Monopolistic Competition and Optimum Product Diversity: Reply

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John Pettengill's comment on our 1977 paper has given us a welcome opportunity to give further thought to the problem. We find that his comment leads on to interesting issues, even though the substantive points he raises are invalid as stated.

I. Welfare Effects of Changing the Number of Firms

Pettengill tests whether there is an excessive number of firms in a monopolistically competitive equilibrium by a device of considerable expository merit. He removes one firm, and redistributes the resources thus released equally over the remaining firms in the sector, to see if welfare can be improved. To do this correctly, we write n_e for the equilibrium number of firms and x_e for the output of each. With fixed cost a and constant average variable cost c, removing one firm releases $(a + cx_e)$ of resources, and this enables the output of each of the remaining $(n_e - 1)$ firms to be increased $(a + cx_e)/\{c(n_e - 1)\}$. The quantity x_0 of the numeraire good is unaffected by this, and the utility function (equation (31) of our paper) is

$$u = x_0^{1-\gamma}\{\sum_i v(x_i)\}^\gamma$$

Therefore welfare increases if and only if

$$(n_e - 1)v(x_e + \frac{a + cx_e}{c(n_e - 1)}) > n_e v(x_e)$$

This can be written

$$(n_e - 1)\{v(x_e + \frac{a + cx_e}{c(n_e - 1)}) - v(x_e)\} > v(x_e)$$

Using the first-order Taylor approximation for large n_e:

$$(n_e - 1)\frac{a + cx_e}{c(n_e - 1)}v'(x_e) > v(x_e)$$

Pettengill's equations (2) and (3) defining the equilibrium are

$$p_e\{1 - 1/g(x_e)\} = c$$
$$p_e = (a + cx_e)/x_e$$

where p_e is the equilibrium price and g is defined in equation (13) of Pettengill:

$$g(x) = -v'(x)/\{xv''(x)\}$$

Substituting, the criterion for welfare improvement is

$$\{x_e/(1 - 1/g(x_e))\}v'(x_e) > v(x_e)$$

or

$$(1) \qquad \rho(x_e) > 1 - 1/g(x_e)$$

where ρ is defined in our equation (35) $\rho(x) = xv'(x)/v(x)$.

Taking logarithmic derivatives of the definition of ρ,

$$\begin{aligned} x\rho'(x)/\rho(x) &= 1 + xv''(x)/v'(x) \\ &\quad - xv'(x)/v(x) \\ &= 1 - 1/g(x) - \rho(x) \end{aligned}$$

Therefore (1) holds if and only if $\rho'(x_e) < 0$, which is precisely the result contained in our equations (41) and (43).

Where, then, does Pettengill go wrong? It is clearly in making an invalid use of consumer's surplus. In fact $v(x_e)$ is not equal to

$$\int_0^{x_e} p\, dx$$

nor do the two stand in any fixed relation. If we maximize the utility function restated above, and evaluate the result in a symmetric case with zero pure profit, we find the relation

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$$p = \frac{1}{nx} \frac{\gamma\rho(x)}{\gamma\rho(x) + (1 - \gamma)}$$

This can be found in our 1975 paper. Now we use the definition of $\rho(x)$ above to write this as $p = v'(x)\ h(x)$, where

$$h(x) = \gamma / \{nv(x)[\gamma\rho(x) + (1 - \gamma)]\}$$

Integrating, we find that Pettengill's criterion bears no clear relation to $v(x)$, and is therefore not valid.

II. Presumptions Concerning Relation between g and ρ

Pettengill is quite right to realize that there are problems in using the family functions

$$v(x) = (k + mx)^j; \quad m > 0, 0 < j < 1, k > 0$$

but the problems are not what he thinks them to be. He thinks that since $v(0) > 0$, utility can be increased by introducing fictitious commodities with zero outputs. However, the original utility function was defined over all potential commodities, so they are all already there. But this tells us what the real problem is: for this family applied to a case with infinitely many potential commodities, the utility sum diverges. Matters might be rescued by defining

$$v(x) = \begin{cases} 0 & \text{for } x = 0 \\ (k + mx)^j & \text{for } x > 0 \end{cases}$$

This is a perfectly respectable increasing and concave function. It is discontinuous (and of course nondifferentiable) at $x = 0$, but that is immaterial since the demand functions for produced commodities are continuous. We only need its derivatives at x_e for the above argument. And here $\rho'(x) > 0$, i.e., the equilibrium has too few firms.

Pettengill's modification is

$$v(x) = (k + mx)^j - k^j$$

which has $\rho'(x) < 0$, i.e., the equilibirum has too many firms. This does weaken the "counterintuitive" presumption we claimed. But it does not suffice to establish the "intuitive" conclusion either. Therefore, albeit with somewhat greater diffidence, we continue to maintain that "the common view concerning excess capacity and excessive diversity in monopolistic competition is called into question" (1977, p. 304).

III. Which Model?

Pettengill concludes by expressing his view that our framework is inappropriate for the problem. His first point in this connection, that our approach must assume that each consumer consumes a small proportion of each product on the market, is untrue. As we clearly stated what is at issue is the convexity of Samuelsonian social indifference curves, and that can arise just as easily (and probably more commonly) because different consumers use different product types. Of course we need not rule out diversification by individual consumers: there are people who sample all thirty-one varieties of ice cream as well as those who stick to one favorite flavor.

Pettengill's preferred approach is the product characteristics model popularized by Kelvin Lancaster. For some purposes, particularly that of providing an intuitive feel for the kind of commodities that will be discriminated against in a market, that approach is extremely attractive. However, it suffers from the disadvantage that the derived demand functions are complex, and do not yield results in terms of parameters like the elasticity of demand that most economists have found intuitively helpful. By allowing such an understanding, the approach taken in our paper, and that of Michael Spence, should serve to complement the Lancaster approach.

The two approaches do differ in an important way, and provide different approximations that are valid in different situations. In most reasonably complex markets, the numbers of firms, consumers, products, and characteristics are all large but finite. Various idealized models take some of these to be infinite, or at least of a different order of magnitude compared to others. The case where the limit is competitive has attracted most attention. When the number of consumers and firms is allowed to increase, while

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holding that of commodities fixed, there is increasing crowding in the commodity space (measured, for example, by cross elasticities), and the limit is evidently competitive. The Lancaster approach with a fixed finite characteristics space yields the same limit, since there must be increasing crowding in commodity space as the number of firms increases. These models may be good approximations in some cases. But there are several important instances where the observed market equilibrium is far from competitive, and a different idealization must be sought. This is the case with our approach, where the degree of crowding in commodity space does not increase, since the numbers of commodities and characteristics are of the same order of magnitude as the number of firms. We would not wish to claim that our approach is the uniformly best approximation, but we do not believe that such a claim can be made by the advocates of models which approximate to a competitive outcome either. Thus Pettengill's preferred approach must be seen as having restrictive validity, and the two approaches as complements rather than alternatives.

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