. \frac{d\ln(\rho)}{dr} \right|_{r_s} = -\frac{GM_{\text{enc}}}{r_s^2}.
\]

results in the following expression for the stagnation radius:

\[
r_s = \frac{GM_{\bullet}}{\nu \tilde{v}_w^2} \left( 4 \frac{M_{\bullet}|_{r_s}}{M_{\bullet}} + \frac{13 + 8\Gamma}{4 + 2\Gamma} \right) \\
= \frac{GM_{\bullet}}{\nu(v_w^2 + \sigma_0^2)} \left[ 4 \frac{M_{\bullet}|_{r_s}}{M_{\bullet}} + \frac{13 + 8\Gamma}{4 + 2\Gamma} - \frac{3\nu}{2 + \Gamma} \right],
\]

where we have assumed \( \gamma = 5/3 \). This relationship can be reparameterized as

\[
\frac{r_s}{r_{\text{inf}}} = \frac{1}{3\nu \zeta^2} \left[ 4 \frac{M_{\bullet}|_{r_s}}{M_{\bullet}} + \frac{13 + 8\Gamma}{4 + 2\Gamma} - \frac{3\nu}{2 + \Gamma} \right],
\]

where \( r_{\text{inf}} = 3GM_{\bullet}/\sigma_0^2 \) and we have defined \( \zeta \equiv \sqrt{1 + (v_w/\sigma_0)^2} \). Because \( M_{\bullet}|_{r_s} = M_{\bullet}(r_s/r_{\text{inf}})^{2-\Gamma} \), in general equation (A.7) must be solved implicitly for \( r_s/r_{\text{inf}} \). However, when \( M_{\bullet}|_{r_s} \ll M_{\bullet} \) or
equivalently $r_s \ll r_{\text{inf}}$, equation (A.7) simplifies considerably:

\[
    r_s = \frac{13 + 8\Gamma GM_{\odot}}{4 + 2\Gamma \nu \tilde{v}_{\text{w}}^2}
\]  

(A.8)
Appendix B

Analytic model for dependence of wind heating on stellar age

B.1 Single Burst

In the case of a single impulsive burst of star formation, the mass and energy injection rate per unit stellar mass at time $t$ after the burst are given, respectively, by

$$\dot{m}(t) = f_{\text{TO}} \dot{m}_{\text{TO}} + f_{\text{OB}} \dot{m}_{\text{OB}} \quad \text{(B.1)}$$

$$\dot{e}(t) = f_{\text{OB}} \dot{e}_{\text{OB}}(t) + 0.5 f_{\text{MS}} \int_{m_0}^{m_{\text{MS}}(t)} \frac{\bar{v}^2 w(M_*, t) \dot{m}(M_*, t) \mu |M_*|}{\bar{m}_*} \, dM_* \quad \text{(B.2)}$$

where the $f$'s represent the efficiency with which each source of mass/energy injection is thermalized (we take all of the $f$'s to be 1). In the top line, the first term corresponds to mass injection due winds from post-main sequence stars. The second term corresponds to mass injection due to stellar winds from massive stars. For the latter we use population synthesis calculations by Voss et al. (2009). From the bottom panel of their Figure 7, the stellar wind mass loss rate per...
massive star can be approximated to within a factor of a few by

\[
\dot{M} = \begin{cases} 
10^{-6.5} M_\odot \text{yr}^{-1} & t < 4 \text{ Myr} \\
10^{-5.4} M_\odot \text{yr}^{-1} \left( \frac{t}{4 \times 10^6 \text{yr}} \right)^{-3} & 4 \text{ Myr} \leq t \leq 40 \text{ Myr.} \\
0 & t > 40 \text{ Myr.}
\end{cases}
\]  

(B.3)

\[ \dot{m}_\text{OB} = f_8 \dot{M} / \bar{m}_\odot, \]
where \( f_8 = 2.6 \times 10^{-3} \) is the fraction of the stellar mass with \( M_* > 8 M_\odot \)
and \( \bar{m}_\odot = 0.35 M_\odot \) is the mean stellar mass for our assumed Salpeter IMF, \( \mu \sim M_*^{-2.35} \).

For the mass loss rate from post-main sequence winds, we take

\[
\dot{m}_\text{TO} = \frac{\Delta M(t) |\dot{M}_\text{TO}(t)|/\mu |\dot{M}_\text{TO}(t)|}{\bar{m}_\odot} \quad t \geq 40 \text{ Myr,}
\]

and 0 for earlier times. By truncating \( \dot{m}_\text{TO} \) at 40 Myr, we are ignoring the non-steady mass injection by core collapse supernovae.

The quantity of mass lost in post-main sequence winds \( \Delta M(t) \) is estimated from the expression given by Ciotti & Ostriker (2007) (their eq. [10]),

\[
\Delta M = \begin{cases} 
0.945 M_\text{TO} - 0.503 & M_\text{TO} < 9 M_\odot \\
M_\text{TO} - 1.4 M_\odot & M_\text{TO} \geq 9 M_\odot,
\end{cases}
\]  

(B.5)

where \( M_\text{TO} \) is the turn-off mass, which at time \( t < t_h \) is calculated as

\[
\log(M_\text{TO})/M_\odot = 0.24 + 0.068 x^2 - 0.34 x + 4.76 \times 10^{-6} e^{-4.58 x},
\]

(B.6)

where \( x = \log(t/10^9 \text{yr}) \). This functional fit is designed to reproduce the results of Maeder & Meynet (1987) (their Table 9) for massive stars while asymptoting to the formula provided by
Ciotti & Ostriker (2007) (their eq. [9]) for intermediate and late times \( t \gtrsim 10^8 \) years. This fit is valid up to \( t \sim 10^{10} \) years.

The first term in equation (B.2) corresponds energy injection from massive stars, while the second term accounts for energy injection from stars on the lower main sequence. Both terms have a thermalization efficiency, \( f \), which we take to be 1. Note we do not account for energy from core collapse supernovae.

The MS wind mass loss rate \( \dot{m}(M_\star, t) \) is calculated based on the generalization of Reimer’s law

\[
\dot{m} = 4 \times 10^{-13} \frac{L_* R_*}{M_*} M_\odot \text{yr}^{-1},
\]

where \( R_* \), \( L_* \), \( T_{\text{eff}} \) and \( g_* \) are the stellar radius, luminosity, effective temperature, and surface gravity, respectively, the latter normalized to its solar value \( g_\odot \). The stellar radius and luminosity are estimated as (Kippenhahn & Weigert (1990); Figs. 22.2) 22.3)

\[
L_* = \begin{cases} 
L_\odot (M_\star / M_\odot)^{3.2} & M_\star > M_\odot \\
L_\odot (M_\star / M_\odot)^{2.5} & M_\star \leq M_\odot 
\end{cases}
\]

\[
R_* = \begin{cases} 
R_\odot (M_\star / M_\odot)^{0.57} & M_\star > M_\odot \\
R_\odot (M_\star / M_\odot)^{0.8} & M_\star \leq M_\odot 
\end{cases}
\]

The wind velocity of main sequence winds is assumed to equal \( v_w(M_\star, t) = v_{w,\odot} (M_\star / M_\odot)^{1/2} (R_\star / R_\odot)^{-1/2} \), i.e. scaling as the stellar escape velocity and normalized to the velocity of the solar wind \( v_{w,\odot} = 430 \) km \( s^{-1} \); this produces an effective wind heating velocity for main sequence winds alone of \( \sim 100 \)
km s$^{-1}$ for $\tau_\star \sim t_h$, close to the value found by Naiman et al. (2013) for globular clusters based on a more sophisticated population synthesis treatment.

Winds from lower main sequence stars only dominate the energy injection a late times 10 Gyr after an impulsive burst of star formation. Even with 100% thermalization efficiency these winds could not thermally stabilize the CNM.

The rate of energy injection due to winds from massive stars, $\dot{e}_{\text{OB}}(t) = f_8 \dot{E} / \dot{m}_\star$, where $f_8 = 2.6 \times 10^{-3}$ is the fraction of the stellar mass with $M_\star > 8 M_\odot$ for our assumed Salpeter IMF. Here $\dot{E}(t)$ is the energy injection rate per massive star, which we estimate as

$$\dot{E}(t) = 1.3 \times 10^{36} \text{erg s}^{-1} \begin{cases} 1 & t < 4 \times 10^6 \text{yr} \\ \left( \frac{t}{4 \times 10^6 \text{yr}} \right)^{-3.73} & t \geq 4 \times 10^6 \text{yr}, \end{cases}$$

(B.10)

based on the results of Voss et al. (2009) (their Fig. 7, top panel), who use a population synthesis code to simulate the mass and energy injection into the ISM from an OB association. Although equation is valid only for $t \lesssim 10 \text{ Myr}$, in practice the precise functional form of $\dot{e}_{\text{OB}}(t)$ is generally unimportant for our purposes, so we adopt equation as being valid for all times.

The effective wind velocity in the limit of an impulsive star formation may then be written as

$$\bar{v}_w(t) = 2 \dot{e}(t) / \dot{m}(t)$$

(B.11)

while

$$\eta = \dot{m}(t) t_h$$

(B.12)

Figure B.1 shows the values of $\bar{v}_w(t)$ and $\eta(t)$ as a function of stellar age, $\tau_\star$.  

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Figure B.1: Effective heating rate, $v^*_w$, and mass loss parameter, $\eta$ (eq. [2.8]), resulting from stellar winds from a stellar population of age $\tau_*$. The dashed lines shows the effective $\eta$, accounting for (non-steady) mass-injection from core collapse supernovae.
B.2 Continuous star formation

Generalizing to an arbitrary SFH $S(t)$, the total rate of mass and energy input can be written as

$$\dot{M}(t) = \int_0^t S(t_1)\dot{\dot{m}}(t - t_1)dt_1$$

(B.13)

$$\dot{E}(t) = \int_0^t S(t_1)\dot{\dot{e}}(t - t_1)dt_1,$$

(B.14)

resulting in a wind heating parameter of

$$v_w^2(t) = 2\dot{E}(t)/\dot{M}(t).$$

(B.15)

The mass return parameter will be

$$\eta = \frac{\dot{M}(t)}{\int_0^t S(t_1)dt_1}.$$

(B.16)

We estimate the stellar wind heating provided by the average star formation history of galaxies of a given $M_\bullet$ using the results of Moster et al. (2013, eqs. 17-20). Note that the star formation histories in Moster et al. (2013) are in terms of halo mass. For a given $M_\bullet$ we assign the halo mass whose star formation history would produce a bulge consistent with the $M_\bullet - M_{\text{bulge}}$ relationship from McConnell & Ma (2013). A slight complication occurs for the largest mass halos, where much of the $z = 0$ stellar mass has been acquired through accretion of satellite halos rather than in situ star formation. To accommodate this, we incorporate analytic fits for mass accretion histories, taken from Moster et al. (2013, their eqs. 21-23), assuming that the age distribution of the accreted stars is equal to the age distribution of those formed in situ. This assumption may be conservative if the primary galaxy’s accretion history is dominated by minor mergers with younger stellar populations.
Figure B.2: Effective wind velocities for nonstandard star formation histories. The black curves shows, for reference, $v_w$ calculated using the halo-averaged $S(t)$, and gray curves show wind heating resulting from perturbed star formation histories given by equation (B.24). In the top panel the star formation rate declines for a time, $\delta t_* = 10^7$ years to fractions $\iota = 0.001$ (dashed), $\iota = 0.1$ (dotted), and $\iota = 0.3$ (dot-dashed) of the halo averaged value. The bottom panel shows results for $\delta t_* = 10^8$ years and $\iota = 0.001$ (dashed) and 0.1 (dotted).
On the other hand, the dynamical friction inspiral time for small satellite galaxies is quite long, generally much greater than the $\sim 10^7$ yr for which young stars can dominate the heating budget. The mass of stars accreted for halo masses, $M_{\text{halo}} < 3 \times 10^{12} M_\odot$, and redshifts, $z > 4$, is small and is neglected.

To find the total mass ($\dot{M}'(t)$) and energy ($\dot{E}'(t)$) injection rates, including the contribution of accreted stars, we add a convolution of the specific mass and injection rates with the accretion history $A(t)$ to the mass and energy injection rates from star formed in-situ. Thus,

$$
\dot{M}'(t) = \dot{M}(t) + \int_0^t A(t_{\text{acc}}) \frac{\dot{M}(t_{\text{acc}})}{\int_{t_{\text{acc}}}^{t_{1}} S(t_1) dt_1} dt_{\text{acc}} \tag{B.17}
$$

$$
\dot{E}'(t) = \dot{E}(t) + \int_0^t A(t_{\text{acc}}) \frac{\dot{E}(t_{\text{acc}})}{\int_{t_{\text{acc}}}^{t_{\text{1}}} S(t') dt_1} dt_{\text{acc}} \tag{B.18}
$$

The corresponding wind heating parameter $v'_{\text{w}}$ and mass return parameter, $\eta'$, will be given by

$$
\eta' = \frac{\dot{M}'(t)}{\int_0^t S(t_1) dt_1 + \int_0^t A(t_1) dt_1} t_h \tag{B.19}
$$

$$
v'_{\text{w}} = 2 \frac{\dot{E}'(t)}{\dot{M}'(t)} \tag{B.20}
$$

Figure B.2 shows how the wind heating varies as star formation histories deviate from their
In these equations,

\[ S(t) = S(t) \times \left( \frac{2}{\pi} (1 - \iota) \arctan(t/\delta t_* + \iota) \right) \]  

(B.24)

This function convolves the recent \((z \approx 0)\) halo-averaged star formation history with local variation to give a more pessimistic estimate for the value of \(\tilde{v}_w\). In particular, replacing \(S(t)\) with \(S(t)\) reduces the recent star formation rate to a fraction \(\epsilon\) of its halo-averaged value, and does so for a characteristic time \(\delta t_*\) into the past. As we can see in Fig. B.2, this dramatically lowers the effective wind speed when both \(\delta t_* \gtrsim 10^7\) yr and \(\epsilon \lesssim 0.1\), but otherwise has too modest of an effect to change the thermal stability properties of the flow (although the location of \(r_s\) and the value of \(\dot{M}\) may change significantly).
Fig. 3.8 compares the results of radio light curves from jets propagating in core and cusp like gas density profiles (Fig. 2.2). We use the following analytic expression to approximate the core galaxy CNM profile in Fig. 2.2

\[
\begin{aligned}
  n &= n(r_s)k(x) && 0.4 \leq x \leq 2.0 \\
  n &= 2.0n(r_s)(x/0.4)^{-0.95} && x < 0.4 \\
  n &= 0.75n(r_s)(x/2.0)^{-0.26} && x > 2,
\end{aligned}
\]

where

\[
x = r/r_s
\]

\[
k(x) = \frac{45}{19/x^{3/2}} \left( \frac{1}{9 - 19x^{2.99-1}/x^{1.9-1}} \right)
\]

To isolate the effects of the shape of the density profile, we consider a core density profile with a stagnation radius \(r_s = 10^{18}\) cm and density normalization \(n_{18} = 2000\) cm\(^{-3}\) which match those of
our high density cusp model.
Appendix D

Peak Luminosities and times

Leventis et al. (2012) present analytic scaling relations for the synchrotron flux of a spherical blast wave propagating through a medium with a power law density profile, \( n \propto r^{-k} \). Here we make use of their results to estimate the peak radio flux of the slow (sheath) component of the jet.

During the late-time, Newtonian stage of the jet evolution, synchrotron self absorption is important for frequencies below

\[
\nu_{sa} = C_1(p,k) E_{54}^{10-4k-p-6k} n_{18}^{30-5p} \epsilon_e^{(p-1)} \epsilon_b^{p+2} t^{10-8k-15p+4kp} (4+k(5-k))^{54-n} .
\]

\text{(D.1)}

where \( E = 10^{54} E_{54} \) erg is the blast wave energy and \( C_1(p,k) \) is a normalization factor. Equation (D.1) is valid only if self-absorption frequency is greater than the synchrotron peak frequency,

\[
\nu_m = C_2(p,k) E_{54}^{10-k} n_{18}^{5} \epsilon_e^{1/2} \epsilon_b^{4k-15} t^{4k-15} .
\]

\text{(D.2)}

The light curve will peak at the deceleration time (eq. 3.4) in case the emitting region is optically
thin then. Otherwise, it will occur after the deceleration time, when the self-absorption frequency crosses through the observing band. The peak time for these two cases is

\[
    t_p \approx \begin{cases}
        0.5 \left(50(3 - k) E_{54}\right)^{1/(3-k)} \\
        \times \Gamma(2k-8)/(3-k) n_{18}^{-1/(3-k)} \text{ yr} & \text{Opt. Thin} \\
        C_1(p, k) \frac{(5-k)(4+p)}{10-8k-15p+4kp} E_{54}^{7/(3-k)} \\
        \times \frac{30-5p}{2(4kp-8k-15p+10)} n_{18}^{30-5p/(2(5k-8k-15p+10))} \\
        \times \epsilon_b^{-(5-k)(p+2)/(4kp-8k-15p+10)} \epsilon_e^{2(5-k)(p-1)/(4kp-8k-15p+10)} & \text{Opt. Thick, (D.3)}
    \end{cases}
\]

where \( \Gamma \) is the initial jet Lorentz factor.

The unabsorbed flux at the peak frequency is given by

\[
    F_{\nu_m} = C_3(p, k) E_{54}^{8-3k/(248-k^2)} n_{18}^{7/(248-k^2)} \epsilon_b^{1/2} \epsilon_e^{2-2k/(3-k)} 
\]

Extrapolating to the observer frequency gives

\[
    \nu_{\text{obs}} F_p(\nu_{\text{obs}}) = \nu_{\text{obs}} F_{\nu_m} \left( \frac{\nu_{\text{obs}}}{\nu_m} \right)^{-1/2}. \quad (D.5)
\]
Combining equations (D.2), (D.3), (D.4), and (D.5), we find

\[ \nu_{\text{obs}} F(p(\nu_{\text{obs}})) \propto \begin{cases} 
E_{54}^{\frac{k(p+5)-12}{4(k-3)}} n_{18}^{\frac{3(p-1)}{4(k-3)}} \nu_{\text{obs}}^{\frac{3-p}{2}} \epsilon_b \epsilon_e^{p-1} \quad & \text{Opt. Thin} \\
E_{54}^{\frac{k(-p-2)-10p+3}{4k(p-2)-15p+10 \nu_{\text{obs}}}} \times n_{18}^{\frac{11(p-2)}{4k(p-2)-15p+10}} \nu_{\text{obs}}^{\frac{14k(p-2)-47p+57}{4k(p-2)-15p+10}} \\
\times \epsilon_b^{\frac{k(-p-2)+p-8}{4k(p-2)-15p+10}} \epsilon_e^{-\frac{11(p-1)}{4k(p-2)-15p+10}} \quad & \text{Opt. Thick}
\end{cases} \quad (D.6)

After peak, we expect that the flux scales as

\[ F_\nu \propto t^{\frac{21-8k-15p+4kp}{10-2k}}. \quad (D.7) \]
Appendix E

Reverse shock

Here we estimate the fraction of the kinetic energy of the jet that is dissipated by the reverse shock, as opposed to the forward shock whose contribution is the focus of this chapter. From continuity, the comoving density of a relativistic jet is given by (e.g. Uhm & Beloborodov 2007)

\[ n_j = \frac{L_{j,\text{iso}}}{4\pi r^2 \Gamma_j^2 c^3 m_p (1 + r\dot{\Gamma}/c \Gamma^3)} \approx \frac{L_{j,\text{iso}}}{4\pi r^2 \Gamma^2 c^3 m_p}, \]  

(E.1)

where \(L_{j,\text{iso}}\) is the isotropic equivalent luminosity. The second term in the denominator can be neglected if the jet Lorentz factor changes slowly (\(\dot{\Gamma}_j \ll c \Gamma^3/r\)), a condition which is satisfied at radii \(r < r_{\text{dec}}\) if \(\Gamma\) changes slowly on a timescale \(\gtrsim t_0\), where \(t_0\) is the jet duration.

The common Lorentz factor of the shocked CNM and the shocked jet can be estimated using the relativistic shock jump condition and pressure equality between the forward and reverse shocks. In the ultra-relativistic limit this gives,

\[ \Gamma_{\text{sh}} \bigg|_{\Gamma_{\text{sh}} \gg 1} = \Gamma \left[ 1 + 2\Gamma f^{-1/2} \right]^{-1/2}, \]  

(E.2)
where

\[ f \approx 40 L_{\text{jet}} n_{18}^{-1} \Gamma_{10}^{-2} \left( \frac{r}{10^{18}\text{cm}} \right)^{-1} \quad (E.3) \]

is the ratio of the density of the jet to that of the CNM. Equation (E.2) is inaccurate for mildly relativistic or non-relativistic flows, in which case we apply the more general expression for \( \Gamma_{\text{sh}} \) given by Beloborodov & Uhm (2006) (their eq. 3, see also Mimica & Aloy 2010)

\[ \frac{\Gamma_{\text{sh}}^2 - 1}{\Gamma_{43}^2 - 1} f^{-1} = 1, \quad (E.4) \]

where

\[ \Gamma_{43} = \Gamma_{\text{sh}} (1 - \beta_{\text{sh}} \beta_j), \quad (E.5) \]

is the Lorentz of shocked jet in the frame of the unshocked jet. Combining equations (E.4) and (E.5), we obtain

\[ \Gamma_{\text{sh}}(f) = \sqrt{\frac{f (\Gamma^2 f - 3) - 2 (\Gamma^2 - 1) \Gamma \sqrt{f} + 1}{(f + 1)^2 - 4 \Gamma^2 f}} \]

\[ \Gamma_{43}(f) = \sqrt{\frac{4 \Gamma f^{3/2} + f^2 + \Gamma^4 f + 4 \Gamma^3 \sqrt{f} + 2 \Gamma^2 (2 f + 1) + f - 1}{(2 \Gamma \sqrt{f} + f + 1)^2}} \quad (E.6) \]

In the lab frame the reverse shock moves with a velocity

\[ \beta_{\text{rs}} = \frac{\beta_{\text{sh}}(f) - \beta_{43}(f)/3}{1 - \beta_{\text{sh}}(f) \beta_{43}(f)/3}. \quad (E.7) \]

Equations (E.6) and (E.7) can be used to determine the radius of the shocks when the reverse shock crosses the trailing edge of the jet and the value of \( \Gamma_{\text{sh,rs}} \) at this time. This involves numerically integrating \( \beta_{\text{rs}}/\beta_j = dr_{\text{rs}}/dr_{\text{ej}} \), where \( r_{\text{rs}} \) is the position of the reverse and \( r_{\text{ej}} \) is the position of the
back of the jet. The latter allows us to calculate what fraction of the initial kinetic energy of the jet is dissipated at the reverse shock, instead of being transferred to the shocked external medium via the forward shock. This is approximately given by

\[ f_{ke} \approx \frac{\Gamma - \Gamma_{sh,rs}}{\Gamma - 1} \quad (E.8) \]
Appendix F

Tidal coupling constants

The energy deposited into a star of mass $m_*$ after a close encounter with a compact object of mass $m_c$ is given by

$$
\Delta E = \frac{G m_c^2}{r_*} \left( \frac{m_c}{m_*} \right)^2 \sum_{l=2}^{\infty} T_l(\eta) \left( \frac{r_*}{r_p} \right)^{(2l+2)} \eta \equiv \left( \frac{m_*}{m_* + m_c} \right)^{1/2} \left( \frac{r_*}{r_p} \right)^{-3/2},
$$

where $r_p$ is the pericenter distance of the encounter, $r_*$ is the radius of the star, and $T_l$ is the tidal coupling constant of multipole order $l$, which depends on the stellar structure and orbit.

For fixed stellar structure, the tidal coupling constant is a function of the ratio $\eta$ of the star’s dynamical time to the time spent near pericenter. For the dominant $l = 2$ modes the energy deposited in the star is

$$
\Delta E = T_2(\eta) \left( \frac{r_p}{r_t} \right)^{-6} \frac{G m_*^2}{r_*},
$$

where $r_t = r_* (m_c/m_*)^{1/3}$ is the tidal radius.
Fig. F.1 compares the $l = 2$ tidal coupling constants for both polytropic and MESA stellar models (Paxton et al. 2011, 2013).\(^1\) Calculating the tidal coupling constant requires a summation over discrete stellar eigenmodes, which we calculate with the open source stellar oscillation code GYRE (Townsend & Teitler 2013).\(^2\) Following Lee & Ostriker (1986), we include the f-mode, the five lowest order p-modes, and the eighteen lowest order g-modes (if they exist) in the summation.

For stars of mass $\lesssim 0.3\,M_\odot$, g-modes are not excited at all and most of the energy is deposited into the f-mode. Larger stellar masses and larger values of $r_p$ result in greater energy transfer into g-modes, while p-modes are always subdominant. Fig. F.2 shows the fraction of energy placed into different modes as a function of pericenter for $m_* = 0.3M_\odot$ and $m_* = 1M_\odot$ stars. An $n = 3/2$ polytropic model accurately reproduces the mode spectrum of the low mass star. However, the mode spectrum of the solar type star is poorly approximated by a polytropic model: the $n = 3$ polytropic model underestimates the energy in g-modes, and overestimates that in the f-mode, for small pericenter distances.

The tidal coupling constant of low mass stars ($m_* \lesssim 0.5M_\odot$), is close to that of an $n = 3/2$ polytrope. The tidal coupling constant approaches that of an $n = 3$ polytrope as the stellar mass aproaches $1M_\odot$.

\(^1\)http://mesa.sourceforge.net, version 9575
\(^2\)https://bitbucket.org/rhdtownsend/gyre/wiki/Home, version 5. We assume adiabatic oscillations.
Figure F.1: Comparison of tidal coupling constant as a function of η (eq. F.1) for different stellar models as labeled. The dashed, red lines show the tidal coupling constants for polytropic stellar models. We have assumed a parabolic orbit.

Figure F.2: Top panel: Fraction of oscillation energy deposited into p-, f-, and g-modes for a star of mass 0.3$M_\odot$. For this calculation we use a MESA model evolved for 5 Gyr, but the results are indistinguishable from that of an $n = 3/2$ polytrope. The g-modes do not contribute. Bottom panel: Same as the top panel, but for a star of mass 1$M_\odot$. The mode decomposition is not accurately reproduced by a polytropic model, as can be seen by comparing the solid and dashed lines.
Appendix G

Corrections for non-linear effects

Linear theory underestimates the energy deposited in the star by a factor of a few for the close pericenters of interest. Non-linear corrections have been calculated by Ivanov & Novikov (2001) for polytropic stellar models. We adopt their prescriptions for the tidal coupling constant for close pericenters.

Fig. G.1 compares tidal coupling constants for polytropic models from linear theory and from Ivanov & Novikov (2001) (see also their Figures 13 and 15). The following expressions reproduce tidal coupling constants from Ivanov & Novikov (2001) for small pericenters, while approaching the results of linear theory at large pericenters.

\[ n=3/2 \text{ polytrope:} \]

\[
T(\eta) = C 2^{(b-g)/s} \left( \frac{\eta}{\eta_0} \right)^{-g} \left( 1 + \left( \frac{\eta}{\eta_0} \right)^s \right)^{(g-b)/s} \times \left( \frac{1}{2} - \frac{1}{2} \tanh \left[ k \left( \frac{r}{r_1} - 1 \right) \right] \right)
\]

\[ C = 2.58, \eta_0 = 1.73, g = -4.36, b = 2.82, s = 9.91, \]

\[ r_1 = 4.5, k = 4 \]  

(G.1)
Figure G.1: Fitted tidal coupling constants $T(\eta)$ from the nonlinear results of Ivanov & Novikov (2001). Results are shown for $n = 3/2$ (top panel) and $n = 3$ (bottom panel) polytropes.

$n=3$ polytrope:

$$T(\eta) = C 2^{(b-g)/s} \left( \frac{\eta}{\eta_0} \right)^{-g} \left( 1 + \left( \frac{\eta}{\eta_0} \right)^s \right)^{(g-b)/s} \times \left( 1 + \frac{\eta}{\eta_1} \right)^{s_2} \left( b-b_2/s_2 \right)$$

$$C = 0.17, \eta_0 = 1.07, \eta_1 = 1.92, g = -3.83,$$

$$b = 5.5, b_2 = 3.49, s = 3.59, s_2 = 6.68,$$  \hspace{1cm} (G.2)

where we have adopted eq. (G.1) for low mass stars with $m_* \leq 0.7M_\odot$, and eq. (G.2) for higher stellar masses.
Appendix H

Binary exchange interactions

H.1 Binary fraction

When soft binaries interact with field stars in the GC they gain energy, become more loosely bound, and eventually dissociate (Heggie 1975; Binney & Tremaine 1987). A binary is soft if its binding energy is less than the kinetic energy of a typical field star, i.e. if,

\[ \frac{Gm_1m_2}{a\langle m \rangle} < \sigma^2, \]

where \( m_1 \) and \( m_2 \) are the masses of the binary components, \( a \) is the semi-major axis, \( \langle m \rangle \) is the mean stellar mass, and \( \sigma \) is the 1D velocity dispersion. Binaries that do not satisfy eq. (H.1) are hard. Interactions with field stars shrink the separation of a hard binary over time, making it a smaller target. Thus, it is much easier (and faster) to dissolve a soft binary than to push a hard binary to coalescence.

The black lines in Fig. H.1 shows the hard-soft boundary in our Fiducial model of the Galactic Center for two different binary masses. For a binary distribution that is flat in \( \log(a) \), from the
semi-major axis of Roche-Lobe contact $a_{\text{roche}} \approx r_t \sim R_\star$ to $a = 900$ AU, we find that $\sim 73\% \ (87\%)$ of binaries with two solar mass (0.3 solar mass stars) are soft at 1 pc. By contrast, in a globular cluster with $\sigma \sim 10$ km s$^{-1}$, only 40–50% of the primordial binaries are soft (Ivanova et al. 2005).

Soft binaries can be ionized in two different ways:

a Direct collisions with field stars, as occurs on a timescale

$$
\tau_{\text{collide}} = \frac{1}{\pi n_\star \sigma a^2 \left( 1 + \frac{2G(m_1 + m_2)}{\sigma^2 a} \right)}.
$$

In our fiducial models, the collision rate of binaries with stars exceeds the collision rate of binaries with compact objects.

b “Evaporation” due to perturbations from distant field stars. For an equal mass binary this occurs on a timescale (Alexander & Pfuhl 2014; their eq. 3)

$$
\tau_{\text{evap}} \approx 0.07 \frac{(m_1 + m_2) \sigma}{G n \langle m^2 \rangle \ln \Lambda}.
$$

where $m_{\text{bin}}$ is the total mass of the binary, $n$ is the number density of perturbers, $\langle m^2 \rangle$ is the second moment of the mass function, $\sigma$ is the 1D velocity dispersion, and $\ln \Lambda \approx 15$ is the Coulomb logarithm.

The red lines in Fig. H.1 show the semi-major axes for which the collision and evaporation times are equal to $10^{10}$ years. Any primordial binaries with semi-major axes $\gtrsim 0.1$ AU within the central parsec would be evaporated on a timescale of $\lesssim 10^{10}$ yr.

On the other hand, binaries with particularly small semi-major axes can be destroyed by magnetic braking. Following Ivanova & Kalogera (2006) (their eq. 4), we find that two stars of mass
Figure H.1: Hard-soft boundary for 1+1 M⊙ (top panel) and 0.3+0.3 M⊙ (bottom panel) in our fiducial model for the GC. The red lines show the semi-major axis for which the time-scale for direct collisions (eq. H.2) and evaporation (eq. H.3) is $10^{10}$ years. Binaries in the gray region are either contact binaries or unphysical as the semi-major axis of the binary would be smaller than the Roche limit.

$m_*$ with semi-major axes obeying

$$ a < 3 \left( \frac{m_*}{M_\odot} \right)^{0.16} \left( \frac{t}{10 \text{ Gyr}} \right)^{0.41} a_{\text{roche}}, \quad (\text{H.4}) $$

are brought into Roche-Lobe contact after time $t$. Solar mass stars in a two day orbit would thus come into Roche-Lobe contact within $\lesssim 5$ Gyr (see also Andronov et al. 2006).

The binary fraction is

$$ f_b \equiv \frac{N_b}{N_s + N_b} = \frac{(1 - f_d)f_{b,o}}{1 + f_d f_{b,o}} \quad (\text{H.5}) $$

where $N_b$ and $N_s$ are the numbers of single stars and binaries respectively, $f_{b,o}$ is the initial binary fraction and $f_d$ is the fraction that are destroyed due to the effects of evaporation and/or magnetic braking. Figure H.2 shows the expected binary fraction at 1 pc after 5 and 10 Gyr, as a function of stellar mass (assuming equal mass binaries). Weighting each mass bin by a Kroupa PDMF, we find
that the binary fraction is $\sim 4\%$ ($3\%$) after 5 (10) Gyr. Our estimate for the binary fraction of solar mass stars accounting evaporation alone ($\sim 10\%$) is comparable to previous estimates (Hopman 2009).

Kozai-Lidov (KL) oscillations induced by the central SMBH can turn some soft binaries into hard binaries, effectively increasing the binary fraction. In particular, KL oscillations can excite binaries to very large eccentricities. Tides can then dissipate energy, creating a tight stellar binary (Antonini & Perets 2012; Stephan et al. 2016). In practice, for the Galactocentric radii of interest ($\sim 1$ pc), the time-scale to excite the binary to very large eccentricities (the octupole Kozai time scale) is generally longer than the evaporation time-scale. Additionally, for a $1M_\odot$ binary, GR precession will damp KL oscillations for binary separations

$$a_1 < 2 \text{au} \left(\frac{a_2}{1 \text{pc}}\right)^{3/4} \frac{(1 - e_2^2)^{3/8}}{(1 - e_1^2)^{1/4}},$$

where $e_1$ is the eccentricity of the inner binary orbit, while $a_2$ and $e_2$ are the semi-major axis and eccentricity of the binary’s orbit around the SMBH (see equation 59 in Naoz 2016).

### H.2 Binary exchange rates

Finally, the rate of compact objects exchanging into existing stellar binaries is

$$\dot{n}_{2+1} = \int_0^\infty n_c f_b n_\star \Sigma v_\infty f(v_\infty) dv_\infty,$$

where $n_c$ and $n_\star$ are the densities of compact objects (BHs or NSs) and stars, respectively, and $\Sigma$ is the total cross-section for the compact object to be captured into a binary with an ordinary star.
This may either occur via a prompt exchange or a resonant capture. In the former case the exchange occurs quickly, while in the latter case a metastable triple system is formed first. The cross-sections for these processes have been calibrated from binary-single scattering experiments as (Valtonen & Karttunen 2006)

\[
\begin{align*}
\Sigma_{\text{ex}} &\approx 0.51 \frac{2m_c \pi a^2}{m_* v^2} (1 - P_c) \\
\Sigma_{\text{cap}} &\approx 1.18(n - 1)(1 - v^2)^{n-2} \frac{2m_c \pi a^2}{m_* v^2} (1 - P_s) \\
P_c &\approx 0.25(n - 1)(1 - v^2)^{n-2} \\
P_s &= \frac{m_c}{2m_*^q + m_c^q} \\
v^2 &= \frac{2m_c v_\infty^2 a_o}{M G m_*}
\end{align*}
\]
where \( m_c, m_\star, \) and \( M \) are the masses of the compact object, the stars in the binary (assumed to be equal in mass), and the three-body system, respectively. The cross-sections go to 0 for \( v \gtrsim 1 \). The power law index \( n (q) \) depends on the angular momentum of the system, and is expected to vary between 4.5 and 3 (1 and 3) as \( v \) goes from 0 to 1. We choose \( n = q = 3 \), but the results are not very sensitive to this choice.

Using densities profiles of our Fiducial model, the rate of 2+1 encounters per unit volume at radii \( \lesssim 1 \) pc is approximately given by

\[
\dot{n}_{\text{bh}, 2+1} = 5 \times 10^{-11} \left( \frac{r}{1 \text{ pc}} \right)^{-2.5} \left( \frac{f_b}{0.01} \right) \left( \frac{m_\star}{m_\star} \right) \text{pc}^{-3} \text{yr}^{-1}
\]

\[
\dot{n}_{\text{ns}, 2+1} = 3 \times 10^{-11} \left( \frac{r}{1 \text{ pc}} \right)^{-1.9} \left( \frac{f_b}{0.01} \right) \left( \frac{m_\star}{m_\star} \right) \text{pc}^{-3} \text{yr}^{-1}
\]

(H.13)

where \( \bar{m}_\star = 0.3 M_\odot \). Integrating over volume and a Kroupa PDMF \( (m_\star = 0.2 - 1 M_\odot) \), we find that the total rate of 2+1 encounters inside of 1 pc is

\[
\dot{N}_{\text{bh}, 2+1} = 8 \times 10^{-10} \left( \frac{f_b}{0.01} \right) \text{yr}^{-1}
\]

\[
\dot{N}_{\text{ns}, 2+1} = 4 \times 10^{-10} \left( \frac{f_b}{0.01} \right) \text{yr}^{-1},
\]

(H.14)

where we have truncated the volume integral where \( a_{\text{hs}} \) equals the stellar radius. Comparing to the tidal capture rates (Fig. 4.11), we see the rate of 2+1 encounters is sub-dominant for binary fractions of \( \lesssim 50\% \) for BHs and 15\% for NSs, as expected in the GC from the above considerations. We stress that these calculations are generous to the 2+1 formation channel, as we have assumed that every exchange interaction involving a main sequence binary and a compact object will lead to XRB formation, while in reality this is only true for a subset of these interactions. For example, three-body interactions can result in a physical stellar collision (Fregeau et al. 2004). Thus, for
the low binary fractions expected in the GC, binary-single exchange interactions should be highly sub-dominant to tidal capture in the formation of XRBs.