Monopoly and the Rate of Extraction of Exhaustible Resources

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In some recent discussions of the "energy crisis," the suggestion has been put forward that the oil producing countries or the oil companies have been acting collusively and have forced the price of oil to a level far higher than it would have been in competitive equilibrium. The object of this paper is to compare the rate of exploitation of an exhaustible natural resource in competitive markets with that of a profit maximizing monopolist. The basic result of my analysis is that there is a very limited scope for the monopolist to exercise his monopoly power; indeed, under the natural "first approximation" of constant elasticity demand schedules, with zero extraction costs, monopoly prices and competitive equilibrium prices will in fact be identical. In other cases there is some tendency for a monopolist to be more "conservation minded" than a competitive market would be.[1]

I. A Two-Period Model

The basic intuition behind my result may easily be seen. First, consider a two-period problem. Assume that there are zero extraction costs. We have a fixed stock of oil to divide between two periods. That part of the stock which we do not consume the first period will be consumed the second. In Figure 1, I have plotted the demand curve this period, from the left, and the demand curve for next period, deflated by $1+r$, where r is the rate of interest, from the right. In competitive equilibrium, an individual must be indifferent between selling a unit of oil today or tomorrow, so $p_t = p_{t+1}/(1+r)$. Thus, market equilibrium is the point of intersection of the two demand curves.

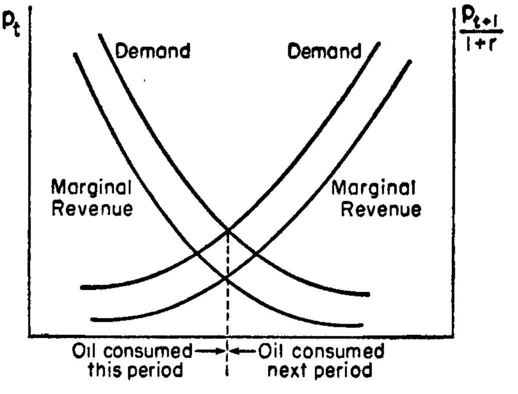

FIGURE 1

A monopolist, on the other hand, compares the marginal revenue he obtains this period with the marginal revenue, discounted by $1+r$, he would obtain next period by transferring a unit of sales from this period to next. In Figure 1, I have drawn the corresponding marginal revenue schedules, and the monopoly equilibrium is the intersection of the two. As I have drawn the curves, they intersect at exactly the same value of sales this period as did the price schedules, i.e., the monopoly equilibrium and the competitive equilibrium are identical. Clearly, if we have constant elasticity demand schedules, then price will be

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[1] This result is referred to in Robert Solow without a precise statement of the conditions under which it obtains. Milton Weinstein and Richard Zeckhauser establish the optimality of the competitive market's depletion of natural resources.

proportional to marginal revenue, and the two equilibria will be the same. If the elasticity of demand next period is higher than this period, the ratio of marginal revenue to price will be higher next period than this period which means that at the competitive price, discounted marginal revenue next period exceeds marginal revenue this period, so it pays to sell more next period: the monopolist is more conservationist than the competitive market. Conversely if the elasticity next period is lower than this period.

With extraction costs, the condition for competitive equilibrium is that rents, i.e., price minus extraction costs, c, rise at the rate of interest, i.e.,

$$p_t - c = \frac{p_{t+1} - c}{1 + r}$$

while the corresponding monopoly condition is that net marginal revenues rise at the rate of interest,

$$MR_t - c = \frac{MR_{t+1} - c}{1 + r}$$

Clearly, with constant elasticity demand schedules, since marginal revenues are a fraction of price, net discounted marginal revenue next period is greater than that for this period, when discounted rents (price minus extraction costs) next period equal that of this period. Again it pays to contract sales this period and expand them next period; with positive extraction costs and constant elasticity demand schedules, a monopolist is more conservation minded than is socially optimal.

As I shall show below, these basic results admit of considerable generalization. The basic argument is a simple one: the monopolist, like the competitor, eventually will exhaust all of the natural resource. It is not like a conventional commodity, where the total amount that will eventually be sold is smaller for a monopolist than for a competitor. Here, the only question is whether a monopolist can rearrange the patterns of sales over time to increase the present discounted value of his profits. My analysis suggests that his power to do this may be severely limited.

In a multiperiod model with zero extraction costs, competitive equilibrium will entail price rising at the rate of interest, while monopoly will require the marginal revenue to rise at the rate of interest. But if there is a constant elasticity of demand, price is proportional to marginal revenue, so price also is rising at the rate of interest. Since equilibrium entails exhaustion of the stock of resources as time approaches infinity, the competitive market equilibrium and the monopoly are described by exactly the same set of equations: the two equilibria are identical. This will be shown more formally in the next section.

II. The Basic Model: Zero Extraction Costs, Infinite Time Horizon

Let the demand function for a quantity, q, of the natural resource, be of the form,[2]

$$(1) \qquad p = f(t)q^{\alpha-1}, \qquad 1 > \alpha > 0$$

where $1/(1-\alpha)$ is the elasticity of demand. The monopolist wishes to

$$(2) \qquad \max \int_0^\infty p(t)q(t)e^{-rt}dt$$

subject, of course, to the constraint on the total stock of the resource, S_0,

$$(3) \qquad \int_0^\infty q(t)dt \leq S_0$$

Substituting (1) into (2), and introducing λ as the Lagrange multiplier on the constraint (3), our maximization problem may be reformulated as

$$(4) \qquad \max \int_0^\infty [f(t)q^\alpha e^{-rt} - \lambda q]dt$$

implying that we set $q(t)$ so that

$$(5) \qquad \alpha e^{-rt} f q^{\alpha-1} - \lambda = 0$$

which, upon substituting (1) and differentiating logarithmically, yields

[2] Obviously, if $\alpha<0$, one can obtain larger profits by reducing q. Some have suggested that the demand for oil in the very short run has less than unitary elasticity, but whether it is optimal for the monopolist to raise its price in these circumstances depends on the long-run demand elasticity as well. See Edmund Phelps and Sidney Winter.

$$\frac{\dot{p}}{p} = r \tag{6}$$

which is identical to the familiar condition for the time path of the price of a natural resource in a competitive market: the price must rise at the rate of interest. Again using (1), this is equivalent to

$$\frac{\dot{q}}{q} = \frac{r - \frac{f'}{f}}{\alpha - 1} \tag{7}$$

Thus, both the competitive market and the monopolist will satisfy the differential equation (7) and the condition (3).[3] This implies that prices at each moment of time in the competitive and monopolistic markets are identical, and hence so must be the rate of utilization of the natural resource. Because of our assumption of constant elasticity, so long as $f(t) > 0$, $q > 0$. Hence, if $f(t) > 0$ for all time, the resource is used up only asymptotically. On the other hand, if $f(t) = 0$ for $t \geq T$, then the resource is used up—in both the competitive and monopolistic markets—at exactly date T.

III. Increasing Elasticity of Demand

There were two assumptions which were crucial to my result of the previous section—a constant elasticity which did not change over time, and zero extraction costs. In this and the next section, these two restrictions are removed.

If the elasticity of demand increases over time—as we might expect—as a result of the discovery of good substitutes for the given resource,[4] we obtain exactly the same equation for the optimal value of $p(t)$ as before (equation (5)). Differentiating (5) with respect to time, we obtain

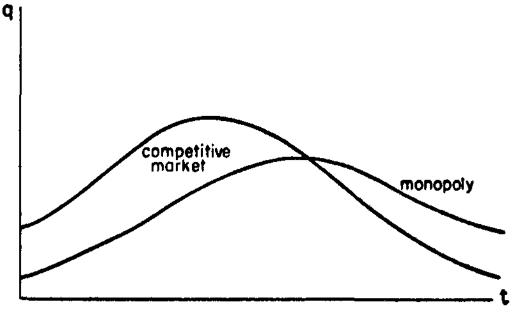

FIGURE 2. INCREASING ELASTICITY OF DEMAND

$$\frac{\dot{p}}{p} = r - \frac{\alpha'}{\alpha} \tag{8}$$

where, by assumption, $\alpha' > 0$. The rate of increase of the price will be slower for the monopolist than that in the competitive market.[5] This in turn implies that if (3) is to be satisfied, the rate of utilization of the natural resource initially will be lower for the monopolist—the monopolist takes a more conservationist policy. Figure 2 compares a possible time profile of the utilization of the resource in the two markets.[6]

IV. Extraction Costs

A similar bias for a monopolist to follow an excessively conservationist policy emerges when extraction costs are taken into account. Let the extraction cost be constant per unit of extraction but be declining with time.

[3] Equation (3) is essentially a boundary value condition.

[4] In the limiting case, where the new substitute is available with an infinite elasticity of supply the moment after discovery, if the date of discovery is known, the competitive and monopoly equilibria are identical. The case we have in mind here is where, say, a substitute is either not a perfect substitute, or does not have a perfectly elastic supply.

[5] Clearly, a more interesting case is that where the change in the elasticity of demand is an endogenous variable, say, a function of the price charged in the market. This turns out to be a far more complicated question, a special case of which is examined by Dasgupta and the author.

[6] Obviously, our formulation still is not as general as it might be, that is, within every period we assume constant elasticity demand curves. More generally, if the revenue function is of the form $R(q, t)$, then while in the monopoly market

$$\frac{\dot{q}}{q} = \frac{\left(r - \frac{R_{qt}}{R_q}\right)}{\frac{R_{qq}q}{R_q}}$$

in the competitive market

$$\frac{\dot{q}}{q} = \frac{\left(r - \frac{R_t}{R}\right)}{\frac{qR_q}{R} - 1}$$

Thus we let $g(t)$ = unit extraction cost at time t, $g' \leq 0$. The monopolist's profits are now just:[7]

(9) $$\int_0^\infty (fq^\alpha - gq)e^{-rt}dt$$

Profit maximization entails

(10) $$e^{-rt}(\alpha f q^{\alpha-1} - g) - \lambda = 0$$

which, upon differentiation and rearrangement, becomes

(11) $$\frac{\dot{p}}{p} = r(1-\gamma_m) + \frac{\dot{g}}{g}\gamma_m$$

where

(12) $$\gamma_m = \frac{g}{\alpha p}$$

γ_m is extraction costs divided by marginal revenue. It is clear that γ_m must be less than unity; if extraction costs are falling rapidly, or γ_m is large, then the market price may actually fall.

In contrast, the competitive solution requires that the individual be indifferent between extracting the oil today, receiving a net revenue (per barrel, say) of

$$p(t) - g(t)$$

or holding the oil one more period and extracting it next period, receiving a net amount of

$$\frac{p(t+1) - g(t+1)}{1+r}$$

i.e.,

$$\frac{p(t+1) - p(t)}{p(t)} = \frac{g(t+1) - g(t)}{g(t)} \cdot \frac{g(t)}{p(t)} + \frac{r(p(t) - g(t))}{p(t)}$$

or, in continuous time

(13) $$\frac{\dot{p}}{p} = r(1-\gamma_c) + \frac{\dot{g}}{g}\gamma_c$$

[7] We revert for simplicity to our assumption that α is constant.

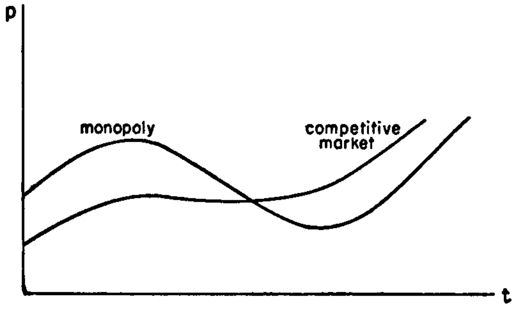

FIGURE 3. DECLINING EXTRACTION COSTS

where

(14) $$\gamma_c = \frac{g}{p}$$

Thus if at any t, $p(t)$ were the same for the competitive and monopoly markets, $\gamma_c(t) < \gamma_m(t)$ so $\dot{p}/p$ for the competitive market is greater than for the monopoly market. Thus, the "price curves" can only cross once, and the monopolist takes a more conservationist policy.[8,9] See Figure 3.

Note that if the monopolist has a lower

[8] It is possible to show that asymptotically both the monopolist and competitor will use up the entire stock.

[9] The above formulation can be made somewhat more general by letting the extraction costs depend on the stock remaining, i.e., the costs of obtaining a flow q from a field with stock S at time t is $g(t)h(S, q)$. Assume we had a large number (N) of identical oil fields. If each owner acted competitively he would

(a) maximize $\int_0^\infty [p(t)q(t) - g(t)h(S,q)]e^{-rt}dt$

so, forming the Hamiltonian

(b) $$He^{rt} \equiv pq - gh(S,q) - vq$$

we obtain the result that q must satisfy

(c) $$p - gh_q = v$$

while

(d) $$\dot{v} = rv + gh_S$$

The corresponding problem for the monopolist is to

(e) maximize $\int_0^\infty [fq^\alpha N^{\alpha-1} - gh(S,q)]e^{-rt}dt$

rate of output initially than a competitive industry, the monopolist's price must eventually be lower, if total supply is to be used up. Thus although the present generation pays higher prices for oil, subsequent generations will benefit. The monopoly equilibrium, however, is dynamically inefficient. If the monopoly were eliminated, the present generation could compensate the future generation for the higher prices, and still be better off.

V. Speculators and Mixed Markets

In some of the cases depicted above, for example, where the elasticity of demand was falling, the price of the natural resource was rising faster than the rate of interest. This would provide an incentive for a speculator to purchase the natural resource and store it, provided storage costs were not too large. In the limit, if storage costs were zero, price would have to rise at the rate of interest, and even though the monopolist would like to be profligate with society's resources, consuming them too quickly, speculators will prevent him from doing so. Thus the monopoly and competitive equilibria are identical.

A converse argument does not hold if price is rising more slowly than the rate of interest. But if there is a mixed market, with one large holder of oil stocks and a large number of small holders, then in equilibrium the small holders will extract their oil first, with the price rising at the rate of interest; subsequently when all of their stocks are exhausted the large producer will extract, with the price rising more slowly than the rate of interest.[10]

VI. Other Biases in the Rate of Extraction

There may, of course, be other differences between a monopoly and a competitive market. In particular, the required rate of return may be different; the monopolist for instance may have easier access to the capital market, and because of his larger size, be better able to pool risks. These suggest that the monopolist might have a lower required rate of return on capital (i.e., r is smaller), which again implies a more conservationist policy for the monopolist than for the competitive market.

In any of the cases where a monopolist is conservation minded, if an industry which was previously competitive becomes cartelized, the effect will be a discontinuous jump in the price.

Whether the recent jump in the price in oil can be attributed to the factors discussed in this paper remains a moot question. It might be argued that in this case, the governments involved have less access to the capital market than do the large oil companies, so that the relevant rate of interest after cartelization was higher; on the other hand, if the oil companies had thought that there was a significant probability of nationalization, they would have pursued a policy of excessively fast extraction.

Similarly, if the rate of interest facing different firms (countries) is different, then the rate at which they would like to extract the natural resource will be different. It is clear that market equilibrium will entail the firm with the highest rate of interest extracting first (with price rising at his interest rate while he is the producer); then the next

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so

(f) $$\alpha p - gh_q = v$$

where

(g) $$\dot{v} = rv + gh_S$$

To see clearly the difference between the two solutions, let $h = \phi(S)q$. Then for the competitive market,

$$\frac{\dot{p}}{p} = r(1 - \hat{\gamma}_c) + \hat{\gamma}_c \frac{\dot{g}}{g}$$

where now $$\hat{\gamma}_c = \frac{g\phi(S)}{p}$$

while for the monopoly (letting $\hat{\gamma}_M = g\phi(S)/\alpha p$)

$$\frac{\dot{p}}{p} = r(1 - \hat{\gamma}_M) + \hat{\gamma}_M \frac{\dot{g}}{g}$$

i.e., with the modification in the definition of γ, we have the same equations as before. Again, it can be shown that the monopolist pursues a more conservationist policy.

[10] This can be viewed as a Stackleberg equilibrium in which the large firm is the leader, and knows that the small firm will behave competitively; for an analysis of the Nash-Cournot equilibrium of this market, see Steve Salant.

highest, etc. If different firms face different extraction costs, the firm with the lowest extraction cost will extract first, then the next, etc. (i.e., it always pays to postpone postponable costs into the future). The competitive market equilibrium ensures that this will happen; the monopolist would behave in an identical way.

The fact that different firms with different extraction costs and apparently different rates of discount are producing simultaneously can then be attributed to: (a) marginal extraction costs are the same, even though average extraction costs are not; (b) offsetting effects of extraction costs and rates of time preference, with low extraction costs being associated with low rates of interest; (c) firms (countries) do not face a constant interest rate at which they can borrow and lend (invest); (d) risk; more particularly, differences in attitudes towards and judgments of the risks involved in postponing extraction.

Tax policy has provided further biases in the rate of extraction between the market solution and the optimal rate of extraction, but the most important provisions—the special treatment of capital gains and the depletion allowances—may not affect the relative rates of extraction of monopoly and competition. If extraction costs were zero, a constant depletion allowance would have no effect on intertemporal allocation (since price is rising at the rate of interest, the value of the depletion allowance, in present discounted terms, is independent of when the oil is depleted); hence, with constant elasticity of demand, the intertemporal resource allocation of monopoly and competition with and without the depletion allowance are all identical. With positive extraction costs, the depletion allowance encourages excessively fast depletion (since rents are rising at the rate of interest, prices are rising more slowly than the rate of interest, and hence the present discounted value of the depletion allowance is declining. Since prices with monopoly are rising more slowly than with competition, there is some presumption that the effect of the depletion allowance in accelerating extraction will be more marked in the former than in the latter. In any case, to the extent that the depletion allowance serves to offset the excessively conservative bias of monopoly, the depletion allowance may actually serve to increase social welfare. Since the return to holding a stock of a natural resource is a capital gain, and capital gains are taxed at a preferential rate, the equilibrium rate of increase of prices[11] is $r(1-t_p)/(1-t_{cg})$ where t_p and t_{cg} are the personal and capital gains tax rate; hence the preferential treatment of capital gains leads to excessive conservation.

A further potential source of bias is related to uncertainty, which we have ignored in this paper. An explicit treatment of the effect of monopoly when there is uncertainty is contained in Partha Dasgupta and the author.

Finally, we note that any analysis of the oil markets in the real world should probably entail an analysis of the behavior of oligopolistic markets. This would clearly take us beyond the scope of this paper; it is my hope, however, that the insights gained from comparing the polar cases of monopoly and competition will be of value in the study of these more realistic market situations.

[11] Assuming the rights for the oil under the land were acquired at essentially zero cost; otherwise, we have to take account of the tax reduction from the write-off of the value of oil rights upon exhaustion of the oil.

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