High-Resolution Seafloor Absolute Pressure Gauge Measurements Using a Better Counting Method

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ABSTRACT

Vibrating quartz force transducers are the critical component of most deep-sea pressure and depth gauges in use in oceanography, producing a frequency output that varies with pressure. Accurate and low drift pressure measurements can be obtained by precisely measuring this frequency. In most implementations, the frequency is determined by counting the number of cycles of a high-frequency standard oscillator occurring during a large number of cycles of the lower-frequency quartz force oscillator. Resolution is limited by the sampling interval (length of counting) and the frequency of the frequency standard. Alternative counting methods can provide significant (20–40 dB) improvements in resolution at sampling rates above 1 Hz. Each counting method can be described as a different filter applied to the output of a counter of the frequency standard gated at each transition of the transducer quartz oscillator. Improvements in resolution can be understood as the result of minimizing the aliasing of higher-frequency counting noise into the spectrum below the Nyquist frequency. A simple multipole infinite impulse response (IIR) filter designed to limit spectral leakage of high-frequency noise minimizes the noise spectrum and thereby optimizes the resolution of the pressure output. The resultant noise spectrum rises as frequency squared above 1 Hz, independent of the sampling rate. At frequencies below 1 Hz, it is limited by noise in the electronics driving the force transducer quartz oscillator. Resolution increases with frequency of the frequency standard up to about 200 MHz, plateauing for higher frequencies due to other noise sources (likely electronic).

1. Introduction

The vibrating quartz fiber force transducer developed by Jerome Paros in 1972 and incorporated into pressure sensors by Paroscientific Inc. enables the acquisition of accurate measurements of ocean depth with small long-term drift. These sensors have been used for decades for observations of ocean currents, tsunamis, ocean mesoscale eddies, and other oceanographic signals, and have recently been used for geodetic purposes (e.g., Watts et al. 2001; Park et al. 2012; Nooner and Chadwick 2009). However, until recently the resolution of these gauges was inadequate at sampling rates high enough to be of practical use for seafloor seismology.

Seismic phases from small local earthquakes and from moderate teleseismic earthquakes produce pressure fluctuations at the seafloor of sufficient amplitude to be observed above seafloor noise levels. In deep water, a low noise, “noise notch” is observed between about 0.02 and 0.1 Hz that permits detection of Rayleigh waves and long-period body waves (Fig. 1). The notch lies between the microseism noise peak (~0.1–0.5 Hz) and noise from long-period ocean waves (infragravity waves). Seafloor seismic noise is described in Webb (1998) and Suetugu and Shiobara (2014). At short periods (>5 Hz), noise levels can be quite low, permitting the detection of short-period body waves from small local earthquakes and airgun sources. The seafloor (pressure) loading from infragravity waves produces significant seafloor
deformation that contaminates seafloor seismic records at long period in deep water (Webb and Crawford 1999, 2010). In shallow water noise from deformation under infragravity wave loading can completely obscure important seismic phases; however, observations of seafloor pressure signals can be used to predict and remove most (up to 30 dB) of the wave loading noise from seafloor seismic observations, enabling useful seismic observations to be made even in quite shallow water (Webb and Crawford 2010). Observations of pressure fluctuations are thus critical to seafloor seismic observations, particularly in shallow water.

A differential pressure gauge (DPG; Cox et al. 1984) was developed to measure pressure fluctuations in the frequency band from 0.1 mHz to 1 Hz at the deep seabed. The DPG has proven quite useful for observing Rayleigh waves and long-period body waves (e.g., Laske et al. 1999; Weeraratne et al. 2007), microseisms, and infragravity waves (e.g., Webb et al. 1991). Many current ocean bottom seismometer (OBS) systems carry a DPG (see www.OBSIP.org) that can provide a long-period band sensor on seafloor instruments that otherwise only deploy short-period inertial sensors, and serve as a backup sensor for OBSs with broadband seismic sensors. However, the DPG is difficult to calibrate precisely, as the gain and long-period response varies with temperature and depth. In shallow water, large signals from ocean waves exceed the range of the gauge (estimated to be <1 m), causing the sensor to clip. Large-magnitude nearby earthquakes can also produce pressure signals that exceed the range of the DPG.

Filloux (1983, 1970) developed a stable Bourdon tube system to measure deep seafloor pressure, employing an optical detector system to measure changes in the tube geometry. The system obtained an impressive 16 bits of resolution (for that time), yielding a resolution of 2 Pa. The data were sampled at a few cycles per hour, but the bandwidth of this type of optical system is likely limited only by the dynamic response of the mechanical system.

The Paroscientific Inc. pressure gauge is based on a curled, oil-filled Bourdon tube within a vacuum cavity (Houston and Paros 1998). A quartz crystal mounted across the bent end of the tube is forced to vibrate at its natural frequency. The Bourdon tube is open at one end to the seawater and unwinds with increasing pressure, applying a strain to the quartz crystal and changing the frequency of vibration of the crystal. Thus, the frequency of vibration of the quartz crystal is proportional to pressure. The frequency also depends on temperature; thus, it is necessary to have an accurate independent measurement of the temperature at the crystal to accurately determine seafloor pressure. Temperature is inferred from measurements of the frequency of vibration of a second quartz crystal mounted adjacent to the crystal on the Bourdon tube vibrating in a different, higher-frequency mode. The frequencies of vibration of the two crystals are combined using an empirically determined set of coefficients to accurately determine both pressure and temperature at the seafloor. Typical accuracy in pressure is 0.01% of the full range of the gauge. Gauge resolution is determined by the “the counting method” used to determine these time-varying output frequencies.

Using standard counting methods, the obtainable resolution of Paroscientific pressure gauges when sampled at useful frequencies for seismology (>1 Hz) is insufficient (by several orders of magnitude) to observe the small pressure fluctuations associated with most seismic phases. This paper describes a high-resolution counting method that provides about a 40-dB improvement in resolution near 1 Hz. The method optimizes the achievable resolution for any sensor that produces a frequency output that varies with an input signal. All current counting methods can best be described as different choices of antialiasing filters applied to the same basic digital data. It is the improved rejection of aliased noise that leads to the higher resolution of the sensor. This paper compares the achievable resolution of each of these methods as a function of sampling rate.

The higher resolution permits usefully raising the sampling rate of the absolute pressure gauge (APG) to at least 100 Hz, enabling observations of short-period

![Pressure spectrum from a differential pressure gauge deployed at site J30A (41°57′N, 128°19′W) in 2791-m water depth showing a typical spectrum of deep-sea noise. At periods longer than 30 s, the spectrum is dominated by signals from long-period ocean waves (infragravity waves). This signal does not reach the seafloor at shorter periods, leaving a noise notch between about 0.025 Hz and the microseism peak (seismic noise) above 0.1 Hz. The noise level in the notch is set by the DPG noise level. Data available online from (http://ds.iris.edu/mda/_CASCADIA_OBS).](http://ds.iris.edu/mda/_CASCADIA_OBS)
seismic phases with the sensor. At higher frequencies the natural resonance of the Bourdon tube and crystal assembly (about 1 kHz) may complicate the sensor response. Hydrophone data could be compared with APG data to investigate the response at higher frequencies, but this has not yet been done. Paroscientific gauges are known to be orientation (direction of gravity) dependent and thus are expected to respond to accelerations of the seafloor. However, Chadwick et al. (2006) have measured the orientation sensitivity of two deep-sea gauges as about 1.5 psi over a full rotation of the sensor (a change in gravity of 2g), corresponding to an acceleration sensitivity of about 525 Pa (m s\(^{-2}\))\(^{-1}\). The relationship between acceleration and pressure in an acoustic wave (which is observed by the pressure sensor) is related to the water density (\(\rho\)), speed of sound in water (\(a\)), and frequency (\(f\)) as \(p/a = \rho a/2\pi f\). The sensitivity of the gauge to acceleration from seismic waves will depend on the orientation of the gauge, but it will always be less by at least a factor of 10 than the direct pressure sensitivity for frequencies less than 50 Hz.

Our improved counting method results in a sensor noise floor for the Paroscientific deep-water pressure gauges that approaches the DPG noise floor at periods near 0.03 Hz. However, the currently achievable noise floor for the APG above 4 Hz exceeds the typical deep seafloor pressure noise background, and thus the sensor is best considered as a strong motion sensor at shorter periods. Other sensors such as geophones and hydrophones will provide better SNR for small-amplitude short-period signals. However, most hydrophone systems are too noisy at longer periods for observations at frequencies below the microseism peak.

This APG system provides several advantages over the Cox et al. (1984) DPG: 1) The measurements are useful from near-zero frequency (limited by drift of the sensor) up to many tens of hertz, enabling observations of depth, oceanographic currents, tides, infragravity waves, the background of microseisms near the 7-s period typical of the seafloor (Fig. 2), and Rayleigh waves and body waves from earthquakes (Fig. 3). 2) The large dynamic range of the APG precludes clipping on any signal at the seafloor, including pressure signals from the largest possible local earthquakes (Fig. 3; also Nosov and Kolesov 2007) or tsunami (e.g., Tsushima et al. 2011). 3) The excellent stability of the gauge allows observation of coseismic and postseismic elevation changes as might be expected from large subduction events (e.g., Ito et al. 2011). 4) The sensors are calibrated over a wide temperature and depth range with a precision of 0.01% (1 part in 10 000) and thus better calibrated than any common device for seismic observations. Most seismometers are calibrated to about 0.5% in amplitude.

2. The counting methods with examples of seafloor data

The Paroscientific pressure gauge produces two time-varying frequency outputs, a channel that is primarily proportional to pressure at a nominal 35 kHz and a second temperature channel at a nominal 120 kHz (Schaad 2009). The temperature channel is used to correct the pressure output for the small temperature dependence of the pressure gauge. The original method for determining the frequencies of these outputs as a function of time is here called “start–stop” counting: zero crossings of the pressure and temperature outputs are used to gate counters that count a much higher-frequency “counting” clock over a fixed number of cycles (\(M\), \(L\)) of the pressure and temperature outputs (Fig. 4). The values of \(M\) and \(L\) are chosen depending on the sampling interval, such that the time required for \(M\) and \(L\) cycles of each of the outputs is slightly less than the desired sampling interval in time. The pressure sensor frequencies are
determined by dividing the counting clock frequency by the total counts on each counter within the \( M \) or \( L \) cycles and multiplying by \( M \) or \( L \). The resolution in frequency depends on the total number of cycles of the counting clock within the sampling interval. The standard counting clock in a Paroscientific gauge from the manufacturer is about 14.7 MHz. For a sampling interval of once per second, the total number of counts for either output \( (N) \)
would be about $14.7 \times 10^5$. The $M$ (pressure) and $L$ (temperature) within the interval would be roughly equal to the frequencies (Hz) of the outputs, or about 35,000 and 120,000 cycles, respectively. The resolution in frequency of either output frequency will be of order 1 part in $14.7 \times 10^5$. The frequency output of the gauges varies by about 7.5% for full scale, so a 10,000-psi (6700 m or 68.9 MPa) gauge should have a resolution when sampled at 1 Hz of about 6 mm or 60 Pa. At higher sampling rates, the number of counts is proportionally less, so the theoretical resolution is proportionally less. Thus, the theoretical resolution might be expected to be order 1 part in $14.7 \times 10^5$ with 10-Hz (600 Pa) sampling and 1 part in $14.7 \times 10^4$ at 100-Hz sampling (6000 Pa).

Davis and Becker (2007) show results from a high-resolution "fractional period" system developed by Bennest Enterprises Ltd. The resolution is described as a few tens of parts per billion (ppb) with a sampling interval of 1 Hz. While not fully described, this appears to be a hybrid counting and analog system, where the system counts over the sampling interval as described above but also measures the fractional count, probably by gating a voltage ramp starting with each cycle of the pressure standard oscillator at the pressure signal zero crossings. The voltage is digitized to obtain the fraction of the frequency standard cycle "left over" from the pressure zero crossings. This system is likely to produce similar performance as the start–stop method when used with a high-frequency clock standard because this measurement also depends only on the start and end of the sample interval. A 20-ppb resolution for the system is equivalent to a 200-MHz clock frequency in a start–stop system with 1-Hz sampling.

RBR-Global Co. produces a system based on a Paroscientific gauge that is described as having a 10-ppb resolution at a 1-s sampling rate using "proprietary technology" (http://www.rbr-global.com/products/bpr).

Figure 5 shows a deep seafloor spectrum from a Paroscientific pressure gauge deployed in 2500-m water depth using the start–stop counting method (S. C. Webb and S. L. Nooner 2008, unpublished data). The sampling interval for the gauge is 10 s. The noise floor of the gauge is about 50 Pa Hz$^{-1}$ for this implementation of a start–stop counter with a counting clock frequency of 17 MHz given the sampling rate and range of the pressure gauge.

One version of a "high resolution" counting method applied to Paroscientific Inc. sensors (developed by Paroscientific Inc.; Schaad 2009) fits straight line segments to the accumulating count between samples (Fig. 4). The high-frequency counting clock is counted, and the value in the counter is temporarily recorded at every positive transition of the pressure signal. The expected improvement in resolution associated with this straight line fit is proportional to $1/\sqrt{N}$, where $N$ is the number of cycles of the pressure transducer clock between data samples. This dependence on $N$ comes from the expected improvement in noise level for a fit of a straight line to data where the noise on each data point is independently distributed and of the same variance. These estimates of the frequency of the temperature and pressure signals are combined using an algorithm with the appropriate sensor calibrations to obtain the pressure and temperature at each sample.

However, the line-fit method is not optimum, primarily because the counting noise is dominated by quantization noise at frequencies above the sampling frequency. A new counting method is based on a simple infinite impulse response (IIR) filter applied to the accumulating count. The IIR filter is chosen to have strong antialiasing properties for rejecting high-frequency noise. The IIR filter method is implemented efficiently in low-power field-programmable gate arrays (FPGAs) that also allow the use of high-frequency counting.
clocks (frequency standards up to 200 MHz) while consuming little power. Our current generation of boards use 100 mW while sampling at 125 Hz.

Within the FPGA, the pressure signal frequency output from the Paroscientific gauge is used to gate a counter of the counting clock to produce the accumulated signal $y_k$ (shown in Fig. 4). Successive values of $y_k$ are differenced to produce the variable $z_k = y_k - y_{k-1}$, which is proportional to the pressure signal output frequency. A series of $K$ stages of cascaded IIR filters of the form

$$w_k = (1 - \alpha)w_{k-1} + \alpha z_k$$
$$u_k = (1 - \alpha)u_{k-1} + \alpha w_k$$
$$v_k = (1 - \alpha)v_{k-1} + \alpha u_k$$

are applied to the variable $z_k$, where the output of each stage is fed into the input of the following stage. The output of the final stage of the filter is then subsampled at the desired sampling rate. The $K$ cascaded filters make a $K$-pole low-pass filter. The number of filters cascaded ($K$) and the variable $\alpha$ are chosen to obtain sufficient antialiasing performance at all sampling rates as explained in section 4. In the current implementation, $\alpha = 2^{-J}$, where $J$ is an integer, so that the multiplication in the filters can be replaced by bit shift operations, which are particularly efficient to implement in the FPGA. Other choices of IIR filter could be implemented, enabling different responses for the system, if desired.

Figure 6 shows spectra from short sections of data from two pressure gauges deployed at the same site and also a spectrum from a shallow-water (78 m) site using the IIR filter counting system. The deep gauges have different maximum ranges (2000 and 4000 m) and thus correspondingly different gains and noise levels due to counting noise. The counting noise in counts is similar between different range gauges; thus, the counting noise pressure level depends proportionally on the range of the pressure gauge. The two spectra from the same deep site overlap over most of the frequency band shown. The spectral rise toward the longest periods is due to infragravity waves (long-period ocean waves) and because of spectral leakage from energy in the tidal band. The roughly flat spectrum at the deep site between 0.002 and 0.02 Hz is the result of infragravity waves, leaving the noise notch between 0.03 and 0.1 Hz (Webb 1998). The drop near 0.02 Hz is due to the hydrodynamic filtering effect of the overlying ocean on the infragravity wave pressure signal (the signal is evanescent from the sea surface, decaying with depth with an e-folding distance equal to the reciprocal of the wave-number). In shallow water, the notch is gone and the pressure spectrum at frequencies below about 0.1 Hz is related to ocean waves. A prominent peak due to ocean swell is seen near 0.07 Hz. The microseism peak is much smaller in shallow water due to the nearness of the free surface, which acts as a pressure release surface for Rayleigh wave propagation (Webb and Crawford 2010).

In most cases, the deep-water noise level in the notch is set by the noise level of the sensors due to counting
noise, but for this deep-water record, signals from teleseismic earthquakes have raised the noise level in the notch. The coherence measured between paired sensors on the deep (>1 km) seafloor is typically high at long period (in the infragravity wave band) and in the microseism peak (0.1–5 Hz) but near zero in the noise notch, reflecting the dominance of the counting noise in that band (in the absence of earthquake signals). In these data, the coherence is above 0.5 within the noise notch because of the teleseismic earthquake signals. The spectrum above 4 Hz is dominated by instrument noise (counting noise) in the absence of large arrivals from earthquakes as demonstrated later in this paper. The spectra are not corrected for the antialiasing filter response and correcting for this response would yield spectra that rise as frequency squared toward the Nyquist frequency.

The noise floor of the 4000-m (6000 psi or 41.4 MPa) gauge can be estimated from the fraction of the spectrum that is incoherent between the pairs of gauges. The coherence squared reflects the coherent energy between the 2000- and 4000-m gauge records. The 2000-m gauge has a lower noise floor than the 4000-m gauge; thus, the noise floor for the deep sensor can be estimated as the fraction of spectral energy in the 4000-m spectrum that is not coherent with the 2000-m gauge (Fig. 6a). The noise floors of the gauges are directly proportional to the range of each gauge, and thus the noise floor for the 2000- and 670-m range gauge spectra (see right panel) can be estimated from the noise floor for the 4000-m gauge. The smallest range gauge exhibits a noise floor that is comparable to the differential pressure gauges (Cox et al. 1984) over most of the range, but the larger range gauges are noisier than what can be achieved with DPGs.

The high counting noise at short period (>1 Hz) reduces the utility of the Paroscientific pressure gauge for seafloor seismology. Hydrophone observations show the background pressure spectrum falls as low as 10–4 Pa Hz−1 near 5 Hz. However, the Paroscientific gauges are useful for detecting short-period arrivals from large nearby events with local magnitude ML > 2 and for regional events with surface magnitude Ms > 3. An advantage of Paroscientific gauges for seafloor seismology is that their large dynamic range precludes clipping on any earthquake arrival no matter how near the source. The sensors thus are excellent “strong motion” sensors. The body waves from a magnitude 8.5 Sumatran earthquake caused pressure signals of order 10 kPa, equivalent to about 1 m of apparent water depth change at sites offshore of Cascadia (Fig. 3b). Signals from a M4.3 event near the Mendocino fracture zone exceeded the dynamic range of nearby OBSs (“clipped”) but are recorded by an APG at the seafloor only 15 km from the event (Fig. 3a). Around the world, only a few OBSs (Toomey et al. 2014) deploy “strong motion sensors” with dynamic ranges of order ±2 g (±20 m s−2) necessary to record large nearby events without clipping. All strong motion sensors have higher noise levels than typical ground noise, so it is necessary to combine these with standard broadband sensors to successfully record small events. APGs can be used as strong motion sensors on OBSs for seismic phases that produce pressure signals.

3. The spectrum of counting noise and high-resolution counting as a digital filter

The methods described above can best be understood as the action of different antialiasing filters applied to the same counting signal. The optimization of the signal to noise across the spectrum at a given sampling rate proves to be an exercise in minimizing aliased noise from counting noise at frequencies above the Nyquist frequency. In this framework, it is possible to predict the spectrum of noise for each of the methods depending on sampling rate and thus compare the performance of each method.

The counting signal yk is defined as accumulated count of a high-frequency stable reference clock gated by positive zero crossing of the pressure signal. The time interval between these measurements varies with the output frequency of the pressure and temperature sensors, but this variation is small compared to the average period of the pressure sensor output, so the time series can be approximated as being sampled uniformly in time. The frequency of the pressure transducers changes 7.5%–12% over the operating range (depending on the gauge depth rating). For a pressure gauge deployed on the seafloor, the largest signals detected will be associated with the ocean tides that are typically about 1 m in elevation. Thus, a pressure gauge rated to 6000-m depth will see a fractional change in pressure of order 1/6000 = 1.6 × 10−5, or a fractional change in the pressure signal frequency or period of about 1.6 × 10−5. Thus, it is justified to consider the interval between sequential values of yk as fixed.

If the high-frequency counting clock rate were infinite and the electronics had no jitter in detecting the transitions of the pressure signal, then yk would be directly related to the pressure signal. However, because the clock rate is finite, the number of “counts,” or cycles of the counting clock between transitions of the pressure clock is finite. The quantization noise for the pressure gauge counting methods is closely related to the quantization noise for many types of analog-to-digital converters. The spectrum of quantization noise was first described by Bennett (1948). A more recent review can be found in Widrow and Kollár (2008). Sleeman et al. (2006) describes digitizing noise in common logging
of the pressure clock and variable pressure signal, the error in pressure signal period varies rapidly within one pressure counts squared per hertz. This is the common result for above centers the distribution for 0 and 1. Here 0.5 is added to the discretized signal to center the error on \(-0.5\) to 0.5.

systems for broadband seismology. The discussion that follows below does not accurately reflect the vast literature on the quantization problem but does account for the behavior of the different counting methods described here.

The counting signal is discrete and can be written as the sum of the error-free signal that would be obtained using an infinite rate counting clock and an error signal \(e_k\) that is due to finite resolution: \(y_k = y(P(t_k)) + e_k + \frac{1}{2}\). This is often called the least count error. Here \(y\) is the count and is a function of the pressure \(P\). The \(y_k\) are obtained at each zero crossing of the pressure clock or at times \(t_k = tk + T_{start}; k = 0, 1, \ldots\) where \(T\) is the period of the pressure clock and \(T_{start}\) is the time of the first data point. Figure 7 illustrates the difference between the variable \(y\) and \(y_k\). The added \(\frac{1}{2}\) in the equation above centers the distribution for \(e_k\) around 0. If the pressure signal period varies rapidly within one pressure signal period, then the number of counts of the high-frequency clock will vary greatly between cycles of the pressure signal, the error in \(y_k\) will be uniformly distributed between \(-\frac{1}{2}\) and \(\frac{1}{2}\), and the error terms \(e_k\) are independent such that \(\{y_k - y(P_k) - \frac{1}{2}\}[y_k - y(P_k) - \frac{1}{2}] = \langle e_k e_k \rangle = \delta_\delta k = \frac{T}{f}\). The variance can be determined if the error is uniformly distributed between \(-\frac{1}{2}\) and \(\frac{1}{2}\), \(\sigma^2 = \frac{1}{12}\). A time series of a variable uncorrelated in time will have a spectrum that is flat (white) in the spectral domain. Given a sampling rate of \(T\) representing a Nyquist frequency of \(f_N = 1/(2T)\), the noise spectrum of the error in variable \(y_k\) will be \(S_y(f) = \sigma^2/(2fN) = T^2/6\) in counts squared per hertz. This is the common result for least count noise. Here \(T\) is the reciprocal of the pressure signal frequency (about 35 kHz).

However, the range of variation in the pressure signal period is typically small, such that the variation in counts between adjacent samples is small unless a very high-frequency counting clock is used. The errors in variable \(z_k = y_k - y_{k-1}\) will thus be correlated between adjacent points but at large enough time difference the errors in the variable will again become uncorrelated, and this spectral model of counting noise seems to approximately fit the observed counting noise spectrum for counting clock frequencies exceeding about 10 MHz. Poorer results are achieved at lower counting clock rates as the errors in \(y_k\) become correlated over large separations.

An estimate of the pressure signal period in each cycle could be obtained by dividing the difference between subsequent values of \(y_k\) by the frequency of the counting clock. Defining a new time series: \(z_k = y_k - y_{k-1}\), the noise spectrum of this new variable is determined from the transfer function in frequency between variables \(z_k\) and \(y_k\). The transfer function between \(z\) and \(y\) can be determined by considering a test signal: \(y_k = Y(f) \exp(i2\pi fT)\). Inserting this into the equation, the result is \(Z(f) \exp(i2\pi fT) = Y(f) \exp(i2\pi fT) - Y(f) \exp(i2\pi (f - 1)fT)\). Thus \(Z(f) = Y(f)(1 - \exp(2\pi fT))\). The spectrum of the error in \(z_k\) is related to the noise spectrum of \(y_k\) by the magnitude squared of the transfer function between the two variables: \(S_z(f) = S_y(f)|1 - \exp(2\pi fT)|^2 = (T/6)|1 - \exp(2\pi fT)|^2\).

The counting noise spectrum at frequencies that are small compared to the pressure sensor frequency \((fT \ll 1)\) will be approximately quadratic in frequency: \(S_z(f) \approx 4(T^2/6)|f|^2\).

An estimate of the seafloor pressure corresponding to the time associated with each point \(z_k\) could in theory be obtained using the relationship between gauge pressure and the pressure signal period provided by the manufacturer (e.g., Fig. 1). It would also be necessary to know the temperature during the same interval to determine pressure precisely. The sampling rate for this time series would be approximately 35 kHz (the frequency of the pressure signal). However, these high-frequency data are not very useful, as the spectrum of the counting noise rises quadratically toward higher frequency.

Pressure gauge data are always sampled at some frequency well below 35 kHz. As noted above, the different counting methods can be thought of as different digital filters applied to the same 35-kHz data stream (the \(z_k\) that is then subsampled down to the selected sampling rate. Whenever a signal is subsampled, the effect on the spectrum is to “fold” the high-frequency spectral components of the original spectrum into the spectrum of the subsampled data in

FIG. 7. Illustration of error from quantization. Blue curve shows schematic time-varying signal. Green curve shows the resulting apparent signal given finite discretization. Red curve illustrates the difference between the two curves (the error) and lies between 0 and 1. Here 0.5 is added to the discretized signal to center the error on \(-0.5\) to 0.5.
a process called aliasing. Because the counting noise rises quadratically with frequency, without good anti-aliasing filtering most of the high-frequency noise is folded down into subsampled data, raising the noise floor of the observed pressure spectrum.

Filtering the original data with a low-pass filter suppresses the high-frequency noise. The choices of corner frequency and number of poles in the filter determine the noise variance in estimates of pressure, and also suppress real signals in the pressure record at frequencies above the corner frequency of the filter. The pressure signal following filtering can—and should—be subsampled with a sampling frequency corresponding to some (small) multiple of the corner frequency of the filter without significant loss of information. There is a trade-off between the corner frequency of the filter (and useful sampling rate) and resolution of pressure. The sharpness of the filter (number of poles) is important to prevent aliasing of the high-frequency noise into lower-frequency components.

In the current implementation of the APG system, multiple stages of an IIR filter of the type \( w_k = (1 - \alpha)w_{k-1} + \alpha x_k \) (where \( x_k \) is the input and \( w_k \) is the output) are applied to the difference in the accumulated clock count between pressure signal transitions \( z_k = y_k - y_{k-1} \). Each filter is initially started with a zero value and is calculated in sequence each time a new value of \( z_k \) is obtained (a new value is obtained for each cycle of the pressure signal from the gauge, or roughly 35 000 times per second). For ease of implementation \( \alpha = 2^{-7} \), where \( J \) is some integer. This allows the multiplications in the filter to be replaced by simple bit shifts in the FPGA.

The transfer function between the input and the output of any stage of these filter stages is \( T_j(f) = \alpha/[1 - (1 - \alpha) \exp(-i2\pi T/j)] \approx 1/(1 - i2\pi fT/\alpha) \).

For small \( \alpha \), the filter is a good approximation to a single-pole filter for frequencies that are small compared to the pressure transducer frequency. The low-pass time constant for this filter is approximately \( T_f \approx T/\alpha \). For \( \alpha = 2^{-7} \), the time constant \( T_f \approx 0.0036 \text{s} \), corresponding to a corner frequency of about 43 Hz for a pressure signal frequency near 35 kHz. For \( K \) identical, cascaded IIR filter stages, the response is the product of the response of each filter multiplied together. The transfer function (response function) for \( K \) stages thus has an \( f^K \) frequency dependence above the corner frequency. The amplitude response for a filter with four poles (\( K = 4 \)) and \( \alpha = 2^{-7} \) is shown in Fig. 8. The phase response is close to linear in frequency at frequencies well below the corner frequency, which can be approximated as a simple time shift (delay) equal to \( KT_f \approx 0.029 \text{s} \).

The power spectrum of the counting noise after applying these filters is then \( S_{\text{countfilt}} = |\alpha/1 - (1 - \alpha) \exp(-i2\pi fT)]^K S_\text{count}(f) \approx |1 + (T_f/\alpha)^2|^K (\alpha/\pi fT)^2 \).

An estimate of the frequency of the pressure signal obtained at each time \( t_k \) is equal to the final stage output (in counts and fractions of counts) divided by the frequency of the counting clock. As noted earlier, the pressure signal frequency varies approximately linearly with the seafloor pressure, changing by 7.5%–12% over the full range of the gauge. Given a full-scale range \( P_{\text{max}} \), a counting frequency of \( F_{\text{count}} \), a pressure signal frequency \( f_{\text{press}} \approx 35 \text{ kHz} \), and a scale factor \( D (D \approx 0.7–0.12) \), the equivalent noise spectrum in power for the counting noise spectrum above is

\[
S_{\text{pressnoise}} \approx S_{\text{countfilt}} F_{\text{press}} P_{\text{max}}^2 F_{\text{count}}^{-2} D^2 = \left( F_{\text{press}} F_{\text{count}}^{-1} \right)^2 \left[ 1 + (T_f/\alpha)^2 \right]^K 2C \pi^2 T^2 f^2 / \beta.
\]

The equation can be simplified by noting \( T = 1/F_{\text{press}} \). The factor \( C \) is an ad hoc correction factor explained below.

The counting noise above the corner frequency of the filters is greatly suppressed and the final variable only varies significantly on time scales larger than \( T_f \). The amplitude of any pressure signals above the corner frequency of the filters is also reduced by the filter; so, it makes sense to resample to a time interval corresponding to some fraction of \( T_f \), such that the Nyquist frequency of the sampling is appropriately above the corner frequency.
of the IIR filters. For example, with the choice $\alpha = 2^{-7}$, $K = 4$, we subsample the data to either a sampling rate of 100 or 125 Hz (or an interval of 0.008 or 0.01 s) corresponding to a Nyquist frequency of 50 or 62.5 Hz. The resampling can be controlled by a precise clock (typically by the same clock producing the counting clock) so that the subsampling is on a constant and precise interval, whereas the time between successive values of the original time series $z_k$ depends on the time-varying frequency of the pressure signal. There will necessarily be some jitter in the relationship between the time of the output of the last filter and the time of the sample acquired, but this will always be less than the period associated with the pressure signal frequency (1/35 kHz or about 0.03 ms).

The IIR filters do produce a delay between the original pressure signal and the output that should be corrected postacquisition using the filter response described above.

The values of $\alpha$ in the filter applied to the temperature frequency signal (nominal 120 kHz) are chosen differently from the values for the pressure frequency signal because the frequencies for the two signals are different. In practice, the temperature is more heavily filtered than the pressure signal because the temperature signal is effectively filtered from changes in seafloor temperature by the time delay of the thermal change through the pressure case containing the pressure transducer and thus only slow (on the order of hundreds) temperature variations are detected by the Paroscientific gauge. It is also necessary to subsample the pressure data in order to obtain synchronized estimates of the pressure signal frequency and the temperature signal frequency. Estimates of both frequencies are required to determine pressure because the pressure frequency signal also depends on temperature.

Given a sufficient number of poles (stages $K > 2$) in the IIR antialiasing filter, the primary effect of aliased noise is to raise only the long-period noise because the counting noise rises quadratically with frequency. For two or fewer stages of IIR filter, the aliasing counting noise contaminates the entire spectrum, bringing up the noise floor at all frequencies, leading to a roughly white spectrum for the noise.

Figure 9 illustrates how the counting noise at higher frequency is aliased into frequencies below the Nyquist frequency when a four-stage (four poles) IIR filter is applied to the counting data. Higher-order filters (and filters with lower corner frequency) suppress the higher-frequency noise to a greater extent, reducing the levels of counting noise aliased into the spectrum. Subsampling the original data at the output of the last stage of the digital data from its (nominal) 35-kHz sampling rate (Fig. 9a) to a slower sampling rate (125 Hz) acts to fold the higher-frequency noise components back into the lower-frequency data as shown in Fig. 9b. Each 125-Hz-wide higher-frequency component band (shown by the different colors in Fig. 9a) is aliased into a band that corresponds to the original frequency either subtracted or added to multiples of the sampling frequency, such that the frequency is less than the Nyquist frequency for the chosen sampling rate. The final spectrum (top black curve in Fig. 9b) is the sum of the counting noise spectrum within the bandwidth of the sampling (less than the Nyquist frequency of 62.5 Hz in this example; shown as the top red curve in Fig. 9b) plus all the aliased components (shown as the pile of folded components from Fig. 9a in the same color scheme). Summing all the noise power from the different folds of the higher-frequency noise determines the total noise spectrum for the counting method (black curve in Fig. 9b).

The effect of aliasing with this four-pole filter example is to raise the noise floor at long period for the gauge and to produce a flat (white spectrum) below 2 Hz. At higher frequency there is little effect. Both the signal and the counting noise are reduced at high frequencies by the antialiasing filter. If one corrects the spectrum for the effects of the filter (and thereby corrects the shape of the signal waveform), then the result would be to reproduce the original quadratic slope of the counting noise spectrum above 2 Hz.

Increasing the number of stages (poles) in the IIR filter reduces the level of the white spectrum of counting noise at long period (Fig. 10); shifting the corner frequency lower also reduces the long-period spectrum of counting noise. The shape of the counting noise spectrum above 1 Hz is constant after correcting for the response of the filter, for four stages or more of filtering. The number of stages determines the level of long-period counting noise. Current implementations use 12 stages of filtering, which puts the counting noise far below other sources of noise at long period. We could likely use as few as six stages without changing the effective noise spectrum for the sensor.

Figure 11a shows the measured spectrum after applying an IIR filter with 24 stages to the pressure output of a “dummy” Paroscientific gauge, where the Bourdon tube has been disconnected from the quartz oscillator; such that the gauge is insensitive to pressure changes (we show data using a range of counting clock frequencies from 10 to 200 MHz). The number of stages is so large that we expect any aliased counting noise to be very small.

At low frequencies, the noise spectrum of the pressure gauges has a spectral shape that depends on frequency as roughly the reciprocal of the square root of frequency. The likely source of this noise is the electronics driving the quartz crystal oscillator and it is intrinsic to the Paroscientific gauge. A model for this noise has been added to the counting noise models in Fig. 11b. There are some differences in the shape of the low-frequency
background electronic noise spectrum between gauges. Figure 5 shows an example where the electronic noise has a flatter spectrum out to about 0.1 Hz before rising toward longer period, although the noise is slightly higher near 1 Hz. This apparently reflects two different implementations of the electronics (T. Schaad 2010, personal communication). Correcting for different gauge parameters, in counts the noise spectrum for the gauge in Fig. 5 is slightly higher at 1 Hz, while the noise at longer period is slightly lower on the gauge in Fig. 10.

Comparing the predicted counting noise levels at long period from different choices of IIR filter to the measurements from the dummy gauge in Fig. 11a, it is apparent that six stages of IIR filtering would be adequate to reduce the aliased counting noise at long periods below the noise floor set by the electronics. However two stages would be inadequate and four stages would also result in a higher noise level near 1 Hz. More stages provide little benefit, except to suppress energetic acoustic signals at frequencies above the Nyquist frequency that could be aliased into the pass band.

The total number of counts is directly proportional to the frequency of the counting clock; thus, a higher-frequency counting clock will produce a greater number of counts for a given pressure signal, and the effective spectral noise floor power for the pressure gauge should be inversely proportional to the square of the counting clock frequency. However, this only holds for counting clock frequencies up to about 200 MHz, after which sources of noise other than the direct counting noise described here also become important, such that higher clock rates produce only small reductions in counting noise (Fig. 11). The likely sources include jitter in the electronics driving the oscillator and in the detection of the zero crossing of the pressure signal used to gate the clock counter (the jitter due to frequency fluctuations of the counting clock is expected to be small compared to the apparent jitter observed in the system—about a few nanoseconds). The effect of other sources of noise in the

![Graph](image-url)
counting system is to add to the least count noise—once the jitter is comparable to the least count error, increasing the counting clock frequency no longer reduces the counting noise floor. This effect is modeled by adjusting the value of the constant $C$ in the equation from about 1 for a counting frequency of 10 MHz, to about 5 for 50 MHz, and to 40 for 200 MHz. In the current system, there is little reduction in noise level for counting frequencies above 200 MHz (Fig. 11). An obvious target for improvement in pressure gauge noise level is this poorly understood source of noise that limits improvements in counting floor at higher counting frequencies. If the full advantage of the higher counting frequencies could be realized, then it would be possible to reduce the rise in noise floor at higher frequencies and permit higher sampling rates.

At low counting clock frequencies (below about 10 MHz), the number of counts within a pressure signal period can become small enough that the number of counts rarely varies between successive zero crossings of the pressure signal. This can lead to poor performance of the counting system, as the error due to quantization becomes correlated over large time intervals (many zero crossing of the pressure output signal). At low clock rates, the performance of the IIR filter system has high noise levels, obtaining a resolution more similar to that of the simple start–stop counting method. For low counting clock frequencies (>$1$ Hz) can exhibit large and variable noise peaks due to counting noise.

4. A comparison of counting methods

The counting noise spectrum rises as $f^2$ before an antialiasing filter is applied. For each of the counting schemes, the count data are resampled to a data rate far below the nominal 35 kHz of the pressure signal. Subsampling leads to aliasing of the counter noise at higher frequencies into lower frequencies, raising noise levels if the antialiasing filter does not sufficiently suppress the counting noise above the Nyquist frequency of the sampling. There is little value in sampling the pressure gauge data at frequencies much above 100 Hz because the rising counting noise swamps most geophysical and oceanographic pressure signals of interest at higher frequencies.

Within this framework, it is easy to understand why the start–stop and line-fit methods for estimating the time-varying pressure signal provide higher counting noise levels and lower resolution for a given sampling rate. The start–stop method of count is equivalent to a finite impulse response (FIR) filter of the boxcar type. Each estimate of the pressure frequency is obtained by differencing the count value at a time just before the end of the sample from the count value just after the start of the sample:

$$q_k = y_k - y_{k-M} = (z_{k-M+1} + z_{k-M+2} + \cdots + z_k)$$

$$= \sum_j b_j z_{k-j}; \ b_j = \begin{cases} 1; 0 \leq j < M \\ 0; j \geq M, j < 0 \end{cases}$$
where $M$ is such that the count extends from just after the previous sampling time point to just before the subsequent sampling time point. The filter response of a boxcar function is a sinc function. The counting noise spectrum converted to the equivalent noise for the pressure spectrum is then

$$S_{\text{linefit}} \approx S_{\text{countfilt}} \left( \frac{F_{\text{press}} P_{\text{max}}}{F_{\text{count}} D} \right)^2 \sin^2(f T_{\text{filt}} \pi)$$

$$\approx \pi^2 T_{\text{filt}}^2 / 3 \left( \frac{F_{\text{press}} P_{\text{max}}}{F_{\text{count}} D} \right)^2 \left[ \frac{\sin(f T_{\text{filt}} \pi)}{f T_{\text{filt}} \pi} \right]^2$$

with a filter response as shown in Fig. 8. The sinc function has a series of zeros at frequencies that depend on the length in time of the boxcar filter, usually defined by the time required for $M$ cycles of the pressure signal. The value of $M$ is usually (but not always) chosen so that the time required for $M$ cycles is slightly less than the sampling interval. Sometimes the pressure signal is counted only over a small fraction of the sampling interval to save power (so fewer batteries are required for long deployments). In either case, the zeros of the sinc function occur at $f_{\text{zeros}} = (l + 1/2)/T_{\text{filt}}; l = 0, 1, \ldots$, where $T_{\text{filt}}$ is the length of the boxcar function in time. By necessity, this time interval must be slightly less than the sampling interval; thus, the first zero of the sinc function does not ever correspond exactly to the Nyquist frequency determined by the sampling interval. The sinc function squared falls off toward higher frequencies roughly as the inverse of the frequency squared ($f^{-2}$), which approximately balances the rise in the unfiltered counting noise spectrum ($f^2$). Thus, subsampling the spectrum to the sampling interval (causing the counting noise to be folded into the noise spectrum below the Nyquist frequency) results in a flat noise spectrum in frequency (Fig. 12). As noted earlier, the spectral level of this white noise in counts is simply $S(f) = \sigma^2 f_N$, where $f_N$ is the Nyquist frequency associated with the sampling rate and $\sigma^2 = 1/12$ is the variance associated with least count noise. The spectral level of the counting noise goes as $1/T_{\text{filt}}$ (proportional to the reciprocal of the time interval of the counting interval). The conversion of this noise to the equivalent pressure noise spectrum will depend on the conversion factor from counts to pascals, which depends on the counting frequency and the counting time interval within a sample.

The line-fit high-resolution method developed by Paroscientific Inc. can also be considered as a form of an FIR filter applied to the total counts at $N$ successive pressure signal zero crossings. The line-fit formula is

$$r_k = \sum_j b_k y_{k-j}; \quad b_j = \begin{cases} (jM - A_1)A_2; & 0 \leq j < M \\ 0; & j \geq M, j < 0 \end{cases} \quad A_1 = M(M - 1)/2; \quad A_2 = (M - 1)M(M + 1)/6.$$  

This is a linear ramp of finite length convolved with the counting data. The derivative of this ramp filter is equivalent to a boxcar function. Thus, the frequency response of this ramp function should go as approximately: $R_{\text{ramp}}(f) \approx \sin(\pi T_{\text{filt}})/(i2\pi f)$. The line-fit filter is
applied to the count data $y_k$ rather than to the difference in successive count values $z_k = y_k - y_{k-1}$, as was used above, but the differencing is equivalent to a simple FIR filter of the kind: $z_k = \sum c_i y_{k-i}; c_0 = 1, c_1 = -1, c_j = 0, k \neq 0, 1$ with a frequency response of $R_{yz}(f) = [1 - \exp(-i2\pi Tf)]$. This response goes as $R_{yz}(f) \approx i2\pi Tf$ at low frequencies, and to 2 near $f = 1/(2T)$. The pressure counting noise spectrum is then expected to go as the square of the product of the $1/R_{yz}(f)$ and the response of the ramp function. At high frequencies, the squared response of the line-fit function thus goes as $f^{-4}$. Given the noise counting spectrum goes as $f^2$, the counting noise spectrum after applying the line-fit algorithm falls as roughly $f^{-2}$ at high frequency (Fig. 13). The aliasing of the high-frequency counting noise into lower frequencies following subsampling leads to a white counting noise spectrum that lies below that of the start–stop algorithm (Figs. 11–14) but above all the IIR filter spectra. The line-fit algorithm provides a factor of $1/M$ reduction on spectral noise over the stop–start algorithm, where $M$ is the number of line-fit datum (cycles of the pressure signal). The value of $M$ is proportional to the sampling interval. The noise spectral level is thus proportional to the reciprocal of the sampling interval squared (Fig. 14).

The simple start–stop counting procedure is equivalent to a boxcar function in time for which the squared response falls like $f^{-2}$, whereas the response with the IIR filtering squared response falls as $f^{-2K}$ at high frequency. The line-fit algorithm provides similar suppression of aliased counting noise above the sampling Nyquist frequency as a two-stage (two poles) IIR filter. The IIR filter is superior to the line-fit method for suppression of counting noise for more stages or poles, $K > 2$. Other choices of low-pass IIR filter than the one described here could be used in filtering the counting data with the performance depending primarily on the number of poles for the filter and secondarily on the shape and choice of corner frequency of the filter (Figs. 11 and 14).

Note that the spectrum of counting noise at long period using an IIR filter is independent of sampling frequency. As long as the IIR filter has a sufficient number of poles

FIG. 13. As in Fig. 9, but for the line-fit counting method. (a) Counting noise spectrum after applying the “line fit” FIR filter. (b) Resultant counting noise spectrum is the summed aliased components following subsampling at 125 Hz (top black curve). The sum is a flat spectrum well above the result for the IIR filter.

FIG. 14. Predicted counting noise spectrum for a pressure gauge with a 6700-m range for different choices of filter and sampling rates. Start–stop method (pink curves), and line-fit method (black lines) at 125 (top curve), 40, and 10 Hz. Counting noise for IIR filters with four stages ($K = 4$) for corner frequencies of 43 (aqua), 11 Hz (red), 2.7 (blue), and 0.68 Hz (green), appropriate for sampling rates of about 125, 40, 10, and 1 Hz. The counting frequency is 10 MHz.
and appropriate choice of corner frequency, the counting noise spectrum is independent of both sampling rate and corner frequency (Fig. 14) (after correcting for filter response). Thus, with the IIR filter method there is no disadvantage to sampling the data at a high sampling rate and later filtering the record to reveal low-amplitude long-period signals.

The effective noise floor of the Paroscientific pressure gauge with sufficient antialiasing filtering rises toward low frequency below 1 Hz because of electronic noise and toward higher frequency because of counting noise, leaving a minimum in the noise near 0.1 Hz (Fig. 14). This minimum is quite useful for seafloor seismology because there is typically a minimum in the deep-water background pressure spectrum between about 0.03 and 0.1 Hz that is useful for detecting seismic Rayleigh waves and long-period body arrivals (see Figs. 4 and 5; also see Webb 1998).

The vibrating quartz force transducer can be applied to other devices. It has been used to develop a strong motion seismometer (Paros 2013; Fukao et al. 2014). Figure 15 shows a spectrum from this sensor using a multipole IIR filter. While the prominent microseism peak is evident in the spectrum between 0.1 and 1 Hz, the spectrum is otherwise controlled by the roughly 1/f noise of this implementation of the vibrating quartz fiber force transducer at long periods and by the same rising counting noise above 1 Hz seen in the pressure gauge spectra. The data shown here are from a system with a counting frequency of 200 MHz, yielding a spectral shape close to the 200-MHz curve shown in Fig. 11. After correcting for response of the IIR filter, the spectrum in Fig. 11 would rise as \( f^2 \) above 20 Hz as expected for counting noise.

5. Conclusions

Counting systems that count cycles of a high-frequency stable clock reference between transitions of a lower-frequency signal to determine the time-varying frequency of that signal can be described as different forms of a digital filter applied to the same basic data. The resolution of the counting schemes increases with the frequency of the counting clock until limited by other sources of noise. The spectrum of counting noise is proportional to frequency squared. The relative noise floor of different counting schemes depends on how effectively the counting schemes reject aliasing of counting noise above the Nyquist frequency for a given sampling rate. A multipole IIR filter with a corner frequency appropriate for the given sampling rate provides the minimum obtainable spectrum noise floor at low frequency, independent of the sampling rate. The IIR filter counting system provides a noise floor at 0.5 Hz, which is about 40 dB lower than the standard start–stop counting system for a sampling rate of 10 Hz, and 30 dB lower for a sampling rate of 1 Hz.

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