Optimal Taxation of Entrepreneurial Capital with Private Information

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ABSTRACT

This paper studies optimal taxation of entrepreneurial capital with private information and multiple assets. Entrepreneurial activity is subject to a dynamic moral hazard problem and entrepreneurs face idiosyncratic capital risk. We first characterize the optimal allocation subject to the incentive compatibility constraints resulting from the private information. The optimal tax system implements such an allocation as a competitive equilibrium for a given market structure. We consider several market structures that differ in the assets or contracts traded and obtain three novel results. First, differential asset taxation is optimal. Marginal taxes on bonds depend on the correlation of their returns with idiosyncratic capital risk, which determines their hedging value. Entrepreneurial capital always receives a subsidy relative to other assets in the bad states. Second, if entrepreneurs are allowed to sell equity, the optimal tax system embeds a prescription for double taxation of capital income â€“ at the firm level and at the investor level. Finally, we show that taxation of assets is essential even with competitive insurance contracts, when entrepreneurial portfolios are also unobserved.

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1. Introduction

A basic tenet in the corporate finance literature is that incentive problems due to informational frictions play a central role in entrepreneurial activity. Empirical evidence on financing and ownership patterns provides strong support for this view. Given that entrepreneurial capital accounts for at least 40% of household wealth in the US economy[1], understanding the properties of optimal taxes on entrepreneurial capital with private information is of essential interest to macroeconomics and public finance. This paper sets forth to pursue this goal.

Our main assumption is that entrepreneurial activity is subject to a dynamic moral hazard problem. Specifically, expected returns to capital positively depend on an entrepreneur's effort, which is private information. Entrepreneurial capital returns and investment are observable. The dependence of returns on effort implies that capital is agent specific and generates idiosyncratic capital risk. This structure of the moral hazard problem encompasses a variety of more specific models studied in the corporate finance literature. The approach used to derive the optimal tax system builds on the seminal work of Mirrlees (1971), and extends it to a dynamic setting. First, we characterize the constrained-efficient allocation, which solves a planning problem subject to the incentive compatibility constraints resulting from the private information. We then construct a tax system that implements such an allocation as a competitive equilibrium. The only a priori restriction is that taxes must depend on observables. The resulting tax system optimizes the trade-off between insurance and incentives.[2]

The paper studies fiscal implementation of optimal allocations in a variety of market structures, allowing for multiple assets and private insurance contracts. This is our main contribution. The properties of the optimal capital income taxes depend on the effects of asset holdings on incentives. Private information implies that the optimal allocation displays a positive wedge between the aggregate return to capital and the entrepreneurs' intertemporal marginal rate of substitution.[3] However, this *aggregate* intertemporal wedge is not related to the entrepreneurs' incentives to exert effort, since the individual intertemporal rate of transformation differs from the aggregate. Hence, we introduce the notion of an *individual* intertemporal wedge, which properly accounts for the agent specific nature of entrepreneurial capital returns. We show that the individual intertemporal wedge can be *positive or negative.* The intuition for this result is simple. More capital increases an entrepreneur's consumption in the bad states, which provides insurance and undermines

[1] Entrepreneurs are typically identified with households who hold equity in a private business and play an active role in the management of this business. Cagetti and De Nardi (2006) document, based on the Survey of Consumer Finances (SCF), that entrepreneurs account for 11.5% of the population and they hold 41.6% of total household wealth. Using the PSID, Quadrini (1999) documents that entrepreneurial assets account for 46% of household wealth. Moscowitz and Vissing-Jorgensen (2002) identify entrepreneurial capital with private equity, and they document that its value is similar in magnitude to public equity from SCF data.

[2] This recent literature is summarized in Kocherlakota's (2005a) excellent review.

[3] Golosov, Kocherlakota and Tsyvinski (2003) show that this wedge is positive for a large class of private information economies with idiosyncratic *labor* risk.

incentives. On the other hand, expected capital returns are increasing in entrepreneurial effort. This effect relaxes the incentive compatibility constraint and dominates when the spread in capital returns is sufficiently large or when the variability of consumption across states is small at the constrained-efficient allocation.

To study optimal taxes, we examine three different market structures. A market structure specifies the feasible trades between agents and the distribution of ownership rights and information. These arrangements are treated as exogenous. A tax system implements the constrained-efficient allocation if such an allocation arises as the competitive equilibrium under this tax system for the assumed market structure. In all of the market structures we consider, the optimal marginal tax on entrepreneurial capital is increasing in earnings, when the individual intertemporal wedge is negative, decreasing when it is positive. The incentive effects of capital provide the rationale for this result. When the intertemporal wedge is negative (positive), more capital relaxes (tightens) the incentive compatibility constraint, and the optimal tax system encourages (discourages) entrepreneurs to hold more capital by reducing (increasing) the after tax volatility of capital returns.

Entrepreneurs can trade bonds in the first market structure we consider. We show that the optimal tax system equates the after tax return on all assets in each state. The optimal marginal tax on risk-free bonds is decreasing in entrepreneurial earnings, while the optimal marginal taxes on risky securities depend on the correlation of their returns with idiosyncratic risk. Entrepreneurial capital is subsidized relative to other assets in the bad states. These predictions give rise to a novel theory of *optimal differential asset taxation*. While in this market structure the set of securities traded is exogenous, in the second market structure, we allow entrepreneurs to sell shares of their capital and buy shares of other entrepreneurs' capital. Viewing each entrepreneur as a firm, this arrangement introduces an equity market with a positive net supply of securities. The optimal tax system then embeds a prescription for optimal *double taxation of capital*- at the firm level, through the marginal tax on entrepreneurial earnings, and at the investor level, through a marginal tax on stocks returns. Specifically, it is necessary that the tax on earnings be "passed on" to stock investors via a corresponding tax on dividend distributions to avoid equilibria in which entrepreneurs sell all their capital to outside investors. In such equilibria, an entrepreneur exerts no effort and thus it is impossible to implement the constrained-efficient allocation. Since, in addition, marginal taxation of dividends received by outside investors is necessary to preserve their incentives, earnings from entrepreneurial capital are subject to double taxation.

The differential tax treatment of financial securities and the double taxation of capital income in the United States and other countries have received substantial attention in the empirical public finance literature, since they constitute a puzzle from the standpoint of optimal taxation models that abstract from incentive problems.[4] The optimal tax system in our implementations is designed to ensure that entrepreneurs have the correct exposure to their idiosyncratic capital risk to preserve incentive compatibility. Holdings of additional assets affect this exposure in a measure that depends on their correlation with entrepreneurial capital returns, and thus should be taxed accordingly. The ability to sell equity

[4]See Gordon and Slemrod (1988), Gordon (2003), Poterba (2002) and Auerbach (2002)

introduces an additional channel through which entrepreneurs can modify their exposure to idiosyncratic risk. A tax on dividend distributions is required to optimally adjust the impact of a reduction in the entrepreneurs' ownership stake on their exposure to idiosyncratic risk. This explains the need for double taxation of capital.

Another important property of these implementations is that optimal marginal taxes do not depend on the level of asset holdings. Consequently, entrepreneurial asset holdings need not be known to the government to administer the optimal tax system, if assets are traded via financial intermediaries who collect taxes at the source, according to a schedule prescribed by the government. This observation motivates the third market structure, in which competitive insurance firms offer incentive compatible contracts to the entrepreneurs and bonds are traded via financial intermediaries. We assume that insurance companies and the government cannot observe entrepreneurial portfolios. It follows that the optimal insurance contracts do not implement the constrained-efficient allocation. We show that the optimal marginal bond taxes relax the more severe incentive compatibility constraint in the contracting problem between private insurance firms and entrepreneurs due to unobserved bond trades and render the constrained-efficient allocation feasible for that problem. Hence, asset taxation is *essential* to implement the constrained-efficient allocation, even *without informational advantages* for the government. Under the optimal tax system, private insurance contracts implement the constrained-efficient allocation with observable consumption, despite the fact that individual consumption remains private in equilibrium.

This finding has important implications for the role of tax policy in implementing optimal allocations. Even under the same informational constraints as private insurance companies, the government can influence the portfolio choices of entrepreneurs through the tax system. This result is most closely related to Golosov and Tsyvinski (2006), who analyze fiscal implementations in a Mirrleesian economy with hidden bond trades. They focus on the optimal allocation with unobserved consumption and show that private insurance contracts do not implement such an allocation, because competitive insurance contracts fail to internalize their effect on the equilibrium bond price. A linear tax on capital can ameliorate this externality. Here, instead, the optimal tax system implements the constrained-efficient allocation with observable consumption, despite the fact that in the competitive equilibrium consumption is not observed.

This paper is related to the recent literature on dynamic optimal taxation with private information. Albanesi and Sleet (2006) and Kocherlakota (2005b), focus on economies with idiosyncratic risk in labor income and do not allow agents to trade more than one asset. They show that the optimal marginal tax on capital income is decreasing in income in economies with labor risk, and this property holds independently of the nature of the asset. Farhi and Werning (2006) study optimal estate taxation in a dynastic economy with private information. They find that the aggregate intertemporal wedge is negative if agents discount the future at a higher rate than the planner and that this implies the optimal estate tax is progressive. Grochulski and Piskorski (2005) study optimal wealth taxes in economies with risky human capital, where human capital and idiosyncratic skills are private information. Cagetti and De Nardi (2004) explore the effects of tax reforms in a quantitative model of entrepreneurship with endogenous borrowing constraints. Finally, Angeletos (2006) studies competitive equilibrium allocations in a model with exogenously incomplete markets and idiosyncratic capital risk. He finds that, if the intertemporal elasticity of substitution is high enough, the steady state level of capital is lower than under complete markets.

The plan of the paper is as follows. Section 2 present the economy and studies constrained-efficient allocations and the incentive effects of capital. Section 3 investigates optimal taxes. Section 3 concludes. All proofs can be found in the Appendix.

2. Model

The economy is populated by a continuum of unit measure of entrepreneurs. All entrepreneurs are ex ante identical. They live for two periods and their lifetime utility is:

$$U = u\left(c_0\right) + \beta u\left(c_1\right) - v\left(e\right),$$

where, c_t denotes consumption in period $t = 0, 1$ and e denotes effort exerted at time 0, with $e \in \{0, 1\}$. We assume $\beta \in (0, 1)$, $u' > 0$, $u'' < 0$, $v' > 0$, $v'' > 0$, and $\lim_{c \to 0} u'(c) = \infty$.

Entrepreneurs are endowed with K_0 units of the consumption good at time 0 and can operate an investment technology. If K_1 is the amount invested at time 0, the return on investment at time 1 is $R\left(K_1, x\right)$, where:

$$R\left(K_1, x\right) = K_1\left(1 + x\right),$$

and x is the random net return on capital. The stochastic process for x is:

$$x = \begin{cases} \overline{x} \text{ with probability } \pi\left(e\right), \\ \underline{x} \text{ with probability } 1 - \pi\left(e\right), \end{cases} \tag{1}$$

with $\bar{x} > \underline{x}$ and $\pi(1) > \pi(0)$. The first assumption implies that $E_1(x) > E_0(x)$, where E_e denotes the expectation operator for probability distribution $\pi(e)$. Hence, the expected returns on capital is increasing in effort.

We assume effort is *private information*, while the realized value of x, as well as its distribution, and K_1 are *public information*. This implies that entrepreneurial activity is subject to a dynamic moral hazard problem. The structure of the moral hazard problem encompasses a variety of more specific cases studied in the corporate finance literature (see Tirole, 2006), such as private benefit taking or choice of projects with lower probability of success that deliver benefits in terms of perks or prestige to the entrepreneur. Moreover, effort can be thought as being exerted at time 0 or at time 1, before capital returns are realized.

We characterize constrained-efficient allocations for this economy by deriving the solution to a particular planning problem. The planner maximizes each agents' lifetime expected utility, conditional on the initial distribution of capital, by choice of a state contingent consumption and effort allocation. The planning problem is[5]:

$$\{e^*, K_1^*, c_0^*, c_1^*(\underline{x}), c_1^*(\bar{x})\} = \arg \max_{e\in\{0,1\}, K_1\in[0,K_0],\ c_0, c_1(x)\geq 0} u(c_0) + \beta E_e u(c_1(x)) - v(e) \tag{Problem 1}$$

subject to

$$c_0 + K_1 \leq K_0,\ \ E_e c_1(x) \leq K_1 E_e(1+x), \tag{2}$$

$$\beta E_1 u(c_1(x)) - \beta E_0 u(c_1(x)) \geq v(1) - v(0), \tag{3}$$

where E_e denotes the expectation operator with respect to the probability distribution $\pi(e)$. The constraints in (2) stem from resource feasibility, while (3) is the incentive compatibility constraint, arising from the unobservability of effort. We will denote the value of the optimized objective for Problem 1 with $U^*(K_0)$.

Proposition 1. *An allocation $\{e^*, K_1^*, c_0^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ that solves Problem 1 with $e^* = 1$ satisfies:*

$$\frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))} = \frac{\left[1 + \mu\frac{(\pi(1)-\pi(0))}{\pi(1)}\right]}{\left[1 - \mu\frac{(\pi(1)-\pi(0))}{(1-\pi(1))}\right]} > 1, \tag{4}$$

$$u'(c_0^*) E_1\left[\frac{1}{u'(c_1^*(x))}\right] = \beta E_1(1+x), \tag{5}$$

where $\mu > 0$ is the multiplier on the incentive compatibility constraint (3).

Equation (4) implies that $c_1^*(\bar{x}) > c_1^*(\underline{x}) -$ there is *partial insurance*. Equation (5) determines the intertemporal profile of constrained-efficient consumption. Equation (5) immediately implies:

$$u'(c_0^*) < \beta E_1(1+x) E_1[u'(c_1^*(x))],$$

by Jensen's inequality. Hence, there is a wedge between the entrepreneurs' intertemporal marginal rate of substitution and the aggregate intertemporal rate of transformation, which corresponds to $E_1(1+x)$. Using the first order necessary conditions for the planner's problem, this intertemporal wedge can be written as:

$$\begin{aligned} \mathrm{IW} &= \beta E_1(1+x) E_1 u'(c_1^*(x)) - u'(c_0^*) \\ &= \beta E_1(1+x)\mu(\pi(1)-\pi(0))[u'(c_1^*(\underline{x})) - u'(c_1^*(\bar{x}))] > 0. \end{aligned} \tag{6}$$

The presence of an intertemporal wedge in dynamic economies with private information stems from the influence of outstanding wealth on the agent's attitude towards the risky distribution of outcomes in subsequent periods, which in turn affects incentives. The intertemporal wedge is a measure of the incentive cost of transferring risk-free wealth with

[5] Given that the investment technology is linear in capital, the efficient distribution of capital is degenerate, with one entrepreneur operating the entire economywide capital stock. Since this result is not robust to the introduction of any degree of decreasing returns, and this in turn would not alter the structure of the incentive problem, we simply assume that the planner cannot transfer initial capital across agents.

return $E_1(1+x)$ to a future period. In repeated moral hazard models, as shown in Rogerson (1985), higher risk-free wealth always has an adverse effect on incentives, because it reduces the dependence of consumption on the realization of uncertainty, and therefore on effort. Golosov, Kocherlakota and Tsyvinski (2003) prove that this logic applies to a large class of private information economies.

In this economy, however, entrepreneurial capital is agent specific and associated with idiosyncratic risk in returns. Hence, the individual intertemporal rate of transformation is given by the stochastic variable $1+x$, and does not correspond to $E_1(1+x)$. It is then useful to introduce the notion of an *individual* intertemporal wedge on entrepreneurial capital, and compare it to the *aggregate* intertemporal wedge defined in (6).

We define the individual intertemporal wedge as the difference between the expected discounted value of idiosyncratic capital returns and the marginal utility of current consumption:

$$\mathrm{IW}_K = \beta E_1 u'(c_1^*(x))(1+x) - u'(c_0^*). \tag{7}$$

By (7) and the definition of covariance, it immediately follows that:

$$\mathrm{IW}_K = \mathrm{IW} + \beta Cov_1(u'(c_1^*(x)), x). \tag{8}$$

Equation (4) and strict concavity of utility imply: $Cov_1(u'(c_1^*(x)), x) < 0$. Then, it follows from equation (8) that $\mathrm{IW}_K <\mathrm{IW}$ and that the sign of IW_K can be *positive or negative*.

The sign of the individual intertemporal wedge is related to the effect of capital on entrepreneurial incentives. An entrepreneur's marginal benefit from increasing capital corresponds to the term $\beta E_1 u'(c_1^*(x))(1+x)$, while her marginal cost is $u'(c_0^*)$. A positive value of IW_K signals an additional shadow cost of increasing entrepreneurial capital- the adverse effect of increasing capital on incentives. By contrast, when IW_K is negative, the marginal benefit of an additional unit of entrepreneurial capital is smaller than the entrepreneur's marginal cost. This signals the presence of an additional *shadow benefit* from increasing capital. In this case, more capital in fact relaxes an entrepreneur's incentive compatibility constraint.

These two opposing forces can clearly be seen by deriving IW_K from the first order necessary conditions for Problem 1:

$$\begin{aligned} \mathrm{IW}_K &= \mu(\pi(1)-\pi(0))\beta\left[u'(c_1^*(\underline{x}))(1+\underline{x}) - u'(c_1^*(\bar{x}))(1+\bar{x})\right] \\ &= \mu(\pi(1)-\pi(0))\beta\left\{\left[u'(c_1^*(\underline{x})) - u'(c_1^*(\bar{x}))\right](1+\underline{x}) - (\bar{x}-\underline{x})u'(c_1^*(\underline{x}))\right\}. \end{aligned} \tag{9}$$

The second line of equation (9) decomposes this wedge into a wealth effect, which corresponds to the first term inside the curly brackets, and an opposing substitution effect. The wealth effect captures the adverse effect of capital on incentives, arising from the fact that more capital increases consumption in the bad state. This provides insurance and tends to reduce effort for higher holdings of capital. The substitution effect captures the positive effect of capital on incentives. This effect is linked to the positive dependence of expected capital returns on entrepreneurial effort and tends to increase effort at higher levels of capital. The size of the wealth effect is positively related to the spread in consumption across states that drives the entrepreneurs' demand for insurance. The strength of the substitution effect depends on the spread in capital returns, which determines by how much the expected return from capital increases under high effort.

By a similar reasoning, the aggregate intertemporal wedge captures the incentive effects of increasing holdings of a risk-free asset with return equal to the expected return to entrepreneurial capital, $E_1 (1+x)$. Clearly, by (6) the marginal benefit is always greater than the marginal cost, due to the fact that higher holdings of such an asset would reduce the correlation between consumption and idiosyncratic capital returns, x, and tighten the incentive compatibility constraint. This observation will play a role in the fiscal implementation of the optimal allocation. We will show in section 3.1 that the after tax return on any risk free asset is equal to $E_1 (1+x)$ in equilibrium. The differential incentive effects of entrepreneurial capital and a riskless asset with the same expected return will lead to a prescription of optimal differential taxation of these assets.

2.1. A Sufficient Condition for $\mathbf{IW}_K < 0$

It is possible to derive an intuitive condition that guarantees a negative individual intertemporal wedge. This condition simply amounts to the coefficient of relative risk aversion being weakly smaller than 1. No additional restrictions on preferences or the returns process are necessary.

To prove this result, we first establish the following Lemma.

Lemma 2. *If* $\{e^*, K_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ *solve Problem 1 and* $e^* = 1$, *then* $(1+\overline{x})K_1^* \geq c_1^*(\overline{x}) > c_1^*(\underline{x}) \geq (1+\underline{x})K_1^*$.

The lemma states that the variance of consumption is always smaller than the variance of earnings at the constrained-efficient allocation. We can then state the following proposition.

Proposition 3. *Let* $\sigma(c) \equiv -cu''(c)/u'(c)$ *denote the coefficient of relative risk aversion for the utility function* $u(c)$. *Then,* $IW_K < 0$ *for* $\sigma(c) \leq 1$.

This result can be understood drawing from portfolio theory. As shown in Gollier (2001), the amount of holdings of an asset increase in the expected rate of return when the substitution effect dominates, for $\sigma(c) < 1$. Since under high effort the rate of return on capital is higher than under low effort, a similar intuition applies in this case.[6]

How relevant is this finding? The value of the coefficient of relative risk aversion is very disputed, due to difficulties in estimation. Typical values of $\sigma(c)$ used in the macroeconomic and financial literature under constant relative risk aversion preferences largely exceed 1. On the other hand, Chetty (2006) develops a new method for estimating this parameter using data on labor supply behavior to bound the coefficient of relative risk aversion. He argues that for preferences that are separable in consumption and labor effort, $\sigma(c) \leq 1$ is

[6] Levhari and Srinivasan (1969) and Sandmo (1970) study precautionary holdings of risky assets and discuss similar effects.

the only empirically relevant case. This finding suggests that low values of $\sigma(c)$, relatively to those used in macroeconomics, may be quite plausible.

Since Proposition 3 is merely sufficient, even if $\sigma(c) > 1$, IW_K can be negative for $\bar{x} - \underline{x}$ large enough. In addition, for given variance of capital returns, it is more likely for IW_K to be negative when $\sigma(c)$ large, since when risk aversion is high the spread in consumption across states at the optimal allocation will be small in this case. Hence, for $\sigma(c) > 1$, the sign of IW_K is a quantitative question and data on the variance of entrepreneurial earnings as well as information on risk aversion is required to provide an answer. In the next section, we turn to some numerical examples to illustrate the possibilities.

2.2. Numerical Examples

To investigate the properties of optimal allocations in more detail, we now turn to numerical examples. We assume $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ for $\sigma > 0$ and $v(e) = \gamma e$, $\gamma > 0$. Here, σ corresponds to the coefficient of relative risk aversion and γ is the cost of high effort. We set $K_0 = 1$ and $\gamma = 0.08$. We assume that the probability a high capital returns depends linearly on effort, according to $\pi(e) = a + be$, with $a \geq 0$, $b > 0$ and $2a + b \leq 1$. The parameter b represents the impact of effort on capital returns. We consider values of a and b such that the standard deviation of x under high and low effort is equalized. This requires $a = 0.25$ and and $b = 0.5$ and implies $\pi(1) = 0.75$ and $\pi(0) = 0.25$. Finally, we set $E_1 x = 0.3$.

We consider three examples. In the first two, we fix $\{\underline{x}, \bar{x}\}$ and let σ vary between 0.95 and 8. The spread in capital returns is greater in the first example than in the second example, leading to a standard deviation of x equal to 14% and 12%, respectively. In the third example, we fix $\sigma = 1.6$ and let $\underline{x}$ vary between 0 and 0.0940 keeping $E_1 x$ fixed, so that SD_1 ranges from 0.17 to 0.12,[7] where SD_e denotes the standard deviation conditional on effort e.[8] The parameters are summarized in Table 1.

[7] If we identify entrepreneurial capital with private equity, then x corresponds to the net returns on private equity. Moskowitz and Vissing-Jorgensen (2002) estimate these returns using the Survey of Consumer Finances. They find that the average returns to private equity, including capital gains and earnings, are 12.3, 17.0 and 22.2 percent per year in the time periods 1990-1992, 1993-1995, 1996-1998. It is much harder to estimate the variance of idiosyncratic returns. Evidence from distributions of entrepreneurial earnings, conditional on survival, suggest that this variance is much higher than for public equity.

[8] The standard deviation of x conditional on high effort is inversely related to $\underline{x}$, for fixed $E_1 x$.

Table 1: Numerical Examples			
Parameters	Example 1	Example 2	Example 3
σ	$[0.95, 8]$	$[0.95, 8]$	1.6
$\underline{x}$	0.05	0.094	$[0, 0.094]$
$\bar{x}$	0.3833	0.3687	$[0.3687, 0.40]$
$E_1 x$	0.30	0.30	0.30
$E_0 x$	0.1333	0.1627	$[0.10, 0.1627]$
SD_1	0.1443	0.1189	$[0.12, 0.17]$
γ	0.08	0.08	0.08
K_0	1	1	1
a	0.25	0.25	0.25
b	0.5	0.5	0.5

Our findings are displayed in figure 1. Each row corresponds to a different example. The left panels display the individual intertemporal wedge (solid line) and the aggregate intertemporal wedge (dashed line). The right panels display c_0^* (dashed line), $c_1^*(x)$ (solid lines) in each state and earnings $K_1^*(x)$ (dotted lines) in each state. In all examples, high effort is optimal for all parameter values reported.

In the first two examples, the individual intertemporal wedge is non-monotonic in σ. It is negative and rising in σ for $\sigma \leq 1.6$, it then declines and starts rising again for σ approximately equal to 4, converging to 0 from below. It is always negative for high enough values of σ, since the spread across states in optimal consumption decreases with σ, for given spread in capital returns, which decreases the wealth effect as illustrated by equation (9).[9] In the first example, the individual intertemporal wedge is negative for all values of σ, while in the second example it is positive for values of σ between 1.3 and 2.2. This is due to the larger spread in capital returns is greater in example 1, which increases the substitution effect isolated in equation (9). The aggregate intertemporal wedge is always positive, but is also displays a non monotonic pattern in σ in both examples, initially rising and then declining in this variable. It tends to 0 for high enough values of σ, since the spread in consumption across states is vanishingly small. For higher values of σ than the ones reported, the optimal effort drops to 0. In that case, entrepreneurs are given full insurance and there are no intertemporal wedges.

The third example σ is fixed at 1.6 -the value that maximizes IW_K in examples 1 and 2- and the spread in capital returns is made to vary keeping the mean constant. All other parameters are as in the previous examples. The individual intertemporal wedge monotonically decreases as the spread in capital returns rises, and turns negative for $SD_1(x)$ greater than 12.5%. The constrained-efficient levels of consumption and capital, as well as the aggregate intertemporal wedge, only depend on expected capital returns and do not vary with the spread in capital returns.

[9] The fact that IW_K starts rising for values of σ greater than 4 is due to the fact that for $\sigma \geq 4$, $c_1^*(\bar{x})$ is approximately costant, while $c_1^*(\underline{x})$ continues to rise with σ. By (9), this causes IW_K to rise.

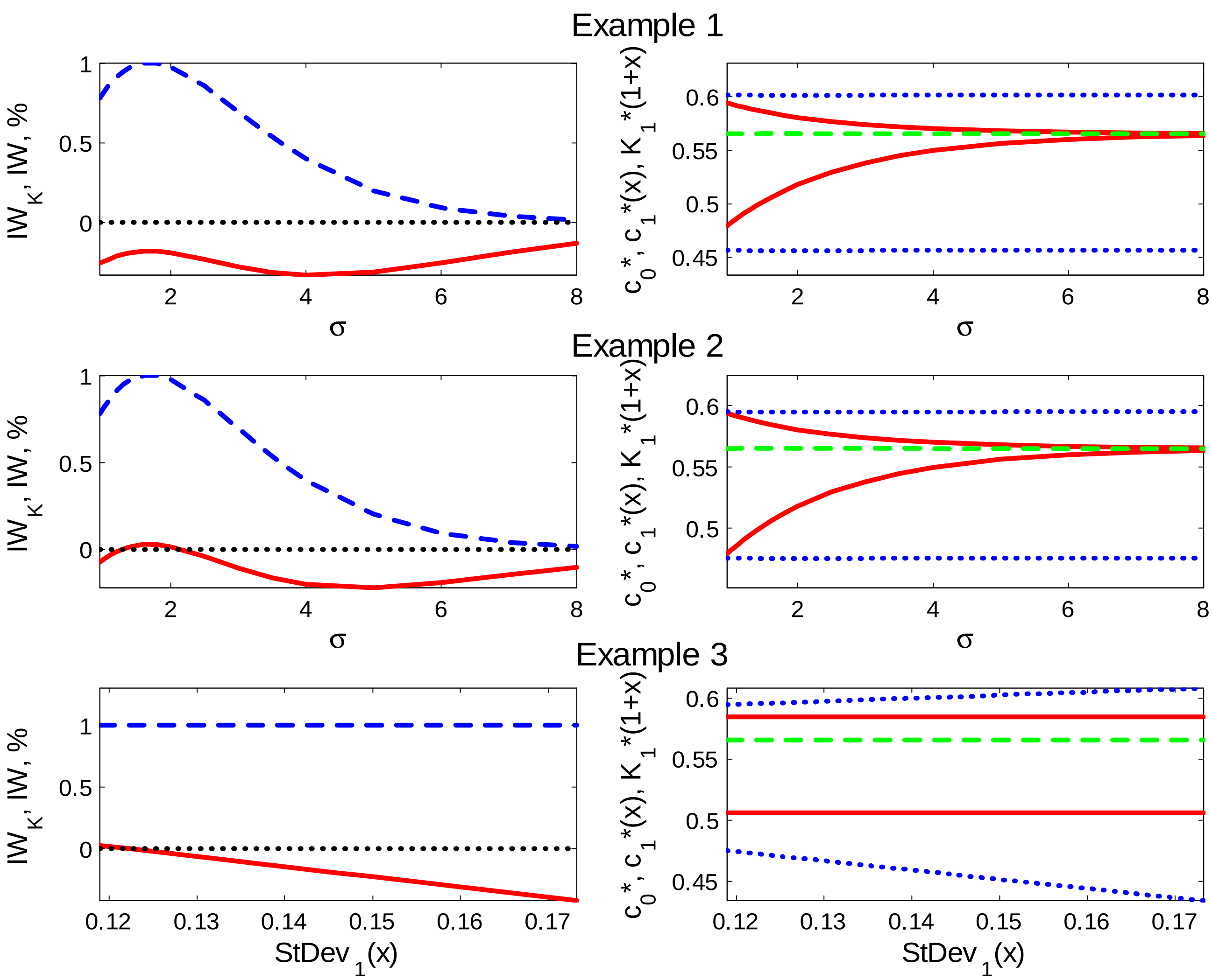

Figure 1: Constrained-efficient allocations in three numerical examples.

3. Optimal Taxes

We now consider how to implement constrained-efficient allocations in a setting where agents can trade in competitive markets. We explore different market structures. A market structure specifies the distribution of ownership rights, the feasible trades between agents and any additional informational assumptions beyond the primitive restrictions that comprise the physical environment. Agents are subject to taxes that influence their budget constraints. A tax system implements the constrained-efficient allocation if such an allocation arises as the competitive equilibrium outcome under this tax system for a particular market structure. This requires that individuals find that allocation optimal given the tax system and prices, and that those prices satisfy market clearing. The optimal tax system is the one that implements the constrained-efficient allocation. The only ex ante constraint imposed on candidate tax systems is that the resulting taxes or transfers must be conditioned only on individual characteristics that are observable.

The first market structure we consider allows entrepreneurs to independently choose capital and effort, as well as trade financial securities in zero net supply. These securities are exogenously introduced and the implicit assumption is that they are costlessly issued. We consider the case of a risk-free bond and also allow for the possibility that these securities are contingent on idiosyncratic capital returns. In the second market structure, we allow entrepreneurs to sell shares of their capital to outside investors giving rise to an equity market. Since capital returns are i.i.d. across entrepreneurs it is also possible to form risk-free portfolios. Both these market structures assume entrepreneurial portfolios to be fully observable. In the last market structure, we allow for competitive insurance markets, as well as financial securities, and assume that the entrepreneur's total security holdings are not observed by insurance companies or the government.

3.1. Optimal Differential Asset Taxation

The first market structure we consider is one in which agents can trade risk-free bonds and independently choose investment as well as effort at time 0. The risk-free bonds yield a return r in period 1, which is determined in equilibrium. Decisions occur as follows. Agents are endowed with initial capital K_0 and choose K_1 and bond purchases B_1 at the beginning of period 0, and they consume. They then exert effort. At the beginning of period 1, x is realized. Finally, the government collects taxes and agents consume. The informational structure is as follows: K_1 and x are public information, while effort is private information. We also assume that bond purchases B_1 are public information. The tax system is given by a time 1 transfer from the agents to the government which is conditional on observables and represented by the function $T(B_1, K_1, x)$. We restrict attention to functions T that are differentiable almost everywhere in their first argument and satisfy $E_1 T(B_1, K_1, x) = 0$, which corresponds to the government budget constraint, given that the government does not have any spending requirements.

An entrepreneur's problem is:

$$\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}(B_0, K_0, T) = \arg \max_{K_1 \in [0, K_0],\ B_1 \geq \bar{B},\ e \in \{0,1\}} U(e, K_1, B_1; T) - v(e), \quad \text{(Problem 3)}$$

where

$$U(e, K_1, B_1; T) = u(K_0 + B_0 - K_1 - B_1) + E_e u(K_1(1+x) + B_1(1+r) - T(K_1, B_1, x)),$$

subject to $K_0 + B_0 - K_1 - B_1 \geq 0$ and $K_1(1+x) + (1+r)B_1 - T(B_1, K_1, x) \geq 0$ for $x \in X$. Here, the debt limit $\bar{B}$ is imposed to ensure that an agent's problem is well defined. The natural debt limit for tax systems in the class $T(B_1, K_1, x) = \rho(x) + \tau_B(x) B_1 + \tau_K(x) K_1$ is $\bar{B} = -\frac{[K_1(1+\underline{x}-\tau_K(\underline{x}))-\rho(\underline{x})]}{1+r-\tau_B(\underline{x})}$. This limit ensures that agents will be able to pay back all outstanding debt in the low state. The initial bond endowment, B_0, can be interpreted as a transfer from the government to the entrepreneurs.

Definition 4. *A competitive equilibrium is an allocation* $\{c_0, e, K_1, B_1, c_1(\underline{x}), c_1(\bar{x})\}$ *and initial endowments* B_0 *and* K_0 *for the entrepreneurs, a tax system* $T(K_1, B_1, x)$, *with* $T : [\bar{B}, \infty) \times [0, \infty) \times \{\underline{x}, \bar{x}\} \rightarrow \mathbb{R}$, *government bonds* B_1^G, *and an interest rate,* $r \geq 0$, *such that: i) given* T *and* r *and the initial endowments, the allocation solves Problem 3; ii) the government budget constraint holds in each period; iii) the bond market clears,* $B_1^G = B_1$.

The restriction on the domain of the tax system is imposed to ensure that the tax is specified for all values of K_1 and B_1 feasible for the entrepreneurs. We now define our notion of implementation.

Definition 5. *A tax system* $T : [\bar{B}, \infty) \times [0, \infty) \times \{\underline{x}, \bar{x}\} \rightarrow \mathbb{R}$ *implements the constrained-efficient allocation, if the allocation* $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$, *the tax system* T, *jointly with an interest rate* r, *government bonds* B_1^G, *and initial endowments* B_0 *and* K_0 *constitute a competitive equilibrium.*

We restrict attention to tax systems of the form: $T(K_1, B_1, x) = \rho(x) + \tau_K(x) K_1 + \tau_B(x) B_1$. Let $B_1^* \geq \bar{B}$ a level of bond holdings to be implemented. Since entrepreneurs are all ex ante identical, if the government does not issue any bonds, $B_1 = B_0 = 0$ in any competitive equilibrium, so that $B_1^* = 0$. Otherwise, $B_1^* = B_1^G$.[10]

We begin our characterization with a negative result and identify a tax systems in the class $T(K_1, B_1, x)$ that does not implement the constrained-efficient allocation. Let B_0 and $T(K_1^*, B_1^*, x)$ respectively satisfy:

$$c_0^* = B_0 + K_0 - K_1^* - B_1^*, \tag{10}$$

$$c_1^*(x) = K_1^*(1+x) + (1+r)B_1^* - T(K_1^*, B_1^*, x). \tag{11}$$

[10] Our definition of competitive equilibrium allows the government to issue bonds at time 0, denoted B_1^G. The government budget constraints at time 0 and at time 1 are, respectively, $B_0 - B_1^G \leq 0$ and $E_e T(K_1, B_1, x) - B_1^G(1+r) \geq 0$, where e corresponds to the effort chosen by the entrepreneurs in equilibrium. Given that the government does not need to finance any expenditures, the amount of government bonds issued does not influence equilibrium consumption, capital and effort allocations, or the equilibrium interest rate. However, if the government did have an expenditure stream to finance, the choice of bond holdings would be consequential.

Then, K_1^* and B_1^* are affordable and, if they are chosen by an entrepreneur, incentive compatibility implies that high effort will also be chosen at time 1. Evaluating the entrepreneurs' Euler equation at $\{1, K_1^*, B_1^*\}$, we can write:

$$u'(c_0^*) = \beta E_1 [u'(c_1^*(x))(1 + x - \tau_K(x))], \tag{12}$$
$$u'(c_0^*) = \beta E_1 [u'(c_1^*(x))(1 + r - \tau_B(x))]. \tag{13}$$

The restrictions on $T(K_1^*, B_1^*, x)$ implied by (10)-(11) and (12)-(13) do not fully pin down the tax system and do not ensure that the constrained-efficient allocation is chosen by an entrepreneur. To see this, let $\tau_K(\bar{x}) = \tau_K(\underline{x}) = \bar{\tau}_K$ and $\tau_B(\bar{x}) = \tau_B(\underline{x}) = \bar{\tau}_B$, so that marginal asset taxes do not depend on x, with $\bar{\tau}_K$ and $\bar{\tau}_B$ that satisfy (12)-(13). Then, $\bar{\tau}_K$ has the same sign as the intertemporal wedge on capital, while $\bar{\tau}_B$ is always positive, since the intertemporal wedge on the bond is positive. Set $\bar{\rho}(x)$ so that (11) holds under $\bar{\tau}_K$, $\bar{\tau}_B$, and let $\bar{T}(K_1, B_1, x) = \bar{\rho}(x) + \bar{\tau}_K K_1 + \bar{\tau}_B B_1$. It follows that:

$$u'(c_0^*) \lesseqgtr \beta E_0 [u'(c_1^*(x))(1 + x - \bar{\tau}_K)] \text{ if } \mathrm{IW}_K \gtreqless 0, \tag{14}$$
$$u'(c_0^*) < \beta(1 + r - \bar{\tau}_B) E_0 u'(c_1^*(x)). \tag{15}$$

Since the incentive compatibility constraint is binding, these equations imply that if agents could only invest in bonds, they would find it optimal to choose bond holdings greater than B_1^* and low effort, while if they could only invest in capital they would find it optimal to choose low effort and a level of capital lower/higher than K_1^* if the intertemporal wedge is negative/positive. However, as shown in the following lemma, since entrepreneurs can invest in both capital and bonds, the optimal deviation under $\bar{T}$ involves an extreme portfolio choice. When the intertemporal wedge is negative, the optimal deviation under $\bar{T}$ is to set entrepreneurial capital equal to 0.

Lemma 6. *Under tax system $\bar{T}$, $\hat{e} = 0$. If $\mathrm{IW}_K > 0$, $\hat{B}_1 = \underline{B}$ and $\hat{K}_1 > K_1^*$; if $\mathrm{IW}_K < 0$, $\hat{K}_1 = 0$ and $\hat{B}_1 > B_1^*$.*

This lemma shows that rather than choose $\{1, K_1^*, B_1^*\}$, which is affordable and satisfies first order necessary conditions, entrepreneurs find it optimal to choose low effort and adjust their portfolio under the tax system $\bar{T}(K_1, B_1, x)$. Hence, it does not implement the constrained-efficient allocation.[11]

We now construct a tax system that does implement the constrained-efficient allocation. The critical properties of this system are that marginal asset taxes depend on observable capital returns and that after tax returns are equalized across all assets, state by state.

[11] The result that non-state dependent marginal asset taxes allow for devations from the constrained-efficient allocation also holds in economies with idiosyncratic labor risk, as discussed in Albanesi and Sleet (2006) and Kocherlakota (2005). Golosov and Tsyvinski (2006) derive a related result in a disability insurance model.

Proposition 7. *A tax system* $T^*(B_1, K_1, x) = \rho^*(x) + \tau_B^*(x) B_1 + \tau_K^*(x) K_1$, *with* $T^* : [\bar{B}, \infty) \times [0, \infty) \times \{\underline{x}, \bar{x}\} \rightarrow \mathbb{R}$, *and an initial bond endowment* B_0^* *that satisfy:*

$$1 + r - \tau_B^*(x) = \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}, \tag{16}$$

$$1 + x - \tau_K^*(x) = \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}, \tag{17}$$

$$c_1^*(x) = K_1^*(1 + x - \tau_K^*(x)) + B_1^*(1 + r - \tau_B^*(x)) - \rho^*(x), \tag{18}$$

and

$$c_0^* = B_0^* + K_0 - K_1^* - B_1^*, \tag{19}$$

ensure that the allocation $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ *is optimal for entrepreneurs for some* $B_1^* \geq \bar{B}$ *and some* $r \geq 0$.

The proof proceeds in three steps. It first shows that the only interior solution to the entrepreneur's Euler equations are B_1^* and K_1^* under T^*, and that local second order conditions are satisfied. It then shows that T^* admits no corner solutions to the choice of K_1 and B_1. Moreover, these results do not depend on the value of effort used to compute expectations over time 1 outcomes. Then, K_1^* and B_1^* are the unique solutions to an entrepreneur's portfolio problem irrespective of the value of effort that she might be contemplating. The last step establishes than $\rho^*(x)$ guarantees that, once K_1^* and B_1^*, have been chosen, high effort will be optimal.

The optimal tax system T^* has two main properties. It removes the complementarity between the choice of effort and the choice of capital and bond holdings, thus removing any incentive effects of the entrepreneurs' asset choice. This guarantees that the necessary and sufficient conditions for the *joint* global optimality of K_1^* and B_1^* are satisfied at all effort levels. Moreover, T^* equates after tax returns on all assets in each state. This renders entrepreneurs indifferent over the composition of their portfolio. The next corollary establishes that the tax system T^* implements the constrained-efficient allocation.

Corollary 8. *The tax system* $T^*(K_1, B_1, x)$ *and initial bond endowment* B_0^* *defined in Proposition 7, jointly with the allocation* $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$, *and government bonds* B_1^G, *with* $B_0^* = B_1^* = B_1^G \geq \bar{B}$, *a return* r, *constitute a competitive equilibrium for the market economy with initial capital* K_0.

The following proposition characterizes the properties of the optimal tax system.

Proposition 9. *The tax system* $T^*(B_1, K_1, x)$ *defined in Proposition 7 implies:*
i) $E_1 \tau_K^*(x) = 0$;
ii) $E_1(x) = r - E_1 \tau_B^*(x)$;
iii) $sign(\tau_K^*(\bar{x}) - \tau_K^*(\underline{x})) = sign(-IW_K)$;
iv) $\tau_B^*(\bar{x}) < \tau_B^*(\underline{x})$;
v) $\tau_B^*(\underline{x}) > \tau_K^*(\underline{x})$ *and* $\tau_B^*(\bar{x}) < \tau_K^*(\bar{x})$.

The average marginal capital tax is zero. Result ii) in proposition 9 implies that the expected after tax return on any risk-free asset is equal to the expected return on entrepreneurial capital. This implies that under T^*, the equilibrium values of r and $E_1\tau_B^*(x)$ are not separately pinned down. This indeterminacy does not affect the dependence of marginal bond taxes on x, which is governed by (17). Hence, without loss of generality we restrict attention to competitive equilibria with $r = E_1(x)$ and $E_1\tau_B^*(x) = 0$.

Result iii) states that the marginal capital tax is decreasing in capital returns, if the individual intertemporal wedge is positive, while it is increasing in capital returns if it is negative. The incentive effects of capital provide intuition for this result. Following the reasoning in section 2, when $\mathrm{IW}_K > 0$, more capital tightens the incentive compatibility constraint. Hence, the optimal tax system discourages agents from setting K_1 too high by increasing the after tax volatility of capital returns. Instead, for $\mathrm{IW}_K < 0$, more capital relaxes the incentive compatibility constraint. The optimal tax system encourages entrepreneurs to hold capital by reducing the after tax volatility of capital returns. By result ii), the intertemporal wedge on the bond is equal to the aggregate intertemporal wedge IW, and hence is positive. Then, higher holdings of B_1 tighten the entrepreneurs' incentive compatibility constraints. This explains result iv), that marginal bond taxes are decreasing in entrepreneurial earnings. The optimal tax system discourages entrepreneurs from holding B_1 in excess of B_1^* by making bonds a bad hedge against idiosyncratic capital risk. Finally, result v) states that capital is subsidized with respect to bonds in the bad state. This results stems from the fact that consumption and entrepreneurial earnings are positively correlated at the optimal allocation, which means that capital returns and the inverse of the stochastic discount factor, which pins down marginal taxes, are also positively correlated. By definition, there is no correlation between bond returns and the inverse of the stochastic discount factor.

To illustrate the properties of optimal marginal asset taxes, we plot them for the numerical examples analyzed in section 2.2 in figure 2, assuming $r = E_1(x)$. Each row corresponds to one of the examples, the left panels plot the marginal capital taxes, while the right panels plot the marginal bond taxes. The solid line plots the intertemporal wedge for the corresponding asset. The dashed-star line corresponds to marginal taxes in state $\underline{x}$, whereas the dashed-cross line corresponds to optimal marginal taxes in state $\bar{x}$. The vertical scale is in percentage points and is the same for all panels.

The first example is one in which the individual intertemporal wedge is always negative. The marginal tax on capital is negative in the low state and positive in the good state, while the opposite is true for the marginal tax on bonds. Hence, the marginal capital tax is increasing in earnings, while the marginal bond tax is decreasing in earnings. The second row corresponds to the example with lower spread in capital returns, which exhibits a positive individual intertemporal wedge for intermediate values of the coefficient of relative risk aversion σ. The third row reports the optimal marginal asset taxes for the third example, in which σ is fixed and we vary the spread in capital returns. In the second and third examples, when $\mathrm{IW}_K > 0$, the marginal tax on entrepreneurial capital is also decreasing in x, positive in the bad state and negative in the good state. However, for all examples, it is always the case that the marginal tax on capital is smaller than the one on bonds in the low earnings state, $\underline{x}$. In the third example, since the constrained-efficient allocation only depends on the expected value of capital returns (held constant here) and not on their spread, the marginal bond tax taxes are constant. Instead, as discussed, the intertemporal wedge on capital is decreasing in the spread of capital returns.

Despite the fact that wedges are everywhere quite small in percentage terms, the magnitude of marginal taxes is significant. The capital tax ranges from 2 to 23% in absolute value, while the bond tax ranges from 0 to 30% in absolute value.

The main finding in the fiscal implementation for the market structure considered in this section is the *optimality of differential asset taxation.* The optimal tax system equalizes after tax returns on entrepreneurial capital and riskless bonds, thus it reduces the after tax spread in capital returns and it increases the after tax spread in the returns to the riskless bond. Consequently, entrepreneurial capital is subsidized relatively to a riskless asset in the bad state.

These results can be generalized to risky securities. Let $r(x) > 0$ for $x = \underline{x}, \bar{x}$, denote the return to a security S_1 in zero net supply. Assume that entrepreneurs can trade this security at price q at time 0. Letting the candidate tax system be given by $T(S_1, K_1, x) = \tau_K(x) K_1 + \tau_S(x) S_1 + \rho(x)$ .Set $\tau_K^*(x)$ and $\rho^*(x)$ as in (17) and (18) for $S_1^* = 0$. Set marginal taxes on the security according to:

$$1 + r(x) - \tau_S^*(x) = \frac{qu'(c_0^*)}{\beta u'(c_1^*(x))}. \tag{20}$$

Following a proof strategy similar to that in Proposition 7, it is possible to show that the resulting tax system implements the constrained-efficient allocation.

The equilibrium price of the security is $q = \frac{E_1\left(1+r(x)-\tau_S^*(x)\right)}{E_1\left(1+x-\tau_K^*(x)\right)}$[12]. Then, (20) implies $E_1\tilde{r}(x) = E_1 x$, where $\tilde{r}(x)$ is the equilibrium rate of return on this security, $\tilde{r}(x) = \frac{1+r(x)}{q} - 1$. The intertemporal wedge on the risky security is:

$$\mathrm{IW}_S = E_1 u'(c_1^*(x))(1 + \tilde{r}(x)) - u'(c_0^*),$$

Let $Corr_e$ denote the correlation conditional on $\pi(e)$. Then, the following result holds.

Proposition 10. *If $Cov_1(\tilde{r}(x), x) > 0$ and $V_1(x) > V_1(\tilde{r}(x))$, then:*

$$E_1 u'(c_1^*(x))(1 + \tilde{r}(x)) > E_1 u'(c_1^*(x))(1 + x),$$

$$\tau_S^*(\bar{x}) - \tau_K^*(\bar{x}) < 0 \text{ and } \tau_S^*(\underline{x}) - \tau_K^*(\underline{x}) > 0.$$

If $Cov_1(\tilde{r}(x), x) > 0$ and $V_1(x) > V_1(\tilde{r}(x))$, $Corr_1(\tilde{r}(x), x) \in (0, 1)$. The proposition states that a security positively correlated with capital with lower variance of returns has

[12] As in the case with risk-free bonds, the equilibrium expected return on this security is not separately pinned down from $E_1\tau_s^*(x)$.

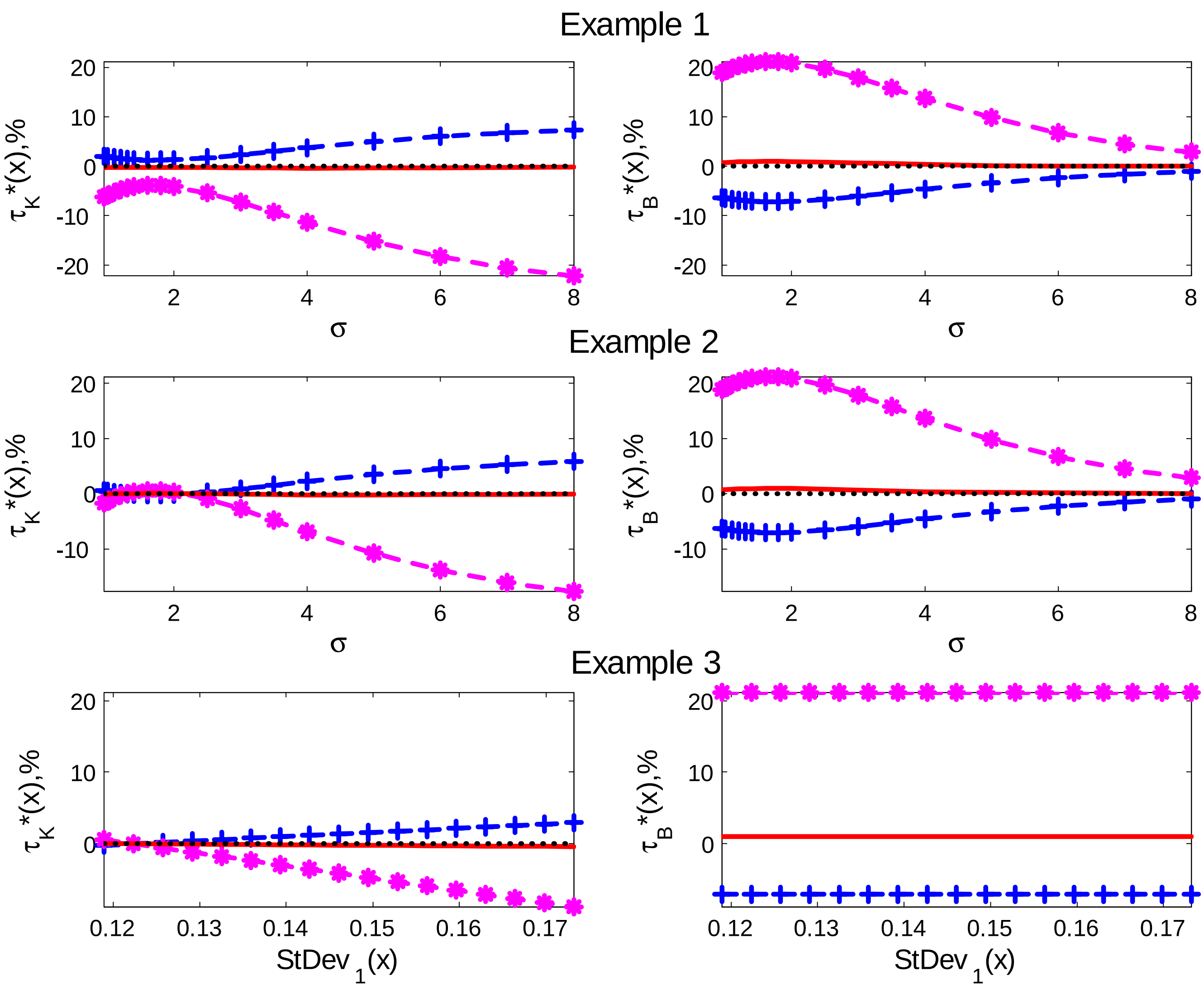

Figure 2: Optimal marginal taxes on entrepreneurial capital and bonds.

a higher intertemporal wedge than capital. An entrepreneur would be willing to hold such a security instead of capital, since it is associated with lower earnings risk. However, this has an adverse effect on incentives. This motivates the higher intertemporal wedge and the fact that $\tau_S^*(x) - \tau_K^*(x)$ is decreasing in x, which implies that capital is subsidized with respect to the risky security in the bad state.

This finding points to a general principle. The correlation of an asset's returns with the idiosyncratic risk that determines the asset's effects on the entrepreneurs' incentives to exert effort and, consequently, the properties of optimal marginal taxes on the asset.

In this implementation, we considered risk-free bonds and other financial securities in zero net supply. In the next section, we consider an implementation in which entrepreneurs can sell shares of their own capital to external investors, thus giving rise to an equity market with a positive supply of securities.

3.2. Optimal Capital Taxation with External Ownership

We now allow entrepreneurs to sell shares of their capital and buy shares of other entrepreneurs' capital. Each entrepreneur can be interpreted as a firm, so that this arrangement introduces an equity market. The amount of capital invested by an entrepreneur can be interpreted as the size of their firm.

An entrepreneur's budget constraint in each period is :

$$c_0 = K_0 - K_1 - \int_{i\in[0,1]} S_1(i)\, di + sK_1, \tag{21}$$

$$c_1(x) = K_1(1+x) - sK_1(1+d(x)) + \int_{i\in[0,1]} (1+D(i))\, S_1(i)\, di - T\left(K_1, s, \{S_1\}_i, x\right), \tag{22}$$

where $s \in [0,1]$ is the fraction of capital sold to outside investors, $d(x)$ denotes dividends distributed to shareholders, $S_1(i)$ is the value of shares in company i in an entrepreneur's portfolio and $D(i,\tilde{x})$ denotes dividends earned from each share of company i if the realized returns are $\tilde{x}$ for $\tilde{x} \in X$. Let $D(i) = E_{\hat{e}} D(i,\tilde{x})$ denote expected returns for stocks in firm i. Gross stock earnings for an entrepreneur with equity portfolio $\{S_1(i)\}_i$ are given by $\int_{i\in[0,1]} (1+D(i))\, S_1(i)\, di$, where $D(i)$ denotes expected dividends from firm i. Since $D(i,\tilde{x}) = d(\tilde{x})$ for all i and $\tilde{x}$ is i.i.d., $D(i) = \bar{D}$ for all $i = [0,1]$. The dividend distribution policy is taken as given by the entrepreneurs and the shareholders. This arrangement should be interpreted as part of the share issuing agreement. Entrepreneurs choose K_1, $\{S_1(i)\}_i$ as well as effort at time 0, taking as given the distribution policy, dividends and taxes. At time 1, x is realized, dividends are distributed, the government collects taxes and the entrepreneurs consume. The variables K_1, x, $S_1(i)$, s and $d(x)$ are public information.

We consider candidate tax systems of the form $T\left(K_1, \{S_1\}_i, x\right) = \tau_P(x)(1+x)K_1 + \tau_s(x)\int_i S_1(i)\, di + \rho(x)$. Here, $\tau_P(x)$ can be interpreted as a marginal tax on entrepreneurial earnings. The marginal tax on stock returns, $\tau_S(x)$, depends only the realization of x for the agent holding the stock and is the same for all stocks since stock returns are i.i.d.

The entrepreneurs' problem is:

$$\left\{\hat{e}, \hat{K}_1, \hat{s}, \left\{\hat{S}_1(i)\right\}_i\right\}(K_0, T) = \arg \max_{\hat{e}, \hat{K}_1, \hat{s}, \left\{\hat{S}_1(i)\right\}_i} u(c_0) + E_e u(c_1) - v(e), \qquad \text{(Problem 4)}$$

subject to (21), (22) and $\int_{i \in [0,1]} S_1(i)\, di \geq \bar{B} = \frac{K_1(1+\underline{x})(1-\tau_P(\underline{x}))-\rho(\underline{x})}{(1-\tau_S(\underline{x}))\int_{i\in[0,1]}(1+D(i))di}$, where $\bar{B}$ is the natural borrowing limit.

The Euler equations for this problem are:

$$\begin{aligned} & -(1-s)\{u'(c_0) - \beta E_{\hat{e}}[(1+x)(1-\tau_P(x))u'(c_1(x))]\} \\ & +\beta s E_{\hat{e}}[(1+x)(1-\tau_P(x)) - (1+d(x))]u'(c_1(x)) \begin{cases} = 0 \text{ for } K_1 > 0 \\ \leq 0 \text{ for } K_1 = 0 \end{cases}, \end{aligned} \tag{23}$$

$$-u'(c_0) + \beta E_{\hat{e}}(1 + D(i) - \tau_S(x))u'(c_1(x)) = 0, \tag{24}$$

$$[u'(c_0) - \beta E_{\hat{e}}(1+d(x))u'(c_1(x))]K_1 \begin{cases} = 0 \text{ for } s \in (0,1) \\ \leq 0 \text{ for } s = 0 \\ > 0 \text{ for } s = 1. \end{cases}. \tag{25}$$

We define a competitive equilibrium for this trading structure and then consider how to implement the constrained-efficient allocation.

Definition 11. *A competitive equilibrium is an allocation* $\left\{c_0,\ \hat{e},\ \hat{K}_1,\ \hat{s},\ \left\{\hat{S}_1(i)\right\}_i,\ \hat{c}_1(x)\right\}$ *with* $\hat{s} \in [0,1]$, *a distribution policy* $\hat{d}(x)$ *and a dividend process* $\hat{D}(i,x)$ *for* $i \in [0,1]$, $x \in X$, *and a tax system* $T(K_1, \{S_1\}_i, x)$, *such that:*

i) the allocation $\left\{\hat{e}, \hat{K}_1, \hat{s}, \left\{\hat{S}_1(i)\right\}_i\right\}$ *solves the entrepreneurs' problem, for given* $\hat{d}(x)$, $\hat{D}(i,\tilde{x})$, *and* T;

ii) the dividend process is consistent with the distribution policy, $\hat{d}(x) = \hat{D}(i,x)$ *for all* i *and* $x \in X$;

ii) the stock market clears, $\hat{s}\hat{K}_1 = \hat{S}_1(i)$ *for* $i = [0,1]$;

iii) the resource constraint is satisfied in each period.

Since all entrepreneurs are ex ante identical, we restrict attention to symmetric equilibria in which s, K_1 and effort are constant for all entrepreneurs. The entrepreneurs face a portfolio problem in the selection of stocks. Given that all stocks have the same expected return net of taxes under the family of tax systems defined by $T(K_1, \{S_1\}_i, x)$. entrepreneurs are indifferent over which stocks to hold. However, they will always hold a continuum of stocks, since this ensures that their portfolio has zero variance. To break the entrepreneurs' indifference over portfolio selection, we assume that all entrepreneurs hold a perfectly differentiated portfolio. Hence, $\bar{D}$ corresponds to gross portfolio returns in equilibrium and we can restrict attention to the case $S_1(i) = S_1$ for all i.

We now construct a tax system that implements the constrained-efficient allocation. Set the marginal profit tax is $\tau_P^*(x)$ as follows:

$$(1+x)(1-\tau_P^*(x)) = \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}. \tag{26}$$

Let $d^*(x) = (1+x)(1-\tau_P(x)) - 1$, so that dividends per share are simply given by after tax profits. This implies: $\bar{D}^* = E_1(1+x)(1-\tau_P^*(x)) - 1$. Set $\tau_S^*(x)$ so that:

$$1 + \bar{D}^* - \tau_S^*(x) = \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}. \tag{27}$$

Lastly, we choose $\rho^*(x)$ to satisfy:

$$\begin{aligned} c_1^*(x) &= K_1^*(1+x)(1-\tau_P^*(x)) - s^* K_1^*(1+d^*(x)) \\ &\quad + \left(1+\bar{D}^* - \tau_S^*(x)\right) S_1^* - \rho^*(x), \end{aligned} \tag{28}$$

for some $s^* \in [0,1)$, with $S_1^* = s^* K_1^*$.

We now prove that the tax system $T^*(K_1, \{S_1(i)\}_i, x)$ implements the constrained-efficient allocation.

Proposition 12. *The tax system* $T^*(K_1, \{S_1(i)\}_i, x) = \tau_P^*(x)(1+x)K_1 + \tau_s^*(x)\int_i S_1(i)\,di + \rho^*(x)$*, where* $\tau_P^*(x)$*,* $\tau_S^*(x)$ *and* $\rho^*(x)$ *satisfy (26), (27) and (28), respectively, implements the constrained-efficient allocation with distribution policy* $1+d^*(x) = (1+x)(1-\tau_P^*(x))$ *and dividend process* $D^*(i)$ *for all* i*. The allocation* $\{K_1^*, s^*, \{S_1^*(i)\}_i, 1, c_1^*(x)\}$ *with* $s^* K_1^* = S_1^*(i)$ *for all* i *and* $s^* \in (0,1)$*, the tax system* $T^*(K_1, \{S_1(i)\}_i, x)$*, the distribution policy* $d^*(x)$ *and the dividend process* $D^*(i,x)$ *constitute a competitive equilibrium.*

The proof proceeds as the one for proposition 7. The values of S_1^* and s^* are not pinned down by the implementation. The setting of marginal taxes ensures that the entrepreneurs' Euler equations (23)-(25) are satisfied as an equality at any $s^* \in (0,1)$ for distribution policy $d^*(x)$, and that local second order sufficient conditions are also satisfied. It follows that the only interior solution to the entrepreneurs' optimization problem is $\{1, K_1^*, S_1^*(i)\}$ for any $s^* \in (0,1)$. In addition, it ensures that the allocation is globally optimal because it rules out any corner solutions to the entrepreneurs' investment and portfolio problems, irrespective of the level of effort. Lastly, the setting of $\rho^*(x)$ ensures high effort is optimal at the appropriate level of capital and portfolio choices. The optimal tax system does not pin down the equilibrium value of s^*. By (23), for $s^* \in (0,1)$, the tax system ensures that entrepreneurs find it optimal to choose K_1^*.

The properties of the optimal tax system can be derived from (26)-(28). First:

$$E_1 \tau_P^*(x) = 1 - E_1\left[\frac{u'(c_0^*)}{\beta(1+x)u'(c_1^*(x))}\right], \tag{29}$$

so that $E_1\tau_P^*(x) > 0$ if $IW_K > 0$ and $E_1\tau_P^*(x) < 0$ if $IW_K < 0$. However, using the planner's Euler equation delivers $E_1(1+x)\tau_P^*(x) = 0$, since:

$$E_1(1+x)(1-\tau_P^*(x)) = E_1\left[\frac{u'(c_0^*)}{\beta u'(c_1^*(x))}\right].$$

This implied that the expected tax paid is zero.[13] In addition, $\tau_P^*(\bar{x}) - \tau_P^*(\underline{x}) < 0$ when $IW_K > 0$ and $\tau_P^*(\bar{x}) - \tau_P^*(\underline{x}) > 0$ when $\mathrm{IW}_K < 0$ from:

$$\frac{u'(c_0^*)}{\beta(1+\underline{x})u'(c_1^*(\underline{x}))} - \frac{u'(c_0^*)}{\beta(1+\bar{x})u'(c_1^*(\bar{x}))} = \tau_P^*(\bar{x}) - \tau_P^*(\underline{x}),$$

since $\mathrm{IW}_K\,\tilde{}\,(1+\underline{x})u'(c_1^*(\underline{x})) - (1+\bar{x})u'(c_1^*(\bar{x}))$. Lastly, by (27):

$$1 + E_1x - E_1\tau_P^*(x) - E_1x\tau_P^*(x) - E_1\tau_S^*(x) = E_1\frac{u'(c_0^*)}{\beta u'(c_1^*(x))}. \tag{30}$$

This implies $E_1\tau_S^*(x) = -E_1\tau_P^*(x) - E_1x\tau_P^*(x) = -E_1\tau_P^*(x)E_1(1+x) - Cov_1(x, \tau_P^*(x))$. If $\mathrm{IW}_K \gtreqless 0$, $Cov_1(x, \tau_P^*(x)) \lesseqgtr 0$ and $E_1\tau_P^*(x) \gtreqless 0$, as discussed above. Hence, the sign of $E_1\tau_S^*(x)$ is typically ambiguous.

Figure 3 plots the optimal marginal asset taxes for this implementation in the three numerical examples analyzed in section 2.2. The left panels correspond to the marginal tax on entrepreneurial earnings, while the right panels correspond to the marginal tax on stocks. The dashed-star line correspond to marginal taxes in the bad state, while the dashed-cross lines correspond to marginal taxes in the good state. The pattern of optimal marginal taxes is consistent with the previous discussion.

3.2.1. Double Taxation of Entrepreneurial Capital

The tax system described in proposition 12 embeds a prescription for *double taxation of income from entrepreneurial capital*: at the firm level thought τ_P^*, and at the level of external investors, through τ_S^*. This property holds for any equilibrium in which $s^* \in (0,1)$ and is jointly determined by the distribution policy and the tax system, since external investors receive a share of earnings *after* tax. We now show that this feature of the tax system necessary to implement the constrained-efficient allocation.

Following the reasoning in section 2, taxation of entrepreneurial earnings and stock portfolios is *required* to ensure that entrepreneurs choose K_1^* and S_1^*, respectively. Absent a marginal tax on capital, entrepreneurs would have an incentive to increase/reduce holdings of K_1 relative to K_1^* and reduce effort. Similarly, given that stock returns are uncorrelated with entrepreneurs' idiosyncratic risk, the wedge on stock portfolios is positive and, by (29) and (30), equal to the aggregate intertemporal wedge IW:

$$\beta E_1\left(1+\bar{D}\right)E_1u'(c_1^*(x)) - u'(c_0^*) = \mathrm{IW} > 0.$$

[13] The constrained-efficient allocation can equivalently be implemented with a marginal tax on tax on capital $\tau_K(x)$ that satisfies (16) and with distribution policy: $1 + d(x) = 1 + x - \tau_K(x)$ and dividend process $1 + D(i, \tilde{x}) = 1 + \tilde{x} - \tau_K^*(\tilde{x})$, so that $1 + D(i) = 1 + E_1(x)$, since $E_1\tau_K^*(x) = 0$. All other results can be derived with a similar reasoning.

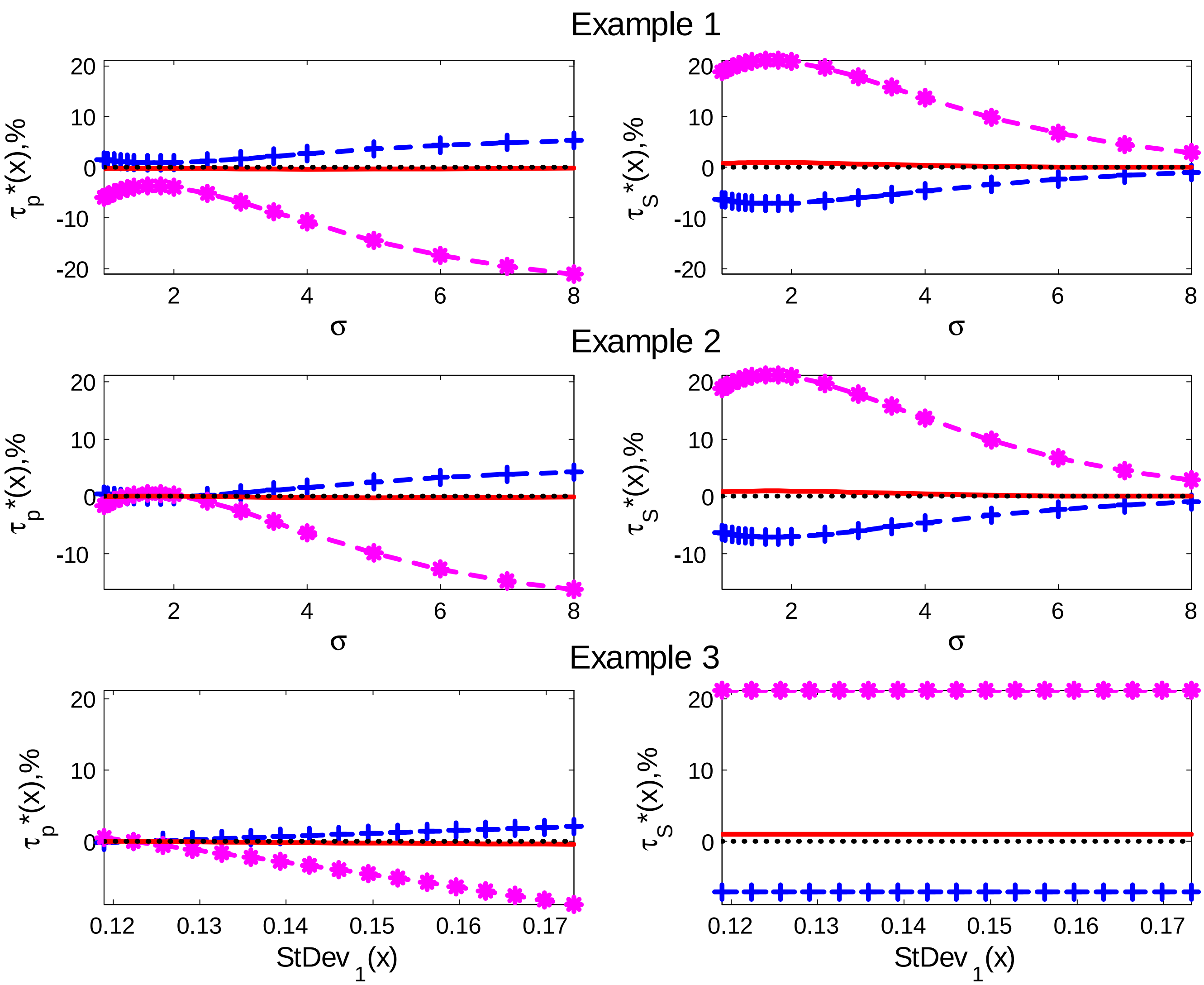

Figure 3: Optimal marginal taxes on entrepreneurial earnings and on stocks.

Hence, absent a tax on stock holdings, entrepreneurs would have the incentive to increase their holdings of stocks and reduce effort.

We now show that it is necessary to tax distributed earnings to ensure that $0 < s^* < 1$ is chosen. To do this, we allow the marginal tax on distributed earnings, $\tau_d(x)$ to differ from the marginal tax on retained earnings, $\tau_P(x)$. Then, the distribution policy can be written as: $1 + d(x) = (1+x)(1-\tau_d(x))$. Setting $\tau_d(x) = 0$ for $x \in X$ avoids double taxation of entrepreneurial earnings. By (25), $1 + d(x) = (1+x)$ implies: $u'(c_0^*) - \beta E_1 (1+x) u'(c_1^*(x)) < 0$ and $s = 0$, if the individual intertemporal wedge is positive. Hence, when $\mathrm{IW}_K > 0$ and there is not tax on distributed earnings, $s^* = 0$ is the only implementable value of s. If the individual intertemporal wedge is negative and $\tau_d(x) = 0$, (25) implies: $u'(c_0^*) - \beta E_1 (1+x) u'(c_1^*(x)) > 0$ and $s = 1$. But equation (23) evaluated at $s = 1$ reduces to:

$$-\beta E_1 \left[(1+x) \tau_P^*(x) u'(c_1^*(x))\right] \leq 0.$$

This is clearly a contradiction since when $\mathrm{IW}_K < 0$, $(1+x)\tau_P^*(x)$ is increasing in x and $Cov_1\left[(1+x)\tau_P^*(x), u'(c_1^*(x))\right] < 0$, which implies $E_1\left[(1+x)\tau_P^*(x) u'(c_1^*(x))\right] < 0$, since $E_1 (1+x)\tau_P^*(x) = 0$. Hence, there is no interior solution to the entrepreneur's choice of K_1. It follows that the constrained-efficient allocation cannot be implemented, since there is no way to set taxes to ensure that a particular value of K_1 will be chosen by the entrepreneur. Even if for any value of K_1 (and S_1), $\rho(x)$ can be set to ensure that high effort will be chosen, there is no way to guarantee that the corresponding value of K_1 will indeed arise.

To understand this property, note that an entrepreneur has three intertemporal margins in this market structure, corresponding to the Euler equation for K_1, the one for S_1 and the one for s. Therefore, there are three potential deviations in her asset position and three intertemporal fiscal instruments are needed to implement the constrained-efficient outcomes. For IW_K positive, an entrepreneur has an incentive to increase her holdings of K_1 relative to K_1^* and therefore would optimally not sell any equity under a tax system in which distributed earnings are not taxed. This simply implies that the only equilibrium is one in which $s^* = 0$. It is still possible to implement K_1^* and e^*. Instead, the case in which the individual intertemporal wedge is negative is particularly problematic. When $\mathrm{IW}_K < 0$, an entrepreneur would optimally reduce her holdings of capital when she reduces effort. This can be achieved directly or by increasing the fraction s sold to external investors. As shown above, if distributed earnings are not taxed, the optimal deviation is $s = 1$, which implies K_1^* and e^* cannot be implemented.

We now characterize the class of tax systems $T\left(K_1, s, \{S_1(i)\}_i\right)$ that rules out $s = 1$ as a possible solution to the entrepreneurs' problem can be characterized as follows.

Proposition 13. *In any competitive equilibrium under a tax system,* $T\left(K_1, s, \{S_1(i)\}_i\right) = \tau_P(x)(1-s)K_1 + \tau_d(x) sK_1 + \tau_s(x)\int_i S_1(i)\,di + \rho(x)$, *and distribution policy* $1 + d(x) = (1+x)(1-\tau_d(x))$, $s \in [0,1)$ *if and only if:*

$$E_{\hat{e}}(1+d(x)) u'(c_1(x)) \geq E_{\hat{e}}(1+x)(1-\tau_P(x)) u'(c_1(x)). \tag{31}$$

This proposition states that the expected discounted value of distributed earnings must be greater than the expected discounted value of retained earnings after tax to ensure that

$s < 1$ in a competitive equilibrium under a tax system $T\left(K_1, s, \{S_1(i)\}_i\right)$. For this condition to be verified at the constrained-efficient allocation, it must be that $\tau_P(x) \geq \tau_d(x)$ if $u'(c_1^*(x))(1+x) > u'(c_1^*(x'))(1+x')$ for $x, x' \in \{\underline{x}, \bar{x}\}$. Then, since $u'(c_1^*(\underline{x}))(1+\underline{x}) \gtrless u'(c_1^*(\bar{x}))(1+\bar{x})$ for $\mathrm{IW}_K \gtrless 0$, this implies $\tau_P(\bar{x}) \leq \tau_d(\bar{x})$ and $\tau_P^*(\underline{x}) \geq \tau_d(\underline{x})$ for $\mathrm{IW}_K > 0$ and $\tau_P(\bar{x}) \geq \tau_d(\bar{x})$ and $\tau_P(\underline{x}) \leq \tau_d(\underline{x})$ for $\mathrm{IW}_K < 0$.

The rationale for this result is simple. When $\mathrm{IW}_K > 0$, entrepreneurs have an incentive to increase holdings of their own capital and reduce effort at the constrained-efficient allocation. A way to discourage this is to make external capital a good hedge. This is achieved by making dividend payouts greater in the good state and smaller in the bad state after tax. Conversely, when $\mathrm{IW}_K < 0$, entrepreneurs have an incentive to reduce holdings of their own capital and effort. To avoid an outcome in which entrepreneurs retain too little ownership, the tax system reduces the hedging value of external capital, by making dividend payments higher in the bad state and lower in the good state.

Obviously, under the optimal tax system $T^*(K_1, S_1, x)$, distributed and retained earnings are taxed at the same marginal rate $\tau_P^*(x)$ defined by (26). Thus, it satisfies (31), which ensures that $s^* < 1$. Moreover, by (25), $s^* > 0$. In general, the first order necessary conditions for K_1 can be rewritten as:

$$\begin{aligned} 0 \;=\;& -(1-s)\left\{u'(c_0^*) - \beta E_1\left[(1+x)(1-\tau_P^*(x))\, u'(c_1^*(x))\right]\right\} \\ & +\beta s E_1\left[(1+x)(\tau_d(x) - \tau_P^*(x))\, u'(c_1(x))\right]. \end{aligned}$$

Then, for equilibria with $s^* \in (0,1)$, it must be that $E_1\left[(1+x)(\tau_d(x) - \tau_P^*(x))\, u'(c_1(x))\right] = 0$, if $\tau_P^*(x)$ satisfies (26), to ensure that K_1^* is chosen.

This argument implies that it is indeed *necessary* for distributed earnings, as well as retained earnings, to be taxed at the firm level to implement the constrained-efficient allocation. Hence, entrepreneurial capital is subject to *double taxation* in the optimal tax system.

The linearity in asset levels of the optimal tax system is an important property in the two previous implementations. It implies that optimal marginal taxes are independent from the individual level of asset holdings. For example, in the first implementation, each unit of the bond B_1 is taxed at the same marginal rate $\tau_B^*(x)$, that depends only on an individual's observable capital returns x. It follows that the government does not need to observe entrepreneurs' portfolios to administer the optimal tax system, if financial securities are traded via competitive intermediaries and taxes on holding of those securities are collected *at the source*. This arrangement is similar to the one in place for consumption taxes in the US, where merchants observe individual units of consumption and apply a mandated consumption tax schedule. They then transfer total tax revenues to the relevant tax authority (the city, county or state for consumption taxes) who only observes the total revenue inflow from each merchant. Similarly, financial intermediaries clearing trades of bonds B_1 could levy marginal tax $\tau_B^*(x)$ on an entrepreneur with returns x. Then, if the entrepreneur's total holdings are B_1^*, the tax paid on bond holdings will be $\tau_B^*(x)\, B_1^*$. Moreover, if each entrepreneurs faces the marginal tax $\tau_B^*(x)$ on bonds and the marginal tax $\tau_K^*(x)$ on capital, we know from Proposition 7, that she will find B_1^* and K_1^* optimal.

We exploit the linearity of optimal marginal asset taxes to implement the constrained-efficient allocation in a market structure with unobserved portfolio holdings in the next section.

3.3. Private Insurance Contracts

We now construct an implementation with private insurance contracts. We assume that there are a continuum of identical insurance companies that behave competitively. Insurance companies can be seen as risk neutral agents that don't exert any effort. Each insurance company writes contracts with a continuum of entrepreneurs and makes zero profits.

Events occur according to the following timing. At time 0, insurance companies offer incentive compatible insurance contracts to the entrepreneurs, denoted with $\mathcal{C} = \{P, R(\underline{x}), R(\bar{x})\}$, where P is the premium paid at time 0 and $R(x)$ is the state contingent transfer at time 1. Entrepreneurs can only purchase one insurance contract. In addition, entrepreneurs buy bonds B_1 which pay a risk-free interest r, and they invest in capital K_1. They then exert effort. In period 1, x is realized, entrepreneurs receive insurance payments and the government levies taxes. Insurance companies are liquidated and their liquidation value, equal to any profits, is rebated to the entrepreneurs. The entrepreneurs then consume.

The informational structure is as follows. The level of investment K_1 and x are public information. We assume that insurance companies and the government do not observe B_1. We restrict attention to candidate tax systems of the form: $T(K_1, B_1, x) = \rho(x) + \tau_B(x) B_1 + \tau_K(x) K_1$.

The optimal insurance contracts solve the following problem:

$$\Pi = \max_{\left[e,K_1,B_1,P,R(x),B_1^I\right]\in\Phi(K_0)} \left\{ P - B_1^I + \frac{B_1^I(1+r) - [\pi(e)R(\bar{x}) + (1-\pi(e))R(\underline{x})]}{1+r} \right\}, \tag{Problem 5}$$

subject to

$$U(e, K_1, B_1; \mathcal{C}, T) - v(e) \geq U\left(\tilde{e}, \tilde{K}_1, \tilde{B}_1; \mathcal{C}, T\right) - v(\tilde{e}), \text{ for } \left[\tilde{e}, \tilde{P}, \tilde{R}(x), \tilde{K}_1, \tilde{B}_1\right] \in \Phi(K_0), \tag{32}$$

$$P - \frac{\pi(e)R(\bar{x}) + (1-\pi(e))R(\underline{x})}{1+r} = 0, \tag{33}$$

where

$$\Phi(K_0) \equiv \left\{ \begin{array}{c} [e, K_1, B_1, P, R(x)] : K_1 \in \left[0, K_0 + \bar{\Pi}_0 - P - B_1\right], B_1 \geq \bar{B}, \\ R(x) \geq K_1(1 + x - \tau_K(x)) + B_1(1 + r - \tau_B(x)) - \rho(x) + \bar{\Pi}_1. \end{array} \right\} \tag{34}$$

$$\begin{aligned} U(e, K_1, B_1; \mathcal{C}, T) &= u\left(K_0 + \bar{\Pi}_0 - P - K_1 - B_1\right) \\ &\quad + \beta E_e u\left(K_1(1 + x - \tau_K(x)) + B_1(1 + r - \tau_B(x)) + R(x) - \rho(x) + \bar{\Pi}_1\right), \end{aligned}$$

$\bar{B}$ is the natural debt limit, and $\bar{\Pi}_t$ denotes aggregate profits from the insurance sector in period $t = 0, 1$, taken as given by each individual insurance company.

Constraint (32) is the incentive compatibility constraint. It requires that the effort, capital and bond allocation specified by the contract is preferred by the agent to any other feasible effort, capital and bond allocation. Insurance companies cannot observe effort and bond holdings, but can induce a particular allocation which is incentive compatible. The entrepreneur takes the tax system, the terms of the insurance contract and the bank's liquidation value as given. Constraint (33) is the zero profit condition imposed on insurance companies. The set $\Phi(K_0)$ describes feasible allocations and contracts. The feasibility requirements reflect the non-negativity constraints in the agents problem.

We assume that entrepreneurs and insurance companies buy bonds from financial intermediaries that collect taxes on bonds at the source. The cash flow of financial intermediaries is denoted with F_t for $t = 0, 1$, with

$$F_0 = B_1 + B_1^I,$$

$$F_1 = -\left(B_1 + B_1^I\right)\left(1 + r - \left[\pi(e)\,\tau_B(\bar{x}) + (1 - \pi(e))\,\tau_B(\underline{x})\right]\right).$$

Definition 14. *A competitive equilibrium with insurance contracts is given by an initial endowment of capital K_0 for the entrepreneurs, an allocation $\{e, K_1, B_1\}$, insurance contracts $\mathcal{C}$, and a tax system $T(K_1, B_1, x)$ such that:*
i) the allocation and loan contracts $\mathcal{C}$ solve Problem 4 given the tax system;
ii) the bond market clears, $B_1 + B_1^I = 0$;
iii) the resource constraint is satisfied in each period.

The first requirement guarantees that the allocation and the corresponding consumption path are optimal for the entrepreneurs, given the tax system and the insurance contracts, since the allocation and contracts are incentive compatible. Insurance companies are optimizing and make zero profits in equilibrium given that they solve Problem 4. In a competitive equilibrium, financial intermediaries obtain a zero cash-flow in each period.

We define the optimal tax system as the one that implements the allocation that solves Problem 1, denoted with $e^* = 1$, K_1^*, c_0^*, $c_1^*(\underline{x})$, $c_1^*(\bar{x})$, in a competitive equilibrium.

Proposition 15. *Let $\tau_K^*(x)$ and $\tau_B^*(x)$ satisfy (16) and (17) and set $\rho^*(x) = 0$ for $x = \underline{x}, \bar{x}$. Then, the tax system $T^*(K_1, B_1, x) = \rho^*(x) + \tau_K^*(x)\,K_1 + \tau_B^*(x)\,B_1$ implements the allocation $e^* = 1$, K_1^*, c_0^*, $c_1^*(\underline{x})$, $c_1^*(\bar{x})$ in the economy with insurance contracts and unobservable bond holdings with $B_1^* = B_1^{I*} = 0$ and $r = E_1(x)$.*

Proposition 15 shows that, by appropriately setting marginal asset taxes, the government can relax the more severe incentive compatibility constraint (32) that arises in the contracting problem between private insurance companies and entrepreneurs, due entrepreneurs' unobserved holdings of financial assets, thus enabling private insurance contracts to implement the constrained-efficient allocation with observable consumption. The proof proceeds in three steps. First, it is shown that the optimal insurance problem with observed B_1 is just the dual of the the planning problem (Problem 1). Then, we show that the incentive compatibility constraint (32) in Problem 5 is equivalent to the incentive compatibility constraint (3) in Problem 1 plus the Euler equations for capital and bonds under T^*. The last step involves proving that the constrained-efficient allocation is feasible for this modified version of Problem 5.

This result has important implications for the role of taxes in implementing allocations. Under this market structure, entrepreneurs entertain an exclusive relationship with one insurance company. If the entrepreneurs' bond holding were observed by the insurer, the optimal contract would implement the constrained efficient allocation in a competitive equilibrium where insurance companies make zero profits. The entrepreneurs' ability to invest in bonds breaks this exclusivity, since it allows for trades that cannot be controlled or observed by insurance firms. This generates an *essential* role for asset taxation at the optimal marginal rates. Even if the government does not have any informational advantages with respect to private insurance firms, it can set and enforce taxes on assets to influence the entrepreneurs' intertemporal decisions. The optimal asset taxes partly relax the entrepreneurs' incentive compatibility constraint in the optimal insurance problem. In the words of Arnott and Stiglitz (1990), the government's "power to tax" and "monopoly on compulsion" allows for the constrained-efficient allocation with *observed* individual consumption to implemented as a competitive equilibrium in a market structure where individual consumption remains private.

These findings are most closely related to Golosov and Tsyvinski (2006), who analyze fiscal implementations in a Mirrleesian economy with *unobserved* consumption due to hidden side trades. They show that private insurance contracts do not implement constrained-efficient allocations in that setting, because private insurers fail to internalize the effect of the contracts they offer on the equilibrium price of the unobservable side trades. A linear tax on capital can ameliorate this competitive equilibrium externality. Their optimal tax system implements the constrained-efficient allocation with unobservable consumption. In the market structure presented here, the tax system implements the constrained-efficient allocation with observable consumption, despite the fact that in the market economy the bond holdings are unobserved by the government and by private insurance firms, which implies that individual consumption remains private.[14]

4. Concluding Remarks

This paper analyzes optimal taxation of entrepreneurial capital in a dynamic moral hazard model with idiosyncratic capital risk.[15] First, we characterize the properties of constrained-efficient allocations and show that the intertemporal wedge on entrepreneurial capital can

[14] Bizer and DiMarzo (1999) derive a related result in a moral hazard model in which agents may borrow. They show that as long as debt repayments can be made state contingent by allowing for default, it is possible to implement the optimal effort allocation with observable savings, even if borrowing is unobserved by the principal, who designs the incentive-compatible salary policy.

[15] This class of environments has not been studied in the recursive contracting literature. An exception is Kahn and Ravikumar (1999) where idiosyncratic capital returns are hidden. They focus on an implementation with financial intermediaries and rely on numerical simulations. They do no provide an analytical characterization of the wedges associated with the constrained-efficient allocation.

be positive or negative. A negative intertemporal wedge signals that more capital relaxes the incentive compatibility constraint. This can occur since the returns from effort are increasing in capital. The main contribution of the paper is to characterize the optimal tax systems that implement the constrained-efficient allocation in different market structures with multiple assets. We derive three results. First, marginal asset taxes depend on the *correlation* of their returns with the entrepreneurs' idiosyncratic capital risk. We also consider whether entrepreneurial capital earnings distributed to outside investors should be taxed at the firm level. We find that entrepreneurial capital should be taxed at the firm level and again when it accrues to outside investors in the form of stock returns. This generates a theory of optimal differential asset taxation and provides a foundation for the double taxation of capital earnings. Lastly, we show that, even if private insurance contracts are available, it is essential to tax assets to implement constrained-efficient allocations when entrepreneurs can trade bonds and their asset holdings are private. This points to an important complementarity between private contracting and taxation.

The empirical public finance literature has documented substantial differences in the tax treatment of different forms of capital income. Specifically, interest income is taxed at a higher rate than stock returns, as discussed in Gordon (2003), while dividends are taxed at a higher rate than realized capital gains. As documented by Gordon and Slemrod (1988), the higher marginal tax rate on interest income is a stable property of empirical tax systems in many industrialized economies. These studies focus mainly on differences in average taxes. Instead, the theory developed in this paper generates predictions on the correlation of marginal asset taxes with individual earnings, and average taxes do not play an important role. Poterba (2002) documents a strong response of household portfolio composition to this differential tax treatment. Auerbach (2002) finds that firms's investment decisions appear to be sensitive to the taxation of dividend income at the personal level and their choice of organization form is responsive to the differential between corporate and personal tax rates. In the economy studied in this paper, the optimal tax system implements the constrained-efficient allocation by influencing portfolio choice and sales of private equity by entrepreneurs. Differential tax treatment of different asset classes is essential to achieve this goal.

The incentive problem that arises with entrepreneurial capital arguably also applies to top executives who hold company stock and other assets. Hence, this analysis could be adapted to such a setting. A quantitative version of the model can be used to provide an assessment of empirical tax systems. Lastly, this model does not consider entrepreneurial entry. By introducing ex ante heterogeneity in private entrepreneurial abilities, it would be possible to analyze optimal selection into entrepreneurship and optimal income and capital taxation in a model with workers and entrepreneurs. We leave these extensions for future work.

References

[1] Albanesi, Stefania and Christopher Sleet. 2006. Dynamic Optimal Taxation with Private Information. *The Review of Economic Studies,* Vol. 73, 1-30.

[2] Angeletos, George-Marios. 2006. Uninsured Idiosyncratic Investment Risk and Aggregate Saving. Forthcoming, *Review of Economic Dynamics.*

[3] Auerbach, Alan, 2002. Taxation and Corporate Financial Policy. *Handbook of Public Economics*, Vol. 3, North-Holland.

[4] Bizer, David and Peter De Marzo. 1999. Optimal Incentive Contracts when Agents Can Save, Borrow and Default. *Journal of Financial Intermediation* 8, 241-260.

[5] Cagetti, Marco and Mariacristina De Nardi. 2004. Taxation, Entrepreneurship, and Wealth. Federal Reserve Bank of Minneapolis, Staff Report 340.

[6] Cagetti, Marco and Mariacristina De Nardi. 2006. Entrepreneurship, Frictions and Wealth. Forthcoming, *The Journal of Political Economy.*

[7] Chetty, Raj. 2006. A New Method for Estimating Risk Aversion. Forthcoming, *American Economic Review.*

[8] Farhi, Emanuel and Ivan Werning. 2005. Progressive Estate Taxation. Manuscript, MIT.

[9] Golosov, Mikhail Narayana Kocherlakota, Aleh Tsyvinski. 2003. Optimal Indirect and Capital Taxation. *The Review of Economic Studies*, Volume 70, Issue 3, Page 569.

[10] Golosov, Mikhail, and Aleh Tsyvinski. 2006. Optimal Taxation with Endogenous Insurance Markets. Forthcoming, *Quarterly Journal of Economics.*

[11] Golosov, Mikhail, and Aleh Tsyvinski. 2006. Designing Optimal Disability Insurance: A Case for Asset Testing. Forthcoming, *Journal of Political Economy.*

[12] Grochulski, Borys and Tomasz Piskorski. 2005. Optimal Wealth Taxes with Risky Human Capital. Manuscript, NYU.

[13] Hassett, Kevin and Glenn Hubbard, 2002. Tax Policy and Business Investment. *Handbook of Public Economics*, Vol. 3, North-Holland.

[14] Gollier, Christian. 2001. The Economics of Risk and Time. MIT Press.

[15] Gordon, Roger. 2003. Taxation of Interest Income. Manuscript, UCSD.

[16] Gordon, Roger and Joel Slemrod. 1988. Do We Collect Any Revenue from Taxing Capital Income? In Tax Policy and the Economy, Vol. 2, ed. Lawrence Summers. Cambridge: MIT Press, pp. 89-130.

[17] Judd, Kenneth. 1985. Redistributive Taxation in a Perfect Foresight Model. *Journal of Public Economics* 28, 59-84.

[18] Khan, Aubhik and B. Ravikumar. 1999. Growth and Risk-Sharing with Private Information. *Journal of Monetary Economics.*

[19] Kocherlakota, Narayana. 2005a. Advances in Dynamic Optimal Taxation. Paper presented at the World Congress of the Econometric Society, August 2005.

[20] Kocherlakota, Narayana. 2005b. Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation. *Econometrica.*

[21] Levhari, D. and T. N. Srinivasan. 1969. Optimal Savings Under Uncertainty. *The Review of Economic Studies* Vol. 36, No. 2, 153-163.

[22] Mirrlees, James. 1971. An exploration in the theory of optimum income taxation. *The Review of Economic Studies* 38:175-208.

[23] Moskovitz, Tobias and Annette Vissing-Jorgensen. 2002. The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle? *The American Economic Review*, Vol. 92, No. 4.

[24] Poterba, James. 2002. Taxation, Risk-Taking, and Household Portfolio Behavior. *Handbook of Public Economics*, Vol. 3, North-Holland.

[25] Rogerson, William. 1985. Repeated moral hazard. *Econometrica* 53:69-76.

[26] Sandmo, Agnar. 1970. The Effect of Uncertainty on Savings Decisions. *The Review of Economic Studies*, Vol. 37, No. 3, 353-360.

[27] Tirole, Jean. 2006. The Theory of Corporate Finance. Princeton University Press.

5. Appendix

Proof of Proposition 1. Letting μ be the multiplier on the incentive compatibility constraint and λ the one on the resource constraint, the first order necessary conditions for the planning problem at $e = 1$ are:

$$-u'(K_0 - K_1) + \lambda E_1 (1 + x) = 0,$$

$$(1 - \pi(1))\,\beta u'(c_1(\underline{x})) - \mu(\pi(1) - \pi(0))\,\beta u'(c_1(\underline{x})) - \lambda(1 - \pi(1)) = 0,$$

$$\pi(1)\,\beta u'(c_1(\bar{x})) - \mu(\pi(0) - \pi(1))\,\beta u'(c_1(\bar{x})) - \lambda\pi(1) = 0.$$

At $e = 0$, the same first order necessary conditions hold with $\mu = 0$. If $e^* = 1$ is optimal, the first order conditions can be simplified to yield (4) and (5). ■

Proof of Lemma 2. Suppose instead that $(1 + \overline{x})K_1^* < c_1^*(\overline{x})$ and $c_1^*(\underline{x}) < (1 + \underline{x})K_1^*$. Consider a class of perturbations to the optimal allocation that increase consumption in the bad state by $\Delta\underline{c}_1$ and reduce consumption in the good state by $\Delta\bar{c}_1$, and preserve incentive compatibility and feasibility. Such perturbations must satisfy:

$$(\pi(1) - \pi(0))\left[-u'(c_1^*(\bar{x}))\,\Delta\bar{c}_1 + u'(c_1^*(\underline{x}))\,\Delta\underline{c}_1\right] = 0,$$

$$-\pi(1)\,\Delta\bar{c}_1 + (1-\pi(1))\,\Delta\underline{c}_1 = \Delta\underline{c}_1\left[1-\pi(1)\left(1+\frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))}\right)\right] \le 0.$$

These conditions imply $\Delta\bar{c}_1 = \Delta\underline{c}_1\frac{u'\left(c_1^*(\underline{x})\right)}{u'\left(c_1^*(\bar{x})\right)} > \Delta\underline{c}_1$, and $\left[\frac{1}{\pi(1)} - \left(1+\frac{u'\left(c_1^*(\underline{x})\right)}{u'\left(c_1^*(\bar{x})\right)}\right)\right] \le 0$, where the latter is always satisfied for $\pi(1) \ge 1/2$. Now let $c_1^*(\underline{x}) + \Delta\underline{c}_1 \ge K_1^*(1+\underline{x})$. Then:

$$\begin{aligned}
c_1^*(\bar{x}) - \Delta\bar{c}_1 &= c_1^*(\bar{x}) - \Delta\underline{c}_1\frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))} \\
&\le c_1^*(\bar{x}) - \left[K_1^*(1+\underline{x}) - c_1^*(\underline{x})\right]\frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))} \\
&= \frac{E_1K_1^*(1+x) - (1-\pi(1))\,c_1^*(\underline{x})}{\pi(1)} - \left[K_1^*(1+\underline{x}) - c_1^*(\underline{x})\right]\frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))} \\
&= K_1^*(1+\bar{x}) + \left[\frac{1}{\pi(1)} - \left(1+\frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))}\right)\right]\left[K_1^*(1+\underline{x}) - c_1^*(\underline{x})\right] \\
&\le K_1^*(1+\bar{x}),
\end{aligned}$$

by $\left[\frac{1}{\pi(1)} - \left(1+\frac{u'\left(c_1^*(\underline{x})\right)}{u'\left(c_1^*(\bar{x})\right)}\right)\right] \le 0$. Hence, this perturbation is incentive compatible, uses fewer resources than the optimal allocation at time 1 and implies that consumption in the good/bad state is smaller/greater than earnings. Since the perturbed allocation uses fewer resources at time 1, it is feasible to increase consumption at time 0 by reducing the level of K_1 by the amount:

$$\Delta K_1 = \frac{\Delta\underline{c}_1}{E_1(1+x)}\left[1-\pi(1)\left(1+\frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))}\right)\right] \le 0.$$

The resulting effect on welfare is:

$$\begin{aligned}
&-u'(c_0^*)\,\Delta K_1 + \pi(1)\left[-u'(c_1^*(\bar{x}))\,\Delta\bar{c}_1 - u'(c_1^*(\underline{x}))\,\Delta\underline{c}_1\right] + u'(c_1^*(\underline{x}))\,\Delta\underline{c}_1 \\
=\ &\Delta\underline{c}_1\left\{-\frac{u'(c_0^*)}{E_1(1+x)}\left[1-\pi(1)\left(1+\frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))}\right)\right] + (1-2\pi(1))\,u'(c_1^*(\underline{x}))\right\} \\
=\ &\Delta\underline{c}_1 u'(c_1^*(\underline{x}))\left\{-\frac{u'(c_0^*)}{E_1(1+x)}\left[\frac{1-\pi(1)}{u'(c_1^*(\underline{x}))} - \frac{\pi(1)}{u'(c_1^*(\bar{x}))}\right] + 1 - 2\pi(1)\right\} \\
=\ &\Delta\underline{c}_1 u'(c_1^*(\underline{x}))\left\{-1 + \frac{u'(c_0^*)}{E_1(1+x)}\frac{2\pi(1)}{u'(c_1^*(\bar{x}))} + 1 - 2\pi(1)\right\} \\
=\ &\Delta\underline{c}_1 u'(c_1^*(\underline{x}))\,2\pi(1)(1-\pi(1))\frac{u'(c_0^*)}{E_1(1+x)}\left\{\frac{1}{u'(c_1^*(\bar{x}))} - \frac{1}{u'(c_1^*(\underline{x}))}\right\} > 0,
\end{aligned}$$

where the third and fourth equality use the inverted Euler equation. Hence, the class perturbations that make consumption in the good/bad state is smaller/greater than earnings is incentive compatible, requires fewer resources and increases welfare. This violates the assumption that $\{K_1^*, c_1^*(x)\}$ was optimal and satisfies $(1+\overline{x})K_1^* < c_1^*(\overline{x})$ and $c_1^*(\underline{x}) < (1+\underline{x})K_1^*$. ■

Proof of Proposition 3. By equation (9), $\mathrm{IW}_K \,\tilde{}\, u'\left(c_1^*\left(\underline{x}\right)\right)\left(1+\underline{x}\right)-u'\left(c_1^*\left(\bar{x}\right)\right)\left(1+\bar{x}\right)$. By Lemma 2, $(1+\overline{x})K_1^* \geq c_1^*(\overline{x}) > c_1^*(\underline{x}) \geq (1+\underline{x})K_1^*$ with at least one inequality strict. Then:

$$\begin{aligned} \mathrm{IW}_K \quad & \tilde{} \quad \left[u'(c_1^*(\underline{x}))(1+\underline{x}) - u'(c_1^*(\overline{x}))(1+\overline{x})\right] K_1^* \\ < \quad & u'(c_1^*(\underline{x}))c_1^*(\underline{x}) - u'(c_1^*(\overline{x}))c_1^*(\overline{x}) \leq 0 \end{aligned}$$

if $u\prime\left(c\right)c$ is weakly increasing in c. Since

$$\frac{\partial u\prime\left(c\right)c}{\partial c} = u''\left(c\right)c + u'\left(c\right) > 0,$$

for $\sigma\left(c\right) \leq 1$, the result follows. ■

Proof of Lemma 6. We first show that under $\bar{T}$, $\hat{B}_1$ and $\hat{K}_1$ cannot both be interior. Suppose not. If optimal bond and capital holdings under $\bar{T}$ are interior:

$$\begin{aligned} u'\left(\hat{c}_0\right) &= E_0 u'\left(c_1^*\left(x\right)\right)\left(1+r-\bar{\tau}_B\right), \\ u'\left(\hat{c}_0\right) &= E_0 u'\left(c_1^*\left(x\right)\right)\left(1+x-\bar{\tau}_K\right). \end{aligned}$$

But:

$$\begin{aligned} & u'\left(\hat{c}_0\right) - E_0 u'\left(c_1^*\left(x\right)\right)\left(1+x-\bar{\tau}_K\right) \qquad (35) \\ = \quad & \beta\left[\frac{E_1 u'\left(c_1^*\left(x\right)\right)\left(1+x\right)}{E_1 u'\left(c_1^*\left(x\right)\right)} - \frac{E_0 u'\left(c_1^*\left(x\right)\right)\left(1+x\right)}{E_0 u'\left(c_1^*\left(x\right)\right)}\right] \gtreqless 0 \text{ if } IW_K \lesseqgtr 0, \end{aligned}$$

since $\frac{E_e u'\left(c_1^*(x)\right)(1+x)}{E_e u'\left(c_1^*(x)\right)}$ is increasing in e if $\mathrm{IW}_K < 0$ and decreasing for $\mathrm{IW}_K > 0$. Contradiction. Then, if $\mathrm{IW}_K < 0$, and $\hat{B}_1$ is interioir, (35) implies $\hat{K}_1 = 0$. If instead $\mathrm{IW}_K > 0$, assume that $\hat{B}_1$ is interior and $\hat{K}_1 = 0$. Then, (35) implies that an entrepreneur would like to increase capital holdings further. Since this can always be achieved by reducing bond holdings, an interior value of B_1 cannot be optimal. Hence, if $\mathrm{IW}_K > 0$ it must be that $\hat{B}_1 = \underline{B}$ and $\hat{K}_1 > 0$ under $\bar{T}$. Moreover, by (14), $\hat{K}_1 > K_1^*$. The binding Incentive compatibility constraint implies that such a deviation obtains higher lifetime utility for the agent. ■

Proof of Proposition 7. We want to show that

$$\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}\left(B_0^*, K_0, T^*\right) = \left(1, K_1^*, B_1^*\right),$$

for some $B_1^* \geq \bar{B}$ and for given r. Suppose that $\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}\left(B_0^*, K_0, T^*\right) \neq \left(1, K_1^*, B_1^*\right)$. If $\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}$ is interior in $\hat{K}_1$ and $\hat{B}_1$, at T^* :

$$1 = E_{\hat{e}} \frac{u'\left(\hat{c}_1\left(x\right)\right)}{u'\left(\hat{c}_0\right)} \frac{u'\left(c_0^*\right)}{u'\left(c_1^*\left(x\right)\right)}.$$

It follows that for any interior $\hat{K}_1, \hat{B}_1$ such that $\hat{K}_1 + \hat{B}_1 \gtrless K_1^* + B_1^*$, then (16) and (17) imply $\frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \lessgtr \frac{u'\left(c_1^*(x)\right)}{u'\left(c_0^*\right)}$ irrespective of the value of $\hat{e}$, a contradiction. Since after tax returns are equated for capital and bonds, agents are indifferent between any portfolio allocation such that total wealth at the beginning of period 1 is $K_1^* + B_1^*$. Moreover, at T^*, the local sufficient conditions for optimality are also satisfied irrespective of the value of $\hat{e}$. To see this, consider the sub-optimization problem associated with the choice of B_1 and K_1 for given e. The elements of the Hessian, H_U, for this problem are:

$$U_{BB}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x))(1 + r - \tau_B^*(x))^2 \leq 0,$$

$$U_{KK}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x))(1 + x - \tau_K^*(x))^2 \leq 0,$$

$$U_{BK}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x))(1 + r - \tau_B^*(x))(1 + x - \tau_K^*(x)),$$

where U_{xy} denotes a cross-partial derivative with respect to the variables x, y.Under (16)-(17), $|H_U| = 0$. Hence, the Hessian is negative semi-definite irrespective of the value of $\hat{e}$. We now consider values of $\hat{K}_1, \hat{B}_1$ that are not interior. The Inada conditions exclude non-interior solutions that result from the non-negativity constraint on time 0 consumption being binding. Hence, there are two candidate non-interior solutions: $\hat{K}_1 = 0$ and $\hat{B}_1 > 0$, and $\hat{K}_1 > 0$ and $\hat{B}_1 = \bar{B}$. In both cases, one of the Euler equations must hold with equality and the other as a strict inequality. Under T^*:

$$1 = E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}, \tag{36}$$

$$1 > E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}. \tag{37}$$

This is a contradiction, since (36) and (37) clearly cannot hold at the same time. Then, K_1^*, B_1^* are globally optimal irrespective of the value of $\hat{e}$. At at K_1^*, B_1^*, $\rho^*(x)$ implies $\hat{e} = 1$ since the constrained-efficient allocation is incentive compatible. Hence, the allocation $\{1, K_1^*, B_1^*\}$ is optimal for the agent given the initial endowments B_0^*, K_0, the tax system T^*, and the interest rate r. ■

Proof of Corollary 8. By Proposition 7, for any $r \geq 0$ and $B_1^* \geq \bar{B}$, the allocation $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ solves the agents' optimization problem in the market economy for initial endowments B_0^* and K_0. In addition, at $B_0^* = B_1^* = B_1^G$ the bond market clears and the resource constraint is satisfied at time 0. The resource constraint at time 1 is satisfied by construction. Hence, by (18), $E_1 c_1^*(x) = K_1 E_1(1 + x) + B_1^*(1 + r) - E_1 T^*(K_1^*, B_1^*, x)$, so that the government budget constraint is satisfied at time 1. ■

Proof of Proposition 9. By (16):

$$E_1 \left[1 + x - \frac{u'(c_0^*)}{u'(c_1^*(x))}\right] = E_1 \tau_K^*(x),$$

which from (5) implies i). ii) follows from the planner's Euler equation, since:

$$E_1 \tau_B^*(x) = 1 + r - E_1 \left(\frac{u'(c_0^*)}{\beta u'(c_1^*(x))}\right).$$

(16) also implies:

$$u'(c_1^*(\overline{x}))\tau_K^*(\overline{x}) - u'(c_1^*(\underline{x}))\tau_K^*(\underline{x}) = u'(c_1^*(\overline{x}))(1+\overline{x}) - u'(c_1^*(\underline{x}))(1+\underline{x}).$$

Since:

$$sign\left[u'(c_1^*(\overline{x}))(1+\overline{x}) - u'(c_1^*(\underline{x}))(1+\underline{x})\right] = sign\,(\text{-IW}_K)$$

and $u'(c_1^*(\overline{x})) < u'(c_1^*(\underline{x}))$, iii) follows. iv) follows directly from (16) and $u'\left(c_1^*\left(\bar{x}\right)\right) < u'\left(c_1^*\left(\underline{x}\right)\right)$. To show v) note that (16) and (17) imply $\tau_B^*\left(x\right) - \tau_K^*\left(x\right) = E_1 x - x$. ■

Proof of Proposition 10. This follows from:

$$\begin{aligned} E_1 u'\left(c_1^*\left(x\right)\right)\left(1+\tilde{r}\left(x\right)\right) - E_1 u'\left(c_1^*\left(x\right)\right)\left(1+x\right) &= Cov_1\left(u'\left(c_1^*\left(x\right)\right), \tilde{r}\left(x\right)\right) - Cov_1\left(u'\left(c_1^*\left(x\right)\right), x\right) \\ &= Cov_1\left(u'\left(c_1^*\left(x\right)\right), \tilde{r}\left(x\right) - x\right). \end{aligned}$$

$Cov_1\left(u'\left(c_1^*\left(x\right)\right), \tilde{r}\left(x\right) - x\right) > 0$ if $\tilde{r}\left(x\right) - x$ is decreasing in x, or $Cov_1\left(\tilde{r}\left(x\right) - x, x\right) < 0$. By the definition of covariance and by the fact that $E_1 x = E_1 \tilde{r}\left(x\right)$:

$$Cov_1\left(\tilde{r}\left(x\right) - x, x\right) = E_1 \tilde{r}\left(x\right) x - E_1 x^2 = Cov_1\left(\tilde{r}\left(x\right), x\right) - V_1\left(x\right). \tag{38}$$

By $V_1\left(x\right) > V_1\left(\tilde{r}\left(x\right)\right)$ and $Cov_1\left(\tilde{r}\left(x\right), x\right) > 0$, $0 < Corr_1\left(\tilde{r}\left(x\right), x\right) < 1$. Then:

$$Cov_1\left(\tilde{r}\left(x\right), x\right) - V_1\left(x\right) = SD_1\left(x\right)\left[Corr_1\left(\tilde{r}\left(x\right), x\right) SD_1\left(\tilde{r}\left(x\right)\right) - SD_1\left(x\right)\right] < 0.$$

In addition, $\tau_S^*\left(x\right) - \tau_K^*\left(x\right) = \tilde{r}\left(x\right) - x$. Since $\tilde{r}\left(x\right) - x$ is decreasing in x and $E_1 \tilde{r}\left(x\right) = E_1 x$, $\tau_S^*\left(\bar{x}\right) - \tau_K^*\left(\bar{x}\right) < 0$ and $\tau_S^*\left(\underline{x}\right) - \tau_K^*\left(\underline{x}\right) > 0$. ■

Proof of Proposition 12. Suppose that $\left\{\hat{e}, \hat{K}_1, \hat{s}, \left\{\hat{S}_1\left(i\right)\right\}_i\right\}\left(K_0, T\right) \neq \left\{1, K_1^*, s^*, \left\{s^* K_1^*\right\}_i\right\}$ for some $s^* \in [0,1)$. If $\left\{\hat{e}, \hat{K}_1, \hat{s}, \left\{\hat{S}_1\left(i\right)\right\}_i\right\}$ is interior, by (26), (27) and (28), (23)-(25) simplify to:

$$1 = E_{\hat{e}} \frac{u'\left(\hat{c}_1\left(x\right)\right)}{u'\left(\hat{c}_0\right)} \frac{u'\left(c_0^*\right)}{u'\left(c_1^*\left(x\right)\right)}.$$

Then, $\hat{K}_1\left(1-\hat{s}\right) + \int_{i\in[0,1]} \hat{S}_1\left(i\right) di \gtrless K_1^*\left(1-s^*\right) + \int_{i\in[0,1]} S_1^*\left(i\right) di$, with $\hat{s} \in (0,1)$, implies $\frac{u'\left(\hat{c}_1\left(x\right)\right)}{u'\left(\hat{c}_0\right)} \lesseqgtr \frac{u'\left(c_0^*\right)}{u'\left(c_1^*\left(x\right)\right)}$, irrespective of the value of $\hat{e}$. Contradiction. Hence, the only interior solution to (23)-(25) is $\left\{K_1^*, s^*, \left\{s^* K_1^*\right\}_i\right\}$ for $s^* \in (0,1)$. In addition, at T^* the local second order sufficient conditions are satisfied. To see this, consider the sub-optimization problem associated with the choice of $\left\{S_1\left(i\right)\right\}_i$ and K_1 for given e. In the symmetric equilibria we are considering, expected returns are the same for all stocks and we can restrict attention to the choice of S_1, where $S_1\left(i\right) = S_1$ for all $i = [0,1]$. The elements of the Hessian, H_U, for this problem are:

$$U_{BB}\left(\hat{e}\right) = u''\left(c_0^*\right) + E_{\hat{e}} u''\left(c_1^*\left(x\right)\right)\left(1 + \bar{D}^* - \tau_S^*\left(x\right)\right)^2 \leq 0,$$

$$U_{KK}\left(\hat{e}\right) = u''\left(c_0^*\right) + E_{\hat{e}} u''\left(c_1^*\left(x\right)\right)\left(1+x\right)^2\left(1-\tau_P^*\left(x\right)\right)^2 \leq 0,$$

$$U_{BK}\left(\hat{e}\right) = u''\left(c_0^*\right) + E_{\hat{e}} u''\left(c_1^*\left(x\right)\right)\left(1 + \bar{D}^* - \tau_S^*\left(x\right)\right)\left(1+x\right)\left(1-\tau_P^*\left(x\right)\right),$$

where U_{xy} denotes a cross-partial derivative with respect to the variables x, y.Under (26)-(27), $|H_U| = 0$. Hence, the Hessian is negative semi-definite irrespective of the value of $\hat{e}$. We now consider values of $\hat{K}_1, \hat{S}_1$ that are not interior. The Inada conditions exclude non-interior solutions that result from the non-negativity constraint on time 0 consumption being binding. Hence, there are two candidate non-interior solutions: $\hat{K}_1 = 0$ and $\hat{S}_1 > 0$, and $\hat{K}_1 > 0$ and $\hat{S}_1 = \bar{B}$. In both cases, of the two Euler equations for K_1 and S_1, one holds with equality and the other as a strict inequality. Under T^*:

$$1 = E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}, \tag{39}$$

$$1 > E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}. \tag{40}$$

Moreover, (25) implies $\hat{s} = 0$. This is a contradiction, since (39) and (40) clearly cannot hold at the same time. Then, K_1^*, S_1^* are globally optimal irrespective of the value of $\hat{e}$ for some $s^* \in [0, 1)$. Moreover, at K_1^*, S_1^*, $\rho^*(x)$ implies $\hat{e} = 1$ since the constrained-efficient allocation is incentive compatible. Hence, $\{1, K_1^*, s^*, \{s^* K_1^*\}_i\}$ is optimal for the agent given the initial endowment K_0, the tax system T^*, and the distribution policy $d(x)^*$, which implies expected return process $\bar{D}^*$. It follows that the resulting allocation, $\{c_0^*, 1, K_1^*, s^*, \{s^* K_1^*\}_i, c_1^*(x)\}$, jointly with the distribution policy $d^*(x)$ and the resulting expected return process $\bar{D}^*$ constitute a competitive equilibrium, according to definition 11. ■

Proof of Proposition 13. Suppose that the distribution policy is $\hat{d}(x)$ and that $E_{\hat{e}}\left(1 + \hat{d}(x)\right) u'(c_1(x)) \neq E_{\hat{e}}(1 + x)(1 - \tau_P(x)) u'(c_1(x))$ for some tax system where (23) holds with equality at $\hat{\tau}_P(x)$. Denote the corresponding competitive equilibrium allocation with $\left\{\hat{K}_1, \hat{s}, \left\{\hat{S}_1(i)\right\}_i, \hat{e}, \hat{c}_1(x)\right\}$, with $\hat{K}_1 > 0$. If $E_{\hat{e}}\left(1 + \hat{d}(x)\right) u'(c_1(x)) > E_{\hat{e}}(1 + x)(1 - \tau_P(x)) u'(c_1(x))$, for some $0 < \hat{s} < 1$, we can write:

$$\begin{aligned} 0 &= -u'(\hat{c}_0)(1 - \hat{s}) + \beta E_{\hat{e}}\left[(1 + x)(1 - \hat{\tau}_P(x)) - \left(1 + \hat{d}(x)\right)\hat{s}\right] u'(\hat{c}_1(x)) \\ &< -(1 - \hat{s})\left[u'(\hat{c}_0) - \beta E_{\hat{e}}\left(1 + \hat{d}(x)\right) u'(\hat{c}_1(x))\right], \end{aligned}$$

which implies $0 > u'(\hat{c}_0) - \beta E_1\left(1 + \hat{d}(x)\right) u'(\hat{c}_1(x))$. But by (25), $\hat{s} = 0$. Contradiction. Similarly, if $E_{\hat{e}}\left(1 + \hat{d}(x)\right) u'(c_1(x)) < E_{\hat{e}}(1 + x)(1 - \tau_P(x)) u'(c_1(x))$ for some $0 < \hat{s} < 1$:

$$\begin{aligned} 0 &= -u'(\hat{c}_0)(1 - \hat{s}) + \beta E_{\hat{e}}\left[(1 + x)(1 - \hat{\tau}_P(x)) - \left(1 + \hat{d}(x)\right)\hat{s}\right] u'(\hat{c}_1(x)) \\ &> -(1 - \hat{s})\left[u'(\hat{c}_0) - \beta E_{\hat{e}}\left(1 + \hat{d}(x)\right) u'(\hat{c}_1(x))\right]. \end{aligned}$$

Then, $u'(\hat{c}_0) - \beta E_1\left(1 + \hat{d}(x)\right) u'(\hat{c}_1(x)) > 0$, which by (25) implies $\hat{s} = 1$. Contradiction. ■

Proof of Proposition 15. We construct a competitive equilibrium in which the allocation is $e^* = 1$, $K_1^*, c_0^*, c_1^*(\underline{x}), c_1^*(\bar{x})$, bond holdings are $B_1^* = 0$ and the equilibrium rate of return on bonds is $r = E_1(x)$. In this equilibrium, $P^* = B_1^I = 0$. To characterize the optimal insurance contracts, we consider a relaxed version of Problem 5, in which the incentive compatibility constraint (32) is replaced by the set of constraints:

$$\begin{aligned} & (\pi(1) - \pi(0)) \left[u\left(K_1(1 + \bar{x} - \tau_K(\bar{x})) + R(\bar{x}) + B_1(1 + r - \tau_B(\bar{x})) - \rho(\bar{x}) + \bar{\Pi}_1\right) \right. \\ & \left. - u\left(K_1(1 + \underline{x} - \tau_K(\underline{x})) + R(\underline{x}) + B_1(1 + r - \tau_B(\underline{x})) - \rho(\underline{x}) + \bar{\Pi}_1\right) \right] \\ \geq \ & \Delta v, \end{aligned} \tag{41}$$

$$\begin{aligned} & u'\left(K_0 + \bar{\Pi}_0 - P - K_1 - B_1\right) \\ = \ & E_1 u'\left(K_1(1 + x - \tau_K(x)) + R(x) + B_1^i(1 + r - \tau_B(x)) - \rho(x) + \bar{\Pi}_1\right)(1 + x - \tau_K(x)), \end{aligned} \tag{42}$$

$$\begin{aligned} & u'\left(K_0 + \bar{\Pi}_0 - P - K_1 - B_1\right) \\ = \ & E_1 u'\left(K_1(1 + x - \tau_K(x)) + R(x) + B_1(1 + r - \tau_B(x)) - \rho(x) + \bar{\Pi}_1\right)(1 + r - \tau_B(x)). \end{aligned} \tag{43}$$

These constraints are the first order conditions for an agent's optimization problem embedded in constraint (32). We refer to the contracting problem under (41)-(43) as Problem 6. We construct a solution to Problem 6 under a candidate optimal tax system and then we show that this solution also solves Problem 5. Then, (42) and (43) will be satisfied at c_0^*, $c_1^*(x)$ and K_1^* and c_0^* are feasible for $B_1^* = 0$. Let $P^* = B_1^I = 0$ and set $R^*(x)$ satisfy:

$$c_1^*(x) = K_1^*(1 + x - \tau_K^*(x)) + R^*(x) + B_1^*(1 + r - \tau_B^*(x)) - \rho^*(x). \tag{44}$$

$R^*(x)$ is clearly feasible for the insurance companies, since (41)-(43) are satisfied at the constrained-efficient allocation. In addition, $\bar{\Pi}_1 = 0$. We need to show that it is indeed optimal. The insurers' problem at $\tau_K^*(x)$ and $\tau_B^*(x)$ and $\rho^*(x)$ can be rewritten with a change of variables as:

$$\max_{e \in \{0,1\}, K_1 \in [0, K_0], c_0 \geq 0, c_1(x) \geq 0, B_1} \left\{ \frac{E_e\left[K_1(1 + x - \tau_K^*(x)) - c_1(x)\right]}{1 + r} - (c_0 + K_1 - K_0) \right\},$$

by substituting the agents budget constraints in each period, since $\bar{\Pi}_t$ is taken as given. The level of B_1 does not matter for the value of this objective. Let $I_0 = K_0 - K_1 - c_0$ and $I_1(x) = K_1(1 + x) - c_1(x)$. Consider the problem:

$$\Gamma(K_0) = \max_{e \in \{0,1\}, K_1 \in [0, K_0], c_0 \geq 0, c_1(x) \geq 0} \left\{ I_0 + \frac{E_e\left[I_1(x) - \tau_K^*(x)\right]}{1 + r} \right\} \qquad \text{(Problem 7)}$$

subject to

$$I_t \geq 0, \ t = 0, 1,$$

$$u(c_0) - v(e) + \beta E_e u(c_1(x)) \geq \bar{U}, \tag{45}$$

and (3). The variables I_t, for $t = 0, 1$ are economy resources net of consumption in each period. Hence, for $1/(1 + r) = 1/E_1(1 + x)$ Problem 7 can be interpreted as a dual planning problem in which the planner minimizes the resource cost of providing a consumption allocation to the agent, subject to an incentive compatibility constraint and a participation constraint.

We now proceed in several steps. First, we show that for $\bar{U} = U^*(K_0)$, the solution of this problem is $e^* = 1, K_1^*, c_0^*, c_1^*(x)$. Suppose not, let $\left[\tilde{e}, \tilde{K}_1, \tilde{c}_0, \tilde{c}_1(x)\right]$ solve Problem 7 at $1/(1+r) = 1/E_1(1+x)$ and $\bar{U} = U^*(K_0)$ with $\left[\tilde{e}, \tilde{K}_1, \tilde{c}_0, \tilde{c}_1(x)\right] \neq [1, K_1^*, c_0^*, c_1^*(x)]$. $\left[\tilde{e}, \tilde{K}_1, \tilde{c}_0, \tilde{c}_1(x)\right]$ is clearly feasible for Problem 1. Moreover, by (45) it attains the maximum for Problem 1. Given that Problem 1 has a strictly concave objective with a convex constraint set, the solution is unique. Hence, $\left[\tilde{e}, \tilde{K}_1, \tilde{c}_0, \tilde{c}_1(x)\right]$ must solve Problem 1. Contradiction. Then, $[1, K_1^*, c_0^*, c_1^*(x)]$ solves Problem 7, which is a relaxed version of Problem 6, since (3) is the incentive compatibility constraint when B_1 is observable. In addition, since $1, K_1^*, c_0^*, c_1^*(x)$ and $B_1^* = 0 = \bar{\Pi}_0 = P^*$ satisfy (42) and (43) under the tax system $T^*(K_1, B_1, x) = \tau_K^*(x) K_1 + \tau_B^*(x) B_1$, K_1^*, B_1^* and $e^* = 1$, given P^* and $R^*(x)$, they solve Problem 6, since they are optimal for Problem 7, which is less constrained. To see that they also solve Problem 5, note that following the arguments the the proof of proposition 7, we can show that an agent's local second order conditions are satisfied and that $1, K_1^*, c_0^*, c_1^*(x)$ and $B_1^* = 0$ are globally optimal. In addition, $R^*(x)$ satisfies (33). Hence, $1, K_1^*, c_0^*, c_1^*(x)$ and $B_1^* = 0$ are feasible for problem 5, given P and $R^*(x)$, and will be optimal for Problem 5, since they are the solution to a relaxed problem. ∎