A framework for contrast enhancement by dyadic wavelet analysis

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ABSTRACT

This paper introduces a method for accomplishing mammographic feature analysis by multiresolution representations of the dyadic wavelet transform. Our approach consists of the application of non-linear enhancing functions $E(z)$ within levels of a multiresolution representation. We show that there exists a simple constraint for $E(z)$ such that image enhancement is guaranteed. Furthermore, a simple case in which the enhancement operator is a constant multiplier is mathematically equivalent to traditional unsharp masking. We show quantitatively that transform coefficients, modified within each level by non-linear operators, can make more obvious unseen or barely seen features of mammography without requiring additional radiation. Our results are compared with traditional image enhancement techniques by measuring the local contrast of known mammographic features.

INTRODUCTION

Many cancers escape detection due to the density of surrounding breast tissue. For example, differences in attenuation of the various soft tissue structures in the female breast are small, and it is necessary to use low levels of X-ray energy to obtain high contrast in mammographic film. Since contrast between the soft tissues of the breast is inherently low and because relatively minor changes in mammary structure can signify the presence of a malignant breast tumor, the detection is more difficult in mammography than in most other forms of radiography. The radiologist must search for malignancy in mammographic features such as microcalcifications, dominate and stellate masses, as well as textures of fibrous tissues (fibroglandular patterns).

A primary breast carcinoma can metastasize when it consists of a relatively small number of cells, far below our present threshold of detection. The importance of diagnosis of breast cancer at an early stage is critical to patient survival. Despite advances and improvements in mammography and mammographic screening programs, the detection of minimal breast cancer (those cancers 1.0 cm or less in diameter) remains difficult. At present, mammography is capable of detecting some cases through indirect signs, particularly through the presence of characteristic microcalcifications. It has been suggested that as normally viewed, mammograms display only about 3% of the information they detect! [1]. The inability to detect these small tumors motivates the multiscale imaging technique presented in this paper.

Digital image processing techniques have been applied previously to mammography. The focus of past investigations has been to enhance mammographic features while reducing the enhancement of noise. Gordon and Rangayyan [9] used adaptive neighborhood image processing to enhance the contrast of features relevant to mammography. This method enhanced the contrast of mammographic features as well as noise and digitization effects. Dhawan et al. [6, 7, 8] have made significant contributions towards solving problems encountered in mammographic image enhancement. They developed an adaptive neighborhood-based image processing technique that utilized low-level analysis and knowledge about a
desired feature in the design of a contrast enhancement function to improve the contrast of specific features. Recently, Tahoces et al. [19] developed a method for the enhancement of chest and breast radiographs by automatic spatial filtering. In their method, they used a linear combination of an original image and two smoothed images obtained from the original image by applying different spatial masks. The process was completed by nonlinear contrast stretching. This spatial filtering enhanced edges while minimally amplifying noise.

Methods of feature enhancement have been key to the success of classification algorithms. Lai et al. [10] compared several image enhancement methods for detecting circumscribed masses in mammograms. They compared an edge-preserving smoothing function [17], a half-neighborhood method [18], k-nearest neighborhood, directional smoothing [5] and median filtering [2], and in addition proposed a method of selective median filtering.

In the fields of image processing and computer vision, transforms such as the windowed Fourier transforms that can decompose a signal into a set of frequency intervals of constant size have been used in many applications, including image compression and texture analysis. Because the spatial and frequency resolution of these transforms are constant, the information provided by such decompositions is not localized in the spatial domain. A wavelet transform [3, 4, 15] is a decomposition of an image onto a family of functions called a wavelet family. In comparison to a windowed Fourier transform, the resolution of a wavelet transform varies with a scale parameter, decomposing an image into a set of frequency channels of constant bandwidth on a logarithmic scale. This variation of resolution enables the wavelet transform to “zoom” into the irregularities of an image and characterize them locally.

In this paper we accomplish mammographic feature enhancement through a dyadic multi-resolution representation [11, 12, 13]. By using a multi-resolution representation, we decompose an image into a multi-resolution hierarchy of localized information at different spatial frequencies. Our approach for mammographic feature enhancement consists of the application of non-linear operators for image enhancement within levels of a redundant multi-resolution representation. We show preliminary results that suggest our method can emphasize significant features in mammography for improved visualization of breast pathology.

**ONE-DIMENSIONAL DYADIC WAVELET TRANSFORM**

The one-dimensional dyadic wavelet transform of a continuous function \( f(x) \) at scale \( 2^k \) and position \( x \) is defined by the convolution of \( f \) and \( \psi \) as follows [14]

\[
W_{2^k}f(x) = f(x) * \psi_{2^k}(x),
\]

where \( f * g \) is defined as \( f(x) * g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)du \) and \( \psi_{2^k}(x) = \frac{1}{\sqrt{2^k}} \psi\left(\frac{x}{2^k}\right) \) is the dilation of a mother wavelet \( \psi(x) \).

A function \( f(x) \in \mathbb{R}^2 \) can then be expressed in term of its wavelet transform as follows

\[
f(x) = \sum_{k=-\infty}^{\infty} W_{2^k}f(x) * \chi_{2^k}(x) = \sum_{k=-\infty}^{\infty} f(x) * \psi_{2^k}(x) * \chi_{2^k}(x),
\]

where wavelets \( \psi(x) \) and \( \chi(x) \) satisfy the condition \( \forall \omega, \quad \sum_{k=-\infty}^{\infty} \hat{\psi}(2^k \omega) \hat{\chi}(2^k \omega) = 1 \), and \( \hat{\psi}(\omega) \) and \( \hat{\chi}(\omega) \) are the Fourier transform of \( \psi(x) \) and \( \chi(x) \), respectively.

For the construction of wavelets, a scaling function \( \phi(x) \) was introduced and defined by \( |\hat{\phi}(\omega)|^2 = \sum_{k=-\infty}^{\infty} \hat{\psi}(2^k \omega) \hat{\chi}(2^k \omega) \), where \( \hat{\phi}(\omega) \) is the Fourier transform of \( \phi(x) \). A scaling function shall satisfy the dilation equation

\[
\hat{\phi}(\omega) = H \left( \frac{\omega}{2} \right) \hat{\phi} \left( \frac{\omega}{2^k} \right),
\]
Figure 1: (a) Structure of the dyadic wavelet transform. (b) A two-level discrete dyadic wavelet transform.

where $H(\omega)$ is the discrete Fourier transform of a discrete filter $h_l$.

Functions $\psi(x)$ and $\chi(x)$ can then be constructed from a scaling function $\phi(x)$ and discrete filters $G(\omega)$ and $K(\omega)$ as follows

$$\hat{\psi}(\omega) = G \left( \frac{\omega}{2} \right) \hat{\phi} \left( \frac{\omega}{2} \right), \quad \hat{\chi}(\omega) = K \left( \frac{\omega}{2} \right) \hat{\phi} \left( \frac{\omega}{2} \right).$$

Figure 1(a) shows a filter-based implementation of the dyadic wavelet transform.

For enhancement purposes, we considered a special class of wavelets which satisfied

$$\psi(x) = \frac{d^2 \theta}{dx^2}.$$ (5)

or in the frequency domain $\hat{\psi}(\omega) = -\omega^2 \hat{\theta}(\omega)$. We further required that $\theta(x)$ be a low-pass function with $\hat{\theta}(0) \neq 0$, and $\lim_{\omega \rightarrow \infty} \hat{\theta}(\omega) = 0$.

A class of wavelet satisfying (5) can be constructed from the following discrete filters [14]

$$H(\omega) = \left[ \cos \left( \frac{\omega}{2} \right) \right]^{2n}, \quad G(\omega) = -4 \left[ \sin \left( \frac{\omega}{2} \right) \right]^2, \quad K(\omega) = \frac{1 - |H(\omega)|^2}{G(\omega)} = -\frac{1}{4} \sum_{i=0}^{2n-1} \left[ \cos \left( \frac{\omega}{2} \right) \right]^{2i}. \quad (6)$$

Using Equations (3), (4) and (6) one can show that

$$\hat{\phi}(\omega) = \left[ \frac{\sin(\omega/2)}{(\omega/2)} \right]^{2n}, \quad \hat{\psi}(\omega) = -\omega^2 \frac{1}{4} \left[ \frac{\sin(\omega/4)}{(\omega/4)} \right]^{2n+2} = -\omega^2 \hat{\theta}(\omega), \quad (7)$$

$$\hat{\chi}(\omega) = -\frac{1}{4} \left[ \frac{\sin(\omega/4)}{(\omega/4)} \right]^{2n} \sum_{i=0}^{2n-1} \left[ \cos \left( \frac{\omega}{4} \right) \right]^{2i} = -\hat{\beta}(\omega).$$

where the filter frequency responses

$$\hat{\theta}(\omega) = \frac{1}{4} \left[ \frac{\sin(\omega/4)}{(\omega/4)} \right]^{2n+2}, \quad \hat{\beta}(\omega) = \frac{1}{4} \left[ \frac{\sin(\omega/4)}{(\omega/4)} \right]^{2n} \sum_{i=0}^{2n-1} \left[ \cos \left( \frac{\omega}{4} \right) \right]^{2i} \quad (8)$$

are low-pass, zero-phase, symmetric and positive.

Figure 2 shows $4\hat{\theta}(\omega)$ and $\sum_{n=1}^{\infty} \hat{\beta}(\omega)$. Note that both functions $\hat{\beta}(\omega)$ and $\hat{\theta}(\omega)$ are close to a Gaussian function. For this specific class of wavelets, Equation (1) can be written as

$$W_{2^k} f(x) = 2^k \frac{d^2}{dx^2} [f(x) * \theta_{2^k}(x)].$$
ENHANCEMENT TECHNIQUES

Linear enhancement

We now consider a linear enhancement operator which multiplies transform coefficients of a single channel \( m \in \mathbb{Z} \) by a constant \( C_m \in \mathbb{R} \). In this case, the output of channel \( m \) is

\[
C_m W_{2^m} f(x) * \chi_{2^m}(x) = -2^m C_m \frac{d^2}{dx^2} [f(x) * \theta_{2^m}(x) * \beta_{2^m}(x)],
\]

and the “enhanced” function \( \tilde{f}(x) \) is

\[
\tilde{f}(x) = f(x) - 2^m (C_m - 1) \frac{d^2}{dx^2} [f(x) * \theta_{2^m}(x) * \beta_{2^m}(x)].
\]

In the frequency domain, this is equivalent to a linear system with a transfer function

\[
T(\omega) = 1 + 2^m (C_m - 1) \omega^2 \hat{\theta}(2^m \omega) \hat{\beta}(2^m \omega).
\]

Note that \( \hat{\theta}(\omega) \) and \( \hat{\beta}(\omega) \) in Equation (8) are real, positive and symmetric. Therefore, such an operator is guaranteed to enhance a specific range of the frequency domain.

Here, we point out a special case for which a single channel \( m \) is enhanced by \( C_m > 1 \), and the bandwidth \( B_{\beta_m} \) of filter \( \hat{\beta}(2^m \omega) \) satisfies

\[
B_{\beta_m} \gg B_f.
\]

where \( B_f \) is the bandwidth of the input signal. For such a case, \( \tilde{f}(x) = f(x) - 2^m (C_m - 1) f''(x) \).

In general, more than one channel may be enhanced (suppressed). Therefore, in its most general form, the system frequency response may be written as

\[
T(\omega) = 1 + \sum_{m=-\infty}^{\infty} 2^m (C_m - 1) \omega^2 \hat{\theta}(2^m \omega) \hat{\beta}(2^m \omega).
\]

Since each channel introduces no phase shifting, the whole system is still guaranteed to be zero-phase, and its frequency response is completely determined by the set \( \{C_m\} \). We call this linear enhancement technique Multiscale Unsharp Masking. A single channel linear enhancement is exactly equivalent to traditional Unsharp Masking.

Figure 2: (a) \( 4\hat{\theta}(\omega) \), (b) \( \frac{2}{n^2} \hat{\beta}(\omega) \), and (c) Overlaid plots of \( 4\hat{\theta}(\omega) \), \( \frac{2}{n^2} \hat{\beta}(\omega) \) and Gaussian \( e^{-0.055\omega^2} \).
Non-linear enhancement

Although linear enhancement methods are traditionally used, non-linear methods are more appealing mainly due to the limited display range \([0 - 255]\) in imaging systems. A non-linear enhancement method can be viewed in two aspects. From the point of dynamic range compression, such a method attempts to bring up features previously difficult to see. From the feature selection point of view, such a method tries to enhance features having certain properties.

In the previous sections, we used Fourier analysis tools to analyze system properties. Unfortunately, for a non-linear enhancement system, such tools can no longer be used. Instead, we need to work directly in the time domain.

For non-linear enhancement, we place an enhancement function (operator) on point \(A\) of Figure 1(a). In general, the output of such a system is

\[
\tilde{f}(x) = \sum_{k=-\infty}^{\infty} E[W_{2k}f(x)] \ast \chi_{2k}(x) = \sum_{k=-\infty}^{\infty} E\left[ 2^k \frac{d^2}{dx^2}[f(x) \ast \theta_{2k}(x)] \right] \ast \chi_{2k}(x).
\]

Consider first a simple case in which a single channel \(p\) is enhanced. In this case, we can write the output as

\[
\tilde{f}(x) = \sum_{k \neq p} \left[ 2^k \frac{d^2}{dx^2}[f(x) \ast \theta_{2k}(x)] \right] \ast \chi_{2k}(x) + E\left[ 2^p \frac{d^2}{dx^2}[f(x) \ast \theta_{2p}(x)] \right] \ast \chi_{2p}(x) \tag{11}
\]

\[
= f(x) + \left\{ E\left[ 2^p \frac{d^2}{dx^2}[f(x) \ast \theta_{2p}(x)] \right] - 2^p \frac{d^2}{dx^2}[f(x) \ast \theta_{2p}(x)] \right\} \ast \chi_{2p}(x)
\]

We further assume that Equation (9) is valid for channel \(p\). Then, we can simplify the above expression as follows

\[
\tilde{f}(x) = f(x) - \left[ E\left( \frac{d^2 f}{dx^2} \right) - 2^p \frac{d^2 f}{dx^2} \right] \tag{12}
\]

Next, consider a soft edge modeled by the function \(f(x) = \frac{1}{1+e^{-\alpha x}}.\) Notice that its second derivative \(f''(x) = -\frac{\alpha}{2} \sinh(\alpha x/2)(\cosh(\alpha x/2))^{-3}\) is antisymmetric. Therefore, an enhancement function \(E(x)\) shall meet the following constraints

- Monotonicity, in order not to change the position of local extrema.
- Antisymmetry, \(E(-x) = -E(x),\) in order not to add a DC component to a band-pass channel output, and to boost edges.
- Continuity, in order to avoid any discontinuities.

We observed that

1. For points \(x\) where \(E\left( \frac{d^2 f}{dx^2} \right) > 2^p \frac{d^2 f}{dx^2},\) the edge is enhanced.
2. For points \(x\) where \(E\left( \frac{d^2 f}{dx^2} \right) = 2^p \frac{d^2 f}{dx^2},\) the edge is unchanged.
3. For points \(x\) where \(E\left( \frac{d^2 f}{dx^2} \right) < 2^p \frac{d^2 f}{dx^2},\) the edge is eroded.
Figure 3: Four-level dyadic edge enhancement by (a) linear operator $E(x) = 2.3x$ and (b) non-linear operator $E(x)$ with $T = 0.2 \max[w(n)]$ and $k = 5$.

For enhancement purposes, case 3 above shall be avoided. Thus, we add an additional constraint to the enhancement function, $E(x) \geq x$. Experimentally, we found the following simple function useful for enhancement

$$E(x) = \begin{cases} 
  x - (k - 1)T, & \text{if } x < -T, \\
  kx, & \text{if } |x| < T, \\
  x + (k - 1)T, & \text{if } x > T,
\end{cases}$$

where $k > 1$. A complete theoretical analysis for a more general case remains to be carried out.

**DISCRETE ALGORITHMS**

**1-D discrete algorithm**

A discrete algorithm can be readily obtained using discrete filters $H(\omega)$, $G(\omega)$ and $K(\omega)$ [14]. Figure 1(b) shows the structure for a two-level discrete dyadic wavelet transform. Note that filter $g[n] = \{1, -2, 1\}$ is a discrete second derivative operator, and $G(2^k \omega)K(2^k \omega) = 1 - |H(2^k \omega)|^2$ is also real, positive and symmetric for all frequencies $\omega$. Therefore, all previous conclusions for continuous systems shall also be valid for the discrete case. Figure 3 shows multi-level edge enhancement results of four-level discrete dyadic wavelet transform using linear and non-linear methods. The parameters for the non-linear enhancement were $T = 0.2 \max[w(n)]$ and $k = 5$. The gain for linear enhancement was $k = 2.3$.

**2-D discrete algorithm**

Mallat and Zhong [14] showed that a two-dimensional dyadic wavelet transform can be easily constructed from one-dimensional wavelets. A two-dimensional discrete dyadic wavelet transform can be implemented using one dimensional dimensional filters $H$, $G$, $K$ and $L$ as in Figure 4(a), where filter $L$ satisfies $L(\omega) = (1 + |H(\omega)|^2)/2$. Figure 4(b) displays the magnitude of the equivalent channel filters for levels 1, 2, and 3, and clearly shows that for the dyadic wavelet transform, orientations are partitioned into horizontal and vertical bands.


**EXPERIMENTAL RESULTS AND DISCUSSION**

Preliminary results have shown that the multiscale processing technique described above, can make more obvious unseen or barely seen features of a mammogram without requiring additional radiation. Our study suggests that the analyzing functions presented in this paper can improve the visualization of features of importance to mammography and assist the radiologist in the early detection of breast cancer. In our study, film radiographs of the breast were digitized using a sampling distance of 200 microns, on a Kodak laser film digitizer, with 10-bit quantization (contrast resolution). Each digital image was cropped to a matrix size of $512 \times 512$ before processing.

Mathematical models of phantoms were constructed to validate our enhancement technique against false positives arising from possible artifacts introduced by the analyzing functions and to compare our methods against traditional image processing techniques of improving contrast. Our models included features of regular and irregular shapes and sizes of interest in mammographic imaging, such as microcalcifications, cylindrical and spicular objects and conventional masses. Techniques for “blending” a normal mammogram with the images of mathematical models were developed. The purpose of these experiments was to test the performance of our processing technique on inputs known “a priori” using mammograms where the objects of interest were deliberately obscured by normal breast tissues. The “imaging” justification for “blending” is readily apparent; a cancer is visible in a mammogram because of its (slightly) higher X-ray attenuation which causes a lower radiation exposure on the film in the appropriate region of a projected image. Our blended mammogram was constructed by adding the amplitude of the mathematical phantom image to a cancer free mammogram followed by local smoothing of the combined image.

Figure 5(a) shows the result after processing the blended mammogram with unsharp masking. Figure 5(b) was obtained after reconstructing the blended mammogram from dyadic wavelet transform coefficients modified by the non-linear enhancing function described in Section 3. Figure 6 shows enlarged areas containing each feature in the processed mammogram for each method of contrast enhancement. Non-linear multiscale enhancement of dyadic wavelet coefficients provided a significant improvement in local contrast for each
Table 1: Contrast values and CII for enhancement by unsharp masking (UNS) and non-linear multiscale enhancement of dyadic wavelet (DYA) coefficients.

<table>
<thead>
<tr>
<th>Feature</th>
<th>$C_{\text{Original}}$</th>
<th>$C_{\text{UNS}}$</th>
<th>$C_{\text{DYA}}$</th>
<th>CII UNS</th>
<th>CII DYA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minute microcalcification cluster</td>
<td>0.0217</td>
<td>0.0312</td>
<td>0.1219</td>
<td>1.4347</td>
<td>5.6093</td>
</tr>
<tr>
<td>Microcalcification cluster</td>
<td>0.0192</td>
<td>0.0239</td>
<td>0.1225</td>
<td>1.2424</td>
<td>6.3623</td>
</tr>
<tr>
<td>Spiccular lesion</td>
<td>0.0295</td>
<td>0.0351</td>
<td>0.1295</td>
<td>1.1866</td>
<td>4.3833</td>
</tr>
<tr>
<td>Circular (arterial) calcification</td>
<td>0.0204</td>
<td>0.0266</td>
<td>0.1219</td>
<td>1.3017</td>
<td>5.9658</td>
</tr>
<tr>
<td>Well-circumscribed mass</td>
<td>0.0277</td>
<td>0.0280</td>
<td>0.0983</td>
<td>1.0101</td>
<td>3.5501</td>
</tr>
</tbody>
</table>

A quantitative measure of contrast improvement can be defined by a Contrast Improvement Index (CII), $\text{CII} = \frac{C_{\text{Processed}}}{C_{\text{Original}}}$, where $C_{\text{Processed}}$ and $C_{\text{Original}}$ are the contrasts for a region of interest in the processed and original images, respectively.

In this paper we adopt a version of the optical definition of contrast introduced by Morrow et al. [16]. The contrast $C$ of an object is defined by $C = \frac{f - b}{f + b}$, where $f$ is the mean gray-level value of a particular object in the image, called the foreground, and $b$ is the mean gray-level value of a surrounding region called the background. This definition of contrast has the advantage of being independent of the actual range of gray levels in the image. With the aid of the mathematical phantom we computed local masks to separate the foreground and background regions of each feature included in the blended mammogram. Table 1 shows the contrast values and CII for the mammographic features shown in Figure 6. Note that non-linear enhancement of dyadic wavelet coefficient performed significantly better than unsharp masking and consistently improved the contrast of each feature. In all cases contrast was improved while preserving the overall shape of each feature profile.

SUMMARY

We have presented a methodology for accomplishing contrast enhancement by multiscale representations. We have demonstrated that features extracted from multiresolution representations can provide for local emphasis of salient and subtle features of importance to mammography. We have compared these techniques to a traditional standard for image enhancement and described a mathematical framework connecting the two methods. The improved contrast of mammographic features make our technique appealing for computed aided diagnosis and screening mammography.

REFERENCES


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Figure 5: Blended mammogram: (a) Enhancement by unsharp masking. (b) Non-linear multiscale enhancement of dyadic wavelet transform coefficients.

Figure 6: Contrast enhancement for features in blended mammogram. (a) Enhancement by unsharp masking. (b) Non-linear multiscale enhancement of dyadic wavelet transform coefficients. Phantom mammographic features from top to bottom: minute microcalcification cluster, microcalcification cluster, spicular lesion, circular (arterial) calcification, and a well-circumscribed mass.